Almagestum Novum astronomiam veterem novamque complectens observationibus aliorum et propriis novisque theorematibus, problematibus ac tabulis promotam. Pars prior tomi primi

New Almagest, embracing both old and new astronomy, advanced by the observations of others and the author's own, and by new theorems, problems, and tables. Volume I, Part 1

[New Almagest, Part 1]

(1651)

by Giovanni Battista Riccioli, S.J. (1598–1671)
Translation by Scott Weisman



Ars Astronomica
2026

”Almagestum Novum astronomiam veterem novamque complectens observationibus aliorum et propriis novisque theorematibus, problematibus ac tabulis promotam. Pars prior tomi primi (New Almagest, embracing both old and new astronomy, advanced by the observations of others and the author's own, and by new theorems, problems, and tables. Volume I, Part 1),” published in 1651, is in the public domain. All translations, original additions, including illustrations, commentaries, and summaries, are copyright.

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Translator Version 2

Upright reason dictates that the recipients of the good, of whatever type and level of beneficence it may be, must show gratitude and blessing to the beneficent person in every way possible, commensurate with the value of the beneficence. And one who has benefited all the people of the world — for example, one who invented a new instrument for the good of the world, or a good book — it is fitting for every discerning person, out of the obligation of love of fellow beings, to at least purchase it, so that the man will profit and his heart will be encouraged thereby to invent yet more good instruments in the world. And similarly, all other wise-hearted people will likewise strive and exert themselves to invent good things and instruments needed for the repair of the world and its perfection.

And therefore, whoever says: ”What do I need this new instrument for?” — he does not act well towards the world. For if not for the man who invented it, where would you be? And what would your city do? And more than this, if the man did not exert himself, what would the world do?

— R. Pinchas Eliyahu Hurwitz, *Sefer HaBris HaShalem* (1797)

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A note on the work

The *New Almagest* (Bologna, 1651) is the great Jesuit encyclopedia of seventeenth-century astronomy — a sprawling survey that takes its name from Ptolemy's ancient masterpiece and aims to do for its own age what the *Almagest* did for antiquity. Its author, the Jesuit Giovanni Battista Riccioli (1598–1671), sets out the whole of astronomy as it then stood. This first part, Books I–VII, covers the sphere and the Earth, the Sun, the Moon, eclipses, the fixed stars and the five planets. It contains the map of the Moon, drawn with his colleague Grimaldi, whose crater and sea names — Tycho, Copernicus, Plato, the Sea of Tranquility — astronomers still use today. The second part, with the comets, the systems of the world and the celebrated 126 arguments on the Earth's motion, is a separate volume.

A note on the translation process

This translation was produced with the translation-pipeline. See the linked repository for details on the process.

A note on the references. A scriptural or bibliographic reference given in brackets or parentheses is, in most cases, an **editorial identification of an allusion rather than a citation printed in the source**. Books of this kind quote and weave scripture constantly without naming it, and the identification is supplied here as a convenience to the reader. Where the source does print a reference of its own, it is given as printed. **An identified allusion is not evidence that the author cited that passage** — look at the original before resting an argument on one.

This text is the product of a highly refined automated translation process. It has undergone only cursory human review and should not be treated as an authoritative or scholarly rendering. It is offered solely as a reference and reading aid — a way to make an otherwise inaccessible work approachable — and any passage of real consequence should be checked against the original source.

Acknowledgements

Translated with the assistance of Claude. The translator thanks the Anthropic team for making this work possible.

Notation and abbreviations

A few conventions have been applied uniformly throughout this translation, so that one quantity is always written one way. They change how a value is *written*, never what it is; where the printed source and this translation differ in notation, the source's own form is recorded in the editorial apparatus.

Numerals are Arabic, even where the source sets Roman ones. Roman numerals are kept only in headings, running headers and signatures, which are transcribed as printed.

Dates are given as year, month, day — *1572 November 11*. Dates are Julian, as the source gives them, and have not been converted to the Gregorian calendar.

Times are written with raised unit letters: $7^{h}\ 36^{m}\ 12^{s}$ for seven hours, thirty-six minutes and twelve seconds. An interval longer than a day carries a raised d — $3^{d}\ 7^{h}$. The degree and minute signs below are never used for time.

Angles are written with the conventional signs — 23° 27′ 30″. Early modern astronomy divides the degree sexagesimally and often runs to places for which modern writing has no common symbol; the orders used in this book are:

sign	order	the source's term
°	degrees	*partes*, *gradus*
′	minutes of arc	*scrupula prima*, *minuta*
″	seconds of arc	*scrupula secunda*
‴	thirds	*scrupula tertia*
⁗	fourths	*scrupula quarta*

A **sign** is 30° — one twelfth of the circle — and is written with a raised s: 5^{s} 3° 57′ is five signs and 3° 57′, that is 153° 57′. The source uses the sign both as a place in the zodiac and as a plain unit of 30° for a motion or a distance, and this translation keeps that distinction by writing the count rather than naming a constellation.

The signs of the zodiac. Longitude is reckoned eastward from the first point of Aries.

	Latin	English	longitude
♈	Aries	the Ram	0°–30°
♉	Taurus	the Bull	30°–60°
♊	Gemini	the Twins	60°–90°
♋	Cancer	the Crab	90°–120°
♌	Leo	the Lion	120°–150°
♍	Virgo	the Virgin	150°–180°
♎	Libra	the Scales	180°–210°
♏	Scorpius	the Scorpion	210°–240°
♐	Sagittarius	the Archer	240°–270°
♑	Capricornus	the Goat	270°–300°
♒	Aquarius	the Water-bearer	300°–330°

	Latin	English	longitude
♓	Pisces	the Fishes	330°–360°

The Sun, Moon and planets.

	Latin	English
☉	Sol	the Sun
☿	Mercurius	Mercury
♀	Venus	Venus
♂	Mars	Mars
♃	Iuppiter	Jupiter
♄	Saturnus	Saturn

Abbreviations used in table headings: **Long.** longitude; **Lat.** latitude; **RA** right ascension; **Asc.** ascension; **Arg.** argument.

Where a table in the source spreads one quantity across separate columns for degrees, minutes and seconds, those columns have been joined into a single column and the value written whole. Where a value runs to thirds and beyond, the columns are kept, since there the columns are the notation.

[Non-book page: front cover of the modern binding — mottled brown calf, raised bands on the spine; no printed text.]

[Non-book page: front pastedown, blank paper, with a library mark in pencil at top left: "1529 (Rar)", struck through. No printed text.]

[Non-book page: front free endpaper, blank, with a library shelfmark in pencil at top right: "Rar 9471: 1/1 fol." No printed text.]

Engraved frontispiece

Diagram. The engraved frontispiece, a full-page copperplate within a ruled border, carrying no title text. It is an allegory of the rival world-systems weighed in a balance.

Apex. At the top centre, in the sky, a short pointed Hebrew word of four letters [?] is set above a hand reaching down out of a cloud from the upper right. Rays stream down from the hand's outstretched fingers, which are labelled in three words: **Numerus**, **Mensura**, **Pondus** ("Number, Measure, Weight").

Upper left. Four winged putti in clouds hold up planetary bodies, each marked with its planetary sign: one holds a spotted globe marked with the sign of Mars; one a globe with a dark limb, marked with the sign of Venus; one, lower down, a globe in phase, marked with the sign of Mercury. Between them shines a radiant Sun drawn with a human face. A ribbon-banner unfurls across this group: **Dies diei eructat uerbum** ("Day pours forth speech to day").

Upper right. Against a darker, star-strewn sky, more putti hold the remaining bodies: Saturn drawn with lateral "handles" and marked with its sign; Jupiter, banded and ringed by small companion stars, marked with its sign; a large Moon showing a mottled, cratered face with a dark crescent of shadow; and below them a comet with its tail. A second banner runs across this group: **Et No[?] nocti indicat scientiam** ("and night to night shows knowledge").

Inscribed rays. Two inscriptions run along the rays descending from the divine hand. On the left, rising towards the hand: **Videbo Cælos tuos, opera digitor tuor** ("I will see your heavens, the works of your fingers"). On the right, descending: **Non Inclinabitur in Sæculum Sæculi** ("It shall not be moved, for ever and ever").

The figures. At the left stands a tall, bearded, half-draped man whose body is covered all over with open eyes — the many-eyed Argus. He grasps a long telescope in his left hand and points with his right towards the inscription and the sky. At the right stands a winged woman — Urania — her gown and mantle strewn with stars and her girdle marked with signs of the zodiac (Virgo and Libra are among those visible). In her right hand, raised high, she holds the beam of a balance; in the crook of her left arm she carries an armillary sphere, whose zodiac band shows the signs of Aquarius, Pisces, Aries, Taurus and Gemini, with one more sign not read [?]. Along the beam of the balance runs the inscription **Ponderibus librata suis** ("balanced by its own weights").

The balance. From the left arm of the beam hangs a pan in the form of a large disc showing a heliocentric scheme: the Sun, with a face, at the centre, surrounded by concentric orbits, with the planets marked by their signs around them: Mercury and Venus on the two innermost orbits, then a small globe of the Earth carrying a crescent Moon on a little circle of its own, then Mars, Jupiter and Saturn. This pan rides higher. From the right arm hangs a second, larger disc, which hangs lower — the heavier side: it shows a scheme in which the Sun, carrying small orbits of its own (planetary signs beside it), revolves together with outer planetary orbits about a central body set lower in the disc — a small globe of the Earth, with a crescent Moon on a little circle round it. Mercury and Venus are marked beside the Sun, Mars on the left, and Saturn and Jupiter on the rim.

Below. Between the two figures, at the foot of the balance, an old bearded man in a mantle reclines on the ground, looking up at the pans. Beside his head is engraved **Erigor dum Corrigor** ("I am raised up while I am corrected"). His arm rests on an ornamental cartouche-shield inscribed **Cl. Ptolem.** (Claudius Ptolemy), whose field shows two globes, one of them drawn with a grid of circles. At the centre foreground stands a crowned cartouche bearing a coat of arms with a lozengy (diamond-chequered) field[1]. At the lower right, cast down on the ground at Urania's feet, lies a third disc showing a geocentric scheme of concentric circles about a central body, with signs around them: the Moon nearest the centre, then Mercury, Venus and the Sun, a sign with a crossed stem at the left [?], then Jupiter and Saturn — evidently the discarded system.

Engraver's signature. At the lower left, inside the border: **F. Curtus Bon. Incid.** ("F. Curtus of Bologna engraved it")[2].

Owner's stamp. Across the plate's lower border, an oval blue library stamp, partly legible: "…POLYTECH…" in its upper arc and "…BIBLIOTHEK…" in its lower arc; the centre is not read [?].

Labels and inscriptions read: Numerus; Mensura; Pondus; Dies diei eructat uerbum; Et No[?] nocti indicat scientiam; Videbo Cælos tuos, opera digitor tuor; Non Inclinabitur in Sæculum Sæculi; Ponderibus librata suis; Erigor dum Corrigor; Cl. Ptolem.; F. Curtus Bon. Incid.; the zodiac signs of Virgo and Libra on Urania's girdle; the planetary signs of Mars, Venus, Mercury, Jupiter and Saturn held by the putti.

Labels not read: the four-letter Hebrew word at the apex [?]; the final letter of "No[?]", hidden in the fold of the banner; the individual planetary signs on the three system-discs (present, but not read one by one); the centre of the library stamp.

[1] supplied identification — the arms of the Grimaldi family; the book is dedicated to Cardinal Girolamo Grimaldi

[2] supplied identification — Francesco Curti, the Bolognese engraver

Title page

THE NEW ALMAGEST [ALMAGESTVM NOVVM]

embracing the old and the new astronomy, advanced by the observations of others and by the author's own, and by new theorems, problems and tables;

DISTRIBUTED INTO THREE VOLUMES,

whose contents the following page will set out.

BY FATHER GIOVANNI BATTISTA RICCIOLI [P. IOANNE BAPTISTA RICCIOLO]

OF THE SOCIETY OF JESUS, OF FERRARA,

Professor of Philosophy, Theology and Astronomy.

BOLOGNA

From the press of the heir of Victorius Benatius [Vittorio Benacci], 1651 [M DC LI].

With the permission of superiors.

VOLUME ONE [TOMVS PRIMVS]

contains ten books, whose subjects are listed below.

- **BOOK I.** On the Sphere of the World in General.
- **II.** On the Elementary Sphere.
- **III.** On the Sun.
- **IV.** On the Moon.
- **V.** On Eclipses.
- **VI.** On the Fixed Stars.
- **VII.** On the Minor Planets. This Book is further subdivided into six Sections.
- **VIII.** On Extraordinary Phenomena. Its Sections:
  - I. On Comets.
  - II. On New Stars.
- **IX.** On the Systems of the World. Its Sections:
  - I. On the Creation and Nature of the Celestial Bodies.
  - II. On the Motions and the Movers of the Celestial Bodies.
  - III. On the Systems of the World that Suppose the Earth Unmoved.
  - IV. On the System that Supposes the Earth Moved.
  - V. On the Harmonic System of the World.
- **X.** General Problems Serving Astronomy. Sections:
  - I. A Compendium of Trigonometry, Old and New, that is, Linear and Logarithmic.
  - II. Problems of the Primum Mobile [Primi Mobilis], Collected and Augmented.
  - III. Problems of Time, Collected and Augmented.
  - IV. Geographical and Cosmographical Problems, Augmented.

- V. Problems of Parallaxes, with Theorems, Collected and Augmented.
- VI. Problems of Refractions, Collected and Augmented.

This first Volume is now being published.

VOLUME TWO [TOMVS SECVNDVS]

contains five books, whose subjects are as follows.

I. THE TRIGONOMETRICAL BOOK [LIBER TRIGONOMETRICVS]

It treats the analysis of plane and spherical triangles, with their theory and practice in various forms. Added to it are Tables of Sines, Tangents and Secants, extended to seconds at the beginning and the end of the quadrant, together with the Logarithms of arcs and of absolute numbers.

II. THE INSTRUMENTAL BOOK [LIBER ORGANICVS]

It treats astronomical instruments, their construction and use, and the method to be followed in observing celestial phenomena.

III. THE OPTICAL BOOK [LIBER OPTICVS]

It treats the optical part of astronomy, and especially refractions in the air, in the eye, in spectacles, and in the lenses of telescopes.

IV. THE GEOGRAPHICAL BOOK [LIBER GEOGRAPHICVS]

It treats the mathematical part of geography, especially the correction of latitude and longitude, hydrography, and the art of navigation.

V. THE CHRONOLOGICAL BOOK [LIBER CHRONICVS]

It treats the reckoning of time, the calendars of the nations, and the epochs of years; and also the year and the day of the birth and of the death of CHRIST the Lord. Added to it is an epitome of chronology, confirmed by astronomical criteria [characteribus Astronomicis], with more correct tables of memorable events and persons.

VOLUME THREE [TOMVS TERTIVS]

contains ten books, whose subjects are as follows.

- **I.** Observations of the Sun.
- **II.** Observations of the Moon.
- **III.** Observations of Eclipses.
- **IV.** Observations of the Fixed Stars.
- **V.** Observations of the Minor Planets.

(Books I–V, braced together:) Those of others and our own, with the astronomical conclusions drawn from them.

- **VI.** The Precepts of the Primum Mobile [Primi Mobilis Præcepta].

- **VII.** The Tables of the Primum Mobile.
- **VIII.** The Precepts of the Second Movers [Secundorum Mobilium Præcepta].
- **IX.** The Tables of the Second Movers.
- **X.** The Remaining Tables, serving Astronomy or Cosmography, with their accompanying Precepts.

Among the tables just named are contained not only those which, on the Author's hypothesis, appear more certain, but also the tables of the other astronomers reduced to an epitome, or the equations to be added to or subtracted from the Author's tables in order to reduce them to the tables of the others.

TO THE GREATER GLORY OF GOD [AD MAIOREM DEI GLORIAM].

Dedication title

TO THE MOST EMINENT PRINCE, HIERONYMUS CARDINAL GRIMALDUS [Girolamo Grimaldi]: THE FIRST PART OF THE FIRST VOLUME OF THE NEW ALMAGEST [ALMAGESTI NOVI PARS PRIOR TOMI PRIMI].

Diagram. Engraved armorial plate: the arms of the dedicatee. A cardinal's hat at the top, its cords knotted and looped out to either side and ending in tassels hanging down both flanks; beneath the hat a cross with trefoil ends stands above the shield. The shield is set in a heavy scrolled cartouche of curling leaves; its upper part (the chief) carries a displayed eagle with a small crown above its head, the lower part is lozengy (a field of diamonds). Engraver's signature at lower left of the plate.

- Signature (lower left, inside the plate border): "IIFontana F." — small engraved script: two initials, the surname, and "F." (fecit, "made [this]").
- No other labels.

TO THE MOST EMINENT PRINCE GIROLAMO CARDINAL GRIMALDI [EMINENTISSIMO PRINCIPI HIERONYMO CARDINALI GRIMALDO].

You have seen, MOST EMINENT PRINCE, at the entrance of this volume, Claudius Ptolemy [Claudium Ptolemæum], who was the renowned author of the old *Almagest*. With one hand he holds out a globe that is at once terrestrial and celestial — the insignia, that is, of his twofold primacy in geography and in astronomy; with the other, leaning as it were on the most noble arms of the GRIMALDI family [GRIMALDAE Gentis], he is raised up to the hope of a New Almagest while the old one is being corrected.[3] For on the very frontispiece of the work I chose to seek a shield and protection from that House, to whose liberality and industry I freely declare that I owe the whole mass of this construction, great as it is. That House contributed its wealth and its own talent; let it therefore contribute, too, both its guardianship and its title — that

[3]the play is on *Erigi* / *Corrigitur*, "raised up" / "corrected", both capitalized in the print.

is, the bulwark of patronage and the ornament of an emblem. And from the great number of the nobles of that family, whom should I single out as my patron, if not You in the first place, MOST EMINENT CARDINAL? For, as the panegyrist Pliny says of Trajan, "it was not a matter of searching for someone to be chosen, but of choosing the one who stood out."

Who, then, unless perhaps a man utterly without knowledge, would deny that You stand out among the members of the Grimaldi House — not only those still living, but also those who went before? And this however golden the branches of that tree on which are counted, besides eighteen bishops or archbishops, also seven and perhaps eight most distinguished Cardinals of the Holy Roman Church [S. R. E.]. Namely: Theobaldus, son of Grimaldus II, Prince of Monaco [Monœci], Cardinal of Saints John and Paul, of the title of St Pammachius, created by Urban II. Guido, nephew of Guido II, Prince of Monaco, Cardinal of St Balbina, created by Paschal II. Lucas, nephew of the same Prince, of Saints John and Paul, of the title of St Pammachius, created by Innocent II. Michael Crespinus, descended from Crispinus Ansgothus, son of Grimaldus I, Prince of Monaco, Cardinal of St Stephen on the Caelian Hill, created by Clement V. Girolamo [Hieronymus], sprung from Oberto Grimaldi [Obertus Grimaldus], Prince of Monaco, of the title of St George, created by Clement VII. Agostino Grimaldi [Augustinus Grimaldus], Prince of Monaco, who, while he was guardian of Prince Luciano [Lucianus] and had received Charles V as his guest, was named Cardinal by the same Clement VII at Charles's wish. To these I would most gladly add Angelicus Grimaldi, created Cardinal by his grandfather Urban V, formerly Guillelmus Grimoaldi [Guillaume de Grimoard] — if only, just as one may form a strong conjecture from the family arms and the surname of both, one could likewise show plainly from history that that thrice-greatest Pontiff and his grandson were of those Grimaldi who sprang from the same trunk from which YOU and the Princes of Monaco have taken your origin. But two things stand in the way, which you have never set below the pontifical mitre itself: Modesty and Truth. The first permits nothing exaggerated in its own commendation; the second admits nothing into this genealogy that is not evident, since it does not need conjecture, however strong, for its own nobility.

But You, GIROLAMO, You — last in time among the Cardinals [Cardines] of the Holy Roman Church just listed, yet at length most eminent in title and in merit — have risen to so great a height by climbing not only the steps of offices and duties, but the grand, heroic leaps [assultibus] of the virtues. You were, to be sure, in 1621 Referendary of both Signatures under Gregory XV; in 1625 Vice-Legate of the Province of the Patrimony; in 1628 Governor of Rome. You were in 1632 Nuncio Extraordinary to the Emperor Ferdinand II; in 1634 Governor of Perugia; and in 1636 Pro-Legate of the Duchy of Urbino. You were at last, in 1641, Archbishop of Seleucia and at the same time Nuncio to Louis XIII, King of the French; and then, in 1643, created Cardinal of the Holy Roman Church, of the title of St Eusebius, by Urban VIII. What brought You

here was not the multitude of your achievements alone, but their greatness; not the number of your dignities, but the dignity of your merits: namely, the highest prudence; a penetrating keenness of mind; a hidden maturity of counsel; a timely dexterity in action; a vigilance that looks round upon everything; an expertise in civil and ecclesiastical affairs, and a profound one; and above all these, a singular uprightness of character that needs no proof. It was by these, surely, by these splendours, that the most wise Pontiff judged the purple should be made illustrious — not You by the purple. In this breast the Most Christian Kings have rightly laid up the sum of French affairs and the treasures of their counsels.

For here indeed is the place to recall with the greatest delight by how close a bond You repay to the royal stock of the Franks the vein of dew from which You once drew the moisture of your birth, by an origin as manifest as that of days from suns. With a noble expenditure, I admit, but a just one, You have consecrated these purple labours [sudores] — yet its own — to blood that is royal, yet your own. And oh, how many have heard of it and how few have understood it: by a harmonic tempering of dissonant negotiations You reconcile civil polity with the hierarchy, the palace of kings with the temple of God — in a word, the Church with the Court! And yet this is both most wonderful and not wonderful at all. For what wonder is there if, in such a council and as it were concert of courtly and of heavenly affairs alike, You are not found degenerate from your ancestors? We are ignorant, I suppose, of the Arnoldi and the Arnulfi [Arnoldos … Arnulphos], and the twin Pippins — one of Landen [à Lauda], the other of Herstal [ab Heristallo]; of the Modualdi and the Clodulfi [Modualdos … Clodulphos], the two Grimoaldi, and the Theobaldi — from all of whom, of course, springs the Grimaldi family! Of these, if you except Saints Modualdus and Clodulfus, all were Counts of Gallia Belgica and Counts Palatine, and with that Mayors of the Palace in Francia or in Austrasia; and all without a single exception were at that time Counsellors of the Kings of East or of West Francia, or Prefects of the Kingdom itself. They administered the supreme power in such a way that those who were bishops — and those three most holy men were bishops: Arnulfus and Clodulfus, prelates of Metz, and Modualdus of Trier — consecrated the court of kings into a church, while the others, as dukes, so conducted themselves in the palaces as bishops do in their own churches. For all of them had set themselves to strive earnestly that what belonged to the subjects and the people should be rendered to the people; what belonged to kings, to kings; what belonged to pontiffs, to pontiffs; what belonged to God, to God. From this stock, therefore, no more noble than it was devout, You drew a sap not foreign but native — one that, by spontaneous imitation, You turned for Yourself into the necessity of a most numerous example.

So France, or rather the Christian world, recognizes revived in You its own and equally your own Pippin of Landen [a Lauda] — never to be praised enough [nunquam satis laudandum], because so often and always praiseworthy

[laudabilem] in You — of whom that most beautiful eulogy of Stephanus, Abbot of Liège [Stephani Abbatis Leodiensis], will forever resound: "For he was" (these are that Abbot's words, in the Life of St Modualdus, the cousin of this Pippin) "of most approved life and most pure reputation, a dwelling-place of wisdom, a trea- sury of counsels, a defence of the laws, an end of controversies, a bulwark of the fatherland, an ornament of the court, a way for dukes, a discipline for kings." Nor did You express in your manner and measure of conducting affairs this Pippin alone, but also the Saints Arnulfus and Modualdus, your forebears [maiores tuos] — yet in You no lesser [non minores]. Posterity, then, will go on with the same Abbot Stephanus, and will repeat what he pointedly remarked about the prelates who moved in the court of Dagobert and of the other kings: "It is pleasing now to consider what that time was like, how acceptable, how serene, when kings indeed reigned, but just and God-fearing men ruled the kings together with the kingdom; when justice, faith, truth, modesty and the other good arts were kept in war as in peace; when all the business of royal power was directed by the line of divine equity." And not stopping there, he soon adds: "What a palace that was, how holy, from which the Church received such renowned and such illustrious priests — from which came men who did not seek the courts of kings, but were rather sought by kings!" Your name will take its place among these names, since You so support kings that You serve God alone and no one else besides; You so prevail by foresight and counsel that You do not domineer; You so turn the rights of arms and the hinges of war that, closing the doors on war, You are able to conclude a stable, perpetual and august peace. And yet, though You do all things worthy of kings, You take offence only at your own praises. For I hear that the modesty of your mind, and hence of your mouth and ears, is so singular that, with a brow otherwise most gracious but drawn almost into severity, You at once shut off access to your praisers alone — even though your Virtue and your Fame have opened a vast theatre to their applause and encomia.

I understand, therefore, that I must fear lest I offend the very man by whom I wish to be defended, and imprudently provoke against me the one I have chosen to invoke as my patron. To obtain this the more easily, I appeal once more to the name of the Grimaldi family as a ground — I had almost said a necessary one — for your protection. For over and above the munificence of that House toward me, Father Francesco Maria Grimaldi [P. Franciscus Maria Grimaldus], of our Society, Professor of Mathematics, has spent his effort on this work of ours — or, more truly, has spent his whole self on it. He stood by me as I observed the celestial phenomena, himself a most skilful observer of the stars. He was at my side as I wrote, suggesting from the vein of his own talent things neither few nor to be regretted. Finally, he oversaw the printing of the volumes, with untiring labour, inviolable fidelity, keen judgement, sincere frankness and the most persistent vigilance. But I add nothing more, lest I provoke to anger the modesty of a man who is, in this respect above all, candid and devout. Nor would I have dared to mention even this much, which I have

disclosed very much against his will, had I not been driven to it by the need to show You in what way the Grimaldi House contributed both wealth and talent to whatever this work amounts to — and by what right of confidence I had to seek from that same House a Protector of the most splendid authority. Here, then, MOST EMINENT ONE, is this first Volume of the New Almagest, cast down at your feet, so that — protected, if not in the fold of your purple, then at least by its outermost hem, and wrapped in it once for all — it may begin to be unrolled by the hands of the learned without shame or fear.

FAREWELL [VALE].

Bologna, in the year 1651, on the day of the Assumption of the Virgin MARY into Heaven [Bononiæ Anno M DC LI. die Virginis MARIÆ in Cælum Assumptæ].

YOUR EMINENCE'S [EMINENTIÆ TVÆ]

most devoted and most obedient Io. Baptista Ricciolus [Giovanni Battista Riccioli], of the Society of Jesus.

Licence of the Reverend Father Vincentius Carrafa [Vincenzo Carafa], Superior General of the Society of Jesus [Facultas R. P. Vincentij Carrafæ SOC. IESV Præpoſiti Generalis.]

Three theologians of our Society have examined the first Volume of the *New Almagest* of Father Giovanni Battista Riccioli [P. Ioannis Baptiſtæ Riccioli], priest of our Society, and have approved it for publication. We therefore grant permission for it to be printed, if it so seems good to those whom the matter concerns. In witness of this we have issued this letter, signed by our own hand and secured with our seal, at Rome, 1647 March 12.

Vincentius Carrafa.

Place of the seal [Locus Sigilli].

Licence of the Reverend Father Alexander Bosellus [Alessandro Boselli], Provincial of the Society of Jesus in the Venetian Province [Facultas R. P. Alexandri Boſelli SOC. IESV Prouincialis in Prouincia Veneta.]

Three theologians of the same Society have examined the additions to the First Volume of the *New Almagest*, divided into two parts, by Father Giovanni Battista Riccioli, priest of our Society, and have approved them for publication. We too, having received authority from our Very Reverend Father Goswin Nickel [R. P. N. Goſuino Nickel], Vicar General of the Society of Jesus, grant permission for them to be printed, if it so seems good to those whom the matter concerns. In witness of this we have issued this letter, signed by our own hand and secured with our seal, at Bologna, 1651 August 17.

Alexander Bosellus.

Place of the seal [Locus Sigilli].

Examined by Father Bonaventura Cavalieri [P. Bonauentura Caualerius], and afterwards by Father Francesco Maria Grimaldi [P. Franciſcus Maria Grimaldus], professors of mathematics, on behalf of the Most Eminent and Most Reverend Cardinal Archbishop.

Don Andreas Cuttica [D. Andreas Cuttica], Clerk Regular of St Paul, and Rector of the Penitentiary [Pœniten. Rect.] [?], on behalf of the Most Eminent and Most Reverend Cardinal Archbishop.

IMPRIMATUR [IMPRIMATVR]

Brother Vincentius Pretus of Serravalle [F. Vincentius Prętus à Serraualle], Inquisitor General of the Holy Office at Bologna.

PRÆFATIO AD LECTOREM IN SEPTEM ARTICVLOS DISTRIBVTA.

Preface to the Reader, Divided into Seven Articles

ARTIC. I. Aſtronomiæ Encomium.

Article I. In Praise of Astronomy

If the heavens stand above the Earth and all the elements; if the stars stand above not stones only but gems; if, finally, that pure, unalloyed and everlasting gold of the celestial lights stands above everything of inferior grade, as above baser metals — that is, above any bodily things whatever below the Moon — then surely ASTRONOMY too, crowned with stars, and those its own, raises its head as Queen among all the faculties or natural sciences. For divine Theology, empress of all disciplines, to which Astronomy and every other branch of learning must yield, is not that natural theology which is usually called by another name the science of metaphysics [Greek: τῶν μεταφυσικῶν (tōn metaphysikōn, "of the things after physics")]. It is the theology that discourses of GOD and divine things by the light of supernatural Faith, divinely infused.

[Margin: The eminence of Astronomy above all the natural sciences.]

With this one exception Astronomy surpasses all the rest — whether you consider the dignity of its subject; or the certainty of its method, and its evidence in so lofty a depth of contemplation; or the number of natural faculties that serve it as handmaids, and which it in turn supplies or assists; or the pleasant tranquillity of a most pure pleasure; or, finally, the princely men, eminent in priestly or royal dignity, who have commended it by private study, by public testimony of honour, or by the vote of their wonder.

[Margin: The chief heads on account of which Astronomy excels.]

So our own Caussin [Cauſſinus noſter], in the preface to his books *On the House of God* [in præfat. ad libb. de Domo Dei], rightly laid it down: *Among the other things by which the human mind is nourished and refreshed, as by a pleasant pasture of wisdom, the first rank belongs to the contemplation of the heavenly bodies, by which we enter into the very house of God, as tent-companions of the Angels and candidates of the first light.* Or, with Manilius, *Astronomica* I [I. Aſtron.]:

> ..... *and we, a part of it, behold the parent of things, and, born of it, draw near to the stars.* [Rerumque parentem / Pars ſua conſpicimus, genitique accedimus aſtris.]

To show the excellence of its subject one needs not a tongue but eyes. To have seen the Sun by day, or the stars by night — above all on a night neither moonless nor overcast — is to have been struck with wonder; the spectacle itself is at once a marvel. The very tongue of God bears witness to it, saying: *The firmament of the height is its beauty, the appearance of heaven in a vision of glory*, etc.

[Margin: The dignity of its subject.]

[Margin: Ecclesiast. 43 [Eccleſiaſt. 43.].]

Let Astronomy be allowed to glory — but in a divine gift — from the same source from which GOD himself commends himself, and to say: *Heaven is my seat*; my wealth is the stars, which neither Abraham, at GOD's bidding, could number, nor Abrachys, that is, Hipparchus — however inflated the style in which Pliny the Elder [Plinius] said: *Hipparchus, never praised enough, for no one proved more fully the kinship of the stars with man*, etc., *who dared a task too bold even for God, to count the stars for posterity*, etc., *leaving the heavens as an inheritance to all.*

[Margin: Book 2, ch. 26 [Lib. 2. c. 26.].]

The heavens, I say, are Astronomy's domain: highest in place of all visible things, everlasting in duration, astonishing in the vastness of their mass, most delightful in the splendour and variety of their lights; awe-inspiring in the majesty of a world that glides silently on, most pleasing in the hidden harmony of like and unlike motions; most fruitful in the secret, steady distillation of perpetual influences; and venerable for the wisdom of the Angels, who by a constant law wheel the celestial bodies round at the motion of the First Intelligence, as under a governor [tanquam Harmoſtæ]. The heavens, finally — whether you call them the curtain of the Tabernacle of the Most High or the outer shell of the Empyrean — are nonetheless a vast token of divine glory and of everlasting happiness; so much so that our most holy Patriarch IGNATIUS [IGNATIO] [Ignatius of Loyola] more than once uttered that sigh, sweeter than any song: *Ah, how sordid the Earth is when I look at the heavens!*

[Margin: The mathematical certainty proper to Astronomy.]

The certainty of Astronomy, moreover, is so great that Plato, in Book 7 of the *Republic*, holds that the eye of the mind, clouded in the other disciplines, is healed by its evidence so that it may discern the truth. For when he had decreed that Astronomy must be kept in the City, he says: *By disciplines of this kind a certain instrument of every soul is purged and revived, which had previously been infected and blinded by other pursuits — though it is more worth preserving than ten thousand bodily eyes. For by it alone is truth seen.* And again, in the same dialogues of the *Republic*, that Platonic Socrates says: *Have you noticed that those who are mathematicians by nature appear sharper at almost all disciplines, while those of duller wit, if they are trained in this one, even if they gain no further advantage, nevertheless usually become cleverer than they were before? For you will not easily find any of the rational disciplines, other than this one, that lays heavier labour and care on the one who learns and thinks it through.*

Astronomers, moreover, have been customarily called, both formerly and now, "the Mathematicians" by antonomasia [Greek: ἀντονομαστικῶς (antonomastikōs, "by antonomasia", the name of the genus given to its pre-eminent member)]. And Ptolemy, although the books of his great *Construction* concern astronomy alone, nevertheless entitled them περὶ μαθηματικῆς συντάξεως [Greek: peri mathēmatikēs syntaxeōs, "On the Mathematical Composition"]. It was because of this certainty that the same author turned himself wholly from natural theology and physical speculations to astronomy and geography; for so he addresses Syrus in chapter 1 of Book 1 of the *Almagest*: *Since, then, we have understood from this that two kinds of speculation deserve the name of conjecture rather than of more certain knowledge — the theological, because it is incomprehensible, and the physical, because owing to the instability of matter it can scarcely be grasped, so that we judge that philosophers can never agree about it — while the mathematical alone (if one uses it rightly) yields firm and unchangeable knowledge*, etc., *we resolved to support this kind as far as our strength allows, and above all that part of it which concerns divine and heavenly bodies. For this part alone considers things everlasting, which are always the same.*

Nor is it so very surprising that simple arithmetic and geometry, separated from all the inconstancy of sensible matter, or those subordinate sciences, at once mathematical and physical, which fall more closely under our senses, attain a greater degree of certainty than bare physics. So much so, that Plato in the *Timaeus* asks of the natural philosopher probable reasons, but of the mathematician and the divine reasons that can be neither refuted nor overthrown; and Aristotle, *Posterior Analytics* 1, text 30 [1. Poſter. tex. 30.], asserts: *It belongs to those who use the senses to know the very "that", but to the mathematicians to know the very "why".*

[Margin: The heroic labours of astronomers.]

But this surpasses all wonder: that astronomers, amid such perplexity and anomaly of the celestial motions, and amid planetary coils so abstruse — coils that stand so far from human sense, mark no trace of their course in the liquid aether, and do not all complete their revolution within the lifetime of a single observer — could, I say, find so happy a way out of the windings of that aetherial labyrinth by the thread of Uranian subtlety, and reach such evidence, that they are able, by a sure foreknowledge that varies only by very few minutes [ſcrupulis], and those mostly negligible, both to foresee long beforehand themselves and to foretell to others without fear the places of the stars, their risings and settings, their conjunctions and digressions, and finally all their configurations and syzygies. There is no art — and I declare this most firmly, having weighed and surveyed everything — there is surely no art that has joined more happily those two things, τὸ χαλεπὸν, καὶ τὸ φανερὸν [Greek: to chalepon kai to phaneron, "the difficult and the evident"], the Arduous and the Evident; none that has advanced so far through so many reefs and shoals of difficulty, and has held, down to this day, harbours so inaccessible. Not even that art which, having conquered the most turbulent monster of the Sea and sailed round the

whole globe, consecrated its Victory as a trophy. For astronomy did not join lands to lands, separated by the dividing Ocean, but heaven to earth — driven through the immensity of the aetherial air by the sails and oars of industry and wit, rather than by wings not given to man, or by Babylonian towers, or by the Giants' ladders of mountains.

Happy the souls whose first care it was to know these things and to climb up to the dwellings above! We may believe that they raised their heads alike above human vices and jests. Neither Venus nor wine broke those lofty hearts, nor the duties of the forum, nor the toil of soldiering; nor did shallow ambition, or glory dyed with false paint, or hunger for great riches trouble them. They brought the distant stars near to our eyes, and set the aether beneath their own intellect. SO IS HEAVEN SOUGHT — not that Olympus should bear Ossa and the peak of Pelion touch the highest stars.

[Margin: Ovid, bk. 1, *Tristia*. [Ouid. l. 1. Trift.]]

Indeed, to accomplish these things one needs not strength and "triple bronze about the breast," but a genius and an intellect that are plainly heavenly — as Plato wisely observes in the *Epinomis*: *"Do not be unaware," he says, "that astronomy is something supremely wise. For the true astronomer must necessarily be, not the one who, following Hesiod and others of that kind, considers setting and rising, but rather the one who will have examined the eight circuits, and how the seven revolve beneath the first, and in what order each completes its circles. And this no nature, unless a wonderful one, will ever easily have looked into."*

[Margin: Plato's praise of astronomers.]

Nor is such a nature reckoned merely wonderful, but something more sublime than human, above all when, by a prediction of eclipses issued in advance, it takes terror away from the minds of mortals. For this reason Livy said of Sulpicius Gallus, who freed from anxiety the army of Paulus Aemilius when it was already dismayed by an eclipse of the Moon: *"When the Moon was eclipsed at the hour he had announced, the wisdom of Gallus seemed almost divine to the Roman soldiers."* Or, conversely, it strikes a salutary terror — the means by which Columbus, when the barbarians of Jamaica denied him access and provisions, threatened them with the eclipse of the following night, and through that portent, once it was seen, obtained not only supplies but obedience and even veneration, as one fallen from Heaven.

[Margin: Bk. 4, Decade 5. [L. 4. Dec. 5.]]

Rightly therefore Pliny the Elder, having recalled Thales, Sulpicius Gallus and Hipparchus — Hipparchus, who *"foretold the eclipses of both luminaries for six hundred years, as the age bears witness, as though he were a sharer in the counsels of nature,"* burst out into that epiphonema: *"Great men, above the nature of mortals, who discovered the law of such great divinities and released the wretched mind of men"*; and a little further on: *"Bravo for your genius, you interpreters of heaven, capable of grasping the nature of things, discoverers of a reasoning by which you have conquered gods and men!"* He had said, moreover, in chapter 11: *"This reasoning draws mortal minds up into heaven, and, as though to those who contemplate from there, it discloses the magnitude of the three greatest parts of the nature of things."*

[Margin: Pliny the Elder, bk. 2, ch. 12. [Plin. l. 2. c. 12.]]

Plato had said this very thing beautifully in that seventh dialogue of the *Republic*. For he had said: *"I went straight on to astronomy, which consists in the motion of depth itself"*; and presently he adds: *"For I think it is plain to everyone that this directs the eyes of the soul to higher things, and leads it from here to there."* — provided, that is, that the astronomer does not stop at the merely apparent motions of the heavenly bodies, but rises to the exemplar of them that shines in the divine Archetype. For he continues: *"Must we not, then, use the variety of the heavenly bodies as a kind of example for the contemplation of things above? — just as if someone were to see figures wrought by Daedalus."* And after a few words he goes on: *"Let us suppose that the true astronomer will do the same when he looks upon the motions of the stars, judging, namely, that the heaven itself and the things that are in it were fabricated most beautifully by the very founder of heaven,"* etc.

[Margin: Astronomy raises the soul to heavenly and divine things.]

To this affinity of astronomy with Heaven and divine things one could adduce that faculty of divining the future, and the power of prognostics [Greek: προγνοστικῶν (prognostikōn, "of prognostics, of foretelling")], on account of which the Athenians publicly set up in the Gymnasium a statue with a gilded tongue to Berosus the astrologer — had not the genethliacs [Genethliaci, casters of nativities] tried, beyond *effects*, to predict *outcomes* as well, and future contingents — a venture always struck out by the sacred Laws, yet never extinguished; or had not the astrologasters, by stooping to every trifle, won infamy instead of fame for this heavenly discipline. Let us nevertheless hear that exordium of Manilius:

[Margin: From Pliny the Elder, bk. 7, ch. 37. [Ex Plinio l. 7. c. 37.]]

By song to draw down from heaven the divine arts, and the stars privy to fate, which vary the diverse fortunes of men — the work of heavenly reason — I undertake.

[Margin: Bk. 1, *Astronomicon*. [L. 1. Aſtronomicon.]]

He called it an *art* in his own judgement, and he also attributed its invention to the Kings of Egypt, when a little later he sang as follows:

It first deigned to move royal souls, souls touching the summits of things, next to heaven, etc. These alone knew such glory, and were the first, by art, to see the fates hanging on the wandering stars, embracing long ages in unremitting labours: through varied practice experience made the art, with example showing the way.

[Margin: The usefulness of astronomy.]

But there is another fruit of astronomy, and a far more manifest one. Just as astronomy advances attended by very many sciences, as if by handmaids — I mean Arithmetic and Geometry, then Optics and Gnomonics, Harmonics and Organics, and finally Geography and Chronology — so in turn it is both an ornament to them and a protection and support, no less than to Medicine, Agriculture, Navigation and Hydrography; but above all to Geography and Chronology, whose parts, which otherwise hold together only with difficulty, it binds with the firmest bonds of the phenomena, and confines within fixed limits of places and times. Moreover the proper distribution of years and months

— without which neither the order of civil business nor the solemnities of sacred rites can stand — depends, in its entirety, on the judgement of astronomers.

[Margin: Bk. 7, *Laws*. [7. de Legib.]]

To this end Plato, in that *Republic* of his, laid down that this part must necessarily be learned, in these words: *"Likewise those things must be thoroughly learned which are useful from the divine circuits of the stars, and of the Sun and the Moon, since with respect to these the whole city must be ordered. Of what things, you will ask? Of the order of days into the month, and of months into the year; and so the seasons, festivals and sacrifices, arranged as is fitting by a kind of natural guidance, will render the city alive and wakeful."* Who arranged the calendars and cycles for the Greeks and Asiatics? Meton, surely, Euctemon, Oenopides, Cleostratus, Eudoxus and Callippus, each of them most expert in the stars. Who took in hand the year, put out of joint by the negligence or the guile of the Roman priests, if not Julius Caesar the Dictator? Although he relied a great deal on Flavius the scribe and on Sosigenes, the Alexandrian astronomer, he was himself, as presider over and director of so great a work, not ignorant of it, and in Lucan he speaks of himself in this way — not more boastfully than truly:

..... In the midst of battles I have always found leisure for the stars, and the regions of heaven, and the gods above; nor shall my year be surpassed by the Fasti of Eudoxus.

That sacred and holy Council of Nicaea, in which, after the condemnation of Arius, so serious an inquiry was held by the three hundred and eighteen Fathers concerning the lawful time for celebrating Easter, entrusted that business above all to the Bishop of Alexandria, since it was agreed that this city had been the Metropolis of astronomy from the time of Hipparchus and Ptolemy. Later, when the Calendar had collapsed — or rather, when the Equinox had slipped back toward the beginning of the year by nearly ten days, and the golden cycle of the Moon and the Sun was degenerating into a leaden one — and when those learned in the stars and the times were protesting from the whole Christian world, the controversy over how to reform it was debated long and much, above all in the Councils of Basel and Constance. Then indeed the very sacred purple of the Cardinals blushed, when it saw so few among ecclesiastical persons initiated into astronomy — even though at that time four distinguished Cardinals were uncommonly learned in it: Petrus Aliacensis [Pierre d'Ailly], Nicolaus Cusanus [Nikolaus von Kues], Bessarion of Nicaea [Bessarion Nicenus], and Nicolaus Schombergerus [Nikolaus von Schönberg]. These men — though at different times — the Supreme Pontiffs judged must be heard, together with all the other most skilled astronomers, especially Io. Lucidus [Giovanni Lucido], Paulus, Bishop of Fossombrone [Paulum Epiſcopum Foroſempronienſem; Paul of Middelburg — identification supplied], Lucas Gauricus [Luca Gaurico], Bishop of Civitate, Albertus Pighius [Albert Pigge], Petrus Pittatus [Pietro Pitati], Io. Stoeflerus [Johannes Stöffler], and Iosephus Moletius [Giuseppe Moletti]. But since, among the others of his own time, Io. Regiomontanus stood out as far

as cypresses usually rise among the pliant viburnum,

Sixtus IV summoned him to Rome above all others for this business, holding out the hope and promise of the Bishopric of Ra- tisbon [Regensburg]. Had Regiomontanus lived — had not either the poison of Trapezuntius [Trapezuntis; George of Trebizond — identification supplied] or a star hostile to him from his very horoscope snatched him away when he was barely in his 41st year — he would surely have undertaken something royal for the correction of the year and of the cycles. Evidently this had been reserved by divine providence for Gregory XIII, so that, with the victory of the art of navigation sailing across the whole globe on the one side, and astronomy now triumphing through the whole Heaven on the other — at nearly the same point of time at which three Japanese princes from the farthest East, imitating as many holy Magi, happily put in at the feet of the Roman Pontiff as at a harbour — he should, having gathered the opinions of distinguished astronomers from every quarter (something the new star in the Chair of Cassiopeia had perhaps foretold ten years before), choose the Lilian form for restoring the Calendar; and that, using our Clavius [Clauio; Christoph Clavius] — or rather his own — as a key [claue] held out to him from heaven, he should open to us a golden age, and that order of the times which God himself was to seal with the most splendid miracles.

[Margin: Miracles in favour of the Gregorian Calendar.]

A witness to what I say is that noble blood which every year at Naples, in the presence of the venerable head of St Januarius the Martyr, liquefies and boils up on September 19 — but on the 19th that belongs to the Gregorian Calendar, the miracle having been moved ten days earlier — just as if blood once shed in testimony to the Roman faith were poured out again and again for the authority of the Roman Pontiff. Again, there is a certain small tree in Swabia which Blessed Birgitta planted in the little garden of her nuns on the night of the Nativity of Christ the Lord, in honour of the threefold Virginity of MARY, ever Virgin before, in, and after childbirth; and by her prayer she obtained that the tree should put forth leaves, flower and bear fruit at distinct intervals of hours within that same night. Afterwards it was accustomed to renew that prodigy every year on the night falling between the 24th and the 25th day of December; and in that very year 1582, which was the first of the Calendar's restoration [reparationus], and ever since, it has followed the Gregorian Calendar. We have learned from Baronius [Cesare Baronio] that by miracles of the same kind, in the fonts of holy Baptism, the Catholics' time for celebrating Easter was confirmed against the Quartodecimans.

Since these things redound in some part to the praise of astronomy, there is no reason why it should try to beg its nobility from the favour or the testimony of princes of every age. For why should it list the Kings and Magi of the Persians together, or the Monarchs of the Chaldaeans, Assyrians and Egyptians — among whom Osymandyes ordered a circle of 365 cubits, as many as there are days in the year, made of solid gold, with the rising and setting of the stars for each day engraved on it, to be placed in his Mausoleum? Nor is there any need to repeat here other dynasts or kings: Atlas of Mauretania, Janus of Italy, Cepheus

of Ethiopia, Numa of Latium, Almamon [al-Maʾmūn] of Arabia, Albategnius [al-Battānī], and Abilfedea [Abū al-Fidāʾ] of Syria, Alphonsus [Alfonso X] of Castile, Matthias of Hungary, Frederick II and Christian of Norway and Denmark. Of these, Alfonso contributed four hundred thousand gold pieces as a kind of dowry for Astraea; Matthias presented Io. Regiomontanus, for a single table of the primum mobile, with a most sumptuous robe and eight hundred Hungarian gold pieces; and Frederick, finally, built Uraniborg [Vraniburgum] for Tycho with royal munificence.

Why, finally, should it appeal to the Caesars — Gaius Julius, I mean, Hadrian, Antoninus, Frederick III, Maximilian I and II, Charles V, Matthias I, Rudolf II, and Ferdinand I and II — who received Urania into their courts as a Queen, with such great honour? Maximilian, indeed, drew mathematical figures with his own hand for his Petrus Appianus [Peter Apian] as he was preparing the *Astronomicum Caesareum*; and Ferdinand I, as Iouius [Paolo Giovio] relates, sent as a gift to Solymanus [Suleiman], Emperor of the Turks, a silver machine of exquisite and admirable workmanship, in which, besides the intervals of the hours, the monthly conjunctions of the Moon and the Sun and the motions of the remaining planets were exactly shown. This work is said to have been the Emperor Maximilian's, and the Turk was wonderfully delighted with the gift, since he had thoroughly learned astronomy and cosmography from Hammon, a physician born in Baetica. Furthermore, how highly the Kings of the Chinese esteem all mathematicians — but astronomers and cosmographers above all — whether by ancestral law or by the custom of the realm, is well known above all to the men of our Society, to whom through these very disciplines an entrance has gradually opened for pouring the light of the Gospel into the darkness of that East; and one of these, Father Matthaeus Riccius [Matteo Ricci], on that account a most honourable Mausoleum from that King earned, even in death. Emulating their munificence, the King of Canciani (he is one of the dynasts of Caucincina [Cochinchina]) is said to repay mathematicians with an estate for each eclipse they foretell — just as King Dionysius once paid a talent of silver to Helicon of Cyzicus for predicting an eclipse of the Sun. Yet more precious than all gold and silver, and more delightful, is to carry the globe of the Earth and the whole heaven itself about with one, enclosed in one's mind, and to contemplate it, as often as one pleases, as if at a single glance. Virgil, certainly, wished for this part of wisdom in the first place, as the way to a happy life, when he sang:

[Margin: Georg.]

> *But me, before all things, may the Muses, sweet above all, whose sacred emblems I bear, struck with a great love, receive, and show me the paths of heaven and the stars, etc. why winter suns hasten so to dip themselves in Ocean, or what delay stands in the way of the slow nights. But if the cold blood about my heart should bar me from approaching these regions of nature, may I love the rivers and the woods, inglorious.*

ARTIC. II. De recto Aſtronomiæ vſu, & Dilutio Obiectorum in illam, præcipuè a SS. Patribus.

Article II. On the Right Use of Astronomy, and the Refutation of the Objections against It, Chiefly from the Holy Fathers

[Margin: 1. Abuſus ſiderum adoratio. — 1. An abuse: the worship of the stars.]

What wonder is it if those who abused heaven — whether impious or ignorant — provoked the arms of heaven against themselves? If, darkening the splendour of the stars with the darkness of superstition, they forged the stars into thunderbolts for themselves? The first crime was to offer, of their own accord, to the stars the worship of latria owed to the one Creator of the stars — as though divinity were divided among them in small portions, or as though all the good and ill of mortals hung from them as from a golden chain of the Fates. Jeremiah laments this folly in chapter 9, in those *who sacrificed to all the host of heaven, and poured out drink-offerings to strange gods*; and in chapter 10 he warned: *Do not fear the signs of heaven, which the Gentiles fear.* God himself threatens them through Sophonias [Zephaniah], chapter 1, saying: *I will destroy from this place the remnant of Baal, and the names of the temple-keepers with the priests, and those who worship the host of heaven upon the housetops.* Holy Job, in chapter 31, judged this impiety the foulest of all, in these words: *If I saw the Sun when it shone, and the Moon moving brightly, and my heart rejoiced in secret* — as though the brightness of day or of night portended happiness to him, or were a sign of a divinity hidden in the Luminaries — *and I kissed my hand with my mouth*, which was a sign of adoration and of giving thanks; *which is a very great iniquity, and a denial against the most high God.* Isaiah mocks them in chapter 47, saying: *Let the augurs of heaven stand and save you, who gazed at the stars and reckoned the months, so that from them they might announce to you what is to come.* The Apostle fears for them in Galatians 4: *You observe days and months, etc. I fear for you, lest perhaps, etc.*

[Margin: 2. Abuſus Genethliacorum temeritas. — 2. An abuse: the rashness of the genethliacs.]

Next to this was the error of the genethliacs [Genethliaci], the casters of nativities, who dared to predict from the position of the stars not merely an inclination — arising from the natural temperament of the humours — toward this character or way of life rather than that, but even future contingents, and events that depend on the mere freedom of the will of one or of several persons. They are a race of men which, as Tacitus says in the *Annals* [in annalib.], *will always be forbidden in the City, and always retained* — he would have said more truly, *in the world*. Though they are false, and deceivers, a thousand times over, if once or twice they have divined by chance and not by art, they gain more fame from a single success of their rashness than infamy from so many utterly manifest lies. For it is not as with *the errors of physicians*, which, as Pliny the Elder observes, *the earth covers*, since the dead, who perished through the physicians' unskill, cannot refute them: *the errors of the astrologasters* [Aſtrologaſtrorum] *are*

not so *covered by the sky* that they cannot everywhere be convicted of manifest falsehood.

[Margin: Concil. Bracar. Can. 9. — Council of Braga, canon 9.]

Against these stand the most wise decrees of the sacred canons, of the councils, and of the supreme pontiffs, above all of Sixtus V and Urban VIII. Hence the phrase so often found in Livy and the other writers of Roman history: *the mathematicians were expelled from the City.* For since astronomy is the most excellent of the mathematical sciences, and the genethliacs twisted it into judicial astrology concerning chance and free events — selling, on the strength of the certain prediction of eclipses, an equal certainty in other matters — therefore of the mathematicians under the name, those astrologers and genethliacs were classed. For magic, as Pliny the Elder says in book 30, chapter 1, *in order to creep in more easily, also mixed in the mathematical arts for show — since no one is not eager to know the future concerning himself, and everyone believes that it is most truly sought from heaven.* Hence the accusations against the mathematicians made long ago by Eudoxus and later by Sextus Empiricus; and afterwards Io. Picus [Giovanni Pico della Mirandola] and our own Alexander de Angelis [Alessandro de Angelis] attacked them in whole volumes. Hence too Aquila of Pontus — although a distinguished mathematician, around the one hundred and thirtieth year of Christ — was driven from the Church because he would not abstain from genethlialogy, as Epiphanius reports in his book *On Measures* [lib. de mensuris]; he deserted to the Jews and translated the Old Testament into Greek far from faithfully, staining the heavenly books, he who had already branded so many marks upon the heavenly art. Hence, finally, the censures of the Fathers against such rogues and against the dregs of the Priscillianists, who, as St Leo says in Letter 71, *believe that the souls and bodies of men are bound to the fatal stars*; who, as St Augustine testifies in book 2 on Genesis, *treat of heaven, and while they treat of it are strangers to the Father who is in heaven.* Indeed, as the same author says in the book of 83 Questions, q. 5 [lib. 83. quæſt. q.5.], they are *men who wish to subject our acts to the heavenly bodies, and to sell us to the stars.* For this whole reasoning, as he also notes in book 2 of *On Christian Doctrine*, chapter 24, *is full of pestilent curiosity, tormenting anxiety and deadly servitude.*

St Ambrose, finally, harries them very often, but above all in book 1 of *On Duties*, chapter 26. There he also rebukes a churchman, or sacred teacher, for excessive study of these matters — one who, in order to overcome the obscurity he supposed to lie in cosmography, fails in his duty or in his own salvation, especially if he goes astray among uncertainties — saying: *What is so unseemly as to venerate pieces of wood which they make themselves? What is so obscure as to treat of astronomy and geometry, as they approve, and to measure the spaces of the deep air, and to enclose heaven also and the sea in numbers — to abandon the cause of salvation and seek that of error? Did not Moses, learned in all the wisdom of the Egyptians, put these things to the proof?* — that is, he tried them by experience. *But he judged that wisdom a loss and a folly, and turning away from it he sought God with his inmost affection; and so he saw him, questioned him, and heard him*

speaking. Who is wiser than he whom God taught, who by the power of his work made void all the wisdom of the Egyptians and all the powers of the arts? He did not hold unknown things for known, and rashly assent to them. You see that what he condemns is the rashness of Egyptian or Chaldaean astrology.

[Margin: Exod.3.& 8. Act. 7. Habr.11. [recte: Hebr.] — Exodus 3 and 8; Acts 7; Hebrews 11.]

Add to this that in the first times of the Church, because of the danger of paganism and of ingrained superstition, the outlines of geometrical figures drawn in the dust or on papyrus were regarded by some as conjuring tricks or necromantic drawings. Then too, as our own Caussinus [Nicolas Caussin] says in book 3 of *De Domo Dei*: *Astrology, of whatever kind it was, was not listened to, while Prophecy resounded and the Spirit poured forth. It would then have been a monstrous sacrilege to read the poets, and to name Jove or Apollo with lips so often consecrated to Christ.* But once Satan had been triumphed over and the idols crushed, the Church was not ashamed to acknowledge God as Lord of all the sciences, and to purge the arts that the errors of the Gentiles had infected. Then not only poetry and rhetoric began to serve as soldiers of Christ, but also philosophy and astronomy — indeed astrology too, restricted to the foreknowledge of natural things — and to recover that dignity which Solomon, in Wisdom 7, proclaims he received from God, saying: *For he himself gave me true knowledge of the things that are, so that I might know the disposition of the world and the virtues of the elements; the beginning and the end and the middle of the times; the alternations of the turnings and the changes of the seasons; the courses of the year and the dispositions of the stars.*

[Margin: Trident. Reg. 9. de lib. prohib. — Council of Trent, Rule 9 on prohibited books.]

[Margin: Auctores pro ſana Aſtrologia. — Authors in favour of sound astrology.]

Therefore, if you learn or teach astronomy for the establishment of the civil and the ecclesiastical calendar; if you seek a deeper knowledge of it so as to defend more solidly the literal sense of Sacred Scripture, which the Copernicans abuse, and the decrees of the Sacred Congregation issued against them or against the genethliacs; if you handle astrology soberly and prudently for the use of agriculture, navigation or medicine, as the Sacred Council of Trent and Sixtus V permit, and abstain from foretelling determinately — even by conjecture — future contingents or free actions: then no Augustines, no Ambroses, no Leos will rebuke you. Indeed St Thomas will be your patron — 1 p., q. 1, art. 5; and 3 *Contra Gentiles*, chapter 85; opusculum 21, or *On the Judgements of the Stars* [de iudicijs aſtrorum]; and opusculum 28; and in book 2 on Genesis — and so will Scotus on book 2, distinction 14, and Panormitanus [Niccolò de' Tudeschi] on book 5 of the Decretals, title on sortilege [de ſortil.], chapter *Ex tuarum tenore*; and those who most recently, on behalf of sound astrology — on its use and its defence — have written: Petrus Ciruellus [Pedro Ciruelo], against Pico, in a large and learned work; Father Carolus Gontranus [?], Doctor of the Sorbonne,

in a little book on this subject; and our own Caussinus, in book 3 of *De Domo Dei*. Indeed, before them, St John Damascene, book 2 *On the Faith*, chapter 7; Boethius, *On the Consolation*; Seneca, *To Marcia*; Chalcidius, on Plato's *Timaeus*; and Plato himself in the *Timaeus* and in book 7 of the *Laws*.

That the astrologers of old introduced fate against freedom, and were condemned for that reason, is taught by St Augustine on Psalm 61; by Bardesanes in Eusebius, book 6 of the *Preparation for the Gospel*; by Saints Ambrose and Basil; and by Tatian, an ancient author — but above all by Tertullian in his book *On Idolatry*, who for that reason concludes:

[Margin: 3. Abuſus inanis gloriatio. — 3. An abuse: vain boasting.]

O divine sentence, persistent even to the earth! The mathematicians are expelled, like their angels; the City and Italy are forbidden to the mathematicians as heaven is to the angels. I too shall conclude this article, finally, against the vainglory of certain astronomers who do not, from the heaven, come to acknowledge Heaven, or the Lord of heaven and the Father of lights. I conclude it with St Augustine, *Confessions* book 5, chapter 3: *Those who know these things exult and are lifted up; and, withdrawing through impious pride and falling away from your light, they foresee an eclipse of the Sun so long beforehand, and do not see their own in the present; for they do not ask religiously from where they have the intellect with which they search these things out.*

ARTICVLVS III. De Origine Aſtronomiæ.

Article III. On the Origin of Astronomy

I would not dare to deny that astronomy arose almost together with the stars themselves, and that, of all that was divinely infused into Adam, it was the first to be brought into act and into experiment. For by what earlier or more powerful spectacle could the theatre of the newborn world, amid so great a variety and novelty of divine works, have drawn the eyes of our first parent to itself, than by the very eyes of the world, the lights of heaven, which at once showed him God already watching over his work? These were the first cradle of this art among the Hebrews. Since the patriarchs lived so long that the revolutions of the planets could be recognized and recognized again through frequently repeated experiment, the art grew up until the descendants of Seth. They, as Josephus asserts in book 1 of the *Jewish Antiquities*, chapters 3 and 8, reduced this art — handed down from Adam himself — to an epitome and carved it on twin columns, so that it should not perish in a cataclysm or an ecpyrosis [ecpyrôſi], a conflagration, of the whole world: one column of brick, against the force of fire; the other of stone, against floods of water. Suidas also affirms that Seth obtained the name "divine" because he had distinguished the stars by their own names.

But whether astronomy flowed first from the Hebrews to the Egyptians, and from them to the Chaldaeans, or Babylonians, and to the Greeks; or rather from the Chaldaeans to the Egyptians and the Greeks; and which of these peoples (leaving the Hebrews aside for the moment) were its inventors — this was an

old controversy even long ago, as Achilles Tatius notes at the beginning of his *Introduction to the Phaenomena of Aratus* [Iſagoges ad Arati Phęnomena]: *They say that the Egyptians were the first of all to measure both the heaven and the Earth, and that they handed down the knowledge of it to posterity carved on columns. The Chaldaeans, however, transfer the glory of this discovery to themselves and attribute it to Belus. But the wiser among the Greeks try to ascribe it partly to the gods, partly to the heroes, and partly to those wise men who came after them.* So Tatius.

[Margin: Chaldaicæ Aſtronomiæ antiquitas. — The antiquity of Chaldaean astronomy.]

Josephus, in book 1 of the *Jewish Antiquities*, chapter 15, seems to stand for the Babylonians or Chaldaeans, as does Diodorus Siculus, book 2, chapter 8, where he says: *All grant this, that the Chaldaeans are skilled in astrology beyond the rest, since they have been occupied with it longer. The number of years for which they claim to have devoted themselves to this kind of study of the stars, no one would easily believe. For they count four hundred and three thousand years, down to the coming of Alexander, from the time when they say their observations of the stars were begun.* But he had said in book 1 that the year was once of 30 days, that is, equal to the lunar period; so 403,000 such years make 33,583 solar years. Epigenes, however — who was among the Babylonians for the sake of study together with Apollonius of Myndus, as Seneca reports in book 7 of the *Natural Questions*, chapter 3 — and Berosus, himself a Chaldaean, reported a much smaller number of years. For Pliny the Elder says, in book 7, chapter 56: *Epigenes, an authority of the first weight, teaches that among the Babylonians there were observations of the stars covering 720 [DCCXX.] years, inscribed on baked bricks; those who give the least, Berosus and Critodemus, 480 [CCCCLXXX.] years: from which an eternal use of letters is evident.* Hipparchus, however, in Ptolemy, observa-

tions made at Babylon concerning eclipses he traces back as far as the year 27 of Nabonassar, which was the year 720 before Christ. But if we believe Porphyry, when Babylon had been taken by Alexander the Great, Callisthenes, at Aristotle's request, carried from that city into Greece observations covering 1903 years. But it is one thing for the Chaldaeans to have existed — and indeed to have been devoted to this science from ancient times — and another for them to have been the first of all. For on behalf of the primacy of the Egyptians, after the Hebrews, there stand the journeys of Thales, Pythagoras, Plato, Eudoxus and others into Egypt, which Laërtius reports, made so that they might be initiated, among other things, into the mysteries of the science of the stars. The same is suggested by the obelisks and the most ancient Pyramids, erected in Egypt not only for ornament or display but also for taking the altitude of the Sun by means of their shadows; the same by the 360 priests in the city of Acanta across the Nile, who used to measure the course of the Sun and the ascensions of the signs with water-clocks; the same by the hierophants of all Egypt, especially those of Memphis and Heliopolis, who had learned this art from Mercury.

[Margin: Antiquity of Egyptian astronomy.]

For this reason Clement of Alexandria says of them, in book 6 of the *Stromata*: *The Egyptians pursue a philosophy of their own; and this is shown above all by their sacred ceremonies. For first comes the Singer, etc. After the Singer comes the Horoscopus, who holds in his hand a clock and a palm, the symbols of astrology. He must always have on his lips the books of Mercury that treat of astrology, which are four in number: one of them is on the order of the fixed stars that are visible; another on the conjunction and illumination of the Sun and Moon; the rest on their risings.* Nor does Diodorus Siculus stand in the way; rather, he unties the knot, in book 1, part 2, chapter 2, when he says: *The Egyptians relate that Belus, son of Neptune and Libya, led colonists into Babylonia, and established priests (the Babylonians call them Chaldaeans) to observe the stars after the manner of the Egyptians.* Lucian confirms this as well, in the dialogue *On Astrology*. Cicero too, in book 1 of *On Divination*, although he seems to favour the Assyrians because of the flatness of their plains and the clearness of their sky, nevertheless adds: *But the Egyptians, as people who profess to be older than the rest of mortals, are said to have acquired this same art of the stars over almost countless ages, that is, over four hundred and seventy thousand years.* But before him Plato touched on a twofold cause in the *Epinomis*, when he says: *The first observer of these things was a barbarian; for an ancient region nourished those who, because of the clear weather of summer, were the first to study them. Such were Egypt and Syria, where all the stars, so to speak, are always seen clearly, since neither rain nor clouds cut off the view of the sky.* Nor does he dare to grant this to the Greeks, but adds: *But because we are further than the barbarians from the clear weather of summer, we came to understand the order of these stars later. Yet whatever the Greeks received from the barbarians, they rendered better.*

Away, then, with the fable of the sons of the Sun, born on Rhodes and brought to Egypt, where they are supposed to have taught astronomy, which Diodorus Siculus brings forward in book 5; and away with the fiction of Lucian in the dialogue *On Astrology*, which ascribes its invention to Orpheus. Besides the authorities already cited, Seneca also condemns him, in book 7 of the *Natural Questions*, chapter 3, saying: *Eudoxus was the first to bring these motions from Egypt into Greece. Yet he says nothing about comets; from which it is evident that not even among the Egyptians, who took greater care of the heavens, was this part worked out. Conon afterwards, himself a diligent inquirer, collected the eclipses of the Sun that had been recorded by the Egyptians.* Finally, that this science flowed from the Egyptians to the Greeks is reported by Herodotus in the *Euterpe* and by Theon of Alexandria in his commentary on Aratus.

Who first taught the Egyptians astronomy is equally disputed. Eupolemus, in Eusebius, book 9, chapter 4, and Nicolaus of Damascus say: *Abraham lived in Heliopolis with the priests, and from him the priests learned astrology; yet he admitted that he had not discovered it himself, but that it had been handed down by succession from Enoch.* Josephus suggests the same thing in book 1 of the *Antiquities*, chapters 8 and 9. But Salianus [Jacques Salian] thinks this scarcely probable, because Abraham's stay among the Egyptians was too short — barely three months — and he assigns this office rather to the patriarch Joseph: see him

at the year of the world 2470, no. 28. Since, however, Mercury was later not only than Abraham but also than Joseph, and indeed than Moses, who is known to have been learned in all the wisdom of the Egyptians; and since that other Mercury called Trismegistus was the grandson of the Mercury who was the grandson of Atlas, as Eusebius testifies in book 9, chapter 4, and St Augustine in book 18 of *The City of God*, chapter 8; and Atlas, the brother of Prometheus, was the maternal

grandfather of the elder Mercury, flourished in the time of Moses — it follows that those are mistaken who attribute to Mercury the first invention of astronomy among the Egyptians. Among them are Iamblichus, in his book *On the Mysteries of the Egyptians*; Firmicus, who adds that his disciples were Petosiris, Nechepso, Anubis and Aesculapius; and Manilius, in book 1 of the *Astronomica* [Aſtronomicωn], in these verses:

> *You, Cyllenian, first and author of so great a sacred art: Through you heaven is now known on earth, and the stars, etc.*

They would certainly have erred less had they conferred this prerogative either on Theuth, under Thamus, king of Egypt, as Plato confers it in the *Phaedrus*, or on Atlas, king of Mauretania, of whom Diodorus Siculus tells the following in book 4 of the *Historical Library*: *They say that Atlas was most skilled in astrology, and was the first among men to discuss the sphere; from which it came about that he was thought to hold up the heavens on his shoulders, the invention of the sphere giving room to the fables.* St Augustine also reports this fable, in the book *The City of God*, chapter 8.

[Margin: Inventors of the sphere.]

[Margin: Atlas.]

Pliny the Elder too says, in book 2, chapter 8: *Anaximander of Miletus is reported to have been the first to understand the obliquity of the zodiac — that is, to have opened the doors of things — in the fifty-eighth Olympiad. Then Cleostratus marked the signs in it, first those of Aries and Sagittarius; the sphere itself Atlas had found long before.* And yet, forgetting this, in book 7, chapter 56, where he relates who invented what, he says: *Astrology was invented by Atlas, son of Libya — or, as others say, by the Egyptians; or, as others, by the Assyrians. The sphere within it, by Anaximander of Miletus, etc. The observation of the stars in navigation, by the Phoenicians.* Some hold that the invention of the sphere is owed to Eunolpus [?]. For Archimedes and Posidonius, whose sphere is praised by Cicero in book 2 of *On the Nature of the Gods*, are judged not so much to have devised it as to have perfected and enlarged it.

[Margin: Or Anaximander.]

[Margin: Perfecters of the sphere.]

It could, however, have happened that different men were the first inventors of this art among different nations. So Prometheus, in Aeschylus, in the *Prometheus*, sings of himself:

[Margin: Prometheus.]

For they had as yet no fixed boundary of winter, nor of flowery spring, nor of fruitful summer: but they did everything at random, until I showed them the risings of the stars, and their settings, hard to discern.

Servius supports this on the seventh Eclogue of Virgil, saying: *Prometheus first disclosed astrology to the Assyrians; he had discovered it, dwelling on the very high mountain Caucasus, by excessive care and attentiveness. This mountain, situated near the Assyrians, is almost neighbour to the stars; and so it also displays the greater stars, and shows their risings and settings carefully.* Others say that Hercules carried astronomy from Egypt into Greece, and that for this reason, in the fables, he alone took Atlas's place in holding up the heavens. Others celebrate Palamedes for this, as Sophocles does in the tragedy *Palamedes*, saying:

[Margin: The height of the Caucasus.]

[Margin: Hercules.]

[Margin: Palamedes.]

He showed and unfolded what was unknown before, and found the measures of the stars, and their revolutions, the guardians of sleep, and the heralds of the dawn.

It is surely fair to believe that observations began from observing the daily revolution and the rising and setting of the stars. Theon, in his commentary on Aratus's *Phaenomena*, teaches that their parapegmata, or hemerologia [day-calendars], with the prognostics for the day noted, were customarily displayed on tablets and posted in public already from the time of Meton. Even so, Plato in the *Epinomis* calls the observers of these things not so much astronomers as [Greek: ἀστρονομοῦντας (astronomountas, "those who star-gaze" — practising astronomy in the loose sense)]; and Geminus, in his *Elements of Astronomy*, says that this part of astronomy — namely astrology — is [Greek: ἄτεχνον (atechnon, "without art" — not a proper art)], and not worthy of discussion.

ARTICVLVS IV. De Progreſſu Aſtronomiæ.

Article IV. On the Progress of Astronomy

Whether astronomy flowed from the Hebrews to the Chaldaeans and from them to the Egyptians, or from the Egyptians first to the Chaldaeans of Babylon and then to the Greeks, it is certain from history that the Chaldaeans and the Egyptians contributed more to establishing it than the Greeks did before the death of Alexander the Great. For, apart from the few things that Thales and Anaxagoras touched on concerning eclipses, the rest of the Greeks were wholly occupied with the rising and setting of the fixed

stars, in observing them, and in adapting cycles of the Moon and the Sun to the establishment of the civil year — the purpose for which they observed the solstices and equinoxes. Such were Oenopides, Cleostratus, Harpalus and Democritus, whose trieteris and octaeteris are famous; such were Meton, Euctemon and Eudoxus, who besides calendars and prognostics devised the cycle of 19 years, by quadrupling which Callippus established the period of 76 years. But we know of nothing that the Greeks of that time knew or set down about

the proper motion of the fixed stars, or their determinate distances from one another; nothing about the revolutions of the planets; nothing about the periods in which the restitution [apocataſtaſis] of the lunar anomaly and of the latitude is completed.

The Chaldaean and Egyptian priests, by contrast — the former at Babylon, the latter at Memphis, at Heliopolis and in the city of Acanta across the Nile, where 360 priests observed with water-clocks the length of the days and the ascensions of the Sun and of the signs — noticed more about the Moon, and much about the other planets as well. Plato admits this in the *Epinomis*, and Diodorus Siculus pursues it more fully in book 2, chapter 8, where he also gives the cause of this difference. For he says of the Chaldaeans: *Both because they are trained in that learning from their tender years, all care for other things set aside, and because, persevering in it for a long time, they turn out most learned. Among the Greeks, for the most part, men come to philosophy late, and after lingering in it a little they turn to making money: very few dwell in it, turning to other occupations for the sake of gain. But the barbarians always dwell in the occupation they have taken up. The Greeks, for the sake of gain, always bent on new opinions and quarrelling among themselves about the greatest disciplines, make their disciples uncertain, and force their minds, doubtful through all their lives, to wander with no settled view. Thus, if anyone carefully examines the schools of the most illustrious philosophers, he will find them very much at odds, and contrary to one another on the greatest matters.*

After Alexandria had been founded by Alexander the Great, however, and the kings his successors had opened a gymnasium in that city for all the disciplines, astronomy returned to Egypt — or rather came back to life — and that glory began to be shared by the Egyptians with the Greeks; for Egypt was by now becoming Greek, no differently from Babylon. So there flourished, at Babylon, Berosus the younger; and at Alexandria, Aristyllus, Timocharis, Eratosthenes, Conon, Hipparchus, Sosigenes and Theon the elder, Ptolemy, Paul of Alexandria, Theon of Alexandria the younger and Hypatia his daughter, Pappus, Diodorus of Alexandria, and the bishops Saints Theophilus and Cyril. Thus from 300 years and more before the birth of Christ to about the year 500 after it — that is, for about eight hundred years — astronomy reigned at Alexandria. Even so, some distinguished men observed the heavens elsewhere too: Aristarchus, Helicon, Archimedes, Cleomedes, both Agrippas, Manilius and Menelaus.

But this discipline owes more to none of these than to Hipparchus and Ptolemy. For Hipparchus was the first to notice the proper motion of the fixed stars about the poles of the ecliptic, and from it the precession of the equinoxes, and the difference between the tropical year and the sidereal year. By comparing eclipses of many centuries observed at Babylon and at Alexandria, he also determined the lunar periods in longitude, latitude and anomaly, and the intervals of eclipses, as close to the truth as possible; he discussed the length of the year and the cycles of the Moon and the Sun far more accurately than Meton and Callippus; he set the places of the fixed stars in order; and he did

much else, which Ptolemy does not deny. Ptolemy, for his part, gathered the parts and members of this science — scattered, as it were, in Hipparchus and others — into the body of a single art, with a truly most beautiful method and symmetry. Just as he was the first to establish geography, so he was the first to establish astronomy, so that the work he himself called the *Mathematical Construction* — by the Greeks, however, [Greek: μεγάλη σύνταξις (megalē syntaxis, "great composition")], the *Great Construction*, and by the Arabs, or the Graeco-Egyptians, the ALMAGEST — was not undeservedly called *that greatest*, or *most perfect, work*. What he handed down about the five minor planets, however, rests almost entirely on Ptolemy's own observations and reflections. Yet he candidly admits, in book 2, chapter 2, book 7, chapter 3, and book 9, chapter 2, that he did not have observations covering so long a time that no notable error could be feared, after many centuries, in the length of the year and in the motion of the fixed stars and planets that he had established. For this reason, since the Ptolemaic tables were by now manifestly at variance with the heavens, al-Battānī, in the

ninth century of Christ; in the tenth, Azophi [al-Ṣūfī]; and in the eleventh, Arzachel [al-Zarqālī] and Geber [Jābir ibn Aflaḥ] of Seville [Hispalenses] — and the Persian mathematicians, the Tybenenses [?], as well — all set their minds to restoring the account of the celestial motions. Alexandria had by then been taken by the Saracens, and astronomy had already been carried off elsewhere, to the Arabs, like a captive. And the Jews contended with the Arabs no less about establishing Cabbalistic superstition, or the ravings of astrology, through the motions of the stars, than about the motions of the heaven themselves. Out of this jumble Alfonso X ordered the Alphonsine Tables to be constructed, and he published them in the year 1252; but he issued them again, revised to the standard of al-Battānī, in the year 1256 — the year in which Johannes de Sacrobosco brought out his *Sphere* at Paris.

But since those Tables rested on opinions gathered from here and there rather than on observations taken over the length of time that was required, not much later they were found to be utterly at variance with the heavens [toto cælo diſſentire], above all in the fixed stars and the minor planets. At the same time it was noticed that the equinox, on which the reckoning of Easter and of the movable feasts depends, was falling many days before 21 March, the day fixed by the Council of Nicaea and the bishops of Alexandria. So work began in earnest on correcting both the Alphonsine Tables and the calendar. Celestial observations, which had lapsed after Thebith [Thābit ibn Qurra], Gulielmus de S. Glodialdo [Guillaume de Saint-Cloud] and Profatius [Jacob ben Machir ibn Tibbon], were taken up again by Rabbi Levi [Gersonides], Petrus Alliacensis [Pierre d'Ailly], Nicolaus Cusanus [Nikolaus von Kues], Io. Blanchinus [Giovanni Bianchini], Georgius Purbachius [Georg Peuerbach], Regiomontanus, Bernhard Walther, Io. Vernerus [Johannes Werner], Abraham Zagutus [Avraham Zacuto], Dominicus Maria [Domenico Maria Novara], Gemma Frisius, Pietro Pitati, and Copernicus, from whose springs the Prutenic

Tables flowed. How many have written about the errors of the Alphonsines and about restoring the calendar can scarcely be told; I shall list the chief of them in the chronicle that follows. In the restoration of the calendar, however, just as the Church chose the Alphonsine length of the year, so in the lunar cycles it preferred to follow the Prutenic Tables; and Stadius, Iunctinus [Francesco Giuntini] and Molletius [Giuseppe Moletti] kept to the Prutenic Tables in their own tables.

Further, since neither set agreed with the heavens at every point, especially in the places of the fixed stars [Affixarum] and of the planets, and since a new and huge star had appeared in the Chair of Cassiopeia in the year 1572, and not a few comets in the following years, a good many astronomers began to turn their eyes to the heavens. The foremost of them were William, Landgrave of Hesse, Mœstlinus [Michael Mästlin], and Tycho Brahe. From Tycho's observations two lights of his school constructed tables: Longomontanus the Danish Tables, Kepler the Rudolphine. At the same time Philippus Lansbergius [Philips van Lansbergen], with Hortensius [Maarten van den Hove], put together astronomical tables partly from the Prutenic and partly from the Danish hypotheses, adding a few observations of his own but comparing them at length with the old and new observations of others; these tables are not inferior to the Danish and the Rudolphine, and in places they are even better.

Just as Maginus [Giovanni Antonio Magini] had gathered the Alphonsine and Prutenic Tables into a better form, and later, before his death, had composed some Tychonic-Copernican ones; and as Nicolaus Mulerius presented tables of the Moon and the Sun drawn from Ptolemy, Alfonso, Copernicus and Tycho — so Vincentius Reinerus [Vincenzo Renieri] brought five kinds of tables together into one volume, in a form that is not without use: namely the Alphonsine, the Prutenic, the Danish, the Lansbergian and the Rudolphine. Lastly Ismael Bullialdus [Ismaël Boulliau], chiefly from the observations of Tycho, of Gassendi and his own, and also from some hidden away in the King's Library at Paris, and with incredible labour of geometrical inquiry, built for us the Philolaic Tables (the newest of all that we have). In ease of use and in agreement with the heavens they are to be put on a level with the Rudolphine, Danish and Lansbergian Tables, and in places preferred to them, though in some places they fall short. I would say the same of the Atlantic Tables of Gotifredus Vuendelinus [Govaert Wendelen], built on eclipses observed from the year 1573 to the year 1643.

Besides these, sunspots and the orbits of the satellites of Jupiter have been observed by Galileo, Simon Marius, Scheinerus [Christoph Scheiner] and Vincenzo Renieri; the lunar spots and librations by Michael Florentius Langrenus [Michael Florent van Langren], Io. Heuelius [Johannes Hevelius] and Petrus Gassendus [Pierre Gassendi], together with the eclipses, on which the astronomy-lovers of this century never cease to fix their attention. And D. Vincentius Mutus [Vicenç Mut], in Majorca, labours almost wholly at astronomical observations. But if anyone wishes to know of a treatment of the

origin and progress of this art other than ours, let him consult the oration of Lucas Gauricus [Luca Gaurico] delivered at Ferra-

ra in praise of astronomy; Petrus Ramus [Pierre de la Ramée] in books 1, 2 and 3 of the *Scholae Mathematicae*; Clavius at the front of his *Sphere*; Barocius [Francesco Barozzi] in the preface to his *Cosmographia*; Io. Roias [Juan de Rojas] at the front of his *Planisphere*; and Longomontanus, Kepler, Lansbergen and Boulliau in the prefaces or prolegomena to their several *Astronomies* or *Tables*. But it will be worth the trouble to run through our *Chronicle*, which is to be placed after this preface and the Index of Chapters.

ARTICVLVS V. Quid in Astronomia Recentiori desiderari videatur, & de Erroribus præcipuis Præcedentium Astronomorum.

Article V. What Seems to Be Wanting in More Recent Astronomy, and on the Chief Errors of the Astronomers Who Came Before.

[Margin: In Fixis quid desideretur. — What is wanting in the fixed stars.]

No part of astronomy handed down so far fails to need something for its completeness, or at least to need the file. I shall point this out briefly, running from the fixed stars down to the Moon.

The longitude and latitude of the fixed stars were corrected by Tycho from right ascensions and declinations he observed himself; but the right ascensions depended on Venus, as is clear from volume 1 of the *Progymnasmata*, from p. 152. Yet Tycho used too large a parallax of Venus in its true declination, because of the excessive parallax of the Sun, as Kepler admits in the foundations of his *Ephemerides*, p. 15, where he says that the dispute between the Landgrave of Hesse and Brahe over a difference of 6′ in the motion of the Sun from the fixed stars is still undecided. Many distances of the stars from one another, besides those Tycho observed, are also wanting, so that the places of the planets could be derived from them more easily. The apparent diameters of the fixed stars, too, are very uncertain, and greater evidence is needed on them. Finally, the number of the fixed stars, which appears far greater through the telescope, must be increased by adding the stars that this instrument discovers in each constellation [astro]. Besides, observed distances of fixed stars are sometimes found to disagree with those Tycho observed with sextants, or with those derived by the working of triangles from their longitudes and latitudes, or from their right ascension and declination. As for their places in the Alphonsine and Prutenic Tables being wrong, sometimes by as much as a whole degree, enough has been said by Cardinal Cusanus in table 53 of the fixed stars arranged by himself and by the cardinals of Nicaea [Niceno: Bessarion] and d'Ailly [Alliacensi]; by Reinhold in the *Theoricae*, p. 248; by Nonius [Pedro Nunes] in his first annotations on the *Theoricae* of Peuerbach; and by Tycho in the *Epistolae*, from p. 54, and in volume 1 of the *Progymnasmata*, p. 146. Nor has anyone after Tycho undertaken this correction except Lansbergen, and he sets

in his tables, p. 102, only 25 stars observed by himself. The rest follow Tycho in this with blind eyes, so to speak; and Boulliau himself adds nothing except the places of certain fixed stars dug out of the older Persian tables.

[Margin: In Saturno quid desideretur. — What is wanting in Saturn.]

Saturn, the slowest of the planets, does not complete its revolutions to the same conditions of celestial place within the lifetime of a single man. It is no wonder, then, that with something like an old man's cunning it has so often deceived astronomers, and that a small error in it has brought forward or put off its aspects with the other planets by many days. The great conjunction of Saturn and Jupiter, which the Alphonsine Tables expected in the year 1563 on 25 September at Leo 4° 25′, was nevertheless seen on 24 August by Iunctinus at Aurange [?] at 14^{h} 30^{m} after noon, with the more northerly Jupiter covering almost the whole of the more southerly Saturn, as he himself relates in the preface to his astronomical tables. There he adds that in the year 1536, on 19 May, at about the second hour of the night, a meeting of Mars with Saturn was observed by his teacher Iulianus Ristoricus [Giuliano Ristori], in which Saturn was almost entirely covered by Mars; and yet the Alphonsine Tables, in Stöffler's *Ephemerides*, had set it down for 25 May. Tycho likewise says in the *Mechanica*, p. 39, that in the year 1563, when he was observing the conjunction of Saturn and Jupiter, the Alphonsine calculation was out by a whole month. Earlier, too, Stoeflerus [Johannes Stöffler] had predicted from the Alphonsine Tables that the conjunction of Saturn and Mars would occur in the year 1536 on 25 May; but Pietro Pitati saw it on the 20th. Tycho reports in the *Epistolae*, p. 56, that in the year 1587, on 9, 14 and 15 January, Old Style, he observed Saturn 2° west of its Alphonsine place, but about 10′ east of its Prutenic place. Longomontanus, for his part, says in book 2 of the *Theorica*, chapter 13, that Saturn was observed at Uraniborg [Vraniburgi] in the year 1591, on 17 March, at $7\frac{1}{3}^{h}$ after noon, at Gemini 22° 43′, and that it was shown by the Da-

nish Tables at 22° 46′, by the Prutenic at 22° 23′, and by the Alphonsine at 25° 7′. Boulliau's tables usually represent the motions of Saturn and Jupiter more exactly than the Rudolphine. The motions of Saturn's satellites, however, and their appearances, occultations and separations [diuaricationes], have not yet been investigated, and centuries are needed before they can be determined with certainty.

[Margin: In Ioue quid desideretur. — What is wanting in Jupiter.]

The star of Jupiter, being faster than Saturn's, has deceived astronomers less. Yet Tycho, as he says in the *Epistolae*, p. 56, found it standing off from its Alphonsine or its Prutenic place by about a third of a degree; and Longomontanus, in book 2 of the *Theorica*, chapter 13, found it 1° [fraction: ?] east of its Alphonsine place but 20′ west of its Prutenic place. As for the periods of its four satellites, whose observation was begun by Simon Marius and by Galileo, D. Vincentius Reinerus [Vincenzo Renieri], an Olivetan and a pupil of Galileo, had observed them for many years. He had gathered all of Galileo's observations and his own into a full volume and had constructed ephemerides of these

motions, when premature death snatched him from us — and someone or other snatched that work, among the spoils, from him and from posterity. This part, therefore, remains for astronomers to complete.

[Margin: In Marte quid desideretur. — What is wanting in Mars.]

The star of Mars, once called unobservable by Pliny the Elder (book 2, chapter 17), has violated the laws of the older tables beyond all measure — those of the Alphonsine and Prutenic Tables by 2, 3, 4 and almost 5°. This is shown from observations compared with both calculations by Gemma Frisius in chapters 19 and 20 of the *Annulus Astronomicus*, by Giuntini in the preface to his tables, by Tycho in the *Epistolae*, p. 56, by Longomontanus in book 2 of the *Theorica*, chapter 13, and by Kepler in the introduction to his *Commentaries on Mars* and in the preface to the Rudolphine Tables. From these we learn that in the year 1544, on 31 October, Mars fell short of its Alphonsine place in the zodiac by 2° 16′; that in the year 1587, on 15 January, it exceeded the Alphonsine place by 2° 30′; that in the year 1593, in August and September, it exceeded the Prutenic place by almost 5°, as it did again in the year 1625 in the month of October; and that in the year 1608 it did so by nearly 4°. And this was not because of the mean motion but because of the prosthaphaereses, which Reinhold wrongly thought permanent and correct. Although after Tycho's observations Kepler, Longomontanus, Lansbergen and Boulliau bound Mars with tighter chains, it still resists at times and does not let itself be wholly bound. Thus in the year 1632, on 5 February, Mars at its acronychal rising [acronychius] should not, by the Rudolphine Tables, have been covered by the Moon; and yet, as Petrus Gassendus observed at Paris and Martinus Hortensius at Leiden, it was covered by it — helped in this, perhaps, by the small parallax of Mars, smaller than Kepler sets it, let alone Tycho.

[Margin: Venus in quo deficiat. — Where Venus falls short.]

We do not, it is true, find errors of *Venus* in the more recent tables as manifest as those of Mars or Mercury. Yet we have very few observations of its retrogradations and greatest digressions taken over several consecutive days before and after. This part must therefore be helped by more observations, especially by observing its conjunctions with the Sun, of which we have only one observed by Tycho. Anyone who wants a specimen of the disagreement between its true place and the Alphonsine or Prutenic place should read Tycho's *Epistolae*, p. 56. Hortensius, too, observed a conjunction of Venus and Mercury in the year 1632, on 31 July, at about the 11½th hour of the night, when by Longomontanus they should have been 2° 53′ apart.

[Margin: In Mercurio quid desideretur. — What is wanting in Mercury.]

Mercury, rather, is a treacherous star, almost always hidden under the rays of the Sun and appearing only among the vapours of the horizon; Copernicus never observed it, because of the vapours [vapiditatem] of the Vistula. It must be caught by the snares of astronomical skill, so that the tables can be cleared of the disgrace under which they have laboured to this day. Giuntini, in the preface to his *Tabulae resolutae*, rightly called intolerable a discrepancy of ten

or eleven degrees in the place of Mercury between the Alphonsine and the Prutenic calculation. For more tolerable is the discrepancy of 2° or 3° between its true place and the place assigned by Alfonso or Copernicus that Tycho found, in the *Epistolae*, p. 56. Longomontanus, in book 2 of the *Theorica*, chapter 20, laments that between the Alphonsine and the Copernican calculation a discrepancy is found that is *immense, and at times rises beyond 7°*. And although he admits that observations of Mercury made by himself or by Tycho were rare, because of the obliquity of the Danish horizon and this planet's short and quick digressions from the Sun, he nevertheless boasts that from 9 observations he has so restored and reformed its hypothesis that the discrepancy between the calculation and the heavens never exceeds a sixth of a degree.

But the conjunction of Mercury with the Sun that Pierre Gassendi [Petrus Gassendus] observed at Paris on 1631 November 7 — seeing it with the help of a telescope in such a way that Mercury appeared beneath the disc of the Sun — made that boast empty. For, as Martinus Hortensius points out in his dissertation with Gassendi on Mercury seen beneath the Sun, the Danish tables, that is Longomontanus's, strayed from the true place of Mercury by 7° 13′ (a discrepancy that Longomontanus had called *immense* [immensam] between the Alphonsine and the Copernican calculation). The Prutenic tables erred by 5°, the Ptolemaic by 4° 25′, the Lansbergian by 1° 21′, the Rudolphine by 14⅔′, and the Philolaic tables of Boulliau differed from these by not even two minutes. Boulliau therefore rightly dedicated to Gassendi, for so fortunate an observation, the whole of book 10 of his *Astronomia*, in which Mercury is treated; and because of this very observation Natalis Duretus [Noël Duret], who had derived ephemerides from 1637 to 1642 from the tables of Lansbergen, went over to the Rudolphine tables for the motion of Mercury.

Yet see Kepler in the *Ephemeridum fundamenta*, pp. 15 and 18, on the extreme difficulty of observing conjunctions of Mercury and Venus with the Sun. There he admits that all the observations of Mercury made by Tycho over 20 years fall within a single third of its eccentric, and that the remaining two thirds stay unobserved in this century and hemisphere. He therefore adds: *In the conjunctions of Mercury, which have great force in meteorology* [in Meteoris]*, I would not dare affirm that my calculation always stays within the limit of that very day, since in so much time the commutation can sometimes vary by five degrees, and the apparent place of Mercury, when it is retrograde, by one degree and a third or a half.*

[Margin: In Sole quid desideretur. — What is lacking in the case of the Sun.]

The theory of the Sun, moreover, which because of the clarity and frequency of its observations might have seemed already complete in every respect, has by no means escaped the hazard of all controversy. For some dispute still remains among astronomers over its apogee and eccentricity, over the greatest declination [maxima declinatione] — that is, the obliquity of the ecliptic — over the length of the year and its uniformity, and over the apparent diameter; but

above all over its parallax and distance. Its parallax — which Hipparchus and Ptolemy once called into doubt, whether it was perceptible at all, and afterwards, from conjectures based on eclipses, extended somehow to about 3 minutes — Copernicus and Tycho assumed without confirming it by any demonstration from their own observations or those of others. Yet the parallax of Mars at perigee, which ought to be greater than the Sun's, since Mars at perigee was found by Copernicus and Tycho themselves to be much nearer the Earth than the Sun, scarcely rises to one minute; from this the Sun's distance is inferred to be three times greater than the Tychonic. And if the lunar dichotomies are considered, the Sun's distance turns out far greater still, and not less than six thousand semidiameters of the Earth. This must therefore be investigated more carefully, since on the Sun's distance depend the distances from the Earth of all the stars (the Moon alone excepted), their parallaxes and true magnitudes, the true magnitude and velocity of the Sun itself, and many other theorems and problems most worth knowing.

I pass over for now the errors of the Prutenic tables for the Sun, greater than those of the Alphonsine, as Clauius [Christoph Clavius] points out in the *Kalendarium Gregorianum*, and Magini in the letters he reports in his *Directiones*, pp. 75 and 95½, adding that the Prutenic tables sometimes err by one whole degree. I pass over the errors of the Alphonsine tables themselves in the equinoxes and in the Sun's apogee, which Nonius pursues in his first annotations on Peuerbach's *Theoricae*. The theory of the solar spots, however, our Scheiner has perfected so far that, on the testimony of Renatus Cartes [René Descartes], Boulliau, Hevelius and many experts, scarcely anything more accurate could be hoped for.

[Margin: In Luna quid desideretur. — What is lacking in the case of the Moon.]

Finally the Moon, which Pliny the Elder (book 2, chapter 9) called the teacher of almost everything we know about the heavens — he is indignant that it is still unknown, though so near the Earth; and Kepler, in precept 171 of the Rudolphine Tables, calls it a stubborn star, still rejecting laws in some places. Writing to Magini on 1601 June 1 [Iunij Kalendis], Kepler also admits that the theory of the Moon gave great difficulty to Tycho, to whom he had by then come in Bohemia, and that it seemed it had to be begun from the parallaxes. He sets out two ways of observing these without a vicious circle, in such a way as to indicate that they had not been observed in that way before that time. But he died in the same year, Tycho. Therefore the parallaxes must be investigated afresh, his and those of others who followed him, especially since the distance and parallax of the Sun also depend on them — and, through the Sun, those of the remaining planets.

There remains, too, a dispute over the motion in latitude and of the nodes, over the lunar eccentricity, and over the apparent semidiameter. Above all, however, it concerns that variation, the third anomaly, which the Moon displays chiefly in the octants, that is, between the syzygies and the quadratures, and

because of which the tables not seldom disagree among themselves, and with the heavens, by almost a whole degree. Moreover, the incredible multitude of the spots observed with the telescope, and the variety of phases in them, require more and more careful observations than have been made so far, in order to perfect the selenography begun by Michael Florentius van Langren and Hevelius — to say nothing of others of lesser note. Still more accurate observations are required by that wonderful libration, by which the Moon, in alternating monthly turns about the centre of its globe, nods towards the Earth and away from it with the same limb. Lastly, the phase of dichotomy, most useful for determining the Sun's distances far more surely than up to this day, and before us left almost untouched and unobserved by everyone, must be weighed with all possible industry and circumspection, and with frequent observations.

[Margin: De Eclipſibus. — On eclipses.]

Eclipses of the Sun and Moon, besides the variety they have separately from the theory of the luminaries, also require their own particular care in the making of observations, especially by the method of the more recent astronomers. That al-Battānī [Albategnius] noticed, in chapter 30, from the observation of two or three eclipses, an error of 45 or 50 minutes of time in the Ptolemaic canons, and the Sun obscured a little more than half, when by Ptolemy's reckoning it ought to have been obscured entirely — that is not at all surprising after an interval of almost 700 years. But is it to be marvelled at or lamented that there is so great a discrepancy among the tables of the more recent astronomers themselves, and between the observations and the calculation of the tables — indeed among the very observations of one and the same eclipse — as the reader will be able to gather from our *Catalogue of Eclipses*, to be placed at the end of book 5, chapter 19? From this, and from chapter 20, he will learn how far Tycho and his followers erred in excluding total eclipses of the Sun, or in not handling annular ones correctly. How little, moreover, the hypotheses of al-Battānī, Copernicus, Tycho, Longomontanus and Kepler agree with one another in fitting together the parallaxes and diameters of the luminaries and the angle and apparent diameter of the Earth's shadow, and how these parts conflict within the hypothesis of one and the same astronomer, will be told in book 3, chapter 11.

Returning now to eclipses: Tycho wondered at the Prutenic error of half an hour in the eclipse of the year 1588, and Kepler, in the *Optics*, chapter 9, at the imperfection of the parallactic tables of Reinholdus [Erasmus Reinhold]. But Vendelinus [Govaert Wendelen] noted Tycho's own errors in several eclipses in his little work on eclipses; and Kepler admitted them in the *Fundamentum Ephemeridum*, p. 11, adducing the testimony of Simon Marius and of other observers at Linz and Munich. Wendelen likewise notes Kepler's errata in this matter, as he does Lansbergen's — whose good faith, too, Wendelen, Phocylides [Johannes Phocylides Holwarda], Boulliau and we ourselves have sometimes found doubtful in these and other observations: he flaunted, rather than showed,

that his tables nowhere and never disagree with the phenomena of the heavens (which he certainly wished too much). Nor did Wendelen, by removing the equation of natural days, bring a sufficient remedy to this discrepancy; indeed his Atlantic calculation sometimes strays further than the Lansbergian or the Tychonic, as D. Vincentius Mutus pointed out very recently. Lastly, Boulliau had scarcely published his *Philolaic Tables* when he was grieved that the eclipse of the Sun of 1645 August 21 differed markedly from his tables.

ARTIC. VI. De Aſtronomiæ nouis ſemper Difficultatibus nouo ſemper ſtudio, & ardore exantlandis.

Article VI. On the Ever-New Difficulties of Astronomy, to Be Overcome with Ever-New Zeal and Ardour.

What has been said so far is aimed not at detracting from this most noble art, or from its practitioners themselves, whom we look up to and revere as the lights of astronomy, nor at casting despair into those who will observe in the future. It is aimed rather at sharpening their industry on this very whetstone of difficulties, and at the same time at making it plain that it will not be labour wasted [actum agere] if anyone is willing to toil in the astronomical dust and, as far as lies in him and as the inconstancy of physical causes permits, to remove its imperfections from this art by filing and polishing — or at least to distinguish between those that can be removed and those that can be tolerated but not removed. For what Kepler admits of his Rudolphine Tables in the dedication, the rest need not blush to admit: *These tables both contain and publicly confess certain imperfections, detected in the very subject of the celestial motions, which cannot be made good or overcome by any art, especially before sufficient experience of the centuries to come.* And in the preface to the reader he says: *The very nature of the mean motions has not yet been thoroughly explored.* And again: *You have below, in the doctrine of eclipses, clear proofs, even from the observations of this time, that the motions of the Sun, the Moon and the primum mobile are not uniform to mathematical exactness, but receive very small physical intensifications out of order.*

The subtlety and difficulty of this art are increased, moreover, when observations over a long time are lacking, or have to be compared with ancient ones that we may doubt. For Ptolemy, most cautiously, in book 2 of the *Magna Constructio*, chapter 2: *We therefore judge it enough if we too try to contribute to these restorations as much as the time elapsed between our own observations and the collected observations of the ancients can contribute; and let us not wilfully neglect an accurate examination. The truth, however, that can be had from the whole perpetuity of time, or from a manifold time of observations, we think must be left to others.* Have you seen the noble resolve of an upright mind, neither despairing nor presuming too much? Hear the same man's candour in book 9, chapter 2, where he says: *The time from which we have observations of the planets is so short, compared with the magnitude of the things, that they make a prediction over a long time weak.*

Al-Battānī recognized things plainly similar in the very slow motion of the eighth sphere, in determining its uniformity or non-uniformity, and the length of the year that depends on it, in chapter 52. His words are: *In this superinduction, if Hipparchus and Ptolemy were deceived because of the instruments with which they observed these things, we too are necessarily deceived in the same way, since the consideration of our observation cannot be completed without their observations. But if this difference arises from some celestial motion still hidden from them and from us, it is fitting that whoever diligently seeks the truth should always observe precisely, and correct what he has found to be faulty in the way our predecessors corrected others.* Johannes Regiomontanus [Io. Regiomontanus], having copied these words almost verbatim in the *Epitome* of the *Almagest*, book 7, chapter 6, himself concluded as well: *It will be expedient to keep an eye constantly on the fixed stars, and for posterity likewise to weigh the considerations that have been written down.*

Here, though, and in all other matters of this kind, one ought to have, if it were possible, as many eyes as Argus has, or as many stars as Astraea shines with all around her. Not in vain did our Scheiner give this warning in book 2 of the *Rosa Ursina*, chapter 10: *He must play Argus who would make faultless observations of the Sun at this time of midday.* Indeed Athanasius Kircher, in book 2 of the *Magnetic Philosophy*, pointing out the causes of slips in observing that arise from many sources — but first of all *from the organs of sight, and the varying dilation of the pupil, and from the medium and its various refractions* — adds: *And if every deception is to be avoided, one must be an angel, not a man.*

To hint at this at the very gateway and frontispiece of the work: on the left side of the picture, Astraea [Astream], furnished with angelic wings and sparkling with stars on every side; on the right, Argus, not content with a hundred eyes but, by a [Greek: προχρονυσμοῦ (prochronismou, "prochronism", i.e. anticipation in time)] [recte: προχρονισμοῦ] and an anticipated use, bringing the telescope to the eye on his knee rather than to the eye of his cheek, so that in seeing the works of God's fingers he seems at the same time about to bend the knee to venerate them.

For God himself so disposed all these things in number, weight and measure as both to fix a cross for the astrologers and genethliacs, who forever wander in the labyrinth of the celestial motions, and to raise the minds of every age to the heavens with ever-new phenomena and subtleties, so that astronomers should not think they may doze securely in future. For those words of God in Job 38 can bear this sense: *Who will declare the reason of the heavens, or who will make the harmony of heaven sleep?* For just as the heavens are said to praise the Lord objectively, because they rouse us to praise him, so they would in a way be sleeping if, bringing forth nothing new from time to time, they invited us to sleep and to neglect of observations. Let us indeed say with Seneca, book 7 of the *Naturales Quaestiones*, chapter 31: *Many things unknown to us the people of a coming age will know. Many things are reserved for ages yet to come, when the memory of us will have faded.*

The world is a paltry thing, unless the whole world finds in it something to investigate. Or rather, let us conclude with our most wise old Clavius [Christoph Clavius], on chapter 4 of Sacrobosco's *Sphere*, p. 452 in my copy: *It is extremely difficult to define the periods of the motions so that they do not deviate from the truth over many centuries of years, since no mortal has ever been able to determine the period of even one planet so exactly that no small fractions are left over or missing — fractions which, over a great interval of years, introduce a noticeable error. So it is truly a wonder that God, Best and Greatest, chose to obstruct the motions of the planets with such great difficulties that no human being can grasp them perfectly, but always finds something to admire in so great an artifice of such noble bodies, and in so great a harmony and concord of their motions, celebrating their founder and mover with perpetual praises. So much so that it seems to have been written by Ecclesiastes, chapter 3 — "And he has handed the world over to their disputation" — above all on account of the constitution of the heavens and their motions, in which there always seems to remain something to be investigated with the utmost diligence by the most skilful searchers of celestial things: lest, that is, if men had ever perfectly understood the number, order, constitution and motion of the heavens, they should cease to investigate and admire the works of God, and their minds, deprived of any occasion for exercise, should grow torpid through idleness.*

This is the same old man who, near the end of his life, having considered the new phenomena of the heavens that had begun to be discovered by Galileo with the Belgian spyglass [perspicillo Belgico] and were being announced through the *Sidereus Nuncius* [Nuntium sidereum], broke out into these words: *Let the astronomers see how the celestial orbs are to be arranged so that these phenomena can be saved.* Empty, therefore, and drawn from poetic fiction is that encomium of Manilius on his Mercury:

Through you the heavens are now [brought] down to earth; now the stars are known. [Per te iam cælum in terras; iam fidera nota.]

For to this very day the heavens have not been brought down to earth, nor are they known as if those celestial things had been called down to earth. Rather, what is true is what Truth itself — namely God — says in Job 38: *Do you know the order of the heavens, or will you set out its reason on earth?*

ARTICVLVS VII. Auctoris in hoc opere Scopus & Ordo indicatur.

[Article VII. The author's aim in this work, and its arrangement, are set out.]

As I reflected both on the matters I set out in Articles 5 and 6 and on what I had repeatedly heard about astronomical subjects, there was put into me, from the very beginning of my studies, a great desire to build an astronomical work which could serve as a kind of library for the men of our Society, and for others who lacked either a large number of books of this kind or the leisure to read them all through — a work into which I might gather, with all the learning I could muster, the whole of astronomy, old and new, together with

the controversies belonging to it. My superiors were at first undecided whether to assign me to teach philosophy publicly or rather mathematics; afterwards they chose to keep me in the chair of philosophy for nearly six years, and in that of scholastic theology for a full ten. Yet that ardour for astronomy, once kindled in my mind, could never be lulled to sleep, much less extinguished. On the contrary, having obtained from them a release from the office of teaching theology, I devoted myself wholly to astronomy; and they themselves, having weighed the reasons for my aim, urged me of their own accord to complete what I had long since begun.

To this was added a spur from certain persons. Some, made bolder by the success of their philosophical talent, scarcely greeted astronomical questions from the threshold, yet did not hesitate to pronounce on them, and fell into shameful errors. Others, by contrast, were of so narrow and cautious a mind that, adhering to Tycho alone, or to some other writer on the sphere, and drawing from him the measures of the motions, distances and magnitudes of the celestial bodies, they thought it a sin to depart from him by even a single unit or the smallest jot. Some, again, so disparaged the observations of the celestial phenomena that they asserted that one could not come even close to the truth by means of them. Others held all astronomical conclusions to be so thoroughly demonstrated that one must go over bodily to the opinion of any approved astronomer, with no prior selection or examination. I noticed, too, that the controversy between the Peripatetics and the Tychonians over the location of comets and new stars was being settled by most people on the mere authority of Tycho and his followers, because no one could, or would, weigh on the scale of trigonometry the prolixity of the Claramontian calculation set out in the *Antitycho* and in the other books of Scipio Claramontius [Scipione Chiaramonti].

Finally, in the question most celebrated in this century, concerning the system of the world — whether one supposes the Earth immobile, with most people, or mobile, with the Philolaics and Copernicans — I saw that many, by hurling weak arguments at their opponents, rather confirmed them in their opinion; and that the Copernicans everywhere kept repeating that the sacred censors, versed only in Peripatetic philosophy and in theology, or perhaps also in knowledge of the Holy Bible and the canons, had issued decrees on astronomical matters against Copernicus that were neither just nor valid, or not at all necessary. These things, I say, and others like them, with which my ears constantly rang, provoked me — it is incredible to say how sharply — to meet them and answer them with all my strength, especially since what is at issue in this Copernican dispute is the literal sense of Sacred Scripture.

Therefore, having called upon the Divine Majesty so that I should not give way under this Atlantean burden, I girded myself for the work. Having explored the heavens for many years with new and very large instruments, and having read through every volume on this subject I could obtain, I built this *New Almagest*. Its first Tome, subdivided into two parts, now appears; the

remaining two will be brought to light in their own time, if God, in whose hands are my lots, prolongs my life. Their subject, and the whole arrangement of the work, I have already indicated sufficiently at the very entrance of this volume.

I would only have the reader warned of this: in this first Tome I have not delivered my final verdict on astronomical numbers and measures, but have merely chosen, as occasion arose, many values that seemed to me more correct for the time being. For I reserve the finished hypotheses for the third Tome, once both the Tychonic observations — which I hear are sweating under the press — and those of others have reached my hands, and our own have become more numerous and more certain.

Whether I have merely stitched together a patchwork from other people's opinions and plundered other people's book-chests, or whether I have rather searched out whatever in this discipline seems worth knowing in such a way that, having gathered it in an easier method and in no more obscure a style, I have from time to time enriched it with a stock of newly found theorems — the reader will judge from the course of the work itself, or from the index of Books and Chapters itself. I hope, too, that I shall be pardoned for a prolixity that was almost necessary: in presenting so great a diversity of opinions, in citing authors, and in repeating nearly the same matters, which I suspected readers would look for in some other place rather than in the one where I had once touched on them. For my errors, if any are detected, I ask correction rather than pardon — but from the expert, and from lovers of the Truth, which, after the greater Glory of God, I have set before myself as my sole end.

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- XIV. On mutual and non-mutual eclipses of the planets, through the interposition of bodies or of shadows; with a discussion of the length of the shadows of the planets. — 720
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CHRONICON DVPLEX ASTRONOMORVM, VEL ASTROLOGORVM COSMOGRAPHORVM, AVT POLYHISTORVM, Qui Astronomica, vel Cosmographica propiùs attigerunt.

[A twofold chronicle of the astronomers, or astrologers, cosmographers or polymaths, who have dealt more closely with astronomical or cosmographical matters.]

I. PARS Ordine Temporum constructa.

[Part I, arranged in order of time.]

Years before Christ	Entry
1990	Zoroaster [Zoroaſtres], king of the Bactrians.
1590	Prometheus, brother of Atlas.
1580	Atlas, king of Mauretania.
1520	Mercury the Elder [Mercurius Maior].
1480	Hermes the Egyptian.
1445	Endymion of Latmus.
1345	Cepheus, king of the Ethiopians.
1012	Solomon, king of the Jews.
640	Thales of Miletus is born. Polemon flourishes.
570	Thales of Miletus dies, aged 70.
560	Oenopides of Chios.
548	Cleostratus flourishes.
544	Anaximander of Miletus flourishes; born in the year 600.
540	Pythagoras flourishes.
530	Anaximenes of Miletus.
520	Harpalus.
479	Pythagoras is killed, aged 63.
480	Anaxagoras begins to philosophize.
470	Democritus of Abdera.
432	Meton and Euctemon.

Years before Christ	Entry
430	Philolaus of Croton.
428	Plato is born.
420	Anaxagoras dies, aged 72.
405	Archytas of Tarentum, and Timaeus of Locri [Timęus Locrus].
404	Helicon of Cyzicus.
390	Euclid the elder [Euclides ſenior].
384	Aristotle is born.
368	Eudoxus.
360	Metrodorus.
348	Plato dies, aged 81.
330	Callippus [Calippus] begins his period.
322	Aristotle dies, aged 63. Theophrastus succeeds him.
320	Pytheas [Pitheas] of Massalia.
300	Aristyllus, Autolycus, Timocharis [Timochares].
289	Archimedes is born. Archelaus flourishes.
286 [recte: 196, per the book's errata]	Eratosthenes dies, aged 80. [Margin: in a later hand, in ink, "19[6?]" in the left margin, the printed 286 struck through and a curved pen stroke running from the figure up to the 289 entry]
285	Dionysius the astronomer.
284	Euclid of Megara, the younger.
281	Berossus [Beroſus] the Chaldean, until 260.
280	Aratus of Soli, and Aristarchus of Samos.
260	Conon.
240	Apollonius of Perga.
230	Isidorus the cosmographer.
214	Archimedes is killed, aged 75.
168	C. Sulpicius Gallus.
162	Hipparchus of Rhodes, until 128.
136	Hipparchus of Bithynia.
86	M. Varro, and Taruntius Firmanus. M. Tullius Cicero [M. Tull. Cicero].
83	Geminus the astronomer.
60	Posidonius [Poſſidonius].
52	Theodosius of Tripoli.
45	Sosigenes of Alexandria.
40	Cleomedes the elder.
38	M. Agrippa.
35	Manilius the astronomer.
30	Dionysius Afer.
20	Strabo the geographer. Artemidorus the cosmographer.

Years of Christ	Entry
15	Strabo is still living, and Germanicus.
40	Straton of Amasea [Straton Amaſenus].
45	Columella.
47	Pomponius Mela.

Years of Christ	Entry
60	Andromachus, and Marinus of Tyre.
68	Seneca is put to death.
78	Plutarch flourishes, until 104.
70	Ptolemy is born; or 69.
34	St Dionysius the Areopagite, aged 25, observes the miraculous eclipse that took place at the death of Christ; in the year of Christ 52 he is converted to the faith; and at over a hundred, or almost 111 years old, in the year 120 he becomes a martyr.
72	Hyginus, author of the *Poeticon Astronomicon* [Poetici Aſtronomici ſcriptor].
80	Pliny the Elder [Plinius maior], who perished at Vesuvius aged 55.
84	Proclus the elder.
92	Agrippa, an observer in Bithynia.
97	Menelaus, or Milleus.
130	Aquila of Pontus.
132	Theon of Alexandria the elder, and Phlegon.
135	Sextus Empiricus.
147	Ptolemy dies, aged 78.
230	Andruzagar the astrologer.
238	Censorinus of Rome.
298	Rabbi Adda.
314	Julius Firmicus Maternus [Iulius Maternus Firmicus].
320	Valens the astrologer.
360	Theon of Alexandria, father of Hypatia.
378	Paul of Alexandria [Paulus Alexandrinus].
380	Philosophus, an astronomer [Philoſophus Aſtronomus].
390	Cleomedes the younger.
395	Sextus Avienus Ruffus. Macrobius.
400	Pappus of Alexandria.
415	Hypatia is torn to pieces.
434	Aëtius of Amida.
444	St Cyril of Alexandria dies; he had begun his cycle in the year 437.
466	St Prosper of Aquitaine dies.
467	Victorinus.
469	Theodoret [Theodoretus].
490	Cassiodorus, until 528.
500	Simplicius, and Thius of Athens.
514	Proclus Diadochus.
526	Dionysius Exiguus.
600	Martianus Capella.
636	St Isidore of Seville dies.
770	Albinus Flaccus, also called Alcuin.
776	Bede dies, aged 106.
827	al-Maʾmūn [Almamon, ſeu Maimon], king of the Arabs.
846	Egmundus the astrologer.
860	Māshāʾallāh [Meſhala] the Arab.
870	Paul, philosopher and astronomer.
880	al-Battānī [Albategnius] of Raqqa [Aracenſis].
890	Achilles Tatius.

Years of Christ	Entry
936	al-Ṣūfī [Azophi, ſeu Elzuphi].
950	al-Farghānī [Alfraganus] the Arab.
956	ʿAlī ibn Abī al-Rijāl [Haly Aben Ragel].
980	al-Fārābī [Alfarabius] the astrologer.
1024	ʿAlī ibn Riḍwān [Haly Aben Rodoan].

Years of Christ	Entry
1030	Campanus of Novara [Campanus Nouarienſis].
1050	Isaac Argyros [Iſaacius Argyrus].
1070	al-Zarqālī [Arzachel] the Spaniard.
1072	Ibn al-Haytham [Alhazen].
1090	Jābir ibn Aflaḥ [Geber] of Seville.
1100	al-Kindī [Alkindus].
1115	Rabbi Avraham [Rabbi Abraham].
1140	Almeon [Almęon].
1141	Robert of Lincoln [Robertus Linconienſis].
1142	Ioannes Hispalensis.
1149	al-Biṭrūjī [Alpetragius].
1150	Avraham ibn Ezra [Abraham Auen Ezra].
1160	Averroes.
1166	Abū Maʿshar [Albumaſar].
1170	Humenus the Egyptian.
1198	al-Fārābī [Alfarabius] the physician.
1199	Omar the astrologer.
1200	Leopold of Austria [Leopoldus de Auſtria].
1220	Nikolaos Kabasilas [Nicolaus Cabaſilla].
1252	Rabbi Yitzchak Hazan [Rabbi Iſaac Hazan].
1255	Albuassin, or Alboazen.
1256	Johannes de Sacrobosco [Ioannes de Sacroboſco].
1256	King Alfonso X [Alphonſus X. Rex] issues his tables for the second time; he was born in the year 1203, and dies in the year 1284.
1269	Witelo [Vitellio], the Thuringian Pole [Thuringo-Polonus].
1275	Thābit ibn Qurra [Thebit Choræ filius].
1282	Guido Bonatti [Guido Bonatus] the astrologer.
1290	Hendrik Bate [Henricus Baten].
1300	Jacob ben Machir ibn Tibbon [Profatius] the Jew.
1310	Yitzchak Yisraeli [Iſaac Iſraelita].
1317	Pietro d'Abano [Petrus de Apono].
1322	Abū al-Fidāʾ [Iſmael Abilfedea].
1322	Until 1325: Cecco d'Ascoli [Cichus Aſculanus].
1335	Rabbi Levi [Rabbi Leui].
1346	Gherardo da Cremona [Gerardus Cremonenſis].
1360	Marco Beneventano [Marcus Beneuentanus].
1377	Giovanni Boccaccio [Ioannes Boccacius].

Years of Christ	Entry
1380	Johannes de Saxonia [Ioannes de Saxonia].
1381	Jean de Lignières [Ioannes de Linerijs].
1397	Heinrich von Hessen [Henricus de Haſsia] dies.
1416	Pierre d'Ailly [Petrus Aliacenſis].
1423	Georg von Peuerbach [Georgius Purbachius] is born.
1436	Regiomontanus [Io: Regiomontanus] is born.
1438	Gazulus of Ragusa [Gazulus Raguſęus].
1440	Georgios Trapezountios [Georgius Trapezuntius] flourishes; born in the year 1396, he will die in 1486.
1442	Ioannes de Egmunda.
1453	Ermolao Barbaro [Hermolaus Barbarus] is born.
1458	Giovanni Bianchini [Ioannes Blanchinus].
1457	Until 1495: Giovanni Gioviano Pontano [Io. Iouianus Pontanus].
1460	Michael Scot [Michael Scotus].
1462	Peuerbach [Georgius Purbachius] dies, aged 39.
1463	Alessandro Achillini [Alexander Achillinus] is born.
	Giovanni Pico della Mirandola [Io. Picus Mirandulanus] is born.
1464	Cardinal Nikolaus von Kues [Nicolaus Cuſanus Cardin.] dies, aged 63.
1468	Johannes Werner [Io. Vernerus] is born.
1473	Cardinal Bessarion [Beſſarion Cardinalis] died.
1474	Avraham Zacuto [Abraham Zagut].
1475	Ludovicus Vitalis [Ludouicus Vitalis].
1475	Until 1504: Bernhard Walther [Bernardus Valtherus].
	Giovanni Battista Capuano [Io. Baptiſta Capuanus].
1476	Regiomontanus [Io. Regiomontanus] dies, aged 40.
1479	Theodoros Gazes [Theodorus Gaza].
1484	Domenico Maria of Ferrara [Dominicus Maria Ferrarienſis] flourishes, and dies in 1514.
1493	Columbus sets sail for America.
1494	Iacobus Faber.
1495	Christianus Molitoris.
	Lucio Bellanti [Lucius Bellancius].
	Pico della Mirandola [Io. Picus Mirandulanus] dies, aged 33.
	Raffaele Volterrano [Raphael Volaterranus] flourishes.
1499	Marsilio Ficino [Marſilius Ficinus] dies, aged 66.
1500	Stephanus Rosinus.
1501	Jacob Milich [Iacobus Milichius] is born.
1503	Ioannes Muntz.
1506	Bartolomeo Vespucci [Bartholomęus Veſpucius].
1508	Henricus Lindauth.
1510	Johannes Stöffler [Ioannes Stœflerus].
1512	Johannes Angelus [Io. Angelus] the Bavarian dies.
1513	Agostino Ricci [Auguſtinus Riccius], and Paulus of Middelburg [Paulus Middelburgenſis].
1514	Georg Joachim Rheticus [Georgius Ioachimus Rheticus] is born; 1550. A condemned author.
1515	Albert Pigge [Albertus Pighius].
1518	Johannes Homelius [Ioannes Homelius] is born.
1520	Andreas Stiborius.
1523	Francesco Giuntini [Franciſcus Iunctinus] is born.

Years of Christ	Entry
1525	Francisco Sarzosa [Franciſcus Sarzoſus].
	Caspar Peucer [Gaſpar Peucerus] is born.
1526	Heinrich Rantzau [Henricus Ranzouius] is born.
1528	Iacobus Homelius is born.
1530	Johannes Vögelin [Io. Vogelinus] flourishes.
	Peter Apian [Petrus Appianus] flourishes.
	Johannes Garcaeus [Ioannes Garcæus] is born.
	Giovanni Battista Benedetti [Io. Baptiſta Benedictus] is born.
1531	Prosdocimo de' Beldomandi [Proſdocimus de Beldomando] flourishes.
	Oronce Finé [Orontius Finæus] flourishes.
	Luca Gaurico [Lucas Gauricus], until 1557.
1534	Gemma Frisius.
1535	Giovanni Lucido [Ioannes Lucidus].
	Girolamo Fracastoro [Hieronymus Fracaſtorius].
1536	Sebastian Münster [Sebaſtianus Munſterus] flourishes, a condemned author; Johannes Schöner [Io. Schinerus], born in 1477, likewise flourishes.
1536	Until 1542: Giuliano Ristori [Iulianus Riſtorus].
1537	Giovanni Battista Amico [Io. Baptiſta Amicus]; Pietro Pitati [Petrus Pitatus].
1538	Alessandro Piccolomini [Alexander Piccolomineus].
1540	Giovanni Pierio Valeriano [Io. Pierius Valerianus], and Pietro Catena [Petrus Catena].
	Francesco Maurolico [Franciſcus Maurolycus], until 1572.
1542	Lilio Gregorio Giraldi [Lilius Gregorius Gyraldus].
1543	Nicolaus Copernicus dies, aged 71.
1544	Michelangelo Biondo [Michael Angelus Blondus].
1545	Sixtus ab Hemminga.
1546	Tycho Brahe is born.
1547	Heinrich Glarean [Henricus Glareanus].
1549	Ariel Bichardus.
1550	Juan de Rojas [Io. Roias], and Johannes Stabius [Io. Stabius].
1552	Giovanni Antonio Delfino [Io. Antonius Delphinus].
1552	Pedro Nunes [Petrus Nonius] is living; he was born in the year 1492.
1553	Erasmus Reinhold [Eraſmus Reinoldus] dies; he was born in the year 1611[4]
1554	Élie Vinet [Elias Vinetus].
1557	Giovanni Battista Carelli [Io. Baptiſta Carellus].
	Gioseffo Zarlino [Ioſephus Zarlinus].
1558	Jean Fernel [Io. Fernelius] dies.
1560	Valentin Naibod [Valentinus Naiboda]. Federico Commandino [Federicus Commandinus].
1561	Michael Neander, and Daniel Santbech.
1562	Giuseppe Moletti [Ioſephus Moletius].
1563	Lucillo Filalteo [Lucillus Philalthæus].
1566	Pierre de la Ramée [Petrus Ramus], Io. Esler.
1568	Titus à Popma, and Christian Wurstisen [Chriſtianus Vurſtiſius].
1570	Abraham Ortelius [Abrahamus Ortelius].
	William, Landgrave of Hesse [Gulielmus Lantgrauius Haſſiæ].

[4]so printed; a death in 1553 cannot follow a birth in 1611 — Reinhold was born in 1511. In this copy a pen stroke runs through the second figure, and a handwritten figure, apparently a 5, is written above it.

Years of Christ	Entry
	Gerardus Mercator.
1570	Jerónimo Girava [Hieronymus Giraua].
	Francesco Barozzi [Franciſcus Baroccius].
1572	There flourish: Adam Ursinus [Adamus Vrſinus], Andreas Nolthius, Bartholomäus Reisacher [Bartholomæus Reiſacherus], Cornelius Gemma, Cornelius Frangipanius, Cyprián Lvovický [Cyprianus Leouitius], Elias Camerarius, Francesco Maurolico [Franciſcus Maurolycus], Guillaume Postel [Gulielmus Poſtellus], John Dee [Io. Dee], Jerónimo Muñoz [Hieronymus Munoſius], Michael Mästlin [Michael Moeſtlinus], Paul Fabricius [Paulus Fabricius], Paul Hainzel [Paulus Hainzelius], Philipp Apian [Philippus Appianus], Reinhold the Younger [Reinholdus Iunior], Tadeáš Hájek [Thaddęus Hagecius], Theodorus Gramineus, Wolfgang Schuler [Vuolfangus Sculerus].
1576	Erasmus Osualdus [Eraſmus Oſualdus], a condemned author; Sisto da Siena [Sixtus Senenſis]. Daniele Barbaro [Daniel Barbarus], Egnazio Danti [Egnatius Dantes]; Girolamo Cardano [Hieronymus Cardanus] dies, aged 75.
1577	There flourish: Bartholomäus Scultetus [Bartholomæus Scultetus], Giovanni Maria Fiornovelli [Io. Maria Fiornouellus], Nikolaus Winkler [Nicolaus Vuinklerus]; Johannes Praetorius [Io. Prætorius].
1582	Luigi Lilio [Aloyſius Lilius] flourishes.
	Christoph Clavius [Chriſtophorus Clauius].
	Mario Bettini [Marius Bettinus] is born on February 6.
1586	Niccolò Cabeo [Nicolaus Cabæus] is born on February 26.
1587	Joseph Justus Scaliger [Ioſephus Scaliger].
	Giovanni Battista Benedetti [Io. Baptiſta Benedictus].
1588	Ioannes Auria.
	Gellius Sasserides [Gellius Saſſerides].
1589	Giuseppe Scala [Ioſephus Scala], and David Origanus [Dauid Origanus] — but the latter is a condemned author.
1590	Mauro Fiorentino [Maurus Florentinus].
1591	Jacob Christmann [Iacobus Kriſtmannus].
1592	Christoph Rothmann [Chriſtophorus Rothmannus] dies.
1598	Giovanni Battista Riccioli [Io. Baptiſta Ricciolus] is born on April 17.
1600	François Viète [Franciſcus Vieta], Bartolomeo Crescenzio [Bartholomęus Creſcentius].
1603	Johann Bayer [Ioannes Bayerus].
1605	Ioannes Heckius.
1606	Georg Henisch [Georgius Heniſchius], Baldassare Capra [Balthaſar Capra].
1608	Bernardino Salino [Bernardinus Salinus].
	Gulielmus Iansonius [Gulielmus Ianſonius].
	Simon Stevin [Simon Steuinus].
1610	Simon Marius.
	Willebrord Snellius [Villebrordus Snellius].

Years of Christ	Entry
1610	Henricus Samerius, S.J. [Societatis Ieſu], dies, aged 70.
	Matteo Ricci [Matthæus Riccius], S.J., dies; he was born in the year 1552.
1611	Nicolaus Mulerius.
1612	Christoph Clavius [Chriſtophorus Clauius] dies, aged 75.

Years of Christ	Entry
	Joseph Lange [Iosephus Langius].
	Giulio Cesare Lagalla [Iulius Cæsar Lagalla].
	Johann Georg Herwart [Io. Georgius Heruartus].
1614	John Napier [Ioannes Neperus].
1615	Henry Briggs [Henricus Briggius].
	David Fabricius [Dauid Fabricius].
	Rudolf Goclenius [Rodulphus Goclenius].
1617	Redento Baranzano [Redemptus Baranzanus].
	Giovanni Antonio Magini [Io. Antonius Maginus] dies, aged 62.
	François d'Aguilon [Franciscus Aguillonius] died, aged 50.
1618	Johann Baptist Cysat [Io. Baptista Cysatus], S.J.
	Jean Leurechon [Io. Leurechomius], S.J.
1619	Carolus Piso [Carolus Piso].
	Orazio Grassi [Horatius Grassus], S.J.
	Camillo Gloriosi [Camillus Glorioſus].
	Benjamin Ursinus [Beniamin Vrſinus].
	Johannes Deckers [Io. Dekerius], S.J., dies, aged 59.
1620	Alessandro de Angelis [Alexander de Angelis], S.J., dies, aged 58.
1621	Christianus Severinus Longomontanus [Chriſtianus Seuerinus Longomontanus].
1622	Georg Schönberger [Georgius Schombergerus], S.J.
1624	Giuseppe Biancani [Iosephus Blancanus], S.J., dies.
	Io. Adamus, S.J.
1625	Andreas Zergol, S.J.
	Pierre Gassendi [Petrus Gaſſendus].
1626	Vincislaus Pantaleo, S.J.
1627	Kepler [Ioannes Keplerus] issues the Rudolphine Tables [Rudolphinas]; born already from the year 1596 [natus iam ab anno 1596].
	Julius Schiller [Iulius Schillerus].
	Albert Curtz [Albertus Curtius], S.J.
1628	Adriaan Vlacq [Adrianus Vlacq].
1630	Jean-Baptiste Morin [Io. Baptiſta Morinus].
	Peter Crüger [Petrus Crugerus].
	Wilhelm Schickard [Vuilhelmus Schickardus].
	Andrea Argoli [Andreas Argolus].
	Charles Malapert [Carolus Malapertius], S.J., dies.
1632	Adam Tanner [Adamus Tannerus], S.J., dies, aged 60.
1632	Philips van Lansbergen [Philippus Lansbergius] issues his Tables.
1633	Galileo [Galilæus] is forced to abjure his opinion on the motion of the Earth.
	Jacob van Lansbergen [Iacobus Lansbergius] writes in favour of the motion of the Earth.
	Melchior Inchofer, S.J., against the motion of the Earth.
	Martinus Hortensius [Martinus Hortenſius].
1634	Andreas Arzet, S.J.
1635	Johannes Phocylides Holwarda [Io. Phocylides Holuuarda].
1636	Henricus Philippi, S.J., dies, aged 71.
	Christoph Grienberger [Chriſtophorus Griembergerus], S.J., dies.
1640	Johannes Hevelius [Io. Heuelius].
	Fortunio Liceti [Fortunius Licetus].
	Carolus Gontranus.

Years of Christ	Entry
	Balthasar Conradus [Balthaſar Conradus], S.J.
	Willem Blaeu [Gulielmus Blaeu].
	Adriaan Metius [Hadrianus Metius].
1643	Paul Guldin [Paulus Guldin], S.J., dies, aged 56.
1644	Giovanni Antonio Roffeni [Io. Antonius Roffenus] dies, aged 63.
	Pierre Hérigone [Petrus Herigonius].
	Michael Florent van Langren [Michael Florentius Langrenus].
	Giorgio Polacco [Georgius Polaccus].
	Govaert Wendelen [Gotifredus Vuendelinus].
	Albert Linemann [Albertus Linemannus].
1645	Lorenz Eichstadt [Laurentius Eichſtadius].
	Marin Mersenne [Marinus Merſennius].
	Anton. Maria de Reitha.
	Emmanuel Maignan [Emmanuel Maignanus].
1647	Bonaventura Cavalieri [Bonauentura Caualerius] dies.
1649	Francesco Montebruni [Franciſcus Montebrunus] dies.
1650	Christoph Scheiner [Chriſtophorus Scheinerus], S.J., dies.
	Niccolò Cabeo [Nicolaus Cabæus], S.J., dies, aged 64.
1651	Nicolas Caussin [Nicolaus Cauſſinus] dies, aged 70.
	Scipione Chiaramonti [Scipio Claramontius] is living, aged 86.
	Mario Bettini [Marius Bettinus], S.J., is living, aged 69.
	Denis Pétau [Dionyſius Petauius] is living, aged 69.
	Carlo Antonio Manzini [Carolus Antonius Manzinus] is living.
	Niccolò Zucchi [Nicolaus Zucchius], S.J., is living.
	Girolamo Sersale [Hieronymus Sirſalis], S.J., is living.
	Albert Curtz [Albertus Curtius], S.J., is living.
	Georges Fournier [Georgius Furnerius], S.J., is living.
	Vicenç Mut [Vincentius Mutus] is living.
	Io. Drie[?]es, S.J., is living, aged 59.
	Giovanni Battista Riccioli [Io. Baptiſta Ricciolius], S.J., aged 54, issues Volume 1 of the *New Almagest* [Tomum 1. Almageſti Noui].

CHRONICI PARS II. ORDINE ALPHABETICO ET AMPLIORI ERVDITIONE DIGESTA.

[Part II of the Chronicle, arranged in alphabetical order and set out with fuller learning.]

A

Abilfedea: see *Ismael Abilfed.*

Abomasus [Abomaſus], a cosmographer somewhat older than Alhazen [Alhazeno].

Abraham Aben-Ezra [Avraham ibn Ezra] — or, as others write it, *Auen-Hezre*, that is, "son of Ezra" — astronomer and philosopher. He flourished in the year of Christ 1144 or 1150. He wrote a book entitled *Mispate hamazaloth*,

that is, *On the Judgments of the Signs*; the book *Taamin*, that is, *Of Astronomical Reasons*; and a book *On the Luminaries and Critical Days*.

Abraham Zaguth [Avraham Zacuto], most learned in astronomy and a public professor at Carthage in Africa, published the *Great Composition* [Magnam compoſitionem]. Afterwards he taught astronomy at Salamanca, and in both places Agostino Ricci [Auguſtino Riccio] attended his lectures. In his treatise *On the Motion of the Eighth Sphere*, chapters 5, 43 and 46, Ricci makes honourable mention of him, and says that in the year of Christ 1474 Zacuto observed Spica [Spicam Virginis] at 17° 10′ of Libra.

Abraham Ortelius [Abrahamus Ortelius] of Antwerp, Royal Geographer, who deserved supremely well of geography, both by his *Theatrum orbis terrarum*, published in 1570, and by his *Thesaurus geographicus*, a very rich dictionary, which he published in 1587.

Achilles Tatius, bishop of Alexandria, wrote a book *On the Universe*, which contains a most learned *Introduction* [Iſagoge] to Aratus's *Phaenomena*; our Pétau [Petauius] has recently translated it from Greek into Latin in his *Uranologion*, from p. 121. He flourished around the year of the Lord 890; but before he became a Christian he had written love stories about Leucippus and Clitophon [de Leucippo, & Clitophonte]. Photius mentions him in the *Bibliotheca*, codex 87, where he calls him not Statius, as the corrupt codex of Suidas has it, but Tatius.

Adam Tanner [Adamus Tannerus], S.J., of Innsbruck [Oenipontanus], a distinguished theologian and a most vigorous assailant of the heretics,

and for that reason [ob idque] very dear to the Emperor Ferdinand II [Ferdinando II. Cæſari Auguſto] and to the Duke of Bavaria; he was most celebrated at Munich, Ingolstadt and Vienna, where he taught publicly. He also wrote a most learned *Dissertation on the Heavens* and an *Astrologia Sacra*, and he died at sixty, of dropsy and lethargy, on 1632 May 25, in the town of Unken [Vncken] in the Tyrolese domain.

Adam Ursinus [Adamus Vrſinus] of Nuremberg wrote a prognostication on the new star of the year 1572; on it see Tycho's critique in volume 1 of the *Progymnasmata*, p. 781.

Adriaan van Roomen [Adrianus Romanus], a Belgian knight, flourished about the year 1615. He wrote an *Uranographia*, an *Idea mathematica*, *Exercitationes cyclicae*, a work on spherical triangles, and one on Archimedes's book *On the Measurement of the Circle*.

Adriaan Vlacq [Adrianus Vlacq] of Gouda published in 1628 the *Arithmetica logarithmica*, or a hundred chiliads of logarithms, with the logarithms of sines, tangents and secants, for solving astronomical, geometrical and arithmetical problems with wonderful brevity.

Adriaan Metius [Adrianus Metius] of Alkmaar wrote learnedly on the use of the globe in 1640.

Aëtius of Amida [Aetius Amidenus], count and physician, wrote a *Hemerologium on the Significations of the Stars*. Cornarius translated it from

Greek into Latin from the third discourse [ſermone 3.] of the *Tetrabiblos*, chapter 164, and Pétau has recently published it in the *Uranologion*, p. 421. This author flourished in the year of Christ 434.

Agrippa observed the stars in Bithynia in the year of Nabonassar 840 and the twelfth of Domitian, as Ptolemy reports in *Almagest* VII.3 — that is, in the year of Christ 92.

al-Battānī [ALBATEGNIVS], called *Mahometus Tinen* or *Mahumedes Aracensis*, son of Geber Auchanus [Gebri Auchani], son of Cruenus [Crueni], ruler [Dynaſta] of Syria, observed the stars both at Raqqa [Aractæ] and at Antioch in Syria. He corrected Ptolemy, since in his own age the Ptolemaic canons already plainly disagreed with the sky; he therefore constructed new tables of the celestial motions. He wrote a book *On the Science of the Stars*, comprised in 57 chapters, which Plato of Tivoli [Plato Tiburtinus] translated from Arabic into Latin, and which Regiomontanus [Io. Regiomontanus] illustrated with some notes. This author flourished in the year of Christ 879 — that is, of Nabonassar 1627, and the first of Antoninus — as he himself testifies in chapter 51.

Albert Pigge [Albertus Pighius], a German, native of Kampen [patria Campenſis], wrote on the observation of the solstices and equinoxes; on the restoration of the calendar, addressed to Pope Leo X in 1515; a defence of astrology; and a work against certain prognosticators.

Albert Curtz [Albertus Curtius], S.J., a Bavarian born at Munich. In 1627 he put forward a new system of the heavens for disputation at Dillingen. Kepler [Io. Keplerus] admired his talent in the theory of the Moon as a rival to his own, and praised it in his Rudolphine Tables, chapter 25. Now, by order of the Emperor, he is in charge of revising and publishing Tycho's observations. I obtained a few of them from him, thanks to the truly noble generosity of spirit with which he is endowed.

Albert Linemann [Albertus Linemannus], a Prussian by nation and a native of Fischhausen [Fiſchhuſinus], public professor of mathematics at the University of Königsberg [Academia Regiomontana] in Prussia. Recently, in 1644, he published a short work of certain astronomical observations, chiefly of eclipses, which he entitled *Memoria saecularis* [Memoriã Sæcularem].

Albinus Flaccus, surnamed Alcuin [Alcuinus], English by nation, a pupil of Bede and the teacher of Charlemagne [Caroli Magni], learned in every branch of knowledge, flourished in the year 770.

Albohazen, or **Albuassin** [Albuaſſin], wrote a book *On the Motion and Places of the Fixed Stars*, which Rabbi Yehudah [Rabbi Iuda] translated from Arabic into the language of Seville [Hiſpalenſem] and presented to King Alfonso. Moved by its authority, the King adopted al-Battānī's [Albategnij] view of the motion of the fixed stars, as confirmed in that book, and in 1256 substituted other, more correct tables for those issued in 1252 — as Agostino Ricci relates in his book *On the Motion of the Eighth Sphere*, chapter 46, on the report of Avraham Zacuto [Abrahami Zaguthi].

Albubater, an Arab astrologer, is recognized by Giuntini [Iunctino] as belonging to the year 500 before Christ.

Abū Maʿshar [Albumaſar], an Arab astrologer called Jaʿfar [Iaphar], wrote eight treatises *On the Great Conjunctions* and *On the Revolutions of the Years*. He is said to have observed a comet above Venus in the year of Christ 844. But since in that work he says that the sect of the Arabs, or of Mahomet — which is known to have begun in the year of Christ 622 — would not last beyond 544 years, and that its end was imminent, he must have flourished not long before the year of Christ 1166 — even though Sanseuinus [Sanſeuinus] draws him back to the year 830, and Biancani [Blancanus] to the tenth century of Christ. But perhaps there were different astrologers of the same name. Much older was that other one — if indeed he is another — whom Giuntini places, by conjecture, in the year 540 before Christ.

al-Qabīṣī [Alcabitius] the astrologer, or **Alchabich**, set out the elements of astrology in four *Differentiae*, or treatises, which Valentin Naibod [Valentinus Naiboda] illustrated with commentaries.

al-Kindī [Alkindus], philosopher and astrologer, wrote a work *On the Changes of the Seasons* [de Temporum mutationibus], and flourished in the year of the Lord 1100.

Alcuin [Alcuinus]: see Albinus Flaccus.

Alessandro Achillini [Alexander Achillinus] of Bologna, philosopher, wrote *On the Celestial Orbs*; he was born 1463 October 29, at 17^{h} 20^{m} after noon.

Alessandro Piccolomini [Alexander Piccolomineus] of Siena wrote four books *On the Sphere of the World* and one *On the Fixed Stars*, in Italian, which he dedicated in 1539 to the lady Laudomia Forteguerri [Laudomiæ Matronæ Forteguerræ]; and also a *Theory of the Planets*, likewise in Italian, addressed to Cosimo de' Medici [Coſmum Mediceum] in 1558.

Alessandro de Angelis [Alexander de Angelis] of Spoleto, S.J., professor of theology and prefect of studies at the Roman College, wrote five books *Against the Conjecturing Astrologers* [in Aſtrologos coniectores]; he died at Ferrara in 1620, aged 58.

Algazel Abukamad, philosopher and astronomer, wrote in Hebrew a book *On the Lights*, called *Haoroth*.

al-Fārābī [Alfarabius], astronomer, lived in the year of the Lord 980; the other, who was a physician, flourished in 1200.

al-Farghānī [Alfraganus Mahumedes] — or, as it stands in the Palatine Library, *Ametus*, or *Ahemed*, or *Muhamed* son of Ametus [filius Ameti] — was called al-Farghānī from the city of Fragana, according to Jacob Christmann [Iacobus Kriſtmannus]. Christmann shows in chapters 6 and 26 of his commentary that he flourished about the year of Christ 950, in which year he observed the Sun's greatest declination as 23° 35′, and the equinox on the 16th day of the month Adar, or March. He wrote elements of astronomy and chronology, compiled chiefly from Ptolemy. Rabbi Yaakov Anatoli [Rabbi Iacob Antolius]

translated these elements into Hebrew, and Ioannes Hispalensis into Latin from the Arabic in 1142; Conradus Gisnerus [Conradus Giſnerus] and Jacob Christmann illustrated them with commentaries.

Ibn al-Haytham [ALHAZEN], or **Alhazenus**, an Arab, son of Alahizenus [Alahizeni] — a name that means "good man" — wrote seven books of *Optics* and one *On Twilight*. He flourished in the year of Christ 1072, or about 1100, as Friedrich Risner [Federici Riſneri] conjectures; Risner illustrated him with figures and commentaries, and records that there were three other distinguished Arabs of the same name.

al-Maʾmūn [Almamon], or **Alamon**, or **Maimon**, emperor of the Arabs, reigned in Baghdad [Baldach]. In the year of Christ 827 he ordered Ptolemy's *Almagest* to be translated into Latin [in Latinum ſermonem]. Fifty years before al-Battānī [Albategnium], that is, in the year 830, he observed the Sun's greatest declination as 23° 35′, as al-Farghānī [Alfraganus] reports in *Differentia* 6 — where Christmann notes that most writers confuse him with Almeon [Almæone], of whom presently. He is the one who, on the testimony of Abū al-Fidāʾ [Abilfedea] and al-Farghānī, summoned the most skilled geometers of the Arabs and ordered them, in the plains of Fingar [Campis Fingar] near the direct route of the Red Sea, to measure how many Arabic miles are contained in one degree of a great circle of the Earth, as we shall explain in its place. Below, where we treat of Ptolemy, we shall show that the first translation of the *Almagest*, made from Greek into Arabic in the year of Christ 827, is owed to this Maimon.

Almeon [Almæon], son of Albumasar [Albumaſaris], as Christmann has it, or of Almansor [Almanſoris], as Reinhold has it in his *Theory of the Eighth Sphere*, is said to have observed the Sun's greatest declination in the year of the Lord 1140 — or, as others say, 1150 — as 23° 33′ 30″.

Luigi Lilio [Aloyſius Lilius] of Verona, and his brother Antonio Lilio [Antonius Lilius]: among the many who devised a form of a perpetual lunar cycle and of a fixed seat for the equinoxes, he carried off the palm with Gregory XIII, who followed him in the reform of the calendar, so that Clavius [Clauio] and Biancani rightly called him a second Sosigenes of our age. He flourished, accordingly, in 1582.

al-Biṭrūjī [Alpetragius] of Morocco [Marrocitanus], astronomer, flourished between the years of Christ 1145 and 1154; his physical *Theory* was translated from Arabic into Latin by Calo Calonymus of Naples [Parthenopæus].

Alfonso X [ALPHONSVS X.], King of Castile and León, the less fortunate son of the most holy and most fortunate King Ferdinand, yet surnamed the Wise for his skill in philosophy and astronomy. While his father was still living, he had from the year 1240 summoned all the most learned Moors, Arabs and Jews to restore the astronomical tables. He is said to have spent four hundred thousand gold pieces on books, instruments and the maintenance of those men, as Ramus reports in book 3 of the *Scholae mathematicae*, Reinhold in the preface to the *Prutenic Tables* [Prutenicas], and Tycho in the preface to Rudolf II in volume 1

of the *Progymnasmata* — although Boulliau [Bullialdus], in the prolegomena to the *Astronomia Philolaica*, puts 4000 in place of 400000. He succeeded his father, who died at Seville, in 1252, on May 30 [3. Cal. Iunij] — the year in which the Alphonsine Tables first appeared. As regards the motion of the eighth sphere and of the apogees [augium], however, the tables relied on Cabbalistic fictions, since the compiling of them had been directed by Rab- bi Isaac Hazat [Iſaac Hazat], cantor of the synagogue of Toledo [Toletanæ Synagogæ cantor]. But when the King had read the book that Abuassim [Abuaſſim] presented to him, he changed his mind, followed al-Battānī [Albategnium], and in 1256 published more correct tables of the said motion and of the places of the fixed stars. Avraham Zacuto [Abraham Zagutus], in Ricci's treatise *On the Motion of the Eighth Sphere*, and Cardinal Nikolaus von Kues [Cuſanus] testify that they saw these tables. Egnazio Danti [Egnatius Dantes] adds, in part 4 of his *Astrolabe*, that he saw Alfonso's *Book of Instruments* [Librum inſtrumentorum], translated from Arabic into Castilian and from Castilian into Latin. In 1257, at the diet of Frankfurt, Alfonso was elected King of the Romans by some of the electors, while others chose Richard the Englishman [Richardum Anglum]. He never ceased to seek and work for his anointing and confirmation, and to petition the supreme pontiffs Urban IV, Clement IV and Gregory X for the diadem; and, having been rebuffed, he remained hostile to them and to Richard until 1275. In 1260 he had suffered a great defeat at the hands of the Moors, yet in 1263 he wrested from them the island of Galaecia [inſulam Galaeciam], settled colonies of Christians there, built a church of the Holy Cross, and chose it for his burial place. In 1266, moreover, he subdued the Moors of Granada, and he promised a hundred thousand gold pieces to his father-in-law James, King of Aragon, who was preparing a crusade [ſacram expeditionem] to the Holy Land.

Whether because he saw his astrological judgements turn out badly, or because he could not master the anomalies of the celestial motions, he is said to have broken out into the blasphemy of saying that he would have ordered the heavens far better than God had made them, if that charge had been his; and when a holy hermit rebuked him for this in vain, he fell into madness — so some report from the *History of Spain* of Juan de Mariana [Io. Mariannæ]. But Sancius, in part 4 of the *History of Spain*, chapter 5, and after him the *Cathechismus historialis* under the word "Alphonsus", say that he was struck and terrified by a sudden storm of rain-clouds and lightning, summoned the hermit who had admonished him, and confessed his sins with great grief; that at the very moment of absolution the storm ceased; and that he amended his life. I do not know whether this happened before he sinned against the Church. For he unjustly seized church possessions from the Archbishop of Compostela, and for that was rebuked by Pope Nicholas, with threats of anathema and interdict added. And although as an old man he took up a dire war with his son Sancho [Sanctio], yet, so as not to be captured by the Moors at Córdoba, he fled secretly to Seville, and struck his son with a dire curse and with deprivation of

the kingdom — all of which Bzovius [Bzouius] recounts at length in the *Annals* under the year 1282. At last he died at Seville in 1284, on April 21 [11. Calend. Maij], fortified by penance and the Eucharist, aged 81; for he had been born at Medina in 1203, at the sixth hour of the morning, as Giuntini [Iunctinus] has it in his *Astrological Calendar* [Kalendario Aſtrologico]. He ordered that his heart be buried on Mount Calvary. He had read the whole Holy Bible ten times, and four times with its glosses, as Génébrard [Genebardus] has it under Urban IV, and Trithemius in the *Catalogue* under the year 1270.

Anaxagoras of Clazomenae [Anaxagoras Clazomenius] began to philosophize in the twentieth year of his age, which was the first year of the 75th Olympiad, or 480 before Christ; and he died aged 72, namely in 428 before Christ, in the 88th Olympiad — granted that the corrupt codex of Laertius has 78. Apollodorus, however, says in the *Chronicles* that he lived only 62 years. On the testimony of Plutarch in the *Nicias*, he incurred the hatred of the Athenians and was thrown into prison because he was the first to teach openly that an eclipse of the Moon is a privation of the light received from the Sun. Laertius, by contrast, says this happened because he had been accused of impiety by Cleon for having said that the Sun is a glowing plate [candentem laminam]; and that, though defended by Pericles, he was nevertheless fined five talents and exiled. He used to say that there are valleys, mountains and dwellings in the Moon.

Anaximander of Miletus [Anaximander Mileſius] was in his 64th year in the second year of the 58th Olympiad, and died shortly afterwards. If we follow Apollodorus in the *Chronicles*, he was born in the third year of the 42nd Olympiad, namely 610 before Christ, and flourished in 544. He said that the Moon shines with light from the Sun, and that the Sun is equal in size to the Earth. He was the first to set up a gnomon, at Sparta [Lacedæmone], for observing the equinoxes and solstices by the shadow of the Sun, as Laertius relates. Pliny the Elder adds, in book 2, chapter 8, that in the 58th Olympiad he was the first to detect the obliquity of the zodiac, and so in a manner opened the doors of astronomy. He was the first to publish a map of the world [de ſitu orbis tabulam], as Strabo says in book 1. Giuntini wrongly draws him back to the year 737 before Christ.

Anaximenes of Miletus [Anaximenes Mileſius], a pupil of Anaximander, said that the stars move in a circle not above the Earth but around it. In a letter he wrote to Pythagoras he complains that he was prevented from searching out the secrets of the heavens by constant fear of death at the hands of the tyrants of Miletus, or of slavery under the king of the Medes. He flourished about 530 before Christ.

Andreas Nolthius of Einbeck [Embeccenſis], mathematician, wrote on the New Star of 1572 at about the same time, addressed to the Landgrave of Hesse; on him see Tycho, volume 1 of the *Progymnasmata*, p. 750.

Andrea Argoli [Andreas Argolus], devoted to astronomy and astrology, wrote *Ephemerides* from 1630 to 1700, and also the *Pandosium sphaericum*; he was still living in 1646.

Andreas Arzet, of the Society of Jesus, of Konstanz [Conſtantienſis], published a *Clavis mathematica* in 1634, and observed the stars, very diligently indeed, for as long as his business and his gout [chiragram] allowed — as I noticed from the observations he most generously shared with me, above all those of eclipses.

Andreas Stiborius [Andreas Stöberl], a Bavarian [Boius], a most acute philosopher, theologian and astronomer, canon of Vienna, wrote an epitome of al-Battānī [Albategnij], of the *Almagest* and of Jābir ibn Aflaḥ [Gebri], and much else. He flourished under the Emperor Maximilian.

Andreas Zergoll, of the town of Santa Croce [S. Cruce] in the territory of the Carni [in finibus Carnorum], of the Society of Jesus, taught mathematics, moral theology and Sacred Scripture at Graz [Græcij] in Styria. In 1625 he put forward *Chronological Theorems* on the years of the birth and death of Christ, maintaining that Christ was born in the Julian year 40 and died in 74, on March 25.

Andromachus of Crete [Andromachus Cretenſis] flourished in the first century of Christ and, if we believe Clavius [Clauio], was the first inventor of the *Theoricae*.

Andruzagar, an astrologer, flourished in the year of Christ 230, as Giuntini has it in his *Catalogue*.

Anton Maria Schyrleus de Rheita [Antonius Maria Schyrlæus de Reitha], a Capuchin preacher and an astronomer of some note, wrote in 1645 a book on astronomy entitled *Oculus Enoch et Eliae*, or *Radius sidereomysticus*.

Apollonius of Myndus [Apollonius Myndius], on Seneca's testimony, studied with Epigenes among the Chaldaeans, among whom, he said, there were observations of 720 years inscribed on baked tiles [coctilibus laterculis].

Apollonius of Perga [APOLLONIVS Pergęus], born at Perga, a city of Pamphylia, called the great geometer, wrote four books of *Conics*, in which he also advanced astronomy. He flourished under Ptolemy Euergetes, about 240 before Christ, as Heraclius says in the *Life of Archimedes*.

Aquila Ponticus, a native of Sinope, the homeland of Diogenes, was a distinguished mathematician under the Emperor Hadrian, about the year of Christ 130, as is gathered from St Epiphanius's book *On Measures and Weights*. But at length, expelled from the Church for the vanity of judicial astrology, he went over to the Jews, was circumcised, and translated the Old Testament into Greek with little integrity.

Aratus, an astronomical poet, son of Athenodorus and Letophila, was not of Tarsus by birth, as Asclepiades held, but of Soli — that is, of the city of Pompeiopolis in Cilicia, formerly called Soli — as most hold of him, with Callimachus. He flourished under Antigonus Gonatas, King of Macedonia, that is, in the 125th Olympiad, which was celebrated in 280 before Christ. He wrote the *Phaenomena*, or appearances of the risings and settings of the stars, with

prognostics, which many have illustrated with commentaries or expositions — above all Germanicus, Rufus Festus, Cicero, Hyginus and Achilles Tatius. Giuntini, however, in his *Catalogue* moves Aratus back as far as 588 before Christ.

Archelaus, a geographer, described the part of the world traversed by Alexander the Great, and flourished shortly after him. Diogenes Laertius [Diog. Laert.].

Archimedes of Syracuse [ARCHIMEDES Syracuſius] was distinguished not only in arithmetic — by which he comprehended the number of the sand even if it filled the whole world — and in geometry, in which he above all investigated the circle, the sphere, the cylinder and the parabola, but also for his skill in astronomy. "For when Archimedes" — the words are Cicero's, *Tusculan Disputations* 1 — "*bound the motions of the Moon, the Sun and the five wandering stars into a sphere, he did the same as the god in Plato's* Timaeus *who built the world, so that a single revolution governed motions most unlike in slowness and speed*." Claudian sings in his celebrated epigram that this sphere was of glass:

Iuppiter in paruo cùm cerneret æthera vitro &c. ["When Jupiter saw the heavens in a little glass," etc.]

The same Tully asserts in *Tusculan Disputations* 5 that when he was quaestor in Sicily he discovered at Syracuse, by the Agraian gates [ad portas Agraianas], the tomb of Archimedes with the sphere and the cylinder. And Livy too, in book 24, describing Syracuse besieged by Marcellus, says: "*And the undertaking, begun with such force, would have succeeded, had not one man been at Syracuse at that time: it was Archimedes, a unique observer of the sky and the stars, but more wonderful as an inventor and builder of engines of war and siege-works*," etc. On his machines and his other distinctions, however, see Pliny the Elder, book 7, chapters 8 and 37; Vitruvius, book 9, chapter 5; Proclus, book 2, chapter 3; Petrus Ramus, book 1 of the *Scholae mathematicae*; and David Rivault [Dauid Riualtus] in the *Life of Archimedes*. It is agreed, moreover, that Archimedes was killed in the storming of Syracuse, against the will of Marcellus, while he was intent on contemplating certain figures drawn in the dust, in 212 before Christ. Archimedes was of the royal stock of Hiero, on Plutarch's testimony; and from the stock of Archimedes sprang St Lucy, the renowned virgin and martyr, as Rivault teaches, who adds that he was born in 289 before Christ and killed in 214, though others say 212. Ob- servations of Archimedes on celestial matters are also mentioned by Ptolemy, *Almagest* III.2.

Archytas of Tarentum [Archytas Tarentinus], a noble Pythagorean and most expert in mathematics, whom Plato sailed to Italy to meet, together with Timaeus, as Cicero reports. He excelled in mechanics, and by their power defeated his enemies in battle five times — to say nothing of the wooden dove that he made fly by means of counterweights [libramentis]. That he also excelled in cosmography and practical geometry is indicated by the lines of Horace, *Odes* book 1:

Te Maris ac Terræ numeroq; carentis arenæ Menſorem cohibent Archyta. ["You, Archytas, measurer of the sea and the land and of the sand that has no number, are confined…"]

Ariel Bicard [Ariel Bicardus] published in the year of our salvation 1549 a collection of his own *Questions on the Sphere of Sacrobosco* [quæſtiones in Sphæram Sacrobuſchi], and dedicated it to his fellow godfather [compatri ſuo] Anton Müller [Antonio Mullero].

Aristarchus of Samos [ARISTARCHVS SAMIVS] flourished 140 years before Hipparchus, that is, about 280 before Christ. For, as Ptolemy relates from Hipparchus in *Almagest* III.2, Aristarchus observed the solstice in the 50th year of the first Callippic period, which began in 330 before Christ. From this I gather that the Cleanthes who, on Plutarch's testimony in the book *On the Face in the Moon*, summoned Aristarchus to trial for having moved the sacred Hearth of Vesta [Sacra Veſtę] from its place — that is, for having asserted that the Earth moves around the Sun — was the one who succeeded Zeno the Stoic in the 129th Olympiad, that is, in 264 before Christ. There survives, moreover, a most learned little work of Aristarchus *On the Distances and Magnitudes of the Sun, the Moon and the Earth*; and there is mention of him, as moving the Earth and removing the fixed stars to an immense distance, in Archimedes's *Sand-Reckoner* [Arenario]. Less correctly, Biancani [Blancanus], in the *Chronology of the Mathematicians*, makes Aristarchus 200 years older than Hipparchus.

Aristyllus [Ariſtillus], an astronomer, observed the fixed stars around the time of Timocharis, as stated in Ptolemy, *Almagest* VII.3 — namely about 300 before Christ.

Aristotle [Ariſtoteles], who deserved not badly of astronomy in his books *De caelo*, was born in the 99th Olympiad, that is, in 384 before Christ, and died at Chalcis aged 63, in the third year of the 114th Olympiad, namely 322 before Christ, as is gathered from Laertius.

Artemidorus, an ancient cosmographer, is mentioned by Pliny the Elder and Strabo, who praises his geographical work; and he was himself a contemporary of Strabo.

al-Zarqālī [Arzachel], a Spaniard, 190 or 193 years after al-Battānī [Albategnium], observed the Sun's greatest declination as 23° 34′ — that is, in the year 1070 or 1073 before Christ [ante Chriſtum]. Ibn Ezra [Aben Ezra] writes of him, in the book entitled *Initium sapientiae* [The Beginning of Wisdom], that no one in that age was to be compared with al-Zarqālī in observing the motions of the heavens, and he acknowledges that he himself was 71 years later. Some make him a native of Toledo; he was also called Abraham Elzara Keel. Georg Joachim Rheticus [Georg. Ioachimus Rheticus], in the preface to his *Ephemerides*, says that he was the author of the Toledan Tables, and that he left 402 solar observations for establishing the Sun's apogee.

Ascanio Martinengo [Aſcanius Martinengus] of Brescia [Brixianus], Abbot General of the Lateran Canons: besides what he most learnedly published, about

the year 1600, in the *Glossa magna* on the heaven and the stars created by God, he also wrote a *Geography of the Holy Land* and composed a notable *Chronology*.

Athanasius Kircher [Athanaſius Kircher], of the Society of Jesus, a native of Fulda [patria Fuldenſis], astonishingly expert in philosophy, in mathematics, and in the Hebrew, Greek, Arabic and Egyptian languages and several other Oriental ones, and now very well known throughout Europe for works of varied and recondite learning, is still living and flourishes at Rome, esteemed by the Emperor and by the Supreme Pontiff. To me he is above all a friend in astronomy [amicus Vranicus], to whom I acknowledge that I owe, among many things, a great many observations collected from everywhere, especially of eclipses. Of his works, those that come closest to astronomy or cosmography are an excellent work *Magnes*, another entitled *Ars magna lucis et umbrae*, and the very recently published *Musurgia*.

Atlas, King of Mauretania, is held by Pliny the Elder, book 2, chapter 8, and by Diodorus Siculus, book 4, to have been the inventor of the sphere, and for that reason to have held up the heavens on his shoulders. If you listen to Eusebius, he flourished about 1640 before Christ; Sansovino [Sanſouinus] puts him in 1715; but more truly he flourished about 1580, when Moses had already been born, according to our *Chronology*.

Averroes [Auerrhoes] would deserve a place in this catalogue on no other ground than that he reduced Ptolemy's *Almagest* to an epitome, in the twelfth century of Christ.

Agostino Ricci [Auguſtinus Riccius] of Casale [Caſalenſis] published in 1513, in the city of Casale Sant'Evasio [Caſalis Sancti Euaſij], a most learned treatise *On the Motion of the Eighth Sphere*, which he dedicated to William, Marquis of Montferrat [Gulielmo Marchioni Montiſferrati]. In that work he declares that he learned astronomy at Carthage and Salamanca from Avraham Zacuto [Abrahamo Zagutho]. He also wrote to Galeotto del Carretto [Galeotum de Carreto], of the marquises of Savona, a letter *On the Authors of Astronomy*, showing that astronomy derived from the Hebrews.

Autolycus of Pitane [Autholycus Pritanęus], from Aeolis [ex Acolide], the teacher of Arcesilaus, flourished in the 120th Olympiad, on the testimony of Laertius in the *Arcesilaus* — that is, about 300 before Christ. He wrote both *On the Moving Sphere* and *On the Various Risings and Settings of the Stars*.

al-Ṣūfī [Azophi], or **Elzuphi**, or **Ebennezophim**, an Arab, author of the Persian Tables, in which the figures and the places of the stars are set out, flourished in the year of the Lord 1062, as some think; but Boulliau [Bullialdus], in the prolegomena to the *Astronomia Philolaica*, draws him back to the year 936.

B

Baldassarre Capra [Balthaſſar Capra] published in 1606 at Padua the *Tyrocinia astronomica*.

Balthasar Conrad [Balthaſſar Conradus], of the Society of Jesus, of Neisse [Niſſenſis], taught philosophy and mathematics at Olomouc [Olomuncij], and in 1640 published a new method of cosmographical maps in a right cone, whose base is the equator. I also have an observation of an eclipse that he sent me.

Bartolomeo Crescenzio [Bartholomæus Creſcentius], a Roman, brought out in 1602 the *Nautica Mediterranea*, a work not without use to cosmographers.

Bartholomäus Pitiscus [Bartholomæus Pitiſcus] of Grünberg [Grumbergenſis] in Silesia: besides the *Great Canon of Sines, Tangents and Secants* [Magnum Canonem Sinuum, Tang. & Secantium], which he extended, at the beginning and end of the quadrant, to every single second, using a radius of 12 figures [Radio cyphrarum 12.], he published in 1612 ten books, partly on the measurement of triangles, partly of geodetic, altimetric, geographical, gnomonic and astronomical problems.

Bartholomäus Scultetus [Bartholomæus Scultetus Gorlicienſis] of Görlitz wrote astronomically and astrologically on the comet of 1577; on it see Tycho's critique in volume 2, p. 369.

Bartholomäus Reisacher [Bartholomæus Reiſacherus], professor of mathematics at Vienna in Austria, wrote on the new star of 1572; on him see Tycho, volume 1 of the *Progymnasmata*, pp. 530 and 746.

Bartolomeo Vespucci [Bartholomæus Veſpuccius] of Florence wrote on the *Sphere* of Sacrobosco and, in 1506, *On the Praises of Astrology*; and he professed astronomy at Padua.

The Venerable Bede [Beda Venerabilis], an Anglo-Saxon from Cambridge [ex Cantabrigia], was steeped in the whole circle [Encyclopedia] of very many sciences, and above all knowledgeable in astronomy, as his books *On the Sphere*, *On the Reckoning of Time* and *On the Cycles of the Moon* show, together with very many other small works of this kind. Trithemius says that he died aged 72, in the year of the Lord's Incarnation 732, on May 31 [pridie Calend. Iunij], and that he was not blind; Sixtus of Siena, however, says that he died in 734, aged 62; but Baronius and Spondanus prove from Bede's own letters that he died in 776, aged 106.

Benjamin Ursinus [Beniamin Vrſinus], of the Mark [Marchicus], an astronomer, flourished in 1619.

Bernardino Salino [Bernardinus Salinus], of the Society of Jesus, from the diocese of Vercelli, died in 1608. Among other things he wrote on various astronomical problems; his works are preserved at Genoa in our College. He taught philosophy three times at Milan, and was a man of wonderful simplicity and of no common zeal against the heretics.

Bernhard Walther [BERNARDVS VVALTHERVS] of Nuremberg, a pupil of Regiomontanus [Io. Regiomontani] and continuer of his observations at Nuremberg, with both armillary spheres [Armillas] and parallactic rules [Regulas Parallacticas], did not cease, from 1475 to 1504, to observe both the meridian altitudes of the Sun and the places of the other planets. The last of these observations was made on 1604 May 30 [30. Maij Anni 1604.] [recte:

1504 — the entry itself runs his observations from 1475 to 1504, and he died a few days after the last], of Mars at about 20° of Leo. Not many days after it he died, an excellent old man — just as Tycho too stepped toward death from an observation of Mars. His observations, together with those of Regiomontanus, were first published at Nuremberg, and afterwards, together with the Hessian and Tychonic observations, by Willebrord Snellius [Villebrordo Snellio].

Berossus the Chaldaean [Berosus Caldæus] flourished in the times of Antiochus Soter, about the 130th Olympiad, namely from 281 to 262 before Christ. On his assertion, the Chaldaeans preserved astronomical observations of 480 years inscribed on baked bricks — granted that Alexander the Great, when Babylon was taken, preferred to all the spoils the observations of 1903 years made and found there. We shall explain his opinion about the Moon in its place, from Vitruvius, book 9, chapter 4, where Vitruvius says that he spread the mathematical disciplines from Chaldaea into Asia. Pliny the Elder, moreover, in book 7, chapter 37, relates that the Athenians, on account of his divine predictions, publicly set up a statue with a gilded tongue to Berossus the astrologer in the Gymnasium. Take care not to confuse him with that most ancient Berossus — whether historian or fabulist — whom Eusebius places in 1272 before Christ, as a contemporary of Orpheus and Musaeus.

Bessarion [Beſſarion Cardinalis Nicænus], Cardinal of Nicaea, Patriarch of Constantinople, among other things wrote a canon of the stars with the Alphonsine numbers corrected. He founded at Venice a notable library, of which Marcantonio Sabellico [Marcus Antonius Sabellicus] is the keeper. He died in 1437 [Anno 1437.].

Boethius [Boetius]: see Severinus [Seuerinum].

Bonaventura Cavalieri [Bonauentura Caualerius] of Milan, a Jesuate [Ieſuatus], a pupil of Galileo [Galilæi], principal professor of mathematics in this University of Bologna. He published an *Uranometria*, a *Practica astrologica*, and a *Centuria problematum*, among which are many astronomical and cosmographical problems, and much else, by which he left the keenness of his talent and judgement on record. He would have left more monuments, had not gout of the hands and feet [chiragra, & podagra] confined him repeatedly to his bed for nearly 12 years, and at last snatched him from us — from us, I say, who often found in that most friendly and most learned man no mean help and encouragement in our astronomical studies. He died at Bologna on 1647 December 3.

C

Callippus of Cyzicus [CALIPPVS CYZICENVS], a distinguished astronomer — on his system of the celestial spheres, Aristotle, *Metaphysics* 12, text 47. He devised a lunisolar cycle or period of 76 years, compounded of four Metonic cycles, but corrected. He began its first year from the beginning of the monarchy of the Greeks, that is, from 330 before Christ, when he himself flourished together

with Alexander the Great and Aristotle. His period is frequently mentioned by Ptolemy in the *Almagest*, especially III.2 and VII.2 and 3.

Camillo Gloriosi [Camillus Glorioſus Gifonenſis], of Giffoni, professor of mathematics at Padua, wrote learnedly on comets in 1619.

Campanus of Novara [Campanus Nouarienſis], an astrologer and astronomer famous in his own age — that is, in the year of Christ 1030, not 1200 — devised a particular method of erecting a celestial figure by division of the prime vertical, which Gazulus followed. He wrote a *Theorica planetarum*, *On the Sphere*, *On the Computus*, etc.

Carlo Antonio Manzini [Carolus Antonius Manzinus] of Bologna, doctor of philosophy, most expert in astronomy, and a very dear friend of mine, published in 1626 *Tables of the Primum Mobile* [tabulas primi Mobilis] with a new art of directing and a compendium of directions. He is now preparing *Theoricae planetarum* with tables, does not cease from astronomical observations, and is still flourishing in 1651.

Carolus Gontranus, doctor of the Sorbonne, General of the Congregation of the Oratory, distinguished for learning and piety, wrote a little book in favour of astrology — but sound astrology — and flourished in 1640.

Charles Malapert [Carolus Malapertius], a Belgian, native of Mons [domo Montenſis], of the Society of Jesus, born in 1581, taught mathematics in Poland and at Douai [Duaci], and was to teach at Madrid [Matriti] had he not died on the journey — at Vitoria [Victoriæ], in November 1630. Among other mathematical monuments he published a little work on sunspots, which he entitled *Austriaca sidera heliocyclia, astronomicis hypothesibus illigata*.

Carolus Piso [Carolus Piſo], a Frenchman, wrote a *Speculum cometae* [ſpeculum Cometæ] in 1619.

Cassiodorus [Caſſiodorus], a man of consular rank and *clarissimus* under the Emperor Anastasius and King Theodoric, flourished from 490 to 528. When Theodoric was dead, he became a monk of the monastery of Classe [Claſſenſis Cęnobij], and wrote a few things on astronomy and on the ecclesiastical computus.

Censorinus [Cenſorinus], a Roman, wrote *On the Birthday* [de die Natali], addressed to Q. Caerellius [Q. Cærellium], in the year 1014 from the first Olympiad, that is, of Christ 238, as is had from his own chapter 8. In that little work, full of learning, some things bear on the harmonic system of the heavens, and more on the reckoning of time.

Cepheus, King of the Ethiopians, was translated into the heavens with his wife Cassiopeia and his daughter Andromeda because he was a distinguished observer of the stars, as stated in Lucian, *On Astrology*, and in Tycho, volume 1 of the *Progymnasmata*, p. 309. He flourished in 1345 before Christ.

Claudius Ptolemy [Claudius Ptolemæus]: see under the letter P, Ptolemy [Ptolemæum].

Cleomedes: there were two. One flourished shortly after Posidonius [Poſſidonium], in the century before Christ, and wrote two books *On the Sphere*,

entitled κυκλικῆς θεωρίας μεθεωρῶν [Greek: kyklikēs theōrias metheōrōn, printed thus for meteōrōn, the usual form of the title, "of the circular theory of the heavenly bodies"], as well as an *Arithmetic* and a *Harmonics*. The other flourished under Theodosius the Younger, about the year 390. But Robert Balfour [Robertus Balforeus], his scholiast, places the former between Posidonius and Ptolemy; although Kepler [Keplerus] and Mästlin [Mœſtlinus] confused the later with the former, and for that reason were rightly charged with parachronism [parachroniſmi] by our Bettini [Bettino], as we shall say in book 5, chapter 5, number 16.

Christianus Molitoris of Klagenfurt [Clagenfurtenſis], a distinguished astronomer, wrote astrological opuscula at Vienna, and died of the plague at Vienna in 1495.

Christian Wurstisen [Chriſtianus Vurſtiſius] of Basel [Baſileenſis], professor of mathematics in the Academy of Zurich [in Academia Tigurina], wrote in 1568 learned *Questions on the Theoricae of Peuerbach* [quæſtiones in Theoricas Purbachij].

Longomontanus [Chriſtianus Seuerini LONGOMONTANVS], a Dane, Tycho's companion and helper in restoring astronomy, and professor of mathematics in the University of Copenhagen [Academia Hafnienſi]. For eight whole years at Uraniborg [Vraniburgi] he not only took part in Tycho's astronomical labours [laboribus Vranicis] but for the most part directed them; and he helped Tycho above all in reforming the catalogue of the fixed stars [canone Fixarum], and in the theory of the Moon, which was then begun and afterwards completed in Bohemia in 1599 and 1600. His *Astronomia Danica* survives, a truly outstanding work, with an appendix on the new phenomena of the heavens, which he published in 1621.

Christoph Clavius [CHRISTOPHORVS CLAVIVS] of Bamberg, of the Society of Jesus, teacher of Grienberger [Griembergeri] and of Biancani [Blancani], revealed his outstanding aptitude for mathematics at Coimbra [Conimbricæ]. Sent from there to Rome, he so excelled in it that certain very learned Germans were glad to go to Rome for this reason too — to meet Clavius in person, among others — while others said that they would rather be blamed by Clavius than praised by others. Tycho by letter, Magini [Maginus], and very many distinguished mathematicians consulted him like an oracle. And Father Petrus Antonius Rauizza, my teacher in theology, used to assure me, when he was urging me to cultivate mathematics as well, that Clavius had brought our Society no less authority and distinction in this science than Suárez [Suario] had in theology. Finally, he alone out of all was chosen by the Supreme Pontiff Gregory XIII to expound the reform of the calendar according to the form of Lilio [iuxta Lilianam formam], which he did most brilliantly; and he defended it admirably against Mästlin [Mœſtlinum] and Joseph Justus Scaliger [Ioſephum Scaligerum]. What he wrote on the *Sphere* of Sacrobosco, on the *Sphaerica* of Theodosius, on the astrolabe, on gnomonics, on arithmetic and practical geometry, on algebra, on the doctrine of triangles, and in other works, is, for

its exceptional clarity joined with solid learning, brighter than the Sun — so that this man's authority has no need of our eulogy. There was in him no less humility of mind and sincerity of most religious conduct than learning. He died at Rome in 1612, on February 5, after midnight, aged 75.

Christoph Grienberger [Chriſtophorus Griembergerus], of the Society of Jesus, born at Hall [Oenohalæ] in the Tyrol, a most deserving pupil of Clavius and afterwards his successor in the Roman chair, contributed much not only to the astrolabe and to sundials, on the testimony of Clavius himself, but also to trigonometry and to astronomy — above all by compiling a catalogue of the fixed stars [Catalogo Affixarum] and comparing their old longitudes and latitudes with the new. He died at Rome on 1636 March 11. He was a man of wonderful candour, gentleness, obedience and humility.

Columbus [Chriſtophorus Columbus], a Ligurian, is called by Biancani in the *Chronologia mathematicorum* the prince of the more recent Argonauts, and is said to have discovered the New World relying on astronomy and geography. Certainly, by threatening the barbarians with an eclipse of the Moon on 1493 October 22, he obtained provisions from them.

Christoph Scheiner [CHRISTOPHORVS SCHEINERVS], of the Society of Jesus, a Swabian, native of Mindelheim [domo Mundelheimenſis], professed the sacred language and mathematics at Freiburg and Ingolstadt. Then, at Rome, for some years he observed with particular care the solar spots that he had earlier discovered at Ingolstadt. Finally, as rector of the College of Neisse [Collegij Niſſenſis] in Silesia, he died in 1650. His more notable works are the *Oculus, sive fundamentum opticum*, the *Sol ellipticus*, the *Disquisitiones mathematicae*, the *Apelles post tabulam*, and the *Rosa Ursina*, in which he wrote on the spots of the Sun so exquisitely and learnedly that Descartes [Renatus de Cartes] and Hevelius [Heuelius] have testified that nothing more excellent could be done in that kind, or be expected from anyone else.

Christoph Rothmann [Chriſtophorus Rothmannus], mathematician of William, Landgrave of Hesse [Vuilhelmi Lantgrauij Haſſiæ], observed the stars at Kassel [Caſſellis] at the same time as Tycho at Uraniborg. In the first volume of Tycho's *Epistolae* there survive many letters of his to Tycho, and of Tycho to him, containing astronomical controversies, besides what he wrote on comets. He died in 1592.

Cecco d'Ascoli [Cichus Aſculanus] wrote on the *Sphere* of Sacrobosco, and taught astronomy publicly at Bologna from 1322 to 1325. He was a most acute physician, philosopher and astrologer — and, as was commonly reported, a necromancer and suspected of heresy — and was burned at Florence in 1328, aged 70. But Sebastiano Antonelli of Ascoli [Sebaſtianus Antonellus Aſculanus], in his *Apology for Cecco*, tries to show that he was unjustly put to death through the envy of a rival.

Cleostratus of Tenedos [Cleoſtratus Tenedius] divided the zodiac into 12 signs, according to Pliny the Elder, book 2, chapter 8, and instituted the octaeteris [Octaeterida] in the 58th Olympiad, that is, in 548 before Christ.

Columella, by forename Moderatus [Moderatus prænomine], in book 9 of *On Agriculture* [de re ruſtica], a calen- dar [darium] of astrology, with prognostics; and he lived about the year 45 of Christ.

Conon, geometer and astronomer, lived under Ptolemy Philadelphus. He invented the story that the Lock of Berenice [Comam Berenices], wife of Ptolemy, had been carried up into heaven, bringing in a new star under her name. He wrote six books on astrology and was a contemporary of Archimedes, who in his book on the quadrature of the parabola laments his death; he therefore flourished about 260 before Christ. Of him Virgil, Eclogue 3: *"In the middle, two signs: Conon, etc."* [*In medio duo ſigna: Conon &c.*]. As Seneca says (*Natural Questions* 7, ch. 3), he collected the eclipses observed by the Chaldaeans.

Cornelius Gemma [Cornelius Gemma Louanienſis] of Leuven, son of Gemma Frisius [Gemmę Friſij], wrote on the new star of 1572; on which see Tycho, *Progymnasmata* vol. 1, p. 553.

Cornelio Frangipane [Cornelius Frangipanus] wrote on the new star of 1572; on which see Tycho's critique, *Progymnasmata* vol. 1, from p. 741.

Cyprián Lvovický [Cyprianus Leouitius], sprung from the Bohemian family of Leowitz [ex Leouitia Bohemorum familia], published astronomical observations together with an astrological judgement on the new star of 1572; on these Tycho should be consulted, *Progymnasmata* vol. 1, p. 705. He is said to have been born in 1524, on July 8, at $21^{h}\ 52^{m}$.

St Cyril of Alexandria [S. Cyrillus Alexandrinus] was made bishop in the year 412 before Christ [ante Chriſtum] [recte: after Christ], and died in 444 on June 9 [5. idus Iunij]. In 437 he began his Paschal cycle of 104 years.

D

Daniele Barbaro [Daniel Barbarus Aquileienſis Epiſcopus], bishop of Aquileia: how knowledgeable he was in astronomy is shown by his most learned commentaries on Vitruvius, above all on Book 9 and toward the end of Book 10, which he published in 1576.

Daniel Santbech of Nijmegen [Nouiomagus] published in 1561 excellent *Problemata Astronomica et Geometrica*, divided into seven sections.

David Fabricius, a Frisian astronomer, wrote prognostications for the years 1615, 1616 and 1617, on which see Kepler in his *Ephemerides*, from p. 16.

David Origanus of Glatz in Silesia [Glacenſis ex Sileſia], professor of mathematics at Frankfurt on the Oder [Francofurti ad Viadrum], constructed new ephemerides from 1595 to 1654, prefacing volume 1 with a very full introduction setting out the chronological, astronomical and astrological foundations. But because of the rashness of his judicial astrology and his assertion of the motion of the Earth, he is listed among the condemned authors.

Democritus of Abdera [Democritus Abderita], or of Miletus, was a contemporary of Anaxagoras. He wrote on the Sun and Moon, on the Great Year, and on astronomy, with a parapegma of prognostics; and he was one year older than Socrates.

Dionysius the Astronomer [Dionyſius Aſtronomus], in Ptolemy, flourished in the year 463 of Nabonassar, that is, 285 before Christ.

Dionysius Afer, surnamed the Geographer, described the situation of the world in a Greek poem, which Priscian translated or imitated. Augustus sent him ahead of his son Gaius, who was being dispatched to the East, to write commentaries on each of those provinces.

St Dionysius the Areopagite [S. DIONYSIVS Areopagita] will be shown by us to have been skilled in astronomy, in Book 5, ch. 18, where we treat of the eclipse at the death of CHRIST, which by its darkness lit his way to the splendour of the Gospel.

Dionysius Exiguus — so surnamed, but great — an abbot, came from Syria to Rome and flourished there in the year of Christ 526. He introduced the cycle of 532 years, and named the years not from the beginning of the most savage tyrant Diocletian but from the Incarnation of the Saviour, and began that reckoning; from him the epoch Christians commonly use is called the Dionysian.

Denis Pétau [DIONYSIVS PETAVIVS Aurelianenſis] of Orléans, of the Society of Jesus, the Varro of our age and most learned in every field, has deserved best of all of theology, chronology and the whole reckoning of times, and of astronomy, as his two volumes *De doctrina temporum*, the *Rationarium temporum*, the *Uranologion*, and the work that has most recently seen the light, the *Theology of the Fathers* [Theologia Patrum], show. And as he is wonderfully accomplished in Greek and Latin letters for humane learning, so through his humanity, modesty and gentleness of manner he lives at Paris most welcome to all, both of our Society and outside it, still in this very year 1651, in his 68th year.

Domenico Maria Novara [Dominicus Maria Nouara Ferrarienſis] of Ferrara, born in 1464, was a distinguished professor of astronomy at Bologna and the teacher of Copernicus. He taught it from 1484 to 1514, and by the words and examples of his own observations spurred his pupils to restore astronomy. He is said to have observed the greatest declination of the Sun as 23° 29′. He died at Bologna aged 50, in 1514 on September 1 [Kalendis Sept.], and was buried in the church of the Friars of Santa Maria Annunziata, where his epitaph is read.

E

Egmundus, an astrologer, lived in the year of Christ 846.

Ignazio Danti [Egnatius Dantes] of Perugia, of the Order of Preachers, public Master of Mathematics in the University of Bologna, flourished in 1576, when he erected the great gnomon in the church of San Petronio for observing the declination of the Sun, as will be described in Book 3, ch. 14. He wrote on the use and construction of the astrolabe, and reduced astronomy together with the other mathematical disciplines to a compendium of tables. At Florence too he erected equinoctial armillae [Armillas Æquinoctiales] on the wall of Santa

Maria Novella for observing the equinox, with the support of Grand Duke Cosimo [Coſmo M. Duce].

Elias Camerarius, professor of mathematics at Frankfurt on the Oder, published a work on the new star of 1572; for Tycho's critique of it read *Progymnasmata* vol. 1, p. 692.

Élie Vinet [Elias Vinctus Santon] [recte: Vinetus, per the book's errata] of Saintes added scholia to the *Sphere* of Sacrobosco in 1554.

Emmanuel Maignan [Emmanuel Maignam], of the Order of St Francis of Paola, recently published an excellent work, *Perspectiva Horaria*, in Books 1 and 4 of which he treated much on refractions that is worth knowing and useful to astronomers.

Manuel Dias [Emmanuel Diaz] of the Society of JESUS observed the comets of 1618 at Cochin in India, and published a treatise against those who hold that comets are sublunary and elementary.

Endymion of Latmus [Endymion Latmius] was so captivated by the contemplation of the Moon, as Pliny the Elder reports (book 2, ch. 9), that he was held to be her lover. He flourished about 1445 before Christ. He is believed to have been the first to observe the course of the Moon, on Mount Latmus in Caria, for 30 whole years — just as Phaethon, and Daedalus and Icarus, observed the course of the Sun, but too inquisitively, and Prometheus, all of whom the fables pretended were punished for their excessive curiosity in searching out heavenly things.

Epigenes, on whom see Seneca in his inquiry on comets, drew astronomy from the Chaldaeans together with Apollonius of Myndus.

Erasmus Oswald Schreckenfuchs [Eraſmus Oſualdus Schreckenfrichſius] of Austria, pupil of Heinrich Glarean [Henrici Glareani], but an author condemned by the sacred censors of the Index, was professor of mathematics at Freiburg, and published *Primi Mobilis Problemata* in 1576 and commentaries on Peuerbach's *Theoricae* [Theoricas Purbachij].

Erasmus Reinhold [ERASMVS REINHOLDVS Salueldienſis] of Saalfeld in Thuringia, son of Johannes Reinhold [Ioannis Reinholdi], pupil of Jacob Milich [Iacobi Milichij], and professor of mathematics at the University of Wittenberg. Johannes Garcaeus [Io. Garcęus], Erasmus's pupil, in his *Methodus Astrologiae*, pp. 38, 126 and 323, places his nativity [Geneſim] in 1511 October 21 at $13^{h}\ 30^{m}$ after noon, under a pole altitude of 50° 46′, with Virgo 28° 29′ ascending, and with a platic conjunction of the Moon with Mercury, which in any sign makes men gifted. But because the luminaries were conjoined in the third house, and in Scorpio — that is, in the house and term of Mars, who was lord of the horoscope — and Saturn was in the same house, the same Garcaeus testifies (pp. 39 and 325) that he died of phthisis [φθίσι], or consumption, in 1553 on February 19, aged 42, that is, in his sixth climacteric. Consequently in Giuntini's *Calendarium Astrologicum*, where he is given as born in 1521 on October 22, in the morning, half an hour before the second hour of the night, the error must be corrected and the year 1511 restored. Erasmus

had published most learned scholia on the *Theoricae Planetarum* in 1542; then in 1551, on July 18, he dedicated the Prutenic Tables to Albert, Margrave [?] of Brandenburg [Alberto Marc[?] i Brandeburgico], Duke of Prussia, of Stettin and of Pomerania — namely to his most generous patron [Mecenati] — tables in the building and correcting of which, and of Copernicus's numbers, he had spent seven whole years. Finally, in 1553, which was his last, he published the *Tabulae Directionum* with the *Canon fecundus* [Canone fecundo], that is, of tangents, and a complete table of oblique ascensions, and new tables of the climates with shadows. He was also at work on confirming and illustrating chronology from eclipses and great conjunctions; but having undertaken to advance the *Canons* of Regiomontanus, he prolonged his own life beyond Regiomontanus's by no more than one year — for Regiomontanus died in his 41st year, as we shall say below. An epicedium for Erasmus, addressed to his surviving father, was written in elegant verse by Villebrochius of Danzig [Villebrochius Dantiſcanus], a pupil of Erasmus.

Erasmus Reinhold the Younger [Eraſmus Reinholdus iunior], son of the Erasmus above, a mathematician and physician of no common sort, as Tycho testifies of him in *Progymnasmata* vol. 1, p. 699. In 1574 he wrote on the new star of 1572; on this writing see Tycho's judgement in the same place.

ERATOSTHENES of Cyrene was born, as Suidas says, in the 126th Olympiad, that is, 276 before Christ, and in his 80th year died of starvation; he was therefore a contemporary of Archi- medes, and he flourished under the Ptolemies Euergetes, Philopator and Epiphanes. He was librarian to Euergetes, that is, prefect of the Alexandrian library, and was called by some "Plato the Younger" [Plato minor], by others "Beta", because he held second place in every kind of knowledge. He observed the greatest declination of the Sun as 23° 51′, whose fallacy we shall discuss in Book 3, ch. 27; on the way in which he measured the circumference of the Earth, see Book 2, ch. 7. Of his poetic, historical, philosophical and mathematical writings nothing survives except a book *On the Zones*, or the arithmetical sieve in Nicomachus, certain harmonic matters in Ptolemy, and the letter to King Ptolemy on the duplication of the cube, with an epigram, and the mesolabe for doubling the cube; these survive in Eutocius on Book 2 of Archimedes' *On the Sphere*, and in Pierre de La Ramée [Petrum Ramum], *Scholae Mathematicae*, book 1. Eratosthenes was so pleased with himself over the duplication of the cube that he consecrated a tablet expounding his splendid invention in the temple of the gods — an ἀνάθημα [Greek: ἀνάθημα (anathēma, "votive offering")] of his genius, forever. Boulliau [Bullialdus] says that the armillae in the portico at Alexandria were set up at his urging.

EUCLID [EVCLIDES Megarenſis] of Megara was a hearer of Socrates, as Cicero says (*Academica*, book 2), and a contemporary of Plato. For, as Laertius relates in his *Socrates*, when his teacher saw him inclined toward lawsuits, he rebuked him with the words: "O Euclid, you will be able to deal with sophists, but not with men." Plato, as Valerius Maximus reports (book 8, ch. 13), sent

the contractors for the sacred altar of Delos to him, to learn its dimensions and form from him. But if we listen to Proclus, Euclid the Geometer flourished as late as Ptolemy, first king of Egypt, who began to reign in the 18th year after the death of Alexander the Great and died in the 124th Olympiad, or 284 before Christ. Accordingly La Ramée (*Scholae Mathematicae*, book 1) and our Clavius, in his prolegomena to Euclid, judge that the Euclid whose geometrical *Elements* survive is a different man from the Megarian — chiefly on this argument, that Diogenes Laertius, speaking of the works of Euclid of Megara, nowhere mentions so outstanding a monument as the books of the *Elements*, nor the *Optics*, *Catoptrics*, *Music*, *Data* and *Phaenomena*. Grant that Euclid, as Proclus held, collected these elements from others, above all from Eudoxus and Theaetetus, adding certain demonstrations where they were lacking: yet this itself is great praise, says La Ramée — to complete what was begun, to make certain what was uncertain, but above all to put in order what was undigested. Nor does Proclus detract from him in calling him not the inventor of the *Elements* but their στοιχειώτης [Greek: στοιχειώτης (stoicheiōtēs, "compiler of elements")], that is, their constructor and demonstrator; by that title many other ancient στοιχειῶται [Greek: στοιχειῶται (stoicheiōtai, "compilers of elements")] have won the palm, and will win it among sensible men in time to come.

EUCTEMON, observer of the solstices with Meton in the 108th year before the death of Alexander the Great, that is, 432 before Christ, is celebrated by Ptolemy (*Almagest*, book 3, ch. 3).

Eudemus, as Theon of Smyrna relates in his *Astronomica*, wrote a *History of Astrology*, which Laertius also confirms in his *Thales*.

EUDOXUS of Cnidus [EVDOXVS Gnidius], as Diogenes Laertius relates in his life, was a pupil of Archytas in geometry, of Philistion the Sicilian in medicine, and of Plato in philosophy. In his 23rd year, though hard pressed by poverty, he moved to Athens, drawn by the fame of the Socratics. From there he went to Egypt, with letters of recommendation from Agesilaus, to King Nectanebus; having stayed there 16 months with chin and eyebrows shaved, he returned to Athens with many disciples, to the vexation of Plato, who had at first sent him away because of his poverty. Then he returned to his homeland, where he was received most warmly. He wrote *On the World*, *De caelo*, and *Phaenomena*, or astrological fasti, with prognostics. Laertius adds that when he was at Memphis he consulted Apis about his fate, and when his garment was licked by the calf Apis, the priests interpreted this to mean that he would be ἔνδοξον μὲν, ἀλλ' ὀλιγοχρόνιον [Greek: ἔνδοξον μὲν, ἀλλ' ὀλιγοχρόνιον (endoxon men, all' oligochronion, "famous, but short-lived")] — that is, famous in name but brief of life; and for this reason he was called Endoxus instead of Eudoxus. And on the authority of another Eudoxus, a physician of Cnidus, he affirms that he flourished about the 103rd Olympiad and died aged 53. Strabo (book 2) says that a watchtower used to be shown at Cnidus from which Eudoxus observed Canopus. If, then, he flourished in Olympiad 103, that is, in 368 before Christ, and was then more than 23 years old,

and lived no more than 53 years, he cannot have been born before 391 before Christ, the second year of Olympiad 97. Eusebius in his *Chronicle* therefore seems to need correcting, when he says at Olympiad 89, year 3, *"Eudoxus of Cnidus is held famous"* [*Eudoxus Gnidius clarus habetur*], and at Olympiad 97, year 1, *"Eudoxus the astrologer is recognized"* [*Eudoxus Aſtrologus agnoſcitur*]. Although Pétau seems to follow him in *De doctrina temporum*, book 2, ch. 6 — where he refutes Scaliger, who among other things ascribes to Eudoxus an age of 103 years — yet in the *Rationarium temporum*, book 3, ch. 13, he follows Diogenes, and corrects what he had said, following Eusebius, in book 13 of *De doctrina temporum*, p. 578. Hipparchus of Bithynia, moreover, expounded the *Phaenomena* of Eudoxus at length; these, together with the octaeteris he instituted [Octieteni-de] [recte: Octieteride], were valued so highly that Caesar in Lucan boasts:

Nec meus Eudoxi vincetur faſtibus Annus. ["Nor shall my year be outdone by the fasti of Eudoxus."]

On the system of the heavens according to Eudoxus's view, see Aristotle, *Metaphysics* 12, text 47. Tully [Tullius], in *On Divination*, says that Eudoxus argued against the predictions of the Chaldaeans.

Eutocius of Ascalon [Eutocius Aſcalonita], later than Theon and Pappus, wrote on the *Conics* of Apollonius and on Archimedes' *On the Sphere and Cylinder*, and flourished toward the end of the 5th century of Christ, about 480.

F

Federico Commandino [Federicus Commandinus Vrbinas] of Urbino rendered select monuments of the Greeks into Latin and expounded them — namely the *Conics* of Apollonius, the works of Archimedes, Aristarchus of Samos, Pappus of Alexandria, the *Analemma* of Ptolemy, etc. He flourished in 1560.

Fortunio Liceti [Fortunius Licetus Genuenſis] of Genoa wrote on comets and on the light of the Moon, and much else besides, about 1640.

François d'Aguilon [Franciſcus Aguillonius] of the Society of JESUS, of Brussels, taught philosophy at Douai and theology at Antwerp, and was the first to introduce mathematical studies into the Belgian provinces. He was a man outstanding in prudence and patience, and died a holy death at Antwerp, where he had been rector, in 1617, aged 50. How much he indirectly helped astronomy is evident from his six books of *Optics*.

Francesco Barozzi [Franciſcus Barocius], a Venetian patrician, flourished in 1570. Besides other notable mathematical works he wrote four books of *Cosmography*, in whose preface he exposed 84 errors of Sacrobosco [Ioannis de Sacroboſco] and his followers.

Francesco Giuntini [Franciſcus Iunctinus], doctor of sacred theology, a Florentine, was born in 1523 on March 7 at $13^{h}\ 20^{m}$ after noon, and in his 51st year published a *Speculum Astrologicum*, packed with astronomical tables and manifold learning, and also a *Sphere*.

Francesco Maurolico [FRANCISCVS MAVROLYCVS Meſſanenſis Abbas], abbot, of Messina, the brightest light of Sicily, shone from 1540, when he published his *Cosmographia*, to the year of Christ 1572, when the new star blazed out in Cassiopeia, on which he also set out his judgement in writing. He was a man of wonderful candour and most polished genius, and deeply learned in the Greek language. To him we owe the canon of secants, or *Tabula Benefica*; dialogues on cosmography; the *Sphaerica* of Theodosius and of Menelaus; the construction of the astrolabe and of other instruments; and much else, partly published, partly to have been published had he lived. We have a long catalogue of these at the beginning of the *Cosmographia* and at the end of the *Opuscula*.

Franciscus Montebrunus, a Genoese patrician, doctor of both laws [I.V.D.], whose friendship I have been granted to enjoy here at Bologna, computed excellent ephemerides for its meridian from the tables of Lansbergen [Lansbergij], from 1641 to 1660. He died at Venice in 1649.

François Viète [Franciſcus Vieta Fonteneenſis] of Fontenay, a distinguished ornament of French mathematics, excelled toward the end of the preceding century, as is evident from the *Cyclometrica*, the *Pseudomesolabum*, the *Apollonius Gallus*, the *Canon Mathematicus* and other monuments of a most acute and profound genius. But neither Clavius [Clauio] nor more judicious men approved of his being so pleased [ita tibi placuerit] [recte: sibi] with that period of his of 3,400 years that, for this very reason, he offered Clement VIII in 1600 a new form of the calendar, asserting that not the Lilian calendar but his own was truly Gregorian. His *Harmonicum caeleste* is named by Boulliau in the Prolegomena to the *Astronomia Philolaica*, p. 21, who says that it is in the hands of Marin Mersenne [Marini Merſennij].

Francisco Sarzosa [Frãciſcus Sarzoſus Cellanus Aragoneus] of Cella in Aragon wrote two books on the *Equatorium of the Planets*, built on the Alphonsine hypothesis, in 1525.

G

GALILEO GALILEI [GALILÆVS GALILÆVS Florentinus], a Florentine, a geometer of sagacious genius and very knowledgeable in astronomy, would have been greater still had he been content to advance the opinion of the motion of the Earth as a mere hypothesis in his *Dialogues* on the system of the world, and had he obeyed the sacred censors once he had been warned by Cardinal Bellarmine — whom, however, he did obey afterwards, when already 70 years old, publicly abjuring that opinion. To him we certainly owe, if not the invention, then surely the perfection and use of the telescope, and many other things hidden from the ancients that were discovered by means of it. He wrote on the spots of the Sun, and on the other things detected through the optic tube, in the *Sidereus Nuncius*, and other short works.

Caspar Peucer the Elder [Gaſpar Peucerus Senior], in medicine and mathematics most learned [ſcientiſſi-lmus], was born at Bautzen [Budiſſinæ] in Germany in 1525 on January 5 at 12^{h} after noon, and had a son of the same

name, on whose skill in astronomy see what Tycho reports in *Progymnasmata* vol. 1, p. 633.

Gazulus of Ragusa [Gazulus Raguſeus], an astrologer, flourished in 1438.

Geber of Seville [Geber Hiſpalenſis], son of Attla [filius Attla] [recte: Affla, per the book's errata], expounds and corrects Ptolemy in nine books on astronomy, written in Arabic but rendered into Latin by Gerard of Cremona [Gerardum Cremonenſem; Gerardo da Cremona]. He was, however, later than Arzachel, whom he names and ranks after Albategnius; and Arzachel lived in the 12th century of Christ. Geber therefore did not live in the 9th century, whatever Blancanus [Giuseppe Biancani] may say in his *Chronologia Mathematicorum* [Chronol. Math.].

Gellius Sasserides [Gellius Saſſerides Hafnianus] of Copenhagen was one of those who assisted Tycho in his observing; his letters survive in Magini's *Directiones*, from p. 78.

Geminus, in Greek γεμῖνος [Greek: γεμῖνος (geminos, "Geminus")], an ancient author, was a Rhodian and lived in the time of Sulla and Cicero, about 83 years before Christ. He wrote a splendid *Isagoge* to the *Phaenomena*, or *Elements of Astronomy*, and is praised more than once by Proclus, whose *Sphere* seems to have been compiled from Geminus's chapters 2, 3, 4 and 12. Edo Hildericus translated him from Greek into Latin; but our Pétau corrected him from two copies and published him at the beginning of his *Uranologion*, where he places him before Christ. Biancani, it is true, in his *Chronologia Mathematicorum*, reports two men named Geminus: one older, 300 years and more before Christ; the other, namely the Rhodian, whom he asserts to have been the teacher of Proclus Diadochus. Proclus, moreover, somewhere ranks him above Euclid. On his date see Pétau, *De doctrina temporum*, book 2, ch. 6, and his notes on Geminus in the *Uranologion* — although Scaliger makes him 120 years later than the Eudoxian solstice.

GEMMA FRISIUS [GEMMA REGNERI FRISIVS], physician and mathematician of Leuven, began to flourish from 1534 onward. Among other things he published books on the use of the globe, on the use of the astronomical ring, on the principles of astronomy and cosmography, on the division of the world, on islands and lands recently discovered, and on the Catholic Astrolabe. He died in 1555, as Genebrardus [Gilbert Génébrard] says under Marcellus II [in Marcello 2.].

Georges Fournier [Georgius Furnerius Gallus], a Frenchman of the Society of JESUS, formerly a most celebrated professor of mathematics at Paris, published in 1643 an outstanding work of *Hydrography*, from which a learning deeper than the sea itself may be drawn — the sea that he made illustrious by so noble a monument. Of his skill in observing phenomena, especially eclipses, it will be shown in its place, also from his letters to me.

Georg Henisch [Georgius Henischius], at the beginning of this century, illustrated the *Sphere* of Proclus with excellent commentaries.

Georg Hartmann [Georgius Hartmannus Norimbergenſis] of Nuremberg, a distinguished instrument-maker [Organopœius], was born in 1489 on February 9, after sunset.

Georg Joachim Rheticus [Georgius Ioachimus Rheticus] was born in 1514 on February 15 at 13^{h} 30^{m} after noon. A follower of Copernicus, he set out his hypotheses in a long *Narratio*, and on his foundations published at Leipzig an ephemeris for the year 1550; there, as also at Wittenberg, he professed mathematics. He also constructed a *Magnus Canon*. But when he despaired of mastering the motion of Mars, he is said to have invoked his Genius, who seized him by the hair, dashed his head against the ceiling above, then flung him down to the floor and mocked him bitterly, saying *"Hic est motus Martis"* ["This is the motion of Mars"] — though Kepler, in the dedicatory epistle of the *Commentaria* on Mars, thinks rather that he rose up raving in a disordered state of mind and dashed his head against the wall. Whatever the truth of it, he himself is numbered among the condemned authors for his astrological vanity and for other reasons.

Giorgio Polacco [Georgius Polaccus Venetus], a Venetian, published in 1644 an *Anticopernicus Catholicus*, in which he defended the rest of the Earth and the motion of the Sun.

Georg Peuerbach [GEORGIVS PVRBACHIVS], born at Peuerbach [Peurbach] on the borders of Bavaria and Austria in 1423 on May 30 at 3^{h} 5^{m} after noon, under a pole altitude of 58° [gr. 58.], died in 1462 on April 8 [ſexto Idus April.], not yet forty years old. He had taught mathematics at Ferrara in Italy and at Vienna; and after the *Theoricae Planetarum*, which he had constructed partly on Ptolemy's principles and partly on those of the Alphonsines, and the *Tables of Eclipses*, he was undertaking the restoration of astronomy by astronomical observations. But as he was dying he bequeathed to his pupil Regiomontanus [Ioanni Regiomontano], as by testament, the task of completing the *Epitome of the Almagest* that he had begun, and he ordered this epitaph to be cut for him in the Cathedral Church at Vienna, where he is buried:

Extinctum dulces quidnam me ſletis amici? Fata vocant: Lacheſis ſic ſua fila trahit. Deſtituit terras animus, cælumque reuiſit: Quæ ſemper coluit liber & aſtra colat.

["Why, sweet friends, do you weep for me, now I am gone? / The Fates call: so Lachesis draws her threads. / My spirit has left the earth and returns to heaven: / free, let it dwell among the stars it always cherished."]

Georg Schönberger [Georgius Schombergerus Oenipontanus] of the Society of JESUS, of Innsbruck, taught moral philosophy and mathematics, and in 1626 published at Freiburg im Breisgau [Friburgi Briſgoiæ] the *Sol illustratus*, in which he defends all the accidents of the Sun, the solar stars, and the fluidity of the heavens; and also a *Centuria* of optical problems. In 1622, moreover, he had published the demonstration and construction of clocks indicating all the hours by a direct ray, by a ray refracted in water, by one reflected in a mirror, and by the magnet alone.

Georgios Trapezountios [Georgius Trapezuntius] was born in Crete in 1396 on April 4 at 12^{h} 20^{m}, under a pole altitude of 36°. He flourished in 1435, when he was Apostolic Secretary. He translated Ptolemy from Greek into Latin; he also wrote on the hundred aphorisms of Ptolemy [Aphoriſmos centum Ptolemæi], on antiscia, and on the falsity of astrological judgements. In extreme old age he forgot all his learning, and he died in 1486, as Génébrard has it under Sixtus IV.

Gerard of Cremona [Gerardus Cremonenſis] translated Geber from Arabic into Latin and expounded the *Theoricae Planetarum*; but Regiomontanus [Io. Regiomontanus] attacked them most sharply in a dialogue devoted to them. He flourished in the 14th century of Christ.

Gerardus Mercator of Rupelmonde [Rupelmundanus], father and son, leading cosmographers of the preceding century, flourished in the year of the Lord 1570.

Germanicus Caesar rendered Aratus's *Phaenomena* into Latin, although Julius Firmicus, in book 2 of his *Astronomicon*, attributes it to Julius Caesar. He was the son of Drusus and was adopted by Tiberius, and he died at Antioch in his 34th year, not without suspicion of poison.

Govaert Wendelen [GOTıFREDVS VVENDELINVS Luminarcanus ex Belgio], from Belgium, a singular astronomer, published in 1644 the *Idea of the Atlantic Tables*, based on very many eclipses observed by himself and by others; and in the preceding year he had published the *Lamp of the Celestial Mysteries* [arcanorum cæleſtium lampadem]. From these small works — great in merit, not in bulk — and from his letters to me, the man's remarkable learning can be discerned, together with the elegance of his style; and I regret that he has been drawn away to other studies and business from the observation of the Moon, to which he devoted nearly all his time. He was born in 1580 on June 6, and was still observing eclipses in 1643, namely in his climacteric year.

Guido Bonatti [Guido Bonatus Foroliuienſis] of Forlì, an astrologer, was living in the year of the Lord 1282.

Guidobaldo del Monte [Guidubaldus ſeu Guidus Vbaldus Marchio ex nobiliſſima familia de Monte], Marquis, of the most noble family del Monte, published, among other notable monuments of mathematics, an *Astrolabium* and *Problemata Astronomica*, and flourished toward the end of the 15th century of Christ [ſub finem ſæculi Chriſti 15.].

Willem Blaeu [Gulielmus Blaeu Hollandus], a Hollander, in imitation of Mercator's *Atlas*, illustrated the whole of geography with very many and most accurate maps, and flourished about 1640.

Gulielmus Ianſonius [Gulielmus Ianſonius Alcmarienſis] of Alkmaar, most celebrated for his geographical maps, flourished in 1608 and after.

William, Landgrave of Hesse [Gulielmus Lantgrauius Haſſiæ], famous for his astronomical observations begun at Kassel [Caſſellis] and kept up for very many years — which Snellius [Willebrord Snel] published, and Tycho in part in his volume of *Epistles* and in part in *Progymnasmata* vol. 2, where he treats

of comets — gave princes a notable example of promoting and enlarging this science. He flourished in 1570 and after.

Guillaume Postel [Gulielmus Poſtellus Barentonius] of Barenton in Normandy, whose geographical volumes Barozzi [Baroccius] mentions, and whose writing on the star of 1572 Tycho subjected to his critique in *Progymnasmata* vol. 1, p. 716, was distinguished not only in mathematics but in philosophy and medicine, and in his knowledge of very many languages. But because of Cabbalistic trifles and astrological ravings he was expelled once and again from the Society, to which he had vowed himself, by our holy Patriarch Ignatius; he fell into open heresies, for which he was sent to Rome bound in chains. He lived to nearly a hundred, and at last is believed to have recovered his senses and to have died in the Catholic faith. Wonderful, but dreadful, are the things that our Daniello Bartoli [Daniel Bartolus] relates of his talent — unstable, however, and steeped in superstitions — in vol. 1 of the *History of the Society of JESUS*, book 3, from p. 371.

Wilhelm Schickard [Gulielmus Schikardus], at Tübingen professor of oriental languages and of astronomy, besides the *Astroscopium*, also published some eclipses he had observed himself. He flourished about 1630.

H

Haly aben Ragel was born in 956 on February 12 at 10^{h} before noon; he is therefore a different man from the one who is said to have lived in 1200.

Haly aben Rodoan flourished in 1024, if we believe Giuntini [Iunctino] and Sansovino [Sanſouino; Francesco Sansovino].

Harpalus instituted the octaeteris [Octoeteridem], or lunisolar cycle, not long after Cleostratus, as Censorinus teaches; on which see Pétau, *De doctrina temporum*, book 2, ch. 3 and 4.

Hecataeus of Miletus [Hecateus Mileſius], shortly after Thales, first, on the situation of the world published a book, as Proclus attests in his commentary on Euclid.

Helicon of Cyzicus [Helicon Cyzicenus], a friend of Plato, was given a talent of silver by King Dionysius for having foretold an eclipse of the Sun to him, as Biancani relates in his *Chronologia*. I think it was the eclipse of the year 404 before Christ, of which we shall speak in the Catalogue of Eclipses.

Henricus Baten of Mechelen [Mechlinienſis] wrote in 1290 on the errors of the Alphonsine tables [tabularum Alphonſi].

Henry Briggs [Henricus Briggius], Savilian professor of geometry in the University of Oxford, altered Napier's logarithms [Neperianos Logarithmos], with Napier's agreement, and elucidated them; on this account alone he deserved outstandingly well of astronomical computation. He flourished in the year of the Lord 1615.

Heinrich Glarean [Henricus Glareanus], geographer, chronologer, musician and physician, a native of the canton of Glarus [ex pago Clarona] in Switzerland, flourished in 1547.

Heinrich von Langenstein [Henricus de Haſſia], a German, died in 1397 on February 11 [3. Idus Februarij] at Vienna, where he had taught theology and astronomy; his skill in astronomy is shown by book 1 of his commentary on Genesis and by his *Theoricae Planetarum* [in Theoricis Planetarum].

Henricus à Lindhout, a physician of Brussels, wrote a *Speculum Astrologiae* in 1508.

Henricus Philippi, of the Society of JESUS, an Ardennais from Saint-Hubert [Arduennas ex S. Huberto], taught philosophy, theology and Sacred Scripture at Graz [Græcij], Vienna and Prague, and in very many works elucidated and corrected the whole of chronology, sacred chronology above all. He was most welcome to Ferdinand and his courtiers, yet took on nothing of the court; he was in the habit of reciting psalms even in his sleep, and was of no common poverty, obedience and patience. He died at Regensburg [Ratiſbonæ] in 1636, on November 30, aged 71.

Heinrich Rantzau [Henricus Ranzouius], viceroy of the Cimbric peninsula [Produx Cimbricus], in 1596, when he was in his 71st year, published a calendar very useful to physicians and astronomers, in which he says he was born in the castle of Steimbrorch in Holstein in 1526, on March 11, at $10^h\ 31^m\ 26^s$. But through his study of judicial astrology he incurred ecclesiastical censures.

Henricus Samerius of Luxembourg, of the Society of JESUS, an outstanding computer of the times from the creation of the world to Christ, died at Luxembourg on January 5, 1610, aged 70.

Hercules Musagetes [Hercules μουσαγητις], that is, the leader of the Muses — so called to distinguish him from the other Hercules — so advanced the doctrine of the celestial sphere that he was said to have succeeded Atlas in holding up the heavens. He flourished in the year 1560 before Christ.

Hermes Trismegistus [Hermes Trimegiſtus], grandson of the elder Mercury, a most ancient astrologer, lived about the year 1480 before Christ, as Giuntini [Iunctinus] places him; but Agostino Ricci, in his treatise on the motion of the eighth sphere [in tract. de oct. ſphæræ motu], says he was older than Ptolemy by 1985 years, so that he flourished in the year 1850 before Christ.

Ermolao Barbaro [Hermolaus Barbarus], a Venetian, patriarch of Aquileia, a man most learned in astronomy as well, was born in 1453 at $12^h\ 45^m$ after noon.

Girolamo Cardano [Hieronymus Cardanus] of Milan was born in 1501, on September 24, at $6^h\ 40^m$ after noon. He flourished above all at Bologna, where he publicly professed medicine and mathematics. As regards astronomy, his writings on Ptolemy and his books *De subtilitate* show well enough what he was; but in many other things he was a suspect author, and was summoned to Rome more than once on grounds of religion, as Alessandro de Angelis says, book 4 [of his work] against astrologers [in Aſtrolog.], ch. 14. He died at Rome in 1576, as Génébrard [Genebrardus] has it under Gregory XIII.

Girolamo Fracastoro [Hieronymus Fracaſtorius] of Verona, poet, physician and distinguished astronomer, published his *Homocentrica* in 1535. To gaze upon

him, Charles V, as he passed along Lake Garda [lacum Benacum], ordered his carriage to stop. He died of apoplexy.

Jerónimo Girava [Hieronymus Giraua] of Tarragona [Tarraconenſis] in 1570 published two books in Spanish, which contain the geography of the whole world, but principally of the New World.

Jerónimo Muñoz [Hieronymus Munoſius], a Spaniard, professor of Hebrew and of mathematics in the University of Valencia, observed the new star of 1572; on which consult Tycho's *Progymnasmata* vol. 1, p. 565.

Girolamo Ruscelli [Hieronymus Ruſſellius] of Ferrara elucidated Ptolemy's *Geography* with notes in the last century.

HIPPARCHUS OF RHODES [HIPPARCHVS RHODIVS], called by some Abrachys, observed the motions of the luminaries and their eclipses, first at Rhodes and then at Alexandria. He collected and weighed the observations of them made long before by the Chaldaeans, corrected the cycle of Callippus, and Eratosthenes' measurement of the Earth's circumference. On the occasion of a new star he observed all the fixed stars and reduced them to a catalogue; and he was at length the first to notice that, besides the motion of the primum mobile [primi Mobilis], they are moved slowly by a motion of their own toward the east about the poles of the ecliptic; and he distinguished the sidereal year from the tropical. He was a man φιλόπονος καί φιλαλιθής [recte: φιλαλήθης] [Greek: philoponos kai philalēthēs, "fond of labour and of truth"], that is, a lover of labour and of truth, as Ptolemy attests (book 3, ch. 2), adding there and in book 7, ch. 1, that his observations seemed to him exquisite. In the same book 7, ch. 2 and 3, Ptolemy reviews Hipparchus' book on the shift of the solstitial and equinoctial points, as also in book 3, ch. 2. Hipparchus' most ancient observation in Ptolemy (book 3, ch. 2) is of the autumnal equinox in the 17th year of the third Callippic period, that is, 162 before Christ. But if the eclipse of the Moon observed at Alexandria in the 54th year of the second period of Callippus, that is, 200 before Christ, was Hipparchus' rather than another's, he would already have begun observing from then as a very young man; for in book 7 of the *Almagest*, ch. 2, he is said to have observed Regulus in the 50th year of the third period of Callippus, that is, 129 before Christ. Astronomy owes Hipparchus so much that it would be unjust to pass over here the double encomium with which Pliny the Elder praised him. For in book 1, ch. 12, after praising Thales and Sulpicius Gallus, he says: *"After them Hipparchus foretold the courses of both luminaries for six hundred years, comprehending the months of the nations, the days and hours, the situations of places and the districts of peoples — time bearing witness — in no other way than as a sharer in the counsels of nature. Mighty men, and above the nature of mortals,"* etc. And in ch. 26: *"Hipparchus, never praised enough, as one than whom no one did more to prove the kinship of the stars with man and that our souls are a part of heaven, detected a new star, and another one born in his own age; and from its motion on the day it shone he was led to doubt whether this happened more often, and whether those stars also move that we think fixed. And he likewise dared a thing presumptuous even for a god — to number*

the stars for posterity and to check off the heavenly bodies by rule, with instruments he devised by which the places and magnitudes of each might be marked, so that from it one might easily discern not only whether they perished or were born, but whether any of them passed by or moved at all, and also whether they grew or diminished — leaving the heaven as an inheritance to all, if anyone should be found to take up that reckoning."

Hipparchus of Bithynia [Hipparchus Bithynus] wrote three books to Aeschrion [ad Æſchrionem] of commentaries on the *Phaenomena* of Aratus and Eudoxus, which our Denis Pétau [Dionyſius Petauius] translated from Greek into Latin and inserted into his *Uranologion* from p. 171. From the many marks of the places in which he records the stars as they were in his time, it is certain that he lived before Christ. For Hipparchus asserts that he observed the last star in the tail of Ursa Minor [poſtremam caudæ vrſę minoris] at a distance of 12° 24′ from the pole of the world, which Marinus of Tyre also affirms of Hipparchus. From this complement of the declination, and from the latitude of this star, which is 66° 2′, and the distance of the pole of the ecliptic from the pole of the equator, 23° 30′, it follows by problem 49 of section 2 of our Book 10 that this star was at Taurus 28° 53′. But we found it, at the completion of the year 1644, at Gemini 23° 38′. It has therefore advanced from Hipparchus to us by 24° 45′, for which, on our motion in longitude, 1780 years are required. Subtracting 1644, there remain 136 years before Christ. Hipparchus of Bithynia was therefore a contemporary of Hipparchus of Rhodes — which was worth knowing, and could not have been investigated by us without the doctrine of triangles and of astronomy.

Orazio Grassi [Horatius Graſſus], of the Society of JESUS, a Ligurian from the city of Savona, long professed mathematics at Rome in our College with great praise. In 1619 he published an astronomical disputation on the three comets of 1618, and the *Libra Astronomica ac Philosophica*, in which he weighs Galileo's opinion on comets. He is, moreover, most expert in architecture [Architecturę, as printed], and a man of great judgement.

Humenus the Egyptian [Humenus Ægyptius] wrote astronomical tables in Arabic in the twelfth century of Christ, which are preserved in the Palatine Library, as Jacob Christmann [Kriſtmannus] reports in his [commentary on] al-Farghānī [in Alfraganum].

Hyginus flourished in the year of Christ 72; his *Poeticum Astronomicum*, on the parts of the world and of the sphere, survives.

Hypatia [HYPATIA], daughter of Theon of Alexandria, flourished under Arcadius and Honorius, and among other things constructed an astronomical canon. Synesius boasts that she was his teacher. But by her eminent learning in mathematics she drew on herself such envy that she was slaughtered by certain Alexandrians, dragged through the whole city and torn to pieces, as Hesychius Illustrius relates in his *Lives of the Philosophers*, and the *Suda* [Suidas]. Her father Theon flourished in the year of Christ 365. But Socrates (book 7, ch. 13) relates that she was killed in 415 by the Alexandrians in a popular riot and

torn apart, because she was thought to have drawn the prefect Orestes away from his friendship with Cyril.

I

Jacques Lefèvre d'Étaples [IAcobus Faber Stapulenſis] wrote a commentary on Sacrobosco's *Sphere*, at the end of which he relates that in the year of Christian salvation 1494, on November 9 [quinto idus Nouembris], he was present at the solemnity at which the body of St Denis [S. Dionyſij], with the bodies of Rusticus and Eleutherius, was brought up [depoſitum fuit] from the crypt in which for

110 years they had lain unexposed. For at that time Charles VIII [Carolus Octauus] had reached the borders of Tuscany with his armies, pressing on toward his Neapolitan expedition; and the kings of France, when they undertook an expedition into foreign provinces, used to order the aforementioned relics of the saints to be exposed.

Jacob Christmann [Iacobus Kriſtmannus] of Johannisberg [Ioannisbergenſis], professor of mathematics and of languages in the University of Heidelberg, besides solar observations wrote in 1590 a commentary on the chronological and astronomical elements of al-Farghānī [Alfragani].

Iacobus Homelius was born in 1528, on July 31, at 15^{h} after noon.

Jacob Milich [Iacobus Milichius], professor of mathematics in the school of Wittenberg [in Schola Vuittembergenſi] and teacher of Erasmus Reinhold, published commentaries on book 2 of Pliny the Elder, which contain much astronomy. He was born in 1501, on February 20, at $12^{h}\ 52^{m}$ after noon, at a pole altitude of 46° 25′.

Jacob van Lansbergen [Iacobus Lanſbergius], a physician, wrote in 1633 an *Apology* for Philips van Lansbergen's [Philippi Lãſbergij] commentaries on the diurnal and annual motion of the Earth, against Froidmont [aduersùs Fromondum].

Johann Adam Schall von Bell [Ioannes Adamus Sciabi], a German of the Society of JESUS, sailed to China [ad Sinas] about the year 1620 with Father Nicolas Trigault [P. Nicolao Trigautio] on his return voyage. He published a treatise on the computation of the lunar eclipse of 1624, and a table of the stars in Chinese.

Johannes Angelus [Ioannes Angelus Bauarus ex Aichem], a Bavarian from Aichach, professor of astronomy at Vienna, died in 1512 on September 29. He wrote ephemerides, prognostications, and a work on the equations of the planets.

Giovanni Antonio Delfino [Io. Antonius Delphinus Caſalenſis è Caſali Maiore] of Casalmaggiore, a Franciscan, wrote in 1559, addressed to Camillo Paleotti [Camillum Palęotum], senator of Bologna, a book on the celestial globes and their motions, full of learning. He was Provincial of the Franciscans and principal Lecturer at Bologna, and died in 1560, aged 54 years, 6 months and 10 days.

GIOVANNI ANTONIO MAGINI [IO. ANTONIVS MAGINVS de Paſqualibus Patauinus] of Padua, professor of mathematics at Padua, published in 1583, in his 28th year, ephemerides derived from the Prutenic Tables for 1581 to 1610, first in Latin and then in Italian, with a very full introduction. Then in 1585 he published *Tabulae secundorum mobilium caelestium*, agreeing with the Prutenic Tables, with the Alphonsine canons added. Afterwards, called in 1588 to Bologna, where he flourished as principal professor of mathematics until his death, he published in 1589 his *Theoricae Planetarum* [Theoricas Planet.]; in 1592 one book on plane triangles and five books on the method of measuring [de Ratione dimetiendi], the fifth of which serves astronomy; in 1596 commentaries on Ptolemy's *Geography* with geographical maps both ancient and modern; in 1604 the tables of the primum mobile [Primi Mobilis] which they call tables of directions; and in 1607 he resumed his ephemerides, for 1610 to 1630, built indeed on the Prutenic Tables, but preceded by a supplement of *Isagogica*, that is, of the reductions of the Sun's motion to the Tychonic, and on the Tychonic equation, with astrological precepts useful for agriculture and navigation, and on the change of the weather. In the same year 1607 he published two books, one on critical days, the other on the legitimate use of astrology in medicine. In 1609 he published the tables of the primum mobile and *Problemata*, and in the same year brought out a description of Italy, which he afterwards enlarged. In 1614 came a supplement to the ephemerides and to the tables of the second movables, containing the Tychonic-Copernican tables with a compendium for calculating eclipses in the Tychonic manner. Finally, in 1620, his son Fabio Magini [Fabius Maginus], in accordance with his father's last will, published the *Italia* with 60 maps, for the most part either constructed or collected by Giovanni Antonio and reduced to a suitable form by twenty years' labour. This distinguished astronomer and cosmographer died at Bologna in 1617, on February 11 — a time he had foreseen from his astrological figure would be fatal to him — and was buried in the portico of the convent of the Dominican Fathers, with an epitaph that Giovanni Antonio Roffeni [Io. Antonius Roffenus] set up to him, adding that he died *"under the hostile aspects of the stars, of the Sun to the body of Mars, which he had foreknown for himself, the Sun running near the opposition [diametrum] of Mars, about the sextile [hexagonum] of Saturn,"* and that he lived 61 years, 7 months, 28 days and 1 hour. Tycho and Kepler had invited him to Germany to build new tables resting on new observations, but his age prevented him and he declined.

Giovanni Antonio Roffeni [Ioannes Antonius Roffenus] of Bologna, a pupil of Giovanni Antonio Magini and a celebrated astrologer, published many astrological discourses. He died at Bologna in the 63rd year of his age and of our salvation 1644. When he had taken to his bed sick with a semitertian fever [ex hemitritęo] and I visited him — for I had formed a friendship with him long before — he declared that he would die of that illness, since his nativity [Geneſim ſuam] and his climacteric year required it; that he was ready for the will of the divine power, and had lived enough and more than enough.

He asked, however, that he might be allowed to open all the hiding places of his conscience to Father Giorgio Giustiniani [P. Georgio Iuſtiniano] of our Society. I promised him this and obtained it at once. And he, knowing that I was already engaged in the labour of the *New Almagest*, out of regard for me bequeathed to our College of Santa Lucia a great part of his books, which have been of great use to me.

Giovanni Battista Benedetti [Io. Baptiſta Benedictus], a Venetian patrician, called from Parma to Turin by the Most Serene Duke of Savoy in 1566, tried after 19 years, that is in 1585, to answer the arithmetical, geometrical, astronomical, optical and musical questions put to him by the Duke and others, by publishing questions and most learned letters. He was born in 1530, on August 15, at 1^{h} 13^{m} after midnight.

Giovanni Battista Amico [Io. Baptiſta Amicus] of Cosenza wrote in 1537 on the motion of the heavenly bodies according to Peripatetic principles.

Giovanni Battista Capuano [Io. Baptiſta Capuanus Sipontinus], of Siponto, or of Manfredonia, a Canon Regular of the Lateran: while he was still a secular and bore the name Francesco, he professed astronomy at Padua in 1475, and then published an exposition of Sacrobosco's *Sphere*, unrevised, which after becoming a bishop he revised and dedicated to his hearers, his fellow canons. He also wrote on Peuerbach's *Theoricae* [in Theoricas Purbachij].

Giovanni Battista Carelli [Io. Baptiſta Carellus] of Piacenza constructed ephemerides from 1557 to 1580, with an introduction and a treatise on astrology.

Johann Baptist Cysat [Io. Baptiſta Cyſatus], of the Society of JESUS, of Lucerne in Switzerland, professor of mathematics in the University of Ingolstadt, besides a revolving cosmographical table [tabulam coſmographicam verſatilem] published in 1619 *Mathemata Astronomica* on the comet of 1618. He is living in this very year 1651.

Jean-Baptiste Morin [Io. Baptiſta Morinus], a Parisian mathematician, wrote on the science of longitudes and a new astronomy, and flourished in 1630.

GIOVANNI BATTISTA RICCIOLI [IO. BAPTISTA RICCIOLVS], of the Society of JESUS, of Ferrara, was born in 1598 on April 17 and entered the Society of JESUS in 1614 on October 6. He taught our members first rhetoric and poetics, then philosophy for nearly six years and scholastic theology for about 10 years, partly at Parma and partly at Bologna, always, however, most devoted to restoring geography, chronology and astronomy by his own observations. At length, having obtained from his superiors exemption from the chair of theology, and as Prefect of Studies at Parma [& Prefectus Studiorum Parmę], he applied himself wholly to the publication of the *New Almagest*, which he had been preparing for many years before, and published its first volume, divided into two parts, in this year 1651, its printing having been begun, however, in 1648.

Giovanni Battista Della Torre [Io. Baptiſta Turrianus] of Verona, a young man of lofty and noble talent and a devotee of the more recondite philosophy,

had already begun to devise the *Homocentrica* that Fracastoro afterwards perfected, as Fracastoro himself attests; dying, he commended to Fracastoro the task of taking up that work. See Fracastoro in the dedication of the work to Paul III.

Johann Bayer [Ioannes Bayerus Rhenanus], from the Rhineland, learned in law and astronomy, published at Augsburg [Auguſtæ Vindelicorum] in 1603 the *Uranometria*, or figures of the stars, with a nomenclature collected from ancient and recent tables and enlarged and corrected by his own astronomical night-watches; of which we shall speak again when we come to Julius Schiller [Iulio Schyllero].

Giovanni Bianchini [Ioannes Blanchinus] of Bologna, a great computer of astronomical tables. When he was general factor, as they say, of the Most Serene Borso d'Este [Borſij Eſtenſis], Duke of Modena and Reggio, Marquis of Este, Count of Rovigo [Rhodigij] and Lord of Ferrara, he dedicated by Borso's order the tables of the celestial motions he had constructed to the Emperor Frederick III, to whom he was known and most welcome. But because the art of printing had not yet prevailed — though invented in 1440 — and few could enjoy those tables, Luca Gaurico later, in 1526, published them corrected and enlarged from the press of Lucantonio Giunta [Lucæ Antonij Iunctæ]. Bianchini flourished at Ferrara in 1458, for which reason Biancani called him a Ferrarese in his *Chronologia* — although Biancani, himself a Bolognese, ought not to have been ignorant of a fellow Bolognese — and there he also wrongly places his floruit after 1500; I believe because Bianchini's tables were afterwards reprinted through Gaurico, as I said.

Giovanni Boccaccio [Ioannem Boccatium] is numbered among the astronomers by our Gualterus in his *Chronicon*, p. 699. He died in 1377.

John Dee [Ioannes Dee] of London, on the occasion of the new star of 1572, published a little book he called *Parallacticae commentationis praxeosque nucleus*, much approved by Tycho, *Progymnasmata* vol. 1, p. 690.

Johannes Decker [Ioannes Deckerius], of the Society of JESUS, a Belgian, native of Hazebrouck [patria Haeſbrouckanus], a pupil of Leonard Lessius [Leonardi Leſſij] in philosophy, taught philosophy at Douai [Duaci], and there and at Leuven scholastic theology. He was chancellor at Graz, and there died on January 10, 1619, aged 59. He was a man of outstanding modesty, obedience and piety. He wrote theorems on the year of the Lord's birth and death, etc.; a chronographical table from the capture of Jerusalem by Pompey to its destruction by Titus; and very many books of sacred chronology not yet published, which are preserved at Graz.

Ioannes Drienes [recte: Derienes, per the book's errata], of the Society of JESUS, a Frenchman, native of Dieppe [patria Diepenſis], was teaching mathematics at Paris [Luteriæ, as printed] in 1643, in his 51st year. He wrote the *Tabulae Canorienses*, or the doctrine of the luminaries.

Johannes von Gmunden [Ioannes de Egmunda], a German, a celebrated theologian and astronomer, taught astronomy at Vienna and died and was

buried there in 1442. He wrote tables of the motions of the planets, and on the eclipses of the luminaries.

Ioannes Essler wrote a *Speculum Astrologicum*, published at Basel in 1568.

Jean Fernel [Ioannes Fernelius Ambianas] of Amiens, first physician to the King, an excellent geometer and astronomer, published among other things in 1528 the *Cosmotheoria*, in which he explains the motion, position, size and theory of the heavenly bodies; he had published the *Monalosphaerium* in 1526. He died in 1558, on April 26, as Génébrard has it under Paul IV.

Johann Georg Herwart von Hohenburg [Ioannes Georgius Heruartus Hohemburgenſis], doctor of both laws [I. V. D.], in 1642 published a *New Chronology* weighed against the astronomical computation of eclipses, etc., which, however, both Pétau and we, in our chronological work, have been compelled to refute.

Johannes Garcaeus [Ioannes Garceus Brandeburgenſis] of Brandenburg wrote a *Methodus Astrologiae*, which he illustrated with nearly 400 nativities; but because he extended his judgements to free actions or future contingents, he was counted among the forbidden authors. He was born in 1530, on December 13, at $13^{h}\ 28^{m}$ after noon.

Ioannes Gazulus of Ragusa [Raguſinus] was an astrologer of great name in 1438.

Ioannes Heremita of Ferrara, an uncommon geometer and astronomer, was born in 1485, on December 23, at $3^{h}\ 32^{m}$ after noon.

Johannes Hevelius [IOANNES HEVELIVS Gedanenſis ſeu Dantiſcanus], of Danzig, or Danzig, professor of mathematics, very recently published the *Selenographia*, or on the spots and libration of the Moon, with copper figures engraved by himself most handsomely and accurately and verified with an exquisite telescope; he flourished from 1630 to 1651.

Ioannes Hispalensis, an astrologer, lived in 1142.

Johannes van Heeck [Ioannes Heckius Dauentrienſis], a physician of Deventer, wrote in 1605 a not inelegant little work on the new star in Ophiuchus [Serpentarij].

Ioannes Homelius, professor of mathematics at Leipzig, was born in 1518, on February 2, about sunrise. For his opinion on the star of 1572, see Tycho, *Progymnasmata* vol. 1.

Giovanni Gioviano Pontano [Ioannes Iouianus Pontanus] of Naples wrote 14 books on celestial matters, and on Ptolemy's *Centiloquium*, and the *Meteora*, these last in most elegant verse. He flourished from 1457, when as a youth he observed a comet, until 1495.

Johannes Kepler [IOANNES KEPPLERVS Vuirtembergicus] of Würtemberg, a pupil of Mästlin [Moeſtlini] and afterwards mathematician to three emperors — Matthias, Rudolf and Ferdinand II — a man of most sagacious and most ardent genius and a most acute investigator of astronomical subtleties, left to posterity various monuments of recondite learning. For he published in 1596 the *Prodromus dissertationum cosmographicarum*, or *Mysterium Cosmographicum*;

in 1604 the *Paralipomena ad Vitellionem*, or the optical part of astronomy; in 1605 on the new star in Ophiuchus; and in 1609 the *Astronomia Nova*, or celestial physics, in commentaries on the motions of the star Mars from the observations of Tycho; in 1610 the *Dissertatio cum Nuncio Sidereo* of Galileo; in 1616 new ephemerides, with their foundations, from 1617 to 1620; in 1618 the first three books of the *Epitome Astronomiae Copernicanae*; and in 1619 the five books of the *Harmonices Mundi* and three books on comets. Again, in 1621, he reprinted the *Mysterium Cosmographicum* revised and elucidated with notes; in 1622 he published books 4, 5 and 6 of the *Epitome*; in 1625 the *Hyperaspistes* of Tycho against Scipione Chiaramonti [Scipionem Claramontium], with an appendix on Galileo's *Assayer* [ex trutinatore Galilæi]; and at last, in 1627, the Rudolphine Tables, built upon Tycho's observations, over which he had toiled for 26 years, but polished to the archetype of his own ideas. In their preface he admits that he was with Tycho only a few months of the years 1600 and 1601. That appendix, too, which is attached to vol. 1 of Tycho's *Progymnasmata*, is Kepler's, as he himself admits on p. 1 of his *Ephemerides* and in his letters to Magini; and so are the *Eclogae Chronologicae*, the *Stereometria*, and the *Trigonometria Logarithmica*.

Jean Leurechon [Ioannes Leurechomius], of the Society of JESUS, a Lorrainer from Bar-le-Duc [ex Barroduco], taught philosophy for six years and mathematics for ten. He wrote the *Hilaria Mathematica*, on sundials, and on the comet of 1618.

Jean de Lignières [Ioannes de Linerijs] and **Johannes de Saxonia** [Io. de Saxonia], German astrologers, flourished about the year of the Lord 1380.

Giovanni Lucido [Ioannes Lucidus Samotheus], from France [ex Gallia], wrote on the emendation of the times, and completed it in 1535.

Io. Maria Fiornouellus of Ferrara wrote a learned little work on comets on the occasion of the comet of 1577.

Ioannes Muntz from Plabeirn [ex Plabeirn], a theologian and astronomer famous in his day and a canon of Vienna, wrote prognostications from the stars, and died at Vienna in Austria in 1503, on December 3 [3. Nonas Decembr.].

JOHN NAPIER [IOANNES NEPERVS Scotus], a Scot, Baron of Merchiston [Baro Merchiſtonij], published at Edinburgh [Endiburgi, as printed] in 1614 that excellent invention of logarithms, that is, of artificial numbers, by which the rule of three is performed by subtraction alone, indeed even by addition alone, and spherical triangles too are resolved by a wonderful shortcut, with such ease of computation that I have quite often managed to work out within one hour 30 or more triangles of this kind, which I could scarcely have finished in six or seven hours by the usual tables of tangents and secants. It is therefore incredible how much he has contributed by that ease to astronomy and to the construction of tables.

Johannes Phocylides Holwarda [Ioannes Phocyllides Holuuarda] wrote an epitome and examination of the *Astronomia Reformata*, and flourished in 1635.

Giovanni Pico della Mirandola [Ioannes Picus Mirandulæ Comes], Count of Mirandola, the phoenix of talents, and amid such loftiness of genius pious and most religious, attacked the judicial astrologers most sharply in 12 books and in disputations. He was, however, challenged to a trial by Lucio Bellanti [à Lucio … Bellancio], the champion of astrology, and after a dispute on this subject he was warned, with a threat, of his death in the very next year, that is, in the 33rd of his age and 1495 of Christ, because of the direction of the horoscope to the body of Mars; and in fact he died aged 33. He had been born in 1463, on February 24, at 2^{h} 42^{m} after noon. Cardano describes his nativity in his book of a hundred nativities, no. 65, but tries to disgrace so great a man with unworthy calumnies — an astrologer, that is, against the enemy of astrologers.

Pierio Valeriano [Ioannes Pierius Valerianus Bellunenſis] of Belluno, the noble expounder of *Hieroglyphica*, wrote an elegant compendium on the sphere, addressed to Cardinal Alessandro Farnese [Alex. Farneſium], still a youth but most devoted to these arts, and Vice-Chancellor of the Holy Roman Church. This author flourished in 1540.

Johannes Praetorius [Ioannes Prætorius Ioachimicus] of Joachimsthal, professor of mathematics at Wittenberg and astronomer of the Nurembergers in the school of Altdorf [in ſchola Altorfiana], wrote, on the occasion of the comet of 1577, a history of comets and on their causes and effects. On him see Tycho's critique, *Progymnasmata* vol. 1, p. 639.

JOHANNES DE MONTEREGIO [IOANNES DE MONTEREGIO Franconiæ], of Franconia, hence called Regiomontanus, the glory of Germany, surnamed the Great, was born in 1436, on June 6, at 4^{h} 40^{m} after noon, at Königsberg [Regiomonti], with the Moon and Saturn in platic conjunction [coniunctis platicè] in Gemini — a conjunction that portends a wonderful sharpness of genius — but with the luminaries conjoined in the same sign, and Mars in the eighth house, so that they were afflicted by Mars and signified a life by no means long. At Vienna he was a pupil of Georg Peuerbach [Georgij Peurbachij], who by his last will charged him with completing the *Epitome of the Almagest*. By King Matthias of Hungary, to whom he dedicated the table of the primum mobile, he was enriched with eight hundred Hungarian gold pieces and a most precious robe, and afterwards with an annual stipend from him and gifts from other princes. Moving to Nuremberg, where he was received with great honour, he observed the heavens for some years, and especially the Sun's declinations. At last he was called to Rome for the correction of the calendar by the Supreme Pontiff Sixtus IV, who had already, in his absence, designated him bishop of Regensburg [in Epiſcopum Ratiſponenſem]. But before the business was done he died — of a pestilential fever, or (as others have suspected) killed by poison by the sons of Georgios Trapezountios [Georgij Trapezuntij] out of envy — in 1476, aged 41, and was buried in the Pantheon. It is incredible how much the whole of mathematics, and astronomy above all, owes to this truly Great man: both for the *Epitome of the Almagest*, the book on comets, and the tables and problems of the primum mobile and of directions;

and for the new editions and translations made or procured by him, especially of Manilius, Ptolemy, Theon, Proclus, Menelaus, Theodosius, Julius Firmicus and Hyginus, and likewise of Archimedes, Apollonius, Serenus, Hero, Vitellio [Vitellionis], Leopold of Austria and Jordanus. So Ramus [Petrus Ramus], in book 2 of the *Scholae Mathematicae*, rightly said that Nuremberg could boast of Regiomontanus more justly than Tarentum of Archytas, Syracuse of Archimedes, Byzantium of Proclus, or Alexandria of Ctesibius. On this occasion he adds that the men of Nuremberg, rivalling the dove of Archytas, had made an iron fly which, released from the craftsman's hand and flying round the guests, flew back into its master's hand; and an eagle, which went out a very long way high in the air to meet the arriving Emperor and escorted him to the city gate.

Juan de Rojas [Ioannes Roias], a Spaniard, in the year 1550 wrote, and dedicated to Charles V, commentaries on the astrolabe, or planisphere.

John of Sacrobosco [Ioannes de Sacroboſco], an Englishman and doctor of the University of Paris, composed his *Sphere*, comprising four books, out of Ptolemy, al-Battānī [Albategnio], al-Farghānī [Alfragano] and the other more ancient authors. Although Barozzi detected 84 errors in it, which he lists at the beginning of his *Cosmographia*, it nonetheless acquired such authority in the schools that a great many men more learned than he illustrated it with commentaries or expositions; I shall list them at the beginning of book 1. But not everyone agrees about when he flourished. Clavius, in the preface to the *Sphere*, says he flourished around the year 1232; Gordon [Gordonus], in his *Chronicon*, in 1245; Biancani between 1200 and 1300. Giuntini, however, in his *Catalogue*, assigns him to the year 1393, and Sansovino [Sanſouinus] in his *Chronologia* to 1389. But these two are beyond doubt mistaken. For Élie Vinet of Saintes [Elias Vinetus Santon], in the dedication of his scholia on this *Sphere*, says that Sacrobosco wrote on the sphere, the astrolabe and the algorism in the year of the Lord 1256 [M CC LVI.], and he confirms it from the verse with which this John's little book on the ecclesiastical computus closes. He adds that John was buried at Paris in the cloister of the Mathurin friars [in Clauſtris ſodalium Matturinalium], and that in the middle of the tomb a sphere can be seen carved, with this epitaph, the token of a barbarous age:

> Of Sacrobosco, John the computist, who divided the times, here lies, by time snatched away. You who follow the times, remember that you will die. If you are wretched, weep; in pity, I beg, pray for me.
> [De Sacro Boſco qui Compotiſta Ioannes / Tempora diſcreuit, iacet hìc à tempore raptus. / Tempora qui ſequeris, memor eſto quòd morieris. / Si miſer es plora : Miſerans pro me precor ora.]

Génébrard, too, under Innocent IV, and our Gualterius in his *Chronologia* place him in the thirteenth century of Christ.

Johannes Schöner [Ioannes Schonerus], mathematician of Nuremberg, was born at Karlstadt [Caroloſtadij] in 1477, on January 16, seven minutes before

the eleventh hour of the night. He published astronomical tables in 1536, which were called the *Resolutae*, and the *Use of the Celestial and Terrestrial Globe* in 1551, and an *Astronomical Equatorium*. But on account of suspect opinions he was rejected by the Sacred Censors among the condemned authors. He had a son, the heir of his father's learning.

Johannes Stabius [Ioannes Stabius], an Austrian, a distinguished cosmographer and astronomer at Vienna under the Emperor Maximilian, flourished around the year 1550.

Johannes Stöffler [Ioannes Stoffler ſeu Stoflerinus, ſeu Stoeflerus Iuſtingenſis], or Stoflerinus, or Stoeflerus, of Justingen, an outstanding doctor of mathematics at Tübingen, published in 1510 an explanation of the construction and use of the astrolabe; in 1518 a Roman calendar; and in 1533 a commentary on the *Sphere* of Proclus, written in the year in which he was in the 79th year of his age; and ephemerides from 1532 to 1551.

Johannes Werner [Ioannes Vuernerus], successor at Nuremberg to Regiomontanus and Bernhard Walther [Bernardi Valtheri], was born in 1468, on January 15, at $5^{h}\ 44^{m}$ after midnight. He wrote on Ptolemy's *Geography*, on the motion of the eighth sphere, and astronomical tables, in which he set out the places of the stars; and in 1514 he gave the Sun's greatest declination as 23° 28′.

Johannes Vögelin [Ioannes Vogelinus] observed the comets of the years 1527 and 1532; he was a pupil of Regiomontanus.

Giuseppe Auria [Ioſephus Auria], a Neapolitan, expounded Autolycus on the sphere and on the risings of the fixed stars, Euclid's *Phaenomena*, and Theodosius of Tripoli on habitations and on days and nights, in 1588.

Giuseppe Biancani [Ioſephus Blancanus] of Bologna, of the Society of JESUS, my teacher in the elements of geometry and of the sphere, once a pupil of Clavius at Rome, taught the mathematical disciplines at Parma up to the year 1624. On June 7 of that year he died, of sheer dissolution and failure of the natural heat, all his organs being found unharmed by the anatomists. He wrote on the mathematical passages of Aristotle and on the nature of the mathematical sciences; a *Chronology of the Mathematicians*; a *Sphere of the World* from more recent observations; an *Introduction to Geography*; an *Apparatus for the Mathematics*; an *Echometria*; and an *Instrument of Sundials* [inſtrumentum Horologiorum], which I was the first to see into print. He was a man of slow but solid judgement, mature in weighing the demonstrations [Apodixibus] of others, of religious candour and charm, and dear to all for the grace of his wit [eutrapeliæ].

Joseph Lange [Ioſephus Langius] of Kaysersberg [Cæſaremontanus], ordinary professor of mathematics and of the Greek language at Freiburg, published in 1612 an *Elementale Mathematicum* of logistic, of both kinds of [vtriuſque] geometry, of astronomy, of the theory of the planets, and of geography.

Giuseppe Moletti [Ioſephus Moletius], professor of mathematics at Padua, wrote commentaries on Ptolemy's *Geography* and enlarged it with new maps in 1562; then, in 1580, the *Gregorian Tables* derived from the Prutenic, for the motion

of the eighth sphere and of the luminaries, on the correction of the calendar and the ecclesiastical computus; and an introduction to the ephemerides.

Giuseppe Scala [Ioſephus Scala], a Sicilian, computed ephemerides from the year 1589, in which he flourished, up to the year 1600.

Joseph Justus Scaliger [Ioſephus Scaliger], son of Julius Caesar [Scaliger] à Burden, flourished at the end of the last century and the beginning of this one. He wrote the *Cyclometrica*; a *Diatribe on the Anticipation of the Equinoxes*; and a most learned work, *De emendatione temporum*, in the year of the Lord 1587. But through the heat of his genius and excessive self-confidence he stumbled in many things, and was deservedly thrashed by the most solidly learned Fathers of our Society, Clavius, Guldin [Guldino] and Pétau; see the judgement on him in Pétau, *Prolegomena* to the *De Doctrina temporum*, chapters 5 and 6. His father had died at Agen [Agenni] in 1558, as Génébrard has it under Paul IV.

Gioseffo Zarlino [Ioſephus Zarlinus], of Fossa Clodia, commonly *da Chioggia*, wrote in 1557 the *Institutioni Harmoniche* [Inſtitutiones Harmonicas], in which he also treats of the harmonic system of the celestial spheres.

Isaac Argyros [Iſaacius Argyrus], a monk, in the eleventh century of Christ wrote on the cycles of the Sun and Moon and on the ecclesiastical computus. Our Pétau translated this work from Greek into Latin and inserted it in the *Uranologion*, p. 359.

Yitzchak ibn Sid [Iſaac Hazan] — *Hazan* meaning cantor of the synagogue of Toledo — a Jew, was the foremost of those whom King Alfonso put in charge of drawing up the Alphonsine Tables, which first appeared in 1252, as Agostino Ricci relates in his treatise on the motion of the eighth sphere, chapter 38.

Yitzchak Yisraeli [Iſaac Iſraelita], of whom Ricci speaks in the same work, chapters 3, 36 and 45, flourished in the year of grace 1310 and wrote the book *Iessod-holam* [Ieſſod-holam], that is, *On the Foundation of the World*, in which he quite often mentions the motion of the eighth sphere.

Isidorus, a geographer and distinguished philosopher mentioned in the Suda [apud Suidam] and in Pliny the Elder, and the teacher of Hypsicles of Alexandria, is set down as having flourished in the time of Apollonius of Perga; see *Apollonius*.

St Isidore of Seville [S. Iſidorus Hiſpalenſis], the Younger, bishop, son of Severianus, duke of Cartagena [Carthaginis], and brother of St Leander, touches on the elements of astronomy in his books of *Origins* and wrote a treatise *On the Sphere*. He died in 636, on January 15 [18. Calend. Februarij].

Abū al-Fidā' [ISMAEL ABILFEDEA ſeu Abulfeda], prince of Syria, Assyria and Persia, as Conrad Gessner [Conradus Giſnerus] says in his *Bibliotheca*, and Christmann after him on al-Farghānī, chapters 10 and 11, was a distinguished cosmographer and eminent in geography. In describing the world subject to the Ishmaelites he takes the beginning of longitudes from the Pillars of Hercules. At the end of the codex Abū al-Fidā' attests that he wrote this *Geography* in the year of the Hegira 721, therefore in the year of Christ 1322. Guillaume Postel

[Gulielmus Poſtellus] brought it from the East and sold it, with other Arabic books, to Otto Henry, Elector Palatine [Ottoni Henrico Principi Palatino]; it is preserved in the Palatine Library.

Ismaël Boulliau [ISMAEL BVLLIALDVS], son of Ismaël Boulliau of Loudun [Iulioduni], who was in the habit of observing the stars, an astronomer of profound research, after his *Philolaus* called up from the dead in 1640, published at Paris in 1645 the *Astronomia Philolaica*. It is built on the hypothesis of the motion of the Earth and of an elliptical orbit described by the planets about a cone, is made illustrious by solid demonstrations, and has the Philolaic tables appended: a work for all students of astronomy, but not to be read without attention. He represented the motions of Saturn, Jupiter and Mercury more accurately than the Rudolphine Tables, and in many respects presented easier tables.

Giuliano Ristori [Iulianus Riſtorius] of Prato, a Carmelite, doctor of sacred theology and an excellent astronomer, and the teacher of Giuntini, observed the planets in 1536 and 1542, as Giuntini relates in the preface to his *Resolved Tables* [tabulas Reſolutas].

Giulio Cesare Lagalla [Iulius Cæſar Lagalla] in 1612 published a disputation on the new phenomena seen by the telescope in the body of the Moon.

Julius Firmicus Maternus [Iulius Firmicus Maternus iunior] the Younger, a Sicilian, flourished under Constantine Augustus, as is clear from his own preface addressed to the proconsul Mavortius Lollianus [Macortium Lollianum] [recte: Mauortium]. He is therefore not absurdly placed in the year of Christ 314 by Sansovino in his *Chronologia*, or in 328 by Giuntini. He wrote eight books of *Astronomica* [Aſtronomicῶn], full of astrological precepts.

Julius Schiller [Iulius Schillerus] of Augsburg in Vindelicia [Auguſtanus Vindelic.], I. V. C., in 1627 published the *Coelum stellatum Christianum*, that is, the stars brought into images of the saints, their profane names being changed into sacred ones; Johann Bayer [Io. Bayero] supplied a new and more accurate *Uranometria*.

L

Lorenz Eichstadt [Laurentius Eichſtadius], doctor of medicine and physic at Stettin [Stetini], and professor of mathematics at Danzig, or Gedanum [Dantiſci ſeu Gedani], flourished in 1645. We shall report some of his observations of eclipses in the *Catalogue of Eclipses*.

Leopold of Austria [Leopoldus de Auſtria], an astrologer of great fame, flourished around the year 1200.

Lilio Gregorio Giraldi [Lilius Gregorius Gyraldus] of Ferrara, whom Lipsius calls the Varro of his age, wrote in 1542 a most learned book on the years, months and other divisions of time of the Romans and the Greeks, with the calendar of both.

Luca Gaurico [Lucas Gauricus], of Giffoni [Geophnenſis] in the Kingdom of Naples, bishop of Civitate [Epiſcopus Ciuitatenſis], professed mathematics in 1531 at the University of Ferrara, and there delivered an oration in praise of

astrology. In the same year, 1531, there appeared at Venice, by his labour and care, many expositors both of the *Sphere* of Sacrobosco and of the *Theoricae* of Peuerbach. In 1552 he wrote, addressed to Paul III, an ecclesiastical calendar; and in 1557 tables of the primum mobile, called tables of directions, with astrological precepts and problems.

Lucillus Philalthaeus [Lucillus Philalthæus] in 1563 wrote splendid commentaries on the books *De caelo*.

Lucio Bellanti [Lucius Bellantius] of Siena wrote 20 questions on astrological truth, and 12 books of a defence of astrology against Giovanni Pico [Io. Picum], whose death he had foretold, in the year 1495, from a celestial figure; to these is annexed a *Dialogue* of Gabriele Pirovano [Gabrielis Pirouani] on the truth of astrology.

Lodovico Vitali [Ludouicus Vitalis], astronomer of Bologna, was born in 1475, on August 23, at 17^{h} 8^{m} after noon.

M

Macrobius Ambrosius Aurelius Theodosius [Macrobius Ambroſius Aurelius Theodoſius], a man of consular rank, by birth of Parma [patria Parmenſis], shows himself not unskilled in astronomical matters in his two books on the *Dream of Scipio*. At the beginning of the fifth book of the *Saturnalia* he indicates that he was a contemporary of Symmachus, who flourished under Theodosius and his sons in the fourth century of Christ, as Jodocus Coccius [Iodocus Coccius] also notes in volume 1 of his *Thesaurus*, in the book on the signs of the Church, article 2. He is therefore wrongly assigned to the second century by Génébrard, Sansovino and Thevet [Theueto], or to the tenth century in the index of the Vatican Library.

Marcus Manilius [M. Manilius] of Antioch wrote for Caesar Augustus five books of *Astronomica* [Aſtronomicω̃n] in heroic verse not to be despised, in which he embraced astronomy and astrology alike. This is the Manlius (the name contracted) who set a gilded ball on the obelisk erected by Augustus in the Campus Martius, so that the shadow, gathered together, might be better bounded and might mark the days of the equinox and the solstice, as Pliny the Elder relates, book 36, chapter 10. Giuntini is therefore not wrong to say that he flourished about the year 48 before Christ.

Martianus Capella [Marcianus Capella], also called Felix Mineus Afer, a little before Heraclius, or about the year of the Lord 600, wrote in the *Marriage of Mercury and Philology* on geometry, arithmetic, astronomy and music.

Marcus Agrippa, son-in-law of Augustus, displayed to the Roman people a map of the world, described in his own commentaries and afterwards painted in a portico, as Pliny the Elder relates, book 3, chapter 2, and book 7, chapter 8. He died aged 51 in the 31st year of Augustus, as Vignier [Vignerus] has it, that is, in the year 12 before Christ.

Marcus Beneventanus [Marcus Beneuentanus] wrote on Thābit [Thebit], on the motion of the eighth sphere; he flourished in the fourteenth century of Christ.

Marin Mersenne [Marinus Merſennius], a Frenchman of the order of St Francis of Paola, touched on many astronomical matters in his most learned commentaries on Genesis; he flourished in 1645.

Marinus of Tyre [Marinus Tyrius], a distinguished geographer, praised by Ptolemy, flourished under Nero. But he must have lived on to Ptolemy's time, that is, to the year of the Lord 69 or 70, since Ptolemy was born in that year, and in book 1 of the *Geography*, chapter 6, says: *"Marinus of Tyre, then, the latest of those who have lived in our time, appears to have applied himself to this part with the greatest zeal; for it is plain that he unrolled very many histories, beyond even those known in earlier times, and read through with the greatest diligence the writings of almost all who preceded him."*

Mario Bettini [Marius Bettinus], of the Society of JESUS, was born at Bologna in 1582, on February 6. Very learned in humane letters and indeed in sacred ones, he taught moral philosophy at Parma, with me among his hearers, and afterwards mathematics publicly. Among other works he wrote the *Apiarium*, or paradoxes of the whole of mathematical philosophy, in whose eighth *Apiarium* he treats chiefly of astronomy, leaving to posterity monuments of his most acute genius; and in the *Euclides applicatus*, or *Aerarium* of mathematics, he touches on the same matters from time to time. He was living in this year 1651, aged 69.

Marsilio Ficino [Marſilius Ficinus], physician, astrologer and philosopher, in writing on Plato's *Timaeus* and his other dialogues discusses astronomical matters in no light way. He died in 1499, aged 66.

Martinus Hortensius [Martinus Hortenſius] of Delft, the colleague and helper of Philips van Lansbergen [Philippi Lansbergij] and no common observer of the stars, as Lansbergen himself declares in his preface to the reader, flourished in the year of Christ 1633. In that year he also published, with Pierre Gassendi [Petro Gaſſendo], a dissertation on Mercury seen in the Sun and Venus unseen.

Matteo Ricci [Matthæus Riccius] of Macerata, of the Society of JESUS, was born in the year of salvation 1552, on October 6, and entered the Society in 1571, on August 15. Having obtained a mission to the East Indies, he left Italy in 1577 and reached Goa in 1576, on September 13. He landed at Macau in 1582, in the month of August, and the following year, entering the kingdom of China, drew a cosmographical map of the whole world in such a way that the kingdom of China occupied the greater part in the middle, while the other kingdoms appeared small at the edges of the oval map. In this he indulged the temper of a nation that thinks magnificently of itself and narrowly of other kingdoms, without damage to geographical truth. He wrote in the Chinese language cosmographical and astrological rules, enriched the cosmographical map with commentaries, and wrote on the sacred calendar and much else in aid of that nation. He died a holy death, as he had lived, at Peking [Pachini], the royal city of the Chinese, in 1610, on May 11, and the King decreed him a most honourable tomb.

Maurus Florentinus, at the end of the last century, described the sphere in the Italian language.

Melchior Inchofer, of the Society of JESUS, of Vienna, who long professed philosophy, mathematics and theology at Messina with praise, published in 1633 a *Tractatus Syllepticus* on the rest of the Earth and the motion of the Sun according to Sacred Scripture and the Holy Fathers; and also an examination of the celestial figures [examen Thematum cęleſtium] of various astronomers up to Tycho, a method of computing eclipses, and a theory of the planets — but these three under another's name, that of the Academicus Vertumnius [Academici Vertumnij]. He died at Milan in 1650.

Menelaus, geometer and astronomer, also called *Mileus* or Milesius, observed the stars at Rome in the first year of Trajan, that is, in the year of Christ 97, as Ptolemy relates, *Almagest* book 7, chapter 3. He wrote six books on chords [de Subtenſis] and three books of *Spherics* on triangles [Sphęricorum triangulorum].

Mercurius the Elder [Mercurius Maior], grandson of Atlas, flourished around the year 1520 before Christ; his grandson Trismegistus around the year 1460.

Māshāʾallāh [Meſshala Arabs], an Arab, a most celebrated astrologer, flourished in the year of Christ 860. He wrote a book on the reckoning of the circle and of the stars and how they act in this world, and on the elements and the celestial orbs.

Meton [METON], astronomer and astrologer of Athens, an observer of the solstices cited by Ptolemy, *Almagest* book 3, chapter 3, wrote *Prognostica*; and he first introduced, or restored, the enneadecaeteris [Enneadecaterida], that is, the cycle of 19 years, which was called the Year of Meton; he made its beginning in the 4th year of the 86th Olympiad, that is, 432 before Christ. But we shall say more of Meton in book 4, chapter 19, number 5.

Metrodorus, philosopher and astronomer, wrote five books on the reckoning of the zones; he was the teacher of Anaxarchus, as Laertius says, and flourished around the 105th Olympiad, or 360 before Christ.

Michelangelo Biondo [Michael Angelus Blondus], a physician, wrote in 1544, addressed to Paul III, on critical days, and, addressed to Rodolfo Pio, Cardinal of Carpi [Rudolphum Pium Cardinalem Carpenſem], on the anticipation of the fixed stars, with their significations.

Michael Florent van Langren [MICHAEL FLORENTIVS LANGRENVS], cosmographer and mathematician to Philip IV, King of Spain, in the Low Countries [in Belgio], published in 1644 a treatise on the true longitude by land and sea through observation of the lunar spots as they are darkened or illuminated, with the censures and approbations of most learned men. In 1645 he began, with plates engraved by himself for this purpose, to publish a selenography, that is, figures of the lunar spots and of the whole lunar disc, to which he gave this title: *Selenographia Langreniana, sive Lumina Austriaca Philippica*. He sent me two copies of it from Brussels, to Parma and to Bologna, to be examined with a special telescope, together with various observations

of eclipses, of the planets and of the fixed stars, and of the diameters of the Moon, in several most courteous letters written to me. He also promises many other figures of the phases of the Moon, with a theory of the other motions and especially of the libration. But, nearly destitute of patrons [Mecœnatibus] and hindered by the great number of children he had to support, he could not publish these sooner and so forestall Hevelius. He is a most excellent man, worthy of a better fortune, to whom, as I owe a very great deal, so I pray from God, best and greatest [D. O. M.], a very great deal of happiness.

Michael Mästlin [Michael Mæſtlinus] of Göppingen [Gæppingenſis], parish priest of the town of Backnang [Bachnang], then professor of mathematics in the University of Heidelberg [Hoidelbergę] and at Tübingen, and Kepler's teacher, wrote on the star of the year 1572, which Tycho reports and strongly approves in volume 1 of the *Progymnasmata*, from p. 544; and on the comet of the year 1577, which Tycho likewise reports in volume 2, from p. 245; then ephemerides from 1577 to 1590 from the Prutenic Tables; and in 1606 *Theses on Eclipses*. He also wrote additions to Rheticus's *Narratio prima*, and an appendix on the dimensions of the celestial orbs according to the opinion of Copernicus. But since he was infected with the Ubiquitarian heresy and hostile to the Roman Pontiff and the Catholic Church, he dared to bark at the Pontiff's authority and at the Gregorian calendar. His barking was silenced, however, by our Clavius, in the *Apologia of the Roman Calendar* addressed to Rudolf II, Emperor of the Romans, written in 1588, which forced him to fall dumb, or to mutter with a secret groan within the sheepfolds of the sectaries.

Michael Scot [Michael Scotus], a diligent observer of the stars, at the request of the Emperor Frederick III published in 1460 questions on the *Sphere* of Johannes de Sacrobosco.

Michael Neander, of Joachimsthal [ex Valle Ioachimica], published in 1561 the *Elementa doctrinae sphaericae* and the *Computi Astronomici materia*.

N

Hicetas [Nicetas ſeu Hicetas Syracuſanus], or Nicetas, of Syracuse, of whom Laertius speaks in his *Philolaus*, said that the Earth moves; his date is not known to me.

Nikolaos Kabasilas [Nicolaus Cabaſilla], a Greek, published a commentary on Ptolemy's *Great Syntaxis* around the year of the Lord 1220.

Niccolò Cabeo [Nicolaus Cabæus] of Ferrara, of the Society of JESUS, was born in 1586, on February 26. Public professor of philosophy at Parma, he wrote, besides the *Philosophia Magnetica*, a commentary on Aristotle's *Meteorologica*, and questions full of recondite learning, in which he repeatedly treats what it is worth an astronomer's or cosmographer's while to know, especially where he deals with comets, the Milky Way, and so on. He died at Genoa, where he was publicly teaching mathematics to great applause, aged 64, in 1650, in the month of July, leaving there a great sense of his loss; and by a death proper to a religious, solidly pious man, constant to himself to the very end, he set the seal on the sincerity of his life.

Copernicus [NICOLAVS COPERNICVS] of Toruń [Torinenſis], canon at Frombork [Fruemburgi] in Warmia [Vuarmia], a region of Prussia, was born in 1472, on January 19, at 4^{h} 48^{m} after noon, as Francesco Giuntini [Franciſcus Iunctinus] records in his *Astrological Calendar*. But Johannes Garcaeus [Io. Garcæus], in his *Astrologiae methodus*, p. 138, says that he was born in 1473, on February 10, at 4^{h} 38^{m} after noon, at a pole altitude of 55°, with the Moon in 7° of Sagittarius regarding by an exact [partiliter] trine aspect Mercury, placed at the beginning of Aries, and with the 16th degree of Libra ascending; he affirms that this aspect was also in Peuerbach's nativity [in Geneſi Purbachij], and that it makes men ingenious. Mästlin, however, in his addition to Rheticus's *Narratio prima*, says that he was born in 1473, on February 19, at 4^{h} 48^{m}, on the Friday before the feast of the Chair of St Peter, and died in 1543, on January 19 [Ianuarij 19.], aged 70, in the very year in which he had seen to the publication of the books of *De revolutionibus*. So either Giuntini is wrong, or the astrologers worked back from the birth to the moment of conception and set that down as the first birth.

Copernicus was, moreover, first a pupil of Domenico Maria Novara of Ferrara [Dominici Mariæ Ferrarienſis], and helped him in his astronomical observations at Bologna, where in 1497, on March 9 [ſeptimo Idus Martij], at the age of 24, he observed the bright star of the Hyades, or Palilicium, occulted by the Moon, as he himself relates in book 4, chapter 27. Then in 1500, now aged 27, he professed mathematics at Rome before a great crowd of students and a circle of great men, as Rheticus reports; and in the same year 1500, in the same place, he observed an eclipse of the Moon on the night after November 5, as he himself reports in book 4, chapter 14. Returning thereafter from Rome to his homeland, he devoted himself wholly to observing the stars and restoring astronomy, choosing Frombork in Prussia for his observations — which lies at the mouth of the river Istula, or Vistula, and which, from eclipses observed in both places, he had found to lie under the meridian of Kraków, as he says in book 3, chapter 19, and book 4, chapter 7 — even though he admits, in book 5, chapter 30, that because of the vapours of the Vistula Mercury could not be observed by him. He teaches in book 3, chapter 2, that he found the pole altitude of Frombork to be 54° 19½′ [gr. 54. min. 19. cum dimidio], and the Sun's greatest declination 23° 28′. But because he omitted refractions, and because he used parallactic rulers — four cubits long, indeed, but of fir wood, with divisions made in ink not very finely — he erred by 2′ and more, as Tycho teaches in volume 1 of the *Progymnasmata*, pp. 34 and 36, where he says that this instrument was sent to him as a gift by Johannes Hannow [Ioanne Hannouio], canon of Warmia [Canonico Var-lmienſi].

For, as Rheticus admits in his preface to the *Ephemerides*, *he wished his researches to be moderate, not excessive*; and when Rheticus quarrelled with him about pursuing the minutes, he replied: *"If I can bring the truth to within sixths of degrees, which are ten minutes, I shall exult in spirit no less than Pythagoras, we are told, did on finding the rule of the right angle [ratione normæ reperta]"*; and he

affirmed that it was possible to get even that far only with difficulty. The first of his observations made at Frombork I find in 1509, concerning an eclipse of the Moon, from his book 4, chapter 13; for the other eclipses, of which he speaks in book 4, chapter 5, were observed in 1511, 1522 and 1523. In 1512 he observed Mars at acronychal rising [acronychium], in 1514 Saturn, and in 1520 Jupiter, likewise acronychal, from book 5, chapters 6, 11 and 16. In 1515 he observed Spica and the Sun's greatest declination, from book 3, chapter 2; in 1522 and 1524 the parallaxes of the Moon, from book 4, chapter 16; in 1525 Spica again; and finally, in 1539, a conjunction of the Moon with Venus. Urged by letters sent from Rome in 1536 by Nikolaus von Schönberg [Nicolao Schombergio], Cardinal of Capua, and also by Tiedemann Giese [Tidemanno Giſio], Bishop of Kulm [Culmenſi], he permitted in the last year of his life, which was 1543, the publication of the books of *De revolutionibus*, which he had kept back for 36 years, as he himself admits in the dedication to Pope Paul III. Tycho, in his *Letters*, p. 240, reports that Copernicus painted his own portrait with his own hand from a mirror; Tycho had it from the chest upwards only, and so he closed an epigram on him thus:

> But the body, you will say, is seen in an image only half-length, not enough for so great a man. Of course: he who carried the whole Earth into the aether — not even the whole Earth itself can hold the whole of him.
> [At corpus, dices ſpectatur imagine ſaltem / Dimidiâ, haud tanto ſufficiente Viro. / Scilicet is, totam qui geſſit in Æthera Terram, / Tota nec hunc totum Terra vel ipſa capit.]

Nicolas Caussin [Nicolaus Cauſſinus], of the Society of JESUS, of Troyes in France [Trecenſis ex Gallia], wrote most recently *De Domo Dei*, that is, on the heaven, and wrote against the astrologers very acutely and learnedly. He died in this year 1651, aged 70.

Nikolaus von Kues [Nicolaus Cuſanus] was born at Kues [Cuſæ], a village on the bank of the Moselle in the diocese of Trier [Teuerenſi]. He was Cardinal of the title of St Peter in Chains, papal legate to Germany and Bishop of Brixen [Brixinenſis]: a philosopher, theologian and mathematician of most subtle genius, skilled in three languages, Latin, Greek and Hebrew. Besides other works that do not concern us, he wrote on the reform of the calendar, on the canon of the fixed stars, and on mathematical complements. He died in 1464, on August 11, aged 63, and was buried at Rome; his heart, however, was sent to Kues, to the hospital of St Nicholas which he had built, and is preserved in the middle of the choir.

Nicolaus Mulerius [Nicolaus Mulerus], physician and rector of the gymnasium at Leeuwarden [Leouuardiæ], published in 1611 the *Tabulae Frisicae Lunae-Solares quadruplices*, namely of Ptolemy, Alfonso, Copernicus and Tycho, with the old Roman calendar.

Nikolaus Winckler [Nicolaus Vuincklerus], physician of Schwäbisch Hall [Hale Sueuorum], wrote in 1577 on the comet of that year.

Niccolò Zucchi [Nicolaus Zucchius], of the Society of JESUS, of Parma, taught philosophy and theology with great praise; at Rome he was the helper and instigator of Father Scheiner in observing the spots of the Sun. He will shortly send to the press the first part of his *Optical Philosophy*, in which he also treats of celestial refractions from appearances through the telescope. Of the keenness and depth of so great a man it is better to be silent than to say a little.

O

Oenopides of Chios [Oenopides Chius] was so great in mathematics that Plato commended him in the *Lovers* [in amatoribus]. Proclus, propositions 12 and 13 [pr. 12. & 13], and Theon of Smyrna in his *Astronomica* attribute to him praise in astronomy [Aſtrologiæ laudem], Theon adding that Eudemus asserts that he first noticed the obliquity of the zodiac and the constitution of the great year. Aelian, in book 7 of the *Varia historia*, chapter 10, relates that when he dedicated a table of laws at Olympia, he inscribed on it an astronomy [Aſtrologiam] of 59 years, affirming this to be the great year; Censorinus, chapter 7, calls him Vaenopides [Vęnopidem]. The obliquity of the zodiac [ſigniferi], however, is said to have been discovered by Anaximander, as Pliny the Elder says, book 2, chapter 8, or by Pythagoras, as Plutarch relates, book 2 of the *Placita*, chapter 12.

ʿUmar al-Ṭabarī [Omar, ſeu Haomar Tiberinus], an astrologer — that is, from Tiberias — son of Belnalfargdianus [Belnalfargdiani], wrote three books on nativities, around the year of the Lord 1200.

Oronce Finé [Orontius Finæus] of the Dauphiné [Delphinas] flourished in 1531 and was Royal Professor of Mathematics at Paris. He wrote on the sphere of the world, or *Cosmographia*, the *Theoricae* of the planets, and astronomical canons, or problems of the primum mobile, and on finding the difference of longitude by the Moon, and many other small works; but Pedro Nunes [Petrus Nonius] published a book on the errors of Oronce.

Osymandias [Oſymandies], king of Egypt, as Dionysius of Halicarnas- -sus, in his books of *Antiquities*, ordered his tomb to be girded with a circle of solid gold, 365 cubits in circumference — as many as there are days in the year — with the risings and settings of the stars and their yearly effects.

P

Pappus of Alexandria [PAPPVS Alexandrinus], around the year of the Lord 390 or 400, wrote eight books of *Collections*, which Federico Commandino [Federicus Commandinus] translated from Greek into Latin and illustrated with commentaries; also a commentary on book 5 of the *Almagest*, and a universal description of the world.

Paul of Alexandria [Paulus Alexandrinus] wrote an *Introduction to Astronomy* [iſagogem Aſtronomiæ] in the year of Christ 378.

Paul Fabricius [Paulus Fabricius], imperial mathematician and physician, wrote a few things on the star of the year 1572, about which see Tycho, volume 1 of the *Progymnasmata*, p. 528.

Paul of Middelburg [Paulus Middelburgenſis], bishop of Fossombrone [Foroſempronieſis], wrote in 1513 to Pope Leo X on the correct celebration of Easter, and to the Emperor Maximilian a most learned work on the day of the Passion of Christ the Lord.

Paul Guldin [Paulus Guldin], of the Society of JESUS, a native of St. Gallen [Sancto-Gallenſis], was born in 1577, on June 12, and was called Habakkuk [Habacuc]. He was brought to the Catholic faith at Freising [Friſingę] and admitted at Munich in 1597 as a temporal coadjutor, taking the name Paul. But when his outstanding aptitude for mathematics was discovered, he was called to Rome, studied philosophy and mathematics, and taught mathematics at Graz [Græcij] and Vienna. He wrote in defence of the Gregorian calendar against Sethus Calvisius [Sethum Caluiſium], where he also wrote against Scaliger's *Diatribe on the Precession of the Equinoxes*; a *Geographical Problem* on the discrepancy in the counting of days between those who sail to the New World and those who stay there; the *Centrobaryca*; and other works. He died in 1643.

Paul Hainzel [Paulus Hainztlius] [recte: Hainzelius, per the book's errata], consul of Augsburg, observed the stars from the year 1572 at Göggingen [Geggingę] near Augsburg of the Vindelici with a very large quadrant of 14 cubits. For his praises see Tycho, volume 1, from p. 352 and at p. 444.

Paulus Philosophus [Paulus Philoſophus] wrote an *Introduction to Astrology* in the 9th century of Christ.

Petosiris [Petoſiris, ſeu Petoſciris], or Petosciris, an Egyptian, treated astrological matters out of the sacred books, at the end of the 9th century of Christ.

Pierre d'Ailly [Petrus Aliacenſis], cardinal of the Holy Roman Church [S. R. E. Cardinalis] and bishop of Cambrai, chancellor of the University of Paris and teacher of Jean Gerson [Ioannis Gerſonis], wrote questions on the *Sphere* of Sacrobosco, on the reform of the calendar, and a *Concordance of Theology and Astronomy*, besides theological and ecclesiastical treatises. He took part in the Council of Constance in 1416, and urged it to repair the calendar.

Peter Apian [Petrus Apianus], of Leipzig [à Leypziga], flourished chiefly at Ingolstadt from the year 1530. He wrote a *Cosmographia*, revised and enlarged by Gemma Frisius, with observations or notes of many eclipses; an *Instrument of the Primum Mobile* [inſtrumentum primi Mobilis], with a hundred propositions or problems; and the *Opus Caesareum*, in which, with most ingenious wheels and automata, he teaches how to solve almost all astronomical problems — with an added treatise on the comets he had observed, and on the torquetum.

Pietro d'Abano [Petrus Aponus], called the Conciliator, flourished in 1317.

Pietro Catena [Petrus Catena] wrote a *Sphere* at about the same time as Giovanni Pierio Valeriano [Io. Pierius Valerianus].

Peter Crüger [Petrus Crugerus], mathematician of Danzig [Dantiſcanus], besides logarithmic tables wrote an *Astronomia Dantiscana*, which however he

did not bring to completion, as Hevelius, his own pupil, testifies on p. 167 of the *Selenographia*. He also wrote a *Uranodromus cometicus*.

Pedro Ciruelo [Petrus Ciruellus], a theologian, wrote learnedly in favour of astrology — but a sound astrology — against Pico. I have not seen him, but Caussin names him in book 3 of *De Domo Dei*.

Pierre Gassendi [Petrus Gaſſendus], canon of Digne, distinguished for his astronomical observations made at Paris, Digne, Aix [Aquis Sextijs] and Marseille, and a man of wonderful candour in style, in mind and in character, is still flourishing, and has flourished from the year 1625. In 1642 he wrote on the apparent size of the Sun low and high; on motion impressed by a mover that is itself carried [de motu impreſſo à motore tranſlato], where there is much in favour of the Earth's motion; likewise a judgement on the new stars seen around Jupiter; and two books of *Institutiones Astronomicae*, on the occasion of the chair of mathematics conferred on him at Paris.

Pierre Hérigone [Petrus Herigonius], professor of mathematics at Paris, wrote a *Cursus Mathematicus*, in whose volume 4, published in 1644, he treats of the sphere of the world and of the use of the geographical map, and in volume 5 the theory of the planets.

Pedro Nunes [Petrus Nonius] of Alcácer do Sal [Salacienſis] wrote on twilights and on the errors of Oronce Finé [Orontij]; then astronomical problems, and rules for observing maritime matters and celestial phenomena, and on the motion of a ship; and on Peuerbach's *Theoricae* [in Theoricas Purbachij], where in note 1 he says: "In our time, that is, in the year 1552 from the birth of Christ, etc.," and yet in the same notes he says he was born in 1492; so at that time he was in his 60th year.

Pietro Pitati [Petrus Pitatus] of Verona wrote an *Isagoge to the Ephemerides*, and on establishing a new calendar, addressed to Pope Paul III in 1537; to this end he had made some observations.

Pierre de la Ramée [Petrus Ramus], a pupil of Oronce Finé [Orontij Finei], published in 1569 two books of *Arithmetic* and 27 of *Geometry*, and in 1566 31 books of *Scholae Mathematicae*, in the earlier of which he treats much concerning the origin and growth of astronomy. Is this the physician whom Giuntini, in his *Astronomical Calendar*, says was born in 1532, on February 22, at $3^h\ 23^m$ after noon?

Phlegon, an eminent chronographer, a freedman of Hadrian, according to Génébrard, book 2 of the *Chronographia*, p. 117, and Volaterranus [Volaterrano] in the *Anthropologia*, book 18, col. 567.

Philipp Apian [Philippus Appianus], son of Peter Apian, declared his judgement on the star of the year 1572 in a letter, which Tycho quotes in volume 1 of the *Progymnasmata*, p. 643.

Philips van Lansbergen [PHILIPPVS LANSBERGIVS] of Ghent [Gandauenſis], in the year of the Christian era 1632 — which he thinks is truly 1635 — when he was in the 71st year of his age, published at Middelburg tables of the celestial motions accommodated to the meridian of Goes [Goeſę], with

a *Treasury of Observations* and *Theories of the Planets*, assembled by the labour of almost 44 years. He also published an *Uranometria*, *Progymnasmata* on the motion of the Sun, a work *On Triangles*, and *Commentationes* on the diurnal and annual motion. These are held in great esteem, and rest partly on the observations of others and partly on his own, and on those of Martinus Hortensius [Martini Hortenſij].

Philolaus of Croton [Philolaus Crotoniata], a pupil of Pythagoras, one of those who said the Earth moves and that all things come about by a kind of harmonic necessity, was of such renown that Plato bought his books from his kinsmen for forty Alexandrian minae, and from them transcribed the *Timaeus*; but see Laertius on Philolaus. He flourished in the year 430 before Christ.

Philosophus [Philoſophus], a man called by this as his proper name in the Suda [apud Suidam], a pupil of Plato around the year 380 before Christ, wrote on the eclipse, on the distance and size of the Sun, Moon and Earth, and on the planets.

Pytheas of Massalia [Pitheas Maſſilienſis], a cosmographer cited by Strabo, flourished under Alexander the Great around the year 320 before Christ.

Plato [PLATO Athenienſis] of Athens, who treats astronomical matters and the system of the heavens in the *Timaeus*, the *Epinomis* and others of his dialogues, was born in Olympiad 88, year 1, on day 7 of the month Thargelion — that is, in the year 428 before Christ. He died in his 81st year, in Olympiad 108, when Theophilus was archon at Athens, namely in the year 348 before Christ, as Laertius records, while reclining at a wedding feast; granted that Neanthes says he died at the age of 84. Speusippus succeeded him.

Pliny the Elder [Plinius Senior], who has much on celestial matters in book 2 of the *Natural History*, flourished under Vespasian, and perished on Vesuvius at the age of 55, in the year of Christ 80, while examining its crater too inquisitively, as Pliny the Younger relates in book 6, letter 16; [see also] Volaterranus [Valaterranus] in the *Anthropologia*, and Génébrard.

Plutarch of Chaeronea [Plutarchus Cheronenſis] shows himself not ignorant of astronomy in the little work *On the Face of the Moon*, in the books *On the Opinions of the Philosophers*, and elsewhere. He flourished from the year of Christ 70 to 104, and was the teacher of Trajan. See Eusebius, Génébrard, Volaterranus [Valaterranum] and Vigarius [Vigarium] [?].

Polemon, a geographer, a hearer of Panaetius of Rhodes, completed a description of the world, and flourished in the year 640 before Christ, according to the Suda.

Pomponius Mela, who wrote on the situation of the world, flourished in the year of Christ 47 under Claudius, and Génébrard assigns him to the first century; but the man who prefixed a preface to Pomponius doubts whether he flourished under Claudius or under Augustus.

Posidonius [Poſſidonius], a pupil of Panaetius, a mathematician according to Pliny the Elder and an astronomer, a geographer according to Strabo, observed Canopus at Rhodes. He flourished under Pompey, who did not blush to knock

at his door and to listen to him; so he lived in the year 50 or 60 before Christ. Cicero, too, in book 2 of *On the Nature of the Gods*, calls him his friend, and commends the sphere he built, which displayed the motions of all the planets.

Proclus Lycius, that is, from Lycia, surnamed Diadochus, was, as the Suda testifies, a Platonic philosopher and a pupil of the great Syrianus, and taught at Athens for a long time. He wrote commentaries on Euclid, on the sphere, on the astrolabe, and on the *Hypotyposes of Astronomy*, in book 2, chapter 23 of which he has these words: *"Taking the parallax in the Sun, since the Earth does not have at all the ratio of a centre and a point with respect to the sphere of the Sun, as Hipparchus and Ptolemy themselves held."* So he was later than Ptolemy, who flourished under Hadrian and Antoninus. And yet Stöffler [Stœflerus], on the *Sphere* of Proclus, p. 1, says that he was the teacher of Marinus of Neapolis [Marini Neapolitani], who wrote the life of Proclus under Hadrian; by this conjecture he holds that Proclus lived under Trajan — though it could still be said that he lived some few years after Ptolemy. Barozzi, however, in the preface to his scholia on Proclus on book 1 of Euclid, says that he was the teacher not only of Marinus but also of Marcus Antoninus [Marci Antonij], and was raised by him to the consulship, on the report of Spartianus. There were therefore two men named Proclus. One was older than Ptolemy, whom Giuntini, in his *Catalogue of Illustrious Mathematicians*, not wrongly places in the year of Christ 84. The other was later than Ptolemy: he wrote the *Astronomical Hypotyposes* and was given the surname Diadochus. And this, unless I am mistaken, is that Proclus the eminent mathematician who, as Zonaras relates in part 3 of the *Annals* — and from him Pierre de la Ramée in book 1 of the *Scholae Mathematicae*, and Baronius at the year of Christ 514 — burned with bronze burning-mirrors the fleet of Vitalian [Vitaliani] as it besieged Constantinople, and was the interpreter of the Emperor Anastasius's dreams; and he is the same man who, as the Suda testifies, in imitation of Porphyry the Platonist spewed out 18 *Epicheiremata* [Epicheremata] against the Christians, which were outstandingly refuted by John the Philosopher [Ioanne Philoſopho]. Biancani, in the *Chronology of the Mathematicians*, placing him within the fifth century of Christ, and Georg Henisch [Georgius Heniſchius], on the *Sphere* of Proclus, take him to be the author of the Proclan *Sphere*. Stöffler [Stoeflerus], however, on the same *Sphere*, p. 1, attributes it to the older Proclus, who flourished under Trajan, and thinks he was called Diadochus by a family surname, from a gem resembling beryl, described by Pliny the Elder in book 37, chapter 10, which gave the family its surname.

Jacob ben Machir ibn Tibbon [Profatius, ſeu Prophatius Iudæus], called Profatius or Prophatius, a Jew, observed the Sun's greatest declination as 23° 32′, 230 years after al-Zarqālī [Arzachelem] and 160 after al-Maʾmūn [Almæonem] — that is, in the year of Christ 1300 — as Copernicus relates in book 3, chapter 2, and Reinhold in the *Theoricae*, p. 238. Kues [Cuſanus] relates that he observed the equinox in 1303.

Prometheus, as Aeschylus has it in the tragedy of Prometheus, and Servius on the 7th Eclogue, first made astronomy known to the Assyrians, having acquired it on the very lofty Mount Caucasus, around the year 1590 before Christ.

Prosdocimo de' Beldomandi [Proſdocimus de Beldomando] of Padua, a little before Giovanni Battista Capuano [Io. Bapt. Capuanum], wrote commentaries on the *Sphere* of Sacrobosco, which came out in 1531 through the work of Luca Gaurico [Lucæ Gaurici].

St Prosper of Aquitaine [S. Proſper Aquitanus], bishop of Reggio [Rhegienſis], wrote an Easter cycle of 532 years, and died in the year of Christ 466.

Claudius Ptolemy [CLAVD. PTOLEMÆVS], prince of astronomers, geographers and astrologers. Whether he was of the royal stock of the Ptolemies, and an Alexandrian, is disputed. That he was is affirmed by Vincent in his *Speculum historiale*, and by Andreas Trapezuntius, son of Georgios, in the preface to his translation of the *Almagest*. Having listed the men illustrious for learning whom Alexandria produced, he adds: *"But by everyone's leave, let me say it: in the reign of Antoninus she brought forth this Ptolemy, sprung of royal stock, easily the first of all the philosophers she nourished, in letters, in talent and in virtue. When the kingdom of the Ptolemies had ended in Cleopatra under Octavian, and Egypt had been reduced to a province, he himself, a private man but of royal spirit and talent, gave himself not to sordid crafts, not to an idle life, not to retreat into solitude; but in the then most celebrated gymnasium of Alexandria, with no mean resources, he devoted himself wholly to letters — first to philosophy, then to mathematics," etc.* Tycho held the same view in the epigram he placed beneath the portrait of Ptolemy that he kept in his museum, as you have it in the *Epistolae*, p. 239. For thus he sang:

> Such were the eyes, the face, the beard, such the limbs borne by that Claudius whose name was Ptolemy. He was indeed sprung from the illustrious line of kings, where the Nile wanders through the meadows with its watering streams. Yet the Roman soldiery, raging through the whole world, did not deem him worthy of royal honour. This he bore patiently, though unwilling, and nursed the hope that he might be one whom a greater honour perhaps awaited. Nor was he disappointed in this — although the Egyptian fields, although the wealth of the Nile, he did not restore. Indeed, still a private man, he held a greater empire than Rome, and was greater than the whole world. Etc.
>
> [Sic oculos, faciem, barbam, ſic membra gerebat / CLAVDIVS is, nomen cui PTOLEMÆVS erat. / Ille quidem illuſtri Regum de ſtemmate cretus, / Nilus vbi irriguis prata pererrat aquis. / Regali tamen hunc non eſt dignatus honore / Romanus toto miles in orbe furens. / Id tulit inuitus patienter, ſpemque fouebat / Se fore, quem maior fortè maneret honos. / Nec fruſtratus in hoc, quamuis Ægyptia rura, / Quamuis Niliacas non reparauit opes. /

Quin priuatus adhuc maius, quàm Roma, gerebat / Imperium, toto maior & orbe fuit. &c.]

Luca Gaurico, however, in the letter before the *Almagest* addressed to Domenico Pallavicino [Dominicum Pallauicinum], speaks thus: *"Our Claudius Ptolemy of Pheludia [Pheludienſis], easily the prince of all the mathematicians who have been, who are, and who will be in years to come, published at Alexandria in Egypt — the city founded by Alexander the Great — the* Almagest*, that is, the work of the great construction, a truly divine work, and, I believe, the* Geography*. He ended his last day when he was in the 78th year of his age, in the year 147 of the saving Incarnation. Whether he also wrote the four books of the* Apotelesmata *and the hundred aphorisms, and whether he was one of the kings of Egypt, I would not dare to affirm."*

Cardano [Cardanus], in the proem to Ptolemy's *Quadripartitum*, calls him a native of Pelusium [Peluſienſem], called an Alexandrian because Pelusium is not far from Alexandria. Magini, however, in his commentary on Ptolemy's *Geography*, notes that Pelusium is 160 miles from Alexandria, that it was once called Eliopolis [Eliopolim], and that today it is not called Damietta [Damiatam], as Castaldo [Caſtaldus] and Ziegler [Zieglerus] think, but rather *Tenesse* [Teneſſe]; and that Ptolemy was called an Alexandrian because he lived long at Alexandria and observed there. Magini denies, moreover, that he was descended *from the ancient kings, the Ptolemies*, since he did not live in the times of the Ptolemies, and in his *Geography* he gives place-names instituted by the Roman emperors, such as Iuliopolis, Forum Tiberii, etc. But these arguments, which Magini himself calls evident, are unworthy of Magini, otherwise a man of keen judgement: for the question is not whether Ptolemy was in fact a king of Egypt, but whether he was descended from their stock.

The authority of the prince Albuguate [principis Albuguate] seems to press harder, in the book *On the Choice of Sciences and the Beauty of Words* [de ſcientiarum electione, & verborum pulchritudine], which is prefixed to the old translation of the *Almagest* published at Venice in 1515 under the direction of Peter Liechtenstein of Cologne [Petri Liechteſtein Colonienſis]. Albuguate affirms that Ptolemy was born and raised at Alexandria, that his lineage was from the land of Shem [de terra Sem] and from the province called Pheuludia, and that he was not one of the kings of Egypt who were called Ptolemies, granted that he had that name. He adds that he was moderate in stature, white in complexion, long in stride, with slender feet; marked with a red sign on the right jaw; with a thick black beard; with the front teeth exposed; with a small mouth; of good and sweet speech; strong in anger and slow to be appeased; that he was accustomed to walk about much and to ride; that he ate little and fasted much; that his breath was fragrant and his clothing neat; and that he died in the 78th year of his age. After his apophthegms, in which he depicts Ptolemy's truly royal mind and lofty wisdom, he adds that Ptolemy, invited by a certain king to a banquet, excused himself and told his friends that kings are like certain paintings, which please when viewed from afar but displease when

seen closer. From this it seems to follow that this was not the Ptolemy who foretold the empire to Otho and accompanied him into Lusitania, as Tacitus relates.

Whether, then, he was of the stock of kings and an Alexandrian by birth or not, it matters little to the glory of a man who obtained the primacy of geography and of astronomy. By his geographical labour he brought the whole world into the dominion of his mind; and by restoring astronomy he raised his head, crowned with stars, to the heavens, and his name to the ages. Suidas, however, plainly errs when he writes that this Ptolemy lived under Augustus and Mark Antony. For if he died in the year 147 of the Christian era, aged 78, he was certainly born in the year 69 or 70, when Galba and Otho reigned; and from the same fact I again infer that he is not the Ptolemy who, as Tacitus relates, foretold the empire to Otho and accompanied him into Lusitania. Ptolemy himself, moreover, in the *Almagest*, often records his observations in the time of Hadrian and Antoninus, and we have no observation of his older than the one in book 4, chapter 9, where he reports an eclipse of the Moon observed by himself in year 9 of Hadrian, that is, of Christ 125, when he was in his 56th year. And in book 7, chapter 3 of the *Almagest* he says: *"From the times of Hipparchus's observation to the reign of Antoninus, in which we observed very many advances of the non-wandering stars, about 265 years are reckoned."* He had said, moreover, that Hipparchus's last observation, near Regulus, was made in year 50 of the third Callippic period, namely 128 before Christ; subtracting these from 265 years, these Ptolemaic observations fall in the year of Christ 136 or 137. So much for Ptolemy's stock, birthplace, and the time in which he lived.

As for his books, Pierre de la Ramée, in book 1 of the *Scholae Mathematicae*, judges thus: *"How great and how celebrated a mathematician was Ptolemy! For the volumes he published on celestial matters contain a kind of library of all earlier ages and schools (as I have often testified of the books of Aristotle), but defined and distributed by a more convenient method and art. We see that he condensed the twelve books of Hipparchus, and the six of Menelaus on chords [de ſubtenſis], into five theorems. He comprises Egyptian astrology — indeed the Latin astrology brought under the empire of Caesar — in an equal compendium; and in the* Quadripartitum *he names the observations of the Chaldeans expressly. The* Music*, the* Itinerary of the Stars*, the* Planisphere *or* Analemma*, and the* Geography *are as many ornaments of a noble mind."* He could have added the little work *On Mirrors*. But there are three principal works of Ptolemy: eight books of the *Geography*; thirteen books of the [Greek: μεγάλης συντάξεως (megalēs syntaxeōs, "of the great composition")], that is, of the *Great Construction*, or *Almagest* — a word that is either compounded of the Arabic *al* and the Greek [Greek: μέγι[?]ον (megiston?, "greatest")], or from the Egyptian *Megasite* [Megaſite], and signifies "the greatest" or "the perfect", as if you were to say that greatest and perfect work. And finally, the [Greek: τετράβιβλος σύνταξις (tetrabiblos syntaxis, "four-book composition")], that is, the *Quadripartite Syntaxis*, or *Quadripartitum*, in which he treats of the *Apotelesmata*, or of the judgements of the stars; to it is

attached the book called [Greek: καρπὸν (karpon, "fruit")], that is, the *Fruit* — namely the *Centiloquium*, or hundred aphorisms, of Ptolemy.

That the Ptolemy who composed the *Almagest*, the *Geography* and the *Quadripartitum* is one and the same is gathered from Ptolemy himself. For at the beginning of the *Quadripartitum*, text 1, he says: *"There are, Syrus, two chief means by which one arrives at astronomical predictions: one, which is first in order and in power, by which we grasp the motions of the Sun, the Moon and the other stars, and their configurations with one another and with the Earth; the other, by which, from the natural property of those configurations, we consider the changes of the things subject to them."* And shortly after, in text 2, he adds: *"And the first of these has its own study, worthy in itself, even if the end is not reached by joining the other to it; and we have explained it in particular in another book, with its reasoning examined by as certain arguments as we could."* By this he indicates clearly enough that he had already treated separately the matters contained in the *Almagest*.

Again, in book 8 of the *Geography*, he says: *"But since we have shown in the Mathematical Syntaxis that the sphere of the fixed stars also moves in the order of the signs [in consequentia Mundi] with respect to the tropical and equinoctial signs, and not about the equinoctial poles but about those equidistant from the circle drawn through the zodiac,"* etc. And conversely, at the very end of the last chapter of book 2 of the *Almagest*, he says: *"With the business of the angles now completed, it would follow that we compute the distances of the illustrious cities in longitude and latitude according to the phenomena. But since this business is of chief importance in itself and is connected with the geographical method, we shall show it separately, following those who, so far as was possible, have handed down this kind of record; and we shall write down by how many degrees each of the illustrious cities is distant from the equinoctial along the meridian described through it, and by how many degrees the meridian of each city is distant on the equator, to the east or west, from the meridian drawn through Alexandria. For it is to this meridian that the times of our computations are accommodated."*

So what Ptolemy promised in this place he carried out in book 8 of the *Geography*, writing down the distances of places from Alexandria in Egypt, and in both works placing Alexandria at a pole-height of 30° 58′ — for so he has it in book 5 of the *Almagest*, chapter 13, and in book 4 of the *Geography*, chapter 13 — just as in both he puts the obliquity of the ecliptic, or the distance of the tropics from the equator, at 23° 50′; and he keeps the same method in finding the sides and angles of triangles. Nor is it an objection that in book 7 of the *Almagest*, chapter 3, he makes Rome west of Alexandria by 1^{h} 20^{m}, or 20°, while in the *Geography* he gives Alexandria a longitude from the Fortunate Isles of 60°, and Rome 36° 40′, and so makes Rome west by 23° 20′. For when he completed the *Geography*, having become more learned in this matter than his former self, and following better sources, he changed his opinion, and corrected what in the *Almagest* he had perhaps assumed from Marinus of Tyre [Marino Tyrio]. Indeed, it is gathered from book 8 of the *Geography* that he changed even this

second opinion, since there he puts an hour and a half between Rome and Alexandria, that is, 22° 30′ — unless there is a fault in these numbers.

Finally, it is no objection that Ptolemy, in book 1 of the *Geography*, chapter 8, mentions Julius Maternus [Iulij Materni]; for this is not the younger Julius Firmicus Maternus, who flourished under Constantine, and who, in the preface to book 2, in chapter 32 and elsewhere, mentions our Ptolemy. I admit, however, with Pétau in the preface to the *Uranologion* and in book 2 of the *Dissertationes*, chapters 4 and 8, that Ptolemy's *Parapegma* on the appearances of the fixed stars and their significations is twofold: one, which Pétau himself translated from the Greek, is not unworthy of Ptolemy, and is printed from p. 71 of the *Uranologion*; the other was translated by Niccolò Leonico [Nicolao Leonico], and is not Ptolemy's but a rhapsody gathered out of a miscellany of various parapegmata that do not agree with one another at all.

Lastly, on the translations of the *Almagest*. Joseph Justus Scaliger [Ioſephus Scaliger], in book 5 of *De emendatione temporum*, p. 394, asserts that he saw a very ancient translation from Greek into Arabic, in whose preface were these words: *"This book was translated by order of Maimon, king of the Arabs, who reigned in Baghdad [Baldach], by Alhazer son of Joseph son of Maire the arithmetician, and by Serigus son of Elbe, a Christian, in the year 212 of the sect of the Saracens [in Anno CCXII. ſectæ Saracenorum]."* He adds that the new moon of Muharram, the first month of the year 212 of the Saracens, fell on 827 April 2, and therefore that both al-Maʾmūn [Almamonem], or King Maimon, and this translation were older than al-Battānī [Albategnio]. Boulliau [Bullialdus] has the same in the prolegomena to the *Astronomia Philolaica*, and adds that by order of the Emperor Frederick II the *Mathematical Syntaxis* was translated from the Arabic language into Latin around the year 1230. In book 3, chapter 2, besides the Greek codex of Basel, he also cites a manuscript of the Royal Library of Paris. Five editions, moreover, circulate widely. The first is the Greek, published at Basel in 1538 with the commentaries of Theon, in which the general title is [Greek: κλ. πτολεμαίου μεγάλης συντάξεως βιβλ. ιγ. (Kl. Ptolemaiou megalēs syntaxeōs bibl. 13, "Claudius Ptolemy's Great Syntaxis, 13 books")], that is, *Claudii Ptolemaei Magnae Syntaxis libri 13*; but in the particular titles of the books it has [Greek: μαθηματικῆς συντάξεως (mathēmatikēs syntaxeōs, "of the mathematical composition")], that is, *of the Mathematical Syntaxis*, which is how Ptolemy himself names it in book 8 of the *Geography*. The second is the Latin, which came out at Venice in 1515 through the work of Peter Liechtenstein of Cologne. The third, likewise Latin, is that of Georgios Trapezountios [Georgij Trapezuntij], revised by Luca Gaurico, which came out at Venice in 1527. The fourth, of the same Trapezountios, was published at Basel in 1541 through the work of Hieronymus Gemusaeus [Hieronymi Gemuſæi]. The fifth, again of Trapezountios, was published at Tübingen by Schreckenfuchs [Schreckenfuxio] in 1551; Kepler also consulted it in the *Catalogue of the Fixed Stars* [Catalogo Affixarum], p. 114 of the Rudolphine Tables, where he says that the Alphonsines and Copernicus followed the Arabic version. Besides

these I have seen no other. Whether it is entitled the *Mathematical Construction* by antonomaſticῶς], because astronomy is the most excellent of the mathematical disciplines, or the *Great*, because it is a great — indeed the [Greek: μέγιστον (megiston, "greatest")] — work: Gemusaeus, in his dedicatory letter, holds that just as nature produced Aristotle in philosophy, so it produced Ptolemy in astronomy, *"to form a supreme man, whom it might set up as an example of all its powers, and show how far the mind of man may advance in this kind, so that it can go no further, in the method of speaking and the manner of inquiry which he was the first to perfect in this discipline."*

Pythagoras of Samos [PYTHAGORAS Samius], son of Mnesarchus [Mneſarchi], an engraver of rings, learned astrology among other things in Egypt and from the Chaldeans, and discovered music in the heavens. As Laertius testifies, he flourished in Olympiad 60, that is, in the year 540 before Christ, and was killed at the age of 80 — or, as most have it in Laertius, at 90. He also showed that Lucifer and Hesperus, which were thought to be two planets, are one and the same, namely the star of Venus, according to Pliny the Elder, book 2, chapter 8. He came to Italy in the year 245 from the founding of the City, or 509 before Christ, according to Solinus, chapter 17. He died in the year 497, after he had been at Croton for 12 years and had led the Italian school, as Eusebius has it. The people of Metapontum worshipped him as a god, as Justin says in book 20. Of the Pythagorean intervals of the stars we shall say more in their place. Plutarch, in book 2 of the *Placita*, chapter 12, says that the obliquity of the zodiac was first detected by him, and yet that Oenopides of Chios claimed this discovery for himself; but Pliny the Elder, book 2, chapter 8, attributes it to Anaximander.

Pytheas: see Pitheas.

R

Rabbi Avraham [RAbbi Abraham] wrote on the sphere in the year of Christ 1115.

Rabbi Adda, son of Ahabac [Rabbi Adda filius Ahabac], flourished at the end of Diocletian's reign and the beginning of Constantine the Great's. He arranged the Hebrew calendar and handed down rules for finding the *tekufot* [Tekuphas], or revolutions of the equinoxes — although many have preferred to follow Rabbi Shmuel [Rabbi Samuelem].

Rabbi Yitzchak [Rabbi Iſaac], an astrologer, in the eleventh century of Christ, according to Génébrard.

Rabbi Levi [Rabbi Leui], full of all the sciences and most devoted to astronomy, wrote the book *Milhamot Hashem* [Milhamot-Heſsem], that is, *Of the Defences of God* [defenſionum Dei], in the year of the Incarnation 1335. In treatise 5 of it he says that in his time Spica and Regulus held the places in the heavens that were due to them according to al-Battānī [Albategnium], as Agostino Ricci [Riccius] relates in the treatise *On the Eighth Sphere*, chapter 43.

Rabbi Moshe [Rabbi Moyſes], in the book *Mishneh Torah* [Miſnetora], that is, *of the Law Repeated*, treated of the double motion of the eighth sphere.

Raffaele Maffei of Volterra [Raphael Volaterranus], chronographer and cosmographer, flourished in the year of Christ 1495.

Redento Baranzano [Redemptus Baranzanus], of Serravalle [Serrauallenſis] in the diocese of Vercelli, a Clerk Regular of St Paul, taught physics and astrology at Annecy [Anneſij] in Savoy, and wrote an *Uranoscopia* in 1617.

Robert Grosseteste [Robertus Linconienſis], bishop of Lincoln, wrote a *Compendium of the Sphere*, published in 1531 by Luca Gaurico. He flourished under the Emperor Conrad in 1140, and was called by the surname *Grossa testa* [Groſſa teſta]; he was a most subtle theologian, as Sisto da Siena [Sixtus Senenſis] says in the *Bibliotheca*.

Rudolph Goclenius [Rodulphus Goclenius], doctor of medicine at the University of Marburg [in Academia Marpurgenſi], published in 1615 his *Urania*, with her twin daughters Astronomy and Astrology.

Rumold Mercator [Rumoldus Mercator], son of Gerard [Gerardi], flourished in 1600.

S

Solomon [SAlomon], who by wisdom divinely infused knew *the courses of the year and the dispositions of the stars*, as is said in Wisdom 7, flourished in the year 1012 before Christ.

Scipione Chiaramonti [SCIPIO CLARAMONTIVS], a knight of Cesena [Cæſenas Eques] and formerly professor of philosophy at the University of Pisa, highly expert in mathematical matters, published many excellent works. The chief of them are: *Antitycho*, for sublunary comets, 1621; *Apologia pro Antitychone*, published 1626; *De tribus novis stellis* [De tribus nouis Stellis], against Tycho and others, 1628; a second *Defensio* of the *Antitycho*, in Italian, 1633; *Supplementum Antitychonis*, on the sublunary seat of comets, 1636; *Antiphilolaus*, 1643; a *Defensio* against the attacks of Fortunio Liceti [Fortunij Liceti], 1644; and sixteen books *De Universo* [de Vniuerſo], in the same year 1644. He is still living, in this year 1651 in the 86th year of his age; for in 1572 he was seven years old, as he himself relates in the preface to his book on the new stars.

Sebastian Münster [Sebaſtianus Munſterus], condemned among the heretics of the first class, 1535; he wrote notes on Ptolemy's *Geography* and a *Cosmographia universalis* [Coſmographiam vniuerſalem].

Seneca flourished under Nero, and wrote on comets in Book 7 of the *Natural Questions* [quæſtionum naturalium]; he was put to death in the year of Christ 65.

Sextus Avienus Ruffus [Sextus Auienus Ruffus], a learned poet, a Spaniard, as some suspect according to Pietro Crinito [Petrum Crinitum], book 5. Besides Aratus's *Phaenomena*, which he rendered elegantly in Latin verse, he completed a book on the circuit of the earth in hexameters, imitating Dionysius; and in iambics he wrote *De Oris Maritimis* [On the Sea-Coasts]. Giraldi [Gyraldus], however, in dialogue 4, says that he did not so much imitate Dionysius as translate him from Greek into Latin. Crinito says he flourished under Diocletian; but Giraldi, from St Jerome [S. Hieronymo] and from the inscription of the

poem addressed to Probus, a man of consular rank, to whom Claudian wrote, proves that he flourished with Claudian under Theodosius, about the year of Christ 395. Génébrard [Genebrardus] places him under Pope Melchiades.

Sextus Empiricus, nephew of Plutarch of Chaeronea [Plutarchi Chęronenſis nepos] and disciple of Herodotus, flourished under Antoninus about the year of the Lord 135. He wrote sharply against the mathematicians, that is, the judicial astrologers [Aſtrologos Iudiciarios].

Simplicius, born in Phrygia, a distinguished Platonist and Peripatetic philosopher and an excellent commentator on Aristotle's *De caelo* [de cælo], flourished at Alexandria from the year of Christ 500 — but of Diocletian 216. Some, not distinguishing these years, assign him to the year of Christ 216 or 224. See Génébrard under Silverius [in Syluerio], and the Suda under the word *Damascius*, who was a friend of Simplicius.

Simon Marius wrote a book on the companions of Jupiter [de Iouis Comitibus], entitled *Mundus Iovialis* [Mundus Iouialis]; he flourished about the year 1610.

Simon Stevin [Simon Steuinus], of Bruges, flourished at the beginning of this century; he set out the theorics of the planets [Theoricas Planetarum] and tables of the celestial motions.

Sixtus ab Hemminga, a Frisian, had at first supported judicial astrology [Aſtrologiæ iudiciariæ] and studied it long and much; later he opposed it. He undertook to demonstrate its falsity from the nativity of Adrianus Junius [Adriani Iunij] and of Charles V [Caroli V.], under whom he flourished, and from many others, and he pronounced the art impossible and useless to mankind.

Sisto da Siena [Sixtus Senenſis], of the Order of Preachers, wrote one book of astronomical questions and one book of geographical questions on Sacred Scripture; he flourished in 1576.

C. Solinus Polyhistor wrote *De situ orbis* [de ſitu orbis]. Biancani [Blancanus] includes him in the first century of Christ; his scholiast places him a little after Vespasian; Sansovino [Sanſouinus] in his *Chronologia* assigns him to the year of Christ 114, with whom Raynaudus [?] agrees in his *Breviarium chronologicum* [breuiario Chronolog.], and Génébrard under Evaristus [in Euariſto].

SOSIGENES, an Alexandrian astronomer, was brought to Rome by C. Julius Caesar [C. Iulio Cæſare] and restored the Roman calendar, introducing a solar year of 365 days and, every fourth year, an intercalary year of 366 days; on which see Pliny the Elder [Plinium], Book 18, chapter 25. He lived long under Augustus and again gave his effort to the correction of the year. He flourished in 45 before Christ and afterwards. Macrobius, however, *Saturnalia* Book 1, chapter 14, relates that Flavius the scribe noted down the individual days and handed them to Caesar, and urged him to that restoration; on which see also Solinus chapter 3, Dio Book 43, Plutarch in the *Caesar*, and Censorinus chapter 8.

Stephanus Rosinus [Stephanus Roſinus], of Augsburg [ex Auguſta Vindelicorum], a philosopher and bachelor of Sacred Scripture at Vienna, where he was a canon, taught astronomy, and in 1500 published a table of the declination of the fixed stars, with prognostications.

Strabo, a Cappadocian and a distinguished geographer, described the world, a great part of which he had travelled through. Biancani places him within the first century of Christ. He flourished, however, not only under Tiberius but also at the end of Augustus's reign, and died in the 11th or 12th year of Tiberius, as Casaubon [Caſaubonus] has in his *Life of Strabo*; therefore before Christ he was:

Straton of Amasia [Straton Amaſenus], a philosopher, wrote seven books of *Geographica* in the first century of Christ, on the testimony of the Suda.

C. Sulpicius Gallus [C. Sulpitius Gallus], a military tribune, was the first among the Romans to predict an eclipse of the Moon, under Paulus Aemilius [Paulo Æmylio], and set out a booklet about it, in 168 before Christ.

T

Taruntius Firmanus, an astrologer and friend of Varro, flourished in 86 before Christ.

THALES of Miletus [THALES Mileſius], the first among the Greeks to predict an eclipse of the Sun, of the stock of Agenor and Cadmus, and the first of the Sages. He wrote on the solstices and equinoxes [de Solſtitijs, & Æquinoctijs]; he was the first to observe that the apparent diameter of the luminaries is the 720th part of its circle; he measured the pyramids of Egypt by their shadow. He was born in the first year of Olympiad 35, that is, 640 before Christ, and died at the age of 70, as Apollodorus has in his *Chronicles*, or 90, as Sosicrates has, or 92, since he died in Olympiad 58, as Laertius has. Callimachus, in a poem, says that he was the first to have measured out the little stars of the Wain [plauſtri ſtellulas], by which the Phoenicians steered their navigation.

Tadeáš Hájek [Thaddæus Hagecius ab Hayck], a Bohemian, wrote accurately on the new star of the year 1572; on which see Tycho, Volume 1 of the *Progymnasmata* [prog.], from p. 505.

THEBIT Ben-Chora, that is, son of Chore, preferred the sidereal year to the equinoctial. He introduced a tenth, immovable sphere, and the motion of trepidation from north to south. He observed the Sun's greatest declination [maximam declinationem] as 23° 33′ 30″, at about the same time as Almaeon [Almæon], as Jacob Christmann [Kriſtmannus] has in his commentary on al-Farghānī [in Alfraganum], chapter 6. He therefore lived in the 12th century of Christ, in which Almaeon also lived, as Génébrard also has, and our Gualterus in his *Chronicon*; and Giuntini [Iunctinus] sets him in the year 1198. But Biancani moves him forward into the 13th century of Christ, as does Clavius [Clauius] in the preface to the *Sphere*, where he places him 1140 [M C XL] years after Ptolemy, that is, in 1287 after Christ.

Theaetetus of Athens [Theætetus Athenienſis] was of great fame, especially in astronomy. He fitted the five regular bodies to the world and to the elements.

He was a contemporary and friend of Plato, who accordingly wrote a dialogue under his name.

Theodoret the Bishop [Theodoretus Epiſcopus] is counted among the astronomers by Giuntini, in the year of Christ 469.

Theodorus Gaza wrote a book in Greek on the months, and on the year, in which he showed himself not unacquainted with astronomy. He flourished about the year of the Lord 1478.

Theodorus Graminaeus [Theodorus Graminæus], professor of mathematics at Cologne, published a *Commentary* on the star of the year 1572 — but one of little astronomical worth, as Tycho judges, Volume 1, from p. 775.

Theodosius of Tripoli [Theodoſius Tripolita], in the year 52 before Christ, wrote *On Days and Nights* [de diebus, & noctibus], three books of *Spherics* [Sphæricorum], and sceptical astrological chapters [ſceptica capita Aſtrologica].

Theon of Alexandria [Theon Alexandrinus], mathematician: there were two of this name. One was a contemporary of Ptolemy and a little older; his observation of Venus, made in the 16th year of Hadrian, that is, the year of Christ 132, is adduced by Ptolemy in Book 10 of the *Almagest*, chapter 1, and from him by Copernicus, Book 5, chapter 20. The other was much later than Ptolemy: he published commentaries on Ptolemy's *Almagest* and on Aratus, was the father of Hypatia, and wrote on the rising of the Dog Star [Caniculæ] and on the small astrolabe. The Suda makes him a contemporary of Pappus. But Denis Pétau [Petauius], in the dissertation of his *Uranologion* [Vranologij], p. 96, shows that he flourished in the year of Christ 360, and Giuntini puts him in 390. Theon himself, in Book 6 on the *Almagest*, p. 334, reports an eclipse of the Sun observed by himself in the year of Nabonassar 1112, that is, of Christ 365.

Theophrastus of Lesbos [Theophraſtus Lesbius], surnamed Eresius [Eriſſius], wrote six books on astrology and succeeded Aristotle in the Lyceum, in the year 322 before Christ.

Thius of Athens [Thius Athenienſis], about the year of the Lord 500, devoted himself to observing the stars, as Boulliau [Bullialdus] relates in the prolegomena to the *Astronomia Philolaica* [Aſtronomiæ Philol.].

Thomas Digges [Thomas Diggeſeus], of Kent in England, on the occasion of the new star of the year 1572 wrote a booklet which he entitled *Alae seu Scalae Mathematicae* [alas ſcalaſque Mathematicas: Mathematical Wings and Ladders]; on which consult Tycho, Volume 1 of the *Progymnasmata*, from p. 653.

Thomas Fincke [Thomas Finkius], of Flensburg, flourished in 1583, when he published the *Geometria rotundi* with tables of sines; afterwards he observed stars and comets, as is clear from his letters, which Giovanni Antonio Magini [Maginus] cites in his *Tabulae directionum* [tabulis directionum], from p. 77, and from Tycho, Volume 2.

Timaeus of Locri [Timæus Locrus, ſeu Locrenſis], a Pythagorean, wrote on the nature of the world; from this work Plato took not only the name of his

dialogue *Timaeus* but in large part its content. Timaeus flourished when Plato was young, and Plato, at the beginning of that dialogue, affirms that he was most expert in astronomy.

TIMOCHARES, astronomer, observed the fixed stars in particular around the year 300, or 294, before Christ, as is gathered from Ptolemy, *Almagest* Book 7, chapters 2 and 3 — namely in the 36th year of the first Callippic period [primæ Calippi periodi].

Titus à Popma, a Frisian [Phryſius], wrote in 1568 tables on the *Sphere* [tabellas in Sphæram] and *Elements of Astronomy* [Elementa Aſtronomiæ].

TYCHO BRAHE, restorer of astronomy, first-born son of Otte Brahe [ex Othone Brahe], of eminent nobility in Denmark, was born in the year of restored salvation 1546 in the castle of Knudstrup [in arce Kundſtorpiana] in Scania. His astronomical life, and the sequence of his observations and works, we have gathered from his own books, but chiefly from the *Mechanica Astronomiae* [Mechanica Aſtronomiæ], from the first book of the *Epistles* [ex Epiſtolarum prima] to Moritz, Landgrave of Hesse [ad Mauritium Haſſiæ Lantgrauium], from the Appendix to Volume 1 of the *Progymnasmata*, from Kepler [Keplero] in the *Hyperaspistes*, p. 104, and in the preface to the Rudolphine Tables [ad Rudolphinas], and lastly from the Bohemian observations published by Willebrord Snellius [Villebrordo Snellio].

Tycho, then, came to Leipzig [Lipſiam] in 1560, at the age of 14, for the sake of humane letters. Having stayed there three years, with his genius urging him on to astronomy — though his tutor, who had set him to jurisprudence, opposed it — he secretly began to accustom himself to astronomical observations. He used a somewhat larger pair of compasses as a sextant, taking the distances of the stars with his eye set at the joint of the compasses, and a small globe of the stars. From these rudiments, without a master, he advanced so far that he noticed manifest errors in the Alphonsine and Prutenic calculations [in Alphonſinis, & Prutenicis Calculis], especially in 1563, when the great conjunction of Saturn and Jupiter took place in the first degrees of Leo, around the end of September; he saw that it departed by a whole month from the Alphonsine reckoning and by some days from the Prutenic table [Prutenico abaco], being then in his 17th year. In 1564, accordingly, he secretly obtained from Bartholomäus Scultetus [Bartholomæo Sculteto] a wooden astronomical radius [Radium Ligneum Aſtronomicum] made on the pattern of the radius of Gemma Frisius [Gemmæ Friſij], divided by transversal points according to the prescription of Homelius [Homelij]; and with it, on clear nights, unknown to his tutor, he took many distances of stars. He did the same at Wittenberg [Vuitebergæ] and Rostock [Roſtockij].

Afterwards, in 1569 and 1570, he stayed at Augsburg [Auguſtę Vindelicorum], where he persuaded Paul Hainzel [Paulo Hainzelio], consul for the year [Conſuli anniuerſario], to have a very large quadrant made, showing single minutes, which Tycho describes in Volume 1 of the *Progymnasmata*, p. 355, and in the *Mechanica*, p. 30. They had it set up at Göggingen [Geggingæ] on a hill

half a German mile from Augsburg, and observed the stars together. Tycho, however, was at that time also using a wooden sextant, but an imperfect one, and at the same time he was toiling at pyronomic labours [Pyronomicis laboribus]. Then, going to Herrevad [Herritzuadium] to his maternal uncle Steen Bille [Stenonem Bille], he obtained a place suitable for chemical and spagyric experiments. While he was there, in 1572 November 11, he began to take notice of the new star in Cassiopeia [Caſſiopeę]; he was using a different sextant, indeed, but he lacked instruments fit for the necessary precision, as he admits in Volume 1 of the *Progymnasmata*, p. 335.

Later he procured one instrument after another, which he carried with him through Germany so as to choose a seat suited to observations. Thus in 1575, with Italy in mind, he passed through Kassel [Caſſellis] around the Easter festival and stayed there more than a week, making some observations with Wilhelm, Landgrave of Hesse [Gulielmo Lantgrauio Haſſiæ]. Then, travelling through the northern parts of Italy, he had resolved to fix his astronomical seat at Basel [Baſileæ Rauracorum], for the healthiness of the air and its borders with Italy, France and Germany, and so as not to be drawn away from the sky again and again by his kinsmen and friends in his homeland. From there he came to Regensburg [Ratiſponam] for the Diet, at which Rudolf, son of the Emperor Maximilian [Rudolphus Maximiliani Imper. Filius], was crowned King of the Romans. From there he returned to his homeland before winter; and while he was preparing his journey to Basel, as though intent on something else, he was invited by letters and by a messenger from Frederick II [Friderico II.], King of Denmark and Norway, who offered him the island of Hven [Huennam] in the Danish Sound — which, Tycho says, may be called Scarlatina by foreigners and Venusia in Latin — and offered him funds for astronomical and pyronomic instruments and works.

Therefore in that same year 1576 he began to build the castle of Uraniborg [Vraniburgi], suited to astronomical work, and new instruments; and, having gathered others skilled in these matters, who were at least 8 or 10, he observed there over many years the fixed stars, the planets, and the comets of the years 1577, 1580, 1585, 1590 and 1593. In 1582 he determined the places of the fixed stars more accurately from the distance of Venus compared with the Sun; and in 1584, having detected refraction, he corrected the equinoxes and solstices, sending also Elias Olai [Elia Olai] to Frombork [Fruemburgum] in Prussia to examine the altitude of the pole taken by Copernicus. In 1585, when he was 40 years old, he observed the winter solstice exactly, and began to write chapter 10 of Volume 2; and in 1588 (up to which year he had extended his observations of the fixed stars) he began to write on the Sun and the Moon and to arrange their tables. In 1590 he wrote on the new star of the year 1572, and obtained a privilege [diploma] from Rudolf II that no others might print what he was permitted to print, etc. In the same year he wrote more accurately on the comet of the year 1577, and up to the year 1592 he applied himself to the treatise on the Sun, the Moon and the fixed stars. In 1593 he obtained another privilege, from

James, King of Scotland [Iacobo Rege Scotiæ], for the printing of his books, which he was printing at Uraniborg at his own expense. Further, in 1596 he published Volume 1 of the *Epistles* [tomum 1. epiſtolarum vulgauit].

Then, embittered and despairing of the expenses to be supplied by the successor of Frederick II, King of Denmark, he nevertheless meanwhile prepared part of the *Mechanica Astronomiae*, polished the catalogue of a thousand fixed stars, and began to print the second volume. So in 1597 he moved, with such instruments as he could take, from the island of Hven to Copenhagen [Hafniam], then to Rostock, and then, as the plague was raging, to Wandsbek [Vuandesburgum] in Holstein [in Holſatiam], at the invitation of Rantzau [à Ranzouio]. In 1598, summoned by the Emperor Rudolf II, he set out from Holstein for Prague but halted at Wittenberg until the plague at Prague should abate; and he completed and published the *Mechanica Astronomiae instauratae* [Mechanicam Aſtronomiæ inſtauratæ]. Then in 1599 he came to Prague, and from there to the castle of Benátky [in arcem Benaticam], granted to him by the Emperor for observing the stars; but on returning to Prague he changed house twice, and meanwhile received his larger instruments from the journey, carried by ship. While he was living in Bohemia, with Longomontanus assisting, he worked more intensively at perfecting the theory of the Moon.

To him, in 1600, in the month of February, Kepler [Keplerus] came at Benátky, invited from Styria by frequent letters from Tycho, on the occasion of the *Mysterium Cosmographicum* [Myſterij Coſmographici]; and Longomontanus was also there. An agreement having been made with Tycho, Kepler returned to Styria in June to fetch his family and his household of books; and in October of the same year, when Longomontanus had already left, Kepler was with Tycho at Prague. But, seized by a quartan fever and hindered by it until the solstice of the following year 1601, he was of no use to him; and since he had gone back to Graz [Gratium] on account of an inheritance and returned to Prague in September, he was with Tycho no more than two months.

For Tycho, in 1601, on the mornings of October 10 and 11, observed the distance of Mars from Pollux, Procyon and Regulus (which was his last observation, as it was also for Walther [vt & Vualthero]). On October 13, at a dinner, having held his urine beyond his custom while he drank too freely, and feeling a tension of the bladder, he put his health below courtesy; and when he had returned home he could not pass water. At the onset of this illness (says Snel) the Moon was in opposition with Saturn and in quartile aspect with Mars in Taurus, and Mars was in the same place that Tycho had set as his own Ascendant. With Mars thus hostile, and Venus giving no help, after five sleepless days the urine at last began to come drop by drop, with very severe torment. There followed continual sleeplessness, fever and delirium; and as his diet [ratione victus], from which he could not be restrained, aggravated the illness, he kept chanting in his delirium that saying of his, *Ne frustra vixisse videar* [Let me not seem to have lived in vain]. At last, on November 24, when

the delirium had abated, he died, having observed the stars for 38 years — but more accurately for 20 — and having set a thousand fixed stars in order.

After his death his heirs published Volumes 1 and 2 of the *Progymnasmata*; the third perished. At length, when his observations had been found in the possession of Kepler's nephew [apud nepotem Kepleri], they were handed over, by order of the Emperor Ferdinand, to Father Albert Curtz [P. Alberto Curtio] of the Society of Jesus [Societ. IESV], who is in charge of their edition. Happier indeed would Tycho have been had he gained heaven and CHRIST; for then he would truly not have lived in vain. But that he was too much inclined toward Luther, Philipp Melanchthon [Philippum Melanchthonem] and David Chytraeus [Dauidem Chytręum], pests of the human race, is shown plainly enough by what he has in Volume 1 of the *Progymnasmata*, pp. 712, 776, 777 — granting that Johannes van Heeck [Io. Heckius] of Deventer, in his disputation on the new star of the year 1604, p. 20, calls him a Calvinist: perhaps because at Prague he migrated from Luther to Calvin, or mixed both poisons for himself. Heckius was at Prague when he wrote this.

V

Valentin Naibod [Valentinus Naiboda] published in 1580 three books of *Astronomical Institutions* [Aſtronomicarum inſtitutionum]; but in 1560 at Cologne, where he taught mathematics, he published an *Enarratio elementorum astrologicorum* [enarrationem elementorum Aſtrologicorum], collected from al-Qabīṣī [Alcabitio] and other Arabs, on account of which he was listed among the condemned authors by the sacred censors.

Valens, an astrologer of Antioch, flourished under Constantine the Great, on the testimony of Zonaras.

M. Varro flourished in 86 before Christ.

Victorinus of Aquitaine [Victorinus Aquitanus], celebrated author of the Paschal Cycle [Cycli Paſchalis], and an astronomer, was called by Pope Hilary to the correction of the Roman calendar in 467.

Willebrord Snellius [Villebrordus Snellius à Royen] flourished at the beginning of this century. He wrote on the comet of the year 1618; a *Doctrina triangulorum* [doctrinam Triangulorum]; the *Eratosthenes Batavus* and the *Tiphys Batavus* [Eratoſthenem ac Typhim Batauum]. He published the Hessian and Bohemian observations, with some notes, together with the observations of Regiomontanus and Walther [Vualtheri], also in 1618. And he translated Ludolph of Cologne [Ludolphum Colonienſem] *De circulo*, with its additions, from the vernacular into Latin, and illustrated it with notes, in 1619.

Vicenç Mut [Vincentius Mutus], of Majorca [Maioricenſis], most expert in astronomy, observes the stars diligently at Majorca [Maioricæ], and wrote an excellent little work, *De Sole Alphonsino* [de Sole Alphonſino]. To him I owe a very great deal.

Witelo [VITELLIO], born of a Thuringian mother and a Polish father, is said to have flourished in 981 according to Giuntini [Iunctino], or in 984 according to Sansovino. But Friedrich Risner [Federicus Riſnerus], in the preface to

Witelo, shows, together with Erasmus Reinhold [Eraſmo Reinholdo] and Caspar Peucer [Gaſpare Peucero], that he lived in 1269, when there also lived Willem van Moerbeke [Gulielmus de Morbera], to whom Witelo dedicated his work, as penitentiary of the Supreme Pontiff. That he was a Pole, that he came to Italy and lived at Cubalum between Padua [Patauium] and Vicenza, and afterwards was at Viterbo [Viterbij], Witelo himself intimates in Book 10, theorems 42, 67 and 74.

M. Vitruvius Pollio [M. Vitruuius Pollio] wrote for Augustus ten books *De Architectura*, in the ninth of which he touched on astronomical matters. He served in the camp of Julius Caesar [in Caſtris Iulij Cęſaris] as prefect of the engines of war, as is gathered from his Book 8, chapter 4, and Book 10, chapter 16. He flourished, therefore, about the year 45 before Christ.

Václav Pantaleon Kirwitzer [Vuenceslaus Pantaleo], of the Society of Jesus, from the province of Austria, set out for China [in Sinas] with Father Nicolas Trigault [P. Nicolao Trigautio], and died at Macao [Macai] in 1626 May 22. He had written up observations of the comets of the year 1618 made by members of our Society in the East Indies [à noſtris in India Orientali].

Wolfgang Schuler [Vuolfangus Schulerus], professor of mathematics at the University of Wittenberg, treated in 1573 of the star of the year 1572; his reply to Peucer is reported by Tycho, Volume 1, p. 621.

Z

Zoroaster [ZOroaſtres], king of the Bactrians, wrote on predictions from the stars about 500 years before the Trojan War, that is, in 1990 before Christ.

End of the Chronicle [FINIS CHRONICI.]

LIBER PRIMVS DE SPHÆRA MVNDI IN COMMVNI ASTRONOMIÆ IMMO' ET COSMOGRAPHIÆ ELEMENTA, ET PRINCIPIA VNIVERSALIA COMPREHENDENS.

Book One: On the Sphere of the World in General, comprising the Elements and Universal Principles of Astronomy, and indeed of Cosmography.

Diagram. A woodcut armillary sphere, set obliquely, standing beneath the book title. The rings are drawn as bands with chequered (alternately black and white) edges. A small globe of the Earth, with land masses shown, sits at the centre. The axis runs from a short cylindrical pole-stub at the upper left to another at the lower right. The broad horizon ring crosses the sphere horizontally; the meridian ring is the outer circle; the equator, the two tropics, the two polar circles and a broad zodiac band carrying sign glyphs are set obliquely to the horizon. Labels, as printed:

- POLVS BOREAL. (upper left, beside the upper pole-stub): the North Pole
- MERIDIANVS. (along the outer ring, top): the meridian

- CIRC. AR CTICVS. (upper left, along a small circle near the north pole): the Arctic Circle
- TRO PICVS CANCRI (upper left of centre, along an oblique ring), followed by a glyph like "69", read as the sign of Cancer ♋: the Tropic of Cancer
- AEQVATOR (right of centre, along an oblique ring): the equator
- HORIZON. (along the horizontal ring, below the globe): the horizon
- AEQVAT (lower left, on the other side of the same oblique ring): the equator
- TROPICVS CAPRI COR (lower centre, along an oblique ring): the Tropic of Capricorn
- ANTA RCTICVS. (lower right, along a small circle near the south pole): the Antarctic Circle
- POLVS AVSTR. (lower right, beside the lower pole-stub): the South Pole
- Zodiac band: sign glyphs along the two visible arcs of the band. Front arc, from the top: ♋ (at the end of TROPICVS CANCRI), ♌, ♍, one sign left of the globe that cannot be made out [?], ♏, ♐, and ♑ partly hidden at the lower left. Rear arc, from the bottom: ♒, ♓, ♉. No sign is visible for Aries or Gemini.

AVCTORES QVI DE SPHAERA IN COMMVNI, Et Aſtronomiæ, vel Coſmographiæ Elementis tractarunt.

Authors Who Have Treated of the Sphere in General, and of the Elements of Astronomy or Cosmography.

The chief writers on this subject have been: Geminus, in his *Elements of Astronomy* [Elementis Aſtronomicis]; Cleomedes, *On the Circular Theory of the Heavens* [de Mundi cyclica theoria]; Alfraganus [al-Farghānī], *On the Rudiments of Astronomy* [de Rudimentis Aſtronomicis]; Meshala the Arab [Māshāʾallāh], *On the Elements and the Celestial Orbs* [de Elementis, & Orbibus Cœleſtibus]; Martianus Capella, in Book 8 of *On the Marriage of Philology* [de Nuptijs Philologiæ]; Bede [Beda], in volume 1, the treatise *On the Sphere* and *On the Constitution of the World* [de Sphæra, & de Mundi Conſtitutione]; and Proclus, in his *Sphere*, which Io. Stoeflerus [Johannes Stöffler], Erasmus Osualdus [Erasmus Oswald Schreckenfuchs] and Georgius Henischius [Georg Henisch] have elucidated with annotations or commentaries.

Ioannes de Sacrobosco [Johannes de Sacrobosco] also won the name of a classic author with his *Sphere*. It was made illustrious by the paraphrase, commentaries or questions of Petrus de Aliaco [Pierre d'Ailly], Cichus Asculanus [Cecco d'Ascoli], Io. Bapt. Capuanus [Giovanni Battista Capuano], Prosdocimus de Beldomando [Prosdocimo de' Beldomandi], Michael Scotus [Michael Scot], Iacobus Faber [Jacques Lefèvre d'Étaples], Maurus Florentinus [Mauro Fiorentino], Bartholomæus Vespuccius [Bartolomeo Vespucci],

Elias Vinetus [Élie Vinet], Ariel Biccardus [Ariel Bicard], Franciscus Iunctinus [Francesco Giuntini], and Christophorus Clauius [Christoph Clavius] of our Society. Robertus Linconiensis [Robert Grosseteste], Campanus [Campanus of Novara], Pierius Valerianus [Pierio Valeriano], Alexander Piccolomineus [Alessandro Piccolomini] and Iosephus Blancanus [Giuseppe Biancani] of our Society likewise wrote on the sphere.

On cosmography, Orontius Finæus [Oronce Finé], Petrus Apianus [Peter Apian], Gemma Frisius, Franciscus Maurolycus [Francesco Maurolico] and Franciscus Baroccius [Francesco Barozzi] wrote, and Io. Fernelius [Jean Fernel] on cosmotheory [de Coſmotheoria]. Finally, the following treat of the elements of this subject, or of astronomy: Ptolemy, in Books 1 and 2 of the *Great Construction* [Magnæ conſtructionis], which in Graeco-Arabic is called the *Almagest*, and there Theon [Theon of Alexandria] in his commentary, and Regiomontanus in the *Epitome*; Geber [Jābir ibn Aflaḥ], in Books 1 and 2 of his *Astronomy*; Nicolaus Copernicus, in Book 1 of *De revolutionibus* [Reuolutionum]; Michael Neander, in the *Elements of Spherical Doctrine* [Elementis Doctrinæ Sphæricæ]; Titus à Popma [Titus van Popma], on the sphere and the first principles of astronomy; Iosephus Langius [Joseph Lange], in the *Elementale*; Christianus Seuerinus Longomontanus, in the *Astronomia Danica*, Book 1, *Of Spherics* [Sphæricorum]; Io. Keplerus [Kepler], in the *Epitome of Copernican Astronomy* [Epitome Aſtronomiæ Copernicanæ]; Adrianus Metius [Adriaan Metius], *On Spherical Doctrine and the Use of the Globe* [de Doctrina Sphærica, & de vſu Globi]; Petrus Herigonius [Pierre Hérigone], in volume 4 of his *Course of Mathematics* [curſus Math.]; and Petrus Gaſſendus [Pierre Gassendi], in the *Institutio Astronomica* [inſtitutione Aſtronomica]. Besides these, there are the older writers reviewed at the beginning. From them, and from others who touch on these matters here and there, we have compiled this book, with the favour of God, Best and Greatest [Deo O. M.].

CAPVT I. Quid ſit Aſtronomia, & Coſmographia, & quibus Scientijs, aut Artibus indigeat.

Chapter I. What Astronomy and Cosmography Are, and What Sciences or Arts They Require.

[Margin: Aſtronomia Definitio. — Definition of astronomy.]

Astronomy is a *physico-mathematical science of the bounded quantity of the celestial bodies, a quantity that also bounds their sensible accidents*. It is subordinate to physics insofar as it considers the sensible changes of the heavens and the stars, or the variety of the sensible accidents in them — such as shape, colour, light, shadow, place, position, order, distance and motion. But it is subordinate above all to mathematics, because it does not consider these accidents precisely as natural affections, or under any other aspect, but insofar as they fall under a definite and bounded quantity, whether that quantity is continuous or discrete, and whether permanent or successive. Astronomy, moreover, is the noblest

species of the whole of mathematics, and it requires no other minds than celestial ones, or *an admirable nature*, as Plato judges *in the Epinomis*.

[Margin: Aſtrologia. — Astrology.]

[Margin: Aſtronomiæ diſcrimen. — How it differs from astronomy.]

Astrology is *the art of divining natural effects from the position and revolutions of the stars*, and so it is a certain part of practical astronomy. I said "natural effects", not events — that is, future contingents that depend on free causes as such, or on chance, or on Fortune as it stands relative to us. For a certain and evident foreknowledge of these belongs by nature to God alone, and this science of divining is truly divine. No human art of these things exists, since from mere and general inclinations determinate events follow neither always nor for the most part. Those who boast of this part of judicial astrology [Aſtrologiæ Iudiciariæ], or practise it, have therefore rightly incurred the censure of Holy Scripture and the sacred canons. Even though, as Tacitus keenly says, that breed of men is one *which will always be forbidden in the City, and always retained*, he would have said better: in the world [in Orbe]. Astronomy is therefore far broader than astrology taken strictly, and it is only through want of skill that some use these names without distinction.

[Margin: Geographia. — Geography.]

Geography, taken more narrowly, is *the art of measuring and describing the globe of the Earth, and its inhabited or habitable surface*. It is concerned with the bounded quantity of the terraqueous globe — compacted, that is, of earth and water — not only absolutely but also relatively: both with respect to its parts, situated in various ways within it and set far apart, and with respect to the regions of the sky, from which come the various properties of lights and shadows, of days and nights, and of the seasons of the year. Its parts are hydrography, which undertakes the description of seas and islands; topography, which describes single places, say one country house or village; and chorography, which describes particular regions only, say Etruria or Italy. Both of the last are content with their description without reference to the whole surface of the Earth. Histiodromy [Hiſtiodromia], moreover, which is the art of navigating by means of the magnetic compass, presupposes hydrography rather than being a part of it.

[Margin: Geogr. — Geography.]

Geography as defined above is as necessary to astronomy as a base to a column, for other reasons and above all because of the difference of longitude and latitude between the places of celestial observations, or of phenomena to be predicted.

[Margin: Coſmographia. — Cosmography.]

Cosmography is *a science compounded of astronomy and geography together*. It differs from spherics — the treatment of the celestial and elementary sphere — only in brevity and compression: what cosmography does amply and expansively, spherics does strictly and succinctly, so as to set before beginners a kind of introduction [Iſagogem] to astronomy and geography. We too take this up

in this first book of ours and in the second, and shall expound the theories of the stars [Theoricas ſyderum] in other books.

The sciences or faculties that a complete astronomy chiefly requires are, further, seven: namely arithmetic, geometry, optics, gnomonics, organics, geography and chronology [Chronica].

[Margin: Arithm[?]tica. — Arithmetic.]

Arithmetic is *the science of numbers, or of discrete quantity, partly speculative and partly practical*. The part of it at least that the astronomer needs is the one that teaches how to compose, or gather into one sum, several numbers; how to subtract, multiply and divide numbers both whole and fractional; how to find a fourth proportional number by the golden rule of proportions, or the rule of three [Regulam Trium], as they call it; and how to extract the square and cube root of a given number. But the special portion of arithmetic that is even more necessary to him is the one that belongs to logistic [Logiſticæ], which applies the rules of reckoning, subtracting, multiplying and dividing to degrees of signs, of circles and of angles, and also to days and hours, and to the minutes of degrees or of hours, that is, sixtieths and sixtieths of sixtieths. We shall give it below, in chapter 5.

[Margin: Logiſtica — Logistic.]

[Margin: Logarithmica. — Logarithmic arithmetic.]

Logarithmic arithmetic is not necessary, yet it is in the highest degree to be sought after for solving triangles, above all spherical ones, and for constructing astronomical tables by that solution. With its help the rule of three is completed by the mere addition of artificial numbers, and things are dispatched much more quickly that are otherwise accomplished — with the greatest trouble, and scarcely — through the usual tables of sines, tangents and secants by repeated multiplication and division, at a cost in time of more than four times as much, and much more with larger numbers. We shall therefore give it very briefly in the second volume of the *Almagest*, in the trigonometrical book.

[Margin: Geometri[?] — Geometry.]

Geometry is *a science partly practical and partly speculative, of magnitudes, or of continuous bounded quantity as such*. The part of it the astronomer must learn thoroughly is the one that treats the more notable properties of triangles, circles, ellipses and spheres. From practical geometry, the portion the cosmographer and the astronomer ought to know better is the one that teaches how to measure, by means of visual rays, either surfaces and distances — this is called geodesy [Geodæſia]; or heights — this is called altimetry [Altimetria], in Greek ὑψομετρία [Greek: hypsometria, "height-measurement"]; or floors — this is called ἐμβαδομετρία [Greek: embadometria, "area-measurement"]; or depths — this is named βαθεομετρία [Greek: batheometria, "depth-measurement"]. These at least must be sampled, and we shall use them in various problems, especially in the geographical book.

[Margin: Trigonmetria. [recte: Trigonometria.] — Trigonometry.]

But the part that deals with the analysis of triangles, and teaches how to solve triangles both plane and spherical, must be learned thoroughly by the astronomer, since he needs to use it constantly and very often indeed. Accordingly, from the various forms of trigonometry in circulation, I have decided to give the one that seemed to me at once the safest and the quickest, in a separate book, which will be the first of volume 2 of the *Almagest* and is entitled *Trigonometricus*.

[Margin: Optica. — Optics.]

Optics is *the science of the manner of vision, and of visibility insofar as it can come about through visual rays and through shadows bounded by rays*, whether those rays are direct and freely propagated from the object to the eye or not. If they are transmitted directly, or unreflected, through openings or slits, it is called dioptrics [Dioptrica]. If they are reflected from some surface partly transparent and partly opaque, it is called catoptrics [Catoptrica] or anacamptics [Anacamptica]. If they pass along refracted lines in a medium of differing density, it is called anaclastics [Anaclaſtica]; yet insofar as anaclastics considers rays not reflected toward the illuminating object but propagated through a transparent medium to the visual power, it is a part of dioptrics. Of these, the parts the astronomer needs are those that teach how to distinguish the true shapes, distances, positions and magnitudes of celestial phenomena from the apparent ones. We shall therefore treat that part of optics which the astronomer needs in Book 3 of volume 2 of the *Almagest*, the *Opticus*, together with much on parallaxes, refractions and shadows. In that part we shall touch on gnomonics [Gnomonicen], which deals with the shadows of gnomons; for the science that deals with shadows and their projections universally is called sciaterics [Sciaterica], from "shadow", which in Greek is σκιὰ [Greek: skia, "shadow"].

[Margin: Organica. — Organics.]

Organics, in its subject matter, is *the art of making and applying mechanical instruments suited to observing the celestial phenomena*. For whatever appears chiefly in the heavens is called in Greek φαινόμενα [Greek: phainomena, "things that appear"], and the observations themselves τήρησες [Greek: tērēseis, "observations"] [recte: τηρήσεις]. No one, however, can be a perfect instrument-maker unless he is thoroughly versed in astronomy; for when the end and the proper use of these instruments are unknown, many points of exactness in their construction must also go unrecognized. I therefore judged it worth my while to set out, in an entire book which I have named the *Organicus*, the way to construct the choicest of all the instruments that are necessary or very useful to astronomy, together with a method for handling and applying them without error, and an order for choosing [them] in observing the phenomena of the heavens. This will be Book 2 of volume 2 of the *Almagest*.

[Margin: Chronologia. — Chronology.]

Chronology is *the art of computing times and their intervals*. It has two parts. The first is the doctrine of times according to the different forms of years and months, and the calendars of the different nations, which comprises the

reckoning of the civil and the sacred computus. The second is the knowledge of the different epochs or eras — that is, of the starting points from which years are counted forward or backward — from which is derived the method of properly connecting them with one another. The astronomer cannot examine as he should the observations made by others at different times, nor derive the celestial motions from those observations for other nations, without knowing those times themselves; and not merely ordinary astronomers but some of the foremost have slipped in this matter. I have therefore thought it necessary to share with the readers of the new *Almagest*, at the end of volume 2, a book, the *Chronologicus*, which I had put together from my youth onward for other uses in sacred and profane history, with tables added that are most convenient for computing dates.

CAPVT II.

De Cœli Rotunditate, Motuque Circulari Cœlorum, aut Syderum.

On the roundness of the heavens, and the circular motion of the heavens or of the stars.

That the heavens are spherical, and that their motion and the motions of the stars manifest to us are circular, is affirmed, with the ancient astronomers, by Ptolemy in Book 1 of the *Almagest*, chapter 3; by Theon in the same place; by Regiomontanus in the *Epitome*, Book 1, conclusion 1; by Copernicus, Book 1, chapters 1 and 4; by Johannes de Sacrobosco, chapter 1, and there by Clavius, p. 76; by Barozzi, Book 1 of the *Cosmographia*, p. 11 — and who does not? It is shown in three ways, as follows.

[Margin: Argumentum 1. — Argument 1.]

I. *A priori*, from fittingness. Just as *the circle is greater than all the regular rectilinear figures isoperimetric with it, so too the sphere is greater than all the bodies isoperimetric with it* — as Theon and Clavius demonstrate in part in the places cited above, and Pappus in part. To the heavens, then, which contain all things within themselves, the most capacious figure was owed, and so the spherical one. Next, of all bodies the sphere is the most nimble for the motion of revolution, as the circle is, because it suffers less resistance from the medium: any of its parts moves into the place of the parts next to it, with no external body rushing in or giving way. And it is evident that a motion of revolution, and a circular one, befits the heavens, since it is the noblest and simplest motion and returns into itself most regularly, and so suits the noblest of inanimate bodies, the simplest, and the most regular in its motions — at least in its more evident ones. Finally, the circle and the sphere, by their capacity and by their figure, in which neither beginning nor end actually appears, in a certain way imitate the divine immensity and infinity. Therefore this sensible world, other things being equal and preserved, represents the archetypal world — that is, God Most Good and Greatest [Deus O. M.] — better by that figure; as do the other, more perfect parts of the upper world as well: the Sun, the Moon and the other stars. Granted, other bodies too, especially liquids, tend toward roundness — drops

of water, seeds as well, and the trunks of trees — and any agents that are not impeded act spherically: that is, they spread their force from one point along lines in every direction, equally distant from it; hence the space to which their force can extend is called the *sphere of activity* [Actiuitatis Sphæra]. Lastly there comes the fittingness drawn from the roundness of the lower world and of the terraqueous globe, which had to be contained by a like body and to be ruled and nourished equally by its influences — not, indeed, by absolute necessity, but on the supposition of what God and Nature are accustomed to do, namely what is best.

[Margin: Argumentum 2. circularis motus. — Argument 2, for circular motion.]

II. *A posteriori*, from the experience of the senses. For to those who watch the heavens on clear days or nights, they appear vaulted like an arch, that is, spherically concave — even though the parts nearer the horizon seem farther from us than those above the zenith, because of a deception of sight, which is accustomed to measure intervals and distances by the intervening bodies, between us and the horizontal part of the heavens. Moreover, the stars, in the daily motion from rising through the meridian toward setting, ascend little by little with uniform motion until they reach the meridian and their highest altitude; then they descend step by step with uniform motion until they set from our sight, and after a proportional time they rise again and begin to ascend. And in this motion they keep among themselves the same distance that first appeared, and so they are seen to be moved equidistantly about the Earth as about a centre, and consequently to describe circles, since an equal distance of lines from a centre is the mark of the circular form. Of the fixed stars, those which always appear to us — such as the Greater and Lesser Bear for the Greeks — describe perfect and manifest circles about one point of the heavens, which is called the pole of the world; those nearer to it are also hidden, but for a shorter time, and those farther away are hidden for a proportionally longer time, according to the ratios of the circles that they describe about the pole of the world. The projections of shadows, too, their increments and decrements, and the figures they describe with their tips, come about exactly as the circular motion of the stars that illuminate the Earth requires. Sundials [Horologia … sciaterica], too, and the instruments with which astronomers observe the altitudes of the stars, their distances and their motions, suppose that the motions of the heavens or of the stars are circular, and many of them require the figure of a whole or a partial circle in order to exhibit the celestial phenomena more nearly to the truth. In the planets' proper motion from west to east, the circular motion prevails, whether it consists of one circle or of several; and if any irregularity appears in it, it is reduced to regularity by adopting the hypothesis of motion through several circles, or through forms depending on the laws of the circle, such as ellipses.

[Margin: Argumentum 3. — Argument 3.]

III. The third argument is taken from the absurdities that would follow if the heavens were not spherical, or if all circular motion were denied to the heavens and the stars. For then the things we spoke of in argument 2 would not appear at all, and many would appear in the opposite way. In the first place, if the heavens were flat, the distances between any two fixed stars would appear unequal to the same observer at different times, or to different observers at the same time; and the stars would be nearer to some than to others, and consequently would look larger at the zenith, or at their meridian altitude, than at the horizon — whereas the contrary happens. You will understand these arguments better once you have learned the circles of the sphere.

SCHOLIA.

[Margin: Isoperimetræ figuræ quæ? — What are isoperimetric figures?]

I. *Isoperimetric figures are those contained by an equal perimeter; regular figures are those that are equilateral and equiangular in themselves. Thus, if a circle is given whose circumference is ten palms, and a triangle whose sides taken together equal ten palms, the two figures will be isoperimetric, or of equal perimeter. And an equilateral triangle — each of whose lines, for the sake of example, is one palm — is a regular figure, since it also has equal angles. That among isoperimetric and regular plane figures the circle is the most capacious, and among bodies the sphere, some argue from the fact that figures with more angles are more capacious, while the circle and the sphere exceed without limit the finite angles of a figure inscribable in them. But if the circle is compared with a regular figure inscribable in it, it will indeed be greater, since it contains that figure and something more; yet this reasoning is not universal. Neither are the demonstrations brought by Theon and Clavius, since they hold only for the sphere compared with a body in which a sphere can be inscribed, and which is contained by plane or conical surfaces. For this reason Clavius admits that one must have recourse to [no.] 70 of Pappus of Alexandria [ad 70. Pappi Alexandrini], who demonstrated this universally of every body.*

II. The spherical figure did not suit man — although he is an image of the archetypal world, and in some respects a more perfect one than the heavens — because it was not suited to the variety of the operations of the soul [animasticarum operationum], as it is to the operations of the heavens. That is why in argument 1 we say "other things being equal and preserved"; and so the objection is answered that Clavius and Barocius made against this argument from fittingness, put forward by Sacrobosco, and indeed by Manilius too, in these verses of Book 1:

> This form endures eternal, and most like the gods, which has no beginning anywhere, nor end in itself, but remains like to the whole, and is equal throughout.

III. Some, in Ptolemy, seem to have said (as above) that the stars are moved along infinite straight lines. They are easily refuted by the stars' return and revolution to the same starting point, which takes place in a finite time. Others said that the stars are extinguished at the western horizon and kindled at the eastern, and born anew,

so as to remove a perfectly circular motion and grant at most a semicircular one. But since the appearances dealt with in argument 2 occur all round the whole globe of the Earth, and what is the horizon for some is the meridian for others, the stars would have to be extinguished and rekindled at every single moment, and the same stars at that. Nor can any sound reason be given why the fixed stars that always appear are not extinguished while the rest are. Indeed, since the same fixed stars that always appear to some — as Cynosura does to the Greeks — lie hidden from others at the same time — as from the Peruvians — the same stars would have to be extinguished and not extinguished; which is absurd.

IV. The vacuum which Aristotle (De Caelo, Book 2, text 22), and Sacrobosco [Sacrobuſcus] with him, says would result if the heavens were not spherical lacks any sound reason. For it would also be avoided if the heavens had an oval figure, as Clavius notes; and the argument supposes that the side of a cube, for example, is a place deprived of the body due to it. But if the world were of cubic figure, its surface would not have to contain any other body, nor would there be in it a privation of such a body — except on the hypothesis of a sphere to be completed, which is the very point in dispute here — unless it is said, in a vicious circle, that it must be spherical because it must be spherical.

V. The absurdities dealt with in argument 3 would be avoided, and many of the appearances dealt with in argument 2 would be saved, if one could say with the Copernicans that the Earth moves; or with others, that the distance of some of the planets from the Earth is, if not infinite, at least so great that the difference in the phenomena arising from a figure circular on the one side and not circular on the other would become insensible. But it is enough for us here that the motion is circular to the senses, and that those who devise these things depart from the senses.

VI. Those who try to reduce the motions of the planets to spirals or ellipses are nonetheless compelled, in those very curves, to depend on the ratios and properties of the circle; so on this point they are not opposed to the common opinion.

VII. Lactantius (Divine Institutes, Book 3, chapter 23) and Procopius of Gaza [Procopius Gazæus] (on Genesis, chapter 1) deny the roundness of the heavens; and Saints Justin, Basil, Chrysostom and Ambrose seem to have doubted it, according to Pererius [Benito Pereira] (on Genesis, Book 2, question 3). But other Fathers stand for it, especially Saints Jerome and Augustine (De Genesi ad litteram, Book 2, chapter 9). Augustine there shows that God stretched out the heavens like a tent and a vault, according to Isaiah, chapter 40, or like a skin, according to Psalm 103 — since a skin too can be stretched spherically, as happens with inflated wineskins, bladders and bellows; and every horizon has above it a hemisphere of the heavens vaulted like an arch. Moreover, the Sun is said in Ecclesiastes 1 to return into its circles; and the divine Wisdom in Ecclesiasticus 24 says "I alone have gone round the circuit of heaven" — in Greek ἐκύκλωσα [Greek: ekyklōsa, "I encircled"], that is, "I have rounded about"; and in Proverbs 8, that it enclosed the deeps with a circuit — that is, the watery matter that is between the heavens and the Earth — or, as the Hebrew reading has it, that it drew a circle upon the face of the deep. From these Pererius teaches that the roundness of the heavens agrees wonderfully with divine Scripture.

CAPVT III.

Suppoſitiones de Terræ Figura, Situ, Magnitudine.

Suppositions on the figure, position and size of the Earth.

I. We suppose (1) that the mass of the Earth and of the water is spherical to the senses — which will be shown more conveniently in Book 2, on the elementary sphere; (2) that the Earth is, to the senses, in the middle of the world, as the centre about which the circular and more manifest motion of the heavens takes place; (3) that the Earth, relative to the heavens, especially the highest heaven, is like a point. Although Ptolemy and Copernicus demonstrate these things at the beginning of their works, and the writers on the sphere have imitated them in their first chapters, we shall show them more opportunely in the book on the elementary sphere; for then, once the circles of the sphere and many other things have been set out, the arguments for these hypotheses will be better grasped. But since we had to treat the circles of the sphere, we had first to show that the heavens are round and their motions circular, so that we should not seem to have chosen these figures arbitrarily. Nor, however, do we here set the different systems of the world before beginners, since knowing them presupposes that many other things about the planets are explained first; they will nonetheless be indicated in Book 3, where the place of the Sun is treated.

CAPVT IV.

Circuli Definitio, Proprietates inſigniores, ac Dimenſiones.

Definition of the circle; its more notable properties; and its dimensions.

[Margin: Circuli Definitio. — Definition of the circle.]

I. *A circle is a plane figure contained by a single line, to which all straight lines drawn from one point within the figure are equal to one another.* Such is the figure ABGD: all straight lines drawn from the point C to the curved line ABGD are equal to one another, as CA, CB, CG, CD, and likewise the others, infinite in number.

Diagram. A circle set into the right column beside the definition. A vertical diameter runs from B at the top to D at the bottom, crossing a horizontal diameter from A at the left to G at the right at the centre C. Below the horizontal diameter a second horizontal chord runs from K at the left to M at the right. A straight line runs from the centre up and to the left to the point F on the circumference, left of B.

- F (upper left on the circumference): point
- B (top): end of the vertical diameter
- A (left): end of the horizontal diameter
- C (centre, just right of the crossing): the centre
- G (right): end of the horizontal diameter
- K (left, below A): end of the lower chord

- M (right, below G): end of the lower chord
- D (bottom): end of the vertical diameter

II. *The circumference of a circle* — in Greek περιφέρεια [Greek: periphereia, "circumference"] — *is the curved line containing, or bounding, the circle.* It is not seldom called the circle by synecdoche; properly, however, the circle is not a line lacking breadth but a plane surface bounded by a curved line.

III. *The centre of a circle is the point from which all straight lines, any whatever, drawn to the circumference are equal to one another* — as the point C in the figure above.

IV. *The diameter of a circle is the straight line drawn from one point of the circumference, through the centre of the circle, to another point of the same circumference,* as ACG or BCD. It is called the diameter, or *dimetiens*, because it divides the circle in two, or measures it through the middle. It is also called the greatest of the chords that can be subtended in a circle by arcs of the same circle.

V. *The semidiameter of a circle, or radius, is half the diameter,* as AC or CG. In trigonometry and in tables of sines it is called the *whole sine* [Sinus totus], because it subtends a whole and entire quadrant of the circle and measures a right angle.

VI. *A quadrant of a circle is the fourth part of the circle, contained by a fourth part of the circumference and by two semidiameters that make a right angle at the centre of the same circle,* as the portion BCG of the circle.

VII. *The greater segment of a circle is a portion of the circle greater than a semicircle,* as KBM; *and the lesser segment is a portion less than a semicircle,* as KDM.

VIII. *A circle is said to be touched,* whether by a straight or by a curved line, when it is touched at a single point of the periphery; or when it is touched in such a way that the periphery is not cut, and the tangent line, if produced, cannot enter the circle.

PROPRIETATES CIRCVLI.

Properties of the circle.

The properties of the circle are most beautiful and innumerable, but candidates in astronomy need to know only a few of them. The chief of these are the following; their proof [apodixis], or demonstration, is to be sought from the authors cited below.

I. *An arc of the circumference of a circle is the measure of the angle made at the centre of the circle by the straight lines that contain the arc.* For an angle is said to be as great — of so many degrees or minutes — as is the arc of a circle described from the vertex of the angle as centre and bounded by the straight lines drawn from that centre to the circumference. Thus in the preceding figure, if from the centre C the arc FB of the circle ABGD is described, and from the same C the lines CF and CB are drawn, the measure of the angle FCB will be the arc FB; and if this arc is 40° 25′, the angle FCB will also be said to be 40° 25′. Hence a quadrant of the circle, such as BG, is the measure of the right angle

BCG, and each is 90°, since the whole periphery of the circle is divided into 360°. A semicircle such as BGD is the measure of the two right angles BCG and GCD taken together, and of 180°; for the straight line GC, falling on the straight line BD, makes with it two right angles, or angles equal to two right angles, by Euclid, *Elements* I.13. So the arc FAD is the measure of the obtuse angle FCD, which is greater than a right angle and consists of more than 90°. Finally, the whole circle is the measure of four right angles, and of 360°; for about the centre, one and the same point C, four right angles stand in the same plane, neither more nor fewer, by the corollaries to Euclid, *Elements* I.13.

II. *The arcs of several circles described from the same centre, and contained by the same straight lines drawn from that centre, are similar to one another.* Thus in the following figure the arc AB is similar to the arc MN, because both are described from the same centre C and both are intercepted by the same straight lines CMA and CNB, drawn from the same centre C. Similar, I say, not equal: that is, the greater arc has the same ratio to its whole greater circle as the lesser arc has to the lesser circle; and if one is, for example, a sixth part of its circle, or 60°, the other too is a sixth part of its circle, and 60°. Our Clavius demonstrated this in his *Sphaera*, chapter 1, p. 203 in my copy [pag. mihi 203].

Diagram. Two concentric circles set into the left column beside property II. Two straight lines run from the common centre C (its label set just below the vertex) up to the left and up to the right, cutting the inner circle at M (left) and N (right) and the outer circle at A (upper left) and B (upper right). Between M and A the line is printed broken, as two short dashes.

- A (upper left, on the outer circle): end of line CMA
- B (upper right, on the outer circle): end of line CNB
- M (left, on the inner circle, set obliquely along the line): point
- N (right, on the inner circle, set obliquely along the line): point
- C (centre): common centre

III. *A straight line touching a circle makes a right angle with the semidiameter or diameter of the circle drawn to the point of contact.* What it is to touch a circle was stated in definition 8. It is demonstrated by Euclid, *Elements* III.18.

IV. *The angle in a semicircle is a right angle.* The angle in a semicircle is the one whose vertex ends on the circumference of the circle while the lines forming it end at the extremities of the diameter. The angle in the lesser segment of a circle is greater than a right angle; in the greater segment, less than a right angle. Euclid, *Elements* III.31.

V. *In a circle the angle at the centre is double the angle formed at the circumference* — provided, that is, that both stand on the same arc of the circumference as their base. Euclid, *Elements* III.20.

VI. *In the same circle, equal straight lines ending at the circumference are equally distant from the centre;* their distance is measured along the shortest line, namely the one drawn from the centre perpendicular to them. Euclid, *Elements* III.14.

VII. *In quadrilaterals inscribed in a circle, two opposite angles are together equal to two right angles;* so the four angles taken together are equal to four right angles. Euclid, *Elements* III.22.

VIII. *The square of the tangent to a circle is equal to the rectangle one of whose sides is the diameter of the circle produced as far as*

the other end of the tangent, while the other side is equal to the external addition to that diameter. Euclid, *Elements* III.36.

IX. *The diameters of different circles are to one another as the circumferences;* and conversely, the circumferences are to one another as the diameters. Pappus of Alexandria, *Collection*, Book 11, theorem 5. *And the square of the semidiameter is half the square inscribed in the circle:* Clavius, in the lemma to proposition 17 of Book 1 of Theodosius.

X. *Circles are to one another as the squares of their diameters* — understand, as to their area. Euclid, *Elements* XII.2. So if the square erected on the diameter of one circle is three times the square erected on the diameter of another circle, the first circle too will be three times that other circle; and likewise for other ratios.

XI. *The arc whose chord is the semidiameter of the circle is a sixth part of the circumference,* from Euclid IV.15.

DIMENSIO CIRCVLI.

Measurement of the circle.

The measurement of the circle is the finding of the size of the diameter from a given circumference; or of the circumference from a given or known diameter; or of the area from a given circumference and diameter. For these, take the following selected theorems and problems, drawn from Archimedes, *On the Measurement of the Circle*; from Clavius, *Geometria practica*, Book 4; and from Ludolphus Collen [Ludolph van Ceulen], *De Circulo et Adscriptis*, Book 4.

I. *A circle is equal to a right triangle one of whose sides about the right angle is equal to the semidiameter of the circle, and the other side equal to the periphery.* The greater side containing the right angle is called the base by Archimedes (*On the Measurement of the Circle*, theorem 1), although most people call the base the line that subtends the right angle itself.

II. *The area of a circle is equal to the rectangle contained by the semidiameter of the circle and a line equal to the semicircumference.* So our Clavius, cited above, and in the *Sphaera*, p. 85.

III. *A circle has to the square of its diameter the ratio of eleven to fourteen;* so Archimedes, theorem 2, cited above.

IV. *The circumference of any circle is triple the diameter and exceeds it further by less than a seventh part of the diameter, but by more than ten seventy-firsts;* so the same Archimedes, cited above, theorem 3. Iosephus Scaliger [Joseph Justus Scaliger] tried in vain to pick at these theorems in his *Cyclometrica elementa*, and for that he was soundly beaten by many — by Vieta [François Viète], Clavius

and Rivaltus [David Rivault]. Hence it is that, with the fraction dropped, you find everywhere — in Pliny the Elder, Book 2, chapter 23, in Macrobius on the *Dream of Scipio*, and in others — that the diameter of the circle is to the periphery as 7 to 22. But a subtler ratio is the one Ptolemy found (*Almagest*, Book 6), and Viète; and subtler still the one of Ludolph van Ceulen, which he wished to have carved on his monument. Our Christophorus Griembergerus [Christoph Grienberger] examined it in his *De Triangulis* and found it true. They are as set out below.

Circuli Diameter ad Circumferentiam eſt — The diameter of the circle is to the circumference:

Authority	as (Vt)	to (Ad)
Ptolemæo — Ptolemy	10,000,000	31,416,666
Vietæ — Viète	10,000,000,000	31,415,926,535
Ludolpho, & Griembergero — Ludolph van Ceulen and Grienberger	100,000,000,000,000,000,000	314,159,265,358,979,323,846½

From these arise the following problems and rules, worked out with sufficient precision.

1. Data Circumferentia reperire Diametrum Circuli.

1. Given the circumference, to find the diameter of the circle.

[Margin: Diametri Circuli inuentio. — Finding the diameter of a circle.]

Multiply the circumference by 7 and divide the product by 22: the quotient will be a diameter slightly less than the correct one. Then multiply the circumference again by 71 and divide the product by 223: the result will be a diameter slightly greater than the correct one. Choose, therefore, the mean between these, or multiply the circumference by 100 and divide the product by 314: in this way you will obtain, in a single operation, very nearly the value you are looking for.

Probl. 2. Data Diametro Circuli reperire Circumferentiam.

Problem 2. Given the diameter of a circle, to find the circumference.

[Margin: Circumferentiæ Circuli inuentio. — Finding the circumference of a circle.]

Multiply the diameter by 22 and divide the product by 7: the result will be a circumference greater than the correct one. Then multiply the diameter by 223 and divide the product by 71: the result will be a circumference smaller than the correct one. Choose, therefore, the mean between these; or, in a single operation, multiply the diameter by 314 and divide the product by 100, for the quotient will be the circumference sought.

Probl. 3. Datis Diametro, & Circumferentia reperire Aream Circuli.

Problem 3. Given the diameter and the circumference, to find the area of the circle.

[Margin: Areæ Circuli inuentio. — Finding the area of a circle.]

Multiply a quarter of the circumference by the diameter, or half the circumference by the semidiameter of the circle; either way the result will be the area of the circle, that is, its surface in square measure. For example, if the semidiameter was 10 feet and the semicircumference roughly 31½ feet in length, the area of the circle will be roughly 315 square feet.

CAPVT V.

De Circuli diuiſione in Gradus, & Graduum, Dierum, Horarumq; in Minutias, & eorum Logiſtica.

Chapter V. On the division of the circle into degrees, and of degrees, days and hours into fractional parts, and on their logistic.

[Margin: Cur circulus diuiſus in gr. 360. — Why the circle is divided into 360°.]

I. Every circle is divided by cosmographers — indeed by all geometers — into 360 equal parts, which they call *degrees* [Gradus]. The semicircle, accordingly, is divided into 180°, the quadrant of the circle into 90°, the sextant into 60°, the octant into 45°, and the triens into 120°. The reason for this division was the number of days in a year midway between the solar year and the lunar year. The solar year is 365 days, or 366 if it is a leap year [Biſſextilis], whereas the lunar year is 354 days; and between these numbers lies this round number, 360. So it was into that many degrees that the zodiac was divided first — the circle which the Sun runs through in roughly the same number of days — then the equator [Æqnator], and finally every circle. This number also contains a most beautiful blend of six and ten, which the Pythagoreans rightly held to be the most fertile numbers: six multiplied by itself gives 36, and this multiplied by ten gives 360. And since the zodiac, from which this distribution arose, is divided into twelve months, as the Moon indicated, every other circle is likewise divided by geometers into twelve segments, which are also called signs, and each sign into 30 degrees [gtadus], or equal parts.

[Margin: Gradus ſubdiuiſio. — Subdivision of the degree.]

II. Every degree — like the day for Ptolemy, and the hour for other astronomers — is subdivided into sixty minutes [minuta], or *scrupula prima*. This is done because of the convenience of the number sixty: among the numbers made up of two figures there is none that has as many aliquot parts as sixty — that is, none that can be divided into as many of its own parts, each measuring it by repetition without a fraction. For it has 10 aliquot parts, and it is produced from six and ten multiplied together. Its half is 30 degrees, its third part 20, its quarter 15, its fifth 12, its sixth 10, its tenth 6, its twelfth 5, its fifteenth 4, its twentieth 3 and its thirtieth 2, as you can see more distinctly in this small table:

Sexagenarij Numeri Partes aliquotæ cū Valore in Gradib. — The aliquot parts of the number sixty, with their value in degrees.

Partes (Parts)	2	3	4	5	6	10	12	15	20	30
Valor (Value)	30	20	15	12	10	6	5	4	3	2

[Margin: Minuti ſubdiuiſio. — Subdivision of the minute.]

III. Every minute, or *scrupulum*, is subdivided into sixty equal small parts, which are called *secunda* (understand *scrupula*: "second" minutes); each second into 60 *tertia* [thirds]; each third into 60 *quarta* [fourths]; each fourth into 60 *quinta* [fifths]; each fifth into 60 *sexta* [sixths]; and so on to infinity, as far as one cares to subdivide into sixtieths of sixtieths. The characteristic signs, or notations, of these fractional parts come in four forms, as can be seen in the small table, or tablet [laterculus], added here. Further on, at the end of the work, I will give a table converting minutes of a day into hours, and hours into minutes of a day.

Notæ	M	S	T	Qua.	Qni.	Sex.	Sept.	Oct.	Non.	Dec.
Vel	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
Vel	I	II	III	IV	V	VI	VII	VIII	IX	X
Vel	′	″	‴	⁗	v	vi	vii	viii	i^{x}	^{x}

Notation key: Notæ = signs; M = minutes; S = seconds; T = thirds; *Qua.* = fourths; *Qni.* = fifths; *Sex.* = sixths; *Sept.* = sevenths; *Oct.* = eighths; *Non.* = ninths; *Dec.* = tenths; *Vel* = "or".

LOGISTICA.

Logistic.

Logistic [Logiſtica] is the part of arithmetic that teaches how to add up, subtract, multiply and divide astronomical or cosmographical numbers — namely signs, degrees, days, hours and their fractional parts, or *scrupula*, and sixtieths of sixtieths. Its rules are the following.

PRÆCEPTVM I. DE ADDITIONE.

Rule I. On addition.

First, arrange the denominators of the numbers so that signs stand properly under signs, degrees under degrees (or days under days), hours under hours, minutes under minutes, and seconds under seconds. Second, begin the adding-up, or reckoning, from the right, proceeding towards the left. Third, for every sixty seconds carry one minute into the column of minutes; for every sixty minutes carry one degree, or one hour, into the column of degrees or of hours; likewise for every thirty degrees carry one sign into the column of signs; and for every twelve signs throw away the circle so made, keeping in the sum the number of signs [Signognorum], if any remainder was left after the twelves were thrown away. In the same way, for every 24 hours put down one day.

EXEMPLVM ADDITIONIS.

Example of addition.

Denominatores	Sig.	Gr.	′	″	vel Dies	Hor.	′	″
Numeri Colligendi	6	25	38	46	13	19	45	38
	8	7	42	51	2	7	15	22
		2	15	20		3	5	12
Summa	3	5	36	57	16	6	6	12

Denominatores = denominations; *Numeri Colligendi* = the numbers to be added; *Summa* = sum; Sig. = signs; Gr. = degrees; Dies = days; Hor. = hours; *vel* = "or".

You see that in the first example the sum of the seconds 46, 51 and 20, which is 117, contains 60 once, that is, one sixtieth. For that sixtieth, therefore, I mentally carry one minute into the column of minutes, and under the seconds I write only the remaining 57, which are left when 60 is taken from 117. Next, since the sum of the minutes 38, 42 and 15, with that minute arising from the seconds added, would be 96, which contains one sixtieth, I carry a unit into the column of degrees and write under the minutes the remaining 36. Third, since the sum of the degrees 25, 7 and 2, with that one unit arising from the minutes added, would be 35, and thirty degrees make one sign, I write only 5 degrees, and for the other 30 I carry one sign into the column of signs. Finally, since the sum of the signs 6 and 8, with that unit added, would be 15, in which twelve signs — that is, one whole circle — are contained once, I throw away the 12 signs and write in the sum only the remaining 3 signs. And so the useful sum, the one to be kept after the whole circles, is $3^{s}\ 5^{\circ}\ 36'\ 57''$.

[Margin: Diſcrimen ſummæ Logiſticæ, a vulgari. — The difference between the logistic sum and the common one.]

Likewise in the second example: if the sum had been reckoned not by logistic but by common arithmetic, it would be 15 days, 29 hours, 65 minutes and 72 seconds; but if the rule of logistic I have stated is observed, it is $16^{d}\ 6^{h}\ 6^{m}\ 12^{s}$.

PRÆCEP. II. DE SVBTRACTIONE.

Rule II. On subtraction.

First, write the numbers to be subtracted under numbers of the same kind — that is, seconds under seconds, minutes under minutes, degrees under degrees, and so on. Second, begin the subtraction from the right and proceed in it towards the left. Third, if the upper numbers are greater in each kind, carry out the subtraction in the usual way. But if they are smaller, so that the subtraction cannot be made from them taken by themselves, borrow one unit from the next kind to the left, resolve it into the following kind, add it, so resolved, to the number that was smaller, and from the total subtract the number you wanted to subtract. And if the signs from which the subtraction is to be made

are fewer, add to them a whole circle, that is, 12 signs, and so you will complete the subtraction.

EXEMPLVM SVBTRACTIONIS.

Example of subtraction.

	Sig.	Gr.	′	″	Sig.	Gr.	′	″
Numerus à quo	8	21	34	49	4	7	20	10
Numerus ſubtrahendus	5	6	25	10	3	20	15	40
Reſiduum	3	15	9	39	0	17	4	30

Numerus à quo = the number from which; *Numerus ſubtrahendus* = the number to be subtracted; *Reſiduum* = remainder.

In the first part of the example there is no difficulty. In the second part, however, since 40 cannot be taken from 10 seconds, I take one minute from the 20 minutes, resolve it into 60 seconds, and add it to the 10 seconds to make 70, from which I subtract 40; the remainder is 30 seconds. Next, when 15 is taken from the remaining 19 minutes, 4 minutes are left. Third, since 20 degrees cannot be subtracted from 7 degrees, I take one sign from the number of signs, resolve it into 30 degrees, and add it to the 7 degrees to make 37, from which I now subtract 20, and 17 degrees are left. Finally, since of the 4 signs, after one sign was taken from them, 3 remained, if I take 3 signs from these nothing is left; so I write a cipher, 0, under the signs. The whole remainder is thus 0^{s} 17° 4′ 30″.

[Margin: Quid tum Signa ſunt pauciora. — What to do then when the signs are fewer.]

But if 3 signs had had to be subtracted from 2 signs, I would have taken a circle of 12 signs and added it to the 2 signs, making a sum of 14 signs; and with 3 subtracted from these, 11 signs would have been left.

PRAECEP. III. DE MVLTIPLICATIONE.

Rule III. On multiplication.

Logistic multiplication has two uses in astronomy. The first is when artificial, or astronomical, numbers are to be multiplied by natural and common numbers. Each kind is then multiplied separately, and afterwards the fractional parts are reduced to degrees or hours by division by sixty, degrees to signs by division by thirty, and hours to days by division by twenty-four. The numbers to be multiplied could, however, be resolved into the lowest kind and so multiplied by natural numbers, and the product of the multiplication afterwards reduced to the earlier kinds by means of division. The second use is when both the multiplier and the multiplicand are logistic and artificial — that is, when degrees, minutes, seconds, etc. are to be multiplied in turn by degrees, minutes, seconds, etc. The operation is then most troublesome, although it can be carried out

somewhat more easily by means of the Canon of Sexagenes, or of Hexacontads. But we are very well off, in that this use is not at all necessary to the astronomer; and so it seems wiser to me not to detain candidates in this entrance-hall with trifles of this sort. Anyone who delights in them should consult Erasmus Reinoldus [Erasmus Reinhold] in the *Prutenic Tables* [Prutenicis Tabulis], from p. 5; Biancani in the *Sphaera*, book 9, chapter 13; Vincentius Renerius [Vincenzo Renieri] in the *Tabulae Mediceae*, precept 3, who gives a table of Hexacontads after p. 60; and Magini in the *Tabulae secundorum mobilium* [tabul. fecund. mob.].

EXEMPLVM MVLTIPLICATIONIS PRIORIS CASVS.

Example of multiplication of the first case.

	Sign.	Gr.	′	″
Multiplicandi	10	20	30	40
Multiplicans fit 3.	3	3	3	3
Summæ fingulæ	30	60	90	120
Summæ Reductæ	32	1	32	0
Sed abiectis circulis.	8	1	32	0

Multiplicandi = the numbers to be multiplied; *Multiplicans fit 3.* = let the multiplier be 3; *Summæ fingulæ* = the separate products; *Summæ Reductæ* = the reduced products; *Sed abiectis circulis* = but with the circles thrown away.

Let $10^{s}\ 20^{\circ}\ 30'\ 40''$ be multiplied by three. I begin from the right and multiply 40″ by 3, which gives 120″. Next I multiply 30′ by 3, which gives 90′. Third, I multiply 20° by 3, which gives 60°. Finally I multiply 10 signs by 3, which gives 30 signs. Now since 120 seconds contain exactly two sixties, that is, two minutes, if these are added to the 90 minutes they make 92′. But since this includes one sixty, that is, one degree, I add it to the 60 degrees, making 61°, and keep 32′. And since these contain two signs, or two thirties of degrees, I keep a single degree and add 2 to the 30 signs, making 32. So the sum comes out at $32^{s}\ 1^{\circ}\ 32'\ 0''$. But if the whole circles, which are contained twice in the 32 signs, are to be thrown away, then with 24 thrown away there remain $8^{s}\ 1^{\circ}\ 32'\ 0''$.

[Margin: Multiplicationis Logiſticæ altera forma. — Another form of logistic multiplication.]

Had I resolved the $10^{s}\ 20^{\circ}\ 30'\ 40''$ into the lowest kind given, namely into seconds, they would be 1153840 seconds. For 10 signs multiplied by 30 make 300 degrees; with 20° added these make 320°. Again, 320° multiplied by 60 make 19200 minutes; with 30′ added these make 19230 minutes. Finally, these multiplied by 60 make 1153800 seconds; with 40″ added they make, as I said, 1153840 seconds. If you multiply these by 3 they make 3461520 seconds. These, however, must be reduced to the usual kinds, as follows. When 3461520″ are divided by 60, the result is 57692 minutes without any fraction, and therefore

without any seconds. Next, when 57692 minutes are divided by 60, the result is 961 degrees and 32/60, that is, 32′. Finally, when 961 degrees are divided by 30, the result is 32 signs and 1/30, that is, 1 degree, and so on, as above. For the analysis of this, consult the last scholium written below.

PRAEC. IV. DE DIVISIONE.

Rule IV. On division.

Logistic division likewise has two uses. The first is when astronomical numbers are to be divided by common ones — for example, if signs, degrees and fractional parts are to be divided by three. This is done either by resolving the given astronomical numbers into the lowest kind and carrying out the division in the usual way, or by dividing each kind separately and, if any fractions are left over, adding them to the remainders of the kind immediately following. The second is when astronomical numbers are to be divided by other astronomical numbers. This is not commonly used either, and the whole of astronomy can be completed without it; so I pass it over. The curious reader may consult on it the authors named in Rule III. For the first case, here is an example.

EXEMPLVM DIVISIONIS.

Example of division.

	Sig.	Gr.	′	″
Diuidenda	10	20	30	40
Diuisor 3.	3	3	3	3
Quotiens	3⅓	6⅔	10	13⅓
Hoc est	Sig. Gr. 3 10	Gr. ′ 6 40	′ 10	″ ‴ 13 20
Ergo Quotiens Logisticè.	3^{s}	16°	50′	13″ 20‴

Diuidenda = the numbers to be divided; *Diuisor 3.* = the divisor, 3; *Quotiens* = quotient; *Hoc est* = that is; *Ergo Quotiens Logisticè* = therefore the quotient, reckoned logistically.

So, when 10 signs are divided by 3, they make 3⅓ signs, that is, 3 signs and 10 degrees. Next, when 20 degrees are divided by 3, the result is 6⅔ degrees, that is, 6° 40′. Third, when the

30 minutes are divided by 3, the result is 10 minutes. Finally, when the 40″ are divided by 3, the result is 13⅓ seconds, that is, 13″ 20‴. Reducing this quotient, therefore, to the usual logistic form, the quotient is 3^{s} 16° 50′ 13″ 20‴.

[Margin: Diuisionis alia forma — Another form of division.]

But someone may perhaps find it easier first to resolve the number to be divided into the lowest of the given kinds — in our case, into seconds. Now, as I said in the second part of the example for Rule III, 10^{s} 20° 30′ 40″ resolved into seconds are 1153840. When these are divided by 3, the quotient is 384613⅓

[printed: 384613′. & ⅓]. And when 384613″ are divided by 60, the result is 6410′ 13″. When the 6410 minutes are divided by 60, the result is 106 degrees and 50 minutes. Finally, when the 106 degrees are divided by 30, the result is 3 signs and 16 degrees. So, as above, the quotient is 3^{s} 16° 50′ 13⅓″, or 13″ 20‴.

PRAECEP. V. DE RADICE QVADRATA.

Rule V. On the square root.

If a square root is to be extracted from an astronomical number, the number must be resolved into the lowest kind given, as I taught above, and from the number so resolved the square root — or the root nearest to a square — is to be extracted by the usual rules of arithmetic.

SCHOLIVM.

Scholium.

I. Among the Greeks sixtieths [Sexagenæ] are called ἑξηκόνταδες [Greek: ἑξηκόνταδες (hexēkontades, "sixties")]; minutes [scrupula] are called λεπτὰ ἢ ἑξήκοστα [Greek: λεπτὰ ἢ ἑξήκοστα (lepta ē hexēkosta, "small parts, or sixtieths")]. If these are first parts, or minutes, they are called πρῶτα [Greek: πρῶτα (prōta, "firsts")]; if seconds, δεύτερα [Greek: δεύτερα (deutera, "seconds")]; if thirds, τριτὰ [Greek: τριτὰ (trita, "thirds")]; if fourths, τετάρτα [Greek: τετάρτα (tetarta, "fourths")]; if fifths, πεντὰ [Greek: πεντὰ (penta, "fifths")]; if sixths, ἑκτὰ [Greek: ἑκτὰ (hekta, "sixths")], and so on. Ptolemy, however, indicates in book 1 of the *Almagest*, chapters 10 and 11, that the circle was divided by Eratosthenes and Hipparchus into 83 parts.

II. The Hebrews, the Persians and formerly the Arabs, as Scaliger affirms in book 1 of *De emendatione temporum*, and as appears from the Jewish calendars, divide the hour into 1080 points [puncta], and each point into 76 moments [Momenta]. So one point equals 3 of our seconds and 20 thirds, and a moment equals 2 thirds and 48 fourths.

III. If either the whole circle of 360 degrees, or a degree, an hour or a day, is to be divided into the parts of the as [Aſſis], in the way the as, or pound [Libra], was divided by the ancient Romans into 12 ounces [vncias], or 12 equal parts, consult the following small table. It is useful to those who read Ptolemy regularly, since, as Keplerus [Kepler] noted in the Rudolphine Tables, chapter 14, Ptolemy very often used this division. Thus the sextant of the circle has 60 degrees, the sextant of an hour or of a degree has 5 minutes, the sextant of a day 4 hours, and so on.

[Margin: Diuiſio Aſſis, ſeu Libræ. — The division of the as, or pound.]

Partes Aſſis	Circuli (Gr.)	Gra. vel Hor. (M.)	Diei. (Hor.)
Vncia	30	5	2
Sextans	60	10	4
Quadrans	90	15	6

Partes Aſſis	Circuli (Gr.)	Gra. vel Hor. (M.)	Diei. (Hor.)
Triens	120	20	8
Quincunx	150	25	10
Semiſſis	180	30	12
Septunx	210	35	14
Bes	240	40	16
Dodrans	270	45	18
Dextans	300	50	20
Deunx	330	55	22
As	360	60	24

Partes Aſſis = parts of the as; *Circuli* = of the circle, in degrees (Gr.); *Gra. vel Hor.* = of a degree or an hour, in minutes (M.); *Diei* = of a day, in hours (Hor.).

IV. The analysis of a degree or an hour into fractional parts down to tenth minutes, and likewise the analysis of the circle itself into minutes down to tenths, is very useful for logistic multiplication and division, and so I give both here in the two following small tables.

Gradus vnus / Hora vna — continet	
Minuta	60
Secunda	3600
Tertia	216000
Quarta	12960000
Quinta	777600000
Sexta	46656000000
Septima	2799360000000
Octaua	167961600000000
Nona	100776960000000000
Decima	6046617600000000000

Gradus vnus / Hora vna continet = one degree, or one hour, contains: minutes, seconds, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, tenths.

Circulus vnus continet	
Gradus	360
Minuta	21600
Secunda	1296000
Tertia	77760000
Quarta	4665600000
Quinta	279936000000
Sexta	16796160000000
Septima	1007769600000000
Octaua	6c4661760000000000 [recte: 60466176000000000]
Nona	362797056000000000000
Decima	21767823600000000000000

Circulus vnus continet = one circle contains: degrees, minutes, seconds, thirds, and so on to tenths.

CAPVT VI.

Sphæræ Definitio, Proprietates insigniores, & Dimensiones.

Chapter VI. The definition of the sphere, its more notable properties, and its dimensions.

I. According to Theodosius, *Sphaerica* book 1, definitions 1 and 2, a sphere [SPHAERA] is a solid body enclosed by a single surface, such that all the straight lines drawn to it from one point within the figure are equal; and that point is the centre of the sphere. Or, as Euclid says in book 11, in a causal definition: a sphere is what results when, the diameter of a semicircle remaining fixed, the semicircle is carried round until it returns to the place from which it began to be carried round. Thus, in the figure placed here, if the semicircle ABG is understood to move about the diameter ACG — which stays fixed, supported on the props AD and GF — and to be carried round in one complete revolution, it will generate the sphere ABKG, or will mark out a space equal to the sphere, whose centre will be C.

Diagram. A small woodcut set into the right column beside definition I: a circle, shaded on its right side to model a sphere, crossed by a horizontal diameter whose two ends rest on two vertical props standing on short bases, like a globe on a stand.

- B (top of the circle): the summit of the semicircle ABG
- A (left end of the diameter): end of the axis; top of the left prop
- C (centre, on the diameter): centre of the sphere
- G (right end of the diameter): end of the axis; top of the right prop
- D (foot of the left prop): base of prop AD
- K (bottom of the circle): lowest point of the sphere ABKG
- F (foot of the right prop): base of prop GF

Labels read: B, A, C, G, D, K, F. Labels not read: none.

II. The axis of the sphere, so called from ἀξίω [Greek: ἀξίω (axiō; glossed in the text as *voluo*, "I turn")], which means "I turn", is a straight line drawn through the centre of the sphere and ending on both sides at the surface of the sphere, about which line, while it stays at rest, the sphere can be turned round — in the preceding figure, the line ACG. Therefore the axis of the sphere is also a diameter of the sphere; but not every diameter of the sphere is its axis, since the sphere does not turn round every diameter, but round one only.

III. The poles of the sphere are the two points that end the axis of the sphere at its surface, and that, like the axis, are immovable — as A and G. They are named from πολέω [Greek: πολέω (poleō, "I turn")], which means "I turn", and so they are also called vertices [Vertices] and hinges [Cardines].

IV. The circuit of the sphere [Ambitus], in Greek περίμετερ [Greek: περίμετερ (perimeter, "perimeter")], is its convex surface; whereas in the circle the circuit and the circumference are the same.

PROPRIETATES SPHAERAE INSIGNIORES.

The more notable properties of the sphere.

[Margin: Triplicata proportio quid? — What is a triplicate ratio?]

I. Spheres are to one another in the triplicate ratio of their diameters: so Euclid, *Elements* book 12, proposition 18. A triplicate ratio is when a proportion of the same kind is continued as far as the fourth term. For example, between 2 and 4 there is the simple double proportion; but if it is made "as 2 to 4, so 4 to 8", the ratio between 2 and 8 will be duplicate; and if it is made again "as 4 to 8, so 8 to 16", there will be between 2 and 16 the triplicate ratio of the double proportion. Thus, if the diameter of the smaller sphere were 2 feet and that of the larger 4, the solidity of the smaller sphere will stand to the solidity of the larger as 2 to 16. Hence, when the solidity of one and the diameters of both are known, the solidity of the other is easily found.

II. The surface of any sphere is four times the great circle in the sphere: so Archimedes, *On the Sphere and Cylinder*, book 1, proposition 30.

III. The rectangle under the diameter and the circumference of a sphere is four times the great circle in the sphere [maximi in Sphæra Circuli]: from Clavius, *Sphaera*, p. 100. Therefore, by property 2, such a rectangle is equal to the convex surface of the sphere itself.

IV. The solid rectangle contained under the semidiameter of a sphere and a third part of its convex surface is equal to the whole sphere. From Clavius, *Geometria Practica*, book 4, and in the *Sphaera*, where he treats isoperimetric figures, proposition 16, or p. 100, where he calls the surface of the sphere its *Ambitus*, and its solidity or total capacity its *Area*.

V. A sphere is equal to a cylinder whose base is equal to the great circle of the sphere, and whose altitude or axis is equal to two-thirds of the sphere's diameter. Archimedes, *On the Sphere and Cylinder*, proposition 32.

VI. A cylinder has to the sphere inscribed in it a sesquialteral ratio [ſeſquialteram proportionem]: Archimedes, *On the Sphere and Cylinder*, book 1, proposition 32. It is therefore as 3 to 2. Moreover, this cylinder has a base equal to the great circle of the inscribed sphere, and an axis or altitude equal to the diameter of that same sphere.

VII. A sphere is equal to a cone whose axis or altitude equals the sphere's diameter, and whose base has a diameter twice as large as the diameter of the sphere. Archimedes, as above.

VIII. A sphere is four times a cone whose base equals the great circle of the sphere and whose altitude equals the sphere's semidiameter. Archimedes, as above.

IX. The surface of a sphere is equal to a circle having a semidiameter equal to the diameter of the sphere. This is a corollary of Archimedes, from property 2, discussed above.

X. A cube has to its inscribed sphere a ratio greater than 21 to 11, but less than 426 to 223. Clavius, from Archimedes, *Geometria Practica*, book 5, chapter 5.

XI. The square on the diameter of a great circle of the sphere has to the surface of the sphere a ratio greater than 7 to 22 and less than 71 to 223. Clavius, as above; he also shows that the said square has to the inscribed circle a ratio greater than 14 to 11, but less than 284 to 223.

XII. The rectangle contained under the diameter of a great circle in the sphere and under its periphery is equal to the convex surface of the sphere, and four times that same circle. Clavius, *Geometria Practica*, book 5, chapter 5, proposition 2.

The remaining properties of the great or non-great circles of a sphere will be reviewed presently, in the next chapter.

DIMENSIONES SPHAERAE

Dimensions of the Sphere

Probl. 1. Datis Diametro, & Circumferentia Circuli in Sphæra maximi, inueſtigare Superficiem Sphæræ conuexam.

Problem 1. Given the diameter and the circumference of a great circle in the sphere, to find the convex surface of the sphere.

[Margin: Superficiei Sphæricæ inuentio. — Finding the spherical surface.]

Multiply the circumference of the great circle by its diameter, and the total will be the surface of the sphere in square measure. Alternatively, find the area of the great circle by Problem 3 of chapter 4 and quadruple it, for you will obtain the same result. This follows from properties 2, 9 and 12 of the sphere, discussed above.

Probl. 2. Datis Semidiametro, & Superficie Sphæræ, inquirere Soliditatem Sphæræ.

Problem 2. Given the semidiameter and the surface of a sphere, to find the solidity of the sphere.

[Margin: Soliditatis Sph. inuētio. — Finding the solidity of a sphere.]

Multiply a third part of the convex surface of the sphere by the semidiameter, and you will have the solidity of the sphere, though in cubic measures. You will obtain the same result if you multiply a sixth part of the surface by the diameter of the sphere. This follows from property 4 above.

Probl. 3. Cubum inuenire ſoliditati Sphæræ datæ æqualem.

Problem 3. To find a cube equal to the solidity of a given sphere.

Find the solidity of the sphere by Problem 2, and find the cube root of that number; for it will give the side of a square on which, if a cube is raised, the cube will be equal to the sphere, or very nearly so if the cube root is not exact but approximate.

Probl. 4. Conum inuenire æqualem Sphæræ datæ.

Problem 4. To find a cone equal to a given sphere.

Let a right cone be made whose altitude is equal to the diameter of the sphere, and whose base is a circle having a semidiameter equal to that same diameter of the sphere; for it will be equal to the sphere, by property 7.

Probl. 5. Cylindrum inuenire æqualem Sphæræ datæ.

Problem 5. To find a cylinder equal to a given sphere.

Let a right cylinder be made whose base is a circle having a diameter equal to the diameter of the sphere, and whose altitude is to the diameter of the sphere as 2 to 3; and this cylinder will be equal to the solidity of the sphere, by property 5.

CAPVT VII.

De Circulis Sphæræ Naturalis, & Artificialis in Communi, eorumque Proprietatibus.

Chapter VII. On the circles of the natural and the artificial sphere in general, and on their properties.

[Margin: Sphæra Naturalis. — The natural sphere.]

The natural sphere, among cosmographers, is the celestial, the elementary, and the terrestrial or terraqueous sphere, since these were made by Nature herself. From what was said in chapter 2, and from what is to be said in the Book on the Elementary Sphere, it is assumed that the heavens, the mass of the elements bounded by the concavity of the lunar heaven, and the globe compacted of earth and water are of spherical shape.

[Margin: Sphæra Artificialis. — The artificial sphere.]

The artificial sphere is one made by art in imitation of the natural sphere. It is either solid, as with the celestial globe, on which the fixed stars are marked and which is called the Aratean sphere [Sphæra Aratæa] after Aratus, who first used or perfected it; and as with the terraqueous globe, on whose surface are drawn the peripheries of the principal circles, imitating in site and position the circles of the celestial sphere.

[Margin: Sphæra Armillaris. — The armillary sphere.]

Or else it is made of rings or armillae representing the circumferences of the circles of the natural sphere, and is called the armillary or material sphere. It is built in such a way that an eye contemplating it from outside can see its parts within, along with the circles of the secondary movables [fecundorum Mobilium] and of the planets that are usually inserted, and in such a way that it may be lighter and easier to handle. Of whatever form and workmanship, in the end, was the sphere that Archimedes enclosed in glass and that Claudian praised in that celebrated epigram:

Iupiter in paruo cum cerneret Aethera vitro,
When Jupiter saw the heavens in a small glass,

Although the image of either sphere can be projected onto a plane in many ways and represented on a plane, an art treated above all by Guidubaldus de Monte [Guidobaldo del Monte] and many others in their planispheres, it is nevertheless better at this point to set before beginners its simplest image, so that they are not frightened off at the start by the multitude and confusion of the circles; provided only that they grow used to imagining, under straight or curved lines, the circles usually signified by their diameters or circumferences, whether whole or merely sketched [adumbratis].

Although, therefore, at the beginning of this book, on p. 1, I set out a figure of the oblique sphere to be viewed with the eye turned to the east, with the divisions of the four principal circles into degrees indicated; and although it seemed right here to set out, roughly in imitation of it, another figure, but of a right sphere, with the eye however turned to the west, and with the beginning of Cancer placed in mid-heaven at C, and so the north pole of the ecliptic below the horizon at M: nevertheless, for greater simplicity and clarity, we shall use a third schema to explain the circles of the sphere, in which the eye, turned to the west, sees a right sphere, so that the north pole of the ecliptic is above the horizon at T, and the beginning of Capricorn in mid-heaven at K.

Let the second schema, then, be of an armillary sphere, but a right one, as you see in the following figure.

Diagram. Woodcut of an armillary sphere in the left column: a heavy outer ring (the meridian) enclosing crossing armillary bands, with a broad graduated band running obliquely across the figure, a small dark globe at the centre, and a horizontal ring through the centre. Labels read on the block:

- E (top)
- K (upper left of E) and C (upper right of E)
- N (upper left) and T (upper right)
- A (left end of the horizontal) and B (right end of the horizontal)
- G (above the central globe)
- F (left of the globe) and I (right of the globe)
- O (lower left) and M (lower right)
- P, Q, R (along the bottom, left to right)

I. Let the third schema, then, be as in the following figure. With centre G and at the interval of the semidiameter AG, let a circle AEBQ be described; it will represent the meridian and at the same time the solstitial colure [Colurum Solftitiorum], while its diameter AGB will represent both the horizon and the axis of the equator. Next let EGQ be drawn, equal to AGB, cutting it in two equal parts and at right angles at the centre G; for it will represent the plane of the equinoctial circle, whose axis will be AGB. The poles of the equator, which are also those of the world, will be the points A and B, namely the southern and the northern. Next let one quadrant of the circle, say AE, be divided into 90°, and from it let 23½° be taken with the compass. On the solstitial colure AEBQ, starting from point E of the equator, let an arc of the same number of degrees, 23½°, be taken, and let it be arc EK; and from point Q let another arc

equal to it be taken toward the other side, and let it be QR. Having marked the points KR, let them be joined by two parallels KR, distant from each other by 12°, or 16°, or 20°; for the small zone enclosed by them, KGR, will represent the zodiac, through the middle of which runs the ecliptic line. Through the centre G let a straight line OGT be drawn, equal to AGB, and let it cut in G, in two equal parts and at right angles, the ecliptic, that is, the middle line of the parallels KGR; for OGT will represent the axis of the ecliptic, whose poles will be O and T. These will be as far from the poles of the world as the ecliptic is from the equator, so that arcs TB and AO will always be as large as EK and QR.

[Margin: Zodiacus. — The zodiac.]

Diagram. Linear woodcut in the left column: a circle with horizontal diameter, vertical diameter and an oblique diameter through the centre, two close oblique lines running from upper left to lower right, and lines near the left and right of the circle joining points on the circumference. Labels read on the block:

- E (top), with K to its left and C to its right
- N (upper left) and T (upper right)
- A (left) and B (right)
- F (left of centre) and G (at centre)
- O (lower left)
- M (lower right)
- P, Q, R (along the bottom, left to right)

II. With this done, take arc PQ equal to arc QR, and EC equal to arc EK, and also arc AN equal to AO, and arc BM equal to TB, marking the points PC NM on the solstitial colure. Through points K and P let a line KFP be drawn, either straight or curved, representing the circle parallel to the equator EGQ; for it will show the Tropic of Capricorn.

[Margin: Tropici. — The tropics.]

A line similarly drawn through points C and R, namely CR, will represent the Tropic of Cancer. Thus point K will be the beginning of Capricorn, point R the beginning of Cancer, and point G the beginning of Aries on one side and of Libra on the other. Finally, through T and M let a straight line be drawn, or if you prefer a curved one, parallel to CR; and through N and O a line parallel to KP. For these will represent the polar circles: TM the Arctic circle, and NO itself the Antarctic circle.

[Margin: Polares circuli. — The polar circles.]

The spaces enclosed by these will represent zones. For the space enclosed within the tropics KP and CR will represent the Torrid Zone, whose breadth KEC, or PQR, will be 47°.

[Margin: Zona. — Zone.]

The space enclosed between the Tropic of Cancer CR and the Arctic circle TM will represent the North Temperate Zone, and the space enclosed between the Tropic of Capricorn KP and the Antarctic circle NO the South Temperate

Zone. The breadth of each of them, CT and KN, will be 43°. The remaining spaces, enclosed between the polar circles and the nearer poles of the world, will represent the Frigid Zones: the northern TBM etc., and the southern NAO etc. Their semidiameter, TB and NA, will be 23½°, while the diameter, TM and NO, will be 47°, which is, of course, the breadth KC of the Torrid Zone.

QVID SINT CIRCVLI IN SPHAERA

Maximi, & non Maximi; Paralleli, & non Paralleli.

What great and non-great circles, and parallel and non-parallel circles, are in a sphere.

I. Those circles are said to be in a sphere whose peripheries lie on the surface of the sphere itself. Others that can be considered within it are nevertheless not considered by cosmographers, except for the circles of the planets; but those are considered in the theorics of the planets, not here, although the chief of them surround their spheres. Further, the circles of a sphere are divided into great and non-great, or small; and into parallel and non-parallel, or inclined.

II. Great circles in a sphere are those that pass through the centre of the sphere; non-great circles are those that do not [non] pass through the centre of the sphere, by proposition 6 of book 1 of the *Sphaerics* of Theodosius of Tripoli.[5]

III. Parallel circles are those that are equally distant either from one another or from some great circle in the sphere; non-parallel circles are those that are inclined to one another and are not equally distant from one another.

IV. The poles of any circle in a sphere are the points on the surface of the sphere from which all straight lines drawn to the circumference of their circle are equal to one another, from definition 5 of book 1 of the *Sphaerics* of Theodosius. This holds true, however, also of arcs on the surface of the sphere drawn perpendicularly from the pole to the circumference, for these too are equal to one another.

V. The distance of small circles from the centre of the sphere is taken along the perpendicular, that is, the straight line drawn from the centre of the sphere to the plane of the circle, from definition 6, added by Clavius to the definitions of Theodosius in his book 1 of the *Sphaerics*.

QVOT, ET QVI SINT CIRCVLI

Maximi Sphæræ Cœleſtis, & Terreſtris.

How many and which are the great circles of the celestial and the terrestrial sphere.

[Margin: Circuli maximi. 13. — Great circles: 13.]

VI. The great circles in the celestial sphere are 13, namely: the horizon; the meridian; the equator; the ecliptic with the zodiac; the equinoctial colure; the solstitial colure; the vertical circles, or circles of altitude; the circles of

[5] the "non" before the second "transeunt" stands between the lines, not in the line of type, and looks like a later addition in ink; the line of type alone reads "non Maximi ſunt, qui tranſeunt"; the errata of the book make the same correction.

declination; the circles of latitude; the hour circles [Horarij]; the circles of position; the circles of the celestial houses; and the circles of distances. Of these the meridian, the horizon and the vertical are immovable, and the rest movable. In the artificial and armillary sphere, however, only the first six are usually set. It would be useful, though, to add one circle turning about the poles of the equator, to serve for the hour circles and the circles of declination; one turning about the poles of the ecliptic, to serve for the circles of latitude; and a third drawn through the poles of the horizon, that is, through the zenith and the nadir, a vertical one, for altitudes. For by moving these around, beginners would more easily learn the doctrine of the primum mobile and many theorems and cosmographical problems.

[Margin: Circuli n[?] Maximi. — Circles n[?] great.]

VII. The circles in the celestial sphere that are not great, or are smaller than the great circles, are 10, namely: the Tropic of Cancer; the Tropic of Capricorn; the Arctic circle; the Antarctic circle; the circles of the climates; the circles of perpetual visibility [ſemper Apparentium]; the circles of perpetual occultation [Semper Latentium]. These seven are parallel to the equator when they are distinct from it; for the equator itself, too, is one of the circles at once of perpetual visibility and of perpetual occultation. The remaining three kinds are:

circles parallel to the ecliptic; the almucantar circles [circuli Almuncatarath], or those drawn through altitudes, which some therefore do not distinguish from the circles of altitude, and these are parallel to the horizon; and the physical or sensible horizon, which is itself also parallel to the horizon simply so called, or the astronomical and rational horizon, and is one of the almucantar circles. In the armillary sphere, however, only the tropics and the polar circles are usually set.

VIII. Circles of longitude, moreover, are not distinguished from the ecliptic or from the parallels of the ecliptic, just as circles of right ascension are not in reality distinguished from the equator or from the parallels of the equator. And although among these are the tropics and the polar circles, the circles of perpetual occultation and of perpetual visibility, and those of the climates, nevertheless there are besides these innumerable other circles parallel to the equator, which can serve to determine the right ascension of stars or of points of the ecliptic, even though the equator alone suffices to measure it. Finally, the circles bounding the zodiac are in truth small circles, belonging to the parallels of the ecliptic, even though the zodiac, because of the ecliptic, is counted among the great circles.

SCHOLION.

The remaining circles that can be imagined in a sphere either belong to the circles of the planets and lie within the sphere, or are not usually considered by cosmographers, such as the parallels of the meridians and of the verticals. Along

these, as we shall show in Book 4 of Volume 2 of the *Almagest*, in the section on Histiodromia, one can travel and sail on the surface of the terraqueous globe. This kind of circle, though less often named, could be added to the small circles, at least those of the terrestrial sphere.

PROPRIETATES INSIGNIORES

Circulorum Sphæræ in Communi.

The more notable properties of the circles of a sphere in general.

I. Great circles in a sphere pass through the centre of the sphere, and conversely. Theodosius, *Sphaerics*, book 1, proposition 6.

II. Great circles in a sphere cut one another in two, that is, into two equal portions; and conversely, if they cut one another in two, they are great. Theodosius, book 1, propositions 11 and 12.

III. Great circles in a sphere are equal to one another, for they have equal diameters, by the corollary of the preceding.

IV. Of parallel circles, those equally distant from the centre of the sphere are equal to one another, and those unequally distant are unequal; and of these, those farther from the centre of the sphere are smaller. Theodosius, book 1, proposition 6.

V. In any circle in a sphere, a straight line drawn from the centre of the circle to the centre of the sphere falls perpendicularly on the circle, that is, makes right angles with the plane of the circle. Theodosius, book 1, proposition 7.

VI. A straight line drawn from the centre of the sphere through the centre of a circle placed in the sphere, if produced, falls on the poles of that circle. Theodosius, book 1, proposition 8. The Arabs and Maurolico add this converse of it: If there is a circle in a sphere, and from one of the circle's poles a straight line is drawn to the centre of the circle, it will be perpendicular to it, and when produced will pass through the other pole; I myself add: and through the centre of the sphere.

VII. A straight line drawn from one pole of a circle in a sphere perpendicularly to the plane of the circle falls on the centre of the circle, and if produced falls on the centre of the sphere and on the other pole of the same circle. Theodosius, book 1, proposition 9.

VIII. A straight line drawn through the poles of a circle in a sphere falls perpendicularly on the plane of that circle, and passes through the centre both of the sphere and of the circle. Theodosius, book 1, proposition 10.

[Margin: Corollaria. — Corollaries.]

From the said propositions V to VIII it follows that a great circle which passes through one of the poles of any circle in a sphere also passes through the other; and therefore that the two poles of any circle in a sphere are diametrically opposite, that is, that a semicircle of a great circle lies between them; and that these four points, namely the two poles of any circle in a sphere, the centre of the circle, and that of the sphere, lie on one and the same straight line, which is

also the diameter of the sphere, as Clavius gathers in his corollaries to the said propositions of Theodosius.

IX. If in a sphere a great circle cuts some circle at right angles, it will cut it in two and will pass through its poles. Theodosius, book 1, proposition 14.

X. If in a sphere a great circle cuts a non-great circle in two, it will cut it at right angles and through the poles. Theodosius, book 1, proposition 14.

XI. If in a sphere a great circle cuts some circle in the sphere so as to pass through its poles, it will cut it in two and at right angles. Theodosius, book 1, proposition 15.

Four other theorems are added here by Clavius from another version, and they are these. 1. If in a sphere a great circle passes through the poles of another great circle, the latter in turn passes through the poles of the former. 2. If in a sphere a circle cuts a circle through the poles and is great, it cuts it in two and at right angles. 3. If in a sphere a circle cuts a circle in two and at right angles, it is a great circle and cuts it through the poles. 4. If a straight line drawn from the pole of a circle in a sphere perpendicularly to the plane of that circle is equal to the semidiameter of the sphere, the circle is great. Clavius, in his scholia on Theodosius.

XII. A straight line drawn from the pole to the circumference of a great circle in a sphere is equal to the side of the square inscribed in a great circle. Theodosius, book 1, proposition 16.

XIII. If in a sphere there is a circle such that a straight line drawn from its pole to its circumference is equal to the side of the square inscribed in a great circle, that circle will itself be great. Theodosius, book 1, proposition 17.

XIV. If in a sphere a straight line drawn through the centre cuts in two another straight line in the sphere not drawn through the centre, it will cut it at right angles; and conversely, if it cuts it at right angles, it will cut it in two. Theodosius, book 1, proposition 22.

XV. In a sphere, parallel circles are described from the same poles; and conversely, if they are described from the same poles, they are parallel. Theodosius, book 2, propositions 1 and 2.

XVI. If in a sphere a great circle touches one of the circles described on the spherical surface, it will also touch another equal and parallel to it. Theodosius, book 2, proposition 6.

XVII. Any arc drawn from the pole of a great circle in a sphere to the circumference of that same circle is a quadrant of the circle, or 90°, from Regiomontanus, Clavius and others on spherical triangles; and that same arc makes right angles with the circumference both of the great circle and of a circle parallel to that great circle: by the same authors.

We shall have to return to these properties, or to the others set down by Theodosius in books 2 and 3 of the *Sphaerics* and explained there by Maurolico or by Clavius, for many of the things to be said below about the circles of the sphere in particular, if anyone wishes to know their more universal reasons.

[Margin: Proportio circuli maximi ad nō maximū. — The ratio of a great circle to a non-great one.]

The ratio of the circumference of a great circle to a non-great one is given by Clavius in the *Sphaera*, p. 384; by Magini on Ptolemy's *Geography*, book 1, chapter 20; and by us in the Geographical Book. See meanwhile Scholium 3 of chapter 16, and the table of it at the end of the work.

CAPVT VIII.

De Horizonte Rationali, & Senſibili.

Chapter VIII. On the rational and the sensible horizon.

[Margin: Horizontis ſpecies. — The kinds of horizon.]

The HORIZON, in Greek [Greek: ὁρίζων (horizōn, "bounding")], from the Greek verb [Greek: ὁρίζομαι (horizomai, "to bound")], which means "I bound" or "I limit", is the circle called *Finitor* [the Bounder], because it bounds and limits day and night, and the sight of an inhabitant on the sphere, etc. It is divided into the rational horizon, which is also called the natural and the astronomical, and the horizon simply so called. Proclus and Geminus, however, call it [Greek: λόγῳ θεοριτὸν [recte: θεωρητόν] (logō theoriton, "contemplable by reason")], contemplable by reason, since our senses do not reach it, at least not the whole of it, but find it by reasoning. It is divided, I say, into the rational and the sensible or physical, which for Proclus and Geminus is [Greek: αἰσθητὸν (aisthēton, "perceptible")], perceptible, that is, by sense; Clavius calls it apparent, and Maurolico and Neander artificial. So much for the name.

[Margin: Horizon Rationalis. — The rational horizon.]

II. The *rational horizon* [Horizon Rationalis] is a great circle of the sphere whose poles are at the inhabitant's vertex and at the point opposite the vertex. It divides the sphere into an upper and a lower hemisphere with respect to someone living on the sphere. To understand this, let C in the accompanying figure be the centre of the world and of the Earth, and BAEN, etc., the globe of the Earth; let HVRK be the heaven, or the celestial meridian; and let an inhabitant A stand on the surface of the Earth. Above his head let there be in the heaven the vertex or vertical point V, and diametrically opposite it the point K. From the points V and K let the great circle HCR be conceived as described: this will be the rational horizon. Its poles will be V, which in Arabic is called *Zenith*, and K, the pole or point that is called *Nadir*, and in Latin *Imum Cæli* [the lowest of the heaven]. If a straight line VK is drawn through these points, it will be the vertical line, or the axis of the horizon, if the horizon were imagined as movable. The horizon will divide the sphere into an upper hemisphere, which in the universe is HVRC and on the terrestrial sphere is BAEC, and into a lower one, which there is HKRC and here is BNEC. The upper is painted white and the lower black, because the Sun placed above the

horizon makes day above and in the meantime is not in the lower hemisphere, or because the upper is visible to the inhabitant A and the lower hidden.

Diagram. Woodcut inset at the right of the left column: a large outer circle with a vertical and a horizontal diameter, and a small inner circle about the centre whose lower half is solid black. Labels read on the block:

- V (top, outside the circle) and K (bottom, outside the circle)
- H (left, outside the circle) and R (right, outside the circle)
- A (on the vertical, above the inner circle)
- B (left), C (centre, just above the horizontal), E (right), along the inner circle's horizontal diameter
- N (on the vertical, below the black half of the inner circle)

[Margin: Horizon Rectus, & obliquus. — The right and the oblique horizon.]

III. The *right horizon* [Horizon Rectus] is the one that passes through the poles of the world and of the equator, as HCR does if we suppose the poles of the world to be H and R, and the equator to be represented by the line VCK. This horizon makes right angles with the equator, and so those living on it are said to live in a right sphere, whose properties are given below. The *oblique horizon* [Horizon Obliquus] is one that does not pass through the poles of the world, but has one of them raised above itself and visible to the inhabitant, and the other below and hidden; of this we shall speak when we treat of those living in an oblique sphere. See its position in the figure placed on page 1.

[Margin: Proprietates Horizōtis. — Properties of the horizon.]

IV. The chief *properties of the horizon* [Horizontis Proprietates] are these. 1. It makes right angles with the meridian. 2. It divides the upper hemisphere from the lower. 3. It is the limit from which the altitudes of the pole and of the stars are taken. 4. It determines the true rising and setting of the stars, which are said to rise when they ascend above the horizon and to set when they descend below it; but above all the Sun, and therefore. 5. In it are marked, besides the four cardinal points, which are *Septentrio, Meridies, Ortus Occaſuſque Aequinoctialis* [north, south, and the equinoctial rising and setting], four other points, namely the solstitial rising and setting at each solstice. In the figure of Chapter 7 these are RP, CK, if the circle AEBQ in that figure is supposed to be the horizon and the line AB the meridian. The point of north and of south is the one at which the meridian intersects the horizon, as in the same figure B and A. The points of equinoctial rising and setting are those at which the equator intersects the horizon, as there Q and E; and lastly the points of solstitial rising and setting are those at which the tropics intersect the horizon. 6. It determines the stay of the stars above and below it; this stay, if it is of the Sun and above, is called the artificial day; if below, the artificial night. 7. It is the circle in which the ortive and occasive amplitude of the stars is measured. This amplitude is as great as the arc of the horizon intercepted by its points at which stars can rise or set, according to their different declination and latitude; half of it is the arc of the horizon between the said points and the points at which the equator intersects it. 8. It is the circle in which are marked the regions from

which the naming of the winds is taken. 9. It shows with which degree of the ecliptic any star rises or sets, and which degree of the equator ascends or descends with a given point of the ecliptic or a star, rightly in a right sphere and obliquely in an oblique one, if that point of the ecliptic or the star is placed on the horizon according to the appropriate altitude of the pole. In this way it determines the right or oblique ascensions and descensions.

[Margin: Azimutha, & Almuncatarath. — Azimuths and almucantars.]

10. It is the circle in which *azimuths* [Azimutha] are taken; these are nothing other than arcs of the horizon intercepted between the meridian and another vertical circle in which the star is located. 11. It is the circle with respect to which the *almucantars* [Almuncatarath] are taken, namely the parallels of the horizon passing through the stars according to their different altitudes. 12. It is the plane with reference to which craftsmen determine the sensible horizon, as being parallel to the rational horizon, when they true floors to the level.

DE HORIZONTE SENSIBILI.

On the sensible horizon.

[Margin: Horizon Senſibilis. — The sensible horizon.]

V. The *sensible horizon* [Horizon Senſibilis], or physical, or artificial, is a non-great circle of the sphere, parallel to the rational horizon, which divides the segment of the sphere visible to the inhabitant from the invisible one. It is called artificial because craftsmen use it in levelling buildings, waters and instruments that are to be set perpendicular to the horizon or so as to stand equidistant from it, although by means of this horizon reference is also had to the rational horizon. In the following figure, then, let C be the centre of the Earth and of the world; FAG the globe of the Earth; HVRK the heaven or the celestial meridian; A an inhabitant on the surface of the terrestrial globe, whose zenith is V and nadir K, and whose vertical line is VAK; and HCR the rational horizon, drawn at right angles to that line. Let the straight line SAP be drawn parallel to HCR and tangent to the Earth at A. It will represent the sensible horizon of the inhabitant A, because it divides the visible segment of the heaven SVP from the remaining invisible one SKP.

[Margin: Diſcrimen Rationalis, & Senſibilis Horizōtis. — The difference between the rational and the sensible horizon.]

VI. The sensible horizon therefore differs from the rational. 1. Because it is a non-great circle. 2. Because it does not divide the sphere into two hemispheres or equal segments, but into two unequal segments, of which the upper, SVP, is smaller and the lower, SKP, larger. 3. Because all the other properties that it shares with the rational horizon it has only as regards appearance, except the first: for it makes right angles with the meridian strictly. But altitudes, risings and settings and their amplitudes, days and nights, ascensions and descensions, azimuths and almucantars, when taken from this horizon or in it, are called

apparent. For example, if HVRK is imagined as a great circle of altitude, and the Sun rises for the eye at A at the point R of the rational horizon, it nevertheless does not of itself, setting aside the refractions of rays, rise visibly for that eye, since the eye at A cannot see what lies below the horizontal line SAP if it is in no way raised above the Earth. But when the Sun has reached P it can appear and rise sensibly for the eye at A. Conversely, when it sets sensibly at the point S, it has nevertheless not yet truly set with respect to the rational horizon, until it descends below the point H.

Diagram. Woodcut inset at the right of the right column: a large circle with a vertical and a horizontal diameter and a smaller circle about the centre; a horizontal line through the top of the smaller circle runs to the large circle on both sides, a shorter horizontal line lies below it within the smaller circle, and two lines from a point on the vertical above the smaller circle pass the smaller circle on either side to the large circle at lower left and lower right, where a horizontal line joins them. Labels read on the block:

- V (top, outside the circle) and K (bottom, outside the circle)
- B and A (on the vertical, B above A)
- S (left, outside the circle) and P (right, outside the circle), at the ends of the upper horizontal
- F (left) and G (right), at the ends of the shorter horizontal
- H (left, outside the circle) and R (right, outside the circle), at the ends of the horizontal diameter
- C (below the centre of the horizontal diameter)
- M (lower left, outside the circle) and N (lower right, outside the circle)

[Margin: Proprietates Senſibilis Horizōtis. — Properties of the sensible horizon.]

VII. The specific properties of this horizon are these. 1. As I said, it separates the visible and smaller segment of the sphere from the larger and invisible one. 2. It determines the invisible portion of the upper hemisphere of the heaven, whose measure is the arc PR or HS, subtended by the Earth's semidiameter CA; so that the eye A sees almost the whole upper hemisphere, namely SVP, except of course the small part HS or PR, which is the smaller the greater the distance CR of the heaven from the Earth's centre C. 3. It is the limit of horizontal parallaxes and refractions. 4. It is the limit from which winds truly reach the inhabitant with a perceptible blast; for they are not judged able to reach him from the rational horizon. 5. It is the plane to which craftsmen directly align their levels and plumb lines.

SCHOLIA.

[Margin: An cuilibet ſuus ſit Horizon. — Whether each person has his own horizon.]

I. The question is often asked whether every inhabitant has his own horizon, distinct from the horizon of another person living on the surface of the Earth.

There is no doubt that he has, if astronomical rigour is kept, since to each person there corresponds in the heaven his own distinct vertical point or zenith. Physically, however, those people have the same horizon for whom the same point of the heaven appears to hang perpendicularly above the head, as when some fixed star appears to many people to be at the vertex of each. For its distance from the vertex of another inhabitant cannot be noted as one minute unless he moves away by about one Italian mile, etc.

[Margin: Horizon oculi à Terra eleuati quis? — What is the horizon of an eye raised above the Earth?]

II. I have not found it examined what the sensible horizon is of an eye or of an inhabitant raised above the Earth, as when the eye climbs from A to the top of a mountain B. For if you say that its horizon is SAP, or another line parallel to it and passing through B, it will not have the principal properties of the physical horizon: it does not cut off from the hemisphere the visible part from the invisible, since an eye so raised can see a large part of the heaven below that line, and indeed sometimes below the rational horizon. Let the straight visual lines BGN on one side and BFM on the other be drawn from the eye B, touching the Earth at F and G: the eye will then see more than a hemisphere, namely the portion MVN, and a star can rise for it at N before reaching the point R of the rational horizon. But if you say that it is the straight line FG, parallel to the rational horizon and drawn from contact to contact of the visual line touching the Earth, because such a horizon divides the smaller and visible segment of the terrestrial sphere, namely FAG, from the larger and unseen one, then this will be a horizon only with respect to the terrestrial sphere. If you say that the physical horizon is the lines BM and BN, or the space enclosed by them all round, it does indeed divide the part of the heaven visible to the eye B, which is MVN, from the invisible MKN, but it is not a circle described from the vertex V as from a pole. It remains, therefore, that it be the circle marked by the straight line MN. Although in the preceding figure this line touches the Earth, that is not necessary: it can cut the Earth, or neither cut nor touch it; and it can also coincide by accident with the astronomical or rational horizon HCR, as the height of the eye AB may require. It is of this horizon, then, with respect to the heaven, that Maurolico must be understood in the *Cosmographic Dialogues*, p. 36, when he says: "Indeed it will be able to view even more than a hemisphere, if it is set in so elevated a place." On the Earth, however, it will be the straight line FG, parallel to it, because this cuts off the visible segment FAG.

[Margin: Diſtantia viſus quāta? — How great is the distance of sight?]

III. One can ask here how great the distance of sight is of an eye raised above the Earth. But the solution of this question depends not only on the eye's height AB, but also on the Earth's semidiameter CA, on whose size depend the arcs AF and AG, which take in the space of the Earth visible from the eye B. Meanwhile let it suffice to know that if the eye B is 6 feet high, which is about

the stature of a man, the arc AF does not reach four Italian miles. The rest I shall say in Book 2, Chapter 10.

[Margin: Portio Cæleſtis Hemiſphærij latens. — The hidden portion of the celestial hemisphere.]

IV. It can likewise be asked how great the portion of the celestial hemisphere is that is hidden from the eye A, namely the arc PR or SH; and likewise how great the portion is that the eye raised at B sees beyond the hemisphere. But both depend on the distance of the heaven from the centre of the Earth, which is CR. Meanwhile let it suffice to know that the arc PR is as great as the horizontal parallax of the star to which the heaven in question belongs. Both will therefore be treated more fully in the Optical Book, Section 1, where parallaxes are discussed; or consult the tables of the horizontal parallax of the Sun, the Moon and the fixed stars in Books 3, 4 and 6.

CAPVT IX.

De Meridiano Circulo.

Chapter IX. On the meridian circle.

[Margin: Definitio Meridiani — Definition of the meridian]

I. The MERIDIAN circle, in Greek [Greek: μεσημβρινὸς (mesēmbrinos, "of midday")], for Ptolemy the circle of midday, or the circle that runs through the middle of the heaven, is a great circle of the sphere passing through the poles of the world and the vertex and lowest point of the inhabitant, and its poles are at the horizon's points of equinoctial rising and setting. In the figure of Chapter 7 such a circle is AEBQ, if the horizon there is AGB, the poles of the world A and B, and so the zenith E and the nadir Q. It is called the meridian because, when the centre of the Sun reaches it by the motion of the primum mobile, it becomes midday for all who are under that circle, if the Sun is above the horizon, just as it becomes midnight if the Sun is below.

[Margin: Meridiani proprietates. — Properties of the meridian.]

II. The properties and functions of the meridian are these. 1. It cuts the equator and the horizon in half and orthogonally, that is, at right angles, by property 11 of Chapter 7, since it passes through the poles of both and is a great circle. 2. It divides the sphere into an eastern and a western hemisphere; signs or stars placed in the eastern are said to ascend, and those in the western to descend, as regards the motion of the primum mobile. 3. It divides equally and in half the diurnal arc into two semidiurnal arcs and the nocturnal arc into two seminocturnal arcs; on this depends the length of the time before noon and after noon, reckoned from rising and setting. 4. It measures the greatest altitude of any star that it can have on any given horizon on a given determinate day; for this altitude is as great as the arc of the meridian between the horizon and the star's centre, not greater than a quadrant. 5. It makes midday, when the Sun's centre has reached the meridian altitude. 6. It measures the altitude of the pole in an oblique or a parallel sphere (for in a right sphere the pole is not

raised at all); for the altitude of the pole is as great as the arc of the meridian between the horizon and the visible pole of the world or of the equator. 7. By cutting the equator in half it divides the natural day into two equal parts, and so into twelve hours before noon and twelve after. 8. It is equivalent to the horizon of the right sphere, and therefore determines right ascensions, even for an oblique horizon or for those living in an oblique sphere; for the same point of the equator that rises or sets together with some point of the ecliptic in a right sphere crosses the meridian together with that point in an oblique sphere. And so astronomers, to avoid the variation of oblique ascensions, begin the natural day from noon. 9. It is therefore the limit from which astronomers count the hours and moments of the natural day, because the stars stand in the same relation to the equator on the meridian, but not on the horizon, as we shall say when we treat of the inequality of natural days. 10. It determines the beginning of geographic longitude, and together with other meridians the difference of longitude between any two places on the Earth.

SCHOLIA.

[Margin: Quot sint Meridiani — How many meridians there are]

I. The meridians worth considering in the sphere are far fewer in number than the horizons. For every inhabitant and every point of the sphere has its own horizon, but not everyone has his own meridian: many live under the same meridian, from one pole to the other. And although as many meridians could be drawn on geographic globes or maps as there are degrees of the equator, and indeed as many as there are minutes, they are usually drawn only as 24, taking in 15° of the equator each, or the time of one hour; or 36, taking in 10° each; or 72, taking in 5° each. The rest on these matters is given below, when we treat of geographic longitude and latitude.

[Margin: Lauretani Templi B. Virg. Situs. — The site of the Temple of the Blessed Virgin at Loreto.]

II. Biancani, *Sphaera*, Book 1, Chapter 4, adds that both the Temple of Solomon in former times, and now the House of the MOST BLESSED VIRGIN at Loreto, were divinely so placed that two opposite faces of each stood toward the equinoctial rising and setting, and the remaining two in the plane of the meridian circle, one to the south and the other to the north.

But because the meridian line and the altitude of the pole belong to the meridian, I have decided to speak of each separately, because of the universal and outstanding use of both in cosmography; but of the altitude of the pole and of the equator I shall speak in Chapter 12, after the exposition of the equator.

CAPVT X.

De inuentione Lineæ Meridianæ, & Declinationis Magneticæ.

Chapter X. On finding the meridian line and the magnetic declination.

From the very many methods that we shall set out, either in the Geographical Book or in the Book of General Problems, we have selected this one as the simplest and clearest for beginners.

I. On some white plane, exactly parallel to the horizon (which is done with a chorobates [Chorobatem], or with a plumb line and an exact square, or with a level), taking C as centre, describe several circles, at least three, about one finger's breadth apart. The smallest of them should have a semidiameter CD large enough to be exceeded by the noon shadow of the style to be used, which you can determine by trial on the previous day or by estimate. It is best to mark these circles not in ink, but only with a dry compass or in yellow colour. Then, at the same centre C, erect the gnomon CG, or style, normally, that is, at right angles to the horizontal plane on every side, which will be done with the help of a very exact square. With these preparations made, about two hours before noon, as roughly judged, when the Sun is beyond the hazard of sensible refractions, or very far from the horizon, and while the weather of the day stays perfectly clear, observe the moment at which the tip of the morning shadow of the style reaches the periphery of one of the described circles, and then with the help of a needle mark the point on that periphery that is indicated by the tip of the shadow. As in the preceding figure, with the style CG set up and the Sun at K, shining through the tip of the gnomon G to B, if the tip of the decreasing shadow has reached B, let the point B be marked. You will mark this more finely and clearly if the style is not longer than half a Roman foot and is not so sharply pointed that the tip of the shadow vanishes, but is very slightly blunted. After noon, too, returning to the observation in good time, observe the moment at which the afternoon shadow, as it grows, reaches the periphery of the same circle on which you marked the point before noon, and then with the needle mark another point on the same periphery; as in the figure given, when the Sun is at P and the shadow CA has reached A, let the point A be marked.

Diagram. Woodcut inset at the right of the left column: three concentric circles with a dotted vertical line through the centre; a small rayed sun stands outside the circles at upper left and another at upper right, a dotted line runs from each through a point on the vertical above the centre toward the opposite lower side, and heavy solid arms run from the centre to that upper point and to two points at lower left and lower right on the circles. Labels read on the block:

- K (upper left, beside the left sun) and P (upper right, beside the right sun)
- M (top, on the vertical, between the suns)
- G (on the vertical, at the top of the upper arm)
- C (at the centre, below the junction of the arms)
- A (lower left, at the end of the left arm) and B (lower right, at the end of the right arm)
- D (on the vertical, below C, on the innermost circle)
- S (bottom, on the vertical, outside the circles)

With these points marked, divide the arc ADB in half at D, by the corollaries to 9 of Book 1 of Euclid's *Elements*; and through the point of division D and the centre of the style C draw as straight a line as you can, either with a ruler or with a silk thread stretched and coated with colour, and let it be SM. This will be the meridian line, and it lies in the very plane of the meridian circle.

[Margin: Declinationis Magneticæ obſeruatio. — Observation of the magnetic declination.]

II. Suppose that before or afterward you divide one of those circles, or another described from the centre C, into degrees, and some of the degrees nearer the line into single minutes or into tens of minutes, and that at the centre C you place a style supporting a magnetic needle, half of which is equal to the semidiameter of the circle divided into degrees. The needle itself either finally comes to rest over the meridian line, and will show that it has no declination, or, with the end that faces north, it will decline toward the west or the east, and you will note how many degrees and minutes it declines. Mariners express declination toward the west by the verb *Maestrizzo*, from the wind called *Magister*, and toward the east by the verb *Græcisso*, from the Greek wind. But to obtain a proper declination, the magnetic needle must be very freely turning, very far from iron objects that could draw or incline it toward themselves, and very well protected from every breath of air and from any shaking of the ground or floor.

SCHOLIA.

[Margin: Probatio 1. lineæ Meridianæ. — Proof 1 of the meridian line.]

I. If before and after noon you mark several points on several circles as the tip of the shadow reaches all of them, and if, when the arc they intercept is divided in half, the same meridian line comes out from all the divisions, you will have strong evidence of a true meridian line.

[Margin: Solſtitio exactior Meridiana linea notatur. — The meridian line is marked more exactly at the solstice.]

II. If you seek the meridian line in the manner described above on the days of the solstice, it will, other things being equal, be more exact and better suited to astronomical work, because on those days the Sun over one whole day hardly varies its declination perceptibly, and therefore the morning shadow is, as far as the senses can tell, just as long as the afternoon one. But on days distant from the solstice, the Sun changes its declination within the same day and the hours of the day, and therefore the morning shadow is longer than the afternoon one as long as the Sun is in the ascending signs, that is, from the beginning of Capricorn through Aries to the beginning of Cancer; for in the descending signs it is shorter.

[Margin: Et Solſtitio probatur. — And it is proved at the solstice.]

III. If on those same solstitial days, or on others, you find by means of an exact azimuthal instrument, at an equal eastern and western azimuth, a morning

altitude of the Sun equal to the afternoon one; or if you count a time before noon equal to the time after noon, and find at the beginning of the former and the end of the latter equal altitudes of the Sun's centre or of a fixed star, you will be able both to draw a new meridian line and to test and correct one already drawn. But you will do this more surely on any clear night, if you obtain for a fixed star an equal altitude on either side together with an equal azimuth, or an equal interval of time.

[Margin: Plures Meridianæ ex vna tranſcriptæ. — Several meridian lines transcribed from one.]

IV. Once one meridian line has been found, you can mark several others on nearby horizontal planes. For with styles set up perpendicularly, as soon as the shadow of the style reaches the meridian line first found, let an assistant be at hand to mark a point at the tips of the shadows that the other styles cast on the horizontal plane; the line drawn through that point and through the centre of its own style will be a meridian line. On walls perpendicular to the horizon, however, it will be enough to fix a style in any way, to mark a point at the tip of the shadow at the moment when the tip of the shadow reaches the first, root or prototype meridian line, and to draw a straight line perpendicularly through that point: this too will be a meridian line. In either case, instead of a style one can use the thread of a plumb line once it is at rest, and at the moment of noon mark two points in its shadow and draw a straight line through them.

CAPVT XI.

De Aequatore, & Motu Primi Mobilis.

Chapter XI. On the equator and the motion of the primum mobile.

[Margin: Aequatoris Nomina. — Names of the equator.]

I. The EQUATOR is called in Greek [Greek: ἰσημερινός (isēmerinos, "of equal day")], in Latin *Aequidialis*, although the name *Aequinoctialis* has prevailed, because when the Sun is placed on it the days equal the nights over the whole world; while for those living under the equator, every day is equal to the night, in whatever point of the ecliptic the Sun stands. The ancients also called it the *Cingulum Mundi* [Belt of the World], and Ptolemy the circle of the *Aequationis diei* [equalising of the day]. Now the equator is a great circle of the sphere whose poles are the same as the poles of the world and of the daily revolution, or of the primum mobile: as in the figure of Chapter 7 the circle EGQ, described from the poles A and B. One of these is called *Australis* [Southern], from Auster, which blows from there, and *Antarcticus* [Antarctic], that is, set opposite the Arctic, and *Meridionalis* [Meridional], because from there the noon shadow is cast toward the opposite region. The other is called *Borealis* [Boreal], from Boreas, which blows from there, and *Arcticus* [Arctic], from the Bears, the constellations placed near it, for the Bear in Greek is called [Greek: ἄρκτος (arktos, "bear")]; and *Septentrionalis* [Septentrional], from the seven brighter stars of Ursa Major, which are called *Triones*, either from their wearing down

(*terendo*) the parts near the pole, or from the fact that [Greek: τρίων (triōn, "ox", as the page itself says)] means an ox, while the constellation of the Bear is like a wagon drawn round the pole at a slow, ox-like pace. No one however sees both poles at once from the surface of the Earth, unless he is raised above it and stands on the equator itself; most people see a single pole, as in the lines of Virgil, Book 1 of the *Georgics*:

This pole stands ever high above us; but that one Black Styx and the deep-dwelling shades see beneath their feet.

[Margin: Proprietates Aequatoris. — Properties of the equator.]

II. These are the properties and functions of the equator. (1) It cuts any meridian at right angles, and not merely in two equal parts; and in the right sphere it cuts the horizon at right angles, because it passes through their poles. (2) It is the measure of the daily motion, or of the primum mobile, which is common to absolutely all the stars, at least in appearance. For the constant agreement of the ages has observed that the wandering and the fixed stars alike are carried round in the space of 24^{h} from east toward west until they return to the east, and that with respect to this motion they complete equal spaces, or equal arcs, in equal times, whether on the great circle [circulus maximus] which for that reason they have also called the equator, or on a circle parallel to the equator; and so in any hour 15° of the equator or of its parallels cross the meridian or rise above the horizon, in 4^{m} of an hour one degree crosses or rises, and in one minute of an hour a quarter of a degree, namely 15′ of the equator or of a parallel. On this principle a small table has been drawn up that converts degrees of the equator into hours and fractions of hours, and conversely hours into degrees, etc., which we shall give at the end of the work. (3) The Sun, moved by this motion, produces the natural day [dies Naturalis], which the Greeks call [Greek: νυκτημέριον (nyktēmerinon, "of a night and a day")], that is, composed of a night and an artificial day [dies Artificialis]; for one complete revolution of the Sun on the equator, or on a parallel of the equator, is itself the natural or composite day. (4) It makes the artificial day equal to the night: always for those who live on the equator itself, but for everyone else only twice a year, namely on the day the Sun enters Aries, which in the Julian calendar, as reformed, is about March 21, when the vernal equinox occurs; and on the day it enters Libra, very nearly September 23, when the autumnal equinox occurs. The reason is that there, or then, while the Sun is carried from rising to setting, 180° of the equator rise, and to these twelve hours correspond. (5) By its common intersection with the horizon it marks the two cardinal points of equinoctial rising and setting. (6) It divides the sphere into two hemispheres, northern and southern; and it distinguishes northern declinations from southern, which are reckoned from it as from a boundary, although they are counted on the meridian: for the declination of a star, or of a celestial point, from the equator is as great as the arc of the meridian between the star's centre and the equator. (7) It is the measure, indeed also the subject, of right and oblique ascensions, since these are nothing other than arcs of the

equator rising together with arcs of the ecliptic, or with parallels of the ecliptic, on which stars may lie, in the right sphere or in the oblique, counted from the vernal intersection of the equator with the ecliptic, or from the beginning of Aries. (8) It is the measure of geographical longitude, which is as great as the arc of the equator intercepted between the first of the meridians and the meridian of the other place. (9) It works together with the ecliptic to determine the remaining artificial days of the year, which are as long as the number of degrees of the equator that rise above the horizon during the time the Sun is carried from rising to setting.

SCHOLIA.

[Margin: Notatio lineæ Aequinoctialis. — Marking the equinoctial line.]

I. On a horizontal plane the equinoctial line cuts the meridian line at right angles, and it is traced by the tip of a style's shadow on the equinoctial days; for on the days when the equinoxes occur, the tip of that shadow travels, to the senses, along one and the same straight line.

[Margin: Diei Solaris, & diei Primi Mobilis differentia. — The difference between the solar day and the day of the primum mobile.]

II. One day of the primum mobile, if it is not referred to the Sun's time — such is the revolution of a fixed star from the meridian back to the same meridian — is exactly 24^{h}. But the solar day is 24^{h} of the primum mobile and about 4^{m} besides, because in that much time one degree of the equator passes through the meridian, while the Sun, by its own motion toward the east, completes very nearly one degree in one day. So, when the point of the equator at which the Sun stood at yesterday's noon has revolved to the same meridian, today's noon will not occur at that same point of the equator, but at the end of one degree which, beyond the 360 degrees, must pass the meridian for the Sun to stand on the same meridian on which it stood yesterday. From this arise two tables, one for the day of the primum mobile, the other for the day of the Sun; see them at the end of the work.

CAPVT XII.

De Altitudine Poli, & Aequatoris Inuenienda.

Here too, out of the very many ways of finding the altitude of the pole [Altitudo Poli] and of the equator which we shall give either in the Geographical book or in the book of General Problems, let us choose for beginners two or three, the most evident of all. The first is by the meridian altitudes of a circumpolar star; the second by the meridian altitude of the Sun, taken with instruments; the third by the same altitude, found from the shadow of a gnomon.

[Margin: Poli Altitudo ex ſtella circumpolari. — The altitude of the pole from a circumpolar star.]

I. For the first, let the horizon be HR, the meridian HZRN, the zenith Z, the nadir N, the vertical line ZN, the equator EQ, the axis of the equator, or

of the world, AB, and the pole of the equator, which is also the pole of the world, northern at B and southern at A. On a night longer than 12^h, then, both meridian altitudes must be taken of the star nearest your pole, of the kind our people call the Polar star, though it is better called Circumpolar; such in this age is Cynosura, the last in the tail of Ursa Minor. Such nights, in this age, are those toward the end of December and the beginning of January; for in the evening as night begins, and the next morning as night ends, both meridian altitudes of that star can be observed without hindrance from twilight. So, with a quadrant or an accurate geometric square set up on the meridian line, take the meridian altitude of that star, both the greatest, when the star is at C, and the least, when it is at P; and add half the difference between the two altitudes to the least altitude, for the sum from that addition will be the very altitude of the pole on your horizon. For example, let the star's least altitude RP be exactly 40°, and the greatest RC 47°. The difference, arc CP, will then be 7°, and half the difference 3° 30′. If you add this to the least altitude of 40°, the result is the altitude of the north pole B, namely arc RB, 43° 30′; and its complement to the quadrant RZ will be arc BZ, 46° 30′.

Diagram. Woodcut inset at the right of the right column: a circle crossed by four diameters through the centre, with a star drawn on the circumference at C and at P. Labels read on the block:

- Z (top) and N (bottom)
- H (left) and R (right)
- E (upper left) and Q (lower right)
- B (upper right) and A (lower left)
- C (star, upper right, between Z and B)
- P (star, right, between B and R)

Labels not read: none. All ten labels are cited in the prose of this page.

Once the altitude of the pole is obtained, you will easily have the altitude of the equator, namely arc HE, since it is equal to the complement of the altitude of the pole, or the remainder to 90°. So in the example given, HE will be 46° 30′, while the distance of the equator from the vertex, namely arc EZ, is always equal to the altitude of the pole. You will learn this plainly on an armillary sphere: for as far as you raise the visible pole from the horizon, so far you will see the equator move away from the zenith Z, counting the degrees on the meridian.

[Margin: Poli Altitudo ex Declinatione Solis. — The altitude of the pole from the Sun's declination.]

[Margin: Tych. To. 1. pag. 79. — Tycho, vol. 1, p. 79.]

II. For the second method, let the meridian and the horizon be as above, but let the equator be AB, its axis EQ, the north pole of the world E and the south pole Q. On a clear day observe the meridian altitude of the Sun with a quadrant or an astronomical square, or with parallactic rulers, or with an astronomical ring, and you will have the seen, or apparent, altitude of the Sun. Convert this into the true altitude by adding the parallax proper to that day

and subtracting the refraction, if it is sensible, consulting the tables of solar parallaxes and refractions, either Tycho's or our own at the end of the work; and thus you will have the true meridian altitude of the Sun. Then from an ephemeris, or from astronomical tables, gather the Sun's place in the zodiac for noon of the day on which the observation was made; and with the Sun's place look up the Sun's declination in a table of solar declinations — such a table is found everywhere, but the most exact is in Clavius, *Sphaera*, from p. 264 — and if the declination is southern, add it to the true meridian altitude of the Sun; if northern, subtract it; and you will have the altitude of the equator on your horizon, whose complement to 90° will be the altitude of the pole. We too shall give an accurate table of declination at the end of the work.

For example, let the Sun be at P, its true and corrected altitude the arc of the meridian RP, 40°, and its southern declination the arc PB, 3° 30′; this, added to the altitude, makes arc RB, 43° 30′, and that therefore will be the altitude of the equator. Its complement to 99° [recte: 90°] will be the distance of the equator from the vertex, namely arc BZ, 46° 30′, and the altitude of the pole will be just as great. But if the declination were CB and the altitude RC, then with CB taken away, RB would remain, the altitude of the equator.

Diagram. Woodcut inset at the right of the left column, the same construction as the figure on the preceding page: a circle crossed by four diameters through the centre, with a star drawn on the circumference at C and at P. Labels read on the block:

- Z (top) and N (bottom)
- H (left) and R (right)
- E (upper left) and Q (lower right)
- B (upper right) and A (lower left)
- C (star, upper right, between Z and B)
- P (star, right, between B and R)

Labels not read: none. E, H, A, N and Q are not cited in the example beside the figure; E, Q and A are cited at the head of this method on the preceding page.

[Margin: Solis altitudo ex vmbra Meridiana. — The altitude of the Sun from the meridian shadow.]

III. If you want to observe the Sun's meridian altitude by the meridian shadow of a gnomon, the height of the style and the length of the meridian shadow must be known in the smallest possible parts, and in parts of the same kind, say in hundredths of one inch [vncia] of the Roman foot. Then, by trigonometry, one must find in the right-angled rectilinear triangle the angle opposite the gnomon. This is done most quickly by logarithms, if you add together the logarithm of the gnomon, or style, and the residual logarithm [Reſiduum Logarithmum] of the shadow; for the sum will give the mesologarithm [Meſologarithmum] of the desired angle, and that angle will be the seen altitude of the Sun's upper limb. If you subtract from it the Sun's apparent semidiameter, there will remain the altitude of the Sun, that is, of its centre, as apparent; and this, corrected by

adding the parallax and subtracting the refraction, will give the true altitude of the Sun, with which, together with the Sun's declination, you will obtain the altitude of the equator and of the pole, as above in the second method.

SCHOLIA.

[Margin: Stellæ idoneæ ad Poli altitud. Capiendam. — Stars suitable for taking the altitude of the pole.]

I. In the first method, other stars nearer the pole may be used, though somewhat larger ones, such as the bright star in the shoulder of Ursa Minor; and among the southern stars those of the CROSS, called *il Crozier* [?]; and of these, the ones nearer the southern pole are those in the ankle and in the left knee of the Centaur — provided that at their meridian altitude they are not near the horizon and do not undergo sensible refraction, or at least that it is established how much refraction must be subtracted from the altitude to obtain the true altitude.

II. In the second and third methods, if the true altitude of the Sun is taken at the time of the equinox, there will be no need to look up its place or its declination, for the Sun's place will be the beginning of Aries or of Libra, and its declination nil. The altitude of the equator will thus be as great as the Sun's true altitude, provided the equinox took place at the very moment of noon with respect to the meridian on which the Sun's altitude was observed. Otherwise, however many hours the equinox preceded or followed, the declination will be that many minutes of arc, and one could err by that many minutes in the altitude of the equator, and of the pole to be deduced from it.

[Margin: Altitudines Poli per Vmbras apud Priſcos fallaces. — Altitudes of the pole found by shadows among the ancients are deceptive.]

III. It was customary for the old practitioners to seek the altitudes of the pole and of the equator by the equinoctial shadows of gnomons. So in Vitruvius, book 9, chapter 8, there is the celebrated observed equinoctial shadow, in ratio to the gnomon, at Rome as 8 to 9, at Alexandria as 3 to 5, at Athens as 3 to 4, and at Rhodes as 5 to 7; and in Pliny the Elder, book 2, chapter 72, at Rome as 8 to 9 and at Ancona as 34 to 35; and in Strabo, book 2, at Carthage as 7 to 11. But the altitudes of the pole obtained this way were uncertain among them, for three reasons. First, because they did not subtract the Sun's apparent semidiameter from the altitude of the Sun's upper edge, just as if the ray passing over the tip of the style and bounding the shadow came from the centre of the Sun and not from its edges. Second, because at the time of observation they believed it was the equinox, which nevertheless had sometimes already preceded by many hours or was yet to follow. Third, because they sometimes took the day of the equinox in the civil and not the astronomical sense. There is the further point that if the style is small, an error in the parts is easily committed, and yet it disturbs the true altitude of the Sun not a little; while if it is longer, the tip of the shadow can scarcely be distinguished because of the penumbra. For

that reason I would believe there is an error in Clavius's *Sphaera*, p. 279, where he calls this way of taking the Sun's altitude more difficult and more certain [difficiliorem, & certiorem]; for I think *incertiorem* ("less certain") should be restored. But more on these things in the Geographical book.

IV. If in the third method the reversed shadow is used, namely that of a gnomon fixed into a wall perpendicular to the horizon, then the angle opposite the gnomon, subtracted from 90°, will give the seen altitude, but of the Sun's lower limb. So the Sun's apparent semidiameter must be added to it to obtain the seen altitude of the Sun's centre.

[Margin: Polaris ſtellæ acceſſus ad Polum quantus. — How far the pole star approaches the pole.]

V. If you have just one meridian altitude of the pole star, add to the least, or subtract from the greatest, this star's distance from the pole, which at the start of this year 1642 is 2° 37′ 30″, and which each year in this century and the next will diminish by 20″, as I have observed, following Tycho, by comparing meridian altitudes of various years taken by us and by others.

CAPVT XIII.

De Latitudine, & Longitudine Geographica.

What was said in Chapter 12 about the altitude of the pole, in connection with the meridian, and in Chapter 11 about the functions of the equator, gives rise to a very timely explanation of the geographical latitude and longitude of places. Without these, places can neither be marked exactly on a globe or on geographical maps, nor, once marked, be found at once.

[Margin: Latitudo Geographica quid? — What is geographical latitude?]

I. *Geographical latitude* is the distance from the equator of a place situated on the surface of the terrestrial globe, measured on that place's meridian circle. So it is exactly as great as the distance of the vertex, or zenith, from the equator; and this is as great as the altitude of the pole, from what was said in Chapter 12. Therefore geographical latitude is known at once if the altitude of the pole is known. It is called *latitude* because the part of the Earth known both formerly and now is narrower from one pole of the world to the other than from west toward east; and custom has established that in a surface bounded by a line other than a circular one, the narrower part is called the latitude [Latitudo]. Next, among astronomers the latitude of the stars is their distance from the ecliptic toward one or other of its poles; in imitation of this, then, the distance of places on Earth from the equator toward one or other pole of the world has been called latitude, although it more closely resembles the declination of the stars, which is reckoned from the celestial equator.

[Margin: Longitudo Geographica quid? — What is geographical longitude?]

II. *Geographical longitude* is the arc of the terrestrial equator intercepted between the meridian of the place and that other meridian, the first of them, which has been adopted and accepted by geographers as the beginning of all

the meridians. It is called *longitude* because the part of the Earth known along the equator and its parallels, from west toward east and conversely, is much greater than what is known from pole to pole of the world; and because the astronomical longitude of the stars is reckoned from west toward east, although it is computed on the ecliptic from the beginning of Aries; and so geographical longitude more closely resembles the right ascension of stars and signs, which is computed on the celestial equator. The first of all the meridians is the one that passes through the Fortunate Islands, and from there Ptolemy, prince of geographers, and very many others have been accustomed to count longitude toward the east, although others count from elsewhere.

III. The *difference of geographical longitude* between two places on Earth is the arc of the equator intercepted between the meridian circles of the two places. The *difference of latitude* is the difference in the altitude of the pole, whether the places lie on the same meridian or on different ones — that is, in equivalence of measure; for formally it is the difference in distance from the equator, reckoned on the meridian of each of the places.

[Margin: Modus prior Longitud. Geograph. inquirendæ. — The first way of finding geographical longitude.]

IV. The ways of finding and determining geographical longitude, or its difference, are very many, and, God willing, we shall give them in the Geographical book. They reduce, however, to two heads. The first is that of the methods which use the latitude of two places and the road distance between them, measured on a great circle of the Earth. These cannot be carried out without trigonometry, or the analysis of triangles, and without certain knowledge of the miles contained in one degree of a great circle of the Earth; so they will be better given after the Trigonometric book, in Book 4 of Volume 2, which will be the Geographical one.

[Margin: Modus poſterior. — The second way.]

The second head is that of celestial phenomena which happen and are observed at the same moment of time in two places on Earth, while their time is observed as well, down to minutes of an hour. For the difference of time, converted into parts of the equator by the method indicated in Chapter 11, property 2, and by the table named there, will show the difference of longitude between those places. The place to which the same phenomenon has appeared more hours and minutes after noon or midnight, or after sunrise or sunset, will be the more easterly. But eclipses of the Moon hold the chief place in this business.

[Margin: Exemplum 1 per Eclipſes. — Example 1, by eclipses.]

To illustrate this, in the following figure let the horizon be HR, the celestial equator HVRD, and beneath it the terrestrial equator AFB, on which there are three observers at three places A, F and B, who observe the beginning of a lunar eclipse and its time, derived chiefly from the fixed stars. Let the beginning of this eclipse occur at the point of midnight with respect to place F, which is to be the farthest of the Fortunate Islands, and let its meridian be VFD, the Moon

being placed very nearly on that meridian; with respect to this meridian let R be the point of rising and H of setting. But let observer B note the same beginning of the same eclipse at exactly the 3rd hour after midnight. This, converted into parts of the equator, will give arc VN in the sky and FB on Earth, of 45°. So the difference of longitude between places F and B will be 45°. Indeed, if F lies under the first of the meridians, from which longitude is begun, the longitude of place B itself will be 45°, and place B will be more easterly than place F by that many degrees, because R is the point of rising, and 3^h after midnight are more than none.

Diagram. Woodcut inset at the right of the left column: a large circle crossed by four diameters through the centre, with a star drawn on the circumference at each of the eight ends; a small circle about the centre; and three short strokes, each ending in a dot, rising from the small circle toward G, V and N. Labels read on the block:

- V (top) and D (bottom)
- H (left) and R (right)
- G (upper left) and K (lower right)
- N (upper right) and M (lower left)
- A (small circle, left), F (small circle, top), and B (small circle, right)

Labels not read: none. K and M are on the block and not cited in the prose of this page.

[Margin: Exemp. 2. — Example 2.]

Again, let the beginning of the same lunar eclipse occur at V at the 6th hour after sunset, the Sun being already at H; but with respect to place B let it occur at the 9th hour after sunset, the Sun for that place being at G; and at the beginning of the eclipse let the Sun have reached D, where the nadir of place F lies. For the difference of longitude will again be 3^h, namely 45°. What I have said of two places on the equator holds for other places off the equator, whether they lie on the same parallel or on different ones: for similar arcs of the parallels are intercepted between those places, that is, of as many degrees as if they were on the equator; and so the difference of the time at which the same phenomena of the sky appear to them is just as great.

[Margin: Exemp. 3. — Example 3.]

Now let the eclipse occur at place F, under the point V of the sky, at the 6th hour after midnight, but at place A, which lies under G, at the 3rd hour after midnight; and let VFD be the first of the meridians. The difference of longitude will appear to be arc GV, 45°, or 3^h, and place A more westerly. Yet because cosmographers proceed from the beginning of the meridians toward the east, and in our case consider the arc of the equator VRDG, which is the complement of arc GV to 360° and is in our case 315°, or 21^h, they therefore say that place A is more easterly than place F, and has in our case a longitude of 315°. So according to them this limitation would have to be added to the definition of geographical longitude: *Reckoned toward the east*. But on this see the Scholia.

[Margin: Vſus Longit. & Latitud. in Globo Geographico. — The use of longitude and latitude on a geographical globe.]

V. Once the longitude of a place is known, a meridian is drawn through the poles of the world, passing through the point of the equator that lies as many degrees toward the east from the beginning of the meridians as the degrees of the longitude found; and on that same meridian, according to the latitude found, the arc of distance from the equator is taken. In this way one obtains the indivisible and determinate point at which the place is to be marked on a globe or on a geographical map, or at which, if it is already marked, it can be found at once. For instance, if you read in a historian who uses the terms of cosmography that Rome is at a northern latitude of 42° and a longitude of 39°, then, counting 39° on the equator from the Fortunate Islands, if the beginning of the meridians is in them, and on the meridian passing through the end of the 39th degree counting 42°, or taking them with compasses, you will at once find the point at which the navel [vmbilicus] of Rome is or ought to be.

In this way tables of the longitude and latitude of places are compiled; of this kind we shall give at the end of the book one far more certain than those constructed so far, deduced from many eclipses.

SCHOLIA.

[Margin: Latitudo, & Longitudo Orbis olim & nũc cogniti. — The latitude and longitude of the world as known formerly and now.]

I. The known latitude of the world, formerly smaller and later greater, did not exceed, in Pliny the Elder, book 2, chapter 108, and in Ptolemy, book 1 of the *Geography*, chapter 10, 80° 19′, from the shore of the Ethiopian Ocean to the northern extreme beyond the mouths of the Tanais; or, as Ptolemy has it, from Agisymba to Thule. But in our age it is 139°, of which 81 are northern and the remaining 58 southern, for navigation has reached that far: northward by Gulielmo Barensonio Belga [Willem Barentsz], southward by Gulielmo Schouten Hollando [Willem Schouten]. On this matter see again below, Book 2, chapter 9.

The known longitude of the Earth's globe, formerly, in Pliny the Elder, book 2, chapter 108, and book 6, chapter 33, and in Ptolemy, book 1 of the *Geography*, chapter 12, and book 7, chapter 5, did not exceed a semicircle, or 180°. But in our time not a few ships' masters have circumnavigated the whole globe, following the ship *Victoria* of Ferdinandi Magellanici [Fernão de Magalhães]; and from their voyages, with the known intervals of overland journeys added, it comes about that the whole circle of 360° is known. But on this, again, Book 2, chapter 9.

[Margin: Longitudo verſus Occidẽtẽ, numerata. — Longitude counted toward the west.]

II. The longitude of places could be counted toward the east so long as it does not exceed 180°; but if it exceeds that, it would better be counted toward

the west, as it is by certain royal cosmographers, and by name by Ludouico Teisera [Luís Teixeira]. Otherwise a place which is more westerly with respect to the place of the first meridian, and in the western hemisphere, could seem more easterly because of a longitude called eastern, as in the third example of no. IV, place A with respect to place F. But if longitude is counted toward the west when it exceeds 180°, and the complement of the longitude to 360° is taken, as arc FA in the same case, the western longitude will agree with the westernness of the hemisphere in which the place lies.

[Margin: Initium Meridianorũ varium. — The beginning of the meridians varies.]

III. Disagreement among authors in determining the longitude of places does not always arise from some error in road intervals or in celestial observations, from which the difference of time, and from that the difference of longitude, has been taken; often it arises from the starting point of longitude. For some begin longitude from the Fortunate Islands, others from the Azores or the Hesperides, others from Thule, others from the Pillars of Hercules or the Strait of Gades; and among the Fortunate Islands themselves some are more westerly than others. So one must first consult the starting points, and what the place of the first meridian is. But more on this in the Geographical book.

[Margin: Solis, an Lunæ Eclipſes meliores ad Longitud. diff. — Whether eclipses of the Sun or of the Moon are better for the difference of longitude.]

IV. Kepler, in the *Optics* and in the Rudolphine Tables, prefers eclipses of the Sun for determining the difference of geographical longitude, because he thinks their beginnings and ends are observed more clearly than those of lunar eclipses, and that the calculation of the parallaxes is no obstacle, since he asserts that it comes out most exact from his tables. But Tycho, Magini and most others, fearing the perplexity of the parallaxes of the Sun and Moon, prefer lunar eclipses in this practice, and especially the beginnings of total eclipses, or the emersion from the shadow of the Earth.

CAPVT XIV.

De Ecliptica, & Zodiaco: vbi de Motu Secundi Mobilis, atque Obliquitate Eclipticæ.

On the ecliptic and the zodiac: where also on the motion of the second mobile, and on the obliquity of the ecliptic.

[Margin: Ecliptica quid? — What is the ecliptic?]

I. The ECLIPTIC is a great circle whose poles stand as far from the nearer poles of the equator as the Sun's greatest declination from the equator. The Sun's greatest declination occurs at either solstice. It is called the ecliptic because eclipses of the Sun and Moon do not happen unless the luminaries are in the plane of this circle or a little distant from it. Frequently, however, the name "ecliptic" refers rather to the line that is the circumference of this circle than to the circle itself. It is also called the Royal Road [Via Regia], because the Sun, king of the planets, always travels along it with the centre of his body, even

granting that the other planets depart from it in latitude, now to the south, now to the north. Aristotle and Geminus also call it the oblique circle, λοξὸς κύκλος [Greek: loxos kyklos, "oblique circle"], or the circle "that runs through the middle of the signs", from the obliquity it has with respect to the equator and the meridians. You can see the position and configuration of the ecliptic in the figure of Chapter 7: there the line KR, drawn through the middle of the parallels, marks the ecliptic.

[Margin: Zodiaci definitio. — Definition of the zodiac.]

II. The *zodiac* is a celestial band or zone, cut in two by the ecliptic into a northern and a southern part, and bounded by the circumferences of two circles that are not great circles. These circles stand from the ecliptic as far as the greatest latitude of the planets, of those that deviate most from the ecliptic, namely 8° or 10°. It is called the zodiac, and by Ptolemy κύκλος τῶν ζωδίων [Greek: kyklos tōn zōdiōn, "circle of the animals"], that is, *the circle of animals*; and also the Signifer, because it is divided into 12 signs named after the stars, whose configurations, by some likeness, resemble the form of men or of other animals, or even their properties in their influences. For Libra is referred to the claws of Scorpius, which grasp the scales; the Arrow to Sagittarius; and the Urn of Aquarius to Aquarius. The ecliptic and the zodiac are, moreover, frequently used for the same thing.

[Margin: Zodiaci Signa quot, & quæ. — How many signs of the zodiac there are, and which.]

III. The *signs of the zodiac* are twelve. Six of them are northern: they begin at the vernal intersection, that is, at the point at which the ecliptic intersects the equator and at which the vernal equinox occurs, and they end at the autumnal intersection, where the autumnal equinox occurs. The other six are southern: they begin at the autumnal intersection and end at the vernal. Their names, characters and qualities, and their power to stir the humours of animals in respect of predominance, you may see in order and in succession in the following table.

SEX SIGNA BOREALIA. — *The six northern signs.*

♈	♉	♊	♋	♌	♍
Aries	Taurus	Gemini	Cancer	Leo	Virgo
Κριὸς	Ταῦρος	Δίδυμοι	Κάρκινος	Λέων	Πάρθενος
Fiery [Igneum]	Earthy [Terreum]	Airy [Aëreum]	Watery [Aqueum]	Fiery [Igneum]	Earthy [Terreum]
Choleric [Cholericum]	Melancholic [Melancholicū]	Sanguine [Sanguineum]	Phlegmatic [Phlegmaticū]	Choleric [Cholericum]	Melancholic [Melancholicū]

SEX SIGNA AVSTRALIA. — *The six southern signs.*

♎	♏	♐	♑	♒	♓
Libra	Scorpius	Sagittarius	Capricornus	Aquarius	Pisces

♎	♏	♐	♑	♒	♓
Ζύγος	Σκόρπιος	Τοξότης	Αἰγόκερος	Ὑδρόχοος	Ἰχθῦς
Airy	Watery	Fiery	Earthy	Airy	Watery
[Aëreum]	[Aqueum]	[Igneum]	[Terreum]	[Aëreum]	[Aqueum]
Sanguine	Phlegmatic	Choleric	Melancholic	Sanguine	Phlegmatic
[Sanguineum]	[Phlegmaticū]	[Cholericum]	[Melancholicū]	[Sanguineum]	[Phlegmaticū]

To help the memory they are also gathered in the following couplet:

Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libraq; Scorpius, Arcitenēs, Caper, Amphora, Pisces.

[There are Aries, Taurus, Gemini, Cancer, Leo, Virgo, / and Libra, Scorpius, the Archer, the Goat, the Amphora, Pisces.]

MOTVS PROPRIVS SYDERVM ORIENTEM VERSVS.

The proper motion of the stars toward the east.

[Margin: Eclipticæ origo. — The origin of the ecliptic.]

IV. The reason for introducing the ecliptic and the zodiac into the artificial sphere, and for recognizing them in the natural and celestial sphere, was the proper motion of the stars from west toward east, which astronomers observed to take place in this oblique circle or in circles parallel to it. This motion appears most plainly in the Moon. If the Moon is seen one night on the meridian at the first hour after sunset, then when 24^{h} have passed from then she will not yet have reached the meridian on the following night; about 50^{m} more will be required, because by her proper motion she has become further east. When she reaches the meridian it will be nearly the second hour after sunset; on the third night it will be the second hour with about 40^{m}, and so on, until she appears on the meridian only at the end of the night. What I say of the meridian holds also of the horizon: if today, as the Sun sets, the full Moon rises on the horizon, tomorrow she will not rise as the Sun sets, but about an hour after sunset, and so on, later and later. Moreover, if tonight the Moon was seen in conjunction with some fixed star, on the following night at the same hour she will appear distant from that star toward the east by about 13°, and so she will continually draw away from it. This is a sign that she has moved toward the east by a proper motion, true or apparent.

On the other hand, the Moon appears to become further east each day in such a way that she now approaches the equator and now recedes from it, now to the south, now to the north, and through one semicircle of her motion she is on this side of the equator, but through the other beyond it. From both motions together, then, the one in longitude toward the east and the one in latitude toward the poles of the equator, there arises a path oblique to the equator, and from this the zodiac.

What has been said of the Moon from daily experience is evident also in the Sun, at least after some days. From the same fixed star with which he was in conjunction he recedes toward the east by nearly one degree each day, and

after three months by about 90°, so that the rising Sun appears on the horizon when that star appears on the meridian. After 6 months he will stand about 180° from it, and as he rises that star will set in nearly the opposite part of the horizon. This is proof that each day the Sun has become, truly or apparently, further east. And since by this motion he declines now to the north, now to the south, this motion, mixed from a motion in length and one in breadth, appears to take place along an oblique circle. The same motion appears in the other stars also, but it does not become evident so quickly in the superior planets, and much less in the fixed stars, since these do not complete one degree toward the east except in about 70 or 72 years, at least in these centuries.

[Margin: Duo motus contrarij — Two contrary motions]

V. How it can actually happen that one and the same star moves continually with two contrary motions, one

[Margin: an possibiles. — whether they are possible.]

diurnal and common, that of the primum mobile, along the equator or its parallels, which are described from the poles of the world, and from east to west; the other proper, that of the second mobile, and this monthly, or annual, or longer than annual, according to the greater or lesser speed of each star's proper motion, striving against the motion of the primum mobile, which is called the sweeping motion [motus Raptus], and tending from west to east: this will be discussed in Book 3. Meanwhile the imagination of beginners is usually helped by the motion of an ant that crawls slowly eastward along the zodiac of an armillary sphere, by one or more degrees of the zodiac, while you with your hands turn the whole zodiac, together with the equator, toward the west. For the present it is enough to conceive the ecliptic, and the zodiac, as described from their own poles, which stand as far from the poles of the equator as the Sun stands from the equator when he declines furthest from it at the solstices; and to conceive all the stars, whether on the ecliptic or on parallels described from the poles of the ecliptic, as moving from west toward east, while by the sweep of the primum mobile they are moved toward the west in a daily revolution of 24^{h}.

[Margin: Eclipticæ munia. — The functions of the ecliptic.]

VI. *The functions and properties of the ecliptic* are these. (1) To cut the equator in two at those two opposite points at which the equinoxes occur: the vernal when the Sun enters Aries, the autumnal when he enters Libra. (2) To cut the equator at these points obliquely, that is, at angles that are not right. The angle that the ecliptic makes with the equator subtends an arc of the colure of the solstices, which colure is one of the movable meridians, and an arc as great as the Sun's declination from the equator at the solstices, which is then at its greatest. This angle is called the *obliquity of the ecliptic*. Of precisely the same quantity, that is, of the same number of degrees, are the said angle and the obliquity of the ecliptic, the Sun's greatest declination, the distance of the tropics from the equator, and the distance of the poles of the ecliptic from the nearer poles of the equator; and according to the truer opinion these are always

23° 30′. (3) With its poles and their daily motion it describes around the poles of the world the Arctic and Antarctic circles, and so determines the frigid zones. (4) By its obliquity it determines the position of the tropics and their distance from the equator, and by means of these it divides the torrid zone from the temperate zones. (5) It is the rule of the motion of the second mobile, by which all the stars are moved from setting to rising, and which is called *motion in longitude* from the beginning of Aries. For the true proper motion of a star is the arc of the ecliptic reckoned from the beginning of Aries according to the series or succession of the signs, as far as the point at which the ecliptic is intersected by the circle of latitude passing through the centre of the star; and the line drawn from the centre of the Earth to that point of the ecliptic is called the line of true motion. (6) It is the boundary from which the latitude of the stars is measured. For the latitude of a star is the arc of a great circle drawn through the poles of the ecliptic and the centre of the star, intercepted between the ecliptic and the centre of the star; and this latitude is northern if the star stands from the ecliptic toward the north, southern if toward the south. (7) It is the cause of the inequality of days and nights in the oblique sphere, and of the alternation of the seasons everywhere, and from there of generations and corruptions, which *Aristotle* rightly attributes (*On Generation and Corruption*, book 2) to the oblique circle: understand, to the Sun and the planets travelling along it, now approaching us, now receding toward the poles. It is likewise the cause of the variety of risings and settings. And so a certain distributive justice is kept: in the same climate where there was winter with the longest nights, there is in its own time summer with the shortest nights; and so that the passage to these extremes may not be immediate, two temperate seasons, spring and autumn, come between. For these reasons God [D.O.M.], by his admirable providence, devised this oblique circle. (8) Together with the equator it is the cause of the oblique ascensions, whose variety depends chiefly on the widely varying angle of the ecliptic with the horizon. For although a whole semicircle of the ecliptic always stands above the horizon, since each is a great circle and so, by *Theodosius*, book 1, theorem 11, they bisect each other, yet they do not always cut each other at equal angles, but now at right angles, now at acute on one side and obtuse on the other. (9) It divides the celestial sphere into a northern and a southern hemisphere with respect to the latitude of the stars, as the equator does with respect to declination; and in Ptolemy, in his book *On the Analemma*, and among the writers on gnomonics, any vertical circle does the same. Hence the same star is said in three ways to be in the northern or southern hemisphere: by latitude, that is, distance from the ecliptic toward the north; or by declination, that is, distance from the equator toward the north; or by departure from some vertical circle chosen for describing the hours, again toward the north; and likewise for the southern hemisphere. So it sometimes happens that one and the same star is at the same time northern by reason of declination and southern by reason of latitude, as when it lies between the equator and the northern semicircle of the ecliptic. (10) It is a partial cause of

eclipses of the Sun at new moons and of the Moon at full moons, when the body of the Moon either is on the ecliptic itself or comes very close to it. For only then can the Moon come between the Sun and our sight, and so the Sun fail, or the shadow of the Earth come between the Moon and us, and so the Moon suffer eclipse.

SCHOLIA.

[Margin: Latitudo Zodiaci. — The latitude of the zodiac.]

I. Sacrobosco in Chapter 2 of the *Sphere*, and Clavius on that passage, give the zodiac a latitude of 12°, because most planets, and most of the time, do not recede from the ecliptic more than 6° of latitude on either side. But Regiomontanus gives the zodiac a latitude of 16°, and others otherwise; for Kepler gives Mars a departure from the ecliptic toward the north of nearly 7° or 6° 47′, and Venus a greatest digression of 10°, or, as Argolus [Andrea Argoli] has it in the *Pandosion Sphaericum*, chapter 29, 9° 3′. Therefore the whole latitude of the zodiac, according to the more recent writers, may for safety's sake be extended to 20°, so as to be the 18th part of the whole circle.

[Margin: Obliquitas Eclipticæ an varia? — Whether the obliquity of the ecliptic varies.]

II. The *obliquity of the ecliptic to the equator*, in Greek Ζωδιακοῦ λόξωσις [Greek: zōdiakou loxōsis, "obliquity of the zodiac"], was and is always the same according to Ptolemy, al-Battānī, Agostino Ricci, Kepler, Gassendi, Kircher, Renieri and ourselves, and according to us indeed 23° 30′. Many others, however, have believed it inconstant and changeable by some anomaly; more on this in Book 3.

[Margin: Duodenaria partitio Zodiaci. — The twelvefold division of the zodiac.]

III. The zodiac was divided not only into six *physical signs* [Signa Physica], by reference to the sixth part of the circle that is taken in by compasses opened to the interval of the semidiameter, as the Alphonsine astronomers and the Arabs did in their sexagesimal tables of motions, but into 12 parts. The reason was both the number of the 12 months, whether lunar or solar, and the four seasons of the year, the four elements and the four humours of animals, in which there is found a threefold degree and notable difference, namely beginning, middle and end, or highest, middle and lowest: and 4 multiplied by 3 give 12. Moreover, the number twelve has five aliquot parts, corresponding to the twelve aspects of the signs, in which they notably exert their power. For the parts of the circle 1 pertain to conjunction, 2 to opposition, 3 to the trine, 4 to the square, and 6 to the sextile aspect.

[Margin: Initium Zodiaci cur à sectione Verna? — Why the zodiac begins from the vernal intersection.]

IV. The reason why astronomers took the beginning of the zodiac and of motion in longitude from the beginning of Aries, that is, from the point of

the vernal intersection of the ecliptic with the equator, was that this beginning had to be sought from one of the four cardinal points, at which either the beginnings or the highest perfection of the seasons of the year occur. It was better, moreover, to begin from the equinoctial points than from the solstitial, because of the equality of the days and the moderate temperateness of the year. It was better, finally, to begin from the vernal equinox than from the autumnal, because the astronomers who introduced these things were in the northern hemisphere, which the Sun enters from the beginning of Aries. In this hemisphere, indeed, there are both more fixed stars in the sky and more inhabited regions, and nations better suited and more inclined to civil culture and to the study of the arts and of religion; and the Sun himself lingers nearly eight days longer there than in the southern hemisphere. Leave aside the question whether the world was created by God [D.O.M.] in spring or in autumn; on this, see our Chronological book, which is Book 5, vol. 2, of the *New Almagest*. Among southerners, however, spring comes when the Sun enters Libra, and for this reason, if those astronomers had been southerners, they could very fittingly have begun the zodiac from the beginning of Libra.

[Margin: Et cur ab Ariete? — And why from Aries?]

V. Finally, the reason for calling the twelve signs of the zodiac by the names of animals, of which above, and the beginning of the first sign the beginning of Aries, was that the dodecatemories [Dodecatemoria], that is, those twelve segments of the zodiac, at the time when astronomers in Egypt and in Greece instituted or completed this division, occupied those regions of the sky in which those stars or constellations of the 12 animals lay: occupied them, I say, either wholly, or with the greater part of themselves, or at least with their beginning. So in about the year 388 before Christ, according to my tables, the first star shining in the horn of Aries was on the colure of the equinoxes and within the zodiac, since its northern latitude does not exceed 7° 7′ 13″. And in the preceding century, or a little before, namely in the year 548, Anaximander of Miletus had flourished, the inventor among the Greeks of the ecliptic and of the solar clock, as Pliny the Elder reports (book 2, chapter 8).

[Margin: Duplex Zodiacus notandus. — The two zodiacs to be noted.]

VI. It must further be carefully noted that there are at least two zodiacs in the sky. One is invisible in itself, merely imaginary, or conceivable by reason. Astronomers commonly speak of this one, and its twelve parts are equal, each divided into 30 degrees. The other is visible, that is, filled with fixed stars and the twelve constellations [Astris], which once gave their names to the parts of the invisible zodiac. The twelve constellations of this zodiac do not occupy equal parts in the sky, nor 30°: the constellation of Aries occupies in longitude 20° 13′, but Taurus 31° 24′, and so some more, some fewer than 30°. Since, however, the fixed stars have by now advanced in longitude, that is, toward the east, by about 28° since the time mentioned above, this zodiac (the "starred" one, to speak barbarously, and visible) differs by almost whole signs from the signs of the invisible zodiac, which is immobile with respect to this second

motion; so much so that now the constellation of Aries lies, with its greatest part, under the Taurus of the invisible zodiac, and Taurus under Gemini, and so on. They therefore call the former the Rational zodiac [Zodiacum Rationalem], and that of the sphere of the primum mobile, or the immobile; the latter the Apparent or Sensible, and mobile, and that of the eighth sphere. I shall speak of it again in Book 3, on the precession of the equinoxes.

[Margin: Domus Planetarum. — The houses of the planets.]

VII. How the 12 celestial houses, which are also called δωδεκατεμόρια [Greek: dōdekatemoria, "twelfth parts"], are related to the 12 signs of the invisible or rational zodiac will be discussed below, in Chapter 34, on erecting a celestial figure. Meanwhile it is enough here to indicate which signs astrologers call the *house of exaltation* of the planets and which that of their *fall* [Casus], and likewise which the principal house and which the less principal. For example, Leo is the principal house of the Sun and Cancer that of the Moon, because the Sun in Leo produces the greatest heat, while the Moon in Cancer moistens most. Aries is the house of exaltation of the Sun, because from there the days begin to lengthen for northerners. You see then, reader, that these things do not hold among southerners: for at that time the days begin to shorten for them, and it is with the Sun placed not in Leo but in Aquarius that they have their greatest heat. But if the power of producing heat was once increased not only by the Sun's distance midway between the summer solstice and the autumnal equinox, but also by conjunction with the Heart of Leo, the Dog Star, etc., this cause at any rate is now separated from the Leo of the rational zodiac and has migrated into its Virgo, beneath which the constellation of Leo is now found. So it is no wonder if the seasons of the years have varied. Nevertheless, here is a table of these houses, as it stands among the astrologers.

Planets and their characters [Planetæ, & eorum character]	House of exaltation [Domus Exaltationis]	House of fall [Domus Casûs]	Principal house [Domus Princip.]	Less principal house [Dom. minus Princ.]
Sun [Sol] ☉	♈	♎	♌	. . .
Moon [Luna] ☾	♉	♏	♋	. . .
Mercury [Mercurius] ☿	♍	♓	♍	♊
Venus ♀	♓	♍	♉	♎
Mars ♂	♑	♋	♏	♈
Jupiter [Iupiter] ♃	♋	♑	♐	♓
Saturn [Saturnus] ♄	♎	♈	♑	♒

[Margin: Eclipticæ in Cœlo obseruādæ ratio. — The way of observing the ecliptic in the sky.]

VIII. The *indicator of the ecliptic in the sky* is the half-full or horned Moon. If you gaze at her horns and, with imagination aiding the eyes, draw a circle through both horns of the Moon, it will pass through the poles of the ecliptic, especially if the Moon has no latitude. And if both horns are on the same vertical circle, which you can observe if they coincide with the thread of a plumb line

hanging perpendicular and still, the Moon will be at the nonagesimal degree of the ecliptic, that is, in the middle of the semicircle of the ecliptic that stands above the horizon. There is also now, in the bend of the third knot of Draco, a star of the 4th magnitude nearest the pole of the zodiac, about three degrees from it, whose longitude is in the 7th degree of Leo, and which shows us the pole of the zodiac as closely as possible.

CAPVT XV.

De Coluris.

On the colures.

[Margin: Colurus quid sit? — What a colure is.]

I. COLURE, in Greek κόλουρος [Greek: kolouros, "docked, curtailed"], does not mean the erect tail of an ox, as Johannes de Sacrobosco ridiculously imagined (for which he was censured by Clavius and by Barozzi), but something imperfect and mutilated. For there are two circles of the sphere that cut each other at right angles at the poles of the world, one of which passes through the equinoctial points of the zodiac and the other through the solstitial. Although in a right sphere they appear whole within the space of 24^h, and do not deserve the name of colures, yet in an oblique sphere they never appear whole, nor uniformly, either at once or successively, as do the other perfect and movable circles, namely the equator and the zodiac, which may be seen whole within the space of 24^h; rather, some portion of them always lies hidden below the horizon. They have therefore been called mutilated and imperfect, and, by antonomasia, *colures*. For besides these there are many other great circles that share this imperfection, namely all those that pass through the poles of the world and are movable: for never do all their parts rise above the horizon in an oblique sphere.

[Margin: Colurus Aequinoctiorū quid? — What the colure of the equinoxes is.]

II. The *colure of the equinoxes* is a great circle whose poles are at the beginning of Cancer and of Capricorn on the equator. It passes through the poles of the world, that is, of the equator, and cuts the equator at right angles at the equinoctial points. In other words, its poles lie on the equator, 90° from the beginning of Aries and of Libra.

[Margin: Colurus Solstitiorū quid? — What the colure of the solstices is.]

III. The *colure of the solstices* is a great circle whose poles are at the beginning of Aries and of Libra. It passes through the poles of the world and cuts the equator and the ecliptic at right angles at the solstitial points. Hence the colures cut each other at right angles at the poles of the world, by 15 of Book 1 of *Theodosius*, since each passes through the poles of the other and through the poles of the equator.

[Margin: Proprietates Coluri Aequin. — The properties of the colure of the equinoxes.]

IV. Besides those indicated in the definition, the properties and functions of the *colure of the equinoxes* are these. (1) It cuts the ecliptic in two, as one great circle does another, but at oblique angles, since it does not pass through the poles of the ecliptic; and its obliquity to the ecliptic is as great as the complement of the obliquity of the ecliptic to the equator, the complement, I say, to 90°, that is, in our own and the truer opinion, 66° 30′. (2) It separates the signs of the ecliptic and the zodiac into six northern signs and six southern. (3) It cuts all circles parallel to the equator in two and at right angles, as can easily be shown from the *Spherics* of Theodosius. (4) It marks the meridian on which noon occurs at the moment of the equinox, and conversely on which the equinox occurs at the moment of noon; for it coincides with the meridian of the place at which these two instants or moments fall together, namely that of the true equinox and that of noon. In this way it marks the equinoxes, whose precession will be discussed below.

[Margin: Proprietates Coluri Solstitior. — The properties of the colure of the solstices.]

V. Besides those indicated above, the properties and functions of the *colure of the solstices* are these. (1) It divides the zodiac into six si- gns that ascend and six that descend.

[Margin: Signa Ascēdētia ac Descendentia quæ? — Which signs are ascending and which descending.]

For northerners the ascending signs are *Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini*, because from the beginning of Capricorn to the end of Gemini the Sun approaches the north pole and, moving away from its least meridian altitude, rises higher above the horizon each day up to the greatest of the meridian altitudes; and from that point the day too, starting from its shortest, begins to grow up to the longest day. But from the beginning of Cancer to the end of Sagittarius the Sun recedes from the north pole, and the day and the Sun's meridian altitude decrease. So the six signs that descend, with respect to those who live on the northern side of the equator, are *Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius*. The latter six ascend directly and descend obliquely; the former ascend obliquely and descend directly for northerners, according to what will be said below about these ascensions. Among southerners, however, everything is the opposite of this: for them the ascending signs are those that are descending for northerners, and vice versa. (2) It cuts in two at right angles not only the equator but also all its parallels; and so by this colure and by the colure of the equinoxes the equator, the parallels of the equator, and the ecliptic are cut into four quadrants, which gives the four seasons of the year. (3) It marks the meridian of the place at whose noon the solstices occur, for the colure of the solstices coincides with that meridian. For that reason, in the figure of chapter 7, we took the same circle AEBQ for the meridian and for the colure of the solstices; and this will also often have to be done in many astronomical problems.

[Margin: Solstitij Etymologia. — The etymology of "solstice".]

In this way it marks the solstices. They are so called both because the Sun, when it has reached the beginning and tropic of Cancer or of Capricorn, declines no further from the equator toward the poles of the world, but is halted within those limits and turns its course toward the equator; and because on those days the difference in the length of the days, and in the meridian altitudes of the Sun and in the shadows, is so imperceptible that the Sun seems to stand in the same parallel. (4) It carries the Sun's greatest declination, that is, the distance of the tropics from the equator, and also the distance of the pole of the ecliptic from the pole of the world; for each of these is an arc of this colure, between the equator and the tropics or between those poles, and in this way it measures those distances. In the following figure the former is the arc EC or EK, and the latter TB or NA.

SCHOLIA.

I. For one at whose noon the equinox falls, the pole of the ecliptic is at its greatest distance from the meridian: toward the west if the equinox is the vernal one, toward the east if the autumnal. But for one at whose noon a solstice occurs, the pole of the ecliptic is on the meridian and higher than the pole of the world by 23° 30′, if the solstice is the winter one for northerners; lower, if the summer one.

[Margin: Præcessio Aequinoctiorum. — The precession of the equinoxes.]

II. The *civil* precession of the equinoxes is the anticipation in time by which the true equinox comes before the day assigned to the equinox in the calendar; I shall speak of it more fittingly in Book 3, in the chapter on this matter. The *astronomical* precession of the equinoxes, on the other hand, is the recession by which the first shining star in the horn of Aries moves away from the point of the vernal equinox, by a motion of longitude toward the east. This holds whether the motion is truly a motion proper to the fixed stars, or, as others prefer, a motion of the equinoctial points themselves toward the west, on account of which the fixed stars, motionless in themselves, appear to move away from those points toward the east. On this subtlety I shall speak in Book 3.

CAPVT XVI.

De Tropicis, Circulis Polaribus, & Zonis.

On the tropics, the polar circles, and the zones.

I. TROPICS are named from the noun τροπή [Greek: tropē, "turning"], which signifies a return or conversion, because when the Sun reaches these circles it returns and bends its course toward the equator, and does not go past these limits toward the poles of the world.

[Margin: Tropicus Cancri q̄ d? — What the Tropic of Cancer is.]

II. The *Tropic of Cancer*, in Greek τρόπικος θερινὸς [Greek: tropikos therinos, "summer tropic"], that is, the summer tropic with respect to northerners,

is not a great circle. It is drawn from the poles of the world, and is therefore parallel to the equator; it stands as far from the equator as the Sun's greatest declination, or the obliquity of the ecliptic, amounts to; and it touches the ecliptic at the beginning of Cancer. Its function, besides those already indicated, is to bound the torrid zone on one side and the northern temperate zone on the other; and then to produce the summer solstice and the longest day for northerners, and the winter solstice and the shortest day for southerners. Such, in the following figure, is the circle CR, drawn from the north pole B.

[Margin: Tropicus Capricorni quid? — What the Tropic of Capricorn is.]

III. The *Tropic of Capricorn* is not a great circle. It is drawn from the poles of the world, is parallel to the equator, stands as far from the equator as the Sun's greatest declination amounts to, and touches the ecliptic at the beginning of Capricorn. Its office is to bound the torrid zone on one side and the southern temperate zone on the other; and then to produce the winter solstice and the shortest day for northerners, and the summer solstice and the longest day for southerners.

[Margin: Dies Solstitiorū quæ? — Which days are those of the solstices.]

The solstices occur when the Sun enters Cancer and Capricorn, namely about the 21st of June and of December in the corrected Julian year, that is, three days before the Nativity of St John the Baptist and of Christ the Lord. Accordingly the saints, Augustine above all, expound that saying of St John the Baptist (*Illum oportet crescere, me autem minui* ["He must increase, but I must decrease"]) as referring also to this growth and shrinking of the days, because Christ was born on the day when the days were already beginning to grow, and St John the Baptist on the day when they were beginning to shrink; for in that age the solstices occurred about the 24th or 25th of June and of December. You have a specimen of the tropics in the figure added here, in which the line CR denotes the Tropic of Cancer and KP that of Capricorn.

[Margin: Polares Circuli qui? — What the polar circles are.]

IV. The *polar circles*, according to the more common acceptation among astronomers, especially the Latin ones, are lesser circles, parallel to the equator and therefore drawn from its poles, and as far from the poles of the world as the tropics are from the equator, namely 23° 30′; in other words, circles whose circumferences are traced by the poles of the ecliptic as they are carried round the poles of the equator in a whole daily revolution. One of them is called the northern, or septentrional, or arctic, ἀπὸ τοῦ ἄρκτου [Greek: apo tou arktou, "from the Bear"], that is, from the Bear, the neighbouring constellation; it contains within it the northern frigid zone and bounds the northern temperate zone. The other is called the southern, or antarctic, that is, set opposite to the arctic; it contains within it the southern frigid zone and bounds the southern temperate zone. So in the adjoining figure the arctic circle is TM and the antarctic NO.

Diagram. A small circle set into the lower part of the right column, the text wrapped around its left side. A vertical diameter runs from E at the top to Q at

the foot, and a horizontal diameter from A at the left to B at the right, crossing at G at the centre. A double line runs from K at the upper left through G to R at the lower right. A further line runs from T at the upper right through G to O at the lower left. Curved arcs stand on the circle between the labelled points named in the text. Fourteen labels were read on the block, and no others were seen; the referents below are the ones the page's own prose assigns.

- E (top): end of the arcs EC and EK
- K (upper left): with P, the Tropic of Capricorn (KP)
- C (upper right): with R, the Tropic of Cancer (CR)
- N (left, above A): with O, the antarctic circle (NO)
- T (right, above B): with M, the arctic circle (TM)
- A (left): the south pole
- F (left of the centre, above the horizontal line): not cited in the text of this page
- G (centre): not cited in the text of this page
- B (right): the north pole
- O (left, below A): with N, the antarctic circle
- M (right, below B): with T, the arctic circle
- P (lower left): with K, the Tropic of Capricorn
- Q (foot): lower end of the vertical diameter, cited in AEBQ for the figure of chapter 7
- R (lower right): with C, the Tropic of Cancer

DE ZONIS.

On the zones.

[Margin: Zonæ quid? — What zones are.]

V. *Zones*, among cosmographers, are bands on the surface of the celestial or terrestrial sphere, but especially the ter- restrial, bounded by the tropics, or by the polar circles, or by both. There are five of them, namely the *Torrid Zone*, the *Northern Temperate Zone*, the *Southern Temperate Zone*, the *Northern Frigid Zone*, and the *Southern Frigid Zone*. Virgil describes these zones superbly in Book 1 of the *Georgics*, starting from the verse *Quinque tenent Cœlum Zonæ* ["Five zones hold the sky"]; so does Ovid in Book 1 of the *Metamorphoses*, starting from *Vtque duæ dextra Cœlum læuaque* ["And as two, on the right, the sky, and on the left"]; and Tibullus, Book 4, Elegy 1.

[Margin: Zona Torrida quæ? — What the Torrid Zone is.]

VI. The *Torrid Zone* is contained between the two tropics, and its breadth is as great as the distance of the tropics from each other, that is, double the distance of the tropics from the equator, namely 47°. Such, in the preceding figure, is the band enclosed between the arcs KC and PR; and it is cut in two by the equinoctial line. Aristotle, and Pliny the Elder (Book 2, ch. 68), judged it uninhabitable on the Earth because of the excessive heat produced by the Sun's rays, reflected more directly toward the air lying over the heads of the

inhabitants; and so it was called *Torrid*, especially since for those inhabitants the Sun passes through the zenith twice a year.

[Margin: Zona Torrida habitabilis. — The Torrid Zone is habitable.]

But in reality it is inhabited in many places. By the gentle providence of God, Best and Greatest, it is blown through by very frequent and almost daily breezes of wind, or sprinkled with rain, so that the heat is tempered; and not rarely men of white colour have been found living in it. Those live in this zone for whom the pole is not raised more than 23° 30′. For its other properties see ch. 20, no. 8.

VII. The *Temperate Zones*, so called from the moderation of heat and cold and of the length of the days, are contained as follows: the northern between the Tropic of Cancer CR and the arctic circle TM, the southern between the Tropic of Capricorn KP and the antarctic circle NO. Each of them is as broad as the complement to 90° that the torrid zone lacks, namely 43°. In the northern temperate zone live the largest part of Europe, of Asia, and of northern America. Those live in the temperate zones for whom the pole is raised no more than 66° 30′ and no less than 23° 30′.

VIII. The *Frigid Zones* are contained, one within the arctic circle TM and the north pole B, the other within the antarctic circle NO and the south pole A; although the poles, being points, can bound rather than contain. And although for those who live within these zones the pole is raised more than 66° 30′, so that they have one continuous day lasting several of our days, indeed even several months, the zones nonetheless are and are called frigid in comparison with the other zones. This is both because they also have a night no less long, made up of the nights of several months, and because the Sun's rays are reflected toward their air only very obliquely and very remotely, and the Sun is very far from the zeniths of the inhabitants; just as, if the Sun stayed always on the horizon or very near it, it would not warm us much. Yet it has been learned from Dutch and English navigators, who have penetrated into the northern frigid zone, that these zones are inhabited.

On the cause of the Sun's greater heat from reflection at less oblique angles, I shall speak in Book 3, ch. 5.

SCHOLIA.

[Margin: Polarium circul. alia notio. — Another notion of the polar circles.]

I. In Proclus, Cleomedes, Geminus, and some Greek astronomers, the polar circles are the same as the circles of perpetual visibility or of perpetual occultation [semper Apparentium, semperue latentium], treated in the next chapter below, because they are drawn from the poles of the world. But taken in this way they are not fit to bound and distinguish the five zones, since for those for whom the pole is raised 66° 30′ the tropics themselves are circles of perpetual visibility, whereas the polar circles as commonly understood are not circles of perpetual visibility except for those living under the tropics.

II. Those who live precisely under the tropics belong to the torrid zone and to the nearer temperate zone; those under the polar circles belong to the temperate and the frigid zone together, since what is in the middle shares in its extremes.

III. If anyone wants the ratio of lesser circles to great circles, let him multiply the sine of the complement of the lesser circle's declination by 360° and divide the sum by the radius; the quotient will be the number of degrees of the great circle to which the whole circumference of the lesser circle is equal. But we shall give a table for this ratio at the end of the book and in the Geographical Book, with the miles corresponding to one degree.

[Margin: Dextra, & sinistra in Mūdo quæ? — Which are the right and the left in the world.]

IV. Since Virgil and Ovid, in their description of the zones, mentioned a right and a left side of the world, Clavius (p. 310) takes the occasion to explain which side of the world is the right and which the left, and Iacobus Mazzonus [Jacopo Mazzoni] more learnedly still, in his defence of Dante, Book 1, ch. 15, and Book 3, ch. 30. The sum of the doctrine is this. Among philosophers and geographers the right is the east, because the daily motion begins from it, as in animals from the right side; or because northern geographers, when about to describe the inhabited world, face the north pole, and so the east remains on their right; and in this way they present their maps to us so that the east is on our right. Among astronomers, on the other hand, the right is the west, because they contemplate chiefly the planets and the stars of the zodiac, and turn to face south (understand this of northerners), so that for them the west is on the right. Among poets, finally, the right is the north, because it is nobler in its multitude of stars and of more civilized and more learned inhabitants, and in the Sun's longer stay in the northern signs, by about 8 days more than in the southern.

[Margin: Tropici cœlestes extra Terræ globum. — The celestial tropics lie outside the globe of the Earth.]

V. Since there is a danger that the unskilled may think that the tropics, the circles of the terrestrial sphere, lie in the same plane as the celestial tropics, and the terrestrial polar circles in the plane of the celestial polar circles, let them remember that the globe of the Earth is, in relation to the sky, like a point, and that the Earth's diameter is therefore far shorter than that part of the sky's diameter which subtends the breadth of the celestial torrid zone. From this it follows that the planes of the celestial tropics lie wholly outside the Earth. It is true that if straight lines are drawn from any points on the circumference of a celestial tropic to the Earth's centre, they will pass through the circumference of its own terrestrial tropic; and I say the same of straight lines drawn from the circumference of a celestial polar circle to the Earth's centre, for they will pass through the circumference of the terrestrial polar circle, as Clavius gathers from the last proposition of Book 6 of Euclid in his *Sphere*, p. 311, where he also adds a figure.

VI. Polybius subdivided the torrid zone into two, as Strabo reports, but he had no followers. The first divider of the zones was Parmenides; but already from the time of Thales they were distinguished by polar circles taken as circles of perpetual visibility. Posidonius, however, made them fixed, by adopting polar circles drawn from the pole of the ecliptic; but if the obliquity of the ecliptic varies, these too vary. Kepler in the *Epitome of Copernican Astronomy* thinks it not improbable that the two temperate zones once contained as much space as the three intemperate ones together, so that there should be equality between intemperateness and temperateness; and accordingly that the obliquity of the ecliptic was greater than 24°. But God did not look to these material Ideas, nor did he practise geometry in mere quantity; he had other ends. See Book 3, ch. 26, Scholium 1.

CAPVT XVII.

De Circulis semper Apparentium.

On the circles of perpetual visibility.

I. CIRCLES of perpetual visibility, that is, of stars or of points of the sky, do not exist in a right sphere, that is, for those who live on the equator. In a parallel sphere, the one in which those living under the poles of the world dwell, the equator itself is the circle of perpetual visibility for one group and of perpetual occultation for the other, just as it also serves as the horizon. So a dweller under the north pole sees perpetually all the points of the northern hemisphere, and all the stars that are within it, for as long as they are within it and keep a northern declination; but he can never see the points or stars of the southern hemisphere, except incidentally by reason of refraction. Conversely, one living under the south pole can always, on every day or night, see all the points and all the stars of the southern hemisphere, but never those of the northern, except incidentally through the refraction of the rays, which lifts the stars.

II. In an oblique sphere, however, the circles of perpetual visibility and of perpetual occultation are lesser circles, drawn from the poles of the equator, and so parallel to it, and touching the horizon. Within them are contained the points of the sky and the stars that of themselves either never set or never rise. There are as many such circles as there are elevations of the pole, and they are the larger the greater the elevation of the visible pole. By means of them one can tell which stars never descend below the horizon, or which never ascend above it, if the altitude of the pole and the declination of the stars are known.

III. The circle of perpetual visibility is equal to the circle of perpetual occultation, by 6 of Book 2 of *Theodosius*, since they are parallel and equally distant from the equator. For those living under the tropics the polar circles themselves serve as circles of perpetual visibility on one side and of occultation on the other; for those living on the polar circles, the circles of perpetual visibility and occultation are the tropics themselves.

IV. But so that you may understand this more easily, let the horizon be HR, the meridian HERQ, the equator EQ, the axis of the equator and of the world AB, the north pole of the world B, the south pole A, and the altitude of the north pole BR, equal to the depression of the south pole HA. Now through the points R and H, where the meridian cuts the horizon, let two circles SR and HL be drawn, parallel to the equator EQ and touching the horizon at R and H. SR will then be the circle of perpetual visibility for a dweller for whom the north pole is raised by as much as the arc RB. Stars on the circumference of the circle SR will, at their least meridian altitude, touch the horizon at R, but will not set; stars within this circle and the pole B will always be above the horizon. Conversely, the circle HL will be, for the same dweller, the circle of perpetual occultation. Stars placed on its circumference will, at their greatest elevation, graze the horizon at H, but will not rise; the others, contained within it and the pole A, will always be hidden below the horizon.

Diagram. A circle set into the left column, the text wrapped down its left side. A horizontal line runs from H at the left to R at the right. A long diagonal runs from E at the upper left to Q at the lower right, and a second long diagonal from B at the upper right to A at the lower left, crossing the first at the centre. Shorter lines stand parallel-looking to E–Q: S to R on the upper right, F to G above the centre, a line from K to N at the foot, and H to L at the lower left. Twelve labels were read on the block, and no others were seen; the referents below are the ones the page's own prose assigns.

- F (top, left of S): with G, the parallel FG, on which a star is supposed
- S (top): with R, the circle of perpetual visibility SR
- E (upper left): with Q, the equator EQ
- B (upper right): the north pole
- K (left, above H): the point to which the star on HL is apparently raised; the page prints "the parallel KH"
- H (left): with R, the horizon HR; with L, the circle of perpetual occultation HL
- R (right): with H, the horizon; with S, the circle SR
- G (right, below R): with F, the parallel FG
- A (lower left): the south pole
- Q (lower right): with E, the equator
- L (foot): with H, the circle HL
- N (foot, right of L): not cited in the text of this page

V. The remaining stars, contained between the equator EQ and those circles, will rise and set, and their stay above the horizon will be the longer, the greater the arc of those circles intercepted between the meridian and the horizon. Yet this arc will always be greater, and that stay longer, than the arc and the stay below the horizon, for stars contained between the equator and the circle of perpetual visibility. This is apparent on the circle FG, parallel to EQ, on which a star is supposed to be: for the arc from F to the horizon HR is greater than the one below, from the horizon to G. Conversely, for stars contained between

the equator and the circle of perpetual occultation, the arc and the stay above the horizon will be less than the arc and the stay below the horizon. Finally, if any stars are on the equator itself, the arc and the stay above will be equal to the arc and the stay below the horizon.

SCHOLIA.

[Margin: Refractio hos circulos variat. — Refraction varies these circles.]

I. What has been said so far of the perpetual visibility or occultation of stars, or of their contact with the horizon, is true of itself, setting aside the physical variation that usually arises from refractions and from the vapours of the horizon, incidentally as regards the astronomical position of the sphere. For suppose that in the preceding figure some star is on the parallel FG. In its greatest depression it would be at G, below the horizon HR, and would of itself set; but since the refraction of its rays can be so great that its image is lifted to R, and the refraction of altitude equals the arc of depression RG, it can appear at the very point R of the horizon, or even above it, if the refraction is greater. Conversely, let another star be on the parallel HL; when it has come to H, and would not of itself rise, it may yet be raised apparently to K by the agency of horizontal refraction. For it will seem to be on the parallel KH and not to be among those that are always hidden, just as the star G, raised to R, will seem to be among those that always appear.

II. I also said in number 1, "Which, and for as long as, etc." because the phrase *semper Apparentium aut Latentiū* must be understood absolutely as regards points of the sky, which do not change their declination; but as regards stars, which do change their declination, only on a hypothesis, namely that their declination is such that they remain within the circle of perpetual visibility or occultation. For in the course of time they leave those circles, and stars that previously never set or never rose begin to set or to rise, because of the changed declination.

[Margin: Modus secernēdi circulum semper Apparentium. — The way of distinguishing the circle of perpetual visibility.]

III. The rule for knowing the size of the circle of perpetual visibility, that is, how far it stands on the meridian from the pole of the world, or how far it declines from the equator, is very easy. The altitude of the pole itself gives its distance from the pole of the world, and the complement of that altitude to 90° gives its declination from the equator, which in the figure above is the arc ES. Thus for us at Bologna, where the north pole is raised 44° 29′ 30″, the circle of perpetual visibility SR stands from the pole B by the arc SB or BR, 44° 29′ 30″, and declines from the equator EQ by the arc SE, of 45° 30′ 30″.

IV. The rule for knowing stars of perpetual or non-perpetual visibility is of this kind. Join the star's northern declination to the altitude of the north pole. If the sum equals a quadrant, that is 90°, the star will touch the horizon but will not set for northerners. If the sum is greater than a quadrant, it will

be one of those that always appear above the horizon. If it is less, it will be one of those that neither always appear nor always lie hidden, but rise and set, unless refraction prevents it. If this refraction is as great as the excess of the quadrant over that sum, the star will incidentally appear on the horizon; if greater than that excess, it will be seen above it; if less, it will still lie hidden below the horizon. And say the same of southern stars and the south pole with respect to southern dwellers.

EXEMPLVM.

Example.

Let the example be Capella, whose northern declination in the year of Christ 1600 was, by my observations, 45° 29′ 35″; at the completed year 1609 it was 45° 30′ 30″; and at the completed year 1644, 45° 34′. The altitude of the pole at Bologna is 44° 29′ 30″. If you add the first declination to this, the sum is 89° 59′ 5″, less than a quadrant. So at that time Capella was, for us, one of the stars that rise and set of themselves. But since the quadrant exceeds that sum by 55″, and the horizontal refraction of the fixed stars is, by Tycho's reckoning, 30′, it was incidentally always visible above the horizon. Now add the second declination to the same altitude of the pole: the sum becomes equal to a quadrant. So in 1609 Capella, at its lowest depression, should have touched the horizon at R, but incidentally, because of refraction, it appeared about 30′ higher. Finally add the third declination, and the sum is 90° 3′ 30″, greater, that is, than a quadrant by 3′ 30″. Therefore in 1644 Capella was already, not only incidentally but of itself, one of those that always appeared above the horizon; and then, at its lowest altitude, it should have been 3′ 30″ higher, but because of refraction 33′ 30″. And it will be always visible for many future centuries, and will appear the higher at its lowest altitude, the greater its northern declination becomes, which in this century at least, and almost in the next, has grown each year by 6″.

V. The rule for knowing the stars that always lie hidden is of this kind. Add the star's southern declination to the altitude of the north pole. If the sum equals a quadrant, the star will touch the horizon but will not of itself rise; if the sum is greater, it will always lie hidden; if less, it will be among those that rise and set. If you are a southern dweller, use the altitude of the south pole but the northern declination, that is, stars declining toward the north, and do the rest as above.

VI. From what has been said you will understand many passages of the poets, and the reason why Virgil, for example, says:

Arctos Oceani metuentes æquore tingi. ["The Bears, fearing to be dipped in the water of Ocean."]

For he is speaking of Ursa Major and Ursa Minor, which for the Greeks and Latin inhabitants never set at that time, just as they do not set even now. Likewise why Ovid said:

Magna minorq; feræ, quarum regit altera Graias; Altera Sydonias, vtraque sicca, rates. ["Great and lesser beasts, of which one guides the Greek ships, the other the Sidonian, and both of them dry."]

For neither of the Bears set for the Greeks, and so either was fit, at any hour of the night, for steering a course at sea. But for the Sidonians and all the Phoenicians, Ursa Major set, and only Ursa Minor stood always above the horizon.

CAPVT XVIII.

De Climatibus, & eorum Parallelis.

[Margin: Clima q̄ d? — What a climate is.]

I. CLIMATE [Clima], in Greek κλίμα [Greek: klima, "ladder" or "step", as the page goes on to gloss it], that is, a ladder or the rungs of a ladder, is a small zone on the surface of the sphere, especially the terrestrial sphere, enclosed by two circles parallel to the equator, or by the equator and one of its parallels. Its breadth, and its difference from the next climate, are as great as competent judges reckon a notable difference between them, in the length of the longest artificial day and in the temperature of the soil or the air. If this difference is taken uniformly in time, so that each climate is distinguished from the next by a difference of half an hour in the longest day of the year, it makes the breadth of the climates ever narrower and narrower toward the poles, as can be seen in the adjoining figure. In it the horizon is ABDE, the equator AGD and the meridian BGE, while the remaining lines indicate the parallels within which the climates are contained. The reason is that for an equal difference in the day, the smaller the difference required in the altitude of the pole, the greater the altitude of the pole itself, as is clear from the theory and the problems of the primum mobile. This is the origin of the name "climates": for if climates are drawn in this way on maps or geographical globes, they will represent ladders by which one climbs from the equator toward the poles, to a greater and greater altitude; and perspective artists draw the more remote rungs of a ladder at a smaller interval from their neighbours than the rungs nearer the eye.

Diagram. A circle set into the lower part of the left column, the text wrapped down its left side. Horizontal lines cross it at intervals, which were not measured. Labels read on the block: five. No others were seen. The referents below are the ones the page's own prose assigns.

- B (top): on the horizon ABDE and the meridian BGE
- A (left): on the horizon ABDE and the equator AGD
- G (centre, on a horizontal line): on the equator AGD and the meridian BGE
- D (right): on the horizon ABDE and the equator AGD
- E (foot): on the horizon ABDE and the meridian BGE

[Margin: Climatum Paralleli. — The parallels of the climates.]

II. *Parallels of the climates*: some bound the climates, giving a beginning to one and an end to another; others are drawn through the middle of a climate. Through the middle, I mean, in proportion to half the difference of the longest day by which one climate differs from the next. For they ought not, as the inexperienced suppose, to be drawn through the middle of the place or space, so as to divide the climate in two. So if the difference of one climate from another is taken from the half hour by which the longest day of one inhabitant exceeds the longest day of another, the parallel through the middle of a climate must be drawn through that part of the climate in which there is a difference of one quarter [of an hour] in the longest day from the day of the parallel bounding the climate. In this way, between two neighbouring parallels drawn through the middles of climates, the difference will be half an hour.

III. *Anticlimate* [Anticlima] is the opposite climate, whose breadth and distance from the equator toward the other pole, and whose longest day of the year, are equal, though not on the same day of the year.

[Margin: Climatum denominatio. — The naming of the climates.]

IV. The *naming* of climates is usually taken from some notable place on the Earth through which the parallel through the middle of the climate passes: so in tables of climates you will note the *parallel through Rome*, the *parallel through Byzantium*, and so on. Usually added are the altitude of the pole, the length of the longest day, the equinoctial and solstitial shadows of gnomons at noon, and the number of miles corresponding to one degree; these belong either to each parallel or at least to the parallels through the middles of the climates. You will see this in our tables of climates, which we shall present more fittingly in the Geographical Book. For it is there that the controversy so far pursued over the number and proportion of the climates, which different people determine differently, and over how many miles correspond to one degree, will have to be settled, and the whole table reformed, for just reasons. Meanwhile see the tables in Clavius's *Sphere*, p. 429; those of Biancani and of Cluverius [Philipp Clüver] in the *Introduction to Geography*; of Langius [?] in the *Elementale Geographiae* [Elementali Geogr.], p. 492; of Barozzi in the *Cosmography*, Book 3, from p. 212; of Hérigone, volume 4, p. 90; of Stöffler on Proclus, p. 30.

CAPVT XIX.

De Triplici Sphæræ Positione, & de Diuersitate Dierum, ac Noctium in illa.

I. The POSITION of the sphere is considered according to the situation of the equator relative to the horizon. Either the equator cuts the horizon orthogonally, that is, at right angles, and from this arises the *right sphere* [Sphæra Recta]; or at oblique angles, and from this the *oblique sphere* [Sphæra Obliqua]; or it does not cut it but coincides with the horizon, so that the parallels of the equator are also parallels of the horizon, and from this comes the *parallel sphere* [Sphæra Parallela]. Look at their diagrams in order in the three following figures: in

the first of them the equator CD cuts the horizon AB orthogonally, etc.; in the second it cuts it obliquely; in the third AB is at once equator and horizon.

Diagram. The first of three circles set in a row across both columns. A horizontal diameter runs from A to B, and a vertical line from C to D. Further vertical lines stand on either side of CD. Eight labels were read on the block, and no others were seen. Which of the vertical lines ends at which letter was read at strip scale and is hedged.

- E, C, F (top, left to right): C on the equator CD; E on the tropic EG and F on the tropic FH, as the prose on this page assigns
- A (left), B (right): the horizon AB
- G, D, H (foot, left to right): D on the equator CD; G and H the lower ends of the tropics EG and FH

Diagram. The second circle of the row, standing over the gutter. A horizontal line runs from H at the left to R at the right. A long diagonal runs from B at the upper right to A at the lower left. A band of parallel lines slants from the upper left to the lower right. The text on the next page assigns all these letters to this figure. Labels read on the block, eighteen in all: H and C together at the upper left, B at the upper right; E, L, K descending the left side; a second H at the left end of the horizontal line, with O just inside it; D and F on the horizontal line; G and R at its right end; N, I, Q, M descending the right side; P and A at the foot. No others were seen. ⚠ H stands twice on the block, and both are needed by the prose (horizon HR, meridian HCRP; parallel HI). The small letters D, F and G were read at strip scale and are hedged.

Diagram. The third circle of the row. A vertical line runs from C at the top to D at the foot, and horizontal lines cross the circle, of which AB is the one through the centre. Eight labels were read on the block, and no others were seen. No line was distinguished on the block as a separate ecliptic FH; line end-points were read at strip scale and are hedged.

- C (top), D (foot): the axis of the world CD
- A (left), B (right): at once equator and horizon AB
- E (upper left), H (upper right), F (lower left), G (lower right): the ends of horizontal lines; the prose on the next page names FH as the ecliptic

[Margin: Sphæra Recta quæ? — What the right sphere is.]

II. The right sphere, then, belongs to those for whom the equator cuts the horizon at right angles, and its properties are these. 1. For them the poles of the world lie on the horizon, while the equator passes through the zenith and the nadir, as the Sun also does on the days of the equinox, since it is then on the equator. 2. All days are equal to the nights, of themselves and setting aside that small portion of the day which refraction adds, when it makes the Sun rise about three or four minutes of time earlier, and set as many later, than it should. The cause of this equality appears in the first figure, in which the horizon is AB, the equator CD and the tropics EG and FH; of these, as of all the parallels along which the Sun travels, the arcs above the horizon are equal to the arcs below the horizon. 3. The shadows, especially the noon shadows, of

gnomons set upright on horizontal planes point south as long as the Sun is in the northern signs, but north when it is in the southern signs. 4. All the stars without exception rise and set for such an inhabitant, stay above the horizon for 12^h, and lie hidden below it for as many. In this sphere are the Island of St Thomas, the middle of Taprobane, and many other places.

[Margin: Sphæra Obliqua. — The oblique sphere.]

III. The oblique sphere belongs to those for whom the equator cuts the horizon at oblique angles, that is, obtuse toward the visible pole and acute toward the other pole. So in the second figure, in which the horizon is HR, the meridian HCRP, the equator EQ and its axis AB; the depressed south pole of the world is A and the elevated north pole is E [recte: B]; the tropics are CN for Cancer and KP for Capricorn; and the rest are parallels of the equator, which the Sun describes by its daily motion when it is at the beginning of the signs.

The properties of this sphere are these. 1. It has one pole always elevated and the other always depressed below the horizon. 2. If an inhabitant is between the equator and a tropic, the Sun passes over his head twice; those outside them never have the Sun at the zenith. 3. All days, except the equinoctial ones, are of themselves unequal to the nights: for northerners they are longer than the nights as long as the Sun is in the northern signs, and shorter than the nights as long as it is in the southern signs. Conversely, for southerners the day exceeds the night when the Sun is in the southern signs, and is shorter than the night when the Sun is in the northern signs. The cause of this inequality is the unequal arcs of the parallels of the equator. Let the Sun be on the tropic of Cancer CN: the semidiurnal arc CG, which the Sun traverses from its rising at G to the meridian at C, is greater than the seminocturnal arc GN, which is below the horizon. So too with the Sun placed at the beginning of Gemini or Leo, as on the parallel HI, the semidiurnal arc HF is greater than the seminocturnal FI. But if the Sun is on the tropic of Capricorn KP, the semidiurnal arc KO is less than the seminocturnal OP; and with the Sun at the beginning of Sagittarius or Aquarius, that is, on the parallel LM, the semidiurnal arc LD is less than the seminocturnal DM; and likewise for the other parallels. All this will become clearer if you turn a material sphere and, placing on the horizon the point of the ecliptic where the Sun is, count the degrees of the equator that rise in the time the Sun takes to reach the meridian, and convert them into times, as was said in Chapter 12, no. 2. For besides this inequality you will also learn the size of the semidiurnal arc with the pole elevated to any altitude you please. 4. For those outside the torrid zone, the shadows of gnomons always point toward the visible, or elevated, pole; for those inside the torrid zone, they point sometimes toward one pole and sometimes toward the other. The Sun never reaches the zenith of the first group, but it reaches that of the second twice a year and passes over their heads. 5. There are some stars, or at least points of the sky, that always appear to them, and some that are always hidden.

[Margin: Sphæra Parallela. — The parallel sphere.]

IV. The parallel sphere belongs only to those two inhabitants who are directly under the poles of the world, as in the third figure, in which horizon and equator together are AB; the meridian ACBD; the axis of the world CD; the ecliptic FH; and the rest are parallels of the equator and of the horizon alike. Its properties are these. 1. For them the poles of the world are at the zenith and the nadir, and the axis of the world is the same as the vertical line. 2. One living under the north pole C has a single day of six months, as long as the Sun is in the northern signs. The Sun then describes parallels around the horizon and never sets for him from the beginning of Aries to the beginning of Libra, and during that time the inhabitant under the south pole D has perpetual night. Conversely, when the Sun runs through the southern signs, the inhabitant under the south pole has perpetual day for about six months, and the other meanwhile a night of six months, that is, from the beginning of Libra to the beginning of Aries. 3. The shadows of gnomons describe with their tips a circle or a helix, because the Sun goes around them on every side in the space of 24^{h}. Yet by the force of refraction the Sun rises for these people before it reaches the equator.

V. Clavius, however, on p. 23 of the *Sphere*, thinks that according to Johannes de Sacrobosco's mind those living under the poles of the world dwell in the oblique sphere, insofar as dwelling in the right sphere does not apply to them. But Sacrobosco does not seem to have made an exhaustive division of the position of the sphere, since he left out the parallel sphere. Common to the oblique and parallel spheres is this: the day for those living toward or under one pole is as long as the night for those living toward the other pole and having an equal altitude of the pole; and for the same inhabitant the day is as long as the night was or will be when the Sun was or will be at the opposite point of the ecliptic. So if with the Sun at the beginning of Leo the day is of 15^{h}, the night will be of as many hours when the Sun is at the beginning of Aquarius. From this appears the admirable compensation and distribution of times made by the infallible Providence of God, Best and Greatest [D.O.M.].

But on the variety of artificial days in these spheres, see what we shall say more fittingly below, in Chapter 26.

CAPVT XX.

Quid sint Antipodes, Antœci, Periœci, Antiscij, Amphiscij, Heteroscij, Periscij: & de Vmbris Gnomonum; earumque diuerso situ, ac figura.

I. So that you may grasp the notions of these names more quickly, in the following diagram or description the horizon is HR and the meridian HARB; beneath it is the meridian of the terrestrial globe KPOM, and the equator AB, and the parallels KM and PO, equally distant from the equator. On these let there be four inhabitants, two in the upper hemisphere, K and P, and two in the lower, M and O. With this laid down:

Diagram. A large circle in the right column, with a smaller circle, the terrestrial globe, inside it at the centre. Four small human figures stand on the

inner circle, feet toward the centre, at K and P above and at M and O below. A horizontal line runs from H to R, a vertical line from A to B, and two diagonals cross at the centre, running from C at the upper left and D at the upper right toward F and E at the lower right and lower left. Twelve labels were read on the block, and no others were seen. The diagonals run from C to F and from D to E. Inside the inner circle two short vertical lines run from K to M and from P to O.

- A (top), B (foot): the printed text calls AB the equator and includes A and B in the meridian HARB
- H (left), R (right): the horizon HR
- C (upper left), D (upper right), E (lower left), F (lower right): not cited in the text on this page
- K (upper left of the inner circle), P (upper right): the two inhabitants of the upper hemisphere
- M (lower left of the inner circle), O (lower right): the two inhabitants of the lower hemisphere

[Margin: Antipodes. — Antipodes.]

II. *Antipodes* of an inhabitant on the Earth's surface are those who live in the place diametrically opposite on the Earth, and have the soles of their feet set against the soles of his. That is why they are called in Greek ἀντίποδες [Greek: antipodes, "with feet opposite"], which you might render in Latin as *Contrapedes*. Such, in the figure above, is the inhabitant M compared with the inhabitant P, and the inhabitant O with K; and in turn P is the antipode of M, and K of O. So they have all their geographical properties entirely opposite. The nadir of one is the zenith of the other; the moment of noon for one is that of midnight for the other; when the day is longest for one, the night is longest for the other, and the day for one is as long as the night for the other; the deepest winter of one is the height of summer of the other; the altitude of the north pole for one is as great as the altitude of the south pole for the other; when the Sun rises for one it sets for the other, and when it sets it rises; and the stars that always appear to one are always hidden from the other. Yet they agree in the size of the altitude of the pole and in having the identical horizon, which is one and the same for both, being described from the same vertical points. They are also called *Antichthones* by Pliny the Elder, Book 16, ch. 22, and *Antigenæ* by Albertus Magnus.

[Margin: Antœci. — Antoeci.]

III. *Antoeci* [Antœci], in Greek ἀντοίκοι [Greek: antoikoi, "those dwelling opposite"], in Latin *Contraccolæ* (for οἶκος [Greek: oikos, "house"] is a house or dwelling), are those who live on the opposite and equal parallel, and so one equally distant from the equator toward the other pole; and in the strict sense of this name, they are those who are also on the same meridian, and for whom noon occurs at the same moment, as K is with respect to P, and M with respect to O. In the broad sense, however, they are all who live on the antiparallel just mentioned, and in this way the antipodes too are included under this name, so

that every antipode is an antoecus, but not the reverse. An antoecus has the opposite equinox and solstice: what is the autumnal for the one is the vernal for the other, and what is the winter solstice for the one is the summer for the other, and vice versa. Likewise the day for one is always as long as the night for the other.

[Margin: Periœci. — Perioeci.]

IV. *Perioeci* [Periœci], in Greek περίοικοι [Greek: perioikoi, "those dwelling around"], in Latin *Circumcolæ*, are those who live on the same parallel and have the same altitude of the same pole, though under a different meridian, as those who live on the parallel PO. They have everything in common in the seasons of the year, in the length of days and nights, and in the position of shadows, but they differ in the reckoning of the hours and moments of the day. Some would have a perioecus, taken strictly, be one who lives at the diametrically opposite point of the same parallel, as O with respect to P, because a whole semicircle of the parallel must be travelled round to reach him; and for him it is midnight when it is noon for the other.

[Margin: Antiscij. — Antiscii.]

V. *Antiscii* [Antiscij], that is, *Contraumbres*, so called from shadow, which in Greek is σκιά [Greek: skia, "shadow"], are those who have shadows turned in the opposite direction. In this way, those in the southern frigid and temperate zones are antiscii with respect to those living in the northern frigid and temperate. But not all who live beyond the equator are antiscii on any given day with respect to those living on this side of the equator; they are so only on certain days. In this way that part of Arabia which lies further south and within the torrid zone is antiscian to Italy. Arabs migrating there to bring help to Pompey were astonished at their shadows, which in their own region they had found to point to the left, toward the south, and afterwards in Latium to point toward the north, which for the poets is the right-hand part of the world, as was said in Chapter 16, Scholium 4. Hence the line of Lucan:

Ignotum vobis Arabes venistis in orbem; Vmbras mirati nemorum non ire sinistras.

["You Arabs came into a world unknown to you, astonished that the shadows of the groves did not fall to the left."]

[Margin: Periscij. — Periscii.]

VI. *Periscii* [Periscij], that is, *Circumbres*, are those whose right shadows go round in a circle and describe with their tip what is almost the circumference of a perfect circle; from this carrying-round of the shadow they are called περίοικοι [recte: περίσκιοι] [Greek: periskioi, "shadowed all round"]. Such are those who live in the parallel sphere, that is, under the poles of the world.

[Margin: Heteroscij. — Heteroscii.]

VII. *Heteroscii* [Heteroscij] are those who have only one kind of shadow, as those who live within the temperate zones: for northerners within the temperate zone shadows extend only toward the north, and for southerners only toward the south.

[Margin: Amphiscij. — Amphiscii.]

VIII. *Amphiscii* [Amphiscij] are those of two shadows, that is, whose shadows are alternately southern and northern, according as they have the Sun now to the south and now to the north of the point overhead. Such are all who live within the torrid zone. Yet when the Sun is at their zenith, gnomons cast no right shadow at noon, and for this reason Pliny the Elder calls them *Ascii* [Ascij], that is, lacking shadows, which happens to them twice a year, as it does once to those who live under the tropics. From this comes Lucan's phrase about Syene, which lies under the tropic of Cancer: *Vmbras nusquam flectente Syene* ["with Syene nowhere bending its shadows"], which Strabo, Cleomedes and Pliny the Elder also affirm. Moreover, those within the torrid zone have two summers and two winters. But those who live in the northern part of the torrid zone have a colder winter with the Sun in Capricorn than in Cancer, and those in the southern part of the torrid zone have a colder winter with the Sun placed in Cancer. They have summer when the Sun is overhead. So for those living under the equator the equinoxes turn into summers, and in this respect they share in the solstitial altitude, that is, the highest. For this reason Lucan calls the equator the *circle of the high solstice*, in these verses:

Deprensum est hunc esse locum, quo circulus alti Solstitij medium signorum percutit orbem. ["It was found that this is the place where the circle of the high solstice strikes the middle of the circle of the signs."]

That is, it cuts the zodiac in two. For the Sun cannot be higher, and also produce greater heat (which are the chief properties of the summer solstice), than when it stands at an altitude of 90° and passes overhead; granted that Gassendi and others have understood by the name "high solstice" the summer solstice, as distinct from the winter one, in which the Sun is lower.

Hence those living within the torrid zone are said to have four solstices, two high and two low. For when the Sun is at their zenith, which happens twice a year, they have the height of summer and the greatest altitude of the Sun above the horizon. But when the Sun is at the beginning of Cancer or of Capricorn, they have two winters, and the Sun then has the least meridian altitude that it can have toward one of the poles. Granted, the etymology of "solstice" may not always hold true for them, and the Sun does not stand still in the manner we described in Chapter 15, no. 5.

SCHOLIA.

[Margin: Antipodas qui negarint. — Who denied the antipodes.]

I. Lactantius (Book 3 of the *Divine Institutes*, ch. 24), St Augustine (Book 16 of the *City of God*, ch. 9), Lucretius (Book 1) and certain other ancient theologians denied that there are antipodes, because they were said to be in another world. It was thought absurd that there should be people not redeemed by CHRIST, since it is commonly held that CHRIST came only to this world of ours and not to other worlds. And because Bishop Virgilius [Virgil of Salzburg] was thought to have spoken in this sense when he asserted that there are antipodes,

Pope Zachary, at the urging of St Boniface, Bishop of Mainz, ordered Virgilius to be suspended from sacred functions if he asserted the antipodes in such a way as to assert several worlds outside this one. This is plain from Zachary's letter, found in Baronius [Cesare Baronio], written to Utilo [Odilo], Duke of Bavaria, which M. Velserus [Markus Welser] also weighs learnedly in Book 5 of the *Rerum Boicarum*. So Zachary did not condemn all who asserted the antipodes, nor did he err on a question of law or of cosmographical history. If he erred, the error was on a question of fact, though not even on that, since he passed a hypothetical sentence, not an absolute one. Antipodes can, however, be said to be in another world in the sense in which the Indians and Americans are said to be in a new world, and Columbus to have discovered a new world, namely new parts of this world, previously unknown to us. But whether someone who asserted that there are human inhabitants on the Moon or on the other planets, and yet added that they belong to this world and that CHRIST died for them too, would deserve any censure other than that of rashness or novelty is not a matter to dispute here. Meanwhile, for retaining the antipodes, consult Cleomedes, *De mundo*, Book 1; Strabo, *Geography*, Book 1; Pomponius Mela, Book 1; Solinus, ch. 66; Macrobius, Book 2 on the *Dream of Scipio*; Martianus Capella, Book 6; Manilius, Book 1, ch. 4; Albertus Magnus, the book *De natura locorum*, dist. 1, ch. 10, and Book 2 of the *Meteora*, tr. 2, ch. 11; Stöffler on Proclus, p. 50.

II. The unlearned, however, think there are no antipodes, because they fear they would fall down. But they do not see that in standing or walking these people press and tend toward the same centre of heavy things, which is at the centre of the Earth, and that the same centre is downward for them and for us. What is the nadir for us is the zenith for our antipodes, and they can no more fall upward to their zenith than we can to ours, for that would be to ascend, not to descend. But O happy fall, if it should befall them and us and everyone, God willing, to fall into Heaven! Against the fantasy of these people see, I beg you, Pliny the Elder, Book 2, ch. 65, and Book 16, ch. 22, arguing it beautifully.

III. It can nonetheless be said that some part of the Earth's surface actually lacks antipodes, because on the opposite side there is no inhabited land, or there are only the waters of the sea, and fish that are *Apodes* [footless] rather than antipodes. In this way, to someone looking round an artificial terrestrial globe, almost all of Italy will appear to lack antipodes, unless perhaps sailors are sailing opposite it at the time.

[Margin: Vmbrarū linea figura diuerſa — The line of the shadows, of differing shape.]

IV. Although the tip of the shadow of a gnomon set upright on a horizontal plane describes an almost circular line for those living under the poles, which is why they are also called periscii, for others, living in the torrid zone and in the temperate zones, it describes a straight line on the day of the equinox, and on the other days a hyperbolic, parabolic or elliptical line. The writers on gnomonics demonstrate this from the *Conics* of Apollonius, and Maurolico

teaches it briefly in Dialogue 2, in these words: "The plane on which the shadow is cast either cuts the parallel or does not. If it cuts it, then the extremity of the shadow describes a hyperbola in the plane. If it does not cut it, then it either touches the parallel or does not. If it touches it, then the top of the shadow describes a parabola in the plane. If it does not touch it, then it either is equidistant from the parallel or is not. If it is equidistant, the shadow describes a circle in the plane; if not equidistant, an ellipse." By the term "plane" he means the whole circle of the horizon. Thus every horizon in which the pole rises less than 66° 30′ has, outside the equinoctial days, a hyperbolic shadow: for such a horizon cuts the parallels on which the Sun stands. But for those living beneath the Arctic or Antarctic circle, that is, those for whom the pole is raised 66° 30′, the shadow is parabolic on their summer solstitial day, because their tropic then touches the horizon; on the other days they have a hyperbolic shadow. Finally, those for whom the pole rises higher than 66° 30′ have at times hyperbolic shadows, at times parabolic and at times elliptical; but if it rises 90°, circular ones.

V. On the ratio of the equinoctial and the solstitial shadow to the gnomon, it will be more fitting to speak in the Geographical Book.

CAPVT XXI.

De Alijs Sphæræ Circulis Capite 7. nominatis.

On the other circles of the sphere named in Chapter 7.

[Margin: Almincantarath. — Almucantar.]

I. ALMVNCATARATH, commonly Almincantarath, circles, so called by an Arabic name, are circles parallel to the horizon, and thus described from the poles of the horizon and drawn through the altitudes of stars or of celestial points. So in the figure added here, let the horizon be HOR and the meridian HVRN, and let a star be at S. Through it let the circle ASL be drawn, parallel to the horizon HOR and described from its pole, the zenith V: this will be an almucantar circle.

Diagram. A circle set into the left column under the opening of Chapter XXI, the text wrapped down its left side. Curved meridians run from top to foot, with a straight vertical between them; horizontal lines cross the circle at intervals, not counted against the prose. A six-pointed star stands at S. Labels read on the block: ten. No others were seen. The referents below are the ones the page's own prose assigns.

- V (top): the zenith and vertex
- A (upper left): on the parallel ASL
- S (upper left, beside the star): the star, on the parallel ASL
- L (upper right): on the parallel ASL
- H (left): on the horizon HOR and the meridian HVRN
- R (right): on the horizon HOR and the meridian HVRN

- Z (left of centre, below the horizontal line HR): on the circle of altitude VZNMV
- O (centre, below the horizontal line HR): on the horizon HOR and the vertical VON
- M (right of centre, below the horizontal line HR): on the circle of altitude VZNMV
- N (foot): the nadir

[Margin: Altitudinis Circulus. — Circle of altitude.]

II. The *circle of altitude* [Altitudinis Circulus], by another name the *vertical* [Verticalis], is a great circle of the sphere drawn through the vertex and the lowest point, that is, through the inhabitant's zenith and nadir, with its poles on the horizon. By 13 and 15 of Book 1 of Theodosius it therefore cuts the horizon at right angles. It is called the circle of altitude because on it is measured the altitude of a star or of any other object visible on the sphere; this altitude is as great as the arc of the vertical circle intercepted between the object and the horizon. So in the figure just given, if a star is at S, and the circle VZNMV is drawn through it, the vertex V and the nadir N, this will be the circle of altitude, and the altitude of the star S will be the arc ZS, while its complement to 90°, the arc SV, will be the star's distance from the vertex. When, however, a star is considered below the horizon, as if it were in the quadrant ZN, the circle is called the circle of depth [Profunditatis] or of immersion [Demersionis]. There are as many verticals as there are points on the horizon. Chief among them are the meridian (for every meridian is a vertical, although not every vertical circle is a meridian) and the *prime vertical* [Verticalis Primarius], namely the one that passes through the points of equinoctial rising and setting, as VON does. Finally, a common vertical is one that passes through the vertices of several places.

III. The *circle of right ascension* [Ascensionis Rectæ circulus] is the same as the equator, or the circle of declination, of which more shortly.

[Margin: Azimuthalis Circulus. — Azimuthal circle.]

IV. The *azimuthal circle* [Azimuthalis Circulus] is the same as the rational horizon, if it is taken as the measure between the meridian and the other vertical circles; otherwise the Arabs call the vertical circles azimuths [Azimutha].

[Margin: Declinationis Circulus. — Circle of declination.]

V. The *circle of declination* [Declinationis Circulus] is a great circle of the sphere passing through the poles of the world and having its own poles on the equator, and therefore cutting the equator at right angles, by 13 and 15 of Book 1 of Theodosius. It is so called because on it are taken the declinations of celestial points or of stars, that is, their distances from the equator: for the declination, say of a star, is as great as the arc of this circle intercepted between the equator and the star's centre. So in the following figure, let the horizon be HGR, the meridian HERQ and the equator EOQ; let the poles of the equator, or of the world, be A the southern and B the northern; and let the circle drawn through the poles of the world and through the star's centre be AOBIA, for

this will be a circle of declination. So if a star is at O or at I, it will have no declination; but if it is at S, on the parallel CN, its northern declination will be the arc OS, and the complement of the declination the arc SB. But if the star is at M, or if the declination of the parallel KP is sought, the southern declination of either will be the arc MO, and the complement of the declination the arc MA. Similarly, if the star were on the horizon, as at G or at F, the northern declination would be the arc FI, and the southern GO.

Diagram. A circle set into the right column, the text wrapped down its left side. A horizontal line, two near-vertical lines, an arc from A to B through the centre region and oblique lines cross it; six-pointed stars stand beside S, O, M, G, I and F. Labels read on the block: seventeen, one of them undecided. No others were seen. Referents are those the page's prose assigns; line end-points were not traced.

- K (upper left): on the parallel KP
- E (top): on the meridian HERQ and the equator EOQ
- C (top, right of E): on the parallel CN
- B (upper right): the northern pole
- S (upper centre, beside a star): the star at northern declination, on the parallel CN
- O (upper centre, beside a star): on the equator EOQ and the circle AOBIA; set small and not cleanly distinct from c on the scan
- M (left of centre, beside a star): the star at southern declination
- H (left): on the horizon HGR and the meridian HERQ
- R (right): on the horizon HGR and the meridian HERQ
- G (left of centre, beside a star): on the horizon HGR
- a small letter at the centre, a turned capital D: not cited in the prose of this paragraph; a page below (Chapter XXII, para. XIV) cites a point D on this figure, and that citation has not been used to settle the letter
- F (right of centre, beside a star): on the horizon
- A (lower left): the southern pole
- I (lower centre, beside a star): on the circle AOBIA
- P (foot): on the parallel KP
- Q (foot, right of P): on the meridian HERQ and the equator EOQ
- N (lower right): on the parallel CN

Gather from the definition that any meridian is a circle of declination, but not conversely; and the same goes for the colures. And because right ascension, discussed below, ends at the point where the equator is cut by a circle of declination, that circle is called the circle of right ascension.

[Margin: Circuli Domorum. — Circles of the houses.]

VI. The *circles of the celestial houses* [Domorum Cœlestium Circuli] are great circles of the sphere drawn through the common intersections of the meridian and the horizon, and dividing into twelve equal parts: either the equator, following the rational way of Avraham ibn Ezra and of Regiomontanus; or the prime vertical, following the way of Campanus and Gazulus; or the semidiurnal

and seminocturnal arcs separately, and by means of them the equator, following the way of al-Qabīṣī and of Johannes de Saxonia; or the equator into 12 parts corresponding to as many temporal hours, or to the intersections of the hour circles, following the way of Ptolemy. On this see below, in the chapter on the celestial theme. Some, however, prefer to draw these circles through the poles of the equator, others through the poles of the zodiac, and in that case they reject the meridian and use another circle.

[Margin: Horarij circuli. — Hour circles.]

VII. *Hour circles* [Horarij circuli] are great circles of the sphere whose poles lie on the equator, drawn through the poles of the equator and through points of the equator that intercept 15° each, that is, the single hourly intervals. All hour circles are therefore in fact circles of declination, but not conversely. For an example of them see the following figure, in which the horizon is HR, the meridian HQBRA, the equator EQ, the poles of the equator, or of the world, B and A, and then the remaining hour circles. The meridian itself is the circle of hour 12, both from noon and from midnight, and so for the remaining 10; for each of them serves for two opposite hours, as for the first after noon and the first after midnight. From this is distinct the circle of the eleventh hour after noon or midnight, although in the figure, to avoid multiplicity, they seem to coincide. When, however, the 24 hours are unequal, or begin from rising or setting, they are drawn otherwise.

Diagram. A circle set into the right column, the text wrapped down its left side. A horizontal line HR crosses it, and a sheaf of curved hour circles converges from A at lower left to B at upper right. A straight line runs from Q at the top to E at the foot, with hour numerals set slantwise along it on both sides. Labels read on the block: six letters, and the numerals below. The referents are the ones the page's prose assigns.

- Q (top): on the meridian and the equator EQ
- B (upper right): a pole of the equator or world
- H (left): on the horizon HR
- R (right): on the horizon HR
- A (lower left): a pole of the equator or world
- E (foot): on the equator EQ
- Numerals, upper half of the line: 12, 11, 10, 9, 8, 7 running down from the top on its left side, and 1, 2, 3, 4, 5, 6 running down on its right side; the sequences were read at the scale of the page and not checked sort by sort
- Numerals, lower half of the line: 7, 8, 9, 10, 11, 12 running down on its right side; on its left side 5, 4, 3, 2, 1 were made out, and a sort at the head of that run could not be settled [?]

[Margin: Latitudinis Circulus. — Circle of latitude.]

VIII. The *circle of latitude* [Latitudinis Circulus] is a great circle of the sphere whose poles lie on the ecliptic, drawn through the poles of the ecliptic and so cutting it at right angles. It is called the circle of latitude because on it the latitude of the stars, that is, their distance from the ecliptic, is measured and

taken: for the latitude of a star is as great as the arc of this circle intercepted between the ecliptic and the star's centre. So in the following figure, let the horizon be HR and the meridian and colure of the solstices HZRQ, the ecliptic ZC, the equator EQ, and the poles of the ecliptic G the southern and P the northern. The circle GIPLG drawn through them will be a circle of latitude. If a star is at I, it will have no latitude; if at K, its northern latitude will be the arc IK and its complement the arc KP; but if at F or at L, its southern latitude will be, for example, the arc FI, and the complement of the latitude the arc FG or LG. From what has been said you will gather that the colures and the meridian can sometimes be circles of latitude, since they can pass at once through the centre of a star and through the poles of the ecliptic.

Diagram. A circle at the head of the left column, the text wrapped down its left side. A horizontal line, an ellipse-like arc and oblique lines cross it; six-pointed stars stand at K, at L, and at a point left of centre near I. Labels read on the block: fourteen. Letters at left of centre are faint on the scan. Referents are those the page's prose assigns; line end-points were not traced.

- Z (top): on the meridian HZRQ and the ecliptic ZC
- E (upper left): on the equator EQ
- B (upper right): not cited in this paragraph
- P (upper right): the northern pole of the ecliptic
- I (upper centre): on the circle GIPLG
- K (right of centre, beside a star): the star at northern latitude
- F (left of centre, below a star; faint): the star at southern latitude
- H (left; faint): on the horizon HR and the meridian HZRQ
- R (right): on the horizon HR and the meridian HZRQ
- G (lower left): the southern pole of the ecliptic
- L (lower centre, beside a star): on the circle GIPLG
- A (lower left, outside the rim): not cited in this paragraph
- C (foot, right of centre): on the ecliptic ZC
- Q (lower right): on the meridian HZRQ and the equator EQ

[Margin: Longitudinis Circulus. — Circle of longitude.]

IX. The *circle of longitude* [Longitudinis Circulus] is taken by some for the ecliptic, because the motion in longitude is counted along it; by others, however, for the circle of latitude, inasmuch as that same circle determines the longitude of a point or star at the place where it cuts the ecliptic, according to what will be said below in Chapter 22, in the definitions.

[Margin: Positionis Circulus. — Circle of position.]

X. The *circle of position* [Positionis Circulus] is a great circle of the sphere passing through the common intersections of the horizon and the meridian, and through a star or a place on the surface of the sphere that lies under a meridian different from the meridian of that star or place with respect to which its position is being considered. It is therefore a kind of movable horizon. So in the following figure, with the meridian HVR etc., the horizon HAR and the star or place P, the great circle HPR drawn through P, passing through the

points H and R at which the horizon and the meridian cut each other, is the circle of position that the star or place P has with respect to the point lying under the vertex V. The poles of this circle lie on some other horizon. When the quadrant of the vertical circle VA is drawn through P, the *angle of position* [Angulus Positionis] HPV or RPV arises.

Diagram. A circle in the left column, the text wrapped down its left side. A horizontal line runs across it from H to R, an arc rises from H through P to R, and a line descends from V through P to A. Labels read on the block: five. No others were seen.

- V (top): the vertex
- P (above centre): the star or place
- H (left): on the horizon HAR, the meridian and the circle HPR
- A (centre, below the horizontal line): on the horizon HAR and the quadrant VA
- R (right): on the horizon HAR, the meridian and the circle HPR

Besides these circles there are many others: namely the parallels to the equator, which the stars describe by the diurnal motion of the primum mobile, and which are described from the poles of the equator; likewise the parallels to the ecliptic, also described from its poles, which the stars describe by their motion in longitude; and the parallels to the meridians, which we will show in the Geographical Book, in the section on histiodromy, can be described by natural motion on the surface of the land or sea. Finally, there is the circle of the distance of the stars, which is a great circle and passes through the centres of two stars.

CAPVT XXII.

In quo Definitiones Arcuum, & Angulorum insignium ad Circulos Sphæræ pertinentium, nec non ipsorum Circulorum ordine Alphabetico.

In which are definitions of the notable arcs and angles pertaining to the circles of the sphere, and of the circles themselves, in alphabetical order.

[Margin: Altitudo syderis qu? — What the altitude of a star is.]

I. The ALTITUDE of a star [Altitudo Syderis], or of a point visible on the sphere, is the arc of a great circle drawn through the vertex, intercepted between the star or that point and the horizon. Its remainder to 90°, that is, to the quadrant, is called the *complement of the altitude* [Complementum Altitudinis]. This is the same as the star's distance from the vertex, and it is the arc of the vertical circle intercepted between the vertex and the star's centre. If the vertical is the meridian, it is called the *meridian altitude* [Altitudo Meridiana], and on that day it is the greatest. See the figure of Chapter 21, no. 2. The altitude is *true* [Vera] if the star's true place is considered, and *visible or apparent* [Visa seu apparens] if its place is seen elsewhere because of parallax or refraction.

[Margin: Almincatarath. — Almucantar.]

II. The *almucantar* [Almincatarath] is for some the altitude of a star itself; for others, a circle parallel to the horizon, drawn through a star that stands high above the horizon. On this see Chapter 21, no. 1.

[Margin: Amplitudo Ortiua. — Ortive amplitude.]

III. The *ortive amplitude* [Amplitudo Ortiua] of a star or celestial point is the arc of the horizon between its most northerly and its most southerly rising: the true rising if the amplitude is true, the apparent if it is apparent. So the Sun's ortive amplitude, the true one, is the arc of the horizon comprehended between the tropics.

[Margin: Amplitudo Occidua. — Occiduous amplitude.]

IV. The *occiduous amplitude* [Amplitudo Occidua] of a star etc. is the arc of the horizon between the most northerly and the most southerly setting of the star etc.: the true setting if the true amplitude is considered, the apparent if the apparent. The amplitude of the other planets is greater than the Sun's ortive and occiduous amplitude by as much as their greatest declination doubled.

[Margin: Angulus Sphæralis. — Spherical angle.]

V. The *spherical angle* [Angulus Sphæralis, seu Sphæricus], taken strictly, is an angle made on the surface of a sphere by the meeting of two arcs that are portions of a great circle of the sphere; or it is the inclination of those two arcs. Taken broadly, it is the inclination or meeting of any two arcs of spherical circles.

[Margin: Angulus Eclipticæ & Meridiani. — Angle of the ecliptic and the meridian.]

VI. The *angle of the ecliptic and the meridian* [Angulus Eclipticæ, & Meridiani] is the spherical angle made by an arc of the ecliptic and an arc of the meridian. At the solstitial points it is a right angle; at the others it is oblique. But once it is known, there are also known both the other angle vertically opposite it, because that is equal to it, and the other that follows next to it, because that is its complement to two right angles, that is, to 180°. Further, such angles at points of the ecliptic equally distant from the equinoctial points are equal to one another; so to construct the table of the angles of the ecliptic and the meridian, which we will give in its place, it is enough to build it for one quadrant of the ecliptic.

[Margin: Angulus Eclipticæ, & Horizōtis. — Angle of the ecliptic and the horizon.]

VII. The *angle of the ecliptic and the horizon* [Angulus Eclipticæ, & Horizontis] is the spherical angle that the portion of the ecliptic standing above the horizon makes with the horizon. Once this angle is known, the one next to it is also known, since it is its complement to 180°. It is also called the angle of the *east* [Orientis] or of the *ascendant* [Ascendentis], meaning the ascendant point of the ecliptic, because it is generally observed in the eastern part of the horizon; although there corresponds to it on the opposite side an equal angle in the west, where the point of the ecliptic diametrically opposite the ascendant point is then setting.

[Margin: Anguli aliorū Circulorum. — Angles of other circles.]

VIII. The *angle of any two circles* [Angulus duorum quorumuis circulorum] is the one made by any two arcs of any two circles, especially great circles. Of this kind are the angle of the equator and the meridian, which is always a right angle; of the equator and the horizon, which in a right sphere is right and in an oblique sphere oblique, and as great as the altitude of the equator, speaking of the acute angle; the angle of the equator and the circle of declination, which is always right; of the equator and the circle of latitude; of the ecliptic and the equator, and this one is 23° 30′; of the ecliptic and the circle of latitude, and this one is always right; of the ecliptic and the circle of declination; likewise the angle of the ecliptic and the vertical; and also of the circles of declination and latitude; of the circles of declination and the vertical; of latitude and the vertical; likewise of the circles of declination and the horizon; of the circles of latitude and the horizon; and of the vertical circle and the horizon, which is always right; and so on for the other comparisons of circles, whether the centres of stars or other points of the sphere are considered in those angles.

[Margin: Ascensio syderis, & Descensio. — Ascension of a star, and descension.]

IX. The *ascension of a star* [Ascensio Syderis] or of a point of the ecliptic is the arc of the equator counted from the vernal section, that is, from the beginning of Aries, toward the east, up to that point of the equator which rises above the horizon together with that star or point of the ecliptic by the motion of the primum mobile. Its *descension* [Descensio] is the arc of the equator counted from the beginning of Aries toward the east, up to that point of the equator which sets below the horizon with that star or point. Thus the simultaneous co-rising or co-setting of parts of the equator with some star is called its ascension or descension, because of the connection it has with the motion of the primum mobile, of which the equator is the measure. For example, if at Rome, at the moment when 3° of Leo rises, degree 106 and minute 40 of the equator rises with it, counted from the vernal section toward the east, that degree and that minute of the equator are called the ascension of 3° of Leo, because they end the arc of the equator that has been ascending from the rising of the beginning of Aries up to the moment when 3° of Leo rises.

[Margin: Ascensionis Recta duplex notio. — The twofold notion of right ascension.]

X. The *right ascension* [Ascensio Recta] of a star or of a point of the ecliptic, and the *descension*, taken as opposed to oblique ascension, in the way commonly used in astronomical tables bearing this title, is the ascension or descension that occurs in a right sphere, because there the equator makes right angles with the horizon, with whatever star or point of the ecliptic it may rise. In the same way *oblique ascension and descension* [Ascensio, & Descensio Obliqua] is that which occurs in an oblique sphere, because of the obliquity of the angles that the equator has there with the horizon, which is indeed very variable. The right ascension of points of the ecliptic rising above a right horizon is as great as their

transit across the meridian in any horizon whatever; that is, as great as the arc of the equator counted from the beginning of Aries toward the east, up to that point of the equator which crosses the meridian, or Midheaven [Medium Cæli], at the moment when that point of the ecliptic crosses the same meridian. The reason is that any meridian is the horizon of someone living in a right sphere. Hence it has come about that astronomers begin the time of the primum mobile from noon rather than from rising or from setting, so that an equal time would correspond for every sphere. But because the stars move toward the east by their own motion, in the time that elapses between rising and transit across the Midheaven the arc of the equator varies somewhat, and the arc that ascends and the arc that crosses the meridian are not strictly the same.

[Margin: Ascensionis Recta altera notio. — The other notion of right ascension.]

XII [recte: XI]. *Right ascension taken as slow*, as it is taken by Pliny the Elder (Book 2, 19), by Manilius and by the ancients, and likewise descension: that is, comparatively, not with respect to the angles of the equator with the horizon but with respect to the arc of the ecliptic counted from the beginning of Aries toward the east up to the point of the ecliptic rising or setting with the point of the equator. It occurs when the arc of the equator is greater than the said arc of the ecliptic, for more time is required; and then that point of the ecliptic, and the whole preceding arc of the ecliptic, is said *to ascend rightly* [rectè ascendere], that is, to ascend more rightly or more slowly. Say the same of a star having the same longitude as that point of the ecliptic which ascends or descends rightly. Conversely, *oblique ascension and descension* taken as less slow is when the arc of the equator ascending or descending with a point of the ecliptic, or with a star having the same longitude, is smaller than the said arc of the ecliptic. For then it ascends or descends more quickly. See the rules to be given separately in the next chapter.

XII. The *right ascension of the Midheaven* [Ascensio Recta Medij Cæli] is the arc of the equator counted from the beginning of Aries toward the east, up to the point of the equator that crosses the Midheaven, or meridian, at any given moment of the day; whether it is considered absolutely, as is often done, or with a point of the ecliptic or a star then crossing the meridian. For brevity, astronomers usually indicate the Midheaven with the letters MC. What the mediation of the heaven [Mediatio Cæli] is, with respect to the stars, I will say below.

[Margin: Horoscopus. — Horoscope.]

XIII. The *ascendant* [Ascendens], or *horoscope* [Horoscopus], is the point of the ecliptic rising at some determined moment of the natural day, say the moment at which an infant is conceived or born. It is so named from two Greek nouns: ὥρα [Greek: hōra, "hour"], which means hour, and σκόπος [Greek: skopos, "target"], which means the target on which one must take aim. For the whole condition of future time is believed to depend on that point; and therefore that moment and point of the ecliptic must be set up as

the chief target at which astrological consideration is to be directed, and must be established before everything else.

[Margin: Ascensionalis Differentia. — Ascensional difference.]

XIV. The *ascensional difference* [Ascensionalis differentia] of a point of the ecliptic or of a star is the arc of the equator intercepted between the point of the equator co-rising with that point of the ecliptic or star and the circle of declination drawn through that point of the ecliptic or through the star's centre. Because this arc is always as great as the difference between right and oblique ascension, some usually define it by saying that it is the difference between the right and the oblique ascension of the point or star. So once tables of ascensional difference have been made, tables of oblique ascensions are easily constructed, as we will teach in its place. To assist the imagination, look at the figure of Chapter 21, no. 5, in which let a star be at F, rising at a point of the horizon, and let the circle of declination drawn through that point be BFIA, etc. For the arc of the equator DI will be the ascensional difference of the star F, and the point co-rising with it will be the point D of the equator.

[Margin: Azimutha quid sint? — What azimuths are.]

XV. *Azimuth*, and with a Latin declension *Azimuthum*, is an Arabic noun signifying the arc of the horizon intercepted between the meridian and the point of the horizon at which a star or point of the ecliptic stands; or through which point of the horizon there passes that vertical circle on which the star or point of the ecliptic stands. If this arc of the horizon is eastern it makes an eastern azimuth; if western, a western azimuth. The complement of the azimuth, which some confuse with the azimuth, is the remaining arc of the horizon needed to complete the quadrant, that is, the arc from there to the point of the prime vertical. So in the first figure of Chapter 21, if a star is at Z or at S, on a point of the vertical passing through Z, the azimuth of that star will be the arc HZ, and the complement of the azimuth the arc ZO.

Colure: see Chapter 15. *Climate*: see Chapter 17.

[Margin: Cōplementum arcus quid? — What the complement of an arc is.]

XVI. The *complement of an arc* or of an angle [Complementum Arcus, vel Anguli] is the remaining arc needed to complete the quadrant, that is, the fourth part of the circle: in other words, what is required to reach 90°. For example, if a latitude, or an arc of latitude, is 30° 20′, its complement will be 59° 40′; for the sum of these makes up the quadrant of the circle, which comprehends a right angle. The complement is obtained at once if the given arc or angle is subtracted from 90°.

XVII. A point of the heaven is said to be *culminating* [Culminans], or to *culminate* [Culminare], as a star also is, that crosses the meridian, or at the time when it crosses it, especially if at the same time it crosses through the zenith.

XVIII. The *declination* [Declinatio] of a star or of a celestial point is the arc of a great circle drawn through the poles of the equator, intercepted between the equator and the centre of that star, or that point.

XIX. The *circle of declination* is a great circle of the sphere having its poles on the equator, and drawn through the poles of the same equator and through the centre of the star, or the point, whose declination is being considered. See Chapter 21, no. 5.

XX. *Descension*, right or oblique: see what was said above, nos. 9, 10 and 11.

XXI. *Direction* [Directio], or the *directory arc* [Directorius arcus], is the arc of the equator between two right ascensions, or two oblique ones, one of the promissor, the other of the significator; that is, of the celestial point that promises or signifies something in the future, say the death of the native.

XXII. The *distance of the poles* [Distantia Polorum], namely of the pole of the equator from the pole of the ecliptic, is the arc of the colure of the solstices intercepted between the pole of the equator and the nearer pole of the ecliptic. It is as great as the obliquity of the ecliptic, or as the Sun's greatest declination. For me, always 23° 30′.

XXIII. The *distance of the tropics* [Distantia Tropicorum] is the arc of the meridian intercepted between the tropics; for me, always exactly 47°.

XXIV. The *distance of the stars from one another* [Distantia syderum inter se] is the arc of a great circle on the sphere, drawn through the centres of two stars at once, intercepted between those two stars. It is true if the true place of the stars is considered; visible or apparent if the visible place is considered, which differs from the true one because of parallax or refraction.

XXV. *Dodecatemorion*, in Greek δωδεκατημόριον [Greek: dōdekatēmorion, "twelfth part"], is a twelfth part: usually of the zodiac, but among some writers sometimes of the equator; or it is one of the 12 celestial houses.

XXVI. *Hour circle*: see Chapter 21, no. 7.

XXVII. *Horizon*: rational, astronomical, natural; or sensible, physical and artificial. See Chapter 8.

XXVIII. *Horoscope*: see *Ascendant*.

XXIX. The *latitude of a star* [Latitudo syderis] is the arc of a great circle drawn through the poles of the ecliptic, intercepted between the ecliptic and the star's centre. It is true if the star's true place is considered, and apparent if its apparent place is considered.

XXX. *Circle of latitude* [Latitudinis circulus]: see Chapter 21, no. 8.

XXXI. *Geographical latitude* [Latitudo Geographica]: see Chapter 13.

XXXII. The *longitude* [Longitudo] of a star or of a point of the ecliptic is the arc of the ecliptic counted from the beginning of Aries toward the east as far as the circle of latitude passing through the star's centre, or as far as the great circle drawn through the poles of the ecliptic and the star's centre. Thus in the figure of Chapter 21, no. 8, if a star is at K and the beginning of Aries at Z, and through K is drawn the circle of latitude PKIG, cutting the ecliptic at I, the longitude of the star K will be the arc of the ecliptic ZI. It is true with respect to the true place, and visible if the visible place lies elsewhere because of parallax or refraction.

XXXIII. The *circle of longitude* [Longitudinis Circulus] is the same as the circle of latitude, since it performs a double office: that of measuring latitude and that of terminating longitude.

XXXIV. *Geographical longitude* [Longitudo Geographica]: see Chapter 13.

XXXV. The *mediation of the heaven* [Mediatio Cæli] is the point of the ecliptic that crosses the meridian together with some star; for some writers, however, it is the same as right ascension.

XXXVI. The *nonagesimal degree of the ecliptic* [Nonagesimus gradus Eclipticæ] is the degree or point of the ecliptic 90° distant from the rising point of the ecliptic. So, once the nonagesimal is known, if 90° are added to it, the rising point of the ecliptic becomes known; and if the rising point of the ecliptic is already known, subtract 90° from it and the nonagesimal of the ecliptic will become known. Note, moreover, that the vertical circle passing through the nonagesimal of the ecliptic is also a circle of latitude.

XXXVII. The *obliquity of the ecliptic* [Obliquitas Eclipticæ] is the angle made by the ecliptic and the equator at the equinoctial points. It is as great as the Sun's greatest declination, and for me it is always 23° 30′.

XXXVIII. *Polar circle* [Polaris Circulus]: see Chapter 16, no. 4.

XXXIX. The *pole of the horizon* [Polus Horizontis] is the same as the vertical point or its lowest point, that is, the zenith and the nadir.

XL. *Circle of position* [Positionis circulus]: see Chapter 21, no. 10.

XLI. The *depth of a star* [Profunditas syderis] is the arc of a vertical circle intercepted between the horizon and the centre of a star placed below the horizon.

XLII. *Circle of those always visible* [Semper Apparentium Circulus]: see Chapter 17.

XLIII. *Vertical circle* [Verticalis Circulus]: see Chapter 21, no. 2.

XLIV. The *zenith* is the vertical point, to which the diametrically opposite point is the nadir.

XLV. *Zone* [Zona]: see Chapter 16, no. 5.

CAPVT XXIII.

Regulæ Selectę de Ascensionibus Signorum, & Punctorum Eclipticæ.

Select rules on the ascensions of the signs and of the points of the ecliptic.

In Chapter 22, nos. 10 and 11, we set out what right ascension and right descension are, and we promised select rules about them, which we now bring out.

I. Rule. *In the right sphere*, Gemini, Cancer, Sagittarius and Capricorn, taken singly, ascend and descend rightly, that is, slowly; the other eight signs ascend and descend obliquely, that is, quickly. The reason is that in the right sphere more than 30 degrees rise with the four signs named, namely 32° 12′, and fewer with the others, as you will learn from the little table below, taken from the complete table of right ascensions.

Ascension and descension of the signs in the right sphere				Degrees and minutes of ascension and descension
♈ Aries	♍ Virgo	♎ Libra	♓ Pisces	27° 54′
♉ Taurus	♌ Leo	♏ Scorpio	♒ Aquarius	29° 54′
♊ Gemini	♋ Cancer	♐ Sagittarius	♑ Capricorn	32° 12′
Quadrant of the ecliptic I	Quadrant of the ecliptic II	Quadrant of the ecliptic III	Quadrant of the ecliptic IV	90° 0′, sum

II. Rule. *In the right sphere*, any quadrant of the ecliptic ascends or descends neither slowly nor quickly, that is, neither rightly nor obliquely in this sense, since 90° of the equator, or a quadrant of the equator, ascend or descend with it.

III. Rule. *In the right sphere*, any two arcs of the ecliptic that are equal and opposite, or equally distant from the prior equinox or solstice, have equal ascensions and descensions, taken singly. Thus the ascension of Taurus equals that of Leo, because they are equally distant from the point of the summer solstice, as you can gather as a sample from the preceding little table. Consequently, once the right ascensions of one quadrant of the ecliptic have been worked out by trigonometry, the ascensions of the remaining signs for the remaining three quadrants easily become known.

IV. Rule. *In the right sphere*, the descension of a sign or of a point of the ecliptic equals its ascension, because descension on one right horizon is ascension on the other right horizon, that of the antipodes.

[Margin: Signa Rectæ & Obliquæ Ascensionis. — Signs of right and of oblique ascension.]

V. Rule. *In the oblique sphere*, among northern peoples, Cancer and the five signs following it ascend rightly, that is, slowly, but descend obliquely; the other six signs ascend obliquely for them and descend rightly. Among southern peoples, however, Capricorn and the five signs succeeding it ascend rightly and descend obliquely, and the other six ascend obliquely for them and descend rightly. This assumes that they rise and set; for if the pole is elevated beyond 66° 30′, certain signs neither ascend nor descend for northern and southern peoples. So those verses of Manilius hold for the oblique sphere, but among northern peoples and on this side of the Arctic circle:

> They go upright, they fall aslant, from the star of Cancer until
> Chiron ends; but the other signs are born prone, and go down on
> a straight path.

[Recta meant, Obliqua cadunt à sydere Cancri / Donec finitur Chiron; sed cœtera signa / Nascuntur prono; descendunt tramite recto.]

Let this stand here as your evidence of these things, from the fuller tables of oblique ascensions, for three altitudes of the north pole.

Ascension of the signs taken separately in the northern oblique sphere.

Altitude of the pole		30°	45°	60°
♈ Aries	♎ Libra	21° 9′	16° 10′	7° 16′
♉ Taurus	♏ Scorpio	24° 22′	20° 3′	10° 56′
♊ Gemini	♐ Sagittarius	29° 55′	28° 1′	22° 56′
♋ Cancer	♑ Capricorn	34° 29′	36° 23′	41° 28′
♌ Leo	♒ Aquarius	35° 26′	39° 45′	48° 52′
♍ Virgo	♓ Pisces	34° 39′	39° 38′	48° 32′
Sum of the semicircle		180° 0′	180° 0′	180° 0′

6

VI. Rule. *In the oblique sphere*, the whole northern semicircle of the signs and the whole southern, among northern and southern peoples alike, ascend and descend in a middling manner, since neither more nor fewer than 180 degrees of the equator ascend or descend.

VII. Rule. *In the oblique sphere*, any two equal arcs of the ecliptic that are equally distant from the nearer equinoctial point have equal ascensions, taken separately from the others and compared with each other. See the little table of Rule 5.

VIII. Rule. *In the oblique sphere*, any two arcs of the ecliptic that are equal and opposite have ascensions, taken together, equal to the ascensions of the same arcs made in the right sphere. For example, at an altitude of the pole of 45°, the ascensions of Aries and Libra, opposite signs, taken together come to 55° 48′, as you will learn from the little table of Rule 5. But if from the little table of the first rule you add up the right ascensions of Aries and Libra, the sum will be exactly 55° 48′.

From this rule and the preceding one there arises a remarkable short cut. It is enough, for any altitude of the pole, to gather the ascensions of one quadrant of the ecliptic; for from these, and from the ascensions of the same quadrant in the right sphere, you will gather the ascensions of the remaining three quadrants in the oblique sphere. To construct tables of oblique ascensions for all altitudes of the pole, moreover, there suffice

5985 analyses of triangles, which would otherwise have to be four times as many. You will obtain the same thing more easily, however, through the ascensional differences, as Clavius teaches in the *Sphaera*, p. 337, and as we do in the Problems of the primum mobile.

IX. Rule. *In the oblique sphere*, signs or arcs of the ecliptic that ascend rightly, that is, slowly, descend obliquely, that is, quickly.

X. Rule. *In every sphere*, right or oblique, the descension of any sign or arc equals the ascension of the opposite sign or arc. So tables of descensions are

[6]The second column of signs is printed in this order, Libra at the top and Pisces at the foot. Rule VII below makes signs equally distant from the nearer equinox equal, and the sum of 55° 48′ for Aries and Libra at altitude 45° in Rule VIII is reached from the first and last rows of the table, so the text uses the signs of this column in the reverse order.

not necessary, since descension can be gathered sufficiently and easily from the tables of ascensions.

XI. Rule. *In every sphere*, right or oblique, the ascension and descension of the same sign or arc, taken together, are equal to the ascension and descension, taken together, of the opposite sign or arc. This in fact follows from Rule 10.

XII. Rule. *In the oblique sphere*, signs that ascend rightly among northern peoples ascend obliquely for southern peoples, and those that ascend obliquely for northern peoples ascend rightly for southern peoples, and so on.

SCHOLION.

All the preceding rules are proved both from the Problems of the primum mobile, by inquiring *a posteriori* through the analysis of triangles into the arcs of ascensions and descensions from their own data; and from the revolution of the armillary sphere, by comparing the degrees of the equator that rise together or set together with the degrees of the ecliptic, and seeing whether they are more or fewer.

Those verses of Lucan, Book 1 (*Non obliqua meant*, etc.), are moreover to be understood of the right sphere, and to be read in the order that our Clavius brings out from Petrus Iacobus Hispanus [?] in the *Sphaera*, p. 325.

CAPVT XXIV.

De Discrimine Dierum Naturalium à Diebus Artificialibus, & Ciuilibus.

On the difference between natural days and artificial and civil days.

[Margin: Diei definitio. — Definition of the day.]

I. The name *day* is equivocal, though the name *night* is not equally so. For sometimes it is taken for the whole day, compounded with that part of itself which is called night. So in Genesis 1, in the Seventy translators, a single day made from evening and morning is called ἡμέρα μιά [Greek: hēmera mia, "one day"], and the second day ἡμέρα δεύτερα [Greek: hēmera deutera, "second day"]. Among most Greeks, however, it is distinguished by a compound name from νύξ [Greek: nyx], which means night, and ἡμέρα [Greek: hēmera], which means day; and it is called either, as a noun, νυχθήμερος [Greek: nychthēmeros, "night-and-day"], or, as an adjective, νυχθημερινὸς [Greek: nychthēmerinos, "of a night and a day"], a name also found in St Paul, 2 Corinthians. The day taken in this way, then, is the time in which the centre of the Sun, by the motion of the primum mobile, or from east toward west, revolves from a meridian back to the same meridian, or from a horizon back to the same horizon, in one whole revolution.

II. Sometimes, however, the day is taken for the simple day, not compounded with the part that is night, or as it is distinguished from night. In this way Genesis 1 says that God, Best and Greatest [DEVS O. M.], divided the light from the darkness and called the one *Day* and the other *Night*. The day so taken is usually defined as the Sun's delay above the horizon, and the night as

its delay below the horizon: not because it delays, that is, does not move, but because it delays there its setting, here its rising. The day is better defined by saying that it is *the time in which the Sun moves from its rising above the horizon until its setting below the same horizon*; and the night, *the time in which the Sun moves from its setting below the horizon until its rising above the same horizon*.

III. It is now disputed by what name, among the Latins, the compound day is to be distinguished from the simple. No one, however, so far as I know, has called the day compounded with the night *artificial*; all call it either *natural*, or *civil*, or *legitimate*. But the simple day, contrasted with the night, some call natural and others artificial.

IV. For the simple day is called *natural*, and the compound day *civil*, by Censorinus, *De die natali*, Chapter 10, and by Gellius, Pliny the Elder, Macrobius, Giraldi, Vinet, Barozzi, Origanus, Goclenius, Boulliau and our own Tirinus; and Origanus adds that there is no artificial day except the one that art makes by lighting lamps to overcome the darkness of night. Isidore, however, calls *legitimate* what others call civil, and it is so called because law, in fixing days, and civil custom, in counting the days of magistracies, contracts and other civil acts, use compound and whole days, so long as they do not expressly distinguish the day from the night, as it is distinguished in Matthew 4: *When he had fasted forty days and forty nights*. They call the day distinguished from the night natural because the motion of the Sun from its rising to its setting does not come from art or from civil institution: nature itself distinguishes light from darkness, to say nothing of God, the author of nature. Moreover the etymology of *dies* is from *dio*, which means open and manifest; so that to be *sub dio* is to be in an open place, exposed to the light of the Sun, and *interdiu* means by day. The connection of this name with the simple day therefore seems more natural.

V. On the other hand, the day compounded with the night is called *natural*, and the simple day *artificial*, by very many, both of the theologians on Genesis Chapter 1, such as Abulensis [Alonso Tostado], Cornelius à Lapide and others, and of the astronomers, such as Regiomontanus, Reinhold, Stöffler, Finé, Sacrobosco, Neander, Maurolico, Clavius, Magini, Tycho, Longomontanus, Kepler, Biancani, Lansbergen and several others, in their tables of the equation of natural days. Their reason is that nature seeks the whole day and intends a complete circulation; whereas they call the simple day artificial because it serves artisans, or because the diversity of such days is produced only by the supreme artifice of God, Best and Greatest [DEI O.M.], and is learned by the manifold use of the artificial sphere.

Although the authors of no. 4 seem older and more elegant, we are nevertheless compelled to follow the authors of no. 5, so as not to seem to speak against the torrent of astronomers. So from here on we shall call *natural days* those that also include the night, and *artificial days* those distinguished from the night.

SCHOLIA.

I. In the frigid zones, the natural day according to the authors of no. 5 would not only be made up of several months of the year, but for those living under the pole it would coincide with the year itself, since there one artificial night lasts six months and one day consists of the other six months.

II. Both the natural day and the artificial day can be taken either as true or as apparent. The true is the one produced by the Sun's true motion with respect to the centre of the Earth; the apparent is the one produced by the motion apparent to us on the surface of the Earth, both because of parallaxes and because of refractions, of which below.

[Margin: Discrimen Diei Solaris à Die Primi Mobilis. — The difference between the solar day and the day of the primum mobile.]

III. Again, the day can be divided into the day of the primum mobile and the day of the second mobile [Secundi Mobilis], that is, the solar day. The former is one whole revolution of the equator; the latter is one whole revolution of the Sun to the same meridian or horizon from which it began to revolve. Besides a whole revolution of the equator, this requires as large a part of a second revolution of the equator as corresponds to the Sun's own motion in longitude, which on average is usually about 1°, that is, 4^m of time. Therefore, if one observes the time in which one conspicuous point of the heaven, but a fixed one, or one moved by no motion other than the diurnal, revolves from a meridian back to the same meridian, and compares it with the revolution of the Sun from a meridian back to the same meridian, the solar day will be found longer than the day of the primum mobile by very nearly 4^m. We have found this by experience more than once, with incredible patience and subtlety, counting both times by means of the vibrations of a pendulum, about which more in the *Liber Organicus* [Libro Organico]. For the former revolution we used the transit of a fixed star through the same meridian from one night to the following night, because the motion of the fixed stars toward the east is imperceptible in one day and is to be counted as nothing in this matter. Astronomers, however, let alone ordinary people, measure the motions of the stars by the solar day and its parts. See what was said in Chapter 12, Second Scholium, and what will be said in Book 3, Chapter 30.

CAPVT XXV.

De Inæqualitate Dierum Naturalium.

On the inequality of natural days.

Although natural days, that is, days compounded of daylight and the darkness of night, are commonly thought to be all equal, whether they are compared with other days of the same year or with days of different years, and although sundials themselves do not make their inequality sufficiently evident, astronomical subtlety has nevertheless detected it. Though this calls for deeper investigation, and is to be examined by us, God willing, in the questions on the primum

mobile, it nevertheless had to be indicated briefly for candidates in astronomy. But it is more conveniently deferred to the end of the Book on the Sun, which is the 3rd; see Chapter 30 there.

[Margin: Inæqualitatis Dier. Natural. Causæ. — Causes of the inequality of natural days.]

The causes are commonly three: the unequal course of the Sun in the zodiac; the inequality of right ascensions for equal arcs of the ecliptic; and the motion of the Sun's apogee.

CAPVT XXVI.

De Inæqualitate Dierum Artificialium.

On the inequality of artificial days.

The inequality of artificial days has two causes. One is common to them and to natural days: for since the artificial day is a part of the natural day, and the natural day is unequal both as a whole and in each of its parts, the artificial day cannot but share in its inequality, which I indicated in the preceding chapter can be made up of three causes, or even of four. The other cause of inequality is proper to artificial days, and it arises from four heads.

[Margin: Inæqualitatis Dier. Artific. Causæ. — Causes of the inequality of artificial days.]

I. The first head is the *horizontal parallax of the Sun* [Parallaxis Horizontalis Solis]. Because of it the Sun's centre, when it really reaches the rational horizon and rises with respect to the centre of the Earth, has not yet reached the sensible horizon of an eye placed on the surface of the Earth, if the eye is not raised at all. But if the eye is raised, whether to the top of a tower or to that of a mountain, it will be able to see the Sun rising even though the Sun is still below the rational horizon. Hence it comes about that even in the right sphere, in which the artificial day in itself is always equal to the night, the day can accidentally, by reason of parallax and of the diversity of the sensible horizon, be less than 12^h if the eye is not raised at all and there is no refraction (of which shortly), or more than 12^h if it is raised sufficiently. On the diversity of the sensible horizon, it has been discussed from Chapter 8, no. 5; on parallaxes I shall speak below, in Chapter 35.

II. The second head is the *horizontal refraction of the Sun* [Refractio Horizontalis Solis], which makes any star appear higher than it is, whereas parallax, on the contrary, depresses it in appearance. Refraction is far greater than parallax: the Sun's parallax does not exceed 3½′, while the Sun's horizontal refraction is about 34′, and with the parallax, which diminishes it, taken away, it is at least 30′. So even in the right sphere the Sun, not yet really risen, can be seen as risen, and when it has already set, can be seen as not yet set; and on both counts the equinoctial day can accidentally be of 12^h and nearly 8^m, and in the parallel or quasi-parallel sphere of more days than it would be without refraction.

III. The third head is the differing obliquity of the horizon, arising from the differing altitude of the pole and of the equator. Because of it, even if the Sun remained always on the same parallel of the equator, the day on one oblique horizon would still equal no day on the other oblique horizons: either because the semidiurnal arc of that parallel would be unequal, owing to the unequal section made by the horizon, or because in the frigid zones, if at times the horizon did not cut the Sun's parallel, it would yet touch it or be unequally distant from it. All this you will learn from Theodosius, *On Days and Nights*, and from figure 2 of Chapter 19 and what was said there in no. 5, or better from the armillary sphere, if, with the Sun placed on one parallel and on the horizon, you count the degrees of the equator up to the Sun's revolution to the meridian, at one altitude of the pole, and then at others and others. This cause, then, operates in the oblique sphere alone.

IV. The fourth head is the obliquity of the Sun's course along the ecliptic. Through it the Sun is placed each day on different parallels, from one tropic to the other, and so on parallels unequally cut by the same horizon or unequally distant from it. Hence it comes about that no days of the first half-yearly course, from the beginning of Cancer to the beginning of Capricorn, are equal to one another on the same horizon, just as no days of the second half-year, from Capricorn to Cancer, are equal to one another. This cause too, moreover, holds only in the oblique sphere.

Setting aside the first and second causes for now, I shall give from the 3rd and 4th certain rules, for a more distinct knowledge of the equality and inequality of artificial days. You will learn that they are true by use of the armillary sphere, or of the table of semidiurnal arcs (on which consult the index of tables at the end of this volume), or from the 2 books of Theodosius of Tripoli *On Days and Nights*.

[Margin: Regula diuersitatis Dierum. — Rule of the diversity of days.]

I. Rule. *In the right sphere*, any artificial days of the same year are in themselves physically equal, both to one another and to their nights, even if mathematically some slight inequality lies hidden in them, arising from the unequal motion of the Sun or from refractions and parallaxes.

II. Rule. *In the parallel sphere*, as the Sun enters Aries, one day begins for northern peoples and one night for southern, which will last 186½ natural days; and the night ends for northern peoples and the day for southern, which lasted 178½ natural days. As the Sun enters Libra, on the other hand, the day begins for southern peoples, and for northern peoples *non vna* [recte: nox vna], one night, which will last 178½ natural days; and the night ends for southern peoples and the day for northern, which lasted 186½ natural days, in these centuries at any rate. So not strictly, but in a rough and broad way, an equinox then occurs for them. On their twilights see the Scholium of Chapter 31.

III. Rule. *In the oblique sphere*, as the Sun enters Aries and Libra, the days on any horizon are physically equal to the nights, that is, of 12^{h}, and consequently equinoxes occur everywhere.

IV. Rule. *In the oblique sphere*, as the Sun enters Cancer, northern peoples have the longest day of the year and the shortest night, and southern peoples the longest night and the shortest day; and the longest becomes so much the longer, and the shortest so much the shorter, as the former peoples are more northerly and the latter more southerly. As the Sun enters Capricorn, on the other hand, southern peoples have the longest day, which is longer for those farther south, and the shortest night, which is shorter for those farther south; while northern peoples have the longest night and the shortest day, the former longer and the latter shorter for those farther north.

V. Rule. *In the oblique sphere*, the day begins to grow and the night to shrink for northern peoples from the Sun's entry into Capricorn, and for southern peoples the day to shrink and the night to grow, and so continuously until the Sun's entry into Cancer. From then the day begins to grow and the night to shrink for southern peoples, and for northern peoples the day to shrink and the night to grow, until the Sun's entry into Capricorn.

VI. Rule. *In the oblique sphere*, as long as the Sun is in the northern signs, the days of northern inhabitants are longer than their nights, and those of southern inhabitants shorter. Conversely, when the Sun is in the southern signs, the days of southern inhabitants are longer than their nights, and those of northern inhabitants shorter.

VII. Rule. *In the oblique sphere*, as long as the Sun is in the northern signs, the day, for northern and southern peoples alike, is the greater the farther they are from the south pole; but when the Sun is in the southern signs, the day, for both, is the greater the farther they are from the north pole.

VIII. Rule. *In the oblique sphere*, any two days of the same horizon, and any two nights, are equal to one another on which the Sun is at degrees of the ecliptic equally distant from the same solstitial point. For example, the Sun at the beginning of Gemini and of Leo makes two equal days, say May 21 and July 23; at Rome it makes them of 15^{h} 48^{m}, because in both places of the ecliptic it is equally distant from the beginning of Cancer, namely 30°, or from the beginning of Capricorn, namely 150°. In the same way, on any other horizon those two days are equal to one another, even though they are not equal to the other days of the year on that same horizon, or to the same days on different horizons. I said *from the same solstitial point* because, although the Sun at the beginning of Leo and at the beginning of Aquarius, for example, is equally distant from the nearer solstice, it is not equally distant from the same solstice: at the beginning of Leo it is 30° from the beginning of Cancer, and at the beginning of Aquarius it is 150° from the beginning of Cancer. And so those two days are not equal to one another, just as the days on which it is at the beginning of Gemini and of Sagittarius are not.

IX. Rule. *In the oblique sphere*, any two days are equal to the future or past nights on the same horizon, as nights are to days, on which the Sun will be, or was, in the other semicircle of the ecliptic, at a degree of it equally distant from the equator. For example, at Rome the day on which the Sun is at the

beginning of Leo is equal to the night that occurred at Rome when the Sun was at the beginning of Aquarius; and the night at the beginning of Leo is equal to the day that occurred there at the beginning of Aquarius. Say the same of the beginning of Gemini compared with the beginning of Sagittarius.

X. Rule. *In the oblique sphere*, any day of northern peoples is equal to the night, in the same natural day, of equally southern peoples, and the night of the former to the day of the latter; just as the day of southern peoples is equal to the night, and their night to the day, of equally northern peoples: that is, of those whose distance from the equator, each toward his own pole, is equal. For example, if for one person the altitude of the south pole and for another the altitude of the north pole is 48° in each case, then if one has a day of 15^h and a night of 9^h, the other will have a night of 15^h and a day of 9^h.

XI. Rule. *In the oblique sphere*, any two days belonging to different natural days among northern peoples are equal to days of equally southern peoples on which the Sun will be, or was, in the other semicircle of the ecliptic, and at a degree of it equally distant from the equator. For example, if for one person the altitude of the north pole and for another that of the south pole is 48°, the day of the one, made by the Sun placed at the beginning of Leo, is equal to the day made, or to be made, for the other at the beginning of Aquarius. Say the same of two of their days, made or to be made, for one at the beginning of Gemini and for the other at the beginning of Sagittarius; and so of similar cases.

XII. Rule. *In the oblique sphere*, outside the cases stated, no artificial days of the same year are equal to one another, and no nights to one another, whether on the same horizon or on different ones.

SCHOLIA.

[Margin: Solis mora in semicirculo Boreali. — The Sun's stay in the northern semicircle.]

I. What I said in Rule 2 was said for the sake of a round number. For if you keep the obliquity of the ecliptic at 23° 30′, and the Sun's apogee and eccentricity as Clavius has them, the Sun spends in the northern signs 187 natural days 6^h 39^m, and in the southern signs 177 days 23^h 21^m. But if you follow the Tychonic obliquity and the rest, it spends in the northern signs 186 days 18^h 24^m 23^s, and in the southern signs 178 days 11^h 24^m 23^s. So the difference between these comes from the different eccentricity of the Sun and obliquity of the ecliptic.

II. In the same Rule 2 I added *in these centuries*, because in them the Sun's apogee is in the northern semicircle of the ecliptic, and the Sun therefore stays more days in that semicircle than in the southern. But if the apogee once was, or ever will be, in the southern semicircle, then the Sun will likewise stay longer in the southern semicircle than in the northern. So if in the year of Christ 1600 the Sun's mean apogee was at Cancer 6° 40′, and the Sun's apogee moves in the order of the signs by very nearly 1′ each year, it follows that it was at the

beginning of Aries in the year 4140 before Christ and will be at the beginning of Libra in the year of Christ 6660; and accordingly that before that year 4140 and after the year 6660 the Sun has stayed, or will stay, longer among southern peoples than among northern. Kepler, however, on p. 42 of the Rudolphine Tables, puts it at the beginning of Aries in the year 3993 before Christ, July 24, on account of other astronomical agreements, because of which he also limited the motion of the solar apogee so that it would fall in that year. On this we speak in the Chronological Book, where the age of the world is treated, and below in Book 3, Chapter 24.

III. The artificial day, no differently from the natural day, can accidentally seem longer or shorter because of a journey toward the west or the east; on this see below, Chapter 28, Scholium 4.

CAPVT XXVII.

De Inuenienda quantitate Semidiurna, & Seminocturna; cognitis Altitudine Poli, & gradu Eclipticæ, in quo Sol versatur.

On finding the semidiurnal and seminocturnal quantity, given the altitude of the pole and the degree of the ecliptic in which the Sun is.

I. The altitude of the pole is known either from the table which we mention in the index of tables at the end of this volume, or from globes or the more correct geographical maps, or from one's own observation or that of others, according to the method given in Chapter 11.

II. The Sun's place at noon on any given day is known either from ephemerides, or from astronomical calculation, or from observing the Sun's declination from the equator and then using the tables of declination. More roughly, it is known from the days of the Sun's entry into the signs of the zodiac, by adding as many degrees of that sign as the assigned day is days distant from the day of entry into that sign. For the entry into the signs of the zodiac the following little table is enough for beginners, using the days of the Gregorian calendar with respect to the meridian of Rome. A leap year of Christ, moreover, is every fourth year of Christ, except the years 1700, 1800 and 1900, in accordance with what will be said in the Chronicle Book. So, when the years of Christ are divided by 4: if nothing is left, the year is a leap year; if 1, 2 or 3 is left in the numerator of the fraction, the year will be the first, second or third after the leap year. A leap year, moreover, is the same as the fourth after a leap year. With this laid down, for these centuries let there be:

Days of the Sun's entry into the signs of the zodiac, in the years after the nearest leap year.

Signs	Months	Year 1	2	3	4
♈ Aries	March	20	20	21	20
♉ Taurus	April	20	20	20	19
♊ Gemini	May	21	21	21	20

Signs	Months	Year 1	2	3	4
♋ Cancer	June	21	21	22	21
♌ Leo	July	23	23	23	22
♍ Virgo	August	23	23	23	23
♎ Libra	September	23	23	23	23
♏ Scorpio	October	23	23	24	23
♐ Sagittarius	November	22	22	22	22
♑ Capricorn	December	21	21	22	21
♒ Aquarius	January	19	20	20	20
♓ Pisces	February	18	18	18	19

III. The first way of finding the semidiurnal quantity of the artificial day is by means of an armillary sphere, or a geographical or star globe, on which the meridian, the equator and the ecliptic are divided into single degrees. For, first, set the pole raised above the horizon according to the altitude of the pole of the place in question, and set on the horizon the degree of the ecliptic in which the Sun is on that day; and with some visible mark, or at least from memory, note the point of the equator, or at any rate the degree, which is then on the horizon. After this turn the sphere or globe toward the west until the Sun, or the degree of the ecliptic in which the Sun is, comes under the meridian, and count how many degrees of the equator have meanwhile ascended above the horizon. Convert these into hours and minutes in proportion, using the little table of Chapter 12, no. 2, and you will have the semidiurnal time of that day for the given altitude of the pole. If you subtract this from 12^h, you will have the seminocturnal time. For example, at Rome, where the altitude of the pole is 42°, let the year be 1650, the Jubilee year, namely the second after a leap year, and the day the 15th of August, on which, by the preceding little table, the Sun will be at Leo 23°. If you raise the pole that is on the meridian of the sphere 42° above the horizon, and set the degree Leo 23° on the horizon, you will see that the degree of the equator on the horizon is 132½°, counted from the beginning of Aries. When the globe or sphere has been turned until the degree Leo 23° is under the meridian, you will note that the degree of the equator on the horizon is 235½°. So 103° have ascended, to which correspond $6^h\ 52^m$. Therefore on that day at Rome the semidiurnal arc is 103°, or $6^h\ 52^m$, and the seminocturnal time is $5^h\ 8^m$.

Where the pole is raised 66½°, the longest day is of 24^h. But where the pole is raised more than 66½°, the arc of the ecliptic that never sets must be carefully noted: for as many degrees as it has, nearly so many natural days will the continuous day have.

IV. The second way is by the tables of oblique ascensions constructed for a given altitude of the pole. From them one gathers the oblique ascension of the degree of the ecliptic in which the Sun is on the given day, and the oblique ascension of the point diametrically opposite to it, or 180° away. The former is subtracted from the latter, and the diurnal arc remains; half of it is

the semidiurnal arc, to be converted into time as above. Let the example be the same as above. At Rome the oblique ascension of Leo 23°, in which the Sun is on July 15 [recte: August], is 132° 30′, whereas the oblique ascension of Aquarius 23°, which is opposite it, is 338° 12′. When the ascension of the degree in which the Sun is has been taken from this, 205° 42′ remain, half of which is 102° 51′, or in round figures 103°, namely about 6^{h} 52^{m}. If the subtraction could not be made, then, so that it can be made, add 360° to the smaller number.

V. The third way is by the ascensional difference defined and explained in Chapter 21, no. 12. For, as Copernicus shows in Book 2, Chapter 7, the ascensional difference is the same as the difference by which the semidiurnal arc exceeds a quadrant of the equator or falls short of it. Therefore for northern peoples when the Sun is in the northern signs, or for southern peoples when it is in the southern signs, the ascensional difference is to be added to 90°, and the semidiurnal arc will result. But when the Sun is in the southern signs it is to be subtracted among northern peoples, or, when it is in the northern signs, subtracted by southern peoples, and the semidiurnal arc will be left.

When the altitude of the pole is greater than 66½°, subtract the altitude of the pole from 90°, and there will remain the declination of the beginning of the arc of what is always visible. Look up this declination in the body of the table of declinations [arealiter], and in the left or right margin you will find the degree of the sign that is the beginning of the arc of what is always visible. From it count the degrees as far as the nearest solstitial point, and double them. Then from the tables of the Sun's motion look up the days and hours in which the Sun completes the degrees of that doubled arc, or at any rate take as many days as there are degrees, and you will have how many natural days the continuous day contains on that horizon. But this second way is not so exact.

VI. The fourth way is by trigonometry, on which see the problems of the primum mobile.

CAPVT XXVIII.

De diuersis Dierum Naturalium Initijs, vnde Horarum quasi species quatuor; & de mutua illarum Conuersione.

On the different beginnings of natural days, from which come, as it were, four kinds of hours; and on their conversion into one another.

I. The same day, begun by different peoples from four chief points of the natural day, as from the hinges of the hours, has produced as many kinds, as it were, or denominations of hours: namely Babylonian hours, counted from sunrise; Italian hours, from sunset; astronomical hours, from noon; and finally European hours, which comprise 12^{h} from midnight and as many others from noon.

[Margin: Initiū Diei variū apud diuersos. — The beginning of the day varies among different peoples.]

II. The Babylonians, Persians, Syrians and Damascenes, with most Eastern peoples, used to begin the day from sunrise; as also, today, do the people of the Balearic Islands and of Nuremberg.

III. The Hebrews began the day from sunset, thinking that night, or evening, preceded light on their horizon, from Genesis 1. So did the Athenians, and so today do the Austrians, Bohemians, Marcomanni, Poles, Silesians, Chinese and Cathayans, but above all, today, the Italians: hence the name *Italian hours*.

IV. The Umbrians, the Arabs and the greatest part of the astronomers started the day from noon: hence *astronomical hours*. So Ptolemy, Peuerbach, the Alphonsines, Regiomontanus, Tycho, Longomontanus, Kepler, Lansbergen, Boulliau, Magini, Biancani and very many others; and rightly, for in this way one avoids the inequality that spills over into the day from the obliquity of ascensions and descensions and from the horizontal vapours at rising and setting; and the moment of noon is in itself conspicuous and of the most evident mark.

V. The Egyptians of old, however, and the Romans and the Mysians began the day from midnight; and among astronomers, Hipparchus, Copernicus, Reinhold, Pétau and Mulerius [Nicolaus Mulerius]. The Roman Church too retains this in its obligations of fasts and of the canonical hours, and in their limits.

VI. But the Spanish, the Portuguese, the French, the Belgians, the English, and a good part of Germany and indeed of Europe count 12^h from noon and as many from midnight: hence *European hours*.

This is gathered from Pliny the Elder, Book 2, Chapter 77; Censorinus, Chapter 10; Macrobius, *Saturnalia*, Book 1, Chapter 3; Isidore, *Origines*, Book 5, Chapter 28; Io. Rosinus [Johannes Rosinus], *Antiquitates Romanae*, Book 4, Chapter 3; Giraldi, *De annis* etc., p. 133; Clavius on the *Sphere*, p. 279; and Origanus, volume 1, p. 95.

Conuersio Horarum in Horas.

The conversion of hours into hours.

VII. Once the semidiurnal time is known from its own table, or from elsewhere, according to what was said in Chapter 27, if you subtract it from 12^h the seminocturnal time will remain. Each of them, doubled, will give the whole: the former the whole day, the latter the whole night, or night-time. In the following rules, moreover, when a subtraction cannot be made, the full sum of 24^h is added to the smaller number, and so the subtraction is made. And when, finally, the sum produced by the prescribed addition exceeds 24, then, 24 being discarded, the remaining hours are kept. With this said in advance:

Babylonicæ Horæ Conuertuntur.

Babylonian hours are converted.

Into Italian hours from the preceding sunset, thus. Add the nocturnal quantity to the Babylonian hours. If, however, the sum exceeds 24, what remains after 24 is discarded will be Italian hours from the following sunset.

Into astronomical hours, from the noon preceding the sunrise, thus. Add to the Babylonian hours the nocturnal and the semidiurnal quantity together. But if the sum exceeds 24, what remains after 24 is discarded is counted from the noon following the sunrise. If, however, the Babylonian hours exceed the semidiurnal quantity, subtract it from the Babylonian hours, and astronomical hours from the following noon will remain.

Into European hours from the preceding midnight, thus. Add the seminocturnal time to the Babylonian hours. But if the sum exceeds 12, they will be European hours from the following noon; and if it exceeds 24, they will be hours from the following midnight, discarding in the former case 12, in the latter 24.

Italicæ Horæ Conuertuntur.

Italian hours are converted.

Into astronomical hours, thus. Add the semidiurnal quantity to the Italian hours, and if the sum does not exceed 24, keep it; if it exceeds, discard 24, and the remainder will be hours from the preceding noon.

Into European hours, thus. Add the semidiurnal quantity to the Italian hours, and if the sum does not exceed 12, they will be European hours after the preceding noon; if it exceeds 12, then with 12 discarded the remainder will be European hours after the preceding midnight.

Into Babylonian hours, thus. Add the diurnal quanti- ty; and if the sum does not exceed 24, they will be hours from the preceding sunrise. If it does exceed it, they will be hours from a sunrise that does indeed precede, but with respect to the sunset from which the Italian hours were counted, it will be the following one.

Astronomicæ Horæ Conuertuntur.

Astronomical hours are converted.

Into European hours, thus. If they do not exceed 12, they coincide with European hours counted from noon. If they exceed 12, then with 12 discarded there remain European hours from the midnight that followed the noon from which the astronomical hours were counted.

Into Babylonian hours, thus. Add the semidiurnal time to the astronomical hours; and if the sum does not exceed 24, they will be Babylonian hours from the sunrise preceding the noon from which you counted the astronomical hours. If they exceed it, then with 24 discarded there will remain Babylonian hours, but from the sunrise following that noon.

Into Italian hours, thus. Subtract the semidiurnal time from the astronomical hours, if it is less than they are, and there will remain Italian hours from the sunset following the noon from which the astronomical hours were counted. But if the semidiurnal time is equal to or greater than the astronomical hours, add 24 to them, and when the semidiurnal time has been subtracted from the

sum, there will remain Italian hours from the sunset preceding the noon from which the astronomical hours were reckoned.

Europæ à Media Nocte Vertuntur.

European hours from midnight are converted.

Into astronomical hours, thus. Add 12^h to the European hours.

Into Babylonian hours, thus. Subtract the seminocturnal time from the European hours, if you can, and you will have Babylonian hours from the sunrise following the midnight from which you were counting the European hours. If you cannot, add 24^h to the European hours and from the sum subtract the seminocturnal time; for there will remain Babylonian hours, but from the sunrise preceding that midnight.

Into Italian hours, thus. Add the seminocturnal quantity to the European hours.

SCHOLIA.

I. Where it is the custom to count hours not from the true sunset but from the civil or artificial sunset, namely half an hour after the true sunset (and this because of the evening twilight, or evening dawn, into which the day is prolonged for the benefit of labourers and artisans), account must be taken of that half-hour. Subtracted from solar Italian hours it will give civil Italian hours, and added to the civil it will give the solar.

[Margin: Europæarū Horarum laus. — In praise of European hours.]

II. The reason for dividing the natural day in two, into 12^h from noon and 12^h from midnight, was the greater convenience of counting them on striking clocks [horisonis horologijs]. Because of this, in the end, those clocks strike only six hours at a time, with as many strokes, since the common people themselves otherwise easily distinguish whether it is six simply or six doubled: for if they hear six strokes at noon, they nevertheless declare it hour 12 from midnight. Moreover, European hours are better suited to civil and ecclesiastical use than Italian hours (forgive me, Italians) or Babylonian: both because they are counted from more fixed and certain points of the day, and because the hour of noon and of midnight is always counted by the same number of hours, being always the twelfth. Consequently business and offices tied to certain hours from midnight or from noon take place throughout the year always at the same hour, that is, at the same number of hours. For example, if a midday meal or a sermon is customarily held at hour 2 before noon, or hour 10 after midnight, there will be no need to change the hour and repeatedly warn the people of the change; it will be settled that these things are to be done always at hour 10. And on one person, the keeper of the public clock, falls the task of regulating the striking clock so that, by inspection of the meridian line, hour 12 is struck at the moment of noon. Those who use Italian or Babylonian hours certainly lack this convenience. For, to make this clear by a single example, at

Rome that hour 2 before noon is hour 16 Italian on equinoctial days, but hour 14½ Italian on the day of the summer solstice, and $17^{h}\ 30^{m}$ on the day of the winter solstice, in Italian reckoning and taking solar hours into account; for in civil reckoning, according to what was said in Scholium 1, the former will be 14 and the latter 17. There are some things, however, which even among those who use European hours cannot take place at the same hour throughout the year: for example, the recitation of Matins of the Roman Breviary, which by privilege may be done as soon as the evening time has begun, that is, as soon as half of the semidiurnal time, counted from noon, has passed; for then the Sun is nearer to setting than to noon, and therefore the evening time is already approaching. So although this office can be begun on the equinoctial day immediately after hour 3 from noon, at Rome it cannot be begun on the day of the summer solstice until $3^{h}\ 43^{m}$ from noon have elapsed, whereas on the day of the winter solstice it can be begun when $2^{h}\ 15^{m}$ from noon have elapsed.

III. Those who are western can be so many hours more western than we are that they count one day fewer than we do; and conversely easterners can be so far east that they count one day more than we do in the same calendar. For suppose a place more western than Rome by 195° of the equator, or 13^{h}, according to what was said in Chapters 12 and 13; such, nearly, is Isabella, one of the Solomon Islands, or Quivira, the city of the kingdom of Quivira. And suppose Easter is celebrated at Rome on March 22, so that the Saturday of Holy Week ends at sunset of the 21st, and the following morning begins at hour 12 Italian of March 22. Then in Isabella or Quivira it will be hour 23 Italian, but of March 21, and so still the Saturday and a time of fasting. And so of similar cases.

IV. Besides, some people can be so much more western or more eastern than others that while the one counts European hours from noon the other is already counting European hours from midnight, and indeed with a disagreement of one day in the number of days of the year. As in the same example of Scholium 3: for the morning hour of Easter, or hour 12 Italian, will for the Romans be hour 6 after midnight of March 22, but for the people of Quivira hour 5 after noon of March 21.

[Margin: Numerus dierum minor eūti ad Ortum. — The number of days is smaller for one going east.][7]

V. Those travelling westward, and measuring time with the usual clocks of 24^{h}, can in the end fall one day short in the number and naming of the days of the same calendar; and conversely those going eastward can count one day more than they otherwise should. The first actually happened to the companions of Ferdinandus Magellanicus [Fernão de Magalhães (Magellan)], who, setting sail from Seville on 1519 August 10, surveyed the whole world by a westward voyage and returned to Seville on 1522 September 7, as the people

[7]as printed; the text beside it gives the smaller count to those travelling west.

of Seville counted it, whereas those shipmasters believed it was September 6, as Pigafetta [Antonio Pigafetta] relates in his history of that voyage, and Ramusius [Giovanni Battista Ramusio] from him.

The cause of this error is that those sailing westward follow the Sun as though it were fleeing, and therefore their day is prolonged every day, so that in the end, when those additions to the day are collected, one whole day is found distributed and, as it were, dispersed among all the other preceding days. For suppose a voyage or journey begun from the island of São Tomé [Insula S. Thomæ], which is on the equator, on January 1 at noon itself, and sailing on the equator westward for 24^{h}, covering 15° (which I say for the sake of clarity, not from practice). So when 24^{h} have elapsed on the striking clock, on the solar clock there will still remain one hour before noon, because the meridian you have reached is one hour farther west than the island of São Tomé. An unskilled shipmaster will therefore think that the striking clock needs correcting, and will think it has run fast by one hour; whereas he ought to say in the log of the voyage: *On the second day we arrived at such-and-such a place, at hour 24 after the noon that occurred on the island of São Tomé, but at that place only at hour 23.* Proceeding in this way, the whole equator would be completed and one would return to the island of São Tomé at the end of day 24, with respect to the meridian on which the island of São Tomé lies; and yet the shipmasters, having followed the Sun, would count the day as only the 23rd. The contrary would happen, for the contrary reason, to those sailing in this way to the east, for in the end they would count one day more, namely the 25th.

Our own Molina [Luis de Molina] treated this problem at the end of the work on the six days, and Biancani in his *Sphaera* at the end of Book 7, proposing the case of two voyagers, one going west and the other east, who meet and disagree by two whole days in the number of days. But many other problems arise from this: whether voyagers of this kind would be obliged to a fast, or a feast day, or the intercalation of the leap day, etc.; or whether they could follow the number and order of days according to the reckoning they made by following solar clocks. It is not fitting for us to digress to these matters here, if the brevity of the introduction [Isagoges] we have undertaken and the end we have set ourselves are to be kept.

CAPVT XXIX.

De Horis Inæqualibus, seu Temporalibus, & earum Commutatione in Horas Aequales.

On unequal or temporal hours, and their conversion into equal hours.

I. Among astronomers, some hours are *equal*: in Greek [Greek: ἰσημεριναὶ (isēmerinai, "equinoctial")], and in Latin you might say *aequidiales* or, as others do, *aequinoctiales*. They are so called because they are the twelfth part of the natural day, and so each is equal to any other; just as on an equinoctial day any night hour is in itself equal to any day hour. It is these we have been speaking of so far, in Chapters 27 and 28. Others are *unequal*, in Greek [Greek:

καιρικαὶ (kairikai, "seasonal")], that is, changeable with the time; and for that reason they are called in Latin *temporales*, or, as Sacrobosco and Giuntini prefer, *naturales*, although they are the twelfth part of the artificial day or night, whose inequality gives rise to the unequal hours; and this by a threefold inequality. The first arises from the difference of oblique horizons, because of which the solstitial day, for example, is shorter at Rome than at Kraków, and so the twelfth part of the Kraków day is greater than the twelfth part of the same day at Rome. The second comes from the difference of the parallels the Sun runs through, with respect to the oblique sphere, because of which the twelfth part of the day of the summer solstice at Rome is longer than the twelfth part of the winter-solstice day, likewise at Rome. The third, finally, arises from comparing a non-equinoctial day with the night of the same natural day and the same horizon, because of which, among northern peoples, the twelfth part of the day is longer than the twelfth part of the night when the Sun is in the northern signs, and shorter when it is in the southern signs. The opposite of this happens among those who live in the south. But the alternation of these inequalities can be judged well enough from what was said about the inequality of artificial days in Chapter 26. So although the hours of the artificial day are equal to the daytime hours of the same natural day and on the same horizon, outside the equinoxes they are nevertheless unequal: to the night hours of the same natural day and the same horizon; to the daytime hours of different natural days, on which the Sun runs through unequal parallels; and to the daytime hours of the same natural day but on different horizons.

[Margin: Conuersio Horarū inæqualium in æquales. — Conversion of unequal hours into equal hours.]

II. Further, daytime temporal hours are counted from sunrise, and night hours from sunset. Daytime hours are converted into equal Babylonian hours by multiplying the hours and minutes of the daytime by the temporal hours and minutes, and dividing the sum by 12. Night temporal hours are converted into equal Italian hours by multiplying the length of the night by the temporal hours, and dividing the resulting sum by 12. *For example*, with the Sun placed at 12° of Cancer, the daytime is 15^{h}. Let temporal hour 10 be given. The 15 hours multiplied by it make 150, which divided by 12 give 12½. So at that moment at Rome temporal hour 10 is hour 12½ from sunrise, but an equal hour. If you add to it the night's length of 9^{h}, the Italian hour will be 21½; if instead you subtract the semidiurnal $7½^{h}$, the astronomical hour will be 5.

Another example. On the same day at Rome let the temporal night hour be 6. Since the night at Rome is then 9^{h}, multiply them by 6, which makes 54; and these divided by 12 give 4½ equal Italian hours. If you add to them the semidiurnal time of $7½^{h}$, the astronomical hour will be 12.

SCHOLIA.

I. *Ancient authors, speaking of unequal hours, call the long daytime hours summer or solstitial hours, and the short ones winter or hibernal hours. To the same hours belongs that saying of* CHRIST, *John 2:* "Are there not twelve hours of the day?" *and the celebrated epigram of Martial, Book 4, Chapter 7, which begins:* "The first and second hour hold those who come to pay their morning calls" [Prima salutantes, atq; altera continet hora.] *Finally, the astrologers use the same hours in assigning the hours over which the planets rule, and they therefore call them* planetary*; of these in the next chapter.*

[Margin: Quatuor Horæ diei partes. — The four Hours, parts of the day.]

II. *The Romans, from about the 300th year after the founding of the City, and the Hebrews and most Orientals, used to divide the artificial day into four equal parts. The* First [Prima] *began at sunrise and lasted to the moment at which three temporal hours were completed; then the signal was given that the First had elapsed and the Third was beginning. The second part was called the* Third [Tertia]*, because it began at the end of three temporal hours, and it lasted to the end of six temporal hours; when these were completed the signal was given that the Third was finished and the Sixth was beginning, which was at midday. The third part, then, was called the* Sixth [Sexta]*, because it began at the end of 6 temporal hours and lasted through three unequal hours, that is, to the end of 9 temporal hours. When these were finished the signal of the Ninth was given, and from there began the fourth part of the day, called from its beginning the* Ninth [Nona] *and from its end* Vespers [Vespera].

[Margin: Et quatuor Vigiliæ. — And the four watches.]

In the same way the night was divided into four equal parts, or military watches, to each of which fell three temporal hours. The First Watch began at sunset and ended when three temporal night hours were completed; from there began the Second Watch, and it lasted until midnight, etc. But for more on these, see St Jerome on Psalm 89; Baronio, Year 34, no. 163; Mazzonus [Jacopo Mazzoni] on Dante, Book 1, Chapter 16; and Fuluius Vrsinus [Fulvio Orsini] in the Appendix to Ciacconus [Pedro Chacón].

CAPVT XXX.

De Horis Planetarijs, & Planetarum Characteribus.

On planetary hours and the characters of the planets.

I. The characters of the planets are these: of the Sun ☉; of the Moon ☾; of Mercury ☿, which is the sign of the caduceus, that is, of a staff entwined by two snakes; of Venus ♀, which is the sign of a mirror with a handle made in the form of a cross; of Mars ♂, which shows a shield-boss with an arrow; of Jupiter ♃, which stands for [Greek: Ζεῦς (Zeus)], that is, Jupiter; and of Saturn ♄, which is the sign of the sickle with which Saturn, that is, Time, mows down all things.

II. The hours over which the individual planets rule are the unequal hours discussed in Chapter 29; or, according to the subtler astrologers, they are those

12 parts in which 15° of the zodiac rise at a time. These are certainly unequal hours, since the halves of six signs rise in unequal times.

[Margin: Horæ Planetariæ, & Hebdomadicorū dierum ordo. — The order of the planetary hours and of the days of the week.]

III. By astrological observation, the order of dominion over the hours and the days of the week begins from the Sun, which is the strongest and the swiftest in acting, and to it is assigned the first hour of the first weekday [prima feria]. From the Sun the descent runs through the planets below the Sun; when these are done one climbs to Saturn, and from him returns through Jupiter and Mars to the Sun. With the 12 unequal hours of the day distributed in this way, Saturn will rule the 12th hour of the first weekday, Jupiter the first hour of the night, and Mercury the 12th hour of the night. The Moon will therefore rule the first hour of the second weekday, and that is why the second weekday is called the day of the Moon, as the first is called the day of the Sun. If you continue in the same order, Mars will rule the first hour of the third weekday, Mercury the first of the fourth, Jupiter the first of the fifth, Venus the first of the sixth, and Saturn the first of the seventh. This is the first cause of the order the pagans kept in assigning the names of the planets to the days of the week. But this order supposes, following the ancient system, that the planets stand so that Saturn is in the highest place, then Jupiter, then Mars, with the Sun below him, Venus below the Sun, Mercury below Venus, and the Moon last of all; whereas it has been discovered that Mars sometimes descends below the Sun, that Mercury and Venus are sometimes carried above the Sun, and finally that Mercury is nearer the Sun than Venus is, as will become clear from what is to be shown in Book 3, Chapter 6, and in Book 9, where the system of the world is treated more fully. Thus there wavers the foundation of this distribution. For it, nevertheless, take the two following little tables.

Daytime hours [Horæ Diurnæ.]	Weekday [Feriæ.] 1	2	3	4	5	6	7
1	☉	☾	♂	☿	♃	♀	♄
2	♀	♄	☉	☾	♂	☿	♃
3	☿	♃	♀	♄	☉	☾	♂
4	☾	♂	☿	♃	♀	♄	☉
5	♄	☉	☾	♂	☿	♃	♀
6	♃	♀	♄	☉	☾	♂	☿
7	♂	☿	♃	♀	♄	☉	☾
8	☉	☾	♂	☿	♃	♀	♄
9	♀	♄	☉	☾	♂	☿	♃
10	☿	♃	♀	♄	☉	☾	♂
11	☾	♂	☿	♃	♀	♄	☉
12	♄	☉	☾	♂	☿	♃	♀

Night hours [Horæ Nocturnæ.]	Weekday [Feriæ.] 1	2	3	4	5	6	7
1	♃	♀	♄	☉	☾	♂	☿
2	♂	☿	♃	♀	♄	☉	☾
3	☉	☾	♂	☿	♃	♀	♄
4	♀	♄	☉	☾	♂	☿	♃
5	☿	♃	♀	♄	☉	☾	♂
6	☾	♂	☿	♃	♀	♄	☉
7	♄	☉	☾	♂	☿	♃	♀
8	♃	♀	♄	☉	☾	♂	☿
9	♂	☿	♃	♀	♄	☉	☾
10	☉	☾	♂	☿	♃	♀	♄
11	♀	♄	☉	☾	♂	☿	♃
12	☿	♃	♀	♄	☉	☾	♂

IV. Another cause of the weekly distribution just described is the aspect nearest to opposition, following seven isosceles triangles whose bases are the sides of a heptagon inscribed in a circle, the circle being divided into seven equal parts and these distributed among the individual planets, beginning from the Moon and proceeding to the right. For this purpose let there be the following circle, with a heptagon inscribed in it; on each of the heptagon's sides let seven isosceles triangles be erected, inscribed in the same circle; and finally at the vertices of the triangles let the characters of the seven planets be placed, beginning from the Moon and proceeding to the right to Mercury, then to Venus, the Sun, etc.

Diagram. A circle with seven points on its circumference, joined by straight chords that form a seven-pointed star; chords joining neighbouring points are also drawn, giving the inscribed heptagon. A planetary sign stands outside the circle at each point. A repaired tear runs down just outside the circle on the right, clear of the signs; the ring of the sign for Venus is printed incomplete.

- Top: ☾ (crescent)
- Upper right: ☿
- Right: ♀ (the ring of the sign is broken by the tear)
- Lower right: ☉ (a ring with a central dot)
- Lower left: ♂
- Left: ♃
- Upper left: ♄

For you will see that the Sun ☉ has opposite it on the right the Moon ☾, and on the left Saturn ♄; and so the first day after the day of the Sun is dedicated to the Moon, while the day of Saturn came before it. When, therefore, you have gone along a straight line from the Sun to the Moon, you will see that ♂ stands opposite the Moon on the right, so the day of Mars follows the day of the Moon. And when you are at the apex of Mars, you will see that Mercury ☿ stands opposite it on the right, so the day of Mercury follows the day of Mars; and likewise for the rest.

But all this, even if it does not lack some plausibility, nevertheless appears to contain much made up at will, and who does not know that it could have been invented otherwise? The harmonic reason handed down by Dio [Cassius Dio] is discussed learnedly by Bettini in *Apiarium* 10, *Progymnasma* 1, Proposition 4. But the diatessaron, the diapente and the 7 intervals of this kind depend on the distances of the stars; and in fixing these GOD had in view a harmony not of the ears but of the intellect, and proportions directed to an end other than sound or the sweetness of sound. It is in vain, then, that we look in the heavens for harmonies other than these.

CAPVT XXXI.

De Crepusculis.

On twilight.

[Margin: Crepusculi Definitio. — Definition of twilight.]

I. Twilight [crepusculum] is an appendix of the artificial day: it is the time between the apparent rising or setting of the Sun and the beginning or end of the solar light reflected to the Earth's surface from the exhalations of the atmosphere. It is called *crepusculum* either from the crackling [crepitus] that the vapours shut in the pores of wood give out, rarefied by the heat of the approaching Sun; or rather from *crepundia*, that is, the first beginnings of a doubtful and uncertain light. This agrees with Censorinus, *De die natali*, Chapter 10, who says: "Uncertain things are called *creperae*, and it is uncertain whether that time belongs to night or to day." If the twilight is morning twilight, it is called *Aurora*, from the golden light [aurea lux] or from the increase of dew [roris], and in Italian *Alba*, from the vapours whitening around the horizon. Four things are usually inquired into at this point: the beginning of morning twilight or the end of evening twilight in a given region and at a given time of year, on which its duration depends; then the Sun's depth below the horizon when morning twilight begins or evening twilight ends; and finally the altitude of the exhalations from which the Sun's direct or refracted light is reflected to the Earth.

[Margin: Modus obseruãdi Crepusculum. — The method of observing twilight.]

II. As to the beginning and end of twilight, each is observed by taking, with the help of a quadrant or a geometrical square, the altitude of some fixed star at the moment when, on a clear night, the illumination of the vaporous exhalations first appears around the eastern horizon, or ceases to appear around the western. For by the problems of the primum mobile, from the star's declination and right ascension together with the altitude of the pole and the Sun's place, one gathers what hour it is, and from that the interval of time before the Sun's rising or since its setting, and thus the duration of twilight. Granted, that moment is doubtful, and that illumination does not become perceptible at once; and so, from the varying temperature both of the year and of the region, different

observers gather a different time and duration, and on that the Sun's varying depth and the altitude of the exhalations depend in part. There is, however, another cause, not so much physical as astronomical, that varies the duration of twilight: namely, the straightness or obliquity of the Sun's ascension or descension. In the right sphere and on equinoctial days, because the Sun rises and sets straight, twilight is at its shortest, other things being equal. If the arc of depth at which twilight begins is, as most writers set it, 18°, those degrees rise in one equinoctial hour and 12^{m}, and twilight there lasts just that long. But in the oblique sphere twilight is longer, and the longer the more oblique the horizon, that is, the greater the altitude of the pole. At the same altitude of the pole, it is generally longer the farther the Sun declines from the equator toward the tropics, and longer still if the declination is toward the visible pole of the world. So the longest twilight occurs at the summer solstice, yet not the longest at the winter solstice. In the parallel sphere, as long as it is night, twilight, if not continuous, nevertheless lasts as morning twilight for more than a month and a half, and as evening twilight for more than another month and a half, while the Sun meanwhile stays below a subhorizontal depth of 18° or 20°. From the differing obliquity of the horizon, then, and the variety of the parallels in which the Sun moves, there arises the astronomical diversity of twilight. And so it happens that evening twilight sometimes and in some places runs on into morning twilight, and that the whole night is not sheer night but twilight, as happens at the summer solstice and on some of the nights nearest it, for those at whose place the pole is raised above 48½°. And although in the common Problems we are going to teach the method of investigating the duration of twilight geometrically, you will nevertheless be able to find it with the help of an armillary sphere, if, at the given altitude of the pole and with the Sun's place in the zodiac given, you set that point of the zodiac 18° below the horizon, or at whatever other depth you think more probable, counted on a vertical circle or with compasses set straight and perpendicularly between that point and the horizon. For when you have noted the point of the equator that is then on the horizon, turned the sphere, counted the degrees of the equator that rise until the point of the zodiac occupied by the Sun has reached the horizon, and converted them into time, you will have the duration of twilight, unless a difference in the air, arising from the condition of the region, stands in the way. For Father Iosephus Acosta [José de Acosta] reports that on the coast of Chile twilight is very short, and that within one quarter of an hour clear day comes out of sheer night.

[Margin: Profūditas Solis Crepusculina. — The Sun's depth at twilight.]

III. The Sun's depth below the horizon on a vertical circle, at the beginning of morning twilight or the end of evening twilight, is investigated from the time of the duration converted into an arc of the equator, and from the complement of the altitude of the pole and the Sun's declination, by the problems treated in the common Problems of the primum mobile and the second mobile, as I shall indicate in Scholium 2. But it comes out differently, because of the physical

causes indicated above. In the same way the altitude of the exhalations depends not only on the varying depth of the Sun but also on the varying quantity assumed for the Earth's semidiameter, and indeed also on the varying opinion about the cause of twilight. For Scultetus, following Cardano, *De subtilitate*, Book 4, thought that the exhalations that reflect the light of twilight to us are above our zenith; and so, using a ray reflected perpendicularly from there to us, he deduced an insane altitude for the exhalations. But who has ever seen the light of twilight above his own zenith, when it girds the horizon by only a few degrees? Nearly all the rest estimate this reflection as coming from exhalations and fumes raised about 40 or 50 thousand paces, but from the Sun's rays striking those exhalations directly. Gassendi, however, holds that that altitude can be less, because that first uncertain light can come from the Sun's ray not directly but by reflection from the lower parts of the atmosphere, so that it is reflected a second or a third time. For my part, with Kepler, and before I had read it in him, I thought and still think that this light is produced from a ray of the Sun first refracted in the surface of the atmosphere and then reflected to us from the exhalations. On this supposition the distance of these exhalations will have to be investigated geometrically by us in a far different way, and with a different figure from the one commonly used, in the Book on Optics, where the effects of refractions will be discussed more subtly. Kepler, however, adds that part of this light comes from the ethereal air spread around closest to the Sun.

[Margin: Auctores qui de Crepuscul aut altitudine Vaporum, &c. — Authors on twilight or on the altitude of vapours, etc.]

In the meantime, however, the chief expositors of twilight may be consulted: namely Ibn al-Haytham and Witelo, Book 10, Proposition 60; Nonius, Book 2, on observation, from p. 82, and in an entire book on twilight; Clavius on the *Sphere*, from p. 131, and in a short work on twilight; Magini, *Primum Mobile*, Book 11, Problem 30; Gemma Frisius in the supplement to the *Astrolabe*; Barozzi in the *Cosmographia*, p. 189; Biancani in the *Sphaera*, Book 6, Chapter 5, together with Book 10, Chapter 15; Tycho, *Progymnasmata*, volume 1, pp. 95 and 733; Scultetus, Book 2 of the *Phaenomena*, Chapter 10; Cardano, *De subtilitate*, Book 4; Rothmann in Tycho's *Letters*, for the year 1588, February 21 and 24; Simon Stevin in the *Cosmographia* of the atmosphere; Abraham Roggembach [?] in Tanner, Question 7 on *De caelo*; the Coimbra commentators, *De caelo*, Book 3, Chapter 5, Question 2; Longomontanus in the *Astronomia Danica*, Book 2 of the *Spherics*, Chapter 11; Kepler in the *Epitome of Copernican Astronomy*, pp. 73, 308 and 369, and also Part 5; Gassendi in the *Institutio astronomica*, Book 1, Chapter 18; Cabeo, *Meteorologica*, Book 1, Question 15, Book 2; Froidmont in the *Meteora*, Book 6, Chapter 4, Article 2; Gloriosi, *De cometis*, Book 2, Chapter 2. Of these, Nonius and Biancani give a table of twilights suited to nearly every elevation of the pole, which, for the reasons indicated, we judge must be reformed in the Book on Optics. Here, however, it seems good to report the opinions of others, and to show beginners, for civil use, a specimen of the table of Nonius and Biancani. The part of that table for the Sun entering the signs,

then, is of this kind, with the Sun's depth set at 18° or 19°. In it, t. n. means that twilight lasts the whole night, or is all-night; d. c. means continuous day; d. p. means pure day; and n. p. means pure night.

Duration of twilight from the astronomical cause, according to Nonius and Biancani [Crepusculi Duratio ex Causa Astronomica, iuxta Nonium & Blancanum.]

Beginnings of the signs [Initia Signor.] / altitude of the pole [Altit. Poli G.]	0° and 5°	10°	20°	30°	40°	45°	48½°	52°	54°	55°	60½°	63°	66½°
Cancer	1h 19m	1h 20m	1h 26m	1h 36m	2h 9m	3h 39m	t.	t.	t.	t.	t.	t.	d.
							n.	n.	n.	n.	n.	n.	c.
Gemini, Leo	1h 14m	1h 15m	1h 21m	1h 31m	1h 50m	2h 19m	2h 40m	t.	t.	t.	t.	t.	t.
								n.	n.	n.	n.	n.	n.
Taurus, Virgo	1h 13m	1h 14m	1h 20m	1h 28m	1h 43m	1h 55m	2h 4m	2h 22m	2h 39m	2h 56m	t.	t.	t.
											n.	n.	n.
Aries, Libra	1h 12m	1h 13m	1h 18m	1h 25m	1h 35m	1h 45m	1h 50m	2h 1m	2h 5m	2h 10m	2h 35m	2h 42m	3h 18m
Pisces, Scorpio	1h 13m	1h 14m	1h 15m	1h 20m	1h 35m	1h 43m	1h 49m	1h 59m	2h 4m	2h 9m	2h 30m	2h 42m	3h 9m
Aquarius, Sagittarius	1h 14m	1h 15m	1h 17m	1h 26m	1h 40m	1h 49m	1h 56m	2h 8m	2h 15m	2h 20m	2h 45m	3h 0m	3h 31m
Capricorn	1h 19m	1h 19m	1h 22m	1h 29m	1h 43m	1h 53m	2h 2m	2h 14m	2h 19m	2h 26m	3h 0m	3h 26m	3h 25m

The rest of the table of twilight [Residuum Tabulæ Crepusculi.]

Beginnings of the signs [Initia Signor.] / altitude of the pole [Altit. Poli G.]	68°	72°	74°	76°	78½°	80°	82°	84½°	86°	88°	88⅘°	89°	90°
Cancer	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.
Gemini, Leo	t.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.	d.
	n.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.	c.
Taurus, Virgo	t.	t.	t.	t.	d.	d.	d.	d.	d.	d.	d.	d.	d.
	n.	n.	n.	n.	p.	c.	c.	c.	c.	c.	c.	c.	c.
Aries, Libra	3h 27m	t.	t.	t.	t.	t.	t.	t.	t.	t.	t.	t.	t.
		n.	n.	n.	n.	n.	n.	n.	n.	n.	n.	n.	n.
Pisces, Scorpio	3h 30m	3h 56m	4h 39m	5h 26m	8h 9m	8h 32m	9h 17m	t.	t.	t.	t.	t.	t.
								n.	n.	n.	n.	n.	n.
Aquarius, Sagittarius	4h 10m	5h 40m	5h 31m	5h 27m	5h 18m	5h 10m	5h 0m	4h 20m	3h 40m	2h 0m	in-	n.	n.
											stās	p.	p.
Capricorn	5h 18m	5h 2m	4h 45m	4h 32m	4h 12m	3h 56m	2h 15m	in-	n.	n.	n.	n.	n.
								stās	p.	p.	p.	p.	p.

This table also serves the southern hemisphere, but with the northern signs exchanged for the southern.

Tabula II. Pertaining to Twilights. Opinions on the depth of the Sun below the horizon at the beginning of morning twilight and the end of evening twilight, and on the altitude of the exhalations or vapours, according to the semidiameter assumed for the Earth. Authors, in the places named above	Depth of the ☉ below the horizon	Altitude of the exhalations, in Italian miles	Earth's semidiameter assumed, in Italian miles
Ibn al-Haytham and Witelo	19° nearly	51 48/60	3850
Pedro Nunes, assumed for the sake of argument against Cardano	19°	68	5010 1/5
But from his own observation and opinion	16° 2′	47 5/8	4978
Clavius in the *Sphere*, with whom Barozzi and the Coimbra commentators [Conimbric.] agree	19°	52	3900
Clavius, the opuscule on twilight	18°	43	3580
Giovanni Antonio Magini and Gemma Frisius	18°	43	3580
Tycho Brahe, and nearly Cabeo	17°	52	3727
Biancani in the *Sphere*	18°	43	3479
Rothmann in Tycho	24°	60	3727
Longomontanus	20°	[?] [recte: 52, per the book's errata, which give the misprint as 56 ⅓; in this copy the printed figure is overwritten in ink and the type cannot be read]	3727
Simon Stevin	18°	41	
Cardano, taking twilight as half an hour [recte: two hours, per the book's errata, which correct *ſemihoræ* to *bihorij*], and Abraham Roggembach [?]	19°	772	5000
Cardano taken absolutely, and Scultetus, though too much so	19°	[?]288 [recte: 288, per the book's errata, which give the misprint as 1288; in this copy the first figure is struck through in ink and cannot be read]	5000
Bodin in Froidmont, far too sparing	3°		
Kepler, from the opinion of others, if a German mile makes 4 Italian, either	16°	34	3727
or	18°	40	3727
or; but in his own opinion the altitude is much smaller	19°	46	3727
Pierre Gassendi, from the opinion of others; but in his own, the altitude of the vapour is smaller	18°	40	4177

		Depth of the ☉ below the horizon	Bolognese Italian miles	Ancient Italian miles
We, from observations at Bologna	At the equinoxes, in the morning	16° 0′	38 0	47½
	At the equinoxes, in the evening	20° 30′	63 1/5	79 1/10

		Depth of the ☉ below the horizon	Bolognese Italian miles	Ancient Italian miles
If twilight is from reflection alone	At the summer solstice, in the morning	21° 25′	69 1/5	86½
But refraction makes the altitude of the exhalations smaller	At the winter solstice, in the morning	17° 25′	45 2/5	56½

[Earth's semidiameter, set beside this block at the right of the table: Bolognese miles, 4000; ancient Italian miles, 5000.][8]

SCHOLIA

I. [Margin: Praxis inuestigandi Altitudinē exhalationum. — The procedure for finding the altitude of the exhalations.] Given the depth of the Sun below the horizon at the beginning of morning twilight or the end of evening twilight, the altitude of the exhalations is easily found, provided twilight is assumed to be the offspring of pure reflection and the Earth's semidiameter is known. The procedure, briefly, is this. From the arc of the Sun's depth subtract 15′, which is the portion by which the Sun, being larger than the Earth, illuminates more than a quadrant of the terrestrial hemisphere on one side. For half of the remaining arc, look up the secant in the tables of secants. Multiply it by the Earth's semidiameter and divide the product by the radius of the tables, or whole sine; you will then have that same secant known in paces or miles, whichever the Earth's semidiameter was known in. From this, therefore, subtract the Earth's semidiameter, and what remains is the excess of the secant, known in the same measures: this is the altitude of the exhalations itself, if it comes from pure reflection from there to us.

For example, the depth of the Sun at the end of evening twilight, at the time of the equinox, observed by me with Father Francesco Maria Grimaldi with the greatest attention and care, was found to be 20° 30′. Taking away 15′, the remainder is 20° 15′. Half of this is 10° 7½′, whose secant is 101582, of the parts of which the radius is 100000. Now the Earth's semidiameter has been found by me to be 5000 ancient Italian miles. The secant 101582 multiplied by this gives 507910000; dividing this by the radius, 100000, leaves 5079 1/10. Taking away, then, the 5000 miles of the terrestrial semidiameter, the altitude of the exhalations remaining is 79 1/10 miles.

II. [Margin: Profūditas Solis Crepusculina, qua Methodo inuestigetur. — The Sun's twilight depth, and by what method it is found.] The depth itself, however, is found from the given altitude of the pole, the observed duration of

[8] In this copy a later hand has written in ink over the table. It has struck through the word for "half an hour" in the Cardano row and written a figure above it; written over the Longomontanus altitude and its fraction; and struck through what stands before "288" in the next Cardano row.

twilight and the Sun's declination, by this method. Add together the semidiurnal arc and the duration of twilight converted into degrees of the equator, subtract the sum from 180°, and call the remainder the argument of the depth. The residual of the second logarithm corresponding to the argument of the depth, added to the mesologarithm of the complement of the altitude of the equator, will then give the mesologarithm of the first arc. Subtract this arc from the complement of the Sun's declination if the Sun is in the southern signs, or from the sum of the declination and 90° if in the northern, and you will have the second arc. Then add together the second logarithm of the complement of the altitude of the pole, the residual second logarithm of the first arc just found, and the second logarithm of the second arc; for the sum will give the second logarithm of the arc to be subtracted from 90°, so as to leave the depth of the Sun that was sought. I am speaking, however, with northerners, and with those who know, or wish to learn, the most expeditious analysis of triangles by logarithms. I shall illustrate this precept with a diagram and with examples in the Optical Book, or in the Book of Common Problems, or in both. Meanwhile, enjoy this indication of mine, such as it is, and do not let twilight observed on a mountain summit, rather than on a plain concentric with the sea and equally distant from the centre, trouble you in the meantime.

III. The altitude of the exhalations and fumes that become the torches of twilight, which is the subject here, is one thing. The altitude of the moister vapours, which fly less high without the wings of the exhalations, is another. Another again is the altitude of the clouds, which are denser vapours and also rise less high than exhalations or thin vapours. And another, finally, is the altitude of the refracting air, that is, the air capable of producing sensible refractions of the celestial rays, which is likewise thought to be much lower, and about which more in the Optical Book.

[Margin: Vespertinū Crepusculū Matutino prolixius. — Evening twilight is longer than morning twilight.] Moreover, the duration of evening twilight, and the depth of the Sun deduced from it, are not the same as those of morning twilight, but greater, as I gather from my own observations with reliable confidence. The cause is that the exhalations, warmed and rarefied during the day, rise higher, but afterwards, around the middle of the night, being cooled, become lower and lower for the following twilight. This is also the physical cause why, in summer twilight, the duration is greater, and so are the depth of the Sun and the altitude of the vapours derived from it. For at Bologna, at a pole altitude of 44° 29½′, I have observed that on the equinoctial days twilight lasts $1^h\ 47^m$ in the morning, but 2^h in the evening; summer morning twilight, around the solstice, $3^h\ 40^m$, for the evening twilight occupies almost the whole half-night; and winter morning twilight $1^h\ 50^m$, for the evening twilight I have not yet been able to observe. Tycho is therefore wrong when, in the *Letters*, p. 139, from observations made in 1589 February 10 and 15, he declares universally that the Sun's twilight depth does not exceed 17°.

IV. [Margin: Hora faciēdi Sacrū Opportunitate Crepusculi. — The hour for saying Mass, by the opportunity of twilight.] Those who by privilege may celebrate the Sacrifice of the Mass immediately from dawn and the beginning of twilight may, if the twilight lasts all night, perform the Sacrifice immediately after midnight. Nor is this surprising, since those whose night is no longer than one hour may say Mass immediately after midnight, and this is not too early.

But what is permitted in this regard to those living in the parallel sphere, for whom twilight lasts up to a month and a half on either side, before one equinox and after the other, is not for this place to determine.

CAPVT XXXII.

De Ortu, & Occaſu Syderum, & de Arcu Viſionis, occaſione Ortus Occaſuſq; Heliaci.

On the Rising and Setting of the Stars, and on the Arc of Vision, on the occasion of Heliacal Rising and Setting.

I. Of the twenty-four configurations, or relations, of the stars to the Sun that Ptolemy lists in *Almagest*, Book 8, Chapter 4, the chief are their rising and setting. Each of these is divided into the astronomical and the poetical. The astronomical is the right or oblique ascent or descent of a star, whether the Sun is then rising or setting, or is on the meridian, or elsewhere. The poetical is so called, not because astronomers do not concern themselves with it too, but because it alone is what poets have spoken of ever since Hesiod, when they marked out for us in verse, by the risings of the stars, the calendars and the seasons favourable to agriculture or navigation, or things of that kind. This poetical rising and setting is referred to the Sun rising or setting, and is subdivided into cosmical, acronychal and heliacal; or into true morning, true evening, and apparent, which is itself subdivided into apparent morning and apparent evening.

True morning rising and setting has been called cosmical, or mundane, because at the Sun's rising the adornment of the world and the face of Nature are renewed. True evening has been called acronychal, with a letter discarded by ectlipsis, when it ought to be called acronychal [Acronyctus], from ἀπὸ τοῦ ἄκρου [Greek: apo tou akrou, "from the extreme"], that is, from the first limit and point, and νυκτός [Greek: nyktos, "of night"], that is, of night; because it occurs accurately and exactly at once, and "punctually", as they commonly say, at the very beginning of the night and the true setting of the Sun; even though the less skilled, with Sacrobosco, call it chronical, or temporal. Apparent rising or setting has been called heliacal from ἀπὸ τοῦ ἡλίου [Greek: apo tou hēliou, "from the Sun"], that is, from the Sun, because the star, though it does not then truly ascend or descend at the horizon, nevertheless begins either to be freed from the neighbouring rays of the Sun or to be hidden by them, and so appears to rise or set.

So that we need not be repeatedly compelled to repeat and compound the Greek words, it is enough to have indicated the simple words first. Thus rising is called here not so much ἀνατολὴ [Greek: anatolē, "rising, east"], which means rising in the sense of the east [Oriens], as ἐπιτολὴ [Greek: epitolē, "rising" of a star]; setting, δύσις [Greek: dysis, "setting"]; morning or cosmical, ἑώα [Greek: heōa, "of the dawn"] or κοσμικὴ [Greek: kosmikē, "cosmical"]; evening or acronychal, ἑσπερία [Greek: hesperia, "of the evening"] or ἀκρονυκτὴ [Greek: akronyktē, "acronychal"]; if it is true, it is called ἀληθινὴ [Greek: alēthinē, "true"]; if apparent or heliacal, it is called φαινομένη [Greek: phainomenē, "apparent"] or ἡλιακὴ [Greek: hēliakē, "heliacal"]. Therefore, by compounding, the true morning rising will be called ἐπιτολὴ ἑώα ἀληθινὴ, and so on for the rest. Now let us explain these kinds one by one.

II. [Margin: Ortus Cosmicus. — Cosmical rising.] True morning rising, or cosmical rising, is the ascent of a star above the eastern horizon at the moment when the Sun rises from the same place, whether it is seen by us or not, because of the nearness of the Sun. This kind of rising belongs to all the stars in general, that is, to planets and fixed stars, unless they are among those that always lie hidden below the horizon or always revolve above it. So in the first of the figures below, in which, with the Sun rising at E, the stars I and L likewise ascend above the eastern horizon AEC.

III. [Margin: Occasus Cosmicus. — Cosmical setting.] True morning setting, or cosmical setting, is the descent of a star below the western horizon at the moment when the Sun rises on the eastern horizon, whether that star is then seen, or a little before, or not. This kind belongs to every kind of star, and to every star that can descend below the horizon and be 180° from the Sun. In the same first figure, for example, are the stars FRG, which descend below the horizon ARC with the Sun rising at the point E of the horizon AEC. And so it does not belong to Venus, which does not depart from the Sun by more than 48°, nor to Mercury, which does not depart by more than 28°.

Diagram. Woodcut, numbered I above the circle, of a sphere divided by two curved horizons running from A at the left to C at the right. The word COSMICI stands above the circle, with B at the top inside the rim and D below the rim at the bottom. The upper horizon arches through three stars labelled F, R and G. A small dark globe sits at the centre on a straight line from A to C, between the two arcs; the lower horizon arches beneath it; the part of the sphere below it is heavily shaded and holds a rayed Sun with a face at E, on that arc, with stars labelled I and L on either side. Labels read:

- I (above the circle): figure number
- COSMICI (above the rim): "cosmical"
- A (left), C (right): ends of the horizons
- B (top, inside the rim), D (below the rim)
- HORIZ. OCCID. (along the upper arc): "western horizon"
- F, R, G (beneath the stars on the upper arc)
- HORIZ. ORIEN. (along the lower arc): "eastern horizon"

• I, E, L (stars and Sun on the lower arc)

Labels not read: none. Letters cited in the prose as AEC, FRG and ARC are all on the block; R and E stand on the two arcs.

IV. [Margin: Ortus Acronyctus. — Acronychal rising.] True evening rising, or acronychal rising, is the ascent of a star above the eastern horizon at the moment when the Sun truly sets, that is, descends below the western horizon. In the second figure below, for example, the stars IKL, if they ascend above the horizon AKC with the Sun setting at E. This kind of rising belongs only to those stars that can be so opposed to the Sun as to stand 180° from it. And so it does not belong to Mercury, which cannot depart from the Sun in the zodiac by more than 28°, nor to Venus, which cannot by more than 48°. But it is not unsuited to the other stars.

V. [Margin: Occasus Acronyctus. — Acronychal setting.] True evening setting, or acronychal setting, is the descent of a star below the western horizon at the moment when the Sun also descends below it and sets. In the second figure, for example, if with the Sun setting at E the stars F and G slip below the horizon AFC. This kind belongs to all kinds of stars, and indeed to any individual star that can descend below the horizon.

VI. [Margin: Ortus Heliacus Matutinus. — Heliacal morning rising.] Apparent or heliacal morning rising is the first appearance of a star on the eastern horizon shortly before the Sun's rising, the star having until then been unable to be seen under the Sun's rays, because of the glare of the too-near Sun. In the third figure below, for example, if, as the Sun E approaches and its rising is imminent, the star F has so emerged from its glare, under which it lay hidden, that it can be seen on the horizon AFC, unless the thickness of the horizontal vapours happens to obstruct it. This kind of rising belongs to all stars that are, or can be, slower than the Sun in apparent motion in longitude, because the Sun, by its proper motion toward the east, can leave them behind it toward the west; and so it belongs to all except the Moon. Moreover, on the following days the star will appear farther and farther from the Sun, as at G:

Diagram. Woodcut, numbered II above the circle, standing at the head of the left column and built like Figure I, but with the Sun on the upper horizon. The word ACRONYCTI stands above the rim. Two curved horizons run from A at the left to C at the right. A rayed Sun with a face stands on the upper arc under the letter E, with a star on each side; F and G stand below those stars. The lower arc has three stars beneath it, with I, K and L above them. A small dark globe sits at the centre on a straight line from A to C. The part of the sphere below the lower arc is heavily shaded. Labels read:

• II (above the circle): figure number
• ACRONYCTI (above the rim): ”acronychal”
• B (top, inside the rim), D (below the rim)
• A (left), C (right)
• E (above the Sun on the upper arc)
• HORIZ. OCCID. (along the upper arc): ”western horizon”

- F, G (below the stars on the upper arc)
- HORIZ. ORIEN. (along the lower arc): "eastern horizon"
- I, K, L (above the stars on the lower arc)

Labels not read: none.

Diagram. Woodcut, numbered III above the circle, standing at the head of the right column. The word HELIACI stands above the rim. Two curves descend from B at the top: one to the left, through the stars at G and F to the rayed Sun beside E, on the lower arc; the other to the right, through the stars at L and I to a second rayed Sun beside H, on the upper arc. A small dark globe sits at the centre on a straight line from A to C, and the part of the sphere below the lower arc is heavily shaded. Labels read:

- III (above the circle): figure number
- HELIACI (above the rim): "heliacal"
- B (top, inside the rim), D (below the rim)
- A (left), C (right)
- HORI. OCCID. (along the upper arc): "western horizon"
- L, I (beside the stars on the right-hand curve), H (beside the Sun on the upper arc)
- G, F (beside the stars on the left-hand curve), E (beside the Sun on the lower arc)

Labels not read: none. No label stands along the lower arc in this figure. The prose's "AFC" and "AIC" are not lettered as such; A, C, F and I are all on the block.

VII. [Margin: Ortus Heliacus Vespertinus. — Heliacal evening rising.] Apparent or heliacal evening rising is the first appearance of a star on the western horizon shortly after the Sun's setting, the star having until then lain hidden under the Sun's rays because of its glare, too great and too near. In the third figure, for example, if, with the Sun set and sunk below the point H, the star I begins to appear on the western horizon AIC, as far as the vapours of the horizon allow, the Sun's glare no longer preventing it. This kind belongs to all stars that can be faster than the Sun in apparent motion in longitude toward the east. It therefore belongs to the Moon, whose first phase after the new moon is noticed in the evening; and likewise to Venus and Mercury after a conjunction with the Sun completed at the apogee of the epicycle, and after descent from the apogee; and Venus is then called Hesperus, or Vesperugo. But it does not belong to the fixed stars or the three superior planets.

VIII. [Margin: Occasus Heliacus Matutinus. — Heliacal morning setting.] Apparent or heliacal morning setting is the first hiding of a star on the eastern horizon under the rays of the Sun, which is near and about to rise shortly afterwards. In the third figure, for example, if the star G, which until then was seen on the horizon before sunrise, has overtaken the Sun E, or has come closer to it because of the star's faster motion in longitude, and no longer appears, the glare of the nearby Sun prevailing and overwhelming the star's rays. This kind belongs to all stars that can be faster than the Sun: thus to the Moon, whose last

phase before the new moon is noted in the morning; and also to Venus, which is then called Lucifer, or Phosphoros, and to Mercury, when they approach the apogee of the epicycle; but not to the fixed stars, nor to the three superior planets.

IX. [Margin: Occasus Heliacus Vespertinus. — Heliacal evening setting.] Apparent or heliacal evening setting is the first hiding of a star on the western horizon under the rays of the Sun, which has set shortly before. In the third figure, for example, if the star L, which the day before appeared on the horizon AIC shortly after sunset, does not appear the next day because of the Sun's approach to it. This kind belongs only to stars that can be slower than the Sun's proper motion in longitude, and thus to all except the Moon; and in the case of Venus and Mercury, only after the first station, as they descend toward the perigee of the epicycle.

X. To help the memory, I have composed for these kinds this octastich, such as it is, which shows which risings or settings belong to which stars.

True risings in the morning and true settings in the evening every star undergoes in its own seasons. Neither true settings in the morning nor risings in the evening has God granted to Venus or to Mercury. Both heliacal rising in the morning and heliacal setting in the evening the Moon alone, among the stars of heaven, lacks. Heliacal settings in the morning and risings in the evening only the Moon, Venus and Mercury possess.

But if this does not help, look at the following table [laterculum].

		Belongs	Does not belong
TRUE	Morning or cosmical rising; evening or acronychal setting	☾ ☿ ♀ ♂ ♃ ♄ ✶	
	Morning or cosmical setting; evening or acronychal rising	☾ ♂ ♃ ♄ ✶	☿ ♀
APPARENT or HELIACAL	Morning rising; evening setting	☿ ♀ ♂ ♃ ♄ ✶	☾
	Morning setting; evening rising	☾ ☿ ♀	♂ ♃ ♄ ✶

XI. Some other rules about the rising and setting of the stars are commonly given here: for example, that stars with northern latitude first rise cosmically with the Sun, then heliacally, then culminate with it, and finally set with the Sun heliacally and afterwards acronychally, while the contrary happens to stars that have southern latitude, or as long as they have it. It is also usual to add examples, either those found in the poets or others. But these require a fuller knowledge of the celestial motions and of the stars, and so I reserve them for the Book on the Fixed Stars, which will be the 6th of this first Volume, or else for Book 4 of Volume 3 of the *New Almagest*.

[Margin: Auctores qui de Ortu, & Occasu Syderum. — Authors on the rising and setting of the stars.] In the meantime I shall point out the chief authors who can be read through on such risings and settings. They are: Autolycus, on the

rising and setting of the fixed stars, with the notes of Giuseppe Auria; Geminus in the *Elements of Astronomy*, Chapters 11 and 16, where there is a catalogue of risings with a Greek calendar; Aratus of Soli in the *Phaenomena* and *Prognostica*, with Cicero, Rufius Festus Avienus and Germanicus Caesar as translators; Achilles Tatius in the *Isagoge* to the *Phaenomena* of Aratus; Hipparchus, but the Bithynian, Book 3 on the *Phaenomena* of Aratus and Eudoxus, with Pétau as translator; the book of Eratosthenes on the *Phaenomena*, with the same translator; Ptolemy in the book *On the Apparitions and Significations of the Fixed Stars*, with an Egyptian calendar, and also *Almagest*, Book 8, Chapter 4, and there Theon and Regiomontanus in the *Epitome*, from Proposition 3; Ovid in the *Fasti* and Virgil in the *Georgics*, with the scholiasts; an old calendar compiled from Ovid's *Fasti*, Columella, Book 11, Chapter 12, and Pliny the Elder, Book 18, from Chapter 26 to 30, which Pétau has in the *Uranologion*; Julius Firmicus, Book 2, Chapter 81; Johannes Stöffler on the *Sphere* of Proclus, from p. 32; Copernicus, Book 2, Chapter 13; Reinhold in the *Directions*, from Precept 34 to 37; Johannes Stadius in the *Canon of the Celestial Motions*, from Chapter 6 to the end, with an ancient calendar and a catalogue of the rising of the stars; Maurolico in Dialogue 3 of the *Cosmographia*, p. 99; Clavius on the *Astrolabe*, p. 655, and on the *Sphere* of Sacrobosco, p. 315; Barozzi in the *Cosmographia*, Book 3, Chapter 1; Biancani in the *Sphaera*, Part 3, Chapter 3, or Book 17, Chapter 10; Giovanni Antonio Magini in the *Directions*, from Canon 17 to 22, and in the *Primum Mobile*, Book 10, Problems 2 and 3, and in the *Theoricae Planetarum*, Book 2, from Chapter 14, and in Canon 98 of the *Second Mobiles*, and in the *Introduction to the Ephemerides*, Part 4, from Chapter 3, with a catalogue of the rising and setting of the stars; Origanus, Volume 1 of the *Ephemerides*, Part 2, Chapter 8, with catalogues; Longomontanus in the *Astronomia Danica*, Book 2 of the *Sphaerica*, Chapter 4, with a catalogue of the stars rising, setting and crossing the mid-heaven, for a pole elevation of 56° and for the year 1630; Johannes Kepler in the *Epitome of Copernican Astronomy*, Book 3, from p. 364 to 399, and Book 5; Dionysius Petavius [Denis Pétau], S.J., in the *Uranologion*, Book 1, and Book 2 from Chapter 8, and Book 7, Chapter 1; Franciscus Montebrunus [Francesco Montebruni] at the end of his latest *Ephemerides*, with a catalogue; Cavalieri in the *Centuria*, Problems 21 and 44; Pierre Gassendi in the *Institutio Astronomica*, Book 1, Chapter 20.

DE ARCV FVLSIONIS, SEV VISIONIS.

On the Arc of Shining, or of Vision.

XII. [Margin: Quid sit arcus Fulsionis. — What the arc of shining is.] The arc of vision [Arcus Visionis], or arc of shining [Arcus Fulsionis], of the stars is the arc of a vertical circle, intercepted between the horizon and the centre of the Sun, that is required for a star to be visible to us despite the Sun's glare. It is therefore the arc on passing which a star rises heliacally; within it the star lies hidden, and on first entering it the star sets heliacally. This arc is not constant,

however, even with respect to the same star. For it varies (1) as the star's latitude and declination vary; (2) as the rectitude of ascension or descension varies, that is, their slowness, and their obliquity, that is, their speed; and (3) as the distance of the planets from the Earth varies. Nevertheless Ptolemy in *Almagest*, Book 13, Chapter 7, and with him Regiomontanus in the same place in the *Epitome*, Propositions 22 and 23, Magini in the *Directions*, Canon 20, and in the *Theoricae Planetarum*, Book 2, Chapter 15, and Kepler in the *Epitome of Astronomy*, p. 369, lay down this arc [Greek: ὁλοσχερᾶς [recte: ὁλοσχερῶς] (holoscherōs, "in general terms, roughly")], as coming very close to the common phenomena, from an ancient observation of the Chaldeans. And it is as follows.

Arcus Visionis Stellarum, seu Profunditas Solis sub Horizonte Requisita ad Stellarum Apparitionem.

The arc of vision of the stars, or the depth of the Sun below the horizon required for the appearance of the stars.

For the planets	Arc	For fixed stars of magnitude	Arc
☿	10° 0′	1	12°
♀	5° 0′	2	13°
♂	11° 30′	3	14°
♃	10° 0′	4	15°
♄	11° 0′	5	16°
		6	17°

Nothing is said here about the Moon, because this arc, necessary for its appearance, is markedly variable, according to the Moon's differing speed and for other causes to be touched on in Book 4. The arcs given above, moreover, apply to stars assumed to be in the ecliptic itself, and were deduced by Ptolemy from arcs of vision observed in Syria by the Chaldeans when the planets were at the beginning of Cancer, near the Sun. For at that time they judged the air to be purer and the vapours of the horizon less harmful to the shining of the stars, and at that time the inclination of the zodiac to the horizon is moderate. So at that time, because of latitude, they found that Saturn rose heliacally in the morning when it was 14° from the Sun in longitude, Jupiter at 12° 45′ and Mars at 14° 30′; but that Venus rose heliacally in the evening at a distance of 5° 40′, and Mercury at 11° 30′. From this distance Ptolemy, by the analysis of triangles, gathered the arc of shining stated above. But more on these matters once the latitude of the planets has been determined.

SCHOLION.

I. [Margin: Ortus, & Occasus notatu faciliores. — Risings and settings easier to notice.] The heliacal rising would better be called an emersion or *phasis* than a rising, as Kepler too notes; and the heliacal setting would better be called [Greek: κρύφις [recte: κρύψις] (krypsis, "concealment")], that is, concealment [occultatio], or [Greek: ἀφανισμὸς (aphanismos, "disappearance")], that is,

disappearance [disparentia], than a setting. Moreover, since the night has two [Greek: ἄκρα (akra, "extremes")], that is, extremes, its beginning and its end, Ptolemy seems to call *acronycti* those stars that shine all night, both rising in the evening as the Sun goes down and setting in the morning as it rises. Chiefly, however, attention is paid to the beginning of night, since they shine all night and can be observed; whereas once the Sun has risen they can scarcely be observed after a quarter [quadrantem]. Again, the morning rising, the true one with the Sun, and the true evening setting with the Sun, cannot be observed, and are not useful for distinguishing the seasons for farming, for medical treatment or for navigation. The better ones, therefore, are the heliacal risings and settings, and after them also the true evening risings and morning settings. And if Pliny the Elder and others mark out the seasons by invisible risings and settings of the stars, perhaps they do so in order to point to hidden and astrological causes of the weather, as the same Kepler observes. But the heliacal risings and settings themselves, and the others, vary somewhat because of parallaxes and refractions, and far more because of the vapours of the horizon, on account of which stars seem to rise later on the eastern horizon and to set earlier on the western than they otherwise should. This gave some people the occasion for differing opinions on the times of these risings and settings, as Pétau notes from Wendelen in the dissertations of the *Uranologion*, p. 99. Certainly stars are very rarely and with great difficulty discerned on the horizon itself, and one must wait until they are raised a little above the horizon.

CAPVT XXXIII.

De Aſpectibus, & Radiationibus Syderum mutuis.

On the Mutual Aspects and Radiations of the Stars.

I. [Margin: Aspectuum numerus. — The number of the aspects.] Mutual aspects, and the radiations of signs or stars, are relations of one to another at a determinate distance at which they are fit to bring something about together. They are commonly counted as five and are attributed to signs and to stars placed within the zodiac, but they are extended to other stars as well. Their characteristic marks, names and the distances required for them are given in the order written below.

			Gr.
☌	Conjunction [Coniunctio]	[Greek: σύνοδος (synodos, "meeting")]	0°
⚹	Sextile [Sextilis]	[Greek: ἑξάγωνος (hexagōnos, "hexagon")]	60°
□	Square [Quadratus]	[Greek: τετράγωνος (tetragōnos, "square")]	90°
△	Trine [Trinus]	[Greek: τρίγωνος (trigōnos, "triangle")]	120°
☍	Opposition [Oppositio]	[Greek: διάμετρος (diametros, "diameter")]	180°

Conjunction and opposition, and indeed all the aspects, are also called [Greek: συζυγίαι (syzygiai, "yokings")], that is, conjugations [coniugationes]. So if two stars are no distance apart in the zodiac, they are said to be conjoined, even

if the heaven of one is separated by a long interval from the heaven of the other, as the Moon from the Sun at new moon. But if they are 60° apart, they are said to regard each other in sextile aspect, and the arc of their distance subtends a chord that is the side of a hexagon, or six-angled figure, inscribable in a circle; by 15 of Book 4 of Euclid that side is a sixth part of the circle, and so the aspect is called sextile, or *sextans*. If they are 90° apart, or a quadrant of the circle, the aspect is square, because the arc of distance subtends a chord that is the side of a square inscribable in a circle. But if they are 120° apart, the aspect is trine, because they are a third [Trientem] apart; or triangular and trigon, because the arc of their distance subtends a chord that is the side of a trigon inscribable in a circle, that is, of an equilateral triangle. And I say the same of the signs of the zodiac. So if, by 15 of Book 4 of Euclid, you inscribe a hexagon in the circle of the zodiac, and by 6 a square, and by 2 of the same an equilateral triangle, you will have figures representing these aspects and radiations, as appears in the following diagram.

[Margin: Figura Aspectuum. — Figure of the aspects.] In it Cancer ♋, Virgo ♍, Scorpio ♏, Capricorn ♑, Pisces ♓ and Taurus ♉ each regard the one nearest to it among these in sextile aspect; but Cancer ♋, Libra ♎, Capricorn ♑ and Aries ♈, in tetragonal aspect, form a square with their rays; and finally Cancer ♋, Scorpio ♏ and Pisces ♓, in trine aspect, form a trigon, or equilateral triangle.

Diagram. Woodcut set into the left column, the text running down its left side: a single circle with the twelve signs of the zodiac outside the rim and straight lines drawn between points on the rim, forming a hexagon, a square and an equilateral triangle that all share the vertex at the top. Labels read, clockwise from the top:

- ♋ (top)
- ♊ (upper right)
- ♉ (upper right, at a vertex)
- ♈ (right, at a vertex)
- ♓ (lower right, at a vertex)
- ♒ (lower right)
- ♑ (bottom, at a vertex)
- ♐ (lower left)
- ♏ (lower left, at a vertex)
- ♎ (left, at a vertex)
- ♍ (upper left, at a vertex)
- ♌ (upper left)

Labels not read: none. The block carries no point-letters and no figure number.

II. The reason for this number was both the observation of the effects and influences that signs or stars produce on sublunary bodies in the positions just named, and the division of the zodiac into harmonic intervals with the integrity of the signs preserved in the dividing. Thus to opposition corresponds

the diapason, because of the double ratio that the whole circle of 12 signs has to the 6 signs by which opposite signs or stars are apart. To trine aspect corresponds the diapente, because of the hemiolic, or sesquialter, ratio between the 6 signs of distance assigned to opposition and the 4 assigned to trine aspect. To square corresponds the diatessaron, because of the sesquitertian, or epitritic, ratio between the 4 signs assigned to trine aspect and the 3 assigned to square. Finally, to sextile corresponds a lesser diapente, because of the two signs of distance attributed to this aspect; for between 2 and the 3 assigned to the next aspect there is a hemiolic ratio, but the interval is smaller than that of 6 and 4 signs. The dodecagon, and the distance of one sign, have been left out because of the weakness of their powers; and the octagon, whose side subtends 45°, and the decagon, whose side subtends 36°, because they divide the circle with damage to the integrity of the signs. On this number of aspects see, if you wish, Ptolemy, Book 1 of *On the Judgments of the Stars*, Chapter 12, and Book 3 of the *Harmonics*, Chapter 8; Julius Firmicus, Book 2, Chapter 29; the *Introduction to Astrology* of Abū Maʿshar, Book 6, Chapter 8; Reinhold on the *Theoricae* of Peuerbach, from p. 179; Maurolico in the *Cosmographia*, p. 46; Magini in the *Directions of the Primum Mobile*, Canon 59; Kepler in the *Harmonice*, Book 4, Chapter 6, in the dissertation on the *Sidereus Nuncius*, in the *Ephemerides*, p. 33, and in the book *On the New Star and the Fiery Trigon*, from Chapter 9, who treats these matters most learnedly. [Margin: Aspectuum Numerus auctus à Recentioribus. — The number of the aspects increased by the moderns.] There, however, he admits among the aspects the decile biquintile aspect, which requires 144°, or two fifths of the circle, and the quintile, which requires 72°, and the semisextile, which requires 30°. And in the same author Fabricius admits the quincunx, 150°; and Mästlin admits, besides the quintile and biquintile, also the semiquintile, 36°, and the sesquiquintile, 108°, and the semisquare, 45°, and the sesquisquare, or *sesquadrum*, 135°. For lower nature is said to be stirred by these aspects. But the most celebrated aspects of all are the Great or Greatest Conjunctions of the superior planets, and their intervals, determined by the trine aspect of the signs of the zodiac, which are called triplicities.

III. [Margin: Triplicitates, seu Trigona Magnarũ Coniunctionũ. — Triplicities, or the trigons of the Great Conjunctions.] The triplicity, or trigon aspect, observable in the recurring Great Conjunctions of Saturn and Jupiter, is divided according to the nature of the 4 elements. The fiery trigon, or fiery triplicity, ends with its angles at the fiery signs, namely Aries, Leo and Sagittarius; the airy at the airy signs, Gemini, Libra and Aquarius; the watery at the watery signs, Cancer, Scorpio and Pisces; and the earthy at the earthy signs, Taurus, Virgo and Capricorn. The conjunction of Saturn and Jupiter takes place every 20th year in the signs of one class, and after every 200 years they pass into three other signs, and in about 800 years they complete all the triplicities with their meetings. Thus in the year of Christ 1583, 1643 and 1703 they have been and will be conjoined in Aries; in the year 1603, 1663 and 1723 in Sagittarius; and

in the year 1623, 1683 and 1743 in Leo. But in the year 1783, the passage to the watery signs having been made beforehand, it will take place in Pisces, etc., as the following figure shows after a fashion. The Greatest Conjunctions of the three superior planets, Saturn, Jupiter and Mars, require more years; this is not the place to treat of them, since there is a controversy over whether true conjunctions are to be considered, or mean ones, that is, according to the mean motions, and from what tables, or how, they are to be computed exactly. See on these matters, if you wish, Kepler in the book *On the New Star and the Fiery Trigon*; Aboassar, or Albumasar [Abū Maʿshar], *On the Great Conjunctions*; Bortholomæus Spina [?] and Io. Goclenius [?] on the same; Valentinus Naiboda [Valentin Naibod], Differentia 4; Origanus, Volume 1 of the *Ephemerides*, Part 3, Section 2, Chapter 1.

Diagram. Woodcut at the head of the right column, below the citations: a circle within a ring divided into twelve compartments, each holding one sign. Inside the inner circle are three bundles of lines running between three regions of the rim, so forming a set of nearly coincident triangles with their vertices near Aries at the top, Leo at the lower left, and Sagittarius at the right. Dotted lines run from the vertices to groups of years, each group set radially, so that the groups at the top and at the right read rotated or inverted. Labels read:

- ♈ (top, left of the head), ♓ (top right), ♒ (upper right), ♑ (right), ♐ (right), ♏ (lower right), ♎ (bottom), ♍ (bottom left), ♌ (lower left), ♋ (left), ♊ (upper left), ♉ (upper left)
- 1583, 1643, 1703, 1763 (near the top vertex, set sideways)
- 1603, 1663, 1723 (near the right vertex, inverted)
- 1623, 1683, 743 (near the lower-left vertex)

Labels not read: the third year of the lower-left group shows only "743" on the scan; no leading figure is visible, and the 1743 of the prose is not supplied to it. The block carries no point-letters and no figure number.

SCHOLIA.

I. [Margin: Reductio Aspectuũ. — The reduction of aspects.] When stars, because of latitude, do not attain the exact distance required for some aspect, one must find the time and the place at which the exact radiation suited to such an aspect will occur. How this is to be done is not agreed among everyone. On this reduction see Regiomontanus in the *Directions*, Problem 31; Magini in the *Directions of the Primum Mobile*, Canon 31; and Kepler in the *Ephemerides*, p. 37.

II. The question is often asked whether the aspect of the planets is to be considered between the bodies of the planets themselves, so that one planet then acts more strongly on another, or rather in their common radiation upon these lower things, through a triangle whose vertex is in the sublunary things and whose base is in the chord subtending the arc of distance required for

aspects distinct from the synod and opposition. On this question see Kepler in the *Fiery Trigon*, Chapter 9, arguing on this matter against Plotinus and Pico.

CAPVT XXXIV.

De Domorum Cœleſtium Diuiſione, & Erectione Thematis Cœleſtis.

On the Division of the Celestial Houses and the Erection of a Celestial Theme.

I. [Margin: Domus Cœlestes quid & quot. — What the celestial houses are, and how many.] The celestial houses, in Greek [Greek: δοδεκατημόρια [recte: δωδεκατημόρια] (dōdekatēmoria, "twelfth-parts")], are twelve segments of the zodiac, divided by the horizon, the meridian and four other great circles of the sphere, whether circles of position or of declination, according to the various systems astrologers have followed. For Avraham ibn Ezra and Regiomontanus use circles of position, which divide the equator into twelve equal parts but the ecliptic into as many parts that are not all equal; this is called the rational method [modus Rationalis], and is used by most of the more recent astrologers. Al-Qabīṣī, however, and his commentator Johannes de Saxonia use circles of declination, which divide the equator into unequal parts, namely the semidiurnal arc into three equal parts and the seminocturnal into three unequal ones; this method appears to have been Ptolemy's, inasmuch as he used hour circles, which divide the equator into 12 parts corresponding to two temporal hours each. But Campanus and Gazulus used circles of position, drawn through the sections of the prime vertical divided into 12 equal parts; for where circles of this kind cut the ecliptic, there are the cusps of the houses, that is, the beginning of one house and the end of another. These methods, with their practice, are carefully taught by Giovanni Antonio Magini in the *Directions of the Primum Mobile*, from Canon 23.

II. [Margin: Thema Cœleste quid. — What a celestial theme is.] A celestial theme [Thema cœleste] is nothing other than a figure representing, for any given moment of time, the position of the heavens, that is, the constitution of the twelve celestial houses. For example, if someone wants to know, at the moment when a certain person was born or when an equinox occurs, which point of the ecliptic was, or is, ascending on the horizon, and which at mid-heaven; and likewise for the other boundaries of the houses. To represent these things it is usual to use either a circle or the diagram below, in the middle of which are written the year, the month and the hour, with the minute after noon, for which the theme is ordered to be erected.

[Margin: Figura Thematis Cœlestis. — Figure of a celestial theme.] The first house begins from the eastern horizon, and its whole twelfth-part [dodecatemorion] lies below the horizon. It is called the Ascendant, because by the motion of the primum mobile its beginning is rising at the moment for which the theme has been constructed. For the names and order of the rest, look at the following little table; but beware of the errors condemned by papal

decrees, where the talk is of signifying future contingents. This, too, someone has enclosed in these little verses:

Diagram. Woodcut set into the right column, the text running down its left side: an outer square containing an inner square, the space between them divided by diagonal rules into twelve triangles, each numbered. The numbers run anticlockwise from the left. Labels read:

- 1 (left), 2 (lower left), 3 (bottom left), 4 (bottom centre), 5 (bottom right), 6 (lower right), 7 (right), 8 (upper right), 9 (top right), 10 (top centre), 11 (top left), 12 (upper left)
- An / M. / H.P.m. (three lines in the inner square)

Labels not read: none. The block carries no figure number.

Vita, Lucrum, Fratres, Genitor, Natiq. Valetud: Vxor, Mors, Pietas, & Munia, Amici, Inimici.

Life, Gain, Brothers, Father, and Children, Health; Wife, Death, Piety, and Duties, Friends, Enemies.

[Margin: Vanitatis Astrologicæ speculum. — A mirror of astrological vanity.]

Order of the houses [Domorum Ordo.]	Names of the houses [NOMINA DOMORVM.]			Significations [Significata]
1	Cardine of the East [Cardo Orientis]	[Greek: ὡρόσκοπος ἀνατολὴ (hōroskopos anatolē, "horoscope, rising")]	Horoscope, Ascendant [Horoscopus, Ascendens]	Life [Vita]
2	First succedent [Succedens prima]	[Greek: ἀναφορὰ (anaphora, "ascent")]	Lower gate [Inferna porta]	Hope, gain [Spes lucrum]
3	First cadent [Cadens prima]	[Greek: δεὰ [recte: θεά] (thea, "goddess")]	Goddess [Dea]	Brothers [Fratres]
4	Cardine of the lowest heaven [Cardo Imi Cœli]	[Greek: ὑπόγειον (hypogeion, "underground")]	Angle of the Earth [Angulus terræ]	Parents [Parentes]
5	Second succedent [Succedens secunda]	[Greek: ἀγατὴ τύχη [recte: ἀγαθὴ τύχη] (agathē tychē, "good fortune")]	Good fortune [Bona fortuna]	Children [Liberi]
6	Second cadent [Cadens secunda]	[Greek: κακὴ τύχη (kakē tychē, "bad fortune")]	Bad fortune [Mala fortuna]	Health [Valetudo]
7	Cardine of the West [Cardo Occidentis]	[Greek: δύσις (dysis, "setting")]	Setting [Occasus]	Spouse [Coniux]
8	Third succedent [Succedens tertia]	[Greek: ἐπικαταφορὰ (epikataphora, "descent")]	Upper gate [Superna porta]	Death [Mors]
9	Third cadent [Cadens tertia]	[Greek: θέος (theos, "god")]	God [Deus]	Religion [Religio]
10	Cardine of the mid-heaven [Cardo Medij Cœli]	[Greek: μεσουβάνημα [recte: μεσουράνημα] (mesouranēma, "mid-heaven")]	Mid-heaven [Medium Cœli]	Kingdom, offices [Regnum, Officia]

Order of the houses [Domorum Ordo.]	Names of the houses [NOMINA DOMORVM.]			Significations [Significata]
11	Fourth succedent [Succedens quarta]	[Greek: ἀγαθοδαίμων (agathodaimōn, "good daemon")]	Good daemon [Bonus dæmon]	Benefactions, friends [Benefacta, Amici]
12	Fourth cadent [Cadens Quarta]	[Greek: κακοδαίμον (kakodaimon, "bad daemon")]	Bad daemon [Malus dæmon]	Prison, enemies [Carcer, Inimici.]

III. Further, once the cusps of the houses have been found, that is, the degree and minute of the ecliptic at their beginning, that degree and minute are written in, with the character of the sign. And once the places of the planets at the same moment have been found, the character of the planet is written in, with the degree and minute of the sign in which it is found. This is done, however, so that if the cusp, or beginning of the domicile, has more degrees of that sign than the planet, the planet is then written before the cusp; but if fewer, it is written after, according, that is, to the succession in the zodiac.

The method of finding the cusps of the houses for a given time is taught by many writers on ephemerides in their introductions, or *Isagoges*, to the ephemerides, but especially by Magini, Stadius, Origanus and Argolus [Andrea Argoli]; let anyone who takes pleasure in these things consult them.

CAPVT XXXV.

De Parallaxibus.

On Parallaxes.

I. [Margin: Parallaxis notio & Definitio. — The notion and definition of parallax.] Parallax, in Greek [Greek: παράλλαξις (parallaxis, "parallax")], from the verb [Greek: παραλλάσσω (parallassō, "to stray")], which means to stray from a place or to vary a place, is a variation of view [diuersitas aspectûs] arising from the different situation of the place from which some phenomenon is observed. It can be defined as the difference between the true place of a phenomenon, observable from the centre of the Earth, and its apparent place, that is, its place as seen from the surface of the Earth. Its species are many, but the first and, as it were, the root of all the others is vertical parallax [Parallaxis Verticalis], or parallax of altitude. By virtue of it the same star or comet, at the same distance from the centre of the Earth and observed from the surface of the Earth, seems farther from the zenith and less high above the horizon than if it were observed from the centre of the Earth; and it appears the more depressed the greater its true distance from the vertex is, so that the parallax is greatest when the phenomenon is on the horizon. Moreover, at an equal altitude above the horizon, that phenomenon has the greater parallax which is nearer to the centre of the Earth. To make this easier to understand:

Diagram. Woodcut in the right column, below the table: a semicircle on a horizontal base line, with two smaller concentric semicircles and a small innermost one about the point A at the centre of the base. A vertical line rises from A to the top of the outer arc, and a second horizontal line crosses the figure above the base, from the outer arc on the left to the outer arc on the right. Straight lines radiate from A and from the point on the vertical where the upper horizontal crosses it, out to the outer arc on both sides. Labels read:

- V (top of the outer arc)
- D, E, and a third letter to the right of E (upper right, outside the outer arc)
- Q, T, X (upper left, outside the outer arc)
- G (upper right, on the second arc), C (on the inner arc)
- I (upper left), D (upper left, to the right of I)
- B (on the vertical, above the upper horizontal line)
- O (left end of the upper horizontal), N (its right end), M (above N), K (above M)
- L (right, on the inner arc), S (right, on the second arc)
- H (left end of the base), R (right end), A (centre of the base, below the line)
- Z (on the base, left of A), Y (on the base, right of A), I (below the base line, under Y)

Labels not read: the third letter to the right of E, which is [?] on the scan. Two letters on the block read as D, one at the upper right and one at the upper left; both are listed as seen. Every letter the prose below cites stands on the block; Q, T, X, E, M and the upper-right D are on the block but not cited in what is printed on this page.

II. In the following figure let the upper hemisphere of the Earth be ZBY, and the semicircle of the vertical circle HVR, passing through the vertex V of the inhabitant or observer B. Let a straight vertical line be drawn from the zenith V to the centre of the world and of the Earth, A, and let it be AV; at right angles to it let the rational horizon HAR and the sensible horizon [Horizon Sensibilis] OBN be drawn. From the centre A let two other semicircles be drawn, one for the heaven of the Moon, which is to be LCD, etc., the other for the heaven of the Sun, SGI; that is, for their vertical circle, which lies below the same vertical of the highest heaven and passes through the vertex of the observer B. Now let the centre of the Moon, L, first appear on the sensible horizon, and through the centre of the Moon let the straight line ALK be drawn from the centre of the Earth, A, as far as the highest heaven. With this laid down it is clear that the Moon L, which seen from the centre A would appear, and would be, raised above both horizons and distant from the vertex V by the arc VK, which is less than a quadrant, will nevertheless, seen from the surface of the Earth at B, appear on the physical horizon and distant from the vertex V by the whole right angle VBN, that is, by a full quadrant of the circle. Its true place, with respect to the centre of the world, will be the point K of the highest heaven;

the line of its true place, ALK; and its true distance from the vertex, the arc VK, which the angle VAK measures. But its seen place will be the point N; the line of its seen place, BLN; and its seen distance from the vertex, VN. Pa- rallax, then — the horizontal parallax [Parallaxis Horizontalis] in this case — will be both the arc KN, inasmuch as it is the difference between the true and the seen distance of the Moon from the vertex, and also the angle KLN: not because the arc KN has been described from that angle as from a centre, but because that angle is as large as the angle ALB, set vertically opposite to it, by the 15th of Book I of Euclid's *Elements*; and the angle ALB is as large as the horizontal parallax, as I shall show in Scholium 3.

III. Next let the centre of the Sun appear at S, a point of the sensible horizon. For, seen from B, it will seem to be on the horizon itself and to be distant from the vertex V by a whole quadrant; but if it were seen from A, it would be raised above the horizon. So its seen place in the highest heaven will be N, but its true place M; the line of its seen place, BSN, and of its true place, ASM; its seen distance from the vertex, VN, but its true distance, VM; and the difference between the true and the seen place, the arc MN. And so this will be the Sun's horizontal parallax: certainly less than the lunar one, because the distance of the Moon L from the Earth's centre A, which is AL, is less than AS, the Sun's distance from the same centre A; and therefore the arc and parallax MN is only a part of the whole lunar parallax KN.

IV. Now let the Sun I and the Moon D appear at the same altitude to the eye at B, seen so that the centres of both bodies are on the same line of the seen place, BX, and the apparent place of both beneath the highest heaven is the point X; and let the lines of the true place be drawn from the Earth's centre A through the centres of the luminaries. Of these, the straight line ADQ, through the Moon's centre D, will show the Moon's true place at Q, and the lunar parallax will be the arc XQ, as large as the angle XDQ. The straight line AIT, on the other hand, through the Sun's centre I, will show the Sun's true place at T, and the solar parallax will be the arc XD [recte: XT], less than the lunar parallax, as being a part of the whole.

V. Now let the Sun G and the Moon C be at the same true altitude, so that the line of the true place, D, is one and the same and passes through the centres of both, namely the line AD. Through these centres let two straight lines of the seen place be drawn from B: the line BCF, drawn through the Moon's centre C, will indicate the Moon's seen place at F and the lunar parallax DF; while the line BGE, through the Sun's centre G, will show the Sun's seen place at E and the solar parallax DE, less than the lunar parallax DF, as a part is less than the whole.

VI. Finally let the bodies be at the greatest altitude and appear at the vertex V; for whether they are seen from A or from B, the straight line of the true place, AV, and that of the seen place, BV, will coincide, and so there will be no parallax. These are the four cases of vertical parallax, that is, parallax observed in a vertical circle. With respect to the vertex it is called parallax of distance

from the vertex; with respect to the horizon it is called *parallax of altitude*, or in altitude. Concerning this species of parallax the following propositions are to be committed to memory. Although they can be demonstrated, they have been accepted by astronomers as certain axioms of a kind, and they can be demonstrated with very little trouble.

[Margin: Regula Parallaxium. — Rule of parallaxes.]

1. *A star situated at the vertex has no parallax.*
2. *A star placed on the horizon has the greatest parallax.*
3. *Of one and the same star, having the same distance from the Earth's centre, the parallax is greater at a greater distance from the vertex than at a lesser.*
4. *Of any two stars whatever, standing at the same true altitude, the parallax of the one that is less distant from the Earth's centre is the greater.*
5. *Of any two stars whatever, appearing at the same seen altitude, the parallax of the one that is less distant from the Earth's centre is the greater.*

SCHOLIA.

I. *The species of parallax are very many, according to the many species of situation in which objects, viewed from different places, can appear; but the chief ones in astronomy are the parallaxes of altitude, of depth, of distance, of longitude, of latitude, of right ascension, and of declination. More about these in our Optical book, in the Section on Parallaxes.*

II. *In no. 6 a fifth case could have been added: namely, when a star is on the rational horizon but appears below it. It has been passed over, however, because with the eye placed at B this cannot happen, since the eye cannot see what is below the sensible horizon unless it is raised above B. It remains, then, either that the star appears on the sensible horizon, and is then truly above it; or that it both is and appears at the vertex at once; or that two stars are at the same true altitude and differ in the seen altitude; or that they appear at the same seen altitude and disagree in the true.*

III. [Margin: Anguli Parallactici demonstratio. — Demonstration of the parallactic angle.] *It was said at no. 2 that the angle made at the centre of the star by the line of the true place and the line of the seen place is the measure of parallax; the angle ALB is of this kind. This is easily shown. For it is the angle by which the true distance from the vertex is exceeded by the seen distance. In that example the seen distance is the angle VBL, which together with the angle LBA makes two right angles, by the 13th of Book I of Euclid's* Elements. *The measure of the true distance, namely of the arc VK, is the angle VAK, that is, BAL, since the arc VK is described from the angle's point A as from a centre; and this angle, together with the angles ABL and ALB, is equal to two right angles, by the 32nd of Book I of* Euclid. *So, taking away the common angle ABL, there remains the angle VBL of the seen distance, equal to the angle BAL of the true distance and the angle ALB taken together. Therefore the angle ALB is the one by which the true distance from the vertex is exceeded by the seen distance. Similarly, in the triangle ACB, the angle at C is the measure of parallax, because it is the one by which the true distance, that is, the angle BAC, is exceeded by*

the seen distance VBF. For the angle ABC is the common complement to two right angles, both of those at B, by the 13th of Book I of Euclid, *and of those in the triangle ABC, by the 32nd of the same Book I. Taking away, then, the common angle ABC, there remains the angle VBC of the seen distance, equal to the angle BAC of the true distance and the angle ACB taken together. Therefore the angle ACB is the one by which the true distance is exceeded by the seen distance.*

IV. *The seen distance from the vertex either becomes known through instruments with which altitudes are taken from B, or is inferred from a given true distance from the vertex, whether by trigonometry or by tables made for this use, which I have in the end thought it better to keep back for the end of the book.*

CAPVT XXXVI.

De Refractionibus.

On Refractions.

I. [Margin: Refractionis notio ac Definitio. — The notion and definition of refraction.] REFRACTION, in Greek [Greek: ἀνάκλασις (anaklasis, "refraction"; literally "a bending back")], is the bent propagation of a direct ray, occurring at the common surface of two transparent media [diaphana] of different density. For by varied and long experiment it has been discovered that the rays of a luminous body and of any visible object whatever, falling on something opaque, are reflected back towards their origin, with the angle of reflection equal to the angle of incidence; but that, falling on something transparent yet of a density different from the medium through which they had so far been propagated, they do not go straight on along the same straight line, but are as it were broken and bent, that is, propagated along another straight line, though they do not return towards their origin; and that if the medium on which they newly fall is denser, they are broken towards the perpendicular, but if rarer, they are refracted away, departing or diverging from the perpendicular.

II. There are as many species of refraction of celestial rays in the air as there are of parallax, but for beginners it will be enough to have pointed out the primary species, from which the others spring: namely *vertical refraction* [Refractio Verticalis], or refraction of altitude, which must be used very frequently in astronomy, so that the true altitude of the stars may be distinguished from the apparent one, altered by the force of refraction. In the following diagram, then, let there be the quadrant COG of a vertical circle, described from the Earth's centre A, and beneath it the quadrant TB of a great circle of the Earth; and from the same centre of the Earth, A, let the spherical surface of the denser vapours be described, of which DRE is a quadrant: denser, I mean, than the air enclosed between DE and GOC. Let ABG be the rational horizon and TRO the physical horizon. Now let the star, as regards its true place or true altitude, be at S, and let its ray SR fall on R, a point of the surface of the denser vapours; to that point

Diagram. Woodcut set into the right column, the text running down its left side: a quadrant with its right angle at A at the lower left, the vertical side rising from A to C and the horizontal side running from A through B and E to G; the outer arc runs from C to G. Within it a heavily inked arc runs from D, on the vertical side, to E on the base; a lighter arc runs from T to B. Near the crossing of the heavy arc stands R: a horizontal line runs from T through R to the outer arc at O, a line runs from L through R, a line runs from A through R, and a line runs from R to the outer arc at S. Labels read:

- C (top of the vertical side)
- D, L, T (down the vertical side, in that order)
- A (the corner, lower left)
- B, E, G (along the base, left to right)
- R (on the heavy arc)
- O, S (on the outer arc at the right, O above S)

Labels not read: none.

let the perpendicular AR be drawn from the centre A. So the ray SR will not go straight to L, as it would if the medium were of the same density, but will be refracted towards the perpendicular RA. Let us now suppose it to be refracted to T. Then, since the eye sees an object along a straight line, and in the position in which it is represented by that straight line which immediately affects the sensory organ, the object will appear to it to be on the line TRO, or at the point O; and so it will appear on the horizon, even though it is in fact below the horizon. Conversely, if the object or luminous body were at T, and the direct ray were TR, here, on leaving the denser medium, it would be refracted from the perpendicular AR, departing from R towards S. In the first case the *ray of incidence* would be SR, the *ray of refraction*, or refracted ray, RT, and the *angle of refraction* LRT; in the second case the *ray of incidence* would be TR, the *refracted ray* RS, and the *angle of refraction* ORS.

Diagram. Woodcut set into the left column at the head of the page, the text running down its left side: the same construction as on the preceding page, a quadrant with its right angle at A, an outer arc from C to G, a heavily inked arc from D to E, a lighter arc from T to B, and lines through R from T to O, from L, from A, and from R to S. Labels read:

- C (top of the vertical side)
- D, L, T (down the vertical side, in that order)
- A (the corner, lower left)
- B, E, G (along the base, left to right)
- R (on the heavy arc)
- O, S (on the outer arc at the right, O above S)

Labels not read: none.

III. From what has been said it is now established that the effect of refraction is altogether contrary to the effect of parallax. For parallax represents a star or similar phenomenon as less high than it really is, whereas refraction represents it as higher: what is at S, for instance, it represents at O. This is also seen

in a coin placed at the bottom of water, in a stick seen broken, and in other things submerged under water, which because of refraction appear higher, and are seen also by those by whom they could not otherwise be seen. There are very many things, moreover, both worth knowing and necessary for the accomplished astronomer, about the refractions of stars and comets, and we shall speak of them at length in the Optical book. Meanwhile let the following propositions, accepted by all as axioms, be enough for the beginner whom we are instructing.

[Margin: Regula [?] Refractionū. — Rule [?] of refractions.]

1. *A star situated at the vertex has no refraction of altitude.*
2. *A star placed on the horizon has the greatest refraction.*
3. *Of one and the same star, retaining the same distance from the Earth's centre, the refraction is greater at a greater distance from the vertex than at a lesser; so much so that at about 45° refraction is scarcely perceptible, while near the horizon it grows with a headlong increase.*
4. *Between the refractions of two stars having the same true altitude above the horizon, that one is the greater which belongs to the star less distant from the Earth's centre, and this of itself.*

For Kepler admits that the fact that Tycho's little table shows refractions of the Moon very slightly smaller than the refractions of the Sun arose from a fallacious observation, though one honestly reported.

As for the tables of refractions, I judged it wiser to put them off to the end of the book, with the remaining astronomical tables.

SCHOLIA.

I. *Besides* the refraction of altitude there is the refraction of depth, of longitude, of latitude, of right ascension, of declination, and of distance *between two stars; these arise from the refraction of altitude or of depth. More on these in the Optical book, in the Section on Refractions, where more exact tables of refractions will also be presented.*

II. *When a true place or a true altitude is compared with the place or altitude seen by the force of refraction, this is understood with parallaxes set aside, or with the seen place and altitude already corrected and reduced to the true by the addition of parallax. So if some star has at once both a parallax of altitude, because of which it appears lower, and a refraction because of which it appears higher, than it really is, then to get the true altitude the parallax must be added to the seen altitude, and from the altitude thus corrected the refraction must be subtracted. But if the true altitude is to be converted into the seen, the parallax will have to be subtracted from it, and the refraction added to the remainder.*

III. *Although refraction differs from reflection chiefly in these respects, that the direct propagation of a ray is impeded in refraction by the density of the transparent medium, but in reflection by opacity, and that a reflected ray turns back towards its origin, whereas a refracted ray does not, this is nevertheless to be understood of*

simple transparency and density taken separately. For if some transparency of a denser medium is mixed with some opacity in a certain respect, reflection and refraction can occur together. Further, there is no contradiction in a ray's being first reflected and then refracted, or first refracted and then reflected. Those who treat of the rainbow or of twilights ought to have considered this, since they exclude refraction on the ground that the sight of the rainbow and of twilight's light comes about through rays reflected to our eye. But against these, below, in the Optical book.

LIBRI PRIMI FINIS.

End of the First Book.

LIBER SECVNDVS DE SPHÆRA ELEMENTARI AC PRÆCIPVÈ DE GLOBO TERRAQVEO.

Book Two: On the Elementary Sphere, and Chiefly on the Terraqueous Globe.

CAPVT I.

De Terræ simul, & Aquæ figura.

On the Figure of Earth and Water Together.

I. [Margin: Opiniones de Figura Terræ. — Opinions on the figure of the Earth.] This terraqueous element in which we live, that is, the one composed of earth and water, *Anaximander* judged to be like a column; *Leucippus*, like a cylinder or a war drum; *Cleanthes*, like a cone or a top; *Heraclitus*, like a bowl [Scaphio]; *Democritus*, like a hollow disc; *Anaximenes* and *Empedocles*, like a flat table; while *Xenophanes of Colophon* held that it rests on a root infinite downwards. So report, in part, Aristotle, *De caelo* Book 2 from text 76, in part Plutarch, *De placitis philosophorum* Book 3, chap. 10, and Laertius in their lives. But that this mass is, to the senses, spherical was first demonstrated by *Parmenides of Elea*, on the testimony of Aristotle; then by *Thales of Miletus*, who added that the Earth floats on the waters; and they were followed by the Stoics, and by the Peripatetics with their Aristotle, above. Among the cosmographers, the same roundness is shown by *Ptolemy*, *Almagest* Book 1, chap. 4, and *Theon* there, and *Regiomontanus* in the *Epitome*, concl. 2; *Cleomedes*, *De Mundo*, p. 376 in my copy; *al-Farghānī*, difference 3 [different. 3]; *Martianus Capella*, Book 6; *Copernicus*, Book 1, chaps. 2 and 3; *Fernel*, *Cosmotheoria* Book 1; *Maurolico*, *Cosmographia*, dialogue 1, p. 7; *Clavius* in the *Sphere*, pp. 3, 117 and 197; *Barozzi*, *Cosmographia* Book 1, p. 22; *Biancani* in the *Sphere*, part 3, tract. 3, chap. 4; and the interpreters of Aristotle, *De caelo* Book 2 from text 104; *Argoli* in the *Pandosion Sphaericum*, chap. 5; and *Kepler* in the *Epitome of Copernican Astronomy*, from p. 17.

II. [Margin: Argumentum 1. Rotunditatis Terræ. — Argument 1 for the roundness of the Earth.] The first argument is from the common name *orb of the earth* [Orbis terræ], which the Vulgate version of Psalms 23 and 92 uses, and which Pliny the Elder also touched on, Book 2, chap. 64, saying: *"We certainly call it the orb of the earth, and we admit that a globe is enclosed by poles; for the form*

of a perfect orb does not exist with such height of mountains and such flatness of plains, but it is one whose curves, if the ends of the lines are taken in by a circuit, would make the figure of a perfect orb." The height of the mountains, however, is not so great that, compared with the Earth's diameter, it can perceptibly impede its globosity; for its diameter is 10348 ancient Italian miles, while the height of the mountains is far below 50 miles, as will be clear on both points from our Geographical book. Some liken its roundness to the shell of eggs or of an orange, whose convexity is not spoiled by the modest unevenness of the pores, which stand for valleys, and of the little bumps, which stand for mountains.

III. [Margin: Argum. 2. Rotunditatis Terræ. — Argument 2 for the roundness of the Earth.] The second argument, a priori, is from the nature of earth and water, which by their very nature seek a spherical figure, because of the equal impetus with which their homogeneous parts tend to the same centre of heavy things, so that they stand at equal distances from it. And so, if the Earth, reduced to physical atoms without any glue of moisture to hold it together, were allowed to move to its own place, all its grains of sand would rush down to the common centre of heavy things, in the way that the parts of water flow together from higher places, until the parts of the surface were equidistant from the centre; and thus they would make up a spherical heap. For the equidistance of the surface from the centre is the property and the proof of a sphere, from what was said in Book 1, chap. 6.

As for the parts of water, all of them, and indeed of all liquids, that they form themselves into a spherical surface if allowed to move naturally until they settle and come to rest is so certain by experiment that it needs no demonstration. Archimedes nevertheless demonstrated this very thing in Book 1 of *On Things Carried in Water* [de his, quæ vehuntur in aquis], in that proposition 2: *"The surface of every fluid that has settled and stays at rest is spherical, and the centre of this sphere is the same as the centre of the Earth."* This can readily be understood in another way, as follows. Let the centre of the Earth be C, the hemisphere of the Earth APB, and two heaps of water MV and KN. For they will flow together towards P, as the lower-lying place, and will not settle until, the surface RGS having been formed, the parts of the watery surface stand at equal distances from the centre C. That they do not settle as long as some are further from the centre of heavy things is plain from daily experience; and that there is no greater reason why some should stand further from it than the rest is established from their homogeneous fluidity.

Diagram. Woodcut set into the right column, the text running down its left side: a semicircle on the base AB with its centre C marked below the base, and a higher arc over it. A straight line runs across the top from M at the left to N at the right, and straight lines run from M and from N down to C. Labels read:

- G (above the top of the higher arc)
- M (left end of the upper line), N (its right end)
- P (below the upper line, at the crown of the semicircle)

- R, V (at the left; V italic, inside the figure)
- S, K (at the right; K inside the figure)
- A, C, B (along the base, left to right)

Labels not read: none.

IV. [Margin: Argum. 3. Rotunditatis Terræ. — Argument 3 for the roundness of the Earth.] The roundness of the Earth together with the sea is gathered from the approach of ships and their withdrawal from shores, or from the sight of towers seen first near and then far off. For

Diagram. Woodcut across the right column: an oval globe with a winding coastline drawn on it, and straight lines radiating from the point P within it towards its upper rim. A straight line runs above the globe from F at the left through A to E at the right. On the upper rim stand small drawn objects: at the upper left two tall narrow pointed ones, one rising from the rim near H to a point beyond K, the other lying from the rim near G with its point at F; at the upper right one with a pennant, by D and B; and at the right another with a pennant, by C, its top towards E. Labels read:

- F (far left), K, A, D, E (far right), along the upper line from left to right
- H (below K), G (below H, on the left rim)
- B (below D), C (on the right rim, below E)
- P (within the globe)
- T (at the foot of the globe)

Labels not read: none.

the eye on the shore at A sees the whole height of the mast and the bottom of a ship setting sail from the shore; then, as the ship withdraws, it sees only the upper parts, and the lower parts, DB, are hidden; and finally the whole ship with its mast CE sinks, for that eye, below the sensible horizon AE. Pliny the Elder too notes this, Book 2, chap. 65. In the same way the eye A, which of the tower KH, equally tall, sees a part of it, yet sees nothing at all of the tower FG, which is just as tall. This is evidence that ships travel over the convex surface of the waters and that their masts stand perpendicular to the water, the line of direction DB and EC tending toward the same centre P — the point toward which the plumb lines of towers and of all buildings also tend. And these things have by now been observed on every sea and on every shore.

Diagram. Set in the left column below the paragraph above. A large circle (the meridian) carries eight letters on or just outside it, each at a small star; inside it, at the centre, a small circle (the terrestrial globe) with three little figures or balls on spokes above it. Straight lines through the common centre join V to D, N to M and H to R; a fourth runs from K through the centre toward G and stops just short of it. The three balls stand on the upper radii, one each on the lines to G, V and N.

- V (top of the outer circle): the zenith star of the argument below
- N (upper right, outer circle)
- R (right, outer circle)
- K (lower right, outer circle)

- D (bottom, outer circle)
- M (lower left, outer circle)
- H (left, outer circle)
- G (upper left, outer circle)
- A (left, on the inner circle's upper arc, beside the ball on the line to G)
- F (top centre of the inner circle, beside the ball on the line to V)
- B (right, on the inner circle's upper arc, below the ball on the line to N)

V Wherever one goes from north toward south along the same meridian, different stars keep coming into view. The southern stars rise higher and higher above the horizon while the northern stars sink lower and lower until at last they are hidden. And the increments of elevation on the one side and of depression on the other keep the proportion that the difference between those stars requires, as measured on the circumference of a circle. So people sailing from Holland or England, or from Portugal, toward the Hesperides and further south see the meridian altitude of the stars of both Bears grow less and less, until, once placed under the equator, they see the Pole Star either grazing the horizon or already hidden beneath its vapors. Conversely, they see the four stars of the Southern Cross, which they had not seen this side of the Torrid Zone, or see them more elevated where before they were less so.

[Margin: Argument 4 for the roundness of the Earth.]

You have a specimen of this experience in the figure above, in which the globe of the Earth is AFB and the meridian is HVRD. An inhabitant at A beneath this meridian sees the star G at his zenith. If he travels toward the south pole R, as far as F, he will now see that star at a distance from his vertex V by the whole arc VG. And if he goes on further under the same meridian to B, he will see the star G at a distance from his vertex by the whole quadrant NG, and now on the horizon GK. Other stars, which stood at a distance from the vertex, he will see at the vertex: the star V when he is at F, and the star N when he is at B. Others, finally, which he did not see while he was at A — such as the star R and K — he will see when he comes to F or B. And this happens not on some one meridian but on all meridians. It is therefore a sign that the Earth is spherical from south to north, since if it departed perceptibly from this figure these things would not appear in this way. This will be evident to anyone who draws the various figures, invented by the ancient natural philosophers, that we listed in number I.

IV [recte: VI] The risings and settings of the stars appear later to those further west, and so do eclipses. Thus for people who are 15° further west, the Sun sets one hour later [than for those to the east of them], and the beginning of the same lunar eclipse occurs one hour later — exactly as the Earth's spherical form requires. Shipmasters sailing around the whole globe from east to west have by now experienced this. This terraqueous mass is therefore spherical all around, and not merely in the direction of the poles of the world.

[Margin: Argument 5 for the roundness of the Earth.]

VII In addition, in every lunar eclipse, anywhere on land or sea, it has been observed that the Earth's shadow — by which, as we here assume, the Moon is darkened — imitates a circular disk. This is a sign that the body casting such a shadow onto the face of the Moon is round. It has rightly been called, then, the *orb of the Earth* [Orbis Terræ]. For figures supporting this argument, see Book 5, on eclipses.

[Margin: Argument 6 for the roundness of the Earth.]

VIII Finally, for those who inhabit the higher parts of the Earth the Sun rises sooner and sets later than for those living low down, and from more elevated places an ever wider prospect of plains and sea opens up. We would by no means experience this if the Earth were flat in the manner of a disk or a table, or of a cubic surface, etc.

[Margin: Argument 7 for the roundness of the Earth.]

SCHOLIA.

I Pliny the Elder (Book 2, Chapter 64) does not deny that the Earth is globular if its circuit is taken through the mountain tops. He held, however, that the mountains on its surface are much higher than Dicaearchus had said. Dicaearchus, having measured Pelion, the highest mountain of Thessaly, and found that its perpendicular height did not exceed 1250 paces, had concluded: "That height is nothing — in proportion, that is, to the whole roundness of the Earth." But Pliny the Elder raises two objections against him, namely "so great a loftiness of mountains, so great a flatness of plains," and adds: "To me this conjecture seems uncertain, since I am not unaware that certain peaks of the Alps rise over a long stretch, no shorter than fifty thousand paces." Pliny therefore thinks it follows from the ascent of the Alps of at least 50 miles that they stand perpendicularly much more than 1250 paces high. This is perfectly true, considering how steep their ascent is in places, as I shall say in the *Geographical Book*. But even if we granted that the Alps are 50,000 paces high, that height, compared with the Earth's circumference of 32512 old Italian miles, would not stand against the roundness Dicaearchus asserted, nor against the phenomena discussed above. I shall touch on opinions about the height of mountains below, in this Book, Chapter 11.

[Margin: Whether the height of mountains stands against the roundness of the Earth.]

II As for the flatness of plains and the level of the sea, a twofold deception of sight is hidden here. Opticians teach one of them when they show, with Ibn al-Haytham (Book 3), that the more distant points of a pavement appear to us more elevated because they are seen by a higher ray; hence the curvature of the sea appears flat, while the flatness of plains appears raised toward the horizon, so much so that the land seems hollow in the manner of a bowl, as Heraclitus said. Maurolico touches on the other in Dialogue 1 of the *Cosmographia*, as follows: "What you survey of land and sea, standing on this hill, is so tiny compared with the whole magnitude that you cannot gauge any curvature in it; just as if you cut from a large circle, of a thousand paces more or less, a circumference two

or three feet long, you would see no bending in it." I myself have tested this on a circumference, with a small nail tied to a rope of fifty feet and carried around: a foot-long part of the circumference had a barely perceptible curvature and looked nearly like a straight line. The whole semicircumference was 157 feet. And yet the proportion of 1 foot to 157 is far greater than that of 70 miles to the 16256 miles contained in the semicircumference of the Earth's upper hemisphere. Now 70 miles of sea cannot be seen unless the lookout or mountain is 600 paces high perpendicularly, as will be shown in Chapter 10 and in the *Geographical Book*, where visual distance is treated; and someone standing on the shore itself sees no more of the sea than 3 miles, or at most 4. What wonder, then, my dear Pliny, if so tiny a portion seems flat to him, for the two causes stated?

[Margin: Why the sea and the plains look flat.]

III As for what Francesco Patrizi [Franciscus Patritius] brings forward in Books 25 and 26 of the *Pancosmia* against Archimedes, Aristotle, Clavius and other defenders of the roundness of Earth and water, it shows so great an ignorance of geometry [ageometriam] that one is reluctant to report it, let alone refute it. Still, I shall point out one or two of his errors. For this, look at the figure set beside number 3 of this Chapter 1. He says, then, that if water lies on the plane MPN it does not flow together to the lowest place, and that the parts M and N are not higher than the point P physically, granted they are so mathematically. He adds that as he withdrew from the town of Comacchio the bottoms of the houses gradually disappeared, and that at length, after ten miles covered on that lagoon, the tower of the principal church, 72 feet high, vanished entirely. And yet the bottom of the lagoon did not exceed two feet, whereas, he says, it should have been 72 feet deep if the tower had to be hidden because of the swelling and roundness of the water.

[Margin: Patrizi's mistaken opinion against the roundness of the Earth.]

But you are hallucinating, my dear Patrizi. Look at figure 2, at number 4 of this Chapter, and let the tower FG in it be 72 feet high, and let the distance FA from the eye A be 10 miles, or 50000 feet. The entire tower below its apex F will be hidden from the eye A, even if it has not sunk at all below the surface of earth or water, as trigonometry will teach you from the true semidiameter of the Earth PG, if you are versed in it. You, however, supposed that the tower FG stands perpendicular to the horizontal line FA. But if that were so, you would have had to be sunk down to 72 feet. Yet you did not descend in that way, and neither do others sailing, from whose view towers standing on the shore — the Lanterna, say, or the Pharos of Genoa — at length pass out of sight;

they are therefore hidden for no other reason than the curved surface of the sea.

IV. But Patrizi presses us with two further experiments in Book 26. He relates that in the year 1562, when he was about to sail to Cyprus and the ship was still moored at the Venetian shore, a mountain of Liburnia overlooking Assuero, two hundred thousand paces from Venice, was seen in the morning; but once the Sun had risen it was no longer seen, while he sailed toward it.

And in the year 1574, when he was sailing beyond Toulon [Tolonem] in the Tyrrhenian Sea, the mountains of Corsica and almost the whole island were sighted, although it is more than 300 thousand paces away. And he concludes: "If over so great a space the surface of the sea had been swollen, it would have taken from our sight either the whole mountain or the greatest part of it." But another would infer more plausibly that those mountains are 12 thousand paces high perpendicularly and the mountain of Liburnia nearly six thousand, since the former could be seen from the surface of the sea at an interval of 300 miles and the latter at 200 — were it not that refraction stands in the way. According to Snel and Kircher, refraction raises the appearances of things, especially near the horizon, and above all for those sailing toward the sunrise; and when refraction ceases, many things lie hidden beneath the physical horizon which by its force appeared above it. But there is more on this in the Optical book.

CAPVT II.

Terram esse Centrum Mundi, & in Centro Cœli supremi Stelliferi, saltem quoad sensum.

That the Earth Is the Centre of the World, and at the Centre of the Highest Starry Heaven, at Least to the Senses.

I. [Margin: Opiniones de Terræ loco in Mundo. — Opinions on the Earth's place in the world.] This is the view not only of the Peripatetics, with *Aristotle*, *De caelo* Book 1 from text 96, and *Pliny the Elder*, Book 2, chap. 69; and of the cosmographers, with *Ptolemy*, *Almagest* Book 1, chap. 5; *al-Farghānī*, difference 4; *Johannes de Sacrobosco* in the *Sphere*, chap. 1, and *Clavius* there, p. 135; *Maurolico*, *Cosmographia*, dialogue 1, p. 9; *Biancani* in the *Sphere*, Book 4, chap. 1; and *Barozzi*, *Cosmographia* Book 1, p. 29. It is also the view of the *Pythagoreans* and *Copernicans*. Although they place the Sun at the centre of the world and say that the Earth moves around it in the orb in which the others hold that the Sun moves — the orb they call the annual orb [Orbem Annuum], from the yearly revolution in the zodiac of the Earth and not of the Sun — they nevertheless admit that the distance of the fixed stars from the Earth is so great, and that not only the globe of the Earth but the annual orb itself is so small, that compared with the distance of the Aplanes, or eighth sphere, it has an imperceptible ratio to it. All phenomena therefore appear to us, to the senses, just as if the Earth were in the middle of the world, so that it can be called the centre of the world in a physical or loose sense, granted that for them it is no such thing in astronomical rigor, the Sun being that centre. Setting sacred authority aside, the arguments by which it is customarily proved that the Earth is in the middle of the world in fact show nothing else than that the Earth is, to the senses, in the middle of the world with respect to the highest starry heaven, as we shall soon see, or that it lies between the Sun on the one hand and the Moon and the three superior planets on the other, when these are

in opposition to the Sun, as the Moon is at full moons. For this is the sum of the arguments.

II. [Margin: Arg. 1. pro Terra in cētro Mundi. — Argument 1 for the Earth at the centre of the world.] First, if the Earth were not in the middle of the world, the horizons of all the inhabitants of the Earth would not cut it in two together with the heaven. Nor would six of the twelve signs of the starry zodiac be seen above the horizon, nor half the heaven. Nor, of two stars diametrically opposite — as the southern eye of Taurus and the heart of Scorpius nearly are — would one be seen on the eastern horizon when the other is on the western horizon. Instead, some would see much more than half the heaven and others a much smaller part, etc., which is contrary to the experience of every age. To make this easier to grasp, let the Earth first be BC, let the centre of the Earth and of the universe be C, let the highest heaven of visible things be HMR etc., and let the rational horizon be HCR and the sensible horizon ABD. In this way the eye B will see half the heaven, if not HMR, at least AMD; and of two stars diametrically or almost diametrically opposite, such as H and R or A and D, when one rises at H or at A, the other will be seen to set at R or at D.

Diagram. Woodcut in the right column near the head of the page: a large circle with four parallel horizontal chords, and two small circles inside it on its vertical centre line, the upper about O and the lower about C. The chord from E to L runs through the top of the upper small circle at N; the chord from F to G runs through the upper small circle's centre O; the chord from A to D runs through the top of the lower small circle at B; the chord from H to R runs through the lower small circle's centre C. Labels read, none unread:

- M (top, above the large circle)
- N (above the upper small circle)
- E (upper left) and L (upper right), ends of the first chord
- F (left) and G (right), ends of the second chord
- O (inside the upper small circle)
- B (above the lower small circle)
- A (left) and D (right), ends of the third chord
- H (left) and R (right), ends of the fourth chord
- C (inside the lower small circle)

Let the Earth next be at O. Its rational horizon will then be FOG and its physical horizon ENL. The horizon, therefore, would not cut the heaven in two, nor would it any longer be a great circle [circulus Maximus] in the sphere of the world, as is commonly supposed. Nor would the eye N see half the heaven, but only the portion EML. Nor would it see two stars 180° apart, such as H and R, opposed to each other on its horizon.

III. [Margin: Arg. 2. pro Terra in cētro Mundi. — Argument 2 for the Earth at the centre of the world.] Fixed stars on the equator or very near it are observed on clear nights to move successively by the diurnal motion, from their rising above the horizon to their setting, in such a way that they are above the horizon for 12 equinoctial hours, when the nights are at least 12 hours long.

This would not happen if the Earth were not, to the senses, in the middle of the universe but were at O. For a star ascending from E would hurry to its setting, through M, at L much sooner than in a space of 12 hours; nor in the meantime would half the equator, HMR, ascend, but a far smaller arc of it, namely EML.

IV. [Margin: Arg. 3. pro Terra in cētro Mundi. — Argument 3 for the Earth at the centre of the world.] The same stars, seen on the same clear night on the horizon, appear either equal to themselves when they have come to the meridian, or rather, because of the vapors of the horizon, larger. This is a sign that they are, to the senses, equally distant from the Earth, and the Earth from them. If you are concerned about the vapors of the horizon, look at the same star — the Fidicula of the Lyre [Lyræ fidiculam], say, or Arcturus — on different meridians, for you will see it of the same magnitude everywhere. But if the Earth were perceptibly nearer the heaven on one side than on the other, say at O, the same star on the meridian, say at M, would appear much larger than at the point E or L of the horizon; for what is nearer appears larger.

By the same reasoning, the distances of the fixed stars from one another would not appear, as they do in fact appear to us, always the same and equal to themselves. For example, I have always observed the distance of Arcturus from Regulus to be 59° 49′, whether near the horizon or far from it, and everyone anywhere will always observe the same. But if the Earth were not physically in the middle of the universe, any two of the fixed stars would appear more distant from each other near their meridian altitude than near the horizon to those who were nearer their zenith than their nadir, as to the inhabitant N. Conversely, to those living in the opposite hemisphere, who were further from their zenith than from their nadir, they would appear more distant near the horizon than near the meridian. For the same distance between two visible things appears greater at a smaller distance from the eye and less at a greater one; and modest refraction near the horizon would not account for so great a difference.

V. [Margin: Arg. 4. pro Terra in cētro Mundi. — Argument 4 for the Earth at the centre of the world.] If the Earth were outside the middle of the universe, that is, outside the plane and axis of the equator, it would be either on the axis or off the axis of the equator. If it were on the axis of the world and of the equator, perceptibly nearer one of its poles than the other, and outside the zodiac, no horizon would cut the zodiac into two equal parts except the right horizon; and so outside the right sphere there would be no equinox anywhere. But then the inhabitant of the right sphere would not be on the equator, and those nearer the pole would have a day always shorter than the night. If, on the other hand, it were within the zodiac and on some parallel that the Sun runs through, the equinox in an oblique sphere would occur on the day on which the Sun runs through that parallel, and not on the days on which it now occurs. And if the Earth were in the plane of the Tropic of Cancer, the equinox would occur on the day on which the Sun, now coming closest to the north pole, makes the solstice:

nor on the equinoctial day would the shadow of a gnomon set up perpendicular to the horizon constantly trace with its tip one and the same straight line. These results run against the established experience of astronomers, as Clavius shows in the *Sphere*, from p. 139, with many figures added. In place of them, look at the figure of the armillary sphere placed at the beginning of the Book, and imagine the Earth in the positions just stated.

But if the Earth were in the plane of the equator but not on its axis, there would be no equinox in the right sphere except on the one horizon that passed through the centre of the world. In the oblique sphere, either there would be no equinox, or it would not fall midway in place and time between the winter and summer solstices; nor would the excess of the longest day over 12 hours equal the shortfall of the shortest day. Finally, if the Earth were located neither on the axis nor in the plane of the equator, we would run into all the absurdities just stated, which conflict with the plainest experience.

VI. [Margin: Arg. 5. pro Terra in cētro Mundi. — Argument 5 for the Earth at the centre of the world.] Since the Earth is the heaviest of all things, it tends simply downward. If it were placed outside the middle, whether whole or cut into parts, it would naturally tend as a whole, as its parts also do, downward and to the centre of the universe, so as to stand equally distant from the place that is, and is called, up, namely the highest heaven. For there would be no greater reason why it should come nearer to one part of the heaven than to another; indeed, there is reason for the opposite, so that it may share equally in the influences and benefits of the heaven. The place naturally due to the Earth is therefore the middle of the world and the centre of the universe. So everyone rightly says and acknowledges that the Earth is in the lowest place of the world, and most true is that saying sanctioned in Holy Scripture, Proverbs 25: *"The heaven above, and the Earth below."* The Copernicans, however, evade this argument.

SCHOLIA.

I. We are not here dealing with the Pythagoreans and Copernicans, who set the Sun motionless at the centre of the world and have the Earth revolve around the Sun in the Ptolemaic orb of the Sun. For to understand their hypothesis and the answers to their arguments is not a matter for a novice but for a more accomplished astronomer. These things are therefore to be treated by us in Book 9, where the systems and hypotheses of all astronomers are discussed, and also in the small work on this subject that we have in hand, entitled *Apology for the Sacred Censure, Passed by the Holy Congregation of Cardinals, against Those Who Assert the Annual and Daily Motion of the Earth* [Apologia sacræ censuræ contra Assertores motus Telluris annui, & diurni, latæ à S. Congregatione Cardinalium].

II. [Margin: Quantam Cœli portionem Terra nostro visui intercludat. — How large a portion of the heaven the Earth shuts off from our sight.] We do

not always see exactly half the heaven. One reason is the vapors of the horizon, which by their opacity obstruct the sight of the fixed stars in it. Another is the position of the spectator, who sometimes is not raised at all above the surface of the Earth and so cannot see what lies below the sensible horizon. This horizon is not a great circle of the sphere [Circulus Sphæræ maximus], but cuts it into a smaller segment, which we see, and a larger, which is hidden from us. Yet if he is raised some paces onto a hill or a tower or a similar lookout, he can see even more than half the heaven. And if he is not raised, that portion of the heaven AH or DR which the Earth's semidiameter takes from us is nevertheless very small and imperceptible. For it is exactly as large as the horizontal parallax of the fixed stars, which in everyone's opinion is imperceptible, and the smaller the further those stars are from the Earth. For this, look at the following little table [laterculum].

In it I have passed over the opinion of the Copernicans, because they do not agree on the distance of the fixed stars from the Earth. Some set it at a minimum of 7850000 semidiameters of the Earth; others with Lansbergen extend it to 41958000 semidiameters; others with Kepler stop at 34077067 semidiameters, or with Hortensius at 10300190. It is certain among them, however, that the said parallax, and the portion of the hemisphere hidden from us, are much smaller than among the Tychonians or Ptolemaics. Against these, Mario Bettini did not hesitate to put the distance of the fixed stars at only 2290 semidiameters of the Earth, on account of certain harmonic intervals; and from that distance follows a sensible parallax, namely 1½′. But there is more on these matters in the Optical book, where parallaxes are treated, and in the Book on the fixed stars.

Distance of the fixed stars from the Earth	In semidiameters of the Earth	Horizontal parallax of the fixed stars ″	‴
Ptolemy and al-Battānī	19000	11	0
al-Farghānī	20110	10	0
Tycho	14000	14	44

III. [Margin: Argum. ab Eclipsibus pro situ Terræ quanti valeant. — What the arguments from eclipses are worth for the position of the Earth.] The reader may perhaps wonder that I have left out the argument from lunar eclipses, which Regiomontanus, Maurolico, Biancani and Clavius — the most brilliant lights of astronomy, that is — use to show that the Earth is situated in the middle of the universe. For they say that if the Earth were not in the middle of the world, lunar eclipses would not always occur in diametrical opposition to the Sun, but sometimes when the Sun and Moon are far fewer than 180° apart in the zodiac, as is shown in the first figure of this scholium. For although with the Earth placed in the middle at A, the Moon L would be eclipsed by the shadow AV, in opposition to the Sun S, yet with the Earth placed outside the centre of the world, as at I, the Moon P would be eclipsed by the Earth's shadow IB, while less than a semicircle of the zodiac from the Sun.

Diagram. Woodcut set into the right column, the text running down its left side: two concentric circles, with a small circle at the right overlapping the outer circle and carrying S within it. From that small circle two long narrow cones run leftward, one through a dark globe in the upper part of the inner circle and out past the outer circle to B, the other through a dark globe in the lower part of the inner circle and out past the outer circle to V. Labels read:

- B (upper left, outside the outer circle, at the point of the upper cone)
- P (top of the inner circle, on the upper cone)
- a small drawn Moon just below P, on the upper cone (a dark disc with a crescent rim)
- I (right of the upper dark globe)
- A (below the lower dark globe)
- V (left, outside the outer circle, at the point of the lower cone)
- L (just right of V, below the lower cone)
- a small drawn Moon just right of L, on the lower cone (a dark disc with a crescent rim)
- S (inside the small circle at the right)

On the other hand, they say, if it is granted, even for two eclipses, that the Moon is in diametrical opposition to the Sun, as astronomical observation and calculation require, it follows necessarily that the Earth is on both of those diameters. And since all diameters of the world intersect at the centre of the world, it must be that the Earth is in the middle of the world. They explain this by the second figure here, in which the Sun in its heaven and the Moon in its own are diametrically opposite in two eclipses, and the diameters intersect at the point C.

Diagram. Woodcut set into the right column, the text running down its left side: two circles one within the other, the inner about the point C. Two straight lines cross at C. At the upper ends of the two lines stand two rayed suns, upper left and upper right, on the upper part of the circles; at the lower ends stand two dark crescent moons, lower left and lower right, on the inner circle. One label read, none unread:

- C (at the crossing of the two lines)

This argument seems to have greater force in horizontal eclipses of the Moon, that is, when the Moon is eclipsed while rising as the Sun sets, or while setting as the Sun rises. Pliny the Elder reports such an eclipse in Book 2, chap. 13, and Mästlin one seen by himself, in Kepler's *Astronomia Optica*; and such, with respect to the horizon of Rome, will be the eclipse of 1668 May 25. So Argoli raises a triumphal song there against Copernicus, believing he has established from this that the Earth, and not the Sun, is at the very centre of the world. Nor did Clavius, in the *Sphere*, pp. 137 and 143, recognize the paralogism; but like good Homer he nodded here.

[Margin: Argoli, & Clauij argum. claudicans. — The limping argument of Argoli and Clavius.]

These authors, however, reason falsely, or at any rate equivocate, from the diameter of the heaven of the luminaries to the diameter of the world. For from the said eclipses it is established only that the Earth comes between the Sun and the Moon, that these three bodies are on the same straight line, and that the Moon is then beneath the part of the ecliptic opposite to the part beneath which the Sun is. All of this can be true even if the Sun were at the centre of the world and the Earth outside it — not only where Copernicus places it, but much further from the centre of the world — provided that the Earth's shadow reached the Moon and the Earth were between the Sun and the Moon, as we shall show in its place. This argument therefore limps on one leg, and does not prove the more important part of the desired conclusion.

CAPVT III.

De Terræ Immobilitate. — On the Immobility of the Earth.

I. [Margin: Motus Terræ quot? — How many motions has the Earth?] Four motions of the Earth are commonly debated. The first belongs to its parts and is called, by a single name made up of two words, an *earthquake* [Terræmotus]. It occurs when an exhalation enclosed in the bowels of the earth and ignited seeks to rarefy and expand; having no room sufficient for itself, it seeks room with great and very swift force through the hollows of the earth, and so shakes parts of the earth and the foundations of buildings until it finds an exit through a vent. In the same proportional way an exhalation ignited within clouds produces thunder while seeking an exit. We are not speaking here, however, of this motion of the earth, but of the motion of the whole Earth.

II. [Margin: Trepidatio Terræ an vlla. — Whether there is any trepidation of the Earth.] The second is the motion of *trepidation* [Trepidationis], whose proponents were *Marsilius* [Marsilius of Inghen], on Book 8 of the *Physics*, tract. 1; *Maior* [John Mair], on Book 2, dist. 24, quaest. 12; *Johannes de Saxonia*, on *De caelo* 2, quaest. 10; *Vasquez* [Gabriel Vázquez], on the *Prima Secundae*, disp. 83, chap. 3; *Scheiner*, in the *Disquisitiones Mathematicae*, no. 15; and *Cottunius* [Ioannis Kottounios], on Book 1 of the *Meteorologica*, lect. 17. For they say that the Earth's centre of gravity must coincide exactly with the centre of the Universe. Since the centre of gravity within the Earth is perpetually changing, because of the collapses and landslips of mountains, diggings of the ground on one side and heapings-up on the other, new buildings, etc., the Earth too, they say, nods this way and that with a perpetual but imperceptible wobbling.

The contrary opinion, however, conforms better to that stability of the Earth which the sacred writings teach, especially Psalm 104: *You who founded the earth upon its stability; it shall not be inclined for ever and ever* [Qui fundasti terram super stabilitatem suam, non inclinabitur in sæculum sæculi], and which the theologians teach in *Raynaudus* [Théophile Raynaud], *Theologia naturalis*, dist. 4, quaest. 3, art. 3. These, with St Gregory (*Moralia* 2, chap. 15) and St Thomas (*De Malo*, quaest. 16, art. 10), say that not even the angels themselves

can naturally move the earth the least bit from its place. And so, finally, do the ancients, who for that reason called the Earth *Vesta*, because it stands by its own force, according to that line of Ovid:

The Earth stands by its own force; from standing by force it is called Vesta [Stat vi Terra sua, vi stando Vesta vocatur.]

The reason is this. The mass of the Earth, with all its weight, exceeds any weight shifted about on its surface by far more than any weight immovable by the motive power of one man, or even of several, exceeds that power. And it is established that there are weights which no human force, no lever and no machine can budge. Therefore neither can weights placed on top of the Earth budge it, even if they were Olympus, Pelion and Ossa. If you say that it moves of itself so as to stand in equilibrium, the case is the contrary. Any weight that is usually added to the Earth on one side has a smaller proportion to exciting its motion than the wing of a fly, or the lightest speck of dust, added to one of two scale-pans that rest in equilibrium with ten equal pounds on each side; especially since, for motion upwards or downwards, the resistance of all the air surrounding the opposite hemisphere has to be overcome. For Cleomedes (*De Mundo*, Book 1) and Seneca (*Naturales Quaestiones*, Book 4, quaest. 11) rightly said that the Earth is to the mountains as a huge ball to a hair [ingentem pilam ad pilum]; whether you say with Simon Stevin (*Statics* [Statices], Book 3) that the weight of the Earth is 2,000,000,400,000,000,000,000,000 pounds, or with *Forerius* [Lorenz Forer], in the *Viridarium disputationum philosophicarum*, on impulse and the centre of gravity, part. 47, that it is 899,564,914,285,714,283,714,285½ pounds. So, just as not any and every effort of ours is enough to move any and every weight, so not any and every heaviness is enough to shift the Earth from its equilibrium, whatever that celebrated boast Archimedes made to Hiero, more magnificent than true:

Tell me where I may stand, and I shall move Heaven and Earth [Dic vbi consistam, & Cœlum Terramq. mouebo.]

For the better theologians judge that this has not been granted by God even to the angels.

III. The third motion is that of daily revolution, which is called the motion of *rotation* [Vertiginis], by which in the space of 24 hours the Earth is said to move about its centre and axis from west to east — [Margin: Vertigo Terræ circa suum cētrum an asseri possit. — Whether a rotation of the Earth about its centre can be asserted.] said so by Plato, *Hicetas* of Syracuse, *Ecphantus* the Pythagorean and *Heraclides* of Pontus, and afterwards by the Copernicans and the Semi-Copernicans. To the second class belong *Gulielmus Gilbertus* [William Gilbert], *Longomontanus*, *Mulerius*, *Origanus* and *Argoli*. For so as not to admit in the stars two continuous and contrary motions, one common and daily from east to west, the other proper from west to east, and so as not to grant the fixed stars an incredible speed, they say that it is the Earth itself which, together with the atmosphere, indeed with almost the whole elementary region, moves towards the east. It therefore goes to meet the stars, and these

seem to move towards the west, though they move by no other motion than towards the east: just as, to people being carried in a boat towards one quarter of the world, the shores and banks seem to move and to recede towards the opposite quarter, although in fact they stand still. And they press on us that line of Virgil, *Aeneid* 3:

We sail forth from harbour, and lands and cities recede [Prouehimur portu, terræq. vrbesq. recedunt.]

But this opinion, when asserted absolutely and not merely hypothetically, was condemned by the Sacred Congregation of Cardinals appointed by Pope Paul V for the Index of Prohibited Books, on 1616 March 5; and again, by order of Urban VIII, by the same Congregation, on 1633 June 22, where it is judged *at the least erroneous in Faith* [Ad minus erronea in Fide]. For in the sacred writings the daily motion is attributed not to the Earth but to the Sun: thus Ecclesiastes 1, *The Sun rises and sets, and returns to its place, and there being reborn*, etc. [Oritur Sol, & occidit, & ad locum suum reuertitur, ibiq. renascens &c.]; and Joshua held it back from this motion (Chapter 10); and very often the Sun is said to have risen, as in Genesis 15 and 19, Judges 19, etc.; and by this motion the Sun went backward on the dial of Ahaz (Ecclesiasticus 48 and Isaiah 38). Nor is there any necessity to depart from the literal sense of Holy Scripture, since the opponents themselves admit that, given the daily motion of the stars, all the phenomena of astronomy are saved in full rigour; and that contrariety of two motions can be produced by a spiral motion, or by the daily motion alone, as we shall show in its place. The said Congregation further calls this opinion *Absurd and false in Philosophy* [Absurda, & falsa in Philosophia]; and rightly. For it was introduced without any necessity, against the common evidence of the senses, to which it is physically evident that the stars move from east to west and that the Earth does not so move, even granting that this is not mathematically evident; nor is it established from any other source that the sense is to be corrected, as it is established in some other cases.

IV. [Margin: Translationis motus an Terræ tribui possit. — Whether a motion of translation can be attributed to the Earth.] The fourth motion of the Earth is that of *translation* [Translationis], which is called the *annual* [Annuus]. By it, with the Sun standing still in the centre of the Universe, the Earth's centre is said to move through that orb in which, according to Ptolemy, the Sun moves with an annual motion of 365 days and about 6 hours; so that the retrogradations and stations of the planets are apparent, not real, and many other astronomical matters are handled better on this hypothesis than if the Earth stood still and the Sun were said to move in the annual orb. So thought long ago, about the annual as well as the daily motion of the Earth, *Philolaus* the Pythagorean and *Aristarchus of Samos*, as is reported of them by Aristotle (*De caelo* 2, text 72), Plutarch (*De Placitis* 3, chap. 13), Laertius in their lives, and Archimedes in the *Sand-Reckoner* [Arenario]. The same was asserted later, not merely hypothetically but absolutely, by Nikolaus von Kues, Nicolaus Copernicus in his *De revolutionibus*, both in the preface and in Book 1, Chapters 8, 10 and 11;

Ioachimus Rheticus [Georg Joachim Rheticus], *Michael Mästlin*, *Schickard*, *Kepler*, *Rothmann*, *Cœlius Calcagninus* [Celio Calcagnini], *Didacus Astunica* [Diego de Zúñiga], *Paulus Antonius Foscarinus* [Paolo Antonio Foscarini], *Galileo*, *Simon Stevin*, *Philips and Jacob van Lansbergen*, *Pierre Hérigone* and *Ismaël Boulliau*. For Gassendi, out of reverence for the decrees of the Sacred Congregation, at the end of Letter 2 *De motu impresso à motore translato* at last acquiesced in the immobility of the Earth.

But this opinion was condemned far more strongly by the same Sacred Congregation, cited above, in whose censure it is called *Absurd and false in Philosophy, and, as regards the immobility of the Sun, formally heretical, as being expressly contrary to Sacred Scripture; but as regards the motion of the Earth, at the least erroneous in Faith* [Absurda, & falsa in Phylosophia, & quoad Solis immobilitatem, formaliter hæretica, vtpote expressè contraria Sacræ Scripturæ; quoad motum verò Terræ, ad minus erronea in Fide]. For not only is the Earth called stable, immobile, etc. in the Sacred Writings (Paralipomenon 16, Psalms 92 and 104, and Ecclesiastes 1), but also, and above all, the Sun is expressly asserted to move with an annual motion, Ecclesiastes 1: *It circles through the south and bends towards the north*, etc. [Gyrat per Meridiem, & flectitur ad Aquilonem &c.] This motion too is against the physical evidence of the senses; nor does any reason compel us to depart either from this evidence or from the literal sense of the Sacred Scriptures. And so in vain are examples heaped up from Holy Scripture in which sense and imagination have to be corrected and in which one must grant that Scripture speaks figuratively, or to suit the grasp of the common people. In those examples, if they hold good, the deception of the senses is already evident from another source: from other, more manifest sensations, or from evident reasoning and necessary, apodictic arguments. Neither of these is found in our case, since the Copernicans admit that all the phenomena of the heavens can be saved on either hypothesis, that of the Earth standing still and the Sun moved, or that of the Earth moved and the Sun and fixed stars standing still. But these matters must be discussed more forcefully and more fully in Book 9.

V. [Margin: Motus Terræ annuus sed Declinationis. — An annual motion of the Earth, but one of declination.] The fifth motion of the Earth, ascribed to it by Copernicus, is the annual motion of *declination* [Declinationis], by which the axis of the terrestrial equator moves in the precedent direction [in præcedentia], contrary to the annual motion of the centre, which takes place in the consequent direction of the signs [in consequentia Signorum]. Its purpose is to account for the Sun's apparent motion obliquely from south to north. Because this motion is a little swifter than the motion of the Earth's centre, it causes the equinoctial and solstitial points to appear to recede in the precedent direction, towards the west; so their motion is called the *precession of the equinoxes*, and by this motion Copernicus accounts for the apparent motion of the fixed stars towards the east. Although the Sacred Congregation of Cardinals issues no express decree on this motion, it nevertheless seems tacitly to have condemned it, since it is no

less repugnant to Holy Scripture than the annual motion of the Earth's centre. For the divine writings attribute the motion of declination too to the Sun, in Ecclesiastes 1, where it is said of it: *It circles through the south and bends towards the north*, etc. [Gyrat per Meridiem & flectitur ad Aquilonem &c.]

VI. [Margin: Motus Librationis duplicis Copernicæ. — The motion of Copernican double libration.] The sixth motion of the Earth is one of double *libration* [Librationis], to account for the anomaly of the precession of the equinoxes and of the obliquity of the ecliptic. Of this and of the preceding one we shall speak more opportunely in Book 3, Chapters 27, 28 and 29, where we shall also speak of the motion of the Earth upwards and downwards, by which it is not the Sun that becomes apogean or perigean, but rather the Earth that becomes aphelial and perihelial [Aphelia & Periphelia]. The slowest motion of the Earth, substituted by *Andrea Cæsalpino* [Andrea Cesalpino] (*Quaestiones Peripateticae*, Book 3, quaest. 4) in place of the proper motion of the fixed stars towards the east, is the one indicated by Copernicus in Book 1, Chapter 11, and it reduces to the Copernican motion of the precession of the equinoxes, although Cesalpino extends its period to 30000 years.

SCHOLIA.

I. [Margin: Argumenta contra motum Terræ quāti valeant. — What the arguments against the motion of the Earth are worth.] Many physical and astronomical arguments are commonly heaped up against the Copernican motion of the Earth, both daily and annual. Even men otherwise skilled in philosophy and astronomy judge them to be apodictic, or wholly demonstrative, yet on inspection they are not so. But it is not for everyone to search out thoroughly the depth of the Copernican hypothesis, and so I have put off the exposition and discussion of these arguments to the end of this work and to a separate book on this subject. I wish only that beginners [tyrones] be warned of three things. The Copernicans attribute that motion, both daily and annual, to all bodies of the atmosphere as a common one, apart from their own motion upwards, downwards and animate [animasticum]. They place the centre of heavy bodies in the centre of the atmosphere, not in the centre of the Universe. And they hold the fixed stars to be at so great a distance from the Earth that the Earth, by its annual motion, experiences no perceptible diversity of aspect. With these points laid down and properly understood, a great many of the arguments hurled against them collapse. And were it not that sacred authority and the evidence of the senses deter us from that opinion, no mathematical arguments have so far been devised, as far as I at least know, that ought to have moved us away from it. I think, however, that with God's help I shall in due time bring forward one or two arguments which, from the phenomena given in astronomy by the Copernicans themselves, will convict their opinion of falsity, in part at least.

II. [Margin: Congregationis Cardinalium auctoritas. — The authority of the Congregation of Cardinals.] The Sacred Congregation of Cardinals, taken apart from the Supreme Pontiff, does not make propositions matters of Faith, even if it defines them to be of Faith or their opposites to be heretical. Hence, since no definition on this matter has yet come forth from the Supreme Pontiff, or from a Council directed or approved by him, it is not yet of Faith that the Sun moves and the Earth stands still, by force of the decree of that Congregation taken precisely; but at most, and only, by force of Sacred Scripture, for those to whom it is morally evident that God so revealed. All we Catholics, however, are bound by the virtue both of prudence and of obedience to hold what that Congregation has decreed, and at least not to teach the opposite absolutely. But I have dealt expressly with this theological subtlety in my Treatise on Faith, where I discuss the Rules of Faith. From this, however, you will understand how Marin Mersenne, who denies that the opinion of Copernicus has been condemned as heresy, can be reconciled with *Iusto Lipsio* [Justus Lipsius], Melchior Inchofer and *Georgio Polacco* [Giorgio Polacco], who say that its opposite is of Faith; which, however, Gassendi denies in the letter mentioned above and at the end of Book 3 of the *Institutio Astronomica*.

CAPVT IV.

Terræ Centrum esse Centrum Commune Omnium Corporum Grauium: Vbi selecta quædam Corollaria ad illud Centrum spectantia. — That the centre of the Earth is the common centre of all heavy bodies; with certain selected corollaries concerning that centre.

I. [Margin: Centrum grauitatis & centrum molis. — Centre of gravity and centre of mass.] The centre of gravity is not of its own kind the same as the centre of figure, or of magnitude. For the centre of magnitude is *that point around which parts of equal magnitude stand on all sides*. The centre of gravity, by contrast, is *a point placed within a body, such that if the heavy body is conceived as hung from it, it rests while it is carried and keeps the position it had from the start, and is not turned about in the carrying itself*. So Pappus of Alexandria defined it (*Mathematical Collections*, Book 8); or, as Federico Commandino says at the start of his book *De centro gravitatis*, it is *a point within a body around which parts of equal moments stand on all sides*. For if a plane is drawn through such a centre, cutting the figure in any way whatever, it will always divide it into parts of equal weight. *Moment* [Momentum] here means the force and power to move downwards, which in inanimate bodies is nothing other than gravity. The *line of direction* [directionis], or *diameter of gravity* [Grauitatis diameter], is a straight line perpendicular to the horizon, drawn through the centre of gravity.

Now in homogeneous bodies, that is, those with wholly similar parts, the centre of gravity is the same as the centre of figure and of magnitude. This is shown more clearly in regular figures: thus in the circle and in the sphere, and in all rectilinear or plane figures with equal sides that can be inscribed

in or circumscribed about a circle or a sphere, the centre of gravity and of magnitude is the same, if they are homogeneous; as when a sphere or a cube is made entirely of the same gold, or lead. In heterogeneous bodies, on the other hand, the centre of gravity is often one thing and that of magnitude another. For if you make a sphere with one hemisphere of wax and the other of lead, the centre of gravity will lie outside the centre of figure and within the lead segment.

II. [Margin: Idem centrum grauit. & molis in Terra. — The centre of gravity and of mass is the same in the Earth.] In the sphere of the terraqueous globe, then, the centre of gravity and the centre of magnitude are judged to be the same, at least physically and to the senses. For although the globe is interspersed with waters and mixed with heterogeneous parts near its surface, these have an imperceptible proportion to the remaining homogeneous mass of the Earth, and are unfit to weigh it down towards one side, as we said in Chapter 3, no. 2, and as will be confirmed by what is to be said below about the depth of the sea and the mass of the waters compared with the Earth. Moreover, as is gathered from the natural history of those who have travelled round almost the whole globe, God, Best and Greatest [DEVS O. M.], has so arranged the mountains and seas around the Earth that the moments around its centre are equal on all sides; whether or not, for that very reason, certain seas are higher, as Cabeo asserts of the Tyrrhenian Sea (*Meteorologica*, Book 1, p. 55), and as others assert of the Red Sea and the Northern Sea [Mari Nortico] with respect to the Ocean *del Sur*. Finally, if the centre of gravity in the Earth were perceptibly distant from the centre of magnitude, plumb lines let down freely above the surface of plains or of the sea would surely show this somewhere, for they would lean to one side rather than the other. Since this has nowhere appeared so far, it would be quite without grounds to suppose that these two centres in the terrestrial sphere stand perceptibly apart from each other.

III. [Margin: Centrum Terræ est centrū omnium grauium. — The centre of the Earth is the centre of all heavy bodies.] From this the question is now easily solved, whether the centre of the terrestrial globe is that common centre towards which all heavy things tend of themselves by an innate appetite, and from which light things withdraw, along straight and shortest lines, unless they are hindered or forced to descend towards it, or ascend from it, by a longer and oblique path. It must indeed be affirmed. For everywhere on land and sea the weights of plumb lines, let down through the air or through still water, descend along that straight line which makes equal angles on either side with the surface of the terraqueous globe, or right angles with a straight line or horizontal plane touching the convex surface of the said sphere; and so along that straight line which, produced either actually or in imagination, continues the Earth's semidiameter and falls on its centre. This is so true that a straight line making equal angles on either side with another has acquired from it the name of *perpendicular line* [Lineæ perpendicularis]. And this becomes more manifest both in waters, whose surface, when still, is also convex-spherical,

for on it the equality of the angles produced on all sides by the thread of a plumb line is apparent, and in drops of water falling straight onto the sand of a shore. For why do they not flee from that sand, which by its dryness will consume them, or at least preserve them less well? Or why do they not turn aside by an oblique path to the nearby sea? Surely because the gravity placed in them, as parts inclined to the good of the whole, prevails, and moves them towards the one common centre of all heavy things, which is in the very middle [meditullio] of the Earth. Finally, we see the masts of ships, the walls of buildings, the trunks of trees, the stalks of grain, the pyramids of flames and the little rods of smoke keeping of themselves, violent motion aside, a position perpendicular to the terrestrial globe in their motion upwards or downwards, or in their rest, wherever in the end they may be. Granted, the smallness of two neighbouring walls, compared with the depth of the Earth, is so slight that they seem parallel and not about to meet perpendicularly at the centre of the earth.

From this there now follow some corollaries worthy of note, from which I shall point out those I judge more worthy of cosmographers and more useful to them.

COROLLARIA

Ex Centro Grauitatis, & Magnitudinis Terræ. — From the centre of gravity and of magnitude of the Earth.

I. The imagination of those must be corrected who think that their antipodes, if there are any, will fall into the opposite part of the heavens. For they do not fall; with gravity inclining them towards the same centre of the Earth, they stand perpendicularly on the earth's surface, no differently from us. Read, please, against such people Pliny the Elder (Book 2, Chapter 65), and what I said in Book 1, Chapter 20.

II. [Margin: Tellus qua vi in medio Mundi pēdeat. — By what force the Earth hangs in the middle of the world.] Likewise to be corrected is the imagination of those who have difficulty grasping how the Earth, unless it rests on a very deep and infinite root below, as Xenophanes imagined, can stand immobile. For if, with God, Best and Greatest, so arranging the element of earth and water, all its parts stand on all sides around the same centre and are in equilibrium, there is no reason why this mass, however great, should be carried elsewhere. But how will they be able to grasp why the whole world stands around the one centre of the Universe, and does not drift this way and that in the divine immensity, which coexists with infinite imaginary spaces, not to say equals them? Let them listen to Manilius, Book 1, Chapter 2:

Nor should the nature of the hanging Earth seem a marvel to you, since the world itself hangs and sets its footsteps on no ground [Nec verò tibi natura admiranda videri / Pendentis Terræ debet; cum pendeat ipse / Mundus, & in nullo ponat vestigia fundo.]

And that line of Ovid, *Metamorphoses* 1:

Balanced by its own weights, it stays motionless [Ponderibus librata suis immobilis hæret.]

Against these people Pliny the Elder too, Book 2, Chapter 65, is to be read.

III. If the Earth were divided into two hemispheres, and only one of them remained, such as the hemisphere AB, on the horizontal plane ACB a single man could stand perpendicularly, at C, while the others would naturally hang inclined more or less to the horizon, according as they were more or less distant from the centre C, or from the point C, terminating the line in which the centre of gravity of the earth would be: for then the common centre of heavy things would not be C, but another point within the earth's hemisphere. But if that centre remained at C, the man B, at the extremity of the horizon, could not stand perpendicularly above the horizon, but would lie down; although he could not even lie there, but would rush downwards towards C. Again, trees, buildings, the stalks of crops, etc., outside C would be inclined.

Diagram. A hemisphere with its flat side upward and shaded curvature below, set into the right column beside the text of no. III. A small human figure stands upright at the middle of the flat top; a second figure at the right end of the flat top leans over or falls.

- A (upper left, at the left end of the flat side)
- C (centre of the flat side, under the standing figure)
- B (upper right, above the leaning figure at the right end of the flat side)

IV. If there were a straight line, or a plain, touching the earth at a single point and produced notably far, such as HPR, touching the Earth's globe PC at the point P, men placed on that plane and distant from the point of contact P, as A and B are, would not stand upright on it, but inclined, as the line of direction AC or BC would require, drawn through their centre of gravity and produced to the centre of the Earth C. And if a heavy sphere were set on the same plane far from the point of contact P, as at H, it would not rest there, but would rush down to the point P, so as to be as close as possible to the centre C. The plains and pavements on which we stand upright, however, although they seem straight and flat, are nevertheless convex, though imperceptibly, for the reasons given in Chapter 1 of this Book, Scholium 2; or if they are flat, our inclination to them is imperceptible.

Diagram. A circle (the Earth) with a straight line tangent at its top, set into the right column beside the text of no. IV. Lines run from the centre of the circle out to figures on the tangent line. Two small figures lean on the right part of the line; a ball or figure stands on its left part.

- P (inside the circle, just below the point of tangency)
- C (lower part of the circle, the centre)
- A (above the tangent, right of P, beside the first leaning figure)
- B (above the tangent, far right, beside the second leaning figure)
- R (right end of the tangent line)

- Not resolved: a small stroke at the extreme left end of the tangent line. The text cites H at that end of the line; no H was read on the block at this magnification.

V. [Margin: Turres inclinatæ nec cadentes. — Towers that lean yet do not fall.] Men standing on the ground on one foot or two, whether leaning or pushed or pulled by another in wrestling, will stay standing for just as long as the line of direction drawn from their centre of gravity to the centre of the Earth falls within their soles; but if it falls outside, they too will fall. The same happens with trees and with towers, in which the root or foundation serves as the sole. From this arose that device of architecture by which a leaning tower, such as the Garisenda Tower at Bologna, does not fall; and that tower of Cologne, leaning so far that men climb over its sloping back. The reason they do not fall is that the line of direction falls within the foundations, as the line EBC or GD would in the attached diagram. So it greatly helps for the foundations to widen downwards with an oblique increase, so that the line of direction, falling outside the projecting part of the tower ABRE, should nevertheless not fall outside the foundations, but should strike within them, even if only slightly, and not be like the line GD. It is also advantageous for the tower, especially in its upper part, to be hollow and empty of stones, and for the rest to be solid, so that the greater part of the weight may the more surely lie in the foundations and in the portion of the tower joined to them, and thus the line of direction may fall within the foundations. An architect will easily determine this, having weighed with due skill the weights of the stones, lime and mortar of both parts: if the weight of the part FDBEA is a thousand, and of the part EBRE 400, he will safely declare that the line of direction will stay within the foundations.

Diagram. A leaning tower drawn as an oblique parallelogram rising to the upper right from a wider, flared base, set into the right column beside the text of no. V. Two dotted verticals descend from the top edge to the base.

- E (top edge, left)
- G (top edge, centre)
- R (top edge, right)
- K (on the right side of the tower, where the dotted vertical from G crosses it)
- A (left, at the junction of the upper tower and the base)
- B (inside the base, on the dotted vertical from E)
- F (bottom left corner)
- C, D (bottom edge, right, at the feet of the two dotted verticals)

VI. The reason why on stairs, or on the slopes of mountains, although they are longer than the planes beneath them, no more men can stand than on the base, is that they must stand perpendicularly on the surface of the earth; and so ascent by steps whose treads are parallel to the horizon is more convenient. Thus in the figure placed here, although the slope of the staircase AK is longer than the base AB, yet if the base AB holds only three men standing and occupying

with their soles the three spaces AC, CD and DB, no more than three will stand on the slope AK either, occupying, in a position upright to the horizon, steps equal to the spaces of the base, such as EI, FL and GK.

Diagram. Three rising rectangular steps over a horizontal base, with a diagonal line drawn from the lower left to the upper right corner, set into the right column beside the text of no. VI.

- A, C, D, B (along the base, left to right)
- E (left, at the top of the first step)
- F (above the second step)
- G (top of the third step, left)
- K (top of the third step, right)
- I (at the upper right of the first step, on the diagonal)
- L (at the upper right of the second step, on the diagonal)

VII. If a single staircase were to be made occupying a whole quadrant of the Earth, by which one had to ascend by equal steps, but with their angles ending on the periphery; for instance from one Pole of the Earth, as far as the Equator, as if in the quadrant ACB a staircase were to be made from A, through D and E, as far as C. The fewer the steps were, and the longer the interval separating them from one another, the more obtuse the angles at which they would meet, and the more they would be raised out of parallel with the horizon; and men leaping up them could stand only at their middle and their end: as FN in the middle of the step AD, GM in the middle of the step DE, and KL in the middle of the step EC. For there the lines of direction, produced straight, would fall on the Earth's centre B, as you see in the lines FNB, GMB and KLB. So too at D, the end of the first step and the beginning of the second, the line BD would coincide with the line of direction, and so one could stand at D. Moreover, from A to N we would experience a descent, and from N to D an ascent; from D to M a descent again, and from M to E an ascent; and D and E would be, as it were, the summits of mountains.

Diagram. A quadrant of the Earth with its centre at the lower right, set into the left column beside the opening lines of the page. Straight chords run along the arc from the lower left to the top right, forming the steps of the staircase, and lines run from the points on them down to the centre, one of them dotted.

- A (lower left)
- B (lower right, the centre)
- C (top right)
- D (left, on the arc)
- F (left, beside N; blurred, close to E in form)
- N (just inside the arc, near F)
- G (upper left, at the end of a line running beyond the arc from B)
- M (on the line from G to B)
- E (on the arc at the top, between G and K)
- K (at the top, above the arc)
- L (on the line from K to B)

Suppose, however, that one staircase were to be made of steps with right angles, each step being one foot wide. Since the Earth's semidiameter AB contains 25870000 feet, that is, 5174 miles, according to what will be said in Chapter 7, the same number of steps would have to be built, but inconvenient ones, each one foot high, since all of them taken together would have to equal the perpendicular height of the staircase CB, which is as great as AB. So it would be better to make it of half-foot steps, 51740000 of them.

VIII. [Margin: Error multorū in Altimetria Montiū ac Turrium. — The error of many in the altimetry of mountains and towers.] Those are gravely mistaken who measure the heights of towers and mountains by altimetry, from the angle of altitude and from a considerable distance between the foot of the mountains and the eye of the measurer. They use a right-angled rectilinear triangle, supposing that of the sides about the right angle in that triangle one is this distance and the other is the very height of the tower or mountain, whereas this side is far from the perpendicular line running to the Earth's centre, along which, nevertheless, the height in question must be reckoned.

For in the next figure let there be the quadrant of the Earth ABC, in which the height of the mountain is PKM, perpendicular and making equal angles on either side at P; and let it be continued in a straight line with the Earth's semidiameter PA. Now almost all the geometers I have read, in order to investigate the height of a mountain from a station at C, take with a geometrical quadrant DEF, or some other similar instrument, through the sight CE, the angle ECF, sighting the summit of the mountain M along the visual line CEM. The arc EF gives this angle. Next they find the distance CK from the eye C to the foot of the mountain, in miles, leagues or stadia; and having constructed from it the triangle CKG, right-angled at K, they gather from the angle C and the distance CK the height KG, which they think to be the very height of the mountain, exactly as if the visual line had ended at G, when in fact it ends at M. But the true height of the mountain is PM itself, far greater than GK. Further, CK is not necessarily a perfect plane; it is often curved, and has to be tested with a level. And often the place of the station C is higher than that curved surface of the Earth which is concentric with the convexity of the sea. Yet you will find that *Oronce Finé*, *Magini*, *Clavius* and *Biancani* did not recognise this false construction. *Cabeo* did recognise it in his *Meteora*, but he too went wrong in the remedy he applied, as I shall show in the Geographical book, in the section on the Height of Mountains, where, God granting, I shall teach how the practice of these geometers is to be corrected.

Diagram. A quadrant of the Earth with its centre at the lower left, set into the left column beside no. VIII. A mountain rises from the arc at the upper right, with the sight lines drawn to it from the station at the upper left and a small quadrant of the instrument at that station.

- A (lower left, the centre)
- B (lower right)
- C (upper left, the station)

- D (above C)
- E (on the small arc, upper left)
- F (on the small arc, below E)
- G (upper right, on the line above K)
- M (upper right, the summit)
- K (right, below G)
- P (on the arc, on the line from A to M)

IX. [Margin: Error aliquorum in Fluminum libramēto. — An error of some people on the gradient of rivers.] Those are equally mistaken who think that the beds of rivers running down from far away into the sea lie in one continuous plane of bottom, and that rivers are borne along a single straight track. If this were true, those rivers would not only be unnavigable, but would rush into the sea with a huge crash and a whirl of waters and a headlong course, and the height of the fall and of the source would be monstrous. Let the proof of this be the Nile, in the following diagram, in which ABC is a quadrant of the Earth's globe, the supposed bed of the Nile is the straight line AM, the source of the Nile in the Mountains of the Moon is M, and the perpendicular height of the spring M

is FM, continued with the Earth's semidiameter FC. Now let the Nile run down from its source to the Canopic mouth A, through, if not 54 degrees, as *Froidmont* will have it in his *Meteora*, then at least 44°, as is gathered from more recent geographical maps; and let these degrees on the Earth's great circle be the arc AF. Then the secant CM, from the tables of sines, would be 13902 of the parts of which the semidiameter CF is 10000. Therefore if the Earth's semidiameter CF is, as I shall show in Chapter 7, 5174 ancient Italian miles, CM would be 7197 48/100 miles; and so, with CF taken away, the height of the Mountains of the Moon, indeed of the source of the Nile, FM, would remain at 2023 48/100 miles. From so great a height, I ask, in how great a plunge would the Nile rush down? For a river whose fall is 6 feet higher in every thousand paces is navigated at the greatest peril; and if it is 2 paces high, it is unnavigable? Yet that is how the summit of the Nile would be raised, since the Nile runs through more than 3000000 paces. Besides, from the mouth A the source M could be seen, and much more the summit of the mountain M, which is contrary to experience. Rivers therefore repeatedly change their bed, and glide, for example, from D to L along a straight or nearly straight line DL, then from L to K, and from K at last to A, describing a line that is nearly a conchoid.

Diagram. A quadrant of the Earth with its centre at the lower left, set into the right column beside the opening of the page. The straight line AM runs horizontally along the top from A to M, outside the arc; lines run from the centre up to the points on the arc, and a broken course runs along the arc.

- A (top left)
- M (top right, at the end of the line from A)
- K (on the top line, near the middle)

- C (lower left, the centre)
- B (lower right)
- D (right, beyond the arc)
- L (right of K, under the top line)
- G (under the top line, left of K; this letter is not cited in the text around the figure)
- I (mid-figure, on one of the radii; not cited in the text around the figure)
- [?] (lower, on the arc, where the text's F should stand; the letter has a stem and a top bar running only to the right, with no middle arm visible: an F whose middle arm has not printed, or a Γ-shaped sort)

X. Here too belongs the fallacy by which some water engineers [Hydragogi] measure the gradient of water badly with levelling instruments. For this purpose let the hemisphere of the Earth be AIB, with its centre C. If from D, through a levelling sight, you look at a visible mark N, and from N, through the sight ER, parallel to the horizon and making right angles with the perpendicular NC, you look at a second mark F, that mark will be much higher than D, the place of the first station, as is clear when the perpendicular CDF is drawn. So if you thought that F was as high as N, and therefore that, with a channel made from N to G, a point a few feet lower than F itself, the water would flow from N to G, you would be badly deceived. Therefore, as our own *Cabeo* also advises in his *Meteora*, Book 2, text 6, question 3, one must place oneself at about the midpoint between the two places or marks D and N whose level is in doubt, as at IM, and through the sight at M look once at the mark D and again at the mark N. For then, if the channel is made from N to a place below D, the water will be able to flow by reason of the slope.

Diagram. A semicircle standing on a base, set into the right column beside no. X, with a line of sight running through the top of the semicircle from E through N, and straight and dotted lines rising to the upper right.

- A (base, left)
- C (base, centre)
- B (base, right)
- I (inside the semicircle, above C)
- E (left, at the lower end of the sight line)
- N (left, above the semicircle)
- R (right of N)
- M (at the top of the semicircle)
- D (right, at the semicircle)
- G (upper right)
- F (top right)

XI. Finally, Scheiner's problem [Ploblema] must be examined, in the *Disquisitiones Mathematicae*, no. 15. [Margin: Scheineri de motu perpetuo hypothesis. — Scheiner's hypothesis on perpetual motion.] For he says that perpetual motion will result if about the centre of the Universe C there is attached a heavy gnomon, at one end of which is the weight G, while the other end of the

gnomon, pierced at C, turns on the axle ACB, supported by the little columns AH and BN. For the weight G, outweighing the rest of the gnomon, will move from G towards P and so describe the circle GPL, and in perpetual motion, since there is no reason why, once moved, it should stop moving. This is confirmed by experience in statics, where a gnomon set up on a floor so that its shorter part is perpendicular to the horizon and its longer and heavier part parallel to the horizon falls over because of the overbalance. But that most acute investigator of the Sun is deceived. For, as Father Francesco Maria Grimaldi argued at once on seeing this figure, that gnomon falls because by its fall it comes nearer to the centre of all heavy things; but the weight G would not fall, because it could not come nearer to the common centre C. Indeed it would not move perpetually even if pushed, but would come to rest when the external impulse died away.

Diagram. A circle with a horizontal axle through its centre, set into the right column beside no. XI. An L-shaped gnomon rises from the centre and carries a large black ball on the circumference at the right of the top. Two small columns stand under the ends of the axle.

- L (top of the circle)
- G (at the ball, upper right)
- D (at the angle of the gnomon)
- A (left end of the axle)
- C (centre)
- B (right end of the axle)
- H (foot of the left column)
- N (foot of the right column)
- P (bottom of the circle)

The letter D is on the figure and is not cited in the text around it.

XII. Yet a perpetual bridge could be made in the air around the Earth, resting on no columns or piers, as Biancani points out in Book 4 of his *Sphaera*: for example, if in the following figure, around the Earth MN and its great circle, say the Equator or some meridian, a bridge were made over the whole circuit on columns of equal height NB, MA, etc., of equal size everywhere and of homogeneous material, as is shown here by the larger circle AB, etc. For with the columns NB and MA removed, it would still stand, because of its equal distance from the centre of heavy things C and the homogeneity of its parts, which balance one another about that centre.

Diagram. Two concentric circles, the outer one broken at the upper left, set into the left column beside the end of no. XII. Two short radial lines run from the centre through the inner circle to the outer.

- A (upper left, outside the outer circle)
- B (upper right, outside the outer circle)
- M (on the left radial line)
- N (on the right radial line)
- C (the centre)

CAPVT V.

In which there is a timely digression on gravitation and on the natural motion of the elements and of mixed bodies [Mistorum] away from or towards the centre of the Universe; and on bodies floating in a liquid.

I. [Margin: Grauitas in specie & Grauitas in indiuiduo. — Gravity in species and gravity in the individual.] I say, first, that a body is *specifically heavier* [Grauius in specie] when it is heavier than another having an equal solid mass or size, as a lead cube one palm across is heavier than a wax cube one palm across. A body is *individually heavier* [Grauius in indiuiduo] when it is absolutely heavier than another, whether it has an equal or an unequal mass, though this is better discerned in an unequal mass, as a pound of feathers is heavier than an ounce of lead.

II. [Margin: Grauiorum ac Leuiorū mutuus motus. — The mutual motion of heavier and lighter bodies.] I say, second, that the elements and the mixed bodies made from them, when placed outside their own place and with the obstacle removed, naturally ascend upward or descend downward by the force of gravity, and not by the force of rarefaction or of preventing a vacuum. I say also that they move not only relative to the common centre of heavy things but also relative to one another, so as to keep their order among themselves: that is, so that the heavier is under the lighter and the lighter stands above the heavier, and the former is less distant and the latter more distant from the Earth's centre, observing a kind of distributive justice. For a wineskin full of air, placed under water on the bottom and released, rises and pushes our hand upward, no less than water is carried down to the bottom; indeed, it shoots a little stick laid on its back upward, above the surface of the water. Nor is there more reason why the heavier should only descend than why the lighter should not ascend.

III. I say, third, that according to the common opinion and system of the Peripatetics and the Ptolemaics, that common centre to which or from which heavy and light things are carried is the centre not only of the Earth but of the whole World, as was proved in Chapter 4. In the opinion and system of the Pythagoreans and the Copernicans, however, it is indeed the centre of the Earth and of the atmosphere, or of the whole elementary sphere, but not, in strict cosmographical terms, of the whole World and of the celestial sphere, since in such a system the centre of the World is the Sun, and the Earth's centre is distant from it by many hundreds of terrestrial semidiameters.

IV. [Margin: Aer aliquid grauitatis habet — Air has some gravity] I say, fourth, that water and our air have some lightness mixed with their gravity, so that by the force of the former they ascend, in order to be above what is heavier than they, and by the force of the latter they descend. Hence inflated playing-balls, well blown up with compressed air, weigh more than when slack, even without any fine flour and wine poured in, as you will detect with a precise balance. Indeed, I myself weighed an ox bladder that when slack was

of 4 scruples and 4 grains, and when inflated was of 4 scruples 6 grains; so the air added by the inflation weighed 2 grains.

V. [Margin: Elementa in suis locis non grauitant. — The elements do not gravitate in their own places.] I say, fifth, that the elements in their own places do not actually gravitate, and that upper homogeneous parts do not press on lower ones. Otherwise slender plants that grow upright from the bottom of waters would be seen bent towards the bottom by the force of the mass of water, and light feathers and awns of grain bent by the whole mass of the surrounding air; and divers swimming under water would feel this gravitation to their great harm. Hence we do not feel a bucket full of water and submerged under well-water to weigh anything except as much as the weight of the bucket itself exceeds an equal amount of water, and we do not feel at all the weight of the water enclosed in the bucket. If, however, the lower parts once begin to move, and access is given for a descent beneath a body lighter than they are, as happens when a pipe is opened at the bottom of casks, then certainly the upper parts move downward. And the motion of all of them together, in the manner of one moving body, is the swifter the greater the mass, and so the more that element or mixed body is heavier than its parts taken separately, as individuals, and than the body below which it is carried, in species. From this some have wrongly deduced that the parts of water or of dust are universally pressed by those above them, and that the elements gravitate in their proper places as long as they are far from the Earth's centre.

VI. [Margin: Cur quædam supernatēt in Aqua, &c. — Why some things float on water, etc.] I say, sixth, with *Archimedes* in the book *De his, quæ vehuntur, aut insident in humido* [On Floating Bodies], and with *Galileo*, *Tartalea* [Niccolò Tartaglia], *Ghetaldo* [Marin Getaldić] and others who have elucidated Archimedes' little work, that the reason why some things float on water or in air is that they are specifically lighter than the body on which they float: that is, because their mass is lighter by its nature than an equal mass of water or of air. And the reason why they descend, and do not float whole, is that they are specifically heavier. Hence a sphere of lead, which cannot float on water, can float on water like a boat if it is beaten out into a very thin plate or leaf and a wide vessel is made from it, because the body made of lead and enclosed air is, in so great a mass, lighter than a mass of water equal to the mass of lead and air together. Hence a ship laden with goods floats for as long as its mass, with the air and the goods it contains, is lighter than an equal mass of water. Nor does it matter for this whether the depth of the water is great or small: even if below the submerged part of the keel there were a depth of only an inch, it would still support the ship in question. Hence ice in water and clouds in air float, even setting aside exhalations levitating upward, because the mass of that aggregate of ice and the enclosed air by whose force it swells, or of vapour and purer air, is lighter than an equal mass of water or of denser and thicker air. This weakens the argument of *Pliny the Elder*, Book 36, Chapter 9, who from the load of ships assigns to the Tiber as great a depth of water as to the Nile.

VII. [Margin: Cur leuissima in Aere supernatent. — Why the lightest things float in air.] I say, seventh, that the reason why some things that are specifically heavier do not descend, such as flakes of gold or specks of dust in air, is not that their shape is in itself less suited to breaking through the medium, as Aristotle thought, granted that shape may help a motion once begun to go more easily and swiftly; it is that, when the shape is changed, a little vessel is made, within which the enclosed air makes up a body with them whose whole mass is lighter than the mass of air on which they float. In the same way a needle, which sinks when set on water by its point, floats if it is laid on the water lightly with its length parallel to the horizon, because within the margins of the water the air clinging to the needle forms a body with it that is lighter than a mass of water equal to the mass of the needle and of the air enclosed within those margins. So if two cones of wax equal in all respects are made, and the vertex of one and the base of the other are set on water, the former will not for that reason penetrate to the bottom because it is sharp, nor will the latter fail to penetrate because its base is blunt and unsuited to dividing the medium; both will float, as you see in the following figure, with MN marking the surface of the water, and in the cones K and L, because their mass is lighter than an equal mass of water.

VIII. [Margin: Cur quædā plus, quædā minùs demergātur. — Why some things are submerged more, some less.] From this one can now catch the scent of the reason why some things descend with their whole mass to the bottom of the water, some float entire, some are wholly submerged below the surface of the water and descend no further, and some, finally, are submerged below the surface of the water by a greater or lesser part of themselves, with the remaining part standing out of the water. For this, take these selected propositions from Book 1 of Archimedes' *De his, quæ vehuntur in aquis* [On Floating Bodies], which stand in him in this order.

Proposition 3 of Archimedes: Solid magnitudes which, having an equal mass, are as heavy as the liquid, when let down into the liquid will be submerged so that nothing of them stands out of the liquid's surface; yet they will not be carried further downward. Archimedes proves this there, as he does the following ones; and the case of this one you may see exemplified in the following figure, in which the water is ABCD, its surface AB, and beneath it the solid F, as heavy as a mass of water as great as the mass of the solid F itself.

4. Any solid magnitude lighter than the liquid, when let down into the liquid, will not be wholly submerged; some part of it will stand out of the liquid's surface. Let the solid E be an example of this.

5. Any solid magnitude lighter than the liquid, when let down into the liquid, will be submerged until a mass of the liquid as great as that of the submerged part has the same gravity as the whole magnitude. Let the solid E be an example, if you understand a mass of water equal to its submerged part to be of the same weight as the whole mass of the solid E.

6. Solid magnitudes lighter than the liquid, when forced into the liquid, are carried upward with as much force as the liquid having a mass equal to that magnitude is heavier than the magnitude itself.

7. Solid magnitudes heavier than the liquid, when let down into the liquid, will be carried downward until they descend; and they will be lighter in the liquid by as much as is the gravity of liquid having a mass equal to that magnitude. Let the solid G be an example.

Diagram. Set into the left column beside proposition 7. Above, a rectangular tank of water holding three solids: one standing partly out of the surface at the left, one at the surface in the middle, and one lower down at the right. Below, a horizontal line with an inverted cone at the left and an upright cone at the right, both crossing it.

- A (left, at the surface of the tank)
- B (right, at the surface of the tank)
- C (lower left corner of the tank)
- D (lower right corner of the tank)
- E (on the left solid, above the surface)
- F (above the middle solid)
- G (on the right solid)
- M (left end of the lower line)
- N (right end of the lower line)
- K (over the inverted cone)
- L (beside the upright cone)

And Book 2, proposition 1: *If some magnitude lighter than the liquid is let down into the liquid, it will have in gravity the same ratio to liquid of equal mass as the submerged part of the magnitude has to the whole magnitude.*

IX. [Margin: Ingenium ponderādi res in Aqua. — A clever way of weighing things in water.] From what Archimedes has demonstrated, it follows that the weight of different liquids can be known by means of a single body whose weight you know beforehand. For if you have a cube, say of lead, weighing one pound in air, and this when submerged in water is found to be of eight ounces, and in wine of six ounces, you will conclude that the water is to that wine in gravity as 4 to 6. In these operations, however, it is best to hang the things to be weighed by horsehairs, since these are as heavy as an equal mass of water; otherwise the cord and its own weight must be taken into account. This way of weighing things in water was used by *Tartaglia*, by *Villalpandus* [Juan Bautista Villalpando] on Ezekiel, by *Getaldić* in his *Promotus Archimedes*, and by ourselves after them. From these sources take the following tables. In the first of them, if a vessel full of oil weighs 9 pounds, the same vessel full of wine will weigh 10 pounds, and full of tin 75 pounds, and so for the rest. *Getaldić* also adds the weight of various spheres of the same mass, namely of gold, lead, etc., and their mass at equal weight; consult him on this.

	Villalpando. Pounds [Libræ.]	*Ghetaldo* Pounds [Libræ.]
Oil	9	1
Wax		1 5/121
Wine	10	1 4/55
Water	10	1 1/11
Honey	15	1 32/55
Tin	75	8 4/55
Iron	80 37/97	8 8/11
Bronze [Aes]	91	9 9/11
Silver	104	11 3/11
Lead	116 1/2	12 6/11
Quicksilver	150	14 62/77
Gold	187 1/2	20 8/11

X. [Margin: Maria quædā cur inscrutabilis fundi. — Why some seas have an unsearchable bottom.] From what has been said there also follows what has been indicated by *Leone Baptista Alberto* [Leon Battista Alberti] and by *Cabeo* in his *Meteora*: namely, that the sea can be so deep that no sounding-line [bolide] one could procure can measure it, even if rope is not lacking. This happens, that is, if the weight of the given lead or iron, together with the rope, is less than a mass of water equal to the mass of the rope and the lead together. For then the rope will not be drawn down perpendicularly by the rope [a fune], but will drift this way and that and float in waves. But these authors do not teach the method of investigating how much lead is to be hung on a given rope so that you may be sure the rope will be drawn straight and perpendicularly down by it. I, however, will teach this practice by one example.

In the second part of the following figure, let the surface of the sea be MR and the sounding-line EGK, whose rope EG is 1000 feet long; and let the weight of the lead GK be sought that is suited to keep the rope EG in the straight position of a perpendicular line under water. Take a ten-foot rope of the same kind and thickness as EG, thoroughly soak it with sea water and weigh it; and let it be, for example, 3 pounds. To it hang the lead GK, weighing 2 1/2 pounds in air, but weighing only 2 pounds when weighed in sea water, so that the difference is half a pound; and let the sum of the weights in air be 5 1/2 pounds. Having noted these on a slip of paper, let the whole ten-foot rope EG, with that lead GK, down below the surface of the sea water MR, and weigh both together with a steelyard; and let their weight together be, for example, 1 pound. Subtracting the weight of 1 pound from their combined weight outside the water, then, 4 1/2 pounds remain: so that depth of water whose mass equals the mass of the whole sounding-line EGK has this weight. And when the half pound, which was the difference between the weight of the lead and the weight of the rope, is taken from the 4 1/2 pounds, 4 pounds remain, which belong to a depth of water equal to the rope EG alone. Let this number be called the argument of the depth of the water. With this argument you will settle the rest of the problem as follows. Multiply by the said argument,

in our case by 4, the number of feet of the given rope, which was 1000, and the result is 4000; dividing this by 10 leaves 400. Say then: if 10 feet of such a rope weigh 3 pounds, a rope of 1000 feet weighs 300 pounds. When these are taken from the 400 pounds, there remain 100 pounds of lead to be hung on the rope of a thousand feet so that it is drawn straight down. Otherwise, if the weight of the lead is less, and the sea deeper than a thousand feet, the rope will drift obliquely, and the depth of the sea cannot be explored with such a sounding-line.

Diagram. Two apparatuses side by side, set into the right column beside the worked example. At the left, a large speckled ball with a short bar and a slanting hook below it. At the right, a horizontal line crossed by a vertical rope, with a pointed lead weight hanging from the rope's lower end. The numerals on the rope and beside the weight give their weights in the example.

- D (above the ball)
- A (left end of the bar)
- C (under the ball, small letter)
- B (right end of the bar)
- F (lower end of the hook)
- M (left end of the horizontal line)
- E (at the crossing of the rope and the line)
- R (right end of the horizontal line)
- 3 (beside the rope)
- G (at the top of the weight)
- 2 1/2 (beside the weight)
- K (at the point of the weight)

XI. [Margin: Leon.Bapt. Alberti fallax modus metiēdi fūdū Maris. — Leon Battista Alberti's fallacious method of measuring the bottom of the sea.] For the other method, which *Alberti* teaches, of exploring the depth of the sea without a sounding-line is most fallacious. As he directs, and *Biancani* after him in the *Sphaera*, Book 5, Chapter 4, a lead hook ABF, of which the longer part is BF, is to be hung from the little loop C on a small ball of oak gall or cork D, and let down to the bottom of the sea, and at the same moment the clepsydra is to be turned. For as soon as that lead strikes the bottom of the sea, the hook will tip back and come out of the loop C, and so the gall or cork will return upward to the surface of the water. When it has returned, the clepsydra is to be stopped and the time measured. Then let the depth of that sea be explored with a sounding-line, and let it be, for example, 300 feet, and that time 10^{m}. Therefore, he says, if a hook of the same weight, and the cork, are let down into another, unknown sea, and the time of descent and ascent together is counted, its depth can be known without a sounding-line: for as 10^{m} are to 300 feet, so this second time is to the depth sought.

However, as *Cabeo* points out in the *Meteora*, Book 2, text 6, question 15, Alberti wrongly supposes that the ratio of depth to depth is the same as that of the one time to the other, and that the waters of the sea are everywhere of the same

thickness and density. But heavy and light things ascend or descend in their natural motion faster and faster towards the end, and so non-uniformly. This takes place, as I shall show elsewhere from our own most certain experiment, made before we had read Galileo on this matter, according to the arithmetical increase of the continually odd numbers. One must therefore find the square number of each time; for whatever proportion there is between these squares, the same will hold between the depth previously known and the one being sought. This assumes, however, that other things are equal in the quality, agitation, etc. of the sea water.

XII. The increase of velocity by which heavy things are carried downward and light things upward in natural motion along a straight line, faster and faster towards the end, will be explained by us more opportunely at the end of this book, in Chapter 21.

CAPVT VI.

That the Earth is like a point with respect to the heavens, although with respect to ourselves it is very large.

I. That this terrestrial globe is very large with respect to ourselves is evident from two things. First, the greatest part of its surface is the part that all the many men and animals do not occupy with their bodies, whether standing or lying down. Second, although the walls of houses are in fact built to the perpendicular, they are not parallel, since the lines of the perpendiculars, if produced, would meet at the centre of the Earth; and yet to the senses they appear parallel, because of the great distance from the centre of the Earth. But if the Earth were small, and the distance of its surface from the centre moderate, the straight lines of the perpendiculars would appear inclined towards one another, in the direction of the centre.

II. [Margin: Argumenta paruitatis Terræ collatæ cum Cœlo. — Arguments for the smallness of the Earth compared with the heavens.] Nevertheless, that this same globe is like a point and exceedingly small with respect to the heavens, especially the highest heaven, is the common assertion of the wise. So say *Aristotle*, *De caelo*, Book 2, text 110; *Ptolemy*, *Almagest*, Book 2, Chapter 6; *Regiomontanus*, in the same place, Conclusion 4; *Copernicus*, Book 1, Chapter 6; *Johannes de Sacrobosco*, in Chapter 1 of the *Sphere*, and *Clavius* on that place, from p. 143; *Maurolico*, Dialogue 1 of the *Cosmographia*, p. 11; *al-Farghānī*, Differences 4 and 22; *Barozzi*, Book 1 of the *Cosmographia*, p. 30.

The arguments for this smallness are almost the same as those for its position at the centre of the universe, on which see Chapter 2. For everywhere on land and sea a whole hemisphere of the heavens is seen, and half of the starry zodiac. And of any two stars diametrically opposed, that is, 180° distant from one another, when one is seen on the eastern horizon the other is seen by us on the western horizon, whether they are fixed stars, as roughly the Heart of the Scorpion and the Eye of the Bull, or wandering ones, as the Sun and the

Moon at full moon. And the same stars, whether viewed on the horizon or on the meridian, appear to us of the same magnitude, and keeping the same distance from the other fixed stars. And stars situated on the ecliptic [recte: the equator, per the book's errata, which correct *Ecliptica* to *Æquatore*], or near it, remain above the horizon for 12 equinoctial hours.

All this would be false if the globe of the Earth were very large in comparison with the heaven of the Sun, let alone the heaven of the non-wandering stars, as one may see in the following figure. In it, if the Earth is small, like A, and the heaven is HZR, the hemisphere HZR can be seen by us, or at least LZM; nor can its portion LH or MR, which lies between the rational horizon HR and the sensible horizon LM, and which is withdrawn from us by the Earth's semidiameter, be noticeable. But if the Earth is imagined as large as the circle BC represents, the sensible horizon would be PBQ, and our eye at B would see of the heavens and the zodiac only PZQ; and stars at Z would appear to us much larger, as being nearer, than at P or at Q, etc.

Diagram. Set into the left column at the right of the text. Three concentric circles: a large outer circle, a middle circle, and a small circle at the centre. Three parallel horizontal lines cross the figure: an upper line touching the top of the middle circle and running out to the outer circle, a second line a little above the centre, and a third through the centre, cutting the small circle.

- Z (top, above the outer circle)
- B (top of the middle circle, on the upper line)
- P (upper line, left end)
- Q (upper line, right end)
- L (second line, left end)
- M (second line, right end)
- H (third line, left end)
- R (third line, right end)
- A (below the small central circle)
- C (bottom, between the middle circle and the outer circle)

III. [Margin: Arg. alia eiusdē paruitatis. — Other arguments for the same smallness.] Astronomical instruments used on the surface of the Earth, and sundials drawn on that surface, show us the motions and positions of the stars just as if their centres, or the tips of their gnomons, were at the centre of the Earth; and on that supposition no sensible error is found. Nor is the variation of view noticeable, except with respect to the Moon, of which shortly. It is therefore a sign that the whole Earth is like a point with respect to the heavens, since it makes no difference whether the observations just mentioned are made on its surface or at its centre.

But if the Earth were very large with respect to the highest heaven, there would also be a great difference between them, and a great variation of view, that is, a great parallax. Nor would the horizontal parallax of the Moon itself be less than 2°, as it now is; nor that of the Sun less than 4′; nor that of the fixed stars less than ½′, as they have been found up to now by all astronomers. For

the horizontal parallax of any star is as great as the arc of the heavens themselves intercepted between the two horizons, namely the rational and the sensible; on this arc see Chapter 2, Scholium 2. Therefore, if the Earth were BC, that parallax would be as great as the arc HP or RQ, and hence of many degrees.

IV. The rays of the Sun reaching the Earth, propagated from the same point of the Sun, are parallel to the senses, as I shall show in Book 3, Chapter 10. And for that reason the meridian shadows of two gnomons set up on the same meridian line are, to the senses, equal on the same day; which would not happen if the Earth were very large with respect to the heaven of the Sun, and therefore close to it. But the very distance of the stars, of which in its proper place, compared with the distance of the Earth's surface from its centre, of which likewise in its proper place, that is, the semidiameter of the heaven compared with the Earth's semidiameter, shows the smallness of this sphere compared with the celestial sphere. And it refutes those who would dare to say that it is by the force of horizontal refraction that a hemisphere is seen by us, and that with this removed a very small portion of the heavens would be seen.

SCHOLIA.

I. On this cosmographical truth is founded that moral and high-minded exclamation [Epiphonesis] of Seneca, found in the preface to the *Natural Questions*: "Is this that point which is divided among so many nations by iron and fire? O how ridiculous are the boundaries of mortals! It is this point on which you sail, on which you make war, on which you dispose kingdoms!" But what is small for a giant is a vast plain for an ant, and the lion's hide that barely covered Hercules could hold many Pygmies. Add here a similar closing exclamation [Epiphonema] of Maurolico, from Scholium 4 below.

II. [Margin: Clauij opinio de mole Terræ, & Cœli. — Clavius's opinion on the bulk of the Earth and of the heavens.] How great a portion of the highest heaven the Earth's semidiameter takes from us out of the hemisphere, that is, how great the horizontal parallax of the fixed stars is according to the opinion of certain authors, we have already said in Chapter 2, Scholium 2. But how great the whole Earth is with respect to the heavens cannot be defined unless the diameters of both are determined. For the sake of argument, however, *Clavius*, on p. 147 of the *Sphere*, following what he himself takes to be the common opinion of astronomers, assumes that the diameter of the firmament, as far as its concave surface, contains 22612½ diameters of the Earth. And since spheres are to one another in the triplicate ratio of their diameters, by proposition 18 of Book 12 of Euclid, he concludes that the solidity of the Earth and sea together is to the solidity of the heaven placed within the concavity of the firmament as 1 to 11562340095703⅛; which is insensible, and like a point, with respect to so vast a number.

But, with Clavius's leave, the comparison that cosmographers intend when they say that the Earth is like a point concerns the diameter or semidiameter; for it is this that makes the variation of view in the phenomena great or small.

Granted, from the smallness of the bulk the smallness of the diameter may be inferred.

III. [Margin: Alfragani argumentum. — Al-Farghānī's argument.] *Al-Farghānī*'s argument from the stronger case [à fortiori], in which he says that many of the fixed stars are larger than the whole Earth and yet are like points of light with respect to the firmament, so that the Earth is much more like a point with respect to the firmament, is indeed excellent, especially if the stars are viewed through the telescope and stripped of that hair of rays that makes them look larger; for they then appear as shining points. But it presupposes a great distance of the stars from the Earth, known and proved from elsewhere; otherwise opponents would say that the stars appear so tiny because they are in fact so tiny, and smaller than the Earth.

IV. [Margin: Quanta Tellus visa ex Cœlis? — How large would the Earth look seen from the heavens?] *Cleomedes*, Book I of the *Cyclical Theory*, says that the Earth viewed from the Sun appears like a point, but from the fixed stars does not appear at all. And *Maurolico* also asks, in Dialogue I of the *Cosmographia*, p. II, how large the Earth would appear if it were viewed from each of the heavens of the planets or of the fixed stars. He could have answered briefly: of as many minutes as the horizontal parallax of those stars, doubled, contains; and that parallax is inferred from the distance of the stars. Nevertheless, assuming the distances of Ptolemy or of al-Farghānī, so as to explain the matter by some perceptible and more familiar comparison, he answers in these words, certainly worthy to be copied out here:

"Placed in the heaven of the Moon, you would see the Earth three times as wide as the Moon is seen from here, and somewhat more. From the orb of the Sun, you would see it twice as wide as Venus appears here. Being in the sphere of Mars, you would say the Earth was equal to a small star, provided it shone. And finally, from the starry heaven you would not easily gauge it by sight, so small would it show itself. I think that, stupefied there, you would say: Goodness! How can it happen that so great a mass, so great a magnitude, looks so tiny? Or how can that little body, or rather that point, hold so many castles, so many estates, so many cities, so many provinces, so many islands, so many mountains, so many forests, so many valleys, so many rivers, so many lakes, so many seas?" And, what is the height of wonder, he adds: "Are there so many envies there? Are so many desires, so many malices, so many frauds practised?" And in these and other words you would laugh at the folly of men.

Yet by the trickery of the demon, who dared to display all the kingdoms of the world and their glory from a very high mountain even to Christ the Lord himself, these things seem greater than can befit the greatness of the human soul. Would that we might ascend that holy mount Zion, on which, according to the prophecy of Isaiah, chapter 33, "the eyes of the just will see the King in his beauty, they will behold the land from afar"; that is, they will look down on it as nothing, if here too they have looked down on it in comparison with heaven.

I now add the words of Pliny the Elder from Book 2, Chapter 68, truly fine ones, on the same argument: "These so many portions of the Earth, or rather, as more have handed down, this point of the universe (for the Earth is nothing else in the universe), this is the material of our glory, this its seat. Here we hold honours, here we wield power, here we covet wealth, here the human race is in tumult, here we renew wars, even civil ones, and by mutual slaughter we make the Earth roomier. And to pass over the public furies of nations, as for this Earth, on which we drive out those who border us and by stealth dig our neighbour's turf on to our own soil, so that he who has marked out his fields most widely and driven his neighbours beyond their bounds, what fraction of the lands does he enjoy? Or when he has extended them to the measure of his greed, what portion of them does he obtain in the end, once dead?"

Finally, to this purpose belongs what Scipio the Elder impresses on the Younger, for despising the glory that is sought from the little men of so small an Earth, as Cicero has it in that truly golden *Dream of Scipio*.

V. Since, however, Maurolico assumed certain distances of the planets that perhaps not everyone concedes, here is how you may learn how large the Earth would appear if it were viewed from this or that distance of a star, that distance being known in semidiameters of the Earth. Look up in our logarithmic tables, or in those of others, the residual of the logarithm of the given distance, and you will have the mesologarithm of the angle under whose double the Earth would be seen. Or else divide the radius by the given distance, and the quotient will be the tangent of the angle under whose double the Earth will be seen.

But if you want to know at what distance the Earth would appear under this or that angle, take half of that angle and look up the secant of its complement to 90°; divide this by the radius, and the quotient will give the distance sought. As a specimen of this, see the following small table; another like it is to be seen in Book 3, Chapter 9, Scholium 4. The use of the following table is this: if the distance of the fixed stars is 20627 semidiameters of the Earth, the Earth viewed from there would measure 20″; and conversely, if it had to be viewed under an angle of 20, the distance would have to be 20627 semidiameters; and so on for the rest.

Angle under which the Earth would be seen	Distance from which the Earth would be seen under the said angle (semidiameters of the Earth)
0′ 20″	20627
0′ 40″	10313
1′ 0″	6876
1′ 20″	5194
1′ 40″	4125
2′ 0″	3438
4′ 0″	1719
6′ 0″	1146
115′ 0″	60

CAPVT VII.

On the absolute magnitude of the Earth, and first on the measures of intervals.

I. [Margin: Mensuræ interuallorum. — Measures of intervals.] Before the threefold magnitude of the Earth is presented, a few points on the measures of intervals must be drawn here from the fuller treatment of them that we give in the *Geographical Book*, section 1. From it we assume that the Italian mile consists of 1000 geometrical paces; the pace of five feet; the foot of four palms; the palm of four common digits in breadth; and finally the digit of four grains of our native barley in breadth; while the cubit is 6 palms, that is, a foot and a half.

Further, from the same source I assume that the Italian stadium was equal to the Olympic or Herculean stadium, and was an eighth part of a mile; or, what comes to the same thing, that it consisted of 600 Herculean feet, but of 625 common Roman feet, or 125 paces. The Greek stadium, however, varied, but more commonly was of 120 Herculean paces, as is gathered from *Pliny the Elder*, Book 2, Chapter 23; *Strabo*, Book 7; *Columella*, Book 5, Chapter 1; *Gellius*, Book 1, Chapter 1; *Polybius*, Book 3; *Censorinus*, Chapter 11; *Martianus Capella*, Book 6, the chapter on the circuit of the Earth; *Isidore*, Book 15 of the *Origins*, Chapter 16; and *Hero the Mechanic*. On the other stadia, miles and leagues we must say much in section 1 of the *Geographical Book*.

II. [Margin: Pes Romanus quinā eligendus. — Which Roman foot is to be chosen.] I likewise assume that, of the many ancient Roman feet of which the paces and Italian miles consisted, we should choose the one in use under Augustus and Vespasian. This is, first, because the Roman consuls and emperors used it when they measured the Roman world through their surveyors, as *Polybius* and *Strabo*, cited above, indicate; and these measures still appear in the *Antonine Itinerary*, corrected by our own *Andreas Schott*. And second, because its measure was kept on the Capitol, from which was taken the height of the twin metal congius still preserved today in the Farnese palace at Rome. This congius was examined by *Poetus* [Luca Peto], the jurist [I. C.], who compared the foot cut on it with several other very ancient bronze Roman feet dug out of the ruins; then our own *Villalpandus* [Juan Bautista Villalpando] and *Grienberger* measured it most carefully, copying the measure of the Roman foot from it.

Poetus has shown us this congius and foot in his book *De antiquis mensuris*, and Villalpando in volume 3 on Ezekiel, part 2, Book 3, Chapter 25. *Marino Ghetaldi* approved and chose that measure in his *Archimedes promotus*; and it is the very one that Grienberger left on a sheet at Rome, which sheet Father *Athanasius Kircher* generously shared with me.

Further, that foot which *Willebrord Snellius* chose in his *Eratosthenes Batavus*, and before him *Budaeus* [Guillaume Budé], *Porcius* [?], *Philander* [Guillaume Philandrier] and *Ermolao Barbaro*, was taken from the Colotian and Statilian marble, as they themselves admit. But it is the foot of the architects, who flourished long after Vespasian, as Poetus points out; and it is to the Villalpandian

foot, as it stands in Snel, as 56 to 54. Snel preferred it, however, because it is equal to the Leiden and Rhineland foot, and to the foot used by the Roman architects in building the fortress of the Catti.

Finally, the more recent Bolognese foot, from the common and authentic measure, which we used in measuring the Earth, is to the ancient Roman foot, that is, the corrected Villalpandian, as 5 to 4, or, more finely, as 1495 to 1200. The true length of the half-foot of the ancient Roman foot just mentioned is as much as you see in the following figure R; and that of the recent Bolognese foot, as much as in figure S.

Diagram. Two long horizontal double-ruled bars at the foot of the page, running across both columns below the text and the table, each with a vertical stroke at both ends and cross-ticks along its length. Each is a printed half-foot, to be read off at its full printed length, so the printed length is the content of the figure. Each bar is divided by five cross-ticks into six equal parts. The lower bar, S, is the longer, beginning further to the left and ending further to the right than R; on the leaf its right end rises with the curve of the page.

- R (left end of the upper bar)
- S (left end of the lower bar)

R: The corrected ancient Roman half-foot.

S: The more recent Bolognese half-foot.

I said "to the corrected Villalpandian" because, as the Dutch printers have observed, and Snel following them, the lengths of lines printed from copper plates become shorter once the sheets, wetted beforehand, have dried, and fall short of their prototype by a sixtieth part of themselves; and each time books are printed again, this defect increases. Taking this into account, Snel corrected and enlarged the Statilian and Colotian foot, copied from the first printing of Philander. But I found at Bologna that sheets, although firmer, when worked under a rotating cylindrical press, do not shrink and contract in the same way after they dry, but only by a fiftieth part of themselves in length. Now it was in this way, from bronze plates, that the Villalpandian congius was published, along with the measure of the Roman foot in his work. We have therefore corrected that measure by adding a 49th part. The Samian and Alexandrian foot, and so too the stadium, was to the common Roman and Greek foot as 6 to 5, as *Hero* of Alexandria the Mechanic testifies, and Snel agrees, Book 2 of the *Eratosthenes*, Chapter 2.

III. [Margin: Modus tutior transmittēdi mēsuras ad exteros, & posteros. — A safer method of transmitting measures to foreigners and to posterity.] But there is another method, more difficult indeed yet safer, of transmitting these measures to foreigners and to posterity, to whom the prototype itself cannot be shown: namely, through measures of liquids. So Villalpando says that if a cubic vessel is made holding precisely 10 pounds of purified cistern water, the height of that cube will be equal to the Roman half-foot taken from the congius of Vespasian. But Snel, Book 2, Chapter 5 of the *Eratosthenes Batavus*, because he uses a larger and less ancient foot, says that a cube equal to the Roman half-foot

contains 125½ ounces of purified water. The present-day Bolognese foot, as I have said, is to the ancient Roman foot, and mile to mile, as 5 to 4. And the Leiden or Rhineland foot, taken from Snel's book and corrected as he himself wishes, is to the corrected Roman foot of Villalpando as 25 to 24, and to the Bolognese as 5 to 6.

IV. [Margin: Duplex modus metiendi Telluris globum. — Two ways of measuring the globe of the Earth.] With these points set out in advance, there are several ways of investigating the magnitude of the terraqueous globe, which we shall present and weigh in the *Geographical Book*; but all come down to two heads. The first contains various ways of investigating the diameter or semidiameter of the Earth, however hidden it may be from our eyes. This kind is presented by *Maurolico*, *Grienberger*, *Bettini* and *Cabeo*. Once the diameter is known, the circumference and the rest of the Earth's magnitude are easily deduced, from what was said in Book 1, Chapters 4 and 6, on the dimensions of the circle and the sphere. But among these ways the most evident and most expeditious is the one that very recently occurred to Father *Paolo Casati* of our Society, and it is as follows.

[Margin: Modus P. P. Casati. — The method of Father P. Casati.] Let there be a tower situated on the seashore, of the kind that is the Lantern of Genoa, and let it be AB, of known height. From its top let the surface of the sea, calm and still, be viewed along the line of sight BD, touching the water, which is concentric with the Earth, at the furthest visible point D. For this tangent, by proposition 18 of Book 3 of *Euclid*, will make with the Earth's semidiameter CD a right angle, namely CDB. While the point D is being viewed from B, let the angle ABD be taken with a dioptra and an exact instrument. And let the straight line AE be understood, perpendicular to AB itself, touching the globe of the Earth at A, and produced to E, until it runs into the secant CE, drawn through the viewed point D; for it will cut the tangent BD at F, and there will arise the triangle ABF, right-angled at A. In this the side AF and the base BF can be found, since there are already known, besides the right angle A, the angle ABF and the height of the tower AD [recte: AB].

Now FB [recte: FD, per the book's errata] is equal to AF itself. For since they belong to the same angle ACD, the tangents BD and AE are equal to each other, and the secants CB and CE to each other. Therefore, when the semidiameters CA and CD are taken away, the excesses of the secants, AB and DE, remain equal to each other. And the angles A and D are right, and the angles at F are vertical and therefore equal by proposition 15 of Book 1, and by proposition 32 of Book 1, B and E are equal complements of two right angles. Hence by proposition 26 of Book 1, FD will be equal to AF itself, which is known. Therefore the whole tangent BD will be known, in parts of which AB was known, since it consists of the known parts BP [recte: BF] and FD. Therefore in the triangle CDB, right-angled at D, with the angle B and the side BD known, the Earth's semidiameter CD will be known by the analysis of

triangles; and when this is doubled, the diameter AG, in parts of the known height AB.

Diagram. Set into the left column at the right of the text. A circle with a vertical line running from a point above it, down through the top of the circle and the centre, to the bottom of the circle. From the upper point a line descends to the right to a point on the circle; a horizontal line runs from the top of the circle to the right, beyond the circle; and a line from the centre runs up to the right through the point on the circle to meet the end of the horizontal line.

- B (top, above the circle)
- A (top of the circle, left of the vertical line)
- F (on the horizontal line, where the descending line crosses it)
- E (right end of the horizontal line)
- D (on the circle, upper right)
- C (centre, left of the vertical line)
- G (bottom of the circle)

V. The second head comprises several ways of investigating how many miles, stadia or leagues are included in a given and known arc of a great circle of the Earth, such as the meridian or any vertical circle. For from this it at once becomes known how many such measures the whole great circle contains; and from this the diameter and the remaining dimensions become known, according to what was said in Book 1, Chapters 4 and 6. Those who have used a method of this kind are *Eratosthenes*, *Posidonius*, *Ptolemy*, *Almæon* [?] with many Arabs, *Fernelius* [Jean Fernel], *Mazzoni*, *Kepler*, and ourselves with Father *Francesco Maria Grimaldi*.

But the methods of others, since they need examination and some of them correction, we reserve for the *Geographical Book*; and for just reasons we present in this place only the one that Father Grimaldi and I, in joint study, not only devised but carried into practice with all the circumspection possible.

[Margin: Modus P. Fr. Mariæ Grimaldi, & Noster. — The method of Father Francesco Maria Grimaldi and our own.] It has two parts. The first lies in investigating the arc of a vertical circle between the tower of the great church of Modena, called the tower of San Geminiano, and the country house that our College of Bologna has on Monte Serra near Paderno [Paternum], where the observation was undertaken during the general vacation, because of the suitability of the place. The second part contains a way of measuring geometrically the paces and miles lying between the tower and that house.

VI. For the first part, let the horizon of Monte Serra be HR, and its meridian HPR, and the north pole B, and the vertex or zenith of that mountain P, and the meridian of the tower of Modena BMH, etc., in which the zenith of the tower is M. Let the altitude of the pole on Monte Serra be BR, 44° 27′, as it has more than once been taken there by us with large quadrants on the occasion of other observations; and so its complement PB is 45° 33′. Likewise, let the altitude of the pole at Modena, taken in repeated observations near the tower

just mentioned by Father Francesco Maria Grimaldi, and precisely established, be 44° 37′; and so its complement BM is 45° 23′.

Diagram. Set into the right column at the right of the text. A circle representing the sphere, with two lines across it from H on the left to R on the right: an upper one on which A stands, and below it the edge of the hatched lower part. From the top point a curved line descends through two points to a point on the horizon; another curved line rises from the left end of the horizon to the upper right point of the circle; and a third curve runs from the lower of the two interior points to the same upper right point.

- P (top of the circle)
- M (inside the circle, upper, where the curves from P and H meet)
- B (upper right on the circle)
- L (inside the circle, lower, on the curve from P)
- A (on the horizon)
- H (left end of the horizon)
- R (right end of the horizon)

Now let there be understood, in a window of our house, a triangle suspended from three threads, in such a way that, with one of them descending perpendicularly by means of a plumb-line hung from it and at rest, the whole triangle would show exactly the plane of the vertical circle PA, the two other threads being directed towards the middle of the tower of Modena. So Father Grimaldi waited, in 1645, at the end of September, with this thread triangle made ready by day and duly directed towards the tower of Modena, for the bright star of Lyra, descending from the meridian, to pass through the plane of the vertical PA. And at once, with a quadrant set up beforehand as was proper, he took its altitude AL, which was 20° 17′, and its distance from the vertex was 69° 43′. At that time the northern declination of this star was, according to Tycho, 38° 29′ 36″; but from our own observations of the fixed stars, carried out with incredible labour, as will be clear from volume 2 of the *New Almagest*, it was 38° 30′. So the complement of the declination of the bright star of Lyra, L, was 51° 30′.

With these points laid down, in the triangle PLB the three sides are given; and so, by proposition 4 of our *Oblique-Angled Spherical Triangles*, the angle PLB is found to be 48° 42′. Then in the triangle BML, with the angle MLB found, which is the same as PLB, and with the sides given as above, BL, the complement of the declination of Lyra, and BM, the complement of the altitude of the pole at the tower of Modena, by proposition 6 of the same *Triangles* the side ML was found to be 69° 25′ 25″. When this was subtracted from the complement of the altitude of Lyra, LP, which was 69° 43′, the arc PM came out at 0° 17′ 35″, lying between the tower of Modena and the house just mentioned, on the vertical circle passing through the vertices of both.

VII. For the second part of the problem, let the tower of Modena be M, and our house on Monte Serra be P. And let the tower at Bologna called the Asinelli be A, and the gate of San Felice with its turret be F, and let the church

and monastery of San Luca be L, on the hill called Custodia, where the most sacred image of the MOST BLESSED VIRGIN stands, painted by St Luke and celebrated at Bologna for its miracles and solemn benefactions; and finally let B, beside the bank of the river Reno, be a certain column or four-sided pier, once the post of some gate. For these stations were chosen by Father Grimaldi as highly suited to this operation.

Diagram. Set in the left column, interrupting the sentence above. A survey network of straight lines. From M at the far left, three long lines run to A, L and P. A stands at the upper right and P at the lower right, joined by a nearly vertical line. F and B lie between M and A, near A; F is joined to A, to B and to L; B is joined to L; L is joined to A and to P.

- M (far left)
- F (upper middle, just below the line MA)
- A (upper right)
- B (middle, left of L)
- L (middle, below F)
- P (lower right)

Then, with a twisted cord stretched straight and supported, the interval FB was measured, so as to have some large side for measuring the remaining triangles more exactly. The angles were taken at the said stations with the help of a geometrical quadrant, and through repeated trigonometry the necessary sides were worked out, until the side MP in the triangle MAP was found: that is, the distance between the tower of Modena and our house on Monte Serra, corresponding to the arc found above. The side BF having thus been found mechanically to be 1088⅖ Bologna paces, the rest of this analysis went as can be seen briefly in the following synopsis. It sets out the order and number of the triangles, the angles both observed and geometrically derived, and the sides of the triangles, with the Bologna paces reduced to ancient Roman ones according to the proportion we stated in no. II to hold between them. Here, then, is the synopsis of this procedure.

Order of the triangles	Angles observed, or derived geometrically: the angles		Value	Sides found trigonometrically, except BF, which was investigated mechanically	Bologna paces	Ancient Roman paces
I.	BFL	Obs.	70° 8′	BF	1088 ⅖	1523
	FBL	Obs.	71° 59′			
	BLF	Coll.	37° 53′	FL	1685 ⅗	2354
II.	LFA	Obs.	116° 9′	FL	1685 ⅗	2354
	LAF	Obs.	43° 16′	FA	864 ⅗	1080
	ALF	Coll.	20° 35′	AL	2204	2755
III.	MLA	Obs.	124° 37′	AL	2204	2755
	MAL	Obs.	50° 17′			
	AML	Coll.	5° 6′	MA	20404	25505
IV.	MAP	Obs.	102° 24	MA	20404	25505
	APM	Obs.	70° 14′			
	AMP	Coll.	7° 22′	AP	2780	3475

Order of the triangles	Angles observed, or derived geometrically: the angles		Value	Sides found trigonometrically, except BF, which was investigated mechanically	Bologna paces	Ancient Roman paces
V.	MAP	Obs.	102° 24′	AP	2780	3475
	APM	Obs.	70° 14′		.	
	AMP	Coll.	7° 22′	MP	21P76 [recte: 21176]	26470

The angles MAP and APM could indeed have been taken, and this operation dispatched with the single triangle APM, if the lie of the land had allowed us to measure the interval AP; but the unevenness of the little valleys and hills did not allow it. It was therefore necessary to use a level plain stretching evenly from the gate F to the pier B, and, sighting the other points mentioned from there, to work out the distance MP, which alone we needed. But Snel needed far more triangles, and his method is much more complicated than ours, as is clear from his *Eratosthenes Batavus*.

[Margin: Nota insignis veritatis, huius mensuræ. — A remarkable mark of the truth of this measure.] [Margin: Distantia inter Turrim Bononiensem & Mutinensē. — The distance between the tower of Bologna and that of Modena.] Two things especially worth noting concur in the measure of the interval MP. For the Roman foot of Villalpando, or Vespasian's foot, corrected as is right from typographical observation, stands to our own Bologna foot as 1200 to 1495, that is, very nearly as 4 to 5. Therefore the distance MA between the tower of Modena and that of Bologna, which Father Grimaldi found to be 20404 Bologna paces without any thought of the Antonine Itinerary, when reduced to paces composed of ancient Roman Vespasianic feet, had to be 25505 paces. And in the most ancient Antonine Itinerary, 25 thousand paces are set down three times between Bologna and Modena, whether because only whole miles are recorded there, or because the distance from the towers to the gates, or to other termini which the Roman surveyors had in view, makes up the difference of half a mile.

Conversely, if the proportion of the ancient Roman foot to that of Bologna is inferred from the distance between these places, taking whole miles, then since that distance is 25 miles in the Antonine Itinerary, and 20 Bologna miles as found by Father Grimaldi and then verified by us, the proportion turns out to be exactly the same as was found from the simple comparison of the two feet, namely as 4 to 5. Amid such darkness and diversity in the ancient measures and distances, I have so far been able to find nothing more evident than this mark of truth among the writers who have laboured in this dust.

VIII. With this foundation laid, then, as a most solid one, the first part of the problem must be joined to the second. So if between the tower of Modena and the summit of Monte Serra, where the house of our College stands, there is an arc of a vertical circle of 17′ 35″, and if in this interval there are 21176 Bologna paces, of which there are 20404 between the tower of Modena and

that of Bologna, then in one degree of a great circle of the Earth, such as a vertical circle is, there are certainly 72256 paces. And if these are converted as 4 to 5, there are in one such degree 90320 Roman paces, measured by the ancient Vespasianic foot. From this measure many of the more accurate ones do not depart very far; on these see the end of Table 1 below, and more in the Geographical Book.

Further, by multiplying 1° by 360 we have the circumference of the whole terraqueous globe, and from it the diameter and its remaining magnitudes, by the rules given in Book 1, Chapters 4 and 6. These measures, finally, are the ones written below.

[Margin: Magnitudo Globi Terraquei trina, immo quina. — The size of the terraqueous globe, threefold, indeed fivefold.] For ease, however, in many if not all of the problems that depend on this, we shall be able to use a semidiameter of the Earth of 4000 Bologna miles, and of 5000 ancient Roman miles. But if, when other stars have been used, something in this measure should have to be changed, we shall not be ashamed to declare it frankly in the Geographical Book.

Meanwhile the distance between the said towers, the altitudes of the pole, and the declination of the bright star of Lyra are most evidently established for us; nor is there reason to doubt about the altitude of Lyra. Even so, we wish to repeat it, to call other stars into counsel, and, God willing, to put into practice other new methods that we have devised for this measurement of the Earth. On another occasion, however, when we supposed the altitude of the pole at Modena, as taken, to be 44° 40′, we deduced from the remaining data above a semidiameter of the Earth of 3897 ancient miles; but it was not evident to us, and from that so great a difference arose.

Measure of the Earth's globe	Italian miles of Bologna, of the year 1645	Ancient Roman Italian miles, in use under Augustus and Vespasian
1 degree of a great circle	72 ¼	90 8/25
Semidiameter of the Earth	4139	5174
Diameter of the Earth	8278	10348
Circumference of a great circle	26010	32512
Area of a great circle	Square miles. 53827695	84108544
Convex surface	Square miles. 215310780	336434176
Solid content of Earth and water together	Cubic miles. 296457106140	580236807150

SCHOLIA.

I. [Margin: Auctores de Quantitate Terræ tractantes. — Authors who treat of the size of the Earth.] The diversity in the size of the terrestrial globe, as great as it is among the authors, arises not only from the different method applied, and from the unequal care taken by different people, but also from the diversity of the measures with which they measured the intervals. If these were brought

back to one better-known measure, the diversity would perhaps not be as great as it appears at first sight. So judge, with Lucillo Filalteo [Philalthæus] wrongly protesting, Pedro Nunes, Book 2 of *De observat.*, Chapter 18, and *De Crepusculis*, Proposition 18; Francesco Maurolico, Dialogue 3 of the *Cosmographia*; Jacopo Mazzoni, in the *Defensio Dantis*, Book 1, Chapter 77; Clavius, on the *Sphere*, Chapter 1, pp. 198 and 211; and the Coimbra commentators [Conimbricenses] on *De caelo* 2, Chapter 14, Question 1. Giuntini, it is true, in his *Sphere*, p. 223, thinks that this came about from the differing swelling of the Earth, measured in different places by different people; and Biancani, in his *Sphere*, Book 4, Treatise 3, Chapter 5, thinks that the Earth has constantly diminished through the wash of the waters, and that its measure has been found ever smaller and smaller, which, however, is false.

The authors just mentioned report various opinions on the size of the terrestrial globe. So too do Aristotle, *De caelo* 2, from the antepenultimate text; Lucillo Filalteo in the same place; Cleomedes, *De Mundo*; Pliny the Elder, Book 2, Chapters 108 and 109; Martianus Capella, Book 6, Chapter 4, and Book 8, p. 192; Finé, Book 5 of the *Cosmographia*, Chapter 4; Fernel, Book 1 of the *Cosmotheoria*, Chapter 1; Daniel Santbech, Problems 15 and 144; Magini, Johannes Werner and Petrus Bercius [Petrus Bertius], in their commentaries on Book 1 of Ptolemy's *Geography*; Francesco Barozzi, Book 1 of the *Cosmographia*, p. 40; Caspar Peucer, in the little work on the measurement of the Earth; Kepler, in the *Epitome of Astronomy*, p. 28; Willebrord Snel, in the *Eratosthenes Batavus*; Mario Bettini, *Apiarium* 2, from p. 78, and *Apiarium* 8, Progymnasma 2, Proposition 3; and Kircher, in the *Ars Magna*, Book 9.

The single opinion of Eratosthenes, on the other hand, is reported by Vitruvius, Book 1, Chapter 6; Strabo, Book 2 of the *Geography*; Censorinus, *De Die Natali*, Chapter 11; and Macrobius, Book 1 on the *Dream of Scipio*. That of Almæon [?] and the Arabs is reported by al-Farghānī, Difference 8, and Ibn al-Haytham, in the little book *De Crepusculis*. Archimedes gives his own in the *Sand-Reckoner*; Ptolemy his own and that of Marinus of Tyre, *Geography*, Book 1, Chapters 7 and 11, and Book 7, Chapter 5; al-Battānī his own, Chapter 6; Tycho his own, *Progymnasmata*, vol. 1, pp. 97 and 733. So also Longomontanus, in the *Astronomia Danica*, Book 2 of the Spherics, Chapter 11; Kepler, in the Rudolphine Tables, Chapter 16, in the *Astronomia Optica*, pp. 115 and 129, and in the *Epitome of Astronomy*, p. 31; William Gilbert, Book 6, Chapter 3; Adriaan Metius, *De Usu Globi*; Stöffler, on the *Sphere* of Proclus, p. 76, and on the *Astrolabe*, Proposition 34; Peter Apian, Part 1 of the *Cosmographia*, Chapter 12; Georg Henisch, on the *Sphere* of Proclus, p. 117; Gemma Frisius, Book 1 of *De Principiis Astronomiae*, Chapter 20; Cornelius Gemma, Chapter 87 of the *Astrolabe*; Mästlin, in the Appendix to Kepler's *Prodromus*; Pierre Gassendi, in the *Institutio Astronomica*, Book 2, Chapter 13; and Mersenne, in the preface to the *Cosmographia Astronomica*.

Lastly, the opinion of the navigators and more recent cosmographers on this matter is adduced by Petrus de Medina [Pedro de Medina], Book 3 of the

Ars Navigatoria, Chapter 7; Nunes, *De observ.*, Book 2, pp. 17 and 27; Gerardus Mercator and Willem Blaeu, at the beginning of the *Novus Atlas*; Clüver, Chapter 7 of the *Introductio ad Geographiam*; Bertius, in the *Tabulae Contractae*, p. 40; Hérigone, vol. 4 of the *Cursus Mathematicus*, p. 164; Bernardinus Ginnarus [Bernardino Ginnaro], Part 1 of *Xaverius Orientalis*, Book 1, Chapter 5; and Ramusio, vol. 1 of the *Navigations*, p. 116, where the opinion of the Portuguese is reported, and p. 374, where that of the Castilians, and in the Voyage of the Dutch to Novaya Zemlya, Part 3, where that of the Dutch. From these, take the following for the present.

II. [Margin: Eratosthenis, & Hipparchi mēsura. — The measure of Eratosthenes and Hipparchus.] Eratosthenes found 694 4/9 stadia in one degree of a great circle of the Earth, and 250000 in the whole circle, as Cleomedes and Strabo report. But if, attracted by a round number, you give 700 stadia to one degree, there are 252000 in the circle; and for that reason this is the number reported by Vitruvius, Macrobius, Censorinus, Capella, Sacrobosco, Clavius, Magini, and indeed Pliny the Elder, who adds that by Roman reckoning these make 31500 miles.

But Eratosthenes erred. Misusing the gnomon of the scaphe, and treating the Sun's rays from different points of the Sun as though they were parallel, he thought that between Alexandria and Syene there were 7° 12′ and 5000 stadia, whereas from the data he himself gives 7° 26′ or 28′ should be put back, as I shall show in Book 3, Chapter 27, no. 7. Therefore he should have put at most 670 stadia in one degree; indeed, since Syene and Alexandria do not lie exactly under the same meridian, he should have put very nearly 600 stadia. If these were Olympic or Roman, they give 75 miles. But if they were Alexandrian stadia, as is more probable, they give 90 ancient Roman miles.

Hipparchus, according to Pliny the Elder, added 25000 stadia to Eratosthenes' figure in the whole circle; therefore 69 stadia to one degree, that is, nearly 9 miles, or, if you follow the Alexandrian reckoning, 10. So with Eratosthenes and Hipparchus corrected, there are 90 or 100 ancient miles in one degree. But more on these points in the Geographical Book, where also on the error of [Margin: Possidonij error. — The error of Posidonius.] Posidonius, who thought there were 5000 stadia and 7° 20′ between Rhodes and Alexandria, when there are only 4° 48′; and on [Margin: Archimedis opinio. — The opinion of Archimedes.] Archimedes, who in the *Sand-Reckoner* affirms that the circuit of the Earth is 300000 stadia.

III. [Margin: Ptolemæi, Albategnij, & Regiomontani opinio. — The opinion of Ptolemy, al-Battānī and Regiomontanus.] Ptolemy, with Marinus of Tyre, gives 500 stadia to one degree. If they had been Roman, these would be 62½ miles. Finé, Barozzi, Filalteo and Giuntini subscribe to this opinion, Giuntini, however, wrongly attributing it to al-Farghānī. But how much more probable it is that the Alexandrian cosmographer used the Alexandrian stadium! For Ptolemy, therefore, one degree has 74 ancient miles, though these are too few. Al-Battānī comes closer to the truth, giving it 85 miles, and nearer to him is

Regiomontanus, giving 80 miles. The cause of the error for Marinus of Tyre and Ptolemy was that they cut the journeys reported by navigators by a third on account of their obliquity, as is clear from Ptolemy, *Geography*, Book 1, Chapters 10 and 12; and I shall show in the Geographical Book that this is too much.

IV. [Margin: Arabum genuina opinio. — The genuine opinion of the Arabs.] Almæon [?], King of the Arabs, as al-Farghānī reports, or rather Alammon [?], as Kircher reads in Abū al-Fidāʾ the geographer, called together a number of wise geometers and ordered them, in the plains of Fingar [?], along the straight route of the Red Sea, to find out how many miles there are in one degree of a meridian circle; and 53⅔ [recte: 56⅔, per the book's errata; corrected by hand to 56⅔ in this copy] miles were found, each of which, however, consists of 4000 cubits, as al-Farghānī says. To this opinion, as the most probable, have adhered Ibn al-Haytham, Thābit ibn Qurra, Ismaël Abilfedea, or Abulfeda [Abū al-Fidāʾ], Abomadus [?] and the rest of the Arabs, as well as Maurolico and Campanus.

Now if you use the Roman cubit, which consisted of a foot and a half, the Arabic mile consisted of 1200 paces. Therefore 5[?]⅔ [the second figure is under an ink blot, with a 6 written over it by hand; the book's errata give 56⅔ for a misprinted 53⅔ in this line] miles are 68 miles, as Fernel, Maurolico and Kepler rightly calculate, although most report those miles indiscriminately as 53⅔ [recte: 56⅔, per the book's errata; corrected by hand to 56⅔ in this copy]. But if the Arabs used the Babylonian cubit, or the ancient Hebrew one, which contained 1 9/12 Roman feet, or the Alexandrian foot, there would be 80 or 81 Roman miles in one degree. Whether Fernel, who drew his opinion of 68 miles from his own experiment, though a very slippery one, as we shall show in the Geographical Book, accommodated it to the numbers of the Arabs, I would not dare to say.

V. [Margin: Iacob. Mazzoni Methodus. — The method of Jacopo Mazzoni.] Jacopo Mazzoni, very ingeniously indeed, starts from the 300 stadia which, on the testimony of Pausanias, Cleomedes and Pliny the Elder, are occupied by the space lacking a meridian shadow of gnomons on the day of the summer solstice at Syene, Cyrene and other places situated under the Tropic of Cancer; and he supposes, from Posidonius, that the apparent diameter of the Sun at that time is the 750th part of the solar heaven in length, that is, 28′ 48″; and he infers 625 stadia in one degree of a great circle of the Earth. But if the diameter of the Sun at apogee is then 30′, 600 stadia in one degree are deduced from this; and if these were Alexandrian stadia, as I shall show to be more probable in the Geographical Book, they make 90 ancient Roman miles, exactly like the corrected measure of Eratosthenes. For it was Eratosthenes too who measured that space under the Tropic.

VI. [Margin: Villebrordi Snellij sententia. — The opinion of Willebrord Snellius.] Willebrord Snel carried out this geometry with incredible labour and industry, as can be seen in his *Eratosthenes Batavus*. In the end he found in one

degree of a great circle of the Earth 68⅖ miles, measured by the foot of Leiden and the Rhineland, which he thought to be the ancient Roman foot. In fact, however, it is the Herculean foot and the greater Greek foot, which Roman architects began to use when the Empire had already been divided, or its seat transferred to Greece. However that may be, I am certain from the data given by Snel himself that the Leiden foot stands to the Roman Vespasianic foot as 25 to 24; and so from his own thesis at least 71 Vespasianic miles are deduced. But his method was too intricate and was composed of too many operations, so that a very small error in each could have produced this sensible and great difference from what we found.

[Margin: Et Gassendi verior. — And that of Gassendi, truer.] Gassendi, who says he is almost certain that 73 Italian miles are contained in one terrestrial degree, is speaking of modern Italian miles. This is as near to the truth as possible, if the talk is of miles such as are used in this district of Bologna. For the diversity of miles within Italy itself is incredible, as I shall show in the Geographical Book.

VII. [Margin: Nauclerorum verior sententia. — The truer opinion of the navigators.] The navigators, as Petrus Medinensis [Pedro de Medina], Nunes and Cornelius Gemma report, give one degree 17½ leagues, each of which, as Pedro de Medina explains, consists of 4 modern Italian miles. In addition, sea-leagues are assigned 4 modern Italian miles by Aloysius Cadamustus [Alvise Cadamosto] in Ramusio, vol. 1, p. 98; by Vasquius Gama [Vasco da Gama], p. 120; by Petrus Aluarus [Pedro Álvares], p. 122; by Simon Maiolus [Simone Maiolo], who was very experienced in voyages to the Canaries, in the tenth Canicular Colloquy; by Io. Gonzales de Mendoza [Juan González de Mendoza], in the index of his *History of China*; and by Fathers Lucena [João de Lucena] and Tursellinus [Orazio Torsellino], in the life of St Francis Xavier. Therefore 17½ leagues make 70 miles of this kind; and if these are equated with Bologna miles, they give 87½ ancient Roman ones. Anyone who prudently weighs the voyages and journeys in Ramusio, Theodorum Bry [Theodor de Bry] and others will see that this is closest to the truth.

To mention just one example, of a very well known and much-travelled voyage: from the port of Sanlúcar, or that of Seville, to the island of Tenerife, a route usually covered on a nearly straight course with following winds, there are 12° on most of the more recent globes. Navigators commonly estimate this route at about a thousand miles, or 250 leagues, although Fernão de Magalhães, as Antonio Pigafetta and Odoardus Barbosa [Duarte Barbosa] state, is said to have traversed it in the year 1519, covering 960 miles. But let these be chosen, so that we may compensate for the obliquity of the route, and let 960 be divided by 12. Here, for one degree there result 70 miles[9]. Nowhere, however, have I been able to read of 53 Italian miles being given to one degree by the navigators, although Clavius affirms it, and following him Biancani, Ginnaro, Bertius and

[9]960 divided by 12 is 80; the page prints 70, and its table below also gives 70 for the recent navigators.

Bettini. But if any have affirmed it, either they were rather inexperienced, or they spoke of longer miles, such as the English ones, for example, which stand to the Italian as 14 to 10, as you will see in the tables and scales of Willem Blaeu, who in the tables of Spain too gives one degree 17 leagues throughout.

VIII. [Margin: Germanorū de Germanicis Milliaribus opinio. — The opinion of the Germans on German miles.] There remain some German geographers or astronomers. Of these, Longomontanus and Mästlin assign 15 German miles to one terrestrial degree; while Peter Apian, Gemma Frisius, Stöffler on the *Astrolabe*, Mercator, Metius, Blaeu, Kepler in the *Epitome of Astronomy* and Georg Henisch assign 15 German and at the same time 60 modern Italian miles, and with these Magini, Hérigone and Clüver agree. But Tycho, and Stöffler on Proclus, assign 15 German and 65 Italian miles. Kepler, however, later says in the Rudolphine Tables, Chapter 16, that the common German mile is two hours' journey for an unencumbered walker, and that there are 15 of these in one degree of a great circle of the Earth. But I, who am not the most robust of men, have more than once easily covered nearly 3 Bologna miles in one hour, and of the smaller kind, such as the ancient Roman miles were, a full 3. Therefore, on Kepler's view, there are 90 ancient Italian miles in one degree. Indeed, in the same place he deduces from Strabo that 800 stadia are due to one degree, that is, 100 ancient Italian miles.

IX. [Margin: Dionysiodori figmentū non omnino vanū. — The fiction of Dionysiodorus, not altogether empty.] Lastly there is Dionysiodorus, famous for his geometrical knowledge. As Pliny the Elder reports in Book 2, Chapter 109, he left in his tomb a letter written in his own name to those above, in which he affirmed that he had reached from the tomb the lowest place of the Earth, that is, the centre, and that it was forty-two thousand stadia to there. Therefore the whole diameter of the Earth, on his opinion, is 84000 stadia, the circumference 264000, and one degree 733⅓ stadia, and so 88 56/100 Italian miles.

This is what Pliny the Elder calls the greatest example of Greek vanity, since Dionysiodorus wanted to attest his geometrical reasoning by an experiment after his death. Yet if you look at the measure, it belongs not to vanity but to truth. For it comes close to the measure that we ourselves have tested among the living, and it agrees with that of Eratosthenes, and nearly with that of Hipparchus, Strabo and the more recent navigators, and likewise with Mazzoni, once corrected. [Margin: Dionysiodori & Dātis iter ad centrū Terræ fictum. — The fictitious journey of Dionysiodorus and Dante to the centre of the Earth.] Mazzoni too, examining the journey of Dante to the underworld, poetically invented, inquires how great the semidiameter of the Earth is, and most ingeniously uses the rays of the Sun, meeting at the centre of the Earth. For where neither Dionysiodorus nor Dante reached, the rays of the Sun are nevertheless supposed, by a more solid fiction, to penetrate, if they are understood as produced straight through gnomons that lack a shadow. Now

follow two tables containing the opinions on the size of the Earth, first corrected, then uncorrected.

Opinions on the size of the Earth, probably corrected or accepted in the preceding Scholia. Authors corrected in the Scholia.	1 degree of a great circle of the Earth: recent Italian miles of Bologna	1 degree of a great circle of the Earth: ancient Roman Vespasianic miles
Dionysiodorus, from Scholium 9.	71	88 56/100
Archimedes, as it stands.	81	104
Eratosthenes, from Scholium 2.	72	90
Hipparchus, from Scholium 2.	80	100
Strabo, from Scholium 8.	80	100
Ptolemy, from Scholium 3, at least. But another correction is needed, as I indicate there.	59	74
Almæon [?] and the Arabs, from Scholium 4.	65	81
Jacopo Mazzoni, from Scholium 5.	72	90
Willebrord Snel, from Scholium 6.	57	71
Pierre Gassendi, from Scholium 6.	73	91 ¼
The more recent navigators, and those named in Scholium 7.	70	87 ½
Some German cosmographers, from Scholium 8.	60	75
Tycho and Stöffler, from Scholium 8.	65	81
Kepler finally, from Scholium 8.	72	90
Or with Strabo.	80	100
Ourselves, as above, for the present, with the fractions thrown away.	72	90

Here we have added either the recent Italian miles from the ancient Roman ones, or those from these, taking the Bologna miles as average modern Italian miles, between those of Piedmont, which are larger, and others, which are smaller.

OPINIONS ON THE SIZE OF THE TERRAQUEOUS GLOBE AS THEY STAND IN THE AUTHORS.

Authors, with the passages cited in Scholium 1.	Measures.	1 degree.	Circumference.	Diameter.
The ancient geometers, in Aristotle.	Stadia.	IIII	400000	127272
Dionysiodorus, in Pliny the Elder.	Stadia.	733⅓	264000	84000
Archimedes, in *The Sand-Reckoner* [in Arenario].	Stadia.	830	300000	95400
Eratosthenes, in Cleomedes and Strabo.	Stadia.	694 4/9	250000	79545 5/11
Eratosthenes, in Vitruvius, Pliny the Elder, Censorinus, Macrobius, and others.	Stadia.	700	252000	80181

Authors, with the passages cited in Scholium 1.	Measures.	1 degree.	Circumference.	Diameter.
	Miles.	87½	31500	10822 8/11
Hipparchus, in Pliny the Elder.	Stadia.	760 8/9	275000	87500
Posidonius, in Cleomedes and Proclus.	Stadia.	666⅔	240000	76363 7/11
Ptolemy, Marinus of Tyre, Capella, Finé, Giuntini, Barozzi, Filalteo.	Stadia.	500	180000	57272 8/11
	Miles.	62½	22500	7159 1/11
Jacopo Mazzoni of Cesena.	Stadia.	625	225000	71591
Mario Bettini, in his earlier opinion.	Stadia.	684 4/9	246396 6/7	78399
al-Battānī.	Miles.	85	30600	9736 4/11
Almæon [?], Abū al-Fidāʾ, al-Farghānī, Ibn al-Haytham, Thābit ibn Qurra, Campanus, Maurolico.	Arabic miles.	53⅔ [recte: 56⅔, per the book's errata; corrected by hand to 56⅔ in this copy]	29400	6491 [recte: 6497, per the book's errata; in this copy the last figure is turned into a 7 in pen, and 97 is written in the margin]
	Italian miles.	68	24480	7789 1/11
Regiomontanus, and Bettini in his later opinion.	Miles.	80	28800	9163 5/8
Fernel, nearly with the Arabs.	Miles.	68 95/1000 [?]	24514½	7800
Willebrord Snel.	Miles.	68⅖	24624	7838
Tycho, and Stöffler on Proclus.	German miles.	15	5400	1720
	Italian miles.	65	23400	7446
Longomontanus, William Gilbert, Mästlin, and sometimes Kepler.	German miles.	15	5400	1720
Peter Apian, Gemma Frisius, Mercator, Magini, Stöffler on the astrolabe, Metius, Clüver, Arnoldus [?], Blaeu, Hérigone.	German miles.	15	5400	1720
	Italian miles.	60	21600	6872 4/11
Some navigators, in Ramusio, using sea leagues and also Italian miles.	Sea leagues.	15	5400	1720
	Italian miles.	60	21600	6872 4/11

Authors, with the passages cited in Scholium 1.	Measures.	1 degree.	Circumference.	Diameter.
More navigators, in Nonius, Pedro de Medina, Cornelius Gemma, and others.	Leagues	17½	6138	1955
	Italian miles.	70	25200	8018
Other navigators, in Clavius, Bertius, Bettini, Ginnaro.	Miles.	53	19080	6070 10/11
Mersenne, giving leagues 3000 paces, but royal Parisian ones.	Royal miles.	60	21600	6872
Pierre Gassendi.	Italian miles.	73	26255	8354
Ourselves, with Father Francesco Maria Grimaldi, using for the present Bologna miles, and those reduced to ancient Roman ones.	Bologna miles.	72¼	26010	8778 [recte: 8278, per the book's errata; corrected by hand to 8278 in this copy]
	Ancient Roman.	90 8/25	32512	10348

CAPVT VIII.

Some corollaries that can be deduced from the size of the Earth are indicated.

[Margin: Time required to circumnavigate the globe of the Earth.]

I First, it is clear from what has been said that what Aristotle contends at the end of *De caelo* 2 holds: the Earth is small with respect to the heaven, and large indeed with respect to human beings, yet not so large that it cannot be travelled around in a few years. And in fact Magellan's ship, as is clear from the narrative of Antonio Pigafetta, who sailed in her, circumnavigated the whole globe in three years, covering 57840 miles on an extremely winding route, or 14460 leagues, to each of which Pigafetta gives 3 miles[10]. And yet they were held up for many months in ports and by dead calms. Now, if it were possible to sail on a straight course 180 Italian miles a day, that is, 2°, then the whole circle could be completed in 180 days, or 6 months. On foot, if you covered 3 miles each hour, you would then complete one degree in 30^{h}, and the whole circumference of the Earth in 450 days.

[Margin: Alexander the Great's couriers.]

But if Anistius and Philonides, Alexander the Great's couriers, covered 1200 stadia a day, or 150 miles, on the testimony of Solinus, Chapter 6, then running in this way they would run around the Earth in 216 days. But woe to their lungs, woe to their chests, woe to their spleens, woe to their legs and soles!

[10] 14460 leagues at 3 miles is 43380 miles; the page prints 3, and its total of 57840 answers to 4.

[Margin: The ingenuity of the Persian kings.]

II If some king wished to learn something as quickly as possible from his antipode, 180° distant, or to announce something to him, then with torches stationed around the semicircle of the Earth, in the manner of the Persian kings and the emperors of Constantinople, or with watchtowers, or with guards shouting in succession and saying, for example, *The King N. is dead*, this could certainly be done within 270^{h}, or 11¼ natural days. For as Cleomedes relates in Book 2 of the *Cyclica Theoria*, and Caelius Rhodiginus [Lodovico Ricchieri] in Book 18, Chapter 8 of the *Antiquae Lectiones*, the king of the Persians, while staying at Susa, received messages from Athens in this way within 48^{h}, although those cities are only 32° apart.

III If Phaëthon had really driven the chariot of the Sun around the Earth at the speed with which the Sun now moves in its daily motion, then, taking the semidiameter of the solar heaven, according to Ptolemy, as 1220 semidiameters of the Earth, and so the circumference of the solar heaven as 7668 semidiameters of the Earth, that is, 38340000 ancient Italian miles, which the Sun runs through in 24^{h}, he would have completed the 32512 miles in the circumference of the Earth in 1^{m} 13^{s}; and so within a single natural day he could have flown around the Earth a great many times.

IV If a shaft were dug right down to the centre of the Earth, a stone let down through the air enclosed in the shaft would reach the centre by natural motion within 25^{m} of time, provided it weighed at least one pound. For in the treatise on the speed of the natural motion of the elements, which we will discuss at the end of this Book 2, Chapter 21, we have shown that a ball of chalk of 8 ounces would cover nearly 4000 miles within 18^{m}, let alone 20^{m}. But the Earth's semidiameter contains about 5000 ancient miles. Yet the stone would swing past the centre, up and down, like a pendulum, as Galileo thinks in the second dialogue on the system of the world, and would finally come to rest at the centre of the Earth.

V If the surface of the Earth's uncovered land is about as great as that of the waters, and if the whole Earth were levelled, or its mountains and valleys were reduced to one convex surface, and each person were allowed one square pace, so that they could stand and lie down, then one square mile would hold 1000000 people. Therefore the Earth's surface, not counting the waters, would hold 168217088000000 people, since half of the terrestrial surface is

168217088 square miles. But whether people descended from Adam, had they not died, would fill this number cannot easily be determined.

VI What Archimedes shows in *The Sand-Reckoner* about the number of the sand, which is not altogether innumerable, and, following him, Clavius in the *Sphere*, p. 218, supposes an Earth's diameter no greater than 10000 miles. But he does not determine the true diameter there. Clavius, however, taking that quantity for convenience, shows that in the whole world, within the concave of the firmament, there would not be grains of sand whose number would exceed unity with 51 zeros, and so that this number can be written, which some denied

in Archimedes' time. But let it be enough to have played at these matters, or played a prelude to them for others.

VII As for a table converting degrees, both of the equator and of the parallels, into miles, or miles into degrees, look for it in the Geographical Book.

CAPVT IX.

On the extent of the Earth known formerly and now.

[Margin: How much of the globe has been navigated so far?]

I Pliny the Elder deals with ancient voyages in Book 2, Chapter 67, with the inhabited portion of the Earth in Chapter 68, and with the measure of the known globe in Chapter 108. So too do Ptolemy, in Book 1 of the *Geography*, Chapters 10 and 11, and Book 7, Chapter 5; Strabo, Books 1 and 2; and Martianus Capella, Book 6; but each for his own age.

[Margin: Distinguished navigators.]

In our own time, not only did the ship of Fernão de Magalhães, called *Victoria* from the conquest of the whole ocean, set sail from Seville on 1519 August 10 and, having circumnavigated the globe westward, return to Seville on 1522 September 7, having covered 14460 sea leagues, though by a winding route, as Antonio Pigafetta relates in his account of this voyage, in Ramusio; but after her Gulielmus Scouthen [Willem Schouten] and others surveyed the whole globe along its longitude. For Thomas Candysh [Thomas Cavendish], an English nobleman aged 28, set sail from England on 1586 July 21 and, having circumnavigated the globe, returned to England on 1588 September 5. But before him Franciscus Dracus [Francis Drake], a noble knight, leaving England on 1577 December 13 [Idibus Decēbris], returned to his native country after going around the globe on 1580 September 28 [4.Kal.Octobris], when he was 43 years old.

II As for latitude, the same Schouten, a Hollander, reached Cape Horn, which is at 58° south latitude. Another man, Gulielmus Barensonius [Willem Barentsz], a Netherlander, reached 81° north latitude. Together these make up 139° of latitude; and 41° are lacking to the semicircle between the poles of the world, or of the Earth. If anyone wishes to know the longitude and latitude of the globe as formerly known, here is a small table for you, with the degrees of the equator or meridian reduced to ancient Italian miles, according to our opinion in Chapter 7; multiplied by 8, these will give stadia.

KNOWN LATITUDE OF THE GLOBE OF THE EARTH.	Lati-tude.	Ancient Italian miles.
For Hipparchus, in Strabo, Book 1, from Meroë to the Borysthenes.	42° 28′	1822
For Eratosthenes, in Strabo, Book 2, from the equator toward the north.	70° 20′	6600
For Pliny the Elder, from the shore of the Ethiopian Ocean to beyond the mouths of the Tanais.	80° 20′	7230
For Isidorus, in Pliny the Elder, from the Ethiopian Ocean to Thule.	80° 31′	7246
For Martianus Capella, from the Ethiopian Ocean to beyond the mouths of the Tanais.	89° 24′	8046

KNOWN LATITUDE OF THE GLOBE OF THE EARTH.	Lati-tude.	Ancient Italian miles.
For Marinus of Tyre, in Ptolemy, from the Tropic of Capricorn to Thule.	87° 0′	7830
For Ptolemy himself, from Agysimba to Thule; 79° 26′, but in round figures.	80° 0′	7200
Now, from 58° south to 81° north.	139° 0′	12510

KNOWN LONGITUDE OF THE GLOBE OF THE EARTH.	Longi-tude.	Ancient Italian miles.
For Eratosthenes, in Strabo, Book 2, from the farthest part of Spain to India.	128° 42′	11583
For Artemidorus, in Pliny the Elder, from Gades to the Ganges.	127° 45′	11497
For Isidorus, in Pliny the Elder, from Gades to the eastern mouth of the Ganges.	144° 24′	12960
For Pliny the Elder himself, from India to Gades.	126° 10′	11355
For Martianus Capella, from the longitude of Africa and of Asia together.	148° 30′	13365
For Marinus of Tyre, in Ptolemy, from the Fortunate Isles to the Seres.	225° 0′	20250
For Ptolemy, correcting Marinus, from the Fortunate Isles to the Seres.	177° 15′	15952
For Ptolemy himself, from the meridian of the Fortunate Isles to the Serican meridian, on the equator.	180° 0′	17200
For on the parallel of Thule, at 63° north altitude.	180°	7300
Of Rhodes, at 36° north altitude.	180°	13100
Of Syene, at 23° 50′ north altitude.	180°	14400
Of Agisymba, at 16° 26′ south altitude.	180°	15460
In our time, the whole circumference.	360°	32400

CAPVT X.

On the distance of sight, or the amplitude of the sensible horizon: where also on the altitude of an object visible on Earth, and on the levelling of waters.

I Many things worth knowing depend on exact knowledge of the Earth's semidiameter. Among them are the four indicated in the title of the chapter, which belong to the same inquiry; chiefly the distance of sight [Distantia visus], to which human sight extends on the surface of the Earth or of the sea, or the portion of the convex surface of the terraqueous globe that it can see, which others call the amplitude of the sensible horizon [Amplitudo Horizontis sensibilis]. Once this has been found, the remaining things mentioned in the title will at once be known, as I will soon show. Moreover, the distance of sight can be taken either as simple, that is, looking toward one part of the world, say the east; or as composite, looking toward both opposite parts, say the east and then, with the eye turned round, the west.

[Margin: Opinions on the distance of sight.]

II I find the opinions on the distance of sight, collected with the greatest care, to be strongly at odds. For:

For Eratosthenes, in Clavius's *Sphere*, p. 297, the simple distance of sight is 352 stadia, or 44 Italian miles.

For Proclus, in the *Sphere*, the simple distance is 250 miles, unless you read in him 100 stadia, namely [Greek: δισχιλίων (dischiliōn, "of two thousand")]; for then it would be 12½ miles.

For Macrobius, *On the Dream of Scipio*, Book 1, the simple distance is 180 stadia, or 22½ miles.

For Albertus Magnus, in Clavius, the distance of sight, I believe the composite one, is 125 miles.

For Clavius, *Sphere*, p. 297, and Barozzi in the *Cosmographia*, the simple distance of sight is 62½ miles, because Ptolemy assigns that many miles to one degree of the terrestrial equator; as if it were certain that a person sees one degree of a great circle of the Earth, whatever his stature and height, which is nevertheless false.

For Goclenius, in *Urania*, that same distance is 20 Italian miles.

For Mario Bettini, *Apiarium* 2, Progymnasma 3, Proposition 7, the average distance of sight is 280 stadia, or 35 miles.

But for Kepler, in Book 1 of the *Astronomia Copernicana*, p. 23; Giordano Bruno, p. 283; Denis Pétau, Book 7 of the *Uranologion*, in the *Variae Dissertationes*, Chapter 10; Pierre Hérigone, vol. 4 of the *Cursus Mathematicus*; and Niccolò Cabeo, in the *Meteora*, it is relative to the height of the eye. If human stature is 1⅕ paces and a person stands on the seashore, according to them he cannot see more than, as nearly as possible, 3½ Italian miles. But if he were 12 paces high, he would see 9 Italian miles toward one side; if 131 paces high, 30 miles; if 524 paces, 60 miles; if 4721 paces, 180 miles. For Kepler there assumes that the Earth's semidiameter contains 3440 Italian miles, and 860 German. Finally, Geminus, in the *Elements of Astronomy*, Chapter 4, said that the diameter of the sensible horizon, as FG in the following figure, does not exceed 200 stadia, or 25 miles, and the semidiameter, or the distance of sight, 100 stadia, or 12½ miles.

[Margin: The error of the geometers about the distance of sight.]

But the opinion of Kepler, Bruno, Pétau, Hérigone and Cabeo is true: namely, that this distance depends on differences of stature and of the height of the eye; and I add, on differences in the size of the Earth's semidiameter. And it is a wonder that so many other geometers, and of such great ability, have been so mistaken in this that they defined this distance absolutely.

Diagram. Set in the left column, interrupting the sentence that follows between "in se-" and "quenti". A large circle with a smaller circle inside it, the two sharing one centre, C. A vertical diameter runs from V at the top of the large circle to K at its foot, through B, A and C. Four horizontal lines cross it: SP, touching the small circle at its top, A; FG, a little below; HR, through C; and MN, at the foot of the small circle. From B two straight lines descend, through F to M and through G to N, so that BMN stands as a triangle around the small circle.

- V (top of the large circle)
- K (foot of the large circle)

- B (on the vertical, between V and A)
- A (on the vertical, at the top of the small circle)
- S (left end of the upper horizontal)
- P (right end of the upper horizontal)
- F (left, on the small circle)
- G (right, on the small circle)
- H (left end of the horizontal through the centre)
- R (right end of the horizontal through the centre)
- C (centre)
- M (lower left, on the large circle)
- N (lower right, on the large circle)

Labels read: all thirteen above. Labels not read: none.

For in the following diagram let FAG be the Earth, whose rational horizon is HCR, and whose sensible horizon is FG with respect to the eye at B, but SP if the eye is at A, a point of the Earth's surface concentric with the surface of the water. For it will see nothing except what is nearest the eye, and it will not see anything of the convexity of the terraqueous globe below the line SAP. But because the eye of a standing person is raised about as much as a person's stature, say AB, it sees something of the convexity of this globe: as much, that is, as the visual line tangent to this globe takes in, namely on this side the arc AF, bounded by the tangent BF, and on that side the arc AG, bounded by the tangent BG. But if AB is not a mere human stature, but includes the height of a mountain or a tower, etc., a far wider space of land or sea all around can be seen; and all the wider, the greater the height AB of the eye, or, at an equal height, the shorter the Earth's semidiameter assumed. Granted, with respect to the heaven an eye at B may see more than a hemisphere, namely the arc MVN. Since, therefore, ordinary human stature is about 5 feet, or one pace, then if the Earth's semidiameter is 3440 Italian miles, Kepler, and with him Pétau, Hérigone and Cabeo, rightly state that a person placed on the seashore cannot see, of the convex surface of the sea, toward one region, more than 3½ miles, and that sight cannot reach even to 4. But if with us you take the Earth's semidiameter as 5000 ancient Roman miles, he will see only 3 miles, as will be clear from the analysis of the triangle, of which presently.

[Margin: Problem of finding the distance of sight.]

So, in order that, given the height of the eye B above the surface at A and the Earth's semidiameter CA, the arc AG can be known, which, converted into miles, gives the distance of sight, let there be a second, simpler figure, in which the Earth's hemisphere is HAR, the rational horizon HCR, and BAC the perpendicular to it, erected from the Earth's centre C. On this the eye is at B, and its height AB is given, which, joined with the Earth's semidiameter AC, likewise given, will give the whole BC.

Diagram. Set at the right of the right column, with the text beside it running short. A semicircle HAR on the base HCR, with C at the middle of the base. A vertical rises from C through A, at the top of the arc, to B above

it. A horizontal chord joins F and G, both on the arc, crossing the vertical. A straight line runs from B down to G, and another from C up to G.

- B (top of the vertical, above the arc)
- A (on the vertical, at the top of the arc)
- F (left, on the arc)
- G (right, on the arc)
- H (left end of the base)
- C (middle of the base)
- R (right end of the base)

Labels read: all seven above. Labels not read: none.

Now let the visual line be drawn from the eye B, touching the Earth or the sea at G, and let this be BG; and to the point of contact G let the straight line CG be drawn from the centre C. For by Euclid III.18 it will make a right angle with the tangent at the point G, and the triangle CGB will be right-angled at G. In this triangle, since the base BC is given, as I said, and the Earth's semidiameter CG, the angle BCG, which measures the arc AG, can then be found by 11 of our Right-Angled Plane Triangles; and this arc, converted into miles according to the opinion chosen on the miles in one degree of a great circle of the Earth, will show the distance to which sight extends toward one side.

But if, conversely, this distance is given, and the arc AG, that is, the angle BCG, together with the Earth's semidiameter CG, and the height AB of the eye is sought, the base CB must be sought by 3 of our Right-Angled Triangles; and when the semidiameter CA has been subtracted, the height of the eye B will be known; or, if the eye were at G, looking at the top B of a mountain, a tower or the like, the height AB of the mountain, tower, etc. will be known, in the same kind of measure in which the Earth's semidiameter CG was given.

[Margin: Abridgement of the problem.]

Therefore, in the first case, make: as the base CB is to the radius of the tables, so the given side CG, or the Earth's semidiameter, is to the sine of the angle CBG, whose complement to 90° will be the angle BCG, that is, the arc AG. Or join together the residual of the logarithm of the base CB with the logarithm of the side CG; and, with unity removed from the left, the result will be the logarithm of the angle CBG, whose complement to 90° is the angle BCG.

In the second case, make: as the radius is to the given semidiameter CG, so the secant of the angle BCG, that is, of the given arc AG, is to the base CB; if you subtract the Earth's semidiameter CA from this, the known height AB will remain. But when the arc AG is less than 1°, or 90 ancient miles, it is expedient first to reduce the Earth's semidiameter CG to paces, or rather to feet, since a pace consists of five feet; and if the arc is less than one mile, the same semidiameter must be reduced to inches, of which 12 make one foot, so that the operation for the levelling of waters [libramento aquarum], discussed below, may be safer and finer. And the secant of the angle BCG, or of the given arc AG, must be multiplied by the semidiameter so resolved, and the

product divided by the radius, or whole sine [Sinum Totum]. For the quotient will give the base CB in the parts into which CG was resolved, and when CG, that is, CA, is subtracted from the whole CB, the height AB will be known.

EXEMPLVM.

[Margin: Example of the second case.]

Let the eye be at G, say in the bottom of a ship, and let it see the summit B of a mountain at a distance of 4°, the distance from which navigators commonly see Mount Pico, which is on the island of Tenerife, one of the Canaries; and let the Earth's semidiameter CG be, according to our opinion, 5000 ancient Italian miles. Now the secant CB of the arc AG of 4° is 10024419 parts, which, multiplied by 5000, makes 50122095000. When this sum is divided by the radius 10000000, there remain for the base CB 5012 miles and in addition 2095/10000, that is, 209½ paces. Therefore, with the 5000 miles, or CA, subtracted, there remains the height AB of the mountain, 12 ancient Italian miles and 209½ paces; and with this set down in turn, the arc AG, or the distance of sight, of 4° would be found by the first part of the problem. By this device the following small table was constructed by us, with 2^{h} of labour.

The distance of sight and of an object, one of the two being placed on the surface of the terraqueous globe, at the extreme visible or visual limit, and the other raised above that surface, the Earth's semidiameter being taken as 5000 ancient Roman miles.

Height of the eye, or of the object, AB: Paces	Feet	Inches	Distance of sight and of the object, or arc AG: Deg., Min.	Anc. Ital. miles	Paces
0	0	2 1/3	0° 0½′	0	750
0	1	0	0° 1′	1	500
1	0	0	0° 2′	3	0
2	0	0	0° 3′	4	500
3	2	6	0° 4′	6	0
5	2	6	0° 5′	7	500
7	2	6	0° 6′	9	0
10	2	6	0° 7′	10	500
13	2	6	0° 8′	12	0
17	0	0	0° 9′	13	500
21	0	0	0° 10′	15	0
30	2	6	0° 12′	18	0
41	2	6	0° 14′	21	0
54	0	0	0° 16′	24	0
68	2	6	0° 18′	27	0
84	2	6	0° 20′	30	0
126	0	0	0° 25′	37	500
190	2	6	0° 30′	45	0
338	2	6	0° 40′	60	0
529	0	0	0° 50′	75	0
761	2	6	1° 0′	90	0
Miles	**Paces**		**Deg., Min.**	**Anc. Ital. miles**	

Height of the eye, or of the object, AB: Paces	Feet	Inches	Distance of sight and of the object, or arc AG: Deg., Min.	Anc. Ital. miles	Paces
3	45		2° 0′	180	
6	850		3° 0′	270	
12	209		4° 0′	360	
19	100		5° 0′	450	
27	550		6° 0′	540	
37	550		7° 0′	630	
49	150		8° 0′	720	
62	350		9° 0′	810	
77	150		10° 0′	900	
111	700		12° 0′	1080	
320	900		20° 0′	1800	
773	500		30° 0′	2700	
1527	50		40° 0′	3600	
1778	600		50° 0′	4500	
5000	0		60° 0′	5400	
9619	0		70° 0′	6372	
23793	850		80° 0′	7200	
41027	550		83° 0′	7470	
71677	950		86° 0′	7740	
286493	450		89° 0′	8010	
Semidiameters of the Earth			**Deg., Min., Sec.**		
☾ 60			89° 3′ 40″	80,16	
☉ 1210			89° 57′ 10″	8095	
7000			89° 59′ 30″	8098	
✶ or ☉ 14000			89° 59′ 45″	8099	

[Margin: The distance of 14000 semidiameters is that of the Sun according to Wendelen, but of the fixed stars according to Tycho.]

SCHOLIA

And the use of the preceding table.

[Margin: To find the distance of sight, given the height of the observer.]

I Given the height of the eye, either by itself or placed on the summit of a mountain or tower, as AB in the preceding figure, and with that height looked up in the first column of the table, you will find the distance, or arc of sight, expressed in minutes or degrees in the second column, and in ancient Italian miles in the third column. For example, if the height is one pace, the eye can see from it an arc of 2′, or 3 miles. But if the exact height is not in the table, use the proportional part between the greater and the lesser. Thus, the height of the Lantern of Genoa is 480 geometrical feet down to the foot of the crag, or 96 paces; in the first column of the table the next lower number is 84 paces, to which correspond 20′, or 30 miles, while the next higher is 126, to which correspond 25′, or 37½ miles. You will therefore infer that a distance of sight of nearly 22′, or 33 miles, corresponds to the height of 96 paces. The operation

will, however, be more accurate if you work according to the rule and the example given before the table.

[Margin: The height of an object, given the distance, with the eye not raised.]

II If the eye is not raised above the Earth, and you wish to know how great the height of the object must be from the given distance of the visible thing, look up the distance in the second or in the third column, and opposite it you will find the required height in the first column. For example, if the eye is on the northernmost shore of the island of Corsica, which may be supposed to be 3° distant from one of the Alps, say from Mount Vesulus [Monte Viso], you will find in the first column that a height of 6 miles and 850 paces is required. But if no such mountain is seen from there, either the assumed distance is false, or intervening mountains stand in the way, or Mount Vesulus is not so high.

[Margin: The distance of sight between an object and an eye, both raised.]

III But if both the eye and the object are raised, and you seek the greatest distance from which the object's summit can be seen, look up separately the distances corresponding to each height and add them together. For example, if a ship's mast is given as 27½ feet high, that is, 5 paces 2½ feet, a distance of sight of 5′, or 7½ miles, corresponds to it. But if the Lantern of Genoa is 96 paces high, then from what was said in no. 1, 22′, or 33 miles, correspond to it. Therefore, by combining these, the distance within which the summit of the Lantern will begin to be seen from the summit of the mast, or the summit of the mast from the summit of the Lantern, and beyond which they will vanish from the eyes, will be 27′, or 40½ miles. But an eye one pace high will not see the ship's mast beyond 10½ miles.

IV Given the distance of a sluice-head [Incilis] or spring from which water is to be led through a single straight channel to a place on a surface of the Earth concentric with the sea (river mouths are of this kind), you will find in the first column its fall [libramentum], that is, the perpendicular height that the sluice-head or spring must have. Otherwise the water will either not run down from there to the intended place, or will do so only by a curved path or through several straight channels not set in a direct line, as was said in Chap. 4, no. 9. From this you will learn that Leon Battista Alberti in his *Architecture*, and others, came as close as possible to the truth when they said that if a straight channel or conduit is a thousand paces long, the height of the sluice-head or spring must be at least 9 inches, but that for the water to flow from there more securely to the desired place, a fall of one foot is required.

CAPVT XI.

On the height of mountains.

[Margin: Opinions on the height of mountains.]

I We said in Chap. 1 of this Book, with Dicaearchus and most cosmographers, that the height of mountains is no obstacle to the physical roundness of the Earth.

But because Pliny the Elder judged that height to be greater than Dicaearchus did, and therefore held a somewhat different view of the Earth's roundness, as we said there in Scholium 1, it is useful to know how great the maximum height of mountains is, especially since some parallax of altitude or depth of the stars, visible even below the rational horizon, ultimately depends on it. But these matters involve many points that are indeed worth knowing, yet rather subtle and not sufficiently examined so far; and therefore we defer a full treatment of them to a complete section on the subject, to be given in the Geographical Book. From it we shall sample a few things here, so as not to send candidates in cosmography away altogether hungry.

II The ancient geometers, then, assigned to no mountains a height greater than 4 Italian miles, though most assigned only 10 or 15 stadia, or 1250 or 1875 paces. I believe because they measured them by a method that I have shown to be fallacious in Chap. 4, Corollary 8, and in Book 10, sect. 4, probl. 32. [Margin: The height of mountains.] Thus Dicaearchus, in Pliny the Elder, Book 2, Chap. 65, and in Geminus, *Elements of Astronomy*, Chap. 14, found Pelion to be 10 stadia high, Satabyrium 14 and Cyllene 15, that is, in paces, Pelion 1250, Satabyrium 1680 and Cyllene 1875. [Margin: Of Pelion, Satabyrium, Cyllene. Of Olympus.] Likewise, as Plutarch has it in the *Aemilius*, Xenagoras measured Olympus at 10 19/125 stadia, or 1269 paces. But in Strabo, Book 11, the Rock of Sisimitres was found to be 15 stadia, or 1875 paces, high, and that of the Sogdians 30 stadia, or 3750 paces. Ibn al-Haytham, and Pererius [Benet Perera] on Genesis, Book 12, Disp. 9, allowed the highest mountains 4 miles. Boterus [Giovanni Botero] in his *Report on the Sea*, Scorcia [?] in *On the Nile*, Book 1, Chap. 2, and Biancani in his *Sphere*, Part 3, Treatise 3, Chap. 5, believe that no mountain is higher than one and a half Italian miles, because no sea is deeper either; since God, in the opinion of St Damascene, *On the Faith*, Book 2, Chaps. 9 and 10, and of Philoponus on Book 2 of *On Generation*, t. 37, made the mountains out of the hollow of the sea. As if some mountain higher than the deepest sea could not have been heaped up from the earth of several hollows, just as many bricks are dug out of clay pits, and from them are built structures far higher than the depth of the pits! Besides, in some places the sea is much deeper than one and a half miles, as I shall say in Chap. 14. Cleomedes, too, thought that the height of neither mountains nor sea exceeds 15 stadia; indeed Plutarch, cited above, not even 10. Kepler, in *Astronomia Optica*, pp. 129 and 135, and in the *Epitome of Copernican Astronomy*, Book 1, p. 26, allowed the loftiest mountains half a German mile, or 2 Italian miles, the same amount he also allowed to the air suited to refractions. Furnerius [Georges Fournier], *Hydrography*, Book 19, Chap. 8, denies that any mountain is 7358 paces high.

[Margin: Of Pico, Etna, Lilybaeum.]

But Willebrord Snel, in *Eratosthenes Batavus*, Book 2, from Chap. 17, shows that Mount Pico of Tenerife, one of the Fortunate Islands, is 10, or at least 8, thousand paces in perpendicular height, Etna 4 thousand paces, and the promontory of Lilybaeum a single mile. Fromondus [Libert Froidmont] in

Meteora, Book 1, Chap. 2, Pétau in the *Uranologion*, Book 7, Chap. 10, and Cabeo on Book 1 of the *Meteorologica*, t. 63, q. 2, subscribe to this. But in Book 10, sect. 4, probl. 30, and sect. 6, probl. 50, I shall demonstrate that Pico is 10 Italian miles high, and Etna 5 8/10, even if refractions assist its being seen from afar.

[Margin: The height of the Caucasus.]

On the Caucasus, it is astonishing how Jacopo Mazzoni, in *The Comparison of Plato and Aristotle*, sects. 3 and 4, Chap. 5, Biancani, on the mathematical passages of Aristotle, at Book 1 of the *Meteorologica*, no. 48, and Cabeo on Book 1 of the *Meteorologica*, t. 63, q. 2, twist themselves, because Aristotle was not sufficiently understood, or because inquiry into deeper astronomy and geometry was neglected. Mazzoni, for his part, infers from the data given by Aristotle a perpendicular height of the Caucasus of 78 miles, but from Magini he reports 276 miles. Biancani gives 270, or 78, or 52, all of which, however, he considers absurd. Cabeo concedes the Caucasus 15 miles at most. But from the data given by Aristotle, fully and plainly understood, I shall demonstrate in Book 10, sect. 4, probl. 30 and 36, that the Caucasus must be granted a height of 58 Italian miles, if no account is taken of refractions; but if account is taken of them, I shall show in Book 10, sect. 6, probl. 50, that the Caucasus is 51 Italian miles high.

[Margin: The height of Mount Athos.]

As for Mount Athos, if what Pliny the Elder, Solinus, Plutarch and others consistently report about its shadow is true, I shall show most evidently in Book 10, sect. 4, probl. 35, that it is 28 Italian miles high. Those skilled in deeper geometry have approved these demonstrations.

[Margin: Mount Casius of Seleucia.]

III. There remain other mountains of no common height, treated in my Geographical Book. For Mount Casius formerly saw the Sun from its summit from the fourth watch of the night, on the testimony of Pomponius Mela, Book 1, Chap. 49, Solinus, Chap. 37, Pliny the Elder, Book 5, Chap. 22, and Ammianus Marcellinus, Book 22; and from this I deduced a height of at least 20 miles. [Margin: Ida.] On the summit of Ida too, a mountain of Phrygia, the Sun shone long before it did on those at the mountain's foot, as Solinus reports, at Chap. 16, and Diodorus Siculus, Book 17. [Margin: Haemus.] Haemus, a mountain of Thrace, was climbed for 6 thousand paces, as Pliny the Elder, Book 4, Chap. 11, and Solinus, Chap. 15, relate, and it was asserted to Philip, King of Macedon, that the Graian Alps could be seen from its peak. [Margin: The Alps.] The Italian Alps rise over a stretch of no less than 50 miles and are climbed in scarcely 5 days, as Polybius, Books 3 and 4, Strabo, Book 4, and Pliny the Elder, Book 2, Chap. 65, say; but for Polybius the ascent and descent comprise 1200 stadia, or 150 miles, from which I estimated a height of 10 or 12 miles. And yet several Peruvian mountains, compared with these Alps, seem like very tall buildings set beside low huts, as José de Acosta affirms, *Historia Indica*, Book 3, Chap. 9. [Margin: The mountains of America.] And the mountains of Beragua

[Veragua] in the province of Paria in America have an ascent of 50 miles, on the testimony of Peter Martyr [Pietro Martire d'Anghiera] in Ramusio, vol. 3, p. 15. [Margin: Mount Atlas.] Pomponius Mela, Book 3, toward the end, Capella, Book 6, the chapter on Africa, and Pliny the Elder, Book 5, Chap. 22, tell marvels of Atlas, a mountain of Mauretania, which on account of its height is sung of as rising to the confines of the Moon and holding up the sky. Pliny the Elder adds that the Consul Suetonius Paulinus reached its summit only at the tenth camp, that is, by a military march of ten days, from which a height of 15 miles and more is inferred, even with the windings taken into account. [Margin: Stolp. The highest of the Riphaeans.] The highest of the Riphaean mountains, called Stolp by the Tartars, that is, the column of the world, can scarcely be climbed in 17 days [recte: scandi], as Sigismund von Herberstein asserts, in Ramusio, vol. 1, p. 169; from which I showed in my Geographical Book that it must be granted a minimum height of 25 miles. [Margin: The mountains of Norway.] The higher mountains of Norway are climbed in no more than four days, on the testimony of Olaus [Olaus Magnus], Book 2, Chap. 2; from this Simone Maioli, in Colloquy 6 of the *Dog Days*, infers a journey of 40 miles, but that is too short; from this I deduced a height greater than 6 miles. [Margin: The Mountains of the Moon.] The Mountains of the Moon, from whose lake rise huge rivers, the Nile, the Niger and the Zaire, which traverse in their course almost the whole length or breadth of Africa and discharge into the sea: from the sure rules on the fall of rivers, to be set out in Chap. 13, I have established for these a height no less than 12 or 15 miles. [Margin: The mountains of Chile.] Father Alonso de Ovalle has recently told me, in speech and in writing, in Chap. 5 of his *Chilean Report*, of the remarkable height of the mountains of Chile, and has affirmed that their ascent takes no less than four days. I am therefore certain that mountains or hills, however high, which we look up at as they extend from the Apennine, are to the very high mountains that Sacred Scripture calls *Mountains of God* as low plants are to very tall pines or oaks.

[Margin: A twofold objection is met.]

But if there are any to whom so great a height would, so to speak, bring on dizziness, let them, I ask, first examine our foundations, and then, if they can, overturn them. As for the fear of violating the Earth's roundness, which moves some, let them know that this Cybele is of such vast stature that not only towers, but not even mountains of 50 miles, set around her head a crown unfitting to it; on this see the appendix to Chap. 1 of this Book. But how preposterous it is to wish to estimate the height of mountains, in some places geometrically evident, from the height of the regions of the air, which is varied and uncertain and to be distinguished not absolutely but relatively to the mountains themselves, I shall show in the Appendix to Chap. 19 of this Book, where mountains that raise their heads above the clouds are treated.

CAPVT XII.

On the diminution of mountains, and on the change in the roundness of the Earth, of shores, etc., and on the argument from this for a finite duration of the world.

Above all, we have three arguments for a perpetual change in the roundness of the Earth and for the decrease of mountains: 1. in the mountains themselves; 2. in the plains; 3. in the shores of the sea.

[Margin: The diminution of mountains.]

I. As to the first: that saying of Job 14 is certain from experience: *By flooding the Earth is little by little consumed.* For rain waters, river waters, and those of torrents and melted snows gnaw away the sides of mountains; so that the foundations of houses on mountains are seen laid bare, or the houses themselves hang from the highest banks of the mountains, about to fall at any moment, or half-ruined. Hence the foundations of the Capitol at Rome are now visible, and others discussed by Georgius Agricola, *De subterraneis*, Book 3, Chap. 1. Moreover, these same mountains are everywhere bored through and undermined by the hidden tunnels of insidious waters, until they give way and great masses of earth slide down, which the Italians call *Labinas*, from their sliding [a labendo]. These have sometimes been so great as to drag entire villages and towns with them into the grave. This happened within our memory to the village of Piuro, within the borders of the Swiss, the Clavena river being blocked as a result; and to others in Pliny the Elder, Book 2, Chap. 91.

[Margin: The increase of the Earth's plains.]

II. By contrast, plains lying below mountains grow from time to time from the mountains' sand and clods, carried down by the force of waters or of carts, to whose wheels they cling; so that the Earth's roundness, which decreases at the summit of the mountains, increases at their feet and in the fields. And therefore ancient buildings that once stood wholly exposed now lie hidden below the pavement, wholly or in great part; and the higher the piled-up earth has shod them in buskins, the greater the proof of their antiquity. Who does not know that ancient Rome must now be sought beneath Rome; that the Arch of Septimius has now become an underground bridge; and that one descends into the Pantheon, into which one formerly climbed by many steps? Or what architect is unaware that this skin of earth heaped on top is what the common people call *Cōmotam* [gloss: "moved" or "disturbed" ground], and is by no means fit to bear the foundations of houses? From this, finally, you may see the ancient windows and door-leaves blocked up at Modena, Ravenna, Benevento and other more ancient cities.

[Margin: Changes of shores.]

III. From the same flooding, and from banks gradually brought together toward the sea, and from earth thrust out by rivers and torrents, inasmuch as the waters' tribute does not suffice, shores grow, and not only valleys but estuaries of the sea, and seas, are filled up. And those exchanges of Earth and Sea take place which Aristotle treats in Book 1 of the *Meteorologica*, Pliny the Elder in

Book 2 from Chap. 85 to Chap. 102, Strabo in Book 12, Polybius in Book 4, and Bercius [?] in Book 1 of the *Geography*. For the part of Egypt called the Delta is a gift of the Nile, and a great part of the territory of Ferrara is a gift of the Eridanus [Po] and the Rhenus [Reno], because they consist of sand and clods carried down by these rivers. The Ammonian marsh and that of Maeotis, on the other hand, were formerly much deeper and could bear larger ships than afterwards, when mud had been brought into them, on the testimony of Aristotle. But at the very shores of the sea five noteworthy things result from this. First, the shores themselves are extended, and cities formerly on the shore are now many stadia distant from the sea. Indeed, where Guelders now is, there was formerly sea; and already in Pliny the Elder's time the sea had receded from the port of Ambracia by 10 miles, from Athens by 5, and now from Padua by 25, from Barcelona by 30, from Ravenna by 6. And there was formerly sea above Memphis as far as the mountains of the Ethiopians, and around Ilium and all Teuthrania, where there are now plains thanks to the Maeander.

Second, islands have been joined to the mainland and, as it were, glued to it, by earth carried in by rivers and torrents or by the collapse of promontories. Thus Pliny the Elder, Chap. 89, Book 2, relates that Antissa was joined to Lesbos, Zephyrium to Halicarnassus, Ethusa to Myndus, Dromiscos and Perne to Miletus, and Narthecusa to the promontory of Parthenium; indeed, that Hybanda, once an island of Ionia, afterwards stood 200 stadia from the sea, within the mainland; that Syrites, in the interior of Ephesus, and Derasidae and Siphonia, in that of Magnesia, which were formerly islands, are inland; and finally that Epidaurus and Oricum have ceased to be islands. To this refers that oracle in Ovid, *Metamorphoses* 15:

The time will come when one day the Pyramus, with its rapid waters, will reach sacred Cyprus by its heaped-up shore.

[Tempus erit, rapidis olim cùm Pyramus vndis, / In sacram veniet congesto littore Cyprum.]

That is, to that Cyprus which, formerly joined to the mainland of Syria, was afterwards torn from the mainland by the force of an earthquake or of the sea; just as Sicily was torn from Italy, Euboea from Boeotia, Atalante and Macris from Euboea, Besbycus from Bithynia, and Leucosia from the promontory of the Sirens, on the testimony of Pliny the Elder, Book 2, Chap. 88.

[Margin: New islands, or islands swallowed up.]

Third, by a hidden addition of earth, with nature working at this under the waters of the sea, new islands arise, which were formerly like hidden hills or mounds of the waters; or certainly they emerge when the sea is thrust elsewhere. Of this kind were the Echinades, heaped up by the river Achelous, and others, of which Pliny the Elder treats, Book 2, Chaps. 86 and 87.

Fourth, the sea having swollen and risen from the soil brought into it, the Venetian, Baltic, Dutch and Prussian estuaries have been filled up; and the sea, overflowing from there, has either swallowed certain islands or peninsulas, or looms over them and threatens them, as Pliny the Elder relates, Book 2, Chaps.

90 and 92, and the geographers here and there concerning Zeeland and certain neighbouring places.

Fifth and last, when earth has been mixed with the roots of trees and with papyri, rushes, seaweeds and other longer and aquatic plants, and thus heaped up, certain great masses form, yet lighter than water and therefore floating. The people of Ferrara, in their valleys, call these *Cuoras*, as if *Coria* [hides]; poets and historians, on the other hand, call them floating islands in the sea; Pliny the Elder treats them, Book 2, Chap. 95. Unless you should say that they are carried here and there resting on the backs of whales, or that they waver and sway, rather than float, from the nearby tremor of volcanoes, or by the force of subterranean fires.

[Margin: An argument against the eternity of the world.]

IV. Therefore, from these frequent exchanges, Biancani, in Book 4 of the *Sphere*, Chap. 4, and on the mathematical passages of Aristotle, p. 103, concludes that the Earth neither has been from eternity with this shape, nor will naturally last for eternity. And he judges it necessary that, with earth brought into it, the sea has at length overflowed, or will wholly overflow, and has overwhelmed or will overwhelm the whole face of the Earth, if the world is from eternity or will be for eternity. But Bartholomaeus Mastrius [Bartolomeo Mastri] and Bonaventura Bellutus [Bonaventura Belluto], *De caelo*, Disp. 4, q. 4, art. 6, teach that this is by no means necessary, because by the force of winds or of cold lands can be heaped up elsewhere, or turn to stone, or new hills be produced. And Simon Stevinus [Simon Stevin], Book 2 of the *Geography*, says that lands, from the decrease of mountains, are levelled in such a way that on one side they rise evenly and on the other hold in the waters; and that it is not necessary for all the earth of the mountains to slide down into the hollows of the sea, but that part of it is spread over the terrestrial surface, so that it never becomes everywhere lower than the sea. Finally Cabeo too, on Book 1 of the *Meteorologica*, text 72, q. 1, infers from this perennial diminution of the mountains that the world has not been from eternity in the way in which it now is; but about a future eternity, nothing. [Margin: Alfonso Pandolfi on the end of the world.] You can see more against Biancani's argument in the most illustrious and reverend Alfonsus Pandulfus [Alfonso Pandolfi], in the treatise *On the End of the World*, which began to be printed this year, 1648, here in Bologna.

CAPVT XIII.

Some select matters for cosmographers: on springs and rivers, and their origin, measure, velocity, overflow, etc.

I. From mountains to springs, from these to rivers, and through these to the seas, the passage is easy for us. But it is not part of our plan to treat fully here what natural philosophers, or hydraulic engineers [Hydragogi], and the measurers and levellers of waters teach about these. Still, a few things must

be pointed out that savour of mathematics and that, if unknown, could lead cosmographers into shameful errors.

DE ORIGINE FONTIVM ET FLVMINVM.

On the origin of springs and rivers.

[Margin: Opinions on the origin of springs.]

All Catholics must indeed accept that saying of Ecclesiastes 1, not as a popular proverb but as an utterance of divine Wisdom: *All rivers enter the sea, and the sea does not overflow: to the place from which [?] the rivers go out they return, to flow again.* But as to how this happens, there are three heads of opinion. For some teach that it happens by local motion alone; others, by a new production of waters from clouds raised from the sea; others, by a new production of waters in the subterranean caverns of the Earth and by a hidden rain, with vapours raised from Tartarus, which communicates with the sea.

[Margin: 1st opinion.]

II. Saints Basil (Homily 4 on the *Hexaemeron*), Jerome (on Chap. 1 of Ecclesiastes), Damascene (*On the Faith* 2, Chap. 9), Isidore (*Origines* 3, Chap. 20) and Philo (*De mundi opificio*), and Hugo Victor. [Hugh of St Victor] (on Chap. 2 of Genesis), Maioli (Colloquy 11 of the *Dog Days*) and the Coimbra commentators [Conimbricenses] (on Book 1 of the *Meteorologica*, at text 62), seem to attribute this solely to local motion, and that perpetual. Add to these Albertus Magnus, who in Book 2 of the *Meteorologica*, tr. 2, Chap. 11, says that vaporous air in caverns of the Earth draws waters upward; I believe, when it is cooled at night or in winter, condenses and occupies a smaller place, in the way that we see air enclosed in thermoscopes and in oblong glass phials condense and, to prevent there being a vacuum, draw water upward. Add St Thomas, in 2, dist. 14, q. 1, and Zanardus [Michele Zanardi] on Book 1 of the *Meteorologica*, who attribute this to the stars, which by a hidden power draw waters upward for the benefit of plants and animals. Add Scaliger [Julius Caesar Scaliger], *Exercitationes* 46, and Geraldinus [?] on Book 1 of the *Meteorologica*, who attribute this to the thicker and heavier water of the sea, which compresses and propels the subterranean waters, made lighter by percolation itself, and drives them to burst out through narrower channels, as through the goose-neck of phials, above the level of the sea. This would be helped if in some place the sea were higher than the spring. Add finally Seneca, *Natural Questions* 7, Chap. 25, who thinks that, just as the upper part of animals draws blood upward, so the upper parts of the Earth draw waters upward, lest they crack from excessive dryness; in much the way that a sponge or a filter draws water upward by its absorbent dryness. Unless here you prefer to take refuge in a Machine and in Intelligences, which maintain this perpetual motion by their impulse or traction.

[Margin: 2. Opinion.]

III. Molina [Luis de Molina], disput. 11 on the work of the six days, has recourse to external rain, which, coming mostly from clouds that arose from

the sea, is supposed to penetrate the earth and produce springs. But it has been observed that rainwater does not penetrate the earth beyond 12 or 15 feet, and much less the rocky mountains of Peru, Chile and the like, under which, however, there are deep sources of springs. Certainly from the beginning, as is said in Genesis 2, "The Lord had not yet rained upon the earth", and yet "A spring rose from the earth, watering the whole surface of the earth."

[Margin: 3. Opinion.]

IV. Others, finally, attribute it to an internal rain and a perpetual dripping of waters. So Aristotle, Book 1 of the *Meteorologica*, from text 63 on, and the Peripatetics there, the Parisians, Kottounios and Cabeo. For that there is under the earth an Abyss of waters, or Tartarus, is handed down not only by Plato in the *Timaeus* and the *Phaedo*, Seneca, *Natural Questions* 3, Chap. 11, Pliny the Elder, Book 2, Chap. 103, Contarenus [Gasparo Contarini], 2 *De elementis*, Virgil, *Georgics* 4, and Fracastorius [Girolamo Fracastoro] in his *Siphyllis*; Sacred Scripture also hints at it, in Genesis 2 and 49, Psalm 134, Proverbs 8 and Ecclesiasticus 24. Those waters, then, on this view, are raised into vapours by the force of subterranean fires; these vapours, cooled in a colder region of the Earth, are turned into drops, which at length run down through higher passages to the mouths of springs. For we see, in the digging of wells, the earth sweating out these drops like a sponge, and in vaulted mountain caverns drops trickling from above, so much so that in one of them I counted more than 300 drops, which were filling the little cistern of a single spring.

V. Yet physico-mathematics [Physicomathesis] could hardly endorse this opinion, if there is brought to calculation on the one hand the water that any river discharges into the sea daily, and on the other the multitude of all the rivers, torrents, streams, springs and lakes that contribute their waters to one and the same river through very many branches. The Eridanus [Po], near its mouths, is about two perches, or 4 paces, deep in the middle, and less deep near the banks; but let it be 15 feet, or 3 paces, all round, and let it be a thousand paces wide there, and cover six miles in one hour. From this it follows that every hour it discharges into the Adriatic 18000000 cubic paces of water, and every day 432000000. Grant now that 60 rivers flow directly into the Eridanus, as Pliny the Elder affirms; of these the Mincius [Mincio] flows from Lake Benacus [Garda], the Oleum [Oglio] from Iseo, the Adda from the lake of Como, and the Ticinus [Ticino] from Verbanus [Maggiore]. How many myriads of drops, then, do you suppose serve for producing and maintaining even one of these lakes? What of so many other rivers and streams flowing together into one of those 60? Take the Athesis [Adige] alone, which flows directly into the sea: let it take in 50, partly rivers, partly streams, before it reaches Trent, and afterwards about as many again, as I myself counted on the chorographic maps of Trent, Verona and Ferrara. All this considered, 432000000 drops would have to labour for the Eridanus alone, each of which, by frequent dripping, would produce one cubic pace of water every day. Now let there be 5 cubic drops in one inch of a foot, reckoned by length; a cubic pace will therefore

require 8100000000 drops. One drop will not make up this number even if it falls at every second of the hour, for at the end of one day there will be only 86400 such drops. But here, for this matter, I only indicate the method, which is to be set out fully in the Geographical Book.

DE MENSVRA AQVARVM.

[Margin: Caution in the measuring of water to be diverted.]

VI. As to the measure of waters to be diverted into meadows, gardens and other similar uses, it is not enough to look at the width of the mouth or opening, and to obtain, say, an obol of water, or to write it into a contract. One must attend to the greater or lesser velocity of the water, arising from brooks, small veins, rains, melted snows, the rarefaction and condensation of enclosed air, winds and so on, but chiefly from the slope. And so at the least the same inclination of the pipe must be written into the agreement; the greater it is, the greater the supply of water it furnishes. As to the measure of the water that rivers carry into the sea, one must attend not to the number of mouths, which are often hardly to be distinguished from the sea's estuaries, but to the trunk of the river before it is dispersed into those mouths; there its width and depth are to be measured. That depth is always smaller near the sea, and it is not uniform but shaped like a shell, and smaller near the banks. Finally the velocity too must be attended to, of which I shall speak at no. 8. On the measuring of depth enough has been said in Chap. 5, Coroll. 10.

DE LIBRAMENTO CANALIVM ET FLVMINVM.

[Margin: Fall of streams and rivers to be led off.]

VII. Fall [libramentum], in this context, is the height at one end, above the convex surface of the Earth, of the horizontal line along which the water descends, and its depression at the other, whether that line is a portion of the sensible horizon or equidistant from it. In a word, it is the slope or inclination of the straight channel along which water-engineers [Hydragogi], for the sake of easier excavation, lead waters over some thousands of feet or paces, although longer rivers do not run down the same straight channel throughout, but along one channel after another, as I said in Chap. 4, Corollary 9. The question is how great the height of the place must be from which water is to be led to a given distance. But one height is sufficient for the water not to stagnate but at least to flow slowly, another for it to run, another for it to run rapidly; hence the differing opinions of the authors. For the water to run, and for the irrigation to be done swiftly and securely, Vitruvius, Book 8, Chap. 7, requires 5 feet of height for every thousand feet of distance, since he says: "And let the bed of the channel have a graded fall, of not less than half a foot per hundred feet." From this I conjecture that in that passage of Palladius, "If it comes over level ground, let the structure slope gradually by a foot and a half between every sixty or hundred feet, so that it can have the force to run", one should read half a foot. For how could it be "gradually" if it were a foot and a half, and in a

thousand feet there were 15 feet, or 3 paces, and in a thousand paces the height were 15 paces? That would be an intolerable rapidity, or rather a precipice of water. But let us hear Leon Battista Alberti, Book 10 of the *Architecture*, and in his stead Daniel Barbarus [Daniele Barbaro] on Book 8 of Vitruvius, Chap. 6: "Geometers, moreover, say that if a straight line touches the globe of the Earth and is extended for a thousand paces, the interval between that line and the circumference and globe of the Earth does not exceed ten digits. So it has come about that, if anyone wishes to lead water in a furrow, the water does not move in each thousand unless at a level a whole foot lower than the place from which the furrow first began to be made. This place is called the *Incile* by jurists, from that first incision made for the leading of water. But if the place declines six feet in a thousand paces, the course of the water will be rapid, so as to be inconvenient and dangerous for ships." For although a height of 10 inches, which the physical horizon has after one mile according to what was said in Chap. 10, no. 6, may seem to suffice for descent, nevertheless, lest the water stagnate and stick because of the fibres and roughness of the channel and the very sluggishness and viscosity of the liquid, a whole foot at the least is required. Hence Kepler in the *Astronomia Optica*, p. 135, said that Vitruvius requires in aqueducts a fall of the two-hundredth part of the space run, that is, half a foot per hundred feet; but he adds: "There is no doubt that so great a height would be ruinous to those sailing. Therefore, if you assign a hundred miles to the windings of the Albis [Elbe], much less than half a German mile will go to the elevation of the Norican land from which it rises and flows into the Ocean, because it flows gently and stagnates." And Froidmont, Book 5 of the *Meteorologica*, Chap. 3, following Daniele Barbaro, said: "The swifter rivers decline toward the centre of the Earth by three, four, five or more feet per mile; but if by six feet, the course is so rapid and torrential that it becomes inconvenient for steering ships in it." Cardanus [Girolamo Cardano], however, in Book 1 of *De rerum varietate*, Chap. 6, says that for diverting canals a height of 8 inches per thousand feet suffices. Therefore, if a river is slow and stagnant, of the kind the Elbe is in Germany and the Arar [Saône] in France, its fall per thousand paces of channel will be much less than five paces, which are the two-hundredth part of a thousand; but if it is so rapid as to be wholly unnavigable and intolerant of ships, its fall per Italian mile will be higher than the two-hundredth part of a mile, that is, more than 5 paces or 25 feet, according to Vitruvius and Kepler; but according to Daniele Barbaro and Froidmont it will be higher than 9 feet, unless Barbaro meant to write a thousand feet in place of a thousand paces. In that way he would be better reconciled with the others, and on that supposition Froidmont, who follows him word for word, would have to be emended. For why do Vitruvius and Palladius require at the minimum 5 feet, or one pace, per thousand feet, unless because the water flows too gently and slowly if the height is less? But if the height stays within 5 paces after a thousand paces, the river will not be unnavigable. We can therefore establish by probable conjecture at least six degrees of the height of the fall,

corresponding to six degrees of velocity of navigable rivers, as we have set out in the following little table. It can happen, however, that the same river shares in several of the velocities listed below, or even in all of them, since, as I said, rivers do not always run along the same channel, or one of the same fall. Thus our Eridanus is unnavigable above Roccelum [?], navigable above the Ticinus and very swift, swift at Cremona and Mantua, and at Ferrara and beyond of moderate velocity; and there is no doubt that the same happens proportionally with the others. For the levelling of waters, moreover, and for finding the height sufficient for the water's outflow, see what we advised in Chap. 4, Coroll. 10.

Degrees of velocity of navigable rivers. Degree.	Height of the fall after each thousand paces of channel. Paces.
Unnavigable	7 8 or more
1 Navigable with danger	6
2 Navigable, very swift	5
3 Navigable, swift	4
4 Navigable, moderate	3
5 Navigable, slow	2
6 Navigable, very slow	1

DE VELOCITATE AQVARVM.

[Margin: Method of measuring the velocity of waters.]

VIII. We have now, then, slid down to the velocity of waters running naturally along sloping channels. Benedictus Castellus [Benedetto Castelli] teaches how to measure it by counting the time in which a gall-nut or a wooden ball, floating on a river or canal, covers a fixed distance. But this method is deceptive. Lighter things, such as cork, straw and gall-nuts, do not take up as much impetus as heavier things, and they are more easily retarded by the cross-current of the waves and by eddies; hence it can happen that the water itself covers twice as long a distance in the same time as that ball. Then again, the lower parts of a river move more slowly than the upper, and near the banks more slowly than in the middle of the channel. So Cabeo, Book 1 of the *Meteorologica*, from text 60, p. 336, suggests another method, carried out with a pole that has a weight hung on it below, so that in its course it is kept by the weight in a perpendicular position, with gourds tied around the pole at the top, which do not allow it to sink down entirely. For if the time is counted in which the gourds cover a fixed distance, he thinks that a more certain pronouncement can be made about the velocity of the water. But here too, because of the resistance, and an unequal one at that, of the lower weight and of the gourds, the water is carried faster than that pole. I should therefore think it better either to tinge the water with so much blood or ink that it keeps the colour in its course, and to count the time in which the coloured water reaches a fixed distance; or, in canals or pipes previously dry, to count the time from when the water begins to flow from an opened stopcock [epistomium]

or opening and runs through a fixed interval; or, for navigation on rivers, to rely on the tradition and experience of the riverside dwellers or shipmasters, who know very well how many miles are usually made in such a stretch of such a river in each hour or day. The rest concerning the velocity of rivers can be reduced to these 5 points worthy of note. The first is that rivers are much swifter near their springs, and mostly unnavigable, than in the middle stretch or near their mouths and outlets. Hence millers place their mills far from the mouths. So the Nile as far as the island of Meroë is unnavigable, and so rapid that it was called the Eagle; so the Danube above Vienna, the Po above the Ticinus, and the Rhône above Avignon are much swifter than afterwards. In the Po, at the Ticinus, about 10 miles are usually made each hour, so that the banks seem to fly; and in the Danube above Vienna in some places 10, in others 12; whereas in the Po, from Francolino to Loredum [Loreo], only 6 or 7 miles are usually covered in one hour. Secondly, the cause of the greater slowness near the mouths is twofold: one is the lesser height of the fall, the other the greater width of the channel. Because of these the banks too are less high there. And so in the Po, for 10 miles near the sea, a height of embankments of 6 feet is enough, but above Ferrara it requires higher banks, of 12 or 15 feet, and above Mantua up to 20; otherwise it readily overflows on the arrival of new waters, unless somewhere far from the sea it is wider, which rarely happens. So rivers compensate for depth with width, and they move more slowly where they are wider, not faster, as Castelli thought, deceived by the smallness of the embankments. For he thought that the embankments were made lower there because the water, passing straight through at greater velocity, does not have sufficient time to overflow towards the sides. Thirdly, the cause why water is the slower the wider the channel is, and again the swifter the narrower the channel or place through which it passes, as when it is forced to pass between the piers and arches of bridges or through narrow enclosures, is this: there the oncoming water is spread out from a straight course into an oblique one, and the quantity of water that holds the straight course is smaller, since a good part of it diverges to the sides. In the second case the cause is that just as much water comes on from behind, and all that water which was being carried dispersed, in a course mixed of straight and oblique, must pass in a straight course; therefore the water must yield place to it so much the sooner. Hence, if water was passing through a channel 100 paces wide in one hour, it will afterwards pass through a channel 50 paces wide in half an hour. In this matter Castelli was likewise deceived; against him, Cabeo, 1 *Meteorologica*, text 60. Fourthly, if two rivers flow down from the same fall and source, one by a straight and very short course, without windings and meanders, the other by an oblique course and through windings, the one that descends more directly is, other things being equal, the swifter. For at an equal interval, or distance from the source, the one carried more directly descends to a lower place, and the inclination of its channel is greater. For example, after the first thousand paces, from a place 100 paces high it descends to a place 80 paces high; but a winding

river, from a place 100 paces high, after a thousand paces of channel does not descend to a place less high than 90 paces. Hence it is that rivers arising from mountains near the sea, like some of those of Peru and Chile, because they have no windings and reach the sea by a very short route, are almost wholly unnavigable; and that, at equal velocity, a more winding river gives evidence of a greater height at its source. In estimating that height it is not enough to consider how many degrees of a great circle it covers on Earth, as some do in the case of the Nile; one must attend to the miles it makes in its oblique course, and weigh one by one the velocity it has in the separate stretches of straight channel. [Margin: Velocity of rivers.] Fifthly and finally, from the history and narration of many, whom I consider it an offence not to believe, I gather that the most rapid rivers, in which, going downstream without sails and oars, 12 miles are made each hour, are the most rapid of the navigable ones, and can indeed be navigated, but not without some danger, and that great attention is needed lest the ship, driven crosswise, strike somewhere. Of this kind in some places are the Danube, the Tigris and the Rhône. After these, rivers are held most rapid without such danger when 10 thousand paces are made each hour, as in the Po above the Ticinus, where the banks seem to fly away behind one; and rapid, when 8 miles. Conversely, they are held slowest when in one hour, without oars and sails, only 2 thousand paces are made, as near the mouths of the Elbe and the Saône; less slow, with 4 thousand paces. A river, therefore, or that part of a river, will be of velocity midway between these in which 5 or 6 thousand paces are covered each hour, as in the Po after Ferrara as far as Loredum. And these things are to be understood of the ordinary course, one not urged on by an extraordinary quantity of water from rains or melted snows, or by winds driving from behind. From this it is permissible, with a balancing that is certainly probable, to compose from these degrees and from the table of no. 7 this second table.

Degree of velocity.	Height of the fall.	Thousands of paces in 1 hour.
	Per thousand paces.	
* Unnavigable	Paces 7. 8. or more	more than 12
1 Navigable with danger	Paces 6	12
2 Navigable, very swift	Paces 5	10
3 Navigable, swift	Paces 4	8
4 Navigable, moderate	Paces 3	6
5 Navigable, slow	Paces 2	4
6 Navigable, very slow	Paces 1	2

DE INTVMESCENTIA, ET EXVNDANTIA FLVMINVM.

[Margin: Swelling and overflow of rivers: from what cause?]

IX. The adequate cause of the swelling of rivers, on account of which they overflow above their embankments into the fields, is not the wind blowing against the mouths, driving the waters back and raising them only as much as

the velocity of the course is retarded, as Benedetto Castelli, with some Roman architects and water-measurers [Hydrometræ], supposed. For that reason it is not a sufficient remedy, for avoiding the floods of the Tiber, the Po and similar rivers, to excavate other mouths not exposed to the wind. For it is impossible for those mouths to meet these two conditions: to be as near the old mouths as is required by the time for the driven-back waters to burst out there through the new channels, and not to be exposed to the same wind, since the same wind occupies nearly the whole stretch of that shore which could hold those mouths. The chief cause, moreover, is new water arriving from rains or melted snows, in such quantity and with such speed that it exceeds what is discharged into the sea in the same time; and if a wind is added that retards its course at the mouths, this concurs accidentally in the flood and the swelling. Therefore either an outlet for the water must be opened by new channels above the place for which a flood is feared, and closed with gates that can be opened when the river swells; or the embankments must be raised only as much as experts judge necessary from the swelling of many years, though always raised more far from the mouths than near them, because there the channel is usually narrower and is filled sooner by the extraordinary water. You will see more on this in our Cabeo, on 1 *Meteorologica*, from text 60, through 9 questions.

CAPVT XIV.

On the threefold dimension of the sea, and the comparison of the bulk of the terrestrial waters with the bulk of the Earth, and their equilibrium.

[Margin: Is the surface of the sea or of the land the greater?]

I. Leaving aside the waters that are above the heavens and those hidden in the abysses of the Earth, since they do not fall within our sight and measurement, there remains a twofold question about the waters that we can see and measure: one about breadth together with length, that is, about the surface; the other about depth. That the land surface is greater than the known surface of the sea is affirmed by the author of Book 4 of Esdras, Chap. 6, though he is apocryphal, who asserts that of the surface of the terraqueous globe a seventh part is water and that the remaining six were granted by God to the land to be inhabited and sown; by Salianus [Jacques Salian], vol. 1 of the *Annals*, at the third day of the world; by Alexander Piccolomineus [Alessandro Piccolomini] in his treatise on this matter; by Io. Baptista Benedictus [Giovanni Battista Benedetti] in his apology for Piccolomini; and by Io. Boterus [Giovanni Botero] in his relation on the sea, who adds that there are probably more stars in the northern hemisphere for this reason, that they may keep in its original dryness the greater part of the uncovered land, which lies in the northern hemisphere. On the contrary, that the surface of all the waters extends more widely than that of the land is the view of Copernicus, Book 1, Chap. 3; Bodinus [Jean Bodin]; Froidmont, Book 5 of the *Meteorologica*, Chap. 1; Cabeo in the same place, text 10, quest. 1; and Antonius Berga [Antonio Berga] in his discourse on this matter

against Piccolomini. Finally, on the more recent geographical maps they seem to Biancani, Book 5 of the *Sphere*, Chap. 4, physically equal, especially if lakes, pools, marshes and rivers are added. Certainly so great a difference does not appear between these surfaces that a longer controversy should be started over it; nor are geographical maps so certain with regard to regions not yet fully discovered that we are compelled to acquiesce in them.

[Margin: Opinions on the depth of the sea.]

II. The depth of the sea, says Scaliger, *Exercitationes* 38, rarely exceeds 80 paces, very rarely 100. And Benedetti, in his letter to Horatius Mutus [?], reports that the greatest depth does not exceed 500 paces, from the account of the shipmasters D. Franciscus Lopez [?] and Rodericus Gusmanus [?]. But Botero too, in the relation on the sea, Scorcia [?] on the Nile, Kottounios on Book 1 of the *Meteorologica*, lect. 17, and Biancani, Book 5 of the *Sphere*, Chap. 4, say that it is rarely found to reach one mile, never to go beyond a mile and a half, and mostly to be below 500 paces. Finally Caesar, having sent explorers through the Mediterranean, the Tyrrhenian and the neighbouring seas, did not find it greater than 15 stadia, that is, 1875 paces; hence that saying of Pliny the Elder, Book 2, Chap. 102: "Palianus [?] reports the deepest sea to be of fifteen stadia."

III. Others, however, hold a deeper view of the sea. Cabeo, Book 1 of the *Meteorologica*, text 20, q. 1, though he says that most frequently it does not exceed 60 or 100 paces, admits that in some places it reaches two or three miles; and Georgius Reisch [Gregor Reisch], Book 7 of the *Margarita Philosophica*, tr. 1, Chap. 44, says that in some places it is of 4 miles. Pliny the Elder, Book 6, Chap. 22, and Solinus, Chap. 54, say that not far from the island of Taprobane there are some channels of the sea "so deep that no anchors settle or have been able to reach the bottom." And Olaus Magnus, Book 2, Chap. 10, relates that near Ulma in Sweden an abyss was found, measurable by no ropes; and that in the ocean of Norway the depth is in some places so great that it could not be reached even by ropes as great as the largest ships could be loaded with, a very heavy mass of lead or iron being added. And our own José de Acosta, Book 3 of the *Indian History*, last chapter, Antonius Pigafetta [Antonio Pigafetta] in his voyages in Ramusio, and Froidmont in the *Meteorologica*, Book 5, Chap. 1, affirm that in some places in the Pacific Sea neither the Castilians nor the Hollanders could find bottom with a sounding-lead, however great. For this reason Maiolo, in the *Dog Days*, Colloquy 10, concludes that the sea is in some places of unsearchable depth. But this could have happened because the bulk of the whole lead was by then lighter than an equal bulk of water; in that case the line of the lead is not stretched straight down but floats obliquely. For see what we said about the method of measuring the depth of the sea in Chap. 5, Coroll. 10. We can, however, plausibly believe that the sea is in some places as deep as 10 miles and more, if we admit mountains so high, and that the sum of their height and the sea's is so great, according to what was said in Chap. 11, even if ships rarely or hardly ever stand above those inverted mountains of

water, and on the perpendicular of those watery peaks, while they explore the bottom with the lead.

IV. Now, if we give the whole sea the depth of one Italian mile, so that excess may compensate defect, and the surface of the sea is taken to be as great as that of the uncovered land, the cubic miles of the sea will not be more than 168217088. But if we give it 500 paces everywhere, they will not be more than 84108544, since the whole surface of the terraqueous globe, from what was said in Chap. 7, no. 8, is 336434176 square miles. Therefore the bulk of this water would be to the bulk of the Earth, on the first hypothesis, about as 168 to 580237, and on the second as 84 to 580237, since the bulk of the terraqueous globe, from what was said in the same place, is 580236807150 cubic miles. So, as regards this part of the element of water, there is no doubt that the bulk of water is much smaller than that of the Earth.

V. But whether, when the water that lies hidden in the receptacles of the Earth is reckoned in, or that which existed from the beginning of the world, before God

Almighty divided the waters from the waters and transferred part of them into heaven, as is found in Genesis 1, in Psalm 148 and in the Song of the Three Young Men; and this is the opinion of so many Fathers that, on the testimony of Delrio [Martín del Río], they carry the force of a single Council, though one not yet confirmed on this point by the Supreme Pontiff, so far as I know: whether, I say, when these are reckoned in, the bulk of the element of water is greater than that of Earth, was once a celebrated controversy. On one side stood Alessandro Piccolomini and his apologist Giovanni Battista Benedetti, who together with Copernicus, Book 1, Chap. 3, fought fiercely for the Earth's excess, and whom Biancani, Book 4 of the *Sphere*, Chap. 4, and Cabeo, Book 1 of the *Meteorologica*, text 11, quest. 1, have endorsed; on the other stood Antonio Berga, in his dissertation on this matter, contending for the water. [Margin: The bulk of the waters compared with the bulk of the Earth.] I judge the question to be unknown to us, and manifest to God Almighty alone and to his Intelligences. For I hear the Lord rebuking in Job 38: "Have you entered the depths of the sea, and walked in the furthest reaches of the Abyss?" I hear Ecclesiasticus, Chap. 1: "The height of heaven, and the breadth of the earth, and the depth of the Abyss, who has measured?"

The arguments of Piccolomini and Berga, moreover, are too merely dialectical [topica] for us to be obliged to lean to either side because of them. Piccolomini's arguments for the most part hold of the uncovered part of the water: as when he says that land is the containing place and water what is placed and contained; that the rays of the Sun and Moon, reaching the bottom of the sea, are the cause of the tides; that vapours of no immense bulk are drawn up from the sea, and so on. Berga's arguments, on the other hand, either rest on gratuitous assertions, as when he says that the whole Earth is like a sponge full of water, and that the bulk of the sponge is smaller than the water enclosed in it, for that is guessing; or they prove too much, as when he concludes the excess

of water from the greater number and bulk of fish than of land animals. For this would prove that the air, which he himself admits to be greater than land and water, is smaller by as much as birds are smaller than whales and elephants; and indeed that the water of the known sea alone is greater than the bulk of the Earth, because only in that water can the fish known to us live, although about this part of the water he holds the same view as Piccolomini. He adds that for the equilibrium of Earth and water, which have the same centre of gravity, it was necessary that water, being lighter than earth, should make up its defect of weight by an excess of bulk: as though water were in one pan of a balance and earth in the other, and as though this equilibrium could not come about if water, although smaller, were suitably arranged around the Earth's centre with that equilibrium preserved. Water's greater affinity with air, in place, in transparency and in moisture, the air being supposed far greater than earth, would at most hold for the whole elementary water created at the beginning. Yet it does not hold, both because liquid things on earth are not greater than dry things for the reason that they are akin to water, if water is supposed greater; and because it is very uncertain whether air, as distinct from the ethereal and celestial aura, is greater than earth, as I shall say below. Finally, Aristotle, 1 *Meteorologica*, text 12, did not say absolutely that a lower element is exceeded by the higher in a tenfold proportion, nor did he teach absolutely, 2 *On Generation*, text 37, that from one cube of earth, if it is rarefied, ten cubes of water come to be; he was arguing against Empedocles, for the sake of a fictitious example. Nor did God in fact make water out of rarefied earth.

[Margin: Is the equilibrium of the seas certain?]

VI. There remains the last controversy, on the equilibrium of the sea with itself and with the Earth. For from what Archimedes demonstrated about the consistency of waters and of every liquid, Biancani, Book 5 of the *Sphere*, Chap. 3, thinks: "All seas are necessarily of exactly equal height, and their surface is equidistant from the centre of the world, because if one were higher than another, it would flow into the lower. It is therefore plain that the counsellors of Sesostris, King of Egypt, were deluded when they dissuaded him from cutting the Egyptian Isthmus, saying that the Red Sea was higher than our Mediterranean sea and that it would accordingly come about that it would submerge all Greece and part of Asia." But notwithstanding these things, St Ambrose, Book 3 of the *Hexameron*, Chap. 2, where he relates that Sesostris and Darius the Mede had the same plan, and Froidmont, Book 5 of the *Meteorologica*, admit that the Red Sea is 3 cubits higher than the Egyptian shore. And Cabeo, Book 1 of the *Meteorologica*, p. 55, contends that the Ligurian Sea is higher than the Adriatic, because he has found by experience that from the pass of the Apennines they call the *Bocchetta* as far as Genoa the slope is, to the senses, as great as from that pass as far as the Ticinus, and that from the Ticinus to the Adriatic shore there still remains the whole of that slope down which the Po runs into the sea. To these is added the Corinthian Sea, which toward the east is believed to be higher than the western one; and this is evident from the

levelling of that Isthmus and from observation at its ends. Two of our Fathers Procurators have affirmed the same thing to me about the Ocean *del Nort* at the port of *Nombre de Dios*, that is, of the Name of God, compared with the Ocean *del Zur* at the port of *Panamà*: for to one looking from the highest mountains along the breadth of that Isthmus which joins the Americas, the Northern Ocean appears higher than the Southern.

Cabeo answers the argument taken from the equilibrium of the waters by saying that this equilibrium must follow if one considers the particular inclination of the waters, by force of which they demand to be equidistant from the centre in the parts of their surface; but not if one considers the inclination toward the good of the Universe, because of which those parts of the sea that lie diametrically opposite the heavier parts of the earth, in the opposite hemisphere, must be higher than the others. But it is not apparent how the land opposite the Ligurian Sea is heavier, since it is plain from the globe and from terrestrial maps that it is covered by the Ethiopic Ocean, at 43° of southern altitude and the neighbouring degrees. I would therefore judge rather that this equilibrium is impeded, both by the course of the sea toward the west, unless this course itself presupposes a slope, and by the variations of the tide, whose reciprocal alternations do not give sufficient time for this equilibrium to be completed. For it is established that in the channels and straits of the sea, the Sicilian, the English and so on, the water goes along one shore and carries ships with it toward the east, and along the other toward the west. Father Grassus [?] observed this very thing in the Tiber, on account of eddies twisting the water back toward the higher part, so that in the same river, with the current, one ship was carried downstream toward Ostia and another toward the Milvian Bridge. What if winds, blowing more frequently in the opposite direction, impede the balancing?

CAPVT XV.

On the Motions of the Sea.

[Margin: First motion of the sea, in longitude.]

I. The motion of the sea imitates the threefold motion of the celestial bodies, in longitude, latitude and depth. First, it moves from east to west. For ship-masters have observed that the voyage from Palestine to Spain, from England to Ireland, from Goa toward the Cape of Good Hope, from Spain toward America, and from the Strait of Magellan across the Pacific Sea to the Moluccas, is completed much more quickly than in the contrary direction, heading east, even when no adverse winds stand in the way and following winds are not lacking. The whole Atlantic and Ethiopic Sea does indeed run toward America; but this course is more evident in the straits and channels themselves, as between Africa and the island of St Lawrence [S. Laurentij insulam], where on account of the multitude of shipwrecks those Current [*Currentes*] waters are called the Valley of Jehoshaphat [*Vallis Iosaphat*]; and in the Gulf of Mexico at the island

called the Dragon's Mouth [*Os Draconis*]; and near Gadalupa [Guadeloupe] and the port of Cartagena and of Santa Marta: so much so that ships cannot struggle against it even with a helping wind, barely complete one whole mile in a day, and have the lead of the sounding-line swept away toward the west. Willebrord Snel adds, in his *Tiphys Batavus*, Book 2, Prop. 8, that between Brazil and Angola this motion of the sea is carried from April 20 to July 20 toward the north-west [Zephyroboream], but from October 20 to January 20 toward the south-west [Austrozephyrum].

I should be inclined to believe that the cause of this motion is the attraction of vapours made by the Sun as it moves with its diurnal motion, placed now in the northern signs, now in the southern; for it is fitting that the air should be moved by their motion, and hence perennial breezes and winds too blow rather toward the west. It is difficult to attribute this to rivers, since many great rivers run toward the east, such as the Maragnon, the Aureliana, the Canada, the Danube and the Yanson, and many to the south or the north. Nor does the diurnal motion of the heavens sweep our air along with it, as Contarenus [Gasparo Contarini] and the Coimbra commentators [Conimbricenses] think, since the whole heaven does not move, nor do feathers or threads suspended in the air indicate this to us.

[Margin: Second motion of the sea, in latitude.]

II. The second motion is from north to south; on it see Aristotle, Book 2 of the *Meteorologica*, Chap. 1, and St Thomas there, and Albertus, tr. 3, Chap. 6. For the Maeotian marsh runs down through the Thracian Bosphorus into the Propontis with such force that it suffices to turn mills, and ships can barely struggle against it. The Propontis in turn rolls down through the Hellespont into the Aegean, whether because of the force of the rivers that break out into the Euxine, as Aristotle thinks, or for some other cause. The Gothic Sea too flows toward France. Granted, those 4 channels [Euripi] near the Arctic pole are said to be carried very rapidly toward the same pole.

[Margin: Third motion of the sea, in height, or the tide of the sea.]

III. The third motion of the sea is flux and reflux, commonly called the tide of the sea [*Aestus Maris*]; because of its swelling upward and subsiding downward, it can be reduced to a motion of height and depth. For Galileo dared in vain to deny this swelling, manifest as it is to so many. Equally in vain does anyone gird himself to investigate the cause of this reciprocating tide who has not first thoroughly learned its history; for its variety is astonishing, both in the quantity of the tide and in the time of its alternations, as I shall presently indicate in brief, going round the shores of the sea in order.

The tides of *Europe* are as follows. In the Adriatic gulf the tide was once so great, on the testimony of Procopius, Book 1 of the *Gothic War*, that it very rapidly occupied many thousands of paces; now it occupies 2 miles of the Venetian shore at most. In the Caspian and in the Tyrrhenian Sea there is no tide, nor in all the rest of the Mediterranean on the shores of Europe, apart from some around Marseille and in the Euripus. Beyond the Mediterranean

a great tide occurs around the western shores of Spain and France. The river Baetis indeed feels it as far as Seville; there, however, on the testimony of Pliny the Elder, was a well that decreased as the tide rose and rose as the tide fell. The Tagus undergoes the tide for 20 leagues, and the Garonne as far as Bordeaux; and the same tide, at its greatest, is experienced by the shores of Flanders, up to 9 miles, and of Holland, Norway and England. The Thames receives the tide for up to 60 miles, and it then rises above 14 royal cubits. Formerly, however, on the testimony of Pliny the Elder, Book 2, Chap. 97, the British tide swelled to 80 cubits; and in the district of Lindisfarne, as Bede asserts in his life of St Cuthbert [S. Gutberti], it became an island twice a day at the approach of the sea, and at its retreat turned back into mainland. Between Norway and Denmark the tide is barely felt, on the testimony of Barthol[?] [recte: Bartolinus, per the book's errata, which correct *Bartholomæo* to *Bartolino*; in this copy the end of the printed name is struck through in ink and the type cannot be read] the Dane.

The shores of *Africa* on the Mediterranean experience a great tide, but those of the Atlantic, on this side of the Tropic of Cancer, almost none. Beyond the Tropic, at Guinea, it is so strong that 3 anchors barely suffice to hold ships against it. It is great too from there as far as the island of St Thomas and around the shores of Ethiopia, and so great in the Red Sea, or Arabian Gulf, that it has given the impious a handle for saying that Moses waited for the reflux of the sea and crossed it dry-shod by no other miracle.

The shores of *Asia* facing the Indian Ocean surge with a frenzied flux and reflux, as Q. Curtius relates, Book 9. At the shores of Cambay it is so rapid that in 2 hours it occupies 30 leagues, and the inhabitants barely escape the deluge unless they are called back by the sound of a cymbal from others keeping watch on higher ground. It is great too in the Gangetic Gulf, in the Erythraean Sea and in the Chinese Sea, so much so that the river Yansù surges daily as far as Nanking [Nanchinum], although at Pachinum [Peking] it is felt only at full and new moons, as Nicolaus Trigautius [Nicolas Trigault] relates.

The shores of *America* near the Baccalaos have a slight tide, and at Cuba and Mexico one barely perceptible; but in the Strait of Magellan, on the testimony of Acosta and of Father Alfonso Ovalle [P. Alfonso Oualio], who told me of it, the tides of the two seas meet each other, with a manifest swelling on both sides. The shores of Paria and of Peru also experience a great tide; at Panama it occupies 2 leagues. The rest of the northern coast of America, except the Vermilion Sea and California, suffers no tide from the Ocean, as Petrus Bercius [Petrus Bertius] relates. The river Maragnon retains saltiness for up to 100 leagues, says Argoli in the *Pandosion*, which is an argument that the Ocean surges nearly that far up it.

[Margin: Periods and alternations of the tide of the sea.]

IV. The alternations of time and the periods of the tides are some monthly, some daily, and some as it were yearly, half-yearly or quarterly. In many places the tide is greater in winter than in summer, and greater at the equinoxes than

at the solstices, though it is great around the rising of the Dog Star; and it is greater at full moon, which is why sailors call it living water [aquam viuam], than at new moon or after the quadratures. Hence that Venetian proverb: *Dalli otto alli noue, l'acqua no se moue; dalli ventuno alli ventidue, la non và ne in sù ne in Zue* [from the eighth to the ninth, the water does not move; from the twenty-first to the twenty-second, it goes neither up nor down]. Yet at Pachinum the tide rises only at new moon, and at the mouths of the Indus likewise it rises at new moon, on the testimony of Bertius; while on the Ethiopian coast and around Calicut it rises at full moon and falls at new moon, as Godignus [Nicolau Godinho] relates, *De rebus Abissinorum*, Book 1, Chap. 11. In most seas, however, there is twice a day a flux and twice a reflux; and in the Adriatic, Belgian, Dutch, English, Portuguese, Pacific and Gangetic seas it flows for 6 hours and ebbs for each following 6. But in the Atlantic Ocean and at the mouths of the Garonne it swells for 7 hours and subsides for 5. Conversely, at the mouths of the river Niger and at the shores of Guinea and of the Erythraean Sea, it rises for 4 hours and falls for 8. At the shore of Cambay it flows very rapidly for 2 hours and ebbs for 2. Where the flux lasts 6 hours and the reflux 6, the flux begins as the Moon ascends above the horizon, the reflux as it crosses the meridian, the flux as the Moon sets, and the reflux as it reaches the lowest point of the heavens.

So much for the history. On the causes, see what I shall indicate in the Scholia, and what we shall add at Book 4, Chap. 1, Scholium 4.

SCHOLIA.

[Margin: Authors who wrote on the motions of the sea.]

I. On the motions of the sea, and especially on the tide, one may consult Aristotle, Book 2 of the *Meteorologica*, text 6, and there Albertus Magnus [Alb. M.], tr. 3, Chap. 6; Froidmont, Book 5; the Coimbra commentators, tr. 8; Zanardus [?], quest. 30 and 31; and Cabeo, Book 2, from quest. 5 to 12 of text 6. Then Pliny the Elder, Book 2, Chap. 97, and Book 3, Chap. 1; Plutarch, Book 3 of *De Placitis Philosophorum*, Chap. 17; Mela, Book 3, Chap. 1; Gemma Frisius in the *Astrolabium*, Chap. 91; Faber [?], theorem 99; Fidelis Daniel [?], Book 1 of *De Providentia*, Chap. 91; Pandulfus Sfondratus [Pandolfo Sfondrati], in his excellent treatise on the cause of the tide of the sea; Io. Taisnerus [Jean Taisnier] and Hieronymus Borrus [Girolamo Borri], in the book on the flux and reflux of the sea; Scaliger, *Exercitatio* 52; José de Acosta, Book 3 of the *Historia Indica*, Chap. 14; Ramusio, vol. 3 of the *Navigations*, Book 2, Chap. 10; Botero, in the *Relation on the Sea*; Simone Maioli, in the *Dog Days*, colloquy 10; Biancani, on the mathematical passages of Aristotle, p. 196, or on the book *De Mundo ad Alexandrum*, Chap. 3, and in the *Sphere*, Book 5, Chap. 2; Laurentius Corvinus [?], Book 11 of the *Geography*; Galileo, in the *Dialogues*, from p. 410; Johannes Kepler, in the commentary on Mars, p. 3 of the introduction, and in the *Ephemerides*, p. 24; Bertius, in the *Tabulae contractae*, Book 1; Simon Stevin, Book 6 of the *Geography*; Levinus [Levinus Lemnius], *De occultis naturae*

miraculis; Ficinus [Marsilio Ficino], Ennead 2, on Plotinus, Book 3, Chap. 6; Pierre Hérigone, vol. 4 of the *Cursus Mathematicus*, p. 412; Antonius Maria de Rheita [Antonín Maria Šírek], in the *Oculus Enoch et Eliae*, Book 4, Chap. 3; Willebrord Snel, in the *Tiphys Batavus*, Book 2, Prop. 8; Argoli, Chap. 10 of the *Pandosion Sphaericum*; Rogerius Baccon [Roger Bacon], Chap. 5 of *De speculis*; Renatus des Cartes [Descartes], Part 4 of the *Principia Philosophiae*, no. 49.

[Margin: The cause of the tide of the sea not yet clearly known.]

II. "But as yet no schools," says Maiolo, "have sufficiently unravelled this question, and Scaliger was ready to call Cardano a god, though in bombast and half as a pagan, if he had brought forward a sufficient cause of the reciprocating tide of the sea." About the simple flow there seems to be less difficulty. For St Augustine says, Book 1 of *De Mirabilibus Sacrae Scripturae*, Chap. 7: "We are permitted to consider the flooding swellings of the sea, but we are deprived of understanding of it as it recedes." And in Book 7 of the *City of God*, Chap. 22, he says that 2 goddesses were consecrated by the Romans for this reason: Venilia, or Malina, for the approach of the sea, which the Italians call *Maream*, and Salacia, or Lidona, for its withdrawal from the shore to the open sea. This alternating motion, moreover, the Greeks call [Greek: πλημμυρίδα (plēmmyrida, "flood-tide")].

III. The Stoics attribute the cause of this tide to the soul of the terrestrial globe, breathing in and out through the cavities of the sea as if through nostrils; Apollonius of Tyana, to spirits panting under the Ocean; Timaeus, in Plato, to rivers rushing from the mountains of the Celts into the Atlantic; Buccaferreus [Lodovico Boccadiferro] and Io. Picus [?], to a sea-bed sloping more or less; Sfondrati, to the motions of the sea discussed in nos. 1 and 2, and to the various whirlpools and regurgitations arising from them. Seleucus, Galileo and some Aristarchus or other, recently revived out of France, attribute it to the agitation of the Earth, carried around the annual orb together with the diurnal motion and that of libration; because of the mixing of these the water, as in a vessel driven round in a circle, is pushed and shaken to one side and repelled to the other at fixed periods. I myself at one time, when I was younger, attributed it to motions arising from subterranean rivers and from the Abyss's eddies, or to an ebullition produced by subterranean fires, or to the alternating dominion of the 7 planets through the temporal hours, assigning them a great part of this cause; but afterwards I hesitated.

IV. The greatest part, however, attribute it to the motions and influences of the Moon and Sun: so Aristotle, Pliny the Elder, Strabo, Heraclitus, St Thomas, Scotus, Albertus Magnus, Ficino, Contarini, Bacon, Zanardus, Gul. Gilbertus [William Gilbert], Fidelis Daniel, the Coimbra commentators, Biancani, Stevin, Botero, Kepler, Rheita, Froidmont and Cabeo. But of these, Kepler with Gilbert, Zanardus and the Coimbra commentators attribute it to a magnetic attraction of the Moon, drawing the waters to itself; Contarini and Faber to occult influences of the Moon; Aristotle and Heraclitus to blasts excited under

the waters by the heat of the luminaries, and to the ebullition that results, especially on account of the heat reflected from the Moon at full moons. Rheita attributes it to a rarefaction produced in the air by the Moon, because of which the compressed waters of the sea swell against the shores, and to a condensation as the air cools, because of which the water, following the contracted air lest a vacuum occur, flows back. Cabeo, who has examined the opinions of others most diligently of all, attributes it to an occult faculty of the Moon, which excites the sulphurous and nitrous spirits at the bottom of the sea and stirs them in nearly the same manner as it moves the catarrhs and humours of animals, even when it is at new moon and placed below the horizon.

V. Although in some places the tide rises for 6 hours and falls for 6, nevertheless, because the Moon advances daily toward the east by about 13° and rises later each day by about 48^m, the beginnings and ends of these motions change daily by nearly one hour. This very time, however, varies with the varying latitude of the Moon, the obliquity of the ascensions, and other anomalies in the Moon. Simon Stevin adds that the highest tide occurs when the Moon is in the nonagesimal degree of the ecliptic, rather than on the meridian and in the degree opposite to it; and that sometimes the tide takes place in $8^h\ 5^m$ but the reflux in $3^h\ 55^m$. But let us conclude these Scholia with those verses of Lucan, Book 1:

> Whether, moved by the second star, the wave of wandering Tethys surges at the lunar hours; whether flame-bearing Titan, to draw up the waters that feed him, raises the Ocean and lifts its billows to the stars: inquire, you whom the labour of the world torments. But for me, whatever cause you are that moves such frequent passages, as the gods have willed, lie hidden ever.

CAPVT XVI.

On Navigation: Some Things Pointed Out to Beginners in Cosmography.

[Margin: Authors on nautical matters.]

I. It is not part of our plan to hand down the art of navigating in this place, nor to pursue everything treated by Bayfius [Lazare de Baïf] in *De re navali*; by Petrus Medinensis [Pedro de Medina] in 6 books *De re nautica*; by Nonius [Pedro Nunes] in his twofold problem on the art of navigating, and in nearly the whole of Book 2 of *De Observatione*, with an annotation on Aristotle's mechanical problem on the motion of a ship, in which certain points have been corrected by Giovanni Battista Benedetti, *Epistolae*, p. 214, where there is also something on the nautical armilla; by Willebrord Snel in his *Tiphys Batavus*; by Gemma Frisius in the appendix to Apian's *Cosmographia*; by Simon Stevin and Pierre Hérigone on *Histiodromia*; by Adrianus Metius [Adriaan Metius], Book 5 of the *Doctrina sphaerica*, from Chap. 4; by Morisetus [Claude-Barthélemy Morisot] in the *Orbis maritimus*; by Dominus de Varuich [?]; by Georgius Fournierus [Georges Fournier] of the Society of Jesus in his *Hydrographie*; and

by Marin Mersenne, Book 2 of *De Histiodromia* and Book 2 of the *Mechanica*. For these things are treated more fully in our Geographical book, which is the fourth of vol. 2 of the New Almagest. And so I have chosen to point out here only very little for beginners, so that they may not be wholly ignorant of what seems from this subject to pertain to Cosmography.

DE NAVIBVS ET EARVM PARTIBVS.

II. Fournier, Book 1 of the *Hydrographie*, gives an excellent account of the structure of ships; and the names of the parts are given both by him and by Morisot, Book 2 of the *Orbis maritimus*, from Pollux, Athenaeus, Baïf and others. Now the mast of a ship is called [Greek: ἱστὸς (histos, "mast")], from which comes *Histiodromia*, that is, the art of navigating. The heel of this mast is called by Macrobius, Book 5 of the *Saturnalia*, *Pterna*, its middle part *Trachelon*, and its summit *Carchesium*; the seat of the mast is the *Edolion*, and the cross-timber from which the sails hang is the *Antenna*, whose horns, as Virgil puts it in *Aeneid* 3, are also called *Ceruchi*, and in Greek [Greek: ἀκροκεραία (akrokeraia, "yard-tip")], the yards themselves being [Greek: κεραία (keraia, "yard")]. Isidore, Book 19 of the *Etymologies*, Chap. 3, gives 5 kinds of sail: namely the *Acation*, the largest sail, in the middle of the ship; the *Epidromon*, of the second size, at the stern, in Italian, if I am not mistaken, *il Trinchetto*; then the *Dolo*, the smallest, fixed at the prow, and the *Artemo*, for steering the ship; and the *Siparum*, which is added when the force of the wind flags. But the *Mendicum*, which is placed at the prow, as Sextus Pompeius would have it, hardly differs from the *Epidromon*.

[Margin: Ranks of rowers.]

III. There were formerly 3 differences among rowers. Some, the *Thalamitae*, were placed in the highest rank and therefore needed longer oars; others, the *Zygitae*, in the middle; others, called *Thranitae*, rowed in the lowest, as Snel holds with the scholiast of Aristophanes. For otherwise Pollux, Chap. 9 of the *Onomasticon*, Baïf in *De re navali*, and Philander [Guillaume Philandrier] on Book 10 of Vitruvius, Chap. 8, place the [Greek: θαλαμίτας (thalamitas, "Thalamites")] in the prow, the [Greek: ζυγίτας (zygitas, "Zygites")] on the sides of the ship, and the [Greek: θρανίτας (thranitas, "Thranites")] in the stern.

[Margin: Distinctions of ships by oars or rowers.]

Hence the differences of ships were taken either from the number of oars or from the ranks of rowers. Those named from the number ended in [Greek: ορος (-oros, "-oared")] if they had at least 20 oars: so [Greek: εἰκόσορος (eikosoros, "twenty-oared")], one consisting of 20, and [Greek: τριακόντορος (triakontoros, "thirty-oared")], one consisting of 30 oars. If they had fewer than 20, they were distinguished by the thole-pins [Scalmis] or pegs on which the oars are supported: hence [Greek: ναῦς τρίσκαλμος (naus triskalmos, "ship of three thole-pins")], a ship with 3 thole-pins, and [Greek: δεκάσκαλμος (dekaskalmos, "of ten thole-pins")], one with 10, and so on. From the line of oars, or rank of rowers, a ship was called [Greek: μονοήρις (monoēris, "single-ranked")] if it was propelled by a single rank of oars, [Greek: διήρις (diēris,

"two-ranked")] if it was a bireme, that is, of a double rank of oars, and [Greek: τριήρις (triēris, "three-ranked")] if it was a trireme, or consisted of a triple rank, and likewise with the others.

[Margin: Remarkable ships of Hiero and Ptolemy.]

Among these the most celebrated was the ship described by Athenaeus, Book 1, Chap. 10, and called the *Moschion*, which could not be launched into the sea except with the screw of Archimedes. It consisted of 20 ranks of oars, and was first called the *Syracusia*, and later, when Hiero sent it to Ptolemy, the *Alexandrina*. But more remarkable still was another, of Ptolemy Philopator, which on the testimony of Callixenus consisted of 40 ranks of rowers and was 280 cubits long, 38 wide and 48 high. The stern decks stood 53 cubits from the sea. The oars of the *Thranitae* were of 38 cubits, with lead added at the handle of the oar, and with a fellow rower; the rowers were 4000, and on each side 50 rowers; the interscalmia, or intervals of the thole-pins, were of 7 feet; the ship's servants 400; the marines 3000; there were 2 prows, 2 sterns and 7 rams; and finally stores of provisions sufficient for 50 triremes. See also another ship of the same Ptolemy in Athenaeus, Book 1, Chap. 9, and one of Demetrius in Plutarch.

[Margin: Estimate of ships by more recent authors.]

IV. More recent authors, however, estimate the capacity of ships from the burden they can carry. So a ship's tun [dolium] is of 2000 pounds; few ships carry 2000 tuns, very few 3000. A naval tun of 2000 pounds, says Mersenne, is reckoned to have 28 cubic feet; if you fill it with water, of which a cubic foot is 72 pounds, the water will be of 2016 pounds. A ship, however, is not said to be of so many tuns with respect to the water that it can hold from the bottom to the deck, but with respect to the water it can carry without inconvenience to navigation, which is commonly reckoned to be half of that water. Otherwise, as great as the weight the ship has in air, so great is the weight it has when borne on the water: so that if a ship is of 6000000 pounds, it can strictly be loaded with 6000000 pounds, but not without inconvenience, whereas with 3000000 pounds it certainly can without inconvenience. To judge the proportion of other ships from the measure of one, take a ship of 300 tuns. Its keel should be about 73 feet, its height 10 feet. Its anchor, whose shank is triple each of the 3 arms, should be about one-thousandth of the weight of the ship, so that no storm can dislodge it, but its cable twice as heavy. A storm, however, that tosses ships of 300 or 600 tuns barely moves a ship of 2000 tuns. [Margin: The ship Corona and the Fortune of Denmark.] See in Fournier, Book 1, Chap. 31, the ship called the *Corona*, of 1500 tuns, 200 feet long but 120 in the keel, with capacity for 72 guns; its mast is as great as the keel; its sails are of 600 ells; its anchor cable is 2 feet in girth and of 14300 pounds; the anchor itself, without axle or cylinder and nails, is of 4855 pounds; the weight of the ship is 4000000 pounds; the awning [tentorium] is worth 14000 gold pieces. Famous too is the ship called the *Fortune of Denmark* [*Daniæ Fortuna*], with 200 guns mounted; its mast with its extensions is of 216 feet, but without them 85. The largest ships of

the Ocean, moreover, the Spanish carracks, require yards so large that they can hardly be raised by 200 men. Finally the sounding-lead [Bolis] is pyramidal in shape, and if it is of lead and of 36 pounds it requires a rope of 24 pounds. There are, however, those who for a bottom of 1200 feet require a lead of 60 pounds, while others are content with 20.

DE REMIGIO.

V. Aristotle, in Question 5 of the *Mechanics* — which have been elucidated by Alessandro Piccolomini, by Biancani on the mathematical passages of Aristotle, no. 246, by Nonius at the end of Book 2 of *De Observatione*, and by Mersenne in the *Synopsis of Geometry*, Book 2, *Mechanics* — recognises three kinds of lever [Vectis]. The first is when the hypomoclion, that is, the fulcrum [fulcimentum], is between the weight and the moving power; the second when the weight is between the fulcrum and the power; the third when the motive power itself is between the weight and the fulcrum. So Aristotle and Piccolomini reduce the oar to a lever of the first kind, so that the sea is the weight and the ship the fulcrum. But Nonius, Biancani and Mersenne reduce it to a lever of the second kind, so that the sea is the fulcrum, the ship's hinges [cardines] the weight, and the helmsman with the rudder the moving power, turning the ship to the right while moving the sea to the left, and conversely, and by a small motion of the stern causing a large motion in the prow. For if the tiller [temo] were at the prow, the force of the water striking the side of the prow would turn the ship round, and the prow would become the stern. Since, however, Aristotle assumes that the blade of the oar always moves back whenever the vessel moves forward, and asks why the ship advances more than the half-blade [semi-palmula], Nonius, in the place just cited, therefore demonstrates the following propositions, worth knowing.

[Margin: Nonius's propositions on rowing.]

1. *If rowers can move a vessel, the handle of the oar always runs through a greater distance than the vessel.*
2. *If the handle of the oar, by its own motion, and the vessel run through equal distances, it will be impossible for the blade to move; it will instead stay unmoved, as if a centre.*
3. *If the handle of the oar, by its own motion, has covered twice the distance the vessel has, the vessel will be carried forward by as much as the blade has moved back; and conversely, if the ship moves by as much as the blade moves back, the motion of the handle is twice as great.*
4. *If the vessel runs through less distance than the handle of the oar, but more than half, it will be advanced more than the blade moves back; but if less than half, it will be carried forward less.*
5. *If the vessel moves faster than the handle, the blade will move forward and will never move back, and it will run through the distance by which the vessel's motion exceeds the motion of the handle.*

Hence it comes about that the distance the blade runs through and that of the vessel, taken together, equal the distance of the handle, when the handle moves more than the vessel; from which you may correct Aristotle's errors [sphalmata] on this point.

DE VELIFICATIONE, Mensura itineris Maritimi, & Velocitate Nauium.

On Sailing, the Measure of a Sea Journey, and the Speed of Ships.

[Margin: The mast of a ship reduced to a lever.]

VI. The mast with its yard and sails is reduced to a lever of the first kind, because the ship is the weight, the heel [Pterna] at the foot of the mast is the fulcrum, and the masthead [Carchesium] is the head of the lever, while the wind is the moving power. So Mersenne judges, with Aristotle, Question 6 of the *Mechanics*; and therefore, he holds, the taller the mast, the faster the ship moves. But Vitruvius, Book 10, Chap. 8, and Biancani on that Question 6, think that the weight is the ship with respect to that part by which the mast stands up from the ship, and that the fulcrum is in the bottom of the ship; otherwise the ship, resisting the wind, would overturn, and with the stern raised the prow would go under. This amounts to saying that the mast with its yard is a lever of the second kind. The remaining matters concerning sailing presuppose what is to be said about winds, the magnetic compass [Pyxis Magnetica], the sea chart and loxodromes, on which see below.

[Margin: Measure of a sea journey.]

VII. Measuring a sea journey far from the shore is difficult. An old instrument is found in Vitruvius, Book 10, Chap. 14. It consists of a wheel with projecting paddles, driven by the water, so that the revolutions of the wheel are completed; and by means of another wheel or a toothed cylinder, which moves a drum [timpanum] and by its sound alerts the counter, the revolutions of the wheel can be counted. For if the circumference of the wheel is known, for example three paces, and 3000 revolutions have been made in one hour, 9000 paces will have been covered. Others, however, hang from a cord of known length a cylinder, or some small beam or small wooden boat, and throw it off the stern. As the ship moves, the cord, suspended from a small cylinder that turns on pivots at the slightest touch, unwinds. So when the whole has run out, and the time meanwhile has been found by some clock, it will be established how many paces the ship has covered under the same steadiness of wind.

[Margin: Speed of ships.]

VIII. The speed of ships using the wind alone, moreover, is as various and uncertain as the winds, the weights of the ships, and the density of the waters or the resistance of running waters. In the Mediterranean, where the winds are closer to their source because the shores are near, sometimes about 500 Italian miles are completed within 24 hours, the number reckoned from Messina to Naples, and thus 20 miles and more in one hour; but for the most part the speed is far less. So too in the Ocean, where the winds, more even and further from land, blow more sluggishly, and ships of far greater bulk and weight have to

be driven. Thus the three Japanese envoys sent to Gregory XIII thought they had sailed very fast when, around the island of St Helena, they covered 180 or 200 miles a day; so too the companions of Fernão de Magalhães in the Pacific Ocean. A voyage there is judged fast if 150, 130 or at least 120 miles are run in a day; slow if, leaving aside calms or waves of the sea running against the ship, only 60 or 50 miles are covered in one natural day. On the speed of rivers, moreover, see what we said in Chap. 13. And so much on these matters, for such learning as it may give.

DE ARTE NAVIGANDI SVB AQVIS.

IX. Mortals have dared this too; whether with happy or unhappy success is not yet clear. The art of this navigation is briefly taught by Marin Mersenne in Book 2 of *De arte navigandi* and in Book 4 of the *Harmonica*, p. 368. There he also commends Ioannes Barrieus [?], an outstanding diver and the inventor of the art of breathing under water for 6 hours or more; whereas with other divers, on whose heads is placed a cask or leather bell of about 8 cubic feet, breathing fails beyond a quarter of an hour, the air being by then tainted. Within an hour we need 360 breaths, each of which lasts 10^{s}, and breathing cannot be put off longer without discomfort. But on these things see Mersenne: for I have no time to set them out, and it is enough to have pointed out the sources, to be tasted and drunk more fully elsewhere.

CAPVT XVII.

On the Winds and Anemography.

I. The number, naming and distribution of the winds are given in many places: in the commentators on Aristotle, Book 2 of the *Meteorologica*, Chap. 6; in Seneca, Book 5 of the *Natural Questions*, Chap. 16; Varro, Book 3 of *De re rustica*; Vitruvius, Book 1, Chap. 6; Vegetius, Book 4, Chap. 38; Pliny the Elder, Book 2, Chap. 47, and Book 18, Chap. 34; Aulus Gellius, Book 2, Chap. 12; Peter Apian and Gemma Frisius, Part 1 of the *Cosmographia*, Chap. 15; Stöffler, Part 2 of the *Astrolabe*, prop. 44; Oronce Finé, Book 5, Chap. 6 of the *Cosmographia*; Maurolico, Dialogue 2 of the *Cosmographia*; Barozzi, Book 2 of the *Cosmographia*; Egnazio Danti in the *Anemographia*; Daniel Santbechius [Daniel Santbech], proposition 155; Kepler in the *Epitome of Astronomy*, p. 191; Gassendi in the *Institutio Astronomica*, Book 1; Petrus Bertius in the tables *contractae*, p. 19; Pierre Hérigone, vol. 4 of the *Cursus Mathematicus*, p. 166; Gulielmus Blaeu [Willem Blaeu], vol. 1 of the *New Atlas*, Chap. 9; Philibertus Monetus [Philibert Monet] in the *Parallel of the French Language with the Latin*, under the word *Ventus*; and Metius, Book 5 of the *Doctrina sphaerica*, Chap. 4. The first observer of the winds is said to have been Aeolus, King of Sicily, as Strabo and Pliny the Elder have it; but others afterwards observed more winds, as physics, agriculture and seafaring advanced.

[Margin: 4 winds.]

II. Aeolus and Homer set four cardinal winds, which are:

1. *Solanus*, blowing from the equinoctial rising. It was once called *Eurus*, because it flows from the rising, and *Apeliotes* from the Sun, which in Greek is [Greek: ἥλιος (hēlios, "sun")]. In Italian, *Levante*.
2. *Favonius*, blowing from the equinoctial setting; also called *Zephyrus*, from its bringing life, which is [Greek: ζωὴ (zōē, "life")], as if *Zoipheros*. In Italian, *Ponente*.
3. *Septentrio*, blowing from the northern pole, or rather from that intersection of the horizon and the meridian to which this pole is nearer. It was once called *Boreas*, from a stormy sound, which in Greek is [Greek: βοὴ (boē, "shout, sound")], and *Aparctias* from Arctos, that is, the star the Bear. In Italian, *Tramontana*.
4. *Auster*, or *Notus*, diametrically opposite *Septentrio*; [Greek: ἀπὸ τῆς νοτίδος (apo tēs notidos, "from moisture")], that is, from clouds or from humidity. In Italian, *Ostro*. Ovid describes these excellently in Book 1 of the *Metamorphoses*, from the verse:

   Eurus withdrew to the Dawn and the Nabataean kingdoms. [*Eurus ad Auroram, Nabathæaq. regna recessit.*]

[Margin: 8 winds.]

III. Andronicus Cyrrhestes, on the testimony of Vitruvius, set eight winds, which he also marked out at Athens on an octagonal marble tower, with images and with a rotating bronze Triton that was to point out with a rod the wind that was blowing. Their names and order are as follows.

1. *Solanus*, or *Subsolanus*, blowing from the equinoctial rising.
2. *Eurus*, or *Vulturnus*, from the winter rising.
3. *Auster*, from the south, or *Notus*.
4. *Africus*, from the winter setting.
5. *Favonius*, or *Zephyrus*, from the equinoctial setting.
6. *Caurus*, or in contracted form *Corus*, from the setting at the summer solstice.
7. *Septentrio*, from the Arctic Pole.
8. *Aquilo*, or *Boreas*, from the rising at the summer solstice.

[Margin: 12 winds.]

IV. Philosophers, ship-masters and cosmographers afterwards recognised twelve winds, in Varro, Manilius and Pliny the Elder; granted that Aristotle names only eleven and yet recognised 12, on the testimony of Olympiodorus, for he set none opposite *Aquilo*, or *Meses*. Manilius indicates these in Book 1, from the verse:

The sphere of the sky is marked out into four parts. [*Quatuor in partes Cœli describitur orbis.*]

Their names and order are as follows.

1. *Subsolanus*, blowing from the equinoctial rising.
2. *Eurus*, or *Vulturnus*, from the winter rising; distant from *Subsolanus* 23½°.
3. *Euroauster*, *Euronotus*, *Phœnicias*, from the eastern intersection of the horizon with the Antarctic Circle; distant from *Auster* 23½°.

4. *Auster*, from the south.
5. *Austroafricus*, or *Libonotus*, from the western intersection of the horizon with the Antarctic Circle; distant from *Auster* 23½°.
6. *Africus*, or *Libs*, from the winter setting; distant from *Favonius* 23½°.
7. *Favonius*, from the equinoctial setting.
8. *Caurus*, or *Argestes*, from the summer setting; distant from *Favonius* 23½°.
9. *Circius*, named from the whirling of the clouds, blows from the western intersection of the horizon and the Arctic Circle. It is also called *Thrascias*, not from harrying Thrace, as Barozzi thinks, but from breaking the shadows of the clouds; for [Greek: θραύω (thrauō, "I break")] means "I break" and [Greek: σκιά (skia, "shadow")] "shadow". It is distant from *Septentrio* 23½°.
10. *Septentrio*, from the Arctic pole.
11. *Boreas*, or *Aquilo*, or in Aristotle *Meses*, from the eastern intersection of the horizon and the Arctic Circle; distant from *Septentrio* 23½°.
12. *Hellespontius*, or *Cæcias*, from Cæcia, a river of the Hellespont; blowing from the summer rising and distant from *Subsolanus* 23½°.

[Margin: 24 winds of Vitruvius.]

V. Vitruvius, in his second distribution, sets twenty-four winds, placing the twelve already mentioned differently, as can be seen in the following order. In it, disregarding the solstitial points, he divides the horizon into 24 equal parts, giving each 15°. With this interval established, they can easily be compared with the winds of Varro and Pliny the Elder, which stood 23½° from the four nearest cardinal points: as great, that is, as the distance of the tropics from the Equator, and of the polar circles from the poles of the world. Let the order and nomenclature of the Vitruvian winds, then, which differ in places from the Varronian and Plinian, be as in the following table. On it I note in advance that *Ornithias* is so called because it stirs birds to fly; *Etesiæ* from the year, which in Greek is [Greek: ἔτος (etos, "year")], because they blow at fixed times of the year; *Gallicus*, because it raged in Narbonese Gaul; and *Carbas* from [Greek: καυρόω (kauroō, glossed "I disturb" on the page)] "I disturb" and [Greek: βοὴ (boē)] "sound", as Maurolico conjectures. But it is surprising how Vitruvius placed *Argestes* and *Cæcias* across the Equator, when Varro and Pliny the Elder better located them on this side, since the latter takes its name from Cæcia, a river of the Hellespont. And he differs greatly from them in many other things too, as we shall soon see.

VENTI EX VITRVVIO.

The Winds according to Vitruvius.

Order	Names	Distance from the cardinal points
1	*SOLANVS*, from the equinoctial rising	0°
2	*Ornithias*	15° from Solanus
3	*Cæcias*	30° towards

Order	Names	Distance from the cardinal points
4	*Eurus*	45° Auster
5	*Vulturnus*	60°
6	*Euronotus*	75°
7	*AVSTER*, from the south	90° from Solanus
8	*Altanus*, *Leuconotus*	15° from Auster
9	*Libonotus*	30° towards
10	*Africus*	45° Favo-
11	*Subvesperus*	60° nius
12	*Argestes*	75°
13	*FAVONIVS*, from the equinoctial setting	90° from Auster
14	*Etesiæ*	15° from Favo-
15	*Circius*	30° nius
16	*Caurus*	45° towards
17	*Corus*	60° Septen-
18	*Thrascias*	75° trio
19	*SEPTENTRIO*	90° from Favonius
20	*Gallicus*	15° from Septen-
21	*Supernas*	30° trio
22	*Aquilo*	45° towards
23	*Boreas*	60° Solanus
24	*Carbas*	75°

Diagram. Caption: "Diagram of the Winds according to Barozzi." A circle in the right-hand column with numbered dots round its circumference, running clockwise from point 1 at the right-hand end of the Equator. One vertical diameter crosses five horizontal lines, each label being divided by the vertical: the Arctic chord at the top, the Tropic of Cancer, the Equator, the Tropic of Capricorn and the Antarctic chord at the bottom.

- Interior, top: "Ar | ct"
- Interior, upper line: "Trop | ♋"
- Interior, middle line: "Aequ | ator"
- Interior, lower line: "Trop | ♑"
- Interior, bottom: "Anta | rct."
- Circumference, clockwise from the right end of the Equator: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 [?], 19, 20, 21, 22, 21 [recte: 23], 24, 25, 26, 27, 28, 29, 30, [?], 32.

[Margin: 32 winds.]

VI. Finally, from as far back as the time of Charlemagne, cosmographers and ship-masters have used thirty-two winds. Barozzi, in his *Cosmographia*, places them so that, following the distribution of the ancients, he assigns twelve of them to the cardinal points of the horizon and to the solstitial points and those of the polar circles, and the others to the remaining points, which divide the intermediate intervals equally; as can be seen in the figure on the preceding page. But most writers, imitating Vitruvius in this respect, divide the horizon into 32 equal parts, giving each interval between the winds 11¼°. Of these,

besides the four cardinal winds, others standing equally on either side of them have been called Lateral or Collateral: namely *Scirocco*, *Garbino*, *Maestro* and *Greco*, to use the Italian words. But after Flavius Melfitanus [?] painted these winds on the rose of the nautical compass, they were called rhumbs [Rhumbi] from the shape, whose point marks the region of the wind on the horizon, as can be seen in the following diagram, which comes after the 3 tables that follow.

TAB. I ORDO, & NOMINA VENTORUM ANTIQUA, RECENTIBUS PROXIMO SITU RESPONDENTIA.

Table I. The Order and Ancient Names of the Winds, Corresponding to the Modern Ones by Nearest Position.

Order of the winds	Greek names	Latin or Graeco-Latin names	Distance from the preceding cardinal wind, by the division of the horizon: Varro and Pliny the Elder	Vitruvius	The moderns
1	Απηλιώτης.	*Apeliotes*, *Solanus*, *Subsolanus*, formerly *Eurus*.	From Solanus. Deg.	Deg.	Deg.
2	Υπεύρος.	*Hypeurus* or *Hypereurus*, in Vitruvius roughly *Ornithias*.		15°	11¼°
3	Εύρος.	*Eurus*, *Vulturnus*, very nearly in Varro and Pliny the Elder.	23½°		22½°
4	Μεσεύρος.	*Meseurus*; in Vitruvius nearer to *Cæcias*.		30°	33¾°
5	Νοτοαπηλιώτης.	*Notapeliotes*, in Vitruvius *Eurus*.		45°	45° 0
6	Υποφοινίκιος.	*Hypophœnicias*, nearer to Vitruvius's *Vulturnus*.		60°	56¼°
7	Φοινίκιας.	*Phœnicias*, in Varro and Pliny the Elder roughly *Euroauster*.	66½°		67½°
8	Μεσοφοινίκιος.	*Mesophœnicias*, *Mesophœnix*, nearer to Vitruvius's *Euronotus*.		75°	78¾°
9	Νότος.	*Notus*, *Auster*, blowing from the south.	From Auster.	0°	0°
10	Υπολιβόνοτος.	*Hypolibonotus*, *Leuconotus*, nearer to Vitruvius's *Altanus*.		15°	11¼°
11	Λιβόνοτος.	*Libonotus*, *Austroafricus*, very nearly in Varro and Pliny the Elder.	23½°		22½°
12	Μεσολιβόνοτος.	*Mesolibonotus*, nearer to Vitruvius's *Libonotus*.		30°	33¾°
13	Νοτοζέφυρος.	*Notozephyrus*, in Vitruvius *Africus*.		45°	45°
14	Υπολίψ.	*Hypolibs*, nearer to Vitruvius's *Subvesperus*.		60°	56¼°
15	Λίψ.	*Libs* and *Africus*, very nearly in Varro and Pliny the Elder.	66½°		67½°
16	Μεσολίψ.	*Mesolibs*, nearer to Vitruvius's *Argestes*.		75°	78¾°
17	Ζέφυρος.	*Zephyrus*, *Favonius*.	From Favonius.	0°	0°

Order of the winds	Greek names	Latin or Graeco-Latin names	Distance from the preceding cardinal wind, by the division of the horizon: Varro and Pliny the Elder	Vitruvius	The moderns
18	Ὑπαργέστης.	*Hypargestes*, *Hypocorus*, in Vitruvius *Etesiæ*.		15°	11¼°
19	Ἀργέστης, Καύρος.	*Argestes*, *Caurus*, very nearly in Varro and Pliny the Elder.	23½°		22½°
20	Μεσαργέστης.	*Mesargestes*, *Mesocorus*, nearer to Vitruvius's *Circius*.		30°	33¾°
21	Ἀρκτοζέφυρος.	*Arctozephyrus*, in Vitruvius *Caurus*, now *Magister*.		45°	45°
22	Ὑποκίρκης.	*Hypocircius*, *Hypothrascias*, nearer to Vitruvius's *Corus*.		60°	56¼°
23	Κίρκιος, Θρασκίας.	*Circius*, *Trascias*, very nearly in Varro and Pliny the Elder.	66½°		67½°
24	Μεσοκίρκης.	*Mesocircius*, *Mesothrascias*, nearer to Vitruvius's *Thrascias*.		75°	78¾°
25	Ἀπαρκτίας.	*Septentrio*, *Aparctias*, formerly *Boreas* and *Aquilo*.	From Septentrio.	0°	0°
26	Ὑπερβορέας.	*Hyperboreas*, nearer to Vitruvius's *Gallicus*.		15°	11¼°
27	Βορέας.	*Boreas* and *Aquilo* and *Meses*, very nearly in Aristotle, Varro and Pliny the Elder.	23½°		22½°
28	Μεσοβορέας.	*Mesoboreas*, *Mesaquilo*, nearer to Vitruvius's *Supernas*.		30°	33¾°
29	Ἀρκταπηλιώτης.	*Arctapeliotes*, in Vitruvius *Aquilo*; now *Græcus*.		45°	45°
30	Ὑποκαικίας.	*Hypocæcias*, nearer to Vitruvius's *Boreas*.		60°	56¼°
31	Καικίας.	*Cæcias*, *Hellespontius*, very nearly in Varro and Pliny the Elder.	66½°		67½°
32	Μεσοκαικίας.	*Mesocæcias*, in Vitruvius roughly *Carbas*.		75°	78¾°

Tab. II. Names of the Winds according to the Authors listed below.

	Cardano and Egnazio Danti	Ioannes Keplerus [Kepler]	Philiberto Moneto [Philibert Monet], of the Society of Jesus
1	*Subsolanus*	*Subsolanus*	*Solanus*
2	*Hypeurus*	*Hypeurus*, *Hypereurus*, *Ornithias*, *Cæcias*	*Subsolanus*; *Hypeliotes*
3	*Eurus*	*Eurus*, *Vulturnus*	*Ornithias*, *Meseurus*
4	*Meseurus*	*Meseurus*, *Vulturnus*	*Vulturnus*, *Helioteurus*
5	*Notapeliotes*	*Notapeliotes*, *Euroauster*, *Euronotus*	*Eurus*
6	*Hypophœnix*	*Hypophœnix*, *Hypereuronotus*	*Vulturnus*, *Phœnicias*
7	*Phœnix*	*Phœnicias*, *Euronotus*	*Altanus*
8	*Mesophœnix*	*Mesophœnix*, *Meseuronotus*	*Mesophœnix*
9	*Notus*	*Auster*	*Notus*, *Auster*

	Cardano and Egnazio Danti	Ioannes Keplerus [Kepler]	Philiberto Moneto [Philibert Monet], of the Society of Jesus
10	*Mesolibonotus*	*Mesolibonotus*, *Altanus*	*Hyponothus*, *Subauster*
11	*Libonotus*	*Libonotus*, *Austroafricus*	*Libonothus*
12	*Hypolibonotus*	*Hypolibonothus*, *Hyperlibonotus*	*Mesolibs*
13	*Notolibycus*	*Africus*, *Libs*	*Africus*, *Libs*
14	*Mesafricus*	*Mesafricus*, *Mesolibs*	*Subvesperus*
15	*Africus*	*Africus*, *Subvesperus Libshyphesperos*	*Libozephyrus*
16	*Hypafricus*	*Hypafricus*, *Hypolibs*	*Mesozephyrus*, *Etesiæ*
17	*Zephyrus*	*Favonius*	*Zephyrus*, *Favonius*
18	*Mesocorus*	*Mesocorus*, *Mesargestes*	*Circius*
19	*Corus*	*Corus*, *Caurus*, *Argestes*	*Argestozephyrus*, *CauroZephyrus*
20	*Hypocorus*	*Hypocorus*, *Hyperargestes*	*Leuconothus*, *Albicaurus*
21	*Borrholibycus*	*Corus*, *Etesiæ*, *Olympias*	*Argestes*, *Caurus*, *Corus*
22	*Hypocircius*	*Hypocircius*, *Mesothrascias*	*Hypargestes*, *Sciron*, *Olympias*
23	*Circius*	*Circius*, *Thrascias*	*Thrascias*
24	*Mesocircius*	*Hyperthrascias*, *Corus*, *Thrascias*	*Supernas*
25	*Septentrio*	*Septentrio*	*Aparctias*, *Septentrio*, *Boreas*
26	*Hypaquilo*	*Hypaquilo*, *Hyperboreas*, *Boreas*	*Gallicus*, *Hyperboreas*
27	*Aquilo*	*Aquilo*, *Boreas*, *Gallicus*	*Aquilo*, *Meses*
28	*Mesaquilo*	*Hypocæcias*, *Hypercæcias*, *Aquilo*, *Boreas*	*Hypomeses*, *Subaquilo*
29	*Borrhapeliotes*	*Borrhapeliotes*, *Arctapeliotes*	*Cæcias*, *Hellespontius*
30	*Hypocæcias*	*Mesaquilo*, *Mesoboreas*	*Hypocæcias*
31	*Cæcias*	*Cæcias*, *Carbas*, *Hellespontius*	*Cæciheliotes*
32	*Mesocæcias*	*Mesocæcias*	*Carbas*

TAB. III. MORE RECENT NAMES OF THE WINDS.

Order	Italian	German, Dutch	English	French	Spanish
1	LEVANTE	Oost-vel-Oosten	East	Est	Est, vel Leste
2	Quarta di Leuan. Scir.	Oost-ten-Suden	East-to-South	Est-quart-ausudest	Est 4. à suest
3	Leuante Scirocco	Oost-Sud-Oost	East-South-East	Est-Sud-Est	Lesueste
4	Quarta di Scir. Leuan.	Sud-Oost-ten-Oosten	South-East-to-East	Sud-est-quart-al-est	Sues 4. à leste
5	SCIROCCO	Sud-Oost	South-East	Sud-est	Sueste
6	Quarta di Scirocc. Ostro	Sud-Oosten-Sud	South-East-to-South	Sud-est quart-ausud	Sueste 4. al sur
7	Ostro Scirocco	Sud Sud Oost	South-South-East	Sud-Sud-est	Su sueste
8	Quarta di Ostro Sciroc.	Sud ten-Oosten	South-to-East	Sud-quart-au sud-est	Suest 4. à sueste

Order	Italian	German, Dutch	English	French	Spanish
9	OSTRO	Sud-vel Zuyden	South	Sud	Sur
10	Quarta d'Ostro Garb.	Sud ten-Vuesten	South-to-Vuest	Sud-quart-au-Sud-Oest	Sur 4. al sudest
11	Ostro Garbino	Sud-Sud Vuesten	Sout-South-Vuest	Sud Sud-Oest	Su sudueste
12	Quarta di Garb. Ostro	Sud-Vuestgen-Suden	South-Vuest-to-South	Sud-Oest-4 au-Sud	Sudueste 4. al sur
13	GARBINO; Lebeccio	Sud-Vuest	South-Vuest	Sud-Oest	Sudueste
14	Quarta di Garb. Ponen.	Sud Vuest-ten-Vuesten	South-Vuest-to-Vuest	Sud-Oest-4-Oest	Sudueste 4. al Oes
15	Ponente Garbino	Vuest-Sud-Vuest	Vuest-South-Vuest	Oest-Sud-Oest	Oes sudueste
16	Quarta di Ponen. Garb.	Vuest-ten Suden	Vuest-to-South	Oest-4-aud-Sud-Oest	Oes 4. sudueste
17	PONENTE	Vuest-vel-Vuesten	Vuest	Oest-vel-Ouest	Oeste, vel Oes
18	Quar. di Ponen. Maest.	Vuest-ten Norden	Vuest-to-North	Oest-4-au-Nordoest	Oes 4. al Nor Oeste
19	Ponente Maestro	Vuest-Nord Vuest	Vuest-North-Vuest	Oest-Nord-Oest	Oesnoroeste
20	Quar. di Maest. Ponen.	Noort-Vuesten-Vuest	North-Vuest-to-Vuest	Nord-Oest-4-Oest	Noroeste 4. al Veste
21	MAESTRO	Noort-Vuest	North-Vuest	Nord-Oest	Noroeste
22	Quar. di Maest Tram.	Noort-Vuesten-Noort	North.Vuest-to-North	Nord-Oest-4-au-Nord	Noroeste 4. al Nort
23	Tramontana Maestro	Noort-Noordt-Vuest	North-North-Vuest	Nord-Nord-Oest	Nor noroeste
24	Quar. di Tram. Maest.	Noordt-ten-Vuesten	North-to-Vuest	Nord-4-au-Nord-Oest	Nort 4. à Noreeste [recte: Noroeste]
25	TRAMONTANA	Noort-vel-Noorden	North	Nord	Nort, vel Norte
26	Quar di Tramon. Greco	Noort-ten-Oosten	North-to-East	Nord-4-au-Nord-est	Nor 4. à Nordeste
27	Tramontana Greco	Noort-Noort-Oost	North-North-East	Nord-Nord-est	Nor Nordeste
28	Quar. di Greco Tramon.	Noort-Oosten-Noort	North-East-to-North	Nord-est-4-Nord	Nordest 4. à Nort
29	GRECO	Noort-Oost	North-East	Nord-est	Nordeste
30	Quart. di Grecoleuante	Noort-Oosten-Oost	North-East-to-East	Nord-est-4-est	Nord 4. aleste
31	Grecoleuante	Oost-Nordt-Oost	East-North-East	Est-Nord-est	Lesnordeste
32	Quar. di Leuante Greco	Oost-ten-Noorden	East-to-North	Est-4-aut Nord-est	Les 4. à Nordest

Diagram. A full-width woodcut compass rose of 32 pointed rays, shaded along one side, inside a numbered ring. At the centre a small circle is quartered

by two crossed lines; within it a vertical arrow with a small square at its midpoint points north, and a small trefoil ornament stands at the arrow's foot. The northern ray carries a fleur-de-lis at its tip. The cardinal names are set in capitals outside the ring, and the 32 rim figures, set radially, run clockwise from the east.

- Top, outside the ring: SEPTENTRIO [north]
- Right, outside the ring: ORIENS [east]
- Bottom, outside the ring: MERIDIES [south]
- Left, outside the ring: OCCIDENS [west]
- Rim figures: 1 at the east, then 2–8 down the south-eastern quadrant, 9 at the south, 10–16 up the south-western quadrant, 17 at the west, 18–24 up the north-western quadrant, 25 at the north, 26–32 down the north-eastern quadrant. All 32 are legible.

From the first table above it is clear out of what affinity of places and names there has arisen among the moderns such great licence, or confusion, in the nomenclature of these 32 winds. As a favour to the learned, I have decided to give a specimen of it in the 2nd table, drawn not from all authors but from the principal ones, keeping the same order of the winds as regards the numbers, but setting down the names according to other writers. I admit, however, that the wind called *Hypeurus*, because it lies under *Eurus*, can on another reckoning be called *Hypereurus*, that is, *Supereurus*, and indeed *Meseurus*, that is, midway between *Eurus* and *Solanus*; and so with similar cases.

VII Finally, modern shipmasters subdivided each of the rhumbs of the winds, or the intervals between rhumbs, into 4 parts, assigning to each 2° 48′ 45″. In line with this division, when they assigned to the quarters of the rhumbs their inclination from the meridian line drawn from Auster to Septentrio, or from *Sur* to *Nort*, they considered the four quadrants of the horizon separately. They began the order of the rhumbs and quarters not from the equinoctial rising but from Auster, counting from there 7 rhumbs towards the east and another 7 rhumbs towards the west; likewise from Septentrio 7 towards the west and 7 towards the east. For this distribution, look at the following figure with its small table, in which the division of one quadrant is to be applied to the others. The table's last column gives the winds to which, in the order of the 32 winds used until now and begun from the equinoctial rising, the same inclination from the meridian applies.

Diagram. A small circular woodcut at the foot of the left column: three concentric rings crossed by a vertical and a horizontal diameter, with radial lines dividing the space between the outer rings. In the central circle a vertical arrow points north. The outermost band carries the rhumb numbers, counted in each quadrant from the vertical diameter towards the horizontal one.

- Top, outside the ring: Nort
- Right, outside the ring: Est (a blot lies over the last letter)
- Bottom, outside the ring: Sur
- Left, outside the ring: Vee[?]

- Rim figures, north-eastern quadrant: 1.2.3.4.5.6.7 clockwise from Nort
- Rim figures, south-eastern quadrant: 7.6.5.4.3.2.1 clockwise from Est
- Rim figures, south-western quadrant: 1.2.3.4.5.6.7 clockwise from Sur
- Rim figures, north-western quadrant: 7.6.5.4.3.2.1 clockwise from the left-hand label

Order of Rhumbs	Order of Quarters of Rhumbs	Inclination of Rhumbs and Quarters from the Meridian	Winds with the same Inclination: Order of the Winds
	1	2° 48′ 45″	
	2	5° 37′ 30″	
	3	8° 26′ 15″	
I	0	11° 15′ 0″	8, 10, 24, 26
	1	14° 3′ 45″	
	2	16° 52′ 30″	
	3	19° 41′ 15″	
II	0	22° 30′ 0″	7, 11, 23, 27
	1	25° 18′ 45″	
	2	28° 7′ 30″	
	3	30° 56′ 15″	
III	0	33° 45′ 0″	6, 12, 22, 28
	1	36° 33′ 45″	
	2	39° 22′ 30″	
	3	42° 11′ 15″	
IV	0	45° 0′ 0″	5, 13, 21, 29
	1	47° 48′ 45″	
	2	50° 37′ 30″	
	3	53° 26′ 15″	
V	0	56° 15′ 0″	4, 14, 20, 30
	1	59° 3′ 45″	
	2	61° 52′ 30″	
	3	64° 41′ 15″	
VI	0	67° 30′ 0″	3, 15, 19, 31
	1	70° 18′ 45″	
	2	73° 7′ 30″	
	3	75° 56′ 15″	
VII	0	78° 45′ 0″	2, 16, 18, 32
	1	81° 33′ 45″	
	2	84° 22′ 30″	
	3	87° 11′ 15″	
	4	90° 0′ 0″	1, 17

SCHOLIA.

[Margin: On the *Etesiæ*.] I. The *Etesiæ* do not seem to belong among the ordinary and general winds, either because they blow only at one fixed part of the year, or because they have no fixed seat. For in Spain and in Asia they blow from the east; in Pontus from Aquilo; elsewhere from elsewhere. Pliny the Elder and Strabo themselves grant that they blow from quarters other than Aquilo and the west, namely from the east too, and from the south. Again,

all winds that blow infallibly at a fixed season of the year, especially if they are wholesome and moderately cooling, can, by extending the Greek word, be called *Etesiæ*, or adjectivally *Venti Etesii*, that is, annual winds, from "year", which in Greek is [Greek: ἔτος (etos, "year")]. In this way, for Strabo, the wind *Eurus*, which blows from the Iberian coast into the Sea of the Sardinians at a fixed season of the year, is Etesian. Moreover, those *Aquilones* that precede the *Etesiæ* are called *Prodromi Etesiorum* [forerunners of the Etesiæ]. There were also formerly *Etesiæ* of 40 days after the Dog Star, called the Canicular *Aquilones*.

[Margin: What is *Ornithias?*] *Ornithias* too, that is, "of the birds", or the enticer of birds to flight or to breeding, could have been a name for *Zephyrus*, since it is the harbinger of the birds' brood; just as the same *Zephyrus* has been called *Chelidonias*, because it rouses and entices swallows when it blows in spring.

[Margin: Provincial winds.] II. Besides the general winds there are many other provincial winds, particular to some one region. Such are *Iapyx*, which blows in Calabria towards Greece; *Olympias*, which blows from Mount Olympus; *Sciron*, from the Scironian rocks; *Corinna*, which harasses Flaminia and the district of Bologna; *Catogis*, which harasses Pamphylia, and *Atabulus*, Apulia; and the Peruvian winds, called by the Spanish *della Cordigliera*, most delightful breezes to breathe, but a most immediate plague, since once breathed they kill living creatures on the spot. Furthermore, the winds that stir up storms in whirlwinds are called *Typhones*, and those that, shut up in clouds, suck waters upward are called *Scyphones*; those that blow back from the sea towards the shore are called *Tropæi*; and finally those suited to sailing farther from the shore, because of their continuous and long-sustained tenor of blowing, are called *Apogei*, and those otherwise, *Perigei*.

[Margin: The sphere of the winds.] III. If the winds blew along straight lines only, as their gusts are drawn for us by painters, one and the same wind could extend only as far as the sight itself reaches, looking all round the horizon from the place the winds blow from. Therefore, if we suppose that no winds blow from a higher region of the air than the highest mountains, and that no mountains are perpendicularly higher than 20,000 paces, the same wind will not extend beyond 460,000 paces towards one side, as is clear from what was said about the distance of sight in chapter 10 of this book. But if the mountains, and the height of the second region of the air, discussed below in chapter 19, are much less, the dominion of the same wind will be much smaller. If, however, the winds blow along curved lines, whether parabolic, hyperbolic, elliptical, circular or irregular, or if one wind is sown and propagated from another, the argument does not hold. On the force of the winds, and on the speed of navigation achieved by winds alone, I said a little at ch. 16, no. 4; more will be said in the Geographical Book.

CAPVT XVIII.

Some Matters Pointed Out for Beginners on the Declination of the Magnetic Needle, the Compass Box or Nautical Compass, the Marine Chart, Histiodromy and Loxodromies.

I. Very many writers have rightly extolled the praises and properties of the magnet stone, or Heraclean stone. For the magnet is a kind of little Earth, or in Greek [Greek: μικρογὴ (mikrogē, "little earth")], which can be called a sort of microcosm: in its motions and its turnings towards the regions of the world there is a kind of exemplar of the Copernican Earth, if that motion could be granted to the Earth, and certain sympathetic motions of the planets themselves are indicated in them; and they offer a splendid specimen of the other sympathies that exist in all of nature, and finally a compendium of natural magic. [Margin: The power of the magnet.] But there is no time to linger over these things, nor is it fitting for us to do so. [Margin: When was it applied to navigation?] If you wish, consult on them Pliny the Elder, book 36, chapter 16; William Gilbert, on magnetic philosophy; Georgius Agricola [Georg Agricola], book 5 of *De fossilibus*; Niccolò Cabeo in the *Philosophia Magnetica*; Athanasius Kircher in the *Magnes*; Bernardus Cæsius [Bernardo Cesi], *De Mineralibus*, book 4, part 1, chapter 6; Scaliger, *Exercitation* 131; Renatus des Cartes [Descartes], *Principles of Philosophy*, part 4, no. 145.

Some have doubted whether the magnet's power of turning itself to the poles, and of communicating this to steel, was known in antiquity. Giraldi, Calcagninus [Celio Calcagnini] and Leuinus [?] affirm it, on account of those lines of Plautus in the *Menecho*: *Cape versoriam* ["Take the *versoria*"]. Others commonly deny it, with Cabeo, book 1, chapter 6, and Pineda [Juan de Pineda], *De rebus Salomonis*, book 4, chapter 15. For they say Solomon was ignorant of it, at least in practice, so that the Jews should not be dispersed by navigating, and the origin of the Messiah might be more certain, and so that it might be reserved for the promulgators of the Gospel. Therefore the ancients, as Strabo, Pliny the Elder and Martianus Capella relate, steered by Cynosura, like the Phoenicians and Sidonians, or by Ursa Major, like the Greeks, or by birds, like those who sailed to Taprobane.

The first to mention the property of facing the north with one of its faces and the south with the other was Albertus Magnus, about the year 1245, and Vincentius Bellovacensis [Vincent of Beauvais], who calls the north *Zoron* and the south *Afron*. The one who first observed this turning in the magnet was Petrus Peregrinus [Pierre de Maricourt]. But the one who observed it in the magnetic needle, and who invented the nautical compass, was either Io. Goias, or Giras, of Amalfi, in 1302, as Blondus [Flavio Biondo] and Ortelius, tab. 6, relate, or Flavius of Amalfi at the same time, as Gilbert asserts. Some, however, think the inventor was older, as reported in Kircher, book 2, part 6, chapter 1. At p. 189 of the same author, read the method of rubbing a needle on a magnet so that it keeps a stable verticity; for it is well established that the

verticity is easily destroyed if the faces are interchanged in the rubbing, or if iron implements are close by, drawing the needle towards them.

[Margin: Magnetic declination.] II. The declination of the magnetic needle from the meridian line, in Greek [Greek: καλιβόκλισις (kaliboklisis, "inclination of the steel")], is the angle made by the meridian line and the magnetic needle, which has the centre of its motion on the meridian line. If it declines towards the east it is said to *Græcissare*; if towards the west, to *Magistrissare*; if to neither side, then it is [Greek: καλιβόδειξις (kalibodeixis, "pointing of the steel")]. On the island of Corui [Corvo], on Flores and Maij [Maio], at Messina by Charybdis, at Malta, at Vienna in Austria; at the Island of the Trinity, at the mouth of the river *Cantani* [?], in the Terzeræ or Tertiariæ Islands, and those of Boni aspectûs [Boa Vista]; at Cape *dellas Agullias* near the Cape of Good Hope, and near Constantinople it does not decline at all. In other places it declines, as you will learn from the small table to be set at the end of this chapter. The cause of declination is the greater or lesser abundance, or the nearness, of magnetic mines. That is why the declination is so enormous near the Arabian Gulf and Ilua [Elba], and in the Davis Strait: namely because magnet is dug out there. As for the other causes that Cabeo adduces, in book 1, chapter 7, we struck them out long ago in our *Philosophy*, since the greater tract of uncovered land does not always lie in the part towards which the magnetic needle tends, and so the declination cannot be attributed to that land.

[Margin: The nautical compass.] III. The nautical compass [*Compassus nauticus*] is a box in whose base lies a circle representing the horizon, divided into 32 rhumbs for as many winds; from these rhumbs is made the figure called the *Rosa Ventorum* [rose of the winds]. At the centre of this horizon a pin stands perpendicularly upright, carrying a pointer, or magnetic needle, which is also called a *versorium* from its verticity. The whole box, though, can be called a *versorium*, because it too is suspended from two or three rings crossing one another, so that whatever agitation of the ship occurs, its horizon remains parallel to the true horizon. A turning vane or pennant is usually added above, whose position shows the wind blowing from the opposite side.

[Margin: The marine chart.] IV. The *Charta Marina*, or nautical or hydrographic chart, is a chart on which the surface of land and sea is drawn, with places according to their proper geographical latitude and longitude. In very many places, where the wind is apt to change, images of the rose of the winds are added, of the kind seen in the second figure of the preceding chapter. That chart is very imperfect which has parallel meridians; better is the one in which the meridians meet at the poles of the world, and the lines drawn from the rhumbs of the winds to different places are curved and helicoid; of this kind the best charts were drawn by Gherardus Mercator [Gerard Mercator], Edouardus Vuricht [Edward Wright], Robertus Hues [Robert Hues], Iansonius [?], and other more recent makers, on new globes or maps.

[Margin: What is loxodromy?] V. Loxodromy is the oblique course that a ship holds when it follows the guidance of the magnet. For one can indeed sail

along the same circle, whether a great circle on the sphere or a lesser one; and if the voyage is continued along the circumference of the same great circle, it is shorter, safer and less costly. Yet that demands more ingenuity, and the analysis of very many spherical triangles, or a great mass of tables. Ship-captains have therefore chosen an easier method of navigating. It consists in following the same inclination of the rhumb on one meridian after another for as long as the same wind blows, though with that inclination corrected according to the declination of the magnetic needle, known beforehand; and in steering the ship by means of the rudder so that it follows the line marked out by the rhumb, either from the start of the voyage or chosen later. It follows necessarily that the line the ship's keel describes is curved and imitates the form of a helix. But more on these matters in the Geographical Book.

[Margin: What is histiodromy?] VI. Histiodromy is the art of navigating, so called from the ship's mast, which in Greek is called [Greek: ἱστὸς (histos, "mast")]. It has two principal parts. One is that of conjecturing, from the nautical chart, from the measure of the sea journey completed and from the latitude of places, where you are, and hence which rhumb and which wind are to be used next, so as to reach the desired place by the shortest or the most convenient navigation. The other consists in continually steering the ship along the appropriate rhumb, whether this is done by loxodromy or by orthodromy. But for more on this art see Pedro Nunes, in a twofold problem and in almost the whole of Book 2 of *De observatione*; Pedro de Medina, in six books on the art of navigation, addressed to Philip, King of Spain; Bartholomæus Crescentius [Bartolomeo Crescenzio] in his *Nautica Mediterranea*; Willebrord Snel in his *Tiphys Batavus*; Adriaan Metius in his art of navigation; Simon Stevin in *Imenereutice*; Father Georges Fournier of the Society of Jesus, in a French volume on this subject; Father Marin Mersenne in his *Histiodromia*; and Pierre Hérigone in the *Histiodromia* found in volume 4 of his *Cursus Mathematicus*, where there are also many tables for this purpose, together with fuller tables of magnetic declination taken from Io. Teliero [?]. We too shall give notable theorems and problems of histiodromy in the Geographical Book, in a section on this subject, where we shall set out a fuller table of magnetic declinations. For now the following small table will suffice, for the sake of some learning.

The Eastern Declination of the Magnetic Needle.

Place	Declination
Aleppo in Syria	3° 0′
Alexandria in Egypt	5° 45′
Amsterdam	9° 30′
Bologna in Italy	3° 30′
At the Cape of Good Hope	2° 30′
Ceylon	15° 30′
Cochin	17° 0′
Cologne	5° 0′
Crete, in its eastern part	15° 0′

Place	Declination
In Davis Strait	50° 0′
Ferrara	5° 50′
Florence	3° 30′
Ghent	1° 40′
Genoa	5° 58′
On the island of Elba, at Portoferraio [Iluæ insulæ in portu ferreo]	20° 0′
Ingolstadt	4° 30′
London, formerly	11° 16′
but now	4° 3′
Leuven	9° 0′
Lyon in France	4° 30′
Macau, on the borders of the Chinese	1° 30′
Madrid	5° 0′
Mantua	0° 30′
Marseille	2° 40′
Milan	2° 30′
Mainz	6° 7′
Eastern Declination.	
Narsinga [Narsingæ]	12° 0′
Nuremberg	8° 0′
The island of Palma in the Canaries	6° 10′
Palermo	6° 0′
Parma	6° 0′
Plemuda in England [Plemudæ in Anglia]	3° 24′
Prague	5° 30′
Rome, now	2° 45′
Between the islands of Tristan and Trinidad	19° 0′
Venice	5° 0′
Vilnius	3° 40′
Near Nova Zembla	17° 0′

Western Declination.

Place	Declination
Antwerp	9° 0′
In the Canary Islands	1° 0′
On the island of Frisland	28° 0′
Goa	17° 0′
Loreto [Laureti]	4° 0′
On the island of St Lawrence	25° 0′
Mozambique	12° 0′
Naples	0° 30′
Opposite the Arabian Gulf	16° 0′

Another, longer table is in our Geographical Book.

[Margin: Anomaly of magnetic declination.] VII. Some hold that magnetic declinations vary at one and the same place on the Earth. Among them is Marin

Mersenne, in his art of navigating, in the tract *De Magnete*. For he relates that at London it was observed by the Englishman Burrosius [William Borough], in the year 1580, as 11° 15′; but at the same place by Gontero [Edmund Gunter], in the year 1622, as 6° 13′; and by Gellibrando [Henry Gellibrand], in the year 1634, as 4° 6′. He adds that at Paris in the year 1644 it is 3°, whereas 30 years before it was 8°; and finally that at Aix it was recently observed by Gassendi as 5°, whereas long before the same man had found it to be 9°. Hence, he says, if it is observed for at least 50 years, the period of this reciprocation can be found. But I should like to be certain whether, while observing, they avoided iron implements, stones akin in nature to iron, and other things that usually draw the needle to themselves. Certainly, if it were true that the needle used to decline 6° in Italy, as Biancani relates in book 1 of the *Sphere*, chapter 4, then, since I found after almost 30 years that it declines 3° 20′, this difference does not seem one to be disregarded, so that timepieces and navigation may be handled and practised more correctly.

CAPVT XIX.

On the Height of the Regions of the Air, of Clouds, of Rainbows, and of the Whole Elementary Sphere.

I. The height of the air in which sensible refractions occur is, for Tycho (volume 1 of the *Progymnasmata*, p. 95), as great as that of the exhalations which, by reflecting the rays of the Sun about to rise, produce twilight; that is, 52 Italian miles, as he himself thinks. Longomontanus subscribes to this in the *Astronomia Danica*, book 2 of the *Spherics*, problem 2, not distinguishing vapours from exhalations, and from the greatest height of the vapours measuring the quantity of the refractive transparent medium, which for him is 56⅓ Italian miles. But Kepler, in the *Epitome of Copernican Astronomy*, p. 69, and in the *Astronomia Optica*, pp. 117, 129 and 135, distinguishes exhalations, smokes rising high and twilight torches from the vapid, thick air, which differs from water only by degrees of gravity. For this air alone, he says, is suited to sensible refractions of the rays of the stars, but it does not rise above the Earth beyond two Italian miles: as if, indeed, on mountains of so slight a height the stars could be observed unrefracted. For my part, I would believe that the air in which clouds can dwell, and in which rains, snows and hails can form, is suited to refractions; that it is indeed less high than 52 miles, but in some places higher than 2 Italian miles, as I shall presently say of both.

[Margin: The height of the regions of the air.] II. The height of the highest region of the air is therefore as great as that of the exhalations, which are hot and dry. For this is how the highest region is distinguished from the middle one by Aristotle (book 1 of the *Meteora*, summa 1, chapter 4), by Kottounios on that place (lecture 18), by Cabeo (text 15, question 2, and text 46, question 11), by Froidmont (book 1 of the *Meteora*, chapter 1; book 3; and book 5, chapter 5) and by other commentators: in the former they place exhalations

and fiery impressions, in the latter vapours and watery impressions. But on the height of the exhalations that produce twilight, opinions differ according to the differences of the air and the quantity assumed for the Earth's semidiameter in investigating this height. I have already reported them in Book 1, chapter 31, on twilight; transfer them here from there.

III. On the height of the middle region and of the clouds, opinion is no less divided. Aristotle, in the place cited, and Kottounios think it as great as the greatest height of the mountains, just as the lowest region ends at the summits of moderate mountains, as far as the rays of the Sun reflected from the Earth reach. And yet there is no less controversy over the greatest height of the mountains; granted that it may be more probable that they do not exceed 20 or 25 [recte: 40 or 50, per the book's errata; the printed figures, unreadable under the ink in this copy, are given as the errata quote them] Italian miles[11] in perpendicular height. But not a few think that sunlight reflected from the Earth reaches as far as the Moon and produces on it that secondary light which is seen at ecliptic new moons, as I shall say when treating of the lunar light. To measure the height of the lowest region of the air from this is therefore a task of uncertain hazard. Posidonius holds that *the height in which cloud-masses, winds and clouds arise is not less than four hundred stadia*, as Pliny the Elder has it, book 2, chapter 23. For the reading from old copies is not forty but four hundred, in Pedro Nunes, *De crepusculis*, proposition 18; in Tadeáš Hájek, in his book on the new star of the year 1572, p. 119; and with him in Tycho, above; and 400 stadia are 50 Italian miles. Yet Pliny the Elder adds in the same place: *Many, however, have reported that clouds rise to a height of nine hundred stadia. These things are unascertained and inextricable.* But Albertus Magnus in the *Meteora* grants 15 miles, unless one should read stadia. Froidmont, in the *Meteora*, book 1, chapter 1, and book 5, chapter 5, teaches that extraordinary exhalations can rise as far as the Moon, as comets make clear, but ordinary ones to 52 Italian miles; while the thicker vapours do not exceed 8 miles, and clouds and snows do not exceed 4 Italian miles. Cardano, on the contrary, in book 17 of *De subtilitate*, denies that clouds are raised beyond two thousand paces, which Kepler, cited in no. 1, thinks of the refractive air; and he says that the denser clouds, heavy with rain, scarcely rise to 500 paces. Kepler adds that on sea coasts they do not exceed an Italian mile. Father N., Rector of the College of the Society of Jesus at Metz, asserted to me in the year 1646 that many clouds had been brought to measurement by him, and that none had been found higher than five thousand paces. Finally Father Cabeo, in book 1 of the *Meteora*, text 15, question 2, and text 46, question 11, rightly teaches that the measure of the height of the clouds and of the middle region is not everywhere the same. For some clouds full of sulphurous and nitrous exhalations are carried higher; and in some places the middle region can be higher than the one that is highest elsewhere, especially

[11] in this copy the 4 of 40 and the whole of 50 are written over in ink by a later hand; the printed figures beneath them cannot be read.

if vapours are raised from mountain lakes. Let there be, however, a synopsis of the opinions in the following small table.

Height of the Middle Region of the Air, or	*Height of the Clouds.* Italian miles
Cardano, *if thick clouds*	½
If thin	2
Froidmont, *if the clouds are thin*	8
If thick, as also snows	4
Kepler	1 or 2
Father N., Rector at Metz	5
Albertus Magnus	15
Posidonius	50
Others, in Pliny the Elder	112½
For Pliny the Elder himself uncertain; for Cabeo and for me, variable	

[Margin: First method of measuring the height of clouds.] IV. But if anyone nevertheless wishes to measure the height of clouds, even if he cannot determine their greatest height from that, here is a threefold problem for you. First, let N be the edge of a stationary cloud, and let its altitude be observed with a quadrant or a geometric square, at the same moment, by two observers, one stationed at A and the other at B, or, if possible, at D. In this way the angle NAC will be known, and either NBC or NDA.

Diagram. A small woodcut set into the right column: a horizontal base line with a cloud drawn above its left and central part, its right end at the apex point. Four straight lines descend from the point at the cloud's edge to points on the base line, the third of them vertical.

- N (top, at the cloud's edge): the observed edge of the cloud
- A (base, far left): first observer
- B (base, left of centre): second observer
- C (base, right of centre): foot of the vertical line
- D (base, far right): alternative station of the second observer

In the first case, then, since the angle NBC is given, the angle NBA is also given, because it is its complement to two right angles, by proposition 13 of the first book of Euclid's *Elements*. Measure, then, the terrestrial interval AB in paces; and so in the triangle NAB the angles NAB and NBA, and the side AB adjacent to them, will be known. Therefore, by rule 8 of our oblique rectilinear triangles, the side AN will not remain hidden. Then, with the perpendicular NC drawn in imagination, making the right angle NCA, in the triangle NCA, right-angled, you will find from the known angle A and the hypotenuse AN the perpendicular height of the cloud NC, by rule 1 of the right-angled triangles. But if the angle at D was observed, measure the interval AD, and from it and the angles A and D find the side AN, and with that, as above, the height NC. In this way I, with Father Francesco Maria Grimaldi, found a white cloud 2177 paces high.

The second method, from Simon Stevin, is as follows. Let DK be the shadow of the cloud, with one observer at D observing the boundary of the shadow, and another at K; and with a third at the physical midpoint I, meanwhile taking the angle EIG with the help of a radius or a sextant, by sighting the extremes of the cloud, E and G. [Margin: Another method.]

Diagram. A woodcut set into the right column: a cloud drawn as a shaded lens across the top, with a straight line through it from E to G. Lines run from E and G down to a point L at the foot, and a vertical line runs from the cloud to L. Lines also run from E and G to a central point on the vertical line, and at that level a shadow is drawn as a small horizontal ellipse crossing the two lines to L.

- E (top left): one extreme of the cloud
- F (top centre, inside the cloud): foot of the vertical on EG
- G (top right): the other extreme of the cloud
- D (left of the small ellipse): one boundary of the shadow
- I (centre of the small ellipse): the third observer
- K (right of the small ellipse): the other boundary of the shadow
- L (foot): meeting point of the Sun's extreme rays

So, having drawn the Sun's extreme rays EL and GL, and given the angle ELG, which subtends the apparent diameter of the Sun, and having drawn EG, the diameter of the cloud, and the perpendicular LF to it, the half-angles of those angles will be known; and accordingly GLF, the apparent semidiameter of the Sun, and GIF from the observation. Once the shadow's semidiameter IK has been measured in paces or feet or perches, then in the triangle KIL, right-angled at I, the side IK and the opposite angle L will be known. Therefore, by rule 6 of our right-angled triangles, the side LI becomes known. Then in the triangle LIG, in which are given the angle L, the side LI, and the angle LIG, made up of the right angle LIK and the complement of the observed angle GIF to a right angle, which is the angle GIK, you will find, by rule 8 of our oblique-angled triangles, the side IG. With this as a base, finally, and with the observed angle FIG, in the triangle FIG, right-angled at F, you will find, by rule 10 of the right-angled triangles, the distance of the cloud FI, and by rule 1 the true semidiameter of the cloud FG.

But if you had investigated LK, then as LK is to LI, so is LG to LF, by proposition 4 of Book 6. So, with LI subtracted, the height IF will be known.

Froidmont teaches a third method, and Cabeo explains and completes it (book 1 of the *Meteora*, text 44, question 11): by means of the time between lightning and thunder, counted by the swings of a pendulum, if you have first observed the time between a gun's flash from the ignited powder and the hearing of its report. This method is very slippery and exposed to great error, unless it is carried out with the utmost and scarcely attainable subtlety; and so it is enough to have indicated it here.

V. The height of the rainbow, moreover, is very small and does not exceed a thousand paces, because its apex is not higher than about 45°. If it is 45°,

a right-angled isosceles triangle results, one side of which is the height of the rainbow and the other the distance of the plane of the rainbow from the measurer's eye. That distance does not usually exceed a thousand paces, if you prudently note the place on which the arc of the rainbow rests, as I have noted more than once; and I have also often found that angle to be less than 45°, as I shall say in Scholium 4.

SCHOLIA.

I. Mario Bettini, in *Apiarium* 8, progymnasma 8, proposition 3, thinks that the height of the refractive air can be demonstrated from the quantity of the refractions alone, known in degrees or minutes. Accordingly, taking from Tycho the Sun's greatest refraction of 34′, and setting the Earth's semidiameter at 3035 5/11 Italian miles, he concludes that the height of this air is only 151 paces. But I shall show the fault of this reasoning in the Optical Book, where refractions are treated. Meanwhile, it is certain enough that on mountains much higher than 151 paces, for instance on Mount Paterno and the like, refractions of the stars are perceived; although Kepler, in the *Epitome of Copernican Astronomy*, p. 72, says that on the highest mountains refraction is sometimes almost nil, especially in winter, when the air is low and less vapid. But he adds that both on mountains and in coastal places it is sometimes prodigious.

II. On Cardano's error about the greatest height of the exhalations of twilight I have said enough in Book 1, Chapter 31, where I also indicated why I have deferred my own opinion on this height to the Optical Book, although I have many observations of the Sun's depth at the beginning or the end of twilight. That Abraham Roggembach [?], in Tanner, question 7 on *De caelo*, and Cardano, book 4 of *De subtilitate*, with twilight set at only half an hour, say the height of the exhalations is 772 miles, comes from the same blunder: namely, that they think the reflected ray of twilight falls perpendicularly above our zenith.

III. From what has been said you see that the quantity of the air as distinct from the fire, and of the fire as distinct from the ethereal and celestial substance, is uncertain. For it is more probable that below the Moon there is something of the ethereal breeze, and that the bounds of the heaven do not end precisely where the body of the Moon at perigee descends to. Tycho teaches at length, in his letters to Christoph Rothmann, that the upper air and the fluid substance of the heaven can be of different species, even if they agree in transparency, fluidity and rarity; and it is not for this place to treat these matters fully. The reader will therefore pardon us if we do not dare to define the quantity of the atmosphere, although Biancani, in book 7 of his *Sphere*, has defined the elementary sphere by the height of the Moon at perigee, namely, following Tycho, by 52 semidiameters of the Earth; and has accordingly set its volume to the volume of the Earth alone as 321992320 to 2290, and the volume of the air (its height being set at 43 miles) to the volume of the Earth as 86 to 2290.

IV. Maurolico asserts that the height of the inner and more vivid rainbow is 45°, and that of the outer 56°. Yet Descartes, as found in his own *Meteora*, chapter 8, observed that the former does not exceed 41° 47′, and the latter 51° 37′. They are speaking, however, of the height taken not from the line of the sensible horizon, which does not always meet the centre of the rainbow, but of the height from the centre of the rainbow itself, and of the angle made at the spectator's eye by two lines, one of which goes to the apse or summit of the rainbow and the other to the centre; and this second line, prolonged straight, goes to the centre of the Sun.

Diagram. A woodcut set across the right column. At the left a rayed Sun with a letter in its disc. A horizontal line runs from the left through a central point to the right, and a straight line from the Sun's centre descends through that central point to the lower right. From the central point a further line rises to the top of a rainbow, drawn as two concentric dashed arcs; a line from the top of the rainbow falls to the lower line, crossing the horizontal. The arcs reach down past the horizontal on both sides.

- S (left, in the Sun's disc, cut in reverse): the Sun
- H (far left, on the horizontal): the horizon
- O (centre, at the junction of the lines): the eye
- R (right of centre, on the horizontal): the horizon
- I (on the horizontal, where the line from the top falls across it): foot of the height from the horizon
- E (far right, at the end of the horizontal): where the rainbow's arcs cross the horizon; RE is the distance between the horns in the table below
- B (top of the arcs): the summit of the rainbow
- C (below the horizontal, on the left arcs): not named in the text
- A (on the lower line): the centre of the rainbow
- D [?] (lower right, at the foot of the arcs, on the lower line): not named in the text

Therefore, the higher the Sun is above the horizon, the lower the centre of the rainbow lies below it, and no straight visual line can be drawn to that centre from our eye unless the Sun is very near the horizon, or we are on a high mountain, or a very deep valley lies between the horns of the rainbow, as can be seen in the figure added here. In it let HOR be the physical horizon; let A be the centre of the rainbow, O the eye and S the Sun, set on the same straight line SOA; let AB be the height of the rainbow from its centre, or its semidiameter, which the angle BOA measures; but let BI be the height from the horizon, which the angle BOI measures, taken with a quadrant or a geometric square. These angles are sometimes found to differ, either because of a less accurate observation, since the altitude of the Sun and that of the rainbow were not taken at the same moment, or from differences in the dewy cloud. The visible altitude of the Sun's centre S, that is, the angle SOH, which by proposition 15 of Euclid's first book equals the vertical angle IOA, must, together with the observed angle BOI, make up the angle BOA. I should like to add here some

observations of Father Francesco Maria Grimaldi and of our own, such as they are, and taken in haste; although I intend, now that I have read Descartes's book, to undertake others more accurate, if God wills. And once, from Mount Paterno, we saw a rainbow in the valleys that was more than semicircular. [Margin: A rainbow greater than a semicircle, against Aristotle, *Meteora* 3, summa 2, chapter 4.]

OUR OBSERVATIONS

Year	Month	Day	Visible Altitude of the Sun, Angle SOH	Altitude of the Rainbow from the Horizon, Angle BOI	Altitude of the Rainbow from its Centre, BOA
1643	August	9, evening	16° 10′	24° 40′	40° 50′
	September	8, evening	28° 40′	12° 32′	41° 12′
1644	May	31, evening	11° 4′	30° 4′	41° 8′
Distance of the horns RE 81° 40′; the colour at the top red, in the middle yellow, at the bottom green; and after 10^{m} a rainbow of inverted colours was added; and then the distance RE, observed with a hemicycle, was seen to be 103°. And the altitude:			9° 50′	40° 30′	50° 20′
1647	June	2, morning	4° 0′	38° 0′	42° 0′
	July	4, evening	28° 40′	12° 32′	41° 12′
	August	3, evening	4° 0′	37° 0′	41° 0′

But on the rainbow, more can be found in Witelo, book 10, from proposition 63 onwards; Maurolico, at the end of book 3 of the *Diaphana*; Biancani, on the mathematical passages of Aristotle, book 3 of the *Meteora*, from no. 162 to no. 182; Froidmont, book 6 of the *Meteora*, chapter 1; Cabeo, book 3 of the *Meteora*; Descartes, in the *Meteora*, chapter 8; the Coimbra commentators and Kottounios on book 3 of the *Meteora*; and many more in them.

CAPVT XX.

On the Oscillations of the Pendulum [Perpendiculum], Suitable for Measuring Other Motions and Times, Both in the Elements and in Mixed Bodies and in the Heavenly Bodies.

[Margin: The occasion for investigating the oscillations of the pendulum.] I. When I was publicly teaching philosophy at Parma, I wanted to test by what increase of speed heavy bodies descend faster and faster toward the end of their fall, and I saw that this required a very fine measure of time, down to

almost its smallest perceptible particles. An occasion for this measurement at last offered itself when Father Niccolò Cabeo asked me by letter to test whether two pendulums, equal in weight and in length, and released from a limit equally distant from the perpendicular toward the same quarter, would continually travel together toward the same sides in both directions. For it was then that I began to suspect that any oscillations of one and the same pendulum are equal in time to any others; and this I afterwards learned thoroughly through repeated and more accurate experiments. For at that time neither Galileo's dialogues on the system of the world, where the same thing is affirmed in Dialogue 2, nor the short work of D. Io. Baptista Baliani [Giovanni Battista Baliani] on the natural motion of solids had yet reached my hands: I read the former only two years later, and the latter only ten years later. But neither in them nor in others did I find a way or measure for determining time by oscillations of this kind. I shall therefore set out what I tested to the point of satiety, by myself and with the help of my colleagues, together with selections from Galileo and Baliani. To do this the more readily:

Diagram. A pendulum and its mounting, set in the left column. At the top a horizontal bar runs from A at the left to B at the right, with three screws or clasps drawn on it. Below the bar hang two plates forming a frame; a small axle or cylinder lies across the frame, and a chain hangs from its middle down to a large shaded sphere. A dotted vertical line drops from the left plate to the left end of the sphere's horizontal diameter. A dotted arc runs from S at the upper left, through two marked points, down to the sphere, and rises on the other side through two more marked points to T at the upper right.

- A (left end of the bar): the iron bar AB
- B (right end of the bar): the iron bar AB
- D (on the bar, left): clasp or screw
- H (on the bar, between D and F, above the left plate): head of the plate HE
- F (on the bar, right of H): clasp or screw
- G (on the bar, right of F): clasp or screw
- L (left of the left plate, at the height of the axle): end of the axle LM
- E (on the left plate): hole in the plate
- K (on the right plate, above the axle): head of the plate KI
- I (on the right plate, at the height of the axle): hole in the plate
- M (right of the right plate): end of the axle LM
- S (upper left, end of the arc): limit of the swing
- Y (on the arc, left): marked point; the prose on the next page names Y as a point of the third and fourth arcs
- Q (on the arc, left, below the last): marked point, not named in the prose on this page
- T (upper right, end of the arc): limit of the swing
- X (on the arc, right, below T): marked point; the prose names X as a point on the return toward T

- Z (on the arc, right, below X): marked point, not named in the prose on this page
- N (top of the sphere): where the chain joins it
- P (left end of the sphere's diameter): end of the diameter
- O (centre of the sphere): centre
- Q (right end of the sphere's diameter): end of the diameter
- R (below the sphere): foot of the sphere

II. In the present diagram let AB be an iron bar, fixed to some beam or support that is wholly immovable and trembles as little as possible, by means of the clasps or screws D, F, G. To this bar let there be attached or soldered, on either side, two quite thick plates of iron or steel, equal, equidistant and set perpendicular to the horizon, such as HE and KI, pierced at E and I. And let a small cylinder C be made, with an axle LM projecting from it and passed through the holes E, I, so that by means of it the pendulum is held in its motion between the plates HL and KI, and so always travels in the plane of the same vertical SOT and cannot wander off sideways. Let the length of the axle LM be as great as the diameter of the sphere POQ, and let it be parallel to the horizon, so that with a thread or rule LP one can know very exactly OC, the height of the pendulum, to be taken from the centre of the sphere to C, the centre of the cylinder set on the axle, and parallel to LP itself. Let the sphere NPRQ be made of metal and not exceed two pounds, lest with too great a weight the axle of the cylinder become less suited to equality of the motions toward the end, and be worn down too much by long use. Let this sphere be hung from C by a small chain CN, made of very small rings or links, but as strong as the weight of the sphere requires, so that they cannot break or stretch. For if you use a cord of thread, or of bristles, or of small cithara strings, it eventually stretches under the force of the weight and of the sphere's motion, and the measure of the height CN varies, although it must be constant. If, on the other hand, you use a metal wire or a slender rod, the pendulum comes to rest after very few oscillations and is unsuited to measuring a rather long time. It is useful, moreover, to know exactly not only the height CO, with LP taken from the pole L and the extremity P of the diameter, but also, by an accurate balance, the weight both of the sphere NR by itself and of the small chain CN.

[Margin: How the pendulum is to be handled.] III. With this pendulum [pendulo seu perpendiculo], or a very similar one, thus prepared, or, which is more advisable, with several of different heights, let the sphere be grasped by hand and, with the small chain extended, drawn upward through the plane of the vertical equidistant from the plates E, I, so that by the force of the motion its centre O describes the circumference SOT; and let it be released from T freely, with no impulse imparted from outside. It will then move along the arc SOT, nodding and oscillating alternately half upward and half downward, receding from the perpendicular line CO and returning to it, until, the impetus it had conceived at T or at S being spent, it at last comes to rest on that line. And if, when it is approaching rest and describing tiny arcs, it is gently grasped by

hand a little before it reaches the perpendicular, at once drawn upward through the same plane, and immediately released, with no perceptible delay between grasping it and completing the pull, its motion can be continued even for a whole day, so that this motion is suited to counting time exactly. But that takes remarkable dexterity and much prior practice. It should not be raised from O toward T or S through an arc OT or OS that reaches a quadrant of the circle; indeed, it is more advisable not to exceed a half-quadrant. And if its motion is to be continued, one should not wait for the pendulum to be close to rest, but repeat its elevations a good while earlier, so that it describes larger arcs. The definitions, further, or notions of the terms that must be known beforehand, are these.

[Margin: Definitions concerning the pendulum.] The *pendulum* is a weight suspended from some height in such a way that it can be moved from its rest without resistance; such is the sphere NR, suspended from C.

The *perpendicular of the pendulum* is the straight line standing at right angles to the horizon, on which the pendulum naturally comes to rest, as in our case COR.

The *centre of the natural motion* of the pendulum is the point about which the centre of the pendulum moves circularly, or along arcs of a circle: such as the point C.

The *height of the pendulum*, or its *length*, is the distance of the centre of gravity of the pendulum itself from the centre of its natural motion: such as the straight line CO.

The *vibrations of the pendulum* are its oscillations, or oscillatory motions, to one side and the other of its perpendicular, made by natural motion. For the sphere, raised to T, while by the force of gravity it strives to go downward along the shortest line it can, seems to be drawn by the chain CN through the plane TOS; and although this motion may seem violent for the sphere NR taken by itself, yet it is not violent for the whole pendulum taken as a complete unit, since each single ring of the chain moves naturally by its own weight, and the whole pendulum together tends to the perpendicular line, so that on it the pendulum may stand as little distant as possible from the common centre of heavy bodies, which is connatural to it.

A *simple vibration* of the pendulum is one single oscillation of it from a terminus a quo, the same in kind, to the terminus ad quem opposite in kind; such as the motion made along the arc TOS, from the terminus T to the terminus S.

A *compound vibration* is an oscillation mixed of approach and withdrawal, or a reciprocal motion from a terminus a quo, the same in kind, to the terminus the same in kind; such as the motion from T to S together with the return from S, toward T, to X.

A *semivibration* is the motion by which the pendulum recedes from the perpendicular as from a terminus a quo, or returns to it as to a terminus ad quem; of this kind is the motion made from O to T, or from T to O. With these

laid down, let there be *selected propositions, confirmed by our own most certain experiments: some of them are asserted by Galileo and D. Io. Baptista Baliani, and the first by Wendelen.*

[Margin: Galileo, Dialogue 2, on the system of the world. Baliani, on the natural motion of solids. Wendelen, in the *Eclipses*, p. 25.]

PROPOSITIO I.

Any simple vibration of one and the same pendulum is equal to the senses to any other simple vibration, and any compound vibration to any other compound one, in the time of its motion taken in itself: that is, it lasts an equal time, or in Greek [Greek: ἰσοχρονῆ (isochronē, "equal in time")].

[Margin: First experiment.] IV. For make yourself a short sand-glass of powdered tin, running, say, for half a quarter of an hour, or 10^{m}, so that there is less tedium at the start of the experiments. Let it have two clepsydras or bulbs, marked with some sign, say one with the letter A and the other with B. At the same moment of time at which you release the pendulum from the limit T, turn over bulb A, in which the powder has stood; and by yourself, or with a companion, count the simple vibrations, or the compound ones if you prefer, until all the powder has run out, and write their number on a slip of paper. It is useful to throw one pebble into a small vessel prepared for the purpose for every ten or every twenty vibrations, so that the imagination is not confused in counting while the eye attends to the reciprocal motions of the pendulum. Better still, have a witness and companion who also counts the vibrations silently. When the powder has run down into bulb B, turn the glass over so that the powder runs back into bulb A, but do not count the vibrations meanwhile. When all the powder has run down into bulb A, if the pendulum's motion continues, wait for it at some limit, say at Z, and at once turn the glass over and count the vibrations as above. For if you carry all this out exactly, you will find that an equal number of vibrations answers to those equal times. And you will find the same if the pendulum has already come to rest and you raise it again. But it will make for greater certainty if the second time you release the pendulum from a different limit: for example, if the first time you released it from T, release it the second time from X. I said the same bulb A should be used because, owing to the difference in their interior shape and the differing edges of the hole on either side, the time of one is rarely equal to the time of the other. If, however, you have not been so careful, there may be a difference of one or another vibration between the vibrations of the first and of the second time.

[Margin: Second experiment.] V. Second, set up in the plane of the meridian two threads suspended from the same point above, one of which stands perpendicular to the horizon and to the meridian line marked on it; for with that line it will form a triangle of thread that marks out, exactly to the senses, the plane of the meridian circle. Then wait until the centre of some fixed star is in the plane of that meridian. This is shown by the two threads, one covering the

other and both covering that centre, with the unmoving eye directing its gaze along those threads. At once let a companion be on hand to begin counting vibrations until another fixed star, notably distant from the first, reaches the same plane of the meridian with its centre, and let the number of vibrations be set aside. On the following night, with the pendulum released from a different limit, in whatever way, let the vibrations be counted again, from the transit of the same first star to the transit of the same second star across the same meridian. For if everything has been done exactly, and the pendulum has travelled in the same plane of its own, you will detect either no disparity in the number of vibrations or a negligible one. Thus, when we wished to weigh whether the pendulum customarily used for measuring astronomical times had suffered any perceptible change through wear of the axle and of the holes E, I, or from some other source, we counted on three nights, namely May 19 and 28 and June 2, with Fathers Francesco Maria Grimaldi and Franciscus Zenus [?], very practised in this matter, as witnesses and helpers, the vibrations from the transit of Spica to the transit of Arcturus across the same meridian. Twice we found 3212 simple vibrations and once 3214, using our custom of throwing single pebbles into a vessel after every thirty compound vibrations. But if the vibrations of one and the same pendulum took perceptibly unequal times, there could not fail to be a great difference in their number, showing itself plainly at least after many vibrations. Yet it nowhere shows itself. Therefore they are equal in time to the senses, in themselves, and leaving aside accidental causes, as when the wind blows from the opposite direction, or something else out of the ordinary speeds up or slows down the pendulum. Wendelen adds that if the pendulum is raised beyond 40° or 45° its vibrations are of longer duration, and likewise longer when the Sun is at apogee; on this matter see below, proposition 15.

PROPOSITIO II.

The earlier vibrations of one and the same pendulum are swifter than the later ones, by a difference that is uniformly difformly unequal.

For they are completed in equal times, as was shown in proposition 1, and yet they traverse unequal spaces, as is established by evident experiment. And if the arc of the first vibration was TS, the arc of the second will be smaller, as SX; that of the third smaller still, as XY; that of the fourth YZ; and that of the fifth ZΦ. Yet the difference is greater between the two earlier ones than between the two later ones: so that if the difference of the first two is the arc XT, the difference of the two later ones will be the arc ZX, smaller than the arc XT; and toward the end it is so small that equal arcs seem to be described in equal times by the centre O.

PROPOSITIO III. IV. V. VI. & VII.

III. Of two pendulums equal in all respects except height, the one of greater height completes a smaller number of vibrations within an equal time, but its whole motion persists for a longer time.

IV. Of two pendulums equal in every way except height, the one of lesser height completes its single vibrations, and its whole motion, more quickly, other things being equal.

From this, Galileo judges, it comes about that, while the upper rings of the chain by which the pendulum is hung, being less distant from the centre of motion C, strive to bring themselves to rest more quickly than the lower and more distant ones, they gradually weaken the pendulum's impetus; and that this is the reason why a motion which, with the air's resistance excluded, would perhaps be perpetual, at last ceases.

V. Of two pendulums equal in everything except heaviness, the heavier persists longer in motion, and within an equal time performs a greater number of vibrations.

VI. Of two pendulums equal in all other respects, the one that is slightly heavier, but notably greater in bulk, completes fewer vibrations in an equal time than the other.

I would believe the cause to be the resistance of the greater amount of air to be broken through by the greater bulk.

VII. Of two pendulums equal in other respects, but with one exceeding in individual heaviness and the other in bulk, the lighter comes to rest sooner and completes fewer vibrations in an equal time.

Propositions 5, 6 and 7 are against Baliani, who, if the height of the pendulums is equal, their vibrations are, he thinks, of equal duration. But I cannot disbelieve my own eyes.

PROPOSITIO VIII.

Of two pendulums equal in everything except height, the lesser height is to the greater as the square of the vibrations of the greater height is to the square of the vibrations of the lesser height, performed within an equal time; and conversely.

So it is according to Galileo, Baliani and many experiments of our own. In other words, the heights of pendulums of different height are to one another as the squares of the times. For let there be two spheres of lead, of one ounce each, one at a height of one foot and the other of two feet; and let the vibrations of the one-foot pendulum be counted, nearly 85, for meanwhile the vibrations of the two-foot pendulum will be counted as 60. Now the square of the number 60, namely 3600, is twice as great as the square of the number of nearly 85, which is 7200, just as the two-foot height is twice as great as the one-foot. The slight small difference in the numbers of the squares arises from our not being able to discern parts of a single vibration; so if you count 85 whole vibrations, their square will be 7225, which is a little more than double 3600. Now if the vibrations of a three-foot pendulum are 60, a pendulum of one-foot height will meanwhile complete very nearly 104; and the square of these, 10816, is very nearly triple the square 3600, and so on. From this arise two notable problems.

PROPOSITIO IX. Et Problema I.

Given two pendulums wholly equal but of different height, and given the vibrations of each, counted separately in an equal time, and the height of one, to find the height of the other.

Multiply each number of vibrations by itself, and you will have the squares of the vibrations. Then multiply the square of the vibrations of the known height by the known height itself, and divide the sum this produces by the square of the vibrations of the unknown height. The quotient will give the unknown height.

PROPOSITIO X. Et Problema II.

Given two pendulums wholly equal but of different height, known in each, and given the number of vibrations of one, to find the number of vibrations of the other.

Multiply the square of the number of the given vibrations by the height of the pendulum whose vibrations are given, and divide the sum this produces by the height of the other pendulum. The quotient will show you a number whose square root will give the number of vibrations.

PROPOSITIO XI. Et Problema III.

To find the time of the primum mobile, or of the solar day, that corresponds not only to the whole motion of a given pendulum but also to its single simple and compound vibrations; and from this, further, to count times exactly by its vibrations.

[Margin: Finding a pendulum for measuring times.] If the time corresponding to all the vibrations, or at least to many taken together, can be established, there is no doubt that from it, by the golden rule of proportions, the time corresponding to each one can be derived. For example, if to 3600 vibrations there corresponds one hour, that is, 60^{m}, then certainly to 60 vibrations there will correspond one minute of time, and to each single vibration one second of the seconds of time, by Proposition 1. Now this can be found in many ways, but chiefly in four, which I shall explain better by example than by a longer precept.

[Margin: First method.] First, I had observed that a pendulum of about one pound, 3 ancient Roman feet and 4 inches [vncias] high, was such that to one simple vibration of it there corresponded one second of time, and to 60 vibrations one minute; for with a sand-glass of one quarter of an hour I had many times counted about 900 of its vibrations, that is, sometimes a little more and sometimes a little fewer, because of the variability to which clocks of this kind are subject. So, to come closer to a more exact measure, once an astronomical and horizontal dial had been made, I and Father Francesco Maria Grimaldi counted, from the third hour before noon to the third after noon, 21706 simple vibrations. Now in six solar hours there are contained six hours of the primum mobile and one further minute, according to what will be said in Book 3, Chapter 30, or 21660^{s}. Therefore one simple vibration of that pendulum contained somewhat less time than one second, and it seemed

that the pendulum's height should be increased, so that, according to what was said in Proposition 3, it would complete fewer vibrations within the same time. But since the arrival of the shadow at the hour lines cannot be discerned so exactly that we need not fear an error of some seconds, I set this method aside and turned to others.

[Margin: Second method.] Second, then, we used a pendulum whose height was 3 ancient Roman feet and 4 inches, with a sphere of iron of 8 ounces and its small chain of 3 3/5 ounces; and we arranged 9 colleagues, well instructed for the purpose, nearly all of whom now publicly teach philosophy, theology or mathematics, to relieve one another after about half an hour of counting. In 1642, from noon on April 2 to noon on April 3, we kept up the counting of the simple vibrations. Their number, from the pebbles thrown into a vessel for each sixty vibrations, was found to be 1466 sixties and 38 vibrations besides. But the day of the primum mobile contains its 1440^{m}, and the day of the Sun 1444^{m} of the primum mobile. Such a pendulum, therefore, in one day of the primum mobile, completes 1462 sixties of vibrations and 38 vibrations more, whereas it should complete only 1440 if a single simple vibration of it corresponded to one second. I therefore added one ring to the small chain, so that the number of vibrations would turn out smaller and come closer, with its single vibrations, to the seconds of the primum mobile.

[Margin: Third method.] Third, then, we used the pendulum of the second method, but now 3 ancient Roman feet, 4 inches and 20/100 of one inch high. But having observed the transit of the Tail of Leo across the meridian by means of the triangle of threads described above at marginal number 5, on the night following May 12, we counted with the same colleagues, from the transit of this star through the M. C. to its next transit on the night following May 13, 1449 sixties of simple vibrations and 58 vibrations besides. So they exceeded the day of the primum mobile by very nearly 10 of its minutes; and by that proportion I made myself a small table of equation. But so that this difference should not be attributed to the variation that can arise in keeping the pendulum's motion going, and so that I should not, by imposing so much tedium in counting through a whole natural day on myself and my colleagues, abuse my own health and their endurance, I chose a fourth method. Keeping the earlier pendulum, I used another, whose height I first tested against that pendulum by trial, to see how great it should very nearly be to satisfy my wishes.

[Margin: Fourth method.] Fourth, then, I used a pendulum whose height is 3 ancient Roman feet, 2 inches and 67/100 of one inch; with a sphere of solid brass of 20 1/2 ounces, and a brass chain of 174 rings, whose weight is 3 2/5 ounces. With this pendulum, from the transit of Spica across the meridian to the transit of Arcturus across the same meridian, we counted in 1645, on three distinct nights with the observation repeated, namely on May 19 and 28 and June 2. Using Fathers Francesco Maria Grimaldi and Zenus, we found that we had counted 3212 simple vibrations twice and 3214 once. In that year the southern declination of Spica, as observed by us, was 9° 15′ 28″, that of

Arcturus, northern, 21° 5′ 22″, and the distance, taken repeatedly with sextants, 33° 22′. Therefore, by the problems of the primum mobile treated in Book 10, the difference of right ascension between Spica and Arcturus, computed on the equator, was 13° 18′. To this there corresponds, in time of the primum mobile, 53^{m} 12^{s} [Horaria Minuta 53′. 12″.]; or, reducing to seconds alone, 3192^{s}. Therefore, if 53^{m} 32^{s} of the already mentioned pendulum, that is, 3212 simple vibrations, are worth 3192^{s} of the primum mobile, then certainly 60^{m} of the pendulum, that is, 3600^{s}, are worth 59^{m} $37½^{s}$ of the primum mobile, or at least 59^{m} 36^{s}. So one simple vibration of it is worth 59‴ 36⁗ of the primum mobile. This is how the small table of equation for time [tabella Æquatoria temporis] was made. I show the reader a specimen of it, so that he may see from it how we have used other similar ones in counting times of the primum mobile. To understand it, remember that we are accustomed to use single pebbles or grains of bean for every sixty simple vibrations.

Pendulum 3 ancient Roman feet, 2 67/100 inches high, whose total weight, brass sphere with chain, is 23 11/12 ounces.

Simple vibrations	Times of the primum mobile: m	s	‴	⁗	Sixties of vibrations, or pebbles	Times of the primum mobile: Hours	m	s	‴	Sixties of vibrations, or pebbles	Times of the primum mobile: Hours	m	s	‴
1	0	0	59	36	1	0	0	59	36	100	1	39	20	0
2	0	1	59	12	2	0	1	59	12	200	3	18	40	0
3	0	2	58	48	3	0	2	58	48	300	4	58	0	0
4	0	3	58	24	4	0	3	58	24	400	6	37	20	0
5	0	4	58	0	5	0	4	58	0	500	8	17	40	0
6	0	5	57	36	6	0	5	57	36	600	9	56	0	0
7	0	6	57	12	7	0	6	57	12	700	11	35	20	0
8	0	7	56	48	8	0	7	56	48	800	13	14	40	0
9	0	8	56	24	9	0	8	56	24	900	14	54	0	0
10	0	9	56	0	10	0	9	56	0	1000	16	33	20	0
20	0	19	52	0	20	0	19	52	0	1440	23	50	24	0
30	0	29	48	0	30	0	29	48	0	1450	24	0	20	0
40	0	39	44	0	40	0	39	44	0	1500	24	51	0	0
50	0	49	40	0	50	0	49	40	0	2000	33	6	40	0
60	0	59	36	0	60	0	59	36	0					
70	1	9	32	0	70	1	9	32	0					
80	1	19	28	0	80	1	19	28	0					
90	1	29	24	0	90	1	29	24	0					
100	1	39	20	0	100	1	39	20	0					

PROPOSITIO XII. Problema IV.

To find a pendulum one simple vibration of which is equal to one second of time of the primum mobile.

[Margin: A pendulum that exactly represents the times of the primum mobile by its motions.] Let a pendulum be made whose brass sphere weighs 20 1/2 ounces, with a brass chain of 174 rings, whose weight is 3 2/5 ounces, and whose height is 3 ancient Roman feet, 3 and 27/100 inches. That is, reducing the height to hundredths of an inch, let it be 3927 hundredths. For by Proposition 9 and Problem 1, the heights of two equal pendulums are to each other as the squares of the vibrations. Now a pendulum 3867 hundredths of an inch high, with its 60 vibrations or its 3600‴, equals 3576‴ of the primum mobile, by what was said in Proposition 11. Therefore, if the square of the number 3600, which is 12960000, is multiplied by the height of 3867 hundredths, the result will be 50116320000. Divided by the square of the 3576‴, which is 12787776, this leaves 3927 hundredths of an inch, that is, 3 feet, 3 27/010 [recte: 27/100] inches.

PROPOSITIO XIII. Problema V.

To derive various pendulums from one, for a finer measure of time.

[Margin: Pendulum for a time of 15‴.] Let a pendulum be made whose height is 9 and 76/100 inches of the ancient Roman foot, and let the small brass sphere, with its small chain, weigh 18 1/3 drachms. For by testing it against the pendulum treated in Propositions 11 and 12, I found that this one completes 180 of its compound vibrations while that longer pendulum completes 180 simple vibrations, which by the foregoing small table are worth 179^{s} 48‴. Therefore this pendulum represents 30‴ by its simple vibration, very nearly one second by its compound vibration, and 15‴ by its half-vibration.

[Margin: Pendulum for the swiftest motions.] Let another pendulum likewise be made, whose height is 1 15/100 inches of the ancient Roman foot, the foot on which see our chapter 7. Let its small brass sphere weigh 4 drachms, but let it hang by a thread from the centre of motion, for its weight is not so great, nor its use so frequent, that the thread could be perceptibly stretched. This pendulum completes 360 of its simple vibrations in the time in which the pendulum immediately preceding completes 60 compound vibrations, that is, 60^{s} of time or 3600‴. So by its single vibration it is equal to 10‴ of the primum mobile. We used a pendulum of this kind to measure the natural motion of heavy bodies. But in counting its vibrations, since they are extremely quick, one must raise one finger of the closed hand after each ten of them and be very attentive; or rather, for greater evidence, one must use two equal pendulums of this sort and have two counters, each noting his number separately, so that it is clear at the end of the operation whether they agree or not. Here, then, are small tables for the two pendulums just described, for adding up their times. Although the weight noted in them is not kept exactly and scrupulously, the same time will still be obtained, provided that the length of the pendulum from its centre to the centre of motion is exactly the same, and that no rather stiff iron wires are used (otherwise the pendulum's motion ceases too soon), but twisted threads, or rather small chains woven of the smallest loops.

Pendulum of 18 drachms, of a height of 9 76/100 inches.

Simple vibrations	Time of the primum mobile: ˢ	‴
1	0	30
2	1	0
3	1	30
4	2	0
5	2	30
6	3	0
7	3	30
8	4	0
9	4	30
10	5	0
20	10	0
30	15	0
40	20	0
50	25	0
60	30	0
70	35	0
80	40	0
90	45	0
100	50	0

Pendulum of 4 drachms, 1 15/100 inches high.

Simple vibrations	Times of the primum mobile: ˢ	‴
1	0	10
2	0	20
3	0	30
4	0	40
5	0	50
6	1	0
7	1	10
8	1	20
9	1	30
10	1	40
20	3	20
30	5	0
40	6	0
50	8	0
60	10	0
70	11	40
80	13	20
90	15	0
100	16	40
200	33	20

PROPOSITIO XIV. Et Problema VI.

Given the weight of a pendulum, to find the height at which one simple vibration of it is completed in one minute of time, or one hour, or one natural day. Provided that its chain does not break, etc.

[Margin: Height of the oscillation of a minute, of an hour and of a day.] I said "provided that its chain does not break, etc." because the excessive length of the chain could either slow the motion too much, or, by the excessive weight of so many rings taken together, overcome the strength of the upper rings and, by loosening them, be broken off. We therefore cannot be certain of this effect at insane heights. Yet speaking in itself, and considering the precise difference of height, Proposition 9 holds. Supposing it, let us take the pendulum of Proposition 12, namely the one that is of 20 1/2 ounces, and with its chain of almost 24 ounces, or 2 pounds. If its height is 3 feet, 3 27/100 inches, and one simple vibration of it is completed in one second of time, as was said in Proposition 12, then by Proposition 9, if nothing else intervenes, at a height of 2356 paces one vibration of it will last one minute of time; at a height of 8,482,320 paces, one hour; and at a height of 4,885,816,320 paces, one natural day. And so, for one simple oscillation of it to last one day, the height of such a pendulum would have to be about 1000 semidiameters of the Earth.

PROPOSITIO XV. Problema VII.

To measure astronomical times more exactly with a pendulum than with any other clock.

We have very often used the pendulum treated in Propositions 11 and 12, with colleagues highly practised in this and most trustworthy, to measure the intervals of time between the transits of fixed stars across the meridian, and so the differences of right ascensions; and to measure the duration of eclipses, and the intervals between the transit of other planets and of some fixed star across the meridian. For from the practice described in Proposition 11, method 4, we were certain that within one or two hours no error greater than 5^s of time could be committed. Moreover, we used the pendulum a dodrans high [dodrantalis altitudinis], treated in Proposition 13, to count the time in which the whole disc of the Sun emerged from a clear horizon, and in which the disc of the Sun and of the Moon, on the equator or outside it, passed across the meridian, both by direct view and by view reflected into the base of the cone of light, received on a white tablet; and so to measure the apparent diameters of the luminaries, and for many other astronomical times, as will be recounted in their proper places.

[Margin: Langren, Mersenne, Wendelen and Kircher measured times with the pendulum.] I was glad, moreover, when some years later I heard that pendulum oscillations had been adopted by other astronomers: namely by Michael Florent van Langren, as I learned from a letter he himself sent to me; by Govaert Wendelen, as is evident from his *Eclipses* and his *Idea of the Atlantic Tables* [Idea tabularum Atlanticarū], p. 25; by Father Athanasius Kircher, as is clear from his *Magnes* [Magnete], p. 450; and by Mersenne and others. But it

is not established that all were equally accurate in determining the measures of the pendulum. For, so that one vibration should be equal to one second of time, Mersenne, according to Wendelen, requires a weight of 8 ounces and a cord of 3 feet, but Parisian feet; Kircher, according to the same author, a thread of 3½ feet, with no weight added. Wendelen nearly agrees with this, saying: "With a suspension of 42 Roman feet, I obtained oscillatory swings of an iron weight of 52 pounds numbering at most 543 per hour; and that is on a fair par with this reckoning of three and a half feet." He then adds: "And my friend Langennus [recte: Langrenus, per the book's errata] too, having long since undertaken to measure times with iron rods [Rhopalis ferreis] of different lengths and weights, has laboured over many other things and over the diameter of the Sun as well, and most successfully indeed." He does not show, however, how what he appends after those words can be true: "Although it is not true that all oscillations of the same suspension are of equal duration, it is true that in winter, that is, with the Sun at perigee, more are made in one hour than in summer, or with the Sun at apogee, etc." [Margin: Whether oscillations vary with the motion of the Earth?] Unless perhaps he thinks that because of the motion of the Earth they are quickened by the rapid diurnal motion on account of the approach to the Sun, and slowed in the recession from it; a view that, as we shall show in Book 3, chapter 30, also pleased Kepler, Longomontanus and Boulliau. Wendelen concedes, however, that "if the pendulum is drawn out on either side beyond its line of plumb to 10°, it completes a very great number of oscillations, isochronous for a very long time, that is, of equal duration, and equal to every sense." For us, however, who do not admit the motion of the Earth, that difference of oscillations between the Earth at aphelion and at perihelion vanishes altogether; nor does it please Mersenne, in Book 4 of the *Harmonia*, p. 369.

[Margin: Comparison of the pendulum with other clocks.] I said, moreover, that we can measure times more exactly with a pendulum than with any clock. For water clocks and the clepsydras used by the ancients, on which see Cleomedes, Book 2 of the *Cyclical Theory*, Proclus in the *Astronomical Hypotyposes*, chapter 3, and Martianus Capella, Book 8 of *On the Marriage of Philology*; and sand-glasses, on which see David Rivault [Dauid Riualtus] on the *Sand-Reckoner* of Archimedes; and mercury clocks [Hydrargyrologia], on which see Tycho, volume 1 of the *Progymnasmata*, p. 149; and *horisonia* [Horiſonia] [?], on which see the same author in the same place; and *scaphia* [Scaphia], on which see Macrobius, Book 1 on the *Dream of Scipio*, chapter 20; and all sundials [Sciaterica]: none of these are measures so exact for measuring a whole hour and its parts, whether because of their uniformly difform motion or because of the uncertain arrival of the shadow at the hour lines, as we shall say in Book 3, chapter 9, method 4, which it is useful to read beforehand at this point.

[Margin: The speed of the pulses of the human artery.] The beats of the human artery, composed of systole and diastole, which some have thought

suitable for a finer measure of time, are subject to many anomalies because of the different constitutions of different people and the same person's different ages, states of health, disturbances of mind, bodily movements, or sleep, etc. Granted that Cardano, in *De proportionibus*, Book 5, Propositions 58 and 218, says that a healthy person's artery beats 4000 times in one hour; and Kepler, in the *Epitome of Astronomy*, p. 278, that in a robust person of mature age and melancholic constitution, or one growing old, a single pulse of the artery is perceived in each second of time, with no distinction of systole or diastole; yet that this slowness is rare, and that commonly 70 beats are counted in one minute, but in the choleric and in women 80, so that commonly there are about 4000 in one hour. Wendelen speaks of 4200 beats of his own artery in one hour, quite often tested. But while very many members of our Society counted their pulses and I counted the time with the pendulum of Proposition 13, I detected a difference within one minute of time, from 50 pulses to 85, in most, however, about 60 or 65 or 70 pulses. But let us now apply the pendulum to measuring the speed of heavy bodies.

CAPVT XXI.

On the speed of heavy bodies descending in natural motion, and the proportion of the increment of their speed.

I had not yet seen Galileo's Dialogue 2 on the system of the world, nor Baliani's little work on the natural motion of heavy solids and liquids, when I was already trying, by means of the motions of the pendulum, to find out in what proportion of speed heavy bodies descending naturally moved faster and faster toward the end. I shall therefore briefly and in perfect good faith set out what I found by experiment, and in what I agree with them, or am compelled to dissent from them.

PROPOSITIO I.

Of two bodies of equal bulk, the heavier descends in natural motion from the same limit to the same limit more quickly than the lighter.

[Margin: Two bodies equally large but not equally heavy do not descend with equal speed.] For twelve globes were made of solid clay, of 20 ounces, and as many others of compressed paper, coated with a shell of clay, equal in bulk to the former but of 10 ounces. They were released two at a time from the tower of the Asinelli at Bologna at the very same moment, that is, one of ten ounces and the other of twenty. The heavier struck the ground of the pavement below while the lighter was still about 15 feet from the same pavement. And with two counters employed, using the smallest pendulum, treated in chapter 20, Proposition 13, the lighter globe struck the pavement later by about three vibrations, that is, by about 30‴. This was confirmed by repeating the experiment twelve times. Neither Galileo nor Baliani denies this proposition, but they say that the interval both of space and of time between the fall of the lighter and that of the heavier is small, because they tested it at a height of only

50 or 100 feet. [Margin: Height of the tower of the Asinelli.] But the tower of the Asinelli is 312 ancient Roman feet high, as we measured it with Father Grimaldi, and it is very well suited to such experiments, since it does not taper to a peak like a pyramid or a cone, but rises four-sided between nearly parallel plane surfaces right up to the top. The distance, however, from the cornice [coronide] to the pavement of the scaffolding [parapegmatis pauimentum], the distance from which the globes were released, is 280 feet. Present at these and the following experiments as witnesses and helpers, though not always all together, were Fathers Stephanus Ghisonus [?], Camillus Rodengus [?], Iacobus Maria Pallauicinus [?], Vincentius Maria Grimaldus [Vincenzo Maria Grimaldi], Francesco Maria Grimaldi and Zenus; Georgius Cassianus [?], and others.

PROPOSITIO II.

Of two bodies of the same species, but of unequal bulk and weight, the heavier descends in natural motion from the same limit to the same limit more quickly than the lighter.

For on 1645 August 4 we released from the tower of the Asinelli a great many chalk globes, in pairs together, one of 4 ounces and the other of 8, the experiments being repeated fifteen times; and we always observed that the heavier struck the pavement while the lighter was still 4 feet or a little more from it. When other globes were used, the distance of the lighter ones from the pavement, at the moment when the heavier ones struck it, was as great as you see in the table below.

Two globes of the same bulk released together from the same height	Globe that descended more quickly	Feet by which the lighter globe stood off from the pavement
Wax, of 6 7/8 ounces; and wood, of 4 6/8 ounces	Wax	15
Wax, of 1 5/8 ounces; and iron, of 11 7/8 ounces	Iron	30
Chalk, of 9 ounces; and wood, of 2 1/2 ounces	Chalk	20
Lead, of 2 1/2 ounces; chalk, of 2 ounces	Lead	25
Lead, of 2 1/2 ounces; wood, of 2 1/2 ounces	Lead	40
Wax, of 15 ounces; and chalk, of 20 ounces	Chalk	[recte: 12, per the book's errata; the printed cell is empty, and 12 is written by hand on the leaf in brown ink in this copy]

Let those semi-empirics, whoever they are, now go and boast, dropping pebbles from a small height and seeing a tiny interval between their impact, or striking of the pavement, that all bodies released from equal height onto the same pavement reach it in the same time to the senses.

PROPOSITIO III.

Of two bodies of unequal gravity, the proportion in descent and speed is not the same as in gravity.

[Margin: Aristotle's error on the motion of heavy bodies.] This is against Aristotle, who is deservedly refuted above all by Galileo in Dialogue 2 on the system of the world and by Baliani, cited above, and by our experiments adduced for Propositions 2 and 3. For if Aristotle's position were true, then in the time in which the globe of 20 ounces completed a space of 280 feet, the globe of 10 ounces should have completed only 140, and should have been as many further feet from the pavement, whereas it was only 15 feet away. The other, lighter globes, moreover, were never distant from that pavement in the proportion by which they were exceeded in gravity, even though all were released from the same distance of 280 feet, as is clear from the table of Proposition 2.

PROPOSITIO IV.

Heavy bodies in natural descent along a perpendicular line move faster and faster toward the end, with that increment of speed which obtains among the evenly odd numbers counted from unity; or such that the spaces traversed in given times are to each other as the squares of the times; or such that the spaces traversed have to each other the duplicate proportion of that which the times have in which those spaces were measured.

The whole assertion is that of Galileo, cited above, and of Baliani, and it has quite often been proved by our experiments. The evenly odd numbers from unity are said to be these: 1, 3, 5, 7, 9, 11, 13, 15, etc. Thus if in the first quarter of an hour some heavy body has completed 1 stadium, in the second quarter it will complete 3, and in the third 5 stadia, and so on for the rest of its progress. To test this, Father Fr. M. Grimaldi and I prepared a great many chalk globes of the same bulk, each of 8 ounces, which we released from different towers or buildings, from windows or little windows better suited to taking measurements; and we used, first of all, the towers of Bologna, namely that of the Asinelli, which is 312 Roman feet high, and San Pietro, 208 feet high, and San Petronio, 200 feet high, and San Giacomo, 189 feet high, and San Francesco, 150 feet high, although we did not use their entire height, but the height that was suited to the above-mentioned proportion. Moreover, to learn more finely the time in which the released globes reached the pavement, we employed two of the smallest pendulums, treated in chapter 20, Proposition 13, in the second place, namely those of which one simple vibration is worth 10‴ of the primum mobile. Among many experiments, however, so as not to grow too long-winded, I set before my reader in the following small table the two given below, the most carefully selected and the most certain of all.

Experiments	Simple vibrations of the pendulum	Time corresponding to the vibrations	Distance completed by the chalk globe of 8 ounces at the end of the time (Roman feet)	Distance therefore completed separately in each of the equal times (Roman feet)	Ratio of the increase of speed, expressed in simple numbers (evenly odd numbers)
	5	$0^{s}\ 50'''$	10	10	1
	10	$1^{s}\ 40'''$	40	30	3
I	15	$2^{s}\ 30'''$	90	50	5
	20	$3^{s}\ 20'''$	160	70	7
	25	$4^{s}\ 10'''$	250	90	9
	6	$1^{s}\ 0'''$	15	15	1
	12	$2^{s}\ 0'''$	60	45	3
II	18	$3^{s}\ 0'''$	135	75	5
	24	$4^{s}\ 0'''$	240	105	7
	about 26	4^{s} about $20'''$	280		

In the first experiment, then, the operation having been repeated several times, we observed that from a height of 10 feet the above-mentioned globe reached the pavement in only five vibrations of the above-mentioned pendulum. We then tried out the height that a globe equal to it would cover in 10 vibrations, and we found that it was 40 feet, and so with the rest. In the second experiment, however, we took the height as given and explored the times themselves. For having found the height of 15 feet that the globe had covered in 6 vibrations, we concluded that, if the above-mentioned proportion were true, 60 feet ought to be covered at the end of 12 vibrations. Having therefore sought out a height of 60 feet that was convenient for the practice, we acknowledged this to be true, and so with the others. We could not, however, find the height due to 30 vibrations, which would have been suitable for the rest of the progression.

You now see that in the first experiment, as the distance 10 is to 40, so the square of 5 vibrations, which is 25, is to the square of 10 vibrations, which is 100; and in the second experiment, as the distance 15 is to the distance 60, so the square of 6 vibrations, which is 36, is to the square of 12 vibrations, which is 144; and so with similar cases.

COROLLARIA.

[Margin: The blunder of Simplicius and Arriaga.] I. Those therefore are wrong by the whole heaven and all the elements who, taking their occasion from Simplicius, *De caelo*, Book 1, text 88, together with our own Arriaga [Rodrigo de Arriaga], *On Generation*, disputation 4, section 5, either deny or call into doubt the ever greater increase in the speed of heavy bodies toward the end, which others commonly admit along with Aristotle, *Physics* 5, text 76, and *De caelo*, Book 1, text 88, and Question 19 of the *Mechanics*. Nor is it an objection that smoke does not rise faster at the end, as bubbles do out of water. For it is hindered by the cold air around it and by the air's constant agitations, by which it is dispersed; and the vaporous part of the smoke is deserted by the drier

exhalations flying out of its cloud. Likewise it is no objection that a leather ball falling from on high does not break a vessel which is broken by a stone released from a lesser height. For it is not the impetus acquired on the way alone that concurs in breaking a vessel, but also hardness, density, etc. Nor in these cases are the other things assumed to be equal.

[Margin: The cause of the ever faster motion.] II. It is probable that the cause of the ever faster motion of heavy or light bodies toward the end is the intensification of the impetus produced by their heaviness or lightness at every moment in which they are away from the term of their natural rest. For it is not fitting that they should lose the impetus once produced, and if it were not increased, they would remain too long in a violent place. More on this matter at some time [olim] in our *Physics*.

III. If a chalk globe of 8 ounces covers 15 feet, or 3 paces, in a single second of time, as is clear from our second experiment, then by the proportion treated in Proposition 4, if nothing else intervened to hinder or vary the motion, such as heat produced by friction and dissolving the globe, or the differing rarity of the air or aether, it would cover 10800 paces in 60 seconds, that is, in 1 minute. For as the square of 1 second, which is 1, is to the square of 60 seconds, which is 3600, so are 3 paces to 10800 paces. In one hour, however, that is, in 60 minutes, it would complete 38880000 paces. For as the square of 1 is to the square of 60 minutes, which is 3600, so are 10800 paces to 38880000. And in two hours it would complete 155520000 paces altogether. For as the square of 1 hour is to the square of 2 hours, which is 4, so are 38880000 paces to 155520000 paces. Finally, in 48 hours, or 2 days, it would complete 89579520000 paces altogether. For as the square of 1 hour is to the square of 48 hours, which is 2304, so are 38880000 paces to 89579520000 paces. But there are 5000000 paces in the Earth's semidiameter, according to what was said in chapter 8. Therefore, dividing the above-mentioned paces by this figure, we shall have the semidiameters of the Earth set down below.

	In the time	Completes ancient Roman paces	Or semidiameters of the Earth
A chalk globe of 8 ounces, of itself, and if nothing else happens, and the space is of the same density as our air.	$0^h\ 0^m\ 1^s$	3	
	$0^h\ 1^m\ 0^s$	10800	
	$1^h\ 0^m\ 0^s$	38880000	7 39/50
	$2^h\ 0^m\ 0^s$	155520000	31
	$3^h\ 0^m\ 0^s$	349920000	70 ☾
	$48^h\ 0^m\ 0$	89579520000	17916, with a star symbol

[Margin: The time in which a heavy body would descend from the Moon or from the firmament to the Earth.] IV. Those people therefore err through the greatest ignorance who, to exalt marvellously the distance of the stars from the Earth, say that a millstone [mola molendini] would hardly reach the Earth

from the firmament after many years. From this, too, the opinion that Galileo reports in Dialogue 2 on the System of the World began to be suspect to me, namely that a cannonball would not reach the Earth from the Moon except in 6 hours; and indeed the opinion of Galileo himself too, who asserts from his own experiment that a cannonball of one hundred pounds covers one hundred cubits, or 150 feet, in five seconds of a minute of an hour, and so will cover 276480 miles in 4 hours. This distance is greater than the lunar distance taken as 56 semidiameters of the Earth, with the Earth's semidiameter at 3500 miles, as he himself assumes; for then the Moon's distance is 196000 Italian miles. But to me a chalk globe of 8 ounces, and one far lighter than that cannon globe, completes 240 paces in 4 seconds of an hour. That cannonball therefore covers many more; but Galileo will have erred in assigning a time of the primum mobile to pendulums, or at any rate let his followers show by what method he attached this time to them. Nevertheless, consistently with his own hypothesis, he says that that cannonball would reach the Earth from the Moon, that is, would cover 196000 miles, or 56 semidiameters of the Earth, in 3^{h} 22^{m} 4^{s}. But for me a chalk globe of 8 ounces would complete 349920 miles in three hours, and would measure out the lunar distance in a little more than two hours. And if for Tycho the firmament is not more than 14000 terrestrial semidiameters from the Earth, the same globe would run through this distance within two days, since for me it will finish nearly 18000 semidiameters of the Earth.

PROPOSITIO V.

[Margin: The wonderful proportion between the straight and the circular motion of heavy bodies.] *Just as the different heights of the same pendulum are to one another as the squares of the times or of the vibrations, in circular motion, so the distances covered by the same weight in natural straight motion are to one another as the squares of the times.*

This is evident from what was said in Proposition 8 of chapter 20 and Proposition 4 of this chapter, taken together. Nor is there any other difference, except that in the motion of pendulums there are, within an equal time, more vibrations of the less high pendulum, whereas in the straight motion of heavy bodies descending from a greater height more times are included. Yet that proportion is admirable, and for so many centuries it was unknown to mortals.

SCHOLIA.

I. If anyone wants more on the motion of heavy solids inclined in various ways upon a plane, let him consult Book 3 of D. Giovanni Battista Baliani's On the Motion of Heavy Solids *[de motu grauium solidorum], which is followed by Book 4, on the motion of heavy liquids, Book 5, on the sections of channels, and Book 6, on the openings of a vessel. For we are not permitted to digress further in weighing the motions of this elementary sphere; we are already being swept off to contemplate the heavens, and now, even now, the Sun with its radiance is seizing us toward itself.*

[Margin: A new reciprocation of the pendulum.] *II. In 1643 there appeared a Letter of Pierre Gassendi to Gabriel Naudé, in whose postscript he treats of a twofold reciprocation of pendulums each day, like the tide of the sea, observed by Alexander Calignonus [?] of the Dauphiné. This man hung a pendulum from a height of 30 feet, with the length of its thread enclosed in a tube so that it should not feel the agitations of the air, and with a conspicuous little point fixed to the lead below. Under this point, while it was at rest, he placed another little point fixed in a cube. And he saw that the pendulum's little point diverged from the point of the cube, for six hours at a time toward the north and for six toward the south, not without a bending from north into east and from south into west; and that the limit of the excursion toward the south was reached at noon and again at midnight. And he measured these digressions, which I know have not yet been published. From this he himself and Gassendi infer, first, that the Earth nods and that this reciprocation arises from the Earth's motion, just as, in Galileo's opinion, the tide of the sea arises from that motion, because of the unequal speed of the diurnal and annual motion; but that the ebb and flow of the sea does not take place from east to west and back, as Galileo holds, but as it happens in pendulums. Second, he infers that the altitude of a star taken at midnight is not the same as that taken at the sixth hour, because of the nodding of the pendulum, and that therefore both altitudes of the Pole Star are to be taken at the sixth hour of the morning and of the evening. Gassendi points out many other things worthy of his sagacity, but things as nodding as the solidity of this experiment itself nods.*

III. For I, with Father Francesco Maria Grimaldi, using a very long pendulum protected from the agitation of the air, etc., hung from a room subject to no tremors, found by experience that after it came to rest it was never moved from the perpendicular line for a whole day, although we visited it every half-hour. And from another quarter I think myself certain, from what is to be said in Book 9 on the systems, that the Earth does not move. Moreover, as our own Fournier reports in his Hydrographie*, between Norway and Canada there are masses of ice which in springtime float from north toward south, and which never, as they would have to because of a like motion, come back toward the north. Nor does the flow between Brazil and Africa, which comes from the east, become a reflow toward the east; nor do ships there, going from Brazil to Africa, feel any resistance from this motion. So Gassendi himself objects to himself, or rather to Calignonus. The cause of a nodding pendulum, then, where it does nod, must be sought elsewhere.*

APPENDIX Ad Librum II.

The Reverend Father Georges Fournier, of the Society of Jesus, recently published a book in French in which he treats hydrography with deep and solid learning. It contains many things with which I would most gladly have enriched this book of mine, if the volume of it, which he had most generously sent me from France in the flourishing days of Father Iacobus Dinettus [Jacques Dinet], had reached my hands in good time. But by what shipwreck or other mishap I do not know: did it perish, or did it put in elsewhere? At last another

copy of it, bought by one of our men, was shared with me, when the last page of this book was already about to go to press. So in this narrow compass of time I wanted at least to point out to the reader the books and chapters in which he treats the things I dealt with above. The rest, which concern navigation or the restoration of geography, I shall review in my Geographical Book.

From the Hydrographie *of Father Fournier.*

Book 1. On the structure and parts of ships.

Book 2, chapter 9. On certain very notable towers built on the seashore, which they call Pharoi.

Book 9. On the ebb and flow of the sea, where there is much against Galileo.

Book 11. On the variation of magnetic declination, and indeed under the same meridian, chapter 12.

Book 15. On winds.

Book 18, chapter 9. On the equilibrium of the sea, where he maintains that one sea is not higher than another.

Book 18, chapter 13. On the way of exploring the depths of seas.

Book 19, chapter 8. On the distance of vision, or the extent of the sensible horizon.

FINIS LIBRI SECVNDI.

LIBER TERTIVS DE SOLE.

CAPVT I.

On the Names, Substance and Encomia of the Sun.

[Margin: The names of the Sun.] I. The Sun [Sol] is so called in Latin because it is alone [Solus] and unique in the world, always shining and imparting day to some. In Hebrew it is *Chammah*, from heat; in Greek *Phoebus*, because it is [Greek: Φῶς Τοῦ βίου (phōs tou biou, "light of life")], that is, the light of life, and [Greek: Ηλιος (hēlios, "sun")]; and it is distinguished by many other names among the Egyptians and the Greeks. For the same is *Osiris* and *Horus* and *Titan* and *Apollo*, father and sower, that is, of the celestial harmony and of music, and also of medicinal plants, and so of medicine itself; and, if we believe Virgil, of wine and the vine, and therefore the same as *Liber*, from that passage of *Georgics* 2:

> *You, O brightest lights of the world, who lead the year gliding through the sky, Liber and nurturing Ceres.*

[Margin: The encomia of the Sun.] II. The encomia of the Sun, for their part, Pliny the Elder touches on in a few words, Book 2, chapter 6, saying: *The Sun is carried in the middle of the wandering stars, most ample in magnitude and power, ruler not only of the seasons and the lands but also of the stars themselves and of the heaven.* What he adds, however, must be cured with Catholic salt. For he says: *It is fitting to believe, weighing its works, that this is the soul of the whole world, or more plainly its mind; that this is the chief government and divine power of nature.*

This supplies light to things and takes away darkness; this conceals the other stars; this tempers the changes of the seasons and the year, always reborn, according to the need of nature; this dispels the gloom of the sky and also clears the clouds of the human mind; this lends its light to the other stars as well; splendid, outstanding, seeing all things, hearing all things too. Indeed, as Philostratus records, the Gymnosophists among the Indians used to contemplate it standing for the whole day with unaverted eye, and the 365 priests of the Egyptians observed the annual course of the Sun in the city of Heliopolis: so great was the care among them for this star that cares for all things. And rightly is the Sun everywhere called the torch of the seasons, the eye of the world, the heart of the heaven, the choragus of the stars, the governor of sensible nature, the fountain of light, the theatre of appearance [theatrum ſpeciei], the parent of colours, the lamp of gladness, the inextinguishable furnace of life-giving heat, the most brilliant trace of Divinity, and truly a splendid argument for divine providence. But all these proclamations are surpassed by what divine scripture brings forth about this coryphaeus of the planets, namely Psalm 18: *In the Sun he has set his tabernacle*, etc.; and Ecclesiasticus 42: *The Sun, giving light, has looked upon all things, and full of the glory of the Lord is his work*; and chapter 43: *The Sun in its appearance announcing at its going forth, a wonderful vessel, the work of the Most High; at noon it burns up the earth, and in the presence of its heat who can endure? Keeping a furnace in works of heat, the Sun threefold burning up the mountains, breathing out fiery rays, and shining with its rays it blinds the eyes. Great is the Lord who made it, and at his words it has hastened its journey.* Lastly, the mantle of the Queen of the stars is sought in the Apocalypse from nowhere other than the Sun, and a crown for the name of Jesus from nowhere other than its rays; nor did Christ himself set out a clearer summary of the divine beneficence toward us than when he said of God: *Who makes his sun rise upon the good and the bad.*

[Margin: The substance of the Sun.] III. The *substance* of the Sun, however very bright it is, is most obscure to us, and therefore disputed. For the Peripatetics, with Aristotle, *De caelo*, Book 2, chapter 7, flatly deny that the stars are of elementary matter, and so not fiery either, but say that they possess a certain fifth substance; and that the Sun is hot not in act but only in virtue, because it produces heat in lower things, whether by light alone or also by motion and friction of the air. And Pererius [Benet Perera] adds, on Genesis, Book 2, question 10, that if the Sun were fiery, fire would be perpetually moved in a violent motion, namely a circular one, whereas the motion natural to fire is both straight and upward; likewise, that all things would have burned up long ago, or that it would long since have exhausted the vapours of the whole Ocean; although Aristotle, Book 2 of the *Meteorologica*, judges childish and unworthy of a philosopher the opinion of those who said that the Sun feeds on moistures drawn up on high.

[Margin: The Sun seems fiery to many.] These things notwithstanding, that the Sun's substance is fiery is an opinion neither new nor yet antiquated. For *Anaxagoras* thought it a glowing stone; *Zeno* of Citium, according to Laertius,

Book 7, the purest fire, taking in from the sea the nourishment of an intelligible, that is, of a very subtle, vapour; *Democritus* and *Metrodorus* a glowing mass; *Anaximander* a portion of fire appearing from an opening in a great wheel full of fire; *Xenophanes* a clot of little fires collected from vaporous breaths; *Epicurus* a pumice or sponge set alight with fire; *Plato* said it was compacted from a great quantity of fire; the *Stoics*, according to Plutarch, Book 2 of the *Placita*, chapter 20, and *Seneca*, Book 7 of the *Natural Questions*, a conflagration endowed with mind. Nor is the Sun called [Greek: Φῶς πυρὸς εἶδος (phōs pyros eidos, "light, a form of fire")] in the *Timaeus* alone; it is also called by the Atlantians, on the evidence of Diodorus Siculus, Book 3, [Greek: πῦρ ἱερὸς ἐν [?] οὐρανῷ (pyr hieros en ouranōi, "sacred fire in heaven")], sacred fire in heaven. And in *Aristotle*, *De caelo*, Book 2, chapter 13, *Pythagoras*, placing the Sun at the centre of the universe, said that fire is at the middle of the world, and called it the guard of Jupiter. That all the stars are fiery, however, the Egyptians thought as well as the Stoics, and that from this would come at last the ecpyrosis of the world, or its conflagration; on this matter Cicero says much in Book 2 of *On the Nature of the Gods*. Sacred Scripture favours this opinion, saying of the Sun: *Keeping a furnace in works of heat*, Ecclesiasticus 43. That line of the Church's hymn favours it: *On the fourth day, who, establishing the flaming wheel of the Sun*, and again: *Now the fiery Sun withdraws*. The Hebrew name *Schemer* or *Schumah*, which means *there fire*, favours it. Finally, very many Fathers favour it, whom our own *Scheiner*, Book 4 of the *Rosa Ursina*, and *Antonius Maria de Rheita* [Antonín Maria Šírek of Rheita], in *Oculus Enoch et Eliae*, Book 4, chapter 2, gather and follow; among whom *Cyril*, in catechesis 9, stated in a beautiful antithesis *a watery heaven, and in it a fiery Sun*. Of the same opinion are *Kepler* in the Paralipomena to Witelo, p. 224, *Boulliau*, Book 1 of the *Astronomia Philolaica*, chapter 7, and *Athanasius Kircher*, Book 1 of the *Ars Magna Lucis et Umbrae*, chapter 1, who call the Sun a fiery sphere, concreted from an ethereal flux, and similar to casting furnaces, or to bronze liquefied and surging in their vats, with black soots mixed in. The larger telescopes certainly show it to us as such, as a kind of ignited ocean of flames, rough and surging with waves and whirlpools. See, however, what I shall say in Book 4, chapter 2, number 2, and in Book 9.

[Margin: Whether the Sun is animate?] IV. *Origen*, moreover, Book 1 of the *Peri Archon*, judged that the Sun and all the stars are *animate*, and indeed with a rational soul, so that they were capable of virtue, vice and redemption. This is erroneous in faith, as *St Bonaventure* points out on 2, distinction 14. For Origen was condemned on this very point by the Second Synod of Constantinople, as *St Jerome* reports in the letter to Pammachius, and *Nicephorus*, Book 17, chapter 128. Nor does the Lateran Council, in the chapter *Firmiter*, acknowledge any intellectual creature other than the angelic and the human. *Pliny the Elder* too, Book 2, chapter 8, attributes a mind to the Sun; but *Achillinus* [Alessandro Achillini], Book 3 of *On the Orbs*, and *Niphus* [Agostino Nifo], *On the Substance of the Orb*, teach that the Sun has an Intelligence in place of a soul, and *Caietanus*

[Thomas Cajetan] thinks that those words of Psalm 135, *Who made the heavens in understanding*, can be taken of an intellective soul of the heavens, and that this is the power of the heavens often expressed in Sacred Scripture. On this matter, according to Aristotle's meaning, *St Thomas*, *Contra Gentes*, Book 2, chapter 70; *Aureolus* [Pierre Auriol], as reported by Capreolus [Jean Capréolus], on 2; *Scotus*, in the same place, distinction 14, question 1, saying: *If the stars are not animate, this has been believed rather than demonstrated.* A soul lower than the intellective, however, was attributed to the stars (that is, one either sentient or vegetative or at least formative), and one that moves itself and others to the Sun, by, besides the Pythagoreans, Platonists and Stoics, *Simplicius*, *De caelo*, Book 1, comment 50; *Avicenna*, *Metaphysics* 9, chapter 2; *Scaynus* [Antonio Scaino], in his *Miscellanies*, chapter 16; *Mirandulanus* [?], *Monomachia* 19, section 2; *Philalthaeus* [Lucillo Filalteo], *De caelo* 2, text 13; and *Kepler*, in his *Mars*, Part 3, chapter 33. That this can be sustained as probable is affirmed by *Ferrariensis* [Francesco Silvestri], *Contra Gentes*, Book 2, chapter 70, and *Martinengus* [?], volume 2 on Genesis, p. 883.

On the contrary, many interpreters of Aristotle, on *Physics* 8 and *Metaphysics* 12, deny any soul whatever to the Sun and to all the stars. For Aristotelian Intelligences, they say, are forms assisting these bodies, not informing them; and there is no differentiation of organs, or any animastic effect [animasticum effectum], from which a soul in them could be inferred. So too *St Thomas*, First Part, question 70, article 3; and, as reported by Pererius, Book 2 on Genesis, question 7, Saints *Basil*, *Cyril of Alexandria*, *Damascene*, *Jerome* and *Ambrose*, although *St Augustine*, according to the same author, was in doubt on this matter. Yet to some, Aristotle seems to have attributed to the heavens, besides the assisting Intelligence, a kind of material soul; but this is not worth discussing here.

CAPVT II.

De Solis Qualitatibus, Lumine præsertim ac Figura Rotunda, & quandoq. Elliptica. [On the Qualities of the Sun, Especially Its Light and Its Round and Sometimes Elliptical Shape.]

The principal QUALITIES of the Sun seem to be *Light, Heat, Colour, Density, Opacity, Motive Force, Productive Force* and *Shape*. About these, much remains for natural philosophers to point out, and little for us, as astronomers.

[Margin: Light comes from the Sun, and is greatest in the Sun.] I LIGHT, which we see in the Sun, is not from the Sun itself but kindled by the fixed stars, said *Leucippus*, according to Laertius, Book 9; or by the Element of Fire, said *Philolaus* and *Empedocles*. But all the most distinguished lights among philosophers and astronomers acknowledge the Sun itself as the source of cosmic light. [Margin: For the opinion of Philolaus and Empedocles see below, number 5.] And not only do all the planets and the Moon, Venus and Mercury in particular, receive their primary light from the Sun, but also the comets

above the Moon and the new stars, as *Giordano Bruno* holds, with many others, in his book *De immenso et innumerabilibus*; and indeed so do the non-wandering stars, according to *Scheiner*, p. 71 of his *Disquisitiones Mathematicae*. Bruno, on the contrary, thinks that the fixed stars are Suns, wholly luminous of themselves and through themselves, and innumerable and infinite besides. But *Kepler*, in his *Dissertatio cum Nuncio Sidereo* of Galileo, does not hesitate to affirm: *The body of this Sun of ours is brighter by an inestimable measure than all the fixed stars together.* This is because the fixed stars, taken together, pour so dim a light into the most open places of the universe, whereas the Sun, through a single tiny opening in a window, lights a room so well that small letters can be read through by that light, which the whole light of the fixed stars does not achieve. To me, however, it does not seem improbable that, if as many fixed stars were to come together into one globe as would equal the bulk of the Sun, and were no less distant from the Earth than the Sun is, their gathered force, made nearer, would light the lands as much as the Sun does now; and, conversely, that a particle torn off the Sun, of the size at which Sirius or the Bright Star of Lyra now appears, would not greatly exceed the light of these stars. And *Descartes*, Part 3 of his *Philosophy*, number 9, holds the fixed stars to be neither smaller nor less bright than the Sun. As for whether all that light which was made by God on the first day of the world was on day 4 imparted to the Sun alone, or distributed among all the stars, or whether in its place another light flowed from the stars, let the interpreters see to that on chapter 1 of Genesis.

[Margin: The colours of the Sun.] II No COLOUR among the real colours is necessarily asserted of the Sun; for the phenomena it is enough to have the colour the Schools call intentional [intentionalem], which varies with the varying mixture of light and opacity, both in the body of the Sun itself and outside it, because of the vapours of the horizon or various clouds. For this is why it appears to us now glowing white, now crimson, now golden, now flame-blue, and sometimes pale, ashen or rust-coloured.

[Margin: The heat of the Sun.] III HEAT is asserted or denied of the Sun by almost the same arguments as a fiery substance is; on these, chapter 1, number 3, above. For it is not enough that heat is produced by the Sun in sublunary things for it to be called hot itself, in act and formally, and not merely in virtue and power. By what means the Sun produces heat in sublunary things, though this is not part of our plan, will be touched on below in chapter 5, in the discussion of parallelism and the Sun's illumination.

[Margin: The density of the Sun.] IV The DENSITY of the Sun, according to *Kepler*, *Astronomia Optica*, chapter 6, is so great that there is as much matter in the Sun alone as in all the rest of the ethereal air of the universe. For he says that so great a density is required by its heat-producing force, so sharp and reaching so far. That is why, he says, coals burn more than flame, and white-hot iron more than a coal: namely because of density, or a greater multitude of parts in the same space. This author adds that the water from which *Moses* indicates the heavens were made was thinned in the expanse of the aether as much as it

was condensed in the Sun, so that the body whose force had to permeate the whole aether should contain within itself as much matter as is contained in the entire aether. But this argument is very much a dialectical one [topicum]; for there is not as much matter in the heart of animals as in all the rest of the body, even though its force spreads to the whole body.

[Margin: The opacity of the Sun.] V. Some OPACITY certainly appears in the Sun, since it bounds our sight and we cannot see through it to what lies behind it, whereas we do see across the flame of our own fire, which is certainly endowed with some opacity. [Margin: But with diaphaneity.] Yet that the Sun is not wholly opaque, but, like very dense crystal, so diaphanous that our sight makes its way into the depth of the solar body and rays reach us not from the surface alone but also from the centre of the Sun, was affirmed by some of the Pythagoreans and by Empedocles. For so *Plutarch* relates, *On the Opinions of the Philosophers*, Book 2, chapter 20: *Philolaus the Pythagorean said that this disc is glassy, receiving the reflected brightness of the cosmic fire and turning the light back towards us; so that the Sun seems to be a fire in the heaven which scatters light to us as if by the reflection of a mirror; and this we call the Sun, as if the image of an image.* Plutarch goes on and says: *Empedocles makes two Suns: one, which is the primordial fire in one hemisphere of the world, placed opposite the reverberation of its own splendour; the other in the other hemisphere, which is the reflected brightness of the first, cast from the roundness of the Earth onto the crystalline Sun, with the motion of the fiery Element; and, to say it in one word, that the Sun is the reflected brightness of the fire that is around the Earth.*

But in *Astronomia Optica*, p. 223, *Kepler* teaches that the Sun was made from water condensed to the highest degree and purified, and is therefore so transparent that rays reach from its centre to the surface, and, refracted at the surface into the rarer aether, reach us from the aether; and that the interior of the Sun is thus seen by us through refracted rays. This is perhaps why the interior of the Sun looks blue to us, although its limb appears yellow: the opacity of so great a depth presents that apparent colour, as happens in the deeper parts of a crystal suffused with light. Near the horizon, moreover, it is seen, with the help of a spyglass [tubospicilli], to be like glass melted in furnaces.

[Margin: The force moving the Sun.] VI. The MOTIVE FORCE by which the Sun is carried around, whether by a motion of spin about its own centre or by one of translation around the Earth: since it does not come from an informing soul, as I consider more probable from what was said in chapter 1, number 4, it remains that it comes from something extrinsic, and very probably from some Intelligence. *Ecclesiastes*, chapter 1, seems to have indicated this Intelligence under the name of Spirit, when it says: *The Sun rises and sets, and returns to its place; and, being reborn there, it circles through the South and bends toward the North: surveying all things round about, it goes on*

Spirit. I am aware, however, that *St Jerome* and some interpreters understand something else by the name of Spirit in this passage. But this question about the motive force of the stars or the heavens is a general one, to be put off to

Book 9, on Systems and Hypotheses. Meanwhile I would have you advised that, if the Sun moves actively and from an intrinsic source, it moves for the good of the other planets and of sublunary things in such a way that it still moves for its own good: not only because it receives in turn from the planets a varied mixture of influences, but also because its own good is to exert and show its force, and to discharge the office to which God destined it, according to that line of the Psalm: *By your ordinance the Day continues, for all things serve you.* But if it is said to be moved from outside by an Intelligence, this motion will still not be perpetually violent for the Sun, because it suits the Sun's passive potency, which demands such a motion; and it does not tend to the Sun's destruction, but to making its perfection manifest without harm to it, and to spreading that perfection into so great a variety of effects. Nor do Intelligences need an animal power to move the stars, whatever *Kepler* may say in Book 4 of the *Epitome of Astronomy*, p. 508.

[Margin: The magnetic force of the Sun.] VII The MAGNETIC FORCE, of drawing the planets to itself by that part in which they are friendly and sympathetic, and of driving them away by that in which they are hostile and antipathetic, is attributed to the Sun by *Kepler*, *Astronomia Optica*, chapter 6, and in the *Commentaries on Mars*, Part 3, chapters 33 and 34. For he says that the Earth and the other planets are seized by the Sun, through its light, as if by a hand; and that while they are attracted and driven away, or while they themselves resist less or more by their agility or their inertia, they are at last turned into a circuit by the Sun, which is spun about the centre of its body; and that therefore those more distant from the Sun move more slowly, and the same planets most slowly when they are aphelion [Aphelij], that is, farthest from the Sun, and most swiftly when they are perihelion [Perihelij], or nearest the Sun. He had also indicated this in Book 4 of the *Epitome of Copernican Astronomy*, Part 3, question 5, saying: *For carrying the planets around towards the East, the Sun has in place of hands the power of its body, sent out in the straight lines of its rays into the whole extent of the world.* But this very motive force, he says, comes from the corporeal soul of the Sun, which is the former of forms in sublunary things, much as the formative faculty of the foetus is in the womb of living creatures. *Boulliau*, on the contrary, *Astronomia Philolaica*, Book 1, chapter 12, and Book 4 of his *Philolaus*, chapter 2, holds that all the planets move from an intrinsic source, because they move of necessity; an argument that is by no means necessary. *Kepler* adds, in the *Epitome*, p. 586, that these fibres pass from sign to sign by the motion that is otherwise attributed to the Sun's apogee, and he shuts the Earth within a crust. Of this fiction, elsewhere.

[Margin: The shape of the Sun.] VIII The SHAPE of the Sun appears round to us when it is raised far from the horizon; and most, with the Stoics, did indeed judge the Sun to be spherical, according to *Plutarch*, *Placita*, Book 2, chapters 20 and 22; where, however, he reports that it seemed flat to *Philolaus* and *Anaximenes*, in the manner of a disc or a circular table, and to *Heraclitus* like a bowl [Scaphij]. *Biancani*, nevertheless, *Sphaera*, Book 10, chapter 11, thinks

that all its phenomena are saved if the Sun is said to be hemispherical. But the spots, narrower at the Sun's limb and wider in its middle, give a probable argument that it is spherical and moves by a spin about its own centre. Add the nobility of the spherical shape, which suits the noblest of the stars, spreading its rays equally in every direction into the upper and lower world.

[Margin: Why the Sun appears flat?] IX Yet the Sun appears flat because of its huge distance. For according to *Euclid*, *Optics*, theorem 25, and *Ibn al-Haytham*, Book 3, proposition 25, *a sphere viewed from afar appears as a circle*, because sight cannot distinguish the greater distance of the edges from us from the lesser distance of the parts in its middle; and so we judge all its visible points to be equally distant from us and therefore to make up a flat surface. And because of the same distance we cannot certainly distinguish the angles of the stars, if perhaps they are hexagonal, or many-angled, or spherico-angular, as *Giordano Bruno* thinks they are. For not even a flame appears pyramidal when it is very distant, since according to *Ibn al-Haytham*, Book 3, propositions 25 and 59, *a many-sided figure seen from a great interval appears without angles, and circular.* [Margin: The roughness of the Sun.] Certainly the Sun, viewed through the larger telescopes, seems to have a rippled and gently uneven surface, as *Scheiner* acknowledged in the *Rosa Ursina*, p. 604, although he had earlier, in the *Disquisitiones Mathematicae*, p. 31, attributed that unevenness, seen especially when the Sun is viewed near the horizon, to vapours of the horizon raised shortly after its rising; while *Athanasius Kircher*, Book 1 of the *Ars Magna*, Part 1, chapter 1, attributes it to the eruption and whirlpools of flames flashing out perpetually from the Sun. But let the Sun at least be for us as round as the globe of the Earth, namely physically, and in such a way that a moderate unevenness of surface does its roundness no more harm than the mountains and valleys of the Earth, or the little valleys and little hills on the surface of an orange,

Diagram. A hatched oval, wider than it is tall, in the right column: the appearance of the Sun grazing the physical horizon, as the text below describes it. A dotted horizontal line AB and a dotted vertical line through C cross at the centre.

- A (left end of the horizontal dotted line): label only
- B (right end of the horizontal dotted line): label only
- C (top, at the head of the vertical dotted line): label only
- K (top edge, left of C): label only
- L (top edge, right of C): label only
- D (just right of the crossing of the dotted lines): label only
- G (lower part, beside the vertical dotted line): label only

Diagram. Directly below the first figure, a second hatched oval, nearly circular and slightly wider than tall: the appearance of the Sun a few degrees above the horizon, as the text below describes it. No lines drawn in it.

- H (centre): label only

[Margin: The elliptical Sun.] X Now, on the very horizon and near the horizon, at rising as well as at setting, the shape of the Sun appears egg-shaped

and as if elliptical; but in such a way that the higher it is raised above the horizon, the more and the sooner that ellipse is reduced to a circular shape, as can be seen in the two figures here. In the first of these, AB, is shown the appearance of the Sun grazing the physical horizon with its lower limb; in the second, H, the appearance of the Sun standing a few degrees above the horizon. This appearance [Emphasis] is not a mere mockery of the eyes, or a deception arising from the humours of the eye and from the contraction of the pupil in height and its dilation in breadth. Nor does it come from the glass of the telescope, since this phase is seen both with the naked eye and on a white tablet, or on walls facing the rising or setting Sun, and this solar ellipse [Ellipsis hæc Helliaca] is the greater the thicker the vapours of the horizon are. Nor is it in the Sun itself, since far from the horizon the Sun appears circular. It is therefore in the image, or likeness, of the Sun formed in the vapours of the horizon, and in its appearance transmitted to the eyes through refracted rays, as our own *Scheiner* learnedly teaches in the little book entitled *Sol Ellipticus*, and, following him, *Bettini*, *Apiaria* 8, *Progymnasma* 4, proposition 3. This is confirmed by the signs and properties of refraction, which in this phenomenon is greater in the part nearer the horizon, and there takes a steep and sudden increase. This phenomenon is therefore an offspring of refraction, born in the atmosphere. We shall give its causes more subtly in our book on optics; for the moment it is enough to have indicated the chief ones.

[Margin: The cause of the Sun's ellipse.] XI Since, therefore, the rays of the Sun fall from the purer air onto the denser spherical surface of the atmosphere, which is concentric with the Earth, some of them perpendicularly and without any refraction, others obliquely, though from the same point of the Sun (of which kind are those that reach our eye, placed outside the centre of this atmosphere), and more obliquely those fall that flow from the lower point of the Sun, say G, than those from the upper, say C. They are therefore refracted towards the perpendicular, in the surface of a vertical plane, say CG, as the common law of refractions requires; and the ray, inclined towards the centre of the atmosphere and of the Earth, is carried higher towards the sky, and also shows us the part of the Sun it flowed from in a higher position. And the refraction upward, whether in the circle of altitude or in the vertical CG, is the greater the more oblique the ray's incidence on the surface of the atmosphere. Therefore the lower limb of the Sun, G, appears higher than the upper, C, and so the diameter of altitude, which is CG, is contracted. But at points A and B this refraction takes place according to altitude in such a way that another slight refraction does indeed follow from it, by which the distance AB is diminished; but this is insensible, and so the Sun's diameter in the transverse length AB is not sensibly contracted. Since, then, the diameter CG is contracted and the diameter AB is not sensibly contracted, the Sun appears to us as an oblong spheroid, and as an ellipse or ellipsoid ACBG. Again, because point G is refracted upward more than point D, and D more than C, the semidiameter of altitude GD comes to appear more contracted than the semidiameter DC, and

this ellipse appears blunter at G than at C. And what has been said of point C relative to G is to be understood proportionally of any point of the Sun relative to lower points, say of points K and L: namely, that they are refracted upward less than the lower points, those nearer the horizon. But if it were refracted equally, the Sun would appear round to us; or if the refractions, as it rises from the horizon, were of the same measure, it would look elliptical to us all day. But because they diminish, this ellipse at length becomes smaller, as in H, and finally vanishes into a circular disc. When the Sun is about to set, however, it is seen first as in H, and while it sets a larger and more oblong ellipse appears, as in ADB. You will see something similar in the flame of a candle if you tilt a curved glass lens placed between it and your eye, so that the lower rays strike it more obliquely than the upper ones.

SCHOLIA.

[Margin: The first observer of the elliptical Sun.] *I Since the elliptical Sun is seen every day at rising and at setting, when the sky at the horizon is clear, it is astonishing that the older writers on optics either did not notice this phenomenon or wrote nothing about it; indeed, Ibn al-Haytham, Book 7, proposition 55, and Witelo, Book 10, proposition 54, declared universally that the stars appear round on the horizon. Yet Strabo, Book 3, seems to have indicated it, when he reports from the opinion of Posidonius:* because of moist vapours the broken line of sight takes in broader images of the Sun and the Moon when rising or setting. *But to Scheiner, to whom, as the prerogative of observing the solar spots, so too the task of bringing this elliptical falling-short of a perfect circle to measurement through observations had been reserved by divine gift.*

[Margin: The difference between the diameters of the elliptical Sun.] *II Since, from Tycho,* Progymnasmata, *Book 1, chapter 1, and his table, which we shall give at the end of the work, the Sun's refraction at the horizon is 34′, such will be the refraction of point G. But when the Sun is half a degree high, that is 30′, which is very nearly the Sun's apparent diameter at apogee, at noon, and so when the whole Sun stands clear of the horizon and G touches the horizon, then, I say, its refraction by Tycho's reckoning is 29′. Therefore the limb G is raised 5′ more than the limb C, and the difference of the diameter GC at the horizon from the diameter of the Sun placed far from the horizon can be at most 5′ or 6′. But the diameter of the transverse length ADB, because of the refraction that diminishes distances, does not reach 1″. For Kepler, in* Astronomia Optica, *p. 130, and we in our book on optics have demonstrated that if two stars on the horizon, or two bright points A and B, were 90° apart, their distance would not be diminished by the force of refraction in altitude by a full 18″; if they are 45° apart, it is not diminished by 10″; and finally, if they are 5° apart, it is not diminished by one whole second. Now the extremes of the Sun, A and B, are not 5° apart, but only half a degree. What wonder, then, if it appears sensibly contracted along the diameter of altitude CG, but not along the length AB? But what if, because of another kind and cause of refraction, the Sun's diameter AB appears greater on the*

horizon than at noon, while the diameter CG is not equally greater? On this part of the phenomenon, however, more when we treat of the apparent diameters of the Luminaries, especially in our book on optics.

[Margin: When and where the solar ellipse vanishes.] *III Although the Sun and the Moon undergo sensible refraction up to 45° of altitude from the horizon, the difference of refraction of the points G and C becomes insensible around an altitude of 18° or 20° according to Scheiner, or at least 25° by Tycho's reckoning. For the difference between the Sun's two diameters lasts somewhat longer from the Tychonic table than from the observations of Scheiner and from ours; for we too have often measured both diameters of this ellipse on a white tablet. But we have weighed both his observations and ours, and will show them to you in the book on optics, volume 2 of the* New Almagest. [Margin: The affinity of the solar ellipse with twilight.] *Meanwhile the Sun's morning ellipse seems to fail at the same altitude as the depth at which evening twilight fails, and its evening ellipse to begin at the same altitude as the depth of the Sun below the horizon at which morning twilight begins, namely around 18° or 20° in both cases. But in the parallel sphere, since the Sun circles the horizon more closely for whole continuous days, it must be seen as elliptical for many weeks and months on end.*

CAPVT III.

De Solis Maculis, & Faculis. [On the Spots and Faculae of the Sun.]

I For 55 centuries and more the Sun had rolled on for the world, with the fame of its whiteness and of its ever undefiled splendour intact. For the eclipses imputed to the Sun are eclipses not of the Sun, which is not deprived of light, but of our eyes, which are cheated of the Sun's light by the interposition of the Moon. But what the living eye of mortals in its simplicity had not suspected, it dared to detect, with its eye doubled, so to speak, and with the fragile transparency of glass lenses, dead ones though they are, taken on within a tube: blemishes in the face of light, *Spots in the Sun*. (For that is what those celestial corpuscles near and around the Sun are called, blackish on the part of them visible to us.) It detected also certain small areas brighter than the rest of the Sun's surface, and flame-vomiting, which they have called *Faculae*; of this kind, in the following figure, are the Spots F, G, D and the Faculae K. For semi-transparent corpuscles, which transmit some of the Sun's light, they have called Shadows rather than spots; although, from our Society, *Carolus Malapertius* [Charles Malapert] has called all these corpuscles the Austrian Stars.

[Margin: Who was the first discoverer of these spots, Scheiner or others?] And in the past, indeed, *Adelmus* noted one spot in the Sun and *Auen Rodan* [?] two, at different times, but they took them for Mercury, as I shall say in Scholium 2. But after these, the first who noticed them and distinguished them from every other known star was Father *Christoph Scheiner* of the Society of Jesus. In 1611 May, at Ingolstadt, on the occasion of investigating the Sun's apparent diameter with the help of the telescope, he noticed these corpuscles

and pointed them out in person to Fathers *Iacobus Gretserus* [Jacob Gretser] and *Adamus Tannerus* [Adam Tanner], and by letter to *Angelo Lynceo* [?] and *M. Velsero* [Markus Welser], as he himself shows in the *Rosa Ursina*, from chapter 1 to chapter 6. But because Father *Theodorus Busaeus* [Theodor Busaeus], then Provincial, did not want the novelty of this phenomenon to be published by him at once, his name had to be suppressed for the time being; and the three letters to Markus Welser on this subject bore in front of them the feigned name of a painter, namely *Apelles behind the picture*, secretly listening to the judgements of others, so that the man who had rather timidly painted for us the genuine face of Apollo, that is, one stained or dark in places, might be called Apelles and yet stay hidden. Welser's letters, spread into Germany and Italy, roused *Galileo* above all to observe these spots and to write back to Welser his judgement on them, weighing in as many letters the sayings of Apelles behind the picture. Nor indeed does any observation of these spots older than 1612 survive in Galileo, although *Kepler* in his *Ephemerides* affirms that they were observed in 1611 by the son of *David Fabricius*. [Margin: The praise of Scheiner.] This at least is certainly agreed: that *Scheiner* once again these spots and the solar faculae observed at Rome for several years, in the presence at times of very learned and very distinguished men, who visited this Apelles, no longer in hiding, and marvelled at the Sun painting on the cleanest paper by means of its own ray and of the images [speciebus] conveyed through the telescope. Father *Nicolao Zucchio* [Niccolò Zucchi] urged him to repeat his [Greek: τηρήσεις (tērēseis, "observations")], and the bitterness of the objections sometimes drove the good old man to tears; yet he carried this through with such diligence, shrewdness and circumspection that he bore with the Sun through more than two thousand observations, and handed down to posterity the position, place, shapes, sizes, risings, perishings, revolutions and periods of the spots so splendidly that, by making better observations a thing to despair of, he has conquered others, himself and the Sun. Nor did *Descartes*, in part 3, number 35, of the Principles of his new Philosophy, undeservedly pronounce: *nothing more diligent could be wished for in this field*. [Margin: Other observers of the spots, or writers on them.] Granted that the same spots have been observed and expounded to us by *Galileo* in his book on the Sun's spots; by *Malapert* in his *Austrian Stars*; by *Biancani* in the *Sphaera*, Book 10, chapter 21; by *Chiaramonti* in the *Anti-Tycho*, part 4, chapter 22; and by many others, of whom in number 9. And granted that certain things about these spots are touched on by *Tanner*, *De caelo*, part 1, disputation 6, question 3; *Martinus Hortensius* in the Dissertation with Gassendi on Mercury seen under the Sun; *Kepler* in the *Ephemerides* from p. 17 to p. 20, and after the *Sidereus Nuncius*; *Io. Cottunius* [Ioannis Kottounios], Book 1 of the *Meteorologica*, lecture 31; *Auersa* [Raffaele Aversa], question 34, section 9; *Mastrius* [Bartolomeo Mastri], *De caelo*, disputation 2, question 2, article 3, from number 91; *Argoli* in the *Pandosion Sphaericum*, chapter 44; *Boulliau*, Book 1 of the *Astronomia Philolaica*, chapter 8; and *Šírek* in the *Radius Sidereomysticus*, Book 4, chapter 2, from member 4 to 7.

From these, and from some observations of our own made long ago at Parma with Biancani and others, we have thought it right to select these few points.

[Margin: Method of observing the Sun's spots.] II The Sun's spots are observed either by direct sight, but with the Sun shaded by thin vapours, clouds, silk veils of rather open weave, or coloured glasses; or by reflection, from a plane mirror that throws the Sun's image on to paper about 20 feet away; or by receiving the Sun's image, through a tiny hole or through the lenses of a telescope, on very clean paper parallel to the hole or lenses and more than 4 palms from them, with the room moderately darkened all round. The best part of the day for this is the one in which the Sun stands at the 90th degree of the ecliptic counted from the horizon. If you use only two lenses and do not add a third to erect the images, they will appear in an inverted position: those on the right on the left part of the paper, the upper ones in the lower part, and so on. [Margin: The reality of this phenomenon.] Now since the spots have been observed by all these methods at the same hour of the same day, and found in the same position and with the same shape and size, it is clear that they are not bubbles or knots in the glass, nor fleeting little clouds of the air, nor mere tricks of the eyes, but a real phenomenon, and one that persists for some days.

Diagram. In the left column: a circle representing the Sun's disc, crossed by two parallel horizontal lines, an upper one between F and D and a lower one, the diameter, between A and C. Dark irregular spots lie scattered over the disc, and hatched spots lie on the limb at F and D; four star-shaped, flame-like figures stand two above and two below the lower line, each beside a K.

- K (upper left, beside a star-shaped figure)
- G (upper centre, above a large dark spot on the upper line)
- K (upper right, beside a star-shaped figure)
- F (outside the circle, left, at the upper line)
- D (outside the circle, right, at the upper line)
- A (outside the circle, left, at the lower line)
- C (outside the circle, right, at the lower line)
- B (centre, just below the lower line)
- K (lower right of centre, beside a star-shaped figure)
- K (lower left, below a star-shaped figure)
- M (lower centre)
- M (lower right of centre)

[Margin: Colours of the spots.] III The colour of the spots, such as D, F, G, M, is black or dark blue; that of the shadows, blackish; that of the faculae K, K, a brilliant white or yellow. The faculae seem to be either brighter small areas of the Sun, or little flames suddenly breathed out from it, or, if you listen to Šírek [Rheitensem], the Sun's light thrown back from the more remote spots on to the spots nearer the Sun. Many of the spots look blacker near the Sun's limbs, if they merge with others as at D and F, than in the middle at G, where, being semi-transparent, they are faintly lit by the Sun; though not so much that they are not darker than the Moon's usual spots and than some clouds,

since in the solar eclipse of 1612 May 29 they looked blacker than the parts of the Moon penetrated by the Sun, and are seen here and there through the rarer clouds themselves. So, at least, *Scheiner*. *Galileo*, however, maintains that near the limbs they are less black, because the segments of them lit by the Sun are uncovered to us and their backs are seen confusedly and at a narrower angle, although the obliquity of the line of sight at which they are seen near the limbs could make them look darker to us, in the way that panes of glass seen with their surface tilted look darker than when they are viewed straight on. He adds that the spots, on the side on which the Sun lights them, are brighter than the brighter parts of the Moon at full Moon, because the spots do not appear less clear and bright than the field around the Sun; yet in that field Venus is not seen, because of the excessive glare of the neighbouring Sun, nor would the bright parts of the Moon themselves be seen. Therefore, if the spots are brighter than the Moon and Venus, how much more so than the darker parts of the Moon! But I flatly deny that the spots are as bright as the atmosphere, or field of aether, immediately surrounding the Sun.

[Margin: Shape of the spots.] IV The *shape* of the spots usually appears irregular, and very unstable in one and the same spot, so that hardly any appears on the following day such as it was seen the day before. Near the limbs, as at D and F, they contract and become oblong; but near the middle of the Sun, as at G or M, they appear broader but less long, other things being equal. And again, near the limbs, several spots that had appeared scattered and separated from one another in the middle of the Sun's disc seem to come together into nearly one spot. This is a strong argument either that the spots move around the Sun's globe, or that the Sun turns by rotation about its own centre and carries such corpuscles around with it, and that therefore, near the Sun's edges, they grow thin individually but become closer to the others in appearance.

[Margin: Number of the spots.] V The *number* of the spots is variable and uncertain. At times, however, 50, and at times 33, have been distinctly counted at the same moment; but sometimes one or another, and sometimes none, and then the weather was, other things being equal, hotter and drier. Thus in 1618, when the Beam and the Comet shone, no spot was observed, says *Argoli* in the *Pandosion Sphaericum*, chapter 44. Nor in 1632, from July 12 or 19 to September 15, a time of remarkable drought, could [poſuit] [recte: potuit] any spot be detected on the Sun, at Rome by *Grienberger*, at Mazzorbo [Mazorbi] by *Argoli*, or at Cesena or Pisa by others, as *Chiaramonti* and *Argoli*, cited above, attest. Moreover *Fortunius Licetus* [Fortunio Liceti], in Book 6 of *De novis astris et cometis*, says that it was often observed at Padua by *Vincentio Fridiano* [?] that in clear and rather dry weather no spots of the Sun, or fewer, appear; and *Šírek* [Antonius Maria Reithensis] says the same, Book 4, chapter 2, member 9, of the year 1642, adding that in June it was cold because of the multitude of the spots. [Margin: Hence the uncertainty of astrology.] Let the astrologers go now and, since these contributory causes [concausis] of meteorological changes were unknown before this time, acknowledge the uncertainty of their predictions.

[Margin: Size of the spots.] VI The apparent *size* of the spots is likewise variable and unstable. *Scheiner*, in *Apelles behind the picture*, p. 14, sets Venus equal to the middling spots and, as he himself judges of Venus, at 3′; and, p. 25, he sets the largest of the spots, observed at the beginning of January 1612, equal to Venus and greater than Mercury. In my observation notes I have met with none larger than a tenth part of the Sun's apparent diameter. *Biancani*, however, says that a certain spot occupied a fifth part of the Sun's diameter and was therefore equal to the whole Earth, since the Earth's diameter is a 5½th part of the Sun's diameter. *Galileo*, for his part, likens the rather large spots to Asia and Africa together with the Mediterranean. *Chiaramonti* attacks this in the *Anti-Tycho*, chapter 23, denying that, even if a spot were as large as the Earth, it could, placed near the Sun, cast a shadow that would reach our eyes, or indeed one whose apex would descend below the heaven of the Sun. That is true of a shadow simply so called, but not of a penumbra, which, while it lacks some direct rays, is still lit by certain other rays. *Scheiner*, again, in the *Rosa Ursina*, acknowledges some spots to be to the Sun's diameter as 10, as 50, as 100 to 2000; some to be larger than Africa, not a few larger than the Moon, and certain compound ones equal to the entire surface of the Earth.

[Margin: Duration of the spots.] VII The *duration* of the spots seems less constant than that of certain small areas and faculae. Some, arising suddenly in the middle of the Sun's disc, and others arising elsewhere, vanish shortly afterwards; some last 1, 2, 3, 10, 15, 20, 30 days; very few, 40. Some return the same after a period of about 27 days. Under the Sun, moreover, these linger longer on the diameter ABC itself than far from the centre, for example in the paral- lel FGD. As for rising and perishing, not only apparent and optical but also real and physical, they are granted to some spots by *Scheiner*, *Galileo* and *Biancani*.

[Margin: In which direction do the spots move?] VIII. The apparent motion of the spots, or the rotation of the Sun carrying the spots around, is like that of Mercury and Venus around the Sun. In the upper part of the epicycle, on the arc of direct motion, in which they linger longer, these planets move from west towards east, but in the lower part towards the west. That is why *Galileo* says, absolutely, that the spots, which move so as to complete a period in 27 or 28 days and return, move towards the east; whereas *Scheiner*, attending to the motion visible to us, had said that they move towards the west, since they are then below the Sun. For in the morning, as the Sun rises, the spots ascend from the lower parts of the Sun, but in the evening they descend to the lower parts of the Sun. [Margin: And when the motion is rectilinear, when curvilinear.] Now those that appear in the winter season travel from the southern rising to the northern setting, and those in the summer from the northern rising to the southern setting. Their paths are similar, and pass from rectilinear into curvilinear. For from Sagittarius into Taurus inclusive, with the lower pole of the solar axis turned towards the Earth, the path of the spots appears curved upwards on the Sun's disc; but from Gemini into Scorpio, with the same pole

turning away from the Earth, their path appears curved downwards. At the intermediate times, about the beginning of December and of June, it looks rectilinear. [Margin: The apparent inequality of the motion.] This motion is uniform in itself but apparently unequal: it looks slower near the Sun's limbs than far from them, and is fastest in the middle of the solar disc, exactly as the motion of corpuscles adhering to a sphere turning uniformly about its own centre requires. This rather confirms that the Sun's body is spherical and carries the spots around by its rotation. From this inequality around the Sun it comes about that, although no spots have properly been seen ever to go backwards, a certain direct motion, a certain retrograde motion with respect to the Sun's centre, and a certain station can nevertheless be granted to them. [Margin: The speed of the spots.] *Scheiner* adds that the speed of the spots that remain under the Sun's disc for 14½ or 15 days is so great that in one natural day they cover 5693 miles on a great circle of the Sun; but this depends on the distance and size of the Sun that he himself assumes, which is not the same among all authors. [Margin: The Sun's axis is mobile.] The whole motion, of 27 days or a little more, he calls monthly, and he says that it takes place around an axis of the Sun that is not immobile but mobile with a slow, annual motion. By this motion the Sun's axis is inclined and its poles move in such a way that the southern pole of the axis around which the monthly motion takes place rises for us about the beginning of December, and the northern then sets, whereas about the beginning of June the southern is hidden and the northern rises; and from this inclination of the axis of the solar body to the ecliptic arises the curved path of the spots. But many matters of subtler inquiry about the motion of these spots, and about the poles and the axis on which that motion takes place, we deliberately reserve for chapter 12 of the ninth book and of its fourth Section. For from this motion *Galileo* draws one of his strongest arguments for the annual motion of the Earth, to be set out in that Section, and so we shall digress on this motion there at the proper time.

[Margin: The distance of the spots from the Earth.] IX. The *place* of the spots, or their *distance from the Earth*, is commonly judged to be above Mercury at perigee, and there is now no one who does not grant that they are either on the Sun's surface or as if contiguous with it, because no parallax from the Sun has been observed in them, let alone one smaller than Mercury's. Rather, on the same day the same spots have been observed in the same part of the Sun, with a motion uniformly proportional to the motion of the Sun and far slower than the motion of Mercury, however remote from one another on the Earth the observers were, as appears from the figures and from the history of the observations. [Margin: The agreement of the observers.] Thus, indeed, they were observed at Ingolstadt by *Scheiner* and *Cysatus* [Johann Baptist Cysat]; at Padua and Florence by *Vincentius Fridianus* [?] and *Galileo*; at Rome, again, by *Scheiner*, *Grienberger*, *Paulus Guldinus* [Paul Guldin] and *Niccolò Zucchi*; at Parma by *Biancani*; at Douai by *Malapert*; in the Low Countries by *Martinus Hortensius*; at Brussels by *Daniel Antoninus* [Daniele Antonini]; at Coimbra by

Gulielmus Velij [?]; in the East Indies by *Gaspar Ruess*. Notwithstanding this, *Chiaramonti* and *Kottounios*, both devoted to the Peripatos, the latter to his own Stagira, maintain that all the spots are below the heaven of the Moon, so as not to admit alterations in the heaven. [Margin: Chiaramonti's weak argument.] *Chiaramonti*, indeed, adduces a few observations of the year 1626, between August 25 and 29, in which some spots were noted by different observers in different parts of the Sun within a few hours, and from this he argues for a large parallax. But those observations ought not to outweigh so many others that are more certain, and those spots were not of the kind that last the whole day but of the kind that rise and perish shortly afterwards, others rising in another part of the solar disc; and accordingly the different place on the Sun did not belong to the same spot. [Margin: And Kottounios's against the place of the spots in the heaven.] *Kottounios*, for his part, heaps together spurious spots, of which I shall speak in Scholium 1, compacted from little globes of cloud; for he says that these are meteorological impressions of the air, and that therefore in moister seasons more and larger spots appear on the Sun. But if the Sun's genuine spots, when more numerous or larger, are not the effect but the cause of colder and moister weather, because they intercept the Sun's rays, Kottounios's argument will turn out frigid.

[Margin: Opinions on the substance of the spots.] X. The *substance* of the spots, then, if they are indeed above the Moon and very near the Sun, seems celestial rather than elementary, and offers vast material for disputing about novelties in the heaven, on which we say more in Books 8 and 9. Here, therefore, I think it enough to indicate the chief opinions. *Raphaël Auersa* thinks that, by a power of celestial causes hidden from us, some parts of the heaven are made more opaque than others, and that the heaven therefore appears sprinkled with spots. But *Malapert*, *Mastrius* and *Šírek* judge that they are perpetual stars lit by the Sun, opaque satellites of the Sun surrounding it, and of irregular shape because of their varied concurrence and their motion through different epicycles; for it is less awkward, they say, to posit this irregularity in the heaven than rising and perishing. The faculae, says *Šírek* [Rheitēſis], are the Sun's light reflected on to the spots nearer the Sun from the more remote spots. By contrast, *Galileo*, *Kepler*, *Boulliau*, and at length, though once of the contrary opinion, *Scheiner* and *Biancani* deny that they are stars, because they are neither round nor of regular shape, and are very like the most inconstant clouds; and because many of them, arising in the very middle of the Sun, vanish before they have reached the edges, or, appearing suddenly in the middle, are extinguished at once. Hence they are generated anew and perish, and are nothing other than soots or vapours boiling up out of the furnace of the solar globe, and some of them, kindled, flash in the manner of faculae. So these men make a Vulcan out of Phoebus. But how do so many remain so long that they move around the Sun, by their own motion or the Sun's, for about 27 days? Is this the nature of soots and of solar little clouds, lasting out from a more compact matter that has

grown upon the Sun? But I would rather withdraw into the shade of silence than speak of the Sun in things unascertained and unworthy of it.

SCHOLIA.

[Margin: A spurious spot on the Sun, and darkness.] I. *It is known from the historians that spurious spots have more than once appeared under the Sun, from globules of denser vapours and clouds. Hence* Cleomedes, *Book 2 of the* Cyclica Theoria, *says:* The Sun very often appears spotted to us, and around it there are as it were cloudy coils, which are nevertheless almost infinite stadia away from it, and yet seem to us to be around it. *But these are of the class of meteors, and a large parallax could be noted in them by different observers at the same time. Hence that passage on which* Plutarch, *in the* Caesar: Now the blinding of the Sun's radiance: for through that whole year its orb came up pale and without brilliance, and sent out a weak and thin heat. *To this, surely,* Virgil *looked back,* Georgics 2, *hinting at a long and spurious half-eclipse of the Sun in these verses:*

He also pitied Rome when Caesar was put out, When he covered his shining head with dark rust, And the impious ages feared an eternal night.

Zonaras also relates, in the Irene, *that because of these clouds gathered beneath it the Sun did not shine for 17 days; and that in* Justinian's *time, for the greater part of the year, though the sky was clear, the Sun lay hidden beneath them, scarcely rivalling the light of the Moon, as* Patritius [Francesco Patrizi] *reports, Book 19 of the* Pancosmia. *But see* Maiolo, *in the* Dies Caniculares, *in the conversation on the Sun. Some of these portents, however, could be referred to an extraordinary accumulation of genuine spots contiguous with the Sun; but not the blood-red colours on the Sun, of which* Génébrard *speaks in the* Chronicle, *years 1547, 1585 and 1592, and others elsewhere.*

[Margin: A spot taken for Mercury.] II. *A certain spot of the Sun was at one time believed to be Mercury. For in the* Annales Francici *published by* P. Pithœo [Pierre Pithou], Adelmus *the Benedictine, in the Life of* Charlemagne, *in the year 807.*

of CHRIST *says:* The star of Mercury, on March 17 [16.Kal. Aprilis], was seen on the Sun as a small black spot, though a little above the middle centre of that star; it was seen by us in this way for eight days, but when it first entered or left could not be noted, because clouds prevented it. *Kepler, in chapter 8 of the* Astronomia Optica, *p. 306, had altered these words, putting in place of "eight days" the adverb "eight times", and in place of the year 807 the year 808. For he believed that in 808, on March 17, Mercury was conjoined with the Sun at perigee, because of its small latitude, supposing the nodes of Mercury's orbit with the ecliptic to be at the end of Aries and of Libra; and yet that it could not have lingered under the Sun for eight whole days, since Mercury is so swift at perigee that it can scarcely be under the Sun for 7 hours. Two years later, at the end of the* Dissertatio cum Nuncio Sidereo *of* Galileo, *he confirmed this view from* Amoino [Aimoin], Bede, Eginardo [Einhard], Vincentio Bellouacensi [Vincent of Beauvais], Polydoro

Virgilio [Polidoro Virgili] *and* Sabellico [Marcantonio Sabellico], *who speak of a spot seen under the Sun in the time of* Charlemagne. [Margin: Kepler's candid palinode.] *But when, in 1612, he had been informed about the solar spots, he denied that what Adelmus speaks of was Mercury, and asserted that it was a spot, just like the one that* Kepler *himself had seen on the Sun in 1607 May 28, thinking it to be Mercury. He also acknowledged that Mercury's nodes are at 13° of Taurus, not at the end of Aries, as is plain from his letters to* Galileo *and from the* Ephemerides, *pp. 17 and 18. For Mercury's latitude on that day, May 28, had been greater than the Sun's semidiameter. Moreover* Galileo, *On the Sun's Spots, p. 54;* Scheiner *in the* Rosa Ursina; Gassendi *in the Life of* Claudij Fabricij [Nicolas-Claude Fabri de Peiresc]; *and* Boulliau, *Book 1 of the* Astronomia Philolaica, *chapter 8, deny that Mercury could have been seen under the Sun for 8 days, since it can scarcely remain under it for 7 hours, and sometimes gets clear of the Sun's disc in 4 hours. To this class, moreover, seem to belong those two spots that their observer* Auen Rodan [?] *took for Mercury, as* Picus Mirandulanus [Giovanni Pico della Mirandola] *reports,* Against the Astrologers; *or* Averroes, *in* Copernicus, *Book 1, chapter 10, who says that at the time of a conjunction of Mercury with the Sun he saw something blackish on the Sun.*

[Margin: Mercury could be taken for a spot by the inexperienced.] III. *Yet the latitude of Mercury at perigee is sometimes so small that it is less than the Sun's semidiameter, and Mercury can therefore be seen under the Sun, whatever may be said by* Ptolemy *and* Copernicus, *and with them* Fernel, *Book 1 of the* Cosmotheoria [Coſmoth.], *chapter 7, in the Scholia, who deny that it and Venus can, because of their excessive latitude, be seen under the Sun at conjunctions, or be interposed between the Sun and our sight.* Kepler *had predicted that this phenomenon would occur in 1631 on November 7, but, mistrusting the calculation of the Rudolphine Tables, he had rather timidly advised astronomers to direct their eyes to the heaven at that time.* [Margin: Pierre Gassendi's good fortune in observing Mercury under the Sun.] *Accordingly* Pierre Gassendi, *in that year 1631, on November 7, at Paris, with a telescope made ready, observed Mercury in the morning. It painted itself on the paper with its own small shade, showing an apparent diameter of not more than 23″ or 20″, whereas it was previously believed to be 2′ or 3′. It left the western edge of the Sun, he says, at* $10^h\ 28^m$ *after midnight, and reached a northern latitude of 6′ 20″, as* Gassendi *himself reports in the* Institutio Astronomica, *Book 2, chapter 14. He had not observed the beginning, because it had begun with the Sun still below the horizon; but the beginning of the observation was at* 9^h *after midnight, and* Gassendi *gathered from the Rudolphine Tables that the true conjunction of Mercury with the Sun was at Paris at* $7^h\ 58^m$ *after midnight, with the Sun placed at Scorpio 14° 36′ and Mercury's northern latitude not exceeding 4′ 30″. So relates his very close friend* Boulliau, *Book 10 of the* Astronomia Philolaica, *chapter 5, who also deservedly dedicated the whole of that Book 10 to* Gassendi, *as one who had deserved so well of Mercury by this and other observations.* [Margin: Boulliau's vote for Gassendi.] Boulliau, *indeed, adds from his own tables that in that synod Mercury was at 14° 37′ 42″, and that when it left the Sun it had a latitude of 6′ 20″; and in chapter 7 he gathers*

that Mercury's northern node was at Scorpio 13° 20′ 52″. [Margin: And that of Martinus Hortensius.] *There is also extant a most learned dissertation of* Martinus Hortensius *with* Gassendi, *published at Leiden in 1633, in which he confirms from its motion that this Gassendian phenomenon was Mercury, not a spot; since there is no spot at such a distance from the Sun's centre that travels in one day beyond a thirteenth part of the solar diameter, and those that stay around the Sun's limbs scarcely traverse in one day a 26th or 30th part of the solar diameter, as those practised in observations of the solar spots know; whereas this corpuscle was under the Sun for very few hours.* Hortensius, *however, having first corrected the polar altitude of Paris, says that the conjunction of Mercury with the Sun took place at Scorpio 14° 34′ 30″, with a northern latitude of ☿ of 5′, and indeed at 7^h 43^m after midnight; that the beginning of the ingress was at 4^h 58^m, the end at 10^h 28^m, and Mercury's stay under the Sun 5^h 30^m; and finally that the northern node was at Scorpio 14° 54′ 43″. But, he says, from the Rudolphine Tables this conjunction ought to have been at Paris at 0^h 47^m after noon on November 7, and so those tables differ from the observation by 5^h 4^m, by 14′ 24″ in the motion of longitude, and by 35″ in latitude;* [Margin: Comparison of the Rudolphine Tables with other tables in the motion of ☿.] *yet they are closer to the truth than the Ptolemaic tables, which differ from this observation by 4° 25′, the Copernican by 5°, the Danish by 7° 13′ and the Lansbergian by 1° 21′.*

CAPVT IV.

De Motu Vertiginis Solis, occasione Macularum: Vbi & de Vertigine aliorum Planetarum, & de Tremore Solis. [On the Motion of Rotation of the Sun, Occasioned by the Spots: Where Also on the Rotation of the Other Planets, and on the Tremor of the Sun.]

[Margin: Those who assert the Sun's rotation.] I The rotation [Vertiginem] of the Sun about the centre of its body, indicated by *Plato* in the *Timaeus*, has at last been recognised by our own and the preceding century, and it has been asserted above all by *Kepler* in the *Commentaries on Mars*, part 3, chapter 34; *Scheiner* in the *Rosa Ursina*; *Galileo*, *On the Sun's Spots*, pp. 32 and 49; *Boulliau*, Book 1 of the *Astronomia Philolaica*, chapter 12; and *Šírek*, Book 4 of the *Radius Sidereomysticus*, chapter 2, member 3. Now *Kepler* grants that this rotation takes 24 hours, if the Earth's daily motion is supposed to come from the magnetic force of the Sun, which drives it around by the Sun's own rotation; but that it takes 3 days, if the same proportion is supposed between the periodic motion of Mercury, which is of 88 days, and the motion of the Sun, as holds between the semidiameter of Mercury's orbit and the diameter of the solar body, which is nearly thirtyfold. This, however, is playing in the heaven without any foundation in phenomena appearing on Earth.

II. *Galileo*, *Scheiner* and *Boulliau*, for their part, hold that this rotation is nearly monthly, that is, of 27, 28 or 29 days, and they gather this from the apparent motion of the spots, which, unless they perish sooner, move under the Sun's disc from one edge to the other in 13½, 14 or 14½ days, lie hidden for as many

days behind the Sun's back, and, having completed the period of their circuit around the Sun, return. [Margin: The ground for asserting this rotation.] This motion appears just such as the rotation of the Sun's sphere about its centre demands, if the spots are supposed to move with its motion. For it appears faster in the middle than near the limbs, and greater on the Sun's diameter than on parallels to the diameter. Furthermore, a motion so constant and regular in so many spots is more fittingly attributed to one body carrying them around than either to a fluid medium around the Sun or to so many epicycles, multiplied in vain, on which each of them would be carried around the Sun. Granted, *Galileo* adds that if they did move in this way, the result would still be that because of their nearness they would communicate this motion to the Sun, which is spherical and resists nothing, and is indeed inclined to circular motion. *Gassendi* judged that the great circle of the solar body, along which, and along whose parallels, this rotation takes place, is so inclined to the ecliptic that the apparent motion of the spots seems to describe straight lines at the equinoxes and curved lines away from the equinoxes. [Margin: Disagreement between Scheiner and Galileo, and Šírek.] *Scheiner*, on the contrary, says that they describe straight lines near the solstices and curved lines at the equinoxes and elsewhere, and that this arises from a certain other annual motion besides the monthly one, by which the poles of the monthly motion move gradually with their axis. *Šírek* [Rheitensis], for his part, denies the Sun a monthly motion of rotation and assigns it to the spots, as being stars, he says, distinct from the Sun. But he grants the Sun an annual rotation of 365 days, from which it comes about that in summer it heats more also for this reason, that it turns its brighter and hotter part towards us.

[Margin: The rotation of the other planets.] III. On this occasion *Šírek* [Rheitensis], in the place cited above, adds that the other planets, the Moon excepted, move with a motion of rotation about the centre of their bodies, but that the Moon slowly oscil- lates; and that, just as the Sun rises 365 times within the annual revolution of its rotation, which is completed in $365^{d}\ 5^{h}\ 48^{m}$, so the other planets are moved 365 times within one revolution of their own periodic motion. And accordingly that a rotation, Jupiter's for example, is obtained by making the proportion: as 365 is to 24^{h}, so 4330^{d}, in which Jupiter's motion of nearly 12 years is completed, is to $284¾^{h}$, or $11^{d}\ 20¾^{h}$. On this foundation, so arbitrary, not to say flimsy, he has built for us the following little table. So easy is it,

Rotation of the Planets, from Šírek [*Vertigo Planetarum, ex Rheitenſi.*]

Planet	Rotation
♄	$29^{d}\ 12^{h}\ 38^{m}$
♃	$11^{d}\ 20^{h}\ 45^{m}$
♂	$1^{d}\ 21^{h}\ 6^{m}$
☉	$365^{d}\ 5^{h}\ 48^{m}$
♀	$0^{d}\ 14^{h}\ 45^{m}$

Planet	Rotation
☿	$0^{d}\ 5^{h}\ 45^{m}$

for those whose wit itches to devise novelties, to invent motions in the heavens from the tiniest spark of analogy. That is a giddiness [vertigo] or an itch of the mind rather than of the stars. What we ourselves have observed in the belts of Jupiter that bears on the rotation of Jupiter we shall say when we treat of Jupiter. The oscillations of the Moon, from Wendelen, and its twin libration, from van Langren and Boulliau, will be dealt with in Book 4.

[Margin: The rotation of the Moon is uncertain.] As for the rotation of the Moon, Gassendi has no doubt about it in his *Institutio Astronomica*, book 2, where he treats of the phases of the Moon. For he says that the Moon always turns the same spots toward the Earth, and yet that according to all authors it moves on an epicycle about the centre of the epicycle carried by the eccentric. On that supposition the front part at apogee ought to become the rear part at perigee; but, he says, these things are reconciled if one posits a motion of rotation by which as much of the lunar globe is turned back as is turned away by the motion of the epicycle. But not everyone uses an epicycle in the theory of the Moon. Magini in particular, in the preface to his *Theorica* of the Moon, professes to preserve this constant position of the spots toward us by an eccentric alone, whose centre, however, is carried round by the circumference of a small circle, as we shall explain in Book 4, chapter 9. Meanwhile, just as the monthly variation of the solar spots seems to indicate a solar rotation, so the monthly invariability of the lunar spots seems to indicate the Moon's immunity from any motion of rotation.

IV. That trembling of the Sun, more evident when it is rising than when it is setting, and indeed more evident shortly after its rising, and a certain jumping, with a torn and as it were serrated limb, appear as they do because of vapours heated by the Sun and running across it with an irregular upward ascent. So Bettini teaches, in *Apiary* 8, Prog. 4, prop. 4, together with Scheiner in the *Disquisitiones Mathematicae*, nos. 30 and 34, and before them Tycho, vol. 1, p. 403. For from this comes also a large part of the cause of the twinkling of the stars; and Witelo, book 10, prop. 55, thought no differently. I myself have often seen a long little cloud of narrow breadth lying across the middle of the Sun's disc after its rising and ascending at an equal pace with it, so that the body of the Sun, because of the ever-changing discharge of its rays, seemed to roll over as a whole in a vertical plane with a very rapid rotation, by an utter deception of the eyes.

[Margin: The Easter dance of the Sun.] Because around the vernal equinox more vapours accompany the rising Sun with this jumping, and around that equinox we celebrate Easter, the common people attribute an Easter dance to the Sun, as Tycho notes in the place cited.

CAPVT V.

De Illuminatione, & Calefactione Solis: vbi de Figura Radij Solaris, & de Parallelismo Radiorum. [On the Illumination and Heating of the Sun: Where Also on the Shape of the Solar Ray, and on the Parallelism of Rays.]

I. Setting aside the physical controversies over the cause of the heat produced in sublunary things by the motion of the Sun, a cause that the friction of the heavens certainly no longer supports in this age, when the heaven of the planets has been found not to be solid; and taking as given the common principles from the opticians' store, namely that from any point of a luminous body rays are propagated in all directions to any point of the illuminable subject, provided that an opaque body does not intervene; that they are propagated along straight physical lines; and that a ray falling perpendicularly on an opaque body is reflected back on itself, whereas one falling obliquely is reflected elsewhere, and always in such a way that the angle of reflection is equal to the angle of incidence: on these terms it will not be difficult to give the evident cause of the excess of the noon and summer heat produced by the rays of the Sun over the heat of other hours and seasons. For the cause indicated in the preceding chapter, from the absence of spots or from the rotation of the Sun turning its hotter part toward us, is either not constant or rests on an uncertain hypothesis.

Diagram. A semicircle standing on its diameter, the horizon BAC, with the arc BMC representing part of the meridian. From the centre A straight lines rise to the arc: to M at the top, to L and N on either side of it, and to F, D and E low on the left. Lines of the same fan run to O and P, stopping short of the arc on either side of M, and to G, H and K on the right, also stopping short of the arc. A small arc joins L and N below M, and another joins F and E to the right of D, marking the Sun's disc at each position.

- L, M, N (top of the arc, left to right): the Sun's limb, centre and limb at the zenith
- O, P (inside the arc, left and right of M): ends of the reflected rays
- F, D, E (left side of the arc, top to bottom): the Sun's limb, centre and limb at the lower position
- G, H, K (inside the arc on the right, top to bottom): ends of the reflected rays
- B, A, C (along the diameter, left to right): the ends of the horizon and its centre

[Margin: The cause of noon and summer heat.] II. Let BAC be the horizon and BMC a portion of the meridian, on which let M be the zenith, and there the centre of the Sun, whose perpendicular ray MA shines on A, as happens on equinoctial days at noon above those who live on the equator. For that ray will be reflected from A, an opaque point of the Earth, back on itself toward M, will become stronger as a result, and will excite greater heat with greater light in the stretch of air AM. Now let the extreme rays of the Sun be LA and NA, making an angle LAN as great as the Sun's apparent diameter LN. For

the incidence of the ray LA will be the angle LAB, of 89° and about 45′, since all it lacks of the right angle MAB is the angle MAL, subtending the apparent semidiameter of the Sun, which is approximately 15′. Therefore the angle of reflection equal to it will be NAC, and so the ray LA will be reflected into the ray NA, toward N, increasing its light and heat, and in turn the ray NA will be reflected from A into L, increasing the light and heat of the ray LA. In this way any two rays of the Sun setting out from points equally distant on either side of the middle M will make each other stronger by mutual reflection, and more so near the point A, because the action in them decreases in a uniformly difform manner [vniformiter difformiter] the further one goes from A, the origin of the reflection. An inhabitant of A will therefore at that noon feel an intolerable heat above and around him, because of the air heated by the direct rays of the Sun and by the reflected rays, with a reflection so close that there is none that does not coincide with some other direct ray. And if the Sun is not at the zenith, but a little beyond or short of it, some rays reflected elsewhere, although they do not run into the direct rays, will still come so close to the direct ones that by an almost joint action they strongly heat the surrounding air: as if the ray LA should be reflected to P, or the ray NA to O.

III. Now let the Sun be far from the zenith or vertex M at its meridian altitude, and let it be at D, distant from M, for example, 60°. Its central ray DA, falling on A with an angle of incidence DAB of 30°, will be reflected to H; and the reflected ray AH, with an angle of reflection HAC likewise of 30°, will be distant from the direct ray DA by the whole angle DAH of 120°. The incidence of the extreme ray EA will be EAB, of 29° and about 45′, and the reflection KAC of the reflected ray AK will be as great; the ray AK will therefore be distant from the ray EA by 120° 30′. Finally, the incidence of the other extreme ray FA will be the angle FAB, of 30° 15′, and the angle of reflection GAC of the reflected ray AG likewise 30° 15′; the ray AG will therefore be distant from its direct ray FA by 119° 30′. Because of so great a divergence and separation of the reflected rays from the direct ones, arising from the obliquity of incidence of the direct rays, they will not be able to intensify one another or assist one another by nearby action. This, then, is the chief cause why the Sun, in summer and especially at the solstice, produces at noon far greater heat on the Earth than at midwinter: because it comes closest to our vertex, and the less oblique incidence of its rays makes the reflected rays nearer to the direct ones. On the same day, moreover, the heat of the noon Sun is greater than that of the rising or setting Sun, for the same cause, namely that the Sun when higher shines on the Earth with less oblique rays.

[Margin: Other causes of greater heat.] IV. Two other joint causes are added, however: one from refrac- tion, which is less when the Sun is higher, so that the rays are less weakened; the other from the Sun's stay above the horizon, which is longer on summer days than on midwinter days. Note meanwhile, reader, how much the shape of the walls, valleys and mountains from which the Sun's rays are reflected to some place contributes to increasing the heat.

For if the places from which the reflection occurs are concave, and much more if they are parabolically or elliptically concave, and the place receiving the reflected rays lies at the other focus [vmbilico] of the ellipse or at the focus of the parabola, a far greater heat results, because of the gathering of the reflected rays at one and the same point. This is established from the properties of the parabola and the ellipse, on which we write in our book on optics, and others, now together with Apollonius, in the conic sections or on the parabolic mirror.

[Margin: Why and when the shape of the solar ray is round.] V. When the rays of the Sun enter through a window or a round opening directly facing the Sun, and end at a wall or a board parallel to the opening, an image [species] of the solar ray also appears, and it is circular. There is no doubt that the roundness of the Sun and of the opening, and the straightness of the plane receiving perpendicularly the cone of the Sun's rays and its base, come together to produce this shape. But when the opening is polygonal, say quadrangular or triangular, and yet a more distant wall parallel to the plane of the opening receives a round or circular image of the Sun, the round shape of the Sun does not suffice as the explanation. For if the opening is fairly large and quadrangular, and near the wall, a quadrangular image of the Sun is painted on the wall. The cause, therefore, is that on a distant wall several bright quadrangular bases are painted about the same centre, about which they are so disposed that their outer angles lie on the circumference of the same circle, but their tips can hardly be discerned because of the weakness of the light that has travelled further, and its smallness from the tiny opening. So, those angles being blunted, a shape that is in reality polygonal appears to the senses circular, as we explain more fully in our *Optics*. Meanwhile one may consult Aristotle in the *Problems*, Problem 5, Section 15, and Biancani on that passage, no. 345, and in his *Sphaera*, book 10, chapter 18; Witelo, book 2, prop. 39; Petsanus [John Pecham] in the common *Perspectiva*, prop. 5; Kepler in the *Astronomia Optica*, chapter 1, prop. 6; and Hérigone, vol. 5, in the *Optica*, prop. 20.

[Margin: The parallelism of the rays of the Sun.] VI. It remains to explain the parallelism of the rays of the Sun. For mathematicians commonly suppose, with Ptolemy in the *Analemma* and Proclus in the *Hypotyposes* of astronomy, that rays, but rays setting out from the same point of the Sun, reach the Earth sensibly parallel; and this is demonstrated by Pedro Nunes, book 2 of *De observatione*, chapter 18; Witelo, book 2, prop. 35; Willebrord Snel, book 1 of the *Eratosthenes Batavus*, chapter 9; and Bonaventura Cavalieri, chapter 24 of the *Burning Mirror* [Speculi Vstorij]. It could moreover be argued a fortiori: two neighbouring walls built to the perpendicular are sensibly parallel, even though in mathematical rigour they converge and, if produced as far as the centre of the Earth, would meet there, as plumb lines would too. How much more, then, will two rays of the Sun setting out from the same point toward the Earth be sensibly parallel, since on the Earth they are much more distant from the Sun than two walls are from the centre of the Earth, as is supposed here from what will be said below in chapter 7. But it is better demonstrated

by a double method, not only for the rays of the Sun but also for those of the Moon or of an even nearer luminous body, given that its distance is so great that the extent of the illuminated subject bears an insensible proportion to it.

Diagram. At the top a shallow curved line, concave upward, represents the surface of the Sun, with the point S at its middle. Below it is the Earth as a semicircle on the diameter ATC with centre T. From S straight lines descend to the semicircle: one through N to P, one through I to B, one to T, and one to R at the left end of the arc near A. A short line runs from P to T. A horizontal line joins N and I; below it an arc and a straight line both join P and B, which stand on the semicircle. A separate line drops from the upper curve straight to B; K stands just to the right of the top of that line.

- G (left on the upper curve), S (middle, above the curve), K (under the curve, right of S), L (right on the upper curve)
- N, I (on the two descending lines, at the ends of the horizontal line, left and right)
- B (right, on the semicircle), P (left, on the semicircle), R (left, at the end of the line from S), A, T, C (along the diameter, left to right)

[Margin: First proof of the parallelism of the rays of the Sun.] VII. First, with Witelo and Nonius, let ABC be the hemisphere of the Earth, whose centre is T and horizon ATC, and on its surface let two gnomons PN and BI be erected perpendicularly, which are supposed to bear no sensible proportion to the distance ST or SP of the Sun from the Earth. From the same point S of the solar surface GL let two rays fall on the said gnomons, SNP on one side and SIB on the other, and let there be drawn to the feet of the gnomons the straight line PB, and to their tips the straight line NI, parallel to PB. The triangle SPB will then be cut proportionally by the straight line NI, by Euclid VI.2, and as SP is to SN, so will PB be to NI. But SP is not sensibly greater than SN, because the proportion of PN to SP is taken to be sensibly nil. Therefore neither will PB be sensibly greater than NI, and hence PB and NI will be parallel and at the same time sensibly equal. Therefore, by Euclid I.33, the straight lines PN and BI, which join on the same sides the straight lines PB and NI, parallel and sensibly equal, will also be equal to each other and sensibly parallel, and so the rays falling on the gnomons PN and BI will be sensibly parallel.

[Margin: Second proof of the same parallelism.] VIII. Second, with Snel, in the same figure let one ray of the Sun be SR, touching the Earth at R and making there, by Euclid III.18, a right angle with the Earth's semidiameter RT, and let another ray from the same point be ST. Let the distance of the star from the Earth be given, and let it be as great as the ray ST; and let us now take the Ptolemaic distance of the Sun, of 1210 semidiameters. For in the triangle SRT, right-angled at R, there will be given RT and the base ST, of 1210 parts of which RT is one, and so the side RT. Therefore by no. 7 of our *Plane Right Triangles* the angle RTS will be found to be 89° 57′ 9″ 35‴, and so falling short of a right angle by only 2′ 50″ 25‴. The angles STR and SRT will therefore be sensibly equal and right, and so by Euclid I.28 the rays ST and RT will be

sensibly parallel, however far apart they stand on the Earth, by almost a whole quadrant of the Earth's circumference. How much more so, then, the rays ST and SP, which are less distant from each other? And so, as Snel concludes, if two places P and B were as far apart as Alexandria in Egypt and Syene, namely 7½°, the angle would fall short of a right angle by no more than 22″ and 36‴.

[Margin: The parallelism of the rays of the ☾.] IX. Now let the Moon be at S and its distance from the Earth ST at least 49 semidiameters, of which TR is one; then by the same rule 7 the angle STR will be 89° 51′ 12″. But let two of its rays SP and SB be supposed to fall on points of the Earth distant by only one terrestrial degree PB, or 90 miles, of which there are 245000 in the Moon's distance SP, since the Earth's semidiameter holds, in our opinion, 5000 ancient Italian miles, and let us put the Moon's distance SP at no less than 49 semidiameters of the Earth. On these suppositions, in the isosceles triangle SPB, with SP and SB given, each of 245000 miles, and PB of 90, either angle at the base, namely SBP or SPB, will be 89° 59′ 22″, and so sensibly right, and by Euclid I.28 SP and SB will be parallel. Cavalieri, moreover, has shown above that if the luminous body S is one mile distant from B or P, and BP is only 10 cubits, as is BI too, IN will be shorter than the straight line BP by one small part of the 3783036 equal parts of which BP consists, and so NP and IB will be sensibly equidistant.

[Margin: The parallelism of rays falling on a lens, a telescope or the pupil from the same point.] X. But so that we may gather some universal benefit from this, we suppose two rays to be sensibly parallel when each falls so nearly perpendicularly that the angle they make falls short of a right angle by no more than 3″. On this supposition we shall inquire by the preceding problem, according to the given distance of the extremes on which the rays shine, namely either on the human pupil, whose diameter PB is taken as 40 parts, of which an inch of the Roman foot is 100, or on the glass lens of a telescope or on a burning mirror, whose diameter PB is two feet, or finally on a space PB of 100 miles, how great a distance SP or SB suffices and is required for the physical parallelism of the rays SP and SB to be preserved, and for the angles SPB and SBP each to be 89° 59′ 57″. To this I reply that there suffices and is required a distance of the luminous point S of 230 paces from such a pupil; of 14 Italian miles from such a lens; and from extremes of the Earth 100 miles apart, a distance of 200000 Italian miles, that is, 50 semidiameters of the Earth, which is about the distance of the Moon at perigee. Safely[12], then, the rays of the higher stars, but rays setting out from the same point, fall sensibly parallel on the Earth.

[Margin: The pseudo-parallelism of rays from different points of the Sun, in certain authors.] For if they come from different points, say from G and L, they do not reach the Earth parallel, as Eratosthenes falsely assumed. On his fallacy there is more from us in Book 3, chapter 27, no. 7, and meanwhile see Snel

[12] the page prints *Secutè*; read as *Securè*

and Nonius above, by whom Pliny the Elder, Giorgio Valla, Ziegler and, in part, Witelo, book 2, prop. 39, are censured for this same pseudo-parallelism.

SCHOLION.

[Margin: A cold and witless cause of summer heat.] The chief and regular cause of greater heat in summer is not, as some have suspected, a greater velocity of the Sun or its greater nearness to the Earth, since at that time the Sun is rather much slower than at midwinter, and by its proper motion completes each day no more than about 57′, whereas around midwinter it completes nearly 61′, and on the meridian the daily motion appears slower. Moreover, once the Sun's apogee [Apogeitate] is admitted, the Sun around the beginning of Cancer will for so many centuries now have been more distant from the centre of the Earth than around the beginning of Capricorn, where it lies at perigee. Nor is it only in summer that the Sun is joined with hotter stars, say with Mars or Regulus; it is joined with some of them in winter too. Nor again is the cause an annual rotation of the Sun turning its hotter part toward the Earth. For whether this rotation is lengthwise, to the east or the west, or breadthwise, to the north or the south, it will not be able to turn its hotter part toward us northerners in such a way that the southerners do not for the most part face the same part; and so it would make summer at the same time for our Antoeci and Antipodes, which is contrary to experience. For it is established, from the Father Procurator of the Province of Chile, which extends to the Strait of Magellan, that it is deepest winter there when the Sun at the beginning of Cancer makes the height of summer for us. The chief cause, then, is the one I set out in no. 1.

CAPVT VI.

De Solis ac Terræ Loco in Mundo, Qua occasione Proponuntur Insigniora Mundi Systemata, & Appēdix de Terræ multiplici motu iuxta Copernici Hypothesim. [On the Place of the Sun and of the Earth in the World, on Which Occasion the More Notable Systems of the World Are Set Out, with an Appendix on the Manifold Motion of the Earth according to the Hypothesis of Copernicus.]

Although more will have to be said about the systems of the world in Book 9, sections 3 and 4, for a fuller understanding of what will be said about the Sun the principal systems of the world must nevertheless be indicated here.

I. Systema Pythagoræ Ptolemæi &c.

[I. The System of Pythagoras, Ptolemy, etc.]

Diagram. Concentric circles about a central Earth, which is drawn with hatched land inside a ring of small flame-like or wave-like marks. Seven rings carry the planetary symbols in a vertical line above the centre. Outside them a ring carries scattered stars, and the outermost band is divided by short rules into twelve compartments, each holding a zodiac sign.

- Planets, from the outermost inward along the vertical line: ♄, ♃, ♂, ☉, ♀, ☿, ☾
- Zodiac band, going anticlockwise from the right side: ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏ (drawn as a small scorpion), ♐ (an arrow), ♑, ♒ (wavy lines), ♓
- Asterisks for stars in the ring just inside the zodiac band

[Margin: The most ancient system.] I. In this system, the oldest of all of which there is any record, the Earth rests motionless at the centre of the world, and around it are the elements and the heavens, in the order indicated by the symbols of the planets and by this series of names: of the Moon, Mercury, Venus, the Sun, Mars, Jupiter and Saturn, and of the eighth sphere, which is also called the sphere of the fixed stars, and ἀπλάνη [Greek: ἀπλάνη (aplanē, "not wandering")], or Aplane, that is, not wandering at all, and by many the Firmament. We shall show in Book 9, section 3, chapter 2, that this system was that of Pythagoras, Archimedes, the Chaldaeans, Cicero, Pliny the Elder, Ptolemy and very many Arabs, of the Alphonsines and the followers of Peuerbach, and that it was commonly accepted by most people down to Magini and Clavius, even though the more recent among them have added other spheres above the eighth for the motions of trepidation and of the primum mobile, according to what will be said in section 3, chapter 1.

The second system, that of Plato and probably of Aristotle, Eudoxus and Callippus, and also of Tycho and Jābir ibn Aflaḥ, though Jābir elsewhere had followed Ptolemy, places the Sun immediately above the Moon, and sets Venus above the Sun, Mercury above Venus, Mars above Mercury, Jupiter above Mars, Saturn above Jupiter, and the fixed stars above Saturn. The author of *De Mundo* to Alexander, however, and Porphyry, Apuleius, Marsilio Ficino and certain Platonists placed Mercury above Venus, keeping the rest; and so you see two series in the following diagram. But more on this system in Book 9, section 3, chapters 3 and 5.

II. Systema Platonis, aut Platonicorum.

[II. The System of Plato, or of the Platonists.]

Diagram. Concentric circles about a central Earth, drawn with an irregular land mass inside a wavy ring and a dotted ring. Two columns of planetary symbols run upward from the centre, side by side, each headed by a name printed in curved letters outside the outermost circle, which carries scattered stars.

- "Plato" (above the left column); symbols from the outermost inward: ♄, ♃, ♂, ♀, ☿, ☉, ☾
- "Porphyrius" (above the right column); symbols from the outermost inward: ♄, ♃, ♂, ☿, ♀, ☉, ☾
- Asterisks for stars in the outermost ring

There are, however, reasons and authorities by which it could be made probable that Eudoxus, Callippus and Aristotle followed the system of Pythagoras; of these in their place. Meanwhile what Clavius says in his *Sphaera* is false, that

the author of the little book *De Mundo* to Alexander is the only one who places Mercury below Jupiter and above Venus. Finally, Plato grants in the *Timaeus* that the Earth is turned about its own centre, which Aristotle denies.

III. Systema Ægyptiorum, Vitruuij, Capellæ, Macrobij, Bedæ &c.

[III. The System of the Egyptians, of Vitruvius, Capella, Macrobius, Bede, etc.]

Diagram. Concentric circles about a small central Earth, drawn with hatched land inside its own small circle. On the next circle stands ☾, and on the large circle beyond it, at the top, stands ☉, rayed. Two smaller circles are drawn about ☉ as centre: the inner carries ☿ at its lowest point and the outer carries ♀ at its highest point. Beyond these, circles carry ♂, ♃ and ♄ near the top, and the outermost ring carries scattered stars.

- ☾ (on the circle nearest the Earth), ☉ (top of the next circle)
- ☿ (below ☉, on the smaller circle about it), ♀ (above ☉, on the larger circle about it)
- ♂, ♃, ♄ (upper left, on successive outer circles, moving toward the top)
- Asterisks for stars in the outermost ring

II. This system is that of the Egyptians, in which, as can be seen in the preceding figure, the Earth is set in the middle of the world with the elements, and the first heaven is that of the Moon, the second that of the Sun, but in such a way that around it Mercury goes on a tighter epicycle and Venus on a wider one, and so they are carried now below the Sun, now above it, though they appear larger and more clearly when they are near their greatest digressions below the Sun than when they approach apogee above the Sun. In fifth place is the heaven of ♂, in sixth of ♃, in seventh of ♄, in eighth of the fixed stars; these four heavens, like those of ☉ and ☾, are described about the Earth. To this system Vitruvius, Martianus Capella, Macrobius, Bede and Argoli have subscribed, as we shall show in Book 9, section 3, chapter 4.

But Argoli has the Earth moving about its own centre. Al-Biṭrūjī followed the same system in part, insofar as he places Venus above the Sun and Mercury below the Sun, for reasons to be explained in the same section, chapter 6.

IV. Systema Philolai Aristarchi & Copernici.

[IV. The System of Philolaus, Aristarchus and Copernicus.]

Diagram. Concentric circles about a central Sun, drawn as a rayed disc. Two inner rings carry the labels for Mercury and Venus. Next comes a dotted circle, the annual orb, on which stands a small circle about the Earth, drawn as a hatched globe, with the Moon's symbol on that small circle. Three circles beyond it carry, at upper left, the symbols of Mars, Jupiter and Saturn with their periods lettered along the arc. The outermost circle carries scattered stars.

- Top, outermost planetary ring: ♄, "Reuolutio Annorum. 30"
- Top, next ring: ♃, "Reuolut. Annor. 12"
- Top, next ring: ♂, "Reuol. Annor. 2."

- Beside the small circle about the Earth, on the left: "Terre"; on the right: "et Lun…" [?] (the ending of the word is not legible)
- Inner rings: "Reuolutio mensium. 9.", with ☿, and "Reuolutio Dierum 80.", with ☿
- Lettered round the dotted circle: "Annuus" (left), "orbis" (lower left), "annua reuolut." (foot), "orbis" (right)

III. This fourth system is called Copernican after Copernicus, who perfected it and propped it up with many arguments and ingenious hypotheses. Otherwise it is usually called Pythagorean, and said to have been born in the Samian or even the Italian school, as regards the annual motion of the Earth. For that motion was asserted long ago by Pilolaus [recte: Philolaus] the Pythagorean, in the part of Italy called Magna Graecia, and by Aristarchus of Samos, as reported by Aristotle, *De caelo* 2, from text 72; Cicero, *Academic Questions* 2, and *Tusculan Disputations* 1; Plutarch, *De Placitis*, book 3, chapter 13; Archimedes in the *Sand-Reckoner* [Arenario]; and Laërtius in their lives. This system revived, moreover, under Nikolaus von Kues, but imperfectly, until Nicolaus Copernicus asserted it, not merely as a hypothesis, as the ignorant keep saying, but absolutely. He was followed by Georgius Ioachimus Rheticus [Georg Joachim Rheticus], Mästlin, Rothmann, Kepler, Simon Stevin, Galileo, Schikardus [Wilhelm Schickard], Lansbergen, Boulliau, Giordano Bruno, Celio Calcagnini, Didacus Astunica [Diego de Zúñiga], Paulus Antonius Foscarinus [Paolo Antonio Foscarini], Pierre Hérigone, Descartes, and Anton. Laurentius Politianus [?]. Not, however, by Wendelen, nor by Gassendi: although they leaned towards it, they held back their opinion on hearing the decrees of the Sacred Congregation of Cardinals. In Gassendi's case this is plain from the end of the last letter *De motu impresso a motore translato*; in Wendelen's, I know it from his own letters and replies, most courteously sent to me. Boulliau and Lansbergen also dissent from Copernicus in this, that they grant the fixed stars a proper motion, which Copernicus denies.

Copernicus, then, places the Sun immobile at the centre of the universe, at least as regards motion of translation. Around it he sets first, nearest to it, Mercury, completing its revolution in 80 days; then Venus, with a nine-month revolution; then the Earth with the whole elementary sphere on the annual orb, completing an annual revolution of 365 ½ days from west toward east. That is, the orb or circle it travels on is like the one on which the Ptolemaic, Egyptian and Tychonic systems have the Sun revolving in an annual period. To this Earth Copernicus also attributes other motions, namely the diurnal, by which the Earth moves about its centre and about an axis inclined to the plane of the ecliptic in the space of 24 hours from west to east, as we shall say shortly in the Appendix. The Moon goes round the Earth in a monthly revolution. Next, again about the Sun as the centre of the universe, Mars completes its two-year revolution, then Jupiter its twelve-year one, and finally Saturn its thirty-year one. The sphere of the fixed stars, for its part, is the outermost and entirely immobile, and stands at so vast a distance from the sphere of Saturn

that the Earth's annual orb is like a point in relation to it. Nor can we perceive any sensible parallax in observing the fixed stars, and hence the altitudes of the pole, whether the Earth is at one point of the annual orb, nearer to one of the fixed stars, or at the opposite point, farther away. The arguments for and against this hypothesis I more prudently defer to the fourth section of Book 9, for by then, after the treatment of all the planets, they will be better understood. The decrees of the Sacred Congregation of Cardinals I shall set out in that same section, together with my own opinion, and I shall weigh the arguments with which many today attack this hypothesis. For granted that a great many arguments, both physical and mathematical, have been hurled against it up to now, they have not yet shaken it among those who have penetrated the depths of the Copernican hypothesis (which is certainly given to few). Indeed, day by day, among the Germans and the French above all who itch to be Copernicans [Copernicoturientes], however many decrees cut it back it sprouts again, and, to use the music of the Venusine lyre,

Through losses, through slaughter, from the very steel it takes strength and spirit.

Nor is there anyone of this kind who, as Plutarch relates of Aristarchus in the essay *De facie in Orbe Lunae*, *fears being charged with violated religion, for having moved the household gods and Vesta of the universe from their place*. But we shall throw no little religious scruple into them.

V. Systema Tychonicum.

[V. The Tychonic System.]

Diagram. A large outer circle carrying scattered stars, centred on the dark globe of the Earth. A small circle about the Earth carries ☾. A larger circle about the Earth is the Sun's path, and on it stands the rayed disc of ☉, drawn above the Earth, with small circles about it carrying ☿ and ♀. Three large circles centred on the Sun, and therefore eccentric to the Earth, carry the outer planets; one of them, the circle of ♂, passes close about the Earth.

- Top, near the outer circle: ♄, and below it ♃
- About the Sun: ♀ (upper), ☿ (close by the Sun), ☉ (centre of this group)
- About the Earth: ☾ (on the small circle)
- Below the Earth: ♂
- Foot: ♃, ♄ (set upside down)

[Margin: Tycho's newest system.] IV. The fifth system is the Tychonic, invented by Tycho himself and indicated in his letters to Rothmann, p. 147, where he also claims it for himself against Nicolao Dithmarso Vrso [Nicolaus Reimers], who had been passing it off as his own; and in volume 1 of the *Progymnasmata*, p. 479, and volume 2, p. 189. Longomontanus followed him, but with the Earth's diurnal motion added (book 1 of the *Theorica*, chapter 1). Scheiner leans towards it in the *Disquisitiones Mathematicae*, no. 22, though there he wrongly says that this system shares nothing with the Copernican; so do Biancani in his *Sphaera* and most members of our Society. As you see, this system has the Earth immobile in the middle of the universe, with the

elements. About the Earth the Moon turns in a monthly motion, and then the Sun in an annual one. Last of all visible things comes the sphere of the fixed stars, which stand at unequal distances from the Earth, a sphere that moves both with a diurnal motion of 24 hours and with a proper and very slow motion toward the east. And so the centre of the Earth is also the centre of the lunar and solar heaven and course, and also of the sphere of the fixed stars. But the centre of the other five planets is the Sun itself: that is, not only of Mercury and Venus, which go around the Sun itself below and above it between the Sun and the Moon, as in the Egyptian system, but also of Mars, Jupiter and Saturn, as the Copernican system has it. [Margin: The affinity or equivalence of the Tychonic system with the Copernican.] The orb of Mars, moreover, goes round the Sun in such a way that, at opposition to the Sun, Mars becomes perigean, descends below the orb of the Sun and comes nearer to the Earth than the Sun itself, and so then appears much larger than it does near apogee. In this respect too the system is equivalent to the Copernican, in which Mars, opposed to the Sun, comes far nearer to the Earth, which lies between it and the Sun, and appears larger than in the Ptolemaic system, or the Egyptian. It also rivals the Copernican system, insofar as with fewer revolutions, and without the solidity and superfluity of the Ptolemaic or rather Peuerbachian spheres, it very easily defends all the phenomena, although the Copernican displays greater simplicity and elegance.

VI. Systema Nostrum olim.

[VI. Our Former System.]

Diagram. A large outer circle carrying scattered stars, within which two further circles, about the dark globe of the Earth near the centre, carry at the top ♄ and ♃, each with small rings of its own beside it. The rayed disc of ☉ stands above the Earth, with small circles about it carrying ☿ and ♀, and a larger circle about the Sun carries ♂, which appears both above and below. A small circle about the Earth carries ☾.

- Top: ♄, with a pair of concentric rings beside it
- Below it: ♃, with four concentric rings beside it; then ♂
- About the Sun: ♀, ☿; the circle to the right of the Sun is drawn with a heavy stroke
- About the Earth: ☾
- Below the Earth: ♂

[Margin: Our former system.] V. With the reader's kind leave, I should like to indicate here a system that came to my mind when I was once publicly teaching philosophy at Parma, while dealing with the heavens. It is semi-Tychonic: it has everything else in common with the Tychonic, except for the orbs of Saturn and Jupiter, whose centre in my version was not the Sun but the Earth itself, as you see in the diagram. I was moved to this because Saturn, attended by two planets or bodyguards [Lateronibus], and Jupiter, by four planets going around it, seemed to me to be primary planets, exercising their

monarchy in their own world and heaven no less than the Sun, attended by Mercury, Venus and Mars, this last as a large and more widely ranging satellite. For if we viewed Saturn or Jupiter from as near as we do the Sun, they would not seem to us so much smaller than the Sun, nor would their satellites seem so much smaller than Venus, Mercury and Mars as they do now. I also thought it probable that there is some intermediate planet, sharing in its motions with the two superior planets and with the inferior ones, and that Mars is of this kind: though in many respects it is more like Saturn and Jupiter, in others it is more like Venus and Mercury. In this way Mars will enter not the foreign heaven of the Sun but its own, or its sovereign's, as Venus and Mercury do. [Margin: The system of Baliani, not yet mature.] Meanwhile I shall finish the chapter by indicating the invention of D. Io. Bapt. Baliani [Giovanni Battista Baliani], a patrician of Genoa, who had begun to construct a new system in which the Moon would hold the centre of the universe and the other bodies would move around it. But it has not yet come to light, and would that Lucina could successfully act as midwife at this birth. For my part I would suppose that he meant to move the Earth on the monthly orb of the Moon, as Copernicus moves it on the annual orb attributed elsewhere to the Sun, as I shall explain shortly in the Appendix. On the jumble of other old systems, see Plutarch, *De Placitis*, book 2, chapter 15; Pliny the Elder, book 2, chapters 8 and 17; and Io. Antonius Delphinus [Giovanni Antonio Delfino], in his book *De Globis Caelestibus*.

APPENDIX.

De Loco Solis ac Terræ, Deq. Multiplici Terræ ipsius Motu, iuxta Hypothesim Nicolai Copernici. [On the Place of the Sun and of the Earth, and on the Manifold Motion of the Earth Itself, according to the Hypothesis of Nicolaus Copernicus.]

I. Taking the occasion of the Copernican system, I should like to set out his opinion in this place, insofar as it concerns the motions of the Sun, which he transferred to the Earth, granted that I have not yet discussed the motions of the Sun. I do so lest this explanation, inserted elsewhere, disturb the order and interrupt what I have indicated is to be treated later, jointly and continuously. Copernicus himself gives his opinion on the place and motion of the Earth in book 1, from chapter 5 to 12, and in book 3, chapters 1 and 3. So do Rheticus in the first *Narratio* on the books of Copernicus, with Mästlin's annotations; Kepler in the *Mysterium Cosmographicum*, chapter 1, in the *Commentaries on Mars*, and in the *Epitome of Copernican Astronomy*; Galileo in the dialogues on the world system; Pierre Gassendi in book 2 of the *Institutio Astronomica* and in letter 2 *De motu impresso*; and Pierre Hérigone in volume 5 of the *Cursus Mathematicus*, from p. 575.

II. In the diagram below, then, let S be the centre of the world, and next to it the centre of the solar globe, which, as the heart of the universe and the source of light, is to rest immobile and free from all motion of translation; for the

motion of rotation, treated in chapter 4, does not conflict with the Copernican hypothesis. About S let the celestial ecliptic ACLK be described, and in its plane let there be the line of the equinoxes LA and, cutting it at right angles at S, the line of the solstices CK. Again, from the centre S, or from a point as near to it as the eccentricity of the annual orb of the Sun among other authors allows, let the annual orb EFBD be described in the plane of the celestial ecliptic. On its circumference let there be the centre of the Earth GHIP, and let this centre be understood to stand successively at the four cardinal points, as the annual revolution requires: namely at B, at D, at E and at F.

Diagram. A large circle, broken at the upper and lower right, is the celestial ecliptic, with its four points marked by zodiac signs outside it. A vertical line CK and a horizontal line LA cross at its centre S, where the Sun is drawn as a large rayed disc. Inside it a smaller circle, the annual orb, carries four small circles of the Earth, each with its own crossed diameters: at the top about F, on the left about B, on the right about E, at the foot about D.

- Top, outside the circle: ♋; below it: C
- Top small circle: P (upper right), G (left), F (centre), I (right), H (lower left)
- Far left: L, and outside it ♎, set sideways
- On the line at far left, inside the ecliptic and below L: ☾
- Left small circle: P (upper right), G (left), B (centre), I (right), H (lower left)
- Between the left small circle and the Sun: ☾ on the line, with N below it
- Centre: S
- Right small circle: P (upper right), G (left), E (centre), I (right), H (lower left)
- Far right: A, and outside it ♈, set sideways
- Foot small circle: P (upper right), G (left), D (centre), I (right), H (lower left)
- Foot: K, and below it ♑

[Margin: 1. The diurnal motion of the Earth.] First, then, let the Earth be at B, where the Sun S appears to Earth-dwellers at the beginning of ♈, under the point A, and let the equator of the Earth be GHIP (for according to Copernicus the equator, with the tropics, the polar circles and the other parallels to the equator, should be conceived on the Earth rather than in the heavens), with its diameter GI coinciding with the diameter BE of the annual orb, as the position of the equinox requires. Let the globe of the Earth, with its whole elementary atmosphere, move about its centre B in a diurnal revolution toward the east, or in the order of the signs, from I to P, from there to G, and through H back to I, describing on its surface the equinoctial or equidial circle and returning to I in the space of 24 solar hours. For in this way all the stars will seem to us to move toward the west, since our eyes go to meet them toward the east, whether they are above the Earth or between the Sun and the Earth, as we said in chapter 6, at system 4. Recall that here. But this holds above all for

the Sun S: for when the point I and its inhabitant have revolved to P, the Sun will seem more westerly by the whole arc PI, whereas before it seemed to the inhabitant of I to be on the meridian. And so, once this single motion of the Earth is posited, the diurnal motion of the fixed stars and of all seven planets is removed, a motion that would otherwise have to be multiplied in each of them, or else placed in some highest sphere, as a primum mobile that carries the lower spheres along with it, or impresses or communicates that motion to the lower ones. There is removed, besides, that contradiction which appears between the diurnal motion toward the west and the proper motion of the stars toward the east; nor is it necessary to recognise in the planets any motion but one to the east, speaking of the motion in longitude. It is no wonder, however, that the stars seem to us rather to move with this diurnal motion, because of a deception of sight similar to the one the Virgilian Aeneas meant, in book 2 of the *Aeneid*, when he indicated the apparent motion of the shores, which truly belongs to the ship and the sailors, in that line:

We are carried out from the harbour, and lands and cities recede.

For the phenomena of the risings and settings of the stars are exactly the same whether the stars themselves are carried round the Earth to the west, or, with them thus unmoved, the Earth revolves to the east about its centre and the axis of its equator in an equal interval of hours.

[Margin: The agreement of the elementary motions with the diurnal motion of the Earth.] III. According to Copernicus's view, the air nearer the Earth moves with the terrestrial globe, to say nothing of the element of water joined to the Earth, and all bodies cognate to the Earth share in this circular motion of diurnal revolution. On the other hand, if heavier things are removed from their place, when let go they naturally tend downward, not to the centre of the universe S but to the centre of the cognate body, or of the whole atmosphere. For this reason their descent is a motion mixed of straight and circular, by virtue of which things thrown perpendicularly upward fall back to the same place, notwithstanding the motion of the Earth. Birds, clouds and other things driven or thrown toward the west still share in this eastward motion of the whole elementary sphere, and are carried less far to the west than they would be if, with the Earth and the atmosphere standing still, they were borne to the west with the same impetus. Copernicus does, however, except from this motion that highest region of the air in which comets or similar phenomena appear, because they seem either to remain suspended in still air or at least to rise and set in the manner of the stars. As for Sacred Scripture, when it ascribes rising and setting to the Sun, the followers of Copernicus contend with him that it speaks in accommodation to the understanding of the common people and to the deception of our sight and imagination, and figuratively rather than properly and to the letter. But in this they certainly err, as we shall teach in its place.

[Margin: The authors of this motion, and the ecclesiastical censure passed upon them.] This, then, is that diurnal motion which Heraclides of Pontus,

Ecphantus the Pythagorean, Plato in the *Timaeus*, and Nicetas or Hicetas of Syracuse, as reported in Cicero, Plutarch and Laërtius, attribute to the Earth; as do now the semi-Copernicans William Gilbert (*De Magnete*, book 6, chapter 3), Origanus (*Ephemerides*, volume 1), Longomontanus (book 1, pp. 161 and 220) and Argoli (*Pandosion*, chapter 3), though they place it at the centre of the world, as I reported in book 2, chapter 3. Their opinion, however, is judged by the Sacred Congregation of Cardinals *at the least erroneous in faith, and absurd and false in philosophy*, as I said in book 2, chapter 3, no. 3, and shall say more fully in Book 9, section 4. Nor does Tycho stand up for this motion anywhere in volume 1 of the *Progymnasmata*; indeed he attacks it sharply in his letters, pp. 147 and 188.

[Margin: 2. The annual motion of the Earth's centre.] IV. Hitherto we have considered the centre of the Earth at B, from which the Sun S appeared under A, the beginning of ♈. Let it now move from there in the order of the signs toward D, each day by as much as the Sun's centre moves on the ecliptic in the hypothesis of others, that is, completing on the annual orb by mean motion about 59′ 8″ 20‴ a day. For when the Earth is at D, the Sun S will appear to us under C, at the beginning of ♋; when the Earth is at E, the Sun S will be seen under the point L, at the beginning of ♎; when it is at F, the Sun S will be seen under the point K, which is the beginning of Capricorn; and finally, when the annual revolution is complete and the Earth has been restored to B, the Sun will again appear under A, at the beginning of Aries. And so this single motion of the Earth will display to us the alternations of the equinoxes and solstices, provided that the diameter HP of the terrestrial equator is inclined to the ecliptic, or the annual orb, by as much as the obliquity of the ecliptic for the time being in the common hypothesis [obliquitas Eclipticę temporaria]; that its diameter GI, at right angles to HP, always lies in the common section of the planes of the annual orb and the terrestrial equator, but at the equinoctial points B and E coincides with the diameter BE, the planes nevertheless being inclined as the obliquity of the ecliptic requires; and finally that the terrestrial equator proceeds through the whole year always parallel to itself, its axis to itself, and so GI also to itself.

[Margin: 3. The Earth's motion of declination.] V. But suppose the parallelism of the equator to itself and of the axis to itself were not kept, and the axis instead followed the motion of the Earth's centre, that is, were inclined to the axis of the ecliptic always in the same way, facing the same point of the axis of the ecliptic or annual orb, and the axis of the terrestrial equator did not point throughout the year to sensibly the same regions of the world. This would be contrary to the most manifest phenomena, and no inequality of days and nights would appear: there would always be either equinox or solstice, either spring or autumn or summer or winter, or some other unvarying temperature and measure of days and seasons. Let P, then, be the limit of the equator's greatest declination to the south, and H to the north. For with the Earth placed at F, the Sun will appear under the beginning of Capricorn to make the winter solstice

and to reach its greatest southern declination, while the equator's limit H there declines most toward the north, because the slope of the equinoctial to the line FS, by virtue of the diurnal revolution, turns out for itself on the surface of the Earth the winter tropic, parallel to the terrestrial equator, according to the obliquity contained under the angle SFH. Now let the centre F go on to B, completing the quadrant FB, but let P, the end of the greatest declination, be carried by about the same amount against the order of the signs, so that BP does not follow the course of the ecliptic, or the same longitude as SB, but is turned back through a quadrant toward C, and so the angle SFI has meanwhile been kept equal to the angle FSB. For there the diameter GI will coincide with the straight line drawn from S, as it also will at E, and the equinoxes will occur. With the Earth placed at D, the Sun will be seen to enter Cancer and to reach its greatest northern declination, while the limit P declines most toward the south, by the angle SDP. On other days, according to the varying inclination of the axis of the equator from the axis of the ecliptic, the Earth by virtue of the diurnal motion traces out on its surface other and yet other parallels to its equator, and produces the difference of artificial days and nights. Since the Earth in its annual motion carries the lunar heaven around with it, it follows that, given the diurnal motion, the changes of the lunar phases are saved, eclipses especially. Thus, with the Earth placed at B, we at the point I see ☾ N conjoined with and set beneath ☉ S; but placed at G, we see the Moon L opposite the Sun S and eclipsed by the Earth's shadow, unless latitude prevents it.

VI. But since the two annual motions just mentioned, one of the centre and the other of the axis of the equator, made in opposite directions and running to meet each other, are almost equal, the annual revolutions of the centre and of the declination are also almost equal. For if they were exactly equal, the equinoctial and solstitial points would not change places under the sphere of the fixed stars, nor would they seem to recede from the fixed stars toward the west, or the fixed stars toward the east, with that slow motion that was at length detected in them. [Margin: 4. The Earth's motion of the equinoctial points, elsewhere ascribed to the fixed stars.] But because they are not exactly equal, and while the centre of the Earth in its annual motion completes about 59′ 8″ 20‴ a day on the annual orb in the order of the signs, the axis of the terrestrial equator is supposed to move 59′ 8″ 20‴ against the order of the signs, the centre of the Earth has therefore not yet been restored to the point B when, the motion of declination having got ahead, the Earth's equator is found to have directed one of its diameters, namely GI, at the ☉ from a more westerly point between B and F. So the line LA of the mean equinox appears shifted, and the mean vernal equinox occurs at a more westerly point between A and K. And hence a fixed star, for example the first of Aries, when the equinoctial point from which we count longitudes deserts it, seems to have receded eastward from A toward C; whereas, as Copernicus would have it, the fixed stars themselves do not move, but rather the equinoctial and solstitial points recede from them toward

the west, completing their period in 1717 Egyptian years, 5 days. And this is, for Copernicus, the mean precession of the equinoxes, which for others would be the mean succession, or the mean motion of the fixed stars according to the succession of the signs. It is a certain fourth motion included in the Copernican motion of the Earth, but an apparent one, since in reality it is the same as the motion of declination, somewhat faster than the annual motion of the Earth's centre.

[Margin: 5. The motion of the Earth by which the obliquity varies.] I said, moreover, that the Earth's equator always faces almost the same part of the world, because if the obliquity of the equator and the ecliptic varies with time, the position of the terrestrial equator varies also, and a certain fifth motion, of the obliquity, must be conceived. Its period from greatest to greatest or from least to least obliquity is twice as long as the mean motion of the precession of the equinoxes, that is, of 3434 Egyptian years and 10 days. This motion occurs because of a certain libration, by which the axis of the terrestrial equator nods toward the sides of the world.

VII. Finally, since each of the motions just mentioned, namely that of the precession of the equinoxes and that of the obliquity, is unequal, now fast and now slow, then to account for the anomaly of both we must conceive two reciprocal motions of the poles of the terrestrial equator, resembling hanging librations. By one of them the poles are carried up and down about the angle at which the equator cuts the ecliptic, and the inclination of the equator to the ecliptic is altered. By the other the precession of the equinoxes is now increased and now diminished, by a transverse motion, from the preceding parts into the following and from the following into the preceding, so that the poles themselves describe certain lines resembling twisted garlands, as we shall set out in a diagram placed at chapter 28, number 9. [Margin: 6 & 7. The motions of the Earth's poles.] So, with this twin libration added, a certain sixth and seventh motion, or rather an affection of the annual motion, is included in the Copernican hypothesis.

VIII. Moreover, an annual motion of the Earth besides the diurnal, as though the Earth were some planet, was attributed long ago to it by Philolaus the Pythagorean, a man of so great a name that Plato came to Italy for the sake of visiting him; and by Aristarchus of Samos; then by Nikolaus von Kues; and, after Copernicus, by Rheticus, Mästlin, Giordano Bruno, Kepler, Celio Calcagnini, Rothmann, Schickard, Stevin, Galileo, both Lansbergens, Boulliau, Pierre Hérigone, Baranzano, Diego de Zúñiga, Paolo Antonio Foscarini, Descartes, and many others here and there in Germany and France. And yet this hypothesis has been pronounced by the Sacred Congregation of Cardinals *formally heretical as regards the immobility of the Sun*, inasmuch as it is *expressly contrary to Sacred Scripture*; I spoke of this in Book 2, chapter 3, and more will be said in Book 9. Granted, it very ingeniously disposes of all the phenomena of the Sun and of the fixed stars by the motions of one body in the world, and that not a very large one, with a truly marvellous economy; indeed it also shows, from the

motion of the Earth alone, the cause of the retrogradations, direct motions and stations that appear in the planets.

[Margin: 8. The motion of the Earth up and down.] IX. Lastly, the Sun seems at some times to recede from the centre of the Earth so far as to be at its greatest distance in apogee, and at other times to approach the Earth so as to be nearest to it in perigee, so that the annual orb is eccentric to the centre of the world; and the Sun's apogee also seems to shift unequally in the order of the signs. To account for these two phenomena we must conceive either a small circle near the Sun, on whose circumference the centre of the Earth's annual orb is carried, or the Earth itself as carried now higher, now lower, and that with a certain anomaly, as Copernicus holds. So much for the fivefold motion of the Earth, namely the diurnal; the annual motion of the centre; the annual motion of the declination; the libration of the poles, mixed from the up-and-down motion and the transverse motion; and the motion of the eccentricity, or of its centre, up and down. The others are not distinct motions, but certain affections of these.

SCHOLIA.

I. The annual motion of the declination and the motion of libration of the equinoxes and of the obliquity, discussed in the preceding chapter from number 5 to number 8, do not necessarily require the Earth to be outside the centre of the world, and they are not necessarily connected with the annual motion of the Earth treated in number 4, granted that in the hypothesis of Copernicus they are connected. [Margin: A new affection of the Earth's diurnal motion.] To the diurnal motion, however, Longomontanus, Kepler and Boulliau add another motion, or affection of motion, which Wendelen also indicates: by it the diurnal motion is evidently slowed in aphelion and quickened in perihelion, and in this way the inequality of the days, which seems to arise from the unequal motion of the Sun along the ecliptic, is amply compensated; according to what is to be said in chapter 31.

[Margin: A compendium of the celestial motions in the motion of the Earth.] II. The Copernican hypothesis of the diurnal and annual motion of the Earth accounts not only for the proper motion of the fixed stars; the common diurnal motion of all the stars; the annual motion of the Sun toward the east; the oblique motion of the Sun toward the poles of the world, or motion of declination; the motion of anomaly of the precession of the equinoxes, and the motion of anomaly of the obliquity of the ecliptic, or of the two librations; and, if you add the motion by which aphelia and perihelia arise, it contains this motion too within itself. Beyond that, it removes the three eccentrics of the Sun, Venus and Mercury, by which in the hypothesis of others these two planets are moved according to the motion of the Sun; it also removes the three epicycles of Saturn, Jupiter and Mars, with their motions. From the Earth's approach to those planets and its recession alone, a reason is given why in opposition to

the Sun they are perigean and in conjunction apogean; why they undergo a certain variation of latitude, for which Ptolemy instituted five other motions; and why the five minor planets seem at times to stand still and to go backward. And if you believe Galileo, the cause of the sea's tide is also given from this. But these things, lightly indicated here, will be said more fully elsewhere; but they will be refuted.

CAPVT VII.

De Solis à Terra Distantia Mutabili, aut Immutabili. [On the Distance of the Sun from the Earth, Whether Changeable or Unchangeable.]

I. It would be the mark of Epicurean stupidity, or madness, to reckon the interval between the Earth and the Sun at as few ten-foot rods as to judge the Sun itself to be a foot across. Its height has so far exceeded the subtlety of all astronomers, and here there would be need of the wings of Astraea herself, and of *Feathers not given to man*. And yet, with the Sun's distance unknown, the distance and parallax of all the stars, except the Moon, are unknown. [Margin: Pliny the Elder's saying, still notable today, Book 2, chapter 23, on the uncertain distance of the Sun.] So that saying of Pliny the Elder about these intervals holds to this very day: *These things are undiscovered and inextricable*; and shortly after: *If anyone should care to pursue these matters more deeply, not so that there may be a measure (for to want that is the idleness almost of a madman), but only so that the mind may hold an estimate for conjecturing.* Kepler confessed something similar in the *Commentaries on Mars*, p. 71. *The Sun*, he says, *is not nearer than 230 semidiameters of the Earth, yet it is not infinitely many distant; but between 700 and 2000 no certain number seems yet to have been demonstrated.* What do you say, Kepler: between 700 and 2000? I for my part say not even between 700 and 7000, and indeed perhaps not even within 15000, as I shall show below. But first I must set out the choicer methods by which the distance of the Sun from the Earth can be measured in some fashion, the other, less certain ones being relegated to the Scholia.

I. PROBLEMA DE SOLIS DISTANTIA PER PARALLAXES.

[I. Problem: On the Sun's Distance by Parallaxes.]

[Margin: The first method, by the horizontal parallax.] II. What parallax is I explained in Book 1, chapter 35, which anyone who has forgotten it should reread; but the following diagram will dispose of this matter more simply. In it let AC be a quadrant of the Earth, and let H be the centre of the Earth and of the solar heaven, and also of the world; about H let YSR be a quadrant of a vertical circle conceived in the Sun's orb. Let HR be the rational horizon, on which the vertical line YH, drawn from the zenith Y to the Earth's centre H, falls perpendicularly. Let the eye be at A, and its sensible horizon AB, parallel to HR; and let the centre of the Sun appear at B, rising on the sensible horizon, or setting.

Diagram. A woodcut quadrant set into the right column: an outer arc runs from the top of a vertical line down to the right end of a horizontal base, and a smaller inner arc concentric with it runs from a point on the vertical to a point on the base. Both straight lines meet at the lower-left corner. A horizontal line runs from the point on the vertical to the outer arc, and straight lines run from the lower-left corner and from that same point on the vertical up to two points on the outer arc.

- Top of the vertical line: V (the text calls this point Y)
- Upper point on the outer arc: S
- Point on the vertical, at the end of the inner arc and of the horizontal line: A
- Lower point on the outer arc, at the other end of the horizontal line: B
- Lower-left corner, the centre: H
- End of the inner arc on the base: C
- Right end of the base: R

For when the straight line HB is drawn from the centre H to the Sun, the greatest and horizontal parallax of the Sun will be the angle BHR, or the arc BR; for its true altitude above the horizon HR will be the arc BR; and yet to the eye at A, the Sun seen at B will appear not at all elevated above the horizon. Now the angle ABH is equal to the alternate angle BHR, made by the line HB falling on the two parallels HR and AB, by 29 of the First Book of Euclid; and by the same proposition the angle HAB is a right angle and equal to the angle AHR, also a right angle, being made by the perpendicular YH. Therefore, given in the triangle AHB, right-angled at A, the horizontal parallax, that is, the angle B, and taking the Earth's semidiameter as one, the distance of the Sun from the Earth, namely the base HB, will be found by 5 of our *Plane Right-Angled Triangles*: that is, if you look up the residual of the logarithm [Residuum Logarithmi] of the angle B, for this will be the logarithm of the base HB; or if you subtract the angle B from 90° and divide the secant of the remainder by the radius of the tables, for the quotient will be the base HB itself, in parts of which the Earth's semidiameter AH is one. Therefore, if the horizontal parallax is as in the following small table, the Sun's distance from the centre of the Earth will also be, as you see in that same table; and a difference of a few seconds in the parallax, let alone of minutes, will bring a great difference in the Sun's distance.

Horizontal parallax of the ☉	Secant of the angle AHB	Distance of the Sun HB, in semidiameters of the Earth
0′ 15″	154698556112	15470
0′ 30″	68754937464	6875½
1′ 0″	34377468732	3438
2′ 0″	17188734366	1719
3′ 0″	11459157557	1146

With the radius set at 10000000.

[Margin: 2. The method by any parallax whatever.] III. Now let the Sun be at S, its true altitude the arc SR, and its true distance from the zenith the arc SY; and let the straight lines HS and AS be drawn. The angle YHS will belong to the true distance from the zenith, and the angle YAS to the distance seen from the eye at A. This angle is greater than the former, by 32 of the First Book of Euclid, and by the same proposition equal to the two interior and opposite angles S and H of the triangle SHA. So the angle S will be the difference between the angle SHA, of the Sun's true distance from the zenith Y, and the angle YAS, of the apparent distance, and hence it will be the measure of the parallax. Therefore, if any parallax of the Sun in the vertical circle, or parallax of altitude, is given, namely the angle S, then added to the Sun's true distance from the zenith it will give the angle SAY, whose complement to two right angles, by 13 of the First Book of Euclid, is the angle SAH. But the same parallax S, subtracted from the Sun's apparent or observed distance from the zenith, will give the angle AHS. So in the triangle SAH, with the Earth's semidiameter AH taken as 1, and given the angle S of the parallax together with either of the angles SAH or SHA,

Diagram. A woodcut quadrant of the same construction as on the previous page, set into the left column at its right: a vertical line and a horizontal base meeting at the lower-left corner, an outer arc from the top of the vertical to the right end of the base, a smaller inner arc, a horizontal line from a point on the vertical to the outer arc, and straight lines from the corner and from that point on the vertical to two points on the outer arc.

- Top of the vertical line: V (the text calls this point Y)
- Upper point on the outer arc: S
- Point on the vertical: A
- Lower point on the outer arc: B
- Lower-left corner: H
- End of the inner arc on the base: C
- Right end of the base: R

the other will also be known, being the complement of the two remaining given angles to two right angles, by 32 of the First Book of Euclid. [Margin: A shortcut of the rule.] So use the angles S and HAS, and by 2 of our *Plane Oblique-Angled Triangles* find the side HS, that is, the distance of the Sun from the centre of the Earth. That is, add together the logarithm of the angle S and the residual of the logarithm of the angle SAH; for their sum, with one taken off at the left, will be the side HS, in parts of which AH is one. Or else divide the right sine of the angle SAH by the sine of the angle S, and at once the side HS will come out, in parts, however, of which AH is one semidiameter of the Earth.

[Margin: The parallax of the Sun, without its distance from the Earth, cannot be observed outside the torrid zone.] But if only the way of finding the parallax of the Sun, independently of its distance from the Earth, were as certain as either part of this problem is easy! Outside the torrid zone, however,

this is simply impossible [adynaton]. One would have to know the Sun's true altitude from its motion beforehand, independently of the parallax, so that the difference between the true altitude and the apparent one, or the one observed with instruments, might reveal the parallax of altitude. But the Sun's true motion cannot be gathered from tables more exactly than from the mean motion deduced from observed equinoxes; and the moments of the equinoxes cannot be determined unless the altitude of the Sun then observed is corrected, by adding the parallax to the observed altitude. In the same way, neither the Sun's greatest declination nor the obliquity of the ecliptic can be known from observation unless the Sun's altitude taken at the solstices, or its rising amplitude, is corrected by the parallax. And so, although Gotefridus Vendelinus [Govaert Wendelen], in a letter sent to me this very year, truly says that *parallaxes of the Sun are assumed and feigned rather than grasped*, and that whether no parallax or one of at most 3′ is assumed at the equinoxes, no error can be committed exceeding 10′, *mocking almost all sense*, nevertheless the intervals of the equinoxes, or the distance of the tropics, cannot be known exactly except through altitudes observed at the equinoxes or solstices and reduced to true ones by adding the parallax; especially since refraction mixes itself into the winter altitude, and refraction itself is not known unless the parallax is known beforehand.

IV. I said *outside the torrid zone*. [Margin: Snellius's method of observing the parallax of the Sun.] For, as the very acute Snel points out in his notes to the Hessian observations, let the Sun's greatest declination be observed with instruments showing at least every ten seconds. This will be done if there, from some circumpolar fixed star or from the very certain declination of some fixed star, the distance of the equator from the zenith, or the altitude of the pole, is observed; and let a second observer, outside the torrid zone, observe on the day of the summer solstice the greatest apparent declination of the Sun, the true altitude of the pole having first been observed from the fixed stars. For the difference between this apparent declination and the true declination observed by the inhabitant of the tropic, and hence true (since the Sun, then standing at his zenith, undergoes no parallax at noon), would be the parallax of the Sun. This holds provided that all these things are done beyond the hazard of refractions, and that you can be certain that no sensible refraction intervenes, without prior knowledge of the parallaxes [Greek: παραλλάξεων (parallaxeōn, "of the parallaxes")]; and likewise that you are certain you are directly under the tropic itself. If precision is looked for here, this last is very difficult, because the space in which gnomons cast no shadow at noon on the solstitial day occupies 300 stadia, as was observed by Eratosthenes and others, in Strabo, Book 17, in Cleomedes, Book 1 of the *Cyclical Theory*, and in Pliny the Elder, Book 2, chapter 73. So, even if wells in that tract are then wholly lit, and styles set up perpendicularly cast no noon shadow, the observer still cannot conclude that he is under the tropic itself, but only in that tract of 300 stadia, which subtends

as large an arc of the heaven as the Sun's apparent diameter, namely about 30′. So this method too is slippery.

[Margin: Another, better method, of Father Francesco Maria Grimaldi.] V. A subtler and sounder method, therefore, is the one that Father Francesco Maria Grimaldi recently suggested to me, the day after I had discussed this matter with him. Let someone, he says, stand directly under the equator, and on the equinoctial day let the Sun's distance from the zenith be taken with a large quadrant showing tens of seconds, about two hours before or after noon, that is, before or after the Sun's passage through the zenith. And let a second companion, using a most exact pendulum [Perpendiculum], count the time between the Sun's passage through the zenith and the moment the altitude was taken, and let that time be converted into an arc of the equator. For if this arc is equal to the observed distance of the Sun from the zenith, the parallax of the Sun will be nil to the senses. But if that arc is less than the observed arc, their difference will be the very parallax of the Sun that is sought, corresponding to the apparent distance. Here, moreover, one must use the table that converts the time, not of the day of the primum mobile, but of the solar day, into parts of the equator, and a pendulum that does not stray by a single second in this time; otherwise, for every second it strays, the parallax will be wrong by fifteen seconds. Finally, you would need to be certain both that the equinox occurs on that day not far from noon and that you are directly under the equator itself. This you will be able to know if the declination of some fixed star is exactly known, observed elsewhere by someone who has observed the altitude of the pole and of the equator from circumpolar stars placed beyond the hazard of refraction; of this kind are the Tychonic declinations of the fixed stars and ours. For if the distance of such a star from the zenith, observed in the meridian, is equal to its declination observed elsewhere, you will be certain that your zenith lies on the equator itself. But for the present we cannot make this trial. So we must rise to this height by other ladders, and the first step must be fixed on the Moon, so that from there you may ascend to the Sun.

II. PROBLEMA DE SOLIS DISTANTIA PER ECLIPSES.

[II. Problem: On the Sun's Distance by Eclipses.]

[Margin: The authors of the second problem.] VI. The author of this second problem is held to be Hipparchus, and its restorer Ptolemy, *Magna Constructio*, or *Almagest*, Book 5. He was afterwards imitated by Jābir ibn Aflaḥ, *Astronomy*, Book 5; al-Battānī, chapter 30; Regiomontanus in the *Epitome of the Almagest*, Book 5, from proposition 19 to 21; Copernicus, Book 4, chapters 19 and 20; Longomontanus in the *Astronomia Danica*, Book 1 of the *Theorica*, chapter 9; [Margin: Three data required for this problem.] Maurolico, *Cosmographia*, Dialogue 3, p. 85; Magini in the *Theoricae Planetarum*, Book 2, chapter 24; Lansbergen in the Prolegomena to the *Uranometria*; Bettini, *Apiaria*, Apiary 8, Progymnasma 3, proposition 11; and Ismaël Boulliau, Book 4 of the *Astronomia*

Philolaica, chapter 1. Granted, most of them have gathered by this method results that contradict their own hypotheses, as Lansbergen shows in the same place. Now three data are supposed here. The first is the apparent semidiameter of the luminaries, observed by radii, dioptras, or otherwise. The second is the Moon's distance from the centre of the Earth by means of parallaxes, which we shall show in the next Book can be known independently of the Sun's parallax. The third is the apparent semidiameter of the Earth's shadow at the place through which the Moon passes at the time of some lunar eclipse; how this is investigated is to be said in Book 5, which is on eclipses. These being given:

In the adjoining figure let A be the centre of the Earth, B that of the Sun, and CD the diameter of the Sun. Let two extreme rays of the Sun be drawn, touching the Sun at C and D and the Earth at M and N, until they meet at E, and let these be CME and DNE. And from the Sun's centre B, through the Earth's centre A, let the straight line BA be drawn as far as E; for AE will be the axis of the Earth's shadow. Now let the centre of the Moon be at F, as it eclipses the whole Sun without delay, and let the straight line AID be drawn, touching the Moon at I and the Sun very nearly at D, so that the angle FAI subtends the apparent semidiameter both of the Moon, FI, and of the Sun, BD, without sensible difference. After this, in AE let AG be taken equal to the Moon's distance AF, G being the place of the Moon's passage in a lunar eclipse. Let the diameter of the Earth's shadow, PL, be drawn, cutting AE at right angles, and let GL be its semidiameter. From A let the straight line AL be drawn to the point L, and let LK be drawn parallel to the straight line AG. With these laid down:

Diagram. A tall, narrow woodcut set into the left column at its right. A large circle at the top has a horizontal diameter. Two long lines run from the ends of that diameter down past a smaller circle, which also has a horizontal diameter, and converge at a point at the foot, forming a long cone. A central vertical axis runs from the large circle to the foot. A small circle stands on the axis between the large circle and the middle circle, and a further small circle with a horizontal diameter stands on the axis below the middle circle. Further straight lines cross the region from the middle circle's centre toward the top and the lower diameter.

- Left end of the upper diameter: C
- On the upper diameter, left of centre: R
- On the upper diameter, at the axis: B
- Right end of the upper diameter: D
- In the small upper circle: F
- Beside the small upper circle, on its right: I
- In the middle circle, left of the axis: T and A
- In the middle circle, right of the axis: K
- Left end of the middle diameter: M
- Right end of the middle diameter: N

- Left end of the lower diameter: P
- In the lower circle: G
- Right end of the lower diameter: L
- Foot of the cone: E

FIRST, in the triangle AGL, right-angled at G, there are given through Ptolemy the distance of the Moon at apogee, AG, of 64⅙ terrestrial semidiameters, and the apparent semidiameter of the Earth's shadow, that is, the angle GAL, of 40′ 45″. Therefore by trigonometry GL, the true semidiameter of the shadow, will be 45 63867/100000 parts, of which, in the Ptolemaic manner, the Earth's semidiameter AN is 60. Now because the straight lines AG and KL are parallel, the straight line GL is equal to AK, by 33 of the First Book of Euclid. Therefore, with the equal AK taken away, there remains KN, of 14 36133/100000 parts.

SECOND, then, in the triangle KLN, right-angled at K, there are given KL, equal to GA, that is, of 64⅙ parts of which AN is one, or of 3850 parts of which AN is 60; and KN, of 14 36/100 such parts. By trigonometry the angle KLN is found to be 12′ 49″. By 29 of the First Book of Euclid this angle is equal to the angle AEN, of the half cone of the shadow; and to this angle and the angle ADE taken together the exterior angle BAD is equal, by 32 of the First Book of Euclid. This is the angle subtending the semidiameter of the Sun and of the Moon at apogee, which for Ptolemy is 15′ 40″.

THIRD, then, with the straight line BN drawn, the angle BND will subtend the same semidiameter of the Sun, and by the same 32 of Euclid it will be equal to the angles NEB and EBN taken together. So, when the angle BEN of 12′ 49″ is taken from the angle BND of 15′ 40″, there will remain the angle ABN of 2′ 51″, which is evidently the horizontal parallax of the Sun. And thus far with Lansbergen; for the others cited at the beginning of this number 6 set up this problem differently, and hunt first for the axis of the shadow, AE, and then for the Sun's distance, AB. But it will be found more briefly through the horizontal parallax, with the help of the First Problem, on which see number 2. So to a parallax of 2′ 51″ there will correspond a distance of the Sun of 1206 16/60 terrestrial semidiameters; or if you take the Ptolemaic parallax of 2′ 50″, AB will be 1210 terrestrial semidiameters.

[Margin: The ageometrical hypothesis of al-Battānī.] VII. Using roughly this method, Lansbergen convicts al-Battānī of inconsistency. For al-Battānī, with Ptolemy, puts the semidiameter of the Sun at apogee at 15′ 40″ and the distance of the new and full Moon from the Earth at 64⅙ semidiameters of the Earth; but the semidiameter of the Moon at that time at 14′ 45″, and of the shadow at 38′ 20″. From these he thinks it follows that the Sun's horizontal parallax is 3′, when only 26″ follows, and that the distance of the Sun at apogee from the centre of the Earth is 1146, when 7936½ semidiameters of the Earth follow. [Margin: And of Copernicus.] Further, Lansbergen refutes Copernicus for a similar inconsistency and want of geometry [ageometria]. For Copernicus puts the distance of the new and full Moon from the Earth at 65½ semidiameters

of the Earth, and the apparent semidiameters as follows: of the Sun at apogee 15′ 50″, of the Moon 15′ 0″, and of the shadow at the place of the Moon's passage 40′ 18″. From these he gathers the Sun's horizontal parallax as 2′ 55″ and the Sun's distance from the centre of the Earth as 1179 semidiameters of the Earth, whereas from those data there follow a parallax of 3′ 39″ and a distance of the Sun at apogee of 941 53/60 terrestrial semidiameters. [Margin: And of Tycho.] Besides this, Tycho too is charged by Lansbergen with a similar inconsistency. For Tycho puts the distance of the new and full Moon, also at apogee, at 58 8/60 semidiameters of the Earth, and the apparent semidiameters of the Sun at apogee at 15′ 0″, of the Moon at apogee at 16′ 0″, and of the shadow there at 43′ 0″. From these it follows that the angle of the half cone of the shadow, AEN, is 16′ 8″, greater than the semidiameter of the Sun, when it ought to be less. [Margin: And of Longomontanus.] Into this absurdity he also throws Longomontanus, who puts the new and full Moon, at the Sun's mean distance, at 57 38/60 semidiameters of the Earth, and the apparent semidiameters of the Sun at 15′ 0″, of the Moon at 16′ 0″, and of the shadow at 43′ 0″, as Tycho also does. From these it follows that the angle of the half cone of the shadow is 16′ 38″, and greater than the semidiameter of the Sun, when it ought to have been less, by 32 of the First Book of Euclid. As for Kepler's inconstancy in gathering the distance of the Sun, we shall speak of it below, in Scholium 2.

[Margin: Two notable corollaries, and] VIII. Gather from what has been said, first, how circumspect an astronomer must be, so that in his hypothesis the apparent semidiameters of the luminaries and of the shadow, the parallaxes and distances, and the angle of the cone of the shadow cohere by a geometrical bond. Second, how little suited this method, or this second problem, is to the precision that astronomy requires. For in observing the semidiameters of the luminaries an error of one half-minute can scarcely be clearly avoided, and much less in the semidiameter of the shadow, in which it is very easy to commit an error of two and three minutes; nor can it be observed accurately enough, as Longomontanus admits, Book 1 of the *Theorica*, chapter 9, p. 298. [Margin: The uncertainty of the second problem.] So, with the phenomena of eclipses preserved, the Sun could be distant from the Earth by far more semidiameters than the astronomers just mentioned have put. Suppose it is as far away as Wendelen wishes, 14000, or indeed 15470 semidiameters of the Earth, so that to this distance, according to what was said in number 2, there corresponds a horizontal parallax of the Sun of 15″, which in the figure of number 6 is the angle ABN; and suppose the Sun's apparent semidiameter to be only 15′. For the angle of the half cone of the shadow, by Lansbergen's method, will have to be 14′ 45″, and the angle KLN will have to be as large. With this angle, and with the Moon's distance AG, that is, KL, set at 64 parts of which AN is 1, or 3840 parts of which AN is 60, there is gathered in the right-angled triangle KLN the side KN of 15⅓ parts, and there remains AK, or GL, of 44⅔. With these, and with AG of 3840 parts, in the right-angled triangle AGL, the angle GAL, subtending the semidiameter of the shadow GL, is gathered as 40′ 54″, that is,

nearly 41′, the amount the Alphonsines put, which could hardly be convicted of falsity. Certainly Wendelen declares that, with so great a distance of the Sun assumed, he will represent eclipses perfectly. And on the uncertainty of this method, see Kepler's words below, at number 10. So let us carry our step over from the ladders of Hipparchus and Ptolemy to those of Aristarchus.

III. PROBLEMA DE SOLIS Distantia per Lunæ Distantiam, & Διχοτομίαν.

[III. Problem: On the Sun's Distance by the Moon's Distance and the Dichotomy.] [Greek: Διχοτομίαν (dichotomian, "dichotomy", the Moon's half phase)]

[Margin: The notable problem of Aristarchus.] IX. The third problem is that of Aristarchus of Samos, a fellow-citizen of Pythagoras, which he set out in an outstanding little book on the *Magnitudes and Distances of the Sun and Moon*, as it is entitled. Pappus of Alexandria, in Book 6 of the *Mathematical Collections*, from Proposition 38, and Federico Commandino have illustrated it with notes. The problem presupposes nothing except that the Moon's distance from the centre of the Earth is known, and also its elongation from exact quadrature, that is, from the quadrate aspect to the Sun, at the time when she shines half-lit for us in the evening before the first quadrature, or in the morning after the second quadrature. This elongation is obtained by observing the Moon's place at the time of the half phase, in the way this is usually done on other occasions: for from the Moon's true place in the zodiac, and from the Sun's place observed at noon or deduced from corrected tables, the arc of the said elongation will be known. This being assumed:

In the following figure let the centre of the Earth be T, from which let the Sun's orbit, or ecliptic, ABSC be described, and the Moon's orbit GLRF. Let the Sun be on the horizon ATS, or about at S, and let the Moon, dichotomous [Greek: διχότομα (dichotoma, "halved, cut in two")], that is, bisected, or shining half-lit for us, be at L, a little before the first quadrature. Let IK be the circle dividing the dark part of the Moon from the bright part, and let a straight line KIT be drawn through the plane of this circle as far as our sight, since at that time that plane inclines toward our sight. Finally, from the centre of the Sun S to the centre of the Moon let the straight line SL be drawn, perpendicular to the said plane. For since the Moon is lit by the Sun, the plane marking off the dark part from the illuminated part receives perpendicularly, at the Moon's centre, the ray coming from the centre of the Sun. In the triangle LTS, then, right-angled at L, there will be given the side TL, which is the Moon's distance from the centre of the Earth, and the angle LTS, which is the Moon's elongation from the Sun in the ecliptic, together with its complement to the 90° required for the Moon's exact quadrature; and this complement, by 32 of the First Book of Euclid, will be the angle S. Given therefore the angle S and the opposite side TL, you will easily find, by the fifth of our *Plane Right-Angled Triangles*, the

base TS, which is the distance of the Sun from T, the centre of the Earth. I shall report a problem of Father Grimaldi that rivals this one in Scholium 5.

Diagram. A woodcut in the left column, with the text wrapped at its left from "Sit in sequēti" to "paulò ante". Two circles share the centre T. The outer circle is the Sun's orbit, or ecliptic; the inner is the Moon's orbit. A horizontal straight line runs from A through T to S. A small circle at S stands on the outer circle and represents the Sun. A small circle on the upper right of the inner circle represents the Moon, its upper left half shaded black. A straight line runs up from T through I to K at the Moon, and a second straight line joins S to the centre of the small lunar circle at L.

- A (left, on the outer circle): end of the horizon line
- B (top, on the outer circle)
- C (bottom, on the outer circle)
- S (right, within the small solar circle): the Sun
- G (left, on the inner circle)
- R (right, on the inner circle, near the horizon line)
- F (bottom, on the inner circle)
- T (centre): the Earth
- K (upper right, at the Moon): end of the line from T
- I (on the line from T to K, below the Moon)
- L (faint, within the small lunar circle): the Moon's centre

[Margin: A closer discussion of this problem.] X. To carry out this problem more scrupulously, it should be used above all when the Moon has no latitude, or very little. Two times are to be noted: one at which you begin to doubt whether she is shining half-lit, having previously been certain that she appeared less than half-lit in the evening, or more than half-lit in the morning; the other at which, after she has already appeared half-lit for some minutes of time, you now begin to doubt whether she exceeds or falls short of the exact phase of dichotomy. Between these two times the midpoint is to be taken as the moment of the half phase, for that phase has a certain breadth. The difference of that moment from the moment of true quadrature, converted into an arc of the ecliptic according to the Moon's hourly motion from the Sun, will give the angle S. Aristarchus, for his part, takes this angle as 3°, certain that it cannot be greater; for that was enough for him to show from it that the Sun is much further from us than Pythagoras, himself also a Samian, had placed it, following harmonic ratios, as I shall say in the Scholia, when he made the Sun's distance now three times, now one and a half times the distance of the Moon. [Margin: Longomontanus's opinion on the angle S.] Longomontanus, however, in Book 1 of the *Theorica*, chapter 6, p. 241, sets this angle at 2° 30′, from selected observations of Tycho with the Moon and the Sun placed at their mean distance, that is, between apogee and perigee; and to this corresponds, according to Longomontanus himself, a mean motion of the Moon from the Sun of $4^{h}\ 55\frac{1}{3}^{m}$. With this angle assumed, therefore, and with the Moon's mean distance at 56 terrestrial semidiameters, he gathers from the analysis of

the triangle LTS a mean distance of the Sun of 1283 semidiameters of the Earth, and indeed even 1288. [Margin: And Kepler's.] Kepler, by contrast, says in his Ephemeris for the year 1618 that the Moon appears halved, or bifalcate [bifalcatam], about 4 hours before the first quadrate aspect or after the second. On the Ephemeris for the year 1619 he has the following, worth noting: *But as for the difference of time between the dichotomous phase and the quadrature, although it has remained here too, its quantity, and that of the Sun's parallax as well, is still uncertain. And what I long ago reasoned from the slightness of the parallaxes of Mars, I am now most of all confirmed in, since I have been engaged in bringing out the Harmonics: that the Sun's horizontal parallax is not greater than one minute. Yet I still use it at two minutes, as in the years 1617 and 1618, because the matter is not yet fully settled. Therefore the interval of time between the dichotomous phase and the quadrature will be less than is gathered from the Ephemeris. This I leave to Galileo and Marius [Simon Marius] to determine with the eyes and the telescope, if so much trust is to be placed in it in this matter, especially given the roughness of the lunar globe. Exert yourselves, you men of philosophy: the matter is not to be despised, and is entirely worthy of having account taken of it in an Ephemeris; namely, from the bare sight of the Moon exactly bisected, and a faithful noting of the time, to the hour and at least its quarter, to pronounce on the ratio of the Sun's orb to the Moon's orb, and hence on the quantity of the Sun's parallax; and lastly on the very ratio both of the altitudes and of the globes of all the planets, compared with the semidiameter of the Earth. All of these have so far depended on the untrustworthy shadow of the Earth, and on the estimation of the digits eclipsed in the Moon, with their most vexing diversity; and lastly on the confusion of very many causes entering into the demonstration, each of which can be emended separately while the rest stand, with an incredible variation in the resulting parallax of the Sun.*

[Margin: The author's opinion on this.] XI. Mindful therefore of the problem of Aristarchus, and spurred on by Kepler's exhortation, I began, as often as I could, to direct at the dichotomous Moon my eye, both bare and aided by the telescope, with Father Francesco Maria Grimaldi striving at this along with me. I gave particular care to those phases in which the Moon held, as nearly as possible, apogee or perigee, or an equal distance from either; and to these, by God's favour, another piece of good fortune was sometimes added: that the Sun was at those times once near perigee and again around apogee, as I shall report in the Observations to be weighed in tome 3 of the *New Almagest*. And although I have not yet fully decided this matter, and am waiting for some other lunar dichotomies so as to pluck almost every scruple from my mind, I nevertheless believe that I have not served astronomy badly, and that I can declare, far more probably than others so far, that this angle S is never much greater or less than half of one degree; and hence that about one hour elapses between the phase of the half Moon and the quadrate aspect with the Sun.

Here I shall select two of our clearer observations of the lunar dichotomy. In the year 1646 October 16 the quadrature of the Moon was observed, from which the parallax was derived, and from that the Moon's distance TL, of 66⅔

terrestrial semidiameters, as I shall say in Book 4, chapter 14, number 11. Taking the time midway between the two extremes of the doubtful period, that is, the period within which, and neither before nor after it, we were certain that the phase of dichotomy occurred, the phase of dichotomy preceded the quadrature by 59^m 48^s. To this time, according to the true hourly motion of the Moon, belong 31′ 34″, which was the size of the angle S. Therefore, by the solution of the triangle TLS, the distance of the Sun TS was 7260 terrestrial semidiameters. The Sun, moreover, was 106° from apogee. Again, in the year 1648 June 13 the quadrature was observed, of which I shall speak in Book 4, chapter 14, number 11, and from the parallax the Moon's distance TL was derived, of 66 42′ terrestrial semidiameters. The time of the dichotomy, midway between the doubtful extremes, by which the phase of dichotomy followed the quadrature, was 58^m 33^s; and to this, from the Moon's true hourly motion at that time, belong 30′ 15″, and such was the size of the angle S. Therefore the distance of the Sun TS was 7572 terrestrial semidiameters, and it was then 15° from apogee. Moreover, at that middle time the section of the line separating the bright part of the Moon from the dark passed, as the telescope showed, through the spots of Amalfi, Pamphili, Mazarin, Leopold and Frederick of Holstein, of which I shall speak in Book 4, chapter 7. Now from the Sun's observed eccentricity, of 3460 parts of which the radius, or semidiameter, of the solar eccentric is 100000, as I shall show in chapter 24, number 4, and from the said solar distances, with the arc of anomaly assumed in which the Sun stood at the time of the said dichotomies, it follows by trigonometry that the radius, or mean distance of the Sun, is 7327 terrestrial semidiameters, the greatest, or at apogee, 7580, the least, or at perigee, 7074, and the eccentricity 253. This precision, however, will not always be needed, just as it is needed in defining the Sun's apparent semidiameters, parallaxes and equations; but sometimes round numbers will be enough: 7300 for the mean distance, 7600 for the greatest, and 7000 for the least, as we shall do in chapters 11 and 12.

XII. Not resting content with this, I wrote, both before and after, to the two explorers of the lunar phases most famous and most sharp-eyed in this century, Govaert Wendelen and Michael Florent van Langren, the Royal Cosmographer. [Margin: Wendelen's opinion on the distance of the Sun from this problem.] Wendelen, then, whom I was astonished to see raising the Sun, in the *Idea of the Atlantic Tables* [Idea Tabularum Atlanticarum], as far as 14656 semidiameters of the Earth at mean distance, and whom I had asked for the reason for this height, at last replied, in the year 1647 November 9, that he had done so because a parallax of the Sun of at most 15″ better represented the equinoxes and their intervals, and eclipses, and better reconciled the altitude of the pole, taken from circumpolar stars at Forcalquier in Provence, with the distance of the tropics taken at the solstices; but chiefly because of the lunar dichotomies [Greek: δικοτομίας [recte: διχοτομίας] (dichotomias, "dichotomies")]. For although it had earlier seemed to him that the Moon, when she shines half-lit for us, is 89° from the Sun, and hence that the Sun is about sixty times the lunar interval

[diastematis Lunaris] away from us, or about 3600 terrestrial semidiameters, yet, he says: *Using greater care after that, and better telescopes, I found, as you too did, that dichotomy [Greek:* διχοτομίαν *(dichotomian, "dichotomy")] occurs at a distance of the Moon from the Sun of about 89½°, from which the semidiameters now came out at 7000, more or less. Finally, however, comparing the dichotomies with one another, morning as well as evening, as far as the roughness of the Moon allowed one to reason, and considering also that the light of the Sun, falling into the lunar air, hastens the evening dichotomy and delays the morning one, I could, with every factor weighed, find nothing else than that dichotomy [Greek:* δικοτομιαν *[recte:* διχοτομίαν*] (dichotomian, "dichotomy")] comes down to a distance of 89° 45′, or even one that is too great. From this the distance of the Sun from the Earth was built up to at least 13740 semidiameters, which matched excellently with the earlier examination by parallaxes.* After this, having weighed the intervals of the planets, even the Jovian ones, and their admirable harmony, of which he speaks elsewhere, he concludes a mean distance of the Sun from the Earth of 14656 terrestrial semidiameters.

To this I reply that neither does that air around the Moon appear in telescopes, the large and very bright ones that I have at last obtained, nor, if it is there, is it to be reckoned of such height and density that by the refraction of the Sun's rays it could vary the phase of dichotomy by a whole quarter of an hour. So too Kepler, in the *Optics*, p. 303, said that the lunar air is of no great depth and very rarely attains a sensible refraction of the Sun's rays, and that one is not to take refuge in it except in the extreme necessity of saving a phenomenon otherwise indefensible. With that air, then, or its effect, set aside, Wendelen's observations of the dichotomy support me, who knew nothing of them (I call heaven to witness) when I first began my own, in order to bring the problem of Aristarchus back into practice, being the first to do so, as I thought. Let us now see what van Langren reported.

[Margin: Van Langren's reply on this, and his noble efforts.] XIII. Van Langren, then, replied to me in a letter dated 1646 August 22 that if the dichotomous Moon were 89° from quadrature, and the Moon's distance from the Earth were 60 terrestrial semidiameters, the Sun's distance would be 3420 terrestrial semidiameters. Although he indicates his opinion here, he nevertheless adds that he is wholly taken up with a certain device for this and for investigating other lunar phenomena more accurately, and that when he has finished it he will communicate everything to me. Meanwhile, the bisection of the Moon near the quadratures occurs now between the spots Isabella and Maria, now between Roma and Harum [?], with an admirable variety in the theory of the Moon, which he hopes will please astronomers. He is alluding to the spots of the Moon marked with those names, and to the lunar libration, of which I shall speak in the Book on the Moon. You see then, reader, that if van Langren asserts this distance, mine is almost the mean proportional between that of van Langren and that of Wendelen: for as 3420 is to 7300, so very nearly is 7300 to 14656. You see, moreover, how great the variety is in this very problem.

To make it more apparent, I have added the following small table, in which, according to the variation of the angle S, or of the distance of the dichotomous Moon from quadrature, and of the Moon's distance from the centre of the Earth, one may survey the variation of the Sun's distance from the Earth, gathered from the solution of the triangle LTS treated in number 9.

Distance of the Sun from the centre of the Earth in semidiameters of the Earth, with the distance of the dichotomous Moon given likewise in semidiameters of the Earth.

Distance of the dichotomous Moon from quadrature	34 terr. semid.	44 terr. semid.	52 terr. semid.	56 terr. semid.	60 terr. semid.	64 terr. semid.	66 terr. semid.	72 terr. semid.
3° 0′	650	841	994	1070	1147	1223	1261	1376
2° 45′	709	917	1083	1167	1251	1334	1376	1501
2° 30′	779	1009	1192	1284	1376	1467	1513	1650
2° 15′	866	1121	1325	1426	1528	1630	1681	1834
2° 0′	974	1261	1490	1605	1719	1834	1891	2063
1° 45′	1114	1441	1703	1834	1965	2096	2161	2358
1° 30′	1299	1681	1987	2139	2292	2445	2521	2751
1° 15′	1559	2017	2384	2567	2750	2934	3025	3301
1° 0′	1948	2521	2980	3209	3438	3667	3782	4125
0° 45′	2598	3361	3973	4278	4584	4889	5042	5501
0° 30′	3896	5042	5959	6417	5876	7334	7563	8251
0° 15′	7790	10084	11917	12834	13751	14668	15126	16501

Aristarchi Suppositiones, & Conclusiones.

The Suppositions and Conclusions of Aristarchus.

XIV. So far we have treated the problem of Aristarchus and adapted it to more recent hypotheses. Let us now return to Aristarchus and set out the suppositions on which this third problem rests, and the distance of the Sun that he gathers from them. They are as follows.

[Margin: The suppositions of Aristarchus.] Supposition 1: *The Moon receives her light from the Sun.* Hence it comes about that of the lunar sphere one hemisphere is bright and the other dark, and that the ray issuing from the centre of the Sun falls perpendicularly on the plane of the circle separating the bright part from the dark, as, in the figure of number 9, the ray SL falls on the plane IK.

Supposition 2: *The Earth has the ratio of a point and centre to the sphere of the Moon.* So it makes no difference whether the phase of the half Moon is viewed from the surface of the Earth or from the centre. It is better, however, in the triangle treated in number 9, to take the lunar distance TL shortened by one semidiameter of the Earth, because that phase is seen by observers on the surface of the Earth.

Supposition 3: *When the Moon appears half-lit to us, the great circle of the Moon that marks off her dark part from the bright one inclines toward our sight*;

that is, the plane of the circle IK lies on the straight line KT; otherwise it falls elsewhere, outside the whole Earth and our sight, toward G or R. But this supposition Aristarchus demonstrates afterwards in Proposition 5. Earlier, however, he shows in Proposition 2 that *the Sun illuminates more of the Moon than a hemisphere*, being larger than the Moon, and in Proposition 4 that *the circle dividing the dark part of the Moon from the bright does not differ sensibly from the great circle of the Moon.*

Supposition 4: *When the Moon shines half-lit for us, she is distant from the Sun by less than a quadrant by the thirtieth part of a quadrant.* So she is 87° from the Sun, and he takes the angle LTS to be that size, and hence the angle S is 3°, to which correspond about 6 hours of the Moon's motion in the zodiac. He assumes this generously, however, since it is certain even to those who take up the Moon carelessly near the quadratures, that no more than 6 hours elapse between her half phase and the quadrature. In much the same way the early astronomers were content to define the times of the equinoxes if they avoided an error of 6 hours. Moreover, in Proposition 6 he demonstrates that when the Moon shines half-lit for us, she is not a whole quadrant from the Sun; otherwise the angle LTS would be right, as TLS also is, and so in a rectilinear triangle there would be three angles together greater than two right angles, against 32 of the First Book of Euclid.

Supposition 5: *The breadth of the shadow is that of two Moons*, namely at the place where the Moon passes through the shadow of the Earth.

[Margin: A knot in Aristarchus is untied.] Supposition 6: *The Moon subtends the fifteenth part of a sign*, that is, 2°, and he repeats the same in Proposition 4. But since the Moon does not subtend even 40′, this has seemed absurd to many, and neither Pappus nor Commandino unties this knot. But Aristarchus may have assumed this in a sense excluding any greater diameter, certain that there would be no one to contend that the Moon ever appears larger, not even on the horizon; and if anyone wanted her to be smaller, that would further confirm his assertion. For since Pythagoras had said that the distance of the Sun was either three times or at most one and a half times the lunar distance, it was more than enough for Aristarchus to purge the Samian school of this error, and to show, from hypotheses that no one, however dull his wits, could deny, that the ratio of the solar distance to the lunar is much greater than triple or one and a half times. Otherwise Aristarchus, as Archimedes reports in the *Sand-Reckoner* [Arenario], held that the diameter of the Sun, and hence of the Moon eclipsing it, was 30′ at apogee.

[Margin: The conclusion of Aristarchus on the distance of the Sun.] The conclusion of Aristarchus, then, in Proposition 7, is this: *The distance at which the Sun stands from the Earth is more than eighteen times the Moon's distance from the Earth, but less than twenty times.* And so it is to the Moon's distance about as 19 to 1. This thesis is also reported by Plutarch, *On the Opinions of the Philosophers* [de Placitis], Book 2, chapter 31, and by Pliny the Elder, Book 2, chapter 21, who says: *Many have also tried to track down the intervals of the stars from the*

Earth, and have declared that the Sun is nineteen parts distant from the Moon, as much as the Moon herself is from the Earth. Aristarchus, however, does not state definitely the absolute distance of the luminaries from the Earth, nor do Pappus or Commandino following him; because Aristarchus did not want to tie his demonstrations to observations still uncertain, but, by assuming what no one could deny, to show that the mere proportions of the distances and magnitudes of the Sun, the Moon and the Earth are greater than many thought.

[Margin: A warning that the variation of the Sun's distance must be properly harmonized with the other things connected with it.] XV. However the distance of the Sun may be investigated and established, if the Sun is supposed to become now apogean, that is, farthest from the Earth, and now perigean, that is, nearest to the Earth, the mean distance will have to be determined from half the difference between the greatest and the least; but these must agree with the difference of the parallaxes and of the Sun's apparent semidiameter at such distances. And if not only does the Sun become now apogean and now perigean within the same year, but these distances themselves vary as the eccentricity of the Sun's orb varies, those who admit this must determine various distances of the Sun agreeing with it. Finally, those who bisect the Sun's eccentricity must make the difference of the greatest distance from the least half as great. All of this will be set out below, each in its own place.

[Margin: The authors from whom the following table was compiled.] Meanwhile it will be worth the effort to set before the eyes the more notable opinions on the distance of the Sun from the Earth. They are supplied, though scattered about, by Pliny the Elder, Book 2, from chapter 21 to 23; Plutarch, *On the Opinions of the Philosophers*, Book 2, chapter 31; Censorinus, *De Die Natali*, chapter 11; Ptolemy, Book 3 of the *Almagest*, chapter 4, and Book 5, chapter 15, and there Theon, and Regiomontanus in the *Epitome*; al-Battānī, chapters 28, 30, 50; al-Farghānī, Difference 21; Copernicus, Book 3, chapter 16, and Book 4, chapters 19 and 21; Neander [Michael Neander] in the *Elementa Sph.*, p. 124; Fernel, Book 1 of the *Cosmotheoria*, chapter 7; Maurolico at the end of the *Cosmographia*; Clavius in the *Sphere*, p. 215; Magini in the *Theoricae Planetarum*, Book 1, chapter 3, and Book 2, chapter 24, and in the *Supplement to the Ephemerides*, p. 58; Barozzi, Book 1 of the *Cosmographia*, p. 48; Mästlin in the Appendix to the *Mysterium Cosmographicum* of Kepler; Tycho, tome 1 of the *Progymnasmata*, pp. 98, 119, 472; Longomontanus, Book 1 of the *Theorica*, chapters 1, 5, 6, 9, that is, pp. 177, 231, 241, 298; Kepler in the *Nova Stella*, chapter 16, in the *Ephemerides*, p. 2, in the *Mars*, chapter 30, and in the *Epitome of Astronomy*, Book 4; Biancani, Book 10 of the *Sphere*, chapter 1; Lansbergen in the Prolegomena to the *Uranometria* and Book 2, Element 11; Bettini, *Apiaria*, Apiary 10, Progymnasma 1; Athanasius Kircher in the *Ars Magna Lucis et Umbrae*, Book 9, Problem 9; Antonín Maria Šírek in his *Radius Sydereomysticus*, Book 4; Boulliau, Book 4 of the *Astronomia Philolaica*, chapter 1; Wendelen in the *Idea of the Atlantic Tables* and in letters sent to me; Michael Florent van Langren in letters sent to me; Renieri and Argoli in their tables; Galileo, Dialogue 3 of

the *System of the World*. The opinion of Hipparchus, however, is gathered as varying, from the distance he gives for the Moon at apogee, now 72, now 83 semidiameters, and at perigee now 62, now 71, as Theon reports on Book 5 of the *Almagest*, chapter 11; and from the angle LTS, of 87°, which he too took as the greatest, as Pappus reports, and following him Commandino, in the notes to the little book of Aristarchus. If he thought that angle was in fact smaller, a smaller distance of the Sun results. On Posidonius, read Scholium 1.

If anyone wants the greatest or least distance of the Sun in parts of which the radius, or semidiameter, of the Sun's orb is 100000 or 10000000, let him multiply the apogean or perigean distance by that radius and divide the total by the mean distance; for the mean distance is as great as the semidiameter of the Sun's orb. On the distances to be assigned to the individual degrees of anomaly I shall speak in chapter 33, number 8.

Distance of the Sun from the centre of the Earth in semidiameters of the Earth.

Authors cited in number 15	Greatest, Sun at apogee	Mean distance	Least, Sun at perigee
From the data of Hipparchus	1586	1472	1357
or	1429	1379	1231
Posidonius	13141	✢	✢
Ptolemy and Maurolico	1210	1168	1126
Clavius, Barozzi	1210	1168	1126
al-Battānī and Alfonso	1146	1107	1068
From the data of al-Battānī, Lansbergen	7936	✢	✢
al-Farghānī	1220	1215	1210
Jean Fernel	1309	1256	1204
Copernicus, Magini	1179	1142	1105

Authors cited in number 15	Greatest, Sun at apogee	Mean distance	Least, Sun at perigee
From the data of Copernicus, Lansbergen	942	☩	☩
Michael Neander	1197	1160	1122
Mästlin, at greatest but at least eccentricity	1208½	1160	1111¾
	1197½	1160	1122¾
Io. Offusius [Jofrancus Offusius]		1152	
Tycho and Biancani	1180 [recte: 1182 1/12, per the book's errata; in this copy the last figure is covered by an ink blot, and a pen has written 2 beside it]	1150	1120 [recte: 1117, per the book's errata; in this copy pen strokes cross the third and fourth printed figures, which are given here as the errata quote them, and a pen has written 7 above the cell]
Longomontanus	1334	1288	1242
Kepler, in the *Ephemerides*	1800	1768	1736
And in the *Commentaries on Mars*			
In the *Nova Stella*	☩	1432	☩
In the *Epitome of Astronomy*	3469	☩	☩
From the parallax in the Rudolphine Tables	3438	3381	3327
Lansbergen, at least eccentricity	1550 52/60	1498 33/60	1446 16/60
Ismaël Boulliau	1485 56/60	1460	1433 52/60

Authors cited in number 15	Greatest, Sun at apogee	Mean distance	Least, Sun at perigee
Athanasius Kircher	1940¾	1906¼	1872⅔
Antonín Maria Šírek	2073	2000	1927
Govaert Wendelen	14905	14656	14407
Galileo	☩	1208	☩
Mario Bettini	☩	1145	☩
Van Langren supposes for the present	☩	3420	☩
We for the present	7580	7327	7074
but sometimes it is enough in round figures	7600	7300	7000

The distance corresponding to the individual degrees of anomaly is obtained from the base of a rectilinear triangle, one side of which is the mean distance, or radius of the eccentric, and the other the eccentricity; and the angle they contain is the complement of the anomaly.

SCHOLIA

[Margin: Opinion of Empedocles and of Eratosthenes on the Sun's distance.]

I. The opinions of the ancients on the distance of the Sun and of the Moon from each other and from the Earth are reported as follows. Plutarch, in Book 3 of *De Placitis Philosophorum*, Chapter 31, writes: "Empedocles holds that the Moon stands twice as far from the Sun as from the Earth; the mathematicians, that the Moon stands nineteen parts from the Sun for the distance it stands from the Earth; Eratosthenes, that the Moon is seven hundred and eighty thousand stadia from the Earth." Therefore, since Eratosthenes gives the Earth's semidiameter 39772½ stadia and its diameter 79545 stadia, as we showed in the table of Book 2, Chapter 7, it follows that he set the Moon's distance at more than 19 and less than 20 semidiameters of the Earth.

[Margin: Opinion of Pythagoras on the same.]

Pliny the Elder, moreover, in Book 2, Chapter 21, after indicating Aristarchus's view with his name suppressed, adds: "Pythagoras, a man of

shrewd mind, inferred that from the Earth to the Moon is one hundred and twenty thousand stadia; from the Moon to the Sun, double that; and from there to the twelve Signs, triple; and our own Gallus Sulpicius was also of this opinion." So the earlier view of Pythagoras, like that of Empedocles, made the Sun's distance triple the Moon's distance from the Earth; but it made the Moon's distance about six times smaller than Eratosthenes later did. Censorinus agrees with Pliny the Elder, in *De Die Natali*, Chapter 11, at p. 59 in my Aldine copy, where he says: "Therefore Pythagoras thought that from the Earth to the Moon is about 126 thousand stadia." I said "the earlier", because he adopted harmonic intervals and preferred another opinion of the Sun's distance, and made it one and a half times the Moon's distance, as we shall report presently from Pliny the Elder, Book 2, Chapter 22, and from Censorinus above, when we speak of the harmonic intervals of the stars.

[Margin: Opinion of Posidonius on the same.]

Pliny the Elder goes on, in Book 2, Chapter 23, and says that according to Posidonius it is at least four hundred stadia from the Earth to the region of clouds and winds; from there to the Moon, twenty times a hundred of stadia, that is, twenty times a hundred thousand stadia, or 2000000 stadia; and from there to the Sun, five thousand times (I understand, in the Roman manner, a hundred thousand) stadia, or 500,000,000 stadia. So, adding those three numbers together, there would be from the Earth to the Sun 5020004000 stadia. Now Posidonius assigns to the Earth's semidiameter 38182 stadia and to its diameter 76364, from the table set at the end of Chapter 7 of Book 2. He therefore gives the Moon's distance as 52 15/38 semidiameters of the Earth, and the Sun's distance from the Earth as 13141.

[Margin: Pliny the Elder's conjecture on the Sun's distance.]

Having recited these, Pliny the Elder offers his own opinion in that same Chapter 23. He does not call it certain, only conjectural, yet such that the reasoning of geometric inference in it, which never deceives, cannot be rejected, if anyone cares to pursue these matters more deeply: namely, that from the Earth to the Sun is a sixth part of the circle of the solar heaven. Why? Because that circle is of about 366 parts, as is clear from the days of the year; and the diameter is to the circumference as about 7 to 22, or nearly a third part; therefore the semidiameter, or the Sun's mean distance, is about a sixth part of the solar circle. (He neglects the Sun's eccentricity.) And since the Moon's period, being monthly, is twelve times shorter than the Sun's annual period, the Moon's distance from the Earth is for that reason twelve times smaller than the Sun's distance from the Earth. With these things laid down, Pliny the Elder continues: "It is a marvel how far the shamelessness of the human heart will go, lured on by some slight success, as in the matters above reason furnishes the occasion for impudence. Having dared to divine the distances from the Sun to the Earth, they apply the same to the heaven, since the Sun is in the middle, so that the measure of the World itself comes straightway to the fingers. For they say that

as many sevenths as the diameter has, so many twenty-seconds has the circle, as though the measure of heaven were plainly settled by the plumb line."

[Margin: Childish opinion of Petosiris and Nechepso on the Sun's distance.]

Finally, Pliny the Elder closes Chapter 23 with the opinion of Petoniridis [Petosiris] and Necepsi [Nechepso], kings of Egypt, who said that each part or degree of the lunar circle contains 33 stadia or a little more, that of the circle of Saturn double, that is, 66 stadia, and that of the Sun's circle half the sum of the two measures, that is, 49½ stadia. So if these are multiplied by 360 degrees, there will be 11880 stadia in the Moon's circle, 17820 in the Sun's, and 23760 in Saturn's; and their distance from the Earth, or the semidiameter of the lunar heaven, is about a sixth part of this, or about 1980 stadia, that of the solar heaven 2970, and that of the Saturnian heaven 3960. "This computation carries a great deal of shame, since at the circle of Saturn, once the interval of the zodiac itself is added, the multiplication becomes countless." These are the words of Pliny the Elder, lashing the childish opinion of those kings and indicating his own opinion of a far greater interval.

[Margin: Tycho's opinion and its basis.]

II. So much for the outdated opinions of the ancients, among which, however, the opinion of Posidonius strongly favours the hypothesis of Wendelen. But the ancients were imitated by those who have tried to assess the distances of the Sun and of the other stars from certain other proportions. Thus Tycho, in Volume 1 of the *Progymnasmata*, p. 472, seeing that the Sun's mean distance was set by Copernicus at 1142 semidiameters of the Earth and by Ptolemy at 1165, chose roughly the mean, namely 1150. He was confirmed in this by the little book of Io. Franci Offusij [Jofrancus Offusius], *De Divina Astrorum Facultate*, where he arranges the distances of the heavenly bodies from the Earth harmonically, and has the Sun stand 576 terrestrial diameters from here, so as to assign to this interval that sacred number celebrated by the Pythagoreans and Platonists, since eclipses do not object to it. In that interval there are 1152 terrestrial semidiameters. But Tycho neglects two semidiameters.

[Margin: Kepler's opinion, long inconstant.]

Kepler too, though only hypothetically, assigns 1432 semidiameters of the Earth to the Sun's distance in his book *De Nova Stella*, Chapter 16, so as to make the Sun's diameter six times greater than the terrestrial diameter. In the *Ephemerides*, however, p. 2, he gives the Sun an apogeal distance of 1800 terrestrial semidiameters and the Moon about 60, so that the Sun's distance is thirty times the Moon's, just as the Sun's revolution of 365 days is about thirty times 12 revolutions of the Moon, which are contained thirty times in the Sun's revolution. But he wavers in fixing the Sun's horizontal parallax. For in the same place he says that he sets it smaller than the Tychonic value by two-thirds [beſſe]. The Tychonic value with the Sun at apogee is 3′ 7″; therefore, with eight-twelfths, or two-thirds, taken away, a parallax of 1′ 2″ remains. And in the preface to the Ephemeris for the year 1619 he says that the slightness of the parallaxes of Mars and the harmonic intervals confirm him in the view that

the Sun's parallax is not greater than 1′. "Yet in the Ephemeris for the years 1617 and 1618," he says, "I use one of 2′, because it is not yet fully settled." Why, then, does he assert on that p. 2 that he has set it smaller than the Tychonic by two-thirds, if he was still retaining 2′? In the *Commentaries on Mars* too, p. 162, he sets the Sun's distances: the apogeal at 101800 parts, the mean at 100000, and the perigeal at 98200, of which parts the radius of the Sun's orbit, or the mean distance, is 100000. So if the apogeal distance is 1800 semidiameters of the Earth, the mean will be 1768 and the distance at perigee 1736.

[Margin: But at last constant.]

But in the Rudolphine Tables, Precept 141, the Sun's parallax is always the fifteenth part of the Sun's semidiameter; and this semidiameter, in the same Tables, p. 92, is 15′ with the Sun at apogee; so the parallax is 1′. Now by the analysis of the triangle treated in number 2 of the preceding chapter, given a parallax of 1′, the Sun's distance is shown to be 3438 semidiameters of the Earth. But if the distance of 1800 semidiameters of the Earth is retained, the parallax will be 1′ 55″. It is not to be retained, however; one must stand by his last work, the Rudolphine Tables, in which the Sun's apogeal semidiameter is 15′ and the horizontal parallax 1′; the mean semidiameter 15′ 16″ and the parallax 1′ 1″; and the perigeal semidiameter 15′ 32″ and the parallax 1′ 2″. Therefore the distances corresponding to these, in order, are 3438, 3381 and 3327 semidiameters of the Earth. Remember, however, that in Kepler's mind the eccentricity is to be bisected, by his own advice in the Appendix to Tycho's *Progymnasmata*, p. 821, and from what will be said below when we treat of the eccentricity.

[Margin: Opinion of Šírek.]

III. Antonín Maria Šírek, in his *Radius Sidereomysticus*, p. 20, says that the Sun's distance from the Earth is 2000 terrestrial semidiameters. On p. 94 he says that, from observations of his own and of others made with the greatest diligence, the Sun stands 200 of its own semidiameters from the Earth, and that its semidiameter is to the semidiameter of the ecliptic as 1 to 10, or as 10 to 100, or as a square root to its square. And on p. 193 he says that the bulk of the Earth is to the bulk of the Sun as a square root, but to the distance as a cube root, the former being as 10 to 100 and the latter as 10 to 1000 diameters; and that the Sun thus stands 1000 diameters, or 2000 terrestrial semidiameters, from the Earth. He proves this mean distance, he says, because the apparent diameter of the Sun that he observed at middling distance is 34′ 41″, to which in the table of sines correspond 100 parts, of which the whole sine is 10000. "Therefore," he says, "the Sun's diameter of 100 parts is the square root to the radius, that is, to the true distance." He adds that, just as the Earth's diameter is contained a thousand times in the Sun's distance, so the diameter of the Sun's body is contained a thousand times in the mean distance of Saturn from the Sun; and that the diameter of the body of Saturn is the square root, in solar diameters, to the radius of the firmament, and that this radius is 1000000 solar diameters, which multiplied by 10 give 10000000 diameters of the Earth, or 20000000 semidiameters. But in this at least Šírek is mistaken: that sine of

100, for the angle of 34′ 41″, would stand even if the Sun were almost infinitely many semidiameters of the Earth away; and unless the distance of the Sun is established with definiteness from elsewhere, nothing can be concluded by this method.

[Margin: Why the harmonic intervals are not given here.]

IV. The harmonic intervals of the Sun and the other planets, which Pliny the Elder, Book 2, Chapter 22, and Censorinus, at the end of Chapter 11 of *De Die Natali*, report on the opinion of Pythagoras; and Macrobius, Book 2 on the *Dream of Scipio*, Chapter 4, on the opinion of Plato; or those of Jofrancus Offusius in *De Divina Astrorum Facultate*; or of Johannes Kepler in his *Harmonice* and in the *Mysterium Cosmographicum*; or of Mario Bettini in *Apiarium* 10, we do not insert here, because they cannot be weighed until the distances of the remaining planets have been explained; nor is there such force in them that these distances could be defined from them alone. I have therefore put all this off to the Book on the Systems, where there will be special chapters on the harmonic system of the world.

[Margin: Father Francesco Maria Grimaldi's problem, a rival to the Aristarchan.]

V. Father Francesco Maria Grimaldi of the Society of Jesus has rivalled Problem 3, or the Aristarchan problem, even outside the dichotomies, by observing the part of the Moon's diameter that shines for us, together with the Moon's place at the same moment, either observed or deduced from the most corrected tables, so that the Moon's elongation from the Sun in the zodiac becomes known from the Sun's known place; here too the Moon's distance from the Earth and her apparent semidiameter are assumed. The observation will be better if the Moon is in the nonagesimal degree of the ecliptic counted from the horizon, or, what comes to the same thing, if both her horns are in the same vertical circle.

Let the centre of the Earth, then, be T, that of the Moon A, and that of the Sun S, and let these be joined by the three straight lines TA, TS, AS. For in the triangle ATS the Moon's distance from the Earth, AT, and the Moon's elongation from the Sun in the zodiac, namely the angle ATS, are assumed as given. So if the angle TAS is found, the third angle S can be known by 32 of the First Book of Euclid, as the complement of two right angles; and in this way the side TS, namely the distance of the Sun from the Earth, can be found by the second of our *Plane Oblique-Angled Triangles*.

Diagram. A woodcut in the left column, with the text wrapped at its left from "recidit, si vtrumq." to "inuestigetur an-". A circle stands for the Moon's disc, with its left part hatched dark and the part on the right, past a curved boundary running from H down to I, beside which stand F, K and C, left white. Curved lines cross the disc between G and E and between H and I, and a further curve runs through D, F and E and through B and C. Straight lines run from T, at the foot, up to I, to C and to E, and from T to S at the top right, and a straight line runs from S down to the disc at F.

- S (top right, at the apex of the long lines): the Sun
- H (top of the circle): one horn of the Moon
- D (upper left within the circle)
- F (upper right, on the lit side)
- G (left end of the horizontal diameter)
- K (right of centre, on the horizontal diameter)
- E (right end of the horizontal diameter)
- A (centre): the Moon's centre
- C (lower right, at the edge of the shading)
- B (lower centre, within the shading)
- I (bottom of the circle): the other horn
- T (foot of the figure): the Earth's centre

So, to find the angle TAS, let there be observed with the radius, or with another more exact instrument, one of those that we treat in the Book on Instruments [Libro Organico], the part of the Moon's diameter that appears shining to us, namely KE, but as seen by means of the arc CE. This is done more accurately if through a fine telescope some small spot is noted that then, in the middle of the Moon's disc and equally distant from the horns H and I, begins to be lit before the first quadrature or ceases to be lit after the second, and if the situation of such a spot is known from the *Selenography* of Michael Florent van Langren, on which we speak in Book 4, which is on the Moon. This done, let GHEI be the boundary of the hemisphere, or quasi-hemisphere, of the Moon apparent to us, whose dark part is GHKI and whose shining part is HKIE; and further let the circle that divides the part of the Moon lit by the Sun from the part not lit be HKID. Let a great circle of the Moon GDEB be understood, whose plane inclines both toward the Sun S and toward our eye T, and in which AB and AF are semidiameters of the Moon. The circle GDEB will be described from the horns H and I, as being virtually diametrically opposite, both because it is a great circle and because the point F is equally distant on either side from the Moon's diametrically opposite horns, since the line AF produced runs to the centre of the Sun illuminating the Moon, by the ray SA perpendicular to the plane of the circle HKID, and this ray SA stands equally distant on one side and the other from the extreme rays that determine the Moon's horns H and I. Therefore both the dark point B and the points C and E, which are luminous and visible to us, as being equidistant from the horns, lie in the same plane of the circle GDEB. So, with TC and TE drawn, and with the observed arc CE subtracted from the known arc of the Moon's semidiameter BCE, the arc BC, that is, the angle ATC, will remain known. With this angle in the triangle ATC, and with the true semidiameter of the Moon AC, find the angle TAC, and add it to the right angle SAC, which the Sun's central ray makes with the plane of the circle dividing the lit part of the Moon from the unlit part; and the angle SAT, which we needed, will be known.

CAPVT VIII.

De Solis Parallaxibus.

I. Reread what we said about parallaxes in general in Book 1, Chapter 35, especially about the parallax of altitude, or in the vertical circle, which we said is the difference between the true altitude of the Sun above the rational horizon, as viewed from the centre of the Earth, and its apparent altitude, or the altitude seen by an eye on the surface of the Earth. Its measure is the angle made at the centre of the Sun by the straight lines drawn to it, one from the centre of the Earth, the other from the sight placed on the surface of the Earth. This parallax is easily found if the Sun's distance from the centre of the Earth is given in parts, of which the Earth's semidiameter is one.

[Margin: ⊙ Finding the horizontal parallax.]

II. For let the figure of Chapter 6, Problem 1, be repeated here, in which VSR is a quadrant of a vertical circle in the heaven of the Sun, AC beneath it is a quadrant of a vertical circle on the globe of the Earth, H is the centre of the Earth, HR is the rational horizon, AB, parallel to it, is the sensible horizon, and the line VAH, perpendicular to both, is drawn from the zenith V. Then the angles HAB and VAB are right angles. Now let the Sun appear at B, a point of the sensible horizon, to which let the straight line HB be drawn from H. Then the horizontal parallax is the angle ABH, in the same triangle, right-angled at A, in which the base HB is given, namely the distance of the Sun from the centre of the Earth, and also HA, as 1 part of those known in HB. So by the second of our *Plane Right-Angled Triangles* you will find the angle B, and very easily too; for here the side HA is unity, and it will therefore be enough to divide the radius, or whole sine of the tables, by the distance HB: the quotient [Quotus, seu Quotiēs] will be the sine of the angle B, that is, of the Sun's horizontal parallax. Or if you use logarithms, the residual of the logarithm of the distance HB will give the logarithm of the angle B. Now since this parallax is equal to the alternate angle BHR, by 29 of the First Book of Euclid, which measures the arc BR described from H, it reveals the Sun's true altitude above the rational horizon HR, which is the arc BR. So if the horizontal parallax is, for example, 3′, you will conclude that the Sun, appearing at B and with its centre not raised at all above the sensible horizon, is nevertheless raised 3′ above the true and rational horizon.

Diagram. A woodcut in the right column, with the text wrapped at its left from "Repetatur enim" to "Horizon Sensibilis AB". A quadrant VSBR is centred on H, with the vertical side HAV and the horizontal side HCR; a smaller arc runs from A to C. Straight lines join A to S, A to B, H to S and H to B.

- V (top left, on the vertical side): the zenith
- S (on the outer arc, upper): the Sun
- A (on the vertical side, below V): the eye on the Earth's surface
- B (on the outer arc, right): point of the sensible horizon

- H (bottom left): the centre of the Earth
- C (on the base, at the foot of the small arc)
- R (bottom right, at the end of the base)

[Margin: Finding the parallaxes that are not horizontal. Given the apparent altitude.]

III. Secondly, let the Sun be above the sensible horizon at S, and let the line of true altitude, HS, be drawn to its centre from the centre of the Earth, and from the eye A the line of seen altitude, AS. And let the angle VAS be given by observation or assumed by hypothesis, namely the complement of the seen altitude to 90°, or the Sun's seen distance from the vertex; for by 13 of the First Book of Euclid the angle SAH will be known, as its complement to two right angles, or to 180°. With this angle, and with the given distance of the Sun, HS, and with the side AH, which here is unity, you will find in the triangle AHS, by the first A[13] of our *Plane Oblique-Angled Triangles*, the angle S, which is the measure of the parallax sought in the circle of altitude, or vertical circle. But since the angle SAH will always be greater than one right angle, and by the rules of trigonometry you must take in its place its complement to two right angles, namely the angle VAS, and since the side AH is here unity, you will therefore easily find the parallax S if you divide the right sine of the Sun's seen distance from the vertex by the Sun's distance HS; for the quotient will be the sine of the angle S. Or, working by logarithms, add the logarithm of the angle VAS, or of the seen distance from the vertex, to the residual of the logarithm of the distance HS; and there will result, with a redundant unit or two taken away on the left, the logarithm of the angle S.

[Margin: Use of this parallax.]

This parallax, further, is to be added to the seen, or apparent and observed, altitude, to obtain the Sun's true altitude, or the arc RS. Conversely, it is to be subtracted from the seen distance from the vertex, to obtain the true distance, or the arc VS. For by 32 of the First Book of Euclid the angle VAS is equal to the two angles AHS and ASH taken together; therefore the angle S is the amount by which the angle VHS differs from the angle VAS.

[Margin: How a table of parallaxes is to be constructed.]

IV. By this art the tables of the parallaxes of the Sun, or of any star, in the circle of altitude must be made, if their distance from the centre of the Earth is given. And it is expedient to construct them for three distances of the star, namely the apogeal, the mean and the perigeal, as we shall show at the end of the Book. For other distances one must use a proportional part, or else, setting those tables aside, have recourse to the problem given here in number 2 or 3.

[Margin: Error of certain astronomers in the use of the parallactic table.]

So far we have spoken of finding parallaxes when the seen or observed altitude of a star is given and it has to be converted into the true altitude. This use is served by the table of parallaxes that Kepler gives in the *Astronomia*

[13] so printed, an unexplained letter

Optica, and after him Boulliau in the *Philolaic Tables*, p. 174, and Dom Vincenzo Renieri in the reprinted *Medicean Tables*, p. 332. But when the true altitude of the star is given and the parallax to be subtracted from it must be found, in order to obtain the visible or apparent altitude, another problem and another table must be used. For although the angle S, for example, may be the same, the same value cannot be inferred if the seen altitude and the true altitude are taken for one and the same; nor can the angle VAS be equal to the angle VHS. Those who seek parallaxes in one and the same table, therefore, using the same degree of altitude whether given or seen, as not a few do, are seriously mistaken, especially in lunar parallaxes, which are larger and more perceptible. I shall nevertheless show below, in number 7, how one may use a single one of these tables without error by a double entry. Meanwhile, however, you must be taught the way to construct and use a separate table when the altitude of the star, or its complement or distance from the vertex, is not seen but true.

[Margin: Horizontal parallax, given a star on the true horizon.]

V. First, in the following figure, let the Sun be at R, a point of the rational horizon with respect to the eye A, so that its true altitude is nil and its true distance from the vertex is the arc VR, of 90°. For the parallax at R will be not one of altitude, as many think, but one of depth. But the Sun cannot first be seen below the horizon HR except by an eye raised at least to C, along the line CIR, which, however, will be the sensible horizon of the eye C; whereas the rational horizon of the eye I will be HN, and each horizon will serve the eye I, not raised. Now since AH, the distance of the eye A from the centre of the Earth H, is equal to IH, the distance of the eye I from the same centre, and the Sun's distance HB is equal to its distance HR; and since the angle HAB, made by the tangent BA, is a right angle by 16 of the Third Book of Euclid, as is also HIR, made by the tangent RI, then by 4 of the First Book of Euclid the angle R, which is the horizontal parallax for the eye I, will be equal to the angle B, or the horizontal parallax of the eye A. In this case, therefore, it is enough to find the horizontal parallax by the problem treated in number 2, for that will be the parallax, though one of depth, namely the angle R; and if the eye were raised to C, it would see the star below the horizon HR by just as much as this parallax amounts to.

Diagram. A woodcut in the left column, with the text wrapped at its left from "Sit Primò in sequenti" to "infra Horizontem HR, nisi". A vertical straight line runs from V at the top through C, A and H to M at the bottom, and a large arc runs from V around the right through B, R and N to M. A smaller circle is centred on H and passes through A. Straight lines run from A to B and from H to R; further straight lines run from H to B and from H to N, a line runs from C through I to R, and a dotted line runs from H up to I.

- V (top, on the vertical line): the zenith
- C (on the vertical line, below V): the raised eye
- A (on the vertical line, on the small circle): the eye
- B (upper right, on the large arc)

- I (small letter, on the small circle, upper right): the eye on the Earth's surface
- H (centre): the centre of the Earth
- R (right, on the large arc)
- N (lower right, on the large arc)
- M (bottom, on the vertical line)

[Margin: Non-horizontal parallax, given the true altitude.]

But if the true altitude of the star, as is usual in the use of parallaxes, raises the star above the sensible horizon as well, and in the earlier figure the Sun is, for example, at S, then in the triangle AHS, treated in number 3, the star's distance from the centre of the Earth is given, as I assume, which is the side HS; and the side AH, as one part of such parts; and the angle AHS, of the true distance from the vertex, or the complement of the true altitude above the horizon. So by the third of our *Plane Oblique-Angled Triangles* the angle S can be found, which is the measure of the parallax sought, to be subtracted from the true altitude to obtain the apparent one, or conversely to be added to the true distance from the vertex to obtain the seen distance VAS.

[Margin: Use of this parallax.]

For the angle S is the amount by which VAS exceeds VHS, by 32 of the First Book of Euclid.

[Margin: In which authors these tables of parallaxes are found.]

VI. By this art the tables of parallaxes in the circle of altitude must be constructed, given the true altitude of the star above the horizon and its distance from the centre of the Earth; and the same tables are to be applied using the true altitude. Most of the tables made so far for the parallax of the Moon and of the Sun in the circle of altitude are of this kind. Since we are now speaking of the Sun, such are those given by Ptolemy, Book 5, Chapter 18 of the *Almagest*; Copernicus, Book 4, Chapter 24; Reinhold in the *Prutenic Canons*, p. 121; Magini in the *Canons of the Second Mobiles*, p. 681, and in the *Supplement to the Ephemerides*, p. 59; Tycho in Volume 1 of the *Progymnasmata*, p. 98, and after him Mullerius [Nicolaus Mulerius?], Adriaan Metius and Pierre Hérigone; likewise Longomontanus in Book 1 of the *Theorics*, Chapter 8; Lansbergen in his tables, p. 46; Wendelen in the *Idea of the Atlantic Tables*; and others here and there.

[Margin: Kepler's self-contradiction.]

Kepler gives no such table in the Rudolphine Tables, but in Precept 141 he says that the Sun's horizontal parallax is always the fifteenth part of the Sun's apparent semidiameter, and on p. 2 of the *Ephemerides* he states that it is smaller than the Tychonic parallax by two-thirds. There, however, he contradicts himself, in assigning to such a parallax a distance of the Sun of 1800 terrestrial semidiameters, as we showed in the preceding chapter, Scholium 2.

[Margin: Use of these tables for either altitude.]

VII. You can, however, use the tables treated in number 6 to convert a seen altitude into a true one, in this way. Look up the seen altitude in these tables

just as if it were true, and add the parallax corresponding to it to that same altitude. Then with this sum look up another parallax in the same table; for this is the one that, subtracted [recte: added, per the book's errata] [Margin: *addita* (”added”), written by hand in ink, with a pen stroke through the printed word] from the seen altitude first given, will give the true altitude.

However the Sun's horizontal parallax has been derived by the authors from a presupposed distance of the Sun from the Earth, treated in the preceding chapter, I wish to set out their opinions on it in the following table, so that they can be compared with the distances set by the same authors and reported at the end of the preceding chapter. For Lansbergen, in his *Uranometria*, p. [no number printed], Book 2, Element 8, asserts that, with the exception of Ptolemy, the parallaxes of the other earlier astronomers disagree with the distances, or with the half-angle of the cone of the Earth's shadow and the apparent semidiameter of the Sun. But of this more conveniently below, in Chapter 11, numbers 7 and 8.

Horizontal parallax of the Sun at the Sun's three distances from the centre of the Earth.

Authors	☉ mean distance	☉ at apogee, max. distance	☉ at perigee, min. distance
Posidonius	1′ 45″	1′ 40″	1′ 50″
Ptolemy	2′ 57″	2′ 51″	3′ 4″
al-Battānī	3′ 7″	3′ 0″	3′ 14″
al-Farghānī	2′ 56″	2′ 50″	3′ 3″
Fernel	2′ 40″	2′ 36″	2′ 46″
Copernicus	3′ 0″	2′ 55″	3′ 6″
Neander	2′ 59″	2′ 54″	3′ 5″
Mästlin	2′ 56″	2′ 51″	3′ 2″
Tycho	3′ 0″	2′ 54″	3′ 7″
Longomontanus	2′ 40″	2′ 35″	2′ 45″
Kepler, from the Rudolphine Tables	1′ 0″	0′ 58″	1′ 2″
But from the distance 1800	1′ 57″	1′ 55″	1′ 59″
Lansbergen	2′ 18″	2′ 13″	2′ 23″
Boulliau	2′ 21″	2′ 19″	2′ 24″
Kircher	1′ 46″	1′ 41″	1′ 51″
Bettini	3′ 0″	☩	☩
Šírek	1′ 44″	1′ 39″	1′ 49″
Wendelen	0′ 14″	0′ 14″	0′ 14″
Van Langren, for the time being	0′ 59″	☩	☩
Ourselves, for the time being	0′ 28″	0′ 27″	0′ 29″

SCHOLIA

I. Another way of inferring the horizontal parallax I shall give very shortly, and more conveniently, in Chapter 11, number 7.

But I have indicated ways of observing the Sun's parallaxes independently of its distance in Chapter 7, nos. 3 and 4; they are, however, very difficult. Ways of deriving the other kinds of parallax from the parallax of altitude, namely those of longitude, latitude, declination and right ascension, we shall indicate in Book 5, where we treat eclipses of the Sun, and in the Precepts on eclipses, and in the Optical Book, in the section on parallaxes. But in the view of Wendelen and of Posidonius, and indeed very nearly in our own, the horizontal parallax is so slight that the other kinds can for the most part be safely neglected.

[Margin: Uncertainty of the Sun's parallaxes.]

II. All the ways of gathering the parallax from the Sun's distance, treated in Chapter 7, nos. 6 and 9, are just as uncertain as the distance of the Sun found by means of them. For that reason Hipparchus, or even Ptolemy (Book 5, Chapter 11), left it in doubt whether the Sun has any sensible parallax at all. Maurolico, in Dialogue 1 of the *Cosmographia*, p. 11 ½ [?], considers this variation of view insensible. And Kepler says in the *Astronomia Optica*, p. 304: "On the length and breadth of the Earth's shadow and on the Sun's parallax, if you look to quantity, deliberation is still timid." In the same work, p. 351, speaking of observing the digits of the Moon's diameter in eclipse, he says: "I wish astronomers would take pains to establish some surer methods of this observation. For on this one thing depends what is commonly valued most in astronomy: the measure of the height, and of the body, of the Sun. For if the Sun's greatest parallax is 3′ and one errs by a third of a digit in estimating the lunar defect, 600 semidiameters of the Earth are gone; but if the Sun's greatest parallax is 2′, we shall go astray by 1700 semidiameters of the Earth through the omission of a single 1′ in the lunar defect." Yet in the *Ephemerides* he prefers another method to this one, namely that of Aristarchus, and thinks little of this one that relies on eclipses, as I showed in Chapter 7, no. 10, toward the end. Therefore this business does not depend on this one thing, as is said here. Still, reader, with everything weighed, you see how far from ignorantly, let alone rashly, Wendelen said, in a letter sent to me, that the hypothesis of a Sun [Greek: ἀπαραλλακτικὴν (aparallaktikēn, "without parallax")], that is, one using no parallaxes, cannot be convicted of falsehood from the phenomena, and that from it motions no less exact, and indeed often more exact, are derived. Recall the foundations on which the Sun's distance rests, from Chapter 7 and its Scholia, and you will give Wendelen a kinder hearing.

CAPVT IX.

De Refractionibus Solis.

On the Refractions of the Sun.

[Margin: The refractions of the stars known to the ancients.]

I. First, what we said about refractions in general in Book 1, Chapter 36 should be read again. The evident cause of refractions appears from it: the rays of the Sun and of the other stars fall into the atmosphere, which is denser than

the aether and the purer air and of a different transparency from theirs. And some astronomers or writers on optics did recognise that the lines of the rays of the stars reach our eye refracted, as is clear from Ibn al-Haytham, Book 7, Propositions 15 and 16; from Witelo, Book 10, Propositions 49 and 50; and from Werner in his notes on Book 1 of Ptolemy's *Geography*. Bernardus Vualtherus [Bernhard Walther] too, as he himself relates in his observations under the year 1489 March 6, recognised that Venus in the evening appeared to him above the horizon because of broken rays, when it was in reality below the horizon. The same thing is reported to have happened with ♀ to the Landgrave of Hesse, in Tycho's *Epistolae Astronomicae*, p. 22, and to Mästlin, in Kepler's *Astronomia Optica*, p. 156.

[Margin: Tycho the first measurer of refractions.]

But no one brought a remedy, and before Tycho no one taught a way of correcting observed altitudes of the stars by subtracting the refraction, let alone of measuring those refractions. By way of the circumpolar stars Tycho had obtained the altitude of the equator at Uraniborg as 34° 5′ 15″. From the Sun's meridian altitude at the solstice, corrected by adding a parallax of 1′ 35″ and brought to 57° 36′ 45″, he had gathered the Sun's greatest declination from the equator to be 23° 31′ 30″. Then at the winter solstice, with one instrument after another, exactly divided and applied, he always found the Sun's meridian altitude to be 10° 39′ 42″; and when its parallax of 3′ 3″ was added he had obtained a true altitude of 10° 41′ 45″ [recte: 10° 42′ 45″]. Subtracted from the altitude of the equator, this left the Sun's greatest southern declination as 23° 22′ 30″, which is less than the northern by a full 9′. He concluded that this arose from no other source than the refraction of the solar ray produced in such close proximity to the horizon, which apparently raised the Sun's centre above the horizon, as he himself relates in volume 1 of the *Progymnasmata*, from p. 14. So he devised a way of bringing to measure the refractions of the Sun, of the Moon and of the other stars, which he gives in volume 1 of the *Progymnasmata*, p. 93. We, however, shall make that way more universal, and useful by another method as well. It is as follows.

[Margin: Tycho's problem for measuring refractions.]

II. Let the horizon be HR and the meridian HVRD, in which are the pole of the equator P and the zenith or vertex V; and let the semicircle of altitude or vertical be VAD, in which let the true place of the Sun, corrected for parallax, be at B, but let it appear by the force of refraction at the point A of the horizon. Let two arcs, complements of the Sun's declination, be drawn to the pole P, one from the seen place A, which is AP, and the other from the true place B, which is BP. On a day, then, but a very clear one, of the summer solstice, when the variation of declination from one day to the next is scarcely sensible, let alone within a few hours, let the azimuth of the rising Sun at A be observed, that is, the arc of the horizon AH between the Sun at A and the meridian at H, an arc measured by the angle AVH, for its complement to 180° will be the angle AVP. Moreover, let there be given in the triangle BVP, from the Sun's true

declination, the complement BP, and the complement of the pole's altitude, VP. From these three givens you will find the arc VB by analysis of triangles; and if you subtract from it the quadrant VA, or 90°, there will remain the known arc AB, which is the difference between the true place of the Sun, B, and the seen place, A, and so the horizontal refraction.

Diagram. Set into the right column beside number II: a circle with a vertical line from V at the top to D at the foot and a horizontal line from H at the left to R at the right, meeting at A. P stands on the circumference at upper right. C lies on the vertical above A and B below it, and lines run from P to C, to A and to B. Labels read: V, P, C, H, A, R, B, D. Labels not read: none.

- V (top of circle): zenith or vertex
- P (circumference, upper right): pole of the equator
- C (on the vertical, above A): the Sun's apparent place in number III
- H (left): horizon, meridian point
- A (centre, on HR): the Sun's seen place in number II and true place in number III
- R (right): horizon
- B (on the vertical, below A): the Sun's true place in number II
- D (foot of circle): foot of the vertical

[Margin: Part 2 of the problem.]

III. Now let the Sun be at A, its true place, but let it appear by the force of refraction at C, so that the seen altitude is the arc AC; and let the arc CP be drawn from C, the complement of the known declination; and let the azimuth AH have been observed as above when the Sun appears at C. For so the angle AVH will be known, as above, and from it the angle AVP in the triangle AVP. Since in that triangle VP, the complement of the pole's altitude, is given, and AP, the complement of the true declination, the arc AV can certainly be found by trigonometry. Therefore if, when you observed the azimuth of the apparent Sun at C, you also observed its seen distance from the vertex, namely the arc VC, then with VC subtracted from the arc VA the refraction AC will become known. And so for the others, by observing on the solstitial day the individual degrees of the Sun's apparent altitudes, with their azimuths, from its rising to noon and from noon to its setting. The same is done proportionally for the Moon and for the fixed stars.

[Margin: Our method in observing refractions.]

IV. We, for our part, did use this method, but we observed the azimuths with the help of a filar triangle [Trianguli filaris], which shows the plane of the vertical circle with the greatest exactness, and we waited for the Sun to reach a determined degree of azimuth. This is far more certain than the usual azimuthal quadrants, in which a modest error in the inclination of their plane to the horizon produces an error in the altitude of the stars near the horizon that is not to be despised. For that reason Tycho confessed more than once that azimuthal observations are slippery and full of suspicion, as we shall show in the *Liber Organicus*, which is the second book of volume 2 of our *New*

Almagest. But we have investigated another method besides this one. Having taken the altitude of a known fixed star before sunrise, and having deduced from it the time before noon, we began, with companions well trained for the purpose, to count the times by means of a pendulum, which showed us the time of the primum mobile with the greatest exactness. In this way, in place of azimuths, we used angles at the pole P. So if the Sun appeared at C, the true time before noon was known and the angle APV was also known; with it, and with VP, the complement of the pole's altitude, and AP, the complement of the declination, the arc VA, of the Sun's true distance from the vertex, was found. From this, when there was de- ducted the seen distance VC, observed with a large quadrant, the refraction AC remained known.

[Margin: Tycho's other method.]

V. The other method, which Tycho suggests in volume 1 of the *Progymnasmata*, p. 94, uses the Sun's declination observed with equatorial armillaries in place of azimuths, and with the remaining quantities, given as above, it reveals the refractions. For in the triangle VCP there are given VP, the complement of the pole's altitude; VC, the Sun's seen distance from the vertex, observed with the astronomical quadrant; and CP, the complement of the Sun's seen declination, observed with the armillaries. So the angle VCP will not stay hidden, and when it is subtracted from two right angles, or from 180°, it will reveal the angle ACP. With this, and with CP, the complement of the seen declination, and AP, the complement of the true declination, you will find in the triangle ACP the arc of refraction AC. But this method is of little use for the horizontal refraction. Nor is observation with armillaries as simple, evident and safe as is required here, as I shall teach in the *Liber Organicus* from Tycho himself; and it demands, besides, the analysis of a double triangle.

[Margin: Use of refraction.]

VI. The primary use of a refraction once found is in correcting a star's apparent altitude. To that altitude the parallax of altitude is to be added but the refraction of altitude subtracted, and in this way the true altitude will remain, since the effect of refraction is entirely contrary to the effect of parallax: parallax apparently lowers the star, whereas refraction apparently raises it. The secondary use is in correcting declination, right ascension, longitude, latitude and distance from another star, and in converting these from apparent into true, or from true into seen and apparent. To indicate this too: if in the triangle VCP there are given VC, the complement of the observed altitude, VP, the complement of the pole's altitude, and CP, the complement of the seen declination, the angle CVP can be found. Then with it, in the triangle AVP, and with VP, and with VCA (that is, with the refraction AC added to CV), AP, the complement of the true declination, can be found. Likewise, in the triangle ACP, given AC, the arc of refraction in altitude, and the complements of declination AP, true, and CP, seen, the angle APC can be found, which is the difference of right ascension. And so for the others, as will be set out more fully in the common Problems of the primum mobile [Pimi] [recte: Primi] and in the Optical Book,

in the section on refractions, where, God willing [D.O.M. volente], we shall teach many subtler things about refractions, worth knowing.

[Margin: Tables of refractions.]

VII. Tycho, then, having measured refractions in Denmark by the method set out in nos. 2, 3 and 5, left us a table of them: the solar refractions at p. 79 of volume 1 of the *Progymnasmata*, the lunar at p. 124, and those of the fixed stars at p. 280. Others have since copied these out, such as Longomontanus, Book 1 of the *Theorica* [Theoricor.], Chapter 8; Kepler in the *Astronomia Optica*, p. 121, and at the end of the Rudolphine Tables; Bettini, *Apiarium* 8, Progymnasma 1, Proposition 2; Hérigone, volume 4 of the *Cursus Mathematicus*, p. 98; Boulliau in the *Philolaic Tables*, p. 179; and Metius in the *Astrolabium*. Lansbergen, however, gives his own in his tables, p. 46, and we shall give ours at the end of this Book. Meanwhile it is worth noting that refractions near the horizon vary by a headlong increase or decrease; and that with Tycho those of the Sun and of the Moon vanish around the 45th degree of altitude, and those of the fixed stars around the 20th degree. With the same author the refractions of the fixed stars are 4½′ less than the solar, and can serve for the refractions of the other planets, as he himself admits on that p. 280. But the refractions of the Moon nearer the horizon are less than the solar, and afterwards at some points greater, whereas on account of the greater inclination of the ray that is refracted they ought all to be greater than the solar, as I shall show in the Optical Book. So for Tycho the horizontal refraction is 34′ for ☉ but 33′ for ☾, and for the fixed stars 30′ or 29′ 30″. Kepler touches on the cause in the Rudolphine Tables, Precept 197, but we say more on this in the *Optics*, where the following point of Kepler's in the *Astronomia Optica*, p. 283, must also be weighed: whether, namely, through an eclipse the total refraction at any place can be 61½′.

SCHOLIA.

[Margin: Variety and inconstancy of refractions.]

I. Since the causes of refractions are various, they also yield various refractions. The density of the air and the thickness of the atmosphere, and its height, are not the same on every horizon, but greater in more humid places and colder ones. Nor are they constant on the same horizon through the whole time of the year, but greater about the borders of winter and spring, and greater at the equinoxes than at the summer solstice, and in rainy or cloudy weather or in snowy. Tycho, however, in the *Epistolae* and in volume 1 of the *Progymnasmata*, p. 280, denies that refractions in Hesse are twice as small as in Denmark (as Rothmann kept saying had been observed); he ascribes that rather to a fault of the observations.

[Margin: Various questions about these refractions.]

We must discuss in our *Optics* whether a difference of light, and of distance from the Earth, and of figure, in the star itself or in the surface of the atmosphere, and of the depth of that same atmosphere, makes for a different refraction.

Likewise, how great is the height of the refracting air, and whether it is as great as that of the exhalations that produce twilight, as Tycho and Longomontanus held, but Kepler otherwise. Likewise, whether the density of the atmosphere, decreasing uniformly difformly toward the heavens, reduces the refracted ray to a curved line, or to one straight line, or to several straight lines disposed circularly or parabolically. Likewise, whether the cause of the headlong difference of refractions near the horizon is the far longer stretch of atmosphere through which the refracted ray must pass, as Tycho, Longomontanus, Metius and Cabeo thought, or only the difference of inclination of the ray falling into the atmosphere, or something else. Again, whether Tycho compiled his table of refractions separately from the parallax or with it included. For this reason I have now obtained, from Father Albert Curtz, S.J., who at the Emperor's order now directs the bringing to light of Tycho's observations, the observation of the solar refractions made by Tycho at the summer solstice. Finally, whether a double table is needed, one for the star's true altitude and the other for its apparent altitude, as I said of the parallactic tables in the preceding Chapter; and whether Tycho's table must be corrected again on account of the perhaps excessive parallax of the Sun that he adopted; and things like these, which could not conveniently be treated in this place. The ignorance of Christmann, who denies that the ray leaving a star's centre is refracted, and who keeps saying that star altitudes are therefore not to be corrected by subtracting the refraction, on the ground that the altitudes of stars are observed by that ray alone, is to be refuted more clearly in the same place.

[Margin: The more striking effects of refractions in astronomy.]

II. It is astonishing how many and how striking are the effects of refraction, especially in the luminaries, so much so that if they are unknown astronomy is almost lacking one eye. For first, by visibly raising a star and all its visible points, refraction changes its ascensions and descensions, and its longitude, latitude and declination. II. It contracts the distance between two stars, or two luminous points, above the horizon, but increases it below. III. In the oblique sphere, among northern peoples, it increases the northern rising and setting amplitude and diminishes the southern, if they are compared with the points of rising and setting of the true equinoctial. IV. In the torrid zone and the temperate zones it increases the artificial day by about 8^m or 10^m, on the one side hastening sunrise by 4^m or 5^m of time, and on the other delaying sunset. But in the parallel sphere and in the frigid zones it sometimes increases the day by several hours, and indeed by days, as I shall presently relate. V. It hastens the vernal equinoxes and delays the autumnal, and sometimes makes the hollow of equinoctial armillae appear equally lit on both sides twice on the same day, so that the equinox seems to occur twice. This astounded Hipparchus, who did not know the cause, and gave Ptolemy a handle for suspecting the Alexandrian armillae, as though they had slipped from their original position. VI. It shrinks the apparent diameters of the stars along the altitude, and so shows us the Sun and Moon at the horizon egg-shaped and almost elliptical. VII. It makes the

luminaries, though diametrically opposite and placed on or below the horizon, appear above the horizon, as Cleomedes observes toward the end of Book 2; so that a lunar eclipse is seen even though the Moon is really below the horizon and the Sun above. This was observed, on the testimony of Pliny the Elder, Book 2, Chapter 13, when the two Vespasians were consuls for the third time [Conſulib. duobus Veſpaſianis tertium], namely in the year of Christ 72 February 22; and by Mästlin in the year 1590 July 17, on the testimony of Kepler in the *Astronomia Optica*, p. 136.

and it will happen in the year 1668 May 25, in the morning, on the horizon of Rome. VIII. It makes several Suns and Moons appear in the atmosphere at the same time, on different horizons; and the same Sun at the same moment is below the horizon for some and above it for others. I say the same of the Moon, especially the full Moon.

[Margin: The remarkable Dutch observation for the refractions of ☉.]

IX. In the frigid zones it makes the Sun rise several hours earlier than it otherwise should, and indeed in some places several days earlier, and set later. This is what happened to the Dutch at Novaya Zemlya [nouam Zemblam] in the year 1596, as is told in the account of that voyage of the Dutch. At a pole altitude of 76°, taken again and often by their navigators, they saw the Sun set on 1596 November 4 and stay hidden below the horizon until 1597 January 24, when its rising was unexpected to them, since according to the situation of that horizon and the Sun's declination it should not have risen until February 11 and should have set on October 17 of the preceding year. So the refractions delayed the setting by 17 whole natural days and brought forward the rising by as many days. That this was the effect of refractions is rightly taught by Kepler in the *Astronomia Optica*, p. 138, by Magini in the *Primum Mobile*, Book 6, Problem 16, and by us in the Optical Book. X. If there is vaporous air around the Moon and it has its own atmosphere, of density sufficient for sensible refractions, its phases must vary in eclipses and outside them, and more of the lunar hemisphere, as also of the terrestrial, must be lit by the Sun than would otherwise be the case; and so the shadow of the Moon and of the Earth must be contracted and cut off, as I shall explain more fully in the proper places.

CAPVT X.

De Magnitudine Solis Apparenti.

On the Apparent Size of the Sun.

[Margin: The difficulty of observing the Sun's diameter exactly.]

I. The Sun's apparent size is estimated from its apparent diameter, which it is either impossible or very difficult to determine evidently without an error of 1′. Archimedes at last perceived this after his own efforts and those of Aristarchus, in *The Sand-Reckoner*, where he says:

[Margin: On the testimony of Archimedes.]

"We assume this, since Aristarchus says that the Sun appears as if it were the seven hundred and twentieth part of the circle of the zodiac. For he himself considered how he might capture with instruments the angle that fits the Sun, having its vertex in the eye. But to take any such thing exactly is not so readily done, since neither sight, nor hands, nor the instruments with which the observation is made, are sufficiently worthy of trust to demonstrate it exactly." Now the 720th part of a circle is 30′, and Sosigenes, in Proclus's *Hypotyposis* [Hippot. Astron.], judged this diameter to be always that many minutes.

[Margin: And of Ptolemy.]

Nor did Ptolemy, as he relates in Book 5 of the *Almagest*, Chapter 14, using Hipparchus's dioptra, [find][14] either an exact measure of this diameter or any sensible difference between the diameter of the Sun at apogee and at perigee. For he says: "We too constructed a dioptra with a rod of four cubits, which Hipparchus indicated, and observing the Sun's diameter by it we found that it is everywhere comprehended under very nearly the same angle, no sensible difference worth attention being produced by the distances." He goes on to say that the Moon at apogee and the Sun appear under an equal angle, but adds: "How great an angle is subtended, however, seems very doubtful to us; for the measurement, which is made by repeatedly moving forward or drawing back along the length of the rod, between the sight and the plate, a little plate set against the luminaries, can lead into error away from the exact truth." So he has recourse to lunar eclipses, from which, as I shall teach when treating the Moon's diameter, he derives the diameter both of the Moon at apogee and of the Sun as 31′ 20″, although this very method relies on an estimate of eclipsed digits, which has no greater certainty than the use of the Hipparchan dioptra.

[Margin: And also of Kepler.]

II. Kepler recognised this difficulty, and at the same time the necessity of determining the diameters of the luminaries, in the *Astronomia Optica*, p. 212, saying: "The more careful astronomers do not trust the eye. This is to be guarded against much more when very slender quantities are called to the measure of an angle, as when we inquire into the diameters of the luminaries; which must be done, as a foundation, by all who have undertaken the restoration of astronomy." And on p. 339 he expressly admits: "In any ray whatever let in through a window of any kind, even a square one, the senses do not follow mathematical subtlety; but, by 29 of the first chapter, they do not apprehend the extremities of the luminous image, to which few particles of the Sun radiate, because of the brightness of the intermediate parts of the image, to which all the particles of the Sun radiate." It is as if he were saying that the penumbra which lies between the image of the Sun received through a hole or little window on a plate and pure shadow cannot be exactly distinguished from pure shadow or from the brightness of the luminous image, as I shall teach more clearly below.

[Margin: Kepler's excessive confidence.]

[14] no main verb is printed here; "find" is supplied, as the book's errata supply *reperit* after *ſenſibilem*

If this is so, how then does Kepler, who professes on p. 340 to have used this method, say so confidently on p. 343: "But the matter is certain and open to anyone to investigate, that the Sun's diameter is 30′ at apogee and 31′ at perigee"? Is it really open to anyone? Certainly not to Archimedes, not to Ptolemy, not to Scheiner, not to other very celebrated astronomers, as I shall presently say; not to us; and not even to your own Tycho.

[Margin: Tycho's opinion.]

For Tycho, using a channel or tube of about 32 feet at the solstices and equinoxes, detected the diameter of the Sun at apogee as nearly 30′, and rather less by a few seconds, and at perigee as exceeding 32′ by very few seconds. On account of the evidence of his own experiment, he could not on anyone's authority set the Sun's diameter at mean distance greater than 31′; and he set the apogeal at 30′ and the perigeal at 32′, as he himself reports most expressly in volume 1 of the *Progymnasmata*, p. 471. What more need be said? Do you yourself not relate in the *Astronomia Optica*, p. 339, that Tycho's people used a square hole, but one too wide for a complete intersection of the rays to be possible? And on p. 341, that Tycho's channel was of 1000 parts, of which the breadth of the square hole was 10; that the Sun's diameter found by him at the summer solstice was 29′ 54″, and that in other observations it stands between 28′ 40″ and 31′ 20″; and that on December 5, with the channels changed three times, it was found to be 31′? And there you add that Tycho sets the perigeal diameter of the Sun greater than the apogeal by 2′, on account of the Sun's eccentricity, found to be 3600 parts of which the Sun's mean distance, or the semidiameter of its orb, is 100000. Finally, on p. 342 you relate the altitudes of the Sun's limbs observed by Tycho on March 15, and the varying diameter found from them, namely 29′ 30″, and 30′ 4″, and 30′ 6″, and 30′ 40″, and 30′ 44″, and 30′ 50″.

But of course Kepler, in whose mind it had already settled that Tycho's eccentricity of 3600 parts must be bisected and reduced to 1800 parts, could not admit a difference of more than 1′ between the apogeal and perigeal diameters of the Sun. Yet on p. 330 of the *Astronomia Optica* he confirms this bisection from a subtly observed diameter of the Sun. But this could not persuade us, who have almost wasted away in investigating this same diameter by several methods. Now the causes of this difficulty cannot be perceived unless the methods by which the Sun's diameter is usually or can be investigated are known. There are indeed several of these, and among them not a few devised by us, as will be clear from our *Liber Organicus*, where we shall describe the old and new instruments for the purpose. In this place, however, at least the chief ones must be indicated, the reader being referred there for inspection of the instruments.

Primus Modus obſeruandi Solis Diametrum.

First Method of Observing the Sun's Diameter.

[Margin: The astronomical radius.]

III. All the methods of this practice are carried out either with the eye, whether naked or covered with plates or coloured glasses, directed at the luminaries themselves, or on their shadow or luminous image received on some plate or wall. Among these the most ancient is the one carried out with the astronomical radius [Radium Aſtronomicum], described by Regiomontanus in *De Cometis*, Problem 13; by Gemma Frisius in an entire little work on the subject; by Nonius, Book 2 of *De Observatione*, p. 73; by Finé in *De Re et Praxi Geometrica*, Book 1, Chapter 6; by Pierre de la Ramée, Book 9; by Longomontanus in the *Astronomia Danica*, Book 2 of the *Sphaerica* [Sphæricor.], Chapter 7, and Book 1, Chapter 9; by Metius, Book 3 of the *Doctrina Sphaerica*, Chapter 2; and by Bettini, *Apiarium* 2, Progymnasma 3; and it is indicated by Tycho at p. 33 of the *Astronomia Mechanica* [Aſtronomiæ Mechanicæ]. This radius was succeeded by the Latin radius [Radius Latinus], so called from its inventor Latino Vrſino [Latino Orsini], and illustrated with a commentary by Egnazio Danti.

[Margin: The unreliability of both radii.]

But in truth both radii are deceptive, and in the diameters of the luminaries are exposed to an error of 3′ and more, as Tycho observed above, and in volume 1 of the *Pro- gymnasmata*, pp. 247 and 672, and in the *Epistolae Astronomicae*, pp. 8, 22, 28, 38; Longomontanus above; Kepler in the *Astronomia Optica*, pp. 215 and 343, where he calls that instrument crude [craſſum]; and Scheiner in his *Oculus*, or Optical Foundation, Book 3, p. 246.

There are four causes of the deception. The first is the difficulty of moving the cross-piece [Tranſuerſarium] forward or back while keeping it at right angles to the index; but the Latin radius removed this one. The second is the cheekbone, which prevents the eye from being applied exactly to the head of the radius. The third is the parallax of the visual angle [anguli viſorij], which is formed not at the surface of the eye but either at its centre or at its back. So the angle under which the diameter of the luminaries is seen is smaller than the one under which it is thought to be seen at the surface of the eye; and unless it is known in which part of the eye this angle is formed, an operation begun with these radii cannot be corrected. The fourth is the straddling [diuaricatio] and pulling apart of the eye this way and that, to the right and left margin of the luminaries, since it cannot attend exactly to both at the same point of time.

[Margin: New kinds of radius.]

For this reason, in the *Liber Organicus*, which is the second book of volume 2 of the *New Almagest*, we set out several other kinds of radius; among these, the ones that need two observers are larger and harder to use, but more certain. And in the Optical Book, which is the third of the same volume 2, in the section on the eye, we have investigated the part of the eye in which the so-called visual angle is formed; and in the same place we have treated the correction of an observation begun with radii.

Secundus Modus obſeruandi Solis Diametrum.

Second Method of Observing the Sun's Diameter.

IV. So, to cheat the cheat of the old radius, Hipparchus and Ptolemy chose the dioptra instead, if we believe Kepler in the *Optics* [in Optic.], p. 216, and Cabeo on the *Meteorologica*, text 9, question 1. This dioptra, with the plate near the eye fixed, is described by Proclus in the *Hypotyposis Astronomica*, Chapter 3; by Maurolico in the *Cosmographia*, Dialogue 3; by Biancani in the *Sphaera*, Part 6, Chapter 5; and by Cabeo above. But Egnazio Danti describes it with the plate near the eye movable, in the *Astrolabe*, p. 293.

[Margin: The Hipparchan dioptra is deceptive.]

But Ptolemy, in Book 5 of the *Almagest*, Chapter 14, admits that it is liable to error and does not yield an exact diameter of the luminaries. We too have used this dioptra, and one of 12 feet at that, so that the error in the length of the rule and in the distance of the plates would turn out smaller. We detected that the centres of the two small bright circles do not unite in one sensibly indivisible point, and that the place of the angle subtending the Sun's diameter is not certain to the point that it does not lie beyond or short by several sensible particles.

Tertius Modus Obſeruandi Solis Diametrum.

Third Method of Observing the Sun's Diameter.

V. Others, therefore, chose to take at the same moment, with two astronomical quadrants, the altitudes of the upper and lower limb of the luminaries, or with two armillae their declinations, by means of two observers; for the difference of altitude or of declination gives the apparent diameter. Kepler relates in the *Astronomia Optica*, pp. 342 and 348, that Tycho used this method, but with astonishing diversity and inconstancy of observations.

[Margin: A blemish of Lansbergen.]

And yet Lansbergen, in the *Treasury of Observations* [Theſauro obſeruationum], pp. 100 to 105, does not hesitate to call them "indubitable"; though when he sees Tychonic observations conflicting with his own tables, he nevertheless corrects them as faulty and deceptive.

The altitude of each margin can also be taken by admitting a ray of the Sun through a chink of known height in a wall, onto a pavement on which the wall stands at right angles, and measuring the length of the ellipse that the ray forms and its distance from the wall. Kepler teaches this method in the *Astronomia Optica*, p. 342, and so do we in the *Liber Organicus*, where we shall report our observations made in the church of San Petronio at Bologna, in which there is a metal plate with a hole for admitting the Sun's ray, at a height of 65 Bolognese feet. But all these methods suppose that the margins of the luminaries, or the boundaries of light and shadow, can be distinguished and reached precisely, without suspicion of sensible error, and without taking in any sensible particle;

and anyone willing to make the trial will find it evident that this can scarcely, or not even scarcely, be done.

Quartus Modus obſeruandi Solis Diametrum.

Fourth Method of Observing the Sun's Diameter.

VI. This method consists in counting the time in which the entire disc of a luminary rises above the horizon. The kings of Egypt, indeed, used for this purpose the running of a very swift horse, which they let go when the upper limb of the Sun was rising and checked when the whole Sun had now risen. For knowing the stadia that the horse usually covered in one hour, and the stade or part of a stade it had completed in the meantime while the Sun was rising, they worked out the time, and from it the parts of the equator corresponding to the Sun's diameter.

[Margin: The Sun's ascent counted by a horse's run.]

Although this method is very crude, because of the horizontal refractions and the varying obliquity of the ascensions, Cleomedes nevertheless uses it, in Book 2 of the *Cyclical Theory*, to blunt the grossness of Epicurus, who thought that the Sun could be said to be one foot across, as large as it commonly appears.

But a slightly better method of counting the same time, of the Sun or the Moon ascending, was by clepsydras, or water-clocks [Hydrologia]. These are discussed by Cleomedes in the same place; by Proclus in the *Hypotyposis Astronomica*, Chapter 3; by Martianus Capella, Book 8 of the *Philology*; by Maurolico in the *Cosmographia*, Dialogue 3; by Egnazio Danti, *Astrolabe*, Part 6, p. 290; by Reinhold in the *Theoricae*, p. 206; by Barozzi in the *Cosmographia*, Book 4, p. 339; by Kepler, *Astronomia Optica*, p. 147; and by Bettini, *Apiarium* 8, Progymnasma 3, Proposition 8.

[Margin: The use of clepsydras is deceptive.]

Now Cleomedes relates that the water of the lower clepsydra, which had flowed down from the upper while the whole Sun was ascending, was found by the Egyptians to be to the water of the clepsydra of the entire day as 1 to 750 or 700; and that the Sun's diameter was accordingly estimated, against the equator, as 1 to 750 or 700, that is, 28′ 48″ or 30′ 52″. Proclus and Macrobius relate that this was usually done at the time of the equinox. But even then, in Egypt, where the equator ascends obliquely, the obliquity of the ascent, the horizontal refractions and the uniformly difform fall of the water made this procedure [pragmatiam] uncertain, and so it was rejected by Ptolemy, Book 5, Chapter 14, and by Proclus, Theon, Pappus and Kepler.

[Margin: And of the sand-glass.]

Now David Rivault, on the *Sand-Reckoner* of Archimedes, would have this done, in place of the Ctesibian clepsydra [Cleopſydræ Creſibianæ], by a sand-glass [horologium arenarium], weighing the sand that has run out while the Sun ascends against the sand run out through an entire hour. But he is mistaken, because more sand falls at the beginning, owing to the weight of the greater

mass, which presses from above and drives the sand downwards; and more than once, from the motion of the pendulum, of which more shortly, I have learned that more sand falls in the first quadrant than in the second.

[Margin: The quicksilver clock is deceptive.]

For this reason neither are these clocks suited to such subtleties, nor are those that Tycho had devised from mercury [hydragiro], as he himself admits in volume 1 of the *Progymnasmata*, p. 149, having found Mercury, or quicksilver, no less cunning in this than in eluding astronomers in the heavens and chemists on Earth.

[Margin: Striking clocks are deceptive.]

Nor are striking clocks [Horiſona horologia] suitable either, because toward the end the weights, by reason of leverage, turn the toothed wheels more than at the start; besides the other inequalities to which they are subject on account of humid or dry air and of winds. Tycho perceived this in the same place, p. 145.

[Margin: *Scaphia* are useless for this.]

Sundials in bowls [Sciaterica … horologia in Scaphijs], which used to be employed for this purpose, as Macrobius relates, *On the Dream of Scipio*, Book 1, Chapter 20, are also useless. For although on the equinoctial day they observed two termini of the Sun's shadow, one when the upper limb was rising, the other when the lower limb of the Sun had now risen, and found that interval to be the ninth part of an hour, that is, 1° 40′, this method was nevertheless most deceptive, because the motion of the shadow is far greater around the rising and setting of the Sun than far from them, and very unequal; whereas the ancients, like the good Macrobius, thought it equal. And in all sundials the arrival of the shadow at the lines cannot be discerned without fear of an error of one or several minutes.

[Margin: The times of a pendulum are far better suited to this.]

VII. I, therefore, having at last been assured, on the occasion of another experiment, that the oscillations of one and the same pendulum, keeping the same height, since it was suspended not from a thread but from a small chain, and travelling through the same plane, are of equal duration among themselves, with no sensible difference ever appearing, acquired for myself, among others, a pendulum of which one simple vibration was of one half-second and the compound vibration of one second, as I said in Book 2, Chapter 20; and with this second pendulum I made fifty observations on the Sun. For with my head resting immovably on a trans- verse little beam [trabeculæ], and with my eye protected by coloured glasses and set in a motionless little tube, I waited until the western limb of the Sun began to pass across the meridian, which was marked by two very fine threads set up perpendicularly in the plane of the meridian. Immediately, at a given signal, my assistant, who was usually either Father Francesco Maria Grimaldi, or his brother Father Vincenzo Maria, or Father Zenus, counted the simple oscillations of the pendulum toward one side until the Sun's centre reached the meridian, and then until the whole Sun had

passed and its eastern margin came, as it were, to optical contact with the two threads mentioned.

I did the same very often with the round image of the Sun, admitted through a tiny hole onto a white plate parallel to the Sun, the room being darkened all round, while my assistant counted the time in which that entire base of the radiant cone crossed a little line drawn very finely in the plane of the meridian. When the days were equinoctial, it was enough to convert that time into parts of the equator for the Sun's apparent diameter to become known; but on other days, the Sun's declination being assumed, the arc of the parallel that had crossed the meridian had to be reduced to an arc of the equator, according to the proportion of the parallel to the equator. We shall bring out these observations in volume 3 of the *Almagest*, together with others.

[Margin: In this method too there is some suspicion of a small error.]

In the meantime I frankly confess that with all this subtlety and patience I did not attain the certainty I had hoped for from it. The cause was the uncertain beginning and end of the transit of the Sun's margins, because the threads, however very fine, nevertheless take off and cover some particle of the Sun on either side; so that about 2 seconds of time are counted before you are sure of that beginning and end. As a result, suspicion of an excess or deficiency of one half-minute cuts off exact knowledge of the solar or lunar diameter (for we have very often examined the lunar one too by this method). In the transit of the bright base, moreover, the doubtful boundaries of light and of pure shadow, on account of the penumbra related to both, raise the same suspicion; for the diameter of the hole was subtracted from the space traversed, and so deception could not creep in on us from this source. Yet each method is far more certain than any of those proposed so far, and sufficient to refute clearly those who foist on us a diameter of the Sun greater than 35′ at perigee, or less than 29′ at apogee. All things considered, I judge the truest of all to be 30½′ at apogee and 33′ at perigee, with a few seconds wanting on either side.

Quintus Modus Obſeruandi Solis Diametrum.

Fifth Method of Observing the Sun's Diameter.

VIII. This method was used by that Atlas of the last age, Tycho, with the help of a channel [canalis], as he himself relates in volume 1 of the *Progymnasmata*, p. 471. It was also used by Kepler in the *Astronomia Optica*, Chapter 11, from p. 340 on; by Scheiner in the *Rosa Ursina*, Book 4, Part 2, from Chapter 4 on; by Father Cysat at Ingolstadt; and by us at Bologna, more than sixty times, both in our observatory and in the church of San Petronio.

The procedure is as follows. Into a room thoroughly darkened on all sides a ray of the Sun is admitted, especially around noon, through a round hole in a small plate [lamellæ], or through a square little window, whose breadth, however, should be known in the smallest possible particles, say in hundredths of one inch [vnciæ]. The diameter of the hole or the breadth of the little

window ought not, however, to exceed about two inches of the Roman foot. That plate must be movable in such a way that its plane receives perpendicularly the central ray of the Sun, that is, the axis of the radiant cone or pyramid. The circular base of the other radiant cone, or the square base of the pyramid, is to be received on a white plate [tabellâ] parallel to the plane of the plate in which the hole or little window is; and the distance of these plates must be known in the parts in which the diameter of the hole or the breadth of the little window is known.

Next, with very sharp compasses, the diameter of the bright base of the radiant cone, or the breadth of the pyramidal base, is to be measured, including the penumbra as well, and circumscribing, that is, excluding, only the simple and pure shadow. From that measure the breadth of the little window or the diameter of the hole is to be taken away, for the reason demonstrated by Kepler, and by us in the Optical Book. And so there will remain the corrected base of the image of the Sun, half of which will be one side of a right-angled rectilinear triangle; the other side about the right angle will be the distance of the plates; and with these, by analysis of triangles, the angle subtending the Sun's apparent semidiameter is found.

[Margin: A twofold method within this method.]

Further, others use plates alone, parallel to one another and receiving the Sun's central ray perpendicularly, especially when their distance is very great, as with Tycho, for whom it was about 32 feet, and with us, sometimes 50, and sometimes, as in the church of San Petronio, even greater than 170 feet. But in these cases the boundaries of light and shadow turn out more doubtful, and the motion of the bright base becomes faster, so that a sudden and very attentive observation is needed. Others, however, such as Kepler, Scheiner, Cysat, and we ourselves more often, use a small beam with the already mentioned plates fixed at its ends, so that they can raise and lower them and move them to the right and to the left, as the opportunity of catching the Sun's ray requires, and, as a short way to greater darkness, can place between the plates a blackened paper tube, which also gathers the fibres of the rays so that they are not scattered.

[Margin: Scheiner's error [Greek: ἀμάρτημα (hamartēma, "error, fault")].]

Now Scheiner's distance of the plates was indeed 10000 parts, but 10 Roman feet; and although he varied the holes from time to time, larger and smaller, he never subtracted from the diameter of the bright base, as he should have, the diameter of the hole; and so he found that diameter both inconstant and almost always larger than it should be. Nor does Bettini mention this correction when he indicates this method in *Apiarium* 8, Progymnasma 3, Proposition 10; but in Scholium 2 he applies another remedy, far more difficult, by which the full crossing of the rays [plenaria radiorum decuſſatio], made far from the hole, has to be found. Of this discovery more fittingly in the Optical Book.

But Kepler's distance of the plates was 10368 parts, of which the diameter of the hole was 16½. We, for our part, have used various tubes and distances of the plates, namely of 8, 10, 12 and 15 feet. But we added two things for a more

exact observation. First, we employed an assistant, namely Father Francesco Maria Grimaldi, who from the right, with a very fine needle, would prick the plate at the boundary of pure shadow, as soon as I, from the left, gave the sign that the eastern margin of the bright base was arriving at a little line drawn in the vertical plane of the plate; a sign, I mean, that was instantaneous, given by pronouncing the monosyllable *Sic*. Besides this, repeating the observation, we generally used two holes, one twice as large as the other, so that it would be clear to us whether, after the diameter of either had been subtracted, the same quantity of the diameter of the bright base remained, as it should.

[Margin: Suspicion of a small error here too.]

And this method is indeed the most excellent of all, if the circumspection I have indicated is applied. But since, after all, the boundaries of light and shadow are not entirely evident, and while the eye is busy removing the doubt the bright base moves quickly with the motion of the Sun, a slight suspicion always sticks in the observer's mind of an excess of about half a minute in the Sun's diameter deduced from it. But we are helped by the deficiency in the following method, so that by choosing the mean we may establish a fairer diameter; otherwise, by this method I generally obtained it as 31′ at apogee and 33½ at perigee.

IX. Finally, Pierre Gassendi suggests to us a sixth method, in his Letters to Fortunio Liceti and in the book *On the Apparent Magnitude of the Sun Low and High*. He fixed at right angles to the ends of a small beam two plates, quite equal and equidistant, each having a breadth of 796 particles, of which the distance between the plates was 38308, that is, 22 Paris or Royal feet; and with the plates turned toward the Sun he measured the breadth of the shadow cast by the front plate onto the rear plate, and from it, by analysis of triangles, he hunted out the Sun's apparent diameter. So as to omit nothing that could be useful, we too used this small beam, with the room darkened on all sides. The method of finding the Sun's diameter from the breadth of the shadow is as follows.

[Margin: Problem for the sixth method.]

Let the extreme rays of the Sun be AC and BC, touching the margins of the more remote plate, DE; and let the shadow, parallel to it but narrower, be FG, and the union of the rays at C. Let GL be drawn perpendicular to DE, and from C a perpendicular to the plates FG and DE, and indeed also to AB, dividing the angle ACB in two, as also FG, DE and AB, by 9 and 10 of Book 1 of Euclid. Since, therefore, the angles at O, K and L are right angles, OG, the half-breadth of the shadow, will be known from measure- ment, and equal to KL, and KO equal to LG, by 33 of Book 1. So when OG is taken from KE, the half-breadth of the plate DE, LE will remain known, in the triangle LGE, right-angled at L; in which LG is also known, being equal to the known distance of the plates, KO. Therefore, by analysis of right-angled triangles, the angle LGE will not stay hidden; and to it the angle ICB is equal, by 29 of Book 1, because BC falls on the parallels IC and LG. But ICB subtends the semidiameter of the Sun, and ACB the apparent diameter, which was to be found.

Diagram. A figure set into the left column beside the demonstration: an inverted triangle with its base AB at the top and its apex C at the bottom. A vertical line runs from I, at the middle of the base, down to C. Two horizontals cross the triangle: DE, the upper, and FG, the lower and shorter. A second vertical runs from L on DE down to G.

- I (top centre, above the base): the middle of AB
- A (upper left), B (upper right): ends of the base
- K, L (on DE, side by side, K on the central vertical and L to its right)
- D (left), E (right): ends of the upper horizontal
- F (left), G (right): ends of the lower horizontal
- O (on the central vertical, at the level of FG)
- C (bottom): the apex

Moreover, here too the boundary of light and shadow is doubtful, and a rather large breadth of shadow is usually taken because of the penumbra, and so a diameter of the Sun somewhat smaller than it should be. But if this method is joined with the fifth, since that one tends to produce an excess and this one a deficiency, there will be a certain compensation; and from this one compared with method 4, the fairer estimate of the Sun's diameter, to my mind, is 30½′ at apogee and very nearly 33′ at perigee. The remaining methods, which I deliberately omit, either reduce to these, or are more uncertain than these, or are derived from eclipses; I shall indicate them in Book 4, Chapter 15, no. 2, where Ptolemy is discussed. As for Lansbergen's extending the diameter of the Sun at perigee to 36′, in Class 4 of the lunar observations, this clearly conflicts with our direct observations; and the eclipses he brings forward there can be accounted for in another way, by adjusting the diameters of the luminaries so that they nevertheless do not conflict with observations.

[Margin: Authors on the Sun's apparent diameter.]

XII. It remains for us to gather the opinions of astronomers on the Sun's apparent diameter into one table. In support of its reliability the following stand and can be consulted: Archimedes in the *Sand-Reckoner*; Pappus and Commandino on Aristarchus, *On the Magnitudes and Distances of the Sun and Moon*; Cleomedes, Book 2 of the *Cyclical Theory*; Ptolemy, Book 5 of the *Almagest*, Chapters 14, 15 and 16, and Theon on that place, and Regiomontanus in the *Epitome*, from Proposition 21; al-Battānī, Chapter 30, and Regiomontanus in his notes on that place; al-Farghānī, Difference 22; the Alphonsines, Canon 18; Peuerbach in the *Tables of Eclipses*; Copernicus, Book 4, Chapters 18 to 20; Reinhold in the *Theoricae Planetarum*, p. 210, and in the Prutenic Tables, Precept 49, with the table on p. 123; Daniel Santbech, *De Observationibus*, Propositions 14 and 23; Magini in the *Theoricae Planetarum*, Book 2, Chapters 19 to 24, and Canon 68 of the *Secunda Mobilia*, and in the Supplement to the Ephemerides, p. 291; Clavius on the *Sphere*, p. 186; Barozzi, *Cosmographia*, Book 4, p. 337; Mästlin in his *Theoricae* of eclipses [Thor. Eclipſium]; Tycho, volume 1 of the *Progymnasmata*, pp. 467 and 485; Longomontanus, Book 1 of the *Theorica*, Chapter 9; Kepler, Chapter 11 of the *Astronomia Optica*, in the

Appendix to Tycho's *Progymnasmata*, at the start of the Ephemerides, and in the Rudolphine Tables, Precepts 141 and 148, and pp. 92 and 98 of the tables; Lansbergen in the *Uranometria*, pp. 49 and 115, in the tables, p. 37, and in the *Treasury of Observations*, pp. 100 to 105; Biancani in the *Sphaera*, Book 10, Chapter 3; Scheiner in the *Rosa Ursina*, Book 4, Part 4; Bettini, *Apiarium* 8, Progymnasma 3; Nicolaus Mulerius in the *Tabulae Frisicae*, p. 420; Redento Baranzano, *Uranoscopia*, Part 2, p. 114; Athanasius Kircher, Book 9 of the *Ars Magna*, and in the *Magnes*, p. 561; Boulliau, *Astronomia Philolaica*, Book 4, Chapter 2, and in the tables, p. 138; Šírek in the *Radius Sidereomysticus*, Book 4, Chapter 2, M. 4; Wendelen in the *Idea of the Atlantic Tables*, p. 29; Hérigone, volume 4 of the *Cursus Mathematicus*, p. 59; Johann Baptist Cysat in his papers [in Schedis]; Vincenzo Renieri in the *Tabulae Mediceae*; Pierre Gassendi, *On the Apparent Magnitude of the Sun Low and High*, and in the *Institutio Astronomica*, Book 2, Chapter 9; Michael Florent van Langren in letters addressed to me; Galileo, Dialogue 3 on the Cosmic System; Georges Fournier, *Hydrographie*, Book 13, Chapter 2, from Gassendi. Of these, Copernicus considers the least eccentricity, but Reinhold and Magini in the *Theoricae Planetarum* a threefold eccentricity; Lansbergen nearly the least; but Magini in the Supplement to the Ephemerides, as also Kepler, bisect the eccentricity, and hence the difference of the diameter. The opinions of the Egyptians, however, are reported by Macrobius, Cleomedes and Proclus.

The Apparent Diameter of the Sun, as Seen from the Centre of the Earth [SOLIS DIAMETER APPARENS Ex centro Terræ ſpectati.]

Authors, on whom see no. XII	At the ⊙'s threefold distance: mean	At apogee: greatest	At perigee: least
The Egyptians in Macrobius	100′ 0″	✢ ✢	✢ ✢
The Egyptians in Cleomedes	28′ 48″	✢ ✢	✢ ✢
And Proclus; or	30′ 52″	✢ ✢	✢ ✢
Aristarchus, Archimedes and Sosigenes	30′ 0″	30′ 0″	30′ 0″
Ptolemy and al-Farghānī	32′ 18″	31′ 20″	33′ 20″
al-Battānī, Maurolico, Barozzi	32′ 28″	31′ 20″	33′ 40″
The Alphonsines, and nearly Thābit	32′ 30″	31′ 20″	33′ 50″
Peuerbach, Regiomontanus, Clavius	32′ 27″	31′ 0″	34′ 0″

Authors, on whom see no. XII	At the ⊙'s threefold distance: mean	At apogee: greatest	At perigee: least
Copernicus	32′ 44″	31′ 40″	33′ 54″
Reinhold and Magini, in the eccentricity: least	32′ 36″	31′ 28″	33′ 44″
mean	32′ 36″	31′ 28″	33′ 54″
greatest	32′ 36″	31′ 20″	34′ 4″
Michael Mästlin	31′ 11″	29′ 36″	31′ 45″
Daniel Santbech	32′ 0″	31′ 0″	33′ 0″
Longomontanus	31′ 4″	30′ 0″	32′ 8″
Tycho, Mulerius, Biancani	31′ 0″	30′ 0″	32′ 0″
Magini in the Supplement to the Ephemerides	30′ 30″	30′ 0″	31′ 0″
Kepler and Hérigone	30′ 30″	30′ 0″	31′ 4″
Galileo the Lyncean	30′ 0″	✢ ✢	✢ ✢
Lansbergen	34′ 42″	33′ 34″	35′ 58″
Boulliau	32′ 52″	32′ 18″	33′ 30″
Gassendi and Fournier	30′ 38″	30′ 12″	31′ 6″
Govaert Wendelen	30′ 0″	29′ 30″	30′ 33″
Antonín Maria Šírek	34′ 41″	✢ ✢	✢ ✢
Athanasius Kircher	30′ 54″	30′ 20″	31′ 28″
Van Langren	30′ 0″	29′ 20″	30′ 24″
or	30′ 10″	29′ 51″	30′ 28″
We, all things weighed	31′ 50″ [recte: 31′ 56″, per the book's errata; corrected by hand to 56″ in this copy]	30′ [?]0″ [recte: 30′ 50″, per the book's errata; in this copy the first figure of the seconds is struck through and overwritten in ink and cannot be read]	33′ 8″

The following are uncorrected, because the diameter of the hole was not subtracted from the diameter of the bright base, and the distinctions of the penumbra were taken variously and inconstantly; and Scheiner used holes now smaller, now larger.

	′ ″	′ ″	′ ″
Johann Baptist Cysat	37′ 50″	36′ 30″	39′ 15″

	′ ″	′ ″	′ ″
or	37′ 0″	36′ 0″	40′ 0″
Scheiner; or	45′ 0″	41′ 0″	48′ 0″
or	53′ 0″	47′ 0″	56′ 0″
And so otherwise at other times			

SCHOLIA.

[Margin: Notable advice for establishing the diameters of the luminaries.]

I. In establishing the Sun's apparent diameter, account must be taken not only of the observations by which it was obtained through one of the six methods described, but also of the horizontal parallaxes and of the distances from the Earth, and much more of the eclipses of the Sun and Moon. For the difference of the diameter between greatest and least must be consistent with the distance of the Sun at perigee and at apogee, and with the difference of the horizontal parallaxes that follows those distances. Next, the record of solar eclipses and the diameter of the Moon must be kept safe; since some of these eclipses are reported as total without delay, some as total with a delay of the Sun in darkness, and some as partial but annular, that is, those in which a bright ring of the Sun, not eclipsed, appeared around the Moon, according to what is to be said in Book 4, Chapter 6, no. 4, and its Scholium 1. For it is not certain that this is to be attributed either to contraction in the total eclipses, or to an optical dilatation, or to refractions of rays in the annular ones, occurring in the air around the Sun, the Moon or the Earth, as some think; more on these in the Book on Eclipses, and Book 4, Chapter 17, no. 8. Furthermore, those who with Kepler bisect the Sun's eccentricity, since the diffe- rence in the Sun's distance is cut almost in half by them, must also reduce the difference between the Sun's diameter at apogee and at perigee. And those who think that the Sun's eccentricity varies in different ages must also admit a variation of the Sun's diameter at any of its three distances from the Earth, and compute it with trigonometric exactness. But whether, as Copernicus (Book 4, Ch. 21) and Magini (*Theoricae Planetarum*, Book 2, Ch. 24) report from others, the Sun's apparent diameter stands to its true hourly motion as 66 to 5, depends on each of the two having been established separately, rather than being a rule for establishing them. This business is therefore of incredible subtlety, and calls for a deeper and more circumspect inquiry than some suppose. Many distinguished astronomers have not reconciled all these things well together in their hypotheses, as Lansbergen shows in the *Uranometria*, pp. 58, 59 and 116, and as we show below, Ch. 11, no. 7.

[Margin: Whether the Sun's apparent diameter is always the same.]

II. It is very probable that the Sun does not keep the same distance from the centre of the Earth, but is sometimes at apogee, that is, at its farthest from the Earth, and sometimes at perigee and nearest, and that it therefore appears now larger, now smaller. Yet this cannot be demonstrated with evidence from

mere and immediate observation of this diameter, or even of eclipses, as I shall show in Book 4, Ch. 17, in the Scholium. Scheiner admits the same in the *Rosa Ursina*, though he thinks this difference is almost demonstrated by the use of the telescope, which must be drawn out by about a palm more at the winter solstice than at the summer solstice in order to show the sunspots distinctly; others, from direct inspection of the winter and the summer diameter. But someone could ascribe this difference to the vapours of the horizon, which show the Sun larger in winter, as Io. Bapt. Amicus [Giovanni Battista Amico] does (*Opusculum de motibus coelestibus*, Ch. 10) and Io. Anton. Delphinus [Giovanni Antonio Delfino] at the end of his book *De Globis Coelestibus*: in the same way as they show the rising or setting Sun larger than the Sun at noon, whether because of refractions or because of the dilatation of our pupil, as I shall say shortly. So if someone were to suppose that the Sun's diameter is perpetually, for example, 32′ 20″, were to refer the variation of eclipses to the variation of the Moon, and were able to defend the inequality of the Sun's motion without an eccentric or an epicycle or any other hypothesis founded on a variation of distances, it would not be easy to convict him of falsehood. Fracastoro, for his part, attributes this difference to refractions, on account of the different parts of the sky that lie between the Sun and our sight.

[Margin: Why the Sun and the other stars appear larger at the horizon.]

III. Since the Sun, the Moon and all the stars appear larger at the horizon than far from the horizon, astronomers and writers on optics dispute over the cause. Is it to be attributed to our estimation, which, because of the tracts of land or sea lying between, judges an arc of the sky near the horizon larger, other things being equal, than when we see no distance by which to estimate the sizes in the sky? So think, with Ibn al-Haytham (Book 7, Prop. 55) and Witelo (Book 10, Prop. 54), Kepler in the *Epitome of Astronomy*, p. 81, and in the *Astronomia Optica*, p. 133, Hortensius in his dissertation with Gassendi, and before him the author of the *Perspectiva communis* (Book 1, Prop. 82) and Rogerius Baccon [Roger Bacon], Part 2 of the *Perspectiva*, Dist. 3, Ch. 6. Or is it that our pupil, shaded by the vapours of the horizon, which blunt and intercept the rays, is dilated, and with its opening takes in the images of the stars under a wider angle? So contend Scheiner in the *Sol Ellipticus* and in the *Oculus, seu fundamentum Opticum*, and Gassendi in his letters on the apparent size of the Sun low and high. Or, finally, is it due to the moist vapours of the horizon, which by the force of refraction show the stars larger in just the same proportional way in which coins are seen larger under water? So it seemed also to Ibn al-Haytham (Book 7, Prop. 51) and to Witelo (Book 10, Prop. 53), and so judged long ago Posidonius, in Strabo, Book 3 of the *Geography*; Cleomedes, Book 2, p. 427 in my copy; Ptolemy, *Almagest*, Book 3, Ch. 3; Johannes de Sacrobosco, *Sphere*, Ch. 1, and Clavius there, p. 108; Longomontanus, Book 1 of the *Theorica*, Ch. 9, p. 290; and Barozzi, *Cosmographia*, p. 17. But we shall have to examine these views one by one in the Optical Book, in the Section on Refractions.

IV. So much for the diameter of the Sun viewed from the centre of the Earth. But if it were viewed from the planets, at their mean distance from the Sun, or at their greatest, when they are said to be at aphelion [Aphelij], that is [Greek: ἀπὸ τοῦ ἡλίου (apo tou hēliou, "away from the Sun")], namely removed from the Sun, or at their least, when they are said to be at perihelion [Perihelij], in Greek [Greek: περὶ τὸν ἥλιον (peri ton hēlion, "around the Sun")], it would be seen under a very different angle. On this, take the opinion of Boulliau, *Astronomia Philolaica*, Book 1, Ch. 9, and of Šírek, Book 4, Ch. 2, part 5, founded on Kepler's distances.

The Sun's apparent diameter, viewed from the planets

At the distance of the planets from the Sun

From	Aphelia	Mean	Perihelia
☿	65′ 3″	82′ 0″	99′ 37″
♀	41′ 53″	42′ 10″	42′ 28″
♂	18′ 23″	20′ 0″	22′ 5″
☄	5′ 48″	6′ 0″	6′ 10⅛″
♄	3′ 3″	3′ 11″	3′ 24″
☾ in ☌	31′ 0″	31′ 30″	32′ 6″
in ☍	29′ 2″	29′ 30″	30′ 0″

CAPVT XI.

De Solis vera Magnitudine, cæteriſq. ex illa & ex diſtantia eius, aut Apparenti Semidiametro dependentibus: Vbi de Cohærentia horum cum Parallaxi Horizontali, Semiangulo Coni & Axe Vmbræ terreſtris, & Portione Terræ illuminata à Sole.

On the true size of the Sun, and the other quantities that depend on it and on its distance or apparent semidiameter; where the agreement of these with the horizontal parallax, the semi-angle of the cone and the axis of the Earth's shadow, and the portion of the Earth illuminated by the Sun is also treated.

I. PROBLEMA.

De vera Solis Semidiametro.

Problem 1. On the true semidiameter of the Sun.

I. The true size of the Sun must be sought from its true semidiameter: doubling this gives the true diameter, and from the diameter the remaining kinds of magnitude are derived by the rules given in Book 1, Ch. 4 and 6. The true semidiameter, in turn, is gathered from the given apparent semidiameter and the distance from the centre of the Earth of any star whatever, not only of the Sun.

[Margin: Let the distance and the apparent semidiameter be given.]

Thus, in the diagram added here, let A be the centre of the Sun at apogee, P at perigee, and C the centre of the Earth, these being joined by the straight line APC; and let BC and DC be the extreme rays of the Sun, touching the Sun at B and D and there making, by Euclid 3.16, a right angle with the semidiameters BA and DP. Then the apparent semidiameter, or the angle measuring it, will be ACB with the Sun at apogee and PCD at perigee. So if either semidiameter is given, namely ACB with the distance AC, or PCD with the distance PC, you can find by the rules of right-angled triangles the true semidiameter AB in the triangle CAB, right-angled at B, or the semidiameter DP in the triangle CPD, right-angled at D. These must always be the same, provided that the apparent semidiameters and the distances have been correctly established.

Diagram. Set into the right column beside the text. Two circles stand one above the other on a single vertical straight line, which descends to a point at the foot; from the same point two straight lines rise to touch each circle on its left side, and a short semidiameter runs from each circle's centre to the point of contact.

- A (centre of the upper circle): the Sun's centre at apogee
- B (left edge of the upper circle): point of contact of the ray, end of the semidiameter BA
- P (centre of the lower circle): the Sun's centre at perigee
- D (left edge of the lower circle): point of contact of the ray, end of the semidiameter DP
- C (foot of the figure): the Earth's centre

[Margin: A shorter form of the rule.]

In brief, then, the logarithm of the given distance, taken as base, added to the logarithm of the apparent semidiameter will yield the logarithm of the Sun's true semidiameter. Or multiply the distance by the sine of the angle subtending the apparent semidiameter, and divide the total so produced by the whole sine: the quotient will be the true semidiameter, in parts of which the given distance is expressed.

[Margin: Authors who treat of the size of the Sun and of the Earth's shadow.]

By this device the first column of the following table was produced, partly by us and partly by the authors who treat of the Sun's true size; the rest of the table was completed from the rules on the measurement of the circle and the sphere given in Book 1, Ch. 4 and 6. The authors who treat of the Sun's true size, and on that occasion of the Earth's shadow and so on, are first of all Aristarchus, in his book on the size of the Sun, Ptolemy, *Almagest*, Book 5, Ch. 14 and 15; Regiomontanus, *Epitome of the Almagest*, Book 5, Props. 19, 20 and 21, and in his notes on al-Battānī, Ch. 30; Copernicus, Book 4, Ch. 19 and 21; Mästlin, in the appendix to Kepler's *Prodromus*, p. 154; Tycho, Vol. 1 of the *Progymnasmata*, p. 473; Longomontanus, Book 1 of the *Theorica*, Ch. 9, p. 295; Lansbergen, in the Prolegomena, Book 2 of the *Uranometria*, from Element 7 to 12, and Book 1, Elements 11 and 12; Maurolico, at the end of Dialogue 3 of the *Cosmographia*; Barozzi, *Cosmographia*, Books 1 and 4, pp. 50 and 338;

Magini, *Theoricae Planetarum*, Book 2, Ch. 24; Clavius, in the *Sphaera*, from p. 186; Biancani, in the *Sphaera*, at the end of Book 18 and Book 4, Ch. 3; Boulliau, *Astronomia Philolaica*, Book 4, Ch. 1 and 2; Gassendi, *Institutio Astronomica*, Book 3, Ch. 11; Athanasius Kircher, in the *Magnes*, p. 561, and in the *Ars Magna*, Book 9, at the end; Šírek, *Radius Sidereomysticus*, Book 4, Ch. 2, part 4. The numbers that are missing in some of these I have supplied myself, using a ratio of diameter to circumference of 100 to 314.

The true size of the Sun compared with the Earth

☼ Authors	True diameter contains (Earth's diameters)	Circumference contains (Earth's diameters)	Area of a great circle contains (square Earth diameters)	Convex surface contains (square Earth diameters)	Solidity contains (times the Earth's solidity)
Ptolemy, Maurolico, Clavius, Barozzi	5 1/2	17 2/7	24 ✢	134	166 3/8
Aristarchus, more than	6 1/3	20 1/7	30 2/3	127	254 1/17
Aristarchus, less than	7 1/6	22 3/7	38 0	155	368 1/11
al-Battānī	5 7/10	18 5/7	26 0	108	186
Copernicus	5 27/60	16 1/2	22	91	161 7/8
Tycho and Biancani	5 14/75	16 2/7	22	85	140
Longomontanus	5 807/1000	18 1/5	26	95	196
Kepler	15 0	47 1/10	176	706	3344 [recte: 3375, per the book's errata; 75 is written in pen beside the figure in this copy]
Lansbergen	7 17/30	24 ✢	46	176	434
Boulliau	7 ✢	22 2/7	39	156	343
Wendelen	64 0	200 96/100	3216	12864	262144
Athanasius Kircher	5 1/5	16 ✢	21	83	140
Šírek	10 0	31 4/10	78	314	1000
We, for the present	33 5/6	106 15/100	885	30056	38600

[Margin: The size of the Sun in miles.]

I have not expressed these measures in miles, stadia or leagues, because of the excessive diversity of opinions on the miles contained in the Earth's diameter,

on which see Book 2, Ch. 7. Following the tables given there, multiply the Sun's diameter by the Earth's diameter, and you will obtain this measure. For example, since the Earth's diameter contains 8278 Bolognese miles, but 10348 ancient Roman ones, if you multiply these numbers by 33 5/6, the Sun's diameter, or by 106 1/7, the Sun's circumference, the Sun's diameter will have 280073 Bolognese miles and 350068 Roman miles, and the Sun's circumference 878651 Bolognese miles and 1097768 Roman miles. Kircher, however, in his magnetic work, p. 561, following other measures, gives the Sun's diameter only 42900 Italian miles, and its solidity 7895358900000 cubic miles. On so great a disagreement among astronomers in this and similar measures, see what I shall say in Book 4, Ch. 18, no. 2.

II. PROBLEMA.

Data vera Solis Semidiametro, inuenire Semidiametrum eius apparentem datæ Diſtantiæ congruam.

Problem 2. Given the true semidiameter of the Sun, to find its apparent semidiameter corresponding to a given distance.

II. In the preceding figure, already explained, let the distance of the Sun be given, for example at perigee, namely CP, and the true semidiameter DP. For in the triangle DCP, right-angled at D, from the given base PC and the side DP you will find, by the analysis of triangles, the angle DCP, which subtends the apparent semidiameter of the star, corresponding to the given distance. Thus, since for us the Sun's true semidiameter is 33 5/6 semidiameters of the Earth, and the perigean distance 7000 semidiameters of the Earth, the apparent semidiameter of the Sun at perigee had to be set at 16′ 34″ out of geometric necessity, even if it should not be gathered from observations to be exactly so much, but greater or less by a few seconds.

[Margin: A shorter form of the problem.]

In brief, the logarithm of the true semidiameter, added to the residual of the logarithm of the distance, will yield the logarithm of the apparent semidiameter. Or multiply the true semidiameter by the whole sine and divide the total by the given distance: the quotient will be the right sine of the angle subtending the apparent semidiameter.

III. PROBLEMA.

Quanta ſit portio Solis à nobis ex Terra viſa.

Problem 3. How large is the portion of the Sun seen by us from the Earth.

III. Since MCLF, the sphere of the Sun, is larger than our eye, the whole of its hemisphere CDL cannot be seen from the eye at B, but only MDF, namely as much as is taken in by the rays BM and BF, which touch the Sun at M and F and there make right angles with the Sun's semidiameters MA and FA, by

Euclid 3.16. One can ask how large the arc MDF is, or how large the true diameter MF is, of a circle that is not a great circle, which separates the visible part of the Sun from the invisible; or how large is the arc NAK, described from B through the Sun's centre A and intercepted by the tangents BM and BF. But this arc is as large as the angle B.

Diagram. Set into the right column. A large circle at the top, with a horizontal diameter through its centre and a chord below it; two lines from the foot of the figure touch the circle on either side at the ends of the chord, and a vertical line from the foot passes through the lowest point of the circle to the centre. An arc passes through the centre and across the tangents.

- A (centre of the circle): the Sun's centre
- C (left end of the horizontal diameter)
- L (right end of the horizontal diameter)
- Q (left of C, outside the circle): not cited in the text on this page
- G (right of L, outside the circle): not cited in the text on this page
- M (left end of the chord): point of contact of the tangent BM
- F (right end of the chord): point of contact of the tangent BF
- N (left end of the arc, on the tangent line)
- K (right end of the arc, on the tangent line)
- D (lowest point of the circle)
- B (foot of the figure): the eye

[Margin: A twofold conception of this problem.]

For the first, let AB be drawn, bisecting the angle MBF, which subtends the Sun's apparent semidiameter, that is, the arc NAK. For since the angle ABF is supposed given, as large as the Sun's apparent semidiameter, and the angle at F is a right angle, the angle BAF will be known by Euclid 1.32, as the complement of the angle ABF to a right angle; and by the same reasoning the angle BAM will be known, and hence the whole angle MAF, which measures the arc MDF seen by us.

[Margin: A shorter form of the rule.]

In brief, then, the Sun's apparent diameter, taken from the table of Chapter 10 and subtracted from 180°, will leave the arc of the Sun that is seen. Thus in our opinion, [Margin: The portion of the Sun seen by us.]

the arc of the Sun seen by us is:

With the Sun	Arc
at apogee	179° 29′ 30″
at mean distance	179° 28′ 10″
at perigee	179° 26′ 52″

For the second part of the problem: in the isosceles triangle MAF the angle at A is known, as I have said, and from it the angles at the base, each of which is half the complement of the angle MAF to two right angles; and the Sun's true semidiameter, both MA and AF equal to it, is supposed given, or found by

Problem 1. So the base MF, that is, the true diameter of the circle separating the visible portion of the Sun from the invisible, cannot remain hidden from a trigonometer. For us, therefore, with the Sun at perigee, MF is 33 and almost 5/6 Earth diameters, and thus imperceptibly smaller than the diameter CL.

IV. PROBLEMA.

Quanta ſit Portio Terreſtris Hemiſphærij illuminata à Solis radijs Irrefractis.

Problem 4. How large is the portion of the terrestrial hemisphere illuminated by the Sun's unrefracted rays.

[Margin: If the distance and the true semidiameter of the Sun are given.]

IV. Let EMF be the hemisphere of the Sun and ABCD the globe of the Earth, and let EG and FG be the extreme rays of the Sun, touching the Sun and the Earth at the points E, F and A, C, and there, by Euclid 3.16, making right angles with the semidiameters EK of the Sun and AH of the Earth, and so on. Let a straight line be drawn through the centres K, H as far as the meeting of the rays at G, this being KG. Then the illuminated part of the Earth will be the arc ABC, and the dark part ADC; AGC will be the cone of the Earth's shadow, and HG its axis or height. Finally let the straight line HL be drawn from H, parallel to EA. By Euclid 1.29 the angle AHI, like HLE and HLK, will be a right angle, and by 33 the side EL will be one semidiameter of the Earth, as being equal to the side AH. So in the triangle HLK, right-angled at L, if the Sun's distance KH is given, and the side LK, obtained by taking EL, one semidiameter of the Earth, from the whole EK, the Sun's true semidiameter, you can find the angle LHK. This angle measures the arc IB, which, added to the quadrant AI (since, as I have shown, AHI is a right angle), will yield the arc AB, to which the arc BC is equal. So the arc of the Earth illuminated by the Sun's rays will be known, these rays being imagined to touch the Earth and to meet, unrefracted, at G.

Diagram. Set into the left column. A large semicircle at the top, under a straight line through K, from which two semidiameters run down to E and F; two straight lines descend from its ends to a point at the foot, touching a small circle on the way. A vertical line runs from K through the small circle's centre to the foot, and a slanting line runs from L on the upper figure down to the small circle's centre.

- E (upper left corner): point of contact on the Sun
- K (top centre): the Sun's centre
- F (upper right corner): point of contact on the Sun
- L (on the upper figure, right of E): foot of the line HL
- M (lowest point of the semicircle)
- I (upper left of the small circle): end of the arc IB
- B (top of the small circle)
- H (centre of the small circle): the Earth's centre

- A (left of the small circle): point of contact of the ray
- C (right of the small circle): point of contact of the ray
- D (lowest point of the small circle)
- G (foot of the figure): the meeting of the rays

[Margin: Why the problem is limited to unrefracted rays.]

I said "unrefracted" because, as Kepler notes in the *Astronomia Optica*, pp. 269, 279, and after him Bettini in *Apiarium* 8, Progymnasma 2, Prop. 4, on account of refraction, by which the Sun's rays falling on the spherical surface of our denser air are broken and bent towards the portion ADC of the hemisphere, they illuminate more of it than if they passed through unrefracted. Kepler demonstrated in the same place, Bettini pointed out the demonstration, and we shall show more clearly in Book 5, in treating of the Earth's penumbra, that refracted rays light more than a quadrant of the hemisphere beyond what unrefracted rays light, by as much as the Sun's horizontal refraction. So if that refraction is 34′, as Tycho holds, 34′ must be added to the arc IB in order to have the arc AB, whose double ABC will be the arc of illumination. Therefore in our opinion, since the arc of illumination by unrefracted rays beyond 180° is 29′ 40″ with the Sun at apogee and 32′ 16″ at perigee, if you add to each the double of the horizontal refraction, the arc of true illumination beyond 180° will be 1° 37′ 40″ with the Sun at apogee and 1° 40′ 16″ at perigee, whereas the ancient astronomers judged it to be far smaller, through ignoring or dissembling the refractions.

[Margin: Authors who treat of the illuminated portion of the Earth.]

First, however, one must determine that part of the Earth which is illuminated by rays supposed, by a fiction, to be unrefracted, according to the Sun's given distances from Ch. 7 and its true semidiameter from the table of Problem 1. This portion is determined by Ibn al-Haytham, Nunes and Clavius, in the opuscule *De Crepusculis*, Prop. 5; by Witelo, Book 10, Prop. 59; and by Reinhold, on Peuerbach's *Theoricae*, p. 165, who take from Ptolemy a true semidiameter of 5 1/2 semidiameters of the Earth and an apogean distance of 1210. But Magini, *Primum Mobile*, Book 11, Prop. 28, takes from Tycho a mean distance of 1150 and a true semidiameter of 5 2/3. Kepler, in the *Astronomia Optica*, p. 233, reports Witelo's demonstration. We shall review their opinions and, from our own calculation, which you will see in the table of Problem 5, shall add what follows from the data of others and from ours.

[Margin: The portion of the Earth beyond the hemisphere lit by the Sun.]

The arc of the Earth beyond 180° illuminated by the Sun's unrefracted rays

Authors	☉ at apogee	☉ at perigee
Ibn al-Haytham	27′ 52″	✝ ✝
Witelo	25′ 36″	✝ ✝
Reinhold	25′ 40″	✝ ✝
Magini	34′ 0″	✝ ✝

Authors	⊙ at apogee	⊙ at perigee
By me, from the data of Ptolemy	25′ 26″	27′ 20″
of al-Battānī	28′ 4″	30′ 4″
of Copernicus	25′ 32″	26′ 48″
of Tycho	24′ 22″	25′ 40″
of Longomontanus	24′ 20″	26′ 26″
of Kepler	28′ 0″	30′ 56″
of Lansbergen	28′ 52″	31′ 0″
of Boulliau	27′ 44″	28′ 44″
of Wendelen	29′ 4″	30′ 4″
of Šírek	29′ 52″	32′ 4″
from our own	29′ 40″	32′ 16″

Add to these twice the Sun's horizontal refraction, and the true arc illuminated beyond 180° will result.

V. PROBLEMA.

Quantus ſit Angulus Coni Vmbræ terreſtris Comprehenſæ à Radijs Irrefractis.

Problem 5. How large is the angle of the cone of the Earth's shadow enclosed by unrefracted rays.

V. In the preceding figure the angle of the cone of the Earth's shadow is AGC, and its half-angle AGH. But this, as Kepler indicates in the *Astronomia Optica*, p. 233, is equal to the angle LHK, and this is easily shown: for by construction HL is parallel to EG, and the straight line KG falls on them; therefore by Euclid 1.29 the external angle LHK is equal to the internal opposite angle AGH. So once the angle LHK has been found by Problem 4, the angle AGH is known, whose double is AGC; and thus the preceding table gives this angle.

[Margin: Why the limitation to unrefracted rays?]

Here too I said "unrefracted", because on account of the refraction of the Sun's rays, just as the arc of illumination grows in the portion ADC, so the angle of the shadow enclosed by refracted rays grows, and the axis of the shadow contracts, as we shall say in Book 5, which is on eclipses.

[Margin: Blemishes or slips of Lansbergen and of other astronomers.]

VI. Now Lansbergen, in the Prolegomena and in Book 2 of the *Uranometria*, Element 7, sets the angle of the Earth's shadow with the Sun at apogee at 29′ 8″[15] from his own principles; but from Ptolemy's data at 25′ 38″, from al-Battānī's at 30′ 28″, from Copernicus's at 24′ 22″, and from Tycho's at 32′ 4″ or 32′ and 16″. He investigates it, however, from the semidiameter of the Earth's shadow and the Moon's distance, and from this he concludes, as I shall relate later, that their hypotheses disagree with themselves. This very disagreement is indeed

[15]the page sets the double mark after 29 as well, "29″. 8″."; the same value stands as 29′ 8″ later in this paragraph

gathered from our calculation, since we have obtained this angle differently from the given distances and the Sun's true semidiameter; but what Lansbergen objects against others is also turned back on Lansbergen himself. For from the distance of the Sun at apogee and the true semidiameter this angle comes out 28′ 52″, whereas from the Moon's distance and the semidiameter of the shadow he gets 29′ 8″. So difficult is it for all astronomers to reconcile these subtleties in such a way that they do not disagree by some minutes. And I believe that some small error comes also from the repeated multiplication and division by sines; hence the analysis of triangles by the addition of logarithms alone is easier and safer, as we shall show in Vol. 2 of the *New Almagest*, Book 1, which is the trigonometric book. So I am pleased to set down our calculation, in which, from the distances given in Ch. 7, that is, from the base HK, and from the side LK, which, with one semidia- meter of the Earth EL removed, is known from the table of Problem 1, we have gathered the angle LHK, which is equal to the angle AGH, whose double you have in the preceding table of Problem 4. In the following table, *Log.* means logarithm; and RL the residual of the logarithm [Residuum Logarithmi], which is obtained by subtracting the logarithm from the radius 1000000; and *Dist.* the distance of the Sun from the centre of the Earth.

	Sun at apogee			Angle of the Cone of the Earth's Shadow			Sun at perigee		
Authors		**Semidiam. of the Earth**	**+**	**Logarithms or their Residual**	**Angle LHK**		**Semidiam. of the Earth**	**Log- a- rith. or Resid.**	**Angle LHK**
Ptolemy	*Dist.*	1210	RL	691721		*Dist.*	1126	RL	694846
	LK	4 1/2	*Log.*	065051		LK	4 1/2	*Log.*	065051
	Sum		*Log.*	756772	12′ 43″	*Sum*		*Log.*	759897	13′ 40″
al-Battānī	*Dist.*	1146	RL	694082		*Dist.*	1068	RL	697143
	LK	4 7/10	*Log.*	066990		LK	4 7/10	*Log.*	066990
	Sum		*Log.*	761072	14′ 2″	*Sum*		*Log.*	764133	15′ 2″
Copernicus	*Dist.*	1179	RL	692849		*Dist.*	1105	RL	695001
	LK	4 27/60	*Log.*	064189		LK	4 27/60	*Log.*	064189
	Sum		*Log.*	757038	12′ 46″	*Sum*		*Log.*	759190	13′ 24″
Tycho	*Dist.*	1180	RL	692812		*Dist.*	1120	RL	695078	

	Sun at apogee			Angle of the Cone of the Earth's Shadow	Sun at perigee		
	LK	4 14/75	*Log.*	062142	LK 4 14/75	*Log.*	062142
	Sum		*Log.*	754954 12′ 11″	*Sum*	*Log.*	75722ɑ2′ 50″
Longomontanus	*Dist.*	1334	RL	687484	*Dist.* 1242	RL	690588
	LK	4 4/5	*Log.*	067959	LK 4 4/5	*Log.*	067959
	Sum		*Log.*	755443 12′ 18″	*Sum*	*Log.*	75854713′ 13″
Kepler	*Dist.*	3438	RL	646369	*Dist.* 3327	RL	647795
	LK	14	*Log.*	114613	LK 14	*Log.*	114613
	Sum		*Log.*	76098214′ 0″	*Sum*	*Log.*	762408[?]′ 28″ [recte: 14′ 28″, per the book's errata; in this copy the second figure is overwritten in pen as 4 and the type beneath cannot be read]
Lansbergen	*Dist.*	150 5/6 [recte: 1550 5/6, per the book's errata; corrected by hand to 1550 in this copy]	RL	680939	*Dist.* 1446 1/4	RL	683983
	LK	6 17/30	*Log.*	081385	LK 6 17/30	*Log.*	081385
	Sum		*Log.*	76232414′ 26″	*Sum*	*Log.*	7653681̈5′ 30″
Boulliau	*Dist.*	1486	RL	682798	*Dist.* 1434	RL	684345
	LK	6	*Log.*	077815	LK 6	*Log.*	077815
	Sum		*Log.*	760613 13′ 52″	*Sum*	*Log.*	76216ɑ4′ 22″
Wendelen	*Dist.*	14905	RL	582667	*Dist.* 14407	RL	584143
	LK	63	*Log.*	179934	LK 63	*Log.*	179934
	Sum		*Log.*	76260114′ 32″	*Sum*	*Log.*	7640775′ 2″
Šírek	*Dist.*	2073	RL	668340	*Dist.* 1927	RL	671512
	LK	9	*Log.*	095424	LK 9	*Log.*	095424
	Sum		*Log.*	76376414′ 56″	*Sum*	*Log.*	76693ɑ6′ 2″
Ourselves	*Dist.*	7600	RL	611919	*Dist.* 7000	RL	615490
	LK	32 5/6	*Log.*	151629	LK 32 5/6	*Log.*	151629

Sun at apogee			Angle of the Cone of the Earth's Shadow	Sun at perigee		
Sum	*Log.*	763548	14′ 50″ [recte 14′ 52″, per the book's errata]	*Sum*	*Log.*	76711916′ 8″

Moreover the investigation of the angle of the Earth's shadow is far simpler by our method, and Problem 4 or 5, than by Lansbergen's, because the semidiameter of the shadow that he uses was deduced from a longer and more drawn-out calculation. Two givens are enough for us, namely the Sun's distance and its apparent semidiameter, from which the true one is gathered.

VI. PROBLEMA.

Parallaxin Solis Horizontalem ex Semiangulo Coni Vmbræ terreſtris & Semidiametro Solis viſuali; & viciſſim Semiangulum Vmbrę ex Parallaxi Horizontali, & Semidiametro Solis apparenti elicere.

Problem 6. To derive the Sun's horizontal parallax from the semiangle of the cone of the Earth's shadow and the Sun's visual semidiameter; and conversely the semiangle of the shadow from the horizontal parallax and the Sun's apparent semidiameter.

VII. Subtract the semiangle of the cone of the Earth's shadow from the Sun's apparent semidiameter, and the horizontal parallax will remain. Conversely, subtract the Sun's horizontal parallax from the Sun's apparent semidiameter, and the semiangle of the cone of the Earth's shadow will remain, without sensible error.

For let the figure set on p. 107 be repeated here, in which the centre of the Sun is B, that of the Earth A, and the apex of the Earth's shadow E, these being joined by the straight line BAE. Let the Sun's ray be DNE, and let NB be a straight line drawn from the point N to the Sun's centre. The angle ABN will then be very nearly the Sun's horizontal parallax. And let the straight line AD be drawn from A: BAD will then be the Sun's apparent semidiameter, or its measure. The angle ADN is sensibly equal to the angle ABN, because they stand on the same base AN, and the sides that form them are sensibly equal on account of the Sun's enormous distance from the Earth. Now by Euclid 1.32 the angle BAD is equal to the interior angles AED, which is the semiangle of the shadow, and ADN, which is sensibly equal to the horizontal parallax. Therefore the angle of the Sun's horizontal parallax is the excess of the Sun's apparent semidiameter over the semiangle of the shadow, without

sensible difference. In geometrical rigour, however, the ray AD touches the Sun at a point below D, the point at which the ray ED touches the Sun; just as the ray BN touches the Earth at a point above N, the point at which the Earth is touched by the ray DE. And so this demonstration lacks the exactness [Greek: ἀκριβεία (akribeia, "exactness")] of geometrical rigour. This is why the parallax so deduced does not agree to a hair with the semiangle of the shadow and the Sun's semidiameter, as will be clear to anyone comparing the horizontal parallax set in the table of Ch. 8, deduced exactly from the Sun's distance, with the parallax obtained by subtracting the angle set in the table of Problem 5 from the Sun's apparent semidiameter set in the table of Ch. 10. So the parallax this gives me is at most 3″ less than the exact one; and conversely, if I join the exact parallax to my semiangle of the shadow, the Sun's semidiameter becomes 15′ 17″ 10‴ at apogee and 16′ 37″ 28‴ at perigee, insensibly greater than the exact semidiameter. Therefore, unless a sensible difference of several seconds arises from this, the astronomers' hypotheses are not to be convicted of inconsistency, even though Lansbergen, in the Prolegomena of the *Uranometria* and in Elements 7, 8 and 9, may refute them. He evidently does so because he both uses this demonstration as though it were exact and uses a semiangle of the Earth's shadow derived from elsewhere, that is, from the Moon's distance and the apparent semidiameter of the shadow at that distance of the Moon, as given by those very astronomers. For from these givens he gathers the semiangle of the shadow, as I shall presently show at no. 8; and by subtracting it from the Sun's semidiameter given by them he gathers a horizontal parallax and a distance of the Sun different from the one they themselves laid down, as can be seen in the table set at the end of no. 8.

Diagram. A tall, narrow construction standing in the left column beside the text. At the head, a large circle (the Sun) crossed by a horizontal diameter; two long rays run down from the ends of this diameter and converge to a point at the foot, and further lines descend from the same diameter through the figure. Lower down, on the axis, stand three smaller circles in descending order: a small one, a larger one crossed by a horizontal line, and a small one crossed by a horizontal line just above the point of convergence, with crossing lines between the second and third.

- C (left end of the diameter of the large circle)
- R (above the diameter, left of centre)
- B (above the diameter, at the centre)
- D (right end of the diameter)
- F (inside the upper small circle); a very small mark at this circle's right edge is not resolved
- M (left end of the horizontal line through the middle circle)
- T, A, R (inside the middle circle, on the horizontal line; the R set smaller than the other labels)
- [?] (right end of the same line: a heavy, blotted letter, M or N by its shape)

- P (left of the lowest small circle)
- L (right of the lowest small circle)
- G (inside the lowest small circle, below its horizontal line)
- E (the foot of the figure, at the point of convergence)

VIII. Next, in the figure immediately preceding, Lansbergen gathers the semiangle of the shadow by this method. Given the Moon's distance AG and the angle PAG, which subtends the apparent semidiameter of the shadow, he gathers in the triangle AGP, right-angled at G, the true semidiameter of the shadow, PG. Then, with PT drawn parallel to AG, the angle MTP arises by Euclid 1.20, a right angle, as being equal to TPG, which by the same proposition equals AGP; and by 33, TA is equal to PG, and PT to AG. Therefore, with PG, that is TA, taken from the whole semidiameter of the Earth AM, the side MT remains known. With this and with the side TP, the angle MPT is sought, which by Euclid 1.29 is equal to the semiangle of the shadow PEG, because the straight line ME falls on the two parallels TP and AGE, by construction.

[Margin: Investigation of the true semidiameter of the shadow.]

Thus from badly assembled theories of the Moon and the Sun these and similar absurdities arise, and in the Tychonic and Longomontanic hypothesis it follows that the semiangle of the shadow, namely AED, is greater than the external angle BAD, that is, than the Sun's apparent semidiameter; whereas by Euclid 1.32 it ought to be smaller.

[Margin: Error of Tycho and of Longomontanus.]

The contradiction that Lansbergen found in the hypothesis of the three astronomers is the one you see in the following small table.

DATA AB AVCTORIBVS

Authors	Distance of the Moon at apogee from the centre of the Earth (Earth semidiameters and ′)	Apparent semidiameter of the Sun at apogee	Apparent semidiameter of the Earth's shadow
al-Battānī	64 10′	15′ 40″	38′ 20″
Copernicus	65 30′	15′ 50″	40′ 18″
Tycho	58 8′	15′ 0″	43′ 0″

COLLECTA A' LANSBERGIO

Authors	True semidiameter of the shadow, in parts of which the Earth's semidiameter is 60 (sixtieths)	Semiangle of the shadow	Horizontal parallax	Distance of the Sun at apogee from the centre of the Earth (Earth semidiameters)
Albat.	43	15′ 14″	0′ 26″	7936 30′
Copern	47	12′ 11″	3′ 39″	941 53′
Tych.	44	16′ 8″	0′ 0″	infinita

Now recall from Ch. 7 the Sun's distance at apogee, and from Ch. 8 the horizontal parallax set by these authors, and you will recognize the contradiction.

VII. PROBLEMA

Semiangulum Coni Vmbræ Terreſtris aliter inueſtigare.

Problem 7. To investigate the semiangle of the cone of the Earth's shadow in another way.

See no. 8 and Lansbergen's method set out there; but our method, which we applied in the problem and at no. 5, is certainly simpler.

VIII. PROBLEMA.

Axem, ſeu Altitudinem Coni Vmbræ Terreſtris à radijs Solis irrefractis comprehenſæ inuenire.

Problem 8. To find the axis, or height, of the cone of the Earth's shadow enclosed by the Sun's unrefracted rays.

IX. The first and simplest way, which is independent of the Moon except so far as the Sun's distance became known from the distances of the Moon, is the one we use, with the semiangle of the cone of the Earth's shadow already given or found through Problem 5. For in the figure of Problem 4 and its triangle AGH, right-angled at A, given the angle AGH and the side opposite it, AH, namely one semidiameter of the Earth, we seek the axis or height HG. So the residual of the logarithm of the angle AGH, already set in the table of no. 6, gives the logarithm of the base HG. Therefore take from the table of no. 6 the logarithm corresponding to the Sum, and subtract it from 1000000; the logarithm of the shadow's axis will remain. Or the secant of the angle AGH, divided by the whole sine, gives the base HG, in parts of which AH is one.

[Margin: 1. Way of deriving the axis of the shadow.]

The second way, that of Ptolemy, Book 5, Ch. 15, of Regiomontanus in the *Epitome*, Book 5, Prop. 20, and of Copernicus, Book 4, Ch. 19, depends on the Moon's distance and the true semidiameter of the shadow. For in the figure of Problem 6, as AN, the semidiameter of the Earth, is to GL, the true semidiameter of the shadow, so is AGE to EG, by Euclid 6.4; and, by separation, by the fraction by which GL is exceeded by AN, the whole AG, the Moon's distance, is exceeded by AE, the axis of the shadow. Or, as Boulliau does in Book 4, Ch. 1: with PG taken from AM, MT will be known; so let MT be to the Moon's distance TP as PG is to GE; and with AG added to this, AE will be known.

[Margin: 2. Way.]

The third way, which Longomontanus uses in Book 1 of the *Theorics*, Ch. 9, p. 295, and Biancani nearly so, Book 4, Ch. 3, depends on the true semidiameter of the Sun and its distance. For in the same figure of Problem 6, with MR drawn

parallel to AB, since the angles at B and A are supposed right by construction, each angle at R is a right angle by Euclid 1.29, and by 33, MR is equal to AB, and MA

[Margin: 3. Way.]

to RB. So with one semidiameter of the Earth MA, that is RB, taken from the Sun's true semidiameter CB, CR remains. Now as CR is to RM, so is CB to BE; and with BA, the Sun's distance, subtracted from the whole BE, the height of the shadow AE will be known. Or, by separation, as CR is to RB, so is BA to AE; hence, by dividing the Sun's distance BA by the excess of its true semidiameter over the Earth's semidiameter, the axis of the shadow results. Now Lansbergen, in the Prolegomena of the *Uranometria* and in Book 2, Element 7, uses the semiangle of the cone of the shadow discussed above in the table of Problem 6. We, however, use the semiangle of the cone of the shadow set in the table of no. 6, while the authors cited a little earlier use other givens and another method. For that reason different quantities for the axis of the Earth's shadow are found with us and with Lansbergen, and with the authors already mentioned and others who treat the quantity of the axis of the Earth's shadow; these are the ones I cited before the table of no. 1. See their opinion in this table.

Axis of the Earth's shadow with the Sun at apogee, unless perigee or something else is added.

Authors	In semidiameters of the Earth: according to the authors themselves	In semidiameters of the Earth: according to Lansbergen
Ptolemy	268	268 1/16 [?]
al-Battānī	254	225 1/2
Copernicus	265	282
Longomontanus	268	215
Lansbergen	236	236
Kircher	264	✝
Maurolico	268 3/4	✝
But with the Sun at perigee	249 1/6	✝
Boulliau	249 5/6	✝
But with the Sun at perigee	237 3/5	✝
Biancani, at the Sun's mean distance	254	✝ ✝

Axis of the shadow from my calculation.

From the data of the authors	Sun at apogee (semidiameters of the Earth)	Sun at perigee (semidiameters of the Earth)
Ptolemy	271	252
al-Battānī	245	229
Copernicus	269	256

From the data of the authors	Sun at apogee (semidiameters of the Earth)	Sun at perigee (semidiameters of the Earth)
Tycho	282	268
Longomontanus	272	206
Kepler	246	238
Lansbergen	239	222
Boulliau	248	239
Wendelen	237	229
Šírek	230	214
Ours	231 1/2	213

[Margin: Why the remaining measures of the shadow are deferred.]

X. I have deferred, as is opportune, the remaining dimensions of the Earth's shadow, namely its true and apparent semidiameter at the place where the Moon passes through, to Book 5, which is on eclipses, since that measure varies with the varying distance of the Moon from the Earth. Meanwhile, however, from what has been said at this point, remember that it must agree with the true semiangle of the cone of the Earth's shadow and with its axis, both now established here, and that one may not stray outside these limits without inconsistency and want of geometry [ageometria]. And indeed an astronomer must be an Argus, keeping watch on all the links by which the theories of the Sun and the Moon are entwined. And so it is good to gather into one place the theorems, to be committed deeply to memory, that are deduced from what has been demonstrated in this chapter, the last one excepted.

THEOREMATA EX VERA SOLIS Semidiametro, & Diſtantiâ ſequentia.

Theorems following from the true semidiameter and distance of the Sun.

[Margin: From Problem 3.]

Theorem 1. The portion of the solar hemisphere not seen is as great as the Sun's apparent diameter; and the portion seen is as great as the complement of the apparent diameter to 180°.

[Margin: From Problem 3.]

Th. 2. The circle of the Sun separating the portion visible to us from the invisible portion, and its true diameter, are insensibly smaller than the Sun's great circle and its true diameter.

[Margin: From Problem 4.]

Th. 3. The arc of the terrestrial globe beyond the hemisphere of 180° that is illuminated by the Sun's unrefracted rays, even with the Sun at perigee, is smaller than the Sun's apparent diameter, and indeed by very nearly one minute.

[Margin: From Problems 4, 5 and 6.]

Th. 4. The angle of the cone of the Earth's shadow is as great as the arc of the terrestrial hemisphere illuminated by the Sun's rays beyond the hemisphere, and it is always smaller than the Sun's apparent diameter.

[Margin: From Problem 6.]

Th. 5. The semiangle of the cone of the Earth's shadow, together with the Sun's horizontal parallax, is sensibly equal to the Sun's apparent semidiameter, and insensibly greater. Or the Sun's horizontal parallax is insensibly greater than the difference between the semiangle of the shadow and the Sun's apparent semidiameter.

[Margin: From Problem 8, way 1.]

Th. 6. The secant of the semiangle of the cone of the Earth's shadow, divided by the radius, gives the axis or height of the shadow.

[Margin: From Problem 8, way 2.]

Th. 7. The excess of the Earth's semidiameter over the true semidiameter of the Earth's shadow is to the Moon's distance as the true semidiameter of the shadow is to the Moon's distance from the apex of the shadow.

[Margin: From Problem 8, way 3.]

Th. 8. The excess of the Sun's true semidiameter over the Earth's semidiameter is to the Earth's semidiameter as the Sun's distance is to the axis, or height, of the Earth's shadow.

[Margin: From Scholium 1, Ch. 10.]

Th. 9. The Sun's apparent diameter is to the Sun's apparent hourly motion very nearly as 66 to 5.

SCHOLIA.

[Margin: The crude opinions of the ancients on the magnitude of the Sun.]

I. Epicurus and Heraclitus held that the Sun is a foot across, as large as it appears, and that the opposite cannot be proved, assigning it the breadth of a human footprint. Anaxagoras, by contrast, judged it much larger than the Peloponnese, and Anaximander judged it equal to the Earth, as Plutarch reports, *De Placitis*, Book 2, Ch. 21; and regarding Epicurus, Cleomedes confirms this in Book 2 of the *Cyclical Theory*, where he refutes him at length, as I shall indicate in Scholium 2. Certainly, in the figure of Problem 3, if the Sun's distance BA were at least 30 semidiameters of the Earth, each consisting of 4000 Italian miles, so that AB were 120000 miles, that is, 600,000,000 feet, and the Sun's diameter GQ were one foot and its semidiameter AF half a foot, trigonometry proves that the angle MBF under which the Sun would be seen would not amount to one second, and so the Sun would be invisible to us, let alone a foot across. How much more so if it stood more than a thousand semidiameters of the Earth away, as in fact it does? Yet Macrobius, on the *Dream of Scipio*, Ch. 20, on the basis of the fallacious observations of the ancients, laid down the Sun's diameter as double the Earth's diameter, and its solid volume to that of the Earth as 8 to 1. So did infant astronomy once babble about this giant. For the Sun is like a giant, as it says in Psalm 18, not only in its course but also in its magnitude; indeed, it is also a colossus among the planets, and therefore worthy that for it the Rhodians should erect that huge Colossus, and be called Colossians [Colossenses] on account of it. Pliny the Elder, too, in Book 2, Ch.

11, confirms by various arguments that the Sun is much larger than the Earth; these can be looked up in his text.

[Margin: Cleomedes' arguments against Epicurus.]

II. Cleomedes, for his part, affirms in Book 1 of the *Cyclical Theory* that the Earth viewed from the Sun either could not be seen or would appear like a point or a tiny star; but that if we tried to see it from the fixed stars it would still not appear, even if it possessed the Sun's brilliance. From this he argues that the Sun, and indeed the fixed stars too, are larger than the Earth, since despite that same distance the Sun appears as large as it does to us, and the fixed stars also appear. The argument about the Sun holds, given the supposition of a very great distance of the Sun. Now if the Earth had to appear at 1′, the Sun would have to stand 6876 terrestrial semidiameters away from it; on this matter see what we said in Book 2, Ch. 6, Scholia 4 and 5.

In Book 2, moreover, Cleomedes refutes at length Epicurus, who judges everything by the senses alone, and therefore asserts that the Sun is in actual fact a foot across, just as it appears. I. Because the Sun appears larger at rising and at setting than at midday: therefore it would truly become now larger, now smaller in itself; and indeed at the same time, since at the moment when it rises for us, it is on the meridian for others; and according to the approach and withdrawal of the eye, it would be larger for one person and smaller for another at the same time; and larger when viewed from the bottom of a well than from elsewhere. II. Because if it were a foot across and stood at so great a distance, it would not appear. III. Because over 300 stadia gnomons cast no midday shadow among those for whom the Sun is at the zenith. IV. That an intervening island, or a mountain far off, does not prevent us from seeing both edges of the rising or setting Sun on either side. V. From the true magnitude of the Sun as defined by the astronomers, since Hipparchus made the Sun 150 times larger than the Earth; besides other arguments of lesser weight, which can be read in Cleomedes.

[Margin: Pliny the Elder's arguments for the Sun's magnitude.]

III. Most of the reasons by which Pliny the Elder tries to show in Book 2, Ch. 11, that the Sun is larger than the Earth are not very firm, though some are firm. For from this magnitude, he says, it comes about that trees standing in the east and in the west cast shadows separated by equal intervals; and that on equinoctial days all who are in the first climate seem to themselves to have the Sun above their own zenith, just as if that whole tract in latitude were one and the same point; and that the rising Sun suddenly embraces the highest and broadest mountains, such as Ida, lighting them with its rays on both sides; and that it approaches the poles and recedes, lest it otherwise burn up the lands by its magnitude. Finally, that the Earth's shadow is conical, as is established from eclipses of the Moon; otherwise, if the Sun were equal to the Earth, that dulling shadow would be cylindrical; but if it were smaller than the Earth, the shadow would be calathoid [Calathoides: basket-shaped], as we shall show in Book 5.

CAPVT XII.

Chapter 12.

De Motu Diurno Solis ab Ortu in Occaſum, eiuſq. aut Terræ, ſi ſic moueretur, Velocitate.

On the diurnal motion of the Sun from east to west, and on its velocity, or the Earth's if it moved in this way.

I. The diurnal motion of the Sun from east towards west, until it returns to the east and completes a whole natural day, is one whole revolution of the Sun from the same point to nearly the same point of the same horizon or of the same meridian, according as some begin the natural day from sunrise and others from noon, as was said in Book 1, Ch. 28. I said "nearly" both because of the Sun's declination and because of refraction, which do not allow it on the following day to rise for us, truly or apparently, or to reach mid-heaven, at the same point as on the day before. But to avoid being too long-winded, let what we said in Book 1 be taken as repeated: in Ch. 11, on the motion of the primum mobile, on the 15° of the equator ascending in each hour, and on the difference between the day of the primum mobile and the day of the Sun; in Ch. 24, on the distinction between natural and artificial days; and in Chs. 25 and 26, on the inequality of natural and artificial days, although the inequality of natural days must be discussed more fully, and more fittingly, towards the end of this book in Ch. 31. See also what was said in Book 1, Ch. 27, on finding the semidiurnal arc; in Chs. 29 and 30, on the various kinds of hours; and in Ch. 31, on the addition to the days made by twilight. For from these you will grasp all the properties of the diurnal motion.

[Margin: Velocity of the Sun in the diurnal motion.]

II. As for the Sun's velocity: although it seems slow when viewed directly, yet if you catch its bright image through an aperture on a board many decempedae [ten-foot rods] away from the aperture, you will see it flash across very swiftly. We have shown this to many people at Bologna in the church of San Petronio, with the board more than 20 decempedae from the aperture. Now, for the Sun's velocity, commonly thought incredible, by which it completes a whole circle in 24 hours, to be established, an equinoctial day must be taken: for on that day the Sun completes a great circle; and in these present centuries it is then at about its mean distance from the centre of the Earth, and this mean distance is the semidiameter of the circle already mentioned, and therefore suitable for investigating the circumference of the circle by the rules of Book 1, Ch. 4.

For when the Sun is in the Tropic of Cancer, its distance, now almost at its greatest, is not the semidiameter of the circle that the Sun goes round on that day; nor is that circle the greatest in its sphere, even granted that it is larger than a great circle of a smaller sphere, and so than the equator, because of its greater distance from the centre of the Earth. Likewise in the Tropic of Capricorn, the distance from the centre of the Earth, now almost the least,

is not the semidiameter of the circle that the Sun traverses; and that circle is smaller than the equator. Further, one of the mean distances of the Sun must be taken from the table in Ch. 7, p. 110, and doubled, to obtain the diameter of the solar equator; this diameter must be multiplied by 314, and the resulting total divided by 100, and there will come out the circumference, in semidiameters of the Earth, that the Sun traverses on the equator in 24 hours. From this, by the rule of proportion, it will be clear, if it traverses so many semidiameters in 24 hours, how many it traverses in one hour, or in one minute or second of time. But if you wish to know this velocity in stadia or miles, you will have to choose from Book 2, Ch. 7, some more probable opinion on the size of the Earth's semidiameter.

III. But in the condemned hypothesis, which assigns this diurnal motion not to the Sun or to any of the stars but to the Earth, turned about its own centre towards the east in a space of 24 hours, the very circumference of the Earth's great circle, completed in that motion by the Earth's rotation, can be found in its table in Book 2, Ch. 7; and by dividing it by 24 one can find the distance covered in one hour, and by dividing this by 60 the distance covered in one minute of time by the Earth's rotation. In the little tables of the following Scholium, however, we have contented ourselves with the opinion that gives the Earth's semidiameter 860 German miles and one degree of a great circle 15 German miles, because German mathematicians commonly use these. But in the last column we have added the measures agreeing with our own opinion, which gives the Earth's semidiameter 4139 modern Italian and Bolognese miles, and one degree 72¼. And from the authors named in the first column of the former table we have taken nothing but the Sun's mean distance from the Earth. It should be understood, moreover, that one second of time is about equal to one and a half beats of a human artery.

SCHOLION.

Scholium.

[Margin: Fernel's less apt comparison.]

Fernel, in *Cosmotheoria*, Book 1, Ch. 7, sets out the Sun's daily and hourly velocity, using its proper motion towards the east; and he says that the Sun accomplishes on the ecliptic each day 84476 Italian miles, in one hour 3519, and in one minute 58. But the velocity of the motion towards the west is far greater, since it completes the whole circle there in one year, here in one day; and to this velocity belongs that verse of Psalm 18: "He rejoiced as a giant to run his course", etc.

[Margin: The Earth's annual velocity, on the hypothesis, greater than its diurnal velocity.]

Conversely, if the Earth moved with an annual and a diurnal motion, the velocity would be greater in the annual motion than in the diurnal: for each day it would traverse about one degree of the ecliptic. Or, according to Copernicus

on Kepler's calculation in the book *De Nova Stella*, Ch. 16, the centre of the Earth would traverse in one hour, by its annual motion towards the east, 740 German miles; but by the diurnal motion a city situated on the equator would traverse in one hour about the centre of the Earth, likewise towards the east, only 240 German miles. Now the mean distance of the Earth and the Sun is 1142 terrestrial semidiameters; therefore the diameter of the annual orb is 2284, and the circumference 6943 semidiameters of the Earth; one degree therefore has 19 103/360 semidiameters of the Earth. But let there be 20: since Kepler puts 860 German miles in a semidiameter of the Earth, there will be 17200 German miles in one degree of the annual orb, of which the 24th part is 717. But Kepler will have used an apogee distance. Now let us have the little table promised in Ch. 12.

The Sun's velocity in the diurnal motion from east towards west, the Sun being supposed on the equator and at that time at its mean distance from the Earth.

Authors	Sun's mean distance	Traverses in one day	In one hour	In one minute	In one second	In one second
Distances	Semidiameters of the Earth	Semidiameters of the Earth	Semidiameters of the Earth	Semidiameters of the Earth	German miles	Bolognese miles
Ptolemy	1168	7335	305 2/3	5 1/10	73 1/10	
al-Battānī	1107	6952	289 2/3	4 5/6	70 4/5	
Copernicus	1142	7071	294 2/3	4 11/12	70 2/3	
Tycho	1150	7222	300 11/12	5 1/60	72 54/60	
Kepler	1768	11103	462 2/3	7 3/4	111 1/10	
Kepler corrected	3381	21033	876 5/12	14 3/5	220	
Longomontanus	1288	8089	337 1/24	5 1/2	80	
Lansbergen	1498 1/2	9411	398 1/8	6 1/2	93 1/6	
Boulliau	1460	8269	344 1/2	5 3/4	82 3/5	
Wendelen	14656	92040	3835	64 2/3	917 1/3	
van Langren	3420	21478	895	14 11/12	223	
Ourselves, for the present	7300	45744	1906 1/12	31 3/4	547 1/2	2190 1/4

The Earth's velocity, if it moved with a diurnal motion.

	Kind	In one day	In one hour	In one minute	In one second
	Of mile	Miles	Miles	Miles	Paces
Copernicus, Kepler, etc.	German	5400	225	3 3/4	250
Ourselves	Italian, Bolognese	26010	1083 3/4	18 1/15	301 1/10

CAPVT XIII.

Chapter 13.

De Motu Solis Annuo, & de Annorum Solarium Speciebus.

On the annual motion of the Sun, and on the kinds of solar years.

[Margin: Motion of the Sun in latitude.]

I. Besides the motion by which the Sun revolves daily to the same horizon or meridian, another motion has been detected in it, compounded of a motion in latitude and in longitude, and therefore oblique to the equator. For as regards latitude, the Sun now approaches our zenith and pole, now recedes from it towards the opposite pole of the world. It approaches most on the day of the summer solstice, when it enters Cancer; in the century of Christ this day was about June 25, but now it is June 21 in the Gregorian calendar. On that day it describes by its motion a circle parallel to the equator, called the Tropic of Cancer, and comes closest to the zenith of those living in the northern temperate and frigid zones; and it approaches the north pole in such a way that it stands 66½° from it on the meridian and has the greatest declination it can from the equator, namely 23½°, as has been found from corrected observations of the Sun's meridian altitudes. From there it bends its course and turns southwards, and for this reason that circle has been called the Tropic of Cancer: for the Greeks call the limits from which the turning takes place, and the turnings themselves, τροπὰς [Greek: τροπὰς (tropas, "turnings")].

But when it has crossed the equator and entered the sign of Capricorn, it recedes furthest from the inhabitants already mentioned. This used to happen in the century of Christ on about December 25, but now on the 21st; and then comes the winter solstice, in Latin *bruma*, perhaps a word contracted from *the shortest day* [breuissima die] of the year, which then occurs for northerners, since at the summer solstice the longest day of the year occurs for them. On the day of the winter solstice, therefore, it describes by its diurnal motion the other tropic, called that of Capricorn, and recedes furthest from the equator towards the south, namely by another 23½°; and at once, its course reversed, it returns northwards.

Therefore the distance of the tropics, and the whole arc of the meridian within which the Sun's recession from and approach to the poles of the world is contained, is 47°. And since it returns from one tropic to the other in about six months, and from a tropic to that same tropic in 12 months, or 365^{d} and about 6^{h}, it thereby unrolls for us the year called tropical. This same motion has been and is evident not only from the Sun's meridian altitudes, but also from the shadows of gnomons at noon, and from the points of the horizon at which the Sun rises and sets, now nearer the north, now nearer the south. By this motion, through the Providence of God, Best and Greatest, it comes about that the heat and cold of the seasons are more tolerable, and that by their alternation the life and senses of living creatures are refreshed and restored. And this, finally, is the motion of which Ecclesiastes 1 speaks: "The Sun rises and sets, and there, born again, it circles through the south and bends towards the north, and returns to its circles; surveying all things in its circuit, the Spirit goes on."

[Margin: Motion of the Sun in longitude, and by what signs it was detected.]

II. The remaining part of the annual motion, namely that in longitude towards the east, is not indeed so commonly evident, yet it became known to astronomers from the Sun's apparent elongation towards the east from one of the fixed stars around the zodiac. Let Spica serve as an example, a star celebrated in the observation of astronomers ever since Timocharis and Hipparchus. In the time of Augustus it used to rise heliacally at Rome shortly after the autumnal equinox, namely on about the first day of October. Suppose, then, that when the Sun's rising is imminent at the autumnal equinox, say half an hour before sunrise, Spica does not appear, because of its excessive nearness to the Sun's brilliance; then, a few days later, about half an hour before sunrise, that Spica rises heliacally and is seen on the eastern horizon. For if, getting up in good time in the morning on one day after another, you looked at the east half an hour before sunrise, you would see Spica more and more raised from the horizon and shifted further west; so much so that after about three months [mentes; recte: menses] you would see it on your meridian shortly before sunrise, and after six months near the western horizon, and already setting with the cosmic or morning setting, when the Sun rises almost diametrically opposite. And when the year has rolled round, you would again see Spica, about the beginning of October, after its conjunction with the Sun, rise heliacally in the morning in the east. From these and similar phenomena, once observed, the astronomers concluded either that the fixed stars are carried westwards by the diurnal motion in a swifter course than the Sun, or at any rate that the Sun advances eastwards at a slow pace, striving against the motion of the primum mobile; because it is established that all the stars, wandering and unwandering, are swept from east towards west in 24 hours until they revolve back to the east. But since that other motion in latitude was apparent in the Sun, they decided to attribute this apparent recession of the Sun from the fixed stars towards the east rather to the Sun.

[Margin: Obliquity of the Sun's motion, and the discovery of the ecliptic and the zodiac.]

III. From the Sun's motion, then, compounded of a bending in latitude towards the poles of the world and of a progress in longitude towards the east, there arises an oblique motion. When the daily declinations are examined, this has been found to take place in one and the same circle, which cuts the equator in two equal parts at the two equinoctial points and at an oblique angle to the equator of 23½°, on poles likewise 23½° distant from the poles of the equator; and this whole circle is completed by the Sun in 365 days and nearly 6 hours, about one degree out of 360 being completed each day. This circle they called the ecliptic, from the eclipses of the Sun and Moon, which have been observed never to occur except when the luminaries, in conjunction or mutual opposition, that is, at new or full Moon, are with the centre of their body in the plane of this circle, or decline little from it. The ecliptic midway among the others which (if its obliquity to the equator varies over the course of the

centuries) the Sun describes by its annual motion, Kepler calls the Royal Road [Viam Regiam].

And because the other planets have been observed to move, they too, eastwards in an oblique motion, just as the Sun does, but to recede from the ecliptic in latitude towards the poles of the world, some by 6°, some by 8°, some by about 10°, but not more, they therefore devised a certain zone in the heavens within which the wandering of the planets would be contained, 12°, 16° or 20° wide and bisected by the ecliptic. This zone they named the zodiac, from the figures of celestial animals, with which the stars shining within or around this zone had already been endowed by the poets or by the initiates of hieroglyphics; or [Greek: ἀπὸ τῆς ζωῆς (apo tēs zōēs, "from life")], that is, from life, which depends chiefly on this motion of the Sun. They also named it the Signifer [Signiferum], from the 12 celestial signs, and, antonomastically [antonomasticῶς], the oblique circle.

[Margin: Anaximander, discoverer of the zodiac. Cleostratus, its divider. Atlas, inventor of the sphere.]

Concerning its discoverer among the Greeks, Pliny the Elder, in Book 2, Ch. 8, records as follows: "Anaximander of Miletus is said to have been the first to understand the obliquity of the Signifer, that is, to have opened the doors of things, in the fifty-eighth Olympiad; then Cleostratus the signs in it, and first those of Aries and Sagittarius, etc.; the sphere itself, much earlier, Atlas." Plutarch, in Book 2 of *De Placitis*, Ch. 23, relates that this view of the Sun's oblique course through the ecliptic was accepted by Plato, Pythagoras and Aristotle. The figure of the zodiac in the armillary sphere you can see at the beginning of Book 1, and its explanation in that same Book, Chs. 7 and 14.

[Margin: Names and kinds of the year.]

IV. Furthermore, just as natural and artificial days arose from the Sun's diurnal motion, so from the Sun's proper oblique motion there arose the year, which we enjoy by the gift of the immortal God, or the period of 365 natural days with almost a quarter of a day. In Hebrew this year is SCHANAH, that is, Revolution; in Greek it is called [Greek: ἔννος (ennos, "year")], as if Renewing all things, and [Greek: ἔτος (etos, "year")], that is, a whole revolution of the Sun. And although we have dealt with years more fully in the Chronological Book, it must nevertheless be indicated here that the year is usually divided into natural, and political or civil or ecclesiastical; that the natural is subdivided into temporal and sidereal; the temporal into tropical and equinoctial, mean or true; and the civil into various kinds used by different nations, but chiefly into the Egyptian, and the Julian and Gregorian. Lunar years are meanwhile omitted here and deferred to the Book on the Moon.

[Margin: The temporal year.]

V. The temporal year is a whole revolution of the Sun, of about 365¼ days, from one point of the ecliptic to the same point. If this is the point of the solstice occurring in either tropic, the year is called tropical, and by Cicero in the *Dream of Scipio* and by Censorinus, Ch. 7, *Vertens* [the turning year]; if it

is one of the two equinoctial points, it is, and is called, the equinoctial year, which, however, I shall show in Scholium 5 can be called tropical.

[Margin: The sidereal year.]

VI. The sidereal year is a whole revolution of the Sun from one fixed star to the same, accomplished in $365^{d}\ 6^{h}$ and about $9\frac{2}{3}^{m}$. It is longer than the temporal year because, while the Sun once completes the whole zodiac, the fixed stars meanwhile, by a motion that is slow and was understood only late, creep forward towards the east by about 50″; and the Sun by its mean motion does not complete these except in about 21^{m}. And so, when the annual motion through the zodiac has been completed, it has not yet reached the same fixed star, and reaches it only after 21^{m}.

[Margin: The synodic and periodic year.]

This year could also be called synodic, just as the temporal year could be called periodic: in the same way as the month in which the Moon runs through the zodiac is called periodic, and that in which it is again joined to the Sun is called the synodic month. The sidereal year could indeed be imagined in various kinds, according as the Sun revolves to one planet or another, at least those slower than itself; but the fixed stars have been chosen because their motion is either uniform or the least irregular of all, and this sidereal year comes closer to the temporal. The others are too inconstant, and were therefore rejected by Ptolemy, Book 3, Ch. 2.

[Margin: The mean and the true year.]

VII. The mean year, or equal or regular year, in Greek [Greek: ἔννος ὁμαλος (ennos homalos, "even year")], is that which either is, or is assumed as being, of perpetually the same quantity and measure, or the mean between the longest and the shortest, if the view is true of those astronomers who affirm that temporal as well as sidereal years are unequal; on this matter see below, Ch. 30. Therefore the true year, that is, the one that truly comes about in the heavens and is observed on Earth, and for that reason called apparent, is irregular, in Greek [Greek: ἔννος ἀνόμαλος (ennos anomalos, "uneven year")].

[Margin: The threefold measure of the year according to Copernicus.]

Thus, for example, for Copernicus the longest temporal year is $365^{d}\ 5^{h}\ 55^{m}\ 37^{s}$ 40 thirds; the shortest, $365^{d}\ 5^{h}\ 42^{m}\ 55^{s}$ 7 thirds; and finally the mean, $365^{d}\ 5^{h}\ 49^{m}\ 16^{s}$ 29 thirds. On this supposition the sidereal year must turn out unequal, even if the proper motion of the fixed stars were equal and uniform; and unequal for a double reason if the motion of the fixed stars is unequal; on this matter see below, Ch. 28.

[Margin: The civil year.]

VIII. The civil year, also called ecclesiastical, is a whole revolution of the Sun accomplished in as many whole days as has pleased the legislators or pontiffs in each state or hierarchy. The best known, and eventually received throughout almost all Europe or the Roman world, have been these two kinds and forms: the one used by the Egyptians, and the one devised, or rather introduced, by

Julius Caesar; hence the Egyptian and the Julian year, and, from its reformer, the Gregorian, prevailed.

[Margin: The Egyptian and the Persian year.]

IX. The Egyptian year formerly, like the Persian, consisted of 360 natural days, as many as there are degrees in a circle, so that each month had 30 days. They reckoned an appendix of 5 days, which the Greeks called [Greek: ἐπαγόμενας (epagomenas, "added")], that is, additional, as lying outside the round circle of the months; they did reckon these before beginning the new year, but nevertheless outside all the months, which among them were counted in this order: Thòth, Paophì, Athyir, Chœàc, Tybì, Mechir, Phamenòth, Pharmuthì, Pachòn, Paynì, Epiphì, Mesorì. So when Mesorì was finished, the first of the intercalary or additional days was called [Greek: πρώτη ἐπαγομένων (prōtē epagomenōn, "first of the added days")], and the 2nd [Greek: δεύτερη ἐπαγομένων (deuterē epagomenōn, "second of the added days")], the 3rd [Greek: τρίτη (tritē, "third")], the 4th [Greek: τετάρτη (tetartē, "fourth")] and the 5th [Greek: πεντή (pentē, "fifth")], understanding [Greek: ἐπαγομένων (epagomenōn, "of the added days")]. Since, therefore, if you look at the days of the months alone, there were 360 days, but if you add the appendix usually entered upon before the beginning of the new year from the first day of Thoth, there were 365 days, the intercalation of the 6 hours being neglected, or of the day that arises every fourth year from the 6 hours taken four times, the Egyptian and the Persian year, taken absolutely, was therefore of 365 days, as Ptolemy has throughout the *Almagest*, and Censorinus, Ch. 6, and Scaliger, *De emendatione temporum*, Book 3.

[Margin: The golden circle of Osymandias.]

And so Osymandias [Osymandies], king of Egypt, as Dionysius of Halicarnassus relates, encircled his tomb with a golden ring of 365 cubits to signify the year, marking out the Sun's golden path on earth; and 365 Egyptian priests were maintained in the city of Heliopolis, each assigned to observing the Sun's course on his own day; and in Quintus Curtius 365 Persian priests are counted, bearing the sacred fire before the image of the Sun.

[Margin: The Canicular year, and its beginning.]

Now from the omission of the almost 6 hours required for the Sun to return after 365 days to the same point of the solstice or equinox, it came about that every fourth year the Egyptian year was completed almost one whole day before the day of the previous solstice or equinox, and the new year was entered upon; and that in 40 years it ran ahead by 10 days, and in 1460 years by 365 days. So when 1460 celestial years had rolled round, 1461 Egyptian years had already passed, and the beginning of the Egyptian year, that is, the first day of the month of Thoth, returned to the same day and season, in agreement with the sky. This period of 1460 Julian and almost celestial years, but of 1461 Egyptian years, was called the *Great Canicular Year* and the *Sothiac* or *Cynic Period*, from Canicula, the Dog Star, which in Greek or Syriac is called [Greek: Σώθις (Sōthis, the Dog Star)] or [Greek: Κυνὸς (Kynos, "of the Dog")], because this

period was entered upon from the conjunction and morning rising of the Sun with Canicula, or Sirius.

[Margin: The beginning of the Sothiac Period.]

And indeed it could have been entered upon, as Govaert Wendelen observes in our own Denis Pétau, *Dissertations of the Uranologion*, Book 2, Ch. 4, in the year 1263 before Christ, July 5, at about 6^{h} after midnight at Heliopolis; for at that time Sirius was joined to the Sun at the beginning of Cancer, the true new moon having occurred 46^{m} after midnight. If this is true, the beginning of this period was most celebrated, from its concurrence with the beginning of the tropical year, or the summer solstice, with the *neomenia* or new Moon, and with the conjunction with the greatest of the fixed stars and the most brilliantly red, namely Sirius.

[Margin: Apparent disagreement between Wendelen and Censorinus.]

However, this ingenious conjecture of Wendelen's does not agree with Censorinus. Speaking in *De die natali*, Ch. 8, of the year of the consulship of Ulpius and Pontianus, which, as we show in the Chronological Book and as most chronographers admit, falls in the year 238 of the common epoch of Christ, he adds: *So it may also be known that of that Great Year, which, as was said above, is called the Solar, the Canicular and the Year of God, the hundredth circling year is now current.* Therefore the preceding Canicular Year had been completed in the year of Christ 138; and when 138 is subtracted from the period of 1460, its beginning falls back to the year 1322 before Christ, and is older than Wendelen's beginning by 59 or 60 years. But Wendelen and Pétau do not assert that it was in fact entered upon in that year 1263, only that it could have been.

[Margin: And of Censorinus from Clement of Alexandria.]

Clement of Alexandria, however, says in Book 1 of the *Stromata* that the Sothiac period began after the migration of the Israelites from Egypt by, as the Greek codex has it, 345 years, not 445 as the translator has it; and that migration, according to the better chronology, was in the year 1491 before Christ. Therefore the beginning of the Sothiac period would have been in the year 1146 before Christ. But more fittingly elsewhere on this disagreement. Meanwhile I suspect that this period was not brought into civil use, but only among the learned and those skilled in astronomy; and it is no wonder if, given their differing opinions on the motion of the fixed stars and on the other phenomena congruent with this beginning, some thought it should be entered upon in one way and others in another.

[Margin: The Julian and Gregorian Years.]

X. The *Julian Year* is so called because it was introduced into the Roman world by *Julius Caesar*, at the urging and with the help of *Sosigenes*, the Alexandrian astronomer, whom he brought with him to Rome when the Alexandrian war was finished. It consists of 365 days and, in every fourth year, of 366, on account of the day that arises from the almost 6 hours taken four times, and that is intercalated after February 24 by pronouncing *Bis-sexto Kalendas Martij*;

from this it is called the *Bissextum* and the *Bissextile Year*. The Roman Church used the form of this year, with the months as ordered by Julius Caesar, until the year 1582, in which the year was reformed by *Gregory XIII*, a truly supreme pontiff, and called *Gregorian*. For since the equinoctial or tropical year is not of $365^{d}\ 6^{h}$, as *Sosigenes* assumed, but shorter by about 11^{m} of time, it came about that in every 133 ½ years the celestial and true equinox preceded by one day the day assigned to the equinox in the calendar; which in the time of Julius Caesar was March 25, but in the year of Christ 325, by decree of the Council of Nicaea and by the observation of the Alexandrians, was March 21. And so in the year 1582 the celestial equinox was observed by astronomers, namely by Tycho, to precede March 21 by those very 10 days, and to occur at about March 11. So that the equinox might therefore be kept at March 21, out of reverence for the Nicene Synod and because of the custom of the peoples, and might no longer differ noticeably from the celestial equinox, Gregory XIII ordered October 5 of the year 1582 to be called the 15th, that year being shortened by the omission of 10 days; and that the intercalation of the bissextile day should be omitted not in every 133rd year, as *Giovanni Lucido* once urged upon Leo X in *De emendatione temporum*, but in the years 1700, 1800 and 1900, and resumed in the year of Christ 2000, and so on, so that in every four hundred years the three days by which the equinoxes would run ahead in the Julian year are used up.

And Gregory XIII could indeed have fixed the day of the equinox at March 11 or 10, just as the Council of Nicaea, following the observations of the Alexandrian astronomers, had brought it back to March 21, even though in the time of the Apostles it was celebrated near March 25; and this would have been less inconvenient for the astronomers' calculations for past ages; but other just causes outweighed it.

[Margin: God, Best and Greatest, supporter of the Gregorian Year.]

And while the sectaries, to whom everything papal stinks, bark in vain, God, Best and Greatest, by His vote and by the tally of miracles, confirmed the decree of His Pontiff, or greatest Vicar on earth. For the miracles that still flourish continually in the Church He transferred to the days of the Gregorian Year, the leap of 10 days having been made. If then God is for us, who is against us?

SCHOLIA.

Scholia.

[Margin: The conflict between the diurnal motion of the primum mobile and the annual motion.]

I. A great dispute still flourishes among astronomers as to how the two motions of the Sun, and indeed of all the stars, are to be reconciled with one another, since they seem entirely contrary and in conflict with each other, being made towards contrary regions of the world and to opposite limits: namely, the diurnal from east to west, and the annual from west to east. For although we commonly help the imagination of beginners with the example of an ant

crawling in the direction contrary to that in which a potter's wheel is carried round, or of someone walking eastward on a ship while the ship is carried westward, so that in one way or another we may explain the sphere to them, nevertheless these examples in no way satisfy men practised in philosophy and astronomy alike. We shall treat this matter more exactly in the Book on the Systems. Meanwhile I point out three hypotheses by which these motions are commonly reconciled.

[Margin: The first way of reconciling these.]

The first is that of those who, with Sacrobosco, Peuerbach and a few others, admit solid orbs. For they say that the sphere of the primum mobile, which some call the eighth, others the 9th, others the 10th, others the 11th, by the motion of rapture turns all the lower spheres in 24 hours from east through west back to the east; but that these, being really distinct from it, notwithstanding that contact and rapture, strive against it, with a partial sphere (on which see Ch. 20, Schol. 2), by the force of their own motion towards the east, and carry with them the body of the planet fixed in them.

[Margin: The second way.]

The second is that of those who, with Plato in the *Timaeus*, Hicetas of Syracuse, Heraclides of Pontus and Ecphantus the Pythagorean, say that the Earth, placed in the middle of the world, is moved about its centre by a diurnal rotation in 24 hours towards the east; and that therefore, as we go to meet the Sun and the other stars, which are carried eastward, they seem to move westward, just as the shores seem to move while the ship moves in the contrary direction. These have been followed by Boulliau, Lansbergen, William Gilbert, Longomontanus, Mulerius, Origanus and Argoli, besides Philolaus, Aristarchus, Copernicus and the Copernicans, who give the Earth both this motion and the annual one. This opinion has indeed been condemned by the Sacred Congregation of Cardinals as erroneous in the faith, as was said in Book 2, Ch. 3.

[Margin: The third way.]

The third hypothesis is that of those who say that there are not in the heavens the two motions in longitude already mentioned, but a single one, and that diurnal, made in a kind of spiral towards the poles of the world; and that the swiftest is the motion of the primum mobile, of exactly 24 hours; that after this the motion of the fixed stars is the swifter, yet does not keep pace with the motion of the primum mobile but falls short by some small part, which in the end becomes perceptible; that after this Saturn is the swiftest of the planets and the Moon the slowest, since she does not return to the same meridian until after almost 25 hours, and that therefore she seems to move eastward because of her slowness, although she moves only westward, but more slowly.

[Margin: Greater diurnal speed in the higher stars.]

So think Averroes, al-Biṭrūjī, Alexander Achillinus [Alessandro Achillini], *De orbibus*, Book 1, Francesco Patrizi and Giovanni Antonio Delfino, and indeed some Peripatetics in Martianus Capella, Book 8; nor does Cabeo reject

it (on *Meteorologica* 1, text 38, question 2). And all those favour this opinion who, in distributing the seven strings among the planets, assign the *nete* to Saturn as the most highly strung, and the *hypate* to the Moon as the slowest: such are Cicero in the *Dream of Scipio*, and Macrobius on that passage, Servius, Philander, Valla, Glareanus, and indeed St Ambrose, as I shall say when treating of the harmonic system of the world. But Agostino Ricci, p. 16, maintains that there is no primum mobile distinct from the 8 spheres.

[Margin: The various forms of years.]

II. The forms of years have varied at different times and among different nations. For the Gauls once and the Saxons, and now the Tartars, Turks, Arabs, Chinese, Japanese, Peruvians and Chileans use a purely lunar year; the Hebrews, however, a solar and lunar year together. But the Mexicans formerly, as José de Acosta relates, and the Greeks before and at the beginning of Solon's time, as is gathered from Aristotle, *History of Animals*, Book 6, Ch. 20, from Herodotus, Book 1, from Pliny the Elder, Book 34, Ch. 6, and from St Augustine, *De Trinitate*, Book 4, Ch. 4, used a year midway between the lunar and the solar, namely of 360 days; and for that reason 360 statues to Demetrius of Phalerum were erected and overthrown at Athens in one and the same day. But on Solon's advice they directed the year by the course of the Moon, yet equalizing it with the solar year by various intercalations; and first Romulus, crudely, then Numa and the Romans up to Julius Caesar imitated them.

That boast of the Arcadians, who [boasted that they were] [Greek: προσελιναίους (proselinaious, "before the Moon")] [recte: προσεληναίους], that is, older than the Moon, and of the Egyptians, who from the number of their years displayed themselves as the most ancient of mortals, is to be understood of years that were monthly among the Arcadians, and afterwards three-monthly, or of four-monthly years among the most ancient Egyptians, or of the six-monthly years of the Acarnanians. But on this diversity, if you lack our Chronological Book, you can consult Pliny the Elder, Book 7, Ch. 48; Solinus, Ch. 3; Macrobius, *Saturnalia*, Book 1, Ch. 12; Censorinus, *De die natali*, from Ch. 7; Giraldi, in the book *De annis et mensibus*; Alexander ab Alexandro [Alessandro Alessandri], *Genialium*, Book 3, Ch. 24; Joseph Justus Scaliger, in the books *De emendatione temporum*; and Denis Pétau, in the books *De ratione temporum*.

[Margin: Cycles, and Great Luni-Solar Years.]

III. Great Luni-solar Years are cycles or periods on whose completion the syzygies of the Moon and Sun at last return to the same moment of the same day. Such a period was judged to consist of 19 solar years by Meton; of 59 by Philolaus and Oenopides; of 76 by Callippus; of 82 by Democritus; of 84 by the Hebrews under Constantine the Great; of 247 by Gamaliel; of 304 or 344 or 441 by Hipparchus; of 3400 by Franciscus Vieta [François Viète] and very nearly by Kepler: on these see Book 4, Ch. 20. So that new moons and full moons might return to the same weekday, given the cycle of the Dominical letters, which computists call the Cycle of the Sun and which is of 28 years, various cycles

were devised: namely of 95 years by St Cyril of Alexandria, which consists of five Metonic lunar cycles; or of 318 by Theophilus of Alexandria; or of 532 by Victorinus of Aquitaine, Victor of Capua and Dionysius Exiguus, which arises from multiplying the lunar cycle of 19 years by the solar cycle of 28 years. But if you multiply the cycle of 532 years by the period of the indictions, or 15 years, there results the period of 7980 years, which Scaliger calls the Julian, and which is commended by him too much, and indeed by Pétau as well, *Rationarium temporum*, Book 1, Ch. 4, as outstanding for marking dates. But in this way, if you multiply Viète's period of 3400 by the solar cycle of 28, a Great Period of 95200 years would result.

[Margin: Greater and Great Years.]

IV. The remaining periods of solar years can be called Greater Years, as was the Jubilee Year among the Hebrews, of 49 completed years, but of 50 when both limits are counted; and the Secular Years among the Romans, of 100, 110 or 120 years, on which our Petrus Taffinus [?] has recently written most learnedly in the book *De anno saeculari*; and the Wheel of 52 years among the Mexicans, at whose end they feared the end of the world, as José de Acosta relates in Book 6; and the period of 470 years among the Japanese; and the Canicular period of 1460 years among the Egyptians, of which I spoke in the preceding chapter, no. 9.

[Margin: The Platonic Year.]

But the Great or Platonic Year is most celebrated among writers: on it see Cicero, *Dream of Scipio*, and Macrobius on that passage; Censorinus and Giraldi, cited above; Manutius, the book *De anno magno* in the *Quaesita per epistolam*; Justus Lipsius, *Physiologia Stoica*, Book 2, Ch. 21 and 22; and the scholiasts of Virgil on Eclogue 3. For the Stoics thought, as did the Platonists, that on the complete *apocatastasis*, or revolution of all the stars to the same point, there would be a restoration of all things and an *anastasis*, or resurrection, but preceded by a universal cataclysm or deluge if the stars came together in Capricorn; or by [Greek: ἐκπυρῶσι (ekpyrōsi, "conflagration")], or a general burning, if they came together in Cancer.

But since some look only to the *apocatastasis* of the planets, others only to that of the fixed stars, others to that of absolutely all the stars, they have defined the Great Year variously, according to their differing opinions on the stars' motions: namely as 1460 years, Julius Firmicus Maternus, misusing the Canicular period; or as 2494, Aristarchus; or as 5552, Aretes of Dyrrhachium; or as 9987, Heraclitus and Linus; or as 9994, Dion; or as 12000, Orpheus; or as 15000, Cicero; others as 28000, others as 3000. The Jews who compiled the Alphonsine Tables said that after a thousand Jubilees, that is, in 49000 years, the fixed stars would complete their course, and that then all things were to be renewed.

[Margin: The incommensurability of the celestial motions.]

Perhaps it was better said by Didymus, in Censorinus, and by Kepler in the *Mysterium Cosmographicum*, Ch. 23, that the motions of all the stars are

incommensurable with one another, and that therefore, even if the world endured, they would never all complete their periods at the same point of the heavens.

[Margin: A more learned notion of the tropical year.]

V. We have defined the *tropical year* as the revolution from one point of the solstice to the same point, at the contact of the tropical circles, following the use of this name among more recent writers. Otherwise the equinoctial year can be called tropical, since not a few have also called the beginnings of Aries and Libra [Greek: τροπὰς (tropas, "turnings")], because of the changes of weather that occur in them, as Pétau learnedly observes in *Rationarium temporum*, Book 1, Ch. 1. In his text Sextus Empiricus, *Against the Mathematicians*, Book 5, Ch. 1, says: "The tropic signs are those in which the Sun, when placed, works a change and makes the turnings of the air; such is Aries, and Libra diametrically opposite it, likewise Capricorn and Cancer. For in Aries there occurs the [Greek: ἐαρινὴ (earinē, "of spring")], the vernal turning; in Capricorn the [Greek: χειμερινὴ (cheimerinē, "of winter")], the winter one; in Cancer the [Greek: θερινὴ (therinē, "of summer")], the summer one; in Libra the [Greek: ὀπωρινὴ (opōrinē, "of autumn")], the autumnal one." So Oppian [Oppinnus], *Cynegetica*, Book 1, names the autumnal turnings [Greek: ὀπωρινὰς τροπὰς (opōrinas tropas, "autumnal turnings")]; and Procopius, *De bello Gothico*, Book 1, for the vernal equinoxes has [Greek: τροπὰς ἐαρινὰς (tropas earinas, "spring turnings")]; and Philo in the *Cosmopoeia*, for the autumn equinox, [Greek: μετοπάρου τροπὰς (metoparou tropas, "turnings of autumn")] [recte: μετοπώρου]; and Manilius, Book 3, speaking of the beginnings of the 4 cardinal signs:

Which they call tropic, because in them the four seasons of the year turn, in those signs.

CAPVT XIV.

Chapter 14.

De Obſeruatione Aequinoctiorum & Solſtitiorum, & de Quatuor Anni Cardinibus.

On the observation of the equinoxes and solstices, and on the four cardinal points of the year.

I. The length of the tropical year and the annual inequality of the Sun's motion, which are to be discussed in the following chapters, depend on the observation of the solstices and equinoxes. This cannot be carried out unless the day assigned to the solstice and to the equinox in some ephemeris or in the calendar, and the way of observing each, are known beforehand. As regards the day, our Pétau, in the *Uranologion*, Book 1, Ch. 8, and Book 2, Ch. 5, having computed the Sun's motion from the more recent tables for the Roman meridian, and for the first Julian year, which was 45 before Christ, gathers the Sun's ingress into the four cardinal signs, that is, into the beginning of ♈ ♋ ♎

♑, as you see in the following table, both according to the mean motions and according to the true ones, in hours after noon.

[Margin: The days of the 4 cardinal points in the first Julian year.]

The Sun's true and mean ingress into the cardinal signs, in the year 45 before Christ, at Rome. [Solis Ingreſſus Verus, ac Medius in Signa Cardinalia Anno ante CHRISTVM 45. Romæ.]

	Months	Mean	True
Vernal equinox and ingress into ♈	March	$24^{d}\ 21^{h}\ 11^{m}$	$22^{d}\ 23^{h}\ 47^{m}$
Summer solstice and ingress into ♋	June	$24^{d}\ 4^{h}\ 39^{m}$	$24^{d}\ 23^{h}\ 3^{m}$
Autumnal equinox and ingress into ♎	September	$23^{d}\ 12^{h}\ 6^{m}$	$25^{d}\ 11^{h}\ 11^{m}$
Winter solstice and ingress into ♑	December	$23^{d}\ 19^{h}\ 34^{m}$	$24^{d}\ 23^{h}\ 54^{m}$

[Margin: The day of the vernal equinox in the first Julian year.]

From this it is clear that Sosigenes, who drew up the Julian calendar, had regard to the mean motions rather than the true ones. For, as Pliny the Elder explicitly has it in Book 18, Ch. 25, 26 and 28, Varro in *De re rustica*, Book 1, Ch. 28, and Columella in Book 9, Ch. 14, and Book 11, Ch. 2, he fixed the cardinal points of the year at the 8th before the Kalends of March, June, September and December [VIII. Kalendas Martij, Iunij, Septembris, & Decembris], that is, on March 25, June 24, September 24 and December 25. And thus in the calendar of Julius Caesar, in Giraldi's book *De annis*, in Johannes Rosinus's book of *Roman Antiquities* and in Macrobius, *Saturnalia*, Book 1, the Hilaria, which was a festival for the beginning of spring, are placed on March 25; and it is clear from the same calendar that the Quinquatria usually ended on March 23. And so the equinox was celebrated on the third day from then, inclusive, as Ovid elegantly sings in *Fasti* 3, after reviewing the end of the Quinquatria:

When the coming Dawn has sent ahead three Morning Stars, she will bring daylight hours equal to the nocturnal.

The Fathers too connect these two things, namely that St John the Baptist was born at the summer solstice on June 24, when the days begin to decrease, and Christ at the winter solstice on December 25, when the days begin to increase, expounding that saying of the same Baptist, *He must increase, but I must decrease*: in particular St Augustine and St John Chrysostom, in the sermon on St John the Baptist.

[Margin: The day of the vernal equinox in the year of Christ's Passion.]

Moreover, the same St Chrysostom and St Augustine, and also Tertullian, Cyril, Jerome, Cassiodorus, Bede, Albert the Great and Marianus Scotus, record that Christ the Lord suffered and died at the vernal equinox and on March 25. Finally, that in that age the vernal equinox was fixed [Greek: πολιτικῶς (politikōs, "civilly")], or for civil purposes, to March 25, is taught by Stöffler on Proclus, p. 15, and in the *Calendar*, Propositions 38 and 39; by Reinhold on Peuerbach's *Theoricae*, p. 232; by Longomontanus, *Theorica*, Book 1, p. 175; and by Pétau, cited above.

But since, according to the rules of Rabbi Adda in the Hebrew calendar, the *Tekupha* of the month of Nisan, that is, the cardinal point and equinox of the spring season and of the first ecclesiastical month, fell in the year of Christ 34 on March 22, or according to the true motions on about the 23rd, perhaps for that reason Nicetas and Lactantius, *Institutes*, Book 4, say that Christ died on March 23, and others otherwise, as we shall say in the Chronological Book.

[Margin: The day of the vernal equinox in other centuries after Christ.]

II. But because the true equinox gradually precedes the day assigned to it in the Julian calendar, in the year of Christ 140 it was therefore observed by *Ptolemy* to fall on March 22, and Anatolius of Alexandria followed him in determining the day of Easter; while in the year 325 it was observed at Alexandria on March 21, and for that reason the Fathers of the Nicene Synod assigned this day to the equinox. Dionysius Exiguus retained it in his cycles in the year 525, although it then in fact fell on the 19th. In the year of Christ 1079, 8 distinguished Persian mathematicians, summoned by the Emperor of the Persians to reform the calendar, observed the vernal equinox and found that it occurred on March 14, as Scaliger reports from the Persian tables, *De emendatione temporum*, Books 3 and 4, and Boulliau, *Astronomia Philolaica*, Book 2, Ch. 3. Further, in the years 1290 and 1303 it was observed in Lower Germany on March 13, as Cardinal Nikolaus von Kues relates in his treatise *De reparatione Kalendarii*. Again, about the year 1400, that common couplet was on the lips of the people:

Lambert, Gregory: the night is made equal to the day; Vitus, Lucy give you the two solstices.

By this it was signified that the equinoxes fell on March 12, sacred to St Gregory, and on September 13, sacred to St Lambert; and the solstices on June 15, dedicated to St Vitus, and on December 13, dedicated to St Lucy. At last, from the year 1470 to 1520, Peuerbach, Regiomontanus, Bernhard Walther, Johannes Werner, Copernicus, Pietro Pitati and others observed that the vernal equinox fell on March 12 or 11; and from the year 1570 to 1590 the mathematicians of the Landgrave of Hesse, and then Tycho Brahe, found it on March 11 or 10. For these reasons the restoration of the calendar, once debated in the Councils of Constance and the Lateran, was carried out by order of *Gregory XIII* in the year 1582, and was set forth and defended by our *Clavius*. The vernal equinox was fixed to March 21, not, as Giovanni Lucido and Johannes Stöffler had once judged should be done, to March 10 or 11, but with 10 days taken from the calendar; and it was decreed that at the end of each of three centuries the day otherwise to be intercalated should be omitted, the bissextile being suspended, namely in the years 1700, 1800, 1900, yet the year 2000 should be bissextile in the usual way.

[Margin: The day of the vernal equinox in the Gregorian calendar is always the same.]

If this rule is kept, the civil vernal equinox will always fall on March 21, and yet it will never differ from the mean celestial equinox by a whole day and a

half, as *Clavius* clearly shows against Mästlin in the *Apologia* of the Gregorian calendar, Ch. 6. So this advantage arises from this calendar, that the day on which, or nearest to which, the true equinox is to be observed is now certain. Let us now come to the way of observing these.

Modus Obſeruandi Solſtitia.

The way of observing solstices.

[Margin: The observation of the solstitial day.]

III. First, if you are seeking only the day of the solstice, disregarding the hours, then around the day assigned to the solstice in the calendar or in an ephemeris you must observe the Sun's meridian altitude with a quadrant, an astronomical square or similar instruments; or else the meridian and direct shadow of a gnomon perpendicular to the horizon. For on the day on which the Sun's altitude has been found greatest, so that it no longer approaches your zenith, or the shadow of the gnomon least, you will conclude that the summer solstice occurred within that natural day. But on the day around midwinter on which the Sun's altitude has been found least among all the meridian altitudes of the year, or the meridian shadow longest, judge that the winter solstice occurred within that natural day.

[Margin: The great gnomon of Marseille.]

But the day of either solstice is observed more evidently at noon if the gnomon is very tall, such as Gassendi describes in the *Life of Peiresc*, used at Marseille from as far back as the time of Alexander the Great by Pytheas of Massalia, who found the solstitial shadow at noon to be 213 ⅛ parts of which the gnomon was 600. In the same place Gassendi himself used a gnomon 52 royal feet high, and found the meridian shadow at the summer solstice to be 31950 parts of which the gnomon was 90000; from this he gathered that the obliquity of the ecliptic is now the same as in the time of Alexander the Great.

[Margin: The great gnomon of Bologna in the church of San Petronio.]

We too here at Bologna have the use of a gnomon erected in the church of San Petronio in the year 1576 by the Reverend Father Master *Egnazio Danti*, at the expense and on the authority of the most illustrious Count *Io. Pepuli* [Giovanni Pepoli], perpetual president of the fabric of San Petronio. Take a specimen of it from the following diagram.

In this AB is the wall of the church facing south, at whose summit, at the edge of the vault, there is a metal plate of a thickness equal to the width T, and in it an aperture exactly equal to the circle Z. The plate is inclined from the perpendicular AX, as you see in XB, namely by 60°, as I measured it with *D. Bartholomæus Provalius* [?]; for that is how the workmen inclined it in repairing that vault. Otherwise *Egnazio*, as I learned from his papers and from the account of it, had inclined it by only about 45 ½°, which is the altitude of the equator here. Further, the height of the aperture, AX, is 65 Bolognese feet and 9 ¼ inches [vnciarum]. The meridian line is AV, from which towards the west

there declines (as I measured accurately with Fathers *Francesco Maria Grimaldi* and *Francesco Zeno* from observation of noon, on which I shall say more in the *Liber Organicus*), there declines, I say, by 9° 6 ⅓′ the line, or rather the marble path, AN, by my measure 169 Bolognese feet and 2 ½ inches long, laid in the pavement from A towards the north door of the church; for, since the columns of the church stood in the way, it could not be laid on the meridian line itself. The twelve signs of the zodiac are cut into this marble band, and LN is the greater diameter of the elliptical solar ray from the aperture XB, projected onto the pavement on the day of the winter solstice. For on the day on which a little after noon its end is at N, or very near it, the winter solstice is concluded to occur within that natural day; for on the preceding and following days the northern end of this ellipse stands off from N towards L by a most evident interval of several inches. In the same way the ellipse FH serves for observing the equinoxes, and C for the summer solstice. But on our correction of this gnomon, and on the very many observations undertaken by means of it, it remains for us to speak both in the *Liber Organicus* and in the solar observations to be reviewed in the last volume of the *New Almagest*. Meanwhile one must know that the angle XAN must be exactly right, if gnomons of this kind are to be brought into practice without error.

[Margin: Other ways of observing the solstices.]

I pass over the way of observing solstices by the rising and setting amplitudes of the Sun, for the vapours of the horizon and refractions hinder those; yet more exact it is made, if the Sun's setting is observed behind a mountain, but so that part of the northern limb stands out on this side of the mountain, towards the north; for the day on which the largest part of the solar limb appears will be the solstitial one. *Galileo* touches on this method in Dialogue 3 on the System of the World. Now let there be the figure of the gnomon set out a little earlier.

Diagram. *The Great Gnomon of Bologna, in the Church of San Petronio, set up by the Reverend Father Master Egnazio Danti on the authority of the Most Illustrious Count Ioannes Pepulus [Giovanni Pepoli], in the year 1576.* A full-width woodcut standing across both columns. At the left, a tall hatched wall in section, its top broken off irregularly, with an arched opening at its foot. High in the wall a small disc (the aperture) lies between X and B. Thin lines run up and to the left from the aperture to the letters R, S, Q, P, O and Y, standing for incoming rays. From the aperture, lines, some solid and some dotted, fan down and to the right to a long horizontal ground line running from A at the wall's foot to N at the far right. A short line rises from A to V. On the ground line stand a small circle under C, near the wall; a lens-shaped ellipse between F and H, at mid-length; and a longer lens-shaped ellipse between L and N, at the far end. Signs of the zodiac are set in pairs along the line, one above and one below. At the upper right, separate from the rays, stand a small rectangle lettered T and a large circle lettered Z.

- R, S (top left, above the wall): ends of the two highest rays
- Q, P, O, Y (left of the wall, descending): ends of lower rays

- X, B (either side of the aperture disc): X at left, B at right
- A (foot of the wall, left end of the ground line)
- V (on the short line rising from A)
- C (in the small circle, upper half), with the sign ♋ in the lower half
- D (on the line), with ♊ below it
- ♌ and ♍ above the line, either side of E; ♉ below E
- F, H (ends of the middle ellipse), with ♎ in its upper half, crossed by two short oblique strokes, and ♈ in its lower half
- ♏ above the line, with K and ♓ below it
- ♐ above the line, with I and ♒ below it
- L, N (ends of the far ellipse), with ♑ in its upper half and M in its lower half
- T (in the rectangle, upper right)
- Z (in the large circle, upper right)

IV. But if you wish to know not only the day but also the hour of the solstice, you will achieve this only with the greatest difficulty; and yet you must strive for it, if you are held by a zeal for cultivating astronomy. First, then, you must know the altitude of the pole with the utmost exactness, not from the Sun's altitude but from the circumpolar stars. Next, on the day assigned to the solstice in the ephemeris, and on the days immediately before and after it, take the Sun's meridian altitude with large quadrants that show at least each single ten seconds. Then correct the altitudes of these 3 days by adding the parallax and subtracting the refraction, if any occurs; and you will have the Sun's true altitudes. If it is the winter solstice, subtract these from the altitude of the equator, that is, from the complement of the altitude of the pole; but if it is high summer, subtract the altitude of the equator from them, and the Sun's declinations will remain.

Now if the altitudes of the day immediately before and the day immediately after the solstitial day are wholly equal, the solstice will have occurred at noon of the intermediate day, and the declination found on that day will be the greatest; and with one and the same labour you will have observed the obliquity of the ecliptic. But if they are unequal, and at the summer solstice the altitude of the preceding day is greater than the altitude of the day following the intermediate day, the solstice will have occurred before noon; if it is less, after noon of the intermediate day. And if you multiply the difference of the altitudes by 24 and divide the product by 15, the result will be the hours before or after noon at which the solstice occurred; for the whole difference of declination over one complete day is then about 15″. But if it is the winter solstice, a lesser altitude on the preceding day will mark a solstice before noon, and a greater one a solstice after noon.

[Margin: The difficulty of observing the hour of the solstice.]

So if there is an error of 16″ in the altitude of the pole, or of the Sun, or in the parallax, the solstice will be wrong by one whole day; if of 8″, the error will be half a day; and if the error is 4″, the error will be a quarter of a day. It was therefore with good reason that *Willebrord Snellius*, in his notes on the Hessian

observations, said it is a labour of Hercules to avoid an error of a quarter of a day in solstices.

[Margin: The confidence of Ptolemy, timid elsewhere.]

But *Ptolemy*, in Book 2 of the *Almagest*, Chapter 2, said too confidently: *But in the solstitial observations, I hope that neither we nor Archimedes erred, in observation and calculation, by as much as a quarter of a day.* For the instruments Ptolemy used, as I shall say in the *Liber Organicus*, did not contain so fine a division that they showed tens of seconds. And since the dispute over the parallax of the Sun is still undecided among astronomers, whether it is 3′, or 2′, or 1′, or 30″, or 15″, as is clear from what was said in Chapter 8, the determination of the solstice also remains uncertain, with a doubt of 2¾′, that is, of 4 whole days and more, let alone hours, if we relied on a single noon altitude. But if we take three, as I said, then even if there has been an error in the parallax, it will still affect all the altitudes equally, and the difference of the altitudes is enough for us. Since, however, by God's favour, we have obtained among several observations of solstices one from which we inferred that the solstice occurred at noon itself, and from it the obliquity of the ecliptic, we shall bring this one forward as an example.

EXEMPLVM. Solstitij Obseruati.

Example of an observed solstice.

[Margin: A notable summer solstice observed by the author.]

V. In the year 1646, at Bologna, where the altitude of the equator had been observed by us from the circumpolar stars more than forty times and found to be 45° 30½′, the Sun's meridian altitude was taken with three large quadrants. On June 20 it was seen at 68° 59′ 45″, exactly as on the 22nd. But on the 21st it was observed at precisely 69°. Therefore, from the equal altitude of the 20th and 22nd, and from the difference of declination of 15″, I concluded that the solstice occurred at noon itself, as far as human diligence could observe. Now, at so great an altitude the refraction was not perceptible at all; and the parallax, from the Sun's distance from us as observed through the dichotomies of the Moon, is 30″ at most. So the true altitude on the 21st was 69° 0′ 30″. Subtracting from this the altitude of the equator, 45° 30′ 30″, leaves the obliquity of the ecliptic, 23° 30′. But if I had applied the Tychonic parallax of 1′ 2″, I should have concluded an obliquity of 23° 31′ 2″. Now let us move on to the equinoxes.

[Margin: For the parallax see Chapter 8.]

Modus 1. Obseruandi Aequinoctia per Vmbras Gnomonum.

Method 1. Of observing the equinoxes by the shadows of gnomons.

VI. This is done in two or three ways. First, one observes the day on which the shadow of a gnomon, by its tip, describes through the whole day one and the same straight line on a horizontal plane; for that day is taken as the

equinoctial day. On other days, the tip of such a shadow describes a hyperbolic line in the Torrid Zone and the Temperate Zones; for those who live under the polar circles, that is, for whom the pole is raised 66½°, a parabolic line; and in the frigid zones an elliptical line, approaching more and more to a circle, until for those living under the poles themselves it describes a circular line. The Sun placed in the equator also does this for them, because on account of refractions it appears higher above the horizon; for this is how *Maurolico* must be corrected, who discusses these figures of shadows subtly in Dialogue 2 of his *Cosmographia*, p. 61.

[Margin: Method 1. What figure the tip of the shadow of gnomons describes.]

[Margin: Method 2.]

Secondly, that day which, on an astronomical and horizontal sundial, is found to have 12 hours, or because of refra- ctions that lengthen the artificial day at both ends by about 4^{m}, as I have quite often observed, to have $12^{h}\ 8^{m}$, is to be taken as the natural day of the equinox.

[Margin: Method 3.]

Thirdly, the noon shadows of gnomons are observed, and from their ratio to the height of the gnomon the Sun's altitude is derived. If this is equal to the altitude of the equator known beforehand, as is assumed from the altitude of the pole, it will show that the equinox occurred at noon; but by as many minutes as it exceeded or fell short of the altitude of the equator, by so many hours was the time of the equinox before or after noon.

[Margin: Equinoctial shadows observed long ago in notable cities.]

So from Strabo, Book 2, Vitruvius, Book 9, Chapter 8, Pliny the Elder, Book 2, Chapter 72, Ptolemy, *Almagest* Book 2, Chapters 2, 3, 5, 6, and Cleomedes, Book 1 of the *Cyclical Theory*, you will learn that the equinoctial noon shadow was observed in relation to the gnomon as follows: at Alexandria in Egypt as 3 to 5; at Ancona as 34 to 35; at Athens as 3 to 4; at Byzantium as 111 to 120; at Carthage as 7 to 11; at Rome as 8 to 9; at Tarentum as 9 to 11; and in the Venetian region equal to the gnomon. *Pliny the Elder*, moreover, Book 36, Chapters 9 and 10, relates that Augustus brought from Egypt an obelisk of 116¾ feet, which had once belonged to King Sesostris, and set it up in the Campus Martius in the role of a great gnomon. A stone pavement was laid, on which the noon shadow would fall, and a gilded ball was added to the tip of the gnomon by Manlius the mathematician, to gather the shadow.

But *Ptolemy* admits how far this method is from accurate, *Almagest* Book 2, Chapters 5 and 6. And the ancients erred in taking the altitude of the Sun's upper edge, whose ray bounds the direct shadow [vmbram rectam], in place of the altitude of the Sun's centre, as I shall explain more fully in both the *Liber Organicus* and the Geographical Book; for they ought to have subtracted the Sun's apparent semidiameter from the altitude found by the direct shadow.

Modus 2. Obseruandi Aequinoctia per Armillas Aequatorias.

Method 2. Of observing the equinoxes by equatorial armillaries.

VII. This method was very famous at Alexandria, used by Hipparchus and Ptolemy and described in *Almagest* Book 3, Chapter 2, and by *Egnazio Danti* in the *Astrolabe*, Part 4,

Diagram. A column-width woodcut of an equatorial armillary instrument. A vertical line runs from F down to G, with a bracket at its foot. Against it stands an upright ring, CD, the meridian, drawn as a flat band in perspective, held by two clasps at its left side. A plumb line hangs from C within the ring and ends in a small ball. A second, horizontal ring in perspective, HK, passes through the first; it is the equator, and the two cross at O at the front. At the lower part of the upright ring a curved strap with a clasp runs down to G.

- F (top, at the head of the vertical line)
- C (top, at the head of the upright ring)
- O (front crossing of the horizontal and upright rings)
- B (upper clasp on the upright ring)
- A (lower clasp on the upright ring)
- I, L (small letters on a short horizontal bar inside the upright ring, between B and A)
- E (right of the upright ring, inside the horizontal ring)
- H (left end of the horizontal ring)
- K (right end of the horizontal ring)
- M (below the plumb ball, inside the upright ring)
- N (lower inner part of the upright ring)
- Q (left of the upright ring, at the fastening above D)
- D (foot of the upright ring, at right)
- P (on the lower curved strap)
- G (foot of the vertical line, at left)

p. 317. These rings are made like the rims of wheels, from four-sided, right-angled rulers bent into hoops, as you see here in the accompanying figure. One of them, CD, represents the meridian. It must be placed exactly in the plane of the meridian with the help of the plumb line CM, and be fixed immovably to a wall or column, fastened with the clasps A and B, and rest on a very firm support G, to which it must be so secured by means of the clasp PQ that it is kept immovable in the plane of the meridian.

[Margin: Description of equinoctial rings.]

The other ring, HK, represents the equator. For its plane must coincide with the plane of the equator and bisect the meridian, and be raised in it by as much as the altitude of the equator, that is, the complement of the altitude of the pole. From this placing it follows that, when the Sun passes from the southern signs to the northern and enters Aries, the shadow of the upper part of the ring HOK, which on the preceding day was shading the northern hollow of the lower part, HMK, shades the southern part, HNK. But when at the autumn

equinox the Sun passes from the northern signs to the southern, the shadow in the hollow of the lower ring, which on the preceding day had been southern, becomes northern.

Indeed, if the equinoxes fall during the day, the passage of the shadow does not appear until the hollow lower part of the ring HK is lit equally on both sides; for the shadow is narrower than the upper part of the ring HOK that casts it, since that part is lit by the Sun, which is far larger than the ring. And when it is lit on both sides, that is the sign of the apparent equinox. But if this passage does not appear by day through equal lighting on both sides, and if instead, in spring, on March 20 for example, the ring's shadow is northern until evening but southern on the following morning, it will mark an equinox that occurred in the night between the 20th and the 21st. In autumn, if the shadow was southern in the evening, say on September 22, and northern on the following morning, it will indicate an equinox that occurred in the night between the 22nd and the 23rd, at least apparently. And from the size of the part lit you will infer the hour of the equinox: if the evening lighting was equal to the morning lighting, it will indicate an equinox at midnight; if unequal, beyond or short of midnight, according to the proportion of the excess.

[Margin: The fallacy of rings, unless it is corrected.]

VIII. I said *apparently*, because two things vitiate this practice unless they are corrected. The first is the parallax of the Sun. For the equinoctial ring HOKM is not concentric with the centre of the Earth, but has its centre on the Earth's surface. Parallax therefore hastens the autumnal equinoxes and retards the vernal ones, by showing us the Sun lower than if it were viewed from the centre of the Earth. So from a vernal equinox observed in this way one must subtract as many hours, or minutes of an hour, as the minutes or seconds of the Sun's parallax proper to that altitude; for an autumnal one they must be added. Granted that, as *Lansbergen* judged in Class 2 of his solar observations, *Ptolemy* omitted this correction, let alone *Hipparchus*.

The other and greater fault comes from refraction, which, by raising the Sun in appearance, hastens the vernal equinoxes and retards the autumnal ones. And it sometimes causes the lower hollow of the ring to appear lit equally on both sides twice in the same day: either first in the morning near sunrise and then around noon, when, the refraction having ceased, the truer vernal equinox occurs; or first around noon without refraction and afterwards near sunset because of refraction, when the autumnal equinox has already occurred.

[Margin: Ptolemy's faulty judgement of a fault in the rings.]

Ptolemy, Book 3, Chapter 2, relates that this happened both to Hipparchus, who was astonished at it, and to himself, and he attributes it to a fault in the ring, as having by then slipped from its original and proper placing. But that this was the effect of refraction is the better judgement of *Longomontanus*, Book 1 of the *Theorica*, Chapter 1, p. 172, *Boulliau*, Book 2 of the *Astronomia Philolaica*, Chapter 2, and *Kepler* in the *Astronomia Optica*, p. 146, who adds there that the same thing happened to *Tycho* while he was observing equinoxes by rings, so

that, the ring being lit equally on both sides twice in the same day, the equinox seemed to occur twice.

But from this way of observing, as *Ptolemy* reports, *Hipparchus* was content to assign the equinoxes only to the cardinal points of the day, namely noon, midnight, sunrise and sunset, and felt sure that he had admitted no error of 6 hours; whereas because of refractions he could sometimes have admitted a greater error, indeed up to almost half a day, if we believe *Longomontanus* above.

[Margin: A method of correcting equinoxes observed by rings.]

IX. Nor shall I pass over in silence the method of correcting the error that has crept, through refractions, into equinoxes observed in this way around sunrise or sunset (for I spoke above about the correction for parallax). *Boulliau* indicates this method in the place already cited. It assumes that the time of the observation made is known, say a quarter of an hour after sunrise or before sunset. For that time one seeks, from the altitude of the equator, the true altitude of the Sun; from the tables of refractions, the refraction proper to that altitude; and from this, by the general problems, the refraction of declination. On these problems we treat in the Optical Book, in the section on the refractions of stars in the air, and perhaps in the Book of Universal Problems. As many minutes as there are in the refraction of declination, so many hours are to be added to the apparent vernal equinox, and subtracted from the autumnal.

As for Hipparchus's equinoxes observed by rings, however, which we need, as being the most ancient, for tracing the length of the tropical year, two are more notable, one vernal and the other autumnal, observed at noon or very close to it, and so beyond the hazard of perceptible refraction. Of these the vernal one appeared twice to Hipparchus: once in the morning, when the ring was lit equally on both sides because of refraction, and again around the 5th hour of the day, when the refraction had ceased. But if he had observed the autumnal one in the evening, he would likewise have seen the ring lit again on both sides because of refraction, and not only at noon. In all there are nine equinoxes of Hipparchus observed in this way, reported by Ptolemy in *Almagest* Book 3, Chapter 2. The Ptolemaic ones, likewise observed by rings but at a more precise time, and reported by him in the same place, are 3. I choose to report these, for their antiquity, without any correction; for we must treat that at greater length in Volume 3 of the *New Almagest*.

Hipparchus's equinoxes observed at Alexandria in Egypt. [Hipparchæa Aequinoctia obſeruata Alexandriæ Aegypti.]

[Margin: Equinoxes observed by Hipparchus, from Ptolemy, Book 3, Chapter 2.]

No.	Year of the 3rd Period of Callippus	Year before Christ	Egyptian month	Day	Julian month	Day	Cardinal point of the day
1	17	162	Mesori	30	September	27	Around sunset

No.	Year of the 3rd Period of Callippus	Year before Christ	Egyptian month	Day	Julian month	Day	Cardinal point of the day
2	20	159	Intercalary	1	September	27	Around sunrise
3	21	158	Intercalary	1	September	27	At noon itself
4	32	147	Intercalary	3	September	26	At midnight
5	32	146	Mechir	27	March	24	In the morning, and again at the 5th hour of the day
6	33	146	Intercalary	4	September	27	At sunrise
7	36	143	Intercalary	4	September	26	In the evening
8	43	135	Mechir	29	March	23	Around midnight
9	49	128	Phamenoth		March	23	Around sunset

Ptolemy's equinoxes observed in the same place. [Ptolemaica Aequinoctia ibidem obſeruata.]

[Margin: Equinoxes observed by Ptolemy, from the same, Book 3, Chapter 2.]

No.	Year of the Roman emperors	Year of Christ	Egyptian month	Day	Julian month	Day	Hours
1	Hadrian 17	132	Athyr	7	September	24	About 2 after noon
2	Antoninus 3	139	Athyr	9	September	25	About 1 after sunrise
3	Antoninus 3	140	Pachon	7	March	22	About 1 after noon

Modus 3. Obſeruandi Aequinoctia Per Inſtrumenta Certiora.

Method 3. Of observing the equinoxes by more reliable instruments.

[Margin: Which instruments are best for this.]

X. By more reliable instruments I mean either parallactic rulers [Regulas Parallacticas] or, better, quadrants or sextants, built with so fine a division that they show, if not each single second, at least each ten seconds, according to what is to be said in the *Liber Organicus*. With instruments of this kind, then, first take the altitude of the pole from circumpolar stars free from refraction, and so its complement to 90°, which is the altitude of the equator itself. Afterwards, on the days assigned to the equinox in the calendar, or better in some well-known ephemeris, the Sun's meridian altitude is to be taken. For greater certainty it is useful to take it with at least two instruments, one showing the Sun's altitudes directly and the other its distances from the zenith.

Next, the observed altitude of the Sun is to be corrected and converted into the true one by adding the parallax and subtracting the refraction, taken from

the tables we shall give at the end of the work. For if the Sun's true altitude is equal to the altitude of the equator, know that the equinox has certainly occurred at noon itself on your horizon. But if at the vernal equinox the Sun's true altitude is greater or less than the altitude of the equator, the equinox was away from noon: before it if greater, but after noon if less. At the autumnal equinox, on the other hand, it was after noon if greater, before if less. And as many minutes as there are in the difference of the altitudes, so great will be the Sun's true declination, and by so many hours before or after noon you will determine that the equinox has occurred or will occur.

If, however, one cares to be scrupulous, from the declination found, and from the angle of obliquity of the ecliptic opposite it, which for us is always 23½°, find by the rules of right-angled spherical triangles the base, that is, the Sun's distance from the equinoctial point. Then find in the tables of the Sun's mean motion the time corresponding to it, to be added to or subtracted from noon, so that you have the exact moment of the equinox. But in a matter of such moment the precept must be illustrated by an example.

EXEMPLVM. Modi 3.

Example of Method 3.

[Margin: Our most exact observation of an equinox.]

In the year 1646, on September 22 by the Gregorian style, together with Father Francesco Maria Grimaldi, I obtained with large quadrants at Bologna the Sun's meridian altitude of 45° 44′ 40″. So, adding my parallax, determined meanwhile elsewhere from the Sun's distance, which is almost 30″, and with no perceptible refraction detected at this altitude, the Sun's true altitude becomes 45° 45′ 10″. Subtracting from this the altitude of the equator, which I found in the same place more than forty times, 45° 30′ 30″, there remains the Sun's northern declination of 14′ 40″. From this, therefore, we deduced that the equinox would occur at 14^h 40^m after noon.

But from this declination, and from the obliquity of the ecliptic of 23½°, trigonometry gives the Sun's distance from the beginning of Libra as 32′ 32″, which the Sun completes by its mean motion in 14^h 56^m. So, more scrupulously, the equinox was at 14^h 56^m after noon on September 22. But *Lansbergen's* Tables give this equinox at Bologna at 15^h 54^m after noon, evidently because of a parallax greater than ours, set by Lansbergen on p. 46 of his Canons at 1′ 36″. On Tycho's reckoning, however, the parallax at this altitude would have been 2′ 5″, and so the equinox at 16^h and at least 15^m after noon.

By this method we have examined a great many meridian altitudes observed at the time of the equinox by notable astronomers, from Hipparchus down to ourselves, gathered with the utmost search and compared with our own, in order to establish the length of the tropical year. These, with the favour of God, Best and Greatest, we shall communicate to you in Volume 3 of the *New Almagest*. This method, moreover, is the simplest of all, and the most

evident, and for that reason this way of observing equinoxes is preferred to the others by *Regiomontanus*, Book 3 of the *Epitome of the Almagest*, Prop. 1; *Alessandro Piccolomini*, Book 1 of the *Theorica Planetarum*, Chapter 3; *Tycho*, *Progymnasmata* vol. 1, p. 14; *Longomontanus*, Book 1 of the *Theorica*, Chapter 2; *Lansbergen*, Classes 2 and 3 of his observations; *Biancani* in his *Sphaera*, Book 10, Chapter 5; and *Boulliau*, Book 2 of the *Astronomia Philolaica*, Chapter 2. Of these, *Regiomontanus* and *Tycho* prefer the autumnal equinoxes, because the temper of the air, more thoroughly baked out by the preceding heat, is less subject to refractions.

[Margin: The excellence of this method.]

[Margin: Autumnal equinoxes better for this than vernal ones.]

SCHOLIA.

Scholia.

[Margin: The ancients' opinion on the annual cardinal points.]

I. Since it is certain that the four cardinal points of the year [Anni Cardines] occur either at the common intersection of the equator with the ecliptic, namely the equinoxes, or at the contact of the ecliptic with the tropics, namely the solstices (that is, at the very beginning of Aries, Libra, Cancer and Capricorn, conceived in the rational and invisible zodiac), it is remarkable on what reasoning not a few of the ancients appear to have recognized these cardinal points at the 15th, or 8th, or 6th, or 4th parts, or degrees, of these signs. For if we believe the younger Hipparchus, or the Bithynian, in Book 2 of his *Commentary on the Phaenomena of Aratus and Eudoxus*, Eudoxus and Meton assigned these cardinal points to the middle parts of the signs, or the 15th degree. Columella, however, Book 9, Chapter 14, places them, on the opinion of the same men, at the eighth parts of the signs, which Manilius, Book 3, Chapter 2, and Pliny the Elder, Book 18, Chapter 28, also retained in their own age. Geminus, Chapter 16, on the opinion of Eudoxus, puts the summer solstice at the 6th part of Cancer, the winter solstice at the 4th of Capricorn, and the vernal equinox at the 6th of Aries; but on the opinion of Euctemon he puts the winter solstice and the autumnal equinox at the 1st part. Finally Ovid, *Fasti* 3, places the vernal equinox after the 4th day from the Sun's entry into the visible Aries.

Here it is incredible with what unreason [Greek: ἀλογι[?]ία (the sixth letter is σ or the στ ligature; "unreason, want of reckoning")] and clash of mutually conflicting statements Scaliger, in his books *De emendatione temporum* and in the *Diatriba on the Equinoxes*, and also Salmasius [Claude Saumaise], in his *Exercitations on Solinus*, have tried to reconcile these opinions. Longomontanus too, Book 1 of the *Theorica*, p. 175, thinks that Pliny the Elder is speaking of influences, which for the most part set in at the eighth degree of the signs. But the true view is the one explained by Stöffler on Proclus, pp. 17 and 18, by Reinhold on Peuerbach's *Theoricae*, p. 231, and by Pétau in the *Dissertations* of the *Uranologion*, Book 2, from Chapter 3 to 8, Book 3, Chapters 1 and 2,

and Book 6, Chapter 10, taken together, where much is said learnedly against Salmasius.

For the ancient astronomers and compilers of calendars, although they knew that the cardinal points of the year occur at the beginning of Aries, Libra, Cancer and Capricorn of the rational and invisible zodiac, nevertheless, in order to serve farmers, sailors, physicians and finally the people, said that these points occur at determinate degrees of the starry and visible zodiac, so that people might learn the times of the cardinal points from the very sight of the stars. Once, moreover, the cardinal points stood 15° from the beginning of the visible signs, which happened, if we go by Wendelen as cited in Pétau, around the year 1263 before Christ, but according to Tycho's table in the year 2600. Later, as the fixed stars advanced towards the east, the cardinal points stood 8° from the beginning of the asterisms (for Tycho, that is, or for Kepler in the Rudolphine Tables, around the year 900 before Christ), and at length the cardinal points reached 4°, 2°, and so on. Now, however, the beginnings of the visible asterisms lie further east than the annual cardinal points by 28° and more.

Now some were unaware of the proper motion of the fixed stars to the east, and thought that the cardinal points stayed forever at the same parts of the asterisms. Others knew of it, but, so as not to seem to disturb the calendars and to strive against the ingrained custom of the peoples with a subtlety not easily persuasive to the crowd, they preferred to accommodate themselves to the old calendars. This should not seem surprising, since even from the Council of Nicaea down to the year 1582 the Church calendars kept the equinox at March 21, although it had by then slipped back to the 11th. For hear Columella, Book 9, Chapter 14: "Nor am I unaware of the reckoning of Hipparchus, which teaches that solstices and equinoxes are completed not at the eighth but at the first parts of the signs. But in this discipline of husbandry I now follow the calendars of Eudoxus and Meton and of the ancient astrologers, which are fitted to the public sacrifices, because that old opinion, formed among farmers, is also the better known," etc. So that diversity arose from a catachresis and misuse of the signs of the two zodiacs, and from the divergence of popular usage from the true reckoning.

[Margin: Pliny the Elder's apparent contradiction on the days of the cardinal points.]

II. There remains another knot in Pliny the Elder, to be either untied or cut. For in Book 18, Chapters 25 and 28, he assigns the winter solstice [Brumam] to December 25 [VIII. Kal. Ianuarij] and the solstice to June 24 [VIII. Kal. Iulij]; yet in the same Chapter 25 he takes from Sosigenes a year of 365 days and a quarter, and puts from the winter solstice to the vernal equinox $90^{d}\ 3^{h}$, from there to the summer solstice $93^{d}\ 12^{h}$, and from the autumnal equinox to the winter solstice $89^{d}\ 3^{h}$, which makes a sum of $272^{d}\ 18^{h}$. When this is taken from the year, there remain from the summer solstice to the autumnal equinox $92^{d}\ 12^{h}$. But if these intervals are kept, the summer solstice and the autumnal

equinox ought to fall on June or September 25 or 26 [VII. aut VI. Kalendas Iulij, aut Octobris], and so Pliny the Elder seems to disagree with himself by an interval of two days. But if the surplus of hours is reckoned in, and the limits are taken inclusively on one side and exclusively on the other, the apparent contradiction will cease; Pétau's solution of it, in Book 2 of the *Uranologion*, Chapter 5, is somewhat more intricate.

Those intervals are therefore to be arranged as follows. From March 25 exclusive to June 25½ inclusive, $93^{d}\ 12^{h}$. From June 25½ exclusive to September 26 inclusive, $92^{d}\ 12^{h}$. From September 26 exclusive to December 24⅛ inclusive, $89^{d}\ 3^{h}$. From there, exclusive, to March 25 and 6 hours, $90^{d}\ 3^{h}$, disregarding some very small difference. Thus according to Sosigenes and Pliny the Elder there will have been from the beginning of Aries to the beginning of Libra 186^{d}, and from Libra to Aries $179^{d}\ 6^{h}$. But these intervals will come up for discussion again below, in Chapter 18, and some things were said in Chapter 14, nos. 1 and 2.

CAPVT XV.

Chapter 15.

De Methodo colligendi Quantitatem Anni Aequinoctialis, ſeu Tropici.

On the method of finding the length of the equinoctial, or tropical, year.

[Margin: Authors on the way of finding the length of the year.]

I. I find among the practitioners of the art four methods in all for investigating the measure of the celestial and natural year, whether it is called equinoctial or tropical. All or some of them are discussed by *Ptolemy*, Book 3, Chapter 2; *Regiomontanus* in the *Epitome of the Almagest*, Book 3, Props. 2 and 3; *al-Battānī*, Chapters 27 and 53; *Geminus* in the *Elements of Astronomy*, Chapter 6; *Nikolaus von Kues*, *On the Reform of the Calendar*; *Copernicus*, Book 2, Chapter 14, Book 3, Chapter 13, and Book 4, Chapter 4; *Tycho*, *Progymnasmata* vol. 1, Chapter 1; *Daniel Santbech*, *De Observatione*, Prop. 11; *Scaliger*, *De emendatione temporum*, Book 2, p. 109, and Book 4; *Pétau*, Book 7 of *De doctrina temporum*, Chapter 25; *Biancani* in his *Sphaera*, Book 10, Chapter 6; *Clavius* on the *Calendar*, Chapters 5 and 6; *Longomontanus* in the *Astronomia Danica*, Book 1 of the *Theorica*, Chapters 2 and 5; *Lansbergen*, Class 4 of his solar observations, towards the end; and *Boulliau*, Book 2 of the *Astronomia Philolaica*, Chapters 2 and 3.

Prima Methodus Colligendi Quantitatem Anni ex Anno Sydereo.

First method of finding the length of the year: from the sidereal year.

[Margin: Copernicus's view set out in his own words.]

II. The Babylonians once, as *al-Battānī* and *Ptolemy* indicate, and in imitation of them *Thābit ibn Qurra* and several Indians and Arabs, as *Nikolaus von Kues* attests, and finally *Copernicus*, Book 2, Chapter 13, considered that the measure of the year should be sought from the Sun's revolution to the same fixed star, that is, from the sidereal year, and that a treatment of the fixed stars should therefore be put before the treatment of the luminaries. "We have therefore thought that this view should be followed by us" (the words are Copernicus's) "because among our principles and hypotheses we have assumed that the sphere of the non-wandering stars is entirely immobile, and the wanderings of all the roaming stars are referred to it on equal terms. But let no one wonder why we have adopted this order, since Ptolemy in his great *Constructio* held that no account of the fixed stars could be given unless knowledge of the places of the Sun and Moon came first, and therefore judged that what concerns the fixed stars should be deferred until then. But if you understand this of the numbers by which the apparent motion of the Moon and Sun is computed, that view will perhaps stand. For Menelaus the geometer too determined most stars and their places from lunar conjunctions, by means of numbers." By this he means by tables of mean motions and then of true ones, by which the apparent motion of the luminaries is found; but this presupposes that the mean motions too are known beforehand, from the Sun's annual motion and the Moon's monthly one. He goes on as follows: "But we shall do much better if, with the aid of instruments, we take any given star by way of the carefully examined places of the Sun and Moon, as we shall presently teach. We are also warned by the vain attempt of those who thought that the length of the solar year should be defined simply from equinoxes or solstices, and not also from the fixed stars; on this they have never, right down to us, been able to agree, so much so that on no point has there been greater discord."

He then adds a way of observing with a quadrant the Sun's declination and the distance between the tropics, so that from the obliquity of the zodiac [Signiferi] and the Sun's meridian declination the Sun's place at noon becomes known, and from that its place at any given hour, since it covers about 2½′ on the ecliptic in each hour. He next directs that the distance of the Moon from the Sun be taken by astrolabe when the Sun is close to the horizon, and on the following night the distance of some fixed star from the Moon, and that so one arrives at knowledge of the fixed star's place, adding: "For without the Moon there will be no way of grasping the places of the stars, since she alone of all of them has a share in both day and night."

[Margin: Copernicus not altogether skilled in observing.]

In this he is mistaken: for Venus and Jupiter and Sirius are also seen in daytime, so that we ourselves have been able to take their distance directly from the Sun, as we shall relate among the observations in Volume 3; and Tycho determined the places of the fixed stars by way of Venus and the Sun, without the Moon.

But in Book 3, Chapter 13, after setting out the way of finding the length of the year from equinoxes, and the variation of the year arising from this, he goes on: "The equality of the solar year is therefore more correctly taken from the sphere of the non-wandering stars, as Thābit, son of Qurra, first discovered, and that its length is $365^d\ 6^h\ 9^m\ 12^s$," etc. And a little later: "So Ptolemy is not to be listened to on this point, who thought it absurd and irrelevant to measure the Sun's annual equality against any restitution of the fixed stars." And at length he adds: "But error can also occur about the *asteroterida*, or sidereal, year, though only in a small way and much less than the one we have now explained." But Tycho too, *Progymnasmata* vol. 1, p. 38, and Longomontanus, above, p. 178, support Copernicus in this, and call him a man of lofty judgement.

[Margin: The view of Hipparchus and Ptolemy defended.]

III. But surely of loftier judgement on this were *Hipparchus* and *Ptolemy*, who, in Book 1, Chapter 2, and Book 3, Chapter 1, judged that the treatment of the luminaries should be put before the treatment of the fixed stars, and that the mean length of the year should be found from the Sun's revolution, above all to the equinoctial points. For we cannot know the place of the fixed stars unless the Sun's motion is known beforehand, as *Copernicus* has already admitted; and *Tycho*, at the start of Chapter 2, acknowledges that "without a precise investigation of the Sun's course the places of the fixed stars, and without the fixed stars those of the other planets, cannot be established." Accordingly he defined the places of the fixed stars from the distances of the fixed stars from Venus and of Venus from the Sun, the Sun's place being known first.

Now the whole of that error which spills over from observing equinoxes into the measure of the equinoctial year can also be committed in any observation of the Sun's place from which the places of the fixed stars are to be defined, whether it arises from an uncertain solar parallax, or from refraction, or from a false altitude of the equator and pole, or from the obliquity of the ecliptic, or from the fallibility of instruments, or from carelessness in calculation. Further, even if the error or the inequality were smaller in the length of the sidereal year, when comparing a sidereal year with a sidereal year found by different authors, or a revolution of the Sun to the same star, say Regulus, made and observed after a long interval of time, nevertheless, in separating the equinoctial, or tropical, or temporal [Temporario] year from the sidereal, it is necessary to know beforehand, separately and antecedently to the sidereal, the Sun's revolution to the equinoctial points and its measure; for the Sun revolves to those points before it revolves to the same fixed star, and if there is any greater anomaly in this conjunction of the equinoxes with the Sun, it cannot be avoided by means of the equality of the sidereal year. Since, then, the sidereal year is composed of the Sun's revolution to the equinoctial points and of the motion by which, having passed those points, the Sun must overtake the same fixed star, there is no way that the length of the temporal year can be distinguished from the length of the sidereal year unless the length of the temporal year is known beforehand, separately from the sidereal.

But why say more? Neither *Copernicus*, nor *Tycho*, nor anyone else, so far as I know, has taught or could teach a practical way of separating the temporal year from the sidereal by the Sun's revolution to the same fixed star alone. And *Copernicus* indeed, in Book 3, Chapter 13, having said "For this reason the reckoning of the annual equality is neither simple nor easy to know," and having said much about this difficulty, in the following chapter, with no observation brought forward, as if the pronouncements of this second Pythagoras had to be believed without proof, says that he has found the length of the year, and its equality, which Thābit ibn Qurra published, to be greater by only 1″ 10‴ of a day, so that it is $365^{d}\ 6^{h}\ 9^{m}\ 40^{s}$. He then finds the Sun's simple mean motion and, adding the motion of the precession of the equinoxes, which in others is the motion of the fixed stars, finds the Sun's compound motion, both daily and annual. *Tycho*, by contrast, from a comparison of the equinoxes observed by *Walther* and by himself, first finds the length of the temporal year, and then, from the annual proper motion of the fixed stars, finds the sidereal year, which is composed of both.

So anyone who wishes to define the length of both years will run into two difficulties, one from the motion of the fixed stars or, as *Copernicus* prefers, from the motion of the equinoctial points, the other from the Sun's own motion and its revolution to the equinoctial points. But whoever inquires into the length of the temporal year separately has no concern with the motion or anomaly of the fixed stars. And it is the length of the temporal year that we are seeking here, because it is rather this that we need for defining the motion of the Sun and of the other planets; and this alone would stand even if there were no fixed stars in the heavens, and it is more observable, and more independently so.

[Margin: The Alphonsine method contrary to the Copernican.]

Let us then leave *Copernicus*, to whom *Alfonso* was opposed on another ground. For from the Sun's motion remaining to the quarter of a day he established the annual motion of the fixed stars at 26″ 26‴ 54⁗, which the Sun traverses in $10^{m}\ 44^{s}$ of time, exactly as much, that is, as it lacks of filling out a year of $365^{d}\ 6^{h}$. We depart from Copernicus the more willingly, the more evident or the more probable it is to us that the fixed stars rather than the equinoctial points move, that there is no anomaly in the motion of the former or of the latter, and that the temporal year is not unequal, at least not with as great an inequality as Copernicus supposed, as will be clear from what is to be said below in Chapter 30. Even granting that these were unequal, the inequality would be greater in the equinoxes than in the motion of the fixed stars.

Secunda Methodus Colligendi Anni Quantitatem ex Cyclis Lunæ ſimul & Solis.

Second method of finding the length of the year: from the cycles of the Moon and the Sun together.

[Margin: The year of Meton, Euctemon and Philippus.]

IV. Nearly all astronomers who held that the cycles and revolutions of the Sun and Moon are commensurable derived the length of the tropical year from the Sun's revolution to the equinoctial or solstitial points, but in such a way that they tested it against the syzygies of the luminaries, as against a Lydian touchstone. So Meton, Euctemon and Philippus, as *Geminus* relates in the *Elements of Astronomy*, Chapter 6, and *Censorinus*, *De die natali*, Chapter 7, defined the year as 365 and 5/19 days, as Geminus has it, or 365 days and five nineteenth parts of a day, as Censorinus does, which comes to the same thing, namely $365^{d}\ 5^{h}\ 18^{m}\ 56^{s}\ 51'''$. For they thought that in 19 years of 6940 days the lunations and the meetings of the Sun and Moon were restored exactly to the same moment; and 6940 divided by 19 gives 365 5/19 days.

[Margin: The year of Callippus.]

But *Callippus*, as *Geminus* relates in the same place, and *Ptolemy*, Book 3, Chapter 2, noticed that in four Metonic cycles, that is, in 76 years, there was one day too many and that new moons came one day sooner, and so he took one day from the 76 years, that is, from 27760 days, and his period was 27759 days. Divided by 76, these make for one year 365 and 19/76 days, that is, a quarter of a day, or 6^{h}. *Geminus* says that this measure of the year fits the appearances best, and so Archimedes, Aristarchus, Sosigenes and many others embraced it; although *Censorinus*, Chapter 7, says that the year of Callippus is 365 days, and that of Aristarchus the same and in addition the 1532nd part of one day, a passage which is surely convicted of an error by *Geminus* and *Ptolemy*.

But *Hipparchus*, comparing eclipses and equinoxes, found

[Margin: The year of Hipparchus.]

that in four Callippic periods, that is, in 304 years, there was one day too many, and so that from a year of $365^{d}\ 6^{h}$ the 304th part of a day, or in round terms the 300th, should be subtracted. Accordingly he set the tropical year at $365^{d}\ 5^{h}\ 55^{m}\ 12^{s}$. But if he had subtracted the 304th part, the result would have been $365^{d}\ 5^{h}\ 55^{m}\ 15^{s}\ 45'''$. So they wanted to hunt two hares with one leap: on the one hand the restitutions of new and full moons, on the other the revolutions of the equinoxes after fixed intervals of time. And since *Ptolemy* kept exactly the same length of the year as *Hipparchus*, and this is sufficiently gathered from *Ptolemy*'s Greek text, *Scaliger*, *Longomontanus* and *Boulliau* affirm for that reason that *Ptolemy* had regard to lunisolar cycles, and not only to equinoxes, in determining the length of the year, and *Copernicus* indicates the same, although *Pétau* denies it.

[Margin: The year of Rabbi Adda.]

But *Rabbi Adda* too, who flourished under Constantine the Great, held that in 19 years, but astronomical ones, the syzygies of the Sun and Moon return to the same part of the heavens and the same point, and that meanwhile 235 lunations are completed. So he took, with the most learned rabbis, a lunar month of $29^{d}\ 12^{h}$ and 793 points [punctorum], that is, $44^{m}\ 3^{s}\ 20'''$; he multiplied

it by 235, and dividing the resulting sum of $6939^{d}\ 16^{h}\ 33^{m}\ 3^{s}\ 20'''$ by 19, he found a year of $365^{d}\ 5^{h}\ 55^{m}\ 25^{s}\ 26'''\ 20''''$, as is given in the Hebrew calendar of Rabbi Adda, of Münster [Sebastian Münster] and of others.

[Margin: The year of Meton, Philolaus, Oenopides and Democritus.]

By a similar method *Meton*, *Philolaus*, *Oenopides* and *Democritus* gathered the measure of the year from cycles of the luminaries, as I shall show in Book 4, Chapter 20, nos. 6, 7 and 9. But although observations of new moons, or rather, so that parallax may not interfere, of full moons, are made more easily and more certainly by way of eclipses, it has not yet appeared that two new moons or full moons have taken place at the same point of the heavens and at the same moment of the same day. And indeed, as *Boulliau* says in the passage cited at the beginning, "Who has yet demonstrated that the Sun's annual motion is commensurable with the lunar cycles?"

[Margin: The period of Viète.]

Not even the period of 3400 years which *François Viète* devised in his *Calendar* restores the turns of the Sun and Moon perfectly; rather it falls short by $1'\ 19''$, as *Kepler* says in the Rudolphine Tables, Precept 135. For this period to achieve it, the lunar month would have to be $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 10'''\ 43''''$ and a little more, "which has not yet been established by any observation," says *Clavius* in the *Calendar*.

Tertia Methodus Inueſtigandi Anni Tropici Quantitatem per Solſtitia.

Third method of investigating the length of the tropical year: by solstices.

[Margin: The unreliable observation of solstices.]

V. *Ptolemy*, Book 3 of the *Almagest*, Chapter 2, admits that the length of the tropical year is gathered from the Sun's revolutions to the same solstitial point not without danger of error, in these words: "For the sake of antiquity, therefore, the summer solstices observed in the times of Meton and Euctemon, and afterwards of Aristarchus, are to be compared with those which took place in our own times. But since observations of solstices are hard to discern, and those which these men handed down do not seem to have been taken exactly, which is also confirmed by the judgement of Hipparchus, we have passed over them." *Copernicus*, Book 3, Chapter 13, affirms however that *Callippus* too and *Archimedes* used summer solstices for this, because they took the beginning of the year from the summer solstice, after the manner of the Athenians. But how easy it is to slip in solstices and to err, not merely by a quarter of a day, as *Ptolemy* thought, but by a whole day or even whole days, I have already shown in Chapter 14 at the end of no. 4, and so I shall add no further word on them here.

Quarta Methodus Inueſtigandi Anni Tropici Quantitatem per Aequinoctia.

Fourth method of investigating the length of the tropical year: by equinoxes.

[Margin: Equinoxes to be preferred for this.]

VI. So we must at last take refuge in equinoxes, or in the Sun's revolution to the equinoctial points, if we wish to obtain the measure of the equinoctial year, as *Ptolemy*, Book 3, Chapter 2, *Regiomontanus* in the *Epitome*, Book 3, Prop. 1, *Tycho*, *Progymnasmata* vol. 1, p. 14, and the others mentioned at the start of the chapter have advised. So that this method may be carried out clearly and exactly, the precepts must first be given one by one, and then illustrated by one or two examples. These, then, are to be the precepts.

[Margin: Precept 1.]

1. Choose two vernal equinoxes or, which would be better, two autumnal ones, observed by two highly skilled astronomers and separated from each other by the greatest possible interval. For speaking of this very thing, *Ptolemy*, Book 3, Chapter 2, says: "This restitution will be taken the more exactly, the longer and greater the time between the observations," etc. And he adds the reason: a small error lying hidden in a few years can turn out and show itself as large after a long time; but if the interval between the observations is large, a small error admitted in the length of the year will turn out smaller in fewer years. So *Longomontanus*, in the *Commentary on the Sun*, rightly denies that *Tycho* could have reached an accurate measure of the solar motion solely from the interval of a hundred years between Walther and Tycho himself. *Tycho*, however, chose so small an interval because he was not attempting a length of the year that would agree with all ages, and because he distrusted the observations of the ancients, as appears from him, *Progymnasmata* vol. 1, pp. 45 and 54.

[Margin: Precept 2.]

2. If each equinox was observed in a different horizon, reduce the time of one to the time of the other's horizon, by a proper table of the difference of longitude, or of the meridians, such as we shall give at the end of the work, drawn from our *Geographical Book* and reformed by way of various eclipses or the intervals of journeys.

[Margin: Precept 3.]

3. If the second equinox has been noted on a date according with the Gregorian calendar, take 10 days from the interval between the equinoxes.

[Margin: Precept 4.]

4. Although it is not strictly necessary, it is nevertheless very convenient to choose two equinoxes that have fallen either in a bissextile year or in years equally distant from the bissextile year immediately preceding.

[Margin: Precept 5.]

5. Subtract the time of the more recent equinox from the time of the older one, and the remainder will be the precession of the equinox. Resolve it

into the smallest fractions of time, and at least into minutes of an hour, and keep it as the number to be divided.

[Margin: Precept 6.]

6. Gather the whole Julian years, with the remaining days, hours and minutes, that have passed from the old equinox to the recent one, but so that those days, etc., count as one whole year, and keep the sum of these years as the divisor.

[Margin: Precept 7.]

7. Divide the precession of the equinox, as in Precept 5, resolved into fractions, by the sum of the years, as in Precept 6, and there will come out the precession of the true equinox ahead of the equinox of the Julian year, or the difference by which the year of Sosigenes, of $365^{d}\ 6^{h}$, exceeds the true temporal year. This, then, subtracted from the year of Sosigenes, will leave the true length of the equinoctial, or tropical, year, but derived from true equinoxes, when it ought to be from mean ones. Hence the eighth precept.

[Margin: Precept 8, where note the depth of astronomical science.]

8. The length of the year drawn out in the way already mentioned would be perfectly right if the Sun's motion were equal; or if, though unequal, both equinoxes occurred at points of the anomaly, or of the distance from the apogee, at which the prosthaphaeresis, or equation, was either nil or equal and of the same kind, that is, both subtractive or both additive. But this cannot happen exactly unless the interval from one equinox to the other is so great that the Sun's apogee, having completed its revolution through the zodiac, has returned to precisely the same point; and for this restitution about 21000 years are required, as will be clear from what is to be said in Chapter 25. Yet 6000 years have not so far elapsed since the beginning of the world. And within this very interval we have no observed equinoxes, at least none extant, older than those of Hipparchus. So each true and apparent equinox must be reduced to the mean equinox, and the rest carried out as above.

[Margin: A vicious circle in astronomy, but an apparent one.]

But this reduction needs knowledge of the prosthaphaeresis, and that needs the eccentricity and the place of the Sun's apogee, as will be clear from what is to be said from Chapter 19 to 21. Yet the eccentricity and the place of the apogee cannot be investigated unless the Sun's mean motion is already known beforehand, as will be plain from the method to be given in Chapter 24. So we seem to fall into a vicious circle, and to be seeking the unknown through the unknown. *Copernicus*, Book 3, Chapter 13, saw this knot, but he untied it by asserting that it is enough, for investigating the solar equations by means of the eccentricity and apogee, to use a length of the year of $365^{d}\ 6^{h}$: "Since what differs so little from the whole circle vanishes entirely when taken up in a smaller magnitude," he says. But *Tycho*, *Progymnasmata* vol. 1, p. 12,

denies, indeed, that the Sun's mean motion, *known with exact precision*, is required for investigating the eccentricity and apogee; but he nevertheless assumes for this purpose a tropical year of $365^{d}\ 5^{h}\ 49^{m}$, and on this supposition, as far as p. 51, he investigates the Sun's apogee and the eccentricity suited to both times, those of the equinoxes observed by Walther and by himself. *Boulliau*, however, in Book 2 of his *Astronomy*, Chapter 1, begins from the investigation of the aphelion and the eccentricity, assuming the Tychonic tropical year and its mean motion.

I, for my part, do not think that one should, out of order, take on at this point as much labour as we shall show in Chapter 24 is required for twice investigating the eccentricity and apogee, for the two equinoxes. It is enough, rather, to use an apogee and eccentricity already found by more expert men, even if it were not exactly true; indeed, to use tables of equations, Tycho's for instance, assuming the place of the apogee handed down by Hipparchus or Ptolemy or whoever it may be whose anciently observed equinox we are going to use. For what Copernicus and Tycho say about the length of the year to be assumed here, I say about the Sun's equation and the place of the apogee: namely, that a modest difference vanishes completely in so great an interval. Or at any rate the precept about investigating the eccentricity and apogee must be put off to another place, so that the order and the imagination of beginners are not disturbed; and afterwards, from there, the final polish must be applied to the length of the year derived here from true equinoxes, by reducing each true equinox to the mean equinox, and deriving the precession from the interval between the mean equinoxes, as above.

EXEMPLVM. Methodi 4.

Example of Method 4.

VII. Ptolemy reports, in Book 3, Chapter 2, of the *Magna Syntaxis*, that Hipparchus observed the autumnal equinox with the Alexandrian armillae in year 20 of the third period of Callippus, on the first of the intercalary days, in the morning about sunrise. And not only is it said τῇ πρώτη ἐπαγομένων [Greek: tē prōtē epagomenōn, "on the first of the intercalary days"]; in another manuscript codex as well, which Boulliau cites from the Royal Library at Paris, and in the Basel edition, there is added τῇ νεομηνία ἐπαγομένων [Greek: tē neomēnia epagomenōn, "at the new moon of the intercalary days"], that is, at the new moon of the intercalary days. Moreover, from our Chronological Book and among all those expert in chronology, that year was, in the Julian form, the current year 159 before Christ, and September 27 was to begin, in the astronomical manner, from the following noon. When Hipparchus observed the equinox, September 26 was still running, begun at the preceding noon, and 18 hours of it had passed. It was also the civil new moon, and the Moon's first phase had appeared the previous evening, although the true new moon had occurred two days before. This is indeed a notable chronological mark of

the truth of this year. But I posit, from what will be said in Scholium 1, that the equinox was at 18^h $11¼^m$ after noon on the 26th; for I shall establish in the Scholia that refraction is not to be taken into account here.

I, however, at Bologna in the year 1646, observed the autumnal equinox most accurately, as I have already recounted at the end of Chapter 13. It fell with September 22 completed according to the Gregorian calendar, but the 12th according to the Julian, at 14^h 56^m after noon. Now Bologna, as I have shown in the Geographical Book, lies further west than Alexandria in Egypt by 1^h 43^m. So the Hipparchan equinox was at Bologna at 16^h $28¼^m$ after noon on September 26. Again, as is clear from our Chronological Book, the year 159 before Christ, like the year 1646 after Christ, was the second after the preceding leap year. So if the year were 365 days and 6 hours, the equinox should have fallen in the Julian year on September 26, at 16^h $28¼^m$ after noon at Bologna. But it came earlier, and was on the 12th, at 14^h 56^m after noon. So, with the 12 days etc. subtracted from the 26 days etc., the precession was 14^d 1^h $32¼^m$, which resolved into seconds of time is 1215135 seconds. Now from the Hipparchan equinox to ours there are 1803 full Julian years, 350 days, 21^h 59^m; if the precession already mentioned were added to these, they would make 1804 years. Say, then: if in 1804 Julian years the precession of the equinox, taken according to the true year, is 14^d 12^h 1^m[16], or was 1215135 seconds, in one Julian year how great is the precession? Evidently, dividing 1215135 by 1804 yields a precession of 673 1043/1804 seconds of time, that is, 11^m 13^s $34‴$, etc. When these are taken from the year of Sosigenes, that is, from 365 days and 6 hours, there remains an equinoctial, or tropical, year of 365^d 5^h 48^m 46^s $32‴$.

[Margin: Delimiting the year from mean equinoxes.]

Now in Hipparchus's time the Sun's apogee was at about 6° Gemini, as is plain from Ptolemy, Book 3, Chapter 1; and so the Sun at the beginning of Libra was 114° from the apogee. To this anomaly there answers, in the tables of Tycho's equations, an equation of 1° 54′, but in ours 1° 50′, to be added to the true place so that it comes out mean at the equinox. So by my prosthaphaeresis, at the time of the true equinox, the Sun's mean place was at 1° 50′ of Libra. But the Sun in its mean motion runs through 1° 50′ in 44^h 30^m 8^s. So the Hipparchan mean equinox was at Bologna on September 24 at 19^h 58^m 7^s after noon. But in the year 1646 the Sun's apogee was at 7° Cancer, by what will be said in Chapter 24; so at the beginning of Libra the Sun was 83° from the apogee. So the equation, given my eccentricity, on which see Chapter 24, was 1° 56′, to which the Sun's motion in 46^h 56^m answers. So the mean equinox was on the 10th at 16^h 0^m after noon. So from the Hipparchan equinox to ours the precession was 14^d 3^h 58^m 15^s, which resolved into seconds is 1223895 seconds; and this, divided by 1804, gives an annual precession of 678 783/1804 seconds, or

[16]so printed; the value given a few lines above is 14^d 1^h $32¼^m$, and it is that value which amounts to 1215135 seconds

11^m 18^s 26‴. Subtracted from the year of Sosigenes, 365^d 6^h, this gives a tropical year of 365^d 5^h 48^m 41^s 34‴.

EXEMPLVM SECVNDVM. Methodi 4.

Second Example of Method 4.

VIII. Because Hipparchus's equinox may appear suspect on account of refraction, since it seems to have been observed in the morning around sunrise, I therefore take the equinox of the following year, which is free of this suspicion, having been observed at noon, and which Boulliau, Book 2, Chapter 2, therefore rightly preferred. So, as Ptolemy reports in Book 3, Chapter 2, Hipparchus observed at Alexandria the autumnal equinox in year 21 of the third Callippic period, which was the year 158 before Christ, on the first intercalary day, namely September 27, at noon itself; for then the lower concavity of the armilla appeared equally lit on both sides. But on account of my parallax of 30″ it was on the 27th at 0^h 30^m; and it was the third year after a leap year. At Bologna, however, that equinox was at 22^h 47^m, with September 26 completed. We, at Bologna in the year 1643, likewise the third after a leap year, observed the equinox most accurately with September 22 completed, at 22^h 25^m after its noon. But in the Julian manner and old style it was the 12th at 22^h 25^m. So the precession was 14^d 1^h 22^m, or 1214520 seconds. This, divided by the 1800 whole years between these equinoxes, if the precession did not detract from their wholeness, gives an annual precession of 674 1320/1800 seconds, or 11^m 14^s 43‴. Taken from 365 days and 6 hours, this leaves an equinoctial year of 365^d 5^h 48^m 45^s 17‴.

Moreover, the apogee in Hipparchus's time was at about 6° Gemini, and in the year 1643 at about 7° Cancer. From these places there comes an equation to be added to the Sun's true place: there 1° 50′, here 1° 56′, as I said at the end of the preceding example. So the difference is 6′, by which our mean equinox preceded our true one further than Hipparchus's mean preceded Hipparchus's true; that is, by 2^h 56^m. So the whole precession over the interval of 1800 years was 14^d 4^h 18^m, or 1225080 seconds, which divided by 1800 gives an annual precession of 680 1080/1800 seconds, or 11^m 20^s 37‴. When this is subtracted from the year of Sosigenes, 365 days 6 hours, there is left a length of the tropical year of 365^d 5^h 48^m 39^s 23‴. And by choosing roughly the mean between this and the length in the preceding example, the length of the tropical year will be 365^d 5^h 48^m 40^s. We shall retain this, as derived from the very long interval of 1804 or 1800 years, in accordance with Ptolemy's advice, Book 3, Chapter 2, and with a parallax applied not gratuitously but drawn from our own observations.

IX. The following table, moreover, has been compiled from the authors listed in number 1, to whom are to be added Censorinus

[Margin: Authors on the length of the year.]

Chapter 7 of *De die natali*, where he reports the opinions of Ennius, Harpalus, Aphrodius, Philolaus and Oenopides; Africanus in his fragments; Rabbi Adda and Samuel in the Hebrew calendar; Isaacius Argyrus [Isaac Argyros] in the *Computus Ecclesiasticus*; Dante, in Mazzoni, Book 3 of the *Defence*, Chapter 26; Pitati in the preface to the *Ephemerides*; the Alphonsine Tables; Reinhold in the Prutenic Tables; Magini in the *Theorica Solis*, Chapter 10; Mästlin, cited by Clavius in Chapter 6 of the *Calendar* and by Kepler in the *Epitome of Copernican Astronomy*, p. 341; Tycho, p. 53; Kepler in the Rudolphine Tables; Origanus, Volume 1 of the *Ephemerides*, Chapter 2; Govaert Wendelen in the *Idea of the Atlantic Tables*. Finally, the opinions of Meton, Euctemon, Aristarchus, Archimedes and Sosigenes are reported partly by Ptolemy, Pliny the Elder and Censorinus, partly by Geminus in the *Elements of Astronomy*, Chapter 6. That of the Persian mathematicians, who are called Gelalaei, is reported by Scaliger, Longomontanus and Boulliau. Leaving aside Philolaus's length of 364½ days, the rest set it at 365 days with a surplus, as in the following table.

ANNI AEQVINOCTIALIS QVANTITAS est Dierum 365. & præterea.
The length of the equinoctial year is 365 days and, in addition:

Authors adduced in nos. 1 and 9	Hor.	′	″	‴	⁗	V	VI
Ennius	24	0	0	0	0	0	0
Harpalus	13	0	0	0	0	0	0
Oenopides	8	57	0	0	0	0	0
Euctemon, Meton	6	18	56	51	0	0	0
Aphrodius	3	0	0	0	0	0	0
Aristarchus	6	1	16	0	0	0	0
Callippus, Archimedes	6	0	0	0	0	0	0
Geminus, Sosigenes	6	0	0	0	0	0	0
Rabbi Samuel, Democritus	6	0	0	0	0	0	0
Africanus	6	3	0	0	0	0	0
Hipparchus, Ptolemy	5	55	12	0	0	0	0
Rabbi Adda	5	55	25	26	20	0	0
Isaac Argyros	5	52	48	0	0	0	0
al-Battānī	5	46	24	0	0	0	0
The Persians, according to Scaliger	5	48	30	0	0	0	0
The Persians, according to Longomontanus	5	48	53	20	0	0	0
The Persians, according to Boulliau	5	48	59	0	0	0	0
The Alphonsines, Peuerbach	5	49	15	58	49	46	26
Daniel Santbech	5	49	15	58	51	12	0
or by his own opinion	5	48	41	42	33	50	10
Dante	5	45	36	0	0	0	0
Pitati and Pétau	5	49	0	0	0	0	0
Bernhard Walther	5	48	50	0	0	0	0
Copernicus and Reinhold, at greatest eccentricity	5	55	37	40	0	0	0
Copernicus and Reinhold, at mean eccentricity	5	49	16	23	30	0	0
Copernicus and Reinhold, at least eccentricity	5	42	55	7	0	0	0
Magini from the Prutenic Tables, at greatest eccentricity	5	55	53	0	0	0	0
Magini from the Prutenic Tables, at mean eccentricity	5	49	16	0	0	0	0

Authors adduced in nos. 1 and 9	Hor.	′	″	‴	⁗	V	VI
Magini from the Prutenic Tables, at least eccentricity	5	42	38	0	0	0	0
Mästlin from the Prutenic Tables	5	49	15	46	0	0	0
Cardano	5	48	41	0	0	0	0
Clavius and Lilius	5	49	12	0	0	0	0
Tycho Brahe	5	48	45	0	0	0	0
Kepler in the Rudolphine Tables	5	48	57	36	0	0	0
Longomontanus, at greatest eccentricity	5	51	29	12	0	0	0
Longomontanus, at mean eccentricity	5	48	55	0	0	0	0
Longomontanus, at least eccentricity	5	46	20	48	0	0	0
Origanus, at greatest eccentricity	5	56	53	0	0	0	0
Origanus, at mean eccentricity	5	49	15	45	52	48	0
Origanus, at least eccentricity	5	42	38	0	0	0	0
Lansbergen	5	48	57	2	22	24	0
Boulliau	5	49	4	21	3	0	0
Wendelen	5	49	5	27	16	0	0
Ourselves for the present	5	48	40	0	0	0	0

SCHOLION.

Scholium.

If the Tychonic year and its annual precession beyond the year of Sosigenes are retained, and in the year 158 before Christ the autumn equinox was at noon itself, or half an hour after noon on the 27th at Alexandria, then in the year 159 it was on September 27, $5^h\ 18^m\ 45^s$ before noon, that is, after noon on the 26th at $18^h\ 41^m\ 15^s$. So Hipparchus was not wrong to conjecture, from his armillary observation of the preceding day, that the equinox would come the following morning about sunrise. From this I gather that he determined the equinox from noon observations, and that on the 26th he saw the northern concavity of the armilla lit four times as much as the southern concavity on the 27th, and so that the Sun crossed the equator after about three quarters of a day. For if he had determined it from an observation near the horizon, refraction would have delayed the equinox by more than 6 hours. So the true equinox would have been before 12^h after noon on the 26th, and thus between this and the equinox of the following year, which occurred at noon itself, there would have been 365 days and more than 12 hours, which is a manifest absurdity. So let us keep, as is proper, the equinox of the year 158 on the 27th at $½^h$ after noon, since no refraction intervened: then even if the year of Sosigenes of 365 days and 6 hours were retained, the equinox in the year 159 before Christ could not have happened before 18^h after noon on the 26th. But since, from the better opinions, as above, the greatest year is $365^d\ 5^h\ 55^m$ and the least $365^d\ 5^h\ 42^m$, leaving seconds aside for now, the equinox of the year 159 could not have occurred either earlier than $18^h\ 35^m$ or later than $18^h\ 48^m$ after noon on the 26th. The time midway between these was $18^h\ 41^m\ 30^s$, or, keeping the Tychonic precession, 15^s. On these foundations, then, we have established that

Hipparchan equinox from the equinox of the following year and from many others, from which the greatest and the least among the probable measures of the year have confirmed its mean length for us.

CAPVT XVI.

Chapter 16.

De Quantitate Anni Syderei.

On the length of the sidereal year.

[Margin: What the sidereal year is.]

I. The sidereal year is the Sun's revolution from a given fixed star back to the same fixed star. It consists of the equinoctial year and, in addition, that surplus of time which is required for the Sun, having passed the equinoctial point, to catch up with the same fixed star, which has advanced eastward in one year. Three things, therefore, are required for determining the length of the sidereal year: namely, the mean length of the equinoctial year, if you think the years unequal, or the single true length if you do not; next, the annual motion of the fixed stars along the zodiac eastward, on which see the book on the fixed stars; and finally, the time in which the Sun in its mean motion can get past the small part of the zodiac completed by the fixed stars in their annual motion, which is derived from the tables of the Sun's mean motions. The Alphonsines, however, determined the sidereal year from 1000 jubilees, or 49000 years, in which the fixed stars would complete the zodiac, and from the surplus of the tropical year remaining up to a quarter of a day. It should be noted, though, from the Rudolphine Tables, p. 115, that 25411 sidereal years are equal to 25412 tropical years. But according to Wendelen in the *Idea of the Atlantic Tables*, 25519 sidereal years equal 25520 tropical years. It is not yet certain, however, that these are commensurable. Now let there be a synopsis of selected opinions on the sidereal year.

[Margin: Authors from whom the sidereal year is drawn.]

II. The authors in whom I have found the measure of the sidereal year are Ptolemy, Book 3, Chapter 2, and Book 7, Chapter 3; al-Battānī, Chapters 27 and 51, who also touches on the opinion of the Babylonians and Egyptians; Cardinal Nikolaus von Kues, *On the Reform of the Calendar*, where he reports the opinion of the Indians and of Avraham ibn Ezra; Copernicus, Book 3, Chapter 14, who adds the view of Thābit ibn Qurra; Reinhold in the Prutenic Tables; Magini in the *Theorica Planetarum*, Book 1, Chapter 4, and in the same work on the Sun, Chapter 10; Giuseppe Moletti in the *Gregorian Calendar*, Canon 7; Tycho, *Progymnasmata* vol. 1, p. 44; Longomontanus, Book 1 of the *Theorics*, Chapter 5; Kepler in the *Epitome of Copernican Astronomy*, p. 713, but otherwise from the motion of the Sun and the fixed stars in the Rudolphine Tables; Lansbergen, from p. 101 of the tables and p. 160 of the *Thesaurus of Observations*; Boulliau in the motion of the Sun and the fixed stars. We, however, from a tropical year

of $365^{d}\ 5^{h}\ 48^{m}\ 40^{s}$, and from an annual motion of the fixed stars of 50″, which the Sun completes in its mean motion in $20^{m}\ 17^{s}\ 25'''$ of hours.

ANNI SYDEREI QVANTITAS est Dierum 365. & præterea.

The length of the sidereal year is 365 days and, in addition:

Authors of no. 2	Hor.	′	″	‴	⁗
Egyptians, Babylonians	6	11	0	0	0
Hipparchus, Ptolemy	6	10	48	0	0
al-Battānī	6	8	32	0	0
Thābit ibn Qurra	6	9	12	0	0
Indians, and Avraham ibn Ezra	6	12	0	0	0
Copernicus and Magini	6	9	40	0	0
Giuseppe Moletti	6	9	39	0	0
Tycho Brahe	6	9	26	43	30
Longomontanus	6	9	13	20	0
Kepler in the *Epitome of Copernican Astronomy*	6	9	26	43	30
but from the Rudolphine Tables	6	9	4	23	0
Lansbergen	6	8	58	0	0
Boulliau	6	9	5	21	3
Ourselves for the present	6	8	57	25	0

CAPVT XVII.

Chapter 17.

De Motu Medio Solis Annuo, Diurno, &c. ex Tropico Anno Eliciendo, & in Tabulas Distribuendo.

On deriving the Sun's mean annual, daily, etc. motion from the tropical year, and arranging it in tables.

I. This is done by the rule of proportions, by reducing to the smallest parts, but of the same kind, first the tropical year, which is to be set as the divisor; then the zodiac, which stands in second place in the rule of three; and finally either the common civil year of 365 days, if you seek the annual motion, or the day, if you seek the daily motion. For when the small parts of the zodiac are multiplied by this, the total, divided by the small parts of the tropical year, will give what you seek. Now the zodiac resolved into seconds is 1,296,000″, while one common year is 31,536,000 seconds, and one natural day 86,400 seconds. Multiplying the seconds of the zodiac by these gives the total to be divided: there, 40,870,656,000,000 from the product with the year; here, 111,974,400,000 from the product with the day. It remains, then, for the tropical year to be resolved into seconds, according to the opinion you prefer to follow.

EXEMPLVM.

Example.

II. For me the tropical year is $365^{d}\ 5^{h}\ 48^{m}\ 40^{s}$, that is, 31,556,920 seconds. Dividing by these the total made from the product of the common civil year by the zodiac, which is 40,870,656,000,000, the quotient is 1,295,140 26631200/31556920 seconds. That is, $11^{s}\ 29^{\circ}\ 45'\ 40''\ 50'''\ 38''''$; and that is the Sun's mean motion belonging to one civil and common year of 365 days.

III. Again, dividing by 31,556,920 seconds the total made from the product of the zodiac by the day, which is 111,974,400,000, the quotient is 3548 1045/3156 seconds, that is, $59'\ 8''\ 19'''\ 51''''\ 46^{v}\ 20^{vi}$. This, then, is the Sun's mean motion belonging to one day. Dividing it by 24 hours yields the hourly motion, $2'\ 27''\ 50'''\ 50''''$; and dividing this by 60 gives the motion belonging to one minute of time, namely $2''\ 27'''\ 50''''\ 50^{v}$. But note, with Tycho, *Progymnasmata* vol. 1, p. 52, that the tables of hourly motion do not exactly contain the measure of the equinoctial year, because of tiny fractions finally neglected. So from the multiplication or aggregation of this motion you will not rightly gather the annual length. By this device, then, the table of the Sun's mean motion is constructed; let a sample of it stand here.

Solis Motus Medius in Zodiaco, seu Aequalis.

The Sun's mean, or equal, motion in the zodiac.

	Signs	Deg.	′	″	‴	⁗	V	VI
One year of 365 days	11	29	45	40	50	38	0	0
One leap year of 366 days	0	0	44	49	10	29	46	20
2 common years	11	29	31	21	41	16	0	0
3 common years	11	29	17	2	31	54	0	0
4 years, the last of which is a leap year	0	0	1	51	42	23	46	20
1 month, of 31 days	1	0	33	18	15	48	56	20
of 30 days	0	29	34	9	55	53	10	0
of 28 days	0	27	35	53	16	9	37	20
of 20 days	0	19	42	46	37	15	26	40
of 10 days	0	9	51	23	18	37	43	20
of 5 days	0	4	55	41	39	18	51	40
Daily, or of 1 day	0	0	59	8	19	51	46	20
Of 1 hour	0	0	2	27	50	50	0	0
Of 1 minute	0	0	0	2	27	50	50	0
Of 1 second	0	0	0	0	2	27	50	50
The Sun's mean daily motion, otherwise			**′**	**″**	**‴**	**⁗**	**V**	**VI**
Ptolemy, Book 3 of the *Almagest*, Chapter 2	0	0	59	8	17	13	12	31
The Alphonsines, in the tables, p. 58	0	0	59	8	19	37	19	13 11/12
Reinhold, from Copernicus, in the Prutenic Tables, p. 40	0	0	59	8	19	37	24	25 2/3
Tycho, *Progymnasmata* vol. 1, p. 55	0	0	59	8	19	49	51	40 1/4
Longomontanus in the *Astronomia Danica*	0	0	59	8	19	45	48	12
Kepler in the Rudolphine Tables, table p. 47	0	0	59	8	19	44	45	44
Lansbergen in the tables, p. 15	0	0	59	8	19	44	59	15
Ismaël Boulliau in the *Philolaica*, Book 2, Chapter 2	0	0	59	8	19	42	1	41 1/4
Wendelen in the Atlantic Tables	0	0	59	8	19	41	34	55

	Signs	Deg.	′	″	‴	⁗	V	VI
Ourselves, as above	0	0	59	8	19	51	46	20

CAPVT XVIII.

Chapter 18.

De Interuallis inter Quatuor Anni Cardines, & de Annua Motus Solis Inæqualitate.

On the intervals between the four cardinal points of the year, and on the annual inequality of the Sun's motion.

[Margin: Arguments for the inequality in the Sun's motion.]

I. Although in astronomical tables the Sun's motion, like that of the other planets, is feigned to be equal and uniform, the motion that truly and in fact appears to us observing it from the Earth — and which is therefore called at once the True and the Apparent motion — is unequal and non-uniform, since in equal times the Sun completes unequal arcs of the ecliptic. This is plain from the meridian altitudes and declinations observed before Hipparchus and from then on by all astronomers, and it cannot be denied by any evasion, or by shifting the blame onto the observers, except perhaps by the most ignorant. Certainly this is how it was found to be by al-Zarqālī the Spaniard, with more than 400 observations; by Regiomontanus and Bernhard Walther, with more than 800; by the mathematicians of the Landgrave of Hesse, by Jacob Christmann and by Tycho, with far more; and finally by ourselves, with so many observations that there is scarcely a day of the year without its own observation to answer to it. From all these taken together it appears that in one semicircle of the ecliptic the Sun is faster than its mean motion and covers in one day more than 59′ 8″ etc., so that when it is fastest it completes 61′ and more in one day; whereas in the other semicircle it is slower, and when it is slowest it barely traverses 57′ of the ecliptic in one day.

II. But this inequality appears most evidently in the intervals of time in which the Sun traverses the four quadrants of the ecliptic, or the two semicircles, namely the northern and the southern; for it crosses these equal arcs in far from equal times. Indeed, from observation of the equinoxes it is established that the Sun spends 186 or 187 days in the northern semicircle, that is, from the start of ♈ through Cancer to the start of ♎; but in the southern semicircle, from the start of ♎ through Capricorn to the start of ♈, only 178 or 179 days.

[Margin: A most powerful argument for the inequality.]

Moreover, from Hipparchus to al-Battānī and beyond, it lingered more days or hours in the first quadrant of the ecliptic than in the second, in the 2nd more than in the 4th, and in the 4th more than in the 3rd; so that it lingered more days in the ascending semicircle — that is, the one from the start of Capricorn to the start of Cancer through Aries — than in the descending semicircle of the

ecliptic, which runs from the start of Cancer through Libra all the way to the start of Capricorn. But from observations from about the year of Christ 1400 up to the present it is evident that the Sun lingers longer in the 2nd quadrant of the ecliptic than in the first, in the first than in the 3rd, and in the 3rd than in the 4th; so that it now lingers longer in the descending semicircle than in the ascending. Astronomers attribute this difference to the Sun's apogee, on the ground that it produces longer stays of the Sun in the quadrant or semicircle of the ecliptic in which it lies; so much so that if it ever happened that the apogee was in the last quadrant of the ecliptic, or if it will ever be in the 3rd, the Sun will also have lingered, or will linger, more days or hours in the southern semicircle.

III. And although astronomers do not agree on the measure of these intervals, even when observed in the same century, they do agree on what I have said so far. Both points will be clear from the four small tables below, which are collected from the authors indicated in them and from observed solstices and equinoxes, according to the rules indicated in Chapter 14, whose scholia on the annual cardinal points should be re-read. And when Copernicus, Book 4, Chapter 21, teaches that the Sun's true hourly motion is to its apparent diameter roughly as 5 to 66, he presupposes the same anomaly.

Table I. The Sun's stay in the northern and southern semicircles [I. TAB. Mora Solis in Semicirculis Boreali, & Australi]

Authors	From the start of ♈ to the start of ♎	From the start of ♎ to the start of ♈
Hipparchus and Ptolemy, Book 3, Chapters 2 and 4	$187^{d}\ 0^{h}\ 0^{m}\ 0^{s}$	$178^{d}\ 5^{h}\ 55^{m}\ 12^{s}$
Ptolemy corrected by Longomontanus, p. 178, and by Boulliau, Book 2	$186^{d}\ 11^{h}\ 50^{m}\ 30^{s}$	$178^{d}\ 18^{h}\ 2^{m}\ 0^{s}$
Pliny the Elder, from Sosigenes, Book 18, Chapter 25	$186^{d}\ 0^{h}\ 0^{m}\ 0^{s}$	$179^{d}\ 6^{h}\ 0^{m}\ 0^{s}$
al-Battānī, from his Chapters 27 and 28	$186^{d}\ 14^{h}\ 45^{m}\ 0^{s}$	$178^{d}\ 14^{h}\ 30^{m}\ 0^{s}$
al-Battānī corrected by Longomontanus	$186^{d}\ 14^{h}\ 15^{m}\ 0^{s}$	$178^{d}\ 14^{h}\ 0^{m}\ 0^{s}$
Pietro Pitati, in Nonius on Peuerbach's *Theoricae*, p. 199	$186^{d}\ 23^{h}\ 38^{m}\ 0^{s}$	$178^{d}\ 6^{h}\ 11^{m}\ 0^{s}$
Copernicus, *De revolutionibus* Book 3, Chapter 16	$186^{d}\ 2^{h}\ 0^{m}\ 30^{s}$	$179^{d}\ 3^{h}\ 48^{m}\ 46^{s}$
Reinhold, on Peuerbach's *Theoricae*, pp. 31 and 39	$186^{d}\ 9^{h}\ 18^{m}\ 0^{s}$	$178^{d}\ 20^{h}\ 37^{m}\ 0^{s}$
Magini, in the *Theorica Solis*, Chapter 9	$186^{d}\ 7^{h}\ 46^{m}$ ✢	$178^{d}\ 22^{h}\ 10^{m}\ 0^{s}$
Clavius, on the *Sphere*, p. 438, but with the hours disregarded in civil fashion	$187^{d}\ 0^{h}\ 0^{m}\ 0^{s}$	$178^{d}\ 0^{h}\ 0^{m}\ 0^{s}$
Tycho, *Progymnasmata* vol. 1, pp. 13, 19 and 107 taken together	$186^{d}\ 18^{h}\ 30^{m}\ 0^{s}$	$178^{d}\ 11^{h}\ 18^{m}\ 45^{s}$
Argoli, nearly agreeing with Longomontanus, in the *Pandosion*, Book 1, Chapter 8	$186^{d}\ 18^{h}\ 30^{m}\ 0^{s}$	$178^{d}\ 11^{h}\ 30^{m}\ 0^{s}$
Goclenius, in the *Urania*, p. 53	$186^{d}\ 8^{h}\ 0^{m}\ 0^{s}$	$178^{d}\ 21^{h}\ 0^{m}\ 0^{s}$
Lansbergen, from the *Thesaurus Observationum*	$185^{d}\ 16^{h}\ 12^{m}\ 0^{s}$	$179^{d}\ 13^{h}\ 36^{m}\ 57^{s}$
Wendelen, in letters sent to me	$186^{d}\ 16^{h}\ 0^{m}\ 0^{s}$	$178^{d}\ 13^{h}\ 49^{m}\ 27^{s}$
Ourselves, for the present	$186^{d}\ 14^{h}\ 48^{m}\ 45^{s}$	$178^{d}\ 15^{h}\ 0^{m}\ 0^{s}$

Table II. The Sun's stay in the ascending and descending semicircles [II. TAB. Mora Solis in Semicirculis Ascendente, ac Descendente]

Authors	From the start of ♑ to the start of ♋	From the start of ♋ to the start of ♑
Pliny the Elder, Book 18, Chapter 25	$183^d\ 15^h\ 0^m\ 0^s$	$181^d\ 15^h\ 0^m\ 0^s$
Magini, in the *Theorica Solis*, Chapter 9	$182^d\ 0^h\ 5^m\ 0^s$	$183^d\ 5^h\ 6^m\ 0^s$
Tycho, *Progymnasmata* vol. 1, pp. 13, 19, 107 taken together	$182^d\ 5^h\ 3^m\ 45^s$	$183^d\ 0^h\ 45^m\ 0^s$
Ourselves, for the present	$182^d\ 13^h\ 22^m\ 45^s$ [recte: $182^d\ 2^h\ 22^m\ 45^s$, per the book's errata; in this copy the printed hours are overwritten in a later hand, and they are given here as the errata quote them]	$183^d\ 3^h\ 26^m\ 0^s$

Table III. The intervals of the four cardinal points [III. TAB. Interualla 4. Cardinum]

Authors	From ♈ to ♋	From ♋ to ♎	From ♎ to ♑	From ♑ to ♈
Pliny the Elder, Book 18, Chapter 25	$93^d\ 12^h\ 0^m\ 0^s$	$92^d\ 12^h\ 0^m\ 0^s$	$89^d\ 3^h\ 0^m\ 0^s$	$90^d\ 3^h\ 0^m\ 0^s$
Magini, in the *Theorica Solis*	$92^d\ 20^h\ 49^m\ 0^s$	$93^d\ 10^h\ 57^m\ 0^s$	$89^d\ 18^h\ 9^m\ 0^s$	$89^d\ 4^h\ 1^m\ 0^s$
Tycho, vol. 1, above	$93^d\ 4^h\ 17^m\ 0^s$	$93^d\ 14^h\ 8^m\ 0^s$	$89^d\ 10^h\ 37^m\ 0^s$	$89^d\ 0^h\ 46^m\ 45^s$
Ourselves, for the present	$92^d\ 23^h\ 52^m\ 45^s$	$93^d\ 14^h\ 56^m\ 0^s$	$89^d\ 12^h\ 30^m\ 0^s$	$89^d\ 2^h\ 30^m\ 0^s$
Ptolemy, Book 3, Chapters 2 and 4, with Hipparchus	$94^d\ 12^h\ 0^m\ 0^s$	$92^d\ 12^h\ 0^m\ 0^s$	✝ ✝ ✝ ✝	✝ ✝ ✝ ✝
al-Battānī, Chapters 27, 28	$93^d\ 14^h\ 0^m\ 0^s$	$93^d\ 0^h\ 45^m\ 0^s$	✝ ✝ ✝ ✝	✝ ✝ ✝ ✝
Pitati, in Nonius	$93^d\ 10^h\ 1^m\ 12^s$	$93^d\ 13^h\ 36^m\ 48^s$	✝ ✝ ✝ ✝	✝ ✝ ✝ ✝
Goclenius, in the *Urania*, p. 53	$92^d\ 21^h\ 0^m\ 0^s$	$93^d\ 10^h\ 0^m\ 0^s$	✝ ✝ ✝ ✝	✝ ✝ ✝ ✝
Lansbergen, in the *Thesaurus Observationum*	$93^d\ 2^h\ 27^m\ 0^s$	$92^d\ 13^h\ 45^m\ 0^s$	✝ ✝ ✝ ✝	✝ ✝ ✝ ✝

Table IV. From Tycho, vol. 1 of the *Progymnasmata*, p. 107: from the Alphonsine and Copernican calculations and his own [IV. TAB. Tychonis Tomo 1. Prog. pag.107. Ex calculo Alphonsino, Copernicæo, & Suo.]

Year of Christ	From ♈ to ♎	From ♎ to ♈	From ♈ to ♋	From ♋ to ♎	From ♎ to ♑	From ♑ to ♈
Alphonsine, 1500	$187^d\ 0^h\ 32^m$	$178^d\ 5^h\ 17^m$	$93^d\ 11^h\ 15^m$	$93^d\ 13^h\ 17^m$	$89^d\ 3^h\ 51^m$	$89^d\ 1^h\ 26^m$
Alphonsine, 1600	$187^d\ 0^h\ 22^m$	$178^d\ 5^h\ 27^m$	$93^d\ 10^h\ 18^m$	$93^d\ 14^h\ 4^m$	$89^d\ 4^h\ 40^m$	$89^d\ 0^h\ 47^m$
Alphonsine, 1700	$187^d\ 0^h\ 19^m$	$178^d\ 5^h\ 30^m$	$93^d\ 9^h\ 33^m$	$93^d\ 14^h\ 46^m$	$89^d\ 5^h\ 29^m$	$89^d\ 9^h\ 1^m$
Copernican, 1500	$186^d\ 8^h\ 52^m$	$178^d\ 21^h\ 3^m$	$92^d\ 23^h\ 32^m$	$93^d\ 9^h\ 20^m$	$89^d\ 15^h\ 30^m$	$89^d\ 5^h\ 33^m$
Copernican, 1600	$186^d\ 7^h\ 50^m$	$178^d\ 22^h\ 5^m$	$92^d\ 20^h\ 26^m$	$93^d\ 11^h\ 24^m$	$89^d\ 18^h\ 32^m$	$89^d\ 3^h\ 33^m$
Copernican, 1700	$186^d\ 6^h\ 50^m$	$178^d\ 23^h\ 5^m$	$92^d\ 17^h\ 30^m$	$93^d\ 13^h\ 20^m$	$89^d\ 21^h\ 20^m$	$89^d\ 1^h\ 45^m$

Year of Christ	From ♈ to ♎	From ♎ to ♈	From ♈ to ♋	From ♋ to ♎	From ♎ to ♑	From ♑ to ♈
Tycho, 1500	$186^{d}\ 17^{h}\ 43^{m}$	$178^{d}\ 12^{h}\ 6^{m}$	$93^{d}\ 5^{h}\ 4^{m}$	$93^{d}\ 12^{h}\ 39^{m}$	$89^{d}\ 9^{h}\ 51^{m}$	$89^{d}\ 2^{h}\ 15^{m}$
Tycho, 1600	$186^{d}\ 18^{h}\ 25^{m}$	$178^{d}\ 11^{h}\ 4^{m}$	$93^{d}\ 4^{h}\ 17^{m}$	$93^{d}\ 14^{h}\ 8^{m}$	$89^{d}\ 10^{h}\ 33^{m}$	$89^{d}\ 0^{h}\ 51^{m}$
Tycho, 1700	$186^{d}\ 19^{h}\ 13^{m}$	$178^{d}\ 10^{h}\ 36^{m}$	$93^{d}\ 3^{h}\ 33^{m}$	$93^{d}\ 15^{h}\ 40^{m}$	$89^{d}\ 11^{h}\ 32^{m}$	$88^{d}\ 23^{h}\ 4^{m}$

CAPVT XIX.

Chapter 19.

De Hypothesibus Antiquis, quibus excusatur, seu explicatur Inæqualitas apparens in motu Solis Annuo.

On the ancient hypotheses by which the apparent inequality in the Sun's annual motion is excused, or explained.

I. To the Peripatetics, in the books *De caelo*, to Ptolemy in Book 3, Chapter 3, to Copernicus in Book 3 from Chapter 15, and to many others, it seems absurd that a celestial body, especially the Sun, should move unequally in its own circle and about the centre of its own circle. Astronomers therefore, considering the cause of the apparent inequality discussed in Chapter 18, attributed it to a difference of circles and centres. From *theoria*, or their contemplation, they called the figures of these *Theoricae*, and, from the supposition of the motion described by them, *Hypotheses*.

[Margin: What are the Theorics and Hypotheses of the planets?]

Now the ancients have two hypotheses, or theorics, of the Sun's motion, equivalent to one another as regards very many properties. The first works by way of an ecliptic of the solar heaven that is concentric with the world, or with the Earth's centre, but carries the centre of a small circle called the epicycle, on whose periphery the Sun moves. The second works by way of a circle eccentric to the centre of the world. On these, and on their equivalence: Ptolemy, Book 3, Chapter 3; al-Battānī, Chapter 28; Geber [Jābir ibn Aflaḥ], Book 3 of the *Astronomy*; Peuerbach in the *Theorica Solis*, and on that work Nonius, Osualdus [Erasmus Oswald Schreckenfuchs], Reinhold and Vurstitius [Christian Wurstisen]; likewise Copernicus, Book 3, Chapter 15; Maurolico, *Cosmographia*, Dialogue 3; Alessandro Piccolomini in the *Theorica Planetarum*, from Chapter 1 to 13; Regiomontanus in the *Epitome of the Almagest*, Book 3, Proposition 4; Longomontanus in the *Astronomia Danica*, Book 1 of the theorics, Chapter 2; Kepler in the *Commentaries on Mars*, Chapter 2; Biancani, Book 10 of the *Sphere*, Chapter 8. Yet only the eccentric was used by Nonius (above); Fernel, *Cosmotheoria*, Book 1, Chapter 7; Magini in the *Theorica Solis*; Clavius at the end of the *Sphere*; Barozzi, *Cosmographia*, Book 4, p. 250; Tycho, *Progymnasmata* vol. 1, Chapter 1; Lansbergen in the planetary *Theoricae*, Chapter 1; and Hérigone, vol. 5, in the *Theoricae*, Book 1, Chapter 4, Proposition 3. And all of them, except Longomontanus, prefer the eccentric to the concentrepicycle [Concentrepicyclus]. Both hypotheses must now be set out.

De Concentrepicyclo.

On the concentrepicycle.

[Margin: What are a concentric, an epicycle, and a concentrepicycle?]

II. In the theorics of the planets a concentric circle, in Greek [Greek: κύκλος ὁ μό κεντρος (kyklos homokentros, "concentric circle")], is a circle having the same centre as the centre of the world. An epicycle, on the other hand, is a circle, usually smaller, attached to the circumference of a larger circle, or having its centre on the circumference of another circle. Now if the circle carrying the epicycle is concentric with the world, the aggregate of these is called a concentrepicycle.

Let the centre of the world — and, as most suppose, of the Earth — now be at C, and from it let the circle BMRNB be described, concentric with the world. On its periphery let the centre B lie, and from B let the little circle AGFE be described as the epicycle, to which the little circles KMRN are to be understood as equal. When the two straight lines AR and MN have been drawn, cutting each other at right angles at the centre C, AB will be the semidiameter of the epicycle, and BC the semidiameter of the concentric. BC must be as great as the Sun's mean distance from the Earth's centre, AC as great as its greatest distance, and CF as great as its least; and so AB is as great as the difference between the mean and the greatest distance.

Diagram. A large circle about the centre C, set into the right column with the text running down its left side. A vertical diameter runs from the small circle at the top, through F, C and P, to the small circle at the bottom; a horizontal diameter runs from the small circle at the left, through C, to the small circle at the right. Four small equal circles stand on the large circle at top, left, foot and right, and a fifth at upper left. From C straight lines run to the upper-left small circle, to the left small circle and to the right small circle, the upper-left and the upper of the two left-hand lines touching the small circles away from their centres.

- A (top, in the small circle's upper half): point on the epicycle
- G, E (left and right of the top small circle, outside it)
- B (inside the top small circle, at its centre): faint, read at magnification
- F (below the top small circle, on the vertical)
- D (above the upper-left small circle)
- Z (left of the upper-left small circle)
- K, I (inside the upper-left small circle, K left, I right)
- L (above the left small circle)
- M (inside the left small circle)
- V (above the right small circle)
- N (inside the right small circle)
- C (the centre)
- P (above the bottom small circle, on the vertical)
- Q (right of the bottom small circle)

• R (inside the bottom small circle)

Therefore, if the body of the Sun, fixed by its centre to the periphery of the epicycle, completes the whole epicycle in one year, or if the whole epicycle completes its revolution carrying the Sun around, the Sun will be once at A, the point of greatest distance from the Earth, which is called the apogee; once at P, the point of least distance, or at perigee; and twice at mean distance, namely at the points L and V. And so, in passing, an account is given of the Sun's apparent approach to the Earth and withdrawal from it.

[Margin: Theory of the Sun's distances.]

Now let the concentric move according to the succession of the signs, toward the east, from B to K, M, etc., with the Sun's mean motion, that is, completing equal arcs in equal times; and meanwhile let the epicycle move in the opposite direction, turning the Sun from A to E, completing its own arcs equal in equal times and physically similar to the arcs of the concentric. Thus when the centre of the epicycle has completed the quadrant of the concentric BM, from B to M, the Sun will have completed a quadrant in the epicycle and be at L; and when the centre of the epicycle is at R, the semicircle BMR of the concentric having been completed, the Sun will have completed a semicircle of the epicycle and be at P; and when the centre of the epicycle is at N, three quadrants of the concentric BMRN having been completed, the Sun will have completed three quadrants of the epicycle and be at V, etc. I said *physically*, because if the place of the apogee shifts in the order of the signs with a very slow motion, as below, when on the apogee [?], the epicycle's motion must be a little slower than the motion of the concentric; and when the concentric's point has returned from B to B, the apogee of the epicycle is not at A, but between G and A.

[Margin: Theory of the Sun's motion in the epicycle.]

With this hypothesis made, it is plain: when the Sun is on the line of apsides AP, that is, at apogee and perigee, there will be no difference between the mean motion and the true or apparent motion, and the Sun will be seen on one and the same line of mean place, B or R, and of true place, A or P. But as long as the centre of the epicycle is in the first semicircle of the concentric, BMR, and so the Sun is in the right-hand and first semicircle of the epicycle, AEF, the Sun will be seen further west than the centre of the epicycle, and therefore slower than the mean motion; whereas in the other semicircle it will be seen further east. And when it is at L, a little beyond the first quadrant of the epicycle, that is, at the point of contact where the line from the centre C touches the epicycle, the difference between the mean motion of the point M and the true motion of the Sun at L will be greatest, and is to be subtracted from the mean motion so as to leave the Sun's true motion in the ecliptic. Likewise it will be greatest, but to be added, at the point of contact V, for there the Sun will be further east than the point N, if the mean motion is to be converted into the true and apparent; for if the true is to be converted into the mean, it will have to be subtracted.

III. From what has been said it is plain how the Sun's motion, although in its own circle, can nevertheless appear unequal to us; how in the first semicircle of

each circle it seems slower than the mean motion, so that something must be subtracted from the mean motion to establish the true and apparent, or added to the true to obtain the mean; and how, conversely, in the other semicircle it is faster than the mean motion, so that something must be added to the mean if the mean is to be changed into the true, or taken away if the true is to be changed into the mean. This alternating addition and subtraction is called in Greek [Greek: προσθαφαίρεσις (prosthaphairesis, "addition-and-subtraction")], *Prosthaphærisis*, as if *Addidemptio*; but the Latins call either one an equation [Aequationem], additive or subtractive.

[Margin: The data required for finding the Sun's equation.]

To find it, four givens are required: 1. The Sun's mean motion. 2. The mean motion of the solar apogee, which, subtracted from the Sun's mean motion in longitude, will leave the Sun's mean motion from the apogee, that is, its mean distance from the apogee and the arc of the epicycle completed. 3. The Sun's mean distance from the Earth's centre, that is, the semidiameter or radius of the concentric. 4. The semidiameter of the epicycle, whether from the given distances of the Sun in the table of Chapter 7, or from what is to be said below on eccentricity, in Chapter 24, where the finding of the place of the Sun's apogee will also be treated. For the present, however, by a kind of prolepsis, we must assume these as if known. With these given:

[Margin: Problem for finding the Sun's prosthaphaeresis.]

Let the concentric's point be at K, and meanwhile let the Sun have come from A, that is, Z, to D. From the centre C let the line of mean motion CKZ be drawn, on which the Sun would be seen if the mean motion were being considered. Then let the line of true place and motion CD be drawn, on which the Sun truly appears to the eye at C, and let it be connected with the centre K by the semidiameter of the epicycle KD. For in the triangle CKD there are given CK, the Sun's mean distance, and KD, the semidiameter of the epicycle, in parts of the same kind, and the angle CKD, since by Proposition 13 of Book 1 of Euclid it is the complement to two right angles of the angle ZKD. That angle is known from the arc measuring it, ZD, that is, from the Sun's distance D from the epicycle's apogee Z. Therefore by trigonometry the angle KCD will not remain hidden, which measures the arc KI, namely the difference of the mean and true motion, or the desired prosthaphaeresis [Prosthaphæresim].

[Margin: How the greatest equation is to be found.]

IV. The greatest equation, or prosthaphaeresis, is gathered in the triangles CML or CNV, and indeed more easily, because they are right-angled at the contact L or V, by Propositions 16 and 18 of Book 3 of Euclid. It is therefore enough that the base MC, or the star's mean distance from the Earth, and the semidiameter of the epicycle ML be known; for from these is gathered the angle MCL, subtending and measuring the arc ML, which is the greatest prosthaphaeresis; and the angle NCV is equal to it.

De Excentrico.

On the eccentric.

[Margin: What is an eccentric?]

V. In the theorics of the planets an eccentric circle [Excentricus circulus] is a circle whose centre lies outside the centre of the world, and the distance of its centre from the centre of the world is called the eccentricity [Excentricitas]. Let us then explain, with a diagram added, how an account of the apparent inequality in the Sun's annual motion is given by a simple circle of this kind.

Let T be the centre of the Earth and of the supreme ecliptic, and let the ecliptic itself be the circle AMPQA, divided into four quadrants by the straight lines AP and MQ, cutting each other orthogonally at T; and let AP be the line of greatest and least distance. On it, taking the centre E, let the eccentric HIRN be described, so that its eccentricity is ET, and as great as the difference between the Sun's greatest distance from the Earth's centre, which is the line TH, and the mean distance, which is the radius of the eccentric EH. Let the Sun's apogee be at H, beneath the point A of the ecliptic, and the perigee at R, beneath the point P; and let the line of the apogee, or of the apsides, be HR in the eccentric but AP in the ecliptic.

Diagram. A large circle with an inner circle set within it, standing in the right column with the text running down its left side. A vertical diameter of the large circle runs from A at the top to P at the foot, and a horizontal diameter from M at the left to Q at the right, crossing at T. The inner circle's centre lies on the vertical above T. Through each of the points B, K, I and N on the inner circle a pair of straight lines runs out from the neighbourhood of the two centres to the large circle.

- A (top of the large circle)
- D, C (upper left, on the large circle, D above C)
- F, G (upper right, on the large circle, F left of G)
- M, L (left, on the large circle, M above L)
- Q, V (right, on the large circle, Q above V)
- P (foot of the large circle)
- H (top of the inner circle, on the vertical)
- B (upper left of the inner circle)
- K (upper right of the inner circle)
- I (left of the inner circle)
- N (right of the inner circle)
- R (foot of the inner circle, on the vertical)
- T (the crossing of the diameters)
- E (above T, at the meeting of the lines): small and faint

And let the centre of the solar body be on the periphery of the eccentric, and let the Sun be carried by it according to the order of the signs, in the direction of succession, eastward, from H to B, etc., with the mean or equal motion of the eccentric. For notwithstanding this equality, to us who are at the centre T

the Sun will seem to move unequally as long as it is off the line of apsides. On that line, indeed, the Sun at H will be seen beneath A, and placed at R will be seen beneath P, and the line of mean motion EH or ER will coincide with the line of true motion TA or TP. Let the Sun then be at B, and through its centre let the straight lines EBC and TBD be drawn from the centres of the Earth and of the eccentric, as far as the ecliptic. The line of mean motion and place will be EBC, and if our eye were at E, the Sun at B would be seen beneath the point C of the ecliptic; but because the eye is at T, the line of the true place and motion, apparent to us, is TBD, and the Sun at B is seen beneath the point D of the ecliptic, and therefore further west than the mean motion would bring it, and so slower. Thus the Sun placed at I is not seen beneath L, along the line of mean place EIL, but from T is seen beneath M, along the line of true place TIM. Hence throughout the first semicircle of distance from the apogee A, which is called the anomaly, namely in AMP, the Sun will be seen further west than its mean place, and its motion slower than the mean motion.

Now let the Sun have passed the perigee R and be at N: the line of mean place and motion will be ENV, but that of the true will be TNQ, and so the Sun at N will be seen further east, that is, beneath the point Q, which is further east than the point of the mean place V. And if it is at K, it will be seen along the line TKF, beneath the point F, which is further east than the point G, marked by the line of mean motion EKG. Accordingly, in the second semicircle of the anomaly, or of distance from the apogee, the Sun will appear further east than its mean place, and its motion faster than the mean motion.

VI. The prosthaphaeresis, or equation, in the eccentric is found from four givens, namely: 1. the Sun's mean motion in

[Margin: How the prosthaphaeresis is investigated through the eccentric.]

the ecliptic; 2. the mean motion of the solar apogee, for this, subtracted from the Sun's mean motion, leaves the mean distance of the Sun from the apogee, which is called the Sun's annual anomaly, or Argument, because from it one argues whether, and how much, is to be added to or taken from the Sun's mean place or motion to obtain the true or apparent one, and conversely; 3. the Sun's mean distance from the Earth, or the radius of the eccentric; 4. the Sun's eccentricity. These being given:

Let the Sun be at B; for TBE will be the angle of the equation, or the angle measuring the equation. It is the angle by which the angle BEH, measuring the mean distance of the Sun at B from the apogee H, exceeds the interior angle DTA, by Euclid I.32; and that angle measures the true distance of the Sun's true place D from the true place of the apogee A. Therefore in the triangle BTE we are given EB, the semidiameter or radius of the eccentric; TE, the eccentricity; and the angle BET, since by Euclid I.13 it is the complement to two right angles of the angle BEH. BEH in turn is as large as the anomaly BH, or the distance of the Sun at B from the apogee H. Hence by the rules of oblique-angled triangles the angle B will become known.

[Margin: The greatest equation in the path of the eccentric.]

The greatest prosthaphaeresis, however, is derived from the triangle ITE, right-angled at T, as will be demonstrated in the next chapter. For given EI, the semidiameter or radius of the eccentric, which is the mean distance of the Sun from the Earth, and ET, the eccentricity, the angle I is sought by the rules of right-angled triangles; this angle measures the greatest prosthaphaeresis, namely the angle TIE.

[Margin: Figure 2, for the hypothesis of the eccentric.]

Diagram. Woodcut set into the foot of the first half of the left column, with the text of section VII running down its left side. A large circle (the ecliptic), with centre T, encloses a smaller circle (the eccentric) drawn about E, which stands above T on the vertical line of apsides; that line runs from A at the top through H, E, T and R to P at the foot. Straight lines run from E and from T up to the left and up to the right, through the inner circle to the outer, and a horizontal line through T meets the outer circle on the left at M/L and on the right at Q/V.

- A (top of outer circle): apogee of the ecliptic
- H (top of inner circle, below A)
- C, D, X (upper left, on or beside the outer circle, reading from the top down)
- B (upper left, on the inner circle)
- F, G, O (upper right, on or beside the outer circle, reading from the top down)
- K (upper right, on the inner circle)
- E (on the vertical, above T)
- T (centre of the outer circle)
- M, L (left end of the horizontal, M above L)
- Q, V (right end of the horizontal, Q above V)
- R (foot of inner circle)
- P (foot of outer circle)

VII. But since in the preceding figure the angle HEB does not measure an arc of the ecliptic, the ecliptic not being described from it as a centre, in the present figure, in which the rest is set out as in the preceding figure, the line TX is therefore drawn parallel to the line of mean motion EC, and the line TO parallel to the line EG. For in this way, by Euclid I.29, the angle HEB becomes equal to the angle ATX, which measures the arc XA of the ecliptic; this arc will accordingly be similar to the arc BH, or of as many degrees. And so the arc of the anomaly is counted on the ecliptic itself rather than on the eccentric.

Moreover, since the arc XD of the ecliptic is as large as the angle DTX, and this angle, because of the already mentioned parallelism, is by Euclid I.29 equal to the alternate angle TBE, which is the measure of the prosthaphaeresis, the arc XD is therefore the prosthaphaeresis to be taken from the arc XA, or from the Sun's mean distance from the apogee reckoned on the ecliptic, so that its true distance DA from the apogee may become known, and from that its distance from the beginning of Aries, since the distance of the apogee A from

the beginning of Aries is now supposed known. Conversely, in the second semicircle of the anomaly, AQP, the prosthaphaeresis, namely the arc FO, is to be added to the distance from the apogee reckoned in the order of the signs, namely to the arc AMPO, so that the Sun's true place in the ecliptic and its true distance from the place of the apogee may be known.

[Margin: The prosthaphaeresis applied to the 2nd figure.]

In this figure the prosthaphaereses are obtained from the triangle TEB, as above, given EB, the semidiameter of the eccentric, ET, the eccentricity, and the angle HEB, namely the Sun's anomaly, or annual Argument as they call it, that is, the Sun's distance from the apogee; for by Euclid I.13 its complement to two right angles is the angle BET. Therefore from these three givens the angle EBT, that is DTX, which is the measure of the prosthaphaeresis XD, is obtained by trigonometric analysis.

Comparatio Hypothesis Concentrepicycli cum Hypothesi Excentrici.

Comparison of the hypothesis of the concentrepicycle with the hypothesis of the eccentric.

[Margin: Reasons for the homocentrepicycle.]

VIII. Longomontanus, in his *Astronomia*, Book 1 of the Theorics, chapters 2 and 5, prefers the hypothesis of the homocentrepicycle to the hypothesis of the eccentric, as most suited to nature. For in the minor planets too, having removed the equant and the motion of a star about a centre not its own, he sets out the phenomena of the planets by epicycles, with Copernicus and Tycho. And indeed this hypothesis would favour the annual rotation of the Sun acknowledged by Šírek and Scheiner, as we said in chap. 4, no. 2.

[Margin: Reasons for the eccentric.]

Nearly all the other astronomers brought forward at the beginning of the chapter, and Copernicus and Tycho themselves, although defenders of epicycles in the theory of the 5 minor planets, nevertheless in the theory of the Sun preferred the eccentric to the epicycle. They did so not so much lest the Sun seem direct in the upper part of the epicycle and retrograde in the lower (for that is not to be feared so long as the motion of the epicycle, as we have laid down, is somewhat slower than the motion of the concentric), nor because it is absurd that the Sun at the apogee A should turn one of its hemispheres toward us and at the perigee P the opposite hemisphere of its body (for that is not manifestly absurd, and if it were, it could be avoided by another annual rotation about its own centre). They preferred the eccentric rather because the Sun's phenomena are explained by it with a single and simple circle, and because they are explained with a single motion in the same direction; whereas the homocentrepicycle requires two circles, and the motion of the epicycle is contrary to the motion of the concentric and slower than it, since otherwise it would not satisfy the appearances, as I shall show in the next chapter to have been demonstrated. Since, however, for the motion of the apogee and of

the eccentricity, of which I shall speak below in their places, a small circle is usually employed, on whose periphery the centre of the eccentric is carried, the argument taken from the simplicity of circles and of motion does not seem to hold. It does hold, however, if we confine ourselves to the annual motion, for the whole motion of the Sun in one year can be explained by the eccentric alone, and that rather slow change of the apogee does not sensibly alter it.

CAPVT XX.

Chapter 20.

Colliguntur in vnum Nomina, Definitiones, deinde Regulæ, ac Proprietates ad Hypothesim Epicycli & Excentrici spectantes.

The names and definitions, then the rules and properties, belonging to the hypothesis of the epicycle and of the eccentric are gathered into one place.

[Margin: What the concentric is.]

I. The *concentric* circle, in Greek [Greek: κύκλος ὁμόκεντρος (kyklos homokentros, "homocentric circle")], in the theories of the planets is a circle whose centre is the same as the centre of the world. The *eccentric*, in Greek [Greek: ἔκκεντρος (ekkentros, "off-centre")], is a circle whose centre lies outside the centre of the world; and the distance of its centre from the centre of the world is called the *eccentricity*, in Greek [Greek: ἐκκεντρότης (ekkentrotēs, "eccentricity")], genitive [Greek: ἐκκεντρότητος (ekkentrotētos)]. Such in the nearest figures is the line ET, which ought to be as large as, in the former hypothesis, the semidiameter of the epicycle, such as KD.

[Margin: What the epicycle is.]

II. The *epicycle*, or *epicyclion*, in Greek [Greek: ἐπίκυκλος (epikyklos, "circle upon a circle")], is a circle, generally smaller, attached or fixed by its centre to the circumference of a larger circle. Ptolemy and al-Farghānī call it the circle of diversity [Greek: κύκλος τῆς διαφορᾶς (kyklos tēs diaphoras, "circle of difference")], or of irregularity [Greek: τῆς ἀνωμαλίας (tēs anōmalias, "of the anomaly")]. Such in the first figure is the small circle GAEF.

[Margin: What the concentrepicycle is.]

III. The *concentrepicycle*, or homocentrepicycle [Hemocentrepicyclus], in Greek [Greek: ὁμοκεντρεπίκυκλος (homokentrepikyklos, "homocentric epicycle")], is the aggregate of a concentric and an epicycle, just as the aggregate of an eccentric and an epicycle is called the *eccentrepicycle*.

IV. *Simply concentric* is said of a circle or orb whose surfaces, the convex as well as the concave, are concentric with the world; *simply eccentric*, of one each of whose surfaces is eccentric to the centre of the world. But *concentric in a certain respect* and *eccentric in a certain respect* is one of which one surface is concentric with the world and the other eccentric, as will be clear from the figure of the scholium.

[Margin: Deferent.]

V. The *deferent* is a circle or orb carrying not so much an epicycle as the body of the Sun itself; and so the eccentric and the deferent are the same in the theory of the eccentric. Al-Farghānī and al-Battānī also call it the circle of the departing cusp [Circulus Egressæ Cuspidis].

[Margin: The highest apsis, the aux and the apogee are the same.]

VI. The *highest absis*, or apsis, from the Greek [Greek: ἀψίς (apsis, "arch, vault")], is the topmost apex of a circle, at which the planet is farthest from the centre of the Earth; the Arabs call this point *Aux*, genitive *Augis*, and in Greek [Greek: Ἀπόγειον (apogeion, "apogee")], because in it the planet is [Greek: ἀπόγεος (apogeos, "away from the Earth")], or, as Ptolemy puts it, [Greek: ἀπογειότατος (apogeiotatos, "farthest from the Earth")], that is, most distant [Greek: ἀπὸ τῆς γῆς (apo tēs gēs)], that is, from the Earth.

[Margin: The same: the lowest apsis, the opposite of the aux, and the perigee.]

The *lowest absis*, or *opposite of the aux*, is the point of the circle at which the planet is least distant from the Earth. It is called [Greek: περίγειον (perigeion, "perigee")], because the planet is there [Greek: περίγεος (perigeos, "near the Earth")], or rather [Greek: περιγειότατος (perigeiotatos, "nearest the Earth")], that is, most [Greek: περὶ τὴν γῆν (peri tēn gēn)], namely around or near the Earth. By Copernicus, however, Book 5, chap. 1, the planet's perigee point [Planetæ Perigęi] is called *Hypaugi*.

[Margin: Aphelion.]

VII. *Aphelion*, in Greek [Greek: ἀφήλιον (aphēlion)], is the point at which the Earth or any other planet is farthest from the Sun. For the Earth is called aphelian, or a planet aphelian, because it is farthest [Greek: ἀπὸ τοῦ ἡλίου (apo tou hēliou)], namely from the Sun, which in Greek is called [Greek: Ἥλιος (Hēlios)].

[Margin: Perihelion.]

Perihelion, in Greek [Greek: περιήλιον (perihēlion)], is the point at which the Earth or some planet is least distant from the Sun; for this reason it is called perihelian, as an adjective, that is [Greek: περὶ τὸν ἥλιον (peri ton hēlion)], around or near the Sun.

[Margin: The line of the apsides, or of the apogee.]

VIII. The *line of the apogee*, or of the auges, or of the apsides, is the straight line passing through the centre of the world, and through the centre of the eccentric or epicycle, drawn from the highest to the lowest apsis, or from the aux or apogee to the opposite of the aux or perigee. Such in the first figure is the line ACR, and in the two following figures the line HETR; but if it is extended to the supreme ecliptic, it will be the line AP.

[Margin: The longer and the nearer longitude.]

IX. The *longer longitude*, for Ptolemy, is the greatest distance of a planet from the centre of the universe, in Greek [Greek: ἀπόστασις μεγίστη (apostasis megistē, "greatest distance")], or [Greek: ἀπόστημα μέγιστον (apostēma

megiston, "greatest interval")]; as in the first figure the line CA, and in the others TH.

The *nearer longitude*, on the other hand, is the least distance of a planet from the centre of the universe, in Greek [Greek: ἀπόστασις ἐλαχίση [recte: ἐλαχίστη] (apostasis elachistē, "least distance")]; as in the first figure the line CP, in the others TR.

[Margin: Mean longitude and mean transit.]

X. The *mean longitude* is sometimes taken for the mean distance of a planet from the centre of the Earth, called in Greek [Greek: ἀπόστασις μέση (apostasis mesē, "mean distance")]; such is the line CM, the semidiameter of the concentric, or in the second figure EI, the semidiameter of the eccentric. Sometimes, however, it is taken for the point of the epicycle or eccentric at which the difference between the mean and the apparent motion is greatest, or the greatest prosthaphaeresis, although Ptolemy more properly calls this [Greek: πάροδος μέση (parodos mesē)], namely the mean transit, because the planet passes from the greatest differences of the motions to smaller ones, or because this point in the ecliptic is equally distant from the points of apogee and perigee, as will be said below.

[Margin: Mean and true motion.]

XI. *Motion*, in Greek [Greek: κίνησις (kinēsis)], is either mean, in Greek [Greek: μέση (mesē)], or regular, in Greek [Greek: ὁμαλὴ (homalē)], that is, equal and uniform, in Greek [Greek: ἴση (isē)]; or it is true, in Greek [Greek: ἀλιθινὴ [recte: ἀληθινὴ] (alēthinē)], and such as it truly appears to us, and therefore called apparent, in Greek [Greek: φαινομένη (phainomenē)]. This last, excepting the points of apogee and perigee, is irregular, in Greek [Greek: ἀνωμαλὴ (anōmalē)], or rather unequal, in Greek [Greek: ἀνίση (anisē)], yet nevertheless regulable, uniformly difform.

[Margin: Motion in consequentia and in præcedentia.]

XII. *Motion in consequentia* (understand "of the signs"), or according to the succession and order of the signs in the zodiac, is called in Greek [Greek: κίνησις εἰς τὰ ἑπόμενα (kinēsis eis ta hepomena, "motion toward what follows")]. *Motion in præcedentia*, or against the order or series of the signs, is called [Greek: εἰς τὰ προηγούμενα (eis ta proēgoumena, "toward what precedes")] by Ptolemy and the other Greeks. In the zodiac the former is reckoned toward the east and the latter toward the west; but in other circles, which are divided after the manner of the zodiac into 12 signs and 360 degrees, there is no need to look to these regions of the world. The less skilled, however, take the phrases *in consequentia* and *in præcedentia* in the sense of the ablative case. [Greek: Ψηφορία (psēphoria, "reckoning")], on the other hand, is not motion but the calculation of the motion itself.

[Margin: Apocatastasis, period, synod, parodus.]

XIII. *Apocatastasis*, in Greek [Greek: ἀποκατάστασις (apokatastasis, "restoration")], is the complete revolution of a star, by which, its whole circle completed, it is restored to the same point from which it began to revolve;

this is also called a *period*, in Greek [Greek: περίοδος (periodos)]. But if after completing the circle it revolves further until it is conjoined with another star, it is called a synod [Greek: σύνοδος (synodos)], while the passage from one notable arc or point to another is called [Greek: πάροδος (parodos)].

[Margin: The line of mean motion.]

XIV. The *line of mean motion or place* is the straight line terminating the mean motion or place of a star. In the hypothesis of the epicycle it is the line drawn from the centre of the world to the centre of the epicycle, such as CK, CM, etc.; but in the hypothesis of the eccentric it is the straight line drawn from the centre of the eccentric through the centre of the star as far as the ecliptic, such as EBC and EKG, likewise EIL and ENV; and so, in the third figure, the lines parallel to them, such as TX and TO.

[Margin: The line of apparent motion.]

XV. The *line of true or apparent motion or place* is the straight line drawn from the centre of the Earth through the centre of the star as far as the ecliptic; as in the first figure the line CD, and in the other two the line TBD or TKF.

[Margin: Anomaly.]

XVI. *Anomaly*, in Greek [Greek: ἀνωμαλία (anōmalia)], is the irregularity of motion; sometimes, however, it is taken for the argument of the irregularity and of the equation to be applied. This argument is the arc between the line of the apsides and the line of mean motion, or it is the distance of the planet's mean place from the apogee: as in the first figure the arc ZD, in the second figure BH, but in the third XA, etc.

[Margin: Equation.]

XVII. The *equation* is the arc of the ecliptic to be added to or subtracted from the mean motion or place so that it is converted into the true, or from the true so that it is converted into the mean. Ptolemy also calls it [Greek: διαφορᾶ τῆς κινίσεως [recte: κινήσεως] ὁμαλῆς, παρὰ τὴν ἀνομαλὴν (diaphora tēs kinēseōs homalēs, para tēn anomalēn, "difference of the regular motion from the irregular")], namely the difference of regular motion from irregular. The angle that is the measure of this arc is also usually called the equation. Thus in the first figure the equation is both the arc KI and the angle KCI, while in the third figure it is both the arc XD and the angle XTD.

[Margin: Prosthaphaeresis.]

XVIII. The *prosthaphaeresis* is the equation spoken of above, so called because it is now to be added, now to be subtracted; for in Greek [Greek: πρόσθεσις (prosthesis)] is addition and [Greek: ἀφαίρεσις (aphairesis)] removal, and from these comes the compound name [Greek: προσθαφαίρεσις (prosthaphairesis)], in Latin *Addidemptio*, since the same equation that is added to the mean place to give you the true is to be subtracted from the true to give you the mean.

Regulæ & Proprietates Selectæ ad Hypothesim Epicycli, aut Eccentrici spectantes.

Selected rules and properties belonging to the hypothesis of the epicycle or of the eccentric.

I. It is impossible for the same star to move circularly and equally about different centres of different circles. Therefore if the Sun moves equally on the epicycle or on the eccentric, it will necessarily move unequally on the concentric or on the ecliptic: so Peuerbach, and Reinhold in the Theorics, p. 32. Otherwise, with a side of the triangle produced, the exterior angle would be equal to the interior and opposite angle, contrary to Euclid I.16 and I.32; as in the first figure the angle ZKD, measuring the arc ZD, would be equal to the angle KCI, measuring the arc KI, and in the other two figures the angle BEH, measuring the arc BH, would be equal to the angle DTA, measuring the arc DA, which is absurd.

[Margin: The motion of the epicycle compared with the motion of the concentric.]

II. If the epicycle moves with a motion equal to the motion of the concentric, the point of the apogee will always, on the completion of a revolution, be restored beneath the same point of the ecliptic. If it moves more slowly than the concentric, the apogee point, after a single revolution of the epicycle, will have advanced in consequentia; but if faster, it will have been drawn back in præcedentia of the signs. Copernicus, Book 3, chap. 15, and Reinhold in the Theorics, p. 31.

III. If the epicycle moves with a motion contrary to the motion of the concentric itself, the planet fixed on the epicycle will appear slower than the mean motion in the first semicircle of the epicycle, reckoned from apogee to perigee, and faster in the second semicircle, reckoned from perigee to apogee. Ptolemy, Book 3, chap. 3. Regiomontanus in the *Epitome of the Almagest*, Prop. 4. Copernicus, Book 3, chap. 15. Reinhold in the Theorics, p. 31. Kepler on Mars, chap. 2.

[Margin: The motion of the eccentric examined.]

IV. If the apparent motion is bisected by the line of mean longitude, the equal motion of that time will be as great as the apparent; and if in some time the mean motion of a star on the eccentric and its apparent motion at the centre of the world are equal, the point of mean longitude will be enclosed within their endpoints. Nonius on Peuerbach's Theorics, Annotation 1, Props. 1 and 2.

V. If the Sun's motion were equal in the ecliptic and unequal on the eccentric, the greatest and swiftest motion of the eccentric would be at the apogee, and from there it would decrease gradually as far as the perigee, where it would be least and slowest. Reinhold, above, p. 33. This will easily be clear to anyone who divides the ecliptic into equal parts and draws straight lines from the centre

of the ecliptic to the points of division: for the arcs of the eccentric will be larger at the apogee than at the perigee.

[Margin: The points of greatest slowness and swiftness.]

VI. If the Sun, as agrees with observations, moves equally on the eccentric and unequally under the ecliptic, the apparent or true motion is slowest at the apogee, and from there increases little by little, and always becomes greater than the apparent motion of the preceding day, until at the perigee it becomes greatest and swiftest; from there, however, it begins to decrease, and smaller and smaller it becomes than the apparent motion of the preceding day. Ptolemy, Book 3, chap. 3, and Theon on that passage; Regiomontanus in the *Epitome*, Book 3, Prop. 5; Copernicus, Book 3, chap. 15; Reinhold in the Theorics, pp. 33 and 34; Kepler on Mars, chap. 2. This will easily be demonstrated by dividing the eccentric into equal parts by straight lines drawn from the centre of the eccentric, and drawing straight lines from the centre of the world to the ecliptic through the points of division of the eccentric. For it will at once appear that in the left-hand semicircle, which runs from the apogee to the perigee, the arcs of the ecliptic intercepted by those lines are larger and larger than the preceding ones as far as the perigee, and from there in the other semicircle smaller and smaller.

[Margin: The beginners' delusion.]

Here, however, the imagination of beginners is usually confused, and they do not at once grasp how in the first semicircle the apparent motion can be slower than the mean motion and yet the apparent motion be faster and greater than the apparent motion of the preceding day. These two things nevertheless stand together and are perfectly true, since the term of comparison is different.

VII. In that semicircle or quadrant of the ecliptic in which the Sun's apogee lies, the Sun's apparent and true motion is slower than in the other. This follows from Prop. 6.

[Margin: The use of the equations.]

VIII. From apogee to perigee in consequentia the mean motion is greater than the apparent motion, but from perigee to apogee it is smaller. And so in the first semicircle of the anomaly the equation is to be subtracted from the mean motion to obtain the true, or added to the true to obtain the mean; conversely, in the second semicircle it is to be added to the mean motion to obtain the true, or subtracted from the true to obtain the mean. This is clear from the exposition of figures 1, 2 and 3.

IX. The equations are equal at points of the eccentric or epicycle equally distant from the apogee, and also at opposite points of the ecliptic. Peuerbach and Reinhold in the Theory of the Sun, no. 11, Rules 2 and 4. Regiomontanus in the *Epitome*, Book 3, Prop. 11. For these, look at figure 3.

[Margin: Where the Sun's mean distance occurs.]

X. The Sun's mean longitude, taken for its mean distance from the Earth, occurs at that point of the eccentric from which a straight line can be drawn perpendicular to the line of the apsides, cutting the eccentricity in half. Reinhold

in the Theorics, p. 42. For in this little diagram let the eccentric be ACP, with centre E; let the centre of the world be T, the line of the apsides AP, and the eccentricity ET, cut in half at B. To this point let CB be drawn perpendicular to AP, and let the point C be connected by the straight lines CE and TC with the centres of the world and of the eccentric. Then by Euclid I.4 the triangle CBT will be equal to the triangle CBE, since the side CB is common, EB is equal to BT, and both CBE and CBT are right. Therefore the base CT too is equal to the base CE, which is the semidiameter of the eccentric, or the mean distance from the Earth. Therefore CT is the mean distance, and it occurs at the point C, etc.

Diagram. Small woodcut in the left column, with the text of section X running beside it: the left half of an eccentric, drawn as an arc from A round through C to P, with the straight line AP as its right-hand side. Three straight lines run from C on the arc to the points E, B and T on AP.

- A (top of the line AP)
- E (on AP, below A)
- B (on AP, below E)
- T (on AP, below B)
- P (foot of the line AP)
- C (left, on the arc)

[Margin: The point of mean distance: at what degree of the anomaly it occurs.]

Further, the distance of the point C from the apogee A becomes known by seeking through trigonometry the angle CEB from the givens CE, the radius of the eccentric, and EB, half the eccentricity, in the right-angled triangle CBE. For the complement of the angle CEB to two right angles is, by Euclid I.13, the angle CEA, measuring the arc CA. Thus if EB is 3460 parts of which CE is 100000, CA will be 91° 59′.

XI. The mean longitude, taken for the mean transit, or the passage of the mean motion from, or to, its greatest difference from the apparent motion, occurs at that point of the eccentric from which a line perpendicular to the line of the apsides can be drawn to the centre of the world. In this sense Peuerbach uses *mean longitude* in the Theory of the Sun, no. 4, and Copernicus, Book 3, chap. 3. This will be demonstrated in the next proposition.

[Margin: Where the greatest equation occurs.]

XII. The greatest equation, or the greatest difference between the mean and the apparent motion, occurs at the point of the eccentric from which a straight line can be drawn to the centre of the world perpendicular to the line of the apsides. Ptolemy, Book 3, chap. 3. Regiomontanus in the *Epitome*, Book 3, Prop. 7. Copernicus, Book 3, chap. 15. Nonius on Peuerbach's Theorics, Annotation 1. Reinhold, in the same place, p. 43. Magini in the Theory of the Sun, chap. 9. Tycho, vol. 1, p. 29. On the eccentric ABPC let there be the line of the apsides AP, on which are the centre of the eccentric E and of the world T, through which let the straight line BC be drawn at right angles, and

let the points B and C be joined with the centre E by the straight lines BE and CE. For by Euclid III.3 the straight line ET will bisect BC at T.

Diagram. Woodcut at the head of the right column, with the opening of the text running down its left side. A circle with a vertical diameter from A at the top to P at the foot, and on it the points E and T, E above T. Straight lines converge on C at the right from K, B and N on the left of the circle; further lines join K and N to E and T, and B to E. A dotted line runs from I, at the left side of the circle, to E, and a second dotted line runs down from I to B.

- A (top)
- K (upper left, on the circle)
- E (on the vertical, above T)
- T (on the vertical)
- C (right, on the circle)
- I (left, at the end of the dotted line, just inside the circle)
- [Z?] (left, outside the circle, below the preceding: a letter whose strokes most resemble Z)
- B (left, on the circle)
- N (lower left, on the circle)
- P (foot)

I say, then, that the angles of the equations EBT and ECT are the greatest of all those that stand on the eccentricity as base, ET, at the circumference of the eccentric. For on ET let there stand toward the apogee any angle at the circumference, such as EKT, and toward the perigee the angle ENT, and join their points K and N to the point C by the straight lines KC and NC. By the definition of a circle the lines EB, EK, EN and EC will be equal, and so the triangles EBC, EKC and ENC will be isosceles, and in them, by Euclid I.5, the angles at the base will be equal, namely EBC to ECB, EKC to ECK, and also ENC to ECN. But by Euclid III.7 the side TK is greater than the side TC; therefore by I.19 the angle KCT is greater than the angle CKT. So when the equals EKC and ECK are taken away, the remaining angle ECT is greater than the angle EKT. It is similar with all the angles between A and B standing on ET, and indeed also with all those between B and P: for they are always smaller than the angle ECT, to which EBT is equal. For by III.7 of the *Elements* TB, or TC which is equal to it, is greater than TN; therefore by I.18 the angle TNC is greater than the angle TCN. When these are taken away from the equals ENC and ECN, the remaining angle ECT, and therefore EBT, is greater than the angle ENT. And so for the others.

XIII. The closer the line of apparent motion is to the line of mean transit, or to the line making right angles at the centre of the world with the line of the apsides, the greater is the prosthaphaeresis. Regiomontanus, above; and it is a corollary of the preceding.

XIV. The point of greatest prosthaphaeresis is equally distant from the apogee and the perigee on the ecliptic, but on the eccentric it is more than 90 degrees from the apogee, by an excess above the quadrant as large as the

greatest equation. For if in the preceding figure the straight line EZ is drawn perpendicular to AP, it will be parallel to TB, by I.28 of the *Elements*. Therefore by I.29 of the same the angle ZEB, by which AEB exceeds a right angle, will be equal to the angle B, the greatest prosthaphaeresis.

SCHOLION.

Scholium.

[Margin: The solidity of the orbs rejected.]

I. Hipparchus and Ptolemy, in the theories of the planets, used bare orbs and mere circles. That neither they nor the ancients asserted solid orbs, but set out the circular path of the planets only under the hypothesis of a homocentrepicyclic circle or of an eccentric, is taught by Thomas Aquinas, Lect. 10 on *Metaphysics* 12; Piccolomini in his Theorics; Philaltæus [?] on *De caelo*, Book 2, text 51; Clavius at the end of the *Sphere*; and many others to be brought forward in the book on the System of the World. That the planetary heaven is fluid, however, many Fathers once taught, and it agrees with Holy Scripture, which nowhere attributes motion to these heavens, but to the planets alone [Solis Planetis]. Nearly all the more recent astronomers affirm it, Tycho above all, Longomontanus and Kepler: both from the trajectories of comets, and from the spirals of Mars, Mercury and Venus and their motion about the Sun, so that Mars too descends into the region of the Sun and in its oppositions with the Sun becomes closer to the Earth than the Sun is; and because that solidity would produce remarkable refractions of the lights, and would serve the motion only with great violence; and finally because so tangled and so vast a farrago of orbs inserted one into another is superfluous.

[Margin: A specimen of solid orbs.]

II. Yet, lest you be wholly ignorant of how Sacrobosco and Peuerbach, who are reckoned the chief asserters of the solidity of the orbs, explain the motion of the Sun, let the orb of the Sun in the adjoined figure be SBTK, concentric as to its outermost and convex surface with the centre of the world C, but hollowed out within, so that the concave surface MPDO is eccentric to the world and described from the centre E. And let there be a second partial orb OMPD, carrying the Sun and consisting of a white wheel, which matches the body of the Sun in its thickness and marks out the Sun's path, and of a black lunate part;

Diagram. Woodcut set in the left column: the solid-orb specimen. Concentric black outer ring, thickest at the bottom; inside it a white ring carrying a small circle with a central dot (the Sun); inside that a black ring, thickest at the top, enclosing a white disc that holds two dotted centres.

- S (top, above the outer ring)
- B (far left, outside the outer ring)
- K (far right, outside the outer ring)
- T (bottom, below the outer ring)

- M (left, in the white ring)
- D (right, in the white ring)
- P (bottom, in the white ring)
- E (upper centre point, in the central white disc)
- C (lower centre point, in the central white disc)

eccentric to the world as to its convexity OMPD, but concentric with the world as to its convex surface, which contains the little white circle. This orb is to carry the Sun uniformly about the centre E, as we said above of the eccentric, striving toward the east, while meanwhile the outermost orb SBTK revolves with the diurnal motion of 24 hours toward the west, and by that same motion sweeps the inner orb along with it toward its setting. Moreover, the entirely white circle, in which the centres C and E lie, represents the elementary sphere together with the heaven of the Moon, of Venus and of Mercury. They therefore devise similar things for the orbs of the other planets. But if the motions of the apogee, of the eccentricity and of the obliquity, and their anomalies, are real, how many segments of these orbs will have to be devised?

CAPVT XXI.

Chapter 21.

De Aequationibus seu Prosthaphæresibus motus Solis inuestigandis, & in tabulas distribuendis.

On investigating the equations, or prosthaphaereses, of the Sun's motion, and on arranging them in tables.

Although, when the opportunity offered, I have already suggested this in chapter 19, both through the epicycle at numbers 3 and 4 and through the eccentric at number 6, and in chapter 20 I gave the definitions or notions of the terms relating to it, from number 15 to 19, and the rules from number 8 to the end, I should nevertheless like to gather into a small compass, and distinctly, the precepts necessary for it, referring the reader for the demonstrations to the diagrams and expositions of the chapters already mentioned. First, then:

[Margin: A short method of finding the greatest equation.]

I. *The greatest equation* is investigated by the rules for right-angled rectilinear triangles if, as I assume, there is given the semidiameter of the concentric or of the eccentric, that is, the radius of the Sun's annual orb, and either the semidiameter of the epicycle or the eccentricity, in parts of which the said radius is known. For the angle opposite the eccentricity is the greatest equation.

[Margin: By sines.]

Therefore make: as the semidiameter of the Sun's orb is to the radius, or whole sine, of the tables, so the semidiameter of the epicycle, or the eccentricity, is to another quantity; and you will have the sine of the angle of the greatest equation.

[Margin: The same by logarithms.]

Or join the residual of the logarithm of the semidiameter of the Sun's orb to the logarithm of the semidiameter of the epicycle, or of the eccentricity; and, with unity taken away on the left, the result will be the logarithm of the angle of the greatest equation. So if the eccentricity is 3460 parts of which the radius of the Sun's orb is 100000, the greatest equation will be 1° 59′.

[Margin: The place of the greatest equation.]

II. The arc of anomaly at whose extremity, or distance from the apogee, the greatest equation occurs is found if the greatest equation is added to 90°, by the 14th rule demonstrated in the preceding chapter. So in the preceding example the arc of anomaly to which the greatest equation corresponds is 91° 59′ in the first semicircle, and 268° 1′ in the second. Through forgetting this precept not a few practitioners place the greatest equations short of or beyond the already mentioned limit of the anomaly, as can be seen in their tables.

[Margin: Method for the other equations.]

III. *The other equations* are investigated in an oblique-angled rectilinear triangle, in which there are assumed as given the eccentricity, or the semidiameter of the epicycle, as one side, the radius of the Sun's orb as the other side, and the complement of the anomaly, or of the distance from the apogee, to two right angles. For the angle opposite the eccentricity, or the semidiameter of the epicycle, is the equation itself or its measure, belonging to the degree of the given anomaly.

[Margin: By sines.]

Therefore make: as the sum of the given sides is to their difference, so the tangent of the half-sum of the angles adjacent to the base is to the tangent of the difference, which, subtracted from the half-sum of the angles at the base, will leave the angle of the desired equation.

[Margin: By logarithms.]

Or add the residual of the logarithm of the sum of the given sides to the logarithm of their difference and to the mesologarithm of the half-sum of the angles at the base; and, with unity or two taken away on the left, the result will be the mesologarithm of the angle which, subtracted from the half-sum of the angles at the base, leaves the equation sought. But know that here the sum of the angles at the base equals the arc of anomaly: so when this is known, that sum is known.

EXEMPLVM.

Example.

[Margin: Example of an equation that is not the greatest.]

Let the equation belonging to an anomaly of 46° be sought. Its complement to two right angles will be 134°. Hence the sum of the angles at the base, which is required for two right angles by proposition 32 of Euclid's first book, will be 46°, and the half-sum 23°, whose tangent is 42447. Let the radius of the

Sun's orb be, as above, 100000 parts, and the eccentricity 3460 such parts. For their sum will be 103460, and their difference 96540. Multiply this difference, then, by the already mentioned tangent, and the total will be 4097833580 [recte: 4097833380]. This, divided by the sum 103460, gives the quotient 39607. To this, as a tangent, corresponds the arc of an angle of 21° 36′ 26″, which, subtracted from 23°, gives the equation of 1° 23′ 34″ belonging to an anomaly of 46°.

IV. When the equations for the first semicircle have been found by this method, those of the second semicircle of anomaly will be known: for at points of anomaly equally distant from the apogee the equations are equal, for example at 46° and 314° of anomaly, that is, at 1^{s} 16° and at 10^{s} 14° of anomaly, by rule 9 of chapter 20.

V. It remains, then, for us to give the method of investigating the eccentricity, which is always equal to the semidiameter of the Sun's epicycle, and the place of the Sun's apogee, so that, when the Sun's mean motion or place is subtracted from the motion or place of the apogee, the anomaly, or the Sun's annual argument, may be known. This will be done in chapter 24. Once these have been declared, we shall give the greatest equations corresponding to the eccentricity, according to the more notable opinions. But first we must explain the distinction between the physical and the optical equation [Aequationem Physicam & Opticam]. For so far we have dealt with the whole equation, resting, as on a base, on the whole eccentricity.

CAPVT XXII.

Chapter 22.

Quid sit circulus Aequans, quid Aequatio Physica, & Optica, & quomodo seorsim inuestigentur.

What the equant circle is, what the physical and the optical equation are, and how they are investigated separately.

I. Ptolemy had completed the theory of the Sun by a simple eccentric, but in the five minor planets he used a double eccentric. One of these carries the planet, or the planet's epicycle; the other, which he calls the equant [Aequantem], is the one in which the measure of the uniform or mean motion is observed, that is, the one with respect to which the planet moves uniformly. This equant, however, was brought into the theory of the Sun by Kepler, and Magini, Boulliau and Wendelen followed him.

[Margin: Description of the equant.]

So that the notion and properties of the equant may be made clear, let T, in the adjoined diagram, be the centre of the Earth and of the world, from which describe the ecliptic HRV, etc., and, having drawn through T the line of apsides HV, take on it a straight line CT as great as the whole eccentricity,

given or found by the methods to be set out below in chapter 24, and bisect it at E; then,

Diagram. Geometrical figure inset in the left column. A large circle (the ecliptic), with a vertical diameter from H at the top to V at the bottom; two smaller overlapping eccentric circles inside it, a small epicycle on the upper left of the inner circles, and straight lines radiating from the centres on the diameter through the epicycle; two dotted horizontal lines from the diameter to the left.

- H (top of the large circle)
- G (top, left of H, on the large circle)
- L, I (upper left, side by side by the epicycle, on the line ends; set close together)
- D, K (inside the small epicycle)
- B, A (on the diameter, below H, in that order)
- N, O (left, at the end of the upper dotted line)
- C (on the diameter, at the upper dotted line)
- F (right, at the crossing of the two eccentric circles)
- S (left, at the end of the lower dotted line)
- E (on the diameter, at the lower dotted line)
- T, M, P (on the diameter, below E, in that order)
- R (left, on the large circle)
- V (foot of the diameter)

from centre E describe the eccentric AOPF, whose apogee is A, its perigee P, and its semidiameter EA; and from centre C, with the interval of the semidiameter CB, equal to EA itself, describe as the equant another eccentric BOMF, whose apogee is B and perigee M; and from these, at the centres C and E, draw two perpendiculars to the line of apsides, namely NC and SE. In this way there will be the quadrant AS in the first eccentric and the quadrant BN in the equant. Now imagine the centre of the Sun's globe at D, or, in the five minor planets, the epicycle LK that carries them. And through the centre D, from the three centres C, E, T, draw the straight lines TDG as far as the ecliptic, and EI and CL as far as the circumference of the equant.

When this is done, the circle in which are to be counted the equal arcs that the Sun is imagined to traverse in equal times by its mean motion is not the eccentric AOPF, in which the Sun's body nonetheless lies, but another eccentric, BOMF; so that the Sun moves uniformly with respect to a circle and centre not its own, and hence non-uniformly in its own circle and with respect to the centre of the circle in which the Sun itself lies. For example, if in 94 days the Sun by its mean motion completes a quadrant reckoned from the apogee, and hence in 47 days a half-quadrant, this quadrant will be BN, determined by the line CN, and not AS, made by the line ES. So when the Sun is on the line CN, it has not yet completed the quadrant AS of its own circle, but is at O; and to obtain the place where it stands when the half-quadrant has been completed, one must draw, as I said, a line from the centre C of the equant through the centre D of the Sun's globe, so that it cuts off from the

equant the half-quadrant LB, namely the line CDL. When this has been done, if a line EDI is drawn from the centre E of the first eccentric through the Sun's centre, the arc DA will be less than a half-quadrant: because, by proposition 32 of Euclid's first book, the angle DEA, which measures the arc DA, is less than the angle LCB, which measures the half-quadrant LB. Since, then, the circle BOMF is the measure of the Sun's uniform motions, although it is not the subject or immediate instrument of the Sun, it is called the *equant*.

[Margin: What the physical equation is.]

II. This being laid down, the cause for which the Sun's motion partakes of inequality is twofold. One is physical and real, and independent of the eyes of observers: namely that in the eccentric in which the Sun's body lies it moves non-uniformly according to this hypothesis, even though with respect to a circle not its own, or external to it, it moves uniformly; and the difference of these motions is hence called the *physical*, or real, *equation*, that is, one proceeding from a physical cause, whose measure in our example is the angle EDC, standing on half the eccentricity, EC, as on a base. The other cause is apparent, arising from the fact that we are not at the centre E, or C, but at T; and so the Sun, placed at D, is seen under G, a point of the ecliptic less distant from the place H of the apogee, that is, at a smaller arc GH than the arc DA, measured by the angle DEA, and smaller by far still than the arc LB, measured by the angle LCB, by proposition 32 of Euclid's first book.

[Margin: What the optical equation is.]

This difference, which increases the former inequality, is in our case the angle TDE, resting as on a base on the other half of the eccentricity, ET, and it is called the *optical equation*, because it arises not from the Sun's own circles in which it lies, or to which it has regard of itself in its motions, but incidentally from the position of our eyes, as we observe the Sun from T. In this way Ptolemy, Book 3, Chapter 3, said that the motions of the Sun [Greek: ἀνίσους φαίνεσθαι ταῖς ὄψεσιν ἡμῶν (anisous phainesthai tais opsesin hēmōn, "appear unequal to our sight")], that is, appear unequal to our eyes. The equation composed of the physical and the optical, moreover, is that whole equation which we taught in the preceding chapter to gather from the whole eccentricity, namely the angle TDC.

[Margin: Method of investigating the physical and the optical equation.]

III. These equations are investigated separately as follows. The physical one, namely the angle EDC, is investigated in the same triangle EDC, given EC, half of the whole eccentricity, and ED, the radius of the Sun's orb, and the angle DCE, namely the complement to 180° of the angle LCB, which measures the given anomaly. When the physical equation has been found, and the two angles DCE and CDE are thus already known, the third, DEC, will be known by proposition 32 of Euclid's first book, as being the complement of two right angles. Hence the angle TED will also be known, which is the complement to two right angles of the angle CED, by proposition 13 of Euclid's first book. So in the triangle DET, with the angle already mentioned, and with half the

eccentricity, ET, and the radius ED of the Sun's orb, the optical equation TDE can be found. Added together, these give the composite equation TDC.

[Margin: Who introduced the equant into the theory of the Sun, and why.]

IV. So the equant circle, or an ellipse equivalent to it, was devised for the theory of the Sun, as I said, by Kepler: in the *Commentary on Mars*, chapters 4, 23 and 31; in the *Astronomia Optica*, chapter 10; in the *Epitome of Astronomy*, Book 4; and in the Appendix to Tycho's *Progymnasmata*, after volume 1. For that reason he insisted that the Sun's eccentricity must be bisected. Kepler was followed by Magini in the *Supplement to the Ephemerides*, who gives a table of the Sun's equations gathered from a halved eccentricity[17] and composed of the physical and the optical equation; likewise by Wendelen in the Atlantic Tables, by Boulliau in Book 2, chapters 1 and 4, and by Cavalieri in the *Directorium Uranometricum*, chapter 4, pages 140 and 149. Finally, as is related in the Appendix just mentioned, Tycho himself had thought of introducing the equant into the theory of the Sun, but had not yet fully decided on it.

[Margin: Arguments for the Sun's equant.]

Kepler's foundation was twofold. First, he detected a difference of only one minute between the Sun's apparent diameter at apogee, 30′, and at perigee, 31′. Second, the approach of Venus and of Mars, but above all of Mars, to the Earth in opposition with the Sun at perigee. He considers that neither of these phenomena is saved if the whole eccentricity is employed: for the difference of the Sun's diameters would be about twice as great, and Mars in such an opposition with the Sun would not approach the Earth so closely, nor appear half as large again as near conjunction, as it does appear.

[Margin: Arguments against the Sun's equant.]

V. Against the equant, by contrast, stand the objections that Copernicus makes in general, Book 5, chapters 2 and 4, thinking it absurd to posit in the heavens a real inequality, or an equality sought from an external circle and a centre not its own. In the theory of the Sun it is also rejected by Longomontanus, who used the whole eccentricity; by Lansbergen in Class 4 of the Lunar Observations, at the end, who denies that the eccentricity should be bisected and asserts a difference of 2′ 24″ between the diameters of the Sun at apogee and at perigee; and by Antonín Maria Šírek, Book 1, chapter 2: if Mars is stripped of its mane of adventitious rays and observed with the telescope, it does not appear so much larger as to force us to bisect the Sun's eccentricity; or at any rate the eccentric of Mars will have its centre outside the centre of the Sun's body.

SCHOLIA.

Scholia.

I. Kepler, to distinguish the physical and optical equations, once judged it sufficient that the whole equation, drawn out in the usual way, be cut in

[17] the print reads *bissexta* for *bissecta*, halved

two, and that one could not err in this way by more than 6″ at the 45th degree of anomaly; and therefore he had corrected the difference, which in fact is 1′ 6″ there when the Tychonic eccentricity is assumed, by striking out a whole minute.

[Margin: Kepler's error.]

But Magini, in the letters to Kepler found in the *Supplement to the Ephemerides* from page 255, shows that at 45° and 135° of anomaly the difference of the equation so obtained from the equation obtained separately from the physical and the optical can reach 1′ 6″. And so lene, and that the axis of the cone is not perpendicular to the planes of the circles. Boulliau demonstrates this further in the same place, adding the way of cutting a scalene cone so that an ellipse arises having the properties already mentioned.

[Margin: This cone cannot be right.]

[Margin: Equality and inequality shown by means of the ellipse.]

VI. Now, to show in the cone and ellipse already mentioned both the equality and the whole apparent first inequality, in which the Sun too, or the Earth, shares, let the following figure be taken from Boulliau, Book 1, chapter 15. In it ABC is the scalene cone, AI its axis, and BIC its base. Parallel to this base are to be understood the planes of the circles which the planet looks to in its revolution around the surface of the cone: such are ESD and POK, with the others in between; and let their centres be on the axis of the cone, AI.

Diagram. Woodcut in the left column: a triangle standing for the scalene cone, apex at the top and a hatched, flattened oval for its base. A tilted ellipse lies across the cone and bulges beyond its right side; five parallel horizontal chords run from the left side of the cone to the right, crossed by the axis from the apex to the base and by the two diameters of the ellipse, which meet at its centre.

- A (apex): vertex of the cone
- B, C (base, left and right ends): the ends of the base
- I (centre of the base; small and partly within the hatching): foot of the axis
- E, F, V, L, P (left side of the cone, top to bottom): left ends of the chords
- N (upper right, outside the cone): end of the minor diameter on the ellipse
- D (just right of M, inside the circle); G, T, Q, K (right side, top to bottom): right ends of the chords; K is the lower end of the major diameter
- S (upper chord): on the circle ESD
- M (next chord, on the axis): umbilicus on the axis
- Z (middle chord): centre of the circle VZT
- X (centre of the figure): centre of the ellipse
- Y (fourth chord, left of the axis): point on the ellipse and the circle FMG
- H (lower right inside the ellipse): the other umbilicus
- O (lower chord region, on the ellipse): lower end of the minor diameter

- R (below the ellipse, on the lowest arc): label only; not cited in the text on this page

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q, R, S, T, V, X, Y, Z. Labels not read: none.

Between the extremes of the circles let there be the ellipse EOKN, whose transverse or major diameter, or axis, is EXK, while its conjugate or minor diameter, or latus rectum, is OXN. Let one of the umbilici, which *Apollonius* calls the *points from the comparison* [puncta ex comparatione], be M, on the axis of the cone itself, and the other, H, off the axis of the cone; and let the centre of the ellipse be X. Further, let the circle VZT, which passes through the centre of the ellipse, have its centre at Z and its diameter VT, so that VZ and ZT are equal. Therefore the plane passing through AI, the axis of the cone, will likewise cut in two all the diameters of the circles, ED, FG, VT, LQ, PK. But since the centre of the ellipse, X, is distant from the axis AI by the quantity MX, the plane that will bisect the ellipse through the conjugate diameter OXN will not bisect the circles, but will cut off a greater part, say VX, towards E, and a lesser, XT, towards K.

Now let the planet move around the axis of the cone uniformly, describing on the surface of the cone an elliptical line, and in equal times equal angles around the axis AI; and let the terminus of the apparent motion, to which it is referred, be the umbilicus H, in which the Sun stands. For the part most remote from the Sun, or aphelion, will be E, and the nearest part, or perihelion, K. And while the planet (for example, the Earth) completes by its mean motion a quadrant of its revolution around the cone, it is still distant from the quadrant of the ellipse by the quantity of the straight line MX, and is at the point Y, which lies both on the ellipse and on the circle FMG. The straight line MY, drawn from the centre of the circle FG, is an ordinate at right angles upon the diameter both of this circle and of the ellipse; and so the angle FMY is right. Therefore the planet has completed a quadrant of mean motion around the axis of the cone when it reaches Y, but on the ellipse it has traversed less than a quadrant, and to complete the quadrant of the ellipse it lacks the portion YO, or MX. Hence a twofold inequality arises, although the ancients reckoned it single: one from a physical cause, namely from the distance of the centre of the ellipse from the axis of the cone and from the inequality of the circles; the other from an optical cause, from the distance of the centre of the ellipse from the Sun, H. And both together show that the difference of the mean motion from the apparent and the distance of the umbilici of the ellipse are the same. I do not say, however, that the planet describes those circles, but only that it takes, with respect to them, the measure of its uniform motion along the elliptical line.

[Margin: Agreement of the elliptical hypothesis with the phenomena.]

VII. With these things posited the phenomena are saved: there is equality in the motion around the cone but apparent inequality on the ellipse, and half of the inequality is attributed to the eccentricity HX, the other half to that

cause which holds the planet back at aphelion and advances it at perihelion, without the equality being disturbed meanwhile. For since the umbilicus M, around which the motion is uniform, is on the axis of the cone, and the planet, descending from aphelion E, has completed a quadrant of the circumference of the cone and has come to the line MY, it is distant from the quadrant of the ellipse by the difference between the ellipse and the circumference of the circles, which MX measures. Next, since from aphelion to perihelion the circles FG, VT, etc. continually grow, and meanwhile the planet makes equal angles in them at the axis of the cone, the motion from aphelion to perihelion on the ellipse also grows, because equal angles formed by smaller lines subtend smaller circles, and those formed by greater lines, greater ones. Towards the apex of the cone, A, and aphelion the circles are smaller than towards the base and perihelion; therefore the motion on the ellipse is slower around aphelion and faster at perihelion. Yet the equality is not disturbed, because in an equal time the planet completes the other quadrant around the cone, namely the angle YMK, equal to the former, while meanwhile on the ellipse it completes more than a quadrant, namely the circumference YOK. Nor meanwhile has the planet been carried elsewhere: it proceeds along the one surface of the cone, always looking, as the measure of its equality, to the centre of its own circle, the circle it holds on the surface of the cone, that centre being placed on the axis of the cone; and not to another's centre, as in the Ptolemaic hypothesis of the equant circle. Nor are motions multiplied, as in other hypotheses, in order to keep that equality intact. And [?] it is clear for what reason the mean motion from aphelion to the mean longitudes is greater than the apparent, and from there to perihelion is less; nor does this change arise from an intensification or remission of the moving power, but from the wider or narrower circuit of the circles which it must run through. Finally, in this hypothesis no motion contrary to the mean motion is supposed, as in the Copernican and Tychonic theory of the five planets, or in the solar theory of *Longomontanus*. Nor is there any need of the fibres invented by *Kepler*, by which the Sun magnetically attracts the planets and, having seized them, drives them back again towards aphelion. *Boulliau*, Book 1, chapter 16, demonstrates that the difference of the semidiameters of the least and the greatest circle in the cone is equal to the distance of the umbilicus from the centre of the ellipse. In this ellipse, however, and in Kepler's, it is conceded from first to last that part of the inequality of motion is physical and real, such as it would be even if we were at the centre of the ellipse; whereas if we were at the centre of the eccentric circle, the whole inequality would cease. Therefore the physical equality of the Sun's motion is better saved by the hypothesis of the eccentric circle than by the ellipse. Let us now come to the practice of the equations, to be investigated by this hypothesis.

Prima Methodus Inquirendi Aequationes in Ellipsis Hypothesi.

First method of finding the equations in the hypothesis of the ellipse.

[Margin: First method of the equations, from Boulliau.]

VIII. Let there be a circle KGDP, as large as the annual orb or eccentric of the Sun usually is for others, in which KD is the line of apsides; and in it let there be described, by the usual rules, the ellipse KEDC, whose transverse or major diameter, or axis, is KD, while its conjugate or minor diameter, or latus rectum, is EC, cutting KD at right angles at B, the centre of the circle already mentioned and of the ellipse. And let one of the points from the comparison, as *Apollonius* says, or one of the foci, as *Kepler* says, or of the umbilici, as others say, be A, at the centre of the world, in which the Sun stands, distant from the centre B by as much as is the planet's bisected eccentricity in the other hypothe-

Diagram. Woodcut in the right column, standing within paragraph VIII: a circle with a vertical diameter from K at the top to D at the bottom and a horizontal diameter from G to P through the centre B; an ellipse on the vertical diameter, narrower than the circle, passing through E and C on the horizontal. Horizontal chords stand near the top (N to M; H to L) and near the bottom (through A; through R to Q). Straight lines run from I through B towards V, from I to A, and from A and R to V.

- K (top), D (bottom): ends of the vertical diameter
- G (left), P (right): ends of the horizontal diameter
- B (centre): centre of the circle and of the ellipse
- E, C (left and right on the horizontal diameter): ends of the conjugate diameter of the ellipse
- N, M (upper chord; N on the circle, M on the vertical diameter)
- O (just below N's chord, on the ellipse)
- H, L (second chord; H on the left of the circle, L on the right)
- I (on the second chord, at the ellipse)
- F (on the vertical diameter, below M)
- A (on the vertical diameter, below B): the focus or umbilicus, centre of the world
- S, T (on the chord through A, left and right)
- V (below T, on the ellipse)
- R (on the vertical diameter, near the foot)
- Q (lower right, on the circle)

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q, R, S, T, V. Labels not read: none.

ses. The other focus, or umbilicus, is to be F, distant from B by just as much as A is from that same B, so that AF is the whole eccentricity and the base of the first inequality, while BA is half of it, although BA is the whole eccentricity of the centre of the ellipse from the centre of the world. And let the aphelion

be at K and the perihelion at D. Now let the straight lines HF and ST be drawn through the umbilici A and F, perpendicular to KD. For SAT will be what *Kepler* calls διήλιος [Greek: διήλιος (diēlios, "through the Sun")], just as EBC is called διάκεντρος [Greek: διάκεντρος (diakentros, "through the centre")]. And let the planet be at I, from which two straight lines, IB and IA, are drawn, one to the centre of the ellipse, the other to the Sun, A. For the angle BIF will be the physical equation, and AIB the optical equation. The former will be found by the rules of triangles in the triangle IFB, right-angled at F, if the sides about the right angle, namely IF and FB, are known; the latter in the oblique-angled triangle IAB, if its three sides are known. You will find these sides as follows. *Boulliau* demonstrated, in Book 1 of the *Astronomia*, chapter 18, that as GB, the semidiameter of the circle and also the major semidiameter of the ellipse, is to EB, the conjugate semidiameter of the ellipse, so is any ordinate in the circle to any ordinate in the ellipse, if they terminate at the same point of the axis: for example, NM to OM, QR to VR, and HF to IF. Therefore if, as is supposed, the conjugate or minor semidiameter of the ellipse, EB, is given, and the arc of anomaly KH, or of the distance from aphelion, and so its sine HF, make the sine HF to the ordinate IF as the whole sine GB is to the semidiameter EB, and thus IF will be known. And if the planet's bisected eccentricity AB is given, FB, equal to it, will now be given; so the two sides about the right angle F will be known, for investigating the physical equation. If the squares of these sides are added together, the result will be a square whose square root will be the line IB, by Euclid I.47. Now let FB and BA be added together, and let the square of the whole FA be added to the square of the side IF: for the sum will be a square whose square root will be the line IA. Thus, together with the eccentricity AB, the three sides of the triangle IAB will be known, for the optical equation. The whole angle AIF will be the composite equation, which in the first semicircle KGD, or KED, is to be subtracted from the mean anomaly, so as to obtain the coequated anomaly and the true distance of the planet I from the aphelion K.

In the same way, in the other semicircle DPK, the additive equation will be investigated. Let the planet be at V; let the ordinate QR be drawn perpendicular to KD, and let V be joined by the straight lines VB and VA to the centre of the ellipse B and to the centre of the world A. Now from the given conjugate semidiameter of the ellipse, EB or CB, and from the arc of anomaly KGDQ, with the semicircle discarded and DQ thus given, and so its sine QR, if you make the sine QR to another as the whole sine PB is to the semidiameter CB, the ordinate VR will be known. And when the eccentricity BA has been taken from the major semidiameter BD, AD will be known; if RD, the versed sine of the arc DQ, is subtracted from it, RA will be known. So from the two sides about the right angle at R, which are VR and RA, you will find by trigonometry the optical equation RVA; and from their squares you will find a square whose square root will be the line A[?]. Now join the eccentricity AB to RA; and add the square of the whole [?] to the square of RV: for the sum [?] a square

whose square root will be the line VB. And so all the sides of the triangle AVB will become known, for investigating the physical equation, namely the angle AVB, which, joined to the angle AVR, will make up the whole equation to be added to the mean anomaly, so as to obtain the equated anomaly and the true distance of the planet V from aphelion. And so for the rest.

[Margin: The foundation of this method is demonstrated.]

IX. It is not difficult to demonstrate the proportion already mentioned, on which that practice rests: because both in the circle and in the ellipse GB and EB are ordinates to the same point of the axis, and also HF and IF. Therefore, as the rectangle DBK is to the square of GB, so is the rectangle DFK to the square of FH; for each rectangle is equal to its square. But also in the ellipse, by *Apollonius*, as the same rectangle DBK is to the square of the conjugate semidiameter EB, so is the rectangle DFK to the square of the ordinate IF. Therefore, *ex aequali*, as the square of GB is to the square of HF, so is the square of EB to the square of IF. Therefore, by *Euclid* VI.22, as the line GB is to the line HF, so is EB to IF; and by alternation, as GB to EB, so HF to IF; and by conversion, as GB to the excess GE, so HF to the excess HI; finally, by alternation and division, as GE to EB, so HI to IF.

[Margin: Correction of the preceding figure.]

Note further that this figure, taken from Boulliau as it stands, is ill-proportioned: for in the theory of the Sun the distance of the foci from the centre of the ellipse is not as great as is represented here, but far less, since according to Boulliau AB is to BD as 1784 to 100000.

Methodus Altera inueniēdi easdem Aequationes in Ellipsis Hypothesi.

Another method of finding the same equations in the hypothesis of the ellipse.

[Margin: Second method of the equations, from Boulliau.]

X. But since the method already mentioned is rather long and rather troublesome, *Boulliau*, Book 1, chapter 20, and Book 2, chapter 4, devised another, in which the calculation is completed by the usual solution of triangles, by resolving the ellipse into the motions of two circles. For suppose both semidiameters of the ellipse are given, and so their difference, and half of this difference is added to the lesser semidiameter of the ellipse; with the interval of this sum, as semidiameter, let a circle be described; on the circumference of this circle let there be the centre of an epicycle carrying the planet, the semidiameter of the epicycle being equal to half the difference between the two semidiameters of the ellipse given above; and let the motion of the epicycle, beginning from aphelion, be contrary to the motion of its centre and double it, so that while the circle completes a quadrant the epicycle completes a semicircle. If these things, I say, are done, the planet describes an ellipse; and so motion along the ellipse is equivalent to a motion compounded of two circular motions. And in chapter 21 he shows that the inequality of the equations is distributed through

the whole semicircle in proportion to the right sines standing on the line of aphelion and perihelion, counted from the beginning at aphelion. Accordingly, given three or four inequalities and differences of the mean motion from the true, along with the mean motion, the position of the diameter and the line of apsides will be given. *Boulliau* shows this in chapter 22, using two problems of *François Viète* in the Appendicula to the *Apollonius Gallus*. From these he gathers a lengthy method of finding the apogee and the eccentricity of the Sun and, in their own place, of the other planets.

[Margin: The semidiameters of the solar ellipse.]

To conceive the two circles into which *Boulliau* resolves the ellipse, inspect here the figure of the concentrepicycle, given in chapter 19, number 2; and in it let the major semidiameter of the ellipse be CA and the minor CF, and let half the difference, and so the semidiameter of the epicycle, be BF. *Boulliau*, Book 2, chapter 1, determines the minor semidiameter CF to the major CA as 99984 to 100000; so AF is 16 and AB 8 of such parts; and therefore this ellipse differs insensibly from the circle.

Diagram. Woodcut in the right column, between "mi-" and "nor": the figure of the concentrepicycle. A large circle with centre C, with a vertical line from the top epicycle to the bottom epicycle and a horizontal line from the left epicycle to the right. Four small epicycles are set at the top, left, right and bottom of the large circle, and a fifth at the upper left. Straight lines run from C to the upper-left epicycle (two lines, to its upper and lower parts), and from C to L and V at the left and right epicycles.

- A, B (top epicycle; A at its head, B within it, on the large circle)
- G, E (on the large circle, either side of the top epicycle)
- F (on the vertical line, just below the top epicycle)
- D, Z, K, I (upper-left epicycle; D above, Z outside to the left, K and I within it)
- L, M (left epicycle; L above it, M within it)
- V, N (right epicycle; V above it, N within it)
- C (centre of the large circle)
- P, R (bottom epicycle; P above it, R within it)
- Q (on the large circle, right of the bottom epicycle)

Labels read: A, B, C, D, E, F, G, I, K, L, M, N, P, Q, R, V, Z. Labels not read: none.

SCHOLIA.

Scholia.

[Margin: The elliptical hypothesis is not necessary for the Sun.]

I. Now by the admission of Kepler *and* Boulliau *the ellipse described by the motion of the Sun differs so little from a circle that the difference is scarcely sensible, and it cannot be demonstrated, as in the case of Mars, by calculations from observations, as is clear from what was said in the preceding chapter, numbers 3 and 4. On the other*

side, many observations of the apparent diameter, and the difference between them at apogee and perigee, urge that the Sun's eccentricity be kept whole. We therefore think it advisable in the theory of the Sun to use a simple eccentric and the customary method of the equation; especially since the whole equation from the elliptical form is never greater or less than the equation gathered by the simple eccentric by 69″. Seconds, and usually less; so that from the data given by Tycho, Magini, *by bisecting the eccentricity, and* Tycho, *by not bisecting it, gather the equations written below, at the ends and middles of the signs of anomaly.*

Deg.		Anomaly 0^s	1^s	2^s	3^s	4^s	5^s
15	Magini	0° 31′ 20″	1° 26′ 2″	1° 58′ 28″	1° 59′ 32″	1° 28′ 14″	0° 32′ 28″
	Tycho	0° 30′ 48″	1° 24′ 56″	1° 57′ 52″	2° 0′ 4″	1° 29′ 23″	0° 33′ 1″
30	Magini	1° 0′ 40″	1° 45′ 46″	2° 3′ 10″	1° 47′ 40″	1° 2′ 33″	0° 0′ 0″
	Tycho	0° 59′ 54″	1° 44′ 46″	2° 3′ 9″	1° 48′ 36″	1° 3′ 33″	0° 0′ 0″

[Margin: The ellipse in the hypothesis of a stationary Earth.]

Moreover, the Sun's parallax is not so certain that the difference between these equations can be proved from observations, as arising from a hypothesis determined in the heavens. Further, this hypothesis is fitted to the annual motion of the Earth, which we cannot admit. Therefore either this hypothesis is to be rejected altogether, or in the hypothesis of a stationary Earth the theory of the ellipse must be arranged differently: for the Earth is to be placed at the centre of the world, A (according to the figure given above at number 8), and the Sun is to be moved on the ellipse, so that it descends from the apogee K through E to the perigee D. In the other planets it will have to be arranged as will be indicated in the proper places.

[Margin: Among which authors is the practice of describing an ellipse found?]

II. The practice of describing an ellipse is taught by Eutocius *on Apollonius, Book 1, Prop. 21;* Commandino *in the Horologia following Ptolemy's Analemma;* Maurolico, *De lineis horariis, Book 3;* Guidubaldus de Monte [Guidobaldo del Monte] *in the Theory of the Planisphere;* Clavius, *Gnomonics, Book 1, at the end of Prop. 8;* Dauid Riualtus [David Rivault] *on Archimedes' book On Conoids, Problem 3;* Bartholomæus Souerus [Bartolomeo Sovero] *in his book On the Proportion of the Curved and the Straight, Advanced;* Franciscus Aguillonius [François d'Aguilon], *Optics, Book 6, Lemma 22;* Kepler *in the Astronomia Optica, p. 95;* Cavalieri, *De speculo ustorio, chapters 43 to 51;* Bettini, *Apiarium 3, Prog. 10, Prop. 13, and Apiarium 10, Prog. 2, Prop. 6;* Claudius Mydorgius [Claude Mydorge], *Prodromus Catoptricorum, Book 2;* Descartes, *Dioptrics, chapter 8, number 2; and* Boulliau, *Book 1, chapter 20.*

[Margin: Boulliau's equations.]

III. Boulliau, *Book 2, chapter 4, teaches how to investigate the equations in the form of the ellipse, from an epicycle of semidiameter 8 parts, of which the semidiameter of the ellipse is 100000, and from the semidiameter of the circle carrying the epicycle, which is 99992 parts, and from the motion of the epicycle in the contrary direction,*

double the motion of the circle already mentioned: first the simple equations, then the optical separately from the physical. From this he builds a table of equations of his own, which we shall perhaps give at the end of the work, so that its difference from the others may appear.

[Margin: The apparent inequality in the Sun's motion set out by means of a spiral line and the diurnal motion alone.]

IV. If, as al-Biṭrūjī, *and* Patrizi, *and other authors not to be despised hold, the Sun does not move towards the east, but by a single spiral motion towards the west, the diurnal one; and if it therefore seems to become more easterly each day because in its diurnal motion it is slower than the fixed stars and the primum mobile, and does not have the ecliptic as the measure or subject of its path; then there is no need to recognise inequality in its motion. For if it stays about 7 days longer on this side of the equator, it also makes more spirals, but less wide or less distant from one another than beyond the equator. And so it can be said to traverse equal spaces in equal times, so that the spiral line made on this side of the equator is to the whole line beyond the equator as the time of its stay on this side is to the time of its stay beyond. And the differences of declination can be saved by the distances of the partial and daily spirals; indeed, even the apogee and perigee states, by wider and narrower spirals. The cause for which the Sun stays longer among the northerners is that on this side of the equator the tract of the inhabited Earth is greater, and indeed inhabited by people of greater intellect, and where religion, the sciences, the liberal arts and civil culture were to flourish according to the decree of Divine Providence: for this reason I would also believe that there are more fixed stars in the northern tract of the sky than in the southern. But since from this hypothesis, just as it very well follows among other things that there are not two contrary motions in the Sun but one simple diurnal motion, so it is difficult to set out by geometrical figures a method of the equations, for this reason it has been neglected, and indeed condemned by* Kepler *in Mars, chapter 1, to the point of branding* Patrizi, *its restorer, with the mark of insanity. But men should be more sparing with such things; for it is not false for the reason that we cannot with our geometrical little machines bring its reasons to calculation by lines. But more on this elsewhere. For what if someone were to show that such is the line by which the Sun, describing it around the cone, emulates the Keplerian, or rather the Boullialdian, ellipse?*

CAPVT XXIV.

Chapter 24.

De Solis Apogeo, & Eccentricitate, aut Epicycli diametro inuestigandis vna eademq. opera.

On investigating the Sun's apogee and eccentricity, or the diameter of the epicycle, by one and the same procedure.

Up to this point the equations of the Sun's motion have presupposed both the eccentricity, as the base of the prosthaphaeretic angle, and the anomaly, or mean distance from the Sun's apogee, which cannot be known unless the place

in the ecliptic under which the Sun's apogee lies is known beforehand. We must therefore investigate these carefully; for once the eccentricity is found, the semidiameter of the epicycle will be known, because it is equal to the eccentricity; and also both semidiameters of the solar ellipse, because the excess of the greater over the lesser is equal to the doubled eccentricity.

Prima Methodus Inuestigandi Eccentricitatem, & Apogeum Solis.

First method of investigating the Sun's eccentricity and apogee.

I. This first method is taught by *Ptolemy*, *Almagest*, Book 3, chapter 4; *al-Battānī*, chapter 28; *Regiomontanus* in the *Epitome of the Almagest*, Book 3, Prop. 13; *Nonius* on the *Theorics* of Peuerbach, Annotation 1; *Reinhold* on the same, p. 41; *Copernicus*, Book 3, chapter 16; *Snellius* at the end of the *Hessian Observations*; *Longomontanus*, Book 1 of the *Theorics*, p. 176; *Biancani*, *Sphaera*, Book 10, chapter 9; and *Metius* in the *Theory of the Sun*, chapter 7. It rests on the observation of two equinoxes and the solstice between them.

Diagram. Woodcut in the right column: a large circle (the ecliptic) with horizontal diameter C to A and vertical diameter B to D, crossing at the centre E. A smaller circle stands inside it, displaced upward and to the right, touching near the upper right. A second vertical line runs through F, from the short chord near the top down to the smaller circle's foot; a second horizontal line runs through F from the smaller circle's left side to K. A straight line runs from E up through F to T and H; a short vertical joins Q and P at the right.

- B (top of the large circle), D (foot of the large circle)
- C (left), A (right, italic): ends of the horizontal diameter
- E (centre of the large circle, below the crossing)
- L (top, small, above G)
- R (on the vertical diameter, near the top), G (on the second vertical, near the top)
- H (upper right, on the large circle), T (just inside H, on the smaller circle)
- M (left, italic, on the smaller circle), S (on the vertical diameter, on the line M to K), F (at the crossing of the second vertical and the line M to K)
- Q, K (right; Q on the smaller circle, K on the large circle)
- Y (left, on the line C to A), V (on the second vertical, below the line C to A), P (right, below the line C to A)
- N (italic, foot of the smaller circle)

Labels read: A, B, C, D, E, F, G, H, K, L, M, N, P, Q, R, S, T, V, Y. Labels not read: none.

Let ABCD be the ecliptic, whose centre is E, in which the line of the equinoxes AC and the line of the solstices BD cut each other at right angles; let A be the point of the vernal equinox, or the beginning of Aries, C the point of the autumnal, and B the beginning of Cancer. And since in Ptolemy's time,

as is clear from the table of Chapter 18, the Sun stayed longer in the northern semicircle ABC than in the southern, and longer in the first quadrant of the ecliptic AEB than in the others, therefore the cen- tre of the Sun's eccentric is to be placed in this first quadrant, say at F. Let the eccentric GMNP be described from it, and let the apogee line EFH be drawn through the centres E and F; for the point of the apogee will be T on the eccentric but H on the ecliptic, and the eccentricity will be EF. Next, draw through the centre F two straight diameters of the eccentric: KM, parallel to CA, and LN, parallel to BD. In this way, by Proposition 29 of Euclid's Book 1, the angles at S and V will be right; by Proposition 34, SF will be equal to EV, and SE to FV, and the triangles ESF and EVF will be right-angled at S and V; and when the right sines RG and QP are drawn, by the same Proposition 34 RG will be equal to SF, and QP to SE. Therefore, if the sines PQ and RG are known from the known arcs PK and RL, SE and SF will also be known. The rest I shall teach by Ptolemy's example.

[Margin: Ptolemy's example.]

Ptolemy had observed that 178 [recte: 187, per the book's errata] days pass from the vernal equinox to the autumnal, and that to these days the Sun's mean motion according to Ptolemy, namely the arc PLY, corresponds at 184° 20′. And the straight line LV, by Proposition 3 of Euclid's Book 3, cuts the arc PLY in two at L; so the arc LP was 92° 10′. He had next observed that the Sun stays 94^{d} 12^{h} in the first quadrant of the ecliptic, to which the arc PR and a mean motion of 93° 9′ correspond; so the arc RY was 91° 11′. Therefore, when the arc PL was subtracted from the arc PR, there remained the arc LR, 59′, whose sine RG is 1716 of the parts of which the radius FL is 100000, and the side SF is just that much. Next, when the quadrant KL, of 90°, was subtracted from the arc PL, there remained the arc PK, 2° 10′, whose sine PQ, that is SE, is 3781 parts. And from the sides SF and SE already mentioned, in the triangle ESF, right-angled at S, the base EF, or the Ptolemaic eccentricity, is gathered trigonometrically: 4152 of the parts of which the eccentric's radius FT is 100000; or, as *Ptolemy* puts it, 2° 29′ 30″ of the parts of which FT is 60°; or, as *Hipparchus* had roughly found, one twenty-fourth of the radius. In the same triangle there is also found the angle SFE, to which by Proposition 29 of Book 1 the angle FEV is equal, of 65° 30′. This angle measures the arc AH, namely the distance of the apogee H from Aries A on the ecliptic. So *Ptolemy* concluded that the Sun's apogee is at 5° 30′ of Gemini, and that it does not move, because *Hipparchus* had found it in that same place 260 years earlier. [Margin: Al-Battānī's observation.] Yet by the same method *al-Battānī*, chapter 28, finds the Sun's apogee at 22° 17′ of Gemini and the eccentricity 3465 15/54, in the year 1194 of Dijlkarnaim[18], or of Christ 883. But *Nunes*, in the year 1552, from data given by *Pietro Pitati*, obtained an apogee at 1° 28′ of Cancer with an eccentricity of 3781 3/10 of the parts of which the eccentric's

[18]supplied identification — Dhū al-Qarnayn, the era of Alexander

radius is 100000; and he admits that this does not agree with the motion of the apogees [Augium] laid down by the Alphonsines.

Secunda Methodus Inuestigādi Eccentricitatem simul, & Apogeum Solis.

Second method of investigating the eccentricity and the Sun's apogee together.

[Margin: And of Pietro Pitati, with whom Nunes.]

II. The method of *Ptolemy* and of *al-Battānī* rests on the observation of the solstices, from which the interval between equinox and solstice is fixed, and that observation is exposed to an error of at least a whole day, as we showed in Chapter 14, numbers 3 and 4. For this reason *al-Zarqālī* the Spaniard used a more exact method, which, besides the observation of two equinoxes, [Margin: Al-Zarqālī's more exact method.] rests on a place of the Sun observed at about the middle parts of Taurus or of Leo, or at those times when the difference of the Sun's declinations is very perceptible. For that reason this method was chosen by *Regiomontanus*, Book 3 of the *Epitome of the Almagest*, Proposition 14; *Copernicus*, Book 3, Chapter 16; *Nunes* on Peuerbach's *Theorics*, Annotation 1, page 201; *Daniel Santbech*, *On Observation*, Proposition 9; *Tycho*, Volume 1 of the *Progymnasmata*, Chapter 1, from page 20; *Longomontanus*, Book 1 of the *Theorics*, Chapter 2; *Lansbergen* in the *Progymnasmata* on the Sun; *Snellius* and *Iustus Byrgius* [Jost Bürgi] in the notes to the Hessian observations; and *Metius* in the *Theoric of the Sun*, Chapter 8.

Diagram. Woodcut at the foot of the left column, with the text wrapped to its left: a circle for the eccentric, with the vertical diameter BD and the horizontal chord LA below the centre. O stands on BD above C. From T on the upper right a straight line runs down through C to P on the lower left; radii run from O to T and to A; a chord runs from P to A; a short perpendicular falls from the line LA to P, with R on LA above it.

- B (top): upper end of the diameter BD
- D (bottom): lower end of the diameter
- L (left end of the chord): under Libra
- A (right end of the chord): under Aries
- T (upper right, on the circle): the Sun's observed place
- P (lower left, on the circle): the point where the line from T through C meets the circle
- O (on BD, above the chord): centre of the eccentric
- C (on BD, at the chord): centre of the Earth and of the ecliptic
- R (on the chord, right of L): foot of the perpendicular from P
- I (small, right of the diameter, between O and C): foot of the perpendicular from O to TC

Labels read: A, B, C, D, I, L, O, P, R, T. Labels not read: none.

Let ABLD be the eccentric, described from the centre O, and let C be the centre of the Earth and of the ecliptic. Through these centres let the diameter

BD be drawn, and through C the line of the equinoxes AL, so that the point A of the eccentric lies under Aries and L under Libra. Let the portion ABL be the greater, corresponding to the time for which the Sun stays longer in the northern semicircle than in the southern; this arc will be known from the mean motion matching the time already mentioned. Let a place of the Sun between solstice and equinox be observed, say T at the middle of Taurus, and to it draw the eccentric's radius OT, and from O the radius OA. For from the time between the equinox A and the observed place T, the Sun's mean motion will be known, and the arc AT, and so the angle TOA. Next, from T through C draw the straight line TP; for CT will be the line of the Sun's true place, which is known from observation. So the angle TCA is known, which measures the Sun's true distance from Aries on the ecliptic. Now from O draw the straight line OI perpendicular to TC, so that the triangle OIC is right-angled at I. Finally, from the point P draw the straight line PA to A, and to AL the perpendicular PR.

When this has been done, the angle at the centre AOT, now known, stands on the arc TA, and the angle APT at the circumference is, by Proposition 20 of Euclid's Book 3, half the angle AOT; so it too is known. But TCA was also known; therefore, by Proposition 13 of Euclid's Book 1, PCA is known, as its complement to two right angles. So when the angles APT and PCA, taken together, are subtracted from the sum of two right angles, the angle PAC, that is PAL, will be known by Proposition 32 of Book 1. By Proposition 20 of Euclid's Book 3 the arc LP is double this angle; and when LP is subtracted from the arc LDA, the complement of the known arc ABL, the known arc PDA remains, whose chord PA is known from the tables of sines. And when the arcs PDA and AT are added together, the arc PAT will be known, whose chord PT will likewise be known from the tables of sines. Now the angle PCR is known, since by Proposition 15 of Euclid's Book 1 it is equal to the angle ACT, already known, and the angle PRC is right by construction. So the sine of the angle PCR will become known, and that sine is to the whole sine as PR to PC. Again, the sine of the known angle LAP is known, and the ratio of the chord PA to PR is as the whole sine to the sine of the angle LAP; therefore PR will become known in the parts in which the chord AP was given. And since PR is to PC as the sine of the angle PCR to the whole sine, and the sine of the angle PCR was known, PC too will become known in the same parts. When PC is subtracted from the chord PT, CT remains known; and when PT is cut in two at the point I by the perpendicular OI, the side CI will be known. Finally, if you imagine a parallel to the chord PT drawn through the eccentric's centre O, to the periphery on both sides, that chord will turn out to be the chord of a semicircle. So when the known arc PAT is subtracted from the semicircle, its complement to the semicircle will remain, and the right sine of half of that will be the right sine OI. Then, with the sides OI and CI and their squares known in the right-angled triangle OIC, by Proposition 47 the square will be known whose square root will be the base OC, namely the eccentricity, in the parts of

which the eccentric's radius OB was taken. And in the same triangle, by the rules of right-angled triangles, you will find the angle OCI at the centre of the ecliptic, which measures the distance of the Sun's apogee from the beginning of Aries.

This method seemed suspect to *Jābir ibn Aflaḥ* in Book 3 of his *Astronomy*, as *Nunes* also noted in the place cited above, because through the many combinations of numbers, often fractions, and the extraction of a square root, [Margin: Jābir's suspicion of this method.] a small error at each step can end up large by the close of the working. Yet for this reason *Snellius*, in the place cited above, calls Jābir [Greek: μισάριθμον (misarithmon, "number-hater")], that is, a hater of numbers and of calculation. In the end this method was used by *Copernicus*, though he took the Sun's place in the middle of Scorpio and did not correct it for refraction; and by *Tycho*, *Longomontanus*, *Iustus Byrgius* and *Snellius* in the places cited above, though they chose observed places of the Sun beyond the hazard of refractions.

Tertia, & Nostra Methodus Inuestigandi Eccentricitatem, & Apogeum Solis.

Third method, our own, of investigating the eccentricity and the Sun's apogee.

[Margin: Our method.]

III. Our method, then, is easier and not exposed to so many small errors; I make it clear with the following diagram and example. Let ABPD be the ecliptic, described from T, the centre of the Earth and of the world, with BD the line of the equinoxes and AP the line of the apogee, on which let the eccentricity be CT and let C be the centre of the eccentric KMVN. Let C be joined to the points of intersection M and N by the eccentric's radii CM and CN. For MCN will be an isosceles triangle, and by Proposition 5 of Euclid's Book 1 the angles at M and N will be equal. And let the apo- gee of the Sun be K, under the point A of the ecliptic, and the perigee V, under P. Let there be observed the moment of the vernal equinox, occurring at D, and of the autumnal equinox immediately before or after it, occurring at B; and, besides these, in the same year a third place of the Sun far from the solstice, say when it is in the sign of Taurus or Leo, if you are on this side of the equator, or in Scorpio or Aquarius, if you live beyond it. Let that place be the point L of the ecliptic, under which the Sun is at S. To L draw from the centre of the Earth the line of the true place TL, and to the Sun S, from the centre of the eccentric, the line of mean motion CS. When this has been done, proceed as in the example below.

Diagram. Woodcut at the head of the left column, with the opening lines wrapped to its left: a large circle for the ecliptic, and inside it a smaller circle for the eccentric, set off-centre towards the top. A horizontal line BD crosses the large circle and cuts the small one at M and N. A line runs from A at the top down through K and C to V and P at the bottom; another runs down from L

through S, crossing MC at I, to T on the horizontal line. C is joined to M and to N.

- L (top, left of A): on the large circle, the Sun's place on the ecliptic
- A (top): on the large circle, under which the apogee stands
- S (just inside the small circle, near the top): the Sun
- K (beside S): the apogee on the eccentric
- C (below K): centre of the eccentric
- I (small, on the crossing below and left of C): intersection
- B (left) and D (right): ends of the equinoctial line
- M and N (left and right, on the horizontal): the eccentric's intersections with that line
- T (on the horizontal, between M and N): centre of the Earth
- V (below T): the perigee on the eccentric
- P (bottom): on the large circle, under the perigee

Labels read: A, B, C, D, I, K, L, M, N, P, S, T, V. Labels not read: none.

EXEMPLVM. Methodi 3.

Example of Method 3.

In 1646 we observed the autumnal equinox at B, on September 22, at 14^{h} 56^{m} after noon at Bologna; and in 1647 the vernal equinox at D, on March 20, at 5^{h} 56^{m} after noon at the same place. So the Sun was in the southern signs, or on the arc BPD of the ecliptic, for 178^{d} 15^{h}, to which, from the table of mean motion given in Chapter 17, the arc MVN of the eccentric corresponds, of 176° 3′ 41″; and the angle MCN is just that much. Therefore, by Proposition 32 of Euclid's Book 1, the angles at the base MN are together 3° 56′ 19″, which is the complement to two right angles; taken singly, M is 1° 58′ 9½″; and the remainder to the whole circle, namely the arc MKN, is 183° 56′ 19″. Further, in 1646, with two great quadrants at Bologna, at noon on July 28, we observed the Sun S under L, at 5° 14′ of Leo. So the arc LD, that is the angle LTD, was 125° 14′, for that was the longitude from the beginning of Aries; and the remaining angle to the semicircle, that is LTB, was 54° 46′. The interval from the noon of this observation to the following equinox at B was 56^{d} 14^{h} 56^{m}, to which the mean motion corresponds, namely the arc SM and the angle SCM, of 55° 48′ 34″. And so the angle ICS is known, since it is the same as the angle SCM. Now add the angle M, 1° 58′ 10″, to the angle MTL, of 54° 46′, and the sum is 56° 44′ 10″. When this is subtracted from 180°, the angle MIT remains, 123° 15′ 50″, and the vertical angle SIC is just that much; the angle TIC will be 56° 44′ 10″, and the angle S, namely the equation, will be 55′ 36″. So we have, without tables of sines or chords, all the angles we need.

First, then, in the triangle CIS, the eccentric's radius CS is given, of 100000 parts, and the angle SIC, 123° 15′ 50″, and ICS, 55° 48′ 34″; so the third angle S, by Proposition 32 of Book 1, is 0° 55′ 36″. From this, by trigonometry, CI is found to be 1913½ parts. When these are subtracted from the radius CM, of

100000 parts, there remains MI, of 98086½ parts. With this side, and the angle M, 1° 58′ 10″, and the angle ITM, that is BTL, 54° 46′, IT is gathered to be 4084½ parts. Finally, with this side, and the side IC, and the angle TIC, 56° 44′ 10″, there is found TC, the eccentricity, of 3431 of the parts of which the radius CK is 100000, and the angle LTA, 27° 47′ 45″. Subtracted from the angle LTD, of 125° 14′, this leaves the angle ATD and the arc AD, 97° 26′ 15″, which is the distance of the place A of the Sun's apogee from the beginning of Aries, D. [Margin: The eccentricity found from this.] [Margin: Place of the apogee.] Therefore in 1646, on July 28, the Sun's apogee was at 7° 26′ 15″ of Cancer.

EXEMPLVM II.

Example 2.

[Margin: A second example.]

IV. Again, in the same year 1646, on August 4, the Sun's meridian distance from the vertex, taken at Bologna with the great quadrants, was 27° 14′ 50″; but with my parallax of 30″ subtracted it was 27° 14′ 20″, and the northern declination 17° 15′ 10″. From this, with the obliquity of the ecliptic set at 23° 30′, the Sun's place was 11° 56′ 40″ of Leo [♌], and the interval to the autumnal equinox $49^{d}\ 14^{h}\ 56^{m}$. So the true motion, that is the angle LTD, was 131° 56′ 40″, and the mean, that is the angle SCI, 48° 54′ 36″. Therefore the angle LTB, or ITM, was 48° 3′ 20″, and M was 1° 58′ 10″, as in the earlier example. So the angles MIT and also SIC are given, 129° 58′ 30″, and MIS and also CIT, 50° 1′ 30″. Hence the equation CSI is 1° 6′ 54″.

First, then, in the triangle SIC, from the side SC, or radius, 100000, and CSI, 1° 6′ 54″, and SIC, 129° 58′ 30″, the side IC is found trigonometrically to be 2577 of the parts of which CS or CM is 100000. So MI is 97423 parts. Next, in the triangle MIT, from the side MI, 97423, and the angles MTI, 48° 3′ 20″, and M, 1° 58′ 10″, the side TI is found to be 4527 parts. Third, in the triangle CIT, from the sides IC, 2577, and IT, 4527, and the angle CIT, 50° 1′ 30″, there is found [Margin: The eccentricity found from this.] CT, the eccentricity, of 3491 parts, and the angle ITC, 34° 27′ 36″. When this is subtracted from the angle LTD, which was 131° 56′ 40″, there remains the distance of the place of the apogee A from the beginning of Aries, D, that is the angle ATD, 97° 29′ 4″. [Margin: And the place of the apogee.] Therefore the Sun's apogee was at 7° 29′ 4″ of Cancer.

[Margin: Determination of the eccentricity and of the apogee.]

On account of these and other observations, from which I now refrain, I have settled, choosing a middle value, on an eccentricity of 3460 of the parts of which the eccentric's radius is 100000: about what *al-Battānī* and al-Zarqālī found by their 402 considerations, as Regiomontanus says, which was 3465. The apogee, however, I have kept at 7° 26¼′ of Cancer, or 7° 26′. And it is almost midway between the one that the Rudolphine Tables give this year, at 6° 30′ 23″ of Cancer, and Lansbergen's, at 7° 48′ 49″.

Quarta Methodus inuestigandi Eccentricitatem, & Apogeum Planetæ.

Fourth method of investigating the eccentricity and apogee of a planet.

[Margin: The method of François Viète and of Boulliau.]

Boulliau, in Book 1 of his *Astronomy*, Chapter 22, and Book 2, Chapter 1, proposes this method, from two problems of *Viète* demonstrated in the second little appendix of the *Apollonius Gallus*; it is suited, indeed, to the hypothesis of the ellipse, but serves others as well.

Diagram. Woodcut in the right column, with the text wrapped to its left: a circle with the horizontal diameter FG. From C on the upper left, D on the upper right and B on the right, normals fall to the diameter at K, H and I, the last two running on below it to N and M on the lower circle. A vertical from L at the top falls to the centre A. Chords run from C to B and from C and B to D, and E stands where DHN crosses CB.

- L (top): on the circle, above the centre
- D (upper right): on the circle, the second observed place
- C (left, on the circle): the first observed place
- B (right, on the circle): the third observed place
- E (inside, right of centre): the point on the chord CB
- F (far left) and G (far right): ends of the diameter, outside the circle's rim
- K (on the diameter, left): foot of the normal from C
- A (on the diameter, centre): centre of the circle
- H and I (on the diameter, right, set together): feet of the normals from D and B
- M (lower right, on the circle): lower end of the normal from B
- N (bottom right, on the circle): lower end of the normal from D

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N. Labels not read: none.

Let there be given three observed places of the planet, C, D, B, on the circle described from A. For from the interval of the observations the mean motions will be given, and so the arcs CD and DB; and then, from the tables of sines, the chords CD and DB; and of the compound arc CDB the chord CB will be given. And since, by Proposition 20 of Euclid's Book 3, the angle at the centre, which measures the arc of the circle, is double the one at the circumference, the angle CBD will be half the arc CD, and the angle DCB half the given arc DB. Now let the diameter FG be understood as drawn, so that, when the normals CK, DHN, BIM are dropped from the given points C, D, B, the segment KI of the diameter has to the segment HI the ratio which the chord CB has to EB, by Viète's problem; that is, from the difference of the mean motion from the apparent, let the part KI on the diameter FG be the difference of the mean motion from the apparent for the interval CDB, while for the interval DB let the difference of the mean from the apparent be HI. Let it be, then: as the sine of the half-difference of the mean and apparent motion corresponding to the interval CB, that is, as KI, is to the half-difference of the mean and apparent

motion corresponding to the interval DB, namely to HI, so is the chord CB to its portion EB. For in this way, in the triangle EBD, the sides EB and DB, the chord of the arc DB, will be given, and the angle EBD, which we said is half the arc CD. So by trigonometry the angle EDB will not stay hidden, and when doubled, by Proposition 20 of Euclid's Book 3, it will give the arc BGN. If the arc NM, which is equal to the arc DB, is subtracted from this, the arc BM will remain, half of which is BG. This arc, added to DB, the apparent motion, will give the arc GD, from which the complement LD to the quadrant will be known. So let the arc LD be subtracted from the planet's apparent place D, and known will be the apogee or aphelion L, whose line is AL. Further, half of the whole first inequality, that is, the distance of the umbilici from the centre of the ellipse, is the line AG, which is found in this way. For the arcs of apparent motion CL, the distance from the aphelion, and LB, find the right sines KA and AI, and add them into one sum for the sine KI, which you keep as the divisor. Then multiply by the whole sine the sine of the half-difference of the apparent and mean motion corresponding to the arc CDB, namely KI; divide the product by the divisor you have kept, and AG will come out. Doubling this, you will have the distance of the umbilici FG or, in the hypothesis of others, the whole eccentricity.

But since *Boulliau* admits that neither the mean nor the apparent motion is to be assigned to the arcs CB and DB, because in the circles the planet undergoes only half of the first inequality, he therefore says that the planet is to be considered as seen not from one of the umbilici but from the centre of the ellipse, and that the arithmetic mean between the mean and apparent motions is to be taken.

EXEMPLVM BVLLIALDI.

Example of Boulliau.

V. An example will perhaps make the practice clearer. In 1588 *Tycho* observed, from C, the middle of Taurus [♉], to D, the middle of Leo [♌], an elapsed time of $94^{d}\ 5^{h}\ 55^{m}$, to which a mean motion of 92° 53′ 34″ corresponds, but the apparent motion was 90°. And from D, the middle of Leo, to the beginning of Libra [♎], B, $46^{d}\ 9^{h}\ 40^{m}$, to which a mean motion of 45° 44′ 16″ but an apparent motion of 45° corresponds; and from the middle of Taurus to the beginning of Libra, $140^{d}\ 15^{h}\ 35^{m}$, whose mean motion is 138° 37′ 50″ but the true motion 135°, corresponding to the arc CB. The arithmetic mean between the apparent and true motion of the arc CD is 91° 26′ 47″, half of which is the angle CBD, 45° 43′ 23″. And the arithmetic mean of the arc DB is 45° 22′ 8″, half of which is the angle DCB, 22° 41′ 4″. So the angle CDB is 111° 35′ 37″. Finally, the arithmetic mean of the arc CB is 136° 48′ 55″. The chords corresponding to the arcs already mentioned are CD, 71597; DB, 38565; CB, 92983, of the parts of which the whole sine is 100000.

Next, the difference of the mean and apparent motion in the first interval CD is 2° 53′ 34″, half of which is 1° 26′ 47″, and its sine is 2524, corresponding to KH. The difference of those motions in the second interval DB is 44′ 16″, and half of it 22′ 8″, whose sine is 644, corresponding to HI. And finally the difference of the mean and apparent motion in the whole interval CDB is 3° 37′ 50″, half of which is 1° 48′ 55″, whose sine is 3168, corresponding to KI.

With these prepared, make: as KI, 3168, to CB, 92983, so HI, 644, to another; and EB will come out, of 18902 parts. With this side, and the side DB, 38565, and the angle DBC, 45° 43′ 23″, you will find by the analysis of triangles the angle EDB, 28° 4′ 42″, whose double is the arc BN, 56° 9′ 24″. If you take from this NM, that is, the equal of DB, 45° 22′ 8″, BM will be given, 10° 47′ 16″, and half of it, BG, 5° 23′ 38″. DB, 45° 22′ 8″, added to this, gives DG, 50° 45′ 46″, whose complement to the quadrant gives the arc LD, 39° 14′ 24″. So when this arc is subtracted from 15° of Leo, there remains 5° 45′ 37″ of Cancer. If you take from this half the difference of the true and apparent motion corresponding to the second interval, which was 22′ 8″, there remains L, the place of the aphelion, at 5° 23′ 29″ of Cancer [♋], for the year 1588.

Lastly, CL is 50° 23′ 29″, whose sine KA is 77042, and LB is 84° 36′ 31″, whose sine AI is 99557, and the sum is the sine KI, 176599; and the half-difference of the mean and true motion corresponding to the arc CDB, that is KI, was found above to be 3168. If you make as 176599 to 100000, so 3168 to another, AG will come out as 1784 parts, which is the distance of the umbilici from the centre of the ellipse, or half of the first inequality. Therefore the whole eccentricity in the hypothesis of a simple eccentric is 3568 of the parts of which the eccentric's radius is 100000.

EXEMPLVM ALTERVM.

Another example.

VI. But since the investigation of the eccentricity and apogee usually proceeds from observations of the equinoxes together with a place of the Sun observed between them and the solstice, [Margin: Example for equinoctial observations, etc.] Father *Paulus Casatus* [Paolo Casati] applies Boulliau's method to our observations listed under number 3, in this way. Let the circle MBLC be described from the centre O, and let BC be the line of the mean equinoxes, so that the vernal equinox is at C and the autumnal at B; and let the third, mean place of the Sun be A, which is to be joined to B and C. Next, since from observation the motions both true and mean are given in Example 1 of number 3, and their arithmetic mean, in accordance with this mean there are given the arcs AC, 126° 40′ 52½″, and AB, 55° 17′ 17″, and the whole arc CAB, 181° 58′ 9½″; [Margin: By Proposition 20 of Euclid's Book 3.] and so the angles ABC, 63° 20′ 26¼″, ACB, 27° 38′ 38½″, and BAC, 90° 59′ 4¾″. And so from the tables of sines there are given the chord AC, 17833796 [?], of the parts of which the radius is 10000000, and AB, 9279537, and BC, 19997046.

Diagram. Woodcut in the right column, with the opening lines of the column wrapped to its left: a circle with the horizontal diameter HG through its centre O, slightly tilted on the block. Chords run from A at the upper left to B on the left, from A down to L, and from A to C at the lower right; a line runs from B to C through D and X and past O; a perpendicular falls from M at the top to the diameter at O; a chord runs from B down to F, crossing the diameter at S; another runs from C up to T, crossing the diameter at N.

- A (upper left, on the circle): the third, mean place of the Sun
- M (top, right of A): upper end of the perpendicular from O
- T (upper right, on the circle): upper end of the perpendicular from C
- B (left, on the circle): autumnal equinox
- H (left, just below B): left end of the diameter
- F (lower left, on the circle): lower end of the perpendicular from B
- L (bottom left, on the circle): lower end of the line from A through D
- S (on the diameter, left): foot of the perpendicular from B
- D and X (set together, just above the diameter): on the line BC
- I (just below D, on the diameter): where AL crosses the diameter
- O (on the diameter, centre): centre of the circle
- N (on the diameter, right): foot of the perpendicular from C
- G (right, on the circle): right end of the diameter
- C (lower right, on the circle): vernal equinox

Labels read: A, B, C, D, F, G, H, I, L, M, N, O, S, T, X. Labels not read: none.

Hence the line BC is to be divided so that BC is to BD as the sine of the half-difference between the mean and true motion belonging to the whole arc BAC is to the sine of the half-difference between the mean and true motion of the arc AB. The former half-difference is 1° 58′ 9½″ and its sine 343641; the latter is 0° 31′ 17″ and its sine 90998. So as 343641 to 90998, so BC, 19997046, to BD, 5295326. Then let AD be drawn all the way to L, and from O, the centre of the circle, let the perpendicular OI be raised to AL; extended on both sides until it reaches the periphery, it will give the diameter HG, on which let the perpendicular OM be erected from O. And from the points B and C let the perpendiculars BF and CT be dropped, which cut off from the diameter HG the segment SN, so that the proportion of BC to BD is the proportion of NS to IS. For since the angles at S and N are right and equal, the lines SB and CN are parallel by Proposition 27 of Euclid's Book 1; and so, by Proposition 29 of the same, the angles SBX and XCN are equal; and the angles at X, by Proposition 15 of Book 1, being vertical, are equal. So the triangles BSX and CNX are proportional by Proposition 4 of Book 6. And since DI falls perpendicularly on the diameter HG and so is parallel to SB, it also cuts the sides of the triangle BSX proportionally, by Proposition 2 of Book 6; and the triangle XDI is proportional to the triangle CNX. So as CX is to XD, so NX to XI; and therefore, by composition, as CD to XD, so NI to XI. But as XD to

DB, so XI to IS; therefore, *ex aequali*, as CD to DB, so NI to IS; and therefore, by composition, as CB to DB, so NS to IS.

Now in the triangle ABD we have the angle ABD, 63° 20′ 26¼″, and the side AB, 9279537, and BD, 5295326, as found above. So by trigonometry the angle BAD is 34° 21′ 43″ 28‴. And therefore, by Proposition 20 of Euclid's Book 3, we have its double, the arc BL, 68° 43′ 26″ 56‴, from which the arc FL is to be subtracted, which is equal to the arc AB. For the radius OH cuts the line AL at I in two and at right angles, by Proposition 3 of Euclid's Book 3, and by the same proposition BF is cut in two at S; so the arc BH is equal to the arc HF, and AH to the arc HL. So when the equals BH and HF are taken away on each side, AB and FL remain equal. Therefore FL is 55° 17′ 17″; subtracted from BL this leaves BF, 13° 26′ 9″ 56‴, half of which, BH, is 6° 43′ 4″ 58‴. If you add to this AB, 55° 17′ 17″, the result is HA, 62° 0′ 21″ 58‴. And so its complement to the quadrant is the arc AM, 27° 19′ [recte: 59′] 38″ 2‴. The true place A was at 5° 14′ of Leo, whose distance from the equinox C is 125° 14′. If you take the arc AM from this, there remains CM, namely the distance of the Sun's apogee from the beginning of Aries, 97° 14′ 21″ 58‴, which by our method was found to be 97° 26′ 15″. But if, following Boulliau, you subtract the half-difference of the true and apparent motion corresponding to the interval AB, which was 0° 31′ 17″, the distance already mentioned remains, 96° 43′ 5″. But this arises from that arithmetic mean, which does not attain geometric rigour.

VII. In this form, however, the eccentricity is found from half the eccentricity, which in the preceding figure is shown by the line OH, the distance of the umbilici from the centre of the ellipse. For from what has been said the angle BAC is given, 90° 59′ 4″ 45‴, and the angle BAD, 34° 21′ 43″ 28‴. When the latter is subtracted from BAC, there remains the angle DAC, 56° 37′ 21″ 17‴, to which the alternate angle ACT is equal, by Proposition 29 of Book 1. Therefore the arc AT is double that, and so 113° 14′ 42″ 34‴. If AM, 27° 59′ 38″ 2‴, is taken from it, there remains MT, 85° 15′ 4″ 32‴, whose sine is ON, 9965673. And since the arc AB is 55° 17′ 17″ and AM 27° 59′ 38″ 2‴, the whole arc BM is 83° 16′ 55″ 2‴, whose sine is OS, 9931338. So if NO and OS are added, the whole of NS will be 19897011. Let it be, then, as NS, 19897011, is to the radius 10000000, so the sine of the half-difference of the true and mean motion corresponding to the interval BAC, which is 343641, is to OH, 172710, half the eccentricity, or the distance of the umbilici from the centre of the ellipse. Doubled, this gives the whole eccentricity, 345420; but with the radius taken as 100000 it becomes 3454, close to the value we set in no. 4 as 3460.

Quinta Methodus Inuestigandi Eccentricitatem.

Fifth method of investigating the eccentricity.

[Margin: Kepler's method.]

VIII. *Kepler*, in his commentaries on Mars, from chapter 23 to chapter 25, assumes three distances of the Sun, and the Sun's apogee, and from them fishes

out the eccentricity. But the Sun's distance, the parallax and the Sun's apogee are more uncertain than the eccentricity. It is therefore enough to have pointed out the passages in Kepler, and it is not worth the effort to tread so slippery a path of his.

IX. It now remains for us to present, after the Scholium, a table according to the more notable opinions, in which will stand not only the place of the apogee but also the greatest equation corresponding to the eccentricity. Some teach that this eccentricity, and so also the greatest equation, is changeable, as we shall recount later in chapter 26. The eccentricity is expressed here in three ways: in parts or degrees of which the radius of the orbit, or semidiameter of the eccentric, or mean distance of the Sun from the centre of the Earth, is 60; then in parts of which the same radius is 100000; and lastly in semidiameters of the Earth.

[Margin: The fabrication of Antonín Maria Šírek.]

Now *Antonín Maria Šírek*, seeing that the eccentricity comes close to 365 parts of which the radius is 10000, chose exactly this figure, so that it should have as many parts as there are days in the common year. That is a very flimsy basis indeed, since when the radius is increased, as can be done, and made 100000, the number of days in the year is not for all that increased tenfold. Yet in order to keep our radius at 100000, in reporting his view we have made his eccentricity one cipher greater than he did.

SCHOLION.

Scholium.

Wendelen, in Lamp 3 of the *Arcana Caelestia*, says: The speeds of the apsides are to one another in the sextantal ratio [ratio sextantaria] of their eccentricities, which he also indicates in this line of verse:

The sextantal ratio gives the apsides their law.

He calls a ratio sextantal when the root of a proposed number, multiplied six times, creates it. Thus the sextantal root of the number 64 is 2, because 2 by 2 makes 4, and 2 multiplied by 4, 8, and 2 by 8, 16, and 2 by 16, 32, and 2 by 32, 64: so by six progressions one reaches 64. Likewise the sextantal root of the number 729 is 3. So he says that if the eccentricity of planet A is to the eccentricity of planet B as 64 to 729, the motion of the apogee of A will be to the motion of the apogee of B as 2 to 3; and if the former completes 2° in one century, the latter completes 3°. And he adds that he has found the Sun's eccentricity, but bisected, to be of 1745 parts.

SOLIS APOGEVM, AEQVATIO MAXIMA, ET ECCENTRICITAS.

The Sun's apogee, greatest equation and eccentricity.

The letter A in the column of years indicates years before Christ.

Authors	Years of Christ	Apogee of ☉ in	Greatest equation of ☉	Eccentricity of ☉, in parts of which the radius of the orbit etc.: radius 60°	Rad. 100000, such parts	Semid. of the Earth	Radius in semid. of the Earth
Hipparchus in Ptolemy, Book 3, chap. 4	A. 140	♊ 5° 30′ 0″	2° 23′ 0″	2° 29′ 30″	4152	41 ½	1168
But corrected by Longomontanus		5° 16′ 0″	2° 5′ 26″	2° 11′ 30″	3648 ⅙	42 ⅗	1168
By Rheticus, however, in the *Narratio*		3° 0′ 0″	☩	☩	☩	☩	☩
Ptolemy, *Almagest*, Book 3, chap. 4	139	♊ 5° 30′ 0″	2° 23′ 0″	2° 29′ 30″	4152	41 ½	1168
But corrected by Longomontanus		10° 3′ 0″	2° 5′ 26″	2° 11′ 30″	3648 ⅙	42 ⅗	1168
By Rheticus, however		4° 30′ 0″	☩	☩	☩	☩	☩
al-Battānī, chap. 28	883	♊ 22° 17′ 0″	1° 59′ 0″	2° 4′ 45″	3465 3/11	39	1107
But corrected by Longomontanus		22° 53′ 0″	0′ 0″	2° 11′ 24″	3470	39	1108
By Rheticus, however		16° 30′ 0″	☩	☩	☩	☩	☩
al-Zarqālī in Copernicus	1076	♊ 17° 50′ 0″	1° 59′ 0″	2° 4′ 45″	3465 3/11	☩	☩
But corrected by Rheticus		22° 0′ 0″	☩	☩	☩	☩	☩
The Alphonsines in Nunes, and in Reinhold on Peuerbach's *Theoricae*	1252	♊ 28° 40′ 0″	2° 10′ 0″	2° 15′ 0″	3465 3/11	☩	☩
But in Mulerius, p. 320		☩	☩	☩	3780	☩	☩
Bernhard Walther, from Tycho	1488	♋ 4° 15′ 0″	2° 1′ 35″	2° 7′ 35″	3548	☩	☩
or from Longomontanus's calculation		☩	2° 2′ 0″	☩	☩	☩	☩
Copernicus, Book 3, chap. 16 and 17	1515	♋ 6° 40′ 0″	1° 51′ 0″	1° 56′ 0″	3230	37 ⅓	1142
But corrected by Longomontanus		5° 8′ 0″	2° 3′ 0″	2° 9′ 36″	3592 ⅗	41	1142
Reinhold in the Prutenic Tables, precept 17, from Copernicus, Book 3, chap. 21, and Magini, *Sec. Mob.*, Canon 23: Greatest	A. 64	♊ 13° 44′ 45″	2° 23′ 20″	2° 30′ 7″	4170	47 ⅗	☩
Mean	795	♊ 23° 7′ 39″	2° 7′ 0″	2° 5′ 0″	3694	42 9/50	1142
Least	1653	♋ 11° 52′ 31″	1° 50′ 30″	1° 55′ 53″	3219	37	☩
Pedro Nunes in the *Theoricae*, from Pitati and the Alphonsines	1552	♋ 1° 28′ 0″	2° 10′ 0″	2° 16′ 7″	3781 3/10	☩	☩
Tycho, vol. 1, pp. 20, 31, 61	1588	♋ 5° 30′ 0″	2° 3′ 15″	2° 9′ 2″	3584 ⅙	40 ½	1150
Kepler, *Mars*, chap. 25, in the Rudolphine Tables	1588	♋ 5° 32′ 0″	2° 3′ 46″	2° 9′ 36″	3600	63 ⅗	☩
But with the eccentricity bisected		☩	2° 3′ 46″	1° 4′ 48″	1800	31 ⅘	1768
From the parallax, however		☩	☩		1800	57	3381

Authors	Years of Christ	Apogee of ☉ in	Greatest equation of ☉	Eccentricity of ☉, in parts of which the radius of the orbit etc.: radius 60°	Rad. 100000, such parts	Semid. of the Earth	Radius in semid. of the Earth
Jost Bürgi in Snel	1588	♋ 5° 53′ 20″	2° 3′ 30″	2° 3′ 32″	3591 ⅓	☩	☩

RESIDVVM TABVLÆ PRÆCEDENTIS.

Remainder of the preceding table.

Authors	Years of Christ	Apogee of ☉ in	Greatest equation of ☉	Eccentricity of ☉, in parts of which the radius of the orbit etc.: radius 60°	Rad. 100000, such parts	Semid. of the Earth	Radius in semid. of the Earth
Boulliau, from Tycho's data	1588	♋ 5° 23′ 29″	2° 2′ 41″	2° 8′ 24″	3568	☩	☩
But with the eccentricity bisected		5° 23′ 29″	1° 4′ 31″		1784	26 3/20	1460
Magini in the *Supplement to the Ephemerides*, with Tycho's eccentricity bisected	1588	♋ 5° 30′ 45″	2° 3′ 10″	1° 4′ 31″	1792	20	1150
Adriaan Metius	1607	♋ 5° 40′ 0″	☩	☩	3567	☩	☩
Jacob Christmann in the observations of the Sun	1590	☩	2° 2′ 48″	2° 8′ 24″	3570	☩	☩
Longomontanus, *Theoricae*, Book 1, chap. 2 and 5	1590	♋ 5° 30′ 0″	2° 2′ 48″	2° 8′ 26″	3571 1/25	43 ½ [recte: 46 ½, per the book's errata; corrected by hand to 46 ½ in this copy]	1288

Authors	Years of Christ	Apogee of ☉ in	Greatest equation of ☉	Eccentricity of ☉, in parts of which the radius of the orbit etc.: radius 60°	Rad. 100000, such parts	Semid. of the Earth	Radius in semid. of the Earth
Lansbergen in the *Theoricae* of the planets: Greatest	1	♊ 5° 9′ 29″	2° 24′ 56″	31′ 48″	4216	63 17/100	☩
Mean	720	♊ 18° 40′ 11″	2° 12′ 30″	18′ 36″	3853	52 ½	1498 ½
Least	1500	♋ 3° 18′ 25″	2° 0′ 0″	2° 8′ 24″	3490	41	☩
Wendelen in the *Atlantic Tables*, with the eccentricity bisected	1600	♋ 5° 42′ 0″	2° 0′ 0″	1° 2′ 42″	1745 ½	249	14656
Šírek in the *Oculus Enoch* etc.	1640	☩	2° 5′ 30″	2° 11′ 30″	3650	73	2000
We, for the present	1646	♋ 7° 26′ 15″	1° 59′ 0″	2° 4′ 48″	3460	25[?] [recte: 253, per the book's errata; in this copy the third printed figure is covered by an ink blot with a 3 written over it by hand]	7250 [recte: 7327, per the book's errata; in this copy the printed figure is struck through in ink, and 7327 is written by hand in the outer margin]

CAPVT XXV.

Chapter 25.

De Motu Apogei Solaris.

On the motion of the solar apogee.

I. I find four controversies about the Sun's apogee. 1. Whether it moves. 2. Whether with a motion always direct. 3. Whether with an equal one. 4. And how great this motion is.

[Margin: Is the Sun's apogee changeable?]

As to the first, *Ptolemy*, in *Almagest* Book 3, chapter 4, when he had found the Sun's apogee at the same place on the ecliptic where Hipparchus had very nearly found it, namely at 5° 30′ of Gemini, did not dare to assign it any motion such as he attributes to the apogees of the five planets. In this he is excused by *Cardano* in *Daniel Santbech*, in the book *De Observatione*, Prop. 9: for he was unwilling to rely on so short an interval of time between himself and

Hipparchus, namely 280 years, or, on account of a tiny difference that could have arisen from the instruments, to devise new motions in the heavens. But *al-Battānī* and the other astronomers after him, having detected that the apogee had plainly advanced in the order of the signs, affirmed that it moves, although with a rather slow motion, not immediately perceptible. We certainly detected it in the year of Christ 1646, as I said in chapter 24, at 7° 26′ 15″ of Cancer; and other astronomers of this and the preceding century, advanced by about one whole sign from the Ptolemaic place, as is clear from the preceding table.

[Margin: Al-Zarqālī's opinion on the regression of the apogee.]

II. The seeds of the second controversy were sown by *al-Zarqālī*, who, though later than al-Battānī by 193 years, nevertheless found the Sun's apogee nearer the beginning of Aries than al-Battānī had found it. For al-Battānī in the year of Christ 883 found it at 22° 17′ of Gemini, but *al-Zarqālī* in the year 1076 at 17° 53′ of Gemini. From this four opinions arose. The *first* was that of *al-Zarqālī* himself: that the Sun's apogee now advances, now regresses, and that with an unequal motion, because from Ptolemy to al-Battānī, in 740 years, it appeared to have advanced 16° 27′, but from al-Battānī to al-Zarqālī, in 193 years, to have regressed 4° 27′. He therefore devised a small circle on whose periphery the centre of the Sun's eccentric should move forward and backward alternately, so that the motions of the apogee would vary together with the eccentricity, as *Regiomontanus* indicates in Book 3 of the *Epitome*, Prop. 13, *Nunes* in Annotation 1 on Peuerbach's *Theoricae*, and *Copernicus* in Book 3, chapters 16 and 20. And *Agostino Ricci*, in his treatise on the motion of the eighth sphere, imputes the same opinion on the regression of the apogee to King Alfonso.

[Margin: To whom al-Battānī is suspect of fallacy.]

The *second* opinion was that the Sun's apogee always advances, but that al-Battānī's observation, resting on the solstices, was fallacious. So *Nunes*, on Peuerbach's *Theoricae*, Annotation 1, and *Reinhold*, p. 41, and *Georg Joachim Rheticus* in the first account of the books of Copernicus, who adds that al-Battānī misuses the mysteries of astronomy too freely, and wrongly chose eclipses around the mean longitudes, not around the apogee, and that therefore the Sun's apogee, placed six degrees from its true place, could not have introduced any noticeable error. But from where does Rheticus divine that al-Battānī looked to eclipses here, when al-Battānī himself relates that he looked to the solstices and equinoxes?

[Margin: To whom al-Zarqālī rather is suspect.]

The *third* opinion was that the Sun's apogee never regresses, and that it was rather al-Zarqālī who erred in his observations, or in the prolixity of his calculation, and in reconciling the 402 observations or considerations which, as *Regiomontanus* attests above, he had in this matter. *Willebrord Snellius* thinks much the same of him in his notes on the Hessian observations [recte: ad; the page prints ab], concluding that he [Greek: παρατηρῆσαι (paratērēsai, "observed amiss")], that is, observed wrongly, even granted that *Avraham ibn*

Ezra, who flourished 71 years after al-Zarqālī, asserts in the book entitled *Initium Sapientiae* that no one in that age was to be compared with al-Zarqālī in observing the motions of the heavens. *Jābir ibn Aflaḥ*, moreover, held al-Zarqālī's calculations suspect, because he deduced the same eccentricity as al-Battānī and yet a far different place of the apogee. And *Longomontanus* too, in Book 1 of the *Theoricae*, in the commentary on the Sun, though he thinks both erred, nevertheless thinks al-Zarqālī did so more, because the apogee's progress from Ptolemy to al-Battānī agrees better with its progress from al-Battānī to us.

The *fourth* opinion, therefore, is that the Sun's apogee always advances, and it ascribes that apparent regression partly to al-Battānī's, partly to al-Zarqālī's fallacy in observing or in computing. So *Longomontanus*, in the passage just cited. *Tycho*, for his part, promises in vol. 1 of the *Progymnasmata*, p. 33, that this controversy will be settled by him; but death removed him from the living before he could make this good.

[Margin: Inequality of the apogee's motion.]

III. As regards the equality of this motion, nearly all those who say that the eccentricity varies, or that the motion of the fixed stars and of the apogees is unequal, deny it; of these in chapters 26 and 28. But what presses us more is to determine how great the motion of the solar apogee is, for that is quite indispensable for the anomaly and the equations of the Sun's motion.

[Margin: The quantity of the motion of the apogee of ☉ is hard to determine.]

How uncertain it is, however, is clear from what has been said so far; and, to leave al-Battānī and al-Zarqālī aside, it is not yet clear whether the Sun's apogee stood at 5½° of Gemini in Ptolemy's time, or rather in Hipparchus's. For if in Ptolemy's, then from the year of Christ 139 to our observation there are 1507 years, in which the apogee has advanced 31° 56¼′, since we found it in the year 1646 at 7° 26′ 15″ of Cancer. Now when the 15″ are discarded and the progress is resolved into units of ‴, 6897600‴, and these are divided by the 1507 years, the apogee's motion corresponding to one year is found to be 1′ 16″ 17‴. But if it was there in Hipparchus's time, namely in the year 140 before Christ, at 5½° of Gemini, then since there are 1786 years, and almost 1800, to our observation, when the same progress of 6897600‴ is divided by these, the annual and mean motion of the apogee becomes 1′ 3″ 52‴. And yet *Tycho*, from an interval of only 100 years between himself and *Bernhard Walther*, gathers an annual motion of the solar apogee of 45″, which, as *Longomontanus* notes in the commentary on the Sun, does not suit our age;

[Margin: Tycho's slip on the apogee's motion.]

nor should so generous a motion have been derived from so short an interval of years, when even from an interval of 280 years Ptolemy did not dare to determine anything about the apogee's motion.

[Margin: The opinion of al-Farghānī and the Arabs on the motion of the Sun's apogee.]

IV. *Ptolemy*, indeed, in Book 9, chapter 5, held that the apogees of the five minor planets move with the motion of the fixed stars, because of their affinity in slowness of motion; and for Ptolemy the fixed stars creep forward one degree in 100 years by their proper motion. So *al-Farghānī*, Differentia 13, and many Arabs, who flourished in the 11th and 12th centuries of Christ, transferred this to the Sun's apogee, and said that it completes one degree in the zodiac in 100 years.

[Margin: The Alphonsines' opinion on the same.]

So the Jews and Arabs who constructed the Alphonsine Tables came over to a share of this opinion, for they affirmed that the apogees of all the planets except the Moon, and so of the Sun too, move with the same motion as the eighth sphere. But they set the annual motion of the fixed stars and apogees at 26″ 26‴ 54⁗, for two reasons. First, because the space of the zodiac that the Sun traverses with its mean motion is such that, when the 365^{d} 5^{h} and the minutes required for their tropical year are completed, it makes up the quarter of a day, and the sidereal year, as they themselves think, is of 365^{d} 6^{h}. Second, because with this motion assumed the whole zodiac will be completed, if the world lasts, in 49000 years, in which are contained one thousand Jewish Jubilee years, which consisted of 49 full years. So they defined the motion both of the apogees and of the fixed stars by a revolution of 49000 years: an ingenious fabrication, to be sure, but a fabrication, without foundation. *Agostino Ricci*, pp. 34 and 40½, attributes the same opinion on the common motion of the apogees and fixed stars to al-Battānī and Thābit ibn Qurra.

[Margin: The ingenious opinion of Longomontanus and Kepler.]

V. Another path, therefore, was taken by *Longomontanus*, in the *Astronomia Danica*, commentary on the Sun, and by *Kepler* in the Rudolphine Tables, p. 42. For having considered the places and progress of the Sun's apogee, and the more probable epoch of the world's creation, they saw that at the world's beginning this apogee was very close to the start of Aries. Why, then, should they not think that God at the beginning placed the Sun's apogee at the very start of Aries and of the rational zodiac? So they placed the Sun's apogee at the start of Aries in the year before Christ *Longomontanus* 3964, *Kepler* 3993, the year they assign to the world's creation, and they set its annual motion at 1′ 1″ 50‴ 14⁗, as Longomontanus does, or 1′ 2″, as Kepler does; for in this way, with a modest little correction applied, it is held to agree with the places of the solar apogee observed by Hipparchus, al-Battānī and Tycho.

[Margin: Our opinion.]

VI. For my part, building on these: in my chronological book I clearly proved that the world was created by God neither beyond the year 4076 before Christ nor after or short of 3993, and that among all the intermediate years of this range the year 4063 was very probably the one in which the world began from the vernal equinox and a new moon. From that year to our observation, discussed in the preceding chapter, no. 3, there are 5709 years, in which the Sun's apogee has advanced from the start of Aries 97° 26′ 15″. Therefore,

resolving these into units of ‴, I find the annual motion to be 1′ 1″ 10‴. I shall confidently keep this for the present, until someone brings forward something better. For an error of one minute in the Sun's place requires an error of about one degree in the place of the solar apogee; and that cannot come about by omitting one second in the apogee's annual motion except after 3600 years.

[Margin: The authors on the apogee's motion.]

Now in our usual way we shall present a table and synopsis of the chief opinions on this matter, which the authors either state or we have gathered from the data they give. The authors are: *Ptolemy*, Book 3, chapter 4; *al-Battānī*, chapter 28, compared with Hipparchus's observation; *al-Zarqālī* in Regiomontanus, *Epitome*, Book 3, Prop. 13, and in Copernicus; *al-Farghānī*, Differentia 13; *the Alphonsines* in the Tables, Proposition 12, and in *Ricci*, on the motion of the eighth sphere; *Copernicus*, Book 3, chapter 22; *Reinhold* in the Prutenic Tables, and *Magini* in the *Theoricae* of the planets, Book 1; *Mästlin* in the *Theorica* of the Sun appended to Kepler's *Prodromus*; *Tycho*, vol. 1 of the *Progymnasmata*, p. 55, and with him *Magini* in the *Supplement to the Ephemerides*; *Longomontanus*, *Theoricae*, Book 1, chapter 5; *Kepler* in the Rudolphine Tables, p. 42, and in the *Epitome of Astronomy*, p. 118; *Lansbergen* in the Tables, Canon 12; *Boulliau* in the *Philolaic Tables*, and *Astronomia*, Book 2, chapter 2; *Wendelen* in the *Idea of the Atlantic Tables*, on whose opinion in gathering the apogee's motion from the sextantal proportion of the eccentricities see what I said in the Scholium of chapter 24.

Apogei ⊙ Motus Annuus, ac Periodus.

The annual motion and the period of the apogee of ⊙.

Authors	Annual motion: ′	″	‴	⁗	Period in years
Ptolemy	0	0	0	0	0
al-Battānī	0	59	4	0	21966
al-Zarqālī	0	42	16	14	30662
al-Farghānī	0	36	0	0	36000
The Alphonsines	0	26	26	54	49000
Copernicus	0	24	20	14	50717
Reinhold	0	25	33	12	50000
Mästlin	✢	✢	✢	✢	54000
Tycho	0	45	0	0	28800
Longomontanus	1	1	50	14	20958
Kepler in the Rudolphine Tables	1	2	0	0	20903
in the *Epitome of Astronomy*	1	3	0	0	20572
Lansbergen	1	7	30	38½	19255
Boulliau	0	57	0	0	22737
Wendelen	1	2	8	21	20864
We	1	1	10	0	21245
Although elsewhere	1	3	52	0	✢

CAPVT XXVI.

Chapter 26.

De Mutatione Eccentricitatis, & Maximæ Aequationis Solis, earumq. & motus Apogei Anomalia.

On the change of the eccentricity and of the Sun's greatest equation, and on their anomaly and that of the apogee's motion.

I. From the table placed at the end of the Scholium of chapter 24 it is clear, if we are to abide by the observations of the astronomers as they have been handed down, that the eccentricity of the Sun's orbit is not the same, nor the greatest equation the same, nor constant in all ages. For in the ages closest to the birth of Christ an eccentricity of 4152 parts was found by *Hipparchus* and *Ptolemy*, with a greatest equation of 2° 23′. Then, at the end of the ninth century of Christ, *al-Battānī* found an eccentricity of 3465 3/11 parts, of which the eccentric's radius is 100000, with a greatest equation of 1° 59′. *Copernicus*, once the 16th century had begun, deduced from his observations an eccentricity of 3230 and a greatest equation of 1° 51′. And so others differently in other ages.

[Margin: The necessary connection of the eccentricity with the Sun's equation and apogee.]

Now it is certain, from the hypothesis of the eccentric or of the Sun's epicycle (whose semidiameter is always as great as the eccentricity in the hypothesis of the eccentric), and from the method of gathering the apogee, of which I spoke in chapter 24, and from the method of gathering the Sun's equations, on which see chapters 20, 21, 22, 23: it is certain, I say, that if the eccentricity, or the semidiameter of the Sun's epicycle, is varied, the greatest equation and all the others vary too, because their measure is the angle standing on the eccentricity as a base; that a different place of the apogee is gathered; and that if the motion or change of the eccentricity is unequal, the motion or change both of the apogee and of the solar equations is likewise unequal. This, accordingly, was the reason why I set these three before you in the same table at the end of chapter 24. And if anyone in astronomy should set one of these as invariable while the others vary, he could easily be convicted of inconsequence and geometric inelegance. But since the root and foundation of these inequalities is the eccentricity, the whole controversy reduces to the change of the eccentricity, namely whether it truly changes, and with a regular or an irregular change: for there are those who affirm both, and those who deny them.

[Margin: Authors asserting the change of the eccentricity of ☉, etc.]

II. That the eccentricity, and so also the greatest equation, changes, and that their motion is anomalous, as is the motion of the solar apogee, was affirmed by *al-Zarqālī* the Spaniard, in *Regiomontanus*, *Epitome of the Almagest*, Book 3, Prop. 13, who therefore introduced into the theory of the Sun a small circle eccentric to the centre of the world, on whose periphery the centre of the Sun's orbit is carried round, because he had found an eccentricity other than

the Ptolemaic, although the same as al-Battānī's. Likewise *Peuerbach*, and before him the *Alphonsines*, as *Nonius* assumes in Annotation 1 on Peuerbach's *Theoricae*; *Copernicus*, Book 3, ch. 5, 20 and 21; *Georg Joachim Rheticus* in the *First Narration* on Copernicus's books, and there *Mästlin* in his annotations, and in the Copernican *Theorica* of the Sun; *Reinhold* in the Prutenic Tables, Precept 17; *Michael Neander* in the *Elementa Sphaerica*, p. 124; *Giuseppe Moletti* in the *Gregorian Calendar*, Canon 10; *Giovanni Antonio Magini* in the *Theoricae* of the planets, ch. 4, *Theorica* of the Sun; *Tycho*, vol. 1 of the *Progymnasmata*, pp. 31 and 38; *Willebrord Snellius* in the notes to the Hessian observations; *Pitiscius* [Bartholomäus Pitiscus], Book 4 of the Astronomical Problems; *Lansbergen* in the *Theorica* of the Sun, ch. 1; and our own *Joseph Lange* in the *Elementale*, on ch. 9 of the *Theorica* of the Sun. Among these, however, the *Copernicans* and *Lansbergen* determine a greatest, a mean and a least; on these consult our table at the end of ch. 24.

[Margin: Authors denying the change of the eccentricity.]

III. Yet the eccentricity of the Sun and the greatest equation, etc., were judged to be UNCHANGEABLE by *Ptolemy*, Book 3, ch. 2; *Scaliger*, Exercise 99 against Cardano; *Kepler* in the notes to his *Mysterium Cosmographicum*, ch. 1, and in the *Astronomia Optica*, pp. 146 and 147, and in the *Commentaries on Mars*, ch. 69; *Longomontanus* in the *Astronomia Danica*, Book 1 of the *Theorics*, ch. 2, in the commentary on the Sun, under the title [Greek: Σύστασις (systasis, "constitution")] of the Eccentricity, and in his work *Holigerius Rosækrantcius* [Holger Rosenkrantz]; likewise *Antonín Maria Šírek* in the solar hypothesis of the *Radius Sidereomysticus*, who sets it always at as many parts as there are days in the Egyptian year, namely 365, of which the radius is 1000; *Ismaël Boulliau*, Book 2 of the *Astronomia Philolaica*, ch. 3; and *Govaert Wendelen* in the *Idea of the Atlantic Tables*, p. 29. We too side with them, since we see that this change cannot be demonstrated, nor asserted any more solidly than it is denied, either from eclipses and the growth or decrease of shadows, or from observations of the Sun, especially solstitial ones; nor should so many inequalities be shifted onto the heavens on the strength of a slight difference that could have arisen from parallax and refraction, or from some small fault in the instruments. This holds especially since, at the very time when according to our opponents' hypotheses such a measure ought to have been found, a far different one has been found by us, and by others, as is clear from the preceding table; nor do those hypotheses agree with themselves, or with a greater number of observations. We shall, however, set out the theory of this change where we deal with the anomaly of the obliquity of the ecliptic and of the precession of the equinoxes.

[Margin: To where is the theory of this motion deferred?]

[Margin: The period and epoch of the anomaly of the eccentricity are the same as those of the obliquity of the ecliptic.]

IV. Note, however, that the *Copernicans* and *Lansbergen* agree that the period of the anomaly of the eccentricity is as long as the period of the anomaly of

the obliquity of the ecliptic, of which we must speak in ch. 27; that both move equally in their circles; and that when one is greatest, the other is also greatest, etc. But they do not agree on the length of the period, nor on the beginning or epoch of the anomaly, nor on the amount of the greatest, or the mean, or the least, as can be seen from the table at the end of Chapter 24. From this, and from the authors themselves, it is gathered that the period of each, in which the revolution from the greatest back to the greatest eccentricity and obliquity is made, is for *Copernicus* 3434 Egyptian years, which make 3431 ⅔ Julian years, and that its beginning was in the year 63 completed before Christ, and 302 days; for *Lansbergen*, however, the period is of 3002 Egyptian years but 3000 Julian, and its beginning in the first year of Christ. But for other years, see these and the rest in the following small table.

		Eccentricity, in parts of which the radius of the orbit is 100000	Greatest equation of the Sun	In the year before Christ		and of Christ		Period of the anomaly of the eccentricity	
For Copernicus	Greatest	4170	2° 23′ 20″	3495	& 64	3369	& 6800	Of years	
	Mean	3694	2° 7′ 0″	6068	2637	795	4226	Egyptian	Julian
	Least	3219	1° 50′ 30″	5210	1779	1653	5085	3434	3431 ⅔
For Lansbergen	Greatest	4216	2° 24′ 56″	6000	3000	1. 3000	6000	3002	3000
	Mean	3853	2° 12′ 30″	3720	720	720	3720	✝	✝
	Least	3490	2° 0′ 0″	4500	1500	1500	4500	✝	✝

SCHOLION.

Scholium.

[Margin: Rheticus's empty prophecy.]

I. *Georg Joachim* Rheticus, in the *First Narration* on the books of Copernicus, affirms that all monarchies began when the centre of the Sun's eccentric stood at some notable point of the small circle on which the motion of the anomaly of the eccentricity takes place. For when the eccentricity was greatest, the Roman Empire passed under Augustus to monarchy; when the first quadrant had been completed and the eccentricity was mean, the Muhammadan monarchy began; when the semicircle is finished and the least eccentricity is at hand, this monarchy will fall; and when the period is finished, the Lord JESUS CHRIST will come. For at the beginning of the world too it was greatest; and this small circle is that celebrated Wheel of Fortune by whose turning monarchies change. But from the small table set out at the end of the preceding chapter it will be evident that this assertion is empty, provided you hold a correct chronology.

Nor indeed was the beginning of the world in the year 3495 or 6000, as we shall show in the chronological book of our *New Almagest*; and the monarchy of the Assyrians began under Ninus in the year 2055 before CHRIST; that of the Medes under Arbaces in 817; and that of the Persians, finally, under Cyrus in the year 535 before CHRIST. See a similar fiction in ch. 27, Scholium 2.

[Margin: On the theory of the anomaly of the eccentricity and of the apogee.]

II. To learn the theory of al-Zarqālī's small circle, and by what method Copernicus investigated its anomaly, as well as that of the Sun's apogee, and the equation of these, see Copernicus himself, if you wish, Book 3, ch. 20 and 21, where he gathers the least eccentricity as 3210 parts and the greatest prosthaphaeresis of the apogee's motion as 7° 28′. But Reinhold in the Prutenic Tables, Precept 17, with Copernicus's numbers and calculation corrected, sets the least eccentricity at 3219 parts. And Magini, in the *Theorica* of the Sun and in the *Tables of the Second Mobiles*, Canon 23, following Reinhold's numbers, says that the semidiameter of the small circle carrying the centre of the Sun's eccentric is 17′ 7″, of which the radius of the eccentric is 60°, that is, 472 5/22 parts, of which the radius is 100000; that the greatest equation of the apogee's motion is 7° 23′ 36″; that the anomaly of the apogee and of the eccentricity is called in the Tables the Centre of the Sun, and its equation the Equation of the Centre; and that this small circle moves against the order of the signs while the eccentric moves in the order of the signs. But Mästlin, in the Copernican *Theorica* of the Sun, gathers from Reinhold that the semidiameter of the already mentioned small circle is 475 ½, and from Copernicus, Book 3, ch. 22, he notes that the annual motion of the Sun's apogee on a circle concentric with the world is 24″ 20‴ 14⁗, and that it is completed under the sphere of the fixed stars in 54000 years. Granted, Reinhold gathers from Copernicus's data that its annual motion is 25″ 33‴ 12⁗ and its period a little more than 50000 years, but under the zodiac 17000.

CAPVT XXVII.

Chapter 27.

De Solis Maxima Declinatione, & Obliquitate Eclipticæ, harumq. Mutatione, & Anomalia.

On the Sun's greatest declination and the obliquity of the ecliptic, and on their change and anomaly.

I. The Sun's greatest declination occurs at the contact of the tropics and the ecliptic, that is, at the beginning of Cancer and Capricorn, and it is reckoned on the arc of the meridian circle intercepted between either tropic and the equator. This arc is measured by the angle made at the equinoctial points by the equator and by the ecliptic inclined to it. This angle is called the obliquity of the ecliptic, in Greek [Greek: λόξωσις Ξωδιακοῦ [recte: Ζωδιακοῦ] (loxōsis

zōdiakou, "obliquity of the zodiac")], and it is as great as the Sun's greatest declination, and as the distance of the pole of the ecliptic from the nearer pole of the world. Let us now see by what method these are observed, and whether they change, and with what change.

Primus Modus obſeruandi Declinationem Solis Maximam.

The first method of observing the Sun's greatest declination.

[Margin: The first method, by the distance of the tropics.]

II. The first method is through the distance of the tropics, taken from the Sun's meridian altitudes at both solstices. For when the true winter altitude is subtracted from the Sun's summer altitude, what remains is the distance of the tropics, half of which is the distance of either tropic from the equator, that is, the Sun's greatest declination.

[Margin: Ptolemy's observation and opinion on the obliquity of the ecliptic.]

In this way *Ptolemy*, Book 1 of the *Magna Constructio*, ch. 11 and 13, relates that he found the distance of the tropics to be 11 parts of which the whole meridian is 83, that is, 47° 42′ 40″ of which the whole meridian is 360, and thus the obliquity of the ecliptic to be 23° 51′ 20″, which is as much as *Eratosthenes* and *Hipparchus* had once found, says Ptolemy. He therefore judged it to be always that great, and unchangeable.

[Margin: Al-Battānī's observation and opinion.]

Al-Battānī, for his part, relates in ch. 4 that he found in the city of Aracta [al-Raqqa] the Sun's least distance from the vertex, at noon of the summer solstice, to be 12° 26′, and its greatest, at noon of the winter solstice, 59° 36′; and he says that he used a very long alidade, namely the parallactic rulers, which we shall treat in the *Liber Organicus*. From this, then, he deduced the distance of the tropics as 47° 10′, and the obliquity of the ecliptic as 23° 35′, which he thought true and stable, since he doubted the observations of the ancients and founded his hypotheses on this one; and the altitude of the pole at Aracta as 36°. For he says: *On this, then, we shall build this book of ours, because this we received with our eyes, but that by hearing.*

[Margin: Regiomontanus's observation and opinion.]

After these, *Regiomontanus*, Book 1 of the *Epitome of the Almagest*, Prop. 17, says that he observed at Nuremberg the Sun's greatest altitude, at the summer solstice, as 65° 6′, and its least, at the winter solstice, as 18° 10′, from which he gathers the obliquity of the ecliptic as 23° 28′. But if you add to the altitudes his parallax, or rather ours, of about 30″, and take from the winter altitude a refraction of 4′, or Tycho's 5′ ¾, you will gather the obliquity of the ecliptic as 23° and 30′ or 31′. Yet *Clavius*, in the *Sphere*, p. 263 in my copy, reports from Regiomontanus the Sun's greatest altitude observed at Vienna, at the summer solstice, as 65° 30′, and its least as 18° 30′; from these, applying our parallax and refraction, the obliquity is gathered as 23° 31′ or 32′. Granted, *Lansbergen* in the *Thesaurus of Solar Observations* deduces the obliquity as 23° 30′ from another

Viennese observation of Regiomontanus. I, however, from the chords of the distances from the vertex taken at Nuremberg by *Regiomontanus* and *Walther* at the solstices, gather the obliquity as 23° and sometimes 29 ½′, sometimes 30′ or 31′, disregarding the small appendages of seconds. Therefore, from all these values bounded on one side and the other, I cannot more prudently establish any obliquity other than the one finally chosen by Regiomontanus himself in his tables and by *Clavius* in the table of declinations, namely 23° 30′.

[Margin: The opinion of Clavius and of Scheiner, Galileo and Finé.]

Scheiner chose the same amount in the *Rosa Ursina*, p. 356, and *Galileo* in the *Dialogues on the System of the World*, and before them *Oronce Finé* in the *Cosmographia*, Book 2, ch. 24.

[Margin: Copernicus's observation, opinion and error.]

After these *Copernicus*, Book 2, ch. 2, says that in the year 1525 at Frombork he observed the Sun's winter altitude as 12° 16′ 6″, and the summer one as 59° 8′ 54″, from which he gathers the altitude of the pole as 54° 19′ 30″ and the obliquity of the ecliptic as 23° 28′ 24″, about as much as his predecessor *Domenico Maria Novara* and his contemporaries *Johannes Werner* and *Gemma Frisius* found. But *Tycho*, in the *Epistolae Astronomicae*, p. 74, and in vol. 1 of the *Progymnasmata*, p. 34, affirms that he sent to Frombork the very well-instructed *Elias Olai* [Elias Olsen Morsing], who from the circumpolar stars found the altitude of the pole there to be 54° 22′ 15″, from which he detected that Copernicus, not indeed by omitting parallax but by omitting refraction, had erred in the altitude of the pole by 2′ 45″. Now subtract from the winter altitude the 8′ 56″ of Tycho's refraction, and add to both my parallax of 27″ or 30″, and you will gather the obliquity of the ecliptic as 23° 30′ 47″, and with the slightest of corrections, 23° 30′.

[Margin: Egnazio Danti's observation.]

[Margin: Peter Apian's method and opinion.]

Finally *Egnazio Danti*, Part 2 of the *Astrolabe*, ch. 34, relates that in the year 1570 at Florence the Sun's solstitial altitude taken by him at noon was 69° 49′, and the winter one 22° 51′, from which he gathers the obliquity of the ecliptic as 23° 29′. If you subtract from the winter altitude a refraction of 2′ or 3′ and add our parallax to both altitudes, the obliquity of the ecliptic will become 23° 30′ or 23° 30′ 30″. But because of a certain uncertainty and inconstancy of the refractions contained in winter altitudes, this method is not of such precise evidence that an error of one or two minutes cannot creep in. Far less evident still is the method of *Peter Apian*, who says in the *Instrument of the Primum Mobile* that he detected the obliquity of the ecliptic as 23° 30′ from the Sun's rising amplitude, observed many times; for the inconstancy of refractions is far greater in these horizontal observations.

Secundus Modus Obſeruandi Declinationem Solis Maximam.

The second method of observing the Sun's greatest declination.

III. It is better, therefore, first to have investigated the altitude of the pole through circumpolar stars, so that its complement to a quadrant shows the altitude of the equator; and then, on the day of the summer solstice, to take the Sun's meridian altitude with the most exact instruments and to correct it by adding the parallax. For the altitude of the equator, subtracted from this, will give the Sun's greatest declination. Within the torrid and temperate zones, if the day is clear, there is no need to fear any sensible refraction at that time; a somewhat larger variation, however, will arise from the variation in the parallax assumed, which must be added to the solstitial altitude. Let us come to examples.

[Margin: Nonius's observation.]

Nonius, in Book 2 *On Observation*, affirms that at Coimbra he observed more than once the Sun's solstitial and greatest altitude as 73°. If you add a parallax of 30″, it becomes 73° 0′ 30″. But the altitude of the pole taken there by our *Clavius*, while he was teaching at Coimbra, as he has it in the *Sphere*, is 40° 30′, and *Nonius* sets the same above on p. 64. So the altitude of the equator is 49° 30′; when this is taken from the solstitial altitude, the Sun's greatest declination becomes 23° 30′ 30″. But how easily can those 30″ be imputed to an observation not so precise, since Nonius admits in the same book that he did not yet have large instruments at hand for observations, and in the table on p. 65 he sets it at 23° 30′.

[Margin: The observation of the Landgrave of Hesse.]

Christoph Rothmann and *Jost Bürgi*, mathematicians of the Landgrave of Hesse, as is related in Tycho's letters and in Snel in the notes to the Hessian and Bohemian observations, finally found the altitude of the pole at Kassel from circumpolar stars as 51° 19′ 20″, which they had earlier obtained as one minute less, with uncorrected instruments and an inexact placement; so the altitude of the equator was 38° 40′ 40″. I find, moreover, in the last and more correct Hessian observations the Sun's solstitial altitude as 62° 11′ and 62° 12′, from which there results, on our assumed parallax, an obliquity of the ecliptic of about 23° 32′ or 33′; but if the earlier altitude of the pole had been right, it would be 30′ or 31′ above 23°.

[Margin: Homelius's observation.]

Ioannes Homelius [Johannes Hommel], as *Tycho* reports in vol. 1 of the *Progymnasmata*, p. 631, in the year 1570 at Leipzig took the Sun's greatest and solstitial altitude as 62° 11′; the altitude of the pole, however, is set by him at 51° 17′, although Tycho says it is 51° 19′. So the altitude of the equator is either 38° 43′ or 38° 41′. Now add to the altitude taken a parallax of 30″, and the obliquity of the ecliptic will become 23° and in addition either 28′ ½ or 30′ ½, of which the mean is 29′ 30″. And if there is doubt here about two minutes, how much more easily can an error of half a minute be conceded, and the obliquity be brought to exactly 23° 30′? Let us now turn to Tycho himself.

[Margin: Tycho's observations and opinion.]

Tycho, then, in vol. 1 of the *Progymnasmata*, p. 17, from the altitude of the equator established at Uraniborg as 34° 5′ 30″, and from the Sun's solstitial altitudes, applying his own parallax, which at an altitude of 57 ⅗° is 1′ 35″, gathers the obliquity of the ecliptic as 23° 31′ 30″ for the year 1587, as is given in the table of declinations in the same place, p. 28, and in the *Epistolae Astronomicae*, p. 40. *Kepler*, however, says in the Rudolphine Tables that too great a parallax was assumed by Tycho, and that we can always use an obliquity of 23° 30′ 30″. But I have very select Tychonic observations made in the year 1589 to explore the refractions of the Sun, which Father *Albert Curtz* of our Society sent me from Tycho's manuscript at my request; he by the Emperor's order presides over Tycho's observations being brought to light. We also have from *Longomontanus*, Book 1 of the *Theorics*, ch. 2, three selected meridian altitudes observed by *Tycho* at Uraniborg at the summer solstice. If you add to these our parallax of 30″ or 27″, and use the altitude of the equator at Uraniborg, mentioned above, the obliquity of the ecliptic is gathered as you see in the following small table.

In the style of the old calendar	June	Year	⊙ Apparent altitude	True altitude	Obliquity of the ecliptic
From *Longomontanus*	12	1586	57° 35′ 36″	57° 36′ 6″	23° 30′ 36″
	14	1587	57° 34′ 30″	57° 35′ 0″	23° 29′ 30″
	11	1593	57° 35′ 0″	57° 35′ 30″	23° 30′ 0″
From Father *Albert Curtz*					
By quadrant: the larger steel one	11	1589	57° 35′ 25″	57° 35′ 55″	23° 30′ 25″
the revolving one	11	1589	57° 35′ 45″	57° 36′ 15″	23° 30′ 45″
the Tychonic one	11	1589	57° 35′ 45″	57° 36′ 15″	23° 30′ 45″
the other one	11	1589	57° 35′ 50″	57° 36′ 20″	23° 30′ 50″

Will you concede, then, in view of these, that an obliquity of exactly 23° 30′ cannot be evidently overturned from Tycho's observations? For from the last, of the year 1593, just that value follows, and from the two extremes, 29′ 30″ and 30′ 50″, very nearly the same results by choosing the mean. But if you adhere to Tycho's parallax, which here is greater than ours by 1′ 5″, the obliquity of the ecliptic will sometimes become 23° 31′ 55″; but on no foundation, or almost none, he determined his parallax, as I have shown in ch. 8.

[Margin: Lansbergen's observation.]

Philips van Lansbergen, in the *Thesaurus of Observations*, first class of the solar ones, observation 9, says that in the year 1589 at Goes in Zeeland, at the summer solstice, he observed with a large quadrant the Sun's meridian altitude as 61° 58′, but that with his parallax added it was 61° 59′ 4″; and the winter altitude as 15° 2′ 30″ apparent, but 14° 58′ 43″ true. From this he gathers the distance of the tropics as 47° 0′ 21″ and the obliquity of the ecliptic as 23° 30′ 10″, *as much*, he says, *as the Landgrave's men found, as nearly as possible*, and the altitude of the pole as 51° 31′ 7″.

[Margin: And his carelessness in observing.]

Now if we apply our parallax of 30″, the solstitial altitude will become 61° 58′ 30″; the altitude of the equator, 31° 29′ [recte: 38° 29′], taken from this, gives the obliquity as 23° 29′ 30″. I should have preferred him to take the altitude of the pole from circumpolar stars, and not to deduce it from a slippery observation of the winter altitude, with which an uncertain parallax and refraction are mixed. So greater care in observing is wanted here.

[Margin: Wendelen's observation.]

[Margin: Gassendi's opinion.]

Govaert Wendelen wrote to me recently that in Provence he detected from circumpolar stars the altitude of the pole at Forcalquier as 43° 54′, and the Sun's meridian distance from the vertex, in the same place, for several years in succession at the solstices, apparent, as 20° 24′. If you take Wendelen's parallax of 15″ from this, the obliquity of the ecliptic becomes 23° 30′ 15″; but if mine, it becomes 23° 30′ 30″. *Pierre Gassendi*, however, in the *Institutio Astronomica*, p. 24, says that it cannot be greater than 23° 31′.

[Margin: Our observations.]

I, however, from among the other observations of the Sun's solstitial altitudes at noon that I made at Bologna with Father *Francesco Maria Grimaldi*, select those taken with four quadrants, but chiefly with two large ones. The parallax of 30″, added to them, gives the true altitudes; and when the altitude of the equator, 45° 30′ 30″, which was taken more than forty times from a circumpolar star, is subtracted from them, there remains the obliquity of the ecliptic, which you see in the following small table.

Obliquitas Eclipticæ ex Noſtris Obſeruationibus Bononiæ habitis.

The obliquity of the ecliptic from our observations made at Bologna.

At the summer solstice: Year	Month	Day	☉ Meridian altitude, by quadrants	☉ Apparent altitude	True altitude	Obliquity of the ecliptic
1643	June	21	Largest	68° 59′ 55″	69° 0′ 25″	23° 29′ 55″
			Large	68° 59′ 55″	69° 0′ 21″ [recte: 25″]	23° 29′ 55″
			Middling	69° 0′ 0″	69° 0′ 30″	23° 30′ 0″
1644	June	20	Largest	68° 59′ 50″	69° 0′ 20″	23° 29′ 50″
			Large	68° 59′ 50″	69° 0′ 20″	23° 29′ 50″
			Middling	68° 59′ 50″	69° 0′ 20″	23° 29′ 50″
1645	June	20	Largest	68° 59′ 50″	69° 0′ 20″	23° 29′ 50″
			Large	68° 59′ 50″	69° 0′ 20″	23° 29′ 50″
1646	June	20	Largest	68° 59′ 45″	69° 0′ 15″	23° 29′ 45″
			Large	68° 59′ 45″	69° 0′ 15″	23° 29′ 45″
		21	Largest	69° 0′ 0″	69° 0′ 30″	23° 30′ 0″
			Large	69° 0′ 10″	69° 0′ 40″	23° 30′ 10″
		22	Largest	68° 59′ 45″	69° 0′ 15″	23° 29′ 45″

At the summer solstice: Year	Month Day	☉ Meridian altitude, by quadrants	☉ Apparent altitude	True altitude	Obliquity of the ecliptic
		Large	68° 59′ 45″	69° 0′ 15″	23° 29′ 45″

[Margin: The author's opinion.]

Led chiefly by these observations, therefore, we set the obliquity of the ecliptic at 23° 30′, laying the very small difference on the instruments and observations, and we find that in the year 1646 the solstice occurred at the very noon of the 21st, from the equal altitude found on the 20th and the 22nd, so far as could be observed by us through human diligence.

[Margin: Authors who write on the obliquity of the ecliptic.]

[Margin: Lansbergen's deceitful good faith.]

IV. It remains, for the purpose of pursuing the controversy over the change of the obliquity of the ecliptic, to relate the obliquity found in different ages. This is also reported, wholly or in part, by *Regiomontanus*, Book 1 of the *Epitome of the Almagest*, Prop. 17, and Problem 63 of the *Primum Mobile*; *Copernicus*, Book 2, ch. 2, and Book 3, ch. 2; *Reinhold* on Peuerbach's *Theoricae*, p. 238, and in the same place *Vurstisius* [Christian Wurstisen], p. 422; *Nonius*, *On Observation*, Book 2; *Clavius* in the *Sphere*, p. 262; *Tycho*, vol. 1 of the *Progymnasmata*, p. 17; *Barozzi* in the *Cosmographia*, p. 70; *Snellius* in the notes to the Hessian observations; *Lansbergen* in the *Thesaurus of Observations*, from p. 37; *Kepler* in the *Epitome of Astronomy*, Book 3, p. 244, and Book 7; *Biancani* in the *Sphere*, Book 1, ch. 8; and *Boulliau*, Book 5 of the *Astronomia*, ch. 5. Among these, *Lansbergen* places Jacob ben Machir ibn Tibbon [Prophatium] only 68 years after Almeon [?], so as to draw his observation to his own tables, although in fact he came about 160 years after Almeon. Some also attribute to Thābit ibn Qurra an obliquity of 23° 33′, but *Regiomontanus* 23° 33′ 30″.

Declinatio Solis Maxima, seu Obliquitas Eclipticæ.

The Sun's greatest declination, or the obliquity of the ecliptic.

Authors	Years before Christ	Obliquity
Aristarchus of Samos, in year 50 of the Calippic [Anno 50. Calippico]	280	24° 0′ 0″
The *Indians* of old, as reported by Māshā'allāh [Meſſalam]	✝	24° 0′ 0″
Eratosthenes of Cyrene	230	23° 51′ 20″
Hipparchus of Rhodes	140	23° 51′ 20″
	Years of Christ	
Ptolemy [Ptolemæus Pheluldenſis], Book 1, ch. 11	140	23° 51′ 20″
al-Battānī [Albategnius Aracēſis], ch. 4	880	23° 35′ 0″
al-Zarqālī the Spaniard	1070	23° 34′ 0″
Almeon [?] [Almeon Almāſoris Arabs], the Arab	1140	23° 33′ 30″
Thābit ibn Qurra [Tebith Choræ filius]	1210	23° 33′ 30″
Jacob ben Machir ibn Tibbon the Jew [Prophatius Iudæus]	1300	23° 32′ 0″
Peuerbach and *Regiomontanus*	1460	23° 28′ 0″

Authors	Years before Christ	Obliquity
But in the table and in corrected observations	✝	23° 30′ 0″
Pedro Nunes [Petrus Nonius]	1500	23° 30′ 0″
Domenico Maria Novara [Dominicus Maria]	1500	23° 29′ 0″
Johannes Werner	1510	23° 28′ 30″
Nicolaus Copernicus	1525	23° 28′ 24″
But as corrected by us	✝	23° 30′ 47″
Peter Apian in the *Primum Mobile*	1530	23° 30′ 0″
Oronce Finé	1555	23° 30′ 0″
Egnazio Danti	1570	23° 29′ 0″
But as corrected	✝	23° 30′ 30″
The *Landgrave's astronomers* [Lantgrauiani] in Hesse	1570	23° 31′ 0″
Johannes Homelius [Io. Homelius] at Leipzig	1570	23° 29′ 30″
Tycho Brahe	1586	23° 31′ 30″
But from his selected observations, and with our parallax, variously; namely	1586	23° 30′ 36″
	1587	23° 29′ 30″
	1589	23° 30′ 45″
	1593	23° 30′ 0″
Philips van Lansbergen	1589	23° 30′ 10″
But as corrected	✝	23° 29′ 30″
Govaert Wendelen, with our parallax	1620	23° 30′ 30″
But with his own parallax	1620	23° 30′ 15″
Pierre Gassendi	1630	23° 31′ 0″
Clavius, *Scheiner*, *Galileo*	1600	23° 30′ 0″
We, from selected observations	1643	23° 30′ 0″
	1646	23° 30′ 0″

An Reuerà Mutetur Obliquitas.

Whether the obliquity really changes.

V. Because of the observations already mentioned, therefore, the following have held that the obliquity of the ecliptic changes from greatest to least, and with an unequal motion at that: *Peuerbach* in the *Theorica* of the eighth sphere, and there his commentators *Erasmus Oswald Schreckenfuchs*, p. 388, *Io. Bapt. Capuanus* [Giovanni Battista Capuano], p. 260, *Christian Wurstisen*, p. 420, and *Reinhold*, p. 238, and in the Prutenic Tables, Precept 13; *Pierre d'Ailly*, q. 9 on the *Sphere*; *Regiomontanus*, Book 1 of the *Epitome*, Prop. 17; *Copernicus*, Book 2, ch. 2, and Book 3 from ch. 2 onward; *Georg Joachim Rheticus*, *First Narration* on Copernicus's books; *Girolamo Fracastoro* in the *Homocentrica*, sect. 3, ch. 8; *William Gilbert*, Book 6 of *De magnete*, ch. 9; *Scaliger* in the *Diatribe on the Equinoxes*, ch. 4, Corollaries 8 and 9, who puts it at 24° at the beginning of the Olympiads; *Clavius* in the *Sphere*, p. 262; *Magini* in the *Theoricae* of the planets, Book 1, ch. 2, and in the *Directions* of the primum mobile, Canon 1; *Tycho*, vol. 1 of the *Progymnasmata*, p. 100; *Longomontanus*, Book 1 of the *Theorics*, ch. 2 and 4; *Snellius* in the notes to the Hessian observations; *Lansbergen* in the

Tables, Precept 5, and in the *Theorica* of the Sun, ch. 1, and in the *Thesaurus of Observations*, from p. 37; *Boulliau*, Book 5 of the *Astronomia*, ch. 5; *Wendelen* in the *Idea of the Atlantic Tables*; and *Baranzanus* [Redento Baranzano] in the *Uranoscopia*, part 2, q. 3.

[Margin: Authors who assert the change of the obliquity of the ecliptic.]

From these, *Copernicus* in Book 2, ch. 2, promises *to show below, by a sufficiently probable conjecture, that it was never greater than 23° 52′, and will never be less than 23° 28′*, and he makes this good in Book 3, ch. 10. The arguments for this change are as follows. First, it was always found smaller from the time of Aristarchus down to Copernicus. Second, in the very century in which the Sun's eccentricity, the Sun's equation and the length of the year were greatest, and the motion of the fixed stars, or of the equinoctial points, was slowest, namely around the birth of Christ, the obliquity of the ecliptic too was found to be greatest. Third, the decreases of the eccentricity and of the obliquity were proportional: in the 743 years between Ptolemy and al-Battānī the eccentricity decreased by 687 parts and the obliquity by 16⅓′, while in the 632 years from al-Battānī to Copernicus the eccentricity decreased by 235 parts and the obliquity by 6⅗′. And as 687 is to 16⅓, so, very nearly, is 235 to 6⅗. These things, therefore, do not seem to have come about by chance, or from a fallacy of the observations, but from some true cause producing these changes.

[Margin: Arguments for the changed obliquity.]

Very many attribute this cause to the trepidation, or first libration towards the sides of the world, that is, a slow approach and withdrawal towards the poles of the world, produced by the motion of sphere 9 (for those who admit ten spheres) or sphere 10 (for those who admit eleven). *Copernicus*, however, and his followers attribute it to a certain libration of the Earth, as will be explained below in ch. 29. On these premises *Copernicus*, *Lansbergen* and *Longomontanus*, whom *Boulliau* follows outright in this, set out the quantity of the greatest, mean and least obliquity, the epoch and period of this anomaly, and the mean annual motion, even though *Longomontanus* does not recognize the change of the eccentricity. *Kepler* too sets these out, not absolutely, since, as we shall say shortly, he denies the change of the obliquity, but on a feigned hypothesis, in the first of the five forms that I shall discuss in the Scholium. And it is truly remarkable that *Longomontanus* puts the greatest obliquity in the first year of the created world, and *Kepler* the least, even though on that beginning they do not differ by more than 25 years. So far is this change, or its anomaly, from being evident from the observations. We shall give the tables of the equation of this motion at the end of the work.

[Margin: The theory of this anomaly is indicated.]

[Margin: A huge disagreement between Longomontanus and Kepler.]

Authors	Obliquity of the ecliptic	Years before Christ	Years of Christ	Mean annual motion of the anomaly of the obliquity: ′	″	‴	⁗	Period of the anomaly of the obliquity (years)
Copernicus	Greatest	23° 52′ 30″ 95 & 64	3369 & 6800	6	17	24	9	Egyptian 3434; Julian 3431 and 244 days
	Mean	23° 40′ 00″ 68 · 2637	795 · 4226					
	Least	23° 28′ 30″ 10 · 1779	1653 · 5085					
Lansbergen	Greatest	23° 52′ 00″ 00 · 3000	1 & 3000 · 6000	7	6	4	56	Egyptian 3002; Julian 3000
	Mean	23° 41′ 30″ 20 · 720	720 · 3720					
	Least	23° 30′ 45″ 00 · 1500	1500 · 4500					
Longomontanus	Greatest	23° 53′ 39″ 68 (Year of the World 1) · 368	3232 · 6832	6	0	0	0	Egyptian 3603; Julian 3600
	Mean	23° 42′ [illegible]0″ 2700	900 · 4500					
	Least	23° 31′ [illegible]7″ · 2168	1432 · 5032					
Kepler, on a feigned hypothesis from observations	Greatest	23° 53′ [illegible]6″ 2661	6 · 2670	8	6	15	0	Egyptian 2666; Julian 2664
	Mean	23° 40′ [illegible]5″ 1995	672 · 3336					
	Least	23° 28′ 30″ 93 (Year of the World 1) · 1329	1338 · 4002					

Auctores, & Argumenta pro Immutabilitate Obliquitatis Eclipticæ.

Authors and arguments for the immutability of the obliquity of the ecliptic.

VI. Yet the obliquity of the ecliptic was judged immutable by *Ptolemy*, Book 1 of the *Almagest*, ch. 11; by *al-Battānī*, ch. 4, who ranks the old observations of it below his own as uncertain; by *Thābit ibn Qurra* according to *Maurolico* in the *Cosmographia*, p. 88; by *Agostino Ricci* in the treatise on the motion of the eighth sphere, pp. 34 and 40; by *Pierre Gassendi* in the *Life of Claude Fabri de Peiresc* [Claudij Fabricij Peireskij], at the year 1636, and in the *Institutio Astronomica*, Book 2, ch. 8, or p. 40; by *Athanasius Kircher* in his *Magnes*, p. 477; by *Stevin* according to *Hérigone*, vol. 5, p. 579; by *D. Vincenzo Renieri* in the *Tabulae Mediceae* newly reissued, ch. 7, no. 7; by *Kepler* in the Rudolphine Tables, ch. 12 and 34; and very nearly by *Galileo* in Dialogue 3 on the world system, when he says that it undergoes a very small and negligible slight change, one not perceptible except after many thousands of years. I agree with them, but I am led to it by arguments from a deeper inspection of astronomical matters. They are content to hold the observations of the ancients suspect of error, and *Kepler* adds only an argument from the latitude of the fixed stars. I, however, shall show that even in the time of Eratosthenes, and indeed of Pytheas of Massalia, the obliquity of the ecliptic was 23° 30′, as great as it is now, and I shall point out the root of the error lurking in the observation of Eratosthenes.

[Margin: Our opinion on the immutability of the obliquity of the ecliptic.]

VII. Eratosthenes, then, intending to measure the globe of the Earth, first established by his own experiment, and not merely from a constant tradition, that Syene lies under the Tropic of Cancer. There, as also at Berenice, at noon on the day of the summer solstice gnomons cast no shadow over a space of 150 stadia on this side and 150 on that, that is, 300 stadia all round. So report *Cleomedes*, Book 1 of the *Cyclical Theory*, *Strabo*, Book 17, and *Pliny the Elder*, Book 2, ch. 73, and Book 6, ch. 29. Pliny the Elder adds that a well made at Syene for this purpose is wholly lit at solstitial noon, and that the Sun is therefore then over the zenith of Syene, and he calls this a place *of immense subtlety, the world having been detected there, since from it Eratosthenes began to publish the measure of the Earth by the indubitable reasoning of the shadows.* Holding this as established, he set up a scaphe [Scaphium] with a gnomon at Alexandria, and at noon on the summer solstice observed the shadow of the gnomon in the spherical hollow of the scaphe. He assumed that the Sun's rays, because of its enormous distance from the Earth, reach us parallel, which we showed in ch. 5 to be true of rays propagated from one and the same point of the Sun. He used the method that *Pedro Nunes*, Book 2 of *On Observation*, p. 132, and *Willebrord Snellius* in the *Eratosthenes Batavus*, Book 1 from ch. 9, report from *Cleomedes*, and it is as follows.

[Margin: Eratosthenes' observations for the measure of the globe of the Earth.]

Diagram. Set into the foot of the left column, with the text running down its left side. At the top a semicircle, curving downward, stands for the Sun, its diameter AD horizontal with C at the middle, and K on its arc. A long vertical line descends from C through E to B. At the foot a semicircle RET stands for

the hemisphere of the Earth, with its diameter RT and its centre B. On its left, at G, a small scaphe FGH sits on the Earth's arc, with the point I at its centre. Long converging lines run from the upper semicircle down to I, and a line runs from I through G to B. Two letters, L and M, stand side by side on the lower arc of the scaphe.

- A (top left, end of the diameter): label A
- C (top centre): label C
- D (top right, end of the diameter): label D
- K (on the upper arc, left): label K
- H (upper end of the scaphe): label H
- I (centre of the scaphe, apex of the gnomon): label I
- F (left end of the scaphe): label F
- G (foot of the gnomon, on the Earth's arc): label G
- L, M (lower arc of the scaphe): labels L and M
- E (top of the Earth's arc): label E
- R (left end of the Earth's diameter): label R
- B (centre of the Earth): label B
- T (right end of the Earth's diameter): label T

Let there be a hemisphere of the Earth with its meridian RET and centre B. Let Syene be at E, and let the central ray CE of the Sun AKD fall perpendicularly over the zenith of Syene, and let the straight line CE be produced to the Earth's centre B. Next let Alexandria be at G, and there let there be a scaphe, or concave spherical vessel, FGH, with a gnomon IG standing perpendicular to the horizon, so that the straight line IGB produced along it falls on the centre of the Earth. Let the extreme ray of the Sun, AIM, be drawn through the apex I of the gnomon to the bottom of the scaphe. At the bottom of the scaphe, then, the shadow will be MG. Now since the quadrant FMG of the scaphe was divided into 90 degrees, and the degrees into some number of minutes, Eratosthenes found the arc MG, and hence the angle MIG, to be 7° 12′. He could also have worked this out from the isosceles triangle MIG, the sides IG (the height of the gnomon) and IM equal to it being already given, since the apex of the gnomon was at the centre of the hemisphere FGH, and the base MG having been measured with compasses. Having found this angle, and treating the ray AIM as if it were the ray KIM, parallel to CB, with IB falling on these parallels, he concluded by Euclid I.29 that the alternate angle IBE, equal to BIM, is 7° 12′. And since the angle GBE subtends the arc GE of the meridian between Alexandria and Syene, he inferred that this arc is 7° 12′. For he thought that Alexandria and Syene lie under the same meridian, or deviate from it imperceptibly, which *Strabo*, Book 2, and *Cleomedes*, Book 1, also assume, although *Ptolemy*, Book 4 of the *Geography*, ch. 5, gives Alexandria a longitude from the Fortunate Isles of 60° 30′, with a latitude of 30° 58′, and Syene a longitude of 62°. He further assumed that there are 5000 stadia between Syene and Alexandria, and as many between Syene and Meroë, which *Strabo*, Book 2, *Cleomedes*, Book 1, and *Pliny the Elder*, Book 2, ch. 73, also

state, although in Book 6, ch. 29, Pliny the Elder says that explorers sent there by Nero found 874 Roman miles between Syene and Meroë, that is, 6992 Roman stadia. But Eratosthenes and others in that region more probably used the Alexandrian stadium, which stood to the common Greek and the Roman stadium as 6 to 5, as I have shown in the Geographical book. Those stadia would therefore be 5827 in Alexandrian stadia. But this in passing.

Now, says Eratosthenes, if the arc GE of the meridian, of 7° 12′, contains 5000 stadia, then one degree of a great circle of the Earth has 694⅘ stadia, and the whole circle 250000 stadia, as *Cleomedes* and *Strabo*, cited above, report of Eratosthenes' opinion. For taking the round number of 700 stadia to one degree, there would be 252000 stadia in the whole circle; and this is the number that *Vitruvius*, Book 1, ch. 6, *Pliny the Elder*, Book 2, ch. 108 and 109, *Censorinus*, *De Die Natali*, ch. 11, *Capella*, Book 6, ch. 4, and Book 8, p. 192, and *Macrobius* on the *Dream of Scipio*, Book 1, prefer to report as Eratosthenes' mind. Such was the method of Eratosthenes, so that *Pliny the Elder* could say, in Book 2, ch. 108: *A shameless venture, but grasped by such subtle reasoning that one is ashamed not to believe it*; and *Macrobius*, in the place cited, confirms that this measure stands *on most evident and indubitable reasons*.

[Margin: The size of the Earth's circumference according to Eratosthenes.]

Yet in fact we have shown in the Geographical book that this method must be corrected, both because Alexandria and Syene are not precisely under the same meridian, and above all because Eratosthenes misused the parallelism of the Sun's rays. For the ray CB, propagated from the centre of the Sun, and the ray AIM, the extreme ray bounding the shadow of the gnomon, are not parallel, not even to sense. He ought therefore to have drawn from the same point C the ray CIL, and to have added to the angle MIG that he found, 7° 12′, the angle AIC subtending the apparent semidiameter of the Sun, which is 15′ or 16′, and hence the angle LIM vertically opposite and equal to it. Thus the angle LIG, and therefore the angle IBE, equal to it to sense, or the arc GE, would be 7° 27′ or 28′, and so great would be the distance of the Sun's centre C from the zenith of Alexandria; or, with my parallax of 30″ taken away, it would be 7° 26′ 30″ or 7° 27′ 30″. Eratosthenes is therefore rightly convicted of this error by *Snellius* and by *Pedro Nunes* in the places already cited, though they had no inkling from it of anything that would overflow onto the obliquity of the ecliptic.

[Margin: The error of Eratosthenes.]

With this premised: the altitude of the pole at Alexandria, or the distance of its zenith from the equator, was observed to be 30° 58′, as *Ptolemy* has it, Book 4 of the *Geography*, ch. 5, and Book 5 of the *Almagest*, ch. 13. If you subtract the arc between Alexandria and Syene, which from the corrected observation of Eratosthenes is 7° 28′, there remains the distance of Syene, and hence of the Tropic of Cancer, from the equator, namely an obliquity of the ecliptic of 23° 30′, as great as is found now also. It was therefore not 23° 51′ 20″ in those times, as is handed down, and that tradition had no other origin than the faulty

observation of Eratosthenes. For Eratosthenes, subtracting from the altitude of the pole at Alexandria, which he perhaps thought to be about 31° 4′, the arc between it and Syene, thought to be 7° 12′, will have found the distance of the tropic from the equator to be about 23° 52′, and hence the distance between the tropics to the whole circle as very nearly 11 to 83; for that is how *Ptolemy* speaks of

[Margin: The obliquity of the ecliptic was 23° 30′ also in the age of Eratosthenes.]

him, and of Hipparchus, in Book 1, ch. 11, indicating that it was not found to be precisely that much. However Ptolemy came to observe the altitude of the pole at Alexandria, it is enough for me to have established, from the pole altitude he gives and from the arc between Alexandria and Syene observed by Eratosthenes but necessarily corrected, that in that age too the obliquity of the ecliptic was no greater than in ours; or else, if you say that they hunted down the distance of the tropics and the altitude of the equator and of the pole by one and the same operation, that all these observations conflict with one another. But we have said more about the altitude of the pole at Alexandria in the Geographical book.

VIII. *Strabo*, towards the end of Book 2, states that at Byzantium the summer solstitial shadow at noon was found to have to the gnomon the ratio of 41⅘ to 120, and from so great a number of small parts I gather that the observation was quite fine. *Snellius*, in Book 1 of the *Eratosthenes Batavus*, reports from Hipparchus that the equinoctial shadow was found to be to the gnomon as 111 to 120. Now let a triangle be made, right-angled at the point where the gnomon stands perpendicular to the horizon, and from the sides about the right angle, namely from the shadow and the height of the gnomon, let the angle opposite the gnomon be found by the analysis of triangles; this angle measures the apparent altitude of the Sun's upper limb above the horizon. The altitude of this limb will be found to be 70° 45′ 25″ at the solstice and 47° 13′ 55″ at the equinox. The difference of these gives the distance of the Tropic of Cancer from the equator, or the obliquity of the ecliptic, 23° 31′ 30″. Before *Strabo*, then, who flourished under Tiberius Nero, that obliquity was very nearly 23° 31½′, and reducible to 23° 30′ by a very small correction of the observation.

[Margin: 2nd ancient observation, for an obliquity of the ecliptic of 23° 30′.]

IX. *Pytheas of Massalia*, who flourished under Alexander the Great, or about 324 years before Christ, observed at Massalia that at noon the solstitial shadow was to the gnomon as 213⅛ to 600, as *Pierre Gassendi* reports from the old records of the people of Massalia in the *Life of Peiresc*, at the year 1635. Gassendi himself, together with the Peiresc already mentioned, in the great hall of a certain college at Marseille, received the Sun's ray at noon on the summer solstice through a hole made in the wall facing south, which was 52 royal feet high. He found that shadow to this gnomon to be 31950 small parts, of which the gnomon, for the sake of greater fineness, was 90000, which is exactly as

213⅛ to 600. From this he inferred that the distance of the tropic from the zenith of Marseille is now exactly the same as it had been in the time of Pytheas and Alexander the Great, and hence that the obliquity of the ecliptic is the same; which, however, *Gassendi* thinks is 23° 31′.

[Margin: 3rd ancient observation, for the immutability of the obliquity of the ecliptic.]

X. *Ptolemy*, Book 2 of the *Almagest*, ch. 6, at Parallel 11, and Book 1 of the *Geography*, ch. 23, at Parallel 10, says that the longest day of $14^{h}\ 30^{m}$ observed at Rhodes is the indication of a pole altitude of 36°. But this cannot hold unless an obliquity of 23° 30′ is assumed: for to this obliquity, in the problems of the primum mobile and in the tables of semidiurnal arcs compiled from them, at a pole altitude of 36°, there corresponds precisely a semidiurnal arc of $7^{h}\ 15^{m}$.

[Margin: 4th argument, from the semidiurnal arc at Rhodes.]

XI. Although a few latitudes of fixed stars favour the opinion of Tycho, who holds that they change latitude from an extrinsic cause, namely from a variable obliquity of the ecliptic, more of them oppose this opinion, as we shall show in the book on the fixed stars, and they suggest to us the immutability of this obliquity, as *Boulliau* also admits in Book 5.

[Margin: 5th argument, from the latitude of the fixed stars.]

XII. The change of the obliquity of the ecliptic is so closely connected with the change of the Sun's eccentricity, of the tropical year, and of the annual motion of the fixed stars, or, as *Copernicus* thinks, of the precession of the equinoxes, that Copernicus may have been most confirmed by this very fact in asserting and investigating the anomaly of these things, as I said at no. 5. But in the book on the fixed stars I shall show that there is no necessity to acknowledge any anomaly in the motion of the fixed stars; and in ch. 30 of this book, that the quantity of the tropical year is more probably called immutable than mutable; and in ch. 26 we showed that there is no necessity to assert a change of the Sun's eccentricity. And *Longomontanus* and *Boulliau* themselves, who admit the change of the obliquity, nevertheless reject the variation of the eccentricity. There is therefore no confirmation from this of a changed obliquity of the ecliptic.

[Margin: 6th argument.]

XIII. Some authors who fight for the change of the obliquity nevertheless fight against themselves, or against the observations from which they deduced that change. Thus *Lansbergen*, in the *Thesaurus of Observations*, p. 41, because his hypothesis gives this obliquity as 23° 32′ in the year of Christ 1207, also states that so much was observed by *Jacob ben Machir ibn Tibbon* the Jew [Prophatio Iudæo] in the year 1207. But *Reinhold*, on Peuerbach's *Theorics*, p. 238, says that this obliquity was observed by *Jacob ben Machir ibn Tibbon* the Jew in the year 1300; and *Copernicus* himself, to whom *Lansbergen* sends us, Book 3, ch. 2, stands against Lansbergen. For he says that *al-Zarqālī* observed the obliquity of the zodiac 190 years after al-Battānī, and Jacob ben Machir 230 years after al-Zarqālī, and so 420 years after al-Battānī. Yet it is established from *Lansbergen*

himself, in the same place, p. 40, that al-Battānī observed in the year of Christ 880; therefore Jacob ben Machir observed in the year of Christ 1300. But more than once I have caught Lansbergen dragging chronology itself by main force towards his hypotheses. *Longomontanus*, too, differs so far from Kepler that he puts the greatest obliquity of the ecliptic in the first year of the world, which for him is 3968 before Christ, while *Kepler* puts the least in the same year, which for him is 3993 before Christ, as we showed above.

[Margin: 7th argument.]

[Margin: Lansbergen's unreliable credit.]

XIV. On the one side, the change of the obliquity of the ecliptic throws very many astronomical tables into disorder and lays an intolerable burden on astronomers, since those tables would have to be altered every century: above all the tables of oblique ascensions, of the latitude of the planets and fixed stars, and of semidiurnal arcs, and the Sun's motion itself as derived from the declination, and finally all problems resting on latitude; and these things surely should not be laid on astronomers without the strongest foundation. On the other side, as *Boulliau* says, Book 5, ch. 5, no physical cause of this change can be brought forward, nor any probable purpose. Indeed, I judge it more probable that God, the Best and Greatest, willed that the royal road, that is, the ecliptic, along which the Sun should travel, be always one and the same, and be a constant limit and boundary of the latitude of the planets; or at any rate that the Sun itself changes latitude rather than that the ecliptic varies, and that from its mean latitude, which our opponents call the mean obliquity of the ecliptic, the declinations and latitudes of the other stars are to be computed, as from a constant boundary. But on this statement, elsewhere. Meanwhile I think I can appropriate what *Kepler* said in ch. 12 of the Rudolphine Tables, without bringing forward any observations: *That the obliquity of the ecliptic was formerly greater is a matter not only doubtful, but overthrown by me with strong arguments from the very observations of the ancients, and beyond doubt plainly false.* On what foundations I have set it at 23° 30′ is clear from no. 4 and from the table placed before no. 5.

[Margin: 8th argument. The change of the obliquity of the ecliptic is most troublesome to astronomers.]

SCHOLIA.

Scholia.

I. Although *Kepler*, as I showed at the end of the chapter, rejects the mutability of the obliquity of the ecliptic outright, yet in the Rudolphine Tables, ch. 34, he offers anyone wishing to retain it five forms of this anomaly. The first is taken from pure trust in the ancient observations, yet limits this motion by a modest correction so that the least obliquity, 23° 28′ 28″, falls at the beginning of the world, or in the year 3993, and again in the year 1329 before Christ and in the year 1338 of Christ; for he gives it a period of 2664 Julian years. The

second comes from what is said in the *Epitome of Astronomy*, pp. 337 and 917, where he says that if the mean obliquity at the beginning of the world were as much greater than that of Eratosthenes as that of Eratosthenes is greater than the obliquity observed in our century, that is, if it were 24° 17′ 42″, then the combined width of the two temperate zones would come to as much as that of the three intemperate ones, namely the torrid and the frigid, and there would be an equal distribution between excess and equality. And this opinion seems to be favoured by what *Māshā' allāh* the Arab reports in Book 2 of the *Astrolabe* from an observation of the Indians, namely that the obliquity of the ecliptic was observed to be greater than 24°. The third form is fashioned from the second, but so that at the beginning of the world the obliquity is set smaller than the mean obliquity of the second form, in order that it may conflict less with the observations thereafter. The fourth form is if at the beginning of the world there is placed that greatest obliquity, which arises from the second form, or from the division of the zones devised there, namely 26° 5′ 20″. Finally, the fifth is mixed from the observations and archetypal causes, or from the distribution of the zones already mentioned, in which the least obliquity is assumed as 23° 30′ 30″, *suited*, he says, *to our century*, the mean as 23° 54′ 5″, and the greatest as 24° 17′ 40″, but in such a way that this mean obliquity held at the beginning of the world.

[Margin: Five forms of the anomaly of the obliquity, feigned by Kepler.]

II. Just as *Rheticus*, from the change of the eccentricity at the cardinal points of the small circle, dared to prophesy about changes of monarchies and the end of the world, as I said in Scholium 1 of ch. 26, so *Longomontanus*, in the *Astronomia Danica*, Book 1 of the Theorics, ch. 4, p. 221, holds that from the change of the obliquity of the ecliptic and of the equinoctial points *one may declare more about the universal state and change of nations, in all the ages of the world.* But we consider this fiction of Longomontanus as empty as that one of Rheticus, especially given such licence in abusing astronomical observations that from the same observations, for nearly the same year and for the beginning of the world, the former ascribes the greatest obliquity of the ecliptic and *Kepler* the least. As for the end of the world, however, our surest rule must be that decree of Christ the Lord in Matthew 24 and Mark 13: *But of that day or hour no one knows, neither the angels of heaven, nor the Son, but the Father alone.* On which St Augustine says, no more acutely than soundly: *This one sentence loosens the fingers of all who calculate.*

[Margin: A vain attempt to predict the end of the world astrologically.]

III. *Girolamo Fracastoro*, for his part, in the *Homocentrica*, sect. 3, ch. 8, thinks it not improbable that the obliquity of the ecliptic was once greater and greater; indeed, on the fabrications of the Egyptians, so great that the equinoctial colure and the ecliptic were once the same, and the Sun rose in the north and set in the south; indeed, that there was once a time when the Sun set where it now rises, as *Herodotus* and *Mela* report from the records of the Egyptians, who boasted such great antiquity that, since they had existed, the stars had traversed their

courses four times, and the Sun had twice set where it now rises. *Girolamo Fracastoro* adds that the obliquity of the ecliptic will diminish more and more, until the ecliptic coincides with the equator without any obliquity; that then, once the Sun's course has been made oblique, the signs that are now northern will turn out southern, and the vernal equinox will be entered from Libra, etc. Anyone might justly call these the ravings of men dreaming ingeniously rather than the inventions of men philosophizing soberly.

[Margin: Fracastoro's fiction about the obliquity of the ecliptic.]

IV. On the change in the latitude of the fixed stars that follows incidentally from the apparent change of the obliquity, I shall have to speak when treating the fixed stars. Meanwhile, if you are in a hurry, see *Pierre d'Ailly*, q. 9 on the *Sphere*; *Tycho*, vol. 1 of the *Progymnasmata*, from p. 233 to p. 247, and in the Letters, pp. 43, 65, 71; *Lansbergen* in the Precepts of the tables, Precept 12; *Boulliau*, Book 5, ch. 4; and *Kepler* in the Rudolphine Tables, chs. 12 and 34, Precept 193. In the meantime, know that this change, established by Tycho from a few stars, has been overthrown by me from many others, as I shall show in the book on the fixed stars, and as I afterwards found in Boulliau.

[Margin: Latitude of the fixed stars.]

V. Scaliger's fiction in Book 4 of *De emendatione temporum*, in the chapter on the year, where he imagines that the pole of the world is different from the pole of the equator, and that the obliquity of the ecliptic must diminish until the two poles are joined, has already been refuted by *Guldin*, *Pétau* and *Boulliau*, as I shall say in ch. 28, no. 10. For Scaliger was one of those who are least wise just when they are wisest.

[Margin: Scaliger's fabrications.]

CAPVT XXVIII.

Chapter 28.

De Præceſſione Aequinoctiorum Motuque Octauæ Sphæræ.

On the precession of the equinoxes and the motion of the eighth sphere.

I. The precession of the equinoxes is taken in two ways. First, in the civil sense, for the time by which the true, celestial or astronomical equinox precedes the day assigned to the civil equinox in the Julian calendar. This precession arises from the fact that *Sosigenes* and *C. Julius Caesar* assumed a tropical year of 365 days and a quarter of a day, or 6 hours, though the quarter is in fact smaller, and ordered that every fourth year a whole day compounded of the four quarters be added, so that the leap year should come out at 366 days, and four years should contain 1461 days. But four tropical years in reality contain only $1460^{d}\ 23^{h}\ 15^{m}$, as is clear from what was said in ch. 15. Therefore in every four years the equinox anticipates by 45^{m}, in 16 years by 3^{h}, and in 128 years by a whole day. It is no wonder, then, if in the year 1582 the true equinox had

come to 10 or 11 March, an anticipation of at least 10 days. But we have already spoken of this precession in chs. 13 and 17.

[Margin: What is the civil precession of the equinoxes?]

II. The precession of the equinoxes, taken astronomically and in the more common sense, is the motion of the equinoctial and solstitial points in *Præcedentia*, that is, against the order of the signs, toward the west, whether true, or apparent on account of the motion of the fixed stars, or of the eighth sphere, in *Consequentia*, that is, in the order of the signs, toward the east. For this mutual recession, namely of the equinoctial points and the fixed stars from one another, some attribute to the fixed stars, others rather to the equinoctial points, that is, to the points at which the equator and the ecliptic intersect; so that however great the motion of the fixed stars in longitude toward the east is for the one party, just so great is the motion of the equinoctial points toward the west for the other. For some this motion is always uniform; by others it is held to be anomalous and irregular. From this arose the anomaly of the precession of the equinoxes [*Anomalia Præcessionis Aequinoctiorum*], which is nothing other than a non-uniform motion of the equinoctial points toward the west, in the opinion of those who think that these points, rather than the fixed stars, move. Among these there was, as it were, a middle opinion, that of those who said that the fixed stars move now against the order of the signs, now in it, and who posited the motion of trepidation, or a second libration in longitude. So that these things may be better understood, I shall briefly indicate the principal opinions and their authors. For although, if this whole motion belongs to the fixed stars, this discussion would pertain to the book on the fixed stars, yet if it belongs to the equinoctial points, it certainly pertains to this book on the Sun, which by its motion, compared with the motion of these points, brings about for us the equinoxes and the equinoctial year.

[Margin: What is the astronomical precession of the equinoxes?]

[Margin: What is the anomaly of the precession of the equinoxes?]

III. First, then, the *Babylonians* and *Egyptians* once thought that the non-wandering stars, or the Aplanes, and the eighth sphere are moved by no other motion than the diurnal one, toward the west, in the space of 24 hours; and so the eighth sphere, glittering with almost innumerable fixed stars, was their primum mobile. This opinion is also attributed to the Greeks, and first of all to Plato, and to Aristotle, Eudoxus, Callippus and most astronomers who flourished in the third century before Christ; although Cicero, *On the Dream of Scipio*, reporting Plato's opinion, and Plato himself in the *Timaeus*, require for the Great Year *that the eight heavens complete their circuits*, by which words they indicate a proper motion of the eighth sphere in longitude. Since, then, the Babylonians thought the fixed stars immobile as regards motion in longitude, it is no wonder if they defined the year by the revolution of the Sun back to the same fixed star.

[Margin: Who thought the fixed stars immobile as regards proper motion.]

IV. *Hipparchus*, however, comparing his own observations with those of Timocharis and Aristillus, found that Spica and all the other fixed stars, which had earlier been farther from the equinoctial points toward the west, were later less distant; and that therefore either they move toward the east, or the equinoctial points toward the west. He wrote a treatise on the transit, or change, of the equinoctial and solstitial points, which *Ptolemy* mentions in Book 3 of the *Almagest*, ch. 2, and Book 7, chs. 2 and 3, where he confirms from his own observations made with the astrolabe that the fixed stars move in an orderly and uniform way in the order of the signs, toward the east, on the poles of the ecliptic, and complete one degree in 100 years, about as much as *Hipparchus* too had found, although he says that Hipparchus once suspected an inequality in this motion and in the equinoctial year.

[Margin: The proper motion of the fixed stars detected by Hipparchus.]

V. Later authors followed Ptolemy, not all of them indeed in the size of the motion, but in its kind and uniformity. That the fixed stars move uniformly in longitude toward the east, in the ecliptic or in circles parallel to the ecliptic, was affirmed by *al-Battānī*, ch. 51; *al-Farghānī*, Differentia 13; *Rabbi Levi*, in the book of the *Defences of God*; *Albuassim* the Moor [?], in the book on the fixed stars; *Avraham Zacuto* and his

[Margin: Uniform motion of the fixed stars.]

disciple *Agostino Ricci*, in the treatise on the motion of the eighth sphere; and, as cited by him, *Rabbi Yehoshua* in the Talmud; *Rabbi Moshe* in the book *Mishneh Torah* [Miſnetora], or of the repeated law; *Rabbi Avraham ibn Ezra* in the book *Taamin*, or of reasons; and *Haly* [ʿAlī ibn Riḍwān] on the *Quadripartitum*, ch. 11; *Tycho*, vol. 1 of the *Progymnasmata*, ch. 2; *Kepler* in the Rudolphine Tables, ch. 34; *Ismaël Boulliau*, Book 5 of the *Astronomia*, ch. 2, and the mathematicians of the Persians cited by him; and King *Alfonso*, as cited by Ricci, p. 29, after he changed his opinion.

VI. Some before al-Battānī, however, as he himself relates in ch. 52, and after him *Regiomontanus* in Book 7 of the *Epitome of the Almagest*, Prop. 6, said that the fixed stars move now toward the east, up to 8°, completing one degree in 80 or 84 years, and then return toward the west through as many degrees; or, as *Peuerbach* has it in the Theoric of the eighth sphere, perhaps following Gerard of Cremona [Gerardo da Cremona], that they advance toward the east only as far as 7°, together with the apogees of the planets, and complete these in 900 years. But *al-Zarqālī* thought that the fixed stars advance toward the east from the head of invisible Aries up to 10° over 750 years, completing one degree in 75 years; that from there they return through as many years and degrees, and through as many again are carried to the west: so that the whole extent of this motion was 20°, and its period from east to west 1500 years, but the whole period from east back to east 3000. *Thābit*, however, if we believe *Erasmus Reinhold* on the Theorics of Peuerbach, p. 251, said that the beginning of the fixed stars cannot recede beyond 22° from the head of Aries of the primum mobile. Further, *Agostino Ricci*, from treatise 2 of *De Mundi fundamento* by

Yitzchak Yisraeli, reports that al-Zarqālī and Thābit were led to this because Hermes, older than Ptolemy by 1985 years, had once found some of the fixed stars at a longer longitude from the beginning of Aries than Ptolemy did later: for example, the Falling Vulture at 24° of Sagittarius, which Ptolemy has at 17° of Sagittarius, and the bright star of Hydra at 7° of Leo, which Ptolemy has at 30° of Cancer. Ricci adds, from the same Yitzchak, that al-Zarqālī thought that in the year of Christ 560 the head of visible Aries was beneath the head of Aries of the invisible zodiac, and that from there it was carried eastward through 10° in 750 years, so that in the year of salvation 1310, when Yitzchak Yisraeli was writing, it should have turned back toward the west. But if this were so, the head of visible Aries ought in the year 1474 to have stood only 7° 11′ from the beginning of invisible Aries; whereas in that year *Avraham Zacuto*, Ricci's teacher and a very great astronomer, observed that it stood 13° 52′ from it, and Spica at 17° 10′ of Libra. Rightly, then, this motion of access and recess, or of trepidation in longitude, was refuted by *Agostino Ricci* in the treatise on the eighth sphere, by *Regiomontanus* against the hypotheses of Thābit, and by others, who attribute that discrepancy between the observations of Hermes and of Ptolemy to equivocal names of stars. The *Alphonsines* too, when in the year 1252 they saw the fixed stars advanced far beyond the limits of al-Zarqālī and Thābit, departed from their opinion.

[Margin: The motion of access and recess of the fixed stars in longitude.]

VII. So the motion of trepidation, or libration in longitude, was asserted or admitted by others in another sense: namely, that the fixed stars always indeed move toward the east by their mean motion, but unequally by their true motion, and at times seem retrograde on account of slowness, and direct on account of speed. This was the opinion of the Alphonsines, that is, of the Jews and Moors who, convened by Alfonso, King of Castile, composed the Alphonsine Tables. Over them, as *Agostino Ricci* reports, and after him *Reinhold* (the former in his treatise, the latter in the Theoric of the eighth sphere, p. 235), presided *Rabbi Yitzchak*, cantor of the Synagogue of Toledo. They defined the period of the eighth sphere as 49,000 years, namely a thousand Jubilees, and the motion of trepidation as 7000 years, namely a thousand Sabbaticals, of which Leviticus 25 and Deuteronomy 15 speak. Although *Reinhold*, p. 163, *Clavius*, p. 56 of the *Sphere*, and *Maurolico* in the *Cosmographia*, p. 88, think that they defined the annual motion of the fixed stars by the precession of the true equinox before the civil one, or from the Sun's residual motion needed to complete the quarter of a day after the tropical year, namely 26″ 26‴ 54⁗, which the Sun completes in the time of 10^{m} 44^{s}. These authors ridicule these fictions, just as *Boulliau* does in Book 2, ch. 3, and Book 5, ch. 2. However, King *Alfonso*, in the fourth year after the publication of the Alphonsine Tables, namely in the year 1256, having read the book of *Albuhassim* [?] on the motion and places of the fixed stars, translated from Arabic into Spanish, in which al-Battānī's opinion was confirmed, followed al-Battānī and published another table of the fixed stars, different from the earlier canon of the motion of the

apogees and fixed stars. So *Avraham Zacuto* relates in his *Magna Compositio*; and it is clear even from this alone, that the northern star in the forehead of Scorpius is placed by Alfonso at 23° 28′ of Scorpius, which *al-Battānī* in the year of salvation 880 had found at 17° 50′. So in 376 years it advanced 5° 38′, and completed one degree in 66 years, which is al-Battānī's very opinion; whereas according to the older Alphonsine canons it ought to have advanced only 4° 32′. The opinion of al-Battānī is followed by *Rabbi Levi*, *Avraham Zacuto*, and *Agostino Ricci* in the treatise on the eighth sphere, pp. 5, 29 and 31.

[Margin: Another sense of the motion of trepidation among the Alphonsines.]

[Margin: Disagreement of the Alphonsine Tables with Alfonso's final opinion.]

VIII. After the Alphonsines, however, *Io. Bapt. Turrianus* [Giovanni Battista della Torre] and *Girolamo Fracastoro*, in the *Homocentrica*, sect. 2, from ch. 9 to ch. 17, said that the motion of trepidation of 7000 years asserted by the Alphonsines is indeed not established, and that if it were admitted in the eighth sphere, more orbs would have to be admitted; but that it is more probable that the fixed stars always advance directly toward the east on the poles of the ecliptic, because from the time of Meton none of the fixed stars has been seen to go back. This motion, however, is anomalous, and the mean motion is one degree in 100 years; from Christ to al-Battānī, over 900 years, it was fast and increasing, until they traversed one degree in 66 years, but from then until the year 1800 it is fast and decreasing, and the whole period of this anomaly is completed in 3600 years.

[Margin: The motion of trepidation according to della Torre and Fracastoro.]

IX. Now *Copernicus*, in Book 3 of *De revolutionibus*, taught that the fixed stars are moved by no motion. The diurnal motion he ascribes to the Earth's revolution about its own centre, completed toward the east in the space of 24 hours; the proper and slow motion that appears in the fixed stars toward the east he attributes to the equinoctial points, always tending against the order of the signs, though on the terrestrial equator rather than the celestial. This motion, he says, is anomalous, and the point of the mean equinox always tends toward the west; but the point of the true equinox sometimes coincides with the point of the mean equinox, as happened in the year 63 before Christ, and sometimes moves on this side of it, sometimes beyond it, and the whole period of this anomaly is completed in 1717 Egyptian years. The anomaly of the obliquity of the ecliptic, however, is completed in an interval twice as long, namely 3434 years; or, as *Magini* has it from the data given by Copernicus, in the Theorics, Book 1, from ch. 2 to ch. 4, the former is completed in 1717 years and 5 days, the latter in 3434 years and 10 days. But the mean motion of the equinoctial point against the order of the signs is daily 8‴ 15⁗, and annual 50″ 12‴ 15⁗, so that the whole period of this motion is 25,816 Egyptian years, but 25,798 Julian. But if the apparent longitude of the first star of Aries is computed from the

true equinox, this motion is at times fast, so much so that when it is fastest one degree is completed in 54 years; at times slow, so much so that at its slowest one degree is completed in 102 years; and at times moderate, and then one degree is completed in 72 years. *Copernicus* adds, in Book 3, ch. 11, that at the beginning of the Olympiads the first star of Aries was more westerly than the point of the mean equinox by 5° 16′, but at the birth of Christ had come to be more easterly by 5° 32′. *Magini*, however, attributes this anomaly and second libration not to a libration of the Earth, as *Copernicus* does, but to the motion of the ninth sphere, just as he attributes the first libration to the motion of the tenth sphere, which varies the obliquity of the ecliptic, and the mean motion of the fixed stars to the motion of the eighth sphere. With Copernicus, however, as regards this libration made in the Earth's equator, agree *Reinhold* in the Prutenic Tables, Precepts 10 and 11; *Lansbergen*, Precepts 4 and 11 of the tables, and in the Theoric of the motions, chs. 1, 7 and 8; and *Longomontanus*, Book 1 of the Theorics, chs. 2 and 4. But they disagree in the size of the motion: *Lansbergen*, with *Reinhold*, fixes the period of this anomaly at 1717 years, but *Longomontanus* at 3600 solar years, as also the anomaly of the obliquity, for he does not make this one twice that period. Both, however, attribute to the fixed stars a proper motion that is uniform from the mean equinox.

[Margin: Copernicus's opinion on this.]

[Margin: And Magini's.]

[Margin: The opinion of Reinhold, Lansbergen and Longomontanus.]

The foundations of this anomaly are, first, the apparent irregularity in the motion of the fixed stars, since they were seen to complete one degree

[Margin: Foundations of this anomaly.]

in Ptolemy's age in 100 years, in al-Battānī's age in 66 years, and in the fifteenth century of Christ in about 72 years; and, second, the inequality of the tropical or equinoctial year, which appeared greater in Ptolemy's time than in al-Battānī's, etc. For both, they hold, arise from the motion of the equinoctial points themselves against the order of the signs, but an irregular motion.

X. In the last place we must point out the fiction of *Joseph Justus Scaliger* in Book 4 of *De emendatione*, in the diatribe on the anticipation of the equinoxes, which *Clavius* tacitly indicates in the *Sphere*, p. 61, but which is refuted by *Pétau*, Book 3 of the Dissertations of the *Uranologion*, from ch. 3; by *Guldin*, in the refutation of the Calvisian *Elenchus* written against the Gregorian calendar, ch. 10; and by *Boulliau*, Book 5, ch. 4. *Scaliger* says that the equinoctial points recede from the fixed stars toward the west, but on mobile poles of the equator, which he further makes different from the immobile poles of the world; and that from this arose the variation of the obliquity of the ecliptic, the change of the meridian line and of the magnetic declinations, and other such things. For if that motion took place, as Ptolemy would have it, on the poles of the ecliptic, the fixed stars ought to have changed their declinations, and some of them to have come closer and closer to the poles of the world and others to have receded. But this, he says, is false, because the Pole Star, which is now near the

north pole and not a full three degrees from it, was near the same pole in the time of Eudoxus, or about the year 367 before Christ. For *Eudoxus* says in his *Phaenomena*: *There is a certain star which always remains in the same place; that star is the pole of the world.* And in *Hyginus*, from Eratosthenes, who flourished 140 years after Eudoxus, he says of Cynosura: *Among the first stars of the tail is the lowest, which is called the pole, as Eratosthenes says, through which place the world itself is thought to turn.* Moreover, Canopus too now grazes the horizon of Rhodes, just as, on the testimony of *Pliny the Elder*, Book 2, ch. 70, it once did; and these are signs of a declination not changed after so many centuries. But about Canopus there is nothing to wonder at, since it is near the solstitial colure, on which the declination of the fixed stars varies insensibly. Nor was the star which is now the Pole Star, and last in the tail of Ursa Minor, the one which in the time of Eudoxus, Eratosthenes and Hipparchus was the Pole Star, or nearest to the pole of the world. For the last star of this tail, as is found in *Ptolemy*, Book 1 of the *Geography*, ch. 7, and *Strabo*, Book 2, stood 12° 24′ from the pole of the world in Hipparchus's time; and it was then the most southerly, as is said in the same place, which is now the most northerly. And all those who with *Magini* emended Ptolemy's text, calling it the most northerly, were deluded, since in the text it is truly called the most southerly. So there was then another star, which then deserved the name of Pole Star; and this one, which is now the Pole Star, is approaching the north pole of the world more and more, as will be shown from the observations of others and from ours in the book on the fixed stars. From this you may also correct *Proclus* in the *Hypotyposes Astronomicae*, and *Giovanni Antonio Delfino*, on the celestial globes, who, moved by the argument from the Bear and Canopus, denied a motion of the fixed stars on the poles of the ecliptic.

[Margin: Scaliger's fabrication.]

[Margin: The error of Proclus and Delfino.]

Noſtra Sententia de Præceſſione Aequinoctiorum, eiuſq. Anomalia.

Our opinion on the precession of the equinoxes and its anomaly.

XI. First, we are certain that from the time of Aristillus and Timocharis down to our own, that is, from the year 330 before Christ to the year of salvation 1640, there exists no observation in which any fixed star has been seen to go back, but that they have always been seen to advance toward the east; and so we consider the fiction of *al-Zarqālī* and *Thābit*, on which see no. 6, completely empty. Second, before Copernicus, *Proclus Diadochus*, in ch. 2 of the *Hypotyposes Astronomicae*, reports that it seemed to some that the equinoctial and tropical points, rather than the stars, move some degrees in either direction and return again to the same places; and *Copernicus*, taking all motion away from the fixed stars, had to transfer this motion too to the equinoctial points of the terrestrial equator. Yet all who are compelled to follow the hypothesis of a stationary

Earth, among whom we shall show in Book 9, sect. 4, that we too stand, with the best right, must recognize this motion in the heavens; and if we rely on the senses, as one must as long as no reason proves the opposite, we must attribute this motion to the fixed stars rather than to the equinoctial points (indeed *Lansbergen* and *Boulliau*, who attribute the annual and diurnal motion to the Earth, still grant the fixed stars a proper motion): whether those stars then, besides the diurnal motion to the west, perform a second, proper motion to the east, or have a single motion to the west, but slower than the Sun's motion which makes the equinoxes, so that the stars seem to move toward the east. Third, although the anomaly or inequality of the apparent motion of the fixed stars toward the east offers at first sight some appearance of probability, when the observations of Ptolemy, al-Battānī and more recent authors are viewed as they lie, yet no necessity has so far appeared for asserting the inequality of this motion as more probable, let alone as evident; and so we prefer to follow on this point the opinion of Ptolemy, and of the others we related in no. 5. For, first, one may rightly have doubts about the unreliability of the ancient observations, since they used the astrolabe and armillae, taking the distance of the Moon from the Sun when it was near the horizon, and then at night the distance of some fixed star from the Moon, so as to determine the places of the fixed stars in this way, as *Ptolemy* teaches in Book 7, ch. 4, and *Copernicus* in Book 2, ch. 14. But that only the tens of minutes were marked on those armillae is gathered from *Ptolemy's* table in Book 7, ch. 5, in which only those tens are indicated. Further, the horizontal refraction of the Sun, omitted by the ancients, gave a distance of the Sun from the Moon differing from the true one by about 30′; and the parallaxes of the Moon and Sun, and the Sun's motion, not so exactly known to Ptolemy, give rise to a suspicion of variation that is not to be despised; so much so that, with everything reckoned, the places of the fixed stars established by them in this way are suspect of an error of at least one degree. From this, I think, it came about that *al-Battānī*, ch. 52, and after him, in nearly as many words, *Regiomontanus*, Book 7 of the *Epitome*, Prop. 6, said: *Whether, then, the multitude of instruments introduced this variation, or nature has placed in the fixed stars some motion still hidden from us, it is very difficult, and will be, to attain the quality of this motion, because of its slowness. For if our forefathers were deceived by their instruments, we too shall necessarily be deceived, since our considerations, unless compared with the considerations of the ancients, can never teach anything. But if we judge that hidden motion to be in the stars, it will be expedient to keep an eye constantly on the fixed stars, and for posterity likewise to weigh the written considerations.* But *Stevin* too, in Hérigone, vol. 5, p. 579, and *Nonius*, Book 2 of *De observatione*, ch. 4, have called the inequality of this motion into doubt, and suspect that it arose from an error of the observations; and *Erasmus Reinhold*, on the Theorics of Peuerbach, begins thus at the opening of the Theoric of the eighth sphere: *Now we enter a disputation in which the very uncertainty and obscurity of the matter seems to contend with the difficulty of the hypotheses, since this motion, because of its excessive slowness, has not yet been sufficiently explored*

even, as I think, in our own age. Finally, *Kepler* in the Rudolphine Tables, ch. 34, Precept 191, says: *This prosthaphaeresis of the equinoxes, of uncertain credit, when taken into the calculation, turns all astronomy upside down.* And *Pierre Gassendi*, in the *Institutio Astronomica*, Book 2, ch. 8, says at the beginning: *perhaps it rests on no firm foundation*, and at the end: *The matter seems too fictitious for us to pursue it more scrupulously.* And *Boulliau*, Book 2, ch. 3, rejects it absolutely.

[Margin: 1. Our assertion.]

[Margin: 2. Assertion.]

[Margin: 3. Assertion.]

[Margin: The suspicion of al-Battānī and Regiomontanus.]

[Margin: And of Stevin and Nonius,]

[Margin: and of Reinhold,]

[Margin: and of Kepler,]

[Margin: and of Gassendi, and of Boulliau.]

Then, from the observations of the ancients themselves, compared with one another and with Tycho's and ours, it is gathered that the motion of the fixed stars was sometimes fast at a time when, according to the Copernican hypotheses, it ought to have been slow, and conversely; which *Tycho* also noted in vol. 1 of the *Progymnasmata*, p. 254, and which will be shown by us in Book 6, ch. 17, from many fixed stars. And we shall add, from *Tycho*, p. 255, that the difference of longitude and the distance of the fixed stars from one another not rarely disagree in the ancients with what he and we have clearly deduced from our observations. Nor is the inequality of the tropical year certain, or more probable, as will be said in ch. 29. Nor do the Prutenic, or the Danish, or Lansbergen's tables, which use the anomaly of the precession of the equinoxes, give us the places of the stars more certainly than other tables; indeed they are sometimes farther from the observations. Why, then, on account of slight suspicions, do we entangle astronomy in one anomaly after another?

Fourth, therefore, we shall choose a uniform annual motion of the fixed stars of 50″, and thus so great that they complete one degree in 72 years, as great as it was in Callippus's time, as *Reinhold* notes on Peuerbach's Theorics, p. 241, and as great as

[Margin: 4. Assertion.]

very nearly, is their true motion according to *Tycho*, *Kepler* and *Boulliau*, or their mean motion according to *Copernicus* and *Longomontanus*, as can be seen in the small table that follows, in which we have set out the principal opinions of others, adding however, in a second table, the opinion on the anomaly of the fixed stars, or of the precession of the equinoxes. The foundation of our opinion consists above all in the observations written below. For *Hipparchus*, on the testimony of *Ptolemy*, Book 7, ch. 2, observed the Heart of the Lion [Cor Leonis], or Regulus, in the fiftieth year of the third Calippic period, that is, 128 years before Christ, at Cancer 29° 50′. But *al-Battānī*, ch. 51, observed the same star in the year of Christ 879, which was of Dijlkarnaim [Dhū al-Qarnayn] 1191 and of Nabonassar 1627, at Leo 14° 0′. *Tycho*, in vol. 1 of the *Progymnasmata*, ch.

2, p. 260, observed it for the year 1600 at Leo 24° 17′; and we, in the year 1644, at Leo 24° 55′ 20″. Therefore if in 1006 years, from Hipparchus to al-Battānī, it advanced 14° 10′, then in one year it certainly completed 50″ 7/10. Again, if in 1729 years, from Hipparchus to Tycho, it advanced 24° 27′, then in one year 50″ 54‴. Likewise, if in 722 years, from al-Battānī to Tycho, it covered 10° 17′, then in one year 51″. And if in 1773 years, from Hipparchus to us, it completed 25° 5′ 20″, then in one year 50″ 1670/1773. But since it is probable that the longitude of the Moon was taken by him about 30′ less than it should have been, on account of the horizontal refraction of the Sun, which diminished the evening distance of the Moon from the Sun; and that therefore Regulus, believed at night, from the lunar distance, to be at Cancer 29° 50′, was at Leo 0° 20′; and that it has thus advanced from that year down to us 24° 35′ 20″ in 1772 years, and in one year 49″ 169/177, and in round numbers 50″. Now let this be Table 1.

[Margin: The foundation of the motion of the fixed stars that we have adopted.]

Motion of the fixed stars, or precession of the equinoxes: AUTHORS	Annual motion of the fixed stars	″	‴	⁗	⁗′	For 1 degree: Julian years	Total period: Julian years
Hipparchus, Ptolemy, al-Farghānī, Proclus	True	36	0	0	0	100	36000
In Callippus's time, according to Reinhold	True	50	0	0	0	72	25920
Formerly the Moors and the Jews, according to Regiomontanus	Mean	45	0	0	0	80	28800
Al-Battānī, Albuassim [?], Alfonso retracting his own view, and likewise Rabbi Levi, Zacuto, Agostino Ricci and Rabbi Yitzchak	True	54	32	43	38 1/3	66	23760
But in the Alphonsines	Mean	26	26	54	0	136 1/9	49000
In the Persian Tables, according to Boulliau	True	52	28	14	7	68	24480
Al-Zarqālī and Thābit ibn Qurra	Mean	48	0	0	0	75	27000
Rabbi Yehoshua, Rabbi Moshe, Avraham ibn Ezra	True	51	25	42	51	about 70	25200
Della Torre and Fracastoro	Mean	36	0	0	0	100	36000
Copernicus, Reinhold, and formerly Magini	Mean	50	12	5	0	about 71	25798
Gassendi	True	51	19	24	0	70	25000
Tycho, Kepler, Boulliau	True	51	0	0	0	70 3/5	25806
Longomontanus and Argoli	Mean	49	45	0	0	72 1/3	26040
Lansbergen	Mean	46	17	0	0	77 7/9	28000
Wendelen, in the *Idea of the Atlantic Tables*	Mean	50	47	0	0	70 8/9	25520
Ourselves and Pétau	True	50	0	0	0	72	25920

Copernican table, from Clavius, ch. 6 of the Calendar, Magini, Mästlin and Reinhold.

Four cardinal points of the anomaly of the equinoxes	Year before Christ	Year of Christ	Tropical year: $365^{d}\ 5^{h}$ and (minutes, seconds, thirds of time)	Equinox, in the form of the Julian year formerly and of the Gregorian afterwards	Years after which the equinox anticipates by one day	Prosthaphaeresis of the equinoxes
Beginning and end of the revolution and of the circle of the anomaly	5211	1653 May 25	55 37 40	March:	329	0° 0′ 0″ 0‴
	3495	1369 February 2	Greatest	Mean, on the 20th; and		
	1780	5084 December 5		true, after midnight.		
	64					
End of the first quadrant and beginning of the second	4783	366 May 2	49 16 29	Mean, on the 20th, after noon.	134	1° 11′ 22″ 30‴
	3068	2082 March 13	Middling	True, on the 21st, about sunset.		Greatest, to be added
	1353	3798 January 10				
End of the first semicircle and beginning of the second	4354	795 April 16	42 55 7	Mean and true, on the 20th, before noon.	84	0° 0′ 0″ 0‴
	2638	2511 March 1	Least			
	922	4226 December 27				
End of the third quadrant and beginning of the fourth	3924	1224 March 31	49 16 29	True, on the 19th, after midnight.	134	1° 11′ 22″ 30‴
	2208	2940 February 16	Middling	Mean, on the 21st, about sunrise.		Greatest, to be subtracted
	493	4655 December 15				

The whole period of the anomaly of the precession of the equinoxes is, for Copernicus, 1717 Egyptian years and 5 days, or 1715 Julian years and 302 or 307 days.

CAPVT XXIX.

Chapter 29.

Qua Hypotheſi, aut Theorica ſolitæ ſint Explicari Anomaliæ Eccentricitatis Solis, Obliquitatis Eclipticæ, & Præceſſionis Aequinoctiorum.

By what hypothesis or theory the anomalies of the Sun's eccentricity, of the obliquity of the ecliptic and of the precession of the equinoxes are customarily explained.

I. Although, from what was said in chs. 26, 27 and 28, we hold that there are no such anomalies, the schemes must nevertheless be indicated by which they are usually set out by those who assert them, or even by those who merely invent them as a hypothesis. For the sake of ease, it is our choice to begin with the more recent authors, so that the theories of the older ones may be better understood.

Lansbergiana Solis Theoria.

Lansbergen's theory of the Sun.

Let ABDCA be the annual orb, or eccentric circle, whether of the Sun or of the Earth, with centre M; let the centre of the world and of the ecliptic be T; let the line of the equinoxes be BMC, which the line of the solstices DMO cuts at right angles at the centre M; and let the line of the apogee be AMTP, on which the apogee or aphelion is A, the perigee or perihelion P, and the eccentricity MT. Now let there be drawn from the centre of the world T a line TR, parallel to DO, as great as the greatest eccentricity of the Sun, which for *Lansbergen* is of 4216 parts of which the radius AM of the eccentric has 100000; and let the least eccentricity TN be of 3490 parts, so that the diameter of the small circle RN is of 726 parts, and the semidiameter ME 363. With centre at E, therefore, let there be described the small circle of eccentricity RMN, carrying on its circumference the centre M of the eccentric, so that while the eccentric carries the Sun, or the Earth, in the order of the signs from A towards B, this small circle carries the centre of the eccentric against the order of the signs from M towards R, completing each day 1″ 11‴ 0⁗ 49⁗′ 19⁗′′, and each year 7′ 12″; since for *Lansbergen* the greatest eccentricity was at the beginning of the years of Christ, when the centre of the eccentric was at R, and the least in the year 1500, when that centre was at N; and the whole period of this small circle is 3000 years. By this small circle, then, first devised by *al-Zarqālī*, as *Regiomontanus* indicates in Book 3 of the *Epitome of the Almagest*, Prop. 13, an account is given of the change in the eccentricity. For once tables of the mean motion of this anomaly have been drawn up, according to the proportion

of daily and annual motion already indicated, and its numbering is begun from the point R, through N, towards M, its equation is found as follows. Let the year of Christ be 1650. The Lansbergian tables of this mean motion will give a motion of 6^{s} 18° 7′ 56″, which is the arc RNM. Discarding, then, the 6 signs for the semicircle RN, there remains the arc MN, 18° 7′ 56″; and so great is the angle MET in the triangle MET, in which the sides are known: ME, the semidiameter of the small circle already mentioned, of 363 parts, and ET, consisting of the semidiameter EN and the least eccentricity NT, that is, of 3853 parts. From these three givens, therefore, by means of trigonometry, you will gather the base MT, namely the eccentricity belonging to the given time, which in our case is of 3512 parts.

[Margin: Anomaly of the eccentricity.]

Diagram. Lansbergen's theory of the Sun, set in the left column, where it interrupts the first sentence of this page. A large circle, the eccentric, is drawn with its top lost behind a small circle and its centre at M. A vertical line runs from the small circle at the top through M to D at the foot; a horizontal line runs from B through M to the small circle at the right; a diagonal runs from A at upper left through M and T to P at the foot. Three small circles stand on these lines: one at the top of the large circle, one at the right end of the horizontal, and one in the interior, to the right of M, passing through M. A vertical line through the interior circle descends past N to T.

- K (top of the upper small circle): upper end of the diameter KL of the small circle of the obliquity
- O (small letter at the centre of the upper small circle): the point O
- L (below the centre of the upper small circle): lower end of the diameter KL
- A (upper left, on the large circle): apogee or aphelion
- B (left, on the large circle): end of the line of the equinoxes
- D (foot of the vertical): end of the line of the solstices
- P (foot of the diagonal): perigee or perihelion
- M (centre of the large circle): centre of the eccentric
- T (on the diagonal below M): centre of the world and of the ecliptic
- R (top of the interior small circle): upper end of its diameter RN
- E (centre of the interior small circle)
- N (foot of the interior small circle): lower end of its diameter RN
- G (top of the right-hand small circle)
- C (centre of the right-hand small circle): the mean equinox
- V (right of the right-hand small circle)
- F (left of the right-hand small circle)
- I (foot of the right-hand small circle)

II. Now the anomaly of the motion of the Sun's apogee must be set out from *Lansbergen*. For the line of the apogee AP moves perpetually: the mean apogee is O, which moves about the centre of the world T, each day, according to *Lansbergen*, with a mean motion of 11‴ 5⁗ 51⁗⁗ 30⁗⁗⁗, and each year 1′ 7″ 30‴

38‴. The true apogee is A, now, when the eccentricity is growing from N to M. For in the semicircle of increase NMR the prosthaphaeresis must be added to the mean apogee to obtain the true one; but when the eccentricity has diminished from R to N, rightwards, the prosthaphaeresis must be subtracted from the mean apogee. In the case already mentioned, therefore, the prosthaphaeresis is the angle AMO, subtending the arc AO; and by Euclid I.29 this angle is equal to the interior and opposite angle ETM, since RT is parallel to OD. This angle is found in the triangle ETM, given, as above, the sides ME and ET and the angle MET contained by them. So in the example already mentioned MTE is 1° 50′ 36″. For the year 1650, the mean motion of the apogee from *Lansbergen*'s tables is 3ˢ 6° 7′ 17″; if you add 1° 50′ 36″ to this, the true apogee results, that is, the arc AOC, 3ˢ 7° 57′ 53″, reckoned from the mean equinox. For *Lansbergen*, the period of the mean motion of the Sun's apogee is 19255 Julian years.

[Margin: Anomaly of the apogee.]

III. In the same diagram the small circle KL contains the anomaly of the obliquity; for its diameter KL is an arc of the solstitial colure, as great as the difference between the greatest and the least obliquity of the ecliptic: namely, for *Lansbergen*, 22′. He posits that in the year 1 of Christ the greatest obliquity of the ecliptic, 23° 52′, was at K, and at that time likewise the greatest eccentricity at R; and that when the right-hand semicircle from K to L had been completed, in the year 1500, the least obliquity was at L, and the least eccentricity at N, 23° 30′. So for him the motion in the small circle KL is as great as in the small circle RN, namely 1″ 11‴ 0⁗ 49⁗′ 19⁗′′ daily and 7′ 12″ annually. Thus the head of Cancer of the mean ecliptic, when the obliquity is 23° 41′, is the point O; but the head of Cancer at the greatest obliquity, and at its most northerly, is the point K, from which the mean motion of this anomaly is numbered rightwards; at the least obliquity, the head of Cancer at its most southerly is the point L. The equation of the obliquity is found by the method to be given below, in Kepler's hypothesis, no. 5. This first libration, therefore, takes place on the diameter KL, although the motion of the anomaly is numbered on the small circle.

[Margin: Anomaly of the obliquity.]

IV. Finally, the small circle FGVI serves the anomaly of the precession of the equinoxes. For the points of the true equinox librate from west to east and from east to west on the diameter GCI, which for *Lansbergen* is of 4320 parts of which the radius MC has 100000, or 2° 28′ 32″ of the degrees of which the ecliptic has 360°; and the mean motion of this anomaly begins from V and proceeds against the succession of the signs towards I: daily 2″ 4‴ 4⁗ 39⁗′ 3⁗′′, and annually 12′ 34″ 48‴ 17⁗ 37⁗′. So when the anomaly is nil, that is, at V, or 180°, at F, the head of Aries of the primum mobile and that of the second mobile coincide at C, and the mean and true equinoxes are the same. But when the anomaly is 90°, at I, the prosthaphaeresis of the equinoxes is greatest and is to be subtracted from the mean equinox C, so as to obtain the point of the true equinox at I, a point of the ecliptic OBDC. But if the anomaly is 270°, and

at G, the prosthaphaeresis is greatest and is to be added, etc. And in the first semicircle VIF the equation is subtracted from the mean equinox to obtain the point of the true equinox, which is therefore farther west than the point C, and so the true equinox occurs sooner than the mean; but in the second semicircle of the anomaly, FGV, the equation is added, and the true equinox occurs later than the mean. Now it is clear from what has been said that the semidiameter CI, or the arc of the greatest equation, is for *Lansbergen* 1° 14′ 16″, which the Sun completes in one day and about 7 hours; and therefore that there cannot be a greater difference between the true and the mean equinox than $1^{d}\ 7^{h}$. Further, the method of finding the prosthaphaeresis of the equinoxes, given the mean motion of this anomaly and the semidiameter of the small circle CI, is similar to that by which the equation of the obliquity is found, namely by sines, as we shall show below in explaining Kepler's theory, at no. 6.

[Margin: Anomaly of the precession of the equinoxes.]

[Margin: The greatest difference between the true and the mean equinox.]

Kepleri Hypotheſis Pro Anomalia Obliquitatis Eclipticæ, & Præceſſionis Aequinoctiorum.

Kepler's hypothesis for the anomaly of the obliquity of the ecliptic and of the precession of the equinoxes.

V. In the Rudolphine Tables, ch. 34, *Kepler* does not explain his invented hypothesis of the anomaly of the obliquity and of the precession with figures, but indicates it by the bare names of small circles, and proposes five forms, of which we spoke above in ch. 26, in the Scholium. Although this is so, it seems that it can be explained as follows by the scheme below. Let AB be a portion of the solstitial colure, as great as the greatest obliquity of the ecliptic; let B be the pole of the equator and C the pole of the mean ecliptic, from which let a circle ADEF be described, carrying round the pole of the moving ecliptic from A, the point of greatest obliquity, against the series of the signs towards G; and let the semidiameter of the circle, AC, be the difference between the greatest and the mean obliquity: namely, in Kepler's first form 12′ 24″; in the second and third 1° 47′ 40″; in the fourth 0° 41′ 10″; and finally in the fifth 0° 23′ 35″. Let the mean obliquity BC in the first form be 23° 40′ 55″, the greatest, BA, 23° 53′ 19″, and the least, BE, 23° 28′ 31″. But in forms 2, 3 and 4 let the mean obliquity be 24° 17′ 40″, the greatest 26° 5′ 20″, the least 22° 30′. Finally, in the fifth let the mean obliquity be 23° 54′ 5″, the greatest 24° 17′ 40″, the least 23° 30′ 30″. Now if the anomaly of the obliquity, or its argument, from the tables of mean

[Margin: Anomaly of the obliquity.]

motion is nil at A, the obliquity will be greatest; if it is 6 signs, or 180°, and at E, it will be least; but if 90° at D, or 270° at F, the mean and true obliquities will coincide. Beyond these limits, the terminus of the anomaly will be either in the first quadrant, as at G; or in the second, as at I; or in the third, as at L; or in the fourth, as at M. With semidiameters drawn, then, to these points, namely

CG, CD, CI, CL, CF, CM, and with the right sines GH, IK, LK, MH, right triangles will arise by the definition of sines; from one of these you will learn the analysis of the others. Let the terminus of the anomaly of the obliquity, then, be at G, and its mean motion the arc AG, which the angle GCA measures. Look up in the sine tables its second sine CH; multiply this by the semidiameter of the circle given according to one of the five Keplerian forms, or others; divide the total by the radius of the sine table, and the arc of the equation HC will result, to be added to the mean obliquity BC, so that the true obliquity BH may be known. But if the terminus of the anomaly were at M, and the arc of the anomaly ADEFM, then, discarding three quadrants, there will remain the arc FM, whose right sine will be CH; use it as above. And so in the first and fourth quadrants of the anomaly the equation will be added to the mean obliquity. Now if the terminus already mentioned is in the second quadrant, as at I, and the arc of the anomaly is ADI, then, discarding the quadrant AD, the arc DI will be known, whose right sine is KC; but if it is at L, and the arc of the anomaly is ADEL, then, discarding the semicircle ADE, the arc EL will be known, whose second sine is KC. In both cases, therefore, multiply the sine KC by the semidiameter of the circle given in whichever form you choose to use, and divide the product by the whole sine; for the equation KC will result, in the second and third quadrants of the anomaly, to be subtracted from the mean obliquity BC, so that the true obliquity BK remains.

[Margin: Equation of the obliquity.]

Diagram. Kepler's hypothesis, set in the right part of the left column with the text wrapped round it. A long vertical line runs from B at the top, through the top of a circle, to A at the circle's foot. A horizontal diameter joins D and F; two horizontal chords join I and L above the centre and G and M below it; diagonals run through the centre joining the points on the circle.

- B (top of the long vertical): pole of the equator
- E (top of the circle)
- K (on the vertical, where the upper chord crosses it)
- I (left end of the upper chord)
- L (right end of the upper chord)
- D (left end of the horizontal diameter)
- F (right end of the horizontal diameter)
- C (small letter, at the centre): pole of the mean ecliptic
- G (left end of the lower chord)
- M (right end of the lower chord)
- H (on the vertical, where the lower chord crosses it)
- A (foot of the circle): point of greatest obliquity

VI. You will find the prosthaphaeresis of the equinoxes in exactly the same way in the scheme already mentioned, supposing AB to be a portion of the ecliptic and C the point of the mean equinox, but F the point from which the anomaly, or argument of the precession, begins to be numbered towards A; for when the anomaly is nil at F, or 180° at D, the mean and true equinoxes will

coincide. But if the anomaly is 90° at A, the prosthaphaeresis of the equinoxes will be greatest, namely the arc AC, to be subtracted from the mean equinox; but if 270° at E, it will be greatest and to be added. By Kepler's invented hypothesis it is indeed 30′ 31″ in the first form, and so great is the semidiameter of the small circle AC. Outside these cases the prosthaphaeresis is sought exactly as was said above of the prosthaphaeresis of the obliquity: by seeking the right or second sines HC or KC from the mean motion of this anomaly, multiplying them by AC, or the greatest prosthaphaeresis, and dividing the total by the whole sine; for from this the prosthaphaeresis will result, to be subtracted in the first semicircle of the anomaly, FAD, and added in the second, DEF.

[Margin: Anomaly of the precession of the equinoxes, and equation.]

VII. *Kepler* and *Lansbergen* differ, moreover, not only in the measures of the semidiameters and in the mean motions, but also in the epochs. For in Kepler's first form the greatest obliquity of the ecliptic, 23° 53′ 16″, with nil prosthaphaeresis of the equinoxes, was in the year 2661 before Christ, and in the years of Christ 6 and 2670. The least obliquity, 23° 28′ 28″, likewise with nil prosthaphaeresis of the equinoxes, was in the years 1329 and 3993 before Christ, at the beginning of the world, and in the year of Christ 1338. And the mean obliquity, 23° 40′ 55″, with the greatest prosthaphaeresis of the equinoxes, 30′ 31″, to be added, was in the year 1995 before Christ and of Christ 672; with it to be subtracted, in the years 663 and 3327 before Christ and of Christ 2004. And the whole period of the anomaly is of 2664 years. *Kepler* notes, moreover, in the Rudolphine Tables, Precept 187, that if there is no change in the obliquity of the ecliptic, or one such that the pole of the moving ecliptic librates on the solstitial colure from A to E, there is no anomaly of the equinoxes; but that if the pole of the ecliptic is carried round the circumference of the circle, say AGEF, an anomaly of the equinoxes arises, and a prosthaphaeresis of them; and that this is to be added in the first semicircle of the anomaly and subtracted in the second. But *Kepler* begins the anomaly from D, towards E; *Lansbergen*, by contrast, from F, towards A.

[Margin: The difference between Kepler and Lansbergen.]

Longomontani Hypotheſis.

Longomontanus's hypothesis.

VIII. *Longomontanus*, in Book 1 of the *Theorics*, ch. 1, from p. 160, and ch. 4, p. 220, rejects the annual motion of the Earth and that of the Earth's libration, while keeping the Earth's daily motion. He assigns the anomaly of the obliquity to a motion of the poles of the terrestrial equator about the axis of the ecliptic. For this, let A be the north pole of the terrestrial ecliptic, from which let two concentric circles be described. Of these, BC is the arctic ecliptic small circle: let it move from C to B, against the order of the signs, and measure the precession of the equinoxes, or the apparent motion of the fixed stars in the order of the signs; and let the arc H be a portion of the mean ecliptic; and

from centre B let the celestial equator be understood as described, of which the arc L is a portion. Now let the true pole of the ecliptic on the surface of the terrestrial globe move about its own axis, in the order of the signs, on the small circle FEDG; and let it be supposed to have changed its apogee with the motion CB. For with the pole of the terrestrial ecliptic set at F, the distance of the ecliptic from the equator will be greatest, by the interval KL, to which FB is equal. But if the pole F ascends through E to D, the obliquity will be least, LI, to which BD corresponds; and at E or G the obliquity will be mean. The greatest prosthaphaeresis of the equinoxes is at E, subtractive, and at G, additive. *The semidiameter of the small circle AF is 10′ 53″, namely in the ratio which the third number in the order of perfect numbers, that is 496, has to the quadrant of the circle on the surface, 5400, both being understood in minutes.* This is evidently why he fixed the difference of the mean obliquity from the greatest and from the least at 10′ 53″. For him the greatest is 23° 53′, in the first year of the world and in 3600; the least 23° 31′, in the year of the world 5400; and the period, both of this and of the precession of the equinoxes, is of 3600 years. So far *Longomontanus*, with his customary obscurity and wrappings of words, which it is neither pleasant nor useful to unfold at greater length, especially in a hypothesis that is not necessary, and without which astronomy proceeds more readily and no less safely.

Diagram. Longomontanus's hypothesis, set in the right part of the right column with the text wrapped round it. A long vertical line runs from L at the top to M at the foot. Near the top it is crossed by a long curved arc just below L, then by two shorter strokes and a third below them. Lower down, two concentric circles are centred on the vertical; a horizontal diameter is drawn across the inner circle.

- L (top of the vertical, above the long arc)
- I (on the vertical, below the long arc)
- H (on the vertical, below the first short stroke)
- K (on the vertical, below the second short stroke)
- B (on the vertical, below the third stroke and at the top of the outer circle)
- D (on the vertical, at the top of the inner circle)
- A (centre of both circles): north pole of the terrestrial ecliptic
- G (left end of the inner circle's horizontal diameter)
- E (right end of the inner circle's horizontal diameter)
- F (on the vertical, at the foot of the inner circle)
- C (left of the outer circle)
- M (foot of the vertical, below the outer circle)

Copernici Hypotheſis.

Copernicus's hypothesis.

IX. *Nicolaus Copernicus*, in Book 1, ch. 11, and Book 3, ch. 3, ascribes the change in the obliquity of the ecliptic and of the equinoctial points, rather

than of the fixed stars, to the motion of the Earth and to a certain flexing of the terrestrial equator and of its poles, by which it is inclined reciprocally to the plane of the celestial ecliptic. For if the annual motion of the axis of the terrestrial equator agreed exactly with the motion of the centre in longitude, no precession of the equinoxes, he says, would appear. But since they differ from one another, and with an unequal difference, the equinoxes must precede the places of the stars with an unequal motion. In the same way, the unequal inclination of the axis of the terrestrial equator to the celestial ecliptic, towards the poles of the ecliptic, alters the apparent obliquity of the ecliptic. Therefore two reciprocal motions must be understood in the poles of the terrestrial equator, resembling suspended librations, quicker in the middle and slowest around the extremes. By one of them the already mentioned poles librate up and down; by the other they deflect crosswise in longitude. The first completes the anomaly of the obliquity in 3434 Egyptian years, 10 days. The second accomplishes the anomaly of the precession of the equinoxes in half that time, that is, in 1717 years, 5 days. So while one restitution of the obliquity takes place, the whole libration of the equinoxes is completed twice, and those two librations, running together, finally make the poles of the Earth describe certain lines resembling a twisted garland, as in the following figure.

[Margin: Copernicus's theory.]

Let the ecliptic ABCD be described from centre E, in which the beginning of Capricorn is A, of Aries B, of Cancer C, and of Libra D. Let the solstitial colure be RAC, and let the celestial and mean equator, described from pole I and substituted here for the terrestrial one, be RBHD, whose moving poles go up and down through the plane of the colure RC. Let the greatest distance of the poles of the ecliptic and the equator, which produces the greatest obliquity, be FE, and the least GE. Let B and D be the points of the mean equinox, which are to be understood as moving with an equal motion in *Præcedentia*, that is, against the order of the signs, slowly beneath the sphere of the fixed stars. Then let there be two motions of the terrestrial poles: one between the limits F and G, the other, twice as fast, crosswise, from the preceding parts towards the following and from the following towards the preceding. For when the Earth's north pole stands under F, the equinoctial will pass through points B and D. But if the pole, on its way from F, the point of greatest obliquity, to I, the point of mean obliquity, does not travel along the straight line FI, but crosswise in the order of the signs, to K, by force of the second motion, then the equinoctial OQP, etc., will be described from pole K, and the true or apparent equinoctial points will be O and P, more easterly than the mean points B and D. Therefore the true equinoxes will occur later than the mean by as much time as the arcs BO and DP of the ecliptic require; the precession of the equinoxes will be diminished; and the apparent motion of the fixed or other stars, computed from the true equinox, will be less than when computed from the mean equinox. Now let the pole of the equator turn from K against the order of the signs towards I: for when it has reached I, the point of mean obliquity, the apparent

equator will return to the mean equator, and the mean and true equinoxes will coincide at points B and D. Now from I, while the pole of the equator makes for G by force of the first libration, let it be carried towards L by force of the second libration, so that at L there is the greatest deflection against the order of the signs. For from pole L an equator will be described that cuts the ecliptic at points more westerly than the mean points B and D; and therefore the true equinoxes will occur sooner than the mean, and the apparent motion of the stars, reckoned in longitude from the true equinox, will be greater than when computed from the mean equinox.

Diagram. Set in the left column, between the words "maxima distantia" and "polorum Eclipticæ". Two large overlapping circles, one standing higher than the other, cross on the left and right; a third, partial arc runs below them from the left, past O and Q, and rises to P on the right. A vertical line runs from the top of the figure to its foot. Around I on that line is a very small twisted band, drawn as two small loops one above the other. Labels read, none unread:

- R (top of the vertical, on the upper circle)
- A (on the vertical, below R, at the top of the lower circle)
- F (on the vertical, above the small loops)
- I (on the vertical, in the upper loop)
- K (left of the upper loop)
- N (right of the upper loop)
- M (left of the lower loop)
- L (right of the lower loop)
- G (on the vertical, below the loops)
- E (on the vertical, below G)
- B (left, where the large circles cross)
- D (right, where the large circles cross)
- P (right, on the partial arc, above D)
- O (far left, at the end of the partial arc)
- Q (on the partial arc, lower right of E)
- H (on the vertical, at the foot of the upper circle)
- C (foot of the vertical, on the lower circle)

Next, let the pole of the equator travel from L in the order of the signs towards G, the point of least obliquity: for the true equinoctial points will again coincide with the mean, and thus one period of the anomaly of the equinoxes will be completed, a whole circle as it were, consisting of the semicircles FKI and ILG. Meanwhile, however, only half of the first libration will have been completed, namely from F, the greatest distance of the poles, to G, the least. The rest will be completed while the period of the other libration is completed from point G to M, from M to I, from I to N, and from N to F; and thus, by force of the two motions already mentioned, the twisted garland FKILGMINF will be described.

X. However, since Copernicus maintains that celestial motions are composed of circular and equal motions, even though they appear unequal to us, he adds

this second diagram in Book 3, ch. 4, in order to demonstrate that this too can happen in the two librations already mentioned. Let the straight line BE be divided into four parts at C, A and D, and from A let two concentric circles, CIDL and BKE, be described. Taking any point F, let the circle HGA be described from it as centre, whose diameter HA will be equal to the diameter CD, and from H let the straight line HG be drawn perpendicular to BE. For if the moving body H in circle HGA is understood to move equally against the order of the signs towards G, with a motion twice as fast as the equal motion of centre F made by circle CID in the order of the signs, the moving body H will be held on the straight line BE, provided that the motion began from point B. For while centre F, starting from C, completes the half-quadrant CF, the moving body that was at B, having completed the quadrant HG in the second circle, will be at G. And when centre F has reached I, the quadrant CI being completed, the same moving body, having completed the semicircle HGA, will be at A. So when that centre has completed the semicircle CID and reached D, the revolution of the second circle, from H through G and A back to H, will be complete. It is therefore clear how a reciprocal and unequal motion in a straight line can be composed from two motions that are circular and equal taken separately but meet one another; and how the anomaly of the obliquity can be produced by four small circles, that is, two at the north pole and two at the south, and the anomaly of the equinoxes by four others, that is, two at the beginning of ♈ and two at the beginning of ♎.

[Margin: Straight motion arising from circular motions.]

Diagram. Set in the right column, with the text running down its left side. A large circle about A, with a vertical diameter BE and a horizontal line from K through A to L and on to the circle. Inside it a smaller circle concentric about A, and a third circle through H, G and A, whose centre F lies on the straight line from H to A. A straight line runs from H to G. Labels read, none unread:

- B (top of the large circle)
- H (upper left, on the large circle)
- G (on the vertical, below B)
- F (on the line HA)
- C (on the vertical, at the top of the inner circle)
- K (left of the large circle)
- I (left, on the inner circle)
- A (centre, just below the horizontal)
- L (right, on the inner circle)
- D (on the vertical, at the foot of the inner circle)
- E (foot of the large circle)

From the data given by Copernicus, Reinhold in the Prutenic Tables, and Magini in his *Theoricae*, Book 1, and in his *Secunda Mobilia*, Canon 18, set the semidiameter AB at 12′ and the diameter BE at 24′, of the units of which the solstitial colure has 21600, and this is the size of the difference between the greatest obliquity of the ecliptic, which for Copernicus is 23° 52′, and the

least, which is 23° 28′. The anomaly of the obliquity is counted in circle BKE, starting from B towards H, and the mean motion of this simple anomaly is, daily, 1″ 2‴ 2⁗ 30⁗′, and, annually, 6′ 17″ 24‴ 9⁗; its period is 3434 Egyptian years and 10 days. Given the mean motion of this anomaly, say arc BH, the angle HAB is given, and its second sine GA. If then one makes the proportion: as the radius or whole sine AB is to the sine AG, so the semidiameter AB of 12′ is to another quantity, the difference in minutes will result. This is to be added to the mean obliquity, which occurs at A, so that the true obliquity of the ecliptic at the given time may be known. But in the semicircle of the anomaly KE, etc., it would have to be subtracted.

[Margin: The commensurations of these small circles, from Copernicus.]

In the circles of the anomaly of the equinoxes, the semidiameter BC, for Copernicus, for the authors already mentioned, and for Clavius in ch. 6 of the *Calendar*, is 35′ 41″ 15‴; or, if you proceed less scrupulously, with Gassendi in the *Institutio Astronomica*, Book 2, ch. 8, and with Deusingius [Anton Deusing] in the *Cosmographia*, it is 35′, and AB is 1° 11′ 22″ 30‴, or 1° 10′. The anomaly is counted in circle BKE, and its mean motion is twice as great as the motion of the anomaly of the obliquity. It is therefore called the doubled anomaly; its daily motion is 2″ 4‴ 5⁗, its annual motion 12′ 34″ 48‴ 18⁗, and it is completed in 1717 Egyptian years and 5 days. The prosthaphaeresis of the equinoxes is found proportionally, as the prosthaphaeresis of the obliquity is, through the sines, from the given mean motion of the anomaly; and its greatest value is AB, namely 1° 11′ 22″ 30‴.

Finally, the mean motion of the mean equinoctial points, namely of point A, against the order of the signs (or, as others would say, the motion of the fixed stars), is for Copernicus 8‴ 15⁗ daily and 50″ 12‴ 5⁗ annually, with a period of 25816 years. But Gassendi, in Book 3 of the *Institutio Astronomica*, ch. 8, judging that these matters, being uncertain, are to be treated less scrupulously, gives the period of the obliquity as 3400 years and that of the anomaly of the equinoxes as 1700 years.

[Margin: In favour of Copernicus: the votes of Rheticus, Mästlin, Gilbert, Galileo, Baranzano, and formerly of Kepler.]

XI. Copernicus's theory has further been followed by Georg Joachim Rheticus in the *Narratio prima* on the books of Copernicus; by Michael Mästlin in his notes to the same *Narratio*, which are found in Kepler's *Prodromus*, or *Mysterium Cosmographicum*; by William Gilbert in *De Magnete*, Book 6, chs. 8 and 9; by Redemptus Baranzano in the *Uranoscopia*, part 2, question 3; by Galileo in Dialogue 3 of the *De Systemate Cosmico*, p. 281; by Hérigone in vol. 5, p. 577; and, formerly, by Kepler in the *Epitome of Copernican Astronomy*, Book 3, from p. 243, for later, in the Rudolphine Tables, he abandoned both anomalies. Of these, Rheticus deduces from Copernicus's hypothesis that in Ptolemy's time the true equinox was after the mean by 47′ of the ecliptic, and in al-Battānī's time before the mean by 22′. Mästlin sets out this difference in the following diagram. In it the small circle of the anomaly of the equinoxes has the

semidiameter CD, of 1° 11′ 22″ 30‴, from the Prutenic Tables; AB is a portion of the ecliptic; the centre C is the point of the mean equinox, at which the mean equator cuts the ecliptic; and A is the first star of Aries, from which point C perpetually recedes against the order of the signs towards B, with an equal motion as great as the motion of the fixed stars in the order of the signs among other authors. The segments or arcs of the remaining circles are portions of the true equator, shifted by the motion of the second libration of the Earth's axis. So at F was the true equinox in Hipparchus's time, and CF was the subtractive prosthaphaeresis of 21′. G was the point of the true equinox in Ptolemy's time, and GC the additive prosthaphaeresis of 47′. And H was the point of the true equinox in al-Battānī's time, and CH the subtractive prosthaphaeresis of 22′, in units of which CD, or the greatest prosthaphaeresis, is 71′ 22″ 30‴.

Diagram. Set in the left column, with the text running down its left side. A small circle crossed by a slightly curved line that runs from a star at the far left, through the circle, to the far right; four parallel oblique lines cross the circle from upper left to lower right, extending beyond it at both ends. Labels read, none unread:

- A (far left, above the star at the end of the line)
- E (below the line, just outside the left of the circle)
- K (upper left, outside the circle)
- G (above the line, in the left part of the circle)
- C (below the line, near the centre of the circle)
- F (above the line, right of C)
- H (above the line, right of F)
- D (below the line, at the right of the circle)
- B (far right, at the end of the line)
- M (below the circle, lower right)

Io. Antonij Magini Hypotheſis : quæ eſt ferè Verneri, & Gaſſendi, & Deuſingij.

The hypothesis of Giovanni Antonio Magini: which is almost that of Werner, Gassendi and Deusingius.

[Margin: Magini's hypothesis, mixed from the Copernican and the Wernerian.]

XII. Magini, in his *Theoricae*, Book 1, and in his *Second Mobile* from Canon 18, adopts Copernicus's hypothesis as regards the numbers and the figures, using the figures discussed in nos. 9 and 10. But he transfers it from the Copernican Earth to the heavens, and follows the number and order of the spheres handed down by Johannes Werner. For Werner, as Erasmus Oswald Schreckenfuchs explains in his work on Peuerbach's *Theorica* of the eighth sphere, posits eleven spheres: the highest for the motion of the primum mobile; the tenth for the first libration, or trepidation, towards the sides of the world, by which he maintains the anomaly of the obliquity of the ecliptic; the ninth for the second libration, in

longitude, or for the anomaly of the equinoxes; and the eighth for the equal and always progressive motion of the fixed stars. He would have the beginnings of Cancer and Capricorn of the 11th sphere be the poles around which the beginnings of Cancer and Capricorn of the 10th sphere describe the small circles of the anomaly of the obliquity; and the beginnings of Aries and Libra of the 10th sphere be the poles around which the beginnings of Aries and Libra of the 9th sphere describe the small circles of the anomaly of the equinoxes. Following this order, therefore, Magini attributes to the heavens whatever Copernicus had attributed to the librations of the Earth, and he uses the figures already set out by us in nos. 9 and 10, and the period and quantity of motion in each anomaly, exactly as the Prutenic Tables and Copernicus's hypotheses require. So there is no need to repeat what has been sufficiently set out in those numbers. Gassendi, however, in Book 2 of the *Institutio Astronomica*, gives the same number of spheres and nearly the same numbers in the motions of each anomaly; but he concludes that these things are fictitious and are not to be treated more scrupulously.

[Margin: Gassendi's judgement on this hypothesis.]

Arzahelis, Thebitij, Alphonſinorum, & Purbachij Theorica.

The theory of al-Zarqālī, Thābit ibn Qurra, the Alphonsines and Peuerbach.

[Margin: Authors who have written on the motions of the eighth sphere.]

XIII. On the theory of the authors already mentioned concerning the motions of the eighth sphere, and especially the anomalies of the equinoxes, there survive, first of all, the treatises on the motion of the eighth sphere by Thābit himself, then by Campanus of Novara, Albert Pigge, Marcus Beneventanus and Agostino Ricci; several questions by Pierre d'Ailly; Regiomontanus's short work against Thābit, and Problem 63 of the *Primum Mobile*; and some few things in Maurolico, Dialogue 3 of the *Cosmographia*, from p. 88, and in Boulliau, Book 5, ch. 1. There is also the theory of the motion of the eighth sphere by Georg Peuerbach, who illustrated the Alphonsine hypotheses. It has been explained, in notes, commentaries or questions, by Erasmus Reinhold, Erasmus Oswald Schreckenfuchs, Capuano, Nonius and Christian Wurstisen, where the hypotheses of Thābit and al-Zarqālī are also touched on, and by Io. Esler [?] in the *Speculum Astrologicum*, where Thābit's theory too is indicated. From these sources, then, take the following in summary.

[Margin: Al-Zarqālī's hypothesis.]

Al-Zarqālī the Moor, as also Yitzchak Yisraeli in his treatise *On the Foundation of the World*, said that the beginnings of both Aries were conjoined in the year of salvation 560; that from there the beginning of the movable Aries advanced towards the east; that the fixed stars do not advance beyond 10°, and complete these in 750 years, taking 75 years for 1°; that they then return through another 10°, and through as many go back from their original point towards the west;

and that thus in four times 750, that is, 3000, years the period of this trepidation is completed.

[Margin: Thābit's hypothesis.]

Then came Thābit ibn Qurra, who allowed that the fixed stars recede back and forth 22° or 21½°, and no further, but go back through as many degrees; and that the points of the true equinox recede from the mean by 4° 18′ 43″, or, in round figures, 4° 19′, which is the size of the semidiameter of the small circle described by the beginning of the movable Aries around the beginning of the mean Aries, the whole diameter being 8° 37′ 26″, or, in round figures, 9°. Finally, he held that this trepidation is completed in 4320 years, according to Capuano, or in 4056 years and 331 days, according to Campanus. As to whether Thābit admitted a variable obliquity of the ecliptic, Regiomontanus affirms it, on the ground that the beginning of the starry Aries revolves and varies the inclinations of the movable ecliptic, and so does Ricci, above, p. 40; d'Ailly, question 9 on the *Sphere*, and Maurolico, in the passage cited, deny it. So it is doubtful whether he posited 9 spheres, as most relate of him, or 10, as Clavius says on p. 44 of the *Sphere*.

[Margin: The Alphonsines' hypothesis.]

XIV. The Alphonsines, however, when in the year of salvation 1250 they noticed that the fixed stars had progressed beyond 22° of longitude, rejected the hypothesis of al-Zarqālī and Thābit and posited 10 spheres. The highest of these was the primum mobile. The ninth accomplished the mean motion of the fixed stars and of the apogees, with an always progressive motion, in a period of 49000 years, that is, a thousand Jewish jubilees. And the eighth, filled with the fixed stars, accomplished the motion of trepidation in longitude in 7000 years, namely a thousand sabbatical years, in accordance with what was said in ch. 28, no. 7. So that what we have said and shall say may be more easily understood by means of a diagram, let A be the beginning of Aries fixed in the highest sphere of the primum mobile, AC a portion of the fixed ecliptic, and B the beginning of Aries of the ninth sphere, which in the first year of Christ was conjoined with A; but in Alfonso's time, that is, in the year 1250, according to the measure of the mean motion of the fixed stars and apogees, it was distant from A by the arc AB, of 9° 18′, and in the year 1508, by 11° 5′. Now in the eighth sphere, beneath point B of the ninth, let a centre be understood, from which is described the small circle KLMN of the anomaly of the equinoxes, of the fixed stars and of the apogees, and let this circle be completed in

7000 years, in which the whole period of the trepidation is accomplished. Let its semidiameter BL be not 4° 18′ 43″, as Thābit would have it, but 9°, as the Alphonsines and Peuerbach have it. This is the greatest equation, to be added to or subtracted from the mean motion of the fixed stars and apogees, so that their true motion may be had, which is computed from A, the beginning of Aries of the primum mobile, although the motion of access and recess is computed from centre B of the eighth sphere, which lies beneath the beginning of Aries of the 9th sphere. Now from D, the pole of the fixed ecliptic, let the arc DBK of the

colure of the mean equinox be drawn, and let K be the most northerly point of the small circle, from which the anomaly begins towards L. For the mean and true motions coincide when the beginning of Aries, at which the true equinox occurs, is at M or at K, as it was, according to the Alphonsines, in the year 15 of Christ, with 137 days completed. But in the year 1508 the beginning of Aries was at H, and the arc KH was 76° 46′, to which corresponds the arc IB, 8° 45½′. This, added to the arc BA, which we said was 11° 5′ in the year 1508, makes the arc IA 19° 50′ 30″, namely the distance of the first star of the starry Aries from A, the beginning of Aries of the primum mobile.

[Margin: The Alphonsines' deviation.]

This, however, is an intolerable error, since in that year it was found by observers to be distant by more than 26°. But the error arose from that Jewish fiction, and from the too-slow mean motion of the fixed stars invented on the basis of it. So if the movable beginning of Aries is at H or P, the arc HP of the colure being drawn, the equation will be IB, and additive in the first semicircle of the anomaly, KLM; and LB will be the greatest equation. But if the beginning is at E or at G, the colure DE being drawn, the equation will be BF, subtractive in the later semicircle MNK, and the greatest will be BN; and likewise for the others. We can treat these small arcs enclosed within the small circle as straight lines. So, given the arc HK, for example, from the motion of the anomaly, its right sine will be given; let this be multiplied by the given semidiameter LB, and the total divided by the whole sine: the arc IB will result, and likewise for the other equations of the equinoxes. More on these in Reinhold, p. 243, Nonius, p. 300, and Wurstisen, 424, on Peuerbach's *Theorica*.

[Margin: The disagreement of Pigge and Beneventanus.]

I say nothing of the now obsolete dispute between Albert Pigge, who in his treatise on the eighth sphere maintains that the celestial motions are computed by the Alphonsines from A, the beginning of Aries of the primum mobile (Reinhold, p. 240, seems to favour this; Io. Esler [?] maintains the same in the *Speculum Astrologicum*; and I think it the truer view), and Marcus Beneventanus, who says that they are computed from the movable beginning of Aries with its true motion, and that the Alphonsine hypotheses therefore contradict themselves. On this dispute Nonius, in Book 2 of *De Observatione*, ch. 4, and Wurstisen, above, p. 407, still remain in doubt. For I am glad to close this chapter with the words of Gassendi in Book 2 of the *Institutio Astronomica*, ch. 8: *The matter seems too fictitious for us to pursue it more scrupulously.*

Diagram. Set in the left column, with the text running down its left side. A small circle crossed by a slightly curved line from C at the far left to A at the far right. From D at the foot of the figure three long lines rise, narrowing to a point at D, and cross the circle to its upper edge. Labels read:

- K (top of the circle)
- H (upper left, on the circle)
- E (upper right, on the circle)

- L (left, on the horizontal line)
- I (on the horizontal line, inside the circle, left of B)
- B (on the horizontal line, at the centre)
- F (on the horizontal line, right of B)
- N (right, on the horizontal line)
- C (far left, at the end of the line)
- A (far right, at the end of the line)
- P (lower left, on the circle)
- a W-shaped letter [recte: M] (foot of the circle; the text names M at this point)
- G (lower right, on the circle)
- D (foot of the figure, where the lines meet)

SCHOLION.

Scholium.

From the last table of ch. 28 you have learned that when the quantity of the year is least, or greatest, there is no prosthaphaeresis of the equinoxes, but the mean and true coincide; but when the quantity of the year is mean, their prosthaphaeresis is greatest, and it is to be added to the true vernal equinox and subtracted from the autumnal to obtain the mean equinox, but subtracted from the mean vernal equinox and added to the autumnal to obtain the true equinox. This greatest prosthaphaeresis is as you see in the following small table. But take care not to confuse this equation with that of the tropical years; and see the scholia of chapter 29.

	Greatest prosthaphaeresis of the equinoxes, in degrees	Greatest prosthaphaeresis of the equinoxes, in days and hours
Copernicus, Book 3, ch. 8, and Gassendi	1° 10′ 0″	$1^{d}\ 4^{h}\ 25^{m}$
Reinhold in the Prutenic Tables, precept 10	1° 11′ 22½″	$1^{d}\ 4^{h}\ 58^{m}$
Longomontanus, Book 1 of the *Theorica*, ch. 4	0° 27′ 5″	$0^{d}\ 10^{h}\ 58^{m}$
Lansbergen in his Tables, p. 12	1° 14′ 0″	$1^{d}\ 6^{h}\ 2^{m}$
Another prosthaphaeresis But from Longomontanus, Book 1 of the *Theorica*, ch. 5	**In the vernal equinox** (days, hours, ′)	**In the autumnal equinox** (days, hours, ′)
In the age of Hipparchus	$1^{d}\ 17^{h}\ 50^{m}$	$2^{d}\ 0^{h}\ 50^{m}$
In the age of Ptolemy	$1^{d}\ 14^{h}\ 45^{m}$	$2^{d}\ 6^{h}\ 45^{m}$
In our own age	$2^{d}\ 4^{h}\ 30^{m}$	$1^{d}\ 22^{h}\ 50^{m}$

CAPVT XXX.

Chapter 30.

De Anni Aequinoctialis ſeu Tropici Inæqualitate, aut Aequalitate.

On the inequality or equality of the equinoctial or tropical year.

[Margin: Those who assert the inequality of the year.]

I. That the temporal year, which we said is called equinoctial and tropical, is unequal, and that its quantity changes by a certain anomaly, has been affirmed first of all, along with al-Zarqālī, by Thābit in his treatise on the motion of the eighth sphere, and by his follower Campanus of Novara; indeed, before these, by the Babylonians long ago, as reported in Copernicus, who for this reason preferred the sidereal year to the tropical as less unequal. It has been affirmed by Pierre d'Ailly in q. 9 on the *Sphere*; by Yitzchak Yisraeli in the book *On the Foundation of the World*, who adds that the year was the shortest of all in the year of salvation 1310; by Henricus Baten Machliniensis [Hendrik Bate of Mechelen]; by Nikolaus von Kues in his treatise on the reform of the calendar; by Georg Peuerbach in the *Theorica* of the eighth sphere, and there by his commentators Capuano, Oswald Schreckenfuchs, Werner, Reinhold, Nonius and Wurstisen; and again by Nonius in Book 2 of *De Observatione*, ch. 4; by Albert Pigge and Marcus Beneventanus in their treatises on the motion of the eighth sphere; by Fracastoro in the *Homocentrica*, sect. 2, from ch. 9 to 17, and sect. 3, ch. 6; by Maurolico, Dialogue 3 of the *Cosmographia*, p. 88; by Giovanni Antonio Delfino in the book on the celestial globe, p. 76; by Copernicus, Book 3, from ch. 1 to 13; by Reinhold, above, and in the Prutenic Tables, precept 21; by Rheticus in the *Narratio prima* on the books of Copernicus, and there by Mästlin; likewise by Peter Apian in the *Astronomicum Caesareum*, proposition 2; by William Gilbert, *De Magnete*, Book 6, ch. 10; by Clavius in the *Calendar*, and in his *Apologia* of it, chs. 3 and 6, and in the *Sphere*, pp. 55 and 62; by Moletti in the *Gregorian Calendar*, canon 7; by Barozzi in the *Cosmographia*, pp. 7, 250, 259; by Magini in the *Theoricae*, Book 1, ch. 4, and in the tables of the *Secunda Mobilia*, from Canon 18; by Origanus in vol. 1 of the *Ephemerides*, ch. 2, precept 4; by Scaliger in the *Diatriba de Æquinoctiis*; by Tycho in vol. 1 of the *Progymnasmata*, p. 37; by Longomontanus, Book 1 of the *Theorica*, chs. 4 and 5; by Lansbergen in the *Thesaurus Observationum*, class 4, p. 54, and in his Tables, precept 4; and by Pierre Hérigone in vol. 5, in the *Theories of the Planets*, Book 2, ch. 4.

[Margin: Arguments for the inequality of the year.]

II. There were chiefly two arguments for this inequality. The first is the unequal apparent motion in the fixed stars. Since this inequality does not seem attributable to the stars themselves, it seems rather to be ascribed to an unequal motion of the equinoctial and solstitial points, now against the order of the signs, now in it. And if those points move back and forth unequally, the year, being a revolution to these points, must be sometimes longer, sometimes shorter. The second argument is the very quantity of the tropical year, derived by comparing different equinoxes and solstices with one another, as may be

seen in the table we gave at the end of ch. 15. For from Hipparchus to Ptolemy the year was $365^d\ 5^h\ 55^m\ 12^s$, but from Ptolemy to al-Battānī $365^d\ 5^h\ 46^m\ 24^s$. By the Alphonsines, moreover, it was put at about $365^d\ 5^h\ 49^m\ 16^s$. And Tycho, in vol. 1 of the *Progymnasmata*, p. 37, has it as follows.

Anni Aequinoctialis diſpares Quantitates.

The unequal quantities of the equinoctial year.

Interval	Length of the year
From Hipparchus to Ptolemy	$365^d\ 5^h\ 55^m\ 12^s$
From Ptolemy to al-Battānī	$365^d\ 5^h\ 46^m\ 20^s$
From Ptolemy to our observations	$365^d\ 5^h\ 47^m\ 52^s$
From al-Battānī to our observations	$365^d\ 5^h\ 49^m\ 29^s$

Nor does this seem to happen through deception or by chance. For at the very time when the longest year was observed, the greatest obliquity of the ecliptic, the greatest eccentricity of the Sun and the slowest motion of the fixed stars also appeared; and at the time when the year was shortest, the fastest motion of the fixed stars appeared as well. Who, then, would not concede that these things arise from some true and connected causes? For these reasons, therefore, the hypothesis of the anomaly of the precession of the equinoxes was introduced, together with its prosthaphaeresis, on which there is more than enough in the preceding chapter and at the end of ch. 28. Reread the Copernican table there, and in the table of ch. 15 the greatest, mean and least quantity of the year according to Copernicus, Reinhold, Clavius, Magini, Origanus and Longomontanus.

[Margin: Causes of the inequality of the year.]

III. Four causes of the inequality of the year are adduced by Copernicus, Book 3, ch. 13; by Rheticus in the *Narratio prima*, and there by Mästlin; by Reinhold in the Prutenic Tables, precept 21; by Moletti, canon 7 of the *Gregorian Calendar*; and by Tycho, vol. 1 of the *Progymnasmata*, p. 38. The first and chief is the unequal precession of the equinoctial points, that is, the unequal approach of the true equinox to the mean and its recession from it, on which see chs. 27 and 28. The second is the annual inequality of the Sun's motion, because of which it lingers longer in the northern semicircle of the ecliptic than in the southern; and longer formerly in the first quadrant of the ecliptic, now in the second, because of the motion of the mean apogee; on this see ch. 16. The third is the inequality of motion of the solar apogee itself, as seems to be gathered from observations; on this see ch. 25. According to the Alphonsines this inequality arises from the unequal motion of the fixed stars, since in their account the apogees of all the planets, the Moon excepted, move with the same motion as the fixed stars. The fourth is the unequal variation of the Sun's eccentricity, because of which the Sun's equations vary, and on which there is enough in ch. 26.

[Margin: Those who assert the equality of the year.]

IV. Notwithstanding all this, the equality of the tropical year was in the end retained by Hipparchus, in Ptolemy, Book 3 of the *Almagest*, ch. 2; by Ptolemy himself, in the same place; by al-Battānī, ch. 52; by Alfonso, where he changed his opinion, as Ricci reports; and by the Alphonsines, though with little consistency, as Reinhold noted from d'Ailly in the *Theoricae*, p. 240; by Agostino Ricci in the treatise on the motion of the eighth sphere, pp. 34 and 36; by Ismaël Boulliau, Book 2 of the *Astronomia Philolaica*, ch. 3, and in him the Persians; by Wendelen in the *Idea of the Atlantic Tables*; and by Antonín Maria Šírek in his *Radius Sidereomysticus*, where he treats the hypothesis of the Sun. This equality is also strongly favoured in the end by Tycho, vol. 1 of the *Progymnasmata*, p. 113, and by Kepler in the *Epitome of Astronomy*, p. 921, and in the Rudolphine Tables, precepts 182 and 189, as we shall soon see.

[Margin: The opinion of Hipparchus.]

For although Ptolemy, in Book 3, ch. 2, said: "But since Hipparchus seems to be disturbed by this restitution of the year, we shall try to show briefly that no firm reason can be gathered from it that ought to disturb us, and on account of which the annual periods should be judged unequal," yet in the same chapter he adds: "Hipparchus, in the book on the change of the solstitial and equinoctial points, after setting out the winter and summer solstices, which he believed had been observed continually and diligently, admits that the difference in them is not so great that an inequality should for that reason be posited in the annual period."

[Margin: The opinion of Ptolemy.]

Ptolemy himself declares his own opinion in the same place in these words: "Nor do we find that the years differ by a quantity that needs attention, but only by about as much as one can err through the construction and placement of instruments." And a little later, without qualification: "From all these things, therefore, and from what we have detected in successive equinoxes, we have not yet found the magnitude of the annual period to be unequal, if it is referred to one and the same point, and not now to the equinoctial or solstitial points, now to the wandering stars." Finally he says: "Since the same quantity is recognized in many years as in few."

[Margin: Tycho's later opinion.]

Tycho, on p. 113, at last saw that the quantity of the year which agrees with his own century and the nearest ones could, with no difference worth attention, fit all centuries, saying: "However, I think it can be gathered both from the findings of Hipparchus and from other very ancient ones that the very quantity of the equinoctial year, considered universally, comes as close as possible to 49^{m} beyond the ascertained days and hours, only a very few small fractions being wanting on either side, which are of scarcely any importance. On this matter I have decided to speak more accurately elsewhere and to set out my opinion more fully, rejecting meanwhile that excessive inequality of the solar year which the moderns, because of the less accurate observations of their

predecessors, think underlies the appearances." And he adds at once that the Copernican calculation, which rests on the prosthaphaeresis of the equinoxes and on the inequality of the year, goes further astray than the Alphonsine.

[Margin: The opinion of Kepler.]

Kepler, for his part, does say in the Rudolphine Tables, precept 188: "By the most certain evidence of eclipse observations I prove that the Sun's approach even to the fixed stars themselves is unequal in the smallest quantities; I shall publish a separate book on this matter, God willing." And from the *Epitome of Astronomy* he urges a motion of the Sun that fluctuates from physical causes; and from Origanus's *Ephemerides*, vol. 1, p. 99, the instability and inconstancy of the celestial motions. And in the Rudolphine precept 189 he says: "Two causes vary the mean quantity of tropical years. The first is certain and lies in the thing itself: the different beginnings of the tropical years through the year, or, as the astrologers say, the different roots of the revolutions. For the shortest tropical year is the one that begins from the Sun's apogee, the longest the one that begins from the perigee, and a middling one that which begins from the mean longitudes, which today are in the 6th degree of Aries and Libra. The other cause, the prosthaphaeresis of the equinoxes, is both uncertain and imperceptible in a short number of years." But of this second cause he says in precept 191: "To such a degree does this prosthaphaeresis of the equinoxes, of uncertain credit, once admitted into the calculation, turn all astronomy upside down."

For as to the variation of the eccentricity and of the solar equations, there is no doubt that Kepler rejects it, and on this head upholds the equality of the year. For he speaks in the *Commentaries on Mars*, ch. 69, of "the analogy of the earlier observations of Hipparchus and the later ones of al-Battānī and of Brahe, all of which conspire in one equality." Not much later he adds: "Nor is there cause just enough for us to say that the proportion of the orbs is different today from what it once was. For an assertion of the greatest moment needs the firmest testimony, which we lack. For those observations cannot be so accurate," etc. Thus, if the different beginnings of the tropical years did not stand in the way, no cause of the inequality of years would remain for Kepler.

Boulliau, moreover, reports from Kepler's *Epitome of Copernican Astronomy*, p. 927, that the equality of the year has been apparent to all observers from Hipparchus on. Boulliau himself, in Book 2, ch. 3, mocks "the ridiculous fiction of the apogees, the fables of the trepidation of the heavens, and the Jewish Jubilee of the stars returning in 49000 years"; and of an unequal precession of the equinoxes or a non-uniform motion of the fixed stars he says: "But these things are not so. The celestial motions are not subject to so many windings and gyrations of little circles, and their equality does not admit so many anomalies." He goes on to confirm the equality of the year from eclipses, and from the Persians, who in Georgios Chrysokokkes, chs. 8 and 15, recognized no inequality in the year or in the motion of the fixed stars.

Noſtra ſententia de Anni Aequalitate.

Our opinion on the equality of the year.

[Margin: The author's opinion.]

V. Indeed, after weighing on both sides not so much authorities as reasons, I at last arrive at this position: although I do not consider the inequality of tropical years wholly improbable, I nevertheless judge by far more probable their perpetual, as far as the senses can tell, or physical equality; and this chiefly by the following arguments.

[Margin: For the physical equality of the year. Arg. 1.]

First: the same quantity of the year, very nearly, that Tycho obtained from his own observations and those of Walther one single century before him, I obtained from my observations compared with those of Hipparchus and corrected, as is now clear from ch. 15, nos. 7 and 8; and the Persian mathematicians found very nearly the same, as is evident from the table in that same chapter. It is $365^{d}\ 5^{h}\ 48^{m}\ 40^{s}$. Among quantities of the year not invented but observed after Ptolemy, there is none that falls short of this quantity by 4^{m} whole, or exceeds it by 1^{m} whole. For the Hipparchan, the Ptolemaic and the Jewish quantities, which exceed it by about 5^{m} or 6^{m}, were not derived from pure observations of the equinoxes but from lunar cycles, as though these were commensurable with the Sun's course; and we showed in that same ch. 15 that this method is less accurate.

[Margin: Ptolemy's candour.]

Nor did Ptolemy cling to that quantity as the most exact; rather, he left it to posterity to correct from a longer interval, when he said in Book 3, ch. 2: "That the annual period, compared with the solstices or equinoxes, is less than 365 days and a quarter of a day is clear to us, also from what Hipparchus demonstrated. But by how much it is less we cannot detect with the utmost exactness, since the addition of the quarter seems to remain unchanged over several years, on account of its very small difference. Therefore it cannot be found except by the calculation of a longer time," etc.; "this restitution, however, will be grasped the more exactly, the longer and greater the time that has elapsed between the observations." So far, moreover, no equinox is extant older than those of Hipparchus or more recent than ours; and the interval from which we derived the quantity of the year is 1804 or 1800 years. The Ptolemaic quantity, which exceeds ours by about 6^{m}, is therefore no obstacle to us. And a difference of 3^{m}, or at most 4^{m}, which is found in some few years observed by others, is not so great that it cannot probably be ascribed either to the deception of observations or to parallax, if not to refraction, either omitted or falsely assumed. So great an affinity, therefore, and so nearly no difference among the observed years, persuades me that their equality is far more likely than inequality.

[Margin: Arg. 2.]

VI. Second: keeping the quantity of the year at $365^{d}\ 5^{h}\ 48^{m}\ 40^{s}$ and going back through the past centuries as far as Hipparchus, I find the equinoxes

observed by the more skilled astronomers without a difference in time that for the most part exceeds 1 hour, and very rarely reaches 2, let alone 3 hours. Who skilled in astronomical observations would not readily concede that this difference arises rather from the imperfection of the observations or the uncertainty of the parallax than from a true non-uniformity of the celestial year? On the contrary, there is no hypothesis of an unequal year among its asserters that satisfies all the observed equinoxes better than ours. Since, then, we have no observations of equinoxes beyond about 1800 years, and our quantity and Tycho's satisfies these as far as a prudent astronomer can require, why should we deny that it can also satisfy the future 1800 years, if the world lasts so long? But the force of this argument will appear more clearly when, God granting, in vol. 3 of the *Almagest*, we weigh the observations of the equinoxes and compare the table of revolutions and of past equinoxes with them.

[Margin: Arg. 3.]

VII. Third: that saying of Ptolemy, Book 3, ch. 2, is well known: "We think it altogether fitting to demonstrate the appearances, as far as possible, by the simplest hypotheses, provided that nothing worth attention conflicts through observations with this proposed method." And Kepler's saying in the *Commentaries on Mars*, ch. 69, must be urged at this point: "An assertion of the greatest moment needs the firmest testimony." But we neither have the firmest testimony for asserting the inequality of the year, nor is the difference of a few minutes distributed over many centuries such as ought to stand in the way of the simplicity of the hypothesis about the quantity of the year.

[Margin: Four invalid causes of the inequality of the year.]

VIII. Fourth: of the four causes of the inequality of the year listed by Copernicus and the rest in no. 3, none in fact has the force for this that the opponents suppose. Not the first and chief, because it is more probable that the equinoctial points, or rather the fixed stars, move equally, as we said in ch. 28, no. 11, Assertion 3; and this will be said again in the book on the fixed stars, and indeed more fully, with the observations compared and corrected, in vol. 3 of the *New Almagest*. The second by itself does not suffice to make one year, compared with another year, unequal, as Copernicus most expressly admits in Book 3, ch. 13, where, speaking of it, he says: "Of all these only the second was known to Ptolemy, and it alone could not have produced the inequality of the year [inæqualitatem annalem], but entangled with the others it does so the more." Nor is the inequality of the parts of the year a sufficient argument for inferring an inequality of one whole year with another whole year. The third is nothing, and rests on no conjectures, or certainly on very weak ones: namely either the variation of the eccentricity, on which presently, or an unequal motion of the fixed stars, which al-Farghānī and the Alphonsines thought to be the same as the motion of the apogee of the Sun and of the other planets, the Moon excepted. Yet the motion of the solar apogee is faster, from what was said in ch. 25, no. 5, and is at least 1′ each year, whereas that of the fixed stars is only 50″, from what was said in ch. 27; nor, if it were the same as the motion of the fixed

stars, would it for all that be unequal. The fourth, namely the variation of the eccentricity, has already been sufficiently disproved in ch. 26, from no. 3 on.

[Margin: A 5th cause, true but imperceptible, of inequality.]

IX. The fifth cause, which Kepler devises in the Rudolphine Tables, precept 189, namely the different beginnings of the tropical years through the year, cannot make the year perceptibly unequal except after many thousands of years, which perhaps the world will not reach. In reality, however, this comes down to the second cause, namely the annual inequality arising from the Sun's varying distance from the apogee; and Copernicus himself conceded in Book 3, ch. 13, that this by itself does not bring in an inequality of the year: understand, an inequality that is perceptible within a few thousand years.

[Margin: Copernicus elucidated.]

But since we have come upon this deep place in astronomy, we must dig this truth out from further down, and call out into the open what Copernicus and Kepler indicated obscurely in the passages just adduced. So when Copernicus had said: "Therefore the reckoning of annual equality is neither simple nor easy to know," he added a little later: "No one will wholly avoid error unless the Sun, on account of the motion of the Earth, then receives either no prosthaphaeresis or one similar and equal at both ends. If this does not happen, and there is some difference according to their inequality, an equal circuit will certainly not seem to have occurred in equal times. But if at both ends the whole variation has been subtracted or duly applied, the work will be perfect." It is as if he were saying: if the magnitude of the tropical year is gathered from the interval between two remote equinoxes, and all error is to be wholly avoided, both equinoxes must occur either with the Sun at apogee, or with the Sun at perigee, or at an equal distance from the apogee but in the same semicircle of the anomaly, so that the prosthaphaeresis is not only equal but also of the same kind, that is, subtractive in both places or additive in both. Otherwise, if one equinox occurs at one point of the anomaly and the other at another, the quantity of the year so gathered will not be perfectly exact; and since the Sun's apogee continually changes place, and the equinoxes occur at ever different points of the anomaly, the quantity of the year gathered from the equinoxes must incur some very slight inequality. Yet since, from what was said at the end of ch. 25, the motion of the apogee does not complete its circle except in 21245 years, while the world has not yet reached the age of 6000 years and perhaps will not last as long again, we are needlessly anxious to seek the quantity of the year with such subtlety, especially since no observed equinoxes older than those of Hipparchus survive.

X. Let us, however, for form's sake, imagine the world made at the vernal equinox with the Sun at apogee, and that after about 21000 years an equinox again occurs with the Sun at apogee. And again, let us imagine the world made at the vernal equinox, but with the Sun at perigee, and after 21000 years the other equinox occurring with the Sun's perigee revolved back to the beginning of Aries. For if the quantity of the year were gathered, by

the method given in ch. 15, no. 6, from the apogee equinoxes, it would be equal to the quantity derived from the perigee equinoxes: the Sun's revolutions would find the apogee or the perigee at the same fixed point of the sky, and the prosthaphaereses and variations between those paired mean and true equinoxes would not differ in any way. But if, before the period of the apogee is complete, an equinoctial year is begun from a vernal equinox occurring at the apogee itself, the following equinox will occur at the beginning of Aries with the Sun's apogee now advanced beyond the beginning of Aries; but the Sun will return to the beginning of Aries, where the slowness of the apparent motion compared with that of the previous day is greatest. But if the year is begun from the perigee, the Sun will return to the same beginning of the ecliptic, in a position where the apparent motion is fastest. But the difference between these revolutions in a few years is nil or imperceptible, as Pietro Pitati also admits in his *Monthly and Paschal Canons*, ch. 5, where he adds that the whole period of this difference is completed in 24000 years, in which the true quantity of the year is restored, or, according to some, in many fewer years, but at least 1800, as many as there are from Hipparchus to us. But then the quantity of the year is not to be derived from a small interval of years; and from an interval of 1800 years, the difference of the year derived from the mean equinoxes does not differ from that derived from the true ones by more than 6^s of time, as we said in ch. 15, Scholium 2.

Let it stand, then, that the tropical year is always physically equal; and that its quantity, derived from a long interval of years, say 1800 or 2000, of which kind ours very nearly is, can serve the past ages of the world, and probably the future ones, without perceptible error, the subtler mathematical inequality that could be gathered from the different beginnings of the years being set aside among imperceptible things.

[Margin: To find the quantity of the year, if it varies.]

XI. If, however, the inequality of the tropical year pleases anyone, and he wants a way of gathering its quantity for a given time, let him consult Reinhold in the Prutenic Tables, precept 21. Or if the inequality admitted only by Kepler pleases him, let him go to Kepler in the Rudolphine Tables, precept 189; on these see us in the precepts of the tables. But for the greatest, least and mean quantity of the year, and for its period and epochs, consult the table taken from Clavius and placed by us at the end of ch. 28.

SCHOLIA.

Scholia.

[Margin: The greatest and least quantity of the year for Copernicus.]

I. The tropical year, says Clavius in ch. 6 of the *Calendar*, is greatest at the beginning of the anomaly and least at the end of the semicircle of the anomaly, even though the prosthaphaeresis of the equinoxes is then nil; because the annual precession of the equinoxes, which must be subtracted from the sidereal

year to obtain the tropical year, is slowest at the beginning and in the middle of this anomaly. And when the motion of the equinoctial points against the order of the signs is slowest, the Sun, coming to meet them towards the east, certainly has the most left to complete in order to reach the point of the true equinox. Further, the equation of the equinoxes is one thing, and the equation of the precession of the equinoctial points, or of the succession of the fixed stars, another.

[Margin: Mästlin's error on the difference between the mean and the true equinox.]

II. From Copernicus and from what was said in ch. 29, no. 10, the greatest equation of the equinoxes is $1^\circ\ 11'\ 22''\ 30'''$, which the Sun completes within $1^d\ 7^h$ even when its motion is slowest. On this hypothesis, therefore, the mean equinox cannot differ from the true by more than $1^d\ 7^h$. So Mästlin was deceived when he said that they could differ by a whole four days, as Clavius refutes him in ch. 6 of the *Apologia of the Calendar*. And the greatest difference between the longest and the shortest tropical year, for Copernicus, does not exceed 13^m of time, while between the mean and the true year it does not exceed 7^m. The equation of the equinoxes, then, is not the same as the equation of the tropical year.

[Margin: Fracastoro's fiction.]

III. Fracastoro, in the *Homocentrica*, Sect. 3, ch. 6, on the basis of an unequal motion of the Aplanes communicated to the lower orbs, thinks that the year was once, 400 years before Christ, of $365^d\ 6^h$, and afterwards diminished. He has no other foundation than the obsolete opinion of the ancients, who, however, measured the year in civil rather than astronomical terms.

CAPVT XXXI.

Chapter 31.

De Inæqualitate Dierum Naturalium & eorum Aequatione.

On the inequality of natural days and their equation.

[Margin: Authors who write on the inequality and equation of natural days.]

I. Now that all the causes, true or invented, of the inequality of the motion of the Sun, which by its daily revolution completes the natural day for us, have been set out, the inequality and equation of natural days will not be so difficult to understand. They are treated in the first place by Ptolemy, Book 3, ch. 10; and there by Regiomontanus in the *Epitome*, Book 3, from Prop. 23 to the end; by the Alphonsines at the end of Proposition 33, pp. 29 and 31; by Peuerbach in the *Tables of Eclipses*; by Copernicus, Book 3, ch. 26; by Reinhold in the *Theoricae*, p. 233, and in the Prutenic Tables, p. 14, or precepts 1 and 48, and canon 5; by Sarzosus [?], Pr. 28; by Fracastoro in the *Homocentrica*, Sect. 3, ch. 3; by Maurolico in Dialogue 2 of the *Cosmographia*, from p. 53; by Magini in the tables of the *Second Mobiles*, canon 7, and in the *Supplement to the Ephemerides*,

canon 6, and in vol. 1 of the *Ephemerides*, part 2, ch. 6; by Origanus, vol. 1 of the *Ephemerides*, p. 1, ch. 6; by Johannes Stadius, canon 8 of his tables; by Giuntini, p. 6, tab.; by Tycho, vol. 1 of the *Progymnasmata*, pp. 113 and 116; by Longomontanus in the *Astronomia Danica*, Book 1 of the *Theorica*, ch. 2, p. 181; by Kepler in the *Epitome of Copernican Astronomy*, Book 3, p. 280, and Book 4, p. 552, and in the Rudolphine Tables, chs. 15 and 31, and folio 32 of the tables; by Lansbergen, precept 8 of the tables; by Mulerius in the *Isagoge* to the Frisian tables, p. 378; by Clavius in the *Sphere*, p. 399; by Barozzi in the *Cosmographia*, p. 156; by Hérigone, vol. 4 of the *Cursus Mathematicus*, in the *Sphere*, p. 111, and at the end of vol. 5, p. 875; by Ismaël Boulliau, Book 2 of the *Astronomia Philolaica*, from ch. 5 to 7; by Wendelen in the *Lampas Arcanorum Caelestium*, light 1; by Ioannes Esler [?] of Mainz in the *Speculum Astrologicum*; by Renieri in the *Tabulae Mediceae*, ch. 3; and by Ariel Bicardus [?] on the *Sphere* of Sacrobosco, p. 60. From these and from our own studies we shall set out the theory and practice of the equality and inequality of natural days.

II. The natural day, in Greek [Greek: Νυχθήμερος (nychthēmeros, "night-and-day")], that is, *Noctidies* ["night-day"], or the day consisting of an artificial day and a night, on which name I said more in Book 1, ch. 25, is threefold: namely the day of the primum mobile [Dies Primi Mobilis], the sidereal day [Dies Sydereus] and the solar day [Dies Solaris].

[Margin: What the day of the primum mobile, the sidereal day and the solar day are.]

The day of the primum mobile is the time in which some point of the equator in the highest heaven revolves, by a motion towards the west, from one point of the meridian or horizon to the same point of the same meridian or horizon, truly or apparently; for the Pythagoreans and Copernicans attribute this motion not to the heavens but to the Earth.

The sidereal day is a whole revolution of the same fixed star from and to very nearly the same point of the same horizon or meridian, completed by a motion towards the west. For this revolution of the other planets, apart from the Sun, has not yet earned the name of natural day. And if the fixed stars moved by no other motion than this, the eighth sphere would be the primum mobile, as the ancient Babylonians thought. But it is established from the observations of Hipparchus and Timocharis down to our century that they move with a proper motion towards the east, so that in our opinion they advance one degree in 72 years. The sidereal day is therefore very slightly longer than the day of the primum mobile; because, for the same fixed star to return to the same point of the same meridian at which it was yesterday, there must pass, beyond the whole circle of the equator, a portion as great as corresponds to the daily proper motion of the fixed star, which has meanwhile advanced towards the east. Since therefore, for me, they advance 50″ in one year, it follows that in one day they creep forward $8‴\ 32⁗\ 53^{v}$; to which there correspond, of the daily motion of the primum mobile, $34⁗\ 11^{v}\ 40^{vi}$. And so great is the small appendage by which the day of the primum mobile is exceeded by the sidereal day. Over 72 Egyptian

years, moreover, the excess is 4^{m}, over 1080 years one hour, and over 25820 years one day.

[Margin: The solar day.]

The solar day is what we commonly understand by the name of day, and it is a whole revolution of the Sun from and to the same horizon or meridian, completed by a motion towards the west. Granted that, for the Pythagoreans and Copernicans, it is not the Sun by a motion towards the west but the Earth by a motion around its own centre towards the east that produces the daily turning for us. Now since the Sun by its proper motion towards the east advances daily, that is, in each day of the primum mobile, by a mean motion of $59' 8'' 19''' 51'''' 46^{v} 20^{vi}$, from what was said in ch. 17, that is, about $59' 8'' 20'''$, to which there correspond, from the motion of the primum mobile on the equator, horary times of $3^{m} 56^{s}$ 32 thirds 1 fourth 20 fifths, one natural day of the Sun, the mean one, therefore contains, besides the 24 hours of the primum mobile, $3^{m} 56^{s}$ 32 thirds 1 fourth 20 fifths; and while one solar day is being completed, 360 degrees of the whole equator pass, and besides that $59' 8'' 20'''$. So it comes about that in the 365 solar days in which the common year is completed, 366 revolutions of the equator are completed, and as many days of the primum mobile, and very nearly as many sidereal ones. The following table therefore shows the increments [Appendices] and differences of these three kinds of day.

[Margin: Difference of these days.]

Difference of the days	*Times of the primum mobile:* H.	m	s	thirds	fourths	fifths	sixths
1 day of the primum mobile	24	0	0	0	0	0	0
1 sidereal day	24	0	0	0	34	11	40
1 solar day	24	3	56	32	1	20	0

The sidereal day and the solar day can, however, each be divided into exactly 24 hours, but hours of their own. One day of the primum mobile would then be 23 sidereal hours $59^{m} 59^{s}$ 59 thirds 25 fourths 48 fifths 20 sixths, and one hour of the primum mobile would contain $59^{m} 59^{s}$ 59 thirds 58 fourths 34 fifths 32 sixths of a sidereal hour. In the same way the day of the primum mobile can be divided into 24 solar hours, or compared with them, as below.

		H.	m	s	thirds	fourths	fifths	sixths
One day of the primum mobile has	sidereal hours	23	59	59	59	25	48	20
And one hour of the primum mobile	of a sidereal hour	0	59	59	59	58	34	32
One day of the primum mobile has	solar hours	23	56	3	27	58	40	0
And one hour of the primum mobile	of a solar hour	0	59	54	46	26	37	0

So although in one hour of the primum mobile 15 degrees of the equator cross the meridian, and one degree crosses in 4 minutes of the primum mobile, and a common table has been made in that proportion for the mutual conversion of parts of the equator into hours and minute parts of hours of the primum mobile, and of these into parts of the equator, another, finer one is nevertheless needed, by which solar hours may be converted into parts of the equator, giving one hour 15° 2′ 28″, and parts of the equator into solar hours, giving one degree 3^{m} 58^{s} 40 thirds [recte: 3^{m} 59^{s} 20½ thirds, per the book's errata]. We shall present both at the end of the work.

III. Leaving aside the inequality of the sidereal day, which others suppose to arise from an unequal motion of the fixed stars, since it is both imperceptible and, in my view, false, from what was said in ch. 28, no. 5 and 11: the natural days of the Sun, compared with one another, are unequal for many causes. If the anomalies refuted in ch. 24, 25, 26, 27 were real, these causes would resolve into 13, granted that the ancient astronomers reduced them to only two, and that they can be brought down to 4.

[Margin: Cause 1 of the inequality of natural days, resolved into 3 causes.]

The first cause is the inequality of the Sun's motion in the ecliptic, since the Sun moves most slowly around the apogee and most swiftly at the perigee. This cause would hold good even if the Sun's eccentricity, on which the equations depend, did not change, or changed, but uniformly. But if it changes by that anomaly discussed in ch. 26, there will now be three causes. So, according to *Copernicus*, the inequality of natural days arising from this threefold cause is as follows.

		Daily motion of the Sun in the zodiac: ′	″	‴	*Natural day of the Sun:* H.	m	s	thirds
At greatest eccentricity, with the Sun	at apogee	56	46	0	24	3	47	4
	at perigee	61	43	0	24	4	6	52
At least eccentricity, with the Sun	at apogee	57	18	0	24	3	49	12
	at perigee	61	6	0	24	4	4	24
Mean		59	8	20	24	3	56	32

According to *Kepler*, however, the Sun today completes 57′ 5″ after the solstice, but 61′ 21″ after midwinter.

[Margin: Cause 2, resolved into 2.]

The second cause is the motion of the Sun's apogee, because of which the Sun's longer days fall on different days of the civil year, and indeed of the tropical year too. For when the Sun's apogee was at the start of Aries and its perigee at the beginning of Libra, namely at the beginning of the world, the vernal equinoctial day was the shortest, because of the Sun's slowest motion, which

least resisted the motion of the primum mobile. The autumnal equinoctial day, on the other hand (we are speaking here of the natural day), was the longest, because of the Sun's swiftest motion towards the east; and at about the beginnings of Cancer and Capricorn the solstitial days were of middling length. The former fell around the end of April and of October, the latter around the end of July and of January in the Julian year. Now, however, the summer solstitial days are the shortest, around the end of June, and the winter ones the longest, around the end of December, because at those times the Sun is in the one case at apogee, in the other at perigee. The middling days, around the mean longitudes, are roughly the equinoctial ones, that is, around the end of March and of September. Now if the apogee moves with an irregular motion, which we do not admit, from what was said in ch. 24, this cause of the inequality of days is not single but double.

[Margin: Cause 3, resolved into 5.]

The third cause is the obliquity of the ecliptic to the equator, because of which equal arcs of the equator do not rise in a right sphere with equal arcs of the ecliptic, nor cross the meridian with them in any sphere. This is clear from the tables of right ascensions, and from the analysis of a right-angled spherical triangle in which, from the given angle of the ecliptic's obliquity and an arc of the ecliptic taken as base, the side adjacent to the given angle is found, namely the right ascension itself or its argument. Therefore, even if the Sun moved of itself uniformly in the zodiac, the arc of the equator, and so the increment of time to be added to the day of the primum mobile, would still not be the same or equal. This cause alone would hold good even if neither the obliquity of the ecliptic nor the precession of the equinoxes changed, or if they did change, but uniformly. But if both change, and by that anomaly against which we argued in ch. 26 and 27, then four others are entangled with this cause, so that there are not this one but five on account of which the right ascensions vary unequally. And although these causes so far stated make one day differ from another almost imperceptibly, after many days they nevertheless produce a perceptible and evident difference of days, as we shall say in no. 6.

[Margin: Cause 4, resolved into 3.]

The fourth cause is the varying obliquity of the horizon in an oblique sphere, because of which days begun from the rising or setting of the Sun become unequal, since with equal arcs of the ecliptic there rise far more unequal arcs of the equator than those which culminate with the same arcs of the ecliptic, that is, cross the meridian, in accordance with the different obliquity of the horizon to the equator. Two others are joined to this cause, namely the Sun's horizontal parallax, which varies according to the Sun's varying distances from the Earth, and the Sun's refraction, likewise horizontal, which is variable because of the inconstancy of the vapours of the horizon in different places and seasons of the year. For because of these the Sun's right ascensions and declinations vary apparently. But to avoid the inequality arising from this fourth cause, astronomers, with *Ptolemy* (Book 3, ch. 10), begin natural days not from the

horizon but from the meridian, where in every sphere, whether right or oblique, the culminations [Mediationes Cæli], other things being equal, are equal to the right ascensions of the right sphere.

IV. Since, then, the natural days of the Sun are unequal, and so cannot be taken as a common and certain measure of the celestial motions, astronomers had to substitute for them mean, equal days of the Sun, and to distribute the celestial motions over these. But once motions have been collected for equal time, that is, time consisting of equal days, that time must be converted into true time, that is, the time apparent to us, who measure and count times by the Sun's true motion. Conversely, if some celestial phenomenon, say an eclipse, observed at apparent time, is to be examined by astronomical tables consisting of mean motions, the apparent time must be converted into equal time; or at any rate, in either case, as much must be added to or taken from the celestial motion as the difference between equal and apparent time requires[19].

[Margin: This equation is more necessary in the case of the Moon and for astrologers than for astronomers.]

If anyone chooses to disregard this equation, he will indeed incur some error in the other planets, sometimes 1^m 20^s in the Sun, 2^m in Saturn, Jupiter and Mars, 4^m in Venus and 9^m in Mercury, at most, in minutes of time; yet in the Moon the error can be 36′ and more. And astrologers have a still greater need of this correction than astronomers, as *Io. Esler* [?] confirms with examples in the *Speculum Astrologicum*.

[Margin: Rule for the equation of natural days.]

V. The universal rule, used from Ptolemy down to Tycho, for finding the equation of natural days, if the epochs of the motions belong to apparent time, as in the tables of Ptolemy, the Alphonsine and the Prutenic, is as follows. First find the difference between the Sun's mean place or motion belonging to the epoch of the motions, say at the beginning of the years of Christ, and the Sun's motion belonging to the given time. Then find the difference of the right ascensions corresponding to the Sun's true motion, both at the already mentioned epoch and at the given time. Third, compare the difference of the mean motions with the difference of the right ascensions. If they are equal, the sum of equal and apparent days will be the same, and no equation will be needed; but if they are unequal, subtract the smaller difference from the greater and convert the residue into time, and you will have the equation of time you want. For its use, observe the following canons.

	THE EQUATION [ÆQVATIONEM]
If the difference of ascensions is greater than the difference of mean motions	*Add* it to apparent time, to make equal time. *Subtract* it from equal time, to make apparent time.
If the difference of mean motions is greater than the difference of ascensions	*Subtract* it from apparent time, to make equal time. *Add* it to equal time, to make apparent time.

[19]the page prints *quarum*; the correlative of *tantum* requires *quantum*

[Margin: What if the epochs relate to mean time?]

But if the epochs of the celestial motions belong to mean time, as in the Rudolphine, Danish, Lansbergian and Philolaic tables: given the apparent time, find the Sun's place both mean and true, and the right ascension of the true place. If this is equal to the mean place, there will be no equation; if unequal, the difference, turned into time, will be the equation sought, whose use will be as follows.

	THE EQUATION [ÆQVATIONEM]
If the right ascension is greater than the mean motion	*Add* it to apparent time, to make equal time. *Subtract* it from equal time, to make apparent time.
If the mean motion is greater than the right ascension	*Subtract* it from apparent time, to make equal time. *Add* it to equal time, to make apparent time.

These rules hold if the calculation is progressive. But if it is retrograde, reckoning the motions backwards before the epoch, everything must be done the opposite way, and where it is said that the equation is to be added, it is to be subtracted, etc. That the second part of the precept is to be preferred to the first part you will learn from Scholium 1 of ch. 31. But if the Sun's true place is the beginning of Aries and the mean place falls short of it, take 360° for the right ascension; if the mean place is beyond it, the mean motion itself will be the equation to be turned into time.

EXEMPLVM.

Example.

[Margin: The time of the birth of Gregory XIII.]

In the year of Christ 1502, January 7, at $6^h\ 54^m$ after noon at Bologna, in apparent time, Pope Gregory XIII is said to have been born. The equal time is sought, so that the calculation from astronomical tables can be begun for it. At the beginning of the years of Christ the Sun's mean motion, from the Prutenic Tables as applied to Bologna by Magini, was $9^s\ 7°\ 34'\ 45''$, and at the already mentioned time of Gregory's birth it was $9^s\ 25°\ 32'\ 49''$. The difference of these motions is $17°\ 58'\ 4''$. Next, the right ascension of the Sun's true place at the epoch of Christ, that is, Capricorn $8°\ 36'\ 30''$, was $279°\ 23'\ 58''$; but at the time of Gregory's birth the Sun's true place was Capricorn $26°\ 49'\ 22''$, whose right ascension was $298°\ 51'\ 53''$[20]. So the difference of the right ascensions from each other was $19°\ 27'\ 55''$, which is greater than the difference of the mean motions; and when the latter is subtracted from the former, the residue is $1°\ 29'\ 51''$ on the equator, that is, $5^m\ 59^s$ of time. This time, added to the apparent time of Gregory's birth, gives the equal time of the current year 1502, January 7, $6^h\ 59^m\ 59^s$ after noon at Bologna.

[20] the page prints *Signor.* (signs) before 298, where degrees are meant; on this copy a line is drawn through the word

[Margin: A shortcut for this procedure.]

For the proposed time, however, one could have collected the motion of the stars, say of the Sun, and then the equation of time; but as much of the Sun's motion would then have to have been added to the true motion already collected as the equation of time required. Thus in our case the Sun completes 14″ 47‴ in the zodiac in 5^{m} 59^{s} of time. These, therefore, would have had to be added to the Sun's true place collected for the apparent time, which was 9^{s} 26° 49′ 22″, and the Sun's true place would have been 9^{s} 26° 49′ 36″ 47‴. And this is indeed the more prudent course, since the calculation of the Sun's place need not be repeated, etc. Conversely, if the equation of time had had to be subtracted, the Sun's place too would have had to be reduced proportionally. A table of *Magini* is adapted to this business, but a temporary one. For

[Margin: Temporary tables.]

VI. Besides this universal and most certain rule, practitioners usually construct temporary tables [tabulæ temporariæ], which fit only one century without perceptible error, since they rest on a fixed place of the apogee and on an eccentricity and an obliquity of the ecliptic belonging to that century. Tables of this kind are given by the *Alphonsines, Reinhold, Sarzosus* [Francisco Sarzosa], *Magini, Origanus, Tycho, Longomontanus, Mulerius, Kepler, Lansbergen, Boulliau* and *Renieri*, at the places indicated in no. 1. *Regiomontanus* teaches how to construct them by means of trigonometry in Book 3 of the *Epitome*, Prop. 29 and 30, and in Proposition 25 he also gives the problem of finding, in a quadrant of the ecliptic, the point of the ecliptic at which the arc of the ecliptic counted from the equinox differs most from its right ascension, given the obliquity of the ecliptic. This point always lies around the midpoints of Taurus, Leo, Scorpio and Aquarius, and is easily found in the tables of right ascensions. But in the Alphonsine tables and those of Sarzosus, and in the second table of Peuerbach, of Regiomontanus and of Reinhold, the equation is always taken from apparent time to make mean time, and added to mean time to make apparent time. In the others their titles are to be observed. According to Ptolemy, however, it is to be subtracted from 15° of Aquarius to the end of Libra, and added from the beginning of Scorpio to 15° of Aquarius.

Three things, moreover, are worth noting in these tables or for them. The first is the limit from which natural days begin to grow above the middling ones, the limit from which they begin to decrease, and what the greatest equation in them is. The second is how great the greatest difference is between the collected mean, shorter and longer days, and where it occurs. The third is where there is none. As to the first: days begin to grow, and to become greater than equal days, from the degrees of Scorpio written below up to the degrees of Aquarius written below; and from those same degrees of Aquarius they begin to decrease, as far as the same degrees of Scorpio. At these points the equations in the astronomers' tables are greatest, as can be seen in the following small table.

	Scorpio: limit	Scorpio: greatest equation	Aquarius: limit	Aquarius: greatest equation
Ptolemy	0°	$33^m\ 20^s$	15°	$16^m\ 40^s$
Alfonso	8°	$32^m\ 52^s$	21°	✢ ✢
Peuerbach	8°	$32^m\ 48^s$	21¼°	✢ ✢
Copernicus	11°	$22^m\ 22^s$	22°	$8^m\ 9^s$
Tycho	8°	$24^m\ 30^s$	22°	$8^m\ 0^s$
Kepler	10°	$17^m\ 0^s$	12°	$15^m\ 0^s$
Lansbergen	8°	$24^m\ 18^s$	22°	$8^m\ 0^s$
Boulliau	13°	$13^m\ 8^s$	18°	$12^m\ 34^s$

The limits of the greatest difference between arcs of the ecliptic and the right ascensions corresponding to them, with their differences, vary among authors. But the Sun's apogee is, for Ptolemy, at Gemini 5°; for Peuerbach, at the beginning of Cancer; for others, beyond the beginning of Cancer.

If the obliquity of the ecliptic is 23° 52′.

Degrees of the signs	Arc of the ecliptic	Right ascension
Taurus 16°	46°	43° 26′
Leo 13°	133°	135° 34′
Scorpio 16°	226°	223° 26′
Aquarius 13°	313°	315° 34′
Difference of right ascension and arc of the ecliptic		2° 34′

If the obliquity of the ecliptic is 23° 30′.

Sign	Arc of the ecliptic	Right ascension
Taurus 16° 4′ 44″	46° 4′ 44″	43° 35′ 39″
Leo 13° 45′ 16″	133° 45′ 16″	136° 14′ 21″
Scorpio 16° 4′ 44″	226° 4′ 44″	223° 35′ 39″
Aquarius 13° 45′ 16″	313° 45′ 16″	316° 14′ 21″
Difference of right ascensions, etc.		2° 29′ 5″

For *Boulliau*, however, this difference is 2° 29′ 44″, since he follows a different obliquity of the ecliptic.

So in these centuries days grow from 16° of Taurus to 14° of Leo, because of the difference of right ascensions alone; from 14° of Leo to 16° of Scorpio they decrease. Again, from 16° of Scorpio they grow up to 14° of Aquarius, and from there as far as 16° of Taurus they decrease. The total difference of the shorter and longer days, collected at an obliquity of 23° 52′, is 10° 16′, but from our obliquity of 23° 30′ it is 9° 56′ 20″. These are composed of the difference of ascensions, 2° 34′ or 2° 29′ 5″, quadrupled; that is, in horary minutes, $41^m\ 4^s$ in the former case and $39^m\ 45^s$ 20 thirds in the latter.

[Margin: Greatest difference of natural days, compounded from all causes.]

Now suppose that the other cause, arising from the inequality of the Sun's motion in the ecliptic, is joined to this one, and that with the greatest difference of right ascensions, which for us is 2° 29′ 5″, there is joined at the same point of the ecliptic, say at Taurus 16° 4′ 44″, the Sun's greatest prosthaphaeresis, which for us is 1° 59′, and that both are additive or both subtractive. The difference in one quadrant, from the middle of Taurus to the middle of Leo, would then be 4° 28′ 5″. Quadrupled, this would be 17° 52′ 20″, that is, in time, 1^h 11^m 29^s 20 thirds between the shorter and longer days, and half of this between the mean days and the longer or shorter ones; but this is with days computed from the meridian. *Copernicus, Reinhold* and *Magini*, however, who follow the anomalies of the obliquity, of the equinoxes and of the eccentricity, say that this whole difference can be 10° between middling and smaller days, that is, 40 horary minutes, and 20° between smaller and greater days, that is, 1^h 20^m. For *Ptolemy*, on the other hand, the former is 33^m 20^s and the latter 1^h 6^m 40^s.

[Margin: Where and when natural and apparent days are equal.]

Conversely, there is no difference between mean and apparent days when the differences of right ascensions and the equation of the solar motion are equal, but one subtractive and the other additive, so that they compensate for or cancel each other; and this happens twice a year. This difference of days also ceased when the Sun with its apogee was at the beginning of Aries or at the beginning of Cancer, and it will cease if both are at the beginning of Libra or of Capricorn. For at these cardinal points no equation is to be applied, either to the Sun's motion or on account of the right ascensions.

CAPVT XXXII.

Chapter 32.

Recentiorum Aſtronomorum Sententia de Inæqualitate Dierum Naturalium, eorumq. Aequatione.

The opinion of more recent astronomers on the inequality of natural days and their equation.

Up to now, from Ptolemy to Tycho, the equation of natural or civil days has seemed necessary to astronomers, and indeed as great an equation as is gathered by the rule given in ch. 31, no. 5, especially in the Moon, according to what was said in the same place, no. 4. Nevertheless, from the time of Tycho it began to be called into doubt. So it will be worthwhile to hear the opinions of the more recent astronomers on this matter.

[Margin: Authors denying the necessity of the equation of days.]

I. The first opinion among them, then, is that of *Jacob Christmann* in his solar observations, of *Vitichius* [Paul Wittich] in Longomontanus below, and of *Govaert Wendelen* in the *Lampas Arcanorum Caelestium* and in the *Idea of the Atlantic Tables*. They judge the equation of natural days, that is, the conversion

of apparent time into equal and of equal into apparent, to be by no means necessary.

[Margin: Christmann's reasoning.]

Christmann does so because he thinks that the inequality of days arising from the Sun's unequal motion is imperceptible, and that the one which would arise from the obliquity of the ecliptic is rendered imperceptible, since the Sun, advancing along the parallels of the equator, behaves, so far as the senses can tell, exactly as if it were advancing along the equator. But he is mistaken, because the Sun does not describe parallels but spirals, or rather spherical helices, and marks out the oblique path of the ecliptic.

[Margin: And Wittich's.]

Wittich, on the other hand, thought that as much is taken from mean time by one of the already mentioned causes as is added by the other. This, however, is false, since the difference arising from right ascensions compared with the mean motion is much greater than the one arising from the Sun's unequal motion, except in only two cases, in which they compensate each other: namely when the Sun with its apogee is at some cardinal point of the equinoxes or solstices, or outside these points, twice a year, according to what was said at the end of ch. 31.

[Margin: Wendelen's foundation.]

Wendelen, having expressed his opinion in this elegant verse:

In the heavens, tomorrow's total is equal to today's. [In Cœlis par eſt hodiernæ craſtina ſummæ.]

that is: *All days* [Greek: Νυχθήμεροι (nychthēmeroi, "of a night and a day")] *are equal to one another,*

has so far put forward no other foundation a priori, but supports his opinion a posteriori, from more than 45 observed eclipses, which he finds answer to calculation without any equation of natural or civil days; using, however, luni-solar tables derived from more than two hundred eclipses, and oscillatory prosthaphaereses of the Moon. Nor does he fear for himself from the eclipses observed by Tycho, since their observation is in places suspect, for reasons indicated in the preface to the Atlantic Tables, which we shall review in the Book on Eclipses.

[Margin: Boulliau's opinion of Wendelen.]

Boulliau, however, in Book 3, ch. 15 of his *Astronomy*, nearly agrees with Wendelen as to some eclipses, but opposes him as to others. Nor can these matters be established and determined with certainty from eclipses observed at different places unless the difference of the meridians is first established; and yet between distant places this itself cannot be gathered by any surer indication than from eclipses observed at both. I nevertheless await a reply from Wendelen himself, a very dear friend and most expert in astronomy. For I have asked him how it can happen that from a necessary cause of the inequality of days the necessary effect, namely the inequality of days itself, does not follow. And if it does not follow, from where is the excess of days above the mean ones offset

by a deficiency? *Boulliau*, Book 2, ch. 6, at the end, denies that Wendelen can demonstrate this offset.

[Margin: Tycho's twofold method of equating time.]

II. *Tycho*, as is clear from vol. 1 of the *Progymnasmata*, pp. 113, 116 and 123, in the example of the lunar calculation, did use the customary astronomical equation of days in the calculation of the Sun, say in the time of the solstices and equinoxes; but in the Moon he also applied a second equation, since he had learned by experiment from many eclipses that the Moon regulates its elongation from the Sun not from the Sun's mean place but from its true place. Later, however, he approved the plan of *Longomontanus*, and in lunar syzygies kept only that difference of days and equation which arises from the difference between the right ascension of the Sun's true place and the true place of the Sun itself, converted into time. By this device was made *Tycho's* small table in vol. 1 of the *Progymnasmata*, p. 116, which *Magini* also gives in the *Supplementum Ephemeridum*, p. 10. The earlier one, according to the eccentricity and apogee of his own time, *Tycho* gives in vol. 1, p. 113. Both are given by *Kepler* in the tables, p. 32; by *Mulerius* in the Frisian Tables, pp. 130, 280 and 281, explaining them on p. 378; and by *Origanus* in vol. 1 of the *Ephemerides*, p. 100, who adds that Tycho in the earlier table nearly neglected the inequality arising from the Sun's unequal motion, because he had rejected the excessive inequality of the year asserted by Copernicus.

[Margin: Longomontanus's opinion.]

III. *Longomontanus*, in Book 1 of the *Theorica*, ch. 2, in the commentary on the Sun, confesses that *in all his astronomical study he never laboured with greater difficulty than here*, namely in seeking a more solid cause for equating time; and that the old method of Ptolemy, which we treated in the first rule, ch. 31, no. 5, is not safe enough, because it is derived variously from various epochs, and because the inequality of the solar motion has no certain starting-point in the ecliptic but the one established by the line of the apogee. So after restoring the Moon's course in Bohemia in the year 1600, he retains a single cause of the inequality, namely the one arising from the obliquity of the ecliptic, that is, the difference of the right ascension and the Sun's true place; because lunar eclipses and the necessity of demonstration demand it, without denying meanwhile a moderate difference arising from the unequal precession of the equinoxes, which for *Longomontanus* does not exceed 4′. The other cause of inequality, flowing from the Sun's unequal motion, he says he abandoned *almost without reason*, except that in respect of this too the Moon conforms itself to the Sun's apparent motion; but that this cause is removed, or rather compensated, if the motion of the diurnal rotation is supposed to be adjusted to the Sun's approach to and recession from the Earth. Thus with the Sun at apogee, for example, where the natural day would be shorter because of the slowness of the Sun's own motion, which resists the motion of the primum mobile least, the day nevertheless becomes longer because of the retarded motion of the primum mobile. Conversely, with the Sun at perigee, when the day ought to be longer,

it nevertheless becomes shorter from another source, because of the quickened motion of the primum mobile, or rather of the Earth's diurnal rotation. For from this, on his account, a great argument can also be drawn in its favour. But this compensation seems assignable no more to the Earth than to the primum mobile itself in the heavens, if one were granted, or to the Sun as it describes its spirals or helices, by which it accomplishes its diurnal and annual motion together.

[Margin: Longomontanus's argument for the Earth's diurnal motion, not a necessary one.]

IV. Mention of the Earth's motion, however, has led us to the opinion of *Kepler*. In the *Epitome of Copernican Astronomy*, Book 3, from p. 279, he ascribes the natural day not to the Sun but, with the Pythagoreans and Copernicans, to the Earth turning in diurnal rotation about its own axis. This, he holds, is the motion of the primum mobile, which with *Longomontanus* he considers irregular, increased by the Earth's annual approach to the centre of the Sun and slowed by its departure. But in the *Ephemerides*, p. 10, no. 30, he teaches that Tycho's equation, derived from a single cause, does not satisfy the eclipses observed by *Simon Marius*, by himself and by others, and that a third equation must be applied. This at times extends to 21′ 41″, but it is of the contrary sense to the one Tycho omitted, is double it and more, and begins from the Sun's apogee. This equation, he says, arises from certain supernumerary times, namely from the 5^{d} 6^{h} remaining beyond 360 days to the end of the solar year, and it is a property of the inequality of the motion of the primum mobile. The common equation, however, arising from the two known causes, he adapts so that it begins from 2° of Cancer. This way of equating, he says, he has not yet been able to refute from observations. Likewise in the Rudolphine Tables, chaps 15 and 32, Precept 71, he sets out three forms of equating time.

[Margin: Kepler's view.]

[Margin: Three forms of equating time.]

The first is the Tychonic, which he calls *empirical*, because it has been confirmed by experiment and by observations of many eclipses since Tycho's time. He therefore prefers it to the others in the calculation of the Moon, especially since it is very simple and serves for every age, being nothing other than the difference between the Sun's true place and its right ascension.

[Margin: The first, empirical, or Tychonic.]

The second is called by Kepler *astronomical* and *demonstrative*. It arises from the inequality of the Sun's motion, which begins from the apogee, and from the inequality of right ascensions, which begins from the equinoctial or solstitial points. The beginnings of both causes coincided, both in the year 3993 before the era of Christ, April 24, when the Sun and the Sun's apogee were at the beginning of Aries, and in the year of Christ 1466 June 14, when the Sun and the Sun's apogee were at the beginning of Cancer; and the mean natural day was then equal to the true and apparent day. This also happens twice in any year, when the one cause compensates the other; that is, the equation of the

Sun is as great as the difference between the right ascensions and the mean motions, but the one is additive and the other subtractive. The universal rule of this equation is the one I gave in chap. 31, no. 5. In imitation of the Prutenic Tables, however, he gives on p. 32 a temporary table adapted to the year of Christ 1616, but serving without perceptible error for more than 50 years before and after. This equation, then, which he says some eclipses support, he affirms must be applied, so that it may afterwards be established from observations whether it suffices, or whether some other form of equating that satisfies more phenomena must be substituted.

[Margin: Second form of the equation: astronomical.]

The third equation is called *physical* by Kepler, as arising from a physical cause, namely from the power of the Sun, which aids the Earth, the body of the primum mobile, and accelerates its diurnal motion through the whole of that semicircle of the annual orb in which the Earth approaches the Sun, but relaxes it in the other, in which the Earth recedes from the Sun. He imagines an archetypal year of 360 days, which is a figured number; but the supernumerary 5 days and 6 hours, reduced to 7560^{m} of time, he divides by 360, from which 21^{m} results in the quotient, and that is the greatest physical equation. He wants the physical equation to be added where the astronomical would have to be subtracted, and subtracted where it would have to be added; and he wants the Sun to aid the Earth by a one-hundred-and-eightieth part of the annual semicircle, distributed through it. So it comes about, he says, that by this not only is that which Tycho had discarded from the astronomical equation compensated, but even more is restored in the contrary direction. He admits that some eclipses favour this form of equating, and that many oppose it. For it he gives on p. 32 a second table, not in time but in parts of the equator, to be converted into time; there the excess of the physical over the astronomical, applied separately, is the physical equation correcting the astronomical. This excess is at its greatest $3^{\circ}\ 21'$, that is, $13^{m}\ 24^{s}$ of time, and is to be applied around the quadrant of the Sun's equated anomaly. The equation, however, that he teaches must be applied to eclipses in chap. 31, Precept 171, is monthly; for he says it has been observed that around the points of the vernal equinox the Moon enters the Earth's shadow later, and around the autumnal one earlier. That physical equation, however, is called by *Boulliau*, Book 2, chap. 7, *altogether chimerical*.

[Margin: Third form of the equation: physical.]

[Margin: Spring and autumn eclipses.]

V. *Lansbergen* too, in Precept 8, besides the astronomical equation (on which, chap. 31, no. 5), affirms that another must be applied in eclipses of the Sun and Moon, arising from a hidden cause in the Moon. It is very irregular, suspect for its awkwardness, and seems rather to have been devised to reconcile with observations the Lansbergian tables, which would otherwise go astray. It is of this kind:

[Margin: Lansbergen's opinion.]

Signs and degrees in which the luminary is eclipsed	*Equation to be applied to mean time*	
♈ 0°	30′	Add
♋ 18°	10′	Subtr.
♌ 7°	18′	Subtr.
♌ 18°	16′	Subtr.
♍ 0°	1′	Subtr.
♎ 0°	5′	Subtr.
♎ 18°	10′	Subtr.
♏ 24°	10′	Subtr.
♓ 20°	*uncertain*	

VI. *Ismaël Boulliau*, an astronomer of no common depth, in Book 2 of the *Astronomia Philolaica*, chap. 6, condemns Tycho's equation as diminished in the calculation of the Moon, on the ground that the diurnal revolution is the measure of all motions, to which the Moon too must be subject. He therefore establishes three causes of the inequality of days. The first is from the right ascensions of the terrestrial equator, which are unequal while the arcs of the zodiac are equal. The second is the difference of those same ascensions, insofar as it comes from the inequality of the annual motion arising from the eccentricity. The third is from the unequal motion of the diurnal revolution about its axis, which represents the primum mobile. For from the very fact that the Earth moves by its annual motion faster at perihelion and slower at aphelion along the ellipse, it follows too that the Earth's diurnal spinning [turbinatio], or rotation about its centre, is swifter when it approaches the Sun and slower when it recedes from it. In the same way, with a sphere descending an inclined plane and meanwhile spinning in rotation about its axis, if the descent is faster on a more inclined plane, the rotation is faster too, as is the return of any one point to contact with the plane.

[Margin: Boulliau's opinion.]

Let then the ellipse of the annual orb be CFDH, with centre A; let the Sun be B, the line of aphelion CD, and that of the equinoxes FH; the aphelion C, the perihelion D; the Earth's globe IOK, and its axis IK, always parallel to itself. While the Earth's centre moves from C towards F under the zodiac in the annual orb, let the Earth spin about the centre E, from Q to E, towards O, etc.; and let the Earth's equator be OEQ, and the terrestrial zodiac OVQ. Now let the Earth come from aphelion C to E, the middle of Aquarius, and in 45 days of equal motion let it have made 44° 31′ 15″. For to the arc VZ of the ecliptic there will correspond the arc EL of the terrestrial equator, 47° 0′ 15″, belonging to apparent time, and the difference between the time and true place, and the equal one, will be 2° 29′, which are lacking to apparent time for it to be reduced to mean time. The same thing happens proportionally if the Earth is at G, 15° of Leo. Now let the arc EX, or GX, on the equator be 45°, the measure of mean time. The place of mean time, IXK, will therefore cross the meridian sooner than the ap- parent-time place ILK, by 2° 29′, or 10^{m} of time.

Another cause of inequality is added, namely the Sun's unequal motion, because of the eccentricity AB and because of the physical acceleration of the Earth along the ellipse.

Diagram. A woodcut in the left column, between the lines of the paragraph. A large ellipse, the annual orb, is crossed by a vertical axis CD and a horizontal axis FH, which meet at A. The point B lies on the axis below A. Two straight oblique lines run up and to the left: one from A towards the first globe, one through B stopping short of that globe. Two small globes stand on the ellipse: one at the upper left, centred on the orbit at E, and one at the lower right, centred at G. Each is drawn with its axis IK, a horizontal equator OQ, a curved zodiac and further curved meridians.

- C (top of ellipse): with the sign ♑ beside it
- H (right end of the horizontal axis): with the sign ♎ beside it
- F (left end of the horizontal axis): with the sign ♈ beside it
- D (bottom of ellipse): with the sign ♋ above it
- A (centre): intersection of the two axes
- B (on the vertical axis, below A)
- First globe, upper left: I (top); K (bottom); O (left); Q (right); E (centre); X and L (on the equator, right of E); V (below E); R (left); S (right); Z (small and oblique, inside the globe beside S); the sign ♒ (near E, to the left)
- Second globe, lower right: I (top); K (bottom); O (left); Q (right); G (centre); L and X (on the equator, left of G); Δ (below the equator, near O); V (above G); S (left, upper); R (right, upper); Z (small and oblique, inside the globe beside S); the sign ♌ (on the equator, right of G)

But we supposed the Sun's equal place to be at 15° of Aquarius, that is, the Earth at G, at 15° of Leo. For Boulliau the equation here is additive, 1° 18′ 6″, so that the Sun's apparent place is at 16° 18′ 6″ of Aquarius, whose right ascension, counted from the tropic of Capricorn, is 48° 47′, at Δ. But the place of mean time was found above on the line IXK, at 45°. Therefore XΔ, the equation of time, is 3° 47′, that is, 15^{m} 18^{s} of time, by which mean time is greater than apparent; for the point X crosses the midheaven sooner than Δ. There remains now the third cause, namely the inequality of the Earth's diurnal revolutions, which are slower around aphelion and in the semicircle HCF than in the semicircle FDH; and these accelerations and retardations are similar. For Boulliau the annual motion has become faster here by 38′ 54″ in the ecliptic. So the place of apparent time, Δ, will be nearer the meridian by 38′ 54″ at the moment when the place of mean time is on the meridian itself. Because of the acceleration, therefore, the difference of mean from apparent, which was 3° 47′, must be reduced by subtracting 38′ 54″, so that 3° 8′ 6″ remain, namely 12^{m} 32^{s} of time, by which in this case mean time precedes apparent; and so for the rest. On this foundation rest the three tables of *Boulliau*, set on pp. 8 and 9 of the tables.

VII. Among so many disagreements of astronomers, no method seems to me more certain than the customary astronomical one, on which already chap. 31, no. 5: both because it rests on sure foundations, not on ones made up at will or by ingenuity, and because, all things weighed, it satisfies the eclipses I have observed more than the others do. The objections brought against this method flow either from false hypotheses about the exact motions of the Sun and Moon, or from an incorrect difference of meridians, or from observations of eclipses that are deceptive for the reasons indicated by *Kepler* in the *Ephemerides*, p. 12, by *Wendelen* in the preface to the *Eclipses* and to the *Idea of the Atlantic Tables*, and by ourselves below, on eclipses. But tables must be used whose epochs pertain to mean time; and thus the second part of the precept, on which chap. 31, no. 5, is to be employed. This will be the simpler for us, the fewer the causes of the inequality of days we acknowledge, having of course rejected the mutations, let alone the anomalies, of the obliquity of the ecliptic, of the eccentricity and of the equinoxes, according to what was said in chap. 31, no. 3.

[Margin: Our opinion.]

SCHOLIA.

Scholia.

I. Boulliau, *Book 2, chap. 7, refutes Ptolemy because he used epochs adapted to apparent time. From this it comes about that, when the epoch is varied, the Ptolemaic equation varies, and that ancient method does indeed show how much some one span of apparent time, elapsed as a whole, differs from mean time, but cannot reduce any point of time from apparent to mean, or conversely. These points also tell against the Alphonsine and Prutenic tables, whose epochs pertain to apparent time. In this, however, this method is not false, if it serves the use for which it was born: namely to show by how great an interval apparent and mean time differ from each other over some given span, but not to equate time by reducing from apparent to mean.* Tycho *too, on p. 56, although he knew that time must be equated, nevertheless omitted the equation with the Sun observed at the beginning of Aries, because there is no right ascension there. But see the end of our second precept, chap. 31, no. 5.*

[Margin: The imperfection of the Ptolemaic, Alphonsine and Prutenic equation.]

[Margin: A blemish of Tycho's in the epochs.]

II. Jean-Baptiste Morin, *in the treatise on longitudes, part 7, chap. 4, admits two causes of inequality,* the Sun's eccentricity and the obliquity of the ecliptic: *for from the eccentricity arises the Sun's unequal apparent motion in the ecliptic. But in chap. 6, forgetting himself, he applies only one, when he says:* "For a given mean time, let the mean and true place of the Sun be taken from the tables, as well as the right ascensions of those places themselves; and let their difference, if there is any, be converted into time." *And earlier, says* Boulliau, *he had concluded in these words:* "Therefore the equation of time is nothing other than the difference of

the right ascensions of the mean and true place of the Sun at the same instant, converted into time." *But this is to admit no other cause than the difference of the mean and true place of the Sun: for, without perceptible difference, in the proportion in which the mean and true place of the Sun differ, the ascensions of the mean and true place differ too. So Morin was deservedly thrashed by* Boulliau, *Book 2, chap. 7, and by* Pierre Hérigone, *at the end of vol. 5 of the* Cursus Mathematicus; *and not without much salt of acrimony was the swelling of his boasting dried up, the boast in which he had gloried that he had found a new method of equating time, unknown to others. He had also accused Tycho and others on a false charge, as if they had used not only the equator as the measure of time but also the ecliptic.*

[Margin: Morin's blunders on the equation of time.]

III. *In chap. 31, no. 2, we compared the solar day with the sidereal and with the pure day of the primum mobile, and indicated two kinds of tables: one for the mutual conversion of parts of the equator and times of the primum mobile, the other for the mutual conversion of parts of the equator and times of the solar day. Someone will perhaps therefore wonder when one should be used rather than the other. We reply to him that the small table of solar times must be used whenever the measures and limits of time are referred to the Sun, and the day and parts of the day depend on its motion, which indeed happens in most cases. But if the times do not depend on the Sun's motion, as when we count the interval between the transit of two fixed stars across the meridian, the other small table must be used. Granted, because of the small difference, most people use the table of times of the primum mobile, in which to one hour there correspond exactly 15° of the equator, and to one minute of an hour exactly 15′ of the equator.*

[Margin: The use of the double table of the equator converted into times.]

IV. *The way of perceptibly distinguishing the time of the primum mobile, and one day of it from one solar day, is this: by means of a pendulum, subtract* 4^m *from the solar day; or count its oscillations exactly from the transit of some fixed star across the meridian to the transit of the same star the following night across the same meridian, and then with the same pendulum count the oscillations from the transit of the Sun's centre to the second transit of the Sun across the same meridian, as I said in Book 3, chap. 20, where the motion of the pendulum is treated. For one revolution of the fixed stars differs imperceptibly from the diurnal revolution of the primum mobile.*

[Margin: A way of distinguishing the day of the primum mobile from the solar day.]

CAPVT XXXIII.

Chapter 33.

De Conſtituendis Epochis, ſeu Radicibus Motuum Solis, Ordinandiſq. Tabulis Solaribus.

On establishing the epochs, or radices, of the Sun's motions, and on arranging the solar tables.

I. *Epocha*, in Greek ἐποχὴ [Greek: epochē, "epoch"], from ἐπέχω [Greek: epechō, "I hold back"], that is, "I restrain, I stop, or I occupy", is a fixed position on which something stands and rests, and from which, as from an immovable starting point, things arise. Hence the epoch of times and motions among astronomers and chronologers is some certain and fixed moment of time from which the other times and motions are counted, backward or forward, with respect to a certain place on Earth. It is also called the radix [Radix] and the era [Aera]. Thus if motions and times are counted from the beginning of the world, or of the Olympiads, or of the reign of Nabonassar, of Augustus, etc., or from the death of Alexander, or from the birth of CHRIST our GOD, or from the making of King Alfonso's tables, it is called the epoch, or radix, of the world, the epoch of the Olympiads, etc. Now, so that the motions of the Sun and of the other stars can be collected for any given time whatever, there must be some certain and known starting point from which the count of the motion is to be begun, and it must be known what place in the heavens that star held at the moment of time assumed as the epoch, or radix. The way of establishing epochs, especially of the solar motion, is indicated by *Ptolemy*, Book 3, at the end of chaps 8 and 10; the *Alphonsines*, Canon 8 and at the end of 33, and pp. 33 and 41; *Reinhold* in the Prutenic Tables, Precepts 3 and 4, and in the tables, p. 1; *Copernicus*, Book 3, chap. 26; *Tycho*, vol. 1 of the *Progymnasmata*, p. 56; *Johannes Stadius*, Canon 3; *Giovanni Antonio Magini* in the tables of the second mobiles, from Can. 14, and in the tables, p. 40; *Mulerius* in the *Tabulae Frisicae*, pp. 90, 132, 294 and 309; *Longomontanus*, p. 188; *Lansbergen*, Precepts 3 and 8; *Boulliau*, Book 2, chap. 7, and in the tables, Prec. 1; *Kepler* in the Rudolphine Tables, chaps 15 and 18. From these, take the following rules.

[Margin: What an epoch is.]

[Margin: Authors on establishing epochs.]

II. *First*, the meridian must be determined, indeed the horizon too, that is, the longitude and latitude of the place to which the epochs of the motions are to be attached. And it is surely best that those who build astronomy not from others' observations but from their own should use the place on Earth in which they themselves carried out the observations, even if it was otherwise obscure; or at least the one in which observations were made by the most eminent practitioners. Granted, the epochs can be transferred to other places, if the difference of meridians is known from eclipses or otherwise. *Secondly*, a definite beginning of the year, starting from the first day of the year, must be fixed: not indeed from the rising or setting of the Sun, for the reason already given in chap. 31, no. 3, cause 4, but from the noon of the first artificial day of the year, or from the midnight before the first artificial day of the year, or from the noon immediately preceding, at which the last natural day of the preceding year was completed. This last pleases me more, because on account of meridian observations of the Sun at the equinoxes and solstices, most astronomers have begun natural days from noon rather than from midnight. And if the beginning of the first day is taken from the preceding noon, it will not be necessary in

computing motions always to subtract one day from the given day, as is done in tables that begin the day from the current noon. Nevertheless, most have preferred to start the day and the year from this point, as is clear from the following catalogue.

[Margin: The choice of meridian and horizon.]

[Margin: The beginning of the year and the day.]

They take the beginning of the year FROM THE NOON OF THE NAMED DAY

[Margin: Table beginning from the current noon.]

Authors	*First day*	*Place*
Ptolemy	Thoth	Alexandria in Egypt
Copernicus in the epoch of the Olympiads;	Hecatombaeon	Frombork
Reinhold in the epoch of Nabonassar and of Alexander the Great.	Thoth	Königsberg in Prussia
And in the epoch of the Olympiads	Hecatombaeon	the same place
Magini in the earlier epochs, from the same source as Reinhold		Venice
Johannes Stadius in the epoch of CHRIST	January	Berge in Belgium
Tycho and *Boulliau* in the epoch of CHRIST	January	Uraniborg
Longomontanus in the epoch of the world and of CHRIST	January	Copenhagen
Giuseppe Moletti in the epoch of CHRIST	January	Rome
Kepler in the epoch of CHRIST, and before CHRIST	January	Uraniborg
Lansbergen in the epoch of Nabonassar	Thoth	Goes
In the epoch of CHRIST	January	Goes
Origanus in the tables and ephemerides	January	Frankfurt on the Oder
Vincenzo Renieri in the epoch of CHRIST	January	Genoa

FROM THE MIDNIGHT PRECEDING *the first day of January*

[Margin: Table beginning from midnight.]

Authors	*Place*
Reinhold in the Prutenic epochs of C. Julius Caesar and of CHRIST	Königsberg
Mulerius in other Frisian epochs of Ptolemy, of Alfonso, etc., pp. 89, 132 and 309	In Friesland, and at Alexandria
Denis Pétau in the Paris tables	Paris
Andrea Argoli in the epoch of CHRIST	Rome
Govaert Wendelen	In the midst of Iceland and of the Atlantic

FROM THE PRECEDING NOON *of the last day of December*

[Margin: Table beginning from the preceding noon.]

Authors	*Place*
The Alphonsines, in the epochs of CHRIST, of Caesar and of Alfonso	Toledo
Giovanni Bianchini in the epoch of CHRIST	Ferrara

Authors	*Place*
Magini in the *Supplement to the Ephemerides*, and in the later epochs of the second mobile, of CHRIST and of Caesar	Venice
Francesco Giuntini in the resolved tables, and the epoch of CHRIST	Florence
We, in the epochs of CHRIST and before CHRIST	Bologna

III. One must have some observation of the Sun's true place, made exactly at apparent time on some meridian, and the Sun's mean place for the same time. And if the epoch is to be set for apparent time, as *Ptolemy, Alfonso* and *Copernicus* did, one must collect the years, months, days, hours and minutes of hours that intervene between the time of the observed true place and the moment of the assumed epoch. For the interval of time just mentioned, one must seek out from the tables, or from elsewhere, the mean motion that corresponds to it, and add it to the mean motion of the apparent time of the observation made, if the epoch follows that time, but subtract it if the epoch precedes; and you will have the mean or equal motion belonging to the epoch that represents apparent time. But if the epoch is to be adapted to equal or mean time, as we showed must indeed be done in chap. 31, Scholium 1, and as *Tycho, Longomontanus, Lansbergen, Ke- pler, Boulliau, Renieri* did, then the apparent time of the observation must be equated and converted into equal time, following the second part of the rule given in chap. 31, no. 5. Next, the interval of time up to the moment of the assumed epoch must be collected. The mean motion corresponding to this interval must then be collected and added to the mean motion of the equal time at which the observation was made, if the epoch is later than that time, but subtracted, if it is earlier; and you will have the mean or equal motion belonging to the epoch of mean time. But suppose what has been observed is the vernal equinox, that is, the Sun's true ingress into Aries, where there seems to be no right ascension and therefore no equation of time. *Tycho* (p. 56) omitted this equation; but it should not have been omitted. Now to an example of the second case.

EXEMPLVM.

Example.

Tycho, as given in vol. 1 of the *Progymnasmata*, pp. 52 and 56, observed the vernal equinox at Uraniborg in the leap year 1588 March 9, Old Style, at $20^{h}\ 45^{m}$ after noon. The Sun's simple or mean motion, according to the same Tycho, was then $11^{s}\ 27^{\circ}\ 57'\ 25''$. From this apparent time to noon of the preceding January 1 there are 68 days $20^{h}\ 45^{m}$. To these, according to the mean quantity of the Tychonic year, there belong $2^{s}\ 7^{\circ}\ 52'\ 35''$ of mean motion. Subtracted from the Sun's mean motion at the equinox, $11^{s}\ 27^{\circ}\ 57'\ 25''$, these leave $9^{s}\ 20^{\circ}\ 4'\ 50''$. So in the epoch of the year 1587 complete and 1588 beginning, Tycho set the Sun's place or mean motion at $9^{s}\ 20^{\circ}\ 4'\ 50''$, with no other correction.

[Margin: Tycho's epoch corrected.]

But proceeding more rigorously, the apparent time of the equinox should have been corrected and changed into equal time, in this way. At the equinox the Sun's mean place was near the beginning of Aries and its true place at the beginning of Aries, that is, at the end of the ecliptic and the completed 360th degree, or 12 signs. Take away from these the mean place, $11^s\ 27^\circ\ 57'\ 25''$, and there remain $2^\circ\ 2'\ 35''$, by which the right ascension taken for the Sun's true place exceeds the mean place. This excess, converted into time, corresponds to $8^m\ 10^s$ of an hour, to be added to the apparent time of the equinox, so that the equal time is March 9, $20^h\ 53^m\ 10^s$. From here to noon of the preceding January 1, therefore, there are 68 days $20^h\ 53^m\ 10^s$, to which there belong $2^s\ 7^\circ\ 52'\ 55''\ 7'''$ of mean motion. Subtracted from the simple motion, or the Sun's mean place, $11^s\ 27^\circ\ 57'\ 25''$, these leave the Sun's mean place at noon on the first day of January of the year 1587 complete, at Uraniborg, as $9^s\ 20^\circ\ 4'\ 29''\ 53'''$. For the rest, let us establish from our own observation our epoch of the solar motion for the year 1646, which will be the second example.

EXEMPLVM ALTERVM.

Second example.

[Margin: See the end of Chapter 14]

In 1646 September 22 we observed the equinox at Bologna at $14^h\ 56^m$ after noon in apparent time, when the Sun entered Libra, whose right ascension is exactly 180°, or 6 signs. But from our prosthaphaeresis the Sun's mean place was $6^s\ 1^\circ\ 58'\ 57''$. So the mean motion was greater than the right ascension by $1^\circ\ 58'\ 57''$. This difference, converted into time, is $7^m\ 53^s\ 48'''$ of an hour, to be subtracted from the apparent time of the equinox, leaving as the equal time September 22, $14^h\ 48^m\ 6^s\ 12'''$. From here to the end of the year and the last noon of December there are 99 days $9^h\ 20^m\ 5^s\ 28'''$. To these, from our mean motion of the Sun, treated in chap. 17, there belongs a simple motion of the Sun of $3^s\ 7^\circ\ 57'\ 24''\ 40'''$. Added to the Sun's mean motion found at the equinox, $6^s\ 1^\circ\ 58'\ 57''$, these give, for the end of the year 1646 complete, at the last noon of December, the Sun's mean motion as $9^s\ 9^\circ\ 56'\ 22''$, at Bologna.

[Margin: Our epoch of the Sun.]

IV. Once one epoch has been properly established in this way, it is easy to establish others before and after it, by subtracting or adding the mean motions corresponding to the intervening years. But in the jump from the Julian years to the Gregorian, account must be taken of the mean motion corresponding to 10 days. Thus, since for me the Sun's mean motion in 46 years is $11^s\ 29^\circ\ 51'\ 50''$, this, subtracted from the mean motion of the epoch of the year 1646 complete, gives the mean motion of the Gregorian year 1600, complete, at Bologna at noon on the last of December, as $9^s\ 10^\circ\ 4'\ 32''$ in equal time. If to this is added the motion of 10 days, which is $9^\circ\ 51'\ 23''$, the result is the root of the year 1600 in the old Julian style, $9^s\ 19^\circ\ 55'\ 55''$. Subtracting from this the motion of 1600 years, which for us is $0^s\ 12^\circ\ 24'\ 43''$, gives the root of Christ, that is, the epoch at

the beginning of the years of the common Christian era, $9^s\ 7°\ 31'\ 12''$. Granted that, from Hipparchus's equinoxes, this root has elsewhere been $9^s\ 7°\ 28'\ 55''$.

V. Let us now compare our epoch with the epochs of others. For those that begin the day from noon of January, in order to bring them back to the preceding noon of the last day of December, we shall subtract the Sun's diurnal motion, $59'\ 8''\ 20'''$; for those that begin it from midnight, the half-diurnal motion, $29'\ 34''\ 10'''$. And we shall add the time corresponding to the difference of meridians between Bologna and the places of the other epochs. The epochs of Nabonassar, Augustus, etc. we shall set out in the chronological book; meanwhile, see them in *Alfonso*, pp. 33 and 41; in the Prutenic Tables, Can. 10 and at the end; in *Gaurico*, in the notes to Ptolemy, bk. 3, chap. 10; in *Kepler*, in the Rudolphine Tables, p. 39; in *Lansbergen*, in the tables, p. 3; in *Boulliau*, in the tables, pp. 5 and 11; and in *Renieri*, in the *Tabulae Mediceae*, p. 15.

Root of the Sun's mean motion at the beginning of the common epoch of Christ, in equal time, at noon on the last of December at Bologna.

Authors of the tables	Place of the epoch	Sun's mean motion from Aries: Sign	Deg.	′	″	‴	Difference of meridians from Bologna: h	m	s	Sun's motion for the said difference: ′	″	‴		Therefore the Sun's Bologna motion and its root: Sign	Deg.	′	″	‴
Ptolemy	Alexandria	9	7	16	25	40	1	43	0	4	13	48	Add	9	7	20	39	28
Alfonso	Toledo	9	8	21	0	30	1	15	20	3	5	38	Subtr.	9	8	17	54	52
Reinhold	Königsberg	9	7	32	41	21	0	34	0	1	23	47	Add	9	7	34	5	8
Tycho	Uraniborg	9	7	38	55	16	0	0	0	0	0	0		9	7	38	55	16
Longomontanus	Copenhagen	9	7	38	1	40	0	0	48	0	1	58	Subtr.	9	7	37	59	42
Kepler	Uraniborg	9	7	50	48	40	0	0	0	0	0	0		9	7	50	48	40
Lansbergen	Goes	9	7	37	25	40	0	34	0	1	23	47	Subtr.	9	7	36	1	53
Boulliau	Uraniborg	9	7	58	1	40	0	0	0	0	0	0		9	7	58	1	40
Ourselves	Bologna	9	7	31	12	0	0	0	0	0	0	0		9	7	31	12	0

[Margin: Epoch of the Sun's apogee.]

VI. Once the epochs of the Sun's mean motion are established, the epochs of the Sun's apogee must be established. In these it is not necessary to equate the time, because the apogee's very slow motion produces no sensible difference

within one month. Therefore, since we found the Sun's apogee in 1646 July 28 at Cancer 7° 26′ 15″, from what was said in chap. 24, no. 3, and chose its annual motion as 1′ 1″ 10‴, from what was said in chap. 25, no. 6, then, keeping this ratio, its root at the beginning of the years of Christ comes out for us at Gemini 9° 28′ 45″. For the motion of 1646 years is 27° 57′ 32″, which, subtracted from Cancer 7° 26′ 15″, leaves Gemini 9° 28′ 45″.

[Margin: On tables of equations.]

VII. Now that the epochs of the mean motion have been established, together with tables showing the mean motions of the Sun and of the apo- gee per hour, day, month and year, tables must be constructed of the prosthaphaereses, or equations, of the Sun, at least for each degree of anomaly, according to the ratio of the eccentricity to the radius of the Sun's orb, following the rules given in chap. 21.

[Margin: On tables of distances, parallaxes and apparent semidiameters.]

VIII. Finally, a table must be constructed in which, for every five or at least every ten degrees of the Sun's anomaly, the Sun's distances are to be placed opposite those degrees. For this task, repeat here the figure treated in chap. 19, no. 5, or p. 143, since the hypothesis of the eccentric is the more common and the simpler. In triangles BET or IET, given the eccentricity ET and the radius of the Sun's orb EB or EI, and the angle of mean anomaly HEB or HEI, and therefore its complement to two right angles, namely angle BET or IET, you will find by the rules of triangles the distance TB or TI. And so for the rest. The semidiameter of the epicycle, substituted in place of the eccentricity, does this in the same way. It is useful, moreover, to express these distances not only in abstract parts, of which the radius of the orb is 100000, but also in semidiameters of the Earth, which are treated in the table of Chapter 7. In addition, opposite the same degrees of anomaly the Sun's horizontal parallaxes must be placed, following the Problem of Chapter 8, and the Sun's apparent semidiameters, following Problem 3, treated in chap. 11, no. 4. And if to this table you add another table of the Sun's parallaxes for each degree of distance from the vertex, both apparent and true, at the greatest, mean and least distance of the Sun from the Earth, following what was likewise said in chap. 8, the work will be complete. We shall give such tables, God willing, at the end of the work. Now let us descend from Phoebus to Phoebe.

LIBRI TERTII FINIS.

End of the third book.

LIBER QVARTVS DE LVNA.

Book Four: On the Moon.

CAPVT I.

Chapter 1.

De Lunæ Nominibus, Encomijs, & Varietate qua exercuit hactenus Aſtronomorum ingenia.

On the Moon's names and praises, and the variety with which she has so far exercised the talents of astronomers.

[Margin: Etymology of the Moon's name.]

I. The name *Luna* seems to have been formed by contraction from *Lucuna*, because she alone and single sometimes shines together with the Sun; or from the name of *Lucina*, with the middle syllable dropped, as Isidore prefers (*Origines*, bk. 8). In Hebrew she is *Lebanah*, from whiteness. In Greek she is [Greek: Σελήνη (Selēnē)], or [Greek: Σελάνα (Selana)], or [Greek: Σελαναῖα (Selanaia)], from radiance and new splendour, as *Cleomedes* notes (bk. 2, chap. 3). For in Greek [Greek: σέλας, σέλατος (selas, selatos)], or [Greek: σελέας (seleas)], is "radiance", and [Greek: νέον (neon)] is "new". Hence the Arcadians, to boast that they were older than the Moon, also called themselves [Greek: προσεληναῖος (proselēnaios)], that is, "Antelunares", those before the Moon.

[Margin: Why Phoebe?]

The same goddess is called *Phoebe*, as the sister of Phoebus and his deputy, a nocturnal Sun as it were, and *Latois*, a patronymic from Latona. *Euripides* in the *Phoenissae* did call her the daughter of the Sun, [Greek: ὦ θύγατερ ἀιλίε σελάνα (ō thygater ailie selana, "O daughter of [?], Selana")]: I believe because, since she borrows her light from him, she may seem to have been begotten by him. But in the myths she is commonly held to be the daughter of Jupiter and Latona. So she is also called [Greek: Ἄρτεμις (Artemis)] and *Diana*, as Jupiter is [Greek: Διός (Dios)]; or because she shines by day and by night, and is as it were *Duana*, says Isidore.

[Margin: Why Diana?]

I pass over the countless names derived from the places where she grew up or was worshipped, above all from Delos and Cynthus, which gave *Delia* and *Cynthia*.

[Margin: Why Hecate and Proserpina?]

She was further held to be *Hecate* among those below, because unless appeased with a hundred victims she ordered the shades of the unburied to wander for a hundred years, as *Servius* attests. Some think she is *Proserpina*, by whose moisture the crops creep forth [proſerpant], as *Isidore* interprets it; or because the same goddess is the Moon in Heaven, Diana on Earth, and [Greek: Περσεφόνη (Persephonē)] under the earth. To this refers that line of *Ovid*, *Fasti* 1:

> *You see the faces of Hecate turned in three directions.*

And that of *Virgil* in the Eclogues: "— the three faces of the maiden Diana." And to that threefold attire, of which, in *Isidore*, bk. 8, the poet sings thus:

> *But when the Moon shines, she gleams in a dim robe; When, girt up, she shoots her arrows, she is the maiden of Latona; When she sits enthroned, she is Pluto's consort.*

[Margin: Why Trivia?]

Hence she is everywhere called *Trivia* and the Three-formed Goddess, and by Virgil *Tergemina*: either from the threefold phase and face in which, as *Cleomedes* teaches (bk. 2, chap. 3), she chiefly appears in the sky, since she is seen now full of light, now sickle-shaped or curved into horns, now bisected and half-full; or because she was believed to preside over fowling, hunting and fishing, and to have taught that three kinds of nets, for birds, wild beasts and fish, should be handled in silence and in shadow, under a dim sky and by a scant light.

[Margin: Why Dictyna?]

So from nets, which are called [Greek: δίκτυα (diktya)], she was named *Dictyna*.

[Margin: Isis and Astarte.]

Lastly, she is the same who among the Egyptians under the name of *Isis*, and among the Sidonians as *Astarte*, as Queen of Heaven, held temples truly profane, with an impious cult often condemned in Holy Scripture; and she who, as Macrobius says (*Saturnalia*, bk. 7, chap. 16), was called among the Latins *Juno* and *Lucina*, from helping births and happily bringing them forth into the light, as Virgil confirms in Ecl. 3 with this verse:

Be favourable, chaste Lucina: your Apollo now reigns.

I am not unaware, however, that the names of Isis and Juno are assigned to Venus by *Pliny the Elder*, bk. 2, chap. 8. But the Moon herself surpasses all variety of names by the admirable variety of her phases and motions.

[Margin: Praises of the Moon.]

II. This, moreover, is that lesser Light which God made and set in the firmament of Heaven to rule the night, and to be the forerunner not only of the New Moon [Neomeniæ] but also of Easter and of the movable feasts that depend on Easter, according to that passage of *Ecclesiasticus* 43: *And the Moon in all things in her season, a showing of time and a sign of the age. From the Moon is the sign of the feast day, a light that wanes at its completion. The month is according to her name, increasing wonderfully at its completion.* There is no contradiction in this passage, granted that she is said both to increase and to wane at her completion: whether because, as the synodic lunar month is completed, she wanes in such a way that she begins to increase immediately after new moon, which is why at that time the Greeks called her old and new Moon; or because, as she approaches new moon, although she seems to us to wane in light, she is nevertheless illuminated from above by the Sun over a greater portion of herself than at other times.

She is also a most splendid symbol: both of the Catholic Church, which receives all the light it shines with from Christ, its Sun; and, by antithesis, of the Virgin Mother of God, who therefore has the Moon under her feet (Apocalypse 12), because she never felt the alternations of sin and grace, but was always clothed with the Sun of original grace. Of the Moon's remarkable power over sublunary things I shall say something in the Scholia. It was certainly for this reason that *Macrobius* (bk. 1 on the *Dream of Scipio*, chap. 11) called her *the*

boundary of life and death, and said in chap. 21: *As she is of aether and air, so the Moon is the boundary of things divine and perishable.*

III. Above all, astronomy seemed to Pliny the Elder to owe so much to the motions and changes of the Moon that he said of her (bk. 2, chap. 9): *I do not know whether she is not the teacher of all that could be learned in the heavens: that the year must be divided into spaces of twelve months, since she herself overtakes the Sun that many times as it returns to its starting point.* Indeed, the apparent proper motion toward the east was taught most plainly of all by the Moon's recession from the meridian, conspicuous at about 13 degrees a day. Moreover, like the chorus-leader of the other stars, she so connects them with the Sun that, as the sole or chief sharer of both day and night, she shows us their right ascensions and declinations, and likewise their longitudes and latitudes: on the one hand through her distance taken from the Sun rising or setting, on the other through the interval between herself and the stars observed at night.

[Margin: Pliny the Elder's encomium of the Moon, in most elegant antitheses.]

I cannot restrain myself from copying into our subject that entire encomium of the Moon from bk. 2, chap. 9 of Pliny the Elder. *But what surpasses the admiration of all is the last star, the most familiar to the Earth, found by nature as a remedy for darkness: the Moon. With her many-formed riddle she has tormented the minds of those who contemplate her and who resent that the nearest star should be the least known, always waxing or waning. Now curved into horns, now divided in equal portion, now rounded into a circle; blotched, and then suddenly shining bright; immense with full orb and suddenly nothing; sometimes up all night, sometimes late, and for part of the day helping the Sun's light; failing, and yet visible in her failure; she hides at the month's end, when she is now believed to be labouring. Again, she is low and high, and not even that in one way, but at times brought close to Heaven, at times touching the mountains; now raised to the north, now cast down to the south. Endymion, first among men, detected each of these things in her, and for that reason, it is recounted, was seized with love of her. We are surely not grateful to those who by labour and care have opened up light for us in this light.*

Or rather, I say, in this darkness. For after her ancient anomalies *Tycho* discovered a new variation in the Moon, and *Boulliau* suspects yet another, either a variation or a libration, but still awaits greater light from the test of observations and begs the help of those who possess Tycho's logbooks. And her monthly equations seemed to *Kepler*, before the Ephemerides, no. 16, suspect of wild behaviour, so much so that in the Rudolphine Tables, before rule 171, he exclaims as if in despair:

[Margin: Many inequalities in the Moon.]

After all the craftsmen's counsels have been exhausted, after so many inequalities of the Moon have been brought to light, she is still a stubborn star, spurning laws, and strays off course by tiny amounts here and there. Above all, for a reason proved by many examples, around the point of the vernal equinox she is found to come under the Sun, or into the shadow, later, and around the autumnal point sooner, than the

calculation so far set out indicates. But so far only eclipses have supplied the tests: what happens to the Moon when she crosses the equinoctial points with the Sun absent, no one, as far as I know, has attended to. So to seek the causes is premature, and no part of this book. Meanwhile, therefore, until time brings them forth, we shall follow the rule shaped from moderate experience.

And in the *Epitome of Astronomy*, p. 558, you say that observations of the Moon still disagree in the smallest quantities with any calculations whatever, and that it is uncertain to what this discrepancy should be referred. But will it ever come about that these things become known to some Endymion, and that what *Seneca* foretold on another occasion (*Natural Questions*, bk. 7) proves true, when he said:

[Margin: Seneca's near-prophecy.]

There are many peoples today who know the heavens only by their face, who do not yet know why the Moon is eclipsed, why she is darkened. A time will come when the day, and the diligence of a longer age, will draw into the light those things that now lie hidden. A time will come when our descendants will wonder that we did not know things so plain.

SCHOLIA.

Scholia.

I. The Moon's power over these lower things is manifest in innumerable effects, of which I should like here to review the more remarkable in some order. First, then, the Moon's power over the conceptions and births of animals is wonderful. That shrewd saying of Pliny the Elder (bk. 18, chap. 32) runs: "Set eggs to hatch at new moon"; and *Columella* (bk. 8, chap. 5) and *Palladius* (bk. 1, chap. 27) confirm it.

[Margin: The Moon's power over eggs.]

But Ptolemy, in prop. 51 of the *Centiloquium*, says: "Whatever sign the Moon is in at the time of birth, make that the Ascendant at conception; and whatever sign she is found in at conception, make that or its opposite the Ascendant at birth." And in bk. 3 of the *Apotelesmatics* he teaches that the position of the heavens, but chiefly with regard to the Moon, contributes to the fetus's moving and coming forth into the light, once it has ripened, in the same position as at conception.

[Margin: The Moon's power in the conception and birth of human beings.]

So it comes about that those conceived at the interlunium also come forth into the light at the interlunium, granted that those so conceived are for the most part weak through lack of moisture. Again, it has been observed in most of Greece, Italy and Spain that infants born in even forties of days, counted from the day of conception, are mortal, while those born in odd forties are viable. This is why a seven-month, nine-month or ten-month birth, and sometimes an eleven-month one, is viable, but an eight-month birth mortal. But these months are lunar, as is taught by Hippocrates in the book *On the Seven-Month*

Birth and the book *On the Eight-Month Birth*, by Aristotle (*On the Generation of Animals*, bk. 4, chap. 10), by Bonaventura (*On the Eight-Month Birth*, bk. 7) and by Turnebus [Adrien Turnèbe] (*Adversaria*, bk. 4, chap. 15), together with the more learned physicians and natural philosophers. Granted, Avicenna (*Canon* 3, Fen 21, chap. 2) and our own Lorinus [Jean Lorin] (on chap. 7 of Wisdom) think that in this matter months of the true or civil solar year should be used.

[Margin: The power of the forty days in the concoction and ripening of the fetus.]

Therefore, according to the observation of the Pythagoreans and of Hippocrates, if an infant is born after the 160th day from conception, when four forties are completed, and before the 200th day, when the fifth forty is completed and the sixth begins, it will be viable, and a child of 7 months. These are not indeed completed months, unless they are all short, since 7 mean lunar months have 206 days 17^{h} 8^{m}; but it is so called because it will have a great part of the seventh lunar month. Again, if it is born after the 240th day, when the sixth forty is completed, and before the 280th, when the eighth begins, it will be viable, and it can be a nine-month or a ten-month child, not, however, so as to complete ten lunar months, but so as to take part of the tenth; for mean lunar months, if nine, amount to 265 days 18^{h} 36^{m}, and if ten, to 295 days 7^{h} 20^{m}. But if it is born after the 200th day and before the 240th, that is, within the sixth forty of days, it is not viable. And since those born within this time are usually and almost always eight-month births, eight lunar months amounting to 236 days 5^{h} 52^{m}, so that they either complete the eighth month or take the greater part of it, an eight-month birth is therefore as a rule held to be not viable.

[Margin: At what Moon hair and sheep are to be shorn.]

II. Among physicians, certainly, ignorance of the Moon's motion and place cannot be tolerated without danger to the sick. For, to pass over critical days, surgery needs the Moon, in support of which Pliny the Elder (bk. 16, chap. 39) says: "Tiberius likewise observed the interlunia in cutting his hair as well. M. Varro prescribed, against falling hair, that it be observed from the full moons." And in Varro (*De re rustica*, bk. 1) it reads thus: "I keep these things too," says Agrasius, "received from my father, not only in shearing sheep but with my own hair, so as not to go bald by cutting it when the Moon is waning." Ptolemy, in the *Centiloquium*, prop. 20, rules as follows: "Do not strike a limb with iron when the Moon occupies the sign that rules that limb."

[Margin: Which signs rule which limbs.]

And which signs of the Zodiac rule which limbs, *Manilius* (bk. 1) expressed elegantly in these verses:

> *For Aries clings to the head; Taurus to the neck; The arms are reckoned under Gemini; the breast under Cancer; The shoulder-blades call you, Nemean, and the flanks you, Virgo; Libra tends the buttocks, and Scorpio reigns in the groin, And the Archer in the thigh; Capricorn too has loved the knees; The Youth guards the shins; Pisces, the feet.*

Macrobius (bk. 1 on the *Dream of Scipio*, chap. 11) calls the Moon the foundress and increaser of mortal bodies; and in bk. 11 of the *Saturnalia*, chap. 19, the Moon is called [Greek: τύχη (tychē, "fortune")] of the human body.

III. The day would fail me if I wished to gather here the laws of agriculture laid down to the Moon's standard. Let Colu- mella be read (bk. 2, chap. 10), and Palladius (bk. 2, chap. 6), and Pliny the Elder passim. For in bk. 16, chap. 39, speaking of the felling of oaks, Pliny the Elder says: "The Moon's reckoning matters enormously, and they will have timber felled only from the twentieth to the thirtieth. All agree, however, that it is laid low most usefully at her conjunction, a day which some call the interlunium and others that of the silent Moon. [Margin: At what Moon trees are to be planted or felled.] Some say that the Moon should be both in conjunction and under the earth, which cannot happen except at night. But if the conjunction falls on the last day of the winter solstice, that timber becomes everlasting." Next he draws this rule from Cato: "Elm, pine, walnut: this and all other timber, when you dig it out, take out at the waning Moon, after midday, with no south wind blowing. Do not touch timber except at the intermenstrual Moon and the half Moon. At that time do not dig it out or cut it off from the ground. In the four days next after the Moon has been full it is best taken out ——" After this, in bk. 18, chap. 25: "Others say that the bean should be sown at full Moon, and the lentil from the 25th to the 30th; the vetch too on the same days of the Moon." The whole of chap. 32 of bk. 18 is worth reading, where Pliny the Elder lays down first of all: "Everything that is felled, picked or stored is done with less harm at the waning Moon than at the waxing." And likewise: "Castrate boars, bullocks, rams and kids at the waning Moon." Again: "Make trenches at night at full Moon, cover the roots of trees at full Moon, sow in moist places at the interlunium and during the four days around the interlunium." Macrobius chimes with this (bk. 7 of the *Saturnalia*, chap. 16), saying that timber brought down at a full or waxing Moon is unfit for building; that grain should be gathered from the threshing-floors only at a waning Moon, so that it stays dry; and that nurses cover infants when they pass under the Moon, so that they are not moistened too much.

[Margin: The Moon, mistress of fluids.]

IV. For it is admitted that fluids grow as the Moon waxes and diminish as she wanes. And so Pliny the Elder says (bk. 2, chap. 99): "From this arises the true conjecture that the Moon's star is not without reason reckoned the star of breath: that this is what saturates the lands, and that in approaching she fills bodies and in withdrawing empties them; that for this reason shellfish grow with her increase, and those creatures that have no blood feel the breath most; and that even a human being grows and shrinks in blood with her light." And in chap. 101 he calls the Moon's star feminine and soft, because "it dissolves moisture, melts ice, reduces the slain bodies of wild animals to putrefaction, and loosens all things with its moistening breath." And in bk. 9, chaps. 7 and 31, and bk. 2, chap. 41: "the bodies of crabs and shellfish grow with the Moon's

increase." Macrobius too (bk. 1 on the *Dream of Scipio*, chap. 19) says that just as the power of sensing in us comes from the Sun, so that of growing comes from the Moon; and in bk. 7 of the *Saturnalia*, chap. 16, that boars killed in the hunt and carried in by night under a full Moon went putrid, while others brought in by day did not, because in the Moon's light there is a hidden warmth, one that diffuses moisture all the more. Yet the distinction Pliny the Elder makes (bk. 18, chap. 28) must be noted: "For at the interlunium the Moon is hottest in summer and icy in winter; conversely, at full Moon she makes the nights icy in summer and mild in winter." He gives the reason: that in summer, at new Moon, the Moon is nearer to us, as the Sun also is, that is, she strikes with more direct rays; but in winter she is nearer at full Moon. And he concludes: "Therefore, being of herself dewy, whenever she is cold she freezes to an immense degree the frosts falling at that time." Alcman too, in Macrobius, calls dew the son of the air and the Moon.

[Margin: Wonders of the tide of the sea, owing to the Moon.]

V. But nothing is more wonderful for showing the Moon's power than the tide of the sea, of which we showed the Moon to be the governor in bk. 2, chap. 15. To what was said there, two points must be added here. The first is what Pliny the Elder notes (bk. 2, chap. 98): "All seas are purged at full Moon." And shortly after: "No animal dies except when the tide is going out. This has been much observed on the Gallic Ocean, and established only in the case of man." The second comes from the same Pliny the Elder (bk. 2, chap. 97): the seas flow in and out twice within 24 hours, and all the periodic variation of the tides is completed in eight years, namely in 2922 days, in which nearly 99 lunations occur and the Moon goes round the Earth 2823 times, that is, 99 times less often than the Sun. Doubling the 2823, then, gives 5646 alternating tides in eight years, and thus 706 alternating tides in one year. But see the commentary on Pliny the Elder for this chapter written by Iacobus Mylichius [Jakob Milich]. On forecasts of winds and storms from the Moon, moreover, see Ptolemy at the end of bk. 2 of the *Apotelesmata*, Pliny the Elder (bk. 18, chap. 5), and Virgil, *Georgics* 1.

To conclude these scholia: I say that many things are brought about by the Moon of which man is shamefully ignorant, though the brute animals are not ignorant of them, since, on the testimony of Pliny the Elder (bk. 2, chap. 41, and bk. 18, chap. 29), [Margin: The ant feels the Moon's changes.] the ant, the smallest of animals, rests at the interlunium, and at full Moon works even by night.

CAPVT II.

Chapter 2.

De Lunæ Subſtantia, & vtrum ſit altera Terra, habeatq. ſuam atmoſphæram ſuoſque incolas.

On the Moon's substance, and whether she is another Earth and has her own atmosphere and her own inhabitants.

I. Apart from the controversy the Moon shares with the other heavenly bodies — whether they are of a substance different from the elementary, as the Peripatetics prefer, or rather elementary, those everlasting fires having been kindled from water on the fourth day of the world by divine command (a matter on which we gave a few indications in bk. 3, chap. 1, and which remains for us to discuss in bk. 9, on the system of the world) — certain things peculiar to this planet must be touched on briefly at this point. For to most writers she has seemed to be of an elementary nature, though to some fiery, and to others rather watery or earthy. [Margin: Opinions of the ancients on the nature of the Moon.] Let us hear first of all Plutarch, *De Placitis Philosophorum*, bk. 2, chap. 25, where he writes this on the Moon's nature: *Anaximander: that she is a circle nineteen times the proportion of the Earth, like the Sun full of fire, and that the failure of her light befalls her through the turning of the wheel. For she is like the wheel of a chariot, which has a hollow rim filled with fire and is furnished with one vent for breathing out. Xenophanes: that she is a compacted cloud. The Stoics: that she is mixed of fire and air. Plato: that the greater part of her is earthy. Anaxagoras and Democritus: a fiery firmament containing within itself plains, mountains and valleys. Heraclitus: an Earth enclosed in a murky cloud. Pythagoras: that the body of the Moon follows a fiery nature.*

[Margin: The Stoics' opinion: by whom and on what reasoning defended.]

Further, one Pharnaces, in the same Plutarch (the essay *On the Face in the Orb of the Moon*), defends the Stoics' opinion with roughly these arguments: that she is the lowest body and the nearest to the Earth, and is therefore fiery in such a way that her fire requires a hypostasis from admixed earth, by right of kinship and neighbourhood; yet she is not heavy as our Earth is, and even if she is, she does not fall downward, either because, like stones whirled round in a sling, she is held in her circle by the speed of her motion, or because she is herself the centre of other small bodies or of parts more akin to her. And even if it were granted that a place in Heaven is given to earthy things against nature, it would still not be shown by this that the Moon is not an Earth, but only that she is an Earth set in a place foreign to her nature, just as the fire of Etna is under the earth against nature, and yet is fire. Nor is it for the good of the universe that its parts should shun communication and mixture; rather, it belongs to the Providence of God the Maker that not each thing should lie where it was born to lie, but that heavy things should at times be carried upward and light things downward, and that, tempered with one another, they should make up a certain harmony and fellowship. After Pharnaces, Apollonides, in the same essay, affirms that the Moon is earthy rather than watery, but *a heavenly Earth, having wonderful beauties of place: flaming mountains, and separate purple*

things [?]*, and gold and silver not scattered in the depths, but such as gleam on her plains or are spread about on her hills.* Finally, a little before the end of this essay it is said, on the opinion of Plato and Heraclitus, that the Moon and all the stars are tempered from fire and earth; according to Xenocrates, however, that the stars and the Sun are composed of fire and the first dense, the Moon of the second dense and an air of her own, the Earth of the third dense, water and fire. Pliny the Elder, on the other hand (bk. 2, chap. 101), seems to have allowed her a watery nature, when he said: *that the Moon's nourishment is in fresh waters, as the Sun's is in salt ones*; although earlier (bk. 2, chap. 9) he had pronounced: *that the stars doubtless feed on earthly moisture, because at half orb she is never seen without spots, the power being evidently not yet sufficient for drawing beyond the due measure. For the spots are nothing other than the dirt of the earth carried up with the moisture.* And in Laertius [Diogenes Laertius], bk. 7, Zeno of Citium affirms that the Moon has more of earth, and is nourished [?] by waters. Macrobius (bk. 1 on the *Dream of Scipio*, chap. 19) says that the Moon is an ethereal and solidified Earth. Theodoret too, in Sermon 4, tells it thus: *The philosophers on the Moon, as follows: Thales thinks she is earthy; Anaxagoras and Democritus, a fiery solid mass containing plains, mountains and valleys set within it; Pythagoras, a stony body; Heraclides, an Earth hedged round with cloud.* After these, Longomontanus (*Theorica*, bk. 1, p. 315) called the Moon an ethereal Earth. Kepler too (*Epitome of Copernican Astronomy*, bk. 1, p. 100) says most expressly that the lunar globe is of the same nature as the Earth in this: that on its outermost surface it has mountains and valleys, continents and seas; and yet heavy things, such as the lunar waters, do not fall either to our Earth or to the middle of the world, but work their way into the places nearest the centre of the lunar globe. And in his dissertation with the *Sidereus Nuncius* he reports the opinion of Giordano Bruno, who says that the fixed stars and the Sun are of a fiery nature, the remaining planets of a watery one, and that by an inviolable law of Nature it comes about that neither can fire do without its water, that is, the Sun without the planets, nor in turn these without it; and that Jupiter is a kind of Earth. He also reports that of Mästlin, in the *Theorems of Eclipses*, who affirms that the Moon is of an earthy nature. Kepler himself, however, thinks that the Moon is another Earth, but one gaping everywhere like pumice with very frequent and very large pores, and rarer and less fit to resist the circular motion by which our Earth moves her round by magnetic force, in proportion as the Sun moves the other planets. On this see his *Commentaries on Mars*, chap. 34. In the *Astronomia Optica* too, p. 228, he says that the Moon's body is dense as the Earth is, and approves the opinion of Posidonius in Macrobius, who says that the Moon's matter is the same as the Earth's. Our own Athanasius Kircher, too (*Ars Magna Lucis et Umbrae*, bk. 1, chap. 4), holds that the Moon is a heavenly Earth and a watery body made by God so that the Sun's heat may be tempered and cooled. Galileo, in the first Dialogue *De Systemate Cosmico*, asserts that there is a great conformity between the Moon and the Earth by reason of proximity, of shape and of the manner of illumination, and because

the matter of both is dense and opaque and the surface rough with mountains and valleys; yet he denies that she is composed of land and sea, or that there are on her rains and other generations like ours. But David Fabricius, as Argoli reports (*Ephemerides*, bk. 1, chap. 10), said that lynx-like eyes are to be found which see men and the other animals on the Moon: but these are old wives' tales.

[Margin: The author's opinion.]

II. For my part, I judge it more probable that the Moon does not have a mixed form composed of our elements, in act or in power, nor one generated from them or resolvable into them, but has a heavenly form, altogether different from the forms of our elements and of the mixtures generated from them. This seems more in keeping, on the one hand, with Divine Omnipotence, so that we do not, as only too narrow a mind would, restrict it in the fashioning of the heavenly bodies to our elements, to which we are accustomed, as though there did not remain many other species of forms nobler than the elementary; and on the other with Providence, lest those heavenly bodies which it willed to be superior in place, in influence and in power of rule should not be of a higher and more excellent nature. Whether, then, they consist of the same prime matter as sublunary things, or of a different one, or of none really distinct from the form, does not much trouble me, since the whole or the chief account of a nature is to be sought from its form. Nor can agreement in certain qualities which can be common equally to elementary and non-elementary forms, such as rarity, density, opacity, transparency, roundness of figure, evenness, roughness, light, colour and the like, be an argument for this rather than that kind of forms. I judge nonetheless that the Moon was placed next to the Earth for this reason: that she has the greatest analogy with it, and, if the Sun were granted to be fiery, she herself should rather be called terraqueous from the effects we experience. But absolutely speaking she was made so that she may transmit to us more quickly and with less harm the influences of the higher stars, especially of the Sun, tempered by the speed of her course through the zodiac, and received and reflected downward by the varied roughness of her surface; and so that, like a cupbearer, she may foretaste for us what, poured on us without the Moon, we could hardly drink in and bear.

[Margin: Whether there is an air around the Moon, and whether it is the Moon's own.]

III. Akin to the preceding question is this other: whether the Moon has her own atmosphere around her, or at least some air spread about her, in roughly the way our Earth is clothed with vaporous and rather thick air, and her own dawn, by which the Moon's compass is enlarged. For not a few authors, and not contemptible ones, affirm it. First of all Plutarch, in the essay *De Facie Lunae*, insinuating his own opinion under another person's mask, acknowledges an air around the Moon that is not rainy, yet is dewy, when he says: *But what wonder is it if on the Moon roots, seeds and plants are born, needing no help from rains or winter, but content with a summery and tender air fitted to their nature? And why is*

it not probable that breaths are given off, which are affected with heat by the Moon, and that gentle airs and light dews and moistures gradually accompany her turning, which, spread around, suffice as nourishment for the plants, since the temperature of the Moon herself is not fiery, not parched, but soft and watery? Mästlin acknowledges the same air, or a denser aether, around the Moon, and with him Kepler, in the *Epitome of Astronomy*, p. 893, in the *Dissertatio cum Nuncio Sidereo*, p. 14, in the *Optica*, pp. 252 and 302, and in the book *De Stella Nova*, chap. 23, not around the Moon only but also around the Sun, and before the *Ephemerides*, pp. 9 and 21; Galileo in the *Sidereus Nuncius*, p. 12; Longomontanus (*Theorica*, bk. 1, chap. 9, p. 315), who concedes this peristasis, denser than the vaporous air surrounding our Earth, because of the greater quantity of moisture on the Moon, from which the Sun's rays draw vapours; Giordano Bruno, in the book *De Immenso*, who holds the same of all the heavenly bodies. David Fabricius too, in Kepler at the place cited from the *Ephemerides*, and Antonius Maria de Rheita [Antonín Maria Šírek] (*Radius Sidereomysticus*, bk. 4, chap. 2, Member 10), admits a vaporous atmosphere around the Moon, though with no clouds visible around her, because they are quickly baked out and scattered by the Sun's rays, which are nearer to the Moon than to us, and by the Moon's swift turning. And Mario Bettini (*Apiarium* 8, prog. 2, prop. 10) does not reject a vaporous circumlunar air, very thin but still apt to refract the Sun's rays. Michael Florent van Langren affirms to me, in a letter most courteously sent to me in 1646, that this same air is seen with telescopes; and so does Wendelen, both in letters written to me and in the *Idea of the Atlantic Tables*, p. 5. From this Longomontanus, Kepler, Fabricius, Bettini and Wendelen infer that some variation passes over from this circumsolar or circumlunar air into eclipses of the Moon and Sun: either the Sun's rays refracted in it enlarge the Sun's image and diminish the solar eclipse, or the Moon appears to be eclipsed sooner, since this air falls into the Earth's shadow before the Moon's body does, and emerges from it later. On this, however, see Kepler's limitation, which we treat below, chap. 16, no. 7. Wendelen adds that from the Sun's rays refracted in the air surrounding the Moon it comes about that the lunar dichotomy, or the Moon shining half-lit to us, appears to us sooner in the first quadrature and later in the second, since, as I interpret it, a greater part of the Moon is lit by the refracted rays than if there were no refraction; just as our Earth is somewhat more lit because of rays refracted by our denser air. But see what I shall say in chap. 18, no. 6. [Margin: The author's opinion on this air, held in suspense.] Meanwhile, with no telescope I have used has this air yet appeared to me so plainly that I should be compelled to acknowledge it near and around the Moon rather than in our own air, in which halos also form. Further, Vincentius Lagalla [?], in his book *De Phaenomenis in Orbe Lunae*, chap. 10, flatly denies that the atmosphere already mentioned exists around the Sun or the Moon.

[Margin: Whether there are inhabitants or dwellers on the Moon.]

IV. Of those who, from the sight of the lunar spots, have recognised mountains, valleys, forests and seas on the Moon like ours, a great part have also

assigned the Moon her own inhabitants, whom they have also named [Greek: Αντίχτονας [recte: Ἀντίχθονας] (antichthonas, "counter-earthers"; rendered by the page itself as "dwellers on another, opposite Earth")], that is, people living on another, opposite Earth. Cicero, *Academic Questions*, bk. 4: *Xenophanes says that the Moon is inhabited, and that she is an Earth of many cities and mountains.* Achilles Tatius reports similar things in the *Isagoge*, saying: *Some think there is on the Moon another region for inhabiting, with rivers and the other things seen on earth.* But let us hear Plutarch, *De Philosophorum Placitis*, bk. 2, chap. 30: *The Pythagoreans say that the Moon appears earthy because, like our Earth too, she is inhabited all round, by animals indeed larger and more beautiful, which contain fifteen times the size of ours, and casting off no excrement.* But in the celebrated essay *On the Face in the Orb of the Moon*, *they say that the Moon hangs over the heads of those who live below as over so many Tantaluses, and that in turn those who inhabit her, bound like Ixions, are rolled round and back with such force that it is no wonder if, by the force of the shaking, the Lion fell into the Peloponnese.* But he counts these among fables. He adds, however, that there are caverns on the Moon, and that the greatest of the caverns is called *the Inner Chambers of Hecate*, where souls render an account of, and pay the penalty for, what they did or suffered once born; that the parts of the Moon turned towards Heaven are called *the Elysian Field*; and that the Genii do not always dwell on the Moon, but come down to us for various tasks, and if they do not perform these rightly, they are thrust back to the Earth and made to struggle with bodies. Hence Macrobius (bk. 1 on the *Dream of Scipio*, chap. 11) places lunar peoples and a Purgatory of souls on the Moon. Nikolaus von Kues (*De Docta Ignorantia*, bk. 2, chap. 11) did not shrink from this opinion about inhabitants of the lunar globe. Bede too, in the book *De Mundi Constitutione*, under the title on the aether, attributes to certain natural philosophers the same opinion about souls and Genii living on the Moon as on an ethereal Earth, or ascending and descending through her. Kepler, moreover (*Astronomia Optica*, p. 250, and the *Dissertatio cum Nuncio Sidereo*, p. 10), though he may seem to place inhabitants on the Moon in jest, seems nevertheless to lean seriously towards this opinion, since he says that *it is not unlikely that there are inhabitants not only on the Moon, but on Jupiter itself*, and he does not despair of their being discovered, if ships or sails are granted for the heavenly breeze. Yet he adds: *Let these things be interposed for pleasure, as a marvel of human daring, which presents itself above all in the men of this age.* What, though, is more daring than Kepler's intellect? That Sagredo, in Galileo (Dialogue 1 on the system of the world), was more sober when he pronounced: *Whether on the Moon or another planet there are generated grasses, plants, animals, rains, thunderbolts, etc., I neither know nor believe, much less that men live there.*

[Margin: Censure of those who assert inhabitants of the Moon.]

Our own Adam Tanner, further, in the dissertation *De Caelis*, q. 4, judges rash the opinion affirming that the Moon is inhabited by men living a mortal and natural life, as resting on no foundation, and indeed as little in accord with Sacred Scripture. For it is evident from Genesis 1 that the stars were destined

by God for a far different office than to be seats of men, and that all men were begotten from Adam, who was produced on our Earth (Genesis 1 and 2). Again, in Deuteronomy 4 it is said: *Lest perhaps, lifting up your eyes to Heaven, you see the Sun and the Moon and all the stars of Heaven, and, deceived by error, adore and worship those things which the Lord God created for the service of all the nations that are under Heaven.* And in Psalm 14 the Earth alone is assigned to men for dwelling: *The Heaven of Heaven is the Lord's, but the Earth he has given to the children of men*; and in Acts 17: *And he made of one every race of men, to dwell upon the whole face of the Earth.* Thus there is one Church militant on our Earth, whose God wills all men to be saved. Whether Enoch and Elijah were carried off to the Moon and are preserved there by divine miracle, Tanner does not discuss there, nor is this the place to weigh these matters with the dignity and diligence that would befit them. If anyone, however, imagines inhabitants on the Moon, I shall at last show, below in chap. 9, in the Scholia, how our Earth would appear to them.

CAPVT III.

Chapter 3.

De diuerſis Lunæ Phaſibus earumq. Varietate, ſed præcipuè de Prima Phaſi.

On the various phases of the Moon and their variety, but especially on the first phase.

I. *Phasis* in Greek is the same as appearance. Now the Moon appears to us at one time full, at another half-full, at another gibbous, at another horned or sickle-shaped, and indeed with her horns turned now to the east, now to the west, and these sometimes erect, sometimes inclined: a truly wonderful variety of illumination through the whole month in which she returns from conjunction with the Sun to conjunction with it again. To put these phases before our eyes, in the following figure let the Sun be at S, illuminating the Moon, as I assume on the basis of what will be said in chap. 5; let the Earth, from which we view the luminaries, be at T; and let the Moon's orbit under the zodiac be AFMG, whose diameter AM is bisected at T, at right angles, by the other diameter FG. Let the Moon now be at A, interposed between the Sun S and our eye T; for she will appear either lit by no light or very dark, like a black round little cloud, and then there will be the synod or conjunction of the luminaries, or new Moon. Then, after about one or two days, she will appear in the evening at the western horizon, a little after sunset, very thinly but very sharply horned, as at B, whereas the previous evening she did not appear; and this is called the first phase. Afterwards, about the fourth or fifth day, increased in light and with a fuller horn, about the first sextile aspect, that is, elongated 60° from the Sun, she will be seen at D. From here, about the seventh day, she will be 90° from the Sun, or a whole quadrant, and will appear at F, half-full or bisected, and there will be the first quadrature. Then, more than half-full,

about day 11 from conjunction, she will appear at H, gibbous, and near the first trine aspect, that is, 120° from the Sun. Finally, about day 13, she will be seen at K so gibbous that she appears almost full, until about day 14 or 15 she appears at M, opposite the Sun, with us at T placed between her and the Sun; and then she will be seen full of light and with a round phase, and there will be the full Moon. She will rise as the Sun sets and shine all night, whereas previously she began to shine after sunset, and shone for us longer and longer the further she was from the Sun, but still not all night. In the first semicircle AFM, in which she waxes, the lit part of the Moon will always face west, and the horns, turned away from the Sun, east.

When the full Moon is over, she will begin to wane, and after sunset will not rise at once at the eastern horizon, but later and later; and in the whole latter semicircle MGA, or the second part of the lunar month, in which she wanes, her lit part will face east and her dark part west. She will appear gibbous one or two days after full Moon at L, then at I, in the second trine aspect, distant from the Sun's place, in the order of the signs, by the arc AFI of 240°, but on the other side by the arc AGI of 120°. Afterwards she will be seen in the second quadrature at G, and in the second sextile aspect at E, and finally in the extreme and last phase at C, and again, after this, in conjunction with the Sun at A. Do not be troubled that the engraver seems to have made a Moon at S in place of the Sun, as he left out the rays; for the Sun seen through a telescope, and without deception of the eyes, appears without the hair of rays, *and an Apollo not only beardless but shorn.*

[Margin: Explanation of the diagram of the lunar phases.]

Diagram. Woodcut set in the right column, below the sentence on the engraver. At the top, a disc bearing a human face and no rays, lettered S beneath it. Below it, a circle (the Moon's orbit) carrying twelve small globes, each shaded in part. Two diameters cross at right angles in the centre of the circle, one upright and one level, with T beside their crossing. The topmost globe, lettered A, is wholly dark; the shading shrinks down each side of the circle through B and C, D and E, F and G (half dark, the dark half below), H and I, and K and L; the bottommost globe, lettered M, carries no shading.

- A (top of the circle): the Moon in conjunction
- B (upper left), C (upper right)
- D (left, below B), E (right, below C)
- F (left end of the level diameter), G (right end of the level diameter)
- H (left, below F), I (right, below G)
- K (lower left), L (lower right)
- M (foot of the circle): the Moon at the full
- S (below the disc at the top): the Sun
- T (at the crossing of the diameters): the Earth

[Margin: The names of the phases.]

II. The Latin and Greek names of the phases already mentioned, used now as adjectives, now as nouns, are those written below, to which add *Syzygy*: for

[Greek: συζυγία (syzygia, "conjunction")] means a conjoining or coupling, and above all the conjunction ☌, when Cicero speaks of its being "in the yoke" [in iugo]; although the word is also used for the diametrical opposition ☍. These names are recorded by Vitruvius, Book 9, chapter 4; Pliny the Elder, Book 2, chapter 14; Marcellinus [Ammianus Marcellinus], Book 20; Macrobius, Book 1 on the *Dream* [Somn.], chapter 6; Geminus, *De Elementis Astronomicis*, chapter 7; and Martianus Capella, Book 8, where he treats the course of the Moon; besides very many more recent writers.

A. New Moon [Nouilunium], the silent Moon [Luna ſilens]; conjunction [Coniūctio], meeting with ☉ [Cōgreſſus cum ☉]; *Neomenia*; synod [Synodus]; kindling of the Moon [Lunæ accenſio]	☌	[Greek: νεομηνία (neomēnia, "new Moon")], or [Greek: νουμηνία (noumēnia, "new Moon", contracted form)]; [Greek: σύνοδος (synodos, "meeting")]
B. First phase [Prima Phaſis]; new Moon [Noua Luna]		[Greek: πρώτη φάσις (prōtē phasis, "first phase")], or
C. Last phase [Vltima Phaſis]; the latest Moon [Luna nouiſſima]. The interlunium [Interlunium] or intermenstruum [intermenſtruum] is the time between the last and the first phase, from C to B		[Greek: νέα σελήνη (nea selēnē, "new Moon")], or [Greek: ἐσχάτη φάσις (eschatē phasis, "last phase")], or [Greek: ἔνη σελήνη (enē selēnē, "last Moon")], or [Greek: ἀφανισμός (aphanismos, "disappearance")]
B, D, C, E. Horned [Corniculata], sickle-shaped [Falcata], curved into horns [curuata in cornua]		[Greek: μηνοειδής (mēnoeidēs, "crescent-shaped")], and [Greek: μηνοειδῆ (the same, another form)]
D. First sextile aspect [Primus Sextilis aſpectus]; and **E.**, second	✶	[Greek: ἑξάγονος [recte: ἑξάγωνος] (hexagōnos, "hexagon")]
F. First quadrature [Prima Quadratura]; **G.**, second; the divided Moon [Luna diuidua], bisected [biſecta], halved [dimidiata], half-full [ſemiplena]	□	[Greek: τετράγωνος (tetragōnos, "square")]; [Greek: διχότομος (dichotomos, "cut in two")]; [Greek: διχότομος ἡμίτομος (dichotomos hēmitomos, "cut in two, half-cut")]
H, K, L, I. The gibbous Moon [Luna Gibba, Gibboſa]		[Greek: ἀμφίκυρτος (amphikyrtos, "convex on both sides")]
H. First, **I.**, second trine aspect [aſpectus Trinus]; the Moon in trine [Luna in triquetro]	△	[Greek: τρίγωνος (trigōnos, "triangle")]
M. Full Moon [Plenilunium], opposition [Oppoſitio], the whole Moon [Luna Totilunis]; in diameter [in diametro], rounded into a full circle [ſinuata in orbem], mid-month [medius menſis]	☍	[Greek: πανσέληνος (pansēlēnos, "full Moon")]; [Greek: διχομηνία (dichomēnia, "mid-month")]
The waxing Moon [Luna creſcens], from A, through F, to M		[Greek: σελήνη αὐξανομένη (selēnē auxanomenē, "waxing Moon")]
The waning or ageing Moon [Luna decreſcens, ſeu ſeneſcens], from M, through G, to A		[Greek: σελήνη φθίνουσα (selēnē phthinousa, "waning Moon")]

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[21] On the leaf the Greek lines do not stand level with the Latin entries line for line. The word *vel* ("or") closing the line for B leads on to the next Greek line, which stands level with the first line of C.

[Margin: Inclination of the horns: what is it?]

III. Besides the phases already mentioned, the inclination or erection of the Moon's horns [Lunarium Cornuum Inclinatio aut Erectio] also deserves consideration. Inclination is when the points of the two horns do not lie in the plane of one vertical circle, but a line drawn through the tips of the horns is inclined to the vertical circle, so that one horn is found on one vertical and the other on another. So it is in the following figure, in which let the horizon be HR, the meridian HBR, the pole of the world P, the pole of the ecliptic Z, the vertex B, and portions of different vertical circles FBZ, or BKN, or BLO. For if the Moon is inclined as at NO, the distance of horn N from the vertex, arc NB, will be equal to arc BO, by which horn O stands from the vertex; but if it is as at KL, one horn, at K, will be farther from the vertex B, namely by the arc KB, than the horn at L, by the arc BL.

[Margin: Erection of the Moon's horns: what it is.]

Erection of the horns, on the other hand, is when both horns are on the same vertical, as when the Moon is at FG, through which points the same vertical circle ZBGF would pass. This variation of phases or positions is not to be despised by an astronomer. Meanwhile, three notable rules are to be handed on to beginners.

Diagram. A woodcut in the left column. The meridian arches from H at lower left over B at the top to R at lower right, with the curved horizon closing the figure below. Arcs of vertical circles descend from B. Three Moons are drawn as dark discs with a pale crescent: one at F–G on the arc towards the upper left, one at K–L in the middle, and one at N–O at the foot. A small circle stands at the upper right around P, with Z on the arc above it. Labels read, all on the block; labels not read: none.

- B (top): the vertex
- F (left of the upper Moon), G (right of the upper Moon)
- L (above right of the middle Moon), K (left, below the middle Moon)
- Z (upper right): the pole of the ecliptic
- P (centre of the small circle): the pole of the world
- H (lower left), R (lower right): ends of the horizon
- N (left of the lowest Moon), O (right of the lowest Moon)

[Margin: Rule for recognising the ecliptic.]

The first is this: if a straight line is drawn on through the Moon's two horns, it will mark the pole of the zodiac and cut the ecliptic at right angles if the Moon has no latitude, or very nearly at right angles if it has some latitude; and so it will show the position of the ecliptic in the sky.

[Margin: Rule for recognising the nonagesimal degree of the ecliptic.]

The second is this: whenever the Moon holds its horns erect, it will be in the nonagesimal degree of the ecliptic reckoned from the horizon, and the vertical circle will be the same as the circle of latitude and longitude. Therefore the Moon will have no parallax in longitude: strictly so if it has no latitude, and nearly so if it has latitude, as al-Battānī teaches (chapter 24), together with

Daniel Santbech (the book *De obſeruatione*, Proposition 14) and Kepler (in the *Optics*, p. 246). You will discern this with a thread hanging plumb, if the horns stand in the same straight line as the plumb-line at rest; or else from chapter 4, Problem 9.

The third: if the new Moon took place in the ascending signs, the Moon's horns point towards the zenith in the first phases; but if it took place in the descending signs, one horn faces the zenith and the other the horizon, for northern observers [apud Boreales], as Barozzi notes (*Cosmography*, Book 4, p. 289).

De Prima Lunæ Phaſi.

On the First Phase of the Moon.

[Margin: Which peoples were accustomed to begin the month from the first phase.]

IV. The first phase of the Moon is very celebrated because of the beginning of the civil month, which many nations were once accustomed to start from it. Even now the Arabs, as al-Battānī (chapter 41) and al-Farghānī report, begin the first day of their lunar months from the evening on which the Moon, rising heliacally in the west, is first seen after sunset. They call this first phase Hay or Helal, and they salute it. From its sickle shape the Turks adopt the form of the horned Moon for the tops of their towers and of their hats [petaſorum]. And, if we believe Joseph Justus Scaliger (*De emendatione temporum*, Book 5), they set the neomenia from the 3rd day after the coming together of the Moon with the Sun, and for that reason, at the neomenia of the month Muharram, in a clear sky, they always see the Moon horned.

Whether the Hebrews once began the neomenia from tables and the periods of cycles, or rather from the first phase of the Moon, we have already discussed in our *Chronological Book*, Book 3, chapter 28. From this I say here briefly, with Paulus Forosemproniensis [Paul of Middelburg] (part 2 of the *Summa*, Book 8) and Scaliger (in the *Prolegomena*, p. 6, and Book 2, p. 150), that the Jews had their own lunar cycles, arranged either by Gamaliel or by someone else, from which they determined the neomenias, and that they began their months from the conjunction, or mean new Moon. So it was truly said by St Dionysius the Areopagite that the fifteenth Moon, on which Christ the Lord suffered, after the fourteenth on which he celebrated the Passover, was at the diameter and opposite the Sun. This would not be right if the Jews had begun the month of Nisan from the first phase, since the first phase sometimes does not appear until the 3rd or 4th day after the new Moon.

Nonetheless, as the same Scaliger says, they used to observe, as they still do today, the first [Greek: φάσιν (phasin, "phase")], not in order to proclaim the neomenia from it, but in order to sanctify it, and on seeing it to exclaim [Greek: ἀγαθὸν τέρας ἔστω ἡμῖν, καὶ παντὶ Ισραὴλ [?] (agathon teras estō hēmin, kai panti Israēl)], *A good omen be to us and to all Israel*. Moreover, a certain few

Jews of the sect of the Sadducees, called Karraim, haters of the other Jews, wait for the first phase so as not to begin their months with them. And perhaps it is of these that a thing is true which Clement of Alexandria (*Stromata*, Book 6, p. 270), Onufrius Panuinus [Onofrio Panvinio] (in his commentaries on the *Fasti*, at the year of the City 786), Salmeron [Alfonso Salmerón] (volume 1, Prolegomenon 38) and Io. Georgius Heruartus [Johann Georg Herwart von Hohenburg] (chapter 171 of his *Chronology*) report of the Jews without qualification: namely, that they used to begin the neomenias from the first appearance of the Moon. But what if the Jewish cycles were once so arranged that the first day of the month, in the greater part of months, fell on the first phase, which usually occurs sometimes one, sometimes two, more rarely three or four days after the new Moon, as I shall say shortly?

That the Greeks, however, began their months from the day of the new Moon is learnedly shown by our Pétau, in the *Uranologion*, Book 4 of the *Dissertations*, chapter 7, especially from the scholiast of Aristophanes on the *Clouds*; by Vitruvius, following Aristarchus, Book 9, chapter 4, whose words we shall give shortly at no. 5; by Scaliger, Book 1, p. 9; and by Varro, *De lingua Latina*, Book 5, and *De re rustica*, Book 1, chapter 37. They teach that the first day of the month is called the Moon [Greek: ἔνην, καὶ νεὰν (enēn, kai nean)], that is, last and new, because *it shares old and new light, and on the same day can be seen as the last and the first Moon.*

De Interuallo inter Nouilunium verum & primam Phaſim vel vltimam.

On the Interval between the True New Moon and the First or Last Phase.

V. How long an interval of time is required, after the moment of the true new Moon, before the Moon's first phase or first illumination can be seen, has rightly been asked since long ago; for the length of this interval is not settled among all writers. For it is true that most, with Pliny the Elder (Book 2, chapter 9), suppose that the Moon when joined to the Sun cannot be seen except in an eclipse of the Sun, because of the excessive glare of the Sun around it, since not even stars much brighter than the part of the Moon turned away from the Sun appear by day.

[Margin: The Moon seen near the Sun, almost in ☌.]

Yet Kepler, in the book *De Nova Stella Serpentarii*, chapter 24, tells the following: *In the year 1553, as a serious and most learned man wrote to me, the Moon was seen at Seville [Hiſpali] standing next to the Sun in Pisces, in the very middle of the sky. The whole city watched and bore witness. But when has anyone seen a similar new Moon? Does not the teaching of the ancients claim two days and more after the union for the first sight of the Moon? This, then, is the fifth testimony to what we contend: that the matter of the heaven, since it is different at different times, differing greatly from itself in subtlety, also at times takes in more brightness than at other times; and so at some times blinds the eyes of men more, at others less, in seeing*

the stars. In the *Epitome of Astronomy*, moreover, Book 6, p. 839, he relates that this happened at Seville on 1553 March 13, with the Sun at 3° ♈ and the Moon at 23° ♓, and so at a distance from the Sun of about 10°. And he says that at such a distance the breadth of the Moon's horn was the 109th part of the Moon's diameter, that is, 19″ or 20″, so that it could hardly have been seen; but that the Moon was nonetheless seen because of the purity of the ether around the Sun, and because of the snows of the Mountains of the Moon in Africa, to which the Sun was then vertical, reflecting the Sun's light on to the Moon, so that the Moon at least, if not the horn itself, could be seen.

And Americus Vespuccius [Amerigo Vespucci] too, in Ramusio, volume 1 of the *Navigations*, p. 132, relates: *Likewise we saw the new Moon on the same day on which it was joined to the Sun*; and Io. de Barros [João de Barros], in the same place, p. 392: *But in summer, on the summit of those mountains, the air is so pure that some of our men saw the new Moon on the day of conjunction.* We ourselves, on 1648 March 25, after sunset, $29^{h}\ 40^{m}$ after the new Moon, saw the first phase so large that it must necessarily have been visible on the same natural day to those further east. Thus, if the rarity and subtlety of the air allow it, the new Moon can be seen a few hours after the moment of new Moon, especially around the meridian. On the other hand, Strabo and Pliny the Elder (Book 6, chapter 22) report that in Taprobane the Moon is not seen except from the 8th day after the new Moon up to the 16th; but this is on account of the position of the sphere.

[Margin: Pliny the Elder's opinion on this interval.]

For the torrid and temperate zones, however, Pliny the Elder says (Book 2, chapter 9): *Having lingered in conjunction with the Sun for two days when it is slowest, it goes out again from the thirtieth day to the same round.* And in chapter 14 he says that the Moon is always hidden within 14° of the Sun. In chapter 17, after saying that the Moon makes conjunction with the Sun twice in no other sign than Gemini, and that only in Sagittarius does it at times make none (in Gemini because of the slowness of the Sun, whose apogee was then in Gemini; in Sagittarius because of its speed at perigee), he adds:

[Margin: Lynceus sees the old and the new Moon on the same day.]

But that the last and the first Moon should be seen on the same day or night, in no other sign than Aries, has also fallen to few mortals, and from this comes Lynceus's fame for keen sight. Nevertheless we saw at the end of no. 4, from Varro and the scholiast of Aristophanes, that the last and the new Moon can be seen on the same day.

[Margin: Vitruvius's opinion.]

Vitruvius, for his part, has this about the Moon (Book 9, chapter 4): *Accordingly, in every month, on the first day, before it passes, it lies hidden beneath the Sun's wheel and rays and is darkened; and since it is with the Sun it is called new. On the following day, on which it is counted second, passing away from the Sun, it makes a slender appearance of the edge of its round.*

[Margin: And Geminus's.]

Now let us hear Geminus, an ancient author, who distinguishes these intervals more carefully in chapter 7 of the *Elements of Astronomy*, in these words: *When it is swiftest the Moon appears sickle-shaped on the very day of the new Moon, when slowest on the third day; and it remains sickle-shaped sometimes up to the fifth day, and sometimes, when it is slowest, up to the seventh. It becomes half when it is swiftest about the sixth, when slowest about the eighth; it becomes full when swiftest about the 13th, when slowest about the 17th.*

[Margin: Al-Battānī's teaching on the first phase.]

But al-Battānī, in chapter 41, after saying that knowledge of this phase was more necessary to the Arabs, since they begin their month from it, than to the ancients, and after touching on the error of those who confused the arc of declination with the arc of latitude in this matter, says that the arc of vision [arcum viſionis] required between the Sun and the Moon, for the Moon to be able to rise heliacally and be seen after conjunction, is 12°, to which a descension on the equator of about 48^{m} of time corresponds. But when the Moon is at apogee and the Sun at perigee, he says, the arc already mentioned is 10° 50′, while with the Moon at perigee and the Sun at apogee it is 13° 30′. So the time required between the new Moon and the first phase is as much as that in which the Moon, then passing through the said degrees, draws away from the Sun.

He adds, besides, rules for knowing whether the Moon's first phase is to be seen on the first or the second day after conjunction on a given horizon. These rules depend on the true place of the luminaries computed for the moment of sunset, on the apparent place derived through the parallaxes of longitude and latitude, and on the oblique or right descension of the Moon, as essential factors; and, as incidental factors, as al-Battānī observes, on the clarity, density and turbulence of the air, and on the sharpness of our sight. For these are the reasons why the Moon frees itself sooner or later from the stretch of the twilight. He assumes, however, that the new and the old Moon cannot be seen on the same day. Anyone who wants examples based on the 12° already mentioned may consult Reinhold above, p. 159.

[Margin: The month of illumination.]

Michael Neander, in the *Elementa sphaerica*, p. 161, says that the month of illumination [Menſem Illuminationis] is the span lying between the first and the last sight of the Moon, which Johannes de Sacrobosco fixes at 28 days. He adds, however, that no definite number of days can be assigned, because the Moon leaves the Sun sometimes sooner, sometimes later, for causes listed elsewhere.

[Margin: More recent authors on this interval.]

These causes are followed out most carefully by Erasmus Reinhold on Peuerbach's *Theoricae*, pp. 153 to 163; by Nonius on the same *Theoricae*, from annotation 3, that is, pp. 279 to 290; by Magini in the *Theoricae Planetarum*, Book 2, chapter 16; by Barozzi in the *Cosmography*, Book 4, from p. 288; and by Kepler in the *Optics*, p. 257.

[Margin: The causes of variation in the first phase are enumerated.]

From these writers I find three causes why the Moon begins to appear sooner or later after the moment of synod, a fourth, incidental cause, from the diversity of the horizontal vapours and of eyes, being passed over; and to these al-Battānī (chapter 21) adds three others. They are: 1. the obliquity of the zodiac and the horizon; 2. the Moon's latitude; 3. the Moon's slower or faster motion; 4. the parallax of the Moon relative to the Sun; 5. the Moon's distance from the Earth; 6. the true interval between the Moon and the Sun.

[Margin: Explanation of the first cause.]

VI. The first cause, then, according to Peuerbach, Reinhold, Magini and Barozzi, is the obliquity of the ecliptic and the horizon, because of which the Moon's descensions at sunset become more right, that is, slower, or more oblique, that is, faster. For if there are more degrees in the arc of the parallel of the equator on which the Moon stands, intercepted between the Moon and the horizon, than in the arc of the zodiac between the Moon and the setting Sun, the Moon will be seen sooner, for it will be above the horizon at the end of twilight. This happens when the Moon is placed in the ascending signs, that is, from the beginning of Capricorn to the end of Gemini, for it is then in signs of right, that is, slow, descension. But if the arc of the Moon's descension is less than the Moon's distance from the Sun in the zodiac, the Moon will set while the evening twilight still lasts, and with such brightness that the Moon's first phase is overwhelmed. This happens when the Moon is in the signs from the beginning of Cancer to the end of Sagittarius, for it is then in signs of oblique, that is, swift, descension; but this holds for northern observers, for among southern observers the opposite occurs, according to what was said in Book 1, chapter 23, Rule 5.

If, however, we are speaking of the morning occultation and the last phase, the Moon will be hidden sooner in signs of oblique ascension and later in signs of right ascension: that is, for northern observers, sooner in the signs from the beginning of Capricorn to the end of Gemini, and later from the beginning of Cancer to the end of Sagittarius; and this all the more, the greater the altitude of the pole. Indeed, as Reinhold acutely notes, this first cause has almost no place in the first, second and third climates. For, as the tables of ascensions show, Pisces and Aries do not descend rightly, that is, slowly, nor Virgo and Libra obliquely, that is, swiftly, unless the pole is raised 11°; nor do Gemini and Capricorn rise obliquely, nor Cancer and Sagittarius set obliquely, unless the pole is raised by 30°.

Nonius, however, above, p. 289, admits with Peuerbach that after conjunction the Moon appears sooner

[Margin: Qualification of the first cause, from Nonius.]

from the beginning of Capricorn to the end of Gemini, and later from Cancer to Sagittarius inclusive; not, however, because of the larger or smaller arc already mentioned, but because of the greater or lesser hiding of the Sun under the horizon, which depends on the larger or smaller angle of the ecliptic and the oblique horizon. On this point he adds excellent demonstrations,

discussed in the problems of the primum mobile, and he concludes that the new Moon in the evening after conjunction, when it stands in Aries, appears sooner, and the ageing Moon in the morning is hidden sooner in Aries, while the opposite happens in Libra. Yet in the opinion of Peuerbach and the Arabs, the Moon appears soonest in ♒ and ♊, and latest in ♎ and ♍; and in the morning it is hidden soonest in ♓ and ♈, and latest in ♋ and ♐.

[Margin: Explanation of the second cause.]

VII. The *second* cause is the differing latitude of the Moon, or its declination from the ecliptic. For the greater the northern latitude, the more slowly, other things being equal, the Moon will descend in the evening and the sooner it will be seen, while in the morning it will hide the sooner, for northern observers. But the more the Moon declines to the south, the sooner it will descend in the evening and the later it will hide in the morning. All the opposite happens for southern observers. This cause, as Nonius and Magini observe, holds in the temperate zones; for in the torrid zone the opposite can happen: namely, that the Moon has a northern latitude and descends sooner, and sometimes descends together with the degree of the ecliptic in which it stands, and sometimes later.

[Margin: Explanation of the third cause.]

VIII. The *third* cause is the greater or lesser speed of the Moon's true or apparent motion. For if the Moon is swift in its proper motion towards the east, it will certainly set later and be seen sooner in the evening, and in the morning will be hidden sooner. But if it is slow, it sets sooner in the evening and hides later in the morning.

[Margin: The three remaining causes.]

IX. The three other causes added by al-Battānī either produce an almost imperceptible difference or reduce to the others. For the parallax of latitude reduces to latitude and the second cause, and the parallax of longitude to the first; in these, authors can be understood to have spoken of apparent longitude and latitude, and from these arises the apparent interval between the Sun and the Moon. The distance from the Earth, for its part, varies either the parallaxes or the speed of the Moon, and so reduces to the other causes.

[Margin: When the Moon begins to be seen on the 1st day after the new Moon, when on the 2nd, 3rd or 4th.]

X. Now, if all the causes, but especially the first three, combine to retard the Moon's descent in the evening, the ageing Moon can be seen on the same day, especially far from the horizon, and in regions where the pole is much raised. If only two causes combine, the Moon can be seen on the second day after its meeting with the Sun, and hidden two days before it. If only one cause, it can be seen on the third day after, and hidden on the third day before the meeting with the Sun. But if the three former causes combine to hasten the Moon's setting, as when the Moon is in the signs from Cancer to Sagittarius inclusive, with southern latitude increased by parallax, and at apogee or slow in its course, then the first sight of the Moon can be delayed up to the 4th day. It is in this way that the statements of the authors, discussed in no. 5, are to

be reconciled or corrected. It should be added, with Reinhold, that both the length of a summer's day and the shortness of the twilight contribute greatly to seeing the last and the first phase on the same day.

[Margin: Kepler's opinion.]

Kepler, however, in the *Optics*, p. 25, despite Reinhold's teaching, judges that, with all causes computed, this matter cannot be comprehended by definite rules, and he lays all the blame on the summer and winter air. He adds that the Moon can be seen at the very moment of conjunction at mid-heaven, if it is at the limit and at a latitude of 5°; for it can then show us about a 70th particle of its body illuminated, and that much more easily if it is viewed from a well, because, I believe, of the example in the *Stella Nova*, related at no. 5. And he adds that if the Moon is at the limits, that is, at its greatest latitude, and swift, the old and the new Moon can be discerned on the same day, and indeed more easily at mid-heaven.

De diuerſitate Phaſium in Quadraturis □.

On the Variation of the Phases at the Quadratures □.

[Margin: The phase of the half Moon.]

XI. The lunar dichotomy, too, is of two kinds: one true, which occurs at quadrature, the other apparent, which occurs when the Moon shines half-lit to us.

[Margin: And its interval from quadrature.]

How long before the first quadrature, or after the second, this phase occurs is not yet settled among astronomers, as is clear from what was said in Book 3, chapter 7, from no. 10 to 13, which need not be repeated here. It is certain, however, that, other things being equal, the interval between the first quadrature and the phase of the half Moon will be the larger, the greater the Moon's apparent latitude towards the spectator's pole and the Sun's declination towards the other pole, or the slower the Moon; but that for these causes the interval between the second quadrature and the phase of the apparent dichotomy will be smaller. Wendelen, moreover, as we said in the same chapter, no. 12, judges that this phase is hastened in the evening and delayed in the morning, because a larger part of the Moon is illuminated by the Sun, through the force of the Sun's rays refracted by the air around the Moon towards the Moon's body, than if there were no refraction. To this is added the greater or lesser distance of the Sun from the Earth, which, however, we taught in the same chapter how to investigate by means of this phase once observed. See also chapter 4 below, Problem 8.

De diuerſitate Phaſium in Syzygijs ☌ et ☍.

On the Variation of the Phases at the Syzygies ☌ *and* ☍.

[Margin: Variation of new Moons and full Moons.]

XII. The phase of the new and the full Moon is without doubt varied by the Moon's latitude, especially its apparent latitude; and its greater or lesser illumination by its greater or lesser distance from the Sun. To begin from this point, let the following paradoxes be held worth noting.

[Margin: 1st paradox.]

The 1st paradox is: *The Moon is never less illuminated by the Sun than when it is full.* So say our Aguillonius [François d'Aguilon] (*Optics*, Book 5, prop. 48), Biancani (in the *Sphaera*, Book 9, chapter 3) and Bettini (*Apiarium* 8, Prog. 2, Proposition 8). The reason is that since the Moon's globe is smaller than the globe of the Sun illuminating it, more than a hemisphere of the Moon is illuminated, as is shown in optics; yet the further the smaller sphere is from the illuminating body, the smaller is that excess of the portion illuminated beyond the hemisphere. And at full Moon the Moon is at its greatest distance from the Sun, if this is compared with its distance from the Sun at new Moon.

[Margin: 2nd paradox.]

The 2nd paradox is: *The Moon is never more illuminated by the Sun than when it is dark to us.* So says Bettini, above; for the Moon is then nearest the Sun. Therefore, even if at ☌ and ☍ a still larger portion of the Moon is illuminated through the force of refraction than if refraction did not intervene, yet, other things being equal, more of the Moon is always illuminated at new Moon than if no refraction of the Sun's rays took place in the air around the Moon, as Bettini noted. How large this portion is will be stated after the Moon's distance and refraction have been determined; meanwhile see chapter 4, Problem 4.

[Margin: 3rd paradox.]

The 3rd paradox: *There has never been a pure full Moon in respect of us.* So Kepler lays down in the *Astronomia Optica*, p. 237, if indeed it pleases us to proceed subtly here and [Greek: ἀκριβολογεῖν (akribologein, "to speak with exactness")]. For if the Moon falls into the Earth's shadow, there will be no whole full Moon: either the whole Moon or a part of it will be cheated of the Sun's light that would otherwise fill it. If, on the other hand, it avoids every eclipse, about 64′ or 66′ must lie between the centre of the Moon and that of the Earth's shadow. But with the Moon at so great a distance from the place of opposition, there is already an intersection of the circle of our vision, as bounded on the Moon, and the circle of illumination by the Sun; for at a distance of the Moon from the place of opposition of 30′ they touch each other. Therefore we shall not then be able to see full illumination by the Sun on the Moon, and so no full Moon either. Yet Kepler admits that the particle of the Moon which, out of the illuminated portion, is hidden from us is imperceptible, and scarcely the 7000th particle of the Moon's semidiameter. But even if there were no intersection, and the Moon, lacking latitude, could be seen illuminated by the Sun, the portion seen would still be less than the portion illuminated.

He teaches very similar things about the new Moon. For wherever the Moon leaves the Sun untouched, it will have a residual horn, for the reasons just given. In a central conjunction, if it appears smaller than the Sun, the circle of

illumination will descend below the circle of vision, and the Moon's limb will be seen lit in a circle, so that it will not appear dark, as a new Moon requires, and the circle of vision will be greater than the circle of illumination. But he leaves out total eclipses of ☉, to which this exception does not apply.

[Margin: 4th paradox.]

4th paradox: *The moment of an apparent full Moon has a breadth of about two hours.* So Reinhold on Peuerbach, from p. 166, in agreement with Witelo, Book 4, Prop. 74. For since at full Moon the Sun illuminates 180° 28′ 12″ of the Moon, if refraction is meanwhile set aside, while we see the Moon under an angle of about 30′, and the arc of a great circle through the pole of vision is less than a semicircle by about 30′: if that excess and this deficit are joined, about 1° will be gathered, and to traverse it about two hours of lunar motion are required. For that whole time, therefore, the Moon will appear full for this cause at least, although strictly it does not appear as full as it is, for the cause stated in paradox 3.

[Margin: 5th paradox.]

5th paradox: *When the Moon has latitude, new Moon and full Moon are threefold.* For it is new Moon when the Moon is distant from the Sun by nothing in longitude, and full Moon when it is 180° distant from it. But these things can be understood as the conjunction or the distance being verified in three ways: by a circle of latitude drawn through the centres of the Moon and of the Sun or Shadow, making right angles with the ecliptic; or by a circle of latitude drawn through the centres of the luminaries, but making right angles with the Moon's orbit; or by a circle so drawn through the centres of the luminaries that they stand equally distant from the nearer node of the ecliptic with the lunar orbit. For although very many think that conjunctions take place in the first way, and therefore reduce the Moon's place from the lunar orbit to the ecliptic, yet, as Kepler says in the Rudolphine Tables, ch. 32, at the end of precept 155, this question is still left undiscussed: When the Moon is not on the ecliptic, the three moments at which the three species of new Moon or full Moon already mentioned occur are different; therefore ☌ and ☍ are threefold.

CAPVT IV.

Chapter 4

In quo Problemata ad Phaſes Lunæ Spectantia.

In which are the problems concerning the phases of the Moon.

PROBLEMA I.

Problem 1

De Lunæ Aetate cognoſcenda præcipuè ex Phaſium quantitate.

On finding the Moon's age, chiefly from the size of its phases.

I. You will achieve this in four ways. The computists [Computiſtæ] hand down the first three of them in the Rules of the Calendar: namely Bede, vol. 1, pp. 187 and 525, and vol. 2, p. 347; Maximus and Isaacius[22], and, drawing on them, Pétau at the end of the *Uranologion*, Book 8, ch. 13, canon 5; Pietro Pitati in his monthly canons, ch. 4; Moletti, Book 2 of the *Calendar*, ch. 4; Clavius, Book 3 of the *Calendar*, ch. 1; and Biancani in the *Sphere*, Book 9, ch. 13, prop. 7.

[Margin: 1st method, by the Golden Number.]

The first method is by the golden number [Aureum numerum] of the given year, which you will find from the tables at the end of this work. Look it up, then, in the Calendar: for opposite it stands the first day of the Moon, and counting inclusively from that day to the given day of the month, you will have the Moon's age.

[Margin: 2nd method, by Epacts or Regulars.]

The second method is by the current Epact [Epactam] of the year. Add it to the number of Kalends elapsed, but counting from March inclusive (for the Kalends of January and February are ignored), and to this sum add the given days of the month; for the whole sum will give the Moon's age, after discarding 30, however, if the sum comes to 30 days or more. As a rule, however, the Moon has some hours beyond the day found, and often it is, for example, the tenth day of the Moon that by this rule would be the ninth. Then again, it matters from where you begin the first Moon: from the kindling of the Moon, that is, the true new Moon; or rather, with the Hebrews, the Greeks and now the Latin Church, from the mean new Moon, at which the luminaries are conjoined according to their mean motions; or from the day following the true new Moon, as Theophilus and Cyril of Alexandria wished, and Bede, *De ratione temporum*, ch. 43; or lastly from the first phase, with the Agarenes and the Jews of the sect of the Karraim.

[Margin: What the Regulars of the Moon are.]

To this method is reduced the one said to go by the Regulars of the Moon [Regulares Lunæ], that is, certain numbers added to the months, which together with the Epact make up the age the Moon has on the Kalends themselves. The Regulars of the months in the new Calendar are as you see in the following small table. For example, if the Epact is 10 and the month April, adding the Regular number 2 makes 12, the days of the Moon that it has on the Kalends of April; from there, then, counting inclusively, you will have the Moon's age for other days, but discarding 30, as above, if they are reached or exceeded.

[22]probably Isaac Argyros

	Regulars
January	1
February	2
March	1
April	2
May	3
June	4
July	5
August	6
September	8
October	8
November	10
December	10

[Margin: 3rd method, by astronomical calculation.]

The third method is by astronomical calculation. For the given time, compute the mean places of the luminaries. If they happen to be conjoined, it will then be new Moon; but if the Moon is elongated from the Sun, divide that elongation by 12° 11′, the amount, that is, by which the Moon is elongated from the Sun each day in mean motion, and the quotient will indicate the Moon's age computed from the mean new Moon, and so also the moment of the mean new Moon. But if you investigate the true new Moon by the rules to be given in Book 10, you will be able to compute the Moon's age from it to the given moment much more exactly.

[Margin: 4th method, by the phases of the Moon.]

The fourth method is by the phases of the Moon and their size. Kepler teaches it in the *Astronomia Optica*, ch. 6, no. 5, or p. 235, taking it from Reinhold, and both of them from that obscure saying of Pliny the Elder, Book 2, ch. 14: *Lucere dodrantes ſemuncias horarum ab ſecunda adijcientem vſq. ad plenum orbem, detrahentemq. in diminutione* [that it shines, adding three-quarters and a semuncia of an hour from the second day up to the full orb, and subtracting them in the waning]; Iacobus Milichius [Jacob Milich], Reinhold's teacher, expounds this excellently in a commentary. And the sense is that from the second day, which according to Pliny the Elder is usually the first phase of the Moon, the Moon adds to its illuminated part a dodrans of an hour, that is 45^{m}, and a semuncia of an hour, that is the 24th part of an hour, which is $2½^{m}$; their sum is $47½^{m}$. From the second day after new Moon, therefore, the Moon shines after sunset for $47½^{m}$ of an hour, but of a temporal hour, and on the following day for $1^{h}\ 35^{m}$, and so on in succession, until it shines for the whole night, that is, for 12 temporal hours. From the time for which the Moon shines after sunset, then, the Moon's age can be known by dividing that time by $47½^{m}$. For the quotient will give the Moon's age; but after full Moon it subtracts just as much time: that is, on the first day after full Moon the Moon rises after sunset only by 47½ hour-minutes, and on the second day only after 1^{h}

35^m, and so on; so from this time too the Moon's age can be known by adding 14½ days to it.

[Margin: Two obscure passages of Pliny the Elder are explained.]

But Pliny the Elder, Book 18, ch. 32, drawing on some other author (for he was a glutton for books), says that this time, which he had called a dodrans and a semuncia of an hour, is *Horæ dextantem ſicilicum* [a dextans and a sicilicus of an hour], that is, $51¼^m$; for a dextans contains ten parts of an as, and in an hour is 50^m, while a sicilicus is a quarter part of an uncia, that is $1¼^m$. For the cause, see Rule 2, Problem 2.

[Margin: Another method, from estimating the digits shining: unreliable.]

One could also observe the digits, or unciae, of the Moon's diameter shining to us, the diameter being on average 32½′. So since the Moon is filled in about 15 days after ☌, it would seem that each day there should shine to us, of the lunar diameter, more than on the preceding day by a fifteenth part of it, that is, that the daily increase should be 2′ 10″, or four-fifths of one uncia. That is not so, however, both because of the Moon's anomaly and its varying latitude and, chiefly, because the illuminated segments of the Moon, other things being equal, even if they otherwise really did increase equally each day, nevertheless appear to us smaller and more contracted near the limbs, because on account of their remoteness they are seen under a smaller angle, just as the remoter columns of a portico look more slender to us and the intervals between them narrower. Hence it is that on the fifth day after ☌, when a third part of the lunar diameter, that is 4 unciae, or nearly 11′, ought to shine to us, nevertheless only about 3 unciae, or 8′, appear. So the certain day of the Moon cannot be estimated from them, especially since the difference of the first phase itself, on which the rest depend, is subject to a variation of nearly 4 days, as was said above.

PROBLEMA II.

Problem 2

Quot Horis Luna Noctu Luceat. Atque adeò quota hora occidat ante ☍, vel oriatur poſt ☍.

For how many hours the Moon shines at night; and so at what hour it sets before ☍, or rises after ☍.

II. This problem is akin to the preceding one. It is handed down by Pliny the Elder, Book 2, ch. 14, and Book 18, ch. 32; Bede, vol. 1, pp. 187 and 528; Isaac the Monk, ch. 7 of the *Computus*; Moletti, Book 2 of the *Calendar*, ch. 18; Clavius, Book 3 of the *Calendar*, ch. 13; Kepler in the *Epitome of Copernican Astronomy*, p. 839, explaining Pliny the Elder; Biancani in the *Sphere*, Book 9, ch. 13, prop. 8; and Hérigone, vol. 4 of the *Cursus Mathematicus*, p. 80.

[Margin: 1st Rule.]

From these, then, the first Rule, from Pliny the Elder, Book 2, ch. 14, as I taught in Problem 1, method 4, is that after sunset the Moon adds to the second day 47½^{m}, that is, very nearly 48 minutes of a temporal hour. So Bede, Isaac, Moletti, Biancani and nearly all the computists say that four-fifths, that is 48 minutes of one temporal hour, are to be added each day. So if they have found the Moon's age by the Epacts, they add one day to it to have a truer age, multiply this by 4 and divide the result by 5. For the quotient gives the temporal hours for which the Moon shines at night, to be counted from sunset, if the Moon is waxing and not yet full, and the Moon's age does not exceed 15 days. But if it does exceed them, they discard 15, multiply the remaining age of the Moon by 4 and divide the result by 5; for the quotient gives the temporal hours after which the Moon rises, counted from sunset, and their complement to 12 hours will give the hours for which it will shine from its rising up to the following sunrise. This Rule, however, gives the time of the Moon's shining somewhat too short, because it rests on the diurnal motion and does not include the time in which the Moon advances by its proper motion toward the east.

[Margin: 2nd Rule.]

The second, therefore, and more exact Rule, from Pliny the Elder, Book 18, ch. 32, is that from the first phase the Moon shines each day for *dextante ſicilico* [a dextans and a sicilicus] of a temporal hour, that is 51¼^{m}, or, following Clavius above, five-sixth parts. So increase by one the Moon's age found by the Epacts, to have a truer age, then multiply this by 5 and divide the result by 6; for the quotient will give the temporal hours for which the Moon shines after new Moon, to be computed from sunset, if the Moon's age has not exceeded 15. For if it has exceeded it, 15 days are to be discarded, the remainder of the lunar age multiplied by 5 and the result divided by 6, to yield the temporal hours after which the Moon rises after sunset; and their complement to 12 hours gives the temporal hours for which the Moon shines before the following sunrise. According to Pliny the Elder, however, the Moon's age would have to be multiplied by 51¼ and the result divided by 60. But since, because of the Moon's unequal motion and because the Moon's age is not exactly known from the Epacts, this rule does not agree with the truth to a hair, it will be enough for civil use to discard 15 days from the Moon's age, multiply it by 4 and divide the result by 5, or, what is better, to multiply it by 5 and divide this by 6. Consult, then, the following small table, and use by preference the last column, which is constructed in the second way.

[Margin: Small table showing the temporal hours for which ☾ shines at night.]

Age of the Moon [Aetas Lunæ], waxing [Creſcentis]: days	Age of the Moon, waning [Decreſc.]: days	Temporal hours for which the Moon shines at night from sunset or after rising [Horæ Temporales quibus Luna noctu lucet ab occaſu Solis vel poſt ortum]: hours	Fifth parts	Hours	Sixth parts
1	29	0	4	0	5
2	28	1	3	1	4
3	27	2	2	2	3
4	26	3	1	3	2
5	25	4	0	4	1
6	24	4	4	5	0
7	23	5	3	5	5
8	22	6	2	6	4
9	21	7	1	7	3
10	20	8	0	8	2
Remainder of the preceding table [Reſiduum Tabulæ Præcedentis]					
Days	Days	Hours	Fifth parts	Hours	Sixth parts
11	19	8	4	9	1
12	18	9	3	10	0
13	17	10	2	10	5
14	16	11	1	11	4
15	15	12	0	12	3
If it is waxing, it shines after sunset; but if it is waning, it shines before sunrise.					

[Margin: The instrument of Giovanni Battista Benedetti.]

As to how temporal hours are to be converted into equinoctial hours, I taught that in Book 1, ch. 29. But Io. Bapt. Benedictus [Giovanni Battista Benedetti], in his *Epistles*, p. 221, taught the way to make an instrument which, at a given altitude of the pole, shows the hour of the Moon's rising and setting. It rests on the semidiurnal arc, with a small wheel added, by which the day of the Moon's age is placed under the given day of the month.

PROBLEMA III.

Problem 3

Quota pars Lunæ noctu luceat, data Lunæ ætate, ſeu quantitatem Phaſeos æſtimare.

What part of the Moon shines at night, given the Moon's age; or, to estimate the size of a phase.

[Margin: The size of a phase.]

III. See what was said in no. 1, method 4; and add to it this rule from Kepler's *Epitome of Copernican Astronomy*, p. 819: As the diameter of a circle is to the sagitta, or versed sine, of the angle between the axes of the lunar shadow and of vision, so, very nearly, is the diameter of the lunar disc to the breadth of the phase. For a little more than half the globe is illuminated by the Sun; and so, when that angle is right and the circle of illumination appears as a straight line, a little more than half the disc is then illuminated.

PROBLEMA IV. V. & VI.

Problems 4, 5 and 6

Quota pars Lunæ à Sole illuminetur, aut à nobis videatur; & quota Terræ portio inumbretur à Luna.

What part of the Moon is illuminated by the Sun, or is seen by us; and what portion of the Earth is shaded by the Moon.

IV. Of the first two I shall speak more opportunely below, ch. 18, nos. 5 and 6, because they depend on the distances of the Moon, which are to be established below. Of the third I shall speak in Book 5, because it concerns eclipses of the Sun.

PROBLEMA VII.

Problem 7

Quanto tempore ante primam Quadraturam, aut poſt ſecundam Luna aimidiata nobis ſplendeat.

How long before the first quadrature, or after the second, the halved Moon shines to us.

[Margin: The time of the halved phase.]

V. Repeat here the figure of Book 3, ch. 7, no. 9, with its explanation, and let there be given the distance of the Moon from the centre of the Earth as the side TL, and the distance of the Sun from the centre of the Earth as the base TS, for the given day or even moment of quadrature. With these, seek in the triangle TLS, right-angled at L, the angle S: for this will be the distance of the Moon's apparent place from the place of quadrature. Convert this distance into time by the Moon's hourly motion, and you will have the time,

Diagram. Woodcut set in the right column, inside the sentence above. A large outer circle carries a horizontal diameter running from A at the left through G, T and R to S at the right. An inner circle centred on T passes through G and R. At the top of the inner circle stands a small circle half blackened, the Moon, with the label K above it and L at its centre, joined to T by a straight line and to S by a second straight line, which together with TS make the triangle TLS. A small circle stands at S, the Sun, on the outer circle's right-hand point.

- A (left end of the horizontal diameter, on the outer circle): A
- B (top of the outer circle): B
- C (foot of the outer circle): C
- G (left, where the diameter meets the inner circle): G
- T (centre, below the diameter): T
- R (right, where the diameter meets the inner circle): R
- F (foot of the inner circle): F
- K (above the small blackened circle): K
- L (small letter inside the small blackened circle): L
- S (inside the small circle at the right end of the diameter): S
- I (on the line from T to the small blackened circle, set aslant): I by which the halved phase, in itself and with refractions in the air around the Moon excluded, precedes the first quadrature or follows the second. Aristarchus, moreover, in prop. 8, teaches that the circle which separates the bright part from the dark in this phase is distant from the Moon's great circle by less than the 3960th part of one right angle, that is, by less than 1′ 22″. And although the phase of the bisected Moon is discerned within a few hours, in which it turns from concave to gibbous, yet an exact full Moon is scarcely distinguished from an inexact one within three days.

PROBLEMA VIII.

Problem 8

Tempus quo Luna erit in Nonageſimo Eclipticæ gradu, & ſi cornuta ſit cornibus erectis lucebit, inuenire.

To find the time at which the Moon will be in the nonagesimal degree of the ecliptic, and, if it is horned, will shine with its horns erect.

[Margin: The time of the nonagesimal phase.]

VI. Longomontanus, Book 1 of the *Theorica*, ch. 9, near the end, gives this instruction on the matter. For the noon of the day after which you want to predict this phase, seek from tables or an ephemeris the true place of the Sun and of the Moon. Next, from the table of houses, under the title of house 10, seek the time from noon corresponding to the given altitude of the pole and to the Sun's place. Third, add to the Moon's place 90°, or 3 signs, and for this aggregate seek in the table of houses, under the title of the first house, the time from noon. Fourth, subtract the lesser time from the greater, and for the remaining difference seek the proportional motion of the Moon from the Sun, with the help of the diurnal motion of each body, which is in the ephemeris. This is to be added to the Moon's motion previously aggregated, so as to make the ascending point of the ecliptic, that is, the point that will ascend at the moment when the Moon will be in the nonagesimal; and for this, again in the table of houses, seek the time under the first house. If you subtract from it the time corresponding to the Sun's place, already found in the second step, there

will be left the time from noon at which the Moon will be in the nonagesimal degree of the ecliptic, counted from the ascending point of the ecliptic. The nonagesimal degree, moreover, will be known if you subtract 90° from the ascending point of the ecliptic. But the precept, because it is of great moment in astronomy, is to be illustrated by an example.

EXEMPLVM Longomontani.

Example of Longomontanus

In the year 1617, on October 4, old style, at noon at Copenhagen, where the pole is elevated 55° 34′, the calculation from Origanus's *Ephemeris* stands thus.

	Arc of the zodiac [Arcus Zodiaci]	House [Domus]	Time from noon [Tempus à Meridie]
Place of the Sun [Locus Solis]	♎ 22° 6′	10	13^h 18^m
Place of the Moon [Locus Lunæ]	♈ 20° 20′		
With 3 signs added [Additis Signis 3.]	♋ 20° 20′	1	23^h 1^m
Difference of the times [Differentia tēporū]			9^h 43^m
Diurnal motion of the Moon from the Sun [Motus diurnus Lunæ à Sole]	14° 14′		
Motion of the difference [Motus differentiæ]	5° 46′		
Ascending point of the ecliptic [Punctum Eclipt. Aſcend.]	♋ 26° 6′	1	23^h 34^m
With the time of the Sun's place taken away [Ablato tempore loci Solis]			
Time of the Moon in the nonagesimal [Tempus Lunæ in Nonageſimo]			10^h 16^m
And the nonagesimal [Et Nonageſimus]	♈ 26° 6′		

Therefore at 10^h 16^m after the noon already mentioned, at Copenhagen, the centre of the Moon was in the nonagesimal degree of the ecliptic, and the ascending point of the ecliptic was then Cancer 26° 6′, while the nonagesimal point was Aries 26° 6′.

PROBLEMA IX.

Problem 9

Diem Primæ & Vltimæ Phaſis Lunæ coniectari in Zonis Temperatis.

To conjecture the day of the first and of the last phase of the Moon in the temperate zones.

VII. From what was said in ch. 3, nos. 6 to 11, we can gather rules for conjecturing the day of the last and of the first lunar phase, whether it will be the first, or the 2nd, or the 3rd, or the 4th before or after new Moon. For the sake of brevity and clarity we have reduced these, as if into a mirror, to the

following small table, whose use is as follows. Look up in the ephemerides the Moon's place and its latitude, whether northern or southern, and lastly whether it is in perigee or in apogee or nearer to these. Then, if the Moon is in the ascending signs, use the table Asc.; but if in the descending, the table Desc. In the first column of these, look for the southern or northern latitude, and in the second the nearness of the apogee or perigee. For opposite these you will have the day of the first phase, computed from new Moon: in the third small column for those living in the northern temperate zone, and in the fourth for the southern. And you will have the day of the last phase, computed before new Moon, in the fifth small column for the northern temperate zone and in the sixth for the southern. The table will be the truer the greater the Moon's latitude and its nearness to the apogee or to the perigee, and where the pole is highly elevated; if the air is clear and the eye lynx-like, where day 1 is given, it will be possible to see the first and the last phase on the same summer day.

If ☾ is in the ascending signs [Si ● in Signis Aſc.] ♑ ♒ ♓ ♈ ♉ ♊

Latitude of ●	Anomaly of ●	Day of the first phase after ☌ [Dies Phaſis Primæ poſt ☌]: for the northern [Borealibus]	Day of the first phase after ☌: for the southern [Auſtralibus]	Day of the last phase before ☌ [Vltimæ ante ☌]: for the northern	Day of the last phase before ☌: for the southern
B	Perig.	1	3	4	2
B	Apog.	2	4	3	1
A	Perig.	2	2	3	3
A	Apog.	3	3	2	2

If ☾ is in the descending signs [Si ● ſit in Signis Deſc.] ♋ ♌ ♍ ♎ ♏ ♐

Latitude of ●	Anomaly of ●	Day of the first phase after ☌: for the northern	Day of the first phase after ☌: for the southern	Day of the last phase before ☌: for the northern	Day of the last phase before ☌: for the southern
A	Apog.	4	2	1	3
A	Perig.	3	1	2	4
B	Apog.	3	3	2	2
B	Perig.	2	2	3	3

SCHOLIA.

Scholia

[Margin: Inclinations of the phases.]

I. The inclinations of the lunar horns, treated in ch. 3, can be applied to the Moon in lunar eclipses, in which, especially if they are total, horned phases of the Moon appear, with the horns turned toward the east on entering the Earth's shadow, but toward the west on leaving it. And in solar eclipses too the Sun becomes crescent-shaped [Lunatus], and the inclinations of the solar horns appear remarkable. Ptolemy calls them [Greek: προσνέυσεις (prosneuseis, "inclinations")], Book 6, and Kepler teaches how to measure them

in the *Astronomia Optica*, from p. 361 to 410. But this is not the place to teach that.

[Margin: Estimate of the diameter of ☾ from the 1st phase.]

II. The same Kepler, in the *Astronomia Optica*, p. 344, teaches how, from the horn of the Moon in the first phase, estimated somehow, to investigate the ratio of the Moon's apparent diameter to the Sun's apparent diameter; and he teaches that the former is usually found greater than the diameter of the Sun, and therefore that the whole Sun can be covered by the Moon. But the estimate of the circumference of the Moon shining to us against the rest not shining is very slippery, as he himself admits, so it is not worth while to linger over these matters.

III. It remains, for describing the phases of the Moon properly, that you should know how to draw the lines of the phases, a matter dwelt on at length by al-Battānī, chapters 30 and 40; Witelo, Book 4, from Proposition 24 to 77; d'Aguilon, Book 5 of the *Optics*, from Proposition 49 to 56; Reinhold, on Peuerbach's *Theoricae*, from p. 166; Rudolf Goclenius, in his *Urania*, chapter 20; and Kepler, in the *Astronomia Optica*, from p. 243. [Margin: How the lines and figures of the phases are to be drawn.] Al-Battānī, for his part, teaches how to draw the shape of the falcate Moon at the first phase from the Moon's longitude and latitude, but Kepler teaches that this method is not very safe. Nor, however, does it much matter to astronomy that these [Greek: σχέματα (schemata, "figures")] or [Greek: σχέσεις (scheseis, "configurations")] be drawn with exactness. Still, so as not to leave this topic wholly untouched, take the following propositions.

[Margin: Rule 1.] 1. The pyramid or cone of the illumination produced in the Moon has its vertex at the tip of the lunar shadow and its base on a lesser circle of the lunar sphere; the cone of vision, by contrast, has its vertex in our eye and its base on a lesser circle of the lunar sphere, since our eye is a smaller sphere than the Moon. But this very circle is usually smaller than the base or circle of illumination, since on the Ptolemaic hypothesis the portion illuminated by the Sun is 180° 31′ 20″, and so the circle of illumination stands 15′ 40″ from the great circle that bisects the Moon. But the portion of the Moon seen by us, when the Moon is not at apogee and joined to the Sun at apogee (for then these two circles, or bases of illumination and of vision, are equal), does not reach 179° 28′ 40″. Therefore the base and circle of vision stands more than 15′ 40″ from the Moon's great circle, and so the circle of vision is usually smaller than the circle of illumination.

[Margin: Rule 2.] 2. The axes of the cones of illumination and of vision, produced in imagination, terminate at the centre of the Moon. In reality, granted, the axis of illumination, if the Moon is opaque, would terminate on the Moon's surface, at the point called the point of incidence, which is the pole on the lunar sphere from which the circle of illumination is understood to be described; and our visual ray would terminate on the Moon's surface, at its own point of incidence, from which as from a pole the circle of vision is

described. But when these axes are produced in imagination, as I said, their common meeting takes place at the centre of the Moon.

[Margin: Rule 3.] 3. If Reinhold is to be heeded (p. 168), our sight takes in only that portion of the lunar body which the base of the cone of illumination has in common with the base of the cone of vision, since of the lunar hemisphere, or quasi-hemisphere, irradiated by the Sun and always turned towards the Sun, no more can be seen by us than the portion turned towards our eyes. Therefore at conjunction these bases have no portion of the Moon's body in common, and so the whole illuminated part is hidden. Understand "usually", my dear Reinhold, for sometimes a circle of illumination is seen around the Moon in a central eclipse of the Sun, when the Sun's apparent diameter is greater than the Moon's apparent diameter; and then the base of illumination, being smaller, is contained within the base of vision, or lies below it. At oppositions, however, the Moon appears full and round, because the base of illumination is larger than the base of vision and extends beyond it.

[Margin: Rule 4.] 4. Near the quadratures, a little before the first or a little after the second, when the circle of illumination inclines its plane towards our sight and the axis of illumination makes right angles with the axis of vision, the Moon appears [Greek: διχότομος (dichotomos, "cut in two")], that is, it appears to shine with half of its hemisphere. Yet the semi-periphery of the illumination appears not as a curve but as a straight line, because of the Moon's great distance from the eye. On account of this distance none of its points appears farther from the eye than the rest; they appear to lie in the same plane as the eye, and all of them appear equally distant from it, as Euclid shows in theorem 22 of the *Optics*.

[Margin: Rule 5.] 5. When the Moon is outside the syzygies and quadratures, the bases of vision and of illumination intersect at oblique angles. If the angle of the axes of the cones of illumination and of vision is acute, or the common portion of the already mentioned bases is less than a quadrant, then the Moon appears illuminated less than hemispherically, that is, it appears falcate. But if that angle is obtuse, and the common portion of the bases greater than a quadrant, then the Moon appears illuminated more than hemispherically, that is, gibbous. In either case, however, the line of illumination appears curved, because it does not lie in the same plane as our eye; and the line of the Moon, whether gibbous or falcate, is not strictly circular but elliptical, as Scheiner notes in the *Disquisitiones*, no. 26, as Kepler shows from conic sections in the place just cited, and as Aristotle had indicated, section 15, Problem 6.

CAPVT V.

Chapter 5.

An Luna proprio, an Solis lumine luceat, & an inſtar ſpeculi ad nos illud reflectat.

Whether the Moon shines by its own light or by the Sun's, and whether it reflects that light to us in the manner of a mirror.

I. That the Moon shines not by its own light but by another's has seemed so obvious to most people that Cleomedes, in Book 2 of the *Cyclical Theory*, chapter 4, confirmed it from the very etymology of the name. He says that it is called [Greek: σελήνην (selēnēn, "Moon", accusative)] because it receives [Greek: σέλας νέον (selas neon, "new light")], that is, a splendour ever new, and he adds: *For this is evidence of a light admitted from outside.* That was the reason why Euripides called the Moon the daughter of the Sun, as I related in chapter 1. Nevertheless there have been those who thought otherwise. For Plutarch, in Book 2 of the *De placitis philosophorum*, chapter 28, reports as follows: *Anaximander teaches that it has its own light, but a thinner one. Antipho* [Antiphon] *that the Moon shines with its own brilliance, and that when this is hidden, it happens through the approach of the Sun, because nature has so arranged that a stronger fire obscures a weaker.* [Margin: Authors who attribute a light of its own to the Moon.] And if you objected the failure of light in a lunar eclipse, they replied, as the same author adds in chapter 29, that this failure comes about: *Anaximander, through the mouth of the wheel being blocked; Berossus, through the part devoid of fire being turned towards us; Heraclitus, through the inverting of the bowl; certain Pythagoreans, through the blocking of a brightness proceeding from the Earth, which they call Antichthon, as if the opposite Earth. More recent thinkers, through the consumption of a fire kindled in due order little by little until it produces the perfect full Moon; after which, its proportion preserved, it decreases by degrees as far as conjunction, where it is entirely extinguished.*

But the opinion of Berossus, because it was more celebrated and needs a more solid refutation, is to be set out more fully from Vitruvius, Book 9, chapter 4. [Margin: The fiction of Berossus the Chaldaean about the light of the Moon.] After saying *Now I shall speak of the light of the waxing Moon and of its waning, as it has been handed down to us by our forebears*, he adds at once: *Berossus, who came from the city of the Chaldaeans and also disclosed his learning in Asia, taught as follows: that it is a ball white-hot in one half, the rest being of a sky-blue colour. When, completing the course of its journey, it passes beneath the orb of the Sun, it is then seized by the rays and the force of the heat, and turned with its white-hot part towards them, because of the affinity of light for light. And when that part, thus called up to the orb of the Sun, looks towards the upper regions, then its lower part, because it is not white-hot, appears dark from its likeness to the air. When it stands perpendicular to the Sun's rays, all the light is held at its upper face, and then it is called the first. When, passing on, it moves towards the eastern parts of the sky, it is released from the force of the Sun, and the outermost part of its white-hot portion sends its brilliance to the earth in an exceedingly thin line, and so from this it is now called the second. And by the daily slackening of its turning it is counted*

third and fourth, day by day. In these words, better suited to an architect than to an astronomer, he describes the monthly motion of lunar rotation about the centre of the Moon's own body, a motion invented by Berossus to explain the succession of the phases. But Cleomedes treats these motions more lucidly in Book 2 of the *Cyclical Theory*, saying in chapter 4: *For Berossus declared the Moon to be fiery in one half and to be moved by many motions: one in longitude, by which it moves with the world; a second in latitude; a third by which it is carried up and down; a fourth by which, like the other planets, it moves with a proper motion; a fifth, by which he thinks it revolves about its own centre and at times turns other parts towards us, and is thereby increased and diminished, a revolution he makes monthly, that is, equal to the synodic month. But his opinion can easily be refuted and disproved, etc.*

II. Very many philosophers and astronomers, however, have always judged that the primary light by which the Moon shines to us is not proper and innate to it, but received from the Sun and reflected from it to the Earth. [Margin: Authors who deny that the Moon's primary light is innate.] Of these I shall list certain chief ones, first from Plutarch, Book 2 of the *De placitis*, chapter 28, who says: *Thales and those who follow his teachings, that the Moon is lit by the Sun; Heraclitus, that the Moon is affected in the same way as the Sun: for these heavenly bodies have the shape of a bowl, and when they take in moist vapour, they are lit, as they appear: the Sun more brightly, since it is carried in purer air; the Moon more dimly, because in murkier air.* And in chapter 29, speaking of the cause of lunar eclipses, he goes on: *Plato, Aristotle, the Stoics and the mathematicians agree in this: that the monthly disappearances are brought about by the Sun, with which the Moon comes together and by whose rays it is overwhelmed; but that it fails in light when it falls into the shadow of the Earth, which stands between the two heavenly bodies; or rather when its illumination is obstructed by the Earth.* [Margin: Thales, Plato, Aristotle, the Stoics, the mathematicians.] This same point Aristarchus laid down as a most certain foundation in the book *On the Sizes and Distances of the Sun and Moon*, supposition 1, saying: *The Moon receives its light from the Sun.* [Margin: Aristarchus and Vitruvius.] Vitruvius subscribes to this in Book 9, chapter 4: *For it is no secret*, he says, *that the Moon has no light of its own and proper to it, but is like a mirror and receives its splendour from the force of the Sun.* Hence the saying of Cicero in the *Dream of Scipio*: *Among these the smallest, which was the last from the heaven and the nearest to the earth, shone with a borrowed light.* [Margin: Cicero and Pliny the Elder.] Pliny the Elder records that not only the Moon but all the stars are lit by the Sun, saying in Book 2, chapter 9: *that it is ruled by the Sun's brilliance, like the other stars, since it shines entirely with light borrowed from the Sun, such as we see flitting in a reflection on water —— and that it is seen with unequal light because only when opposite is it full, while on the remaining days it shows the earth as much of itself as it conceives from the Sun.* And in chapter 10, on the cause of eclipses, he says: *For it is manifest that the Sun is hidden by the intervention of the Moon, and the Moon by the Earth's being set against*

it, and that they repay each other in turn, the Moon by its interposition taking the same rays of the Sun from the Earth, and the Earth from the Moon.

[Margin: Macrobius.] Macrobius, on the other hand, in Book 1 of his commentary on the *Dream of Scipio*, chapter 19, thinks that the Moon indeed shines by the Sun, but the other stars of themselves, and he gives the reason, saying: *That the Moon, which lacks light of its own and borrows from the Sun, must lie below the source of its light. For this reason makes the Moon have no light of its own, and all the other stars shine with their own: that those, being placed above the Sun, are in the very purest aether, in which everything whatever is natural light and its own.* Macrobius holds, moreover, that Mercury and Venus too are carried above the Sun, and a little later he adds: *But the Moon, because it alone is below the Sun and next to the region of perishable things, which now lacks light of its own, could have no light except from the Sun placed above it, to which it shines back.*

But he who made this lesser light to rule the night, and the stars, knows other causes, far more numerous and nobler, in the secrets of his providence: so that the Moon should not, shining with its own splendour, so great and so near, overwhelm the light of the fixed stars and seem to preside over a second day rather than over the night. Therefore, just as the Sun, wholly a day in and to itself, had to preside over our day, so the Moon had to be president of our night, being no day in itself, and to present sheer night through the privation of light of its own; and, just as the Sun had to pour out an abundance of light, so the Moon had, as if only by indicating, to speak the great works of God. To this one might perhaps not inappropriately turn the saying of Psalm 108 [recte: 18]: *The heavens declare the glory of God, etc. Day to day utters speech, and night to night shows knowledge, etc.* But passing over the arguments *a priori*, as they say, let us come to the arguments *a posteriori*, which can be brought chiefly against Berossus.

III. The first argument is that of Cleomedes, Book 2 of the *Cyclical Theory*, chapter 4: [Margin: Argument 1, of Cleomedes, against Berossus.] *First, it cannot be that it is in the aether and is fiery in one half of itself and is not on the whole of a homogeneous nature, as the other stars are.* But this argument savours of physical fittingness more than of necessary proof [apodixim]. Let us therefore hear the second, taken from the lunar eclipse, for he continues: [Margin: Argument 2, from eclipse, of Cleomedes.] *Next, the Moon in eclipse plainly opposes this opinion: for how does its light vanish on running into the earth's shadow, when it ought to have been shown wholly turned towards us? For if it had been such, it would have had to be more splendid on running into the shadow, and not to vanish.* [Margin: And of Tycho.] Tycho pursues this argument more explicitly in volume 1 of the *Progymnasmata*, p. 743: *For who among astronomers or natural philosophers does not know that the Moon is endowed with no light of its own, or very little, and that what it has and reflects to us it merely borrows from the Sun? For as to the fanciful opinion of some, that half of the Moon has its own light and the other half is opaque, and that it so turns itself in order about its centre with respect to us and the Sun that it is therefore thought likely to draw its light from the Sun: this, I say, is*

plainly fictitious, as, if nothing else, lunar eclipses certainly and openly prove, when the Moon is deprived of light by the interposition of the Earth. For if the Moon enjoyed a genuine light, surely when it was in the shadow of the Earth it would not lose it, but would display it all the more evidently. For all light shines more in darkness, when it is not hindered by another, greater brilliance. Nor is the shadow of the Earth anything corporeal, or some dense substance, such that it could darken the Moon's light and snatch it from our sight; it is only a certain privation of solar light, on account of the interposed opaque body of the Earth.

IV. [Margin: Argument 3, our own, likewise from eclipse.] Add to this that, without a great and violent multiplication of motions, no account can be given on this hypothesis of why the Moon in eclipse, beginning its failure from its eastern part, is so deprived of light with respect to us that it turns out as if horned, with its horns turned to the east, while at the emersion from eclipse its horns face west. For either it begins to withdraw its bright hemisphere from us by turning itself eastward about its centre, and then it would fail for us, but from its western part, and yet not so as to appear to us horned towards the east, but gibbous towards the west; or by turning itself westward, and then it would begin to fail from its eastern part, but with a gibbous figure, not a falcate one. In partial eclipses, now, this argument has still more force, for one would have to imagine in the Moon a motion of rotation compounded of motion in longitude and in latitude, by which it would nevertheless appear gibbous towards the ecliptic or the poles of the world, and not, as it does now, falcate towards the ecliptic. But these phenomena, which we now actually see, are explained far more solidly if we posit that an eclipse of the Moon comes from its running into the shadow of the Earth, since it is certain from other sources that the Earth casts this shadow towards the side turned away from the Sun, and it is established that lunar eclipses occur there alone and nowhere else.

V. The fourth argument is drawn from the position of the spots and their successive illumination. [Margin: Argument 4, of Kepler, against Berossus, from the position of the spots.] The spots always appear in the same position towards us, in such a way that at new Moon they are dark, and then one after another they are successively illuminated. Kepler seeks this even from a single spot, in the *Astronomia Optica*, chapter 6, no. 2, and his words are worth quoting. Reporting, from Laërtius [Diogenes Laertius] and Cleomedes, the opinion of Berossus and the contrary opinion of Thales, who says that the Moon shines of itself in no part, he adds: *Berossus could have gathered this even from the fact that it is plain to the eye that the same parts of the Moon do not always shine. For the face we imagine for ourselves in the Moon from the disposition of the spots strives successively into the light, and, always unmoved, is turned downwards towards us, granting the light passage over itself. There is at the western edge of the Moon, a little above the eye towards the top, a very black spot like the Pontus, in the midst of very bright light, separated from the spots as an island from the mainlands, scarcely a finger's breadth from the outermost limb. You will see it there, namely at the western edge and a little towards the top, whether the Moon is halved, or gibbous,*

or entirely full. The circle of illumination therefore first reaches it, afterwards passes it more and more, and after the full Moon leaves it altogether. Therefore the same parts of the Moon are not always in the light, which is what Berossus thought; rather the same parts of its body are perpetually turned towards the Earth, the centre of its gyration. And by going round the Earth, the Moon presents different parts in turn to the Sun, roasting itself monthly at the Sun as if it were being turned on a spit: in much the way that Copernicus said the Earth is turned and roasted at the Sun, or at the fire so called. The comparison taken from the spit and from the rotation of the Earth, however, favours Berossus; but one must attend to Kepler's meaning, and I believe he is speaking of the spot called the Lake of Knowledge [Lacu Scientiæ].

VI. [Margin: Argument 5, our own, likewise from the spots.] The fifth argument is likewise drawn from the lunar spots, but I set it out differently, as follows. At full Moon, especially when viewed through the telescope, there appears in the western part of the Moon a large black spot towards the north-west, called by Langren, Gassendi and Boulliau the Caspian Spot [Macula Caspia], because its figure is almost like that of the Caspian Sea as depicted in geographical hemispheres. In the eastern part, on the other hand, there is so great an expanse of the old spot that it occupies almost half of the lunar disc, and it has earned from Langren the name of Oceanus Philippicus, as can be seen in the diagram to be placed in Chapter 7. Now if the opinion of Berossus were true, at the Moon's first quadrature the Caspian Spot would not appear, but the Oceanus Philippicus; and at the second quadrature the Oceanus Philippicus would no longer appear, but the Caspian Spot, as will be evident to anyone who considers the rotation invented by Berossus. But the opposite happens, for at the first quadrature you see the Caspian Spot towards the west, in the part of the Moon that is halved and illuminated; at the second it appears nowhere, but in its place the Oceanus Philippicus. Let it stand, then, that the Moon does not have in itself a primary light that is proper and inborn; that it does not by its rotation now grant that light to us and now withdraw it; and that it is not always bright in one half of itself and always dark in the other half. Rather, it has its light from the Sun, is successively illuminated as a whole, is as a whole dark of itself, and successively becomes luminous through different parts in turn.

Quomodo Luna Solis lumen ad nos reflectat: vbi de opacitate Lunæ, & ſpeculi analogia.

How the Moon reflects the Sun's light to us: where the opacity of the Moon and its analogy with a mirror are treated.

VI. There are three opinions on this reflection. Cleomedes too touches on two of them, in that most learned little book *On the Cyclical Theory*, Book 2, chapter 4, and Kepler on all three, in the *Optics*, chapter 6, no. 2, from p. 227. [Margin: Opinion 1, of those who assert that the Moon is a mirror.] The first

was that the Moon, like a convex mirror, reflects to us the light of the Sun, received not without some refraction. So Clearchus, in Plutarch's little work *On the Face of the Moon*; on him see below, chapter 6, no. 8. So Pliny the Elder, Book 2, chapter 9, said that the Moon shines with light borrowed from the Sun, *such as we see flitting in a reflection on water*; and more openly Macrobius, Book 1 on the *Dream of Scipio*, chapter 19: *The Earth, on receiving the Sun's light, only brightens, and does not shine back; the Moon, like a mirror, gives out the light by which it is illuminated: for although it is a denser body than the other heavenly ones, yet, as being much purer than the earthly, it becomes penetrable to the light received.* [Margin: The diaphaneity and opacity of the Moon.] Reinhold agrees with these in his work on Peuerbach's *Theoricae*, from p. 165, in the part where he subscribes to Macrobius, and he admits in the Moon many deeply diaphanous parts. For after reporting that there are some who judge that one hemisphere of the Moon, the one inclining towards the Sun, is diaphanous and is filled with the Sun's light like a sponge, while the other, turned away from the Sun, is perpetually dark because of its own opacity and density, he added: *This opinion indeed is by no means to be despised.* For Reinhold had not noticed that what we said above in nos. 4 and 5, on the position of the spots and on different parts of the Moon being illuminated, tells against this opinion. He adds, however: *But Witelo's opinion certainly seems to me more probable, even if it differs little. For he does not so divide the body of the Moon into a diaphanous hemisphere and an opaque one; rather, he holds that it is in any part of it partly rare and diaphanous, partly dense and opaque, yet that the opaque parts are far more numerous than the diaphanous.* Before these, moreover, Posidonius had attributed much diaphaneity to the Moon; for Cleomedes, in Book 2 of the *Cyclical Theory*, chapter 4, reports of him as follows: *Posidonius, then, says not only that the surface of the Moon is flooded with light by the Sun, as happens in solid bodies, but that the Sun's rays penetrate very far into its depth, as it has a rare body.* In this class too, as regards diaphaneity, our d'Aguilon is to be reckoned, Book 5 of the *Optics*, after Proposition 56, since after a long disputation he finally concludes as follows: *It is the opinion of others, which also pleases us more, that one hemisphere of the Moon is perfectly diaphanous, while the other, namely the one that presents to us the appearance of a human face, is only half-diaphanous, that is, sprinkled with a certain opacity, which transmits some of the light but retains the greater part, not only at the outermost surface but also in the depth.* To this our Scheiner also tends, in the *Disquisitiones Mathematicae*, no. 26 (though he, as much as d'Aguilon, openly denies that the Moon is a mirror), and above all Witelo, Book 4, on theorem 77, and Lagalla [Giulio Cesare Lagalla], *De phaenomenis in orbe Lunae*, chapter 11.

VII. [Margin: Opinion 2, of those who deny that the Moon is a mirror.] The second opinion, at the opposite extreme from this one, is that the Moon is not a mirror and does not shine back from the Sun but brightens, and yet by its rough opacity reflects the Sun's light to us, in the way that walls, or the mountains and rocks of the Earth, throw it back. So Plutarch, in the little work *On the Face in the Orb of the Moon*; some Amram [?], an Arab, in Scaliger;

Giovanni Battista Benedetti in his *Letters*, p. 413; Biancani, Book 9 of the *Sphere*, chapter 7; Galileo, Dialogue 1 *On the System of the World*; our Scheiner in the *Disquisitiones Mathematicae*, no. 26; Bettini, *Apiarium* 8, Prog. 6, proposition 1; d'Aguilon, Book 5 of the *Optics*, after Proposition 56; Tanner, *De caelo*, q. 7, par. 3; Cleomedes, Book 2 of the *Cyclical Theory*, chapter 4; and Kepler, in the *Optics*, chapter 6, no. 2. For all agree in this, that the Moon is not a mirror of the Sun. Plutarch indeed, taking the part of Lucius, says: *It is probable, then, that the Moon is not illuminated in the manner of glass or crystal*. And a little later, after saying that light is reflected to us from the Moon as voices are from rocks, which produce an echo, and having raised against himself, at the prompting of Sylla [Sulla], the objection that if this happens by reflection, it happens only at the points at which the angle of incidence can equal the angle of reflection, he adds after various evasions: *Indeed, even if we grant this, and gratify Geometry, so very dear to us, in the first place it is proper for this to happen in mirrors polished to perfection, namely because of the equality of the angles of incidence and reflection. But the Moon has many inequalities, many roughnesses, so that the flashes which come from a great body can shine back on one another from heights of no small size, and can be reflected in every way, be interwoven, and carry on their resplendence among themselves, as if it were being brought to us from many mirrors.* And again below, after saying that there are valleys and mountains in the Moon, and opaque parts, just as on the Earth, and having objected to himself that this seems to rule out the reflection of light from the Moon to us (for if it is granted, the Sun's image must be seen by us at night in the Moon, as it is seen in mirrors and in water), he replies that this by no means follows: both because of the great distance, which would not allow the Sun's image to be seen even if it were in the Moon as in a mirror, and because in reality the reflection does not occur as from a mirror but as from a rough surface, since not even milk, because of a certain roughness, which is nevertheless much less than the roughness of the Moon and the Earth, gives back mirror images.

VIII. [Margin: The argument by which it is established that the Moon is not a mirror.] The argument, then, of Plutarch as well as of Cleomedes, Galileo, Kepler, Scheiner, Biancani, Tanner and d'Aguilon, by which they prove that the Moon is not a mirror of the Sun, and that the Sun's light is not reflected to us from it as from a mirror, is this. If it were so, either we would not see the image of the Sun, because of the enormous distance of the Moon from the Earth (but then we would not see any other light either), or, if we saw the Sun's light, we would also see the Sun's image, though very tiny and like a bright point, both because of the distance and because of the convexity of the lunar globe, since convex mirrors give images smaller than their prototypes. Next, this little image of the Sun would be seen only at one point of the Moon, by one and the same spectator, and the whole Moon would not appear to shine at full Moon, or half of it at the quadratures, but only that small portion in which the Sun's image was seen, as is established from the laws of catoptrics and of the reflection produced in water, glass, crystal and other mirrors, especially convex

ones. The remaining part of the Moon would appear dark. Galileo proves this by a ready experiment and by comparing a mirror and a wall. For from a mirror facing the Sun there is no reflection except from a determinate small part of it, to a determinate place and position of the eye; and thus the image of the Sun is seen by one eye in such a way that it is not seen by another, and the eye that sees the Sun's image at the same time sees other parts of a larger mirror as dark. But from a wall facing the Sun, light is reflected from any sensible part, so that all who are present see together the brightness of the whole lit wall, yet not the Sun's image, because of the roughness of a surface made up of many small surfaces, some like little valleys, others like little hills, which with their sides carry on the reflection towards one part after another. This mode of reflection is excellently described, after Plutarch, by Galileo in the place cited above and by Gassendi, Letter 4 *On the Low and High Sun*, no. 14. Biancani, moreover, confirms the same from spheres on the tops of towers lit by the Sun: for the smoother and more polished they are, the less they reflect light to us, and the rougher they are, the more.

IX. [Margin: Opinion 3.] The third opinion, not wholly different from the second, is that of Cleomedes, Reinhold, Witelo, d'Aguilon, Scheiner, Lagalla, Tanner and Bettini: namely that the Moon, although it is not a mirror, since the unevenness and roughness of its surface stand in the way, nevertheless has some diaphaneity, by virtue of which it admits the Sun's light beyond the surface, and, so affected and altered, reflects it to us: whether in the manner of a half-rare and half-diaphanous cloud, as Scheiner puts it; or, as Cleomedes says, in the manner of iron glowing from fire; or by a certain fourth light, which Kepler, above and in chapter 1, proposition 22, calls communicated light and distin- ct from direct, reflected and refracted light. He confirms this from the fact that the Sun's light enters coloured bodies beyond their surface and, tinged and steeped with their colours, also carries the images [species] of those coloured bodies through a hole into a darkened chamber and paints them distinctly on paper. So it does not follow, if the Moon has a diaphanous surface, that it is for all that a mirror, unless it is polished and smooth and not made uneven by any roughnesses; or, as Bettini adds, if it is not homogeneous in density and rarity, opacity and transparency in a great part of itself, since otherwise a frequent heterogeneity of these can hinder the properties of a mirror.

CAPVT VI.

Chapter 6.

An Luna fecundarium aliquod lumen habeat, & vnde illi lumen, quod in Eclipſibus, aut prope Nouilunia omnia, aut in lucido annulo videtur habere.

Whether the Moon has some secondary light, and from where it gets the light that it seems to have in eclipses, or around all new Moons, or in a bright ring.

I. [Margin: A short summary of the phenomena bearing on the Moon's secondary light.] There is no new Moon near which, in the last and first phases, if the Moon is watched in a clear sky and rather far from the Sun's twilight glow, a certain bright ring is not seen around it. This ring swells and projects on the part turned towards the Sun, while on the rest of the Moon's body there is a certain light resembling burning ash that glows white. In eclipse new Moons, however, the Moon seems to shine on the part turned towards us with some light, though a faint one, and sometimes to be ringed by a bright limb, even when the whole Sun is believed to be covered by it. Finally, in eclipse full Moons, in which the whole Moon undergoes eclipse, it nevertheless often glows faintly with varied light and colours, almost in the depth of the Earth's shadow. From this have arisen controversies among authors: whether this secondary light is found in the Moon, and from where. For we established in chapter 5 that the primary light is derived into it from the Sun. We must therefore discuss these matters in order and distinctly.

De Annulo lucido circa Lunam.

On the bright ring around the Moon.

II. In the morning after the second quadrature, up to the last phase, and in the evening before the first quadrature and after the first phase, the Moon, if watched with the naked and simple eye without lenses, seems surrounded by a bright ring: slender on the part turned away from the Sun, but broader on the part turned towards the Sun, and indeed protruding like a little cap, so that lines drawn out from the centre to this part appear longer. This can be seen in the first part of the figure added here, where the more slender limb of the ring is F E K, and the swelling and projecting one F G K.

Diagram. Two circular woodcut figures set side by side in the left column. Each shows a vertically hatched lunar disc inside a bright outer ring. In the left figure the ring is visibly wider on the right, towards G, than on the left, at E. In the right figure the ring is narrow and runs around the disc. Labels read:

- F (top of the left figure)
- E (left of the left figure, outside the ring)
- I (right edge of the left figure's disc, inside the ring)
- K (foot of the left figure)
- G (between the two figures, at the outer right of the left figure and the outer left of the right figure)
- L (top of the right figure)
- N (right of the right figure)
- M (foot of the right figure)

Labels not read: none.

In some eclipses of the Sun, moreover, with the eye placed on the line through the centres of the luminaries, a certain bright belt appears around the Moon, encircling it, as in the second figure, G L N M. The cause of the

swelling F G K is not in the Moon itself or in vaporous air around the Moon, as Mästlin, theorem 148, and Galileo in the *Sidereus Nuncius*, p. 12, held; nor is the swelling real but apparent, a delusion of the naked eye. This is clear because, if you look at the Moon through a telescope, that protuberance ceases, and the first and last phases are seen as in the third figure, A D B C.

Diagram. A single circular woodcut inset at the right of the left column, the text wrapping around it on its left. A dark, mottled lunar disc stands inside a clear ring, which is wider on the right, at C, than on the left, at D. Labels read:

- A (top)
- D (left)
- C (right)
- B (foot)

Labels not read: none.

[Margin: The cause of the ring's swelling.] The cause of the deception is therefore optical, as Kepler teaches in the *Astronomia Optica*, chapter 5, no. 5, and in the dissertation with the *Sidereus Nuncius*, p. 15; and Scheiner in the *Disquisitiones Mathematicae*, no. 26. For white things, let alone bright ones, set beside black or darker things appear larger and more prominent, although they are not, and enlarge themselves in the eye; and they appear as whole figures, although they are in reality incomplete. This is because of the efficacy of the images produced by white or bright things, which are apter to move the sense organ of sight and to drive, as it were, the images of black and darker things out of possession. So it is that a white column beside a black or red one of equal size, seen from afar, looks larger, just as a sheet of paper bounded by parallel edges and blackened over half of it appears larger in its white half than in its black. So too a thread of spider's web looks ten times larger on the part lit by the Sun than on the remainder not directly lit by the Sun. So too a flame seen from afar looks round; and Venus, although horned if you use a telescope, nevertheless appears round to the naked eyes. Those who are purblind and of weak sight, so that they cannot see distant things, are deluded in this far more. For bright and white things illuminate strongly.

III. [Margin: The cause of the bright ring.] As for the annular limb F E K, or in eclipses of the Sun L G M N, some think its cause is an atmosphere or air, or a circumlunar aether, denser than the rest of the surrounding aether, which was discussed in chapter 2, no. 3. They hold that this shines more than the intervening parts of the lunar disc, by a doubled reflection or by refraction of the solar rays. Among them is Bettini, *Apiarium* 8, prog. 2, prop. 10, and prog. 6, prop. 3; and Fortunio Liceti in the *Quaesita per Epistolas* and in book 2 of *De Lunae luce subobscura*. Galileo, moreover, in the *Nuncius*, pp. 12 and 13, says: *To this reason another can be attached, namely that around the lunar body there is, as around the Earth, a certain orb of a substance denser than the rest of the aether, able to take up and reflect the irradiation of the Sun, although it is not endowed with such opacity that it can block the passage of the sight (especially while it has not been*

illuminated). This orb, illuminated by the solar rays, renders and represents the lunar body in the likeness of a larger sphere, etc. A sign of this is that the part of the Moon flooded with light appears of a wider circumference than the rest of the dark orb. By that very argument Mästlin, from theorem 145 to 149, had taught that air is spread around the lunar globe.

Afterwards, however, Galileo, in Dialogue 1 of *De Mundi systemate*, refuting Scheiner without naming him, teaches that the Moon's secondary light takes the form of a ring, since it appears brighter at the outermost edge than in the middle, although it is not in fact brighter, but appears so because of the difference of the boundaries that limit the Moon's inner disc. For the inner disc, at the boundary F I K in the former of the figures given above, appears darker because of the abundance of splendour of the neighbouring horn F G K, being obscured by it; but on the other side, F E K, where it has for its boundary a tract of twilight darker than the lunar horn, the Moon's secondary light appears brighter around the margin, and from this comes that bright little circle. He proves, moreover, that this is a delusion of the eye, which because of the relation just mentioned judges the limb of the Moon brighter than the rest of the disc: if someone does not see the primary splendour of the lunar horn, it being hidden meanwhile behind the wall of a chimney or of another more distant obstacle, while he does see the rest of the Moon's disc, he sees that whole disc grow bright with secondary light equally, without any difference at the margin, and sometimes with so much light that it seems to rival the light of Saturn. Indeed, as Biancani affirms in book 9 of the *Sphaera*, chapter 4, it appeared to him at such a time to have attained, as if by a miracle, the full Moon, although it was near new Moon. Nor was Scheiner unaware of this experiment, since he says in the *Disquisitiones Mathematicae*, no. 27: *So this second splendour of the Moon is greater than many suppose, since they have sometimes detected it with the Sun still above the horizon, which does not happen even for all stars of the first magnitude. So it is no wonder that it, if the primary lies hidden behind the ridge of some house and the secondary alone shines, is so great that it equals, if it does not surpass, stars of the third magnitude, I add sometimes also of the second, and, what is more incredible, the splendour of Saturn.* And no. 28: *Thus the eye, the dark part of the defective Moon being seen separately, with the bright part hiding behind a roof, beholds it as more brilliant than bright stars.* And yet in no. 27 he attributes the cause of the brighter lunar limb, as of a real effect, to the rays of the Sun that penetrate the Moon's semi-diaphanous body and pass through the Moon's margins by a shorter path, and so are less dulled than those that penetrate the diameter and the whole depth of the Sun, or that are nearer the diameter. Indeed, he tries to confirm the Moon's diaphaneity from this bright ring, saying: *This opinion is proved, first, from experience, because the waning and renascent Moon shines out greatly in the unlit part, especially at the limb.* But if it is a delusion of the eye, which ceases both when a telescope is applied and when the eye looks at the lunar disc from behind a chimney or tower, with the bright horn of the Moon meanwhile shut off, there is no reason to seek a

cause in the object, let alone to refer it either to our air with Lagalla, or to the circumlunar air with Mästlin, Kepler, Bettini, Liceti and formerly Galileo, or to the diaphaneity of the lunar margins with Scheiner. So much, then, for the bright ring that appears to us outside eclipse new Moons.

IV. [Margin: On the bright ring seen around the Moon in eclipses of the Sun.] But what about the ring that has more than once been seen shining around the Moon in eclipses of the Sun? For we cannot put this kind of phenomenon down to a mockery of the eye without force and a certain injury done to astronomical history. Plutarch certainly says in the little work *On the Face in the Orb of the Moon*: *Even if the Moon sometimes hides the whole Sun, yet that eclipse lacks breadth and duration, but a certain splendour shines out around the rim, not allowing the shadow to become deep and excessive.* Cleomedes supports this in book 2 of the *Cyclical Theory*, chapter 4, where, asking how the Moon, although smaller than the Sun as to true diameter, nevertheless optically covers the whole Sun, he says: *Some of the ancients, indeed, held that in perfect eclipses the centres of the luminaries fall on one and the same straight line, and that the rim of the Sun appears all around on every side. Nor is this any kind of invention, for projecting parts have been seen by us, most brilliant and displaying no slight projection, since the Sun is much larger than the Moon. It must be said, therefore, that the Moon is smaller than the Sun. But nothing prevents the whole Sun from being darkened by it optically or apparently, when, that is, it is made optically equal to it.* He therefore admits both kinds of eclipses, namely total ones, and partial ones with the bright circle of the Sun projecting all around the Moon; although others think his words ambiguous and translate the passage from the Greek original so that the second kind is denied.

Moreover, Proclus Diadochus said in the *Astronomical Hypotyposes*, q. 5: *Sometimes the whole Sun seems to be covered from our view; sometimes, at the very moment when the two centres and the eye fall on the same line, the Moon is seen within the Sun's circuit.* And he reports that Sosigenes wrote this very thing in the book *On Revolutions*: namely, that *the Sun at perigee in eclipses is seen not running forward entire, but that at the extremes of its circumference it escapes the circle of the Moon and gives light in no way impeded.* I shall point out examples of these eclipses in Scholium 1.

On these suppositions, the question is the cause of this phenomenon, namely of the bright corona G L N M around the Moon in central eclipses of the Sun. Longomontanus, in book 1 of the *Theorica*, chapter 9, attributes the cause both to rays of the Sun reflected from the Earth, which diminish the lunar shadow (for with the shadow diminished the Sun, which otherwise is then usually seen under the angle of the shadow, is seen under a larger angle), and to refractions of the solar rays made in the vapours of the horizon, by which the Sun's diameter is optically enlarged. But these causes, as we shall say in chapter 16, no. 8, do not hold when such eclipses appear around the meridian far from the horizon, as one appeared to Clavius according to what will be said here in chapter 16, no. 8; and that reflection dilutes the shadow but does not narrow

it. Kepler, however, in the *Astronomia Optica*, from p. 299 to 302, adduces two alternative causes. The first and more frequent, he says, is the varied and alterable matter of the sky, and the air, or if you prefer the aether, around the Sun, which is now thicker and denser than the rest of the aether, now rarer, and so shines more or less from the Sun's rays; this he confirms by the example of the eclipse observed at Naples, in the book *De Stella nova*, chapter 23. The other cause, to be substituted in support if this one should not suffice, he says is denser air around the Moon, and vapours drawn up from its seas, lakes, etc.; in this air the rays of the Sun, refracted towards the Moon's globe, represent the Sun under a wider angle, and thus render its diameter optically larger than it really is. Scheiner here too would have recourse to the diaphaneity of the Moon; but then this ring would have to be seen in every central eclipse of the Sun.

[Margin: Our opinion on this ring.] So when in central eclipses of the Sun a bright circle appears around the Moon, the Moon's apparent diameter is either equal to the Sun's apparent diameter or smaller. If smaller, it is at once clear that the objective cause of the circle just mentioned is in the Sun's disc itself, which extends all around the Moon, so that its direct rays are not all hidden from us by the Moon. But if it is equal, then, because the Sun nevertheless illuminates more than a hemisphere of the Moon, it would not be impossible for some part of the Moon, or some circle of it, to be seen illuminated by us, but through rays reflected from it, if the base of the circle of vision were not smaller than the base of the circle of illumination; or, if it is smaller, for this to come from circumlunar rather than circumsolar air. But since so great a thickness of circumlunar air is uncertain, I therefore hold absolutely that the diameters of the luminaries should be so reconciled that whenever that ring appears, the Moon's diameter is smaller than the Sun's. Granted, the Sun's rays refracted in our air may then make that ring wider. See again on this matter below, chapter 16, nos. 8 and 16.

De luce ſecundaria Lunæ prope Nouilunia, & in Eclipſi Solis.

On the Moon's secondary light near new Moons and in an eclipse of the Sun.

V. In figures 1 and 2 above you have seen that the Moon's disc within the bright periphery is not so dark that it does not show a secondary light. Such a light, then, is seen in the Moon both near new Moons, as anyone can easily observe, especially on the days nearer the new Moon, up to but not including the quadratures, and indeed when the Sun has sunk deeper below the horizon and the light of twilight has lessened; and in many new Moons in which the Sun is wholly, let alone partly, covered by the Moon, as I shall say in Scholium 2. Different authors give different causes of this secondary light.

VI. [Margin: Reinhold's opinion.] The first opinion, conforming to those ancients who attributed a light of its own to the Moon, is that of Erasmus Reinhold, p. 164, on Peuerbach's *Theoricae*. After approving Witelo's opinion

about the Moon's various parts, partly diaphanous and partly opaque, and ascribing the Moon's secondary light to rays of the Sun that penetrate the Moon where it is diaphanous, he nevertheless adds: *Lastly, there is in the Moon a certain peculiar or obscure light, and what this is like is plainly shown by its total eclipses, in which the entire orb is seen in a foul and horrible colour, which, however, is at times ruddy, when the Moon is higher and carried farther outside the ecliptic, at other times blacker, the lower it is, that is, and therefore the more deeply immersed in the Earth's shadow. But in a total eclipse of the Sun, some solar rays also mingle with the light that the Moon has as its own, which, as has been said, penetrate the body of the Moon a little.* So he attributes the chief cause to the Moon's own little light. But if this little light or native whiteness belonged to the Moon, it ought to appear more in the deeper shadow of the Earth, in which, however, sometimes none is seen. So the cause of this light in eclipses of the Moon is another, as we shall say below under the title on this matter.

VII. [Margin: Tycho's opinion.] The second opinion, that of Tycho, *Progymnasmata*, book 2, as cited by Kepler in the *Astronomia Optica*, p. 254, assigns the cause of this light to Venus, which could illuminate the Moon so brightly. But granted that the splendour of Venus, when it shines from that side, could illuminate the margins of the Moon, it could not illuminate the whole disc of the Moon turned towards us, since Venus is much higher than the Moon. And when Venus is above the Sun and joined to it, it can in no way illuminate the Moon on the part facing us; and I say the same of the remaining stars that are absent, or higher than the Moon. As for the fixed stars or planets opposite the new Moon, Kepler in the same work, chapter 7, agrees that the Moon is sometimes illuminated by them, so that through that illumination we see the Moon; but not always, because stars of the first or second magnitude, let alone planets, are not always placed opposite the Moon. And conversely, they are placed opposite the half of the Moon turned away from the Sun, which, however, does not shine. Lastly, the whole hemisphere of the fixed stars on a moonless night does not so illuminate our Earth that we, although nearest to it, see any trace of light reflected from there. Far less, then, can we see the light of the fixed stars reflected by the Moon, at so great a distance of the Moon from us, as Liceti rightly argues in book 1 of *De Lunae subobscura luce*, chapter 10. And if anyone maintains that the Moon is illuminated by the fixed stars, then it would also be illuminated at the quadratures and beyond, which is against experiment.

VIII. [Margin: The opinion of Witelo, d'Aguilon, Scheiner and Lagalla.] The third opinion is that the Moon's secondary light in eclipse conjunctions, or near other conjunctions, is rays of the Sun shining through, but refracted and weakened while they penetrate the Moon's semi-opaque and semi-diaphanous body, or a body opaque in some of its various parts and transparent in others. So think those most brilliant lights of optics: Witelo, *Optics*, book 4, theorem 77; d'Aguilon, *Optics*, book 5, p. 423; Scheiner in the *Disquisitiones Mathematicae*, no. 27, and in *Apelles post tabulam*, p. 53; Vincentius Lagalla, in the book *De*

Phaenomenis in Orbe Lunae, chapter 11. And Caesar Germanicus, in the *Aratea Phaenomena*, towards the end, indicates that it was also the opinion of ancient philosophers. I will not repeat what I said about the Moon's diaphaneity in chapter 5, no. 6. Against this opinion, Kepler in the *Optics*, chapter 6, no. 10, and Galileo in the *Sidereus Nuncius*, from p. 14, and in *De maculis solaribus*, p. 136, adduce the Moon's total opacity, which Kepler says he evidently demonstrated in no. 9, although he only suggested it by mere conjectures. Galileo adds that if the rays of the Sun penetrated the whole depth of the Moon, which is about 2000 miles, they could penetrate far more easily the peaks of the lunar mountains, which are discussed below, and through them the parts of the Moon that follow would be illuminated at the quadratures and at sextile aspects, which is against experience. For so great a transparency, which at a depth of 2000 miles would not impede the Sun's rays, could not prevent those same rays from showing us, with a confused light, the boundary of the directly illuminated part from the remaining part. Anyone can test this in a glass ball exposed to the Sun and viewed from such a position as we view the Moon [Greek: διχότομον (dichotomon, "cut in two", the half Moon)], unless you say that those mountains are opaque there.

Moreover, this secondary light would last at the quadratures and beyond, whereas in fact it vanishes a little before the first quadrature, begins to appear only one or two days after the second, and is always the greater the nearer the Moon is to the Sun; unless you say that this little light does not then appear when compared with the remaining splendour of the Moon. Nor should it seem unlikely to anyone that, if the Moon's globe were as it were glass and transparent like the body of celestial comets, the Sun's rays would produce on the part turned away from the Sun a tail of its own for the Moon, as for a great comet, were it not for the Moon's excessive depth, about two thousand miles, and its density. Because of these Posidonius did not dare to admit rays of the Sun penetrating the whole Moon: for Cleomedes says, in book 2 of the *Cyclical Theory*, chapter 4: *Posidonius, then, says that not only is the surface of the Moon illuminated by the Sun, as solid and opaque bodies are, but that it enters its thickness, as being rare, yet not everywhere nor very deeply, because the Moon has a huge diameter and the Sun is very distant from it; whereas misty air easily admits penetrating rays, as having no depth.* So Scheiner undeservedly adduces Posidonius for himself. Liceti, however, in book 1 of *De Lunae subobscura luce*, book 5, says that the opinion of Witelo and his followers can be defended if channels are posited in the body of the Moon, and certain streaks or veins of diaphanous parts, so disposed that their straightness, when the Moon stands near conjunctions, transmits the Sun's rays to us, while at other, greater distances from the Sun they cannot do this because of their obliquity. But in chapter 6 he rejects Witelo's opinion absolutely, because he considers it arbitrary to posit those channels, or indeed diaphaneity, in the Moon without experiments. Yet even if diaphaneity were present, because of the excessive depth of the Moon,

of more than two thousand miles, the rays would not reach the bottom of the Moon, as I have already said.

IX. [Margin: The opinion of Mästlin, Kepler, Galileo, Gassendi, René Descartes, Hortensius, Šírek and Longomontanus.] The fourth opinion is that this little light comes from rays of the Sun reflected from the Earth right to the Moon, from those parts of the Earth in which it is still day. So think Mästlin, in the *Eclipses*, from thesis 21; Kepler in the *Astronomia Optica*, p. 254, in the *Epitome Astronom.*, p. 832, and in the dissertation with the *Sidereus Nuncius*, p. 15; Galileo in the *Sidereus Nuncius*, p. 15, and in *De Maculis Solis*, p. 136, and in Dialogue 1 of *De Mundi Systemate*; Pierre Gassendi in the book *De Magnitudine apparenti Solis humilis & sublimis*, Letter 2, which is to Liceti, p. 95, and in the *Institutio Astronomica*, chapter 16; Descartes, Part 3 of his *Philosophy*, nos. 12 and 152; Martinus Hortensius in the dissertation *De Mercurio sub Sole viso*, p. 31; Antonín Maria Šírek in the *Radius Sidereomysticus*, book 4, c. 2, member 10; and Longomontanus, book 1 of the *Theorica*, at the end, or p. 315, although, besides the Sun's rays and the remnants of daylight, he also adds light shed by other stars and shining back onto the Moon as onto a mirror. Šírek, however, thinks that this phase of secondary light is helped by the shadow of the lunar body, in which it is more clearly distinguished. [Margin: What of Biancani?] Finally, Biancani, in book 9 of the *Sphaera*, chapter 4, does not dare to settle the dispute between the third and fourth opinions.

The foundation of this opinion is the position of the Moon relative to the Earth and the Sun, which is most apt for catching the Sun's rays reflected from the Earth at just the time when this secondary light in the Moon grows brightest. For in the Moon's extreme and first phase it appears greatest, and overcomes the brightness of the twilight and the density of the horizontal vapours; on the 3rd and 4th day before or after new Moon it is greater than on other days more remote from new Moon; and at the quadratures it now vanishes almost wholly and is seen only when the night is well advanced, because rays reflected from the Earth cannot travel towards the Moon. And yet it ought to appear more at that time, because the Moon is farther from the light of twilight. Hence too it comes about that in total eclipses of the Sun this dim light does indeed appear in the Moon, but not as much as outside eclipses, because the interposed Moon intercepts the greatest part of the solar rays from that tract of the Earth from which, once reflected, they could illuminate the Moon. In partial eclipses of the Sun, by contrast, the very brilliance of the Sun dazzles the eyes and so occupies them that they cannot see the Moon's lesser little light. Hence again is given the reason why, the farther the Moon moves from conjunction with the Sun and the nearer it is to the quadratures, the fainter this secondary light appears: namely, because of the greater obliquity of the reflected rays, and therefore their being fewer. To explain this, what we said in book 3, chapter 5, no. 1, can serve, and the diagrams to be given in chapter 8. And it is also the reason why, nevertheless, during the whole time within the last and first quadrature, some of this light is visible in the Moon: because within that time the Moon is

above the horizon of that terrestrial hemisphere in which it is day, and from which the Moon can be struck by the rebounding rays of the Sun.

Against this opinion, however, Scheiner in the *Disquisitiones*, p. 60, and Liceti in *De Lunae luce subobscura*, book 5, chapter 12, and book 2 throughout, but especially chapters 2, 4, 5, 9, 10, 12, and book 3, chapters 9 and 12, hurl not a few objections that are not to be despised at first sight. Galileo in Dialogue 1 of the *Systems*, and Gassendi in the letter to Liceti, suggest solutions to some of them; we shall indicate the rest summarily.

[Margin: Objections against the 4th opinion, and their solution.] [Margin: Argument 1.] 1. To illuminate is to act and to perfect; to be illuminated is to be perfected, to be acted upon and to be altered. But between things that do not have one matter, as the Earth and the Moon do not, there is no mutual alteration, from *De Generatione* 1, t. 1. And higher things, such as the Moon, ought not to be perfected by lower ones, since rather, from *Meteorologica* 1, chapter 2, this lower world is perfected by the higher, and by the celestial bodies. But these points hold at most of an alteration that is not merely perfective, but corruptive, or perfective connoting a lack; for they prove nothing about an alteration merely perfective as to some accident, one not owed to the Moon but possible.

[Margin: Argument 2.] 2. The distance of the Moon from the Earth is, to the senses, the same during the whole time between the last and first quadrature, and yet the Moon's secondary light varies to the senses; therefore this variation must be sought not from the Earth but from elsewhere. But the inference is invalid, because that variation arises not from the distance but from the varying obliquity of the reflected rays.

[Margin: Argument 3.] 3. The Moon's secondary light decreases as the Moon waxes, in the space or quantity of the illuminated disc, not in intensity, since it lasts the same up to the 8th day; and this would not happen if it came from the Earth. But this objection assumes something false. For it does decrease in intensity; but it must be observed in the first phases, when, with the Sun deeper, the light of twilight does not interfere with this light, so that it can be discerned as greater than the light of the following nights.

[Margin: Argument 4.] 4. If this light came from the Earth, the Moon ought to be illuminated more by the Earth at new Moons than the Moon illuminates the Earth at full Moons, both because the Earth is larger than the Moon, and because the Earth is nearer the Sun at conjunction, and so receives stronger and more intense solar rays than the Moon does at opposition; and yet the opposite evidently happens. I for my part concede the major premise when the Moon is not eclipsing the Sun, for in an eclipse it blocks very many of the Sun's rays that would come directly to the Earth. But I deny the minor premise. The Moon's light at these new Moons seems smaller because we view it through solar rays that reach us not by a primary but by a secondary reflection, since they are reflected first from the Earth to the Moon and then from the Moon to us. At full Moon, on the other hand, we see the light through a single, primary

reflection made in the Moon directly to us. Now if we were on the Moon at the time of conjunction, we would see the Earth like some full Moon; but if we were on the Moon at the time of opposition, we would see the Earth illuminated by the Moon, yet endowed with a lesser light than the Moon's secondary light, because the Moon is smaller than the Earth and casts back fewer solar rays onto it than, other things being equal, the Earth casts onto the Moon.

[Margin: Argument 5.] 5. If this light were reflected from the Earth to the Moon, then in a total eclipse of the Sun there would not be that horrible darkness which we know from the history of every age to have occurred. But the reply has already been given: at that time the greatest part of the Sun's direct rays is taken from the Earth by the Moon, and so very few, and almost none, are reflected to the Moon, and those only from the flanks of mountains or the hollows of certain valleys.

[Margin: Argument 6.] 6. The Moon's secondary light is livelier around the margins than in the middle of the lunar disc; but if it came from the Earth, it would be otherwise. But I have already said, at number 3, that this is an illusion of the eye, and that it disappears if one views the secondary light with the Moon's primary light covered.

[Margin: Argument 7.] 7. Venus is sometimes seen by us even near midday, and very bright, and the Earth is illuminated by the Sun no less than Venus is. So if the Moon is illuminated by the Earth, the Moon, although placed near the Sun, could be seen to shine. But Galileo and Gassendi deny the inference, because the light of Venus is primary and drawn directly from the Sun, whereas the secondary light from the Earth onto Venus is extremely faint because of the excessive distance. This light of which we are speaking, by contrast, is in the Moon secondary and mediate. Nor indeed did Galileo concede, as Liceti supposes on p. 255, that this light is as great as the light at full Moon.

[Margin: Argument 8.] 8. Venus has as much power to illuminate the Earth as the Earth has to illuminate the Moon; but neither Venus nor all the stars together, on a moonless night, impart so much light to the Earth as would suffice for reading, or as is the Moon's secondary light. And surely, who would deny that a person placed on the Moon could not read the characters of any book by the strength of this light? But that thesis is denied, both because Venus is far smaller than the Earth, and because she is far more distant. And so the Moon's primary light at full Moon, by reason of its nearness, supplies us with a quantity sufficient for reading, which Venus can hardly do, especially since she is never full at night with respect to us.

[Margin: Argument 9.] 9. If this light came from the Earth, the Earth would have as much power to illuminate the Moon as the Sun has to illuminate Saturn, because this secondary light in the Moon is so great that it rivals Saturn's light, if it is viewed with the primary light shut off behind the roof of some house, as was said at number 3 from Scheiner, whose objection this also is. But I reply by denying that parity. The Sun illuminates Saturn directly, though Saturn is very

remote; the Earth illuminates the Moon mediately, it is true, but the Moon is distant from it by fewer than 100 of its semidiameters, whereas Saturn is distant from the Sun by more than 10000 semidiameters of the Earth. And it is not unfitting that this nearness should compensate for the lack of a light of its own, which we admit in the Earth.

[Margin: Argument 10. Whether the middle region of the air feels the Sun's reflected rays.] 10. The cold of the middle region of the air does not allow the Sun's rays reflected from the Earth to pass beyond it; otherwise their doubling would hinder the formation of snow and hail. Moreover, the power of the Earth, convex as it is in its parts, and especially in the opaque and dark ones, to reflect the Sun's light is less than it was in the concave mirrors of Archimedes, both because these were hollow and better suited to gathering rays into one, and because they were less opaque; and yet the light reflected by them did not exceed a bowshot, as Tzetzes relates. But since the Sun's direct rays certainly pass through the middle region of the air, and yet hinder neither its cold, which arises from aqueous vapours, nor snow or hail, and since the same rays of the Sun reflected from the full Moon to the Earth overcome the cold not only of that region but not even of the night, and do not warm us, we must say that the argument from the absence of heat that burns or melts, or is strongly perceptible, to the absence of light is not valid, especially since so great a quantity and concentration of rays is not required for illuminating as for burning. Moreover, the mirrors of Archimedes were far too small in comparison with the terrestrial hemisphere illuminated by the Sun, which can therefore reflect far more rays to the Moon; and though these are individually faint, together they can produce this secondary light. And if transparency were required, as in mirrors, it would not be lacking in seas, lakes and ponds.

[Margin: The opinion of Fortunio Liceti.] X. The fifth opinion is that of Fortunio Liceti in the book *De Lunae luce subobscura*, most generously presented to me by the author himself. There he teaches that the Moon's secondary light comes both from light reflected from the circumlunar aether and from some native light, which is awakened proportionally by the Sun's rays, as in the Bologna stone called the light-bearing stone or light-drinker [litheoſphoro, seu lucibibo]. When this stone has been baked and, as the chemists say, calcined, then exposed to light and shortly afterwards withdrawn into a dark place, it appears fiery like live coals, and it keeps that appearance for almost half a quarter of an hour, as we ourselves have often found by experiment here in Bologna. So Liceti saw a great analogy between this stone and the Moon in retaining for some time the light received from the Sun, in the very darkness of a lunar eclipse. Further, in order to assert the same of the secondary light near conjunctions, he had to find a way by which this light would be communicated to the part of the Moon turned away from the Sun, or be awakened if already latent in it; and so he took refuge in the aether adjacent to the Moon and reflecting the Sun's rays into that part. But Gassendi objects to this opinion that this denser aether, which makes its own dawn around the Moon, is of very slight height,

and not so great that the parts nearer the silvery light grow brighter than those more remote, since over the whole of the rest of the disc the brightness is of one kind, and the telescope reveals no other difference than the one due to the spots. Second, just as twilight ceases when the Sun is placed about 18° below the horizon, which is a sign that the height of the twilight exhalations is almost imperceptible compared with the bulk of the Earth, so one must reason about that lunar atmosphere: relative to the Moon's depth it is so small that a solar ray cannot be reflected from it onto the whole disc of the Moon.

[Margin: Our opinion.] So, all things reckoned, Liceti's opinion seems to me ingenious indeed, but the third opinion not improbable, and the fourth much more probable; and when that opinion is properly understood, the phenomena of this light are explained in such a way that the objections against it are more easily and more prudently resolved. If, however, the whole Moon were supposed to be kindled by the Sun as the Bologna stone is, and the effect of this kindling to penetrate to its very bottom, then there would be no need to resort to reflection from the circumlunar atmosphere. It could then be said that this faint glow is not seen at the Moon's quadratures or gibbous phases because the excessive brilliance of the part directly illuminated by the Sun predominates in our eyes; and the cause of this faint glow would be one and the same at full Moons and outside them.

De luce Lunæ in Eclipſibus ipſius Totalibus.

On the Moon's light in her own total eclipses.

[Margin: The Moon, even in her own total eclipse, still glows faintly.] XI. Among the antitheses with which Pliny the Elder (book 2, chapter 9) praises the Moon there is also this: "Failing, and yet conspicuous in her failure." That is, because of that baleful, dark light which, taking on different colours according to the Moon's different immersion in the Earth's shadow, "terrifies the eyes of mortals ignorant of the cause." "For," says Pharnaces in Plutarch's essay *On the Face in the Orb of the Moon*, "she does not lie wholly hidden in eclipses, but shines forth with a certain terrible colour resembling a live coal, which is proper to her." Apollonides, refuting this, adds in the same place: "Therefore that the Moon is earthy and dense could hardly be shown by any other equally clear argument, if that coal-like colour were indeed proper to her. But it is not, my Pharnaces, and we see the Moon in eclipse take on one colour at one time and another at another; and the mathematicians distinguish these by the time, as follows. If the Moon fails toward evening, the Moon appears horribly up to the third hour and more than half an hour beyond; if at midnight, then she gives out this crimson and fiery colour; from the seventh hour and a half a red is shown; toward dawn she now takes on a blue, grim face, from which the poets above all, and Empedocles, also call her *glaucopin*, that is, grey-eyed." Nor is it an obstacle that Scheiner says in the *Disquisitiones*, no. 26: "But the Moon, since she is herself devoid of all light, which has often been noticed both by

us and by others in her full eclipses, where, deprived of all brilliance whatever, she grows black." For this is to be understood of the middle of a total eclipse, as appears from other observations, also of Scheiner, to be touched on in Scholia 2 and 3. Further, I find five causes of this light too in the Moon devised by natural philosophers or astronomers, and from them as many headings of opinions.

[Margin: The opinion of Pharnaces, Theon, Cleomedes, Reinhold, Cardano.] XII. The first was that of the ancients, who argued even from this that some light of her own is in the Moon. So indeed Pharnaces in Plutarch, adduced a little earlier, and Reinhold, cited at no. 6; and Theon in the Commentary on Book 4 of Ptolemy's *Almagest*, where he says of the Moon's body: "But since she has obtained a weaker light of her own, which cannot easily reach our sight, she appears more splendid by another light borrowed from the Sun, so as to illuminate the lands and the air. So when the Earth comes between the solar radiations, then the Moon, deprived of the Sun's light and darkened by the cone of shadow, becomes weaker and darker again." But Cleomedes too, in book 2 of his *Cyclical Theory*, acknowledged this proper light in the Moon; and Cardano, in book 3 of *De subtilitate*, says of the Moon: "Whose light does not (as some think) depend wholly on the Sun, and much less does that of the other stars. For it is agreed that in the greatest eclipses she glows red, like some live coal of fire, and this light is then proper to her. But since at night nothing is seen from afar more dimly than fire, and a flame appears red, and since the Moon is so far from us in eclipses, it is plain that the Moon has a light of her own clearer than any flame or candle. Imagine, then, that the Moon, of herself and aided by no rays of the Sun, is very like a very great and very bright flame." Rising by this argument from distance to the nearness of the Moon, he infers: "Therefore above the aether all things shine and flash so much that if, being there, we looked at the Moon even at the time of an eclipse, we would be dazzled by the brilliance, just as with countless most brilliant wax candles lit and our eyes fixed on them."

Against this opinion, however, there still stand both the varieties of colours which this secondary light takes on, as we heard Plutarch's Apollonides rightly arguing against Pharnaces, and the argument of Tycho in volume 1 of the *Progymnasmata*, p. 743, in that he asserts that this light would shine more in the deeper darkness of the Earth's shadow, whereas in fact it vanishes there as the Moon hastens toward the middle of the eclipse, or can be discerned only with the greatest difficulty.

[Margin: The opinion of Longomontanus, Aversa, Mastri, Belluto and, in part, Gassendi.] The second opinion assigns this light to the fixed stars, or also to some planets, if they stand in a place suitable for it. So indeed Longomontanus in the *Astronomia Danica*, book 1 of the *Theorica*, chapter 9; Aversa [Raffaele Aversa], *De caelo*, q. 32, sect. 2; Mastri and Belluto [Bonaventura Belluto], *De caelo*, disp. 2, q. 5. Gassendi does not deny the same of that faint glow which is scarcely discerned in the middle of a total eclipse, for on the rest he follows the opinion to be reported in fifth place. But against this stands that variety of

colours mentioned above, and the emptying away of this faint glow around the middle, where nevertheless this light ought to have appeared better; since in a clear sky there is no lack of very many fixed stars so placed opposite that they could illuminate the Moon with their rays even at the middle and centre of the Earth's shadow; and sometimes the superior planets are placed opposite together with Venus, and these, with the Earth's shadow no obstacle at all, can direct their rays onto the Moon. But if the faint glow on the Moon, placed so far off, comes from the fixed stars and is visible to us, why do the fixed stars on a moonless night not likewise illuminate papers or clothes near us?

[Margin: The opinion of Galileo.] The third opinion is that of Galileo in the *Sidereus Nuncius*, p. 15, and in the letter which Liceti reproduces in *De Lunae luce subobscura*, p. 262: that this light or coppery redness, which appears around the extremities of the Earth's shadow when the Moon is eclipsed, comes from the Sun's rays illuminating that denser aether which creates an atmosphere around the Moon, and from it pours a kind of dawn into the regions near the Moon. But here Kepler doubts, in the *Dissertatio cum Nuncio Sidereo*, p. 15, whether this dawn, being uniform, could account for this redness of the Moon, so inconstant and of various colours. Further, when the Moon is already wholly in the Earth's shadow, this redness appears also in the parts of the Moon tending toward the shadow's axis, yet the Sun's direct rays do not reach those parts. And if you resort to refracted rays, you will already be running into the fifth opinion, of which presently.

[Margin: The opinion of Wendelen and Liceti.] The fourth opinion is that of Fortunio Liceti in the treatise *De Lunae luce subobscura*, book 2, chapter 6, and book 3, chapters 4 and 11, and of Wendelen in the *Idea of the Atlantic Tables*, p. 5, where they attribute this light partly to the Sun's light reflected from the circumlunar air, partly to the Moon's native light, which is however awakened and brought into second act by the Sun, and preserved in darkness in the way the Bologna stone preserves for a while the light kindled in it by the Sun. But what we have just said against Galileo about light from the circumlunar air holds here too; and the remaining cause I cannot approve, because this light of the Moon which we are disputing gradually weakens from the beginning to the middle of a total eclipse, but shortly after the middle it generally revives and increases as the Moon hastens to emersion, and it is always greater toward the parts of the shadow at both extremes, and of different colours. No sufficient cause is offered for these variations if the Moon retains light conceived from the Sun; for from where does it rekindle that light, while not yet emerging from the shadow?

[Margin: The opinion of Apollonides, Giovanni Battista Benedetti, Gassendi, Kepler and ourselves.] So the fifth opinion, far more probable to my mind, is that this redness, or the other colours of this secondary light, comes from the Sun's rays, but rays twice refracted in the vapours of the air surrounding the Earth's globe: refracted, I say, as the convexity of the atmosphere requires, once on entering and again on leaving, toward the perpendicular and the axis of

the cone of the Earth's shadow. For this is why this light is greater in the parts of the Moon toward the margins of the shadow than toward the axis: there are more refracted rays there. And it varies its colours because the vapours around the Earth, through which the Sun's rays pass, are of different opacity and diaphaneity, from whose mixture with light we also see different colours in the dawn and in clouds illuminated by the Sun. [Margin: You have a diagram of this double refraction below, in book 5.] Finally, this light vanishes in the middle of a total eclipse or near the middle, as we shall say in Scholium 4, because the Moon is so low and so deeply sunk in the shadow, or the vapours so thick, that either almost no rays or only greatly weakened ones are refracted to that part. This opinion was first taught me by Apollonides in Plutarch's essay *On the Face in the Orb of the Moon*, in these words: "Therefore, since we see the Moon in the shadow take on so many colours, they are not right to seize on the coal-like one, which one might most of all say is foreign to her, and rather a mixture of light failing and shining through the shadow, etc." Next, Giovanni Battista Benedetti in the letter to Philibertus Pignonius [?] of Savoy, in my copy p. 255; Pierre Gassendi in the book *De apparenti magnitudine Solis*, letter 2, p. 97; and best of all Kepler in the *Astronomia Optica*, the whole of chapter 7, and in the *Epitome of Copernican Astronomy*, p. 871, and in the *Dissertatio cum Nuncio Sidereo*, p. 20.

[Margin: The cause of the variety of colours in eclipses.] From this you see what judgement is to be passed on the causes of the variety of colours in lunar eclipses that are thrust on us either by Pharnaces in Plutarch, already cited near the beginning at no. 11, or by the Arabs, and following them Cardano in the *Supplementum Almanach* and in the commentary on Ptolemy's *Quadripartitum*, and by the Alphonsine astronomers, partly from the Arabs, partly from Ioannes de Lineriis [Jean de Lignières] at the end of Proposition 32, and from there Magini in canon 89 of the *Secunda Mobilia*. Some derive them from evening or morning time, others from the Moon's approach to apogee or perigee, others from the Moon's varying latitude and nearness to the nodes. For the chief cause, as I have said, is the variety of the vapours encircling the Earth's globe, through which the Sun's refracted rays are coloured. Granted that the differing state of the morning, evening, summer, winter, spring and autumn air contributes to this variety of vapours; and that the place and position of the Moon undoubtedly contributes to receiving the different refracted rays. I say nothing of the prognostics from these colours among the astrologers, for instance Origanus, volume 1 of the *Ephemerides*, part 3, chapter 2, p. 463, and Wendelen, p. 4.

The fringes on the Moon's limb, on which Wendelen writes on p. 6, also belong here.

We shall give figures of the refraction of the rays in a lunar eclipse in book 5, chapter 5, and again in volume 2 of the *Almagest*, in its optical book; and there we shall also resolve what our Bettini objected to Kepler in *Apiarium* 8, Prog. 2, from Prop. 5 to Prop. 9.

SCHOLIA.

Scholia.

[Margin: Solar eclipses with a bright ring around the Moon.] I. I call eclipses of the Sun annular when around the Moon there is left a small circle or a kind of bright ring, that is, a segment of the solar disc not entirely covered by the Moon. Many authors admit eclipses of this kind, and I shall point out examples of them in chapter 16, no. 16, and more fully in book 5, chapters 19 and 20, where they will be treated.

[Margin: The Moon's colour and light in an eclipse of the Sun.] II. It is established that the Moon herself has at times been seen in an eclipse of the Sun, certainly in a black colour if not in another. Granted that Pliny the Elder, book 2, chapter 9, says of a non-eclipsing conjunction: "In conjunction she is indeed not seen, because, turned away, she sends back all the light she has drawn to the place from which she received it." For Longomontanus, in book 1 of the *Theorica*, chapter 9, p. 315, states that in the eclipse of the Sun of 1605 September 23, at Copenhagen, he observed the Moon with a blackish face. And Scheiner, in *Apelles post tabulam*, p. 49, states that on 1612 May 29 he and his companions saw the portion of the Moon that was covering the Sun wholly transparent like crystal or glass, though unevenly, so that in some places it was wholly white and in others was turning white. From this he infers that the Moon is transparent throughout, more or less according to its greater or lesser density. Kepler, in the book *De Stella Nova*, chapter 23, reports that in 1605, in a total eclipse of the Sun, the Moon was seen like a black cloud, but at Antwerp dusky and as if covered with soot, while its upper circumference was wholly white and as if fiery. I myself, on 1630 June 10, saw her swarthy and of a colour mixed from black and crimson. This redness Kepler, in the *Epitome of Copernican Astronomy*, p. 895, attributes to the Sun's rays reflected from the Earth onto the Moon, from that part of the Earth which is not covered by the Moon's shadow.

[Margin: The colours of a lunar eclipse.] III. As regards total eclipses of the Moon and her various colours, Frodoardus [Flodoard of Reims], in his *Chronicon* for 926 April 1, relates that the Moon, undergoing eclipse, turned to pallor, with a small particle of light left, and, as dawn was already beginning, changed wholly to a blood-red colour. And Cornelius Gemma, in book 2 of the *Cosmocritice*, p. 64, says that on 1569 March 3, in the morning at the third hour, Phoebe suffered a horrible eclipse, marked by dreadful colours. For first she shone dusky, then blood-red, soon also crimson, and green, and livid, and at last misshapen by an incredible variety. That is, as Kepler says on p. 274 of the *Optica*, almost with the colours of the rainbow. He adds that when he was a boy in Württemberg, in 1580, at the 10th hour of the night, the Moon in eclipse appeared to him plainly ruddy, and that he saw the same redness in the eclipse of 1599 February 9 and in 1603 on November 18; but in 1598, on August

16, so much of it that the Moon could hardly be believed eclipsed, although she was wholly in the shadow.

[Margin: The Moon extinguished in the middle of an eclipse.] IV. In the middle of a lunar eclipse, however, and in the deeper shadow, the Moon is sometimes barely visible, and to some people more, less, or not at all, according to the variety of the air and of the vapours. This is attested by the eclipses of 1580 and of 1583 March 3, observed by Kepler (from the *Astronomia Optica*, p. 227), and of 1588 by Tycho, and of 1620 June 15 by Wendelen. But above all by the two total lunar eclipses of 1642. For in the one that occurred on April 14, the Moon was seen at Venice by the most illustrious Iulius Iustinianus [Giulio Giustiniani] as of a red colour and like the light of Mars, as I gather from his letter copied to me through Fortunio Liceti; and Doctor Bonuicinus [?] and Master Pieronus [?] saw the same at Vienna at that time. Yet no one in Belgium, as Wendelen testifies on p. 4, nor we with many companions at Bologna, nor Vincenzo Renieri at Pisa, although the sky was clear, could make out the Moon at all in the depth of the shadow, and we searched for her in vain in the sky for almost one hour. On the contrary, in the one that followed on October 7, we saw the Moon crimson almost up to the middle, and pale in the middle. You will wonder less at all this, however, if you first understand what will be said below in book 5, chapter 5, about the refractions that curtail the Earth's shadow more or less, or even dilute it.

CAPVT VII.

Chapter 7.

De Selenographia, ſeu Lunaris faciei Teleſcopijs obſeruatæ Deſcriptione accurata.

On selenography, or the accurate description of the Moon's face as observed by telescopes.

I. What Endymion once noted in the spots of the Moon's face so beautiful that he judged it worthy of his love and of the contemplation of many years has not been handed down. Nor, in Plutarch's most learned essay *On the Face in the Orb of the Moon*, is there any treatment of spots other than the ancient ones, visible to the naked eye. [Margin: Authors of selenography.] But when Galileo first, and Scheiner, noticed new spots, or varieties of parts, with the recently invented telescopes, then Caesar Lagalla [Giulio Cesare Lagalla] also published a not inelegant disputation on them, entitled *De Phaenomenis in Orbe Lunae*. But neither the four figures of the Moon at dichotomy which he used, taken from Galileo, nor Scheiner's figure, likewise of the Moon at dichotomy, which is found in the *Disquisitiones Mathematicae*, nor the figures of the same Moon observed with telescopes [tubospicillis], published by Francesco Fontana of Naples and copied by Argoli into the *Pandosion Sphaericum*, are of such size or care that, beyond a certain first sketch of a new phenomenon, they offer the reader the matter of any complete discussion. The figure of

the Moon set out by Antonín Maria Šírek the Capuchin almost at the very beginning of the *Oculus Enoch et Eliae* is somewhat larger. But all these are surpassed by the selenographic figures, first of Michael Florent van Langren and Johannes Hevelius, then of Eustachius de Divinis [Eustachio Divini] and Father Hieronymus Sirsalis [Girolamo Sersale] of the Society of Jesus.

[Margin: Praise of Langren and Hevelius.] For Langren, long since Cosmographer to the Catholic King, began both at Madrid and at Brussels and elsewhere, with a very large and excellent telescope, to observe one by one even the smallest spots of the Moon, and especially those through which the Moon's section has been seen to pass, whether at dichotomies or in eclipses, and many other things most worthy of note. He distributed these into thirty figures, and large ones at that, engraved in copper with his own hands, with a complete work added, although he has not yet published anything except the figure of the full Moon and certain first leaves of the work itself, in 1645; he generously sent me a copy of these with many other observations. From these, compared with the observations of Hevelius, one may gather that Langren was much earlier in this labour; although Hevelius was more fortunate in bringing forth the whole work of his *Selenographia* and setting it before the public, with a multitude and elegance of figures likewise engraved by his own hand, and an abundance of learning; that work too appeared in 1645, to the great increase of astronomy.

[Margin: The difference between the selenographies of Langren and Hevelius in nomenclature.] They differ, however, both in other things and also in the nomenclature of the parts of the Moon. For Langren used the names of persons distinguished in this age either in mathematics or in ecclesiastical or secular dignity, or joined to him in friendship. The regions and great tracts of the lunar surface, on the other hand, he named with a kind of moral or civil nomenclature. So in him you may read the Land of Dignity, the Land of Virtue, the Philippic Ocean, the Copernican Sea [Terram Dignitatis, Terram Virtutis, Oceanum Philippicum, Mare Copernicanum], etc. Hevelius, however, as though the Moon were another Earth, transferred the geographical names of our Earth to the Moon, although in shape, position, symmetry, etc. there is almost no analogy between the surfaces of the two.

[Margin: The figures of the full Moon by Divini and Sirsalis.] Finally, Divini published a figure of a single full Moon, of 1649 March 28, observed by him with a telescope of 24 palms; and very recently Father Hieronymus Sirsalis of our Society likewise made public another diagram of a full Moon, of 1650 July 13, observed by him with Divini's telescope, with no explanation and no nomenclature of the parts added.

II. As for us, although we once contemplated the Moon's face with optical tubes and had certain diagrams cut in woodblocks, yet after it was granted us to look at the figures of Langren and Hevelius, and concerning the libration of the lunar

[Margin: The beginnings of our Selenography.] globe a rumour had spread, then indeed our spirit was fired to determine, by the most painstaking observations, whether the diagrams already mentioned agreed with the truth in all their details, and what the limits and the periods of this apparent libration were. For this task, besides the telescopes of Galileo, Fontana, Torricelli and Manzini, either given or lent to us, another, sold to us by a Bavarian craftsman, served best of all. It did so not so much by its length, although that is 15 feet, as by the coupling of its lenses, both convex, so happily associated that, although they show the whole disc of the Moon at one time, at least at apogee, they nevertheless so enlarge it and each of its portions that one can make out in them even the smallest details of even small parts, which we noticed had been either not seen or neglected by others.

Since I had long been sure of the diligence, prudence and trustworthiness of Father Francesco Maria Grimaldi in observations of this kind, and since he had more time and health left for it than I had, I unloaded the greatest part of this labour onto him. [Margin: The excellence of Grimaldi's Selenography.] So, having begun the observations afresh, he kept watch for several years with the greatest circumspection and assiduity in contemplating the Moon, a new but happier Endymion of our age. Examining all the large, middling and smallest parts of the Moon's face one by one through the telescope, and comparing them at once with the diagrams of Langren and Hevelius, which he had made a habit of keeping before him, he found that many things had indeed been done excellently by them, yet that not a few remained which had either to be added or to be corrected as to position, size, shape, proportion, and difference of darkness or brightness. He therefore drew new diagrams for himself, one after another, each better than those before it, and did not rest until he seemed to himself to have attained the evidence of the truth for even the smallest marks of the Moon's face. As for the limits, regions and periods of the libration, he has observed so much up to this day that a full volume could be written from it.

Therefore, although he acknowledges that much is owed to Langren, Hevelius, Eustachio and the other selenographers, freely grants them priority in time, and perhaps could not have attempted or accomplished so much had they not gone before, he has nevertheless brought so much of his own skill, trustworthiness and diligence to this study that he cannot set the evidence of his own eyes below what others have handed down. Nor do I, who together with many of our Society was repeatedly present at his observations and admired in him a prudence equal to his patience, think that anything more perfect of this kind can be either desired or hoped for (unless you except the great number of small spots, which seem almost infinite, and the excessive frequency of certain very faint shadings, deliberately neglected to avoid confusion). Descartes and others have judged the same of our Scheiner, too, in the matter of the sunspots he observed.

Would that, just as Father Grimaldi himself drew his diagrams with truly Daedalian fineness and elegance, he had found an engraver on the spot who

had used a more delicate stylus and a gentler hand, incising the copper rather than ploughing it, and had thinned the blackness here and there. For the rest the engraver represented correctly, and the last impressions, which came out when the plate was already worn from repeated printing, show, as regards the difference of dark and light, one more like the Moon's.

III. [Margin: The selection of diagrams for the Selenography.] Now from so many drawings and observations it was decided to choose only six, the others being reserved for a separate volume on this subject. The first, for the whole disc of the Moon, is composed from the more frequent phases, selected so as to represent as much as possible; the second is for the sickle Moon; the third for the gibbous Moon; the fourth and fifth for two dichotomies, such as appear not always, but at the extreme limits of the librations. For if all the variety of the Moon's face had to be drawn in separate diagrams, not even 300 diagrams of phases would suffice, much less (as Langren believed) 30, since the Moon scarcely ever turns exactly the same face to us, or, if she does offer the same one after long intervals, a favourable night and clear skies scarcely ever allow the observer to see it. So great, clearly, is the diversity of this phenomenon, which the Moon's apogee and perigee, the alternation of her latitudes and, most of all, the anomaly of the apparent libration display to mortals in this theatre of divine workmanship. For the libration, and at the same time for the nomenclature of the Moon's parts, we have added a sixth diagram, similar in most respects to the first, which carries the four smaller figures at its corners. These will be explained below, after the figures have been shown.

IV. [Margin: Where the nomenclature of the lunar parts was taken from.] Moreover, I did not take the nomenclature of the lunar parts from the geography of our Earth, as Hevelius did, because of the lack of analogy already pointed out. I took it rather from persons, not of just any standing, nor distributed indiscriminately or as if by chance, as may be seen in Langren's full Moon, but only from those who either were astronomers rather than genethliac astrologers, or from whose older writings or observations some astronomical learning has flowed into our Almagest. We declare, however, that we utterly reject the error of those who have imagined either that the Moon is inhabited by other men, or that the souls of heroes, or even of other mortals, migrate after death into different abodes in the Moon according to their different merits.

As for the regions of the lunar surface that resemble either continents or seas, lakes, marshes and so on, I did not name them from the species of mathematics, as I wished to do (for that would have stood in the way of a more convenient nomenclature of the smaller parts), but from that diversity of effects, mostly meteorological, on which the Moon, most of all the stars, is believed to exert her influence, because of her closeness to the Earth and the multiplicity of her motions and of her parts. Although in this I followed some analogy, almost by mere choice, I am not imbued with any such opinion that I would suspect, much less believe, that those effects in particular from which I drew the naming depend on that portion of the Moon, for instance snows on the Land of Snows

or dreams on the Marsh of Dreams, whether when the boundary of illumination reaches them, or when they are irradiated by other stars in certain aspects. For although great mysteries of nature certainly lie hidden in this treasury, we are nevertheless not permitted, by settling on fictions about them, imprudently to abuse the contemplation of the divine works, which has been granted to us within fixed limits.

V. [Margin: Its distribution was not made by chance.] I said that I did not distribute the names already mentioned over the Moon's disc by chance. In about the upper part I placed the more ancient: in about the first and second octants most of the physico-astronomers, and in the third and fourth, and as far as the beginning of the fifth and sixth, the rest of the ancients. In the lower part, which comprises about the fifth, sixth, seventh and eighth octants, I placed the more recent. Moreover, as far as could be done, I aimed at affinity or likeness of period, of studies or of school, or at correspondences of this kind. For example, to Meton I joined Euctemon, as his partner in observations; to Plato his intimates Theaetetus, Timaeus and Archytas, and I placed Aristotle not far from there. Likewise I set Callippus next to Eudoxus, Mercury to Atlas, Sosigenes to Caesar, Ptolemy to Hipparchus, al-Battānī to him, Alfonso to al-Battānī, al-Zarqālī and Thābit ibn Qurra to him, Peuerbach to these, and directly after, Regiomontanus, Walther and Werner. To Tycho, likewise, I set beside him William of Hesse, Hainzel, Sasserides [Gellius Sascerides] and Longomontanus; just as to Copernicus I set his own Rheticus, Mästlin, Reinhold and most of the others of that school, whom I also deliberately preferred to be tossed in the Stormy Ocean, like floating islands, because of the instability of the Earth they asserted, rather than to stand firm. And so in the other cases, as the learned reader may consider, I aimed at not a few analogies.

The Selenographic Figures follow, to be inserted immediately after this page 204, not sewn in but glued together, or rather enclosed in such a way that they can be drawn out by the reader and surveyed at a single glance.

Blank page (back of the plate that follows).

Diagram. Engraved plate, the sixth Selenographic figure: *VI. Figure for the nomenclature and the lunar libration.* Two mottoes stand above the disc: *Neither do men inhabit the Moon* at the left, *Nor do souls migrate to the Moon* at the right. The Moon's disc is drawn in hatched relief within a double rim. Four straight lines cross at the point A near the centre and divide the disc into eight octants, each named outside the rim, and the ends of the lines carry letters. Seas, marshes, bays, peninsulas and lands are named in larger letters, and the spots with the names of persons in small letters. A scale runs along the foot of the plate from X to Z, numbered from 10 to 200.

Labels as engraved, by region of the plate:

Upper band, left of centre

- VI. FIGVRA PRO NOMENCLATVRA, ET (title, running on into the next region)
- Nec Homines Lunam incolunt.

- II. OCTANS
- I. OCTANS
- P
- Anaximen.
- Anaxagoras
- Euctemon
- Meton
- Philolaus
- Cleoſtratus
- Pythagoras
- Anaximander
- Epigenes
- SICCITATIS
- TERRA
- Xenophanes
- Oenopides
- Harpalus
- Sinus Roris
- PRVINÆ
- Mare Frigoris
- Plato
- Timæus
- Architas
- Terra Grand (cut in this region; see the region below and to the right)
- Sinus Iridû

Upper band, right of centre

- LIBRATIONE LVNARI (end of the title)
- Nec Animę in Lunam migrant.
- III. OCTANS
- Limes Boreus
- B
- G
- Stagnum Glaciei
- Zoroaſter
- Democrit.
- Thales
- Endymion
- TERRA (spaced along the rim)
- Hercules
- Atlas
- Hermes
- Oſymandies
- Mercurius
- Ariſtotel.

- Eudoxus

Upper middle band, left of centre

- p[?]cum (along the rim above the left limb; reading of the second letter unsure, and no letter seen before the p)
- Nodus Orient
- E
- L
- K
- Littus Ecl[?]
- VIII. OCTANS
- TERRA (in the bay beside Sinus Iridû)
- Helicon
- Heraclides
- Ecphantus
- O C E A N V S
- Mare Imbrium
- Palus Nebu / larum
- Archimed.
- Ariſtillus
- Autolicus
- Aratus
- Timocharis
- Palus Putredinis
- Pitheas
- Ariſtarchus
- Seleucus
- Cardanus
- P R O
- C E L
- L A R V M
- Beſſarion
- Cuſanus
- Marius
- Galilæus
- Caualerius
- Reinerus
- Heuelius
- Inſula ventorū
- Keplerus
- Milichius
- Copernicus
- Dom. Maria
- Rheticus
- Stadius

- Dantes
- Hortens.
- Reinhold
- Lansbergius
- Conon
- Eratoſthenes
- Terra Niuiū
- Sinus Æſtuum[?] (over dark hatching)
- Palus Nimborū
- A (point letter at the centre of the disc)

Upper middle band, right of centre

- Terra Grandinis
- Lacus Mortis
- Calippus
- Theætet.[?]
- Lacus Somniorū
- Poſſidonius
- Cepheus
- Meſshala
- Geminus
- Beroſus
- R
- Seneca
- Plutarch.
- Cleomedes
- Macrob.
- Mare Criſium (in two lines, Mare / Criſium)
- Palus Somni
- Proclus
- Firmicus
- VITÆ (large spaced letters)
- IIII. OCTANS
- Mare Serenitatis
- Sulpic. Gal.
- Menelaus
- Vitruuius
- Plinius
- Manilius
- Mare Vaporum
- Iul. Cæſar
- Higinus
- Agrippa
- Soſigen.
- Mare Tranquilitatis

- Taruntius
- Ariadæus
- TERRA (spaced, right of the centre point)
- Hipparchus

Lower middle band, left of centre

- RICCIOLVS
- GRIMALDVS
- T E R / R A (large, in two parts either side of the disc's edge features)
- Rocca
- Euſtachius
- Billy
- Sirſalis
- Crugerus
- Eichſtadius
- S
- Peninſula / Fulminū (Peninſula above, Fulminū below)
- Deriēnes
- Fontana
- Zupus
- Gaſſendus
- Herigonius
- Morinus
- Mare / Humorum
- Merſenius
- Byrgius
- Vieta
- Linemannus
- Moletius
- Peninſula / Deliriorum
- Origanus
- Iunctinus
- Mare Nubium
- Peninſula / Fulgurū
- F
- N (large sort, inside the ring about the central point)
- V (large sort, beside the ring)
- Sinus Epidemiarū
- Bulliadus
- Munoſius
- Profatius
- Alpetragius
- Alphonſus
- Arzachel
- Thebit

- Ptolemæus
- Albategn.
- Blanchinus
- Vernerus
- Purbachius
- Regiomontan.
- Pitatus
- Gauricus
- Apianus
- Aliacenſis
- Valtherus
- Fernelius
- Campanus
- Capuanus
- Cichus
- Rothmanus
- Saſſerides
- Gulielm. Haſſ.
- TER (large)
- Tycho
- Orontius
- Maginus
- Stœfler
- Kriſtmanus
- Schikardus
- Hainzelius
- Phocylides
- C A L O / R I S (large)
- T E R R A S T (large along the rim, running on into the band below)
- O C T A N S (upside down at the lower rim)

Lower middle band, right of centre

- Theon ſen.
- Theon iun.
- S A N I (large)
- T A T I S (large)
- S. Dionyſ. Areop.
- Cenſorinus
- Beda
- Exiguus
- Alcuin
- Hypatia
- Terra
- Mare (large, above Nectaris)
- S. Theophil.

- Alfraganus
- Abilfedea
- Almæon
- Geber
- Abenezra
- Azophi
- Sacroboſc.
- Tahus[23]
- S. Cyrill.
- S. Catharina
- Nectaris
- R A (large)
- Pontanus
- Zagut
- Rab. Leui
- Riccius
- Stiborius
- Neãder
- Reitha
- Metius
- Mulerius
- Fabritius
- Maurolycus
- Barocius
- Gemma Friſ.
- Nonius
- Fracaſtor.
- Piccolomin.
- S. Iſidorus
- Mart. Capella
- Manna
- MARE
- F E C V N D I
- T A T I S
- Goclenius
- Langrenus
- Santbec.
- Vendelin.
- Petauius
- Steuinus
- Snellius
- Furnerius

[23]the third sort is h-shaped; this plate engraves ti in the same joined form in Rheticus and Moletius, and the printed list on p. 276 reads Tatius

- Terra Vigoris
- C
- M
- Nodus Occident.
- H
- Terra … VI[?] … goris (three pieces along the rim from the lower edge up to H; the middle piece is unsure between VI and Vi, and no letter was added between the pieces)
- V. OCTANS (upside down at the lower rim)

Lower band

- X (left end of the scale)
- 10, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200 (scale numbers)
- Z (right end of the scale)
- VII. OCTANS (upside down at the lower rim)
- VI. OCTANS (upside down at the lower rim)
- S T E R I L I T A T I S (along the rim, continuing T E R R A from the band above; the final S is small)
- D
- I
- Limes Auſtr.
- Bayerus
- Schillerus
- Zucchius
- Longomontan.
- Scheiner
- Blancan.
- Clauius
- Gruemberger
- Cyſatus
- Curtius
- Moretus
- Bettinus
- Kircher
- Bartolus
- Caſatus
- Cabeus
- Malapertius
- Schomberger
- Simpelius
- Manzinus
- Tannerus
- Mutus
- Licetus
- Lilij

- Homelius
- Pitiſcus
- Hagecius
- Claramontius
- F E R (large)
- T I L I (large)
- T A T I S (large)
- Q (large, with a swash, at the lower end of the line)

Blank back of a plate leaf: nothing is printed on it, and only the engraving on the other side shows through.

Blank back of a plate leaf: nothing is printed on it, and only the engraving on the other side shows through.

Diagram. Engraved fold-out plate, taken across the full opening. A title stands at the top centre, in two sizes of italic. Beneath it the upper edge of the plate shows three shaded lunar discs side by side: a large disc in the middle, and to either side a smaller figure with a ragged edge facing the centre.

Title as engraved (upper band of the plate):

- SELENOGRAPHIA P. FRANCISCI MARIÆ GRIMALDI SOC. IESV
- Optimo Telescopio ex plurib. Lunę phasib. Selecta, in qua Langreni, Heuelij, Euſtachij, Sirsalis etc. Selenogr. partim firmauit, partim ita
- correxit, et auxit, ut uel minimę particulę ex aliquib. phasibus euidentiam sit assecutus.

Selenography of Father Francesco Maria Grimaldi of the Society of Jesus, chosen with an excellent telescope from many phases of the Moon, in which he has partly confirmed the selenographies of Langren, Hevelius, Eustachio, Sirsalis, etc., and has partly corrected and enlarged them, so that he has secured a clear view of even the smallest particle from some of the phases.

Upper middle band of the plate. The large central disc is cut by no engraved name; its seas are shaded in horizontal hatching and its craters drawn with small shaded rings. The two side figures are partial phases, each with its ragged lit edge turned towards the centre.

- 5 Figª (below the left figure, under its lower limb)
- 2. Figª. (below the right figure, under its lower limb)

Lower middle band of the plate. The central disc continues, its lower half thick with ringed craters; a large rayed crater stands near its lower limb, with long fine streaks running from it across the seas. Two further partial phases stand below the side figures of the upper half, so that a phase figure lies at each corner of the plate.

- 4 Figª (above the lower left figure, over its upper limb)
- 3. Figª. (above the lower right figure, over its upper limb)

Lower band of the plate. Below the discs, inside the plate mark, stand a draughtsman's line at the left, a caption at the centre and an engraver's line at the right.

- Delineauit ipse P. Grimaldus. (lower left, with a swash D)

- I. Figura inserenda Almageſto Nouo P. Io: Bapt. Riccioli (centre)
- Statim poſt 1. tomi pag. 204. (centre, second line)
- Quę non eſt Plenilunij, sed ex plurib. phasib. conſtructa (centre, third line, smaller)
- Dominicus Fontana Sculp. Bononię (lower right)
- añ o 1651. (lower right, second line)

Father Grimaldi drew this himself.

Figure I, to be inserted in the New Almagest of Father Giovanni Battista Riccioli, immediately after p. 204 of Tome 1. It is not a figure of the full Moon, but is constructed from many phases.

Dominicus Fontana engraved this at Bologna, in the year 1651.

The figures that follow, although drawn from the rudiments of the earliest observations, are nevertheless not useless for confirming the roughnesses of the Moon and the light reflected from the Earth onto the dark part of the Moon.

Diagram. *Figure 7. For the 2nd or 3rd day of the Moon.* A woodcut disc of the Moon within a single rim. Nearly the whole disc is dark and closely hatched, with the lunar spots faintly visible through the hatching, and a thin lit crescent runs along the right limb from the top round to the foot, broken by dark oval spots and small ringed craters. The caption is the only lettering; no letters or numerals could be seen on the block.

Diagram. *Figure 8. For about the 5th day of the Moon.* A woodcut disc of the same kind, with the crescent broader than in Figure 7. Large dark spots stand in the lit limb at the upper right, and a row of ringed craters follows the lit edge along the lower right. The rest of the disc is dark and hatched. The caption is the only lettering; no letters or numerals could be seen on the block.

Diagram. *Figure 9. For the gibbous Moon.* A woodcut disc lit over its left part and dark at the right. The lit part shows hatched seas, ringed craters, ray-craters with radiating lines, and clustered rounded spots towards the foot. The boundary with the dark part is irregular. The caption is the only lettering; no letters or numerals could be seen on the block.

Diagram. *Figure 10. For about the 5th day after the full Moon.* A woodcut disc with its left part lit and its right part dark, as in Figure 9. The boundary is sharply toothed, with craters ranged along it, and hatched seas, ray-craters and clustered rounded spots stand in the lit part. The caption is the only lettering; no letters or numerals could be seen on the block.

NOMENCLATVRA LVNARIVM PARTIVM

Nomenclature of the Parts of the Moon

For the Selenography of Father Francesco Maria Grimaldi and for its Figures 1 and 6, constructed by the author of the New Almagest *and explained by him in Book 4, Chapter 7, No. 4. The synonyms added from the Selenography of Langren are indicated in advance by the letter L, and those from that of Hevelius by the letter H.*

I. OCTANS	Anaximenes	Plato	Menelaus	L. *Langrenus*	Fabricius
Anaximander	Aratus	L. *Panciroli lacus*	L. *Maria Imp.*	Macrobius	Fracaſtorius
Ariſtarchus	Archimedes	H. *Lacus Niger*	Mercurius	Plinius	Furnerius *Soc. I.*
L. *Balthaſſar*	L. *Roma*	Thales	Meſſala Arabs	Plutarchus	Geber
Cleoſtratus	H. *Corſica*	L. *Xenophanes*	Oſymandies	Proclus	Hipparchus
Ecphantus	Archytas	Theætetus	Poſſidonius	L. *Puteanus*	Hypatia
Eratoſthenes	Ariſtillus	Timæus	L. *Lafailli*	Seneca	S. Iſidorus Hiſp.
L. *Haro*	Ariſtoteles	3. OCTANS.	Sulpicius Gallus	Soſigenes	Rab. Leui
Harpalus	L. *Brahe*	Atlas	Zoroaſter	Taruntius	Mart. Capella
Helicon Cyzicen[9]	Autolycus	Beroſus	4. OCTANS.	Vitruuius	Metius
Heraclides Pōticus	Calippus	Cepheus	Agrippa	5. OCTANS.	Mulerius
Oenopides	Conon	Endymion	Alcuinus	Abenezra	Neander
Pitheas Maſſil.	Democri-tus	L. *Endymion*	Ariadæus	Abilfedea	Petauius *Soc. I.*
Pythagoras	Epigenes	Geminus	Beda	Alfraganus	Pontanus
L. *Pythagoras*	Euctemon	Hercules	Cleomedes	Almæon	Piccolomineus
Timocharis	Eudoxus	Hermes	Dionyſius Exiguus	Azophi	Reitha
L. *Gaſſendus*	L. *Pozzo*	Hyginus	Firmicus	S. Catharina	Riccius
Xenophanes	Meton	Manilius	Goclenius	L. *Piccolomin.*	Sacrobuſcus
2. OCTANS.	L. *Amalfi*	L. *Iſabella R. Hiſp.*	Iul. Cæſar	S. Cyrillus Alex.	Santbechius
Anaxagoras	Philolaus	H. *Inſula Besbicus*	Langrenus	S. Dionyſ. Areop.	Snellius

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Steuinus	Blanchinus	Malapercius *Soc.I.*	Vernerus	Morinus	Egnatius Dantes
Stiborius	Cabæus *Soc. I.*	Maurolycus	Zucchius *Soc.I.*	Munoſius	Euſtachius
Tatius	Clauius *Soc. I.*	L. *Eſtenſis D.*	7. OCTANS.	Origanus	Galilæus
Theon *Senior*	L. *Maximilianus*	Moretus *Soc.I.*	Bayerus	Phocylides	Grimaldus *Soc. I.*
Theon *Iunior*	Curtius *Soc.I.*	Mutus	Bullialdus	Profatius	L. *Lacus Poſſidonij*

[24] The names are given as printed in the list; the forms engraved on the map differ from them in places. OCTANS is "octant". The expansions that follow are supplied, not printed: Helicon Cyzicen[9], Helicon of Cyzicus; Heraclides Pōticus, Heraclides Ponticus; Pitheas Maſſil., Pytheas of Massalia; Panciroli lacus, Pancirolo's Lake; Lacus Niger, the Black Lake; Inſula Besbicus, the Island Besbicus; Iſabella R. Hiſp., Queen Isabella of Spain; Maria Imp., the Empress Maria; Iul. Cæſar, Julius Caesar; S., Saint; S. Cyrillus Alex., St Cyril of Alexandria; S. Dionyſ. Areop., St Dionysius the Areopagite; S. Iſidorus Hiſp., St Isidore of Spain; Rab. Leui, Rabbi Levi, that is Gersonides; Mart. Capella, Martianus Capella; Soc. I., of the Society of Jesus; L. Piccolomin., Piccolomini.

S. Theophil⁹ Alex.	Cyſatus *Soc.I.*	Nonius	L. *Medicei*	Rothmannus	H. *Palus Mareotis*
Vendelinus	Daniel Bartol.. *S.I.*	Orontius	Byrgius	Schikardus	Hortenſius
Zagutus	Fernelius	Pitatus	Campanus	Schillerus	Heuelius
6. OCTANS.	Gauricus	Pitiſcus	Capuanus	Vietæ	Keplerus
Albategnius	Gemma Friſius	Ptolemæus	Cichus Aſcul.	Zupus *Soc. I.*	Lansbergius
L. *Ferdinand. Imp.*	Gruemberger *S.I.*	L. *Innocentius X.*	Crugerus	8. OCTANS.	Linemannus
Aliacenſis	Gulielm. Haſſiæ	H. *Mons Sypilus*	Deriennes *Soc. I.*	Anton. Rocca	Milichius
Alpetragius	Hagecius	Purbachius	Eichſtadius	Beſſarion	Moletius
Alfonſus Rex	Homelius	Regiomon-tanus	Fontana	Billy *Soc. I.*	Reinerus
L. *Ludouic. XIV.*	Kircher *Soc. I.*	Saſſerides	Gaſſendus	Cardanus	Reinholdus
Apianus	Licetus	Scheinerus *Soc.I.*	L. *Annulus Neptuni*	Caualerius	Rheticus
Arzachel	Lilij *fratres*	Schomberger *S.I.*	Hainzelius	Copernicus	Ricciolus *Soc.I.*
Arzet *Soc. I.*	Longomontanus	Simpelus *Soc.I.*	Herigonius	L. *Philippus IV.*	Seleucus
Barocius	Maginus	Stœflerus	Iunctinus	H. *Mons Aetna*	Simon Marius
Bettinus *Soc. I.*	L. *Viſilij*	Tannerus *Soc.I.*	Kriſtmannus	Cuſanus	Sirſalis *Soc. I.*
Blancanus *Soc. I.*	Manzinus	Valtherus	Merſenius	Dominic. Maria	Stadius

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NOMENCLATVRA LVNARIVM REGIONVM

Nomenclature of the Lunar Regions
With the number added of the octants to which they belong.

Inſula Vētorum 8.	Mare Humorū 7.	Mare Vaporum 3.	Peninſula Fulmi-	L. *Terra Iuſtitiæ*	Terra Sanitatis 5.

[25]The names are given as printed in the list; the forms engraved on the map differ from them in places. The expansions that follow are supplied, not printed: *Senior* and *Iunior*, the Elder and the Younger; S. Theophil⁹ Alex., St Theophilus of Alexandria; Ferdinand. Imp., the Emperor Ferdinand; Alfonſus Rex, King Alfonso; Ludouic. XIV., Louis XIV; Gulielm. Haſſiæ, William of Hesse; Lilij *fratres*, the Lilius brothers; Innocentius X., Innocent X; Mons Sypilus, Mount Sipylus; Annulus Neptuni, Neptune's Ring; Philippus IV., Philip IV; Mons Aetna, Mount Etna; Lacus Poſſidonij, the Lake of Posidonius; Palus Mareotis, the Mareotic Marsh; Soc. I. and S.I., of the Society of Jesus. Eſtenſis D., Medicei, Cichus Aſcul. and Daniel Bartol.. are left unexpanded.

Lacus Mortis 3.	L. *Mare Venetum*	Oceanus Procella-	num 7.	Terra Fertilitatis	L. *Terra Dignitatis*
Lacus Somniorū 3.	Mare Imbrium 1.2	rum 1. 8.	Sinus Æſtuum 7.	5. 6.	H. *Aſiæ pars*
L. *Lacus Scientiæ*	L. *Mare Auſtriacū*	L. *Ocean⁹ Philippicus*	L. *Sinus Medius*	L. *Terra Dignitatis*	Terra Siccitatis
H. *Palus Hyperborea*	Mare Nubium 7.	H. *Mare Eoum*	H. *Mare Hadriaticū*	H. *Aſia*	1. 2.
Littus Eclipticū 1.8	L. *Mare Borbonicum*	Palus Nebularū 2.	Sinus Epidemia-	Terra Grandinis 2	L. *Terra Honoris*
L. *Littus Philippicum*	Mare Nectaris 5.	L. *Terra Virtutis*	rum 7.	Terra Mannæ 4. 5.	Terra Sterilitatis 7
Mare Criſium 4.	L. *Sinus Batauicus*	Palus Nimborū 7.	L. *Mare Populorum*	L. *Terra Temperātiæ*	Terra Vitæ 3. 4.
L. *Mare Caſpium*	Mare Serenitatis 3.	Palus Putredinis 2	Sinus Iridum 1.	H. *Colchis*	L. *Terra Sapientiæ*
H. *Palus Mæotis*	L. *Mare Eugenianū*	Palus Somni 4.	L. *Sinus Geometricus*	Terra Niuium 2.	H. *Sarmatia*
Mare Fecunditatis	H. *Mare Euxinum*	L. *Aeſtuaria Bamelr.*	H. *Sinus Apollinis*	L. *Montes Auſtriaci*	Terra Vigoris 4. 5.
4. 5.	Mare Tranquil[?]i-	Peninſula Delirio-	Sinus Roris 1.	H. *Italia Apenninus*	L. *Terra Pacis*
L. *Mare Langreni*	tatis 4.	rum 7.	L. *Sinus Principis*	Terra Pruinæ 1.	
Mare Frigoris 2.	L. *Mare Belgicum*	Peninſula Fulgu-	Stagnum Glaciei 2	L. *Terra Laboris*	
L. *Mare Aſtronomicū*	H. *Euxinus*	rum 7.	Terra Caloris 7.	H. *Mauritania*	

[26] English renderings of the list above, column by column, with the printed octant numbers; synonyms from Langren and Hevelius follow in parentheses under L and H. Island of the Winds 8. Lake of Death 3. Lake of Dreams 3 (L Lake of Knowledge; H Hyperborean Marsh). Ecliptic Shore 1, 8 (L Philippine Shore). Sea of Crises 4 (L Caspian Sea; H Maeotian Marsh). Sea of Fecundity 4, 5 (L Sea of Langren). Sea of Cold 2 (L Astronomical Sea). Sea of Moisture 7 (L Venetian Sea). Sea of Rains 1, 2 (L Austrian Sea). Sea of Clouds 7 (L Bourbon Sea). Sea of Nectar 5 (L Batavian Bay). Sea of Serenity 3 (L Eugenian Sea; H Euxine Sea). Sea of Tranquillity 4 (L Belgian Sea; H the Euxine). Sea of Vapours 3. Ocean of Storms 1, 8 (L Philippine Ocean; H Eastern Sea). Marsh of Mists 2 (L Land of Virtue). Marsh of Rain-Clouds 7. Marsh of Decay 2. Marsh of Sleep 4 (L Estuaries of Bamelr.). Peninsula of Delusions 7. Peninsula of Lightning Flashes 7. Peninsula of Thunderbolts 7. Bay of Surges 7 (L Central Bay; H Adriatic Sea). Bay of Epidemics 7 (L Sea of Peoples). Bay of Rainbows 1 (L Geometric Bay; H Bay of Apollo). Bay of Dew 1 (L Bay of the Prince). Pond of Ice 2. Land of Heat 7 (L Land of Justice). Land of Fertility 5, 6 (L Land of Dignity; H Asia). Land of Hail 2. Land of Manna 4, 5 (L Land of Temperance; H Colchis). Land of Snows 2 (L Austrian Mountains; H Italy, the Apennine). Land of Hoarfrost 1 (L Land of Labour; H Mauritania). Land of Health 5 (L Land of Dignity; H part of Asia). Land of Dryness 1, 2 (L Land of Honour). Land of Sterility 7. Land of Life 3, 4 (L Land of Wisdom; H Sarmatia). Land of Vigour 4, 5 (L Land of Peace).

Hiſtoria Selenographica ex ſelectis Obſertionibus indicata.

A Selenographic history, set out from selected observations. [recte: Obſeruationibus]

VI. [Margin: Summary of the argument of the Selenographic history.] The spots on the Moon's face are called Old when they can be discerned with the naked eye, without a spyglass [tuboſpicillo], and seem to imitate great seas. The New spots (which from now on we shall call simply spots) are those that can be discerned only with a telescope, because of their smallness. Most of them tend to a round or oval shape, whether they are dark wholly or always, or partly and sometimes. In both kinds the Belgian tube, especially our own, has in recent years detected marvellous things most worthy of note. This concerns both their number, shape, size and position; and above all the colour of their blackness or brightness, the alternation of light and shadows, their height or depth, and so the roughness of the Moon's surface; and finally and most of all the apparent libration of the lunar globe about its centre. Some of these are common to all phases, while others are proper to the Moon when full, or at dichotomy, or horned, or gibbous. These matters are to be explained more fully elsewhere; in this place they are to be touched on more sparingly, as far as is sufficient.

VII. [Margin: Number of the Moon's spots.] The number of new spots so far does not exceed 270 [CC LXX] in Langren's full Moon, 550 [DL] in that of Hevelius, and finally 600 [DC] in ours. This is not because we have not noted more, but because the rest, on account of their slight depth or height, or at any rate their minuteness, can be distinguished rarely, and only when they first or last catch the light of the Sun. These are truly innumerable, so much so that the whole surface of the Moon visible to us is rough with them, as if with little clods, or stony, or full of ledges. Apart from those larger parts that present the appearance of seas, or bays of seas, or lakes, or marshes, or sandbanks, or reefs, or streams and rivers, or islands or peninsulas, or finally the great continents of our Earth, we nevertheless note four things still more notable as regards the number.

[Margin: First notable point in the number of the spots.] *The first* is that some very small spots follow one another so closely, crowded together in a long tract, that when viewed either with a less perfect telescope or far from the boundary of the general illumination, they look like furrows of certain rays. Notable among these are the ones that spread from the spot of Tycho, as from their origin, through the Land of Fertility, of Heat and so on. Some liken them to the sections of melons, beginning from their flower or rose. But if they are viewed at the edge of the illumination with more perfect tubes, they are seen to be spots distinct from one another, just like the mass of stars in the Milky Way.

[Margin: Second notable point.] *The second* is that other, smaller spots adhere to certain spots or are bound into them, as gems are to rings. Such are the spots of *Hipparchus, Ptolemy, Peuerbach, Hainzel, Gassendi, Stöffler* and *Gemma Frisius*. Or they are so set in that a portion of one covers a portion of another nearly

equal to it, as with *Euctemon* with *Meton*, and *Peuerbach* with *Regiomontanus* and *Viète*. Or they are beaded all round with smaller spots clinging to them, or crowned on one part. Among these may be seen *Cleomedes, Riccioli* and above all *Tycho*, around which there are so many little hills and valleys that, according to the alternation of the illuminations, they present sometimes a rose of crowded spots, sometimes one continuous lake.

[Margin: Third notable point.] *The third* is that within certain spots, as if in wombs, there are one or more smaller ones, which cast their little shadow towards the same side as that to which the shadow of the spots enclosing them inclines. Such are *Hercules, Posidonius, Clavius* and some others.

[Margin: Fourth notable point.] *The fourth* is that some are likewise enclosed in larger ones in such a way that they cast their shadow to the side opposite the shadow of those enclosing them: namely *al-Zarqālī, Thābit ibn Qurra, Boulliau, Gassendi, St Catherine, Sts Theophilus and Cyril of Alexandria, Maurolico, Pétau, Langren, Piccolomini, Copernicus, Thābit ibn Qurra* and so on.

VIII. [Margin: Instability of the shapes.] The shape of the new spots mostly tends towards roundness or an ellipse, but in one and the same spot it not infrequently varies in appearance. This happens either because of the libration of the lunar globe, on account of which they appear compressed as they approach the edges of the lunar disc and widened as they recede from them, as you may often see in *Plato, Zoroaster, Endymion, Grimaldi, Riccioli, Langren* and *Pitiscus*, and as happens likewise in the larger, ancient spots, for example in the *Sea of Crises*, the *Sea of Moisture*, the *Bay of Dew* and the *Marsh of Sleep*. Or it happens because of the differing illumination, as a result of which the edges of the spots appear sometimes continuous, sometimes interrupted and separated into particles tending elsewhere, and these either truncated or bisected. Thus the *Lake of Pitati* is sometimes seen not as a lake cut off from the Sea of Clouds by a continuous shore, but as a kind of harbour, and not oblong but round. *Goclenius*, for its part, sometimes imitates an upturned bowl [ſcaphium]. Thus *Aristarchus* sometimes imitates three-bodied Saturn in the sky, with its companions at its sides sometimes joined to it and sometimes separated. And those companions themselves are seen now bounded by continuous edges and like little moons, now like wings, now like outstretched fingers, and that within one or two days. Finally, certain spots that are usually noted as round nevertheless sometimes become hexagonal, namely at full Moon, such as *Copernicus, Eratosthenes* and *Timocharis*. Nor must it go unmentioned that the spot of *Vitruvius* is displayed now as continuously round, now divided into rays, now crowned with the faint splendour of a halo.

IX. [Margin: Variation of the apparent size.] The true size of the spots depends on the true size of the whole lunar globe, or of a great circle on it. On that, as on the height of the lunar mountains, see what we shall say below, namely in the Scholium to chapter 8. The apparent size, on the other hand, varies for the same reasons as the shape, namely because of libration or of differing illumination. So you can draw not a few examples from what was

said a little earlier about the shape, or also from the first notable point on the number of the spots. But besides what has been said about the shape of the *Sea of Crises* and the *Sea of Moisture*, the *Sea of Nectar* too is at one time bounded by a shore extended from *St Catherine* to *Fracastoro*, as is seen in the first figure, and at another time is enlarged, with the same shore receding as it were towards the *Land of Fertility*, as happens in flow and ebb.

X. [Margin: The position of the spots relative to one another varies through the force of libration.] The position of the spots varies in a wonderful way and very frequently through the force of the lunar libration, whether you compare them with one another or with the centre, the limbs or other regions of the lunar disc. Countless examples of this variation are available, but we shall select a very few. First, then, we have quite often seen *d'Ailly, Bianchini* and *Werner* on the same straight line; likewise *Plato, Tycho* and *Clavius* on the same straight line; and on another such line *Maurolico*, set against now *Homelius* and *Pitiscus*, now *Agrippa* and *Aristotle*, now *al-Battānī* and *Meton*. Yet a little later we were astonished to find these off the line already mentioned, by an evident and notable bend. Furthermore, we have noticed the same *Maurolico* not infrequently more distant from *Walther* than from *Homelius*, and more than once equidistant from them; and likewise *Riccioli*, now nearer to *Grimaldi* and more northerly, now more remote and less northerly. And *Harpalus* too at one time shone out in the very middle of the *Bay of Dew*; at another, when the Moon's disc nodded towards *Pythagoras* or *Xenophanes* and narrowed the breadth of the already mentioned bay, it appeared much nearer the upper shore. Finally, the isthmus lying between the *Bay of Dew* and that of *Cold* is sometimes nearer to *Plato*, sometimes farther from it.

[Margin: The position of the same spots varies with respect to the edges of the Moon.] *Secondly*, several spots compared with the nearer edge of the Moon have been observed to change position with an incredible anomaly. To take only striking examples, the *Caspian Sea* or *Sea of Crises* stood off from the right limb of the Moon at its least, yet hardly less than by a quarter of its own breadth; at its greatest, yet hardly more than by three quarters of its breadth likewise. Similarly the spot of *Grimaldi* was seen to stand off from the left limb of the Moon, at its least, by an interval scarcely smaller than its own breadth, but at its greatest by one a little smaller than three times the same breadth. [Margin: Evidence of libration from rising to setting, etc.] By this means the libration of the lunar globe from east to west and back again was detected.

Again, at some times *Tycho* was seen to stand off from the southern edge by the smallest possible interval, namely the distance between its centre and *Pitati*; at others by the greatest possible, namely a little more than the distance between its centre and *Boulliau*. In the same way *Plato* sometimes withdrew from the northern edge of the Moon by the least gap, though one a little smaller than the space between it and *Anaxagoras*; and sometimes by the greatest, that is, beyond the already mentioned space by a further interval as large as twice the breadth of *Plato* itself. [Margin: Evidence of libration from north to south.] This was

the evidence by which the libration of the lunar globe from north to south and from south to north began to reveal itself. Nor is this the end of the marvels, for we have obtained clear evidence of libration towards other regions as well. [Margin: Evidence of libration towards other regions.] Indeed *Zoroaster* and *Endymion* are sometimes carried towards the limb of the third octant, so that *Zoroaster* scarcely appears, or not even scarcely. Indeed, not even *Endymion* is seen whole, but cut off along its upper edge. At times, conversely, both are seen not only whole but also distinctly removed from the Moon's limb, and *Endymion* by an interval as great as that between it and *Hercules*. With a like difference of approach and then of recession, the *Sea of Moisture* and the spots of the *Land of Heat* alternately nod away from the limb of the seventh octant or nod towards it. But at other times the regions of this libration became known towards the first octant or the part opposite it. Thus *Harpalus* several times stood off from the limb of the first octant by exactly the distance between its centre and the nearest shore of the *Bay of Rainbows*, and several times by twice as much. By contrast *Pitiscus*, which was frequently distant from the Moon's limb by about the length of the *Sea of Crises*, nevertheless not infrequently stood off by only a third of that length. Finally, just as, for example, the spot of *Langren* is sometimes distant by as much as the greatest breadth of the *Marsh of Sleep* and sometimes by twice as much, so in the opposite part the *Ecliptic Shore*, with a large part of the adjoining sea, sometimes vanishes from sight and sometimes becomes manifest.

[Margin: A warning about the position of the spots, which differs in different authors or in different figures of ours.] Now although, as regards the constant position of not a few spots and other parts, we have had to correct the *Selenography* of Langren, of Hevelius, of Šírek [Rheitenſis], of Eustachio, of Sirsalis and of others, as will be perfectly clear to anyone comparing figures with figures, the observer should nevertheless take care not to conclude at once that their *Selenography* or ours is faulty because of a differing position of the spots that he may have noted in some phase. For by far the greater part of this diversity arises from the anomaly of the libration in different phases rather than from the error of observers of some diligence; and what one observer did not see in one phase, another may have noticed more opportunely in other phases. It has likewise come about from this that some spots hold one position in the first and sixth of our figures, and another in the 2nd, 3rd, 4th and fifth, as the difference of phases and librations requires. The varying latitude of the Moon is also a reason why, even when the Moon is in the Nonagesimal, neither its whole disc nor the array of its spots holds the same position with respect to the vertical plane.

XI. [Margin: Twofold variation of colours, and its cause.] The colour of the spots is confined to the kinds of blackness and whiteness, but it too varies, whether you compare one spot with another or the same spot with itself at different times. For, other things being equal, some are always whiter than others, and among them *Aristarchus, Menelaus, Proclus, Dionysius Exiguus* and

Linemann gleam most brightly white; or, on the contrary, blacker, among which are *Xenophanes, Plato, Zoroaster, Hermes, Goclenius, Pitati, Christmann* and *Grimaldi*, which therefore seem to be lakes of a sort rather than mountains or valleys. Among the larger parts, the blackest are the *Bay of Surges*, the *Bay of Dew* and the *Lake of Dreams*. Again, the same spots sometimes become darker or brighter than themselves, according as they are lit less or more by the Sun's light, whether direct or oblique or reflected; or according as their hollow is more or less deep, or more or less smooth, polished or rough, and so differently suited to bounding and turning back the light. In this class the variation of *Aristarchus, Plato, Endymion, Posidonius* and *Vitruvius* is more evident, but still more notable is that seen in the *Sea of Vapours* and in the *Marsh of Sleep*. For this sea, when it is freshly lit, differs little in colour from the nearer land, and the isthmus that at other times separates it from the *Sea of Serenity* does not then seem to separate it. The *Marsh of Sleep*, however, when it has just been lit, is seen so furrowed that it nevertheless appears whiter than when it was lit long before. Moreover, those oblong tracts of the *Sea of Tranquillity* and the *Sea of Fecundity*, or of the *Ocean of Storms*, which imitate sandbanks stretched far out, are whiter at full Moons than at other times, and so are observed more evidently. But when the section of the illumination reaches them, they become very dark, which is no slight argument for a greater depth than that of the seas already mentioned. You have a specimen of these in figures 2, 3, 4 and 5.

Finally, some spots appear brighter than the land itself, especially at full Moons. Our figures, however, do not show this, lest it be wholly clouded over with little shadows.

[Margin: Spots visible even in the secondary light.] As for that secondary light which is reflected from the Earth onto the dark part of the Moon, we represent it after a fashion in figures 7 and 8, taken from the rudiments of the first observations. At such a time that part is seen so lit that even the differences of the darker parts, which are usually called seas, appear in that secondary light, and even to the naked eye, provided the Moon is in the signs of right ascension or descension, so that with the Sun sunk far below the horizon the Moon shines raised well above it. The farther the dark part is from the part the Sun lights directly, the less dark it is, and at the other limb it ends in a bright ring.

XII. [Margin: Alternation of the shadows cast by the spots towards the 4 regions of the world.] Moreover, the alternation of the shadows cast by the spots now towards this region, now towards that, proves, like not a few of the things said so far, the roughness and unevenness of the lunar surface, very like that of the Earth. Among the chief spectacles of this phenomenon it draws to itself, not without delight, the eyes and contemplation of those who watch. In general, the shadows that attend the higher parts begin to be cast from that side which faces the Sun as it lights the prominent parts, and they stretch out towards the opposite side. For this reason the great majority of them arise from the western part of the Moon and are cast towards the eastern part, from new Moon up to the following full Moon. But from full Moon to the following

new Moon, the beginning of the shadows is seen in the eastern part of the spots, and the shadows themselves extend towards the western part, until they fail either near the western edge of the spot or near its middle, according as the Moon is nearer to or farther from new Moon. [Margin: Disappearance of the shadows at certain full Moons.] Likewise, if at full Moon the Moon departs from the ecliptic by a marked latitude towards the north or the south, shadows, though thin, arise from that part of the spots which is farther from the ecliptic and pass off towards the part which is nearer. But if the Moon is full of light, and above all if it also then has no latitude or an imperceptible one, then no trace of shadows, or scarcely a perceptible one, appears either. In our Figure 1 and 6, however, since it does not belong to any full Moon, let alone to a single one, but is put together as if from two dichotomies, the shadows of the left part tend to the right and those of the right part to the left, and in the middle some are doubled.

As for that wonderful casting of little shadows by the smaller spots enclosed within the larger ones, whether towards the same side as that to which the shadows of the larger ones incline or towards the opposite side, enough was said when treating of the number, in the 3rd and 4th notable points. Here, however, four further things are to be noted.

[Margin: 1st notable point on the casting of shadows.] *The first* is that most of the edges of a given spot nearer the Sun begin to be lit first, and then the more remote ones, while the interior hollows or valleys do not yet enjoy the light but still lie hidden in a narrower, oblong shadow. Not much later, however, these gradually receive the light in succession, flooded with it either continuously or separately, so that certain points, as it were, of crags begin to emerge into the light from those valleys like bright dots, or else higher fields, as it were, stand out there from the middle of the valleys still in night. But the already mentioned points shine out more evidently and for longer within the surrounding shadow if they belong to those discussed in the third and fourth notable points already mentioned.

[Margin: 2nd notable point.] *The second* is that the very edges of the larger spots do not begin to be lit with continuous light, but are lit separately, like the many summits of hills arranged in a circle, as you will note with a more perfect telescope in *Hipparchus, Ptolemy, al-Battānī, Alfonso, Peuerbach, Regiomontanus, Walther, Plato, Langren* and others of this kind, and sometimes in *Tycho*.

[Margin: 3rd notable point.] *The third* is that while the line of section continuously divides the bright part of the Moon from the dark, many small spots, separated from the continuous bright part of the Moon by a notable interval, are meanwhile already or still bathed in the light of the Sun because of their eminence, while the other, lower parts, whether the hollows of the seas or great valleys, lie between them in darkness. See a specimen of this in figures 2, 3, 4, 5, 8, 9, 10. Thus *St Catherine*, around the 5th day of the Moon, is lit sooner than the portion of the *Sea of Nectar* nearer to it, and that portion is about twice as large.

[Margin: 4th notable point.] *The fourth* is that some tracts of land near the line of section cast so black a shadow onto the neighbouring seas that that part of the seas is by no means visible, while the rest is visible though more remote. We observed this most clearly in the *Land of Snows*, where not undeservedly Langren placed the Austrian Mountains and Hevelius the Apennine. For when the section passes near them, in the first dichotomy (which our figure 3 serves) that tract of mountains casts a most forbidding shadow over the part of the *Ocean of Storms* extending from *Eratosthenes* to *Conon*, or over the nearer part of the *Marsh of Decay*, or of the *Marsh of Mists*, or of the sea lying between. In the second dichotomy, however, the same mountains most blackly darken the part of the *Sea of Serenity* lying below them and stretching from *Calippus* to *Sulpicius Gallus*, as you see shown in figure 4. In the same way, as the section approaches, the shores of the *Sea of Crises* cloud over the nearer parts of that sea, the western shore of the *Land of Health* the eastern portion of the *Sea of Tranquillity*, and the western shore of the *Land of Hoarfrost* the *Bay of Rainbows*, with a more intense shadow. And almost all shores do the same. So it is that in figures 7 and 8 the darkened part of the Moon is darker still near the bright part of the Moon, namely because of the already mentioned shadows, and not only because the light reflected from the Earth fades in comparison with the greater light.

XIII. [Margin: Unevenness and roughness of the lunar surface.] The greater or lesser height and depth of the parts of the lunar surface is made certain by what has been said so far about illumination and shadows. It is confirmed by that wonderful roughness and those windings which are seen on the border of the bright and dark parts, away from full Moons, even with telescopes that are not of the most perfect, and which are indicated in figures 2, 3, 4, 5, 8, 9, 10. At full Moons, however, if the Moon has a notable latitude, the northern edge appears toothed like the land if the latitude is southern, or the southern edge if the latitude is northern. Indeed, at full Moons sometimes, and at other times too, as the libration allowed, we have more than once seen certain inequalities on the shining limbs of the Moon, though using an excellent telescope. They are not so large, because of their multitude and cohesion, and because there is no comparing them with any neighbouring sea, a comparison which elsewhere, far from the limbs, helps in estimating their height. The same prominence is the reason why many shores of the seas, which would otherwise not be lit by the Sun because of their distance from the luminous section, nevertheless shine, standing out like little handles beyond the Moon. The mountain of *St Catherine* rises 8 Italian miles, and some in the *Land of Snows* 12.

XIV. [Margin: Notable points in the horned, full and halved Moon.] As regards the notable points in the individual phases, the many things said so far are enough for the narrow limits of this compendium. It must be added, however, that the southern horn of the sickle Moon sometimes runs out longer than the northern, though very slenderly, in a curve as of the finest little gems, by as much as the breadth of the *Sea of Crises*; and that at full Moons certain spots

are crowned with an unusual little circle of light, or twinkle with a nucleus of more intense light, as in *Boulliau, Menelaus, Vitruvius* and so on. At the dichotomies or quadratures of the Moon, however, the line of section neither always nor usually divides the Moon in two at those parts which in our first or sixth figure lie on the diameter of the lunar disc, but coincides with another line placed off the centre of such figures. It is sometimes nearly parallel to the line G K, and sometimes cut by it, according as the libration carries it, not only towards rising and setting but also towards north and south, or towards other regions.

XV. That the libration so often named here is so great that its whole period is completed within about one month, the same spot returning to the same limb from which it had most receded, and that about 207° of the lunar surface are seen, whereas otherwise no more than 179½° of its hemisphere could be seen at once, will be shown in chapter 9 of this Book. There we shall also deal with the variation of the limits and nodes, with the terms of the period of libration, and with the anomaly of both; it is enough to have indicated these things here.

CAPVT VIII.

Chapter 8.

De Lunæ Figura, & Quid in ea ſint Maculæ Antiquæ ac Nouæ, & Quantæ magnitudinis, Deq. Lunæ Aſperitatibus, Montibus &c.

On the shape of the Moon; what the Old and New spots in it are, and of what size; and on the Moon's roughnesses, mountains and so on.

I. [Margin: Sphericity of the lunar body.] Besides what was said in chapters 2 and 3 about the variety of phases, in which the Moon is seen now full, now sickle-shaped, halved or gibbous, it may be asked whether the whole Moon is spherical or not. That it is spherical is apparently demonstrated from those very phases by d'Aguilon (Book 5 of the *Optics*, after Proposition 56), Biancani (Book 9 of the *Sphaera*, chapter 7), Scheiner (*Disquisitiones Mathematicae*, no. 26), Tanner (*Dissertation on the Heavens*, question 7, §. 3) and many others. For the Moon is lit spherically by the Sun, in the way anyone can test on those gilded spheres usually set on the tops of towers, by observing how the Sun's rays light them successively. You will experience something similar if, in a room otherwise dark, someone carries an opaque sphere around your head, opposite a torch or lamp, so that you have it now between your eye and the torch, now behind, now at the sides. So it is that the line dividing the bright part of the lunar disc from the dark appears straight to the naked eye at the quadratures, but in the other phases curved, a portion of an elliptical circumference, exactly as the successive illumination of a sphere seen from afar requires. From this, then, it is inferred that the Moon is spherical at the very least in that part which faces us, that is, in more than half of itself. This follows both because more

than a lunar hemisphere is lit by the Sun, which is far greater, and because the libration of the lunar globe brings out or hides one spot after another from us just as the shape of a globular body requires, and uncovers a portion of itself beyond the semicircle, by ten degrees on each side of the 360 that go around the whole Moon. Nothing, therefore, is more probable than that the whole Moon is spherical, rather than only half of it or only the two-hundredth part of 360.

II. Now just as this Earth of ours is physically round and yet rough and uneven, because of the hollows of seas and valleys and the height of hills, mountains, forests and towers, so the Pythagoreans thought that the Moon has its own valleys and caverns and its own mountains or hills. So did all those who for that reason called it [Greek: ἀντίχθονα (antichthona, "counter-Earth")], as if an Earth set opposite ours, led to that thought by a conjecture from its spots. Others, it is true, have tried to explain this phenomenon without any roughness or unevenness of the lunar surface, namely through a difference of opacity or density alone. This controversy must be settled here, after hearing the opinions and arguments of others: first concerning the Old spots, then the New.

§.I. Quid ſint Maculæ Lunares Antiquæ, & quid Partes in ea claræ.

§. 1. What the Old lunar spots are, and what the bright parts in the Moon are.

II. We call Old those spots which all antiquity knew, namely the large ones, visible without an optical tube.

[Margin: 1st opinion, of Anaxagoras, the Stoics and Pliny the Elder.] *First*, then, in Plutarch (*De placitis philosophorum*, Book 2, chapter 30), *Anaxagoras* puts the cause of this variety down to an admixture of dark, earthy and cold parts with fiery ones; the *Stoics*, in Laertius, to a tainting by the vapours on which the stars feed. *Pliny the Elder* seems to have followed them in Book 2, chapter 9, where he says: *But the stars are undoubtedly fed by earthly moisture, since at half orb they are always seen spotted, the just force for drawing up more being as yet not available. For the spots are nothing other than the filth of the earth carried off with the moisture.* Yet the constancy of the spots, always in the same shape, proportion and position, refutes the inconstancy and levity of this little tale rather than of an opinion.

[Margin: 2nd opinion, of unnamed persons.] *Secondly*, some in Plutarch (the book *On the Face in the Orb of the Moon*) and in d'Aguilon (Book 5 of the *Optics*, disputation after Proposition 56) attribute these to illusion, or to a condition of the sight yielding to the brightness through weakness. But since they are seen by everyone, whether with the naked eye or with the Belgian spyglass, and always in the same position, shape and so on, this phenomenon cannot, except fallaciously, be attributed to fallacy. Further, as Plutarch argues, we should see something of this kind in the Sun rather than in the Moon, and

should yield most to the Sun's glare, so as to note huge spots in it without any device; whereas the Moon's brightness is gentler and gladdening, and for that reason Empedocles calls it cheerful [Hiladrum] in this verse:

The Sun is sharp in its rays, but the Moon's brightness is cheerful [hiladrus].

[Margin: 3rd opinion, of Clearchus.] *Thirdly*, in the same book of Plutarch *Clearchus* held *that these spots are mirror-likenesses and images of the great sea appearing in the Moon; and that the Moon is of all mirrors the purest in evenness and brightness. Agesianax* supports him in these verses:

Or of the boundless sea, roaring on the part set opposite below, An image is represented in a burning mirror.

If you objected that the Ocean does not appear in the Moon to everyone in the place where it is situated, he answered: *just as the rainbow is seen in a cloud when the sight is refracted to the Sun, so the outer sea is seen in the Moon, not in the place where it is situated, but where the refracted rays of sight reach its brilliance.* For he speaks after the manner of the ancient opticians, who thought that vision occurs by the emission of rays to the object. *Kepler* relates that the Emperor Rudolf Augustus was delighted by this opinion, when he saw an image of Italy offered to him in the Moon, I believe in the place where Hevelius placed the Apennine and we the Mountains of Snows. But it is refuted by *Apollonides* in Plutarch, *Averroes* (2 *De caelo*, part. 49), *d'Aguilon* above, and *Giulio Cesare Lagalla* (*De Phaenomenis in Orbe Lunae*, chapter 11), chiefly by two arguments. First, there is a great dissimilarity between the lunar spots and the seas of the Earth, as also between the bright regions of the Moon and the continents of the Earth, in position, in shape and in size, as will be perfectly clear to anyone comparing the *Selenography* treated in the preceding chapter with geographical hemispheres. Second, if the Moon were a mirror of the terraqueous hemisphere, the spots would be seen in one position after another on this mirror, according to the different stations of the eyes, and the images of many parts of the Earth would be reflected elsewhere, far from our eyes, as is evident from catoptrics and the properties of other mirrors. Yet the spots seem to everyone, whether they look at the Moon at the same time or at different times, to hold the same position in the Moon. What we said against a lunar mirror in chapter 5, no. 7, is relevant here. Nor indeed would I believe that our Antipodes see a different shape and position of the spots in the Moon than we do, even though the terrestrial hemisphere belonging to them has a surface far different from ours; or that someone living in the Moon and seeing our Earth would be philosophizing correctly if he said that the spots of the Earth are images of the seas and continents of the Moon. For they would in fact be parts of the Earth differing either in colour, or in roughness, or in opacity.

[Margin: 4th opinion, as yet of no one.] *Fourthly*, someone could imagine, as *d'Aguilon* notes (Book 5, on Proposition 56), that the Moon's spots are images of the spots of the Sun. But the lunar spots are constant, and perpetually keep the same position, shape, size and number with no motion of their own, apart from the small libration of the Moon; the solar spots, on the contrary, are inconstant,

so much so that often none appear, and they move either of themselves or with the motion of the Sun's rotation.

[Margin: 5th opinion, of Reinhold, Cardano, Giovanni Battista Benedetti, Barozzi, d'Aguilon, Biancani, Bettini, Argoli, Lagalla.] *Fifthly*, then, many consider that the difference between the bright and dark parts of the Moon, although both are directly lit by the Sun, arises from a differing rarity and diaphaneity on the one side and density and opacity on the other. Some speak in this way, as if rarity and diaphaneity did not differ or as if each implied the other. So hold the Peripatetics, whom the following follow: *Benedetti* in his *Letters*, p. 229; *Reinhold* on Peuerbach's *Theoricae*, p. 164; *Barozzi*, Book 4 of the *Cosmographia*, p. 268; *Cardano*, Book 3 of *De subtilitate*; *d'Aguilon*, Book 5 of the *Optics*, Proposition 56; the Scotists *Mastri* and *Belluto*, Book 2 of *De caelo*, q. 2, art. 3; *Giulio Cesare Lagalla*, *De Phaenomenis in Orbe Lunae*, chapter 10; *Bettini*, *Apiarium* 8, Progymnasma 6; *Argoli* in the *Pandosion Sphaericum*, chapter 49; and *Biancani* in Book 9 of the *Sphaera*, chapter 4, where he says that this is the common opinion. [Margin: Are the spots diaphanous and watery parts, or opaque and earthy?] But among these, *Cardano*, *Benedetti*, *Barozzi*, *d'Aguilon*, *Biancani* and *Argoli* think that the spots lie in the rarer and transparent parts of the Moon, because these do not stop the Sun's light at the surface or reflect all of it at once, but absorb it as it penetrates inward, and reflect only a moderate part of it, mixed with refraction. This, they say, is what happens in water and in other mirrors. *Biancani* confirms it by the example of small windows and of openings in walls lit by the Sun: the rarer air in them drinks in the light, and does not reflect it to us with as much brightness as the light reflected from the rest of the wall. They therefore judge that the brighter parts of the Moon lie in its more opaque and denser parts, these being better suited to reflect the Sun's light immediately and in every direction.

[Margin: Opinion of Plutarch, Galileo and Kepler.] Of the same view are *Plutarch*, in the book *On the Face in the Orb of the Moon*; *Galileo*, in the *Sidereus Nuncius*, p. 9, and in Dialogue 1 on the system of the world; and *Kepler*, in the *Dissertatio cum Nuncio Sidereo*, p. 15. There, candidly singing a palinode, Kepler yields the palm to Galileo, saying: *With these arguments you have fully made your case: I grant that the spots are seas, I grant that the bright parts are land.* He confirms the same in the *Epitome of Astronomy*, p. 831; for elsewhere, as we shall shortly report, he had believed the opposite. So too does *Antonín Maria Šírek*, Book 4, chapter 2, member 10. Galileo's words are these: *So that if anyone wished to revive the old opinion of the Pythagoreans, namely that the Moon is, as it were, another Earth, its brighter part would rather represent the land surface, and its darker part the water surface. And for my part I have never doubted that the terrestrial globe, seen from afar and flooded with the rays of the Sun, would present its land surface as the brighter and its water surface as the darker.* Yet *Galileo* does not affirm that there are seas or lands on the Moon. Indeed, in Dialogue 1 on the system he denies it, as *Scheiner* does too, and he speaks only from hypothesis. *Kepler*, however,

seems to affirm it with the Pythagoreans; as for *Plutarch*, we shall speak of him shortly, in no. 4.

III. Against these, however, *Reinhold*, *Mastri*, *Belluto*, *Lagalla* and *Bettini* think that the spots lie in the denser and more opaque parts of the Moon, being parts to which the Sun can impart little light. And *Mastri* adds that they seem to be valleys, because in paintings what is less illuminated appears more sunken, as we were saying above about the ring of the Moon, chapter 6, no. 2. [Margin: Opinion of Scheiner, Giordano Bruno and Longomontanus.] *Scheiner* agrees with them in the *Disquisitiones Mathematicae*, no. 27, corollary 7, concluding: *The darker parts of the Moon are not watery: otherwise they would be more translucent, and, add this too, better suited to the repercussion of light. Kepler* was once of the same opinion, in the *Astronomia Optica*, p. 250, and in the *Epitome of Astronomy*, p. 101; and so is *Longomontanus* in the *Astronomia Danica*, Book 2, chapter 9, p. 315, and *Giordano Bruno* in the book *De Immenso et Innumerabilibus*, p. 289: namely, that the spots of the Moon resemble the continents of the Earth, and the brighter parts its watery or maritime regions. *Fortunio Liceti*, on the other hand, in *De Lunae subobscura luce*, Book 1, chapter 6, says that if the transparent parts of the Moon are bounded within, at the back, by opaque and denser parts, they will appear brighter, because they will reflect a greater and livelier light, as happens in other mirrors.

[Margin: Our opinion.] We, however, are compelled to side with the authors of the former opinion by an experiment open to anyone. When the Sun is rising or setting on the horizon of the sea or of some lake, an enormous reflection of rays scattered this way and that does come from the waves, and the image of the Sun seen in some determinate part of the water, or of a mirror, dazzles eyes placed not far off. Yet if you look from far away, at a distance, at great and entire surfaces of land and water illuminated by a high Sun, you will see that the watery ones give back a certain black and dusky appearance of light, and the land ones a bright one, sometimes so bright that the eyes can scarcely bear it, even if they are not sprinkled with snow or whitened by any other cause. For the mountains of the kingdom of Chile, and other similar whitening rocks and crags, throw back the Sun's rays with much greater splendour. But whether the distinction of the spots from the brighter parts arises solely or chiefly from this, or from roughness, or from the diversity of colours, of which more shortly, I scarcely believe can be decided if you set authority aside.

[Margin: 6th opinion, of Fortunio Liceti.] *Sixthly*, then, *Fortunio Liceti*, in Book 1 of *De Lunae luce subobscura*, chapter 6, attributes the spots to a livid and blackish colour of the Moon, and the brightness of the remaining parts to the Moon's whiteness. I consider this not improbable. In certain pearl shells and matrices, in clouds lit by the Sun facing them, and in white marbles themselves, we see some parts grow white, some grow pale, and some even turn livid; nor is it unfitting to recognise some diversity of colours in the planets, not indeed of colours arising from the elements or from mixed bodies, but of purer ones, akin to light. For these are no more repugnant to the heavenly bodies than

opacity is, as *d'Aguilon* notes in Book 5 of the *Optics*, Proposition 35. And so not a few have admitted these colours in the stars, according to Caelius Rhodiginus [Lodovico Ricchieri], Book 1 of the *Antiquae Lectiones*, chapter 17.

[Margin: 7th opinion, of Father Grimaldi.] *Seventhly*, that darkness or duskiness could be attributed to frequent small holes, or to roughnesses or wrinkles of the Moon, which disturb the reflection. For if someone looks at a whitewashed wall lit by the Sun, part of which is smooth and part rough in the manner of pumice or of a file, the rough part will look less bright, and when compared with the smooth part will seem shadowy and blackish. This appears even more in some bare field only partly ploughed. By this experiment *Father Francesco Maria Grimaldi* thinks that the lunar spots can be defended in this way, granted that he does not reject the two preceding ways. And the observation favours it by which, as in the figures set out above, these spots at the boundary of the lunar section appear somewhat broken and rough, and indeed like acres scored with furrows, as may be noticed in the Marsh of Sleep and of Mists etc. in figure 1. But from here we have gradually slipped down to the question, necessary at this point, of the inequality of the lunar surface.

§. II. De Nouis Maculis Lunæ, eiuſq. Aſperitatibus, Vallibus, Cauernis, Montibus, Collibus &c.

§. 2. On the New Spots of the Moon, and its Roughnesses, Valleys, Caverns, Mountains, Hills, etc.

IV. [Margin: Upholders of the Moon's evenness.] That there is no inequality on the surface of the Moon, but that it is perfectly spherical, and that all the variety of the spotted and bright parts proceeds from a mere difference of colours, or of rarity or transparency, of opacity or density: this is the judgement of *d'Aguilon*, Book 5 of the *Optics*, disputation on Proposition 56; of *Tanner*, *Dissertation on the Heavens*, question 7, paragraph 3; of *Bettini*, *Apiarium* 8, Progymnasma 6; and of *Giulio Cesare Lagalla*, *De Phaenomenis in Orbe Lunae*, chapter 10. Otherwise, says *Tanner*, if there were valleys and hollows in the Moon, they would either be empty, which is held absurd in philosophy, or filled with a diaphanous body, liquid or solid, which levels the roundness but does not prevent the appearance of valleys and hollows. But if the Moon itself is said to be perfectly round, yet to consist heterogeneously of opacity and diaphaneity, exactly the same things can appear on its surface as do in fact appear. So the inequality of the lunar surface is asserted in vain, especially if the heaven through which the Moon is carried is solid. Moreover, on the circumference of the lunar disc that is turned toward the Sun and lit directly by the Sun, inequality of parts nowhere appears, but an exact roundness, especially at full Moons; nor at other times (they say) do any prominent parts appear on its limb, even to one exploring with the telescope. *Cabeo*, for his part, on Book 4 of the *Meteorologica*, t. 14, q. 1, does admit that the opaque parts of the Moon

stand out in many places, but says it is not established whether among them there are diaphanous parts by which the Moon's roundness is levelled.

[Margin: Upholders of the inequality of the lunar globe.] More of the moderns, however, who have observed the Moon more carefully with tube-spectacles, have noted and asserted manifest inequalities on its surface: first of all *Scheiner*, in the *Disquisitiones Mathematicae*, no. 26; *Galileo*, in the *Sidereus Nuncius* and in Dialogue 1 on the system of the world; *Kepler*, in the *Epitome of Copernican Astronomy*, p. 101, in the *Dissertatio cum Nuncio Sidereo*, p. 15, and in the *Astronomia Optica* from p. 248, with his teacher *Mästlin*; *Longomontanus*, Book 2 of the *Theorica*, chapter 9, p. 315; *Biancani*, Book 9 of the *Sphaera*, chapter 7; *Descartes*, Part 3 of the *Philosophy*, no. 152; *Francesco Fontana*, in his observations, tract 2, chapters 2 and 3, and tract 3, chapter 1; *Antonín Maria Šírek*, Book 4 of the *Radius Sidereomysticus*, chapter 2, m. 10; *Athanasius Kircher*, in the *Ars Magna Lucis et Umbrae*, Book 1, chapter 4; *Michael Florent van Langren*, in his *Selenography*; *Johannes Hevelius*, *Selenography*, chapter 6; and *Mastri*, if a vacuum is not admitted and the heaven of the Moon is fluid. But before these, *Agostino Nifo*, on Book 2 of *De caelo*, part 49, and *Patrizi*, in *Philolog.*, Book 20, p. 112. And that same inequality was once asserted not only by the Pythagoreans and by Anaxagoras, according to Plutarch, Book 2 of *De placitis*, chapter 30, who regarded the Moon as another Earth, and by Democritus, Orpheus, Heraclides and Ocellus, but by *Plutarch* himself, in the opusculum *On the Face in the Orb of the Moon.* There he does indeed say: *However, it is not probable that the Moon has the same surface as the sea*; yet he adds: *As to the face that appears in the Moon, just as our earth has certain great bays, so we judge that the Moon too is opened by great depths and ruptures, containing water or dark air, into which the Sun does not penetrate with its light, but, abandoning them, produces a dissipated reflection.* As for the cause of these cavities, *Kepler*, in the *Dissertatio cum Nuncio* and in the *Optics*, though in sport and jest, refers it to men living in the Moon, who hollow out for themselves caverns and very high ramparts to protect themselves from the intolerable heat, since they have a continuous day of 15 days, the Sun's illumination persisting among them in the same region of the Moon. For this reason the Pythagoreans too said, according to Plutarch, Book 2 of *De placitis*, chapter 30, that the Moon is inhabited by animals fifteen times larger than ours. But, dismissing jests, let us come to the arguments by which this inequality is seriously proved.

V. [Margin: Arguments for the inequality of the lunar surface.] *First*, then, the Moon, not only when bisected but also before and after dichotomy, if viewed with the telescope, has the line dividing its bright part from its dark part tortuous and fringed; and that boundary is manifestly full of windings, not to say toothed or serrated, as can be seen in all the figures set out in the preceding chapter. This is an argument that some of its parts are low and others higher, to such a degree that they can be distinguished even at so great a distance of the Moon. For it cannot be said that the dark parts which insert themselves among those windings are impervious to light because of their density or

opacity. Afterwards, as the Moon's illumination grows, they become bright; and conversely those that previously shone turn black as the Moon wanes. And the diaphanous parts, which some insert in order to level the roundness, are too black at that time, and become bright at full Moon.

VI. [Margin: 2nd argument.] *Secondly*, on about the fourth day after New Moon and on the following days, on the left and eastern part of the Moon, which is still black, there appear certain bright points or small areas, like crags or small islands separated from the continental region of the Moon that is continuously illuminated. Between these and that region lie black parts that do not yet enjoy the direct light of the Sun. This appears in the second and eighth figures, and especially in the spot of Saints Catherine, Cyril and Theophilus, in the Mountains of the Snows, and in many other places lit by the Sun long before. Conversely, in the waning Moon there are, on its right and dark part, certain parts which are deprived of the Sun's light later than some others that lie between them and the continuously lit part of the Moon, or that are less distant from its bright portion, as you see in figures 4 and 5, and better still in 9 and 10. Where then does that anticipation of the light received from the Sun in the earlier half-lunation, and the postponement in the later one, come from, if not from the eminence of parts which, like our mountains, are lit by the Sun sooner and cease to be lit later than valleys and lower parts? For this diversity cannot be put down to a mere difference of density or opacity, since those parts that are meanwhile black, though nearer to the illuminated segment of the Moon, while the more distant ones are already or still shining, themselves shine a few hours later, or were shining a few hours before. Nor will you ever be able to imitate this variety of illumination of which we are speaking with any sphere free of all roughness, through a difference of opacity or density alone.

VII. [Margin: 3rd argument.] *Thirdly*, many things were said of the variety of colours and illuminations in chapter 7, no. 11; but everything we said in the same place, no. 12, on the interchanged projection of shadows proves the eminence of some parts of the lunar surface and the depression of others, as we also showed there in no. 13. For mountains and trees or towers cast their shadow to the western side at sunrise and to the eastern side at sunset, while at midday itself, if the Sun stands perpendicularly over them (as happens on solstitial days in the Tropics, and on equinoctial days to those living under the Equator), the shadows cease. In the same way, during the whole time from New Moon to full Moon, the shadows of the new spots and of the lunar shores are carried toward the eastern part of the Moon, and they are smaller and more diluted the nearer the full Moon is. At the full Moon itself they vanish, especially around the middle of the Moon, because those parts are struck more directly and more perpendicularly by the Sun's rays; but once the full Moon has passed, the shadows of those parts begin again, now going toward the western region. And they are always larger and more intense when the parts lying on the line of section, the line that divides the bright part of the Moon from the opaque, first begin to be lit, or first cease to be lit. Finally, those shores of which

we spoke in chapter 7, no. 12, notable point 4, would never overshadow the nearer seas with so manifest and so grim a darkness, now to the right, now to the left, and that shadow would never in turn cease in the same parts of the seas, unless there were tracts of mountains close to those shores, and very lofty ones at that.

VIII. [Margin: 4th argument.] *Fourthly*. It is not only at those full Moons at which the Moon has a notable latitude that its limb, viewed with an excellent telescope, appears roughened and scabrous, or toothed like a saw: namely the northern limb if the Moon's latitude is southern, or the southern limb if the latitude is northern. Outside full Moons too, the eastern and western limb of the Moon, rarely indeed but yet sometimes, when inspected likewise with the best telescopes, seems rough and uneven because of parts protruding on them. This is attested not only by *Hevelius*, above, as observed by himself, but we too have seen it not rarely. *Kepler* also, in the *Astronomia Optica* from p. 249 and in the *Dissertatio cum Nuncio Sidereo*, p. 7, teaches that the Moon's limb has sometimes been seen by him as rough; and to Galileo, who supposes the contrary, he replies: *I do not know how carefully you have examined this, or whether you are not rather asking here from the common opinion?* For *Galileo* on the one hand had recognised a variety of valleys and mountains in the Moon, but had never seen in its limbs lit by the Sun any roughness, which nevertheless ought to be seen. To dispose of this objection he has recourse to two causes. One, which *Kepler* does not refute, is a thicker air around the Moon, like its atmosphere, which always shows the Moon to us as perfectly circular. The other comes from the frequency and crowding of the peaks, which, although in fact separate, yet seen from afar seem continued into a round periphery.

[Margin: Solution of the objection.] But neither is that atmosphere perfectly round, if it is like ours and arises from a like cause; nor is it so thick that some clefts and gaps could not be discerned among the hills of the Moon situated at its margins. The true cause, then, why that roughness of the limbs is rarely seen is partly the imperfection of the telescope, partly the libration of the lunar globe, by force of which there are often no eminent parts on the limb itself, but beyond it or short of it; or if there are, they are among those that stand out least above the rest of the surface. And from what has been said the objection of Bettini and others is resolved: through a defect of the optical instrument, or of a suitable libration, or through the omission of observations at the time when the rough limb of the Moon could have been seen, they nevertheless never saw it as such.

[Margin: 5th argument, but insufficient.] The *fifth* argument, that of Biancani, is as follows: If the Moon, he says, were perfectly spherical, it would not reflect light in every direction, but only into some regions of the air, and we would see very little of its part as bright. But this argument establishes only this one thing: that the Moon is not polished like a mirror, but is at least rough enough to suffice for universal reflection. It does not, however, prove from this that there are valleys or mountains in it.

§. III. Quid ſint Lunæ Maculæ Nouæ.

§. 3. What the New Spots of the Moon are.

IX. What, further, the New spots of the Moon are, which we have shown to imitate our valleys and mountains by their eminence or depression, that is, of what substance, colour, density, etc., it has not yet been given to us to know. Granted that various people have tried to depict them for us by likeness or analogy with various things. *Galileo*, indeed, in the *Sidereus Nuncius*, makes them like crystal drops or icy cups sprinkled with light; *Scheiner*, in the *Disquisitiones*, likens them to large pearls and many-coloured pearls; *Kepler*, to the great pores and holes of a white pumice that has tubercles; *Giulio Cesare Lagalla*, to tufts of cotton, or to clouds in a half-clear sky, partly bright and partly dark; others, to the little eyes of a peacock's tail, but with only the difference of black and white colour. But there is no time to labour in vain on these matters.

SCHOLIA.

Scholia.

I. [Margin: Height of the lunar mountains.] We shall teach how to measure the Mountains of the Moon in Africa, so called because they seem to touch the Moon with their summit, from the fall of the Nile, which has its origin not far from them, in the Geographical book, which is the fourth of Tome 2 of the *New Almagest*. Now we must give the method of measuring the mountains of the Moon that have their roots and substance in the Moon itself. It is taught by Galileo in the *Sidereus Nuncius*, p. 13; by Biancani, Book 9 of the *Sphaera*, chapter 9; and by Bettini, *Apiarium* 8, Progymnasma 6, Proposition 4. They assume the Earth's diameter to be 7000 Italian miles, and the Moon's diameter to be to it as 2 to 7, that is, 2000 Italian miles, with a semidiameter of 1000 miles. They then take the distance of the illuminated lunar peaks from the limit of illumination, or from the lunar horizon, from Galileo's observation, as one twentieth of the lunar diameter, that is, 100 miles. From this they derive a perpendicular height of the lunar mountains of 4 Italian miles, which they think no mountain of our Earth reaches. Kepler agrees with Galileo in this, in the *Dissertatio cum Nuncio Sidereo*, p. 15, namely that the lunar mountains are absolutely greater than the terrestrial ones, whereas earlier, in the *Optics*, p. 250, he had said only that they are greater when the proportion of the globes is considered. Hevelius, however, taking for that distance one twenty-sixth of the lunar diameter, derives 3 Italian miles.

[Margin: Practice of measuring the lunar mountains.] The practice of this measurement is as follows. Let the bright hemisphere of the Moon be ADG and the dark one ABG, and let the nearly great circle separating them, and also the diameter of the Moon, be represented by the line ACG, and the Moon's semidiameter by AC. Then let there be, in the part still dark, a mountain MB, lit at its summit by the Sun's ray SM, which touches the Moon at A and there, by Proposition 16 of Book 3 of Euclid, makes the angle MAC right with the

semidiameter AC; and let the perpendicular height of the mountain, MB, be produced to the centre C, for triangle MAC will be right-angled. So if AC is given, the true semidiameter of the Moon, to which CB is equal, and MA is given, namely the distance of the illuminated summit M from the limit or Horizon of the Moon ACG, in parts of which AC is known, then by the rules of right triangles the base MC will certainly become known; and if BC is subtracted from it, the height of the mountain MB will be left known. Thus in the example assumed, if the side AC is 1000 miles and AM is 100, and these are squared, the sum of the squares will be 1010000, to which, by Proposition 47 of Book 1 of Euclid, the square of the base MC is equal. Its square root is less than 1005 and greater than 1004. So, taking away CB, 1000, there remains BM, of almost 5 miles, but certainly exceeding 4 miles.

Diagram. A small woodcut set into the right side of the left column: a circle representing the Moon, its left half hatched with horizontal lines (the dark hemisphere) and its right half white (the bright hemisphere), divided by a vertical diameter. A horizontal tangent runs across the top of the circle through the upper end of the diameter, extending left beyond the circle to M and right to S. A straight line runs from M down through the hatched area to the centre on the diameter. Dotted lines stand between M and the circle at upper left.

- M (upper left, outside the circle, on the tangent line): the summit of the mountain
- A (top, where the tangent meets the circle): the point of tangency
- S (upper right, on the tangent line): the end of the Sun's ray
- B (within the hatched area at upper left, on the line from M to the centre): the foot of the mountain
- C (centre, on the diameter): the centre of the Moon
- D (right, in the white half): on the bright hemisphere
- G (bottom, at the lower end of the diameter)

II. [Margin: Our opinion on the height of the lunar mountains.] I, however, observed, between the spot of St Catherine, lit on the fourth day after New Moon, and the limit of the usual illumination, the 16th part of the lunar diameter. For me, from what was said in Book 2, chapter 7, the Earth's diameter is 8278 Italian miles of Bologna, but 10348 ancient Roman miles. So if the lunar diameter is to the terrestrial as 2 to 7, it will be 2365 Bolognese miles and 2956 Roman, and its 16th part will be 148 Bolognese miles, but 184¾ Roman. Therefore, by the preceding problem, BM, the height of Mount Sinai or St Catherine, will be 9 Bolognese miles. For the square of the number 148 is 21904, and that of the lunar semidiameter, namely 1182 miles, is 1397124; and the sum, that is, the square of the base MC, is 1419028, whose nearest root is 1191. Taking away, then, BC, 1182, there remains MB, 9 Bolognese miles. But here it is supposed that the ray SA touches the globe of the Moon just as if the point A did not stand out at all; if A were the summit of another mountain, MB would be derived as much higher. This mountain we have elsewhere, with Langren, called Mount Piccolomini, and another, beyond the Mountains of the Snows,

Mount St Xavier, which, because of its distance from the lunar section of 2½′, we think to be about 12 Bolognese miles high. These measures, however, we shall somewhat refine below, chapter 17, problem 2.

III. [Margin: Size of the spots.] The true size of the spots, or of other parts viewed on the lunar surface itself, since it appears to us as if on a flat disc, as a part of the lunar diameter, can be derived from the true diameter of the Moon and from the particles belonging to each spot on the selenographic scale placed at the foot of figure 6 of chapter 7: namely, if the true diameter of the Moon is multiplied by them, and the sum so made is divided by 200. For example, the length of the Sea of Crises by the already mentioned scale is 21 particles, and its breadth, as it lies in figure 6, is 15 particles. Multiplying, then, 2169 Bolognese miles, which is what the Moon's diameter has according to what will be said in chapter 17, now by 21 and now by 15, let the sums be divided by 200, and the length of the Sea of Crises will be found to be 227¾ Bolognese miles, and its breadth 162⅔. However, since the optical measure of this sea varies by force of libration, and since it really lies not on a flat disc but on a spherical surface, this measure too must vary. Šírek [Schirlæus], for his part, in Book 4, chapter 2, member 10, gives the Sea of Crises 100 leagues in breadth, the Ocean of Storms 470, and the tract of mountains in the Land of Snows 100. Let it be enough to have indicated this to anyone with the leisure and the inclination to play in the Moon.

CAPVT IX.

Chapter 9.

De Motibus Lunæ ex ſitu Macularum deprehenſis, præcipuè de Libratione: & de mutuo Terræ Lunæque aſpectu.

On the Motions of the Moon detected from the position of the spots, especially on Libration; and on the mutual aspect of the Earth and the Moon.

I. The position of the lunar spots has seemed to indicate three kinds of motion of the lunar globe around its own centre: namely, of Rotation in latitude toward the poles of the Equator or of the Ecliptic; of Rotation in longitude from West toward East; and of Trepidation or Libration.

[Margin: First rotation of the Moon, idly asserted.] A rotation of the Moon in latitude, in the part visible to us, from north toward south, slightly slower than the daily motion of the Moon in longitude, was asserted by *Francesco Fontana* of Naples, both because he thought it consonant with reason that all parts of the Moon should exert influence on us, and because he seemed to himself to observe some spots becoming more southerly every day until they were hidden. But neither I, nor my colleagues, nor *Galileo*, nor *Scheiner*, nor *Langren* have observed any such thing. The spot of *Tycho* has always appeared to us near the southern limb, 3′ or 4′ distant from it, at full Moons as much as in the other phases in which it is seen lit; and it undergoes only that moderate

variation which a change of latitude, or some libration, of which I shall speak in no. 3, produces. Indeed, the greatest part of the Moon's face perpetually preserves the same position of its spots toward us. Nor can that argument of fitness prevail against the evidence of this experiment, open to everyone. This is especially so since it can itself be countered by saying it is fitting that the Moon, placed on the border of the heavenly and the subcelestial, should always look with one of its parts toward the supercelestial, and from there should drink in through its whole body a varied force of influence, which it pours through its other face into the subcelestial. Much less is it true that the spots themselves travel round the globe of the Moon, as someone has suspected; against that motion, *Langren* says, his more than a thousand observations fight.

II. [Margin: The rotation of the Moon in longitude: in what hypothesis it is necessary.] The motion of rotation in longitude, from the western part of the Moon to the eastern, by which it would revolve on its own centre, invented by *Berosus* the Chaldean, was already refuted in chapter 5, no. 1. For by that motion the same hemisphere of the Moon, and the same face, would not always be turned toward us. Further, in order that the same face of the Moon should always look at us, and that, for example, the *Caspian* or Sea of Crises should always appear to us, when it appears, near the western limb of the Moon, the spot of *Grimaldi* almost on the eastern limb, and the bright valley of *Tycho* at a distance from the southern limb of about 3′, and the spot of *Panciroli* or of *Plato*, a little more than 3′ from the northern limb, should always appear near their limbs, and finally that the spot of *Hipparchus* and the Bay of Heats [*Sinus Aestuum*] should constantly appear to us in the middle of the Moon's disc, no motion of rotation is needed, if the body of the Moon completes its monthly revolution on a pure eccentric, as happens in the hypothesis of *Magini*, *Kepler*, *Lansbergen* and *Boulliau*. But if the eccentric carries an epicycle on its circumference, and the epicycle carries the centre of the Moon, as in the hypothesis of *Ptolemy*, *Peuerbach*, *Copernicus*, *Reinhold*, *Barozzi*, *Tycho*, *Longomontanus*, *Scheiner*, *Biancani* and *Gassendi*, then the already mentioned monthly motion of rotation is certainly needed. Otherwise the same face of the Moon would not always be turned toward us, as can easily be shown by a mere drawing of the epicycle, of which more below; and this has been shown by *Barozzi* in Book 4 of the *Cosmographia*, p. 257, and by *Scheiner* in the *Disquisitiones Mathematicae*, no. 25. Both of these, like *Gassendi* in Book 2 of the *Institutio Astronomica*, chapter 15, assert this lunar rotation for this reason, and *Galileo* asserts in Dialogue 1 that it follows from the epicycle hypothesis; while *Longomontanus*, with little consistency, denied it in Book 2 of the *Theorica*, chapter 9, p. 316. *Magini*, however, in his *Theoricae*, page 70, abandons the lunar epicycles of Ptolemy and Copernicus for the reason that, as *Averroes* in the same place observes, the spots of the Moon that appear at perigee would be withdrawn from our sight when the Moon is at apogee. But the motion of rotation can meet this drawback, if anyone prefers to use an eccentrepicycle. [Margin: Kepler's fiction.] *Kepler*, however, who feared

that the excessive inclination of the lunar fibres would produce an excessive libration of the lunar globe, contrary to the phenomena of the spots, had to invent a globe of the Moon like a kernel inside a crust and rind, which would preserve the inclination of the fibres, while the rind and the surface of the lunar disc meanwhile preserved the position of the spots that we see, as may be seen in the *Epitome of Copernican Astronomy*, p. 613. On this matter see below, chapter 26, Scholion 7.

De Librationis Lunaris Motu per Maculas ope Teleſcopij detecto.

On the Motion of Lunar Libration, detected through the spots with the help of the Telescope.

III. [Margin: Galileo the first discoverer of this libration.] The telescope, as with many other marvels, first indicated to *Galileo* that the globe of the Moon librates about its own centre. With its help he detected that the spot of *Grimaldi* now approaches almost the eastern limb of the Moon, now recedes from it, by a difference in distance three times greater; while the Sea of Crises, which he calls the Corican spot [*maculam Coricam*] because it faces the north-west [*Corum*] when the Moon reaches the meridian, and which stands almost diametrically opposite the spot of Grimaldi, approaches the western limb and alternately recedes from it, by a difference of distance more than double; so that as one approaches its limb, the other recedes from its own. So he says in Dialogue 1 on the System of the World, p. 58 of the Italian and 44 of the Latin edition, where he also touches on the cause and theory of this libration, on which see below. [Margin: Langren observes the same libration.] The same libration was observed, but over more years and more accurately, by *Michael Florent van Langren*, Cosmographer to the Catholic King, as early as the year 1626, as can be gathered from his observations, sent to me with Belgian candour and generosity, in which he promises astronomers an admirable theory of this libration. [Margin: Gassendi likewise, Boulliau and Hevelius.] Later *Pierre Gassendi* also began to note the approach of the Caspian spot (which we call the Sea of Crises) to the western limb and its recession, as well as that of another spot from its own limb, which he calls the Eastern spot and we the spot of Grimaldi. His observations on this subject, from 1636 September 23 to 1642 April 14, are reported by *Boulliau* in Book 3 of the *Astronomia Philolaica*, chapter 13, and there he also lists his own observations of this libration, from 1643 April 1 to 1644 January 18. At about the same years *Johannes Hevelius* diligently brought together very many observations on the appearance of this motion, of which not a little will have to be said below; yet I find in him no observations older than those already mentioned; indeed, the oldest, which he takes from others, are those of Gassendi and Boulliau. The Sea of Crises he himself calls the Maeotic Marsh [*Paludem Męotidem*], and the spot of *Grimaldi* the Mareotic Marsh [*paludem Mareotidem*].

[Margin: Finally Zucchi, Bartoli, Sirsalis and Grimaldi.] Finally, observers of the sky from our Society, especially Fathers Niccolò Zucchi, Daniel Bartolus [Daniello Bartoli] and Hieronymus Sirsalis, confirmed this very libration by the testimony of their own eyes. And we ourselves, who at other times used to attribute this appearance simply to the effect of varied latitude, began to distinguish it from the other apparent motions of the Moon; how far the diligence and skill of Father Francesco Maria Grimaldi has advanced in this field up to the present day will be evident from what is to be said below. Meanwhile, how manifest are the arguments by which we, together with him, have detected this libration, from the approach to the nearer limbs and the recession from them not only of the two spots already mentioned but of others as well, and indeed toward the north and south too, and even from sets of three spots seen now in one straight line and now off it, is clear enough from what was said in chapter 7, from no. 10 on.

Quæſtio I. Quanta ſit prædicta Libratio, ſeu Quotam Lunæ partem vi illius vltra antiquorum opinionem videamus.

Question 1. How great the already mentioned libration is, or what fraction of the Moon we see by its force beyond the opinion of the ancients.

IV. Before we establish how large the portion of the lunar globe is that we can see by force of the already mentioned libration, beyond the portion that astronomers ignorant of this libration thought could be seen by us according to the ancient problem given on p. 234, we must determine how great the difference of distance is between the greatest and the least approach of one and the same spot lying nearer the limb of the Moon, and indeed of such a spot that at its greatest approach it reaches right up to the limb of the Moon, or as close to it as possible. [Margin: Drawing of the circles of the lunar disc.] To this end we shall first of all describe the lunar diagram; that is, we shall teach the method by which, in the sixth of the diagrams set out above after p. 204, we drew the circumferences of the circles that intersect one another, and their diameters.

First, take a diameter of the lunar disc as large as you please, for example as large as the selenographic scale XZ (which we chose to subdivide into 200 equal particles, so that the semidiameter would be of 100 parts). From it cut off with compasses a semidiameter, AB, and at that distance from centre A describe the circle BCDE. Having produced the semidiameter AB as far as D, the end of the whole diameter, take on it a portion AF of 12 parts, of which AB is 100. Then, taking F as a second centre, describe a second circle GHIK; for the circles will intersect at points L and M. A straight line drawn from these points perpendicular to the diameter BD, which is to be produced as far as I, would fall on point N, which bisects the little line AF, although to avoid confusion we have not drawn this line. Next, from point N, at distance AN, describe the little circle ATFV. *Second*, through point A draw the first circle's diameter EC at right angles to the common diameter BAI, touching the already mentioned

little circle at A. *Third*, with both quadrant EB bisected at P and quadrant BC at R, draw from points P and R, through point A, the straight lines PAQ and RAS. In this way the lunar disc will be left subdivided into 8 octants, so that the spots and lunar regions named in alphabetical order in the account given in chapter 7 can be found more easily in a smaller space.

[Margin: Method of observing the amount of the libration.] From the very size of the diameter AF the amount of the whole libration, GB, is apparent; for GB is as large as AF, namely 12 parts of which AB is 100, and DI is equally large. We have found that it is this large by comparing the distance between the greatest and the least recession of spots that reach the limb of the Moon in their approach in such a way that they are not hidden beyond it, with some other fairly large spot nearer to the spot in question; and then, more conveniently, by comparing that fairly large spot with the semidiameter of the lunar disc, already fixed in the diagram of the lunar disc itself. This diagram must be prepared on a small table set beneath, near the telescope, so that when the Moon has been viewed through the tube, and the eye moved repeatedly from the tube to the diagram, what has just been observed can be noted on it. For example, suppose that the spot of *Thales*, whose northern extremity in figure 6 is at point G, has at another time been observed right on the limb at point B, so that it has never appeared either beyond B or short of G. By that spot it will then be established that the whole range of libration on that side is as great as the apparent straight line BG. Therefore, when the Tha- letic spot has returned to G, compare the distance BG with the length of the spot of *Endymion*; for because of its nearness you will clearly see that the whole length of *Endymion*, and a third part of it besides, is contained in GB. We have used the length of *Endymion* rather than its breadth because its breadth varies by force of libration, but its length does not. Once this measure is obtained, it is easy to find with compasses what fraction the length of *Endymion* together with a third of it is of the semidiameter of the lunar disc, already drawn graphically with the due proportions observed. [Margin: Spots more suitable for measuring the libration.] It is assumed, moreover, that the spot at G is in the belly, that is, equidistant from the nodes of libration LM. This is recognized if the other spots, moderately and almost equally distant from it on either side, for example those of *Plato* and *Posidonius*, or *Meton* on one side and *Mercury* on the other, approach the same limb of the Moon together with it, or recede from it, with a similar motion. What has been said of the spot of *Thales* holds in its own way for the spots themselves of *Endymion*, *Meton*, *Mercury*, *Moretus*, *Zucchi*, *Bettini*, *Kircher*, *Riccioli*, etc. Besides these, those spots also serve the purpose secondarily which, although in their approach they do not reach the limb itself, yet come very close to it, and at their greatest recession are still less distant from the limb than from the centre of the Moon, especially if they have such advantages as to strike the observer's eyes more clearly. For us such have been the spots of *Plato*, *Aristotle*, *Atlas*, the Sea of Crises, those of *Langren*, *Pitiscus*, *Tycho*, *Grimaldi*, and the spot of *Harpalus*.

Let no one be surprised, moreover, if, considering libration, as is right, at the limbs of the Moon, namely so as to confine it within its definite extremes, we also employ spots nearer the limbs than the centre. For the variation of the distance between the spots and the centre is greater if the spots are nearer the centre, and more uncertain because of the difficulty of discerning exactly the centre of the lunar disc; as I assume from the laws of optics applied to a librating sphere seen from afar.

V. [Margin: Part 2 of the problem.] It remains for us to inquire how great the arc on the surface of the lunar globe is which corresponds to the quantity BG, that is, to that portion of the semidiameter that we see on the Moon as if on a flat disc. We shall do this by the following problem. From centre A describe BCD, the semicircle of the Moon's body, whose diameter is BAD; to this, from the eye E, placed on the Earth, draw the perpendicular ray EA, and another ray EF, touching the Moon at F. From this point draw FG parallel to BD; for FG will be the diameter of the lunar disc as it appears to us, and HF its semidiameter, which we divided above into 100 equal parts. Now let a spot, for example that of *Thales*, be first seen at point F on the very limb; and then, when it has receded furthest from it, let it be at point K, but appear at point L of the flat disc, by means of the visual ray EKL, which is produced as far as M. Finally, from centre A to point K, draw the semidiameter AK. The proposal now is to find the arc KF that by force of lunar libration we see beyond what we would see without libration. And since we see more of the Moon at apogee than at perigee, we shall have to use the semidiameter and distance of the Moon at apogee.

Diagram. Woodcut set into the right side of the left column, the text running down its left. A horizontal line carries B, A, M, D across the top; a semicircle hangs below it from B to D. Just beneath the top line a second horizontal line carries G, H, L, F. Three straight lines converge downward to E at the foot of the figure — from A through H and C, from M through L and K, and from D through F — and a further line runs from A to K; C stands at the bottom of the semicircle, and K on its right-hand arc.

- B (top line, left end): point of the diameter BAD
- A (top line, centre): centre of the semicircle
- M (top line, between A and D): end of the visual ray EKL produced
- D (top line, right end): point of the diameter BAD
- G (second line, left end): end of FG
- H (second line, left of centre): centre of FG
- L (second line, right of centre): point of the flat disc
- F (second line, right end): point of tangency of ray EF
- C (below the centre on the semicircle): point of the semicircle
- K (right, on the semicircle): position of the spot
- E (foot of the figure): the eye on the Earth

[Margin: Step 1.] *First*, then, from what is to be said in this Book 4, chapter 16, no. 18, the apparent semidiameter of the Moon at apogee, that is, the angle

AEF, is 14′. Therefore, since we have divided the semidiameter HF of the lunar disc, not as it really is but as it appears, into 100 equal parts, it is just as if we had divided the angle HEF into 100 equal angles. And so, since the part LF, which measures the amount of the whole libration, is of 12 parts, if we make: as HF, of 100 parts, is to LF, of 12 parts, so the angle HEF, of 14′, is to the angle LEF, then LEF will be found to be 1′ 41″, and the remainder HEL will be 12′ 19″.

[Margin: Step 2.] *Second*: in triangle AEK the angle AEK has been found, 12′ 19″; from what is to be said below in chapter 14, the distance AE of the Moon at apogee from the Earth is 64¼ semidiameters of the Earth; and from what is to be said below in chapter 17, problem 1, the true semidiameter AK of the Moon is of 26 parts, of which one semidiameter of the Earth is 100. From these three givens, therefore, by the laws of triangles, the angle EAK is found, and hence the arc CK, 62° 14′ 12″; so the remainder KD, to the quadrant, is 27° 45′ 48″. And the arc that we would see by force of libration beyond the arc visible without libration would be this large, if the visual ray EF saw the point D on the Moon. But we do not see the whole hemisphere of the Moon at one and the same time, since from what is to be said below in chapter 17, problem 3, we see of it only 179° 32′, and to complete the visible hemisphere it lacks the arcs FD on one side and BG on the other, each of 14′. If we subtract these from the arc KD, there will remain the arc KF, 27° 31′ 48″: namely, the arc of the Moon's surface that we see thanks to the greatest libration, sometimes toward one quarter of the world, sometimes toward the other, beyond what we would see without libration. [Margin: The portion of the Moon visible by force of libration.] Therefore, adding together everything of the Moon that is successively disclosed to us by means of libration, the visible portion of the Moon comes to 207° 3′ 48″.

Quæſtio 2. Quæ ſint maculæ, quæ ob librationem Lunarem non ſemper apparent.

Question 2. Which spots they are that, because of lunar libration, do not always appear.

VI. I reply that they are those contained within the arcs of the limits, namely, toward the north within the arcs LBM and LGM, and toward the south within the arcs ſtIM [recte: LIM] and LDM of figure 6. But besides these there are some others on the near side of the nodes L, M, because the limits of libration vary in accordance with what is to be said below; from which a conjecture will be made about these remaining ones that are sometimes hidden.

Quæſtio 3. De modis obſeruandi, versùs quas Mundi Lunæue plagas fiat Libratio Lunaris globi, & versùs quas dirigatur Axis huius Librationis.

Question 3. On the methods of observing toward which quarters of the world or of the Moon the libration of the lunar globe takes place, and toward which the axis of this libration is directed.

VII. There are three principal methods of this observation. In all of them the Moon is assumed to be in some determinate vertical, but principally in the meridian, or in the vertical passing through the nonagesimal degree of the ecliptic, so that with respect to these it may be known which quarter of the world a determinate part of the lunar limb faces. For this reason we have represented the Moon in the first figure and in figure 6, and indeed in the rest, almost in the position of the nonagesimal degree. [Margin: First method of observing the quarters of libration.] The first method is as follows: having first observed the approach of some more prominent spot to the nearer limb of the Moon, one then observes toward what point of the lunar limb the other spots situated round it on both sides converge in their approach, as toward a goal they seek. The same is to be done in observing recession. And if testimony is had from several spots, the quarter toward which the libration takes place will be much more evident, especially if, the Moon's phase permitting it, while some spots approach their limb, others on the opposite side recede from the opposite limb, which is nearer to them, and indeed from some common point of the limb diametrically opposite to the point toward which the former were seen to approach. For instance, if in figure 6 the spot of *Aristotle* has been seen to approach the limb LBM, and the spots of *Plato*, *Posidonius*, *Endymion*, etc. also seem to approach the same limb in such a way that the motion of all tends toward point G, recognizable by some indication, for example by the spot of *Thales*, or by a determinate distance from the horns of the Moon, then it will be right to conclude that the libration takes place toward the part of the world that point G faces. And if, on the opposite side, the spot of *Tycho* meanwhile seems to recede from the limb LDM, and similarly the spots *Walther*, *Stöffler*, *Pitati*, *Capuano*, etc. seem to recede, as from a common boundary, from point D, known from the nearer spot of *Zucchi*, the same will be gathered still more certainly. Yet one should not hesitate over this because of the unequal amount of approach compared with recession, since the cause of a moderate disparity can be the variation of latitude, or some anomaly of this libration. [Margin: Hidden cautions for the same observation.] Here too, moreover, one must beware of the apparent approach or recession that can arise from the cause of parallaxes. For if you observe the Moon not far from the rising horizon, and afterward not far from the setting horizon, a spot that at first was seen to be at a distance from the upper limb by, for example, a 10th part of the lunar semidiameter, will afterward be seen, near setting, to be somewhat less distant from the same limb, though now the lower one. This is because the same

limb will not appear to us in the west as appeared in the east, since we view the Moon not from the centre of the Earth but from its surface. Finally, the approach of the spots to the limb of the Moon on days of greatest libration, that is, near the limits, is less perceptible than at other times.

VIII. [Margin: Second method of observing the quarters of libration.] The second method is carried out with three or more spots first observed in one straight line, and then by observing the intermediate ones off it, and noting toward which side their recession from the straight line tends. But it is useful that the outermost of these be as close as possible to the limbs, and the intermediate ones more remote from the limbs; for their recession from the straight line will be more evident in a shorter time. For example, if you have seen *Tycho*, *Ptolemy* and *Aristotle* in one straight line, as they appear in the first figure and figure 6, and then *Ptolemy* has receded toward the limb QMR, this will be an indication of libration having taken place toward that limb. Or if you have observed *Tycho*, *Archimedes* and the western edge of *Plato* in the same straight line, and not much later *Archimedes* has moved away toward the limb SLP, libration toward the same limb will be indicated. And if you have seen in one straight line, for example, *Langren*, *Bede*, *Agrippa*, *Eratosthenes* and *Aristarchus*, and after some time *Bede*, *Agrippa* and *Eratosthenes* have pressed toward the limb PBR, leaving behind them the straight line drawn through *Langren* and *Aristarchus*, it will be a sign of libration begun toward the limb PBR. Conversely, if those which appeared off the straight line afterward come together into one straight line, you will conclude that libration is also taking place in that direction. [Margin: A twofold way of using the second method more evidently.] Further, in order that the determinate part of the lunar limb toward which libration tends may be identified more surely, one must also observe whether those spots that lie in a common straight line, cutting the former line at right angles and passing through the centre of the lunar disc, stay in that same common line; for if they stay, this very line will run into the limits of libration themselves. For example, if, while *Eratosthenes*, *Agrippa* and *Bede* recede toward PBR from the straight line drawn through *Langren* and *Aristarchus*, *Thales*, the eastern edge of *Ptolemy*, *Pitati* and *Bayer* meanwhile stay in the line DAB, it is clear that the limit toward which libration tends is the part B. This same point will be confirmed further by another notable argument, if two spots, one on either side, nearer the limbs of the Moon, stand a quadrant of the lunar circumference away from the limit toward which libration takes place, known beforehand in whatever way through the methods already mentioned, and yet they themselves have no perceptible motion, especially toward the nearest limbs. For from this you will find that the nodes or poles of libration lie on those same nearest limbs. So it would be if in the preceding example the spot of *Langren* toward C, and that of *Aristarchus* or *Seleucus* toward E, scarcely or not even scarcely move meanwhile, while *Eratosthenes*, *Agrippa* and *Bede* move toward B. For it will then be established both that libration takes place toward B and that its poles are near C and E.

IX. [Margin: Third method of observing the quarters of libration.] The third method rests on observing different dichotomies of the Moon, in which one must note through which spots, or other lunar particles, the section passes, that is, the straight line separating the bright part of the Moon from the dark. For although from a few dichotomies observed in this way nothing else can be established than whether libration always takes place toward the same parts or not, yet from more of them one can also gather toward what other parts the globe of the Moon librates. And there is no one who has observed the bisected phases of the Moon, even with a moderate telescope, and can affirm that the already mentioned section always passes through the same spots. Even so, there are definite, determinate courses on the lunar disc within which that section is contained. To represent these in a distinct diagram, as far as it has been possible to observe so far, we have presented the third figure drawn for the former dichotomy and the fourth for the latter. [Margin: Through which spots the outermost sections of the dichotomous Moon pass.] In the third figure the section passes between the centre of *Magini* and its eastern edge, through a point as far from the eastern edge of *Ptolemy* as the diameter of *Ptolemy* itself, through a point east of *Archimedes* by the semidiameter of *Archimedes*, almost through the eastern edge of *Plato*, and finally through the eastern edge of *Anaxagoras*, which is almost on the northern horn itself. In the fourth figure, on the other hand, the section passes almost through the eastern edge of *Maurolico*, through a point distant from the western edge of *al-Battānī* [*Albategni*] by three quarters of the diameter of *al-Battānī*, almost through the eastern limb of *Manilius*, and finally through the western limb of *Archytas*. Langren, indicating the variation of these sections, affirms in a letter to me dated 1649 August 22 that the section of the dichotomous Moon passes now between *Isabella* and the Empress *Maria*, which for us are *Manilius* and *Menelaus*, now between *Roma* and *Haro*, which for us are *Archimedes* and *Timocharis*. [Margin: The lines of sections at different dichotomies are not parallel to one another.] It is also very true, as Hevelius teaches in chapter 26 of his *Selenographia*, that the lines of different sections on the lunar disc are not always parallel to one another, but intersect one another in various ways, though within certain limits; this will also be clear from a careful inspection of his diagrams. Whether what he adds is true, however, that if the Moon's quadrature is about the beginning of Cancer the section is as far west as possible, and if about the beginning of Capricorn as far east as possible, depends on what is to be discussed below. Meanwhile, from the varied position of these sections, the variety of the quarters toward which libration tends will not be able to stay hidden from a shrewd observer. To this end it is convenient to append here some selected observations of the dichotomous Moon.

Obſeruationes Selectæ pro Situ ſectionum variarum Lunæ Dichotomæ prope 1. Quadraturam.

Selected Observations for the Position of various Sections of the dichotomous Moon near the 1st Quadrature.

X. [Margin: Observation 1, from Langren.] 1633 July 14, in the evening, when the quadrature was not yet at hand, the line of the section passed beyond *Eratosthenes* by as much as its diameter, and short of *Timocharis* by as much, likewise, as its diameter.

[Margin: Observation 2.] 1635 October 31, before midnight, the line of the section touched the western boundary of *Isabella*, which for us is *Manilius*, but it still seemed curved, since the dichotomy was not yet complete.

[Margin: Observation 3.] 1636 January 15, in the evening, the lunar section passed at a distance from the eastern edge of *al-Zarqālī* [*Arzachelis*] equal to the diameter of *al-Zarqālī* himself, a spot that Langren calls *Anne*, Queen of France; and it touched the eastern limb of *Roma*, which for us is *Archimedes*. Quadrature took place at the approach of midnight.

[Margin: Observation 4.] 1646 September 16, at 10^h p.m., quadrature having already taken place, the line of the section touched the western edge of *Innocent* and *Pozzo*; at the quadrature itself it had passed through the centre of the Emperor *Ferdinand*. For *Innocent* we substitute *Ptolemy*, for *Pozzo Aristillus*, and for *Ferdinand al-Battānī*.

[Margin: Observation 5.] 1647 June 9, at 10^h p.m., near quadrature, the lunar section passed through the centre of *Ferdinand*, that is, of *al-Battānī*. So far from Langren.

Our observations follow.

[Margin: Observations of ours and of Father Grimaldi.] [Margin: Observation 6.] 1649 August 15, in the evening, an hour and a half after sunset, the section passed near the eastern border of *Archimedes* and *Alfonso* [*Alfonſi*], quadrature having by then been past for some hours.

[Margin: Observation 7.] 1649 September 13, in the evening, quadrature being already past, the section was passing through *Aristillus* and *Autolycus*, and through the eastern extremity of *al-Battānī*, and finally through *Bianchini*, *Werner* and the eastern part of *d'Ailly* [*Aliacenſis*].

[Margin: Observation 8.] 1649 November 10, at the 3rd hour of the night, long before quadrature, the lunar section passed through the eastern edge of *Aristotle* and *Eudoxus*, yet *Menelaus* was not yet appearing. The spot of *St Catherine* was distant from the section toward the west by as much as the interval between its southern edge and the northern edge of *St Theophilus*.

[Margin: Observation 9.] 1649 December 10, at the 1st hour of the night, about the fifth hour before quadrature, the Moon's section was distant from *Aristotle* toward the east by as much as the breadth of *Aristotle*, and from the spot of *Maurolico*, likewise toward the east, by as much as its diameter, and yet nothing of *Manilius* was discerned.

[Margin: Observation 10.] 1650 January 9, at 1½^{h} after sunset, the section was distant toward the east from the western edge of *Aristotle* by as much as the northern edge of *Aristotle* from the southern edge of *Eudoxus*; it was likewise distant toward the east from *Manilius* by as much as the interval between the nearest edges of *Aristotle* and *Eudoxus*; and it passed through *Maurolico*. Although the line of the section appeared slightly curved, it does not necessarily follow from this that quadrature had not yet been completed, since that curvature arose from parallax, in accordance with what is to be said in the appendix to Book 3, chapter 7. At 5^{h} after sunset, however, all of *al-Battānī* and *Hipparchus* now appeared, standing on the line of the section, which now appeared straight, even though at 1½^{h} after sunset nothing of *Hipparchus* and *al-Battānī* had appeared; now, moreover, the section was distant toward the east from *Manilius* by exactly as much as *Manilius* is distant from *Menelaus*.

[Margin: Observation 11.] 1650 February 8, at 3½^{h} after sunset, the section passed through *Ptolemy*, of which only the bright circumference appeared, and half of this stood out toward the east beyond the Moon's bright semidisc. The same section grazed the eastern edge of *Walther*, which was bright but broken up as if into little gems. *Manilius* was distant toward the west from the section by almost double its distance from *Menelaus*.

[Margin: Observation 12.] 1650 March 10, at 4^{h} after sunset, the Moon's section, slightly bent toward the east, passed through the eastern edge of *Archimedes* and through the middle of *Magini*, and it was distant toward the east from *Alfonso* by about two-thirds of the Alphonsine diameter.

[Margin: Observation 13.] 1650 April 9, half an hour after sunset, the section ran through the eastern edge of *Meton*, through the middle of *Plato*, and through *Eratosthenes*.

[Margin: Observation 14.] 1650 May 8, at 1½^{h} after sunset, the Moon, even when seen through the telescope, appeared dichotomous, and its section was scarcely further east than the eastern edge of *Archimedes*. It was distant toward the east from *Ptolemy* by *Ptolemy's* semidiameter, and finally it passed through the middle of *Magini*.

[Margin: Observation 15.] 1650 June 6, at 1^{h} after sunset, although many hours before quadrature, the section was nevertheless observed through the eastern extremity of *Meton*, which occupies the Moon's limb and the horn itself. It was distant just a little toward the east from *Aristillus*, and likewise toward the east from *al-Battānī* by its semidiameter, and it went on through the middle of *Walther*.

[Margin: Observation 16.] 1650 October 2, half an hour after sunset, the Moon appeared through the telescope more than dichotomous, although to the naked eye it appeared otherwise. The section passed through *Walther*, and it was distant toward the east from the eastern edge of *al-Battānī* by its semidiameter. The spots of *Aristillus* and *Autolycus*, moreover, were conspicuous, in the bright part of the Moon indeed, but a little removed from the section.

[Margin: Observation 17.] 1650 October 31, an hour and a half after sunset, the section was distant toward the east from *Aristotle* by a little more than *Aristotle* and *Eudoxus* are distant from one another. It grazed the eastern shore of the *Sea of Tranquillity*, being distant from *Manilius* by half the interval between *Manilius* and *Menelaus*; and finally it was distant toward the east from *Maurolico* by scarcely more than one diameter of *Maurolico*.

[Margin: Observation 18.] 1650 December 29, at 3^{h} after sunset, the section was distant toward the east from *Aristotle* by almost two diameters of *Aristotle*, and from *Menelaus* by as much as the gap between *Menelaus* and *Pliny*; and it passed through *Abū al-Fidāʾ*.

[Margin: Observation 19.] 1651 March 28, at $2½^{h}$ after sunset, a line falling at right angles on the line of the lunar section passed through the centre of *Hipparchus*, and above *St Theophilus* by as much as the latter's diameter; and the Moon was almost dichotomous.

[Margin: Observation 20.] 1651 April 27, 40^{m} after sunset, the Moon was seen already dichotomous, and the section passed through the middle of *Ptolemy* and through the eastern edge of *Walther*, and it was distant toward the east from the centre of *Aristillus* by as much as the diameter of *Ptolemy*; of *Archimedes* nothing was seen except little gems and bright points. The northern edge of *Ptolemy* was equally distant from each of the Moon's horns, and a straight line drawn perpendicular through the line of the section and through the centre of *Ptolemy* passed above the spot of *Langren* by almost one diameter of that spot.

[Margin: Observation 21.] 1651 May 27, at 1^{h} after sunset, since quadrature had occurred almost 6 hours before, the Moon appeared scarcely gibbous, and the line of the dichotomy passed through the eastern edges of *Archimedes* and *Ptolemy*.

[Margin: Observation 22.] 1651 June 25, the section of the Moon, not yet dichotomous, passed through the eastern edge of *Ptolemy*, which had only its circumference bright. Some fragments of the perimeters of the spots of *Alfonso*, *al-Zarqālī* and *Walther* also appeared; through *Walther's* eastern limb the section passed, and it was distant toward the east from *Aristillus* by one diameter of *Ptolemy*; but nothing of *Archimedes* appeared. Finally, the line bisecting at right angles the line of the section, that is, the line passing through the horns, was just a little above *Ptolemy*, and above *Langren* by one diameter of *Ptolemy*.

[Margin: Observation 23.] On the 26th, an hour and a half after sunset, with quadrature now completed, the lunar section was distant toward the east from *Plato* by half of its length, and from *Eratosthenes* by a whole diameter of the same; and it passed through the centre of *Pitati*, and between the centre and the eastern limb of *Tycho*.

[Margin: Observation 24.] 1651 July 25, 8^{m} after sunset, the Moon appeared already gibbous in the largest telescope, but still crescent-shaped to the naked eye. The lunar section, departing little from a straight line, passed through the middle of *Plato*, and it was distant toward the east from *Archimedes* by one of

its diameters, from *Ptolemy* by almost one of the latter's diameters, and from the centre of *Magini* by a quarter of that diameter.

[Margin: Observation 25.] 1651 August 23, at 1^h after sunset, the section of the Moon, not yet bisected, passed through the eastern edges of *Ptolemy*, *Peuerbach* and *Regiomontanus*, and it was distant toward the east from *Aristillus* by as much as the interval between the centres of *Aristotle* and *Eudoxus*.

Obſeruationes Sectionum Lunarium prope 2. Quadraturam.

Observations of Lunar Sections near the 2nd Quadrature.

[Margin: Observation 26. From Langren.] XI. 1633 January 3, in the morning, with the Sun touching the horizon, the line of the lunar section passed over *Magini*, called *Leopold* by Langren, that is, through its western quarter; and it was distant toward the west from *Vladislaus* (who for us is *Tycho*) by the diameter of *Tycho*. It cut *Christiern*, for us *Peuerbach*, so that only a quarter of it was visible, and it touched the eastern edge of *Archimedes*, of which likewise only a quarter appeared; but *Eratosthenes* and *Timocharis* were equally distant toward the east from the line of the section, which touched the western edge of *Plato*.

[Margin: Observation 27. From Father Grimaldi.] 1649 July 31, 1^h before sunrise, that is, about 8 [hours] before quadrature, the line of the lunar section, still curved, passed through the western edge of *Walther* and of *al-Battānī*, and, bending beyond *Aristotle*, it was seen to end between *Democritus* and *Meton*.

[Margin: Observation 28.] 1649 September 28, in the morning at sunrise, the line of the section stood almost through the middle of *Hipparchus*, but more toward the east, and through the middle of *al-Battānī*, and there was not yet a dichotomy.

[Margin: Observation 29.] 1649 November 27, in the morning, at $13½^h$ in Italian civil reckoning, the Moon appeared already slightly crescent-shaped, and its section was between the centre and the western edge of *Plato*. It was distant toward the west from *Eratosthenes* by as much as the length of *Plato*, and from the bright nucleus of *Tycho* by as much as the thickness of its darker corona. The ephemerides, however, put off quadrature to 20^h after noon on the 26th.

[Margin: Observation 30.] 1650 January 25, in the morning, half an hour before sunrise, the Moon appeared to the naked eye, and in the telescope too, already crescent-shaped, but the section scarcely appeared concave at its middle. The line of the section ran through the western edge of *Plato*, and it was distant toward the west from *Eratosthenes* by scarcely more than half the interval between *Eratosthenes* and *Copernicus*; from *Tycho*, moreover, it was distant toward the west by twice the bright diameter of *Tycho* itself.

[Margin: Observation 31.] 1650 March 24, shortly before sunrise, the Moon, standing on the meridian itself, was still just slightly gibbous, for quadrature was to occur at about 2^h after noon. The lunar section, further, passed through

the western cusp of *Meton*, and it was distant toward the west from *Aristillus* by as much as *Aristillus* is from the centre of *Archimedes*; it grazed the western edge of *Hipparchus*, and from *Tycho* it was distant toward the west by as much as *Tycho* is distant from the northern edge of *Pitati*.

[Margin: Observation 32.] 1650 April 23, 1^h before sunrise, with the dichotomy now completed, the line of the section was almost straight in the telescope, although to the naked eyes it appeared curved. This line was distant toward the west from *Plato* by twice the length of *Plato*; from *Aristillus* by one diameter of *Archimedes*; from *Tycho* by the interval between *Tycho's* extremity and the northern extremity of *Pitati*; and it passed through the middle of *Hipparchus*, next to *Ptolemy*.

[Margin: Observation 33.] 1650 September 17, at the very moment of sunrise, the section was distant just a little toward the west from *Hipparchus*, and from *Aristillus* and *Autolycus*, likewise toward the west, by as much as the centre of *Aristillus* from the centre of *Archimedes*, and it passed through the middle of *Meton*. The Moon was still gibbous; yet a straight line drawn from horn to horn passed through *al-Battānī*.

[Margin: Observation 34.] 1650 October 16, at $5\frac{1}{2}^h$ after sunset, a straight line drawn through the horns of the still gibbous Moon passed through the eastern edge of *al-Battānī*. The section, however, stood through the middle of *al-Battānī* and *Hipparchus*.

[Margin: Observation 35.] 1650 December 16, an hour and a half before sunrise, the section of the Moon, then dichotomous and standing on the meridian, was through the western edge of *Plato* and through the middle of *Magini*; but it was distant toward the west from *Eratosthenes* by scarcely one of its diameters. And of the mountains of the *Land of Snows* a tract stretched straight out beyond the Moon's shining semidisc, as long as one and a half diameters of *Eratosthenes*.

[Margin: Observation 36.] 1651 May 12, in the morning, about half an hour before sunrise, the section of the dichotomous Moon was distant toward the west from *Aristillus* by almost one diameter of *Archimedes*, and from *Autolycus* by a little less; and it passed through the middle of *al-Battānī*.

[Margin: Observation 37.] 1651 July 9, at sunrise, the lunar section grazed the western edge of *Hipparchus*, and it was distant somewhat toward the west from *d'Ailly*; from *Tycho*, moreover, it was distant by about one length of the *Caspian* or *Sea of Crises*.

[Margin: Observation 38.] 1651 August 8, 1^h before sunrise, the lunar section ran through the western edges of *d'Ailly* and *Hipparchus*, and it was distant toward the west from *Aristillus* by as much as *Aristillus* itself from *Archimedes*. The Moon appeared crescent-shaped to the naked eyes, but in the telescope evidently dichotomous.

[Margin: Observation 39.] 1651 September 6, at $6\frac{1}{2}^h$ after sunset, the section of the almost dichotomous Moon passed through the western edge of *Walther*, and it was distant toward the west from the centre of *al-Battānī* by a quarter of

the diameter of *al-Battānī*; and from *Aristillus* and *Autolycus* it was distant by as much as they are from one another.

[Margin: Observation 40.] 1651 December 5, in the morning, at about $12\frac{1}{2}^{h}$ in Italian civil reckoning, the Moon was seen markedly crescent-shaped with the naked eyes, but scarcely crescent-shaped in the largest telescope. The Moon's section passed through the western edge of *Archimedes*, and it was distant toward the west from the western edge of *Plato* by almost one length of *Plato*; and it passed through the western edges of *Alfonso* and *Magini*.

Besides the already mentioned observations, observations of other lunar phases are very useful for inferring the various quarters of the libration, by considering the line from horn to horn as it passes through one spot after another. A line falling at right angles on the straight line drawn through the horn is likewise useful for this, if the spots through which it travels are noted, as in Observations 19, 20 and 22.

Quæſtio 4. De Obſeruandis Temporibus Librationum Lunæ maximarum.

Question 4. On observing the times of the Moon's greatest librations.

XII. The moment, or even the hour, of greatest libration, that is, of greatest approach to the Moon's nearer limb or of greatest recession from it, cannot be known from direct observation, because near its time that approach and recession, or their difference from the motion of the preceding days, is scarcely perceptible. Therefore one must observe the time between the last perceptible approach and the first perceptible recession of the same spot, or between the last recession and the first perceptible approach, and divide it in two, so that the almost exact moment of greatest libration may become known, and from it the period from one libration to the other. But if these cannot be observed, one must resort to the days on which the spots seem to stick in a position nearest the limbs. Which spots, further, are suited to discerning the greatest libration has been said sufficiently in Nos. 4 and 7.

[Margin: Spots suited to this observation.] Now although those spots might seem most apt that reach the Moon's very edges, yet because very few of them do not vanish in some part of themselves beyond the edges and do not, by their optical contraction, thrust ambiguity on the observer, it has seemed better generally to use those that do not indeed reach the Moon's edges but are much nearer to them than to the centre of the lunar disc, and that are very conspicuous because of their striking characteristics. Such are *Tycho*, *Pitiscus*, *Langren*, *Aristotle*, *Plato*, *Harpalus*, *Grimaldi*, and, diametrically opposite to the last, the *Sea of Crises*, which is discerned even with the naked eye, and indeed is sometimes recognised even when it stands in the dark part of the Moon, by the force of the light reflected from the Earth onto the Moon, provided the Moon stands clear of the horizon. Yet most have so far used in this only two spots, namely the *Sea of Crises* and *Grimaldi*, under other names. Granted that

the quarters of the greatest librations are not always or generally in that part of the lunar limb which is nearest to the already mentioned spots, nevertheless their approach to and recession from the limb, although oblique, has seemed to those authors sufficient for indicating the times of the greatest librations. Before we determine anything about these times, some observations must be set out first, both those of others and our own.

Obſeruationes Temporis Maximi Acceſſus Macularum binarum ad Lunæ limbum Occidentalem, aut Orientalem ex Gaſſendo, & Bullialdo.

Observations of the Time of Greatest Approach of the two Spots to the Moon's Western or Eastern Limb, from Gassendi and Boulliau.

XIII. Boulliau reports these in Book 3 of the *Astronomia Philolaica*, chapter 13. Although they call the *Sea of Crises* the *Caspian* spot and *Grimaldi* the Eastern spot, we shall nevertheless use our own names too in reporting the observations of others, so that the reader may use our Figure 6.

[Margin: Observations of Gassendi.] 1636 September 25, *Grimaldi* was seen as close as possible to the eastern limb; on October 20, again very near to it; and on November 19 and 20, nearest to it. But on 1642 April 14 it was seen far removed from the same limb. The *Sea of Crises*, on the other hand, on 1636 October 8 and likewise the 9th, was so close to the Moon's western limb that it had never been seen closer by Gassendi; and on November 9, in the evening, it was as close to the same limb as it had ever been at any other time, so that the gap was scarcely a quarter of the already mentioned sea, although the sea was more contracted than usual. On December 5 it was again very close to the same edge; and again, on 1637 February 27, in the evening, it was nearest to the same limb, and on the 28th nearer still. But on 1638 January 24, while the Moon was occulting the Pleiades, the same sea was very thin and near the limb; and on December 20, in an eclipse of the Moon, it was nearest, as also on 1639 February 13.

[Margin: Observations of Boulliau.] 1643 June 3, at 11^h, *Grimaldi* appeared in the same position as before, that is, distant from the eastern limb by a little more than its own length; and on October 1 it became so close that it had never before been seen so close by Boulliau; as also on October 27 it was very near to the same limb. On November 30, however, it was distant from the eastern limb by half its length. The *Sea of Crises*, on the other hand, on 1643 May 3, at $8½^h$, was distant from the western limb by scarcely a quarter of its breadth, and the oblong spots adjoining the western edge of the same sea had disappeared. Again, on May 30 it approached the limb, and on the 31st still more, and on June 2 the western edge of the same sea was less far from the western limb than a 5th part of its breadth. Then, on September 18, at $7½^h$, the western shore of the same sea almost grazed the Moon's western limb, and Boulliau attests that it had not been seen closer by him before, so that the interval scarcely

equalled a 6th part of the breadth of the sea itself, although that breadth was very contracted. But it is expedient to set out a synopsis of these observations in the following small table, adding the Moon's place and latitude, selecting, however, those in which the distance of the spots from the limb was least.

Least distance of the Eastern spot, that is, Grimaldi, or of the Western spot, that is, the Caspian or Sea of Crises, from the nearer limb of the Moon.

Year, Month, Day	Spot	Moon's Place	Moon's Latitude
1636 Sept. 25	*Grimaldi*	♍ 22°	South 2°
Oct. 8 & 9	*Sea of Crises*	♒ 5°	South ½°
Oct. 20	*Grimaldi*	♋ 21°	North 1½°
evening, Nov. 9	*Sea of Crises*	♈ 3°	North 4°
Dec. 5	*Sea of Crises*	♓ 14°	North 3½°
1637 Feb. 27, 28	*Sea of Crises*	♈ 18°	North 4¾°
1638 Jan. 24	*Sea of Crises*	♋ 16°	none
Dec. 20	*Sea of Crises*	♊ 24°	none
1639 Feb. 13	*Sea of Crises*	♊ 27°	none
1643 May 3	*Sea of Crises*	♏ 10°	North 3°
May 31	*Sea of Crises*	♏ 20°	North 3¾°
June 5	*Sea of Crises*	♌ 27°	North 4½°
evening, Sept. 18	*Sea of Crises*	♏ 27°	North 4½°
Oct. 1	*Grimaldi*	♊ 1°	South 4¾°

Obſeruationes Tēporis Maximæ & Minimæ Diſtantiæ inſignium quarumdam macularum à limbo Lunæ propiore, ex P.Grimaldo: addita quantitate prædictæ diſtantiæ.

Observations of the Time of Greatest and Least Distance of certain notable spots from the Moon's nearer limb, from Father Grimaldi, with the amount of the already mentioned distance added.

[Margin: Observations of Father Grimaldi.] XIV. For brevity's sake we shall gather these observations into the following small tables, and we shall add the amount of the spots' distance from the Moon's nearer limb in parts of which the Moon's temporary semidiameter is 100, according to the selenographic scale placed at the foot of Figure 6. Let the reader note, however, that the greatest distance means that which occurs within each single period of the librations; otherwise, if the greatest of one period is compared with the greatest of different periods, they are not all equal to one another, just as the least are not either when compared with other least distances. For what purpose we have added the Moon's place and latitude will be clear from what is to be said; for some have suspected that the greatest librations return whenever the Moon has its greatest latitude, or whenever it returns to the beginning of Cancer or Capricorn. Remember, however, that the difference between the greatest and least distance of *Tycho* from the Moon's southern limb, which sometimes reaches 18 parts, is unsuited to determining the portion of the lunar surface that

we see by the force of libration beyond what we should see without libration, for the reason already indicated at the end of No. 4.

Distance of the western shore of the Sea of Crises, or Caspian, from the Moon's western limb, in parts of which the Moon's semidiameter was 100.

Year	Month, Day	Dist.	Parts	Moon's Place	Moon's Latitude
1649	Aug. 12	min.	6	♎ 13°	B 4°
	Aug. 21	Max.	12	♒ 20°	A 5°
	Sept. 18	Max.	11	♒ 29°	A 5°
	Oct. 14	Max.	13	♒ 13°	A 5°
	Nov. 12	Max.	14	♓ 5°	A 5°
	Dec. 11	Max.	12	♈ 1°	A 3°
1650	Jan. 9	Max.	12	♈ 16°	A 3°
	Mar. 7	Max.	8	♉ 14°	A 0°
	Mar. 16	min.	3½	♍ 6°	B 5°
	Apr. 11	min.	4	♌ 18°	B 5°
	May 8	min.	3	♌ 13°	B 5°
	June 6	min.	3	♍ 4°	B 5°
	July 2	min.	3	♌ 18°	B 5°
	Aug. 1	min.	3	♍ 23°	B 4°
	Aug. 29	min.	3	♎ 3°	B 3°
	Sept. 10	Max.	11	♓ 21°	A 4°
	Oct. 7	Max.	11	♈ 1°	A 3°
	Nov. 4	Max.	11	♓ 27°	A 5°
	Dec. 2	Max.	10½	♈ 7°	A 2°
	Dec. 30	Max.	11	♈ 17°	A 2°
1651	Feb. 25	Max.	8	♉ 3°	B 2°
	Mar. 26	min.	7½	♊ 18°	B 4°
	Apr. 4	min.	3½	♍ 26°	B 3°
	May 4	min.	3¼	♏ 4°	B 1°
	May 31	min.	3½	♎ 26°	B 1°
	June 27	min.	3	♎ 21°	0 0°
	July 25	min.	3	♏ 3°	A 1°
	Aug. 21	min.	3	♎ 25°	A ¼°
	Oct. 27	Max.	11½	♈ 28°	A ¾°

Distance of the eastern limb of the spot Grimaldi from the Moon's eastern limb.

Year	Month, Day	Dist.	Parts	Moon's Place	Moon's Latitude
1649	July 28	min.	4½	♈ 1°	A 4½°
	Sept. 2	Max.	14	♋ 17°	B 4°
	Sept. 30	Max.	11	♋ 24°	B 4°
	Nov. 27	Max.	11½	♍ 7°	B 5°
1650	Feb. 16	Max.	12½	♌ 28°	B 5°
	Mar. 19	Max.	9	♎ 17°	B 2°
	Apr. 26	min.	4	♓ 18°	A 4°
	May 25	min.	4	♈ 10°	A 3°
	June 21	min.	4	♈ 7°	A 3°

Year	Month, Day	Dist.	Parts	Moon's Place	Moon's Latitude
	July 19	min.	3¾	♈ 14°	A 2½°
	Aug. 15	min.	4⅓	♈ 10°	A 2¾°
	Sept. 9	min.	4	♓ 8°	A 4½°
	Oct. 20	Max.	10	♍ 1°	B 5°
	Dec. 17	Max.	12	♎ 11°	B 2°

Distance of the northern limb of Endymion from the Moon's northern limb.

Year	Month, Day	Dist.	Parts	Moon's Place	Moon's Latitude
1650	Apr. 11	min.	3	♌ 21°	B 5°
	June 7	min.	0	♍ 21°	B 4°
	Oct. 8	Max.	10	♈ 4°	B 2½°
	Nov. 5	Max.	9	♈ 14°	A 2°
	Dec. 2	Max.	9	♈ 12°	A 2°
1651	May 26	min.	2	♌ 26°	B 4½°
	June 22	min.	1	♌ 22°	B 4½°
	July 20	min.	1	♍ 1°	B 4°

Distance of the northern limb of Plato from the Moon's northern limb.

Year	Month, Day	Dist.	Parts	Moon's Place	Moon's Latitude
1651	July 4	Max.	28	♓ 28°	A 5°
	Aug. 2	Max.	26	♒ 4°	A 4°
	Aug. 29	Max.	24	♒ 22°	A 4°

Distance of the centre of Tycho from the Moon's southern limb.

Year	Month, Day	Dist.	Parts	Moon's Place	Moon's Latitude
1649	Sept. 23	min.	28	♉ 3°	A 2°
	Nov. 24	Max.	40	♊ 18°	B 4¾°
1650	Feb. 16	Max.	40	♍ 1°	B 5°
	Mar. 18	Max.	36	♎ 6°	B 3°
	Apr. 25	min.	25	♓ 2°	A 5°
	Sept. 8	min.	25	♓ 3°	A 4½°
1651	Feb. 7	Max.	37	♍ 25°	B 3°
	July 9	min.	22	♈ 1°	A 2°
	Aug. 3	min.	24	♓ 12°	A 3°
	Aug. 29	min.	26	♒ 21°	A 4⅔°
	Nov. 29	Max.	38	♋ 7°	A 5°

Quæſtio 5. Quanta ſit Periodus cuiuſuis Librationis ſingillatim ſumptæ.

Question 5. How long is the period of each libration taken singly?

[Margin: Boulliau's opinion on this period.] XV. Ismaël Boulliau, in Book 3 of the *Astronomia Philolaica*, chapter 13, after comparing the observations of Gassendi and his own falling within seven years, of which there are 13 in number, finally concludes that the period of the lunar libration, that is, the time in which the same spot returns to its least distance from the same limb of the Moon from which it had previously been least distant, is of 26 days 14 hours 40′; and so that the daily motion of this libration is 13° 29′ 8″ 54‴ 20⁗ 20⁗′, and the annual motion 8ˢ 2° 19′ 10″ 33‴ 41⁗ 40⁗′. But he admits that this measure does not answer to all the observations, and that this motion includes some anomaly which is still hidden from astronomers because of the small number of observations.

[Margin: Hevelius's opinion on this period.] Johannes Hevelius, in chapter 8 of his *Selenographia*, says that the motion of libration of the lunar disc is completed in about a month's space. For about 14 days, while the Moon moves through the descending signs, its disc librates backward, against the order of the signs, from the south-east toward the north-west [ab Euroauſtro Corum verſùs], that is, toward the upper part, which in the 6th figure is toward the limit B. For the remaining 14 days or so, while the Moon moves through the ascending signs, it librates forward, in the succession of the signs, from the north-west toward the south-east, that is, toward the limit I in our 6th figure. In this way, he says, this motion fits itself entirely to the motion of longitude, although it is made more intricate by the Moon's latitude. And in chapter 9 he affirms that the *Sea of Crises*, which for him is the *Maeotian Marsh*, attains by force of libration its greatest distance from the Moon's limb when the Moon is both in Cancer and has its greatest southern latitude, and its least distance when the Moon is in Capricorn, around the limit of northern latitude. And in chapter 18, speaking of the spaces around the Moon's periphery near the *Maeotis*, he says: *They are extended as far as possible when the greatest libration of all in the Moon stands toward the south-east: that is, when the Moon is, so far as it has been possible to gather up to now, at the beginning of Cancer; and again they are compressed as far as possible when the Moon lingers in Capricorn.* Again, in chapter 54, from various lunations he observed up to the year 1646, he tries to confirm that libration arises from the motion of longitude, of latitude and of the nodes; and that around the beginning of Cancer, especially near the limit of southern latitude, the greatest distance lies between the *Maeotis* and the Moon's western limb, but the least between the *Mareotic Marsh* (we have called this the spot *Grimaldi*) and the Moon's eastern limb. In confirmation he adduces observations of Gassendi and Boulliau taken from Boulliau's Book 3, chapter 13; but he admits that three of them are shaky. Finally, in the same chapter, on p. 438, he concludes in these words: *But whether, in the way just stated, the greatest and least libration persists constantly and perpetually in Cancer and Capricorn, or whether over the course of a longer time the Moon undergoes certain further inequalities, or whether the matter stands otherwise, I cannot for the present affirm with certainty. What I can above all affirm is this: that up to now, in the time*

of my observations, I have detected nothing else than that the limits of libration have been noticed continually in those signs, namely Cancer and Capricorn. I am therefore wholly of the opinion that the limbs of greatest and least libration in the Moon will indeed never vary with respect to the spots; but that the motion of greatest and least libration, or its limits, will not remain perpetually in the same signs, but will in time gradually pass out of Cancer and Capricorn, following the retrogression of the nodes. [Margin: Summary of Hevelius's doctrine.] He therefore urges astronomers to observe this from now on.

[Margin: Hevelius's 1st assertion.] We therefore have three things from Hevelius. *First*, that this period, from greatest to greatest distance of the same spot from the Moon's limb, is of about 28 days. [Margin: 2nd assertion.] *Second*, that the greatest libration, that is, the greatest distance of the western spot from the western limb, does at these times occur when the Moon is at the beginning of Cancer, but with its greatest southern latitude; and so that at that time also the disc of the Moon extends as far as the northern limit of libration, from G to B. Conversely, the least distance of the same spot occurs when the Moon is at the beginning of Capricorn and holds its greatest northern latitude; and so at that time the disc of the Moon extends as far as the southern limit of libration, from D to I. [Margin: 3rd assertion.] *Third*, that *the limits of the librations keep an invariable position on the Moon's disc with respect to the spots*: that is, that the limits of greatest libration are always on approximately the diameter BI of the sixth figure, which passes through the western edge of the spot *Thales* and through the eastern part of the spot *Pitati*; so that the northern limit is always near B and the southern near I, and so that the nodes or poles of this libration are always near the points L and M; but that the greatest librations will not always occur with the Moon at the beginning of Cancer or Capricorn, etc.

[Margin: How certain Hevelius's first assertion is.] As regards the period of this libration, however: compare the earliest observation of the least distance of the spot *Grimaldi* that is on record in the preceding small tables, which from Gassendi was 1636 September 25, with the latest observation of the least distance of the same spot, which for us was 1650 September 9. Between these observations lie 5097 days, which divided by 28 yield 182 periods of libration and 1 day. But compare the earliest observation of the least distance of the *Sea of Crises* or Caspian, which for Gassendi was between 8 and 9 October of the year 1636, with the latest, which for us was 1651 August 21. Between them lie 5440 days, which divided by 28 give 194 periods and 8 days; if you divide by 29 there are 187 periods and 17 days; if by 30, 181 periods and 10 days; if on the other hand you divide by 27, 201 periods and 13 days; and if finally you divide by 26, 209 periods and 6 days. So although in the first example, that of the spot *Grimaldi*, almost no fraction was left over, in the others one was, and therefore a divisor with a fraction would have had to be assumed. But with no divisor of this kind will you find a number of periods without a fraction; or if you do find one, it will still not by itself satisfy each and every one of the periods of the librations, unless you apply some prosthaphaeresis. [Margin: Our opinion

on this period.] Therefore a twofold difficulty in this motion still remains for posterity, if there are any who wish to pursue these matters further. The first concerns the mean motion, or mean period, of this libration; the second concerns the motion of its proper anomaly and the quantity of the equation to be added or subtracted, so that it can be known for a given time how great the distance of the already mentioned spots from the nearer limb of the Moon ought to be. Both inquiries, moreover, require observations not only of more years than we have so far, but even of several centuries. For the quantities of the lunar month, and of the anomalistic and of the draconitic or latitude month, have terms and revolutions that can be observed more surely from lunar eclipses, and without the error of a whole hour; and yet astronomers could not bring them to an exact standard until Hipparchus, Ptolemy and the others skilled in this art brought the observations of many centuries into counsel, as will be clear from what is to be said from chapter 21 to 25. How much more, then, are intervals of this kind needed in this business, since in discerning the terms of this libration by observation it is easy for an error to creep in, not of one or two hours, but of several hours. For the same spot usually lingers at its greatest or least distance from the same limb for two, three or more days before its first withdrawal or approach becomes perceptible. And this itself is often unobservable, not only for the two causes that also sometimes make lunar eclipses unobservable, namely clouds or the brightness of day, but also because lunar spots otherwise suited to this observation are not illuminated by the Sun in such a way that their approach or withdrawal can be noted. Consequently nothing certain has so far been known about this period, except that (speaking of the mean period) it is greater than 26 days and less than 30, and that it is not very unlike the other monthly revolutions of the Moon, nor indeed the monthly rotation of the solar spots, which from what was said in Book 3, chapter 3, no. 8, is of about 27 or 28 days. Finally, if we were to grant, what we shall nonetheless shortly deny, that this period is as long as the time the Moon spends in returning to the same beginning of Cancer or Capricorn, nor that this libration has its own peculiar anomaly underlying it, this period would still be unequal, and indeed with an inequality as manifold as is the anomaly in the Moon's motion of longitude and latitude, to which Hevelius thinks this libration fits itself entirely.

Quæſtio 6. Quibus Anni Menſiſue temporibus contingat Maxima Lunaris diſci Libratio.

Question 6. At what times of the year or month does the greatest libration of the lunar disc occur?

[Margin: Hevelius's 2nd assertion is attacked.] XVI. Now from what was said in the preceding number, especially in Hevelius's 2nd assertion, it is clear that, if you follow that author's opinion, the times of these librations are those when the Moon is at the beginning of Cancer with its greatest southern latitude,

or at the beginning of Capricorn with its greatest northern latitude; although he suspects that, as the nodes of the lunar orbit gradually regress, these greatest librations will at some time occur in other signs as well. But if we are to abide by the observations that exist not only in Hevelius himself, but in Boulliau's Book 3, chapter 13 (the very ones Hevelius adduces in his own favour), and in Father Grimaldi's papers, Hevelius is certainly mistaken. For according to him the western spot, called the *Caspian* by Gassendi and Boulliau and the *Sea of Crises* by us, ought in the years just past to have held its greatest distance from the western limb when the Moon occupied the beginning of Cancer and the greatest southern latitude, and at that time the eastern spot, namely *Grimaldi*, ought to have reached its least distance from the eastern limb. Conversely, when the Moon occupied the beginning of Capricorn and also had its greatest northern latitude, the western spot ought to have acquired its least and the eastern spot its greatest distance from the nearer limb. It is useful to have these alternations before one's eyes in the following synopsis.

Distance from the Moon's nearer limb, according to Hevelius.

Spots		The Moon being
Western, or Crises	Greatest	At the beginning of ♋ with greatest southern latitude
	Least	At the beginning of ♑ with greatest northern latitude
Eastern, or Grimaldi	Greatest	At the beginning of ♑ with greatest northern latitude
	Least	At the beginning of ♋ with greatest southern latitude

Now if you run through the observations of the two already mentioned spots reported in nos. 13 and 14, with the Moon's places in the zodiac and its latitudes added in the small tables of observations, not even one is found that supports Hevelius; indeed almost all oppose him. For in none do those three conditions concur, namely the beginning of Cancer, and southern latitude, and that the greatest, or the beginning of Capricorn with northern latitude, and that the greatest; in most of them, on the contrary, opposite or different conditions are found. You will notice this also in the observations reported above of the remaining three spots, namely *Endymion*, *Plato* and *Tycho*. And they oppose him not by a few minutes in latitude or by a few degrees in the Moon's longitude, but in the former by whole degrees, and even in the very kind of latitude, and in the latter for the most part by whole signs, and even by aggregates of several signs. Moreover, those very few observations that Hevelius sets out from his own stock, or those that he suppresses unexplained or merely indicates, although a great many ought to have been brought to light, were not sufficient to introduce a universal assertion for these times, and one of such moment in astronomy. And if the rest are to be judged from the very ones he brings forward, a great suspicion arises that, because of the novelty of the phenomenon, he brought forth this immature thesis in too hasty a birth. For in chapter 54, from p. 433, laying the foundations of this opinion of his, he says that in the year 1645, December 17, the conjunction of the Moon was near

the beginning of ♈ with a great northern latitude, and that he then observed the least libration at the right-hand limb of the Moon and the greatest at the left; and that in the year 1646, January 1, at 8^h in the evening, the true full Moon was near the beginning of ♋ with a great southern latitude, and that he then observed *almost the greatest* libration at the Moon's right-hand limb and the least at the left. And yet, although at the time of the already mentioned conjunction the Moon was a few degrees from the beginning of ♈, it did not have its greatest northern latitude, since it had one less than 4°; and at the time of the already mentioned opposition the Moon was almost 10° from the beginning of Cancer, and, what stands most in Hevelius's way, the Moon's southern latitude did not exceed 2½°, whereas it ought to have been 5° in order to be greatest. From this very example, moreover, it is clear that Hevelius was content with *almost the greatest* librations, and that he cannot take refuge, to defend his assertion, in the greatest of the greatest; these in turn occur so rarely that the life of one astronomer does not suffice to acquire as large a number of them as the already mentioned assertion would require.

[Margin: Our opinion on the times and places of these librations.] I therefore think it must be admitted that the greatest librations of the Moon, even in this our own time, occur indifferently in any degree of the zodiac; just as neither eclipses nor the nodes or limits of the lunar orbit are tied to fixed degrees of the signs, still less to the beginning of any sign. A sufficient foundation for this opinion is furnished both by the observations reported above and by not a few others.

Quæſtio 7. An in determinata parte limbi Lunaris ſint Limites maximæ Librationis, & Nodi eiuſdem vel Poli.

Question 7. Are the limits of the greatest libration, and its nodes or poles, in a determinate part of the Moon's limb?

[Margin: Hevelius's 3rd assertion is disproved.] XVII. From what was reported in no. 15 under the 3rd assertion, Hevelius held that the limits of each lunar libration keep the same position on the Moon's disc perpetually *with respect to the spots*; and so that the limit of northern libration, in the 6th figure (which we have drawn from Hevelius's plate to represent this very point, as regards the twin circumferences of the lunar disc), is perpetually at B, and that of southern libration at I, and their poles or nodes at L and M, or as nearly as possible. But Hevelius has beyond doubt been deceived, and anyone will be deceived who, in observing the quarters of the lunar librations, has not kept the rules given above in no. 7. That the position of the already mentioned librations and nodes changes *with respect to the spots* we shall show from observations selected for this purpose and carried out according to the already mentioned rules, and from four arguments drawn from them.

[Margin: 1st argument against Hevelius.] *First*, if the limits of the librations clung immovably to the same position on the lunar disc, then when spots very

distant from the limits and nearer the nodes withdraw very perceptibly from the Moon's nearer limb, spots nearer the limits assigned by Hevelius ought also at the same time to move away from the Moon's nearer limb by a perceptible withdrawal. This is said of pairs of spots contained within the same semicircle, that is, one that belongs to the same kind of libration, especially if the spot nearer the limit is nonetheless not so remote from the centre that its withdrawal from the limit is scarcely perceptible for an optical reason. And the same must be said of the approach of two such spots. For example, in the 6th figure, when within the semicircle LDM the spot *Grimaldi* was seen to move perceptibly away from the limb KS, the spot *Tycho*, being nearer the centre, ought to have been seen to withdraw from the limb DQ no less perceptibly, indeed more so. And yet the contrary plainly appeared from 22 to 25 September of the year 1649 and on 17 and 18 March of the year 1650; for at both times *Grimaldi* was seen to withdraw very perceptibly from the limb KS, while *Tycho* meanwhile appeared immovable in the same place on the lunar disc. *Tycho* also stood immovable on 19 and 20 March 1650, while *Grimaldi* was already perceptibly approaching the limb KS on the 20th. Similarly, in the year 1649, from 14 to 18 November, the shore of the *Sea of Crises* was approaching the limb RC with a perceptible motion, while *Endymion* remained immovable that whole time. And in the year 1651, from 27 to 30 August, the shore of the same sea was withdrawing markedly from the limb RC, while *Plato* meanwhile continued unmoved in the same position. We have observed similar things in other spots.

[Margin: 2nd argument.] *Second*, all the spots in the same semicircle, that is, all those in the semicircle EBC of northern libration, or all those in the semicircle EDC of southern libration, ought (if Hevelius's opinion were true) to librate in the same direction, or to agree in approaching the same limit of libration, or in moving away from it as from a common term in withdrawal. And yet in 1649 November 25, 26 and 27, and in 1651 February 11 and 12, while *Grimaldi* was withdrawing from the semicircumference EDC, *Tycho* was approaching it. Conversely, in the same year, on August 6, 7 and 8 and September 1, 2 and 3, while *Grimaldi* was approaching the semicircumference EDC, *Tycho* was withdrawing from it with a perceptible motion. Again, in 1651, from April 28 up to May 3 inclusive, the western shore of the *Sea of Crises* continually approached the limb RC, that is, the semicircumference EBC; and yet *Endymion* and *Aristotle*, placed within the same semicircle, were most plainly seen to withdraw from that same circumference, just as on the 1st, 2nd and 3rd *Plato* withdrew from the same place, having gone unobserved on the preceding days. Conversely, in 1651, from November 22 up to the 25th, the same shore of the same sea withdrew from the already mentioned semicircumference, which *Endymion* and *Plato* had meanwhile been approaching with a notable motion.

[Margin: 3rd argument.] *Third*, if Hevelius's doctrine held, spots in semicircles that differ as to the kind of libration would not at the same time approach, each its own limb of the Moon; rather, while the spots enclosed within the semicircle EBC approached the semicircumference EBC, the spots within the

semicircle EDC would withdraw from the semicircumference EDC; or, while the latter approached EDC, the former would withdraw from EBC. Nor would the opposite appear, and much less in those near the nodes L, M. But the exact opposite happened. For in 1651, from July 7 to 9, from August 5 to 8, and from August 31 to September 4, while *Plato* was approaching the semicircumference EBC, *Grimaldi*, which ought to have been withdrawing from the semicircumference EDC, was nonetheless approaching it. Conversely, in the same year, on March 7, both *Plato* and *Endymion* were seen to withdraw from EBC and *Grimaldi* from EDC. Likewise in the same year, on May 1, 2 and 3, both *Tycho* approached the semicircumference EDC and the western shore of the *Sea of Crises* approached the semicircumference EBC. Finally, also in the same year, from August 5 to 8 and from August 31 to September 3, while *Grimaldi* was approaching EDC, *Plato* and *Harpalus* were approaching EBC, a quarter from which, however, both of them ought to have been withdrawing.

[Margin: 4th argument.] *Fourth*, on Hevelius's hypothesis, while one of the spots in its semicircle moves perceptibly, approaching the Moon's nearer limb or withdrawing from it, another spot situated within the opposite semicircle, especially one far from the Hevelian nodes, ought not to appear immovable. And nevertheless in 1649, from September 22 to 25 and from November 21 to 25, while *Grimaldi* was withdrawing markedly from the semicircumference EDC, *Plato* stood immovable that whole time. Also in 1651, from August 26 up to the 30th, the western shore of the *Sea of Crises* withdrew continually from the limb RC, and meanwhile, through all those days, *Tycho*, which by force of the Hevelian libration should have been approaching the limb IQ, nonetheless remained immovable, in an entirely unvaried position.

These things and more like them, observed not indeed without amazement, but nonetheless with the greatest circumspection, and reported with equal good faith, I find in the records of Father Francesco Maria Grimaldi; and, as he himself pointed them out, I solemnly and scrupulously testify that they happened so. This being the case, the limits of the lunar libration must often be far from the points B and I, as the nodes must be from the points L and M, as these lie in the 6th figure with respect to the position of the spots on the lunar disc. It must nevertheless be granted to Hevelius, and it remains certain from many observations of ours, that the already mentioned limits and nodes are often near the position he assigns.

Quæſtio 8. In qua igitur parte debuerint eſſe Limites Librationis & Nodi, ſaluis obſeruationibus numeri XVII. & ſalua quoad cætera Hypotheſi Heueliana?

Question 8. In what part, then, must the limits of libration and the nodes have been, with the observations of number XVII preserved, and with Hevelius's hypothesis preserved as to everything else?

XVIII. If nothing else is supposed to need changing in Hevelius's hypothesis except the position of the limits and nodes, and if it is admitted that the libration of the lunar disc can actually occur about a movable axis of the Moon, yet in such a way that, with the Moon placed at the nonagesimal or on the meridian, the same spots keep the same position with respect to the sky (that is, for example, that the spot *Meton* is always toward the north in the upper part of the lunar disc, and the spot *Clavius* toward the south in the lower part, as appears in our 6th figure, which imitates Hevelius's as regards this position); if, I say, these things are supposed, we answer in general that the nodes or poles of this libration, and the axis passed through them, must have been near those spots that were seen to be immovable, according to what was reported in the first and 4th arguments. But if two or more spots were moving at the same time also with the same kind of motion, that is, by approach to or withdrawal from the nearest limb of the Moon, the axis of the poles had to be outside them; whereas if they were moving with different kinds of motion, it had to be between them.

[Margin: Places of the nodes from the first argument of Question 7.] Thus, by the examples of the first argument, when *Grimaldi* was withdrawing from the limb SKP or approaching it, while *Tycho* meanwhile was unmoved, the axis of libration had to be near the line BI; as it did also when, with the *Sea of Crises* approaching the limb BC, *Endymion* remained immovable. But when, with the *Sea of Crises* withdrawing from the same limb, *Plato* stayed immovable, the axis of libration had to be near the line PQ.

[Margin: Places of the nodes from the 2nd argument.] From the examples of the second argument, however, the axis of libration had to be between the lines BI and RS. For in this way it would be preserved, through what remains of the Hevelian hypothesis, how, with *Grimaldi* withdrawing from the limb KS, *Tycho* should approach the limb IQ, or, with the latter withdrawing, *Grimaldi* should approach; and likewise how, with the *Sea of Crises* approaching the limb RC, *Plato*, *Aristotle* and *Endymion* should withdraw from the limb PB, or, with that sea withdrawing, they should approach; yet in such a way that the axis of libration was on this side of *Endymion*, that is, between it and the *Sea of Crises*.

[Margin: Places of the nodes from the 3rd argument.] Now, from the observations brought for the 3rd argument, the axis of libration had to be not far from the line BI when, with *Plato* approaching the limb BP, *Grimaldi* also approached the limb KS. When, however, *Grimaldi*, *Plato* and *Endymion* were each withdrawing together from the nearer limb, the axis had to be between the lines BI and RS, but between *Endymion* and *Cleomedes*; just as it had then too, when both *Tycho* to its limb and the *Sea of Crises* to its limb were approaching together. When finally *Grimaldi*, *Harpalus* and *Plato* were approaching together toward the circumference BED, the axis had to be on a line nearer the line BI than the line PQ.

[Margin: Places of the nodes from the 4th argument.] Lastly, both when, with *Grimaldi* withdrawing from the limb SK, *Plato* stood immovable, and when, with the *Sea of Crises* withdrawing from the limb RC, *Tycho* appeared

unmoved, according to what was said in the 4th argument, the axis of libration had to be between the lines BD and PQ.

Once the position of the libratory axis is determined, moreover, its extremities on either side remain determined, namely the poles, which under another aspect can also be called nodes; and so also the limits of libration, since these are a quadrant from the nodes. But in what interval of time the axis of libration returns to the same position, and whether this change itself includes an anomaly, is matter for another investigation, and requires observations over a longer time.

Quæſtio 9. An libratio Lunaris diſci, de qua ſuprà, proueniat ex ſola Parallaxi, aut variatione Latitudinis, vel Diſtantiæ Lunæ à centro terræ.

Question 9. Does the libration of the lunar disc discussed above arise solely from parallax, or from the variation of latitude, or of the Moon's distance from the centre of the Earth?

[Margin: Galileo's opinion on the cause of the lunar libration.] XIX. This question was prompted by Galileo, in dialogue 1 *De duplici mundi systemate*, where he says that the Moon, because of the sympathy it has with the Earth, always looks toward the Earth with one determinate part of itself; and that it is therefore necessary that the straight line joining their centres always passes through the same point of the lunar surface, so that anyone gazing at the Moon from the centre of the Earth would always see the same disc of the Moon precisely bounded by the same circumference. But because the ray of one looking at the Moon's centre from the Earth's surface does not pass through the same point of the lunar surface through which the line joining the centres of the Earth and the Moon passes, unless the Moon is vertical to him, it comes about that, of the rising Moon, some small part of the lower limb is hidden from us which is seen at other times, and just as much on the other lower part of the setting Moon is hidden from us. He therefore attributes the cause of this hiding to parallax, which arises from the Moon's being considered from the centre of the Earth and viewed from its surface. A similar variation, he says, ought to be observed at the northern and southern extremities of the lunar disc: for when the Moon has its greatest or a great northern latitude, some part of the northern limb must be hidden and part of the southern uncovered, and conversely when it has southern latitude. He confirms these consequences from two lunar spots, one of which, when the Moon reaches the meridian, faces the north-west, while the other is diametrically opposite to it; and the first is visible without a telescope, but not the second. He is clearly speaking of the Sea of Crises and of Grimaldi's spot opposite it. And he says that the said variations are observed in both of these, and that they are, in contrary manner to each other, now close to the limb of the lunar disc, now far from it, yet with this difference: that the interval between the Corican spot and the limb of the Moon is more than twice

as great on one occasion as on another, and in the other spot three times as great. Boulliau, however, in Book 3 of the *Astronomia Philolaica*, reporting this opinion of Galileo as if Galileo had said that this libration occurs *so that the Moon may keep a certain direction toward the axis of the Earth* — the Earth being moved, as both of them believe, along the annual orb — brings a demonstration against him, of which we shall speak in the scholium. [Margin: Galileo's opinion is rejected.] To us, meanwhile, Galileo's doctrine seems seriously defective on another head. For, speaking of the already mentioned eastern and western spots, parallax alone cannot bring it about that the space between them and the lower limb of the Moon is two or three times narrower on one occasion than on another. Next, if this phenomenon came from parallax alone, it would have to be the case every day, in one and the same night, at least when the Moon is full, that the Corican spot appears as far as possible from the upper limb of the rising Moon, and that the eastern spot opposite it then appears as close as possible to the lower limb; and conversely that the Corican appears as close as possible to the lower limb of the setting Moon, and the other as far as possible from the upper limb. But this is utterly false. For often, when the Moon is rising, the Corican is at its greatest approach to the upper limb and Grimaldi's spot recedes furthest from the lower limb; and conversely, when the Moon is setting, the Corican is furthest from the lower limb and the Grimaldian nearest the upper limb of the Moon. [Margin: Neither parallax nor latitude are the chief causes of libration.] Parallax, therefore, is not the sole or the principal cause of the lunar libration that is at times seen toward the east or the west.

Nor is parallax combined with the greatest or a great latitude the sole or the chief cause of that libration which appears toward the south or the north. For in our 6th figure the spots of Thales or of Endymion, which according to Galileo ought then to be as close as possible to the upper limb of the Moon when it stands on the meridian, when the Moon has its greatest northern latitude, are nevertheless often very far from that limb; and on the contrary they not rarely come very close to it, even when the Moon has its greatest southern latitude; and similar variations are observed in the southern spots.

[Margin: The Moon's apogee and perigee are not the cause of the Moon's libration.] Gather from what has been said that the chief cause of the lunar libration is not the difference of the Moon's distance from the centre of the Earth, arising from apogee and perigee. For this varies the Moon's parallax, and that by only very few minutes; and of itself, when the Moon is at perigee, it does not hide one limb of the lunar disc rather than the other, but from the whole lunar disc, all round, it hides a scarcely sensible small portion, as will be clear from what is to be said in chapter 17, problem 3.

Quæres igitur 10. Quænam ſit cauſa Librationis in Luna apparentis, ſeu Qua motuum hypotheſi explicari poſſit.

You will ask, then, tenth: What is the cause of the libration apparent in the Moon, or by what hypothesis of motions can it be explained?

XX. Would that it were as easy to construct our own hypothesis as it was to demolish those of others. But the fewness of the years since this libration began to be observed stands in the way; we shall nevertheless indicate what has come to our mind. [Margin: The first hypothesis, by a little circle seen in the middle of the lunar surface.] The first hypothesis, then, seems to rest on the little circle AVFT, marked in the middle of the sixth figure, if the centre of the lunar disc is understood to be carried round through its circumference, alternately now from A through V toward F and T, against the order of the signs, now from A through T toward F etc., in the order of the signs. This holds provided that the centre A of the apparent lunar disc is conceived as in reality becoming, on the surface of the lunar globe, a pole, from which there is described a circle very nearly great, cutting the lunar globe and separating the segment seen by us from the unseen one; and provided next that the little circle described on the spherical surface of the Moon is much larger than the one described in the 6th figure, which, being plane, is drawn only for this purpose, that the quantity of the librations BG and DI can be represented on the same disc near the limb. And if the observations should require the diameter of the said little circle to be now larger, now smaller, this can be managed either by a spiral or helicoid line, or by another little circle, as an epicyclion.

[Margin: The second hypothesis, by 2 little circles around the limbs of the Moon.] The second hypothesis rests on two little circles near the limbs of the Moon, described in much the same way as Scheiner in his diagram describes his little circles around the limbs of the Sun, as will be said in Book 9, section 4, chapter 12, number 21; provided that only that part of the hypothesis is applied here which suffices for libration, and not that which suffices for a complete rotation of the lunar globe, since we have already excluded this rotation from the Moon.

[Margin: The third hypothesis, relying on the centre of the eccentric.] The third hypothesis would rest on the eccentricity of the Moon: namely, if we were to understand that the Moon, either by itself or fixed to an epicycle, always faces with the same face not the Earth but the centre of the lunar eccentric, so that a line drawn from the centre of the lunar body to the centre of the eccentric would always pass through the same point of the lunar surface. [Margin: 1. The case if the Moon has no latitude.] First, then, if the Moon had no latitude and were either at apogee or at perigee, there would be no libration: for the straight line joining the Earth with the Moon would pass through the centre of the eccentric. We here disregard the modest variation that could come from parallax. But if the Moon lacked latitude yet were descending from apogee, then Grimaldi's spot would begin to recede from the nearer limb, and would

recede more and more through the whole first quadrant of the Moon's anomaly, and the Sea of Crises would approach the western limb; then through the 2nd quadrant the distance of Grimaldi would diminish more and more, but that of the Sea of Crises would increase. After perigee is passed, in the 3rd quadrant of the anomaly the nearness of Grimaldi and the distance of the Sea of Crises from the nearer limb would grow; and in the 4th quadrant, finally, both would diminish. [Margin: 2. The case of greatest northern latitude.] Secondly, if the Moon had been carried up to its greatest northern latitude and were at apogee, Plato, Endymion and the other spots of that tract would be at their greatest distance from the northern limb, while Tycho, Longomontanus, Clavius etc. would come nearest to the southern. Conversely, at perigee Plato etc. would approach, while Tycho would recede furthest. But if the Moon were exactly a quadrant from the apogee on either side, no variation from the force of latitude would appear for this libration to explain; there would, however, be another libration at its greatest, namely toward the east or toward the west. [Margin: 3. The case of greatest southern latitude.] Thirdly, if the Moon had been carried down to its greatest southern latitude and were at apogee, Tycho etc. would recede furthest from the southern limb and Plato etc. would approach the northern; but at perigee Plato would recede from the northern and Tycho would approach the southern. But if it were a quadrant from the apogee on either side, there would be no libration of this kind. [Margin: 4. The case of middling latitude.] Fourthly and lastly, if the Moon held a middling latitude and were found neither at apogee, nor at perigee, nor at the end of the quadrants of distance from apogee or perigee, the libration would be greater or smaller according as the Moon shared in different ways in the conditions listed in the preceding cases.

[Margin: The falsity of the third hypothesis on two counts.] XXI. But for the said hypothesis to stand, two things are required. First, that the eccentricity be as great as the greatest libration requires, or so great that by its force we could at some time see of the Moon's disc 12 parts of which its semidiameter is 100, or 28° of the lunar surface beyond what we see without libration, according to what was said in Question 3. But the eccentricity of the Moon deduced by us from selected observations does not exceed, even at the quadratures, where it is greatest, 13053 parts of which the radius of the eccentric is 100000, as will be said in chapter 28; and so, with the laws of trigonometry preserved, the force of the eccentricity cannot represent a portion of the Moon's surface, beyond the usual one, greater than 7°. Second, that the variation set out in the 4 cases above should satisfy all the observations. But even if it satisfies very many of those obtained so far, not a few nevertheless resist it. We are therefore forced to reject this hypothesis as abortive. And indeed, as we shall warn more than once in examining the hypotheses of the planets, the centre of the eccentric does not seem to have in itself the character of an end, or of a desirable good, for the sake of which a planet should direct its motions toward it.

[Margin: What is to be said of the reticle of Hevelius's figure.] Further, the reticle that Hevelius describes in the 6th figure in place of the little circle AVFT does not serve to defend the whole hypothesis of libration, and it rests on a twofold but false foundation: namely, that the greatest librations occur at those times when the Moon has its greatest latitude at the beginning of Cancer or Capricorn, and that the limits and nodes of the libration perpetually keep almost the same position with respect to the spots of the lunar disc.

Quæſt. 11. An Lineæ diuidentis Lunæ ſplendidum ab obſcuro tranſitus non per eaſdem ſemper maculas, oriatur ex ſola Libratione, An etiam ex Latitudine.

Question 11. Does the passage of the line dividing the Moon's bright part from its dark part, not always through the same spots, arise from libration alone, or also from latitude?

XXII. I answer that, whether the question is about the extreme spots on which the Moon's horns fall or about the rest, through which the said line passes, in both a variation arises both from differing libration and from differing latitude; but the variation of the centre in the apparent lunar disc arises from libration alone, or a very little from parallax. [Margin: The line of dichotomy varies with varying latitude.] Considering now only the variation that comes from latitude: since the same plane of a great circle on the lunar globe always lies in the plane of the lunar orbit (if we set aside all libration), therefore, if the dichotomous Moon has its greatest latitude, the plane in the Moon separating the bright from the dark will make right angles with that same plane of the lunar body, always the same, namely the one that lies in the lunar orbit; and whether the latitude is southern or northern, the section will still pass through the same spots. But if the Moon is without any latitude, the said plane will be inclined to the lunar orbit at as great an angle as the lunar orbit is inclined to the ecliptic; and if the Moon is in the northern node, the northern horn will pass through spots further east than those through which it passed when the Moon, likewise dichotomous, was at greatest latitude; but if in the southern node, it will pass through spots further west. The opposite is to be understood of the southern horn. So, in figure 6, if for the dichotomous Moon having its greatest latitude the northern horn falls on *Meton* and the southern on *Moretus*, then, for the same Moon placed in the northern node, the northern horn will pass nearly through *Philolaus* and the southern through *Tanner*; while in the southern node the northern horn will pass through the middle of the *Sea of Ice* [maris Glaciei] and the southern a little below *Biancani*. Consequently the line of dichotomy will also pass through different spots lying outside the horns, according as the latitude is greatest, or is indeed none but the Moon is in the northern or the southern node. [Margin: The section of the other phases also varies for the same reason.] There is, however, excepted in dichotomies the spot or small part which is at the centre of the lunar disc: for, libration set

aside, the same lunar disc, and therefore the same centre of the disc, appears to us; nor would there be any other variation here, except a very little from parallax. From this you will gather that no smaller a variation occurs outside the dichotomies, and indeed a far greater one if the effects of librations and latitudes are combined; although we have given examples of this variation, for dichotomies almost alone, in the 40 observations reported above from number 10. Meanwhile we point out that a straight line drawn through the horns of the waxing or waning Moon does not cut the lunar disc in two, as it does when the Moon is full-grown.

SCHOLIA.

Scholia.

[Margin: How our Earth would appear to inhabitants of the Moon.] I. Just as from the Earth we see the phases of the Moon and its spots successively illuminated, so, if anyone were on the Moon, he would see the Earth illuminated by the Sun, so that on the Earth, where there are seas and other waters, large spots would appear, while the mountains, with their ridges lit, and the valleys, shaded behind the backs of the mountains, would appear as small spots, and the plains of the continents as bright parts. Next, the Earth, like another Moon, would appear now full, but then when it is new Moon for us, and now silent and deprived of the direct light of the Sun, when it is full Moon for us; and alternately sickle-shaped and gibbous. In the first lunar quadrature the Earth would appear to him dichotomous, but shining toward the east of the lunar inhabitant, while in the 2nd lunar quadrature it would appear dichotomous and shining toward the west. Since, however, the same hemisphere always faces the Earth, and the Earth is not carried around the Moon as the Moon is around the Earth, not all the inhabitants who might be on the Moon would see the Earth, but only the inhabitants of the hemisphere turned toward us, whereas all the inhabitants of the Earth see the Moon. Moreover, as I indicated in chapter 6, number 9, the Earth seen from the Moon would appear to shine with a secondary light, because of the Sun's light reflected from the Moon onto the Earth. Galileo says so too, in dialogue 1 of the *Systema Mundanum*.

[Margin: How long a day would the inhabitants of the Moon have?] II. The lunar inhabitants would differ from the dwellers on Earth in this too: that, because the same face of the Moon perpetually faces the Earth, they would have a continuous day of nearly 15 of our days. For example, an inhabitant of the upper hemisphere, in the straight line joining the centres of the Earth and the Moon, would have day from the last quadrature to the first following; and for the remaining 15 days, from the first to the second quadrature, perpetual night; and it would be noon for him at new Moon and midnight at full Moon, and the Sun would rise for him at the last quadrature and set at the first. But for the inhabitant of the hemisphere sloping toward us, everything would happen

the other way round. How large our Earth would appear there depends on the lunar distance, and so we shall speak of this in chapter 17.

[Margin: The Galilean libration attacked by Boulliau.] III. Boulliau, in Book 3, chapter 13 of the *Astronomia Philolaica*, rejects the lunar libration introduced by Galileo in the *Dialogues*, that the Moon may keep a certain direction toward the axis of the Earth moved along the annual orb. For let, he says, the annual orb be LIM, in which the Earth is FEHQ, whose axis, around which it revolves daily, is FIH, and let the Moon's orbit be OPN. Then either the Moon's axis must be kept parallel to the axis FH, as is the axis BC, and then either the face of the Moon turned toward us would change every month, or, for the same face always to be shown to us, a periodic revolution of the lunar disc is necessary. For if the Moon is at N, and BC parallel to FH, unless the Moon's globe is turned, the dwellers on Earth will see the part ASC; but when the Moon has advanced from N to O, the part ASC will be hidden and VDC will be seen, which is against the phenomena. Therefore, lest this follow, a revolution of the lunar body in longitude is necessary. But to introduce this is superfluous, since nature delights in the fewest motions and in economy. Or, secondly, the axis of the Moon keeps the same inclination to the axis of the Earth FH, as if the axis is AD, inclined to FH but parallel to the diameter EQ; and then too, according to this inclination, the face of the Moon would turn; or the axis AD must be turned by a rotation, so that, the Moon being at N, the parts ASD are seen there, and then, when it has moved on to O, the parts AVD are seen.

Diagram. A geometric figure in the right column, standing within Scholium III (the column breaks for it after the word *in*, and the sentence resumes below with *quo Terra*). A central circle (the Earth) carries a vertical diameter and an oblique diameter running from upper left to lower right. A shallow arc runs across the figure through the central circle, from lower left to lower right (the annual orb). A larger arc rises from a small circle on the left, passes over the top of the figure to a small circle on the right, and also curves down beneath the central circle, closing below it (the Moon's orbit). The small circle on each side is crossed by intersecting diameters and curved arcs. Labels read:

- A (upper left of the left small circle): point on the left small circle
- B (top of the left small circle): point on the left small circle
- V (top right of the left small circle): point on the left small circle
- O (centre of the left small circle): point on the Moon's orbit
- S (lower left of the left small circle): point on the left small circle
- C (foot of the left small circle; the sort looks small): point on the left small circle
- D (lower right of the left small circle): point on the left small circle
- L (below and to the left of the left small circle): end of the shallow arc
- F (top of the central circle): axis point
- E (upper left of the central circle): end of the oblique diameter
- I (centre of the central circle): centre

- Q (lower right of the central circle): end of the oblique diameter
- H (foot of the central circle): axis point
- A (upper left of the right small circle): point on the right small circle
- B (top of the right small circle): point on the right small circle
- V (top right of the right small circle): point on the right small circle
- N (centre of the right small circle): point on the Moon's orbit
- D (right of the right small circle): point on the right small circle
- S (lower left of the right small circle): point on the right small circle
- C (foot of the right small circle): point on the right small circle
- M (below and to the right of the right small circle): end of the shallow arc
- P (foot of the large arc, below the central circle): point on the Moon's orbit

[Margin: The uncertainty of judicial astrology.] IV. From the variation and libration of the lunar spots, unknown for so many centuries before, no less than from the companions of Jupiter and Saturn, unrecognised before this century of ours, and their motions, or from the alternation of the solar spots, no small argument can be drawn against the certainty of judicial astrology. For it is most probable that from the variation of these too there are different celestial influences on these lower things.

CAPVT X. De loco Lunæ in Vniuerſo.

Chapter 10. On the place of the Moon in the universe.

I. Since astronomy began to hanker after Copernicus [Copernicoturire], the place of the Moon in the universe has not been so certain, if it is referred to the centre of the world; but if to the centre of the Earth, then Pliny the Elder, Book 2, chapter 9, truly calls the Moon *the last of the stars, and the one most familiar to the Earth*. And in the *Dream* Cicero's Scipio is told: *Furthest from heaven, nearest to the Earth.* It is entirely certain to astronomers that of all the planets, let alone the fixed stars, the Moon is nearest to the Earth, and indeed that it holds the lowest heaven, if the Earth is at the centre of the world. This is shown by four arguments, by which the order of the planets in the universe, but above all of the Moon, is confirmed by Copernicus, Book 1, chapter 10; by Johannes de Sacrobosco in the *Sphaera*, chapter 1, and there by his commentators, listed by us in Book 1, in the preface, chapter 1; above all by Clavius, from p. 39 to 72; by Barozzi in the *Cosmographia*, Book 1, from p. 8; by Maurolico, dialogue 1 of the *Cosmographia*, p. 20; by Finé in the *Sphaera*, Book 4, chapter 8; by Biancani, Book 9 of the *Sphaera*, chapter 1; and by the rest who were cited on the constitution of the world system in Book 3, chapter 6.

[Margin: 1st argument for the Moon's nearness to the Earth.] II. The *first* argument is from the shadow. For that star which, at equal altitude above the horizon, ends with its ray a longer right shadow of the same gnomon is nearer to the centre of the gnomon, and so to the Earth, if the gnomon is erected on

the Earth. But the right shadow of the Moon is longer than the shadow of the Sun having the same altitude, and also than the shadow of Jupiter and of Venus (for it is almost impossible to discern the shadows of the other planets); therefore the Moon is lower than these. So in the diagram added here, if the style CR is erected above the horizon ACB, and the Sun S and the Moon L are at the same altitude above the horizon, though at different times, namely on the line SC, the lunar shadow will be CK, much longer than the solar shadow CG; therefore the Sun moves in a higher heaven. The argument must, however, run the contrary way from versed shadows, which are cast by styles fixed in walls: for such shadows of the higher stars are longer than the shadows of the lower.

Diagram. A geometric figure in the left column, with the text of the argument running in a narrow strip to its left. Two concentric-looking semicircles stand on a common base line; the base is the horizon. An upright stands on the base at C. The Sun, drawn as a small rayed face at S, stands on the outer semicircle at upper left; the letter L stands on the inner semicircle, below and to the right of S. Straight lines run down from the upper left through the head of the upright at R to the base at G and at K, and a line runs down to C. Labels read:

- A (left end of the base): end of the outer semicircle
- D (left end of the base, inside A): end of the inner semicircle
- C (base, foot of the upright): foot of the style
- G (base, to the right of C): end of a shadow
- K (base, immediately right of G): end of a shadow
- E (right end of the base, inside B): end of the inner semicircle
- B (right end of the base): end of the outer semicircle
- S (upper left, in the rayed Sun): the Sun
- L (on the inner semicircle, below S): the Moon
- R (head of the upright): head of the style

[Margin: 2nd argument.] The *second* argument is from eclipses. For the Moon alone is eclipsed by the shadow of the Earth; the other planets do not fall into it, but because of their greater distance from the Earth they pass high above it. Next, the Moon hides from us and eclipses not only the Sun but all the other planets and many of the fixed stars, and indeed in all parts of the zodiac; whereas it has never been seen to suffer eclipse by any other star, or to be covered in any part of itself. Therefore the Moon is of all the stars the nearest to the Earth.

[Margin: 3rd argument.] The *third* argument is drawn from the speed of its proper motion. For, as the writers on optics demonstrate with Euclid, of things carried at equal speed, those that are further away appear to move more slowly. But the Moon completes the zodiac most quickly of all the stars, namely in a monthly course, whereas Mercury takes 88 days, Venus 9 months, the Sun one year, Mars about 2 years, Jupiter 12, Saturn 30, the fixed stars nearly 30000. But the theorem of the writers on optics is wrongly applied to this subject: for the

planets are not in fact moved equally and only in appearance unequally. Next, if the daily motion is considered, the Moon returns to the same meridian more slowly than any of the planets; and yet, because of this slowness, the lowest place seems due to it, since of circles described from the same centre and moved by the same motion, those nearer the centre move more slowly, as they traverse a shorter space in equal time. But this example does not suit the Moon either, because it does not return at the same moment to the same meridian, as the points of those circles do, once placed on the same radius, or on a straight line that can be drawn from the centre; and so this argument is topical rather than apodictic. Which authors, further, have assigned the Nete to the Moon, and which the Hypate, in distributing the harmonic intervals, and for what reason, we have already indicated in Book 3, chapter 7, in Scholium 4, but we shall set it out more fittingly in Book 9, section 5, chapter 7.

[Margin: 4th argument.] The *fourth* argument is taken from parallax, for which consult what was said in Book 1, chapter 35, and the figure placed there. For we showed that the star which, other things being equal, undergoes a greater parallax when viewed from the surface of the Earth is nearer to the Earth; and the Moon's horizontal parallax is, in the opinion of all astronomers, of very nearly one degree, whereas the parallaxes of the other planets scarcely reach 6′ each, as will be established from what is to be said in its place; and for the Sun we have already shown it in Book 3, chapter 8.

[Margin: The place of the Moon in the system of Copernicus and of Baliani.] III. In the Copernican system, however, set out by us in Book 3, chapter 6, the Moon is not nearest to the centre of the universe, at which the Sun is there placed; rather, Mercury and Venus are nearer to it. And in the system that D. Giovanni Battista Baliani, patrician of Genoa, was working out so as better to explain the tide of the sea, the Moon had to be at the centre of the world, and the Earth to be turned around it as it stands still, in a monthly revolution. As to what follows from this hypothesis, both in the phases of the Moon and the position of the spots and in the syzygies of the Sun with the Moon, this is not the place to treat it, but rather the book on the systems of the world. For it is enough here to state that of all the stars the Moon is nearest to the centre of the Earth.

CAPVT XI. De Methodo in communi, qua ſine vitioſo circulo, Lunæ Parallaxes, & inde Solis & aliorum Planetarum diſtantię ac Parallaxes inquiruntur.

Chapter 11. On the method in general by which, without a vicious circle, the parallaxes of the Moon, and from them the distances and parallaxes of the Sun and the other planets, are investigated.

[Margin: Doubt about a vicious circle in this analysis.] I. The parallax of the Moon is the foundation of all stellar parallaxes, since on it depends the knowledge of the Moon's distance from the Earth; on the lunar distance, the

distance of the Sun and its parallax, as is clear from the method given in Book 3, chapters 7 and 8; and on the distance of the Sun depend the distances and parallaxes of the other planets, and indeed of the fixed stars, as we shall show in their places. In this analysis, however, one must take care lest a deceptive circle mock astronomy with giddiness or perpetual uncertainty, as someone has indeed feared: because the motions of the stars cannot be known without observation, and parallax is always involved in observations made on the surface of the Earth and concerning stars placed away from the observer's zenith, and very sensibly so in the Moon, because of its nearness to the Earth. Without knowledge of the parallaxes, moreover, the true places of the stars and their true motions, which must be referred to the centre of the world, cannot be distinguished from those apparent to us who observe them on the surface of the Earth. If, therefore, the parallaxes depend on the motions, and the motions on the parallaxes, as known beforehand, we seem to drive and torment astronomers into a vicious circle, as onto the wheel of Ixion. And lest you think that this fear belongs only to newcomers, hear Kepler in a letter to Giovanni Antonio Magini dated 1601 June 1, which Magini himself reproduces in the *Supplement to the Ephemerides*, p. 253. *The theory of the Moon*, says Kepler, *gives Tycho much difficulty. It seems to me that it must be begun from the parallaxes, which occur because of the sensible distance between the centre and the surface of the terrestrial globe. But the doctrine of parallaxes is confounded with the latitudes of the Moon. There would therefore be need of prior knowledge of the latitudes. Each depends on the other.* He then adds three methods, resting content only with the last of them; but they had not then been brought to practice, nor were they later by Magini or Kepler, so far as I know; nor can they easily be brought to practice, as I shall indicate in the following chapter. What Theseus, then, might hold out to us the thread for escaping from this labyrinth? Yet take heart, reader: for here is the thread.

[Margin: The foundations of astronomical observations, and the first step.] II. *First*, finding the meridian line does not depend on parallaxes, as is clear from what was said in Book 1, chapter 10. Once it has been obtained, the altitude of the equator is taken from the two altitudes of a circumpolar star, according to what was said in Book 1, chapter 12, without any inconvenience from parallax. For the fixed stars have no sensible parallax, since the distance of the pole star from the pole has been found to be the same at the same time by observers at very different altitudes of the pole, for example by Tycho in Denmark and by others in Italy; and I now find it as far from the pole as the observation made by Tycho requires, as I shall say in the book on the fixed stars. This would not happen if it had a sensible parallax, for the difference of parallaxes at different altitudes would bring about a different distance of the star from the pole. Besides, there are various ways of discerning whether a phenomenon is subject to sensible parallax (on these, see the book on new phenomena) that do not depend on prior knowledge of the parallax.

[Margin: Step 2.] *Second*, from the Sun's meridian altitude taken on the day of the summer solstice, when it no longer increases, and compared with the altitude of the equator, one obtains the Sun's greatest declination, or the apparent obliquity of the ecliptic, which suffices for the purpose for the time being. From the Sun's meridian altitude taken on the day of an equinox, and from the apparent obliquity of the ecliptic, the apparent equinox is derived by the method given in Book 3, chapter 14. From two equinoxes separated by a very long interval of time, and from the Sun's place between two equinoxes, according to what was said in Book 3, chapters 15, 17 and 24, one derives both the quantity of the equinoctial year, and from it the Sun's mean motion, and the Sun's apparent apogee, and from it the prosthaphaeresis, by which the Sun's mean place is reduced to the true place without any error that would disturb our business in the meantime, even if merely the apparent equinoxes and the apparent apogee have been used. For in this way one knows in what degree of the ecliptic the Sun is at any given time, even though the exact minutes are not yet established.

[Margin: Step 3.] *Third*, from the beginning and end of a lunar eclipse its middle and the moment of full Moon are discerned; and from the degree of the ecliptic known for that moment, in which the Sun is, the opposite degree, in which the Moon is, is recognised. By comparing eclipses observed in antiquity by the Chaldeans and Egyptians with more recent ones, the Moon's mean motion, of anomaly and of latitude, is investigated by the method to be given below; and in this, because of the very long interval of time, no notable error in the Moon's revolutions can arise. From the true place, known through eclipses of the Moon or through the distance from known fixed stars of the Moon placed in the nonagesimal, the equations are derived, that is, the differences between the mean place and the other [medium vtrumq. locum] [recte: *medium verumq. locum*, "the mean and the true place", per the book's errata].

[Margin: Step 4.] *Fourth*, the Moon's greatest latitude, by which it departs from the ecliptic, is taken as having been observed somewhere when the Moon was at or near the observer's zenith, as once happened for Ptolemy at Alexandria when the Moon was at the beginning of Cancer; or as observed from eclipses of the Moon, which is done independently of the Moon's parallax. Then, once the Moon's altitude has been observed with instruments, and the Moon's place and its latitude have been computed by tables from the motion of latitude and the angle of greatest latitude, one learns how great its true altitude ought to be at such a horizon and such a time. The difference between the true and the observed altitude therefore reveals the parallax of the Moon.

[Margin: Step 5.] *Fifth*, from the Moon's parallax its distance from the Earth is derived, by the problem given in Book 3, chapter 7; and when the Moon's altitudes have been observed at different seasons of the year, its various parallaxes and distances are derived. From these the distance of the Sun is then derived by one of the methods given in Book 3, chapter 7, and when this has been found, the parallax of the Sun, according to what was said in the same place, chapter 8.

Once the Sun's parallax has been found, its motions are corrected; and, going back over the steps, all the observations treated from number 2 here to this number 4 are revised, and the true obliquity of the ecliptic, the true equinoxes, the true place of the Sun at the moment of an eclipsed full Moon, and from it the true place of the Moon, are deduced, according as the method used in these observations is more or less accurate.

III. By this method, after the observations of so many centuries, astronomy has at last advanced so far that we are now certain that the Moon's least distance from the Earth exceeds 30 semidiameters of the Earth and does not exceed 80; indeed, if we rely on the observations of this and the preceding century, that it does not exceed 70 and does not fall short of 50, and that its horizontal parallax is not less than 50′ and not greater than 70′. As for the distance of the Sun from the Earth, it is neither less than 650 semidiameters of the Earth, with a horizontal parallax of at most 5′, nor greater than 16,500 semidiameters of the Earth, with a least parallax of 14″. Astronomy acknowledges, and glories in the fact, that it has advanced far enough in so deep an investigation: for by setting up its calculations within these limits it has up to this day foretold to mortals the motions of the planets, and especially the eclipses of the Moon and the Sun, with a certainty such as it seemed unable even to hope for from so many windings, and such as suffices for human use in this life. Thus it is evident to astronomy itself, a posteriori, that it has not laboured in vain in going round through a vicious circle. Yet it labours still to attain greater precision and subtlety; and what chiefly stands in the way of this is certainly not ignorance of the most exact parallaxes, but the multitude of observations and motions to be reconciled, whose errors, small one by one, nevertheless turn out rather large when taken together. Still, it was worthwhile to know this very thing, so that it should not be unknown how far the hopes and efforts of astronomy can extend. Meanwhile we owe a very great deal to Ptolemy, who observed the Moon's greatest latitude for us, without which the inquiry into parallaxes is undertaken in vain. The rest depends chiefly on the true longitude, known with the help of eclipses, or from the Moon's distance from the Sun or from fixed stars of known longitude when the Moon is in the nonagesimal degree of the ecliptic, where it has no parallax of longitude.

SCHOLION.

Scholium.

[Margin: The back-to-front method of Hipparchus.] We have said that one must begin from the parallax of the Moon, so that from it its distance from the Earth may be determined, through this the distance of the Sun, and from that the parallax of the Sun, following, that is, Ptolemy in Book 5 of the *Almagest*, chapter 11, and the greatest part of astronomers. Yet Ptolemy relates in the same place that Hipparchus tried to investigate first the distance of the Sun from the Earth, and from it the distance of the Moon. Having obtained the

Sun's distance as if by conjectures, first supposing a very small solar parallax, then now a greater one, now none that is sensible, and weighing a certain eclipse, he derived from it various and inconstant distances of the Moon. But these were mere conjectures, exposed to fallacies or to misgiving, since about the Sun it is altogether doubtful not only how much parallax it has, but even whether it has any at all, as Ptolemy says in the same place.

CAPVT XII. De Obſeruanda Latitudine Lunæ maxima in gratiam Parallaxium, cæterisq. latitudinibus ex illa eliciendis: Et de Opinionibus Aſtronomorum circa hanc Latitudinem.

Chapter 12. On observing the Moon's greatest latitude for the sake of the parallaxes, and on deriving the other latitudes from it; and on the opinions of astronomers about this latitude.

[Margin: What the Moon's latitude is.] I. The inquiry into latitude in Ptolemy, Book 5, chapter 12, rightly comes before the inquiry into parallaxes. The latitude of the Moon is the arc of a great circle drawn through the centre of the Moon and the poles of the ecliptic, intercepted between the ecliptic and the centre of the Moon. This arc, like every circle of latitude, makes right angles with the ecliptic. [Margin: And the greatest latitude.] The Moon's greatest latitude, therefore, is the Moon's greatest distance from the ecliptic, reckoned on the arc just mentioned, and measuring the angle at which the Moon's orbit is inclined to the ecliptic. [Margin: What the nodes are, and how many.] For the Moon does not always travel along the ecliptic, but along another circle cutting the ecliptic in two opposite points. One of these is called the Northern Node, or the Ascending, or the Carrying-up [Euehens] Node; in Ptolemy [Greek: σύνδεσμος ἀναβιβάζων (syndesmos anabibazōn, "ascending node")]; in Arabic the Dragon's Head. When the Moon begins to approach it, receding from the opposite node, it is said to ascend, and for brevity this node is indicated by the mark ☊. The other is called the Southern Node, or the Carrying-down [Deuehens] Node; in Ptolemy [Greek: σύνδεσμος καταβιβάζων (syndesmos katabibazōn, "descending node")]; in Arabic the Dragon's Tail. The Moon, on approaching it, is said to de- scend, and it is marked thus: ☋. These nodes are not fixed, but move toward the preceding signs, westward, by 3′ 11″ each day, as we shall show later. At these nodes the Moon has no latitude; at a distance from them of a whole quadrant, or 90°, it has its greatest latitude, and these points are called limits, or bellies of the Dragon. For the lunar orbit AEBD, cutting the ecliptic ACBF at the Southern Node A and the Northern Node B, and having its greatest latitude at the points EC and DF, forms as it were two snakes, with the head at B, the tail at A, and the swelling of the belly at E and F. Hence, when these nodes are joined with the nodes of the ecliptic and the equator, that is, at the beginning of Aries and Libra, the limits of greatest latitude are at the beginning of Cancer and Capricorn. Just as the place of the nodes on the ecliptic can be known from the motion of latitude, gathered from

eclipses independently of parallax, so the place of the limits can be known, since it lies 90° from the place of the nodes; and therefore also when the greatest latitude will fall at the beginning of Cancer and Capricorn.

Diagram. Woodcut set into the left column at the right of the text: two circles of about equal size, slightly displaced, crossing one another at left and right, showing the lunar orbit crossing the ecliptic.

- A (inside, at left): Southern Node, per the prose
- B (inside, at right): Northern Node, per the prose
- E (outside, at top): on the lunar orbit AEBD, per the prose
- C (inside, at top): on the ecliptic ACBF, per the prose
- D (inside, at bottom): on the lunar orbit AEBD, per the prose
- F (outside, at bottom): on the ecliptic ACBF, per the prose

Primus Modus obſeruandi Latitudinem Lunæ.

The first method of observing the Moon's latitude.

II. The first method is through the Moon's meridian altitude observed when the Moon is at the observer's zenith, or as near it as possible, so that it has no parallax or almost none, and when at the same time it has its greatest latitude at the beginning of Cancer and Capricorn, that is, has its nodes at the equinoctial sections. For then the same meridian becomes not only the circle of declination and of altitude, but also the circle of latitude and the solstitial colure, as is certain from the doctrine of the primum mobile. Now if the latitude is northern, subtract from the lunar altitude the sum made up of the altitude of the equator and the distance of the tropic from the equator; if the latitude is southern, subtract the lunar altitude from that sum. The remainder will be the Moon's greatest latitude. This method is given by Ptolemy, Book 5, chapter 12, by Regiomontanus in the *Epitome of the Almagest*, Book 5, Proposition 14, and by Copernicus, Book 4, chapter 15.

EXEMPLVM Ptolemaicum.

Ptolemy's example.

[Margin: Ptolemy, Book 5, chapter 12.] For the sake of the parallaxes Ptolemy built an instrument of three four-sided rulers, which he therefore called parallactic, although it serves for taking the altitude of any star, as will be clear from our *Liber Organicus*. Using this instrument at Alexandria, where, on his own testimony, the altitude of the equator is 59° 2′, he observed the Moon's meridian altitude as 87° 53′, when it was established from the tables of the lunar motions that the Moon had its greatest latitude, a northern one, at the beginning of Cancer. The distance of the tropic from the equator, as he judged from his own observation and that of Eratosthenes, was 23° 51′ 20″. The sum of this and the altitude of the equator was therefore 82° 53′ 20″. When this was taken from the Moon's observed altitude of 87° 53′, there remained the Moon's greatest latitude of 4° 59′ 40″. He considered this accurate enough, because the

Moon was very near the zenith and only 2° 7′ away from it, where the lunar parallax is less than 3′; and he set it at a round 5°.

All astronomers down to Tycho acquiesced in this, together with al-Battānī, Jābir ibn Aflaḥ, Alfonso, Peuerbach, Copernicus and Reinhold, who fixed the Moon's greatest latitude at 5° without carrying out any similar observation, because they were not at a suitable horizon, one at which the Moon would ever be little distant from the zenith and free of notable parallax, but confirmed it in some fashion from eclipses.

Secundus Modus.

The second method.

[Margin: Inhabitants of the torrid zone, or near the tropics, are suited to this.] III. Let the Moon's distance from two fixed stars of known longitude and latitude be observed by inhabitants who at times have the Moon at or very close to the zenith, but at the same moment, by four observers, two using one sextant and two the other. For from these distances the latitude and longitude of the lunar body can be found by the common problems of the primum mobile. But that moment of time must be observed as well, so that, once it has been converted into equal time, the place of the nodes may be investigated by tables, and from it the Moon's distance in longitude from the nearer node. If this is 90°, the Moon's latitude found above will be the greatest; if it is less than 90°, then with this distance as a base, and with the latitude found, in a spherical triangle, the angle opposite the latitude found must be sought by the common rules of triangles, for it will be the measure of the greatest latitude.

Tertius Modus.

The third method.

[Margin: By an eclipse of 6 digits.] IV. The third method requires a lunar eclipse of exactly six digits, out of the twelve of the lunar diameter, which rarely occurs. Let the moment of the beginning and end of the eclipse be observed exactly, so that, half the difference being added to the beginning itself, the moment of the greatest and middle eclipse may be established. For that moment, gather from the tables the place of the nearer node and the place of the centre of the Earth's umbra, which is always 180° from the place of the Sun. Finally, let the semidiameter of the Earth's umbra at the place of the Moon's passage be known, either from tables or from an observation made at the time. With these things set down, let the portion of the ecliptic be AC, and the cen-

Diagram. Woodcut across the right column, interrupting the sentence. A nearly horizontal line runs from A at left to C at right. A large circle, centred at about B on that line, represents the umbra. From C a second straight line slopes down to the left through P, M and F to D. Three small equal circles stand on this sloping line, centred at P (right), M (middle, its upper part inside the large circle and hatched) and F (left). Straight lines run from the point at B

to M and toward O and L; short perpendiculars drop from G toward F, from E toward M and from I toward P.

- A (left end of the horizontal line)
- G (on the horizontal line, inside the large circle, left)
- B (on the horizontal line, centre of the large circle, per the prose)
- E (on the horizontal line, immediately right of B)
- I (on the horizontal line, right, above the small circle P)
- C (right end, where the two straight lines meet: the node, per the prose)
- L (on the line from B toward P)
- O (at the left, where the large circle meets the circle about F)
- M (centre of the middle small circle, the Moon at mid-eclipse, per the prose)
- P (centre of the right small circle)
- F (centre of the left small circle)
- D (left end of the sloping line)

tre of the umbra B; let the Moon's orbit, touching the periphery of the umbra at M, be CMD; from the centre M let the lunar disc be described, whose semidiameter, that is, six digits, is eclipsed at the height of the eclipse; and let the semidiameter of the umbra, BM, be drawn to the centre of the Moon and the point of contact. The triangle CBM will then be right-angled at M. In it, therefore, given the side BM, which is the semidiameter of the umbra, and the base BC, the distance of the node C from the centre of the umbra B, you will find by the rules of trigonometry the angle C, which measures the Moon's greatest latitude. Nor does ignorance of the parallax do harm here: since the centre of the umbra B is at the same distance from the Earth as the centre of the Moon M, they have equal parallax, and so the semidiameter BM is not diminished. It should also be noted from Kepler, *Astronomia Optica*, chapter II, Problem 6, that the eclipsed digits are more easily discerned when they are exactly 6 than when they are more or fewer.

EXEMPLVM. Ex Noſtra Obſeruatione.

An example from our own observation.

[Margin: Eclipse of 6 digits.] In the year 1643 September 27, on the following night, I observed at Bologna, from the altitude of Aquila at the end and using the oscillations of a pendulum from the beginning, the start of a lunar eclipse at $6^h\ 18^m\ 15^s$ after noon and its end at $8^h\ 43^m\ 30^s$; the middle was therefore at $7^h\ 30^m\ 52^s$. The digits at the height of the eclipse were, through telescopes and also with the naked eye, by the agreement of all the witnesses, exactly 6, which is also the number Lansbergen's tables require. The semidiameter of the umbra, observed by the method to be given below on eclipses, and from the distances of the luminaries, was 48′. The Sun's place at mid-eclipse was ♎ 4° 29′ 34″; therefore the place of the centre B was ♈ 4° 29′ 34″. The place of the northern node was ♍ 25° 16′ 28″, and that of the southern node C ♓ 25°

16′ 28″; therefore the distance of the centre, or BC, was 9° 13′ 6″. Let us now use the figure given above, just as if the southern part of the Moon had been eclipsed, for the calculation will come to the same thing. In triangle BMC, therefore, right-angled at M, from the base BC of 9° 13′ 6″ and the side BM of 48 [Gr.48.][27], there results the angle C of 5° 0′ 3″.

Quartus Modus.

The fourth method.

[Margin: Other eclipses suited to this.] V. The remaining eclipses of the Moon, however, serve this procedure, if the moments of beginning and end are observed exactly, so that the middle may be established; if the apparent semidiameters of the Earth's umbra and of the Moon at this time are observed or given; and if the Moon's latitude is of the same kind from the beginning to the end of the eclipse. For in the same figure let the Moon at the beginning of the eclipse be at P, touching the periphery of the umbra at L, and at the end at F, touching it at O. By Euclid, Book 3, proposition 12, the straight line connecting the centres of the umbra and of the lunar disc, such as BP and BF, will pass through the points of contact L and O. Hence, when the known semidiameters of the umbra and the Moon are added together, the straight lines BP and BF will be known; and within the few hours the eclipse lasts these do not vary sensibly. The triangle FBP is therefore isosceles, and the straight line BM, bisecting FP at M, will be perpendicular to the Moon's orbit and will make right angles at M, by the converse of proposition 10 of Book 1 of the *Elements*; and triangle MBP will be right-angled at M. In it the base BP is known, namely the sum of the semidiameters of the umbra and the Moon, and also the side MP, because it corresponds to the Moon's hourly motion from the beginning to the middle of the eclipse, known from tables. So by trigonometry the side BM becomes known. Again, in triangle CBM, right-angled at M, the side BM is now known; and from the moment of mid-eclipse and the tables of the Moon and Sun the place of the umbra B and of the nearer node C, that is, the base BC, is known. So by the rules of spherical and right-angled triangles the angle C, measuring the Moon's greatest latitude, will not remain hidden. In the same way different eclipses can be weighed, and it can be discerned whether the data agree among themselves and with the latitude. I admit, however, that an estimate of the umbra made by eye is unreliable, as Kepler notes at the front of the ephemeris for the year 1619.

I pass over the remaining methods, which presuppose that the Moon's parallax is known, because in this place we are investigating the latitude in order to learn through it the parallaxes, still unknown.

[27] the page sets this side with the degree abbreviation; the same semidiameter is stated just above as 48′

Data Latitudine Maxima Lunæ, reliquas eius latitudines inueſtigare ad quamcunque diſtantiam eius à Nodo.

Given the Moon's greatest latitude, to investigate its other latitudes at any distance of it from the node.

VI. Let A be the node at which the Moon's orbit intersects the ecliptic; let AC be a quadrant of the ecliptic and AD a quadrant of the lunar orbit; and let DC be the arc of greatest latitude, making a right angle with the ecliptic at C. This arc is what angle A measures, and the angle has as many degrees as the arc DC. Let the Moon be, not at D, the limit of greatest latitude, but at E, and from this point let the arc of latitude EB be drawn perpendicular to the ecliptic. This produces the spherical triangle ABE, right-angled at B, in which the angle A is given, being as great as the greatest latitude. If, then, the base AE is given, namely the Moon's distance from the nearer [ptopiore] [recte: propiore] node on its own orbit, or is supposed given, the side EB, namely the Moon's latitude belonging to it at point E, can be found by the rules of trigonometry; and likewise for the others.

Diagram. Woodcut set into the left column at the right of the text: a curved triangle with a cross-arc. An upper arc runs from D at left through E to A at right; a lower arc runs from C at the foot up through B to A; a straight-drawn arc DC closes the left side, and a second arc runs from E down to B.

- D (upper left): limit of greatest latitude, per the prose
- E (above the upper arc): the Moon's place, per the prose
- A (right): the node, per the prose
- B (on the lower arc, where the arc from E meets it): per the prose, foot of the arc of latitude EB
- C (foot): on the ecliptic, per the prose

Opiniones Aſtronomorum, de Maxima Lunæ Latitudine.

Opinions of astronomers on the Moon's greatest latitude.

[Margin: The opinion of the Tychonians.] VII. I have already reported Ptolemy's opinion and its foundation at number 2, in the example, and added that all astronomers down to Tycho judged that the Moon's greatest latitude is always 5°, both in ☌ and ☍ and in □. No wonder they acquiesced in it, since they saw that it satisfies the phenomena of eclipses very nearly. Tycho, however, in volume 1 of the *Progymnasmata*, p. 126, and in the *Letters*, p. 140, agreeing with Rothmann, asserts that up to his own time the Moon's greatest latitude was wrongly thought to be invariably 5°; for it was evident to him from his own most diligent observations that the Moon's greatest latitude at new and full Moons is indeed very nearly 5°, that is, 4° 58′ 30″, but at the quadratures [Quadris] 5° 17′ 16″. Tycho was endorsed by Longomontanus in the *Astronomia Danica*, Book 1 of the *Theorica*, chapter 7; by Magini in the *Supplement to the Ephemerides*, Canon 8; by Nicolaus Mulerius in the *Isagoge* to the *Tabulae Frisicae*, part 2, chapter 6; and by Boulliau in *Astronomia Philolaica*,

Book 3, chapters 12 and 16, who considers it a matter of scruple to depart from the Tychonic latitude, since he has quite often found by experience that *in applications of the Moon to the fixed stars it agrees exactly with the sky*. Lansbergen too dissents very little, in chapter 5 of the *Theoricae* and in precept 9, setting the Moon's greatest latitude at the syzygies [Copulis] at 5° and at the quadratures at 5° 16′, confirming this by no particular observation. Tycho, on p. 114, gives four observations of the lunar latitude taken at Uraniborg, not indeed of the greatest latitude, but ones by which his table and hypothesis of the Moon's latitude can be confirmed; these, however, presuppose that the Moon's parallax is already known.

[Margin: The opinion of Kepler.] VIII. Kepler, moreover, at the front of his *Ephemerides* from number 21 to 24, and in the Rudolphine Tables, chapter 27, retains the Moon's greatest latitude of 5° at other syzygies, but at quadratures 5° 18′. In eclipses of the Sun and Moon, however, he sets the angle of greatest latitude at 5° 18′, constant for almost the whole of that month in which the Sun walks through the places of the nodes, as if the inclination of the lunar eccentric to the ecliptic stayed through that whole month and were as great as at the quadratures. But when the nodes are in quadrature, he sets the greatest latitude at 5°, as great as at non-eclipsing syzygies; and he says he is compelled to this variation both by the form of his physical hypothesis and by eclipses. Yet he adds there and in precept 128 that he has relapsed into the Tychonic equipollence, and that by as much as he increased the latitude in eclipses he diminished the annual prosthaphaeresis of the nodes, which for Tycho is monthly; and that therefore as great a latitude of the Moon is derived from them on his hypothesis as on the Tychonic. However, how much the Rudolphine Tables disagree with eclipses is now acknowledged among recent astronomers; and no wonder, since Kepler indulged his own ingenuity more than the sky, and the mysteries of his own ideas more than inviolate observations. He also notes that Ptolemy and Tycho kept the Moon's greatest latitude at the syzygies equal to the equation at 90° of anomaly.

[Margin: The opinion of Godefridus, and ours.] IX. Govaert Wendelen, in the *Idea of the Atlantic Tables*, p. 30, says that the Moon's greatest latitude at the syzygies is most precisely 5° 0′; but what he himself or van Langren thinks of it outside the syzygies is not yet known. For our part, from many eclipses weighed by the problem treated in methods 3 and 4, as will be clear from volume 3 of the *New Almagest*, it is established that even at eclipsing syzygies the Moon's greatest latitude is so close to 5° that an excess or shortfall of a few minutes, let alone many, cannot be proven by any observation, and it can be kept unchanged with the slightest little correction of the semidiameters and motions. On the quadratures, see Scholia 1 and 3.

LATITVDO LVNAE MAXIMA.

The Moon's greatest latitude.

According to [AVCTORIBVS]	In ☌ & ☍	In □
Ptolemy, al-Battānī, Alfonso, Peuerbach, Regiomontanus, Copernicus, Reinhold.	5° 0′ 0″	5° 0′ 0″
Tycho, Longomontanus, Mulerius, and at length Magini and Boulliau, and nearly Rothmann.	4° 58′ 0″ [recte: 4° 58′ 30″, per the book's errata; corrected by hand to 30″ in this copy]	5° 17′ 30″
Philips van Lansbergen	5° 0′ 0″	5° 16′ 0″
Govaert Wendelen	5° 0′ 0″	☩

RESIDVVM TABVLAE PRAECEDENTIS.
The remainder of the preceding table.

According to [AVCTORIBVS]	[G. /. //.]	[G. /. //.]
Johannes Kepler [Io. Keplero], in ☌ ☍ of eclipses [Eclipt.]	5° 18′ 0″	
Johannes Kepler [Io. Keplero], in other [alijs] ☌ ☍	5° 0′ 0″	in □: 5° 18′ 0″
For us, for the time being [Nobis interim], in ☌ ☍	5° 0′ 0″	5° 18′ 0″

SCHOLIA.

Scholia.

I. Anyone who knows the parallactic rulers [Regulas Parallacticas] and how they are used from experience will recognise how easy it is to err with them by several minutes, especially when the star to be observed is very near the zenith. For the eye to be properly applied to the pinnules [pinnacidijs], one must have the head tilted back, and in fact lie with the whole body; otherwise the eye usually judges the altitude greater than it should. Since, therefore, Ptolemy observed the Moon very near the zenith with this instrument, it is very probable that he assigned it a somewhat greater altitude, or a smaller distance from the zenith, and thus a greater latitude than he would otherwise have given. See what we shall say in the *Liber Organicus* of volume 2 of the *New Almagest*, on the parallactic rulers, and meanwhile what Tycho says about them in the *Mechanica Astronomica*. Relying meanwhile on this probability, I have cut off two minutes and corrected the Moon's latitude, of which I shall speak in Scholium 3, so that it is 5° 18′ at □.

II. Tycho (volume 1 of the *Progymnasmata*, p. 127) and Longomontanus (*Theorica*, Book 1, ch. 7) judge that Ptolemy observed the Moon's greatest latitude as 5° when the Moon was at the beginning of Cancer in such a way that it was also in quadrature aspect to the Sun. This is not improbable, for Ptolemy (Book 5, ch. 13) states that he observed the Moon near the beginning of Capricorn crossing the meridian at sunset, when the Sun was at 5° of Libra, and so very nearly at □ itself, and he says that the Moon's latitude was 4° 59′. Just as it was here almost 5° and at greatest latitude, so it seems to have been when, at the beginning of Cancer, it was seen to have a latitude of 5°. Tycho and Longomontanus add, however, that in this century the greatest latitude of ☾ is 5° 17′ ½ at □, but that the 5° Ptolemy found at those points arose because

the obliquity of the ecliptic was then greater, and 23° 51′ 20″. For had it been, as now, 23° 30′ or 31′, then, added to the altitude of the equator, which at Alexandria is 59° 2′, the meridian altitude of the tropic of ♋ would have been 82° 32′ or 33′; and this, subtracted from the observed altitude of the Moon, 87° 53′, would have left a latitude of 5° 21′ or 20′, about as much as Tycho at □ and Rothmann found.

But how is it probable that at the syzygies [Copulis] the Moon has always kept its greatest latitude of 5°, yet at □ has increased it by just as much as the obliquity of the ecliptic decreases? Either, on the course of its orbit, it has regard to the ecliptic only, and so ought to have kept the same inclination to it at the syzygies as at the quadratures [Quadris]; or it has regard to the equator, and only incidentally to the ecliptic, and so, as the declination of the ecliptic decreased, it ought nevertheless to have kept its own greatest declination constantly, and thus, the latitude at the quadratures being incidentally increased by this, it ought to have increased the latitude at the syzygies too. Although Kepler grants this in part, Tycho and Longomontanus deny it. Should we then rather say what I shall shortly say in Scholium 3?

III. If the obliquity of the ecliptic has always been the same as it is now, 23° 30′, which I showed in Book 3, ch. 27 to be far more probable, then certainly the Moon's greatest latitude observed by Ptolemy at □ must either be corrected, so that it was about 5° 20′, or, because of what was said in Scholium 1, 5° 18′; or else his observation, as regards the altitude of the Moon, must be suspected of error, for the reason indicated in Scholium 1. But if the obliquity of the ecliptic was then 23° 30′ and the greatest latitude at □ 5° 20′, it follows from Ptolemy's other observation that the Moon's parallax is 1° 48′, as I shall show in the next chapter at the end of number 2, and this seems quite absurd. So let the astronomers see what solution they can apply to this knot. For I have not yet been able to observe the Moon's latitude at the quadratures independently of parallax, which, being unknown, is assumed here. For at Bologna the Moon is always far from my zenith, and at □ it has never crossed the meridian at the moment when, according to the ephemerides, it was at the node. Nor do I think this opportunity was ever offered to Tycho himself, whether you run through the ephemerides from the year 1570 to 1601 or what Kepler writes to Magini in the year 1601, to be reported shortly in ch. 13, no. 10. And in the Rudolphine Tables, ch. 27, he too says of this matter: "But the matter is not yet clear in every respect."

CAPVT XIII. De Diſtantijs Lunæ ex Parallaxibus, & de Parallaxibus ex obſeruatione inueſtigandis.

Chapter 13. On finding the Moon's distances from its parallaxes, and its parallaxes from observation.

I. Let the reader recall what I said about parallaxes in general in Book 1, ch. 35, or about the Sun's parallaxes in Book 3, ch. 8. For it follows from what was

said there that the measure of a star's horizontal parallax is the angle B, if the star is at B, on the boundary of the physical horizon AB, which is parallel to the rational horizon HR; and, when the straight line HB is drawn from the Earth's centre H, the triangle HAB arises, right-angled at A. In that triangle, therefore, given the parallactic angle B and from it AHB, as its complement to a right angle, and the opposite side AH, taken as one semidiameter of the Earth, the base HB, that is, the star's distance from the Earth's centre in parts of which AH is one, is easily obtained by the rules for right-angled plane triangles. In brief: if the secant of angle AHB is divided by the radius, or whole sine, or if you look up the residual of the logarithm of angle B, you will have the base HB, or its logarithm.

Diagram. Woodcut set into the right column at the right of the text. A quadrant has its right angle at H (lower left), with one radius rising to V at the top and the other running along the foot to R at the right; an arc runs from V through S and B to R. A stands on the vertical radius HV, and from it a smaller arc descends to C on the foot line HR. Straight lines join H to S and H to B, and A to S and A to B.

- V (top of the vertical radius): the point directly overhead
- S (on the great arc, upper right): the star when its parallax is not horizontal
- B (on the great arc, right): the star on the physical horizon
- A (on the vertical radius): the observer on the Earth's surface
- H (lower left): the Earth's centre
- C (below the foot line, at the foot of the smaller arc): not cited in the prose
- R (lower right): end of the rational horizon HR

However, if the parallax is not horizontal, as with the angle S, then, when the straight lines HS and AS are drawn from the Earth's centre H and from the surface A, since the star's apparent distance from the zenith is given, namely the angle VAS, its complement to two right angles is also given, and so the angle HAS, together with the parallactic angle S corresponding to that distance, and AH as one semidiameter. In the triangle AHS the side HS can therefore be found, which is the star's distance from the Earth's centre. In brief: divide the sine of angle VAS, or of the apparent distance from the zenith, by the sine of the parallactic angle S; or add together the logarithm of angle VAS and the residual of the logarithm of the parallactic angle S. The first gives you the side HS, the second its logarithm. For example, if to an apparent distance of the Moon from the zenith VAS of 50° 55′ there corresponds a parallax S of 1° 7′, as Ptolemy holds, the logarithm of 50° 55′, which is 988999, added to the residual of the logarithm of 1° 7′, which is 171023, gives the logarithm 160022, discarding, as is customary, the redundant unit on the left. With this logarithm you will find in the tables that HS is 39 58′ in parts of which AH is one. You will find the same if you divide the sine of the angle of 50° 55′, which is 77623, by the sine of the angle of 1° 7′, 1949, even though Ptolemy, by a longer calculation, arrives at 39 45′.

Primus Modus obſeruandi Parallaxes Lunæ.

First method of observing the Moon's parallaxes.

II. This first method is taught and illustrated by their own examples by Ptolemy (*Magna Syntaxis*, Book 5, ch. 13), Copernicus (Book 4, ch. 16), Tycho (volume 1 of the *Progymnasmata*, p. 459), and Lansbergen (in the *Thesaurus observationum*, class 5 of lunar observations, and in the *Uranometria*, from p. 3 to p. 11); and by another's example by Regiomontanus (*Epitome of the Almagest*, Book 5, prop. 15) and Daniel Santbech (*de obſeruat.*, prop. 15).

First gather from the tables the time at which the Moon, visible at night or even by day, will be at the beginning of Cancer or Capricorn, or as near to it as possible; for at the tropics the difference of declination is imperceptible within a few hours, and when the Moon is at its meridian altitude, the meridian will be the circle of both declination and latitude. At that time, therefore, observe the Moon's meridian altitude, together with the time. When this has been equated, gather from the lunar motions, by tables or by the problems of the primum mobile, the Moon's latitude and also the Moon's longitude on the ecliptic; and from a table of declinations, the true declination of the Moon's place. If that declination is of the same kind as the latitude, add them; if of a different kind, subtract the latitude from the declination; the sum or the remainder will be the Moon's true declination. Then compare the Moon's observed altitude with the altitude of the equator. If they are equal, the Moon will have no apparent declination; if it is greater than the altitude of the equator, the apparent declination will be northern; if less, it will be southern, among northern peoples. Finally, compare the Moon's apparent declination with its true declination, for their difference will be the parallax of altitude. Using the same observation, however, you can find the Moon's true altitude from its true declination, added to the altitude of the equator if it is northern, subtracted if it is southern; or, subtracting a northern declination or adding a southern one to the altitude of the pole itself, you can find the Moon's true distance from the zenith; and by comparing the true altitude with the apparent, or the true distance from the zenith with the apparent or observed, you can find the parallax in the circle of altitude. Tycho uses the comparison of declinations, Ptolemy and Copernicus that of distances from the zenith, and Lansbergen that of altitudes, but these problems are equivalent to one another. The Moon must, however, be beyond the danger of sensible refraction, and higher than 45° or at least 40°.

EXEMPLVM Ptolemaicum.

Ptolemy's example.

Ptolemy, as he himself relates in Book 5, ch. 13, in the 20th year of the Emperor Hadrian and the year 883 of Nabonassar, on the 13th of the month of Mechir, that is, in the year of Christ 135 October 1, at the apparent hour of equal hours $5^{h}\ 50^{m}$ after noon at Alexandria, observed with the parallactic rulers, the

Sun already setting, the Moon's distance from the zenith on the meridian as 50° 55′. The apparent time reduced to equal time was 5^h 20^m p.m. For that moment he computed from his tables the Moon's place as ♑ 3° 10′; its mean anomaly as 262° 20′, its equated anomaly as 269° 46′; its distance from the northern limit as 2° 6′; its true and northern latitude as 4° 59′; and the true declination of the Moon's true place as 23° 49′ south. Subtracting from this the Moon's true latitude left the Moon's true southern declination as 18° 50′. He added this to the altitude of the pole at Alexandria, 30° 58′, and the Moon's true distance from the zenith came to 49° 48′. But the apparent or observed distance was 50° 55′. The parallax of altitude was therefore 1° 7′, appropriate to the Moon at an anomaly of 269° and nearly at quadrature, for the Sun was then at ♎ 5° 28′. From this parallax he gathered the Moon's distance from the Earth's centre as about 39 45′ semidiameters. However, Ptolemy derived the Moon's latitude from a greatest latitude at the quadratures assumed to be 5°, and the declination of the Moon's place from an assumed declination of the tropic of 23° 51′ 20″. But if, from what was said in the preceding chapter, Scholium 3, it is more probable that the greatest latitude at the quadratures was 5° 20′ and the tropic's distance from the equator 23° 30′, then certainly, at the time of this observation of Ptolemy's, the Moon's northern latitude was 5° 19′ and the declination of degree 3 of ♑ was 23° 28′. The Moon's true southern declination was therefore 18° 9′, the Moon's true distance from the zenith 49° 7′, and the parallax 1° 48′. This has seemed far too great and absurd to everyone, because it would show the Moon's diameter at the quadratures almost twice as great as practitioners have found it down to these times.

Secundus Modus.

Second method.

[Margin: 2. The method of Daniel Santbech.] III. Observe, says Daniel Santbech (book *De observatione*, prop. 15), when the Moon is at the beginning of Cancer or Capricorn and is crossing the meridian together with some star of known declination, the distance between this star and the Moon; this will be done more exactly if at the same moment one observer takes the altitude of the Moon and the other that of the star. For from the star's right ascension you will know the moment of apparent time; and when this has been converted to equal time, you will gather from the tables of the Moon its longitude and latitude, and from them its true declination, and from this the difference between its declination and the star's. Comparing this with the distance taken between them, or with the difference of altitudes, you will gather the parallax of altitude from the difference between these, provided that here too the Moon has been higher than about 45°, so that it is beyond the hazard of sensible refractions. The declination, moreover, is known from the longitude and latitude if the declination of the Moon's place on the ecliptic is known, as we said in the first method.

Tertius Modus.

Third method.

[Margin: 3. Method by observing the time and the Moon's meridian altitude.] IV. In whatever degree of the ecliptic the Moon may be, observe its meridian altitude and the exact moment of time, for here the most precise is required, so that when it has been equated you may gather from the lunar tables its true longitude and latitude. Then, from the tables of declinations of Regiomontanus, or of Magini in the *Directiones* of the primum mobile, which they constructed for each degree of longitude and latitude, look up the Moon's true declination, and from that and the altitude of the equator the true altitude, namely by adding the Moon's declination to the altitude of the equator if the declination is towards your pole, or subtracting it if towards the other. Finally, when the observed altitude has been subtracted from the Moon's true altitude, gather the parallax of altitude. If you lack the tables already mentioned, consult our own or others' problems of the primum mobile, which teach how to find a star's declination from its given longitude and latitude, together with the distance between the poles of the equator and the ecliptic.

Quartus Modus.

Fourth method.

[Margin: 4. Method of al-Battānī and Tycho, by observing the Moon in the nonagesimal.] V. By the example of Tycho (volume 1 of the *Progymnasmata*, p. 461) and the precept of al-Battānī (ch. 40): observe when the Moon is in the nonagesimal degree of the ecliptic counted from the horizon. You will learn this from the erection of the horns, if it is horned, or even gibbous or dichotomous; or from the problem given in this book, ch. 4, Problem 9. Then observe the Moon's apparent latitude, as well as its longitude, which will then be apparent and true at once, and the moment of time; and for that moment, once equated, gather the Moon's true latitude from the tables. For by comparing the true latitude with the apparent, you will detect the parallax not only of latitude but also of altitude, since the circle of latitude passing through the Moon's centre and the nonagesimal of the ecliptic then becomes a vertical circle, or circle of altitude. The latitude and longitude of ☾ are observed, moreover, either with zodiacal armillaries, as I shall teach in the *Liber Organicus*, or by taking the apparent distance from two fixed stars of well-known longitude and latitude, by the common problems of the primum mobile.

Quintus Modus.

Fifth method.

[Margin: 5. Our method, likewise by the Moon in the nonagesimal.] VI. Observe, when the Moon is in the nonagesimal degree of the ecliptic, the exact time and the apparent altitude. When the time has been equated, gather for

that moment from the tables the Moon's true longitude and latitude; and from the right ascension of midheaven, the culminating point of the ecliptic, for the difference in longitude between this and the Moon's place will show the arc of the ecliptic intercepted between the meridian and the nonagesimal. From the table of the angle of the ecliptic and the meridian, you will gather the angle appropriate to the culminating point of the ecliptic, opposite the arc by which the nonagesimal is distant from the zenith. For if, by the rules of right-angled spherical triangles, with the arc of the ecliptic already mentioned and the angle adjacent to it, you find the side opposite it, you will have the distance of the nonagesimal from the zenith, whose complement to 90° is the altitude of the nonagesimal. To this, therefore, add the Moon's northern latitude, or subtract its southern latitude, and the Moon's true altitude will be known; by comparing this with the apparent or observed altitude, you will discern the parallax of altitude, which in this case is also that of latitude. But you can learn the distance of the nonagesimal from the zenith in another way, through its own table, whose use we shall teach, God willing, in the precepts on solar eclipses. If you do not want to depend on tables for the motion in longitude, take at the same moment the distance of the Moon, situated in the nonagesimal, from two fixed stars of known longitude, so as to gather from it the Moon's longitude, which will be true and apparent at once.

You can achieve the same in another way, if a fixed star is then in the same vertical as the Moon when the Moon is in the nonagesimal, and the altitude of each is taken beyond the refractions. For if the latitudes of the Moon and the fixed star are of the same character, the lesser subtracted from the lesser [minor de minore; recte: maiore] will give the difference of latitude, and this, compared with the difference of altitude, will give the parallax. But if they are of different character, the difference of the altitudes taken, subtracted from the sum of the latitudes, will give the parallax, which in both cases is that of latitude and of altitude at once. If because of twilight the star cannot be seen at the time when the Moon is in the nonagesimal, take the azimuth of the Moon together with its altitude and observe the moment of time; and from an Aratean globe [Globo Arateo], first roughly, then by the problems of the primum mobile, find which star must have been in such an azimuth at that moment, and what altitude it must have had there, and carry out the rest as above.

In the methods already mentioned you see a dependence on knowledge of the latitude, or of the Moon's latitude and declination together. Let us now try methods that do not depend on the latitude, or even on the declination.

Sextus Modus.

Sixth method.

[Margin: 6. Our method.] VII. Gather from the tables of the motion of longitude and latitude when the Moon will cross the meridian lacking all latitude, that is, being at one or other of the nodes; and, for that time, the

culminating point of the ecliptic, for that is the very point at which the Moon's centre will be. The declination of this point will be known from the table of declinations of the ecliptic. If you add it to the altitude of the equator when the declination is northern, or subtract it when southern (for I am speaking with northern peoples), the Moon's true altitude will be known. Therefore, if you have observed the apparent meridian altitude and subtracted it from the true, the parallax of altitude will be known. But when will all these things coincide, so that at the time the Moon crosses the meridian it lacks all latitude? Perhaps never in your whole lifetime.

Septimus Modus.

Seventh method.

[Margin: 7. Our method.] VIII. Gather from the lunar tables when the Moon will cross the meridian at a time when it is visible, and will be at one or other of its nodes and at the same time at the node of the ecliptic with the equator, that is, at the beginning of ♈ or ♎. For then it will lack all latitude and declination, and its true altitude will be equal to the altitude of the equator. The Moon's meridian altitude, then observed and compared with the true altitude, that is, with the altitude of the equator, will show the parallax of altitude. But when will these four things coincide? Perhaps never for you, or for many generations of observers. Far less can it be hoped that this should happen not just once, but with the Moon both at apogee and at perigee, and this both at ☌ and at □, so that astronomy may be freed from all doubt about the Moon's parallaxes.

Octauus Modus.

Eighth method.

[Margin: 8. Method of Hájek and Digges.] IX. In Tycho (volume 1 of the *Progymnasmata*, pp. 666 and 668), and indicated by Longomontanus (*Sphaerica*, Book 2, ch. 10, problem 2), there is a general twofold problem of Tadeáš Hájek and Thomas Digges [Thomæ Diggeſei] for finding the parallaxes of any phenomenon. In one of them it is required that the phenomenon be in the same vertical as two known fixed stars, that its apparent distance from each of the fixed stars be taken, and that the azimuth, or the distance of that vertical from the meridian towards the east, be noted; then, when the phenomenon with the fixed stars has been carried by the motion of the primum mobile to a western azimuth equal to the eastern azimuth, that its apparent distance from the same fixed stars be taken again, and the altitude of either of the fixed stars. But as Tycho warns, and Digges does not deny, this can hardly be carried out without error: both because azimuths are hard to observe exactly, unless you use my thread triangle [triangulo filari], discussed in the *Liber Organicus*, and wait for the star at its plane; and because of the slenderness of the sides and angles that occurs in the second observation, since the phenomenon moves little away

from the vertical of the fixed stars within the same night. The second, however, requires the phenomenon's apparent distance in the same vertical with two fixed stars; but in the second observation, with no account taken of verticals, its apparent distance from the same fixed stars and the altitude of either of the fixed stars. Yet this too is liable to error because of the slenderness of the angles, besides the fact that the Moon's centre is found only with great difficulty in the same vertical as two fixed stars, so that the instruments are then set in a suitable position. I have therefore thought it right here to indicate the problem, not to explain it; we shall, however, explain it in the *Liber Opticus*, in the section on parallaxes.

Nonus Modus.

Ninth method.

[Margin: 9. Kepler's method.] X. Lastly, I should like to report the very words of Kepler from his letter to Giovanni Antonio Magini, dated at Graz in Styria, 1601 June 1 [Kal. Iunij], which Magini himself reports in the *Supplement to the Ephemerides*, p. 253. At the end of this letter he says: *"The theory of the Moon causes Tycho much difficulty. It seems to me that it should begin from the parallaxes, which arise because of the sensible distance between the centre and the surface of the terrestrial globe. But the theory of parallaxes is confused with the Moon's latitudes. So prior knowledge of the latitudes would be needed. Each depends on the other. I have therefore considered how parallax might be investigated by observing without knowledge of the latitude. Two methods occurred to me. One, if on the same day the Moon were observed once high and once lower, yet beyond the limit of the refractions, when it is about the limits, where the latitude changes little within two hours. It is also expedient to do this at the beginning of Cancer, where the declination too changes little. But when on the same day after its meridian altitude the Moon becomes sensibly lower, it acquires a parallax in longitude, which must be known beforehand, since that is what is being sought. The other method: that at distinct times the Moon be observed when it is in the nonagesimal degree, at the same limit, at the same distance from the Sun, once higher and once lower. But these three conditions rarely coincide. Add that for each single minute in this observation of altitude an error of one semidiameter of the Earth is committed, and few of these are contained in the eccentricity of the Moon's path. So, in the third place, I take refuge in this: I beg you to observe the Moon as often as you can in the nonagesimal degree, and to share your observations, together with the most exact altitude of the pole at Bologna, with us in Germany. I shall see to it that you have our observations in turn. Thus it will happen that the Moon is sometimes observed at the same time in both places, and so its altitudes at various places of the anomaly will become known; for Bohemia and a good part of Italy are on the same meridian."*

But oh, how uncertain and exposed to an error of several minutes is this very third method! How crude a conception of a confused and entangled mind, which could not be brought to birth in any practice! And so, up to the year

1601, in which Tycho died, there existed in Tycho's journals no observation of lunar parallaxes independent of knowledge of the latitude. I should gladly know, then, by what reasoning he found the greatest latitude at the quadratures to be greater than the latitude at the syzygies by about a third of a degree, for parallaxes derived from the latitude of the syzygies cannot reveal a different greatest latitude.

CAPVT XIV. Referuntur ſelectæ obſeruationes Parallaxium Lunarium, & opiniones inde ortæ Aſtronomorum de Parallaxi Horizontali & Diſtantia Lunæ à Terra in Copulis & in Quadraturis.

Chapter 14. Selected observations of lunar parallaxes are reported, and the opinions of astronomers arising from them on the horizontal parallax and the Moon's distance from the Earth at the syzygies and at the quadratures.

[Margin: 1. The opinion of Hipparchus.] I. With the opinions that are not founded on observations set aside to the scholia that follow, Hipparchus comes first. As Ptolemy reports in part (*Almagest*, Book 5, ch. 11) and Theon in part (in his commentary at the same place), Hipparchus, supposing the Sun's sensible parallax now to be nil, now greater or smaller, tried by certain conjectures taken from eclipses to hunt down the Moon's distances, and from them its parallaxes. For that reason he derived them as varied and inconsistent, and sometimes the Moon's least distance as 71 and its greatest as 83 semidiameters of the Earth, and at other times the former as 62 and the latter as 72½, with an apparent diameter of 35′ at perigee and 30′ at apogee. On Posidonius and other ancients, see Scholium 3.

[Margin: 2. Ptolemy's method and opinion.] II. Ptolemy, for his part, in Book 4, ch. 6 and 10, gathers from eclipses, by the method we shall give in ch. 21, no. 7, that the epicycle's semidiameter is 5 13′ or 5 14′, in units of which the distance of the eccentric's apogee from the Earth's centre is 60. Then, in Book 5, ch. 4, using the already mentioned semidiameter of the epicycle and the greatest equation at the quadratures, 7° 40′, which he had found in ch. 3 from the Moon's true motions as observed and compared with its mean motions, he gathers an eccentricity of 10 19′ and a semidiameter of the eccentric of 49 41′, in units of which the distance of the eccentric's apogee from the centre is 60. Afterwards, in Book 5, ch. 13, relying on the observation I reported in ch. 13, no. 2, in the example, he gathers the Moon's parallax at quadrature as 1° 7′, corresponding to an altitude of 39° 5′ and a mean lunar anomaly of 262° 20′. From this, in the same ch. 13, he gathers the Moon's distance from the Earth's centre at that time as 39 45′ semidiameters of the Earth; the Moon's mean distance at conjunction and opposition, or the distance of the eccentric's apogee from the Earth's centre, as 59 terrestrial semidiameters; the Moon's mean distance at quadrature as 38 43′ semidiameters of the Earth; [Margin: Semidiameter of the epicycle.] and the epicycle's semidiameter as 5 10′ terrestrial

semidiameters. Therefore in ch. 15, by adding the epicycle's semidiameter to the Moon's mean distance from the Earth, [Margin: Distances of the Moon from the Earth.] he establishes the distance of the Moon at apogee from the Earth at conjunction and opposition as 64 10′ semidiameters, which at the quadratures is 43 53′ semidiameters. But by subtracting the epicycle's semidiameter from the Moon's mean distance, the distance of the Moon at perigee comes out as 53 50′ semidiameters at conjunction and opposition, and 33 33′ semidiameters at quadrature. By keeping to the rule of proportions, moreover, the eccentricity is gathered as 10 8′ 41″ semidiameters of the Earth. For if a mean distance of the Moon at the syzygies of 60 units gives an eccentricity of 10 19′ units, then certainly the same distance of 59′ [recte: 59] semidiameters of the Earth [Margin: Eccentricity of the Moon.] gives an eccentricity of 10 8′ 41″ terrestrial semidiameters, that is, almost 10 9′. When the eccentricity is therefore taken away from the Moon's mean distance at the syzygies, [Margin: Semidiameter of the eccentric.] there remains a semidiameter of the eccentric of 48 51′ terrestrial semidiameters. Since the occasion arose, it was worthwhile to set these out here together, as they will serve us elsewhere.

Now, since Ptolemy gives the distance of the Moon at apogee as 64 10′ semidiameters of the Earth at the syzygies, and its visual diameter as 31′ 20″, and from this in Book 5, ch. 14 gathers that the Moon's true diameter is to the Earth's diameter as 16½ to 60; and since he gives the distance of the Moon at perigee at the quadratures as 33 33′ semidiameters of the Earth, it follows by our problem in Book 3, ch. 11, no. 4 that the apparent diameter of the Moon at perigee at quadrature is 56′ 22″. That is nearly twice as great as that of the Moon at apogee at the syzygies, which is evidently contrary to experience. For not Gemma Frisius alone testifies in his *Radius Astronomicus* that he observed, on 1542 December 15, shortly after the quadrature, the Moon's apparent diameter as 30′, at which time according to Ptolemy it should have been at least 50′; many others too have observed a far smaller diameter of the Moon at every quadrature, and for that reason have repudiated Ptolemy's hypothesis on this point: above all Regiomontanus, Book 5 in the *Epitome of the Almagest*, prop. 22; Copernicus, Book 4, ch. 16; and Tycho, vol. 1 of the *Progymnasmata*, p. 118. Nor did Ptolemy confirm so great a diameter by his own observations; otherwise Copernicus would have said that it was so great once but had diminished in the lapse of centuries, and would have devised his own anomaly for it.

[Margin: 3. The opinion of al-Battānī, Alfonso and Peuerbach.] III. Al-Battānī, ch. 39 and 40, follows Ptolemy in all the lunar distances, but diminishes the diameter of the Moon at apogee, though only slightly, so that at perigee at the quadratures a far greater one results than the experience even of someone observing it rather crudely will allow. Nor did the Alphonsines and Peuerbach, in his *Theoricae*, and al-Farghānī, *Differentia* 21, depart from this.

[Margin: 4. The opinion of Copernicus.] IV. Copernicus, Book 4, ch. 16 and 17, derived his parallaxes and distances from two observations, using method 1 of Chapter 13; they are set out below.

EXEMPLA II. Et Obſeruationes Copernici in Quadrato.

Two examples, and observations of Copernicus at quadrature.

[Margin: Moon at apogee.] In the year 1522 September 27, at 5^h 40^m after noon, but 5^h 24^m equated, the Moon's distance from the zenith, observed with the parallactic rulers at Frombork, was 82° 50′. At that time, he says, the true place of the Sun was ♎ 13° 29′ and of the Moon ♑ 12° 33′. The equated anomaly of ☾ was 358° 40′; the southern latitude 4° 47′; the declination of the Moon's place 22° 54′ south; therefore the true declination of ☾ was 27° 41′. The distance of the equator from the zenith, or the altitude of the north pole, was 54° 19′; therefore the Moon's true distance from the zenith was 82°, but its apparent distance was 82° 50′. Therefore the parallax was 50′.

But in the year 1524 August 7, at 6^h, the Moon's distance from the zenith observed at the same place was 81° 55′. The Sun's place was ♌ 24° 14′; the Moon's ♐ 9° 39′; the equated anomaly 239° 40′; the southern latitude 4° 41′; the declination of the Moon's place 21° 55′ south; the Moon's true declination 26° 36′ south; the altitude of the north pole 54° 19′. Therefore the Moon's true distance from the zenith was 80° 55′, but its apparent distance was 81° 55′. Therefore the parallax was 1° 0′.

[Margin: Copernicus's errors in parallaxes.] In these observations there lies at least a threefold error. For the altitude of the pole at Frombork was found by Tycho, through Elias Olai [Elias Olsen Morsing], to be 54° 22¼′, as Tycho relates in vol. 1 of the *Progymnasmata*, p. 34, and in the *Epistolae Astronomicae*, p. 74. And the greatest declination of the ecliptic was not 23° 28′, as Copernicus supposes, but 23° 30′, and so the declination of the Moon's place was 22° 58′ in the first observation and 21° 52′ in the second. And because of the omitted refraction, the distance from the zenith must be corrected so that it was 83° 3′, and the later one 82° 8′. Therefore the parallax was 56′ in the first case and 65′ in the second. But if, as is more probable, the Moon's greatest latitude at quadrature is not 5°, as Copernicus thinks, but 5° 17½′, as Tycho holds, or 18′, each parallax must be diminished by about 16′. Besides which, the Moon was not in such a position that the circle of altitude and the circle of latitude were the same.

Nor does it stand in the way that Copernicus, in Book 4, ch. 27, observed at Bologna in the year 1497 March 9 [7.Idus Martij], at the 5th hour of the night, the Moon eclipsing the southern eye of Taurus, and from this tries to confirm his parallaxes. For, as Tycho points out in vol. 1 of the *Progymnasmata*, p. 245, he used a southern latitude for this star of 5° 10′, whereas it is 5° and nearly 30′; and the Moon was at quadrature, where according to him the greatest latitude is 5°, whereas it is 5° 17½′, that is, greater by almost as much as the latitude of Aldebaran is. The Moon could therefore occult Aldebaran in such a way as to be nearer the southern horn by about a third of the lunar diameter, as Copernicus relates, adding that the Moon then stood 84° from the zenith, and yet omitting refraction. Therefore, from this and similar observations it

cannot be determinately gathered whether the parallax or the latitude is to be increased, but only one or the other.

[Margin: 5. Magini's opinion.] V. Giovanni Antonio Magini, in his *Theoricae Planetarum*, Book 1, from p. 57 to p. 69, takes the measure of the Moon's distances and parallaxes from Copernicus, and so, even though he substitutes an eccentric for the eccentrepicycle, as we shall say later, he labours under the same faults in this matter as Copernicus. But in the *Supplement to the Ephemerides* he builds his parallactic tables and distances on Tychonic foundations. Let us therefore consult Tycho.

[Margin: 6. Tycho's opinion.] VI. Tycho, then, in vol. 1 of the *Progymnasmata*, p. 119, sets out a small table of the lunar parallaxes, both Ptolemy's and Copernicus's and his own, and adds to it two observations at the quadratures, one with the Moon at apogee, the other at perigee, made at Uraniborg, with the days noted in the Old Style. They are these.

Obſeruationes 2. Tychonis in Quadrato.

Two observations of Tycho at quadrature.

[Margin: Moon at perigee.] In the year 1592 August 19, at $18^{h}\ 8^{m}$ p.m., the observed latitude of ☾ was 34′ south, but the true latitude was 1° 12′ 10″. The parallax was therefore 38′ 10″, the Sun's place being ♍ 6° 44′ 18″, the Moon's ♊ 9° 40′, and the equated anomaly 5^{s} 27° 31′ 47″.

[Margin: Moon at apogee.] In the year 1597 March 15, at $5^{h}\ 36^{m}$ p.m., the Moon's meridian altitude was observed as 62° 24′ and its apparent latitude as 4° 48′ north, but the true latitude was 5° 14′ 50″; for the Sun's place was ♈ 5° 7′ 26″, the Moon's ♋ 2° 10′, and its equated anomaly 0^{s} 20° 1′ 27″. Therefore the parallax was 26′ 50″.

After these, on p. 459, he adds six observations of the Moon at quadrature: three at the meridian altitude around the tropics, where the declination does not vary, and three when the Moon was at the nonagesimal of the ecliptic, where the parallax of longitude ceases. They are as follows.

Obſeruationes 3. Tychonis Lunæ in Quadrato in Meridiano prope Tropicos.

Three observations by Tycho of the Moon at quadrature, on the meridian near the tropics.

[Margin: Moon near apogee.] In the year 1583 October 12, at $5^{h}\ 19^{m}$ p.m., the greatest apparent altitude of the Moon's upper limb on the meridian was 13° 38′. Take away the Moon's semidiameter, then 15′, and the altitude of the centre is 13° 23′; and with the refraction of 8′ removed, 13° 15′ remains. This, subtracted from the altitude of the equator, leaves the Moon's apparent declination as 20° 50′. The Moon's place was ♑ 15° 40′ and its true latitude

2° 42′ north; therefore the true declination was 19° 57′. Therefore the parallax was 54′, greater by 1½′ than the canon of Copernicus gives.

In the year 1587 August 4, at 9^h p.m., the meridian altitude of the upper limb of ☾ was observed as 15° 20′ and of the lower limb as 14° 46′; therefore the apparent diameter was 34′ and the apparent altitude of the centre 15° 3′, but with refraction removed, 14° 57′. This, subtracted from the altitude of the equator, gives the apparent declination as 19° 8′. The Moon's place was ♑ 7° 50′ and its latitude 5° 12½′ north; therefore the true declination was 18° 5′ and the parallax 63′, greater than the Copernican by 1′ 30″.

In the year 1587 August 18, at 7^h 26^m before noon, the apparent altitude of the Moon's upper horn on the meridian was 52° and of the lower 51° 29′ 30″; therefore the diameter was 30′ 30″ and the apparent altitude of the centre 51° 44′ 45″. Take from this the altitude of the equator, and there remains the Moon's apparent declination, 17° 39′ 30″. The Moon's longitude was ♊ 27° 30′ and its latitude 5° 14½′ south. Therefore the parallax was 33½′, greater than the Copernican by 30″, because the true declination was 18° 13′.

There follow three other observations of the Moon standing in the nonagesimal degree of the ecliptic from the horizon, where the apparent and the true longitude are the same.

Obſeruationes 3. Tychonis Lunæ in Quadrato in Nonageſimo.

Three observations by Tycho of the Moon at quadrature, at the nonagesimal.

In the year 1586 September 23, at 3^h 9^m after midnight, with the Moon at the nonagesimal of the ecliptic, the apparent latitude was 4° 39½′ south, but the true latitude was 4° 1′; the apparent altitude of the centre was 49½° and the observed diameter 32½′; and so the parallax was 38½′, greater than the Copernican by 30″.

In the same year, October 24, at 6^h 12^m after midnight, when the Moon was at the nonagesimal and in ♋ 27° 5′, the apparent latitude was 5° 50′, but the true latitude was 5° 9′; the apparent altitude was about 49° and the observed diameter 33′; therefore the parallax was 41′, agreeing with the Copernican to the very minute.

On the following day, October 28, at 7^h 29^m after midnight, the Moon was at the nonagesimal and in ♌ 11° 10′; the apparent diameter was about 34′; the apparent altitude very nearly 45°; the true latitude 4° 45′ south, but the apparent latitude 5° 30′. Therefore the parallax was 45′, agreeing scrupulously with the Copernican.

Anyone might wonder, however, why Tycho produces no observations of parallaxes at full Moons. But I wonder more how, on pp. 115 and 119, he rejects the Copernican parallaxes and dissents from them, and yet, because of the agreement of his own with the Copernican ones, says on p. 462: [Margin: Tycho's disagreement with himself.] *"I have now reviewed six particular observations, made with my own instruments, from which I investigated the Moon's*

parallax; and since this agrees as closely as possible with those that the reasonings of Copernicus require at a like altitude, it is consistent that the Moon's parallaxes come out in this way, and consequently that its distance from the Earth, established above, is not unsuitable." But this is surely to dissent from himself. For parallaxes of the Moon cannot be suitable that rest on false Copernican observations and that at quadrature are derived from the assumption of a greatest latitude of 5°, which you think false.

[Margin: 7. The opinion of Longomontanus and Kepler.] VII. Longomontanus, in Book 1 of the *Theorica*, ch. 8 and 9, professes that he follows Tycho. Therefore what we have said about Tycho holds for him too; he does, however, apply a very small correction to these things. But I wish he had published the observations, or stated the reasons for this correction, so that they could be weighed.

[Margin: Kepler's 1st opinion.] Kepler, in the foundations of the *Ephemerides*, p. 5, says that for Tycho the Moon's eccentricity is greatest at the quadratures, 7248, and least at the syzygies; but that for himself, from the form of the physical hypothesis of which I shall speak in ch. 27, the eccentricity is greatest at the syzygies, 6543, and least at the quadratures, 4362, in units of which the radius of the orbit is 100000. But on p. 10 he concludes, from Brahean observations corrected by eclipses, that the Moon's distance from the Earth's centre at the syzygies is about 63 terrestrial semidiameters at apogee, a little more than 55 semidiameters at perigee, and therefore 59 as the mean. From this he concludes that at the syzygies the parallax at apogee is 54′ 41″ and at perigee 62′ 20″. But from these parallaxes there follows an apogee distance of 62 52′ semidiameters of the Earth and a perigee distance of 55 8′.

After this, in the *Epitome of Copernican Astronomy*, p. 482, he says it was fitting that the proportion of the bodies of the Earth and the Moon should be related to the proportion of the Moon's interval and the Earth's semidiameter just as the proportion of the body of the Sun and the Earth was related to the proportion of the solar interval and the Earth's semidiameter; and that from this it follows that, just as the body of the whole Earth is contained in the body of the Sun as many times as the Earth's semidiameter is in the interval or semidiameter of the Sun's orb, so the body of the Moon is contained in the body of the Earth as many times as the semidiameter is contained in the distance or semidiameter of the lunar heaven; and that the semidiameter of the lunar orbit is a mean proportional between the distance of the Sun and the Earth's semidiameter. [Margin: 2nd opinion.] And on pp. 483 and 776 he says that the distance of the Moon at apogee is 59 terrestrial semidiameters and at perigee 54, about as much as Tycho had found at quadrature.

Then on pp. 614 and 820, as also in the Rudolphine Tables, ch. 26, 28 and 31, he affirms that, after observing eclipses and comparing them with parallaxes, and from the proportion of the bodies a priori, on which see p. 482, he found the same distances of the Moon and the same parallaxes at the syzygies and at the quadratures, apogee with apogee and perigee with perigee everywhere

equal, and that its intervals do not vary by reason of the phases; and that the real eccentricity is always 4362, in units of which the mean distance and radius of the orb is 100000. Hence it comes about that for him the distance at apogee is 104362 units, and this is 59 terrestrial semidiameters; therefore at perigee it is 95638 units, or 53 56′ semidiameters, and the mean, which is 100000, becomes 56 28′ semidiameters of the Earth. [Margin: 3rd opinion.] Finally, in the Rudolphine Tables, p. 98, he sets the Moon's horizontal parallax at the syzygies as 58′ 22″ at apogee and 63′ 41″ at perigee. From these, trigonometry gives the distance at apogee as 57 54′ semidiameters of the Earth, at perigee 54 0′, and the mean 55 57′.

[Margin: 8. Lansbergen's opinion.] VIII. Philips van Lansbergen, in Class 5 of the lunar observations and in the *Uranometria* from p. 3, adduces chiefly three observations made at Goes on the meridian near the beginning of ♋, from which he derives his parallaxes, taking the greatest latitude of ☾ as 5° at the syzygies and 5° 16′ at quadrature. They are these.

Obſeruationes 3. Lansbergij Lunæ prope initium Cancri.

Three observations by Lansbergen of the Moon near the beginning of Cancer.

In the year 1600 March 11, at 6^h p.m., the apparent meridian altitude was 63° 51′; the place of ☾ was ♋ 1° 14′; the anomaly 8^s 12° 9′; the latitude 2° 19′ north; the declination 25° 49′ north; the altitude of the equator 38° 29′. Therefore the true altitude of ☾ was 64° 17′ and the parallax 26½′.

[Margin: Moon at opposition, at perigee.] In the year 1601 November 29, at 12^h 15^m p.m., the Moon's apparent altitude was 61° 25′. At that time the Moon's place was ♊ 21° 28′; the anomaly 5^s 27° 27′; the latitude 13′ north; the declination 23° 26′ north; the altitude of the equator 38° 29′. Therefore the Moon's true altitude was 61° 55′ and the parallax 30½′.

[Margin: Quadrature, Moon near perigee.] In the year 1602 September 26, at 16^h 59^m p.m., the Moon's apparent altitude was 59° 39′. The Moon's place was ♊ 27° 25′; the anomaly 5^s 10° 27′; the latitude 1° 45′ south; the declination 21° 43′ north; the altitude of the equator 38° 29′. Therefore the Moon's true altitude was 60° 12′ and the parallax 33½′.

He adds 17 other observations of the Moon in conjunction with the fixed stars, and from his own parallax concludes that Copernicus erred, but Tycho more. But I have more than once insisted that from these applications of the Moon a parallax of determinate measure is not gathered or confirmed, unless the latitude is established independently of parallax; and Lansbergen could not establish this in that climate outside eclipses, unless he appeals to Ptolemy's observations. Moreover, I miss from Lansbergen observations of the full Moon at apogee and of the dichotomous Moon likewise at apogee, so that the parallaxes appropriate to these places might be more certain.

[Margin: 9. The opinion of Wendelen and Kircher.] IX. Govaert Wendelen has so far published no observations on this matter, but in the *Idea of the Atlantic*

Tables he nevertheless brings forward his opinion, which in the table below I shall show along with others. Nor does Athanasius Kircher, in vol. 2 of the *Ars Magna*, Book 9, Problem 9, set down any observation, but he nevertheless gives distances, which you may see in our table at the end of the chapter.

[Margin: 10. Boulliau's opinion.] X. Ismaël Boulliau, in Book 4 of the *Astronomia Philolaica*, supposes from eclipses that the diameter of the Sun at apogee is smaller than the diameter of the Moon at perigee, but the diameter of the Sun at perigee greater than the diameter of the Moon at apogee; and that the Moon's diameters can be known exactly from the Moon's motions in eclipses, because the Moon's motion is known exactly enough, as is also the diameter of the shadow. And from these he examines several distances of the Moon, in order to find the one that of all disagrees least with these diameters. At the quadratures, however, he hunts down the Moon's distance from the form of the ellipse that he employs in the theory of the Moon; for at the quadratures the umbilicus of this ellipse is taken to recede from the Earth by as much as it is itself distant from the centre of the ellipse. But those proportions of the semidiameters could be preserved with the distances altered proportionally. So, in order to have one determinate distance and parallax from which to gather the others, he adduces a single observation of the Moon in the same azimuth as the star preceding the foot of the leading Twin, than which the lower limb of the Moon, in great part eclipsed, appeared higher by 8′. The year was 1638, December 20, and the hour, from the altitude of the Heart of Hydra, $13^{h}\ 40^{m}$ p.m. From this he gathers the altitude of the former star as 56° 42′ and its southern latitude, from Tycho, as 28° 25′, and the Moon's apparent altitude as 57° 7′, whereas from the given latitude and place it ought to have been 57° 40′; therefore the parallax was 37′. And so the parallax for the syzygies, since it is gathered from the latitude, which can be known without parallax, seems more certain; but the one established for the quadratures from the form of the ellipse rather than from observations is not equally certain.

[Margin: 11. Our observations and opinion.] XI. We, for our part, among the many observations that we made at Bologna on this matter, and that, God granting, we shall report in the third volume of the *New Almagest*, have here selected a few, which we trust most, as also the Sun's meridian altitudes taken at the preceding noon. They are as follows.

Obſeruationes 3. Noſtræ in Oppoſit. Lunæ.

Our three observations of the Moon at opposition.

Prima Obſeruatio.

First observation.

[Margin: Moon at apogee.] In the year 1645 May 10, in the following night, from the altitude of Spica in the west, the time at which the Moon's centre passed through the midheaven [M.C.] was $11^{h}\ 53^{m}\ 52^{s}$ p.m., but the equated

time $11^h\ 44^m$. At that time the true place of ☾ on the ecliptic was 22° 30′ 58″; the equated anomaly 0^s 4° 24′ 6″; the true latitude 4° 58′ 22″ north; the declination, from Regiomontanus, 13° 37′ 53″ south; the distance of the equator from the zenith 44° 29′ 30″. Therefore the true distance of ☾ from the zenith was 58° 7′, but the apparent distance, with refraction removed, was 58° 53′ [recte: 58° 53′ 37″, per the book's errata]. Therefore the parallax was 44′ 37″; the distance from the Earth's centre 64 13′ semidiameters of the Earth; and the horizontal parallax 53′ 44″.

Secunda Obſeruatio.

Second observation.

[Margin: Moon at apogee.] In the year 1646 June 27, in the following night, from the distance of Aquila, in the east, from the zenith, the time at which the Moon's centre crossed the meridian was $11^h\ 53^m\ 47^s$ p.m., but the equated time $11^h\ 55^m\ 35^s$. At that time the Moon's place was ♑ 4° 52′ 40″; the equated anomaly 11^s 25° 14′ 27″; the true latitude 2° 16′ 55″ north; the true declination 21° 8′ 5″ south; the distance of the equator from the zenith 44° 29′ 30″. Therefore the Moon's true distance was 65° 37′ 55″, but the distance observed on the meridian was 66° 24′ [recte: 66° 24′ 20″, per the book's errata], and because of refraction 66° 26′ 42″. Therefore the parallax was 49′ 7″, and hence the Moon's distance from the Earth's centre 64 10′ semidiameters of the Earth and the horizontal parallax 53′ 35″. But at the apogee itself the former is 64 15′ semidiameters and the latter 53′ 30″.

Tertia Obſeruatio.

Third observation.

[Margin: Moon at perigee, at opposition.] In the year 1648 March 8, from the altitude of Spica in the east, the time at which the Moon's centre passed across the meridian was, in equated time, $11^h\ 56^m$ p.m. At that time the Moon's place on the ecliptic was ♍ 13° 13′; the equated anomaly 5^s 24° 40′; the latitude 4° 48′ 13″ north; the declination 11° 9′ north; the distance of the equator from the zenith 44° 29′ 30″. Therefore the Moon's true distance from the zenith was 33° 20′ 30″, but the apparent and observed distance was 33° 55′ 30″. Therefore the parallax was 35′, and the distance from the centre of the Earth 54 0′ terrestrial semidiameters, and the horizontal parallax 63′ 43″. But at the perigee itself the former is 53 45′ semidiameters and the latter 63′ 55″.

Obſeruationes 3. Noſtræ in Quadrato Lunæ.

Our three observations of the Moon at quadrature.

Prima Obſeruatio.

First observation.

[Margin: Moon at apogee, at quadrature.] In the year 1646 October 16, from the altitude of Aquila in the west, the time at which the Moon's centre passed across the meridian was $6^h\ 15^m\ 48^s$ p.m., but the equated time $6^h\ 8^m\ 52^s$. At that time the Moon's place on the ecliptic was ♍ 23° 20′; the equated anomaly 0^s 1° 45′ 3″; the latitude 16′ 45″ north. Therefore the declination was 21° 12′ 55″ south. The distance of the equator from the zenith was 44° 29½′. Therefore the Moon's true distance from the zenith was 65° 42′ 25″. But the apparent distance was 66° 26′, and with refraction added 66° 29′ 55″. Accordingly the parallax was 47′ 30″ and the Moon's distance from the Earth's centre 66½ terrestrial semidiameters. Or, correcting it from the following still more refined observation, the distance of ☾ at apogee at quadrature becomes 66⅔ terrestrial semidiameters.

Secunda Obſeruatio.

Second observation.

[Margin: Moon at apogee, at quadrature.] Then in the year 1648 June 13, in the morning, the Moon's centre crossed the midheaven at the apparent time $18^h\ 16^m\ 8^s$ after noon of the 12th, as appeared from the corrected altitude of the Sun. At that time the Moon's apparent distance from the zenith was 52° 50′, but with refraction added it was 52° 50′ 30″. Therefore, when the distance of the equator from the zenith, which at Bologna is 44° 29′ 30″, is subtracted, there remains the Moon's apparent southern declination, 8° 21′. The equated time was $18^h\ 13^m$. At that moment the Sun's place was ♊ 22° 42′ 26″ and the Moon's on the ecliptic ♓ 22° 57′ 5″, its co-equated anomaly 11^s 23° 16′ 30″, and the Moon's southern latitude 5° 17′ 50″. Therefore, by the tables of Regiomontanus, the Moon's true southern declination was 7° 39′ 30″, but the apparent declination was 8° 21′; therefore the parallax was 41′ 29″, corresponding to the Moon's apparent distance from the zenith of 52° 50′ 30″, and hence the Moon's distance from the Earth's centre was 66 42′ terrestrial semidiameters. The quadrature had occurred at $17^h\ 45^m$. The phase of dichotomy, however, when the extremes of the doubtful time were computed and the intermediate one chosen, was at $18^h\ 43^m\ 30^s$, at which moment the Moon was 30′ 15″ on the ecliptic from the place of quadrature. And so, in Book 3, ch. 7, no. 11, using the angle S of the figure set out there in no. 9, and with the Moon's distance found as 66 42′ semidiameters, I derived the Sun's distance, nearly at apogee, as 7572 terrestrial semidiameters.

Tertia Obſeruatio.

Third observation.

[Margin: Moon at perigee, at quadrature.] In the year 1646 March 23, the Moon's centre crossed the midheaven at $5^h\ 30^m\ 36^s$ p.m., as was established from the corrected altitude of the Sun. At that time the apparent distance of the lunar centre from the zenith was 24° 56′ 30″. This, subtracted from the distance of

the equator, 44° 29′ 30″, leaves the Moon's northern and apparent declination, 19° 33′. At that equated moment the Moon's true longitude on the ecliptic was ♊ 26° 10′, the equated anomaly 5ˢ 26° 50′, and the true southern latitude 3° 26′. Therefore, by Regiomontanus's table, the Moon's true northern declination was 20° 1′ 10″, but the apparent declination was 19° 33′. Accordingly the parallax of altitude was 28′ 10″, which, with the Moon's apparent distance from the zenith of 24° 56′ 30″, gives the Moon's distance from the Earth's centre as 51 30′ terrestrial semidiameters. From these and similar observations we establish, at the quadratures, the apogee distance of ☾ as 66 42′ semidiameters and at the perigee itself as 51 28′, so that the mean is 59 0′, such as we found from the syzygies. To this distance there corresponds a horizontal parallax of 66 56′.

It remains for us to set before you, from the authors reviewed above, a synopsis of the lunar horizontal parallaxes and distances, in which three classes of opinions may be seen: for some put the greatest distance of ☾ from the Earth at conjunction and opposition and the least at quadrature; others the greatest and least at conjunction and opposition and the mean at quadrature; and others the greatest and least at quadrature and the mean at conjunction and opposition. Namely:

Distance of ☾:

- Greatest at conjunction and opposition, least at quadrature: Ptolemy, al-Battānī, al-Farghānī.
- Greatest and least at conjunction and opposition, mean at quadrature: Kepler formerly, but later he made them both equal.
- Greatest and least at quadrature, mean at conjunction and opposition: Copernicus, Magini, Tycho, Lansbergen, Longomontanus, Boulliau, ourselves.

Distance of the Moon from the centre of the Earth in semidiameters of the Earth, and horizontal parallax. Left half of the table: in the syzygies, or at ☌ and ☍. Right half: in the quadratures, □. Distances are given in semidiameters and minutes.

Authors	Syzygies: dist. apogee	Syzygies: dist. mean	Syzygies: dist. perigee	Syzygies: parallax apogee	Syzygies: parallax mean	Syzygies: parallax perigee	Quadratures: dist. apogee	Quadratures: dist. mean	Quadratures: dist. perigee	Quadratures: parallax apogee	Quadratures: parallax mean	Quadratures: parallax perigee
Ptolemy	64 10′	59 0′	53 50′	53′ 34″	58′ 16″	63′ 51″	43 53′	38 43′	33 33′	60′ 19″	89′ 0″	103′ 0″
Copernicus	65 30′	60 19′	55 8′	52′ 56″	57′ 0″	62′ 54″	68 21′	60 19′	52 17′	50′ 19″	57′ 0″	65′ 48″
Tycho	58 8′	56 30′	54 52′	59′ 9″	60′ 51″	62′ 39″	60 36′	56 30′	52 24′	56′ 44″	60′ 51″	65′ 36″
Longomontanus	57 38′	56 0′	54 23′	59′ 37″	61′ 26″	63′ 14″	60 4′	56 0′	51 57′	57′ 15″	61′ 26″	66′ 9″
Lansbergen	64 10′	59 5′	54 0′	53′ 34″	58′ 8″	63′ 39″	66 58′	59 5′	51 12′	51′ 20″	58′ 8″	67′ 6″
Boulliau	61 40′	59 5′	56 30′	55′ 56″	58′ 8″	60′ 52″	64 15′	59 5′	53 55′	53′ 30″	58′ 8″	63′ 43″

Authors	Syzygies: dist. apogee	Syzygies: dist. mean	Syzygies: dist. perigee	Syzygies: parallax apogee	Syzygies: parallax mean	Syzygies: parallax perigee	Quadratures: dist. apogee	Quadratures: dist. mean	Quadratures: dist. perigee	Quadratures: parallax apogee	Quadratures: parallax mean	Quadratures: parallax perigee
Ourselves, for the present	64 15′	59 0′	53 45′	53′ 30″	58′ 16″	63′ 55″	66 42′	59 0′	51 20′	51′ 32″	58′ 16″	66′ 56″
Kepler, in the *Ephemerides*	62 52′	59 0′	55 8′	54′ 41″	58′ 16″	62′ 20″	61 36′	59 0′	56 24′	55′ 26″	58′ 16″	60′ 58″
Kepler, in the *Epitome of Copernican Astronomy* and the Rudolphine Tables	59 0′	56 28′	54 0′	58′ 22″	60′ 53″	63′ 41″	59 0′	56 28′	54 0′	58′ 22″	60′ 53″	63′ 41″
Hipparchus	83 0′	77 0′	71 0′	41′ 30″	45′ 0″	48′ 30″						
but later	72 30′	67 30′	62 30′	47′ 30″	51′ 30″	55′ 30″						
Alfonso	61 8′	57 45′	54 23′	53′ 19″	59′ 21″	63′ 17″						
Argoli	60 4′	56 0′	51 57′	56′ 45″	61′ 17″	65′ 36″						
Wendelen	63 56′	60 0′	56 4′	53′ 46″	57′ 18″	61′ 18″						
Kircher	67 50′	62 37′	57 24′									

[Note set inside the table, in the lower right:] On the opinion of Pythagoras, Eratosthenes, Posidonius and Mario Bettini, see Scholia 2 and 3. On the proportions of the lunar orbs, see below, from chapter 23 to chapter 25. As for Don Vicenç Mut, who writes to me that he is certain that the distance of ☾ at apogee is about 62 semidiameters of the Earth, I shall see to that when his most learned letter on the Alphonsine motion comes out.

SCHOLIA.

Scholia.

[Margin: The variation of the lunar distances is most certain.]

I. Although it may be questioned whether the Sun varies its distance from the Earth, there can be no doubt as regards both luminaries, both because of the plain variation in the Moon's apparent diameter and in her parallaxes, and because the differing magnitudes of eclipses require this variation. That is why Pliny the Elder (Book 2, chapter 9) rightly says of the Moon: "And then she is both low and lofty, and not even this in one way, but at times brought close to the heaven, at times touching the mountains, now raised into the north, now thrust down into the south." And in chapter 16 he confirms the same variation from the swifter motion of a planet at perigee, saying: "It is likewise acknowledged that the motion increases so long as they are in the vicinity of

the Earth, a principle most strongly confirmed by the elevations of the Moon." In vain, therefore, does Girolamo Fracastoro (*Homocentrica* Book 3, chapter 23) try to transfer the parallax and the variation of lunar distance onto another circle, which would vary the Moon's latitudes and declinations while the same distance is kept.

[Margin: The Moon's distance in Father Bettini is too great, and that of the fixed stars too small.]

II. It is clear from what has been said that the lunar distance, which Father Mario Bettini (*Apiarium* 10, Progymnasma 1, proposition 3) raised, from the intervals of his monochord, as far as 382 terrestrial semidiameters, far exceeds every measure of the astronomers, and cannot be reconciled with the distances deduced from the Moon's parallaxes. Yet he himself gives and approves this method of deducing them, in volume 2 of the *Aerarium Philosophicum*, p. 73, and in *Apiarium* 8, Progymnasma 3, proposition 9, and Progymnasma, proposition 4. For from his distance there would follow a parallax of only 10½′. What is worse, it would follow that the Moon could never be eclipsed by the Earth's shadow, since no astronomer extends the Earth's shadow beyond 282 semidiameters of the Earth, as is clear from what was said in Book 3, chapter 11, Problem 8; whereas the same author (*Apiarium* 8, Progymnasma 2, proposition 5) maintains, and rightly, that she suffers that eclipse from no other source than the Earth's shadow. Indeed, the whole Sun would never be eclipsed by the Moon, although he grants (*Apiarium* 8, Progymnasma 2, proposition 10) that the whole of it can be covered by her.

Nor does his lunar distance agree with the Sun's distance of 1144 terrestrial semidiameters, which the same author (*Apiarium* 10, Progymnasma 1, proposition 3) established from harmonic intervals; or with the distance derived from Aristarchus's problem applied to the lunar distance (and this method the same Father gives in volume 1 of the *Aerarium*, p. 629). For when the Moon shines on us half-lit, she is evidently elongated from the Sun by less than 87°, or from the moment of quadrature by less than 6^h. But let us suppose (as Aristarchus supposed, for ample safety against the Pythagorean intervals of the luminaries) that the Moon is then 87° from the Sun. By the third problem given in Book 3, chapter 7, if the Moon is 382 semidiameters of the Earth from the Earth, it follows by trigonometric necessity that the Sun is 7299 semidiameters of the Earth from the Earth. This is indeed not altogether false, but it is nevertheless against the distance of the Sun asserted by Father Bettini. Still, he must be pardoned what Pythagoras is pardoned, because they looked only to pure harmonic proportions, so as to show what intervals of the stars followed from them. Thus the same Father, in the same place, puts the distance of the fixed stars at only 2290 semidiameters of the Earth, from which there follows a parallax of 1½′, against all astronomers. Nor indeed does a harmony in which sense and reason do not agree suit Mercury or Apollo, as Ptolemy warns (*Harmonics* Book 1).

[Margin: The opinion of Pythagoras, Eratosthenes and Posidonius.]

III. As for the ancients, I have already pointed out their errors in lunar distances, whether of defect or of excess, in Book 3, chapter 7, Scholium 1. There, from Plutarch (*De placitis philosophorum* Book 2, chapter 31), Pliny the Elder (Book 2, chapters 21 and 23) and Censorinus (chapter 11), I showed that Pythagoras once put 120,000 stadia from the Earth to the Moon, that is 15,000 Italian miles, which scarcely make four semidiameters of the Earth; but that Eratosthenes put 780,000 stadia from us to the Moon, or 97,500 Italian miles, which scarcely reach 25 semidiameters of the Earth. Posidonius therefore, according to the same Pliny the Elder, did better and put 2,000,400 stadia; but of such stadia he gave the Earth's semidiameter 38,182, and so he put the distance from the Earth to the Moon at 52 15/38 semidiameters of the Earth. But Petosiris and Nechepso childishly narrowed this distance to 1980 stadia, on the testimony of Pliny the Elder (Book 2, chapter 23).

[Margin: The variety of parallactic tables, and their authors.]

IV. Once the horizontal parallax has been established for the Moon at apogee and at perigee, both in the syzygies and in the quadratures, several further parallactic tables must be constructed. One sets, against the individual signs and degrees of the anomaly, either the Moon's distance from the Earth or the horizontal parallax proper to that place of the anomaly. Another sets at its head the lunar distances from the Earth, or at least the horizontal parallaxes, and beneath them other parallaxes proper to each degree of the Moon's altitude above the horizon: altitude, I mean, both true and apparent, since it is expedient that this table be double, although we can use a simple one, according to what was said in Book 3, chapter 8, number 7. For these, use the problems given in the same chapter 8. Such tables are given by Ptolemy (Book 5, chapter 18); Copernicus (Book 4, chapter 24); Erasmus Reinhold in the Prutenic Tables, p. 120; Magini in the *Second Mobile*, p. 678, and in the *Supplement to the Ephemerides*, p. 292; Tycho, volume 1 of the *Progymnasmata*, pp. 119, 120; Longomontanus, *Theorica* Book 1, chapter 8, p. 279; Kepler in the *Astronomia Optica*, p. 424; Nicolaus Mulerius in the *Tabulae Frisicae*, p. 421; Lansbergen in his tables, p. 47; Boulliau in his tables, p. 173; and Vincenzo Renieri in his tables, p. 333.

Besides these, there exist tables for the parallax of longitude and latitude, for several degrees of the height of the pole, or several climates. Theon of Alexandria constructed such tables, on the testimony of al-Battānī (chapter 40); and following him the Alphonsine Tables, p. 112; Erasmus Reinhold in the Prutenic Tables, p. 98; and following him Magini in the *Second Mobile*, p. 691, and Lansbergen, from p. 51 to p. 93.

But instead of tables that show the parallaxes of latitude and longitude, one may use Kepler's parallactic table, which Boulliau and Renieri copied out, as noted above, or the solu- tion of a triangle, which we treat in the problems of the primum mobile, Book 10, and in the rules for eclipses; Ptolemy (Book 5, chapter 19); Regiomontanus (*Epitome* Book 5, from proposition 26 to proposition 32); Copernicus (Book 4, chapter 26); Daniel Santbech (*De observat.*,

p. 93); Tycho (volume 1, p. 133); Longomontanus (*Theorica* Book 1, chapter 8, p. 282); Mulerius in the *Tabulae Frisicae*, p. 435; Erasmus Reinhold on Georg Peuerbach's *Theoricae*, p. 193; and Francesco Barozzi in the *Cosmographia*, p. 303.

CAPVT XV.

Chapter 15.

De Refractionibus & Elliptica Lunæ Figura in Horizonte.

On refractions and the elliptical figure of the Moon on the horizon.

I. We dealt with the Sun's refractions in Book 3, chapter 9; but most of what was said there is common to the Moon's refractions. We therefore send the reader back to that place, since we also pointed out there the tables of lunar refractions and their authors. And although I have a great many observations of both luminaries for refractions, taken at different times of the year, I reserve a more subtle examination of them for the Optical Book, for the most just reasons. For in the meantime Tycho's table, which we shall give at the end of this volume, is not unsuitable. So that the present chapter may not be too short, I shall add a few things on the quasi-elliptical figure which the Moon herself takes on at the horizon at full Moon.

[Margin: The Moon an ellipse.]

II. The full Moon, then, rising or setting at the horizon, shows an egg-shaped figure, but at times more pointed than the Sun, almost as you see in the figure attached here. The cause is the same as that of the solar ellipse, already set out in Book 3, chapter 2.

Diagram. An oval, pointed at its left and right ends, set into the left column. A horizontal diameter joins the two points and a vertical diameter crosses it at the centre. Labels read:

- A (left point): end of the long diameter
- B (right point): other end of the long diameter
- C (top, outside the outline): upper end of the short diameter
- G (bottom, outside the outline): lower end of the short diameter
- D (just above the crossing, right of the vertical): no function stated in this passage
- K (inside, upper left): no function stated in this passage
- L (inside, upper right): no function stated in this passage

Labels not read: none.

Take now, from our Georges Fournier (*Hydrographia* Book 13, chapter 2), the measures of the Moon's apparent diameter observed by Pierre Gassendi in 1634, at the full Moons of October and December. Ours do not much conflict with them,

Month	Altitude of ☾, apparent	Diameter AB	Diameter CG
October	4′ 40″	37′ 40″	34′ 0″
	5′ 15″	37′ 15″	35′ 0″
	7′ 0″	37′ 0″	36′ 0″
December	0′ 0″	37′ 0″	29′ 48″

since at times, with the Moon at perigee about the equinoxes, using a sextant, we saw the diameter AB span 38′ or even 40′. Nor can this be ascribed to dilation of the pupil, since one observer was looking at one edge and another at the other.

CAPVT XVI.

Chapter 16.

De Magnitudine Apparenti Lunæ.

On the apparent magnitude of the Moon.

I. Almost everything we said about the apparent diameter of the Sun in Book 3, chapter 10, holds for the diameter of the Moon, except that in the Moon the change of diameter and of distances is more evident. But from the shadow, or from her image received through a hole onto a board parallel to the hole, the measure of this diameter is less evident. Of the six principal methods of observing the diameters of the luminaries, which I described in that same place, the 1st, 2nd, 3rd and 4th therefore suit the Moon better.

[Margin: A method of observing the Moon's diameter by eclipses.]

There remains, however, a threefold special method of investigating the Moon's diameter from eclipses. The first is the one handed down, together with examples, by Ptolemy (Book 5, chapter 14), al-Battānī (chapter 30), Regiomontanus (*Epitome of the Almagest* Book 5, propositions 18 and 19), Copernicus (Book 4, chapters 18 and 19) and Biancani (*Sphaera* Book 9, chapter 8); and I shall bring it forward shortly in reporting the Ptolemaic and Copernican opinion, because it is based on the Moon's latitude and the digits obscured. The second is based on the Moon's hourly motion and the time of incidence, of which I shall speak in number 14. And the third lies in comparing the lunar diameter with the interval between fixed stars, of which I shall speak in number 9.

[Margin: 1. Ptolemy's opinion.]

II. Ptolemy, then (Book 5, chapter 14), rejects the diameter gathered from a time counted by water-clocks, or by Hipparchus's dioptra. By the dioptra, however, he says, the visual diameter of the Sun and the Moon at ☌ and at apogee was found equal to the Sun's, "without any sensible difference that need concern us." He relates that in the year 127 of Nabonassar, on the 27th day of the month Athyr, in the following night, at the 5th temporal hour, at Babylon, the Moon began to be eclipsed, and that at the height of the eclipse a quarter

of the lunar diameter was eclipsed, towards the south. Supposing the Sun then to be at ♈ 27° 5′, he gathers that the middle of the eclipse was at 5^h 50^m equal time after midnight at Babylon, but at 5^h at Alexandria, and in equated time 4^h 45^m; and that accordingly the Moon's mean motion was ♎ 25° 32′, but her true motion 27° 5′, her anomaly 340° 7′, her distance from the node 9° 20′, and so her latitude north 48′ 30″.

Next, in the year 7 of Cambyses, or 225 of Nabonassar, on Phamenoth 17, in the following night, 1^h before midnight at Babylon, but at Alexandria 1^h 50^m equal, and equated 2^h 10^m, the Moon was eclipsed at the middle, and indeed by half of herself, towards the north. From this, supposing the Sun at ♋ 18° 14′, he deduces the Moon's mean place at ♑ 20° 22′, but her true place 18° 14′, her anomaly 28° 54′, her distance from the node 7° 48′, and her latitude south 40′ 40″. Since, therefore, in both cases the Moon was near apogee, from which she stood only 20° in the earlier eclipse and 29° in the later, he gathers the apparent diameter of the Moon at apogee as 31′ 20″, and so that of the Sun as well; and that of the shadow as 40′ 40″, at the place of the transit of the Moon at apogee.

Diagram. A large hatched disc, set into the right column, with a vertical line through its centre running out to a small circle overlapping its top edge and another overlapping its bottom edge. Labels read:

- E (left, outside the disc): point on the shadow's disc
- L (right, outside the disc): point on the shadow's disc
- B (top, inside the upper small circle): centre of the Moon at the first position
- K (on the vertical, within the disc, below B): end of the Moon's semidiameter BK
- ring at the centre of the disc, on the vertical: no letter beside it; the text calls the centre O
- P (on the vertical, within the disc, near the bottom): end of the eclipsed part AP
- A (on the vertical, where it meets the bottom edge): end of the shadow's diameter
- D (inside the lower small circle): centre of the Moon at the second position

Labels not read: none.

For let AEBL be the disc of the Earth's shadow, and AB its diameter. Let B be the centre of the Moon, eclipsed to the north by exactly her whole semidiameter BK, but let her be eclipsed to the south by a quarter of herself, AP, with the Moon's centre placed at D. Let the Moon's latitude OB be 40′ 40″, which is equal to the latitude OB; for it is clear that the apparent semidiameter of the Earth's shadow is also that great, at the place of the transit of the Moon having an anomaly of 29°. For in an eclipse of half the Moon the lunar centre touches the periphery of the shadow, and BO is the Moon's latitude. Therefore,

because the anomaly was almost the same in both cases, OA, the semidiameter of the shadow, was 40′ 40″; but the latitude OD was 48′ 30″. So when OA is taken away, AD remains, 7′ 50″; and this was a quarter of the lunar diameter. Therefore, by multiplying by four, the apparent diameter of the Moon, as though at apogee, was 31′ 20″. Ptolemy adds that in many observations of this kind he found these quantities very nearly in agreement, and therefore used them for investigating the distance of the Sun; and that the diameter of the Moon at perigee in the syzygies was 35′ 20″.

[Margin: A method of gathering from one the other quantities of the apparent semidiameter.]

How the Moon's true semidiameter is gathered from the distance of the Moon at apogee found by parallaxes, as stated in chapter 14, and from her apparent semidiameter, and from that the apparent semidiameter for her other distances from the Earth, is clear from what was said in Book 3, chapter 11, numbers 2 and 4, and Copernicus teaches it (Book 4, chapter 22). But you have already seen that in these eclipses the Moon was not strictly at apogee; indeed, she stood almost a whole sign from apogee. Next, that determination of the quarter part eclipsed rests on an estimate by eye, and it is not so certain as is the estimate of half the Moon, or of the whole Moon eclipsed in darkness without duration.

[Margin: Ptolemy's error.]

Moreover, the Sun's anomaly was not the same in both cases, since in the earlier eclipse the Sun stood about its mean distance, and in the later it was nearer apogee. This reasoning of Ptolemy's is therefore rather crude, and without doubt erroneous.

[Margin: Our method is better than Ptolemy's.]

Better for this purpose are two eclipses, in one of which half the Moon, and in the other the whole Moon without duration, is eclipsed, provided that the Sun and the Moon each hold the same anomaly in both cases. For then the Moon's latitude proper to the later eclipse, if subtracted from the latitude of the half eclipse, will reveal the Moon's apparent semidiameter. But in our whole history of eclipses I could find no such comparisons of eclipses.

[Margin: 2. The opinion and error of al-Battānī, Alfonso, Clavius, Peuerbach and Hérigone.]

III. Al-Battānī (chapter 30), from the observation of eclipses, determines the apparent diameter of the Moon at perigee in the syzygies as 35′ 20″, with Ptolemy; but at apogee as 29′ 30″. Accordingly, since he sets the diameter of the Sun at apogee at 31′ 20″, he denies that ☼ at apogee can be wholly covered by ☾ at apogee. The Alphonsine Tables, in their table of semidiameters, p. 115½, and Peuerbach in his tables of eclipses, give the Moon at apogee 29′ 0″ and at perigee 36′ 8″. Clavius, in the *Sphaera*, favours them, as does Hérigone, who gives the former 30′ and the latter 36′. But Lansbergen, in the Prolegomena

to the *Uranometria*, demonstrates from these and from the distances set by al-Battānī that the distance of the Sun at apogee follows as 7936½ semidiameters of the Earth, and greatly conflicts with the distance of ☼ set by al-Battānī.

[Margin: 3. The opinion of Copernicus, Reinhold and Mästlin.]

IV. Copernicus (Book 4, chapter 19), from unnamed eclipses, corrects the Ptolemaic diameters, saying: "We have judged that these should be put in order and emended thus: we set the apparent diameter of the Sun at apogee at 31′ 40″ (for it must in some way be greater now than before Ptolemy); that of the Moon, full or new, and at the highest apse, at 30′; and also the diameter of the shadow, at her very passage through it, at 80 3/5′. For it is fitting that there should be in them a slightly greater ratio than 5 to 13, but as 150 to 403; and that the whole Sun is not covered by the Moon unless she has a distance from the Earth of 62 parts, of which the radius from the centre of the Earth is one. For these, so set down, seem by a certain reason to cohere both with one another and in everything else, and to agree with the apparent eclipses of the Sun and the Moon." Then in chapter 20 he gathers from these assumptions that the Earth's true diameter is to the Moon's true diameter as 7 to 2, or in a triple sesquialter ratio; and from this in chapter 22 he gathers the Moon's apparent diameter proper to the distances we dealt with in chapter 14: namely, for the Moon at perigee in the syzygies 35′ 38″; but in the quadratures at apogee 28′ 45″, and in the quadratures at perigee 36′ 44″. Mästlin subscribes to this, in the appendix to Kepler's *Prodromus*, p. 154, and so does Reinhold in the Prutenic Tables.

[Margin: And the error.]

But Lansbergen, in the Prolegomena to the *Uranometria*, demonstrates from these and other data given by Copernicus a geometric inconsistency in the Copernican hypothesis: for it follows that the distance of the Sun from the Earth is 941 53′ terrestrial semidiameters, which conflicts with Copernicus himself. From this you see what you should think about p. 123 of the Prutenic Tables, where the Moon's diameter is put at 30′ at apogee and 35′ 38″ at perigee, as Magini also once put it in the *Theoricae*.

[Margin: 4. The opinion of Tycho and Mulerius.]

V. Tycho (volume 1 of the *Progymnasmata*, pp. 119 and 135) puts the Moon's diameter in the quadratures at 32′ 32″ at apogee and 36′ at perigee; but for the Moon at ☌ he establishes 25′ 36″ at apogee and 28′ 48″ at perigee, and at ☍ 32′ at apogee and 36′ at perigee. "Since it has been noticed," he says, "at the ecliptic ☌ of the luminaries, that the Moon does not keep the same visible diameter that she has at other times, but that her edges are thinned by the force of the Sun's light, as some optical reason suggests, so that about a fifth part of the Moon is thus lost." Mulerius followed his table in the *Tabulae Frisicae*, p. 426, and so did Origanus in volume 1 of the *Ephemerides*.

[Margin: Tycho's errors here, or his deception.]

Three things tell against this hypothesis of Tycho. First, from these and the distances of the luminaries that he gives, it follows that the angle of the

Earth's shadow is greater than the angle subtending the Sun's apparent diameter, whereas it ought to be less, as Lansbergen demonstrates in the Prolegomena to the *Uranometria*; and Kepler, in the *Ephemerides*, p. 9, admits that from the hypothesis of the Tychonic distances there follows a diameter of 33′ 9″ at apogee and 34′ 48″ at perigee. Second, a total eclipse of the Sun could never occur, since the diameter of the Sun at apogee, that is its least, is 30′, whereas that of the Moon at ☌ at perigee, that is its greatest, is 28′ 48″. And so Tycho, when he had read in Clavius (*Sphaera*, chapter 4) that a total eclipse of the Sun, with duration and dreadful darkness, was observed at Coimbra in 1560, wrote to Clavius that he marvelled that a total eclipse had been seen in Portugal, and nocturnal darkness by day; for his observations do not allow the Sun to be wholly covered by the Moon, however low she may be. So Kepler relates, in the appendix to volume 1 of Tycho's *Progymnasmata*, and in the *Optics*, p. 285. But that Tycho's little table is not to be relied on was shown, Kepler says in the same place, by the solar eclipse of 1601 December 24, shortly after the death of Tycho, who had died on October 24, as Willebrord Snel relates in the *Observationes Bohemicae*; for that eclipse was greater than the Tychonic lunar diameter. And we shall show below, in number 15, that total eclipses of the Sun have appeared more than once.

[Margin: Tychonic observations of the lunar diameter.]

Third, if we weigh Tycho's observations on this point, their uncertainty and deceptiveness will at once be plain. For Kepler says in the *Astronomia Optica*, p. 218: "In Tycho's observations, with five or six men observing the same Moon, according to the sharpness of each one's sight, the reckoning of the diameter grew, wandering, from 31′ to 36′; and in 1591 February 22 the Moon was observed twice at 31′, six times at 32′, seven times at 33′, six times at 34′, once at 36′." And on p. 348 he goes on in these words: "In 1592 June 14, with an eclipse at hand, Brahe's men, measuring the Moon's diameter with the astronomical radius according to Gemma's teaching, reported 32′ or 31½′: the Moon was at apogee. The method is both uncertain, and always presumed to state more than is right." See here what I said against the radius in Book 3, chapter 10, number 3, and shall say more fully in the *Liber Organicus*. Kepler adds in the same place that in 1592 February 12 the diameter of the Moon, when she was not at her lowest, was observed by the same men as 35′; that in 1587 January 6 the diameter of the quarter Moon at apogee, from the altitude of the horns, was found to be 30′; that in 1588 March 2 the difference of declinations between the edges of the Moon, by the armillaries, was found to be 33′, 32⅓′, 31′ and 30¾′; and that this very method is somewhat less certain. Let us therefore dismiss Tycho.

[Margin: 5. Magini's more recent opinion.]

VI. Magini, then, in the *Supplement to the Ephemerides*, Canon 3 on calculating eclipses, admits that Canon 68 for finding the diameters of the luminaries, which he gave in the tables of the *Second Mobiles*, on the hypothesis of Copernicus, does not satisfy the heavenly appearances. He condemns Tycho's hypotheses in

this matter and their follower Origanus; and finally he gives a table according to Kepler's view, in the appendix to Tycho's *Progymnasmata*, in which the apparent diameter of the Moon, new as well as full, is 30′ 30″ at apogee and 34′ 40″ at perigee.

[Margin: 6. Kepler's inconstant opinion.]

VII. Kepler himself, indeed, has shown himself to us more inconstant than the Moon herself. For in the *Astronomia Optica*, p. 344, he says that in 1600 July 25 the ray of the full Moon at mean distance, let through a hole onto paper, was of more than 105½ parts and of fewer than 100, and therefore greater than 29½′ and less than 31′ 12″; but that by this route it is estimated as smaller than is right, because the extremes of the ray, being of very weak light, are not well caught by the eyes. In 1603 January 26, however, he says the ray of the Moon at perigee was greater than 113 particles and less than 120, and therefore greater than 32′ and less than 34′ 18″. On p. 347, from a comparison with the distance of the Pleiades, of which I shall speak in number 9, he gathers the diameter of the Moon, as she heads towards apogee, as 31′ or 32′. And on pp. 348 and 359 he concludes, from the solar eclipse of 1601 December 24 and the image of the lunar shadow measured on paper, that the diameter of the Moon at apogee is not less than 30½′, so that the Moon at apogee cannot wholly cover the Sun at perigee; but that the diameter of the Moon at perigee is still uncertain to him, whether it is 33⅓′, or 34′, or 34⅔′, and whether the mean is 31′ 55″, or 32′ 15″, or 32′ 35″.

But in the foundations of his *Ephemerides* he restricts the Moon's diameter in the syzygies to 30′, and at perigee to 34⅙′. He admits, however, that he did violence to his own observations, so as to indulge his predecessors, who had affirmed that the Moon, carried beneath the Sun, did not cover the whole of it, and so as to concede to Hipparchus that the diameters of the Sun and the Moon at apogee are equal. And so, although he had wished from his own observations to make the Moon's diameter at apogee 30′ 30″, and even 31′, he nevertheless gave up one minute or half a minute, lest under the compulsion of his hypothesis he should extend the Moon's diameter at perigee too far beyond 34′. Yet he frankly adds: "Nor is the matter closed with me by these figures, because I reserve the liberty of increasing it by one minute, if a greater number of observations should seem to require it." [Margin: David Fabricius's opinion on an increase of the lunar diameter.] Now David Fabricius, because of air or thicker aether surrounding the Moon (which we discussed in chapter 2), held that the lunar diameter in eclipses of the Moon appears greater by 4′ or 5′, and that the duration of the eclipse therefore comes out longer; but in eclipses of the Sun smaller, because of refractions of the Sun's rays occurring in the Moon's atmosphere. Kepler opposes him in the same *Ephemerides*, p. 9. For although he concedes something to this air, it is not so much, nor the whole of this effect. Other causes, he says, contribute to it: namely the deception of sight, which enlarges the image of white and bright things, as he has said in *Astronomia Optica*, p. 286; the diameter of the Moon, which is in reality at times

smaller than the Sun's; and the increase in the duration of an eclipse from the penumbra.

Lastly, in the Rudolphine Tables, precept 131, he says that if the difference of the coequated anomaly is increased by a sixtieth part of itself, the Moon's parallax results; but if that same sixtieth is added to half of the parallax, the Moon's apparent diameter results. In precept 147, however, he refers the reader to the table of semidiameters which he gives on p. 98, where the Moon's semidiameter in the syzygies is 15′ at apogee and 16′ 22″ at perigee. At the foot of that table he appends these words: "Although the Moon's diameter has been deduced very neatly by a demonstration from archetypal causes, observations nevertheless everywhere call for a larger one: perhaps because the height of the lunar air is so great that the demonstrated solidity of the globe is increased by it, all round, by 1′ or 1½′, or even by a larger amount according to the diversity of physical causes." And a little later: "The Moon at the vertical can also appear ½′ greater than on the horizon." So Kepler wavers between observations and the ideas of his hypotheses. In the *Epitome of Astronomy*, too, p. 861, he sets the semidiameter of ☾ at 15′ at apogee and 16′ 22″ at perigee. And on p. 893, so as not to be forced to contract the Moon's diameter, as the annular eclipse of the Sun seen by Clavius in 1567 would require, he shifts the cause of the phenomenon either onto the inflamed substance of a thicker aether around the Sun, or onto circumlunar air refracting the Sun's rays.

[Margin: 7. Longomontanus's opinion.]

VIII. Longomontanus, in *Astronomia Danica*, *Theorica* Book 1, chapter 9, on account of Tycho's observations gives the Moon at apogee in ☌, and taken by itself in ☌, a semidiameter of 16′ at a distance of 57 semidiameters of the Earth 38′. Therefore, for the Moon at perigee, at a distance of 54 semidiameters 23′, he is forced to give a semidiameter of 17′, two minutes smaller than Tycho's. "For although," he says, "it appears greater at full Moon, this nevertheless happens to it incidentally, because of the refraction of the lunar rays in the thick air around the Earth, especially near the horizon." Because of this refraction he thinks that the Sun's diameter too is enlarged, by the refracted solar rays and by optical dilation, and that therefore the Moon's diameter at ecliptic new Moons appears smaller by a fifth of itself and more. This, he holds, is established by the eclipses from 1589 to 1597 observed with a visual instrument; by the bright ring of 1½ digits around the Moon, observed by M. Andrea Fossio [Anders Foss] in the eclipse of the Sun of 1601 December 24 at Bergen in Norway; and by the eclipse of the Sun of 1608 August 10, which at Copenhagen, with parallax allowed for, ought to have been of at least one digit, and yet none was seen, he says, because of the already mentioned refraction, greater there than in Bohemia, where the air is purer. And therefore, he says, the eclipse of the Sun of 1600 June 30, which ought to have appeared scarcely 4 digits, was greater than 5½ digits. But the chief reason why, in an eclipse of the Sun, the Sun's diameter appears greater relative to the Moon, he says, is the diffusion of the Sun's light through the air around the cone of the lunar shadow, from

rays reflected from the Earth. For when the Moon's shadow is diminished and diluted, the edges of the Sun, which are seen under the angle of the shadow, must appear under a greater angle, and dilated. So too in a lunar eclipse the Moon's diameter appears greater, not only by the force of the refraction of the lunar rays that occurs in the thick air around the Earth, but also because the Earth's shadow is diminished by the rays of the Sun refracted in the same air. For these reasons, on p. 301, he sets out a small table for correcting the semidiameters of the luminaries by an optical equation, greater, however, in eclipses than outside them; and it is given below.

Optical equation of the lunar semidiameter, for the polar altitudes listed below: to be subtracted in ☌; to be added in ☍.

Degrees of polar altitude	20	30	40	46	50	53	56	59	62	65	69	74
In eclipses	0″	10″	30″	40″	50″	2′	3½′	4½′	5¼′	5¾′	6′	6⅓′
Outside eclipses	0″	6″	20″	30″	34″	1⅓′	2⅓′	3′	3½′	3⅚′	4′	4⅕′

[Margin: Objections against Longomontanus.]

Against this doctrine of Longomontanus many things occur to me. First, if the Sun's rays are refracted in our air in such a way that they show a greater diameter of the Sun (about which, however, there is doubt from what was said in Book 3, chapter 10, Scholium 3), then it is not the Moon's semidiameter that should be diminished, but the Sun's that should be increased, and indeed outside the syzygies too: for that refraction is not tied to the syzygies of the luminaries. Secondly, the bright ring appears around the Moon in an eclipse of the Sun even when the Sun stands far from the horizon, at an altitude of 40° or 45°, where by Tycho's rule sensible refractions cease. In the eclipse of ☼ of 1567 April 9, observed with a bright ring by Clavius at Rome near noon, the Sun stood 55° above the Roman horizon. Thirdly, when the luminaries are on the horizon or close to it, their apparent diameter in a vertical circle is far smaller than the diameter transverse and parallel to the horizon, as is clear from what was said about the elliptical figure of the Sun in Book 3, chapter 2, and of the Moon in this Book, chapter 15. Therefore Longomontanus's equation ought not to have been applied indiscriminately to the diameter of the luminaries taken in any direction. Fourthly, when the visual diameter of the Moon is in itself greater than the diameter of the Sun, as is often the case in Longomontanus's view (for the Sun's semidiameter is 15′ at apogee and 16′ at perigee, but the Moon's 16′ at apogee and 17′ at perigee), a total eclipse of the Sun can occur, such that none of its rays reach us, whether direct or refracted in our air. Therefore the Sun will not then appear greater, or, if it does, that will rather be because of refraction occurring in the circumlunar air. Fifthly, the Sun's rays reflected from the Earth towards the Moon do indeed dilute and attenuate the Moon's shadow around the edges, but nevertheless they do not change the angle under which we see the Sun's edges, since we do not see those edges by those rays reflected elsewhere than to our eye.

[Margin: 8. Lansbergen's opinion.]

IX. Philips van Lansbergen, on p. 37, determines the Moon's apparent semidiameter in the syzygies as 15′ at apogee and 17′ 49″ at perigee, which he confirms in the *Thesaurus* of lunar observations, Class 4, and in the *Uranometria*, Book 1, Element 10, by this method. For he says that the semidiameter of the Moon at apogee in the syzygies, set at 15′ by Copernicus, agrees exactly with the sky and with observations. From this, and from the distance of the Moon at apogee, he infers that the Moon's true semidiameter is to the Earth's semidiameter as 16 8/10 to 60. From the true semidiameter, and from other distances corresponding to the anomaly, he deduces the other apparent semidiameters by trigonometry. He then confirms these from a threefold observation made by the Braheans [Braheanis] on 1588 March 2, 1591 February 22 and 1592 February 12, in which the Moon's apparent diameter was seen, in the first observation by armillary spheres, at about 33′; in the second by quadrants, at 33′, a middle value chosen between the extremes; and in the third by the radius, at 35′. Likewise from a double observation by Kepler, of 1598 March 29 and 1598 June 17: in the first of these he saw the Moon's diameter a little smaller than the distance of the western stars in the quadrilateral of the Pleiades, which for Tycho is 31′; in the second, equal to the distance of the two bright stars in the quadrilateral of the Pleiades, which is 32′, as Kepler reports in *Astronomia Optica*, p. 347.

[Margin: Doubts about this opinion.]

But Kepler himself admits there that he trusts observations of eclipses more than an ocular estimate of this interval compared with the Moon's diameter. The observation made by the Braheans by the radius, moreover, is unreliable, and the one made by them by quadrants or by armillary spheres is inconsistent, as we said in no. 5. Nor is the middle value chosen by Lansbergen from those extremes necessarily truer, since one of the extremes could have been true and the others false. Whether this diameter actually satisfies eclipses is something we must weigh more fully in volume 3 of the *New Almagest*, in the book on eclipses, and touch on below, in nos. 15 and 16.

[Margin: 9. Boulliau's opinion and method.]

X. Ismaël Boulliau, in Book 4 of the *Astronomia Philolaica*, chapter 1, investigates the Moon's semidiameter by means of the eclipse of 1638 December 21. He reports that the observed time of incidence, that is, from the true beginning of the eclipse to total obscuration, was $1^h\ 1^m\ 30^s$. The Moon's coequated anomaly was 118° 29′ 31″; therefore the hourly motion was 33′; therefore the Moon, in the time of incidence, traverses 33′ 48″ in the zodiac. So great, then, was the Moon's diameter, since it was entirely obscured. [Margin: But uncertain.] But the time of that incidence, as I have it in my history of eclipses, was observed differently by others: namely at Frenequere [Franeker] by Phocylides, $1^h\ 3^m\ 30^s$; at Digne by Gassendi, $1^h\ 1^m$; at Herk [Hercij] by Wendelen, 58^m; at Brussels by van Langren, $57^m\ 41^s$. The Moon's diameter would therefore be 34′ 55″ for Phocylides and 31′ 44″ for van Langren. This method is therefore unreliable,

especially since they do not agree on the hourly motion (for see no. 14), whether this diversity arises from the eyes and diligence of the observers, or from instruments or calculations, or from air refracting the Sun's rays and affecting the penumbra and the shadow of the Earth with differing quantity. Yet he rightly adds that the semidiameters of the luminaries must be reconciled so that the Sun at apogee appears smaller than the Moon at perigee, and can undergo a central eclipse with duration; while the Sun at perigee appears greater than the Moon at apogee, as eclipses attest. Finally, on p. 196, from the proportion of the distances, he determines the Moon's semidiameter in the syzygies as 15′ 35″ at apogee and 17′ 0″ at perigee, but in □ as 14′ 58″ at apogee and 18′ 2″ at perigee. His tables he gives on p. 138.

[Margin: A second example from another eclipse.]

But let us try some eclipse on whose incidence the observers agreed. Out of the whole history of eclipses, then, a single one occurs, that of 1631 November 8, whose incidence Gassendi, Hortensius and van Langren, in widely separated places, observed with remarkable agreement as 66^{m} of time. The anomaly of ☾ was almost 2°, so that it was very close to apogee; but the hourly motion of the Moon from the Sun varies among astronomers, and so the Moon's apparent diameter derived from it varies, as can be seen in the following small table. But I used the true hourly motion of the Moon, with the true hourly motion of the Sun at that time subtracted from it. How much greater, then, will the diversity be if eclipses are used in which the observers agree neither on the interval of incidence nor on the hourly motion?

Time of incidence 66^{m}. Anomaly of ☾ 2°.

Authors	Hourly motion of ☾ from ☼	Apparent diameter of ☾
Copernicus	26′ 36″	29′ 16″
Tycho	27′ 40″	30′ 0″
Kepler	27′ 13″	29′ 48″
Lansbergen	27′ 18″	29′ 50″
Wendelen	27′ 0″	29′ 42″
Boulliau	27′ 10″	29′ 47″
Ourselves	26′ 48″	29′ 28″

[Margin: 10. Gassendi's opinion, and a deceptive procedure.]

XI. Pierre Gassendi, in the book on the apparent magnitude of the low and the high Sun, letter 2, in the *Institutio Astronomica*, Book 2, chapter 9, and in the *Life of Peiresc*, reports that the Moon's apparent diameter was observed as 26′ 36″ at apogee and 31′ 6″ at perigee, by the shadow of a small plate cast onto a second small plate parallel to the first and at a known distance. But on the uncertainty of this kind of procedure see what I said in Book 3, chapter 10, no. 9.

[Margin: 11. Wendelen's opinion.]

XII. Govaert Wendelen, in the *Idea of the Atlantic Tables*, adducing no individual observation but only from eclipses he has weighed, sets the diameter

of the Moon at apogee at 28′ 40″ on the horizon and 29′ 10″ at the zenith, and at perigee at 32′ 42″ on the horizon and 33′ 18″ at the zenith. On p. 5 he suspects that its diameter is increased by the circumlunar air: for this, he says, is perhaps as thick as the air around the Earth, and so the beginnings of eclipses appear earlier, and the ends later, by almost 1^m of time. [Margin: What does van Langren say here?] But Michael Florent van Langren, royal cosmographer, in a letter to me from Brussels of 1647 July 6, writes that he observed the Moon's diameter on many days indeed, but above all that of the Moon at apogee at the full Moons of 1646 May 29, June 27 and July 27, and at perigee in ♌ on 1647 January 20 and February 18; and that the apogee diameter was to the perigee diameter as 68404 to 78146, that is, as 30 to 34 19/68. But he has not yet disclosed how great it is in definite terms.

[Margin: And that of Athanasius Kircher.]

Lastly, Athanasius Kircher, in volume 2 of the *Ars Magna*, Book 9, Problem 9, gives the Moon's diameter as 29′ 22″ at apogee and 34′ 40″ at perigee.

Noſtræ Obſeruationes ſelectæ, & Primum Fundamentum de apparenti Diametro Lunæ.

Our selected observations, and the First Foundation concerning the apparent diameter of the Moon.

[Margin: Our method for refining the Moon's diameter.]

XIII. Since I saw that among astronomers the Moon's diameter in the syzygies is admitted as never greater than 37′ or smaller than 26⅗′, I left untried none of the methods indicated in Book 3, chapter 10, and in this chapter, and to be explained in the *Liber Organicus*, by which I might come as close to the exact measure of this diameter as is permitted to a mortal. Next I went through the eclipses of the Moon and the Sun, of which I have a most carefully assembled history, so that the diameter sketched from other observations should not conflict with any of those eclipses recorded a little more accurately. And finally I bound the distances of the Moon and the Sun, and the rest that follows from them, to the lunar diameter by a geometrical tie, so that no inelegance or geometrical error [ageometria] arising from them could undermine our hypothesis. At the horizon I used, besides sextants, with which it often appeared 40′ across, a tube without lenses, whose opening at a fixed length takes in 45′ of the sky; and at times this scarcely took in the full Moon, so great was the transverse diameter of the ellipse. Further, in deriving the diameter from the time of transit through the M. C., counted by the oscillations of the pendulum, we took account of the Moon's declination and of the ratio of the parallel on which the Moon stood to the equator. Here then are a few out of many, from which the First Foundation of the lunar diameter has been laid, after a fashion. For the Moon, full and at apogee, carefully observed on 1643 February 3, 1644 June 27, and 1647 June 27 and August 22, when it was only 4° or 5° from apogee, showed us an apparent diameter of 29′ or 28½′. But at perigee, on

1643 August 29 and September 27, the apparent diameter was 34′ at the most, and generally less, except for observations by the astronomical radius, which at times gave it as nearly 36′ at perigee. But we are certain that the radius always exceeds by 1′ and more; and so attention had rather to be paid to the times of the pendulum and to the altitudes of the edges, or to the distances of the edges, taken by two persons with a special instrument in the manner of a sextant. In the quadratures, the diameter for me was between 28′ and 27′ with the Moon at apogee, and between 35′ and 34′ at perigee, but always evidently greater here than at apogee full Moons. [Margin: The awkward hypothesis of some astronomers.] But these must be reconciled more accurately by geometry from the distances, so that we do not commit the fault of Ptolemy, Copernicus and Kepler, of which I shall speak in chapter 18, no. 8 [?].

2. Fundamentum Diametri Lunaris ex meo motu Horario.

2. Foundation of the lunar diameter from my hourly motion.

[Margin: Ratio of the Moon's hourly motion to its diameter.]

XIV. The true hourly motion of the Moon is sufficiently known to us from our tables, from the comparison of the motion of two neighbouring days and its difference divided by 24, and at perigee it is 35′ 30″. Astronomers have observed, as Magini attests in the *Theoricae Planetarum*, Book 2, chapter 25, that the Moon's hourly motion is to its visual diameter as 48 to 47, or rather, from Copernicus and Magini, at perigee approximately as 2255 to 2138. It follows that the Moon's apparent diameter is nearly 33′ 15/23, or certainly 33′ 30″. But since at apogee they set it as 29 to 30, from which a Moon's diameter greater than 29 would come out for me, while very many observations require a smaller one at apogee, it was therefore not permissible to follow that apogee proportion. The first to note the proportion of the hourly motion to the lunar diameter as 48 to 47 was al-Battānī, chapters 30 and 43, whom Peuerbach nearly followed in the *Theoricae Planetarum*, and there Reinhold, p. 206. Regiomontanus, in the *Epitome of the Almagest*, Book 5, proposition 21; Barozzi in the *Cosmographia*, p. 341.

3. Fundamentum ex Conſenſu Diametrorum Solis & Lunæ cum Eclipſibus Solis Totalibus.

3. Foundation from the agreement of the diameters of the Sun and the Moon with total eclipses of the Sun.

XV. Tycho, and with him Mulerius, denied total eclipses of the Sun. This is clear from the table of semidiameters drawn up for ecliptic new Moons, in which the Moon's semidiameter, however close to perigee, is always set by them as smaller than the Sun's apparent semidiameter; from the Appendix to the *Progymnasmata*; and from the testimony of Kepler in *Astronomia Optica*, from p. 285. There he also affirms that what gave Tycho occasion to contract the Moon's diameter at new Moons in this way was the optical enlargement of

the solar diameter, or dilation of the solar rays, in eclipses of the Sun, whether you look at the Sun directly at the time, or by reflection on paper or on a small plate that receives the ray and image of the Sun let in through a hole. For in both cases the Sun's diameter appears greater, and the Moon's smaller, than the hypothesis, resting on other foundations, otherwise demonstrates. However, Kepler rightly warns in the same place that the edges of bright things, especially in darkness, or next to dim or black things, are dilated in the eye, because they prevail in moving the sight, and either overwhelm or blunt the image of darker or black things, as he had taught in chapter 5 of his *Optics* from the structure of the eye and from experiments. [Margin: Tycho's manifold deception regarding the semidiameters of the luminaries.] We too touched on these things in chapter 6, no. 3. He adds that because of this deception of sight, which is inconsistent according to the varying structure of the eye and the state of the air, the table of semidiameters of the luminaries should not be altered, nor should one depart from the table that is demonstrated from what truly occurs. Furthermore, when the Sun is not seen, as in a total eclipse of it, that optical enlargement has no place, for things that appear enlarged are presupposed to be seen. Therefore Tycho ought not to have contracted the Moon's diameter universally at every new Moon. The reason why the Moon's semidiameter derived from ecliptic new Moons observed by reflection on a small plate came out smaller than it should, and the Sun's greater, was that the semidiameter of the hole or small window admitting the rays was not subtracted from the semidiameter of the base of the cone of rays, nor added to the portion of the Moon obscuring the Sun's disc, just as if the rays had entered through an indivisible hole, although in fact it was quite wide. So it should have been subtracted there and added here, as Kepler teaches in *Astronomia Optica*, chapter 11, Problems 2, 7, 8 and 9. Otherwise, in a ray or image received in this way, the Moon's diameter appears smaller than outside in the sky, the digits eclipsed fewer than in the sky, and the horns of the eclipse blunter than in the sky, as he had demonstrated in chapter 2, propositions 10, 11 and 12. On this procedure we too shall speak in Book 5, chapter 17, Problem 2. Lastly, Tycho's error is plainly refuted not only by the single total eclipse of the Sun seen by Clavius in 1560, but by several total eclipses which Kepler brings forward in the same place, and of which we treat in Book 5, chapters 19 and 20, where we shall confirm total eclipses of the Sun by examples and by the authority of astronomers, and distinguish the certain from the uncertain. Among these, from those chapters 19 and 20, I have selected the 12 more certain eclipses, leaving out the doubtful ones or those of doubtful date. The astronomer ought to have these before his eyes in reconciling the diameters of the luminaries, but especially the twelfth, because it was indeed total, but without sensible duration, observed at Naples, as we shall relate in Book 5, chapters 19 and 20. For there is greater danger of erring in this one and in those similar to it, nos. 4 and 6, if indeed the Moon's diameter is set sensibly greater or smaller than the Sun's; so that in the place of a central eclipse it comes out either total with notable duration, or annular.

The error, though, appears more evidently in those tables which make annular eclipses of the Sun out of total ones with duration, or total ones with duration out of annular ones, against the bare and sincere history of observations. The remaining eclipses are not so suitable for testing astronomical tables, because if an eclipse is said to have been observed in a given place as of 8 digits, and the tables allow it only 7 or 6½ digits, it will be easy for the author of the tables to condemn the observation and make it out to be deceptive.

[Margin: Total eclipses of the Sun.]

More certain total eclipses of the Sun, selected from the history of observations.

Order		Years	Months	Days
1	Before	585	May	28
2	Christ	310	August	15
3	Of Christ	237	April	12
4		484	January	13
5		840	May	5
6		878	October	29
7		1187	September	4
8		1241	October	6
9		1415	June	7
10		1485	March	16
11		1560	August	21
12		1605	October	12

4. Fundamentum ex Conſenſu Diametrorum Lunæ ac Solis cum Eclipſibus Solis Annularibus.

4. Foundation from the agreement of the diameters of the Moon and the Sun with annular eclipses of the Sun.

XVI. That annular eclipses of the Sun, namely those in which a ring of the Sun's disc appears bright all round the Moon, do sometimes occur, I have already indicated by examples and authorities of astronomers in chapter 6, no. 4, and shall show more clearly in Book 5, chapters 19 and 20, nos. 4 and 5. From this I select the following four, more certain than the others and of more certain date, so that I may also conform the semidiameters of the luminaries to them.

[Margin: Annular eclipses of the Sun.]

Annular eclipses of ☼, selected from the history of observations.

Order	Years of Christ	Months	Days
1	334	July	17
2	1567	April	9
3	1598	February	25
4	1601	December	24

5. Fundamentum ex Conſenſu Semidiametrorum Lunæ cum Semidiametro Solis & Vmbræ Terreſtris in Eclipſibus Lunæ ſelectis.

5. Foundation from the agreement of the semidiameters of the Moon with the semidiameter of the Sun and of the Earth's shadow, in selected eclipses of the Moon.

[Margin: How great the semidiameter of the shadow is.]

XVII. I shall demonstrate in Book 5, chapter 4, that the apparent semidiameter of the Earth's shadow at the place of the Moon's transit, joined to the apparent semidiameter of the Sun, is equal to the sum of the horizontal parallaxes of the Sun and the Moon. Now we have established above, from the firmest foundations, both the semidiameters of the Sun (Book 3, chapter 10) and the parallaxes of the luminaries, and so their distances (Book 3, chapters 7 and 8, and this Book 4, chapter 14). The semidiameter of the Earth's shadow therefore now depends on a necessary predetermination. With these, moreover, the Moon's apparent semidiameter must cohere by a geometrical tie, above all in those eclipses of the Moon in which astronomical tables are given no field to roam, and the goal of truth, standing as it were in an indivisible point, must be confined to a very narrow gap. Such are those observed as exactly total without any sensible duration, or very nearly total, without a barely perceptible remainder of the lunar disc, or those that appeared of only one digit or less, or those, finally, that appeared as none or as insensible, although the ephemerides of the calculations promised them as sensible. Among these, in order to examine my semidiameters and those of others, I have selected the following six from the history of which I shall speak in chapters 20 and 21, leaving out older ones of 2 digits, which some think are not to be trusted as the more recent ones are. I know that account must be taken here too of latitude and of the Moon's motions, but the chief elements are the semidiameters.

Eclipses of the Moon selected from observations for testing the astronomical tables.

Order	Years of Christ	Months	Days	Quantity
1	1580	January	31	Total exactly
2	1598	February	19	Very nearly total
3	1619	June	26	About 1 digit
4	1625	March	24	About 1 digit
5	1626	August	7	None or imperceptible
6	1636	February	20	2⅓ digits

Statuitur Semidiameter Lunæ Apparens.

The apparent semidiameter of the Moon is established.

XVIII. Kepler rightly says in the *Optics*, p. 212, that the investigation of the diameter of the luminaries is the foundation which those who undertake the restoration of astronomy must lay first. It is unbelievable, however, how hard it is to reconcile these diameters with the difference of the parallaxes and

distances, which have a necessary connection with them, and with the more certain observations, and, whenever tables were to be formed and reformed, to make all of this cohere by a geometrical tie, both internally and with what has been observed. Therefore, from several full Moons I observed, the diameters of the Moon that stayed with me as more certain than others were 28′ at apogee, at most 33½ at perigee, and about 30½ at mean distance. From these, and from the distances of the Moon, above all the mean distance of 59 Earth semidiameters, the perigee distance of 53 45′ and the apogee distance of 64 15′, and from the resulting eccentricity of 5 10′ in the syzygies, I found the true semidiameter of the Moon to be 26⅕ of the parts of which the Earth's semidiameter is 100. From this, for the other degrees of anomaly, I determined by analysis of triangles the lunar distances according to what was said in chapter 15, and from these the apparent semidiameters of the Moon corresponding to them, as you will see in our tables. From those tables I have selected the following, for the eclipses listed above. Since our hypothesis agrees wonderfully with these, I thought I should rest on it for the present, especially since, after toiling through many labours, I have found that this table could indeed be constructed somewhat differently, but hardly in such a way as not to offend against certain more evident observations, handed down on more reliable authority.

	Order of eclipse	Equated anomaly of ☼	Equated anomaly of ☾	Apparent diameter of ☼	Apparent diameter of ☾
☼ Total	1	0ˢ 0°	5ˢ 8°	30′ 50″	33′ 10″
	2	2ˢ 14°	6ˢ 21°	31′ 28″	33′ 10″
	3	10ˢ 16°	5ˢ 6°	31′ 4″	33′ 0″
	4	7ˢ 8°	5ˢ 19°	32′ 50″	33′ 24″
	5	10ˢ 25°	4ˢ 20°	31′ 0″	32′ 38″
	6	4ˢ 16°	7ˢ 10°	32′ 42″	32′ 40″
	7	2ˢ 18°	6ˢ 17°	31′ 38″	33′ 16″
	8	3ˢ 20°	6ˢ 9°	32′ 14″	33′ 24″
	9	11ˢ 21°	5ˢ 26°	30′ 50″	33′ 28″
	10	10ˢ 0°	5ˢ 16°	15′ 40″	16′ 39″
	11	1ˢ 4°	4ˢ 10°	31′ 0″	32′ 12″
	12	3ˢ 12°	4ˢ 14°	32′ 6″	32′ 22″
☼ An-nular	1	1ˢ 16°	9ˢ 26°	31′ 8″	29′ 16″
	2	2ˢ 9°	3ˢ 10°	31′ 28″	30′ 54″
	3	3ˢ 22°	4ˢ 10°	32′ 16″	32′ 12″
	4	5ˢ 26°	0ˢ 7°	33′ 8″	28′ 4″
		Equated anomaly of ☼	**Equated anomaly of ☾**	**Apparent semidiameter of the corrected shadow**	**Apparent semidiameter of the Moon**
☾ Eclipses	1	6ˢ 14°	6ˢ 9°	47′ 31″	16′ 42″
	2	7ˢ 25°	1ˢ 10°	38′ 33″	14′ 13″

Order of eclipse	Equated anomaly of ☼	Equated anomaly of ☾	Apparent diameter of ☼	Apparent diameter of ☾
3	11^{s} 27°	0^{s} 16°	38′ 50″	14′ 3″
4	8^{s} 26°	1^{s} 20°	39′ 31″	14′ 24″
5	1^{s} 10°	4^{s} 4°	46′ 5″	16′ 0″
6	7^{s} 24°	9^{s} 23°	40′ 22″	14′ 42″

Consult now the diameters in the tables of other astronomers, which, to make this evident, we shall set out at the end of Book 5 for the already-mentioned degrees of anomaly. You will see that none of the total eclipses is produced by Tycho or by Mulerius, and that numbers 3, 4, 9 and 12 are not produced by Lansbergen, who therefore, to excuse his tables, pretends that the Sun has drawn in its rays: an unheard-of prodigy, and more poetic than astronomical or optical. Of the annular eclipses, Ptolemy delivers none, and Longomontanus almost none, and those only by accident; number 1 is not delivered by Kepler and Wendelen; nor number 2 by Alfonso, Copernicus, Kepler, Boulliau and Wendelen; nor number 3 by Copernicus, Kepler, Boulliau and Wendelen. But they excuse their tables on this point. Kepler does so in the *Optics*, p. 301, attributing that bright ring to air, or to ether, set alight by contact with the neighbouring Sun and shining round about, or else to the circumlunar air or lunar atmosphere, which refracts the Sun's rays as our atmosphere does, and so enlarges the Sun's margins. This is also admitted by Fabricius in Kepler, in the foundation of the *Ephemerides*, p. 22; by Wendelen in the *Eclipses*, p. 5; by Boulliau, p. 210, or Book 4, chapter 5; by Bettini in *Apiarium* 8, *Progymnasma* 2, proposition 10; and by van Langren in a letter sent to me.

[Margin: Errors, or uncertain and forced evasions[28], of certain astronomers.]

Longomontanus, for his part, attributes it both to the air around the Earth, which refracts the Sun's rays and optically enlarges its limbs, above all near the horizon, and to the Sun's rays reflected from the Earth towards the Moon, which diminish the shadow of the Moon, under whose angle we then see the Sun. For otherwise he, like Bettini too, makes the Moon's diameter always larger than the solar one. But since these phenomena can be saved while the other things demonstrated about the Sun and the Moon are also preserved, I did not want to build my hypothesis on uncertain and inconstant causes. This is especially so since inflamed air does not produce a bright ring but suffuses a wide space around the Moon with twilight glow; and telescopes do not plainly reveal air around the Moon, nor is that air to be judged of such density and depth as to produce so great a refraction of the solar rays. What Longomontanus devises, moreover, has either been sufficiently rejected at number 8 or at chapter 6, number 4, or has no force where annular eclipses appear far from the horizon. That Lansbergen extends the diameter of the Sun to 36′ is against the most evident observations. There are also other eclipses of the Sun, either total or

[28]the page prints *effigia*; read as *effugia*

very nearly total, which our tables present far more consistently than those of others, as can be seen in Book 5, chapter 20, and at the end of Book 5, where we shall set out a table of semidiameters, both ours and those of other astronomers, so that, once the quantity of these eclipses has been examined in that chapter 20, the observation can be compared with their hypothesis. There it is truly astonishing that Kepler, in the *Astronomia Optica*, p. 294, contends against the Prutenic Tables that the eclipse of the Sun in the year of the 10th consulship of Constantius and the 3rd of Julian, which Marcellinus records and describes, was total, and yet admits that the Moon was then at apogee, whereas from Kepler's own table this eclipse is partial and annular, as I shall say in Book 5, chapter 19, at the year 360, and in chapter 20, after the eighth eclipse.

Finally, the lunar eclipses I listed above are represented consistently by hardly any tables except Wendelen's. David Fabricius, as Kepler reports in the *Ephemerides*, pp. 9 and 22, says that in the eclipse of ☾ of the year 1613, October 28, a digit seemed to the naked eye to be missing, and by telescope nothing; and that therefore, because of the circumlunar air, the diameter of ☾ is increased to the naked eye by 4′ or 5′, and that eclipses of ☾ seem to begin sooner and end later than they really do and than they do by telescope. Kepler rejects this: for if that air were so dense that it stopped the light of ☼ and could be seen with the naked eye, it would be seen much better by telescope. Likewise Fabricius says that in the smallest eclipses the darkened limb of ☾ is not discerned by the naked eye, and is discerned by telescope. But if the eye sees the eclipsed air of ☾, it will see much more readily both the air and the darkened small part of ☾. The reason why something not seen by the eye is seen by telescope is that the telescope optically enlarges things otherwise invisible. Granted, Kepler may say that it is an optical enlargement of bright objects, beside which dark ones vanish and do not impress a distinct image of themselves on the eye, because the image of the bright objects predominates, as he had taught in the *Optics*, chapter 5. But now let there be a table of opinions on the diameter of ☾ at apogee and perigee.

Synopsis of the apparent diameters of the Moon.

Authors	In syzygies and quadrature aspects	Apparent diameter of the Moon at apogee	Apparent diameter of the Moon at perigee
Ptolemy	in ☌ & ☍	31′ 20″	35′ 20″
	in □	42′ 8″	55′ 0″
al-Battānī	in ☌ & ☍	29′ 30″	35′ 20″
Alfonso and Peuerbach	in ☌ & ☍	29′ 0″	36′ 8″
Copernicus, Reinhold and formerly Magini	in ☌ & ☍	30′ 0″	35′ 38″
	in □	28′ 45″	36′ 44″
Magini in the *Supplement to the Ephemerides*	in ☌ & ☍	30′ 30″	34′ 40″
Clavius and Hérigone	in ☌ & ☍	30′ 0″	36′ 8″
Tycho, Origanus and Mulerius	in ☌	25′ 36″	28′ 48″
	in ☍	32′ 0″	36′ 0″

Authors	In syzygies and quadrature aspects	Apparent diameter of the Moon at apogee	Apparent diameter of the Moon at perigee
	in □	32′ 32″	36′ 0″
Longomontanus physically; but see number 8, optical equation	in ☌ & ☍	32′ 0″	34′ 0″
Kepler in the *Ephemerides*	in ☌ & ☍	30′ 0″	34′ 10″
In the *Epitome*, and in the Rudolphine Tables	in ☌ & ☍	30′ 0″	32′ 44″
Lansbergen	in ☌ & ☍	30′ 0″	35′ 38″
Pierre Gassendi	in ☌ & ☍	26′ 36″	31′ 6″
Ismaël Boulliau	in ☌ & ☍	31′ 10″	34′ 0″
	in □	29′ 56″	36′ 4″
Govaert Wendelen	in ☌ & ☍	28′ 40″	32′ 42″
but at the vertex	in ☌ & ☍	29′ 10″	33′ 18″
Athanasius Kircher	in ☌ & ☍	29′ 22″	34′ 40″
Ourselves for the present	in ☌ & ☍	28′ 0″	33′ 30″
	in □	27′ 0″	35′ 6″

[Margin: Kepler's objection is refuted.]

But Kepler presses us hard. In the foundation of his *Ephemerides*, p. 9, he adjusted his diameters so as to hold to Hipparchus, who from observation makes the diameters of the Sun and of the Moon at apogee equal. Our hypothesis does not hold this, nor indeed does that of any of the astronomers, apart from Ptolemy and Kepler, as is clear to anyone comparing the preceding table with the table of Book 3, chapter 10. But Ptolemy, in Book 5 of the *Almagest*, chapters 11 and 14, describes Hipparchus to us as doubtful and changeable about the Sun's distance, and therefore also about its diameter and parallaxes, and he confesses that he himself was not so certain and accurate about the Sun's diameter either. For he says in chapter 14: *We too constructed the dioptra with the four-cubit rule which Hipparchus described, and, observing with it, we found that the Sun's diameter is contained everywhere under very nearly the same angle, with no perceptible difference worth troubling about produced by its distances; but we found that the Moon's is contained by the same angle as the Sun's only when, at oppositions, it is farthest from the Earth, etc.* So we should not stand by this. Nor has any eclipse of the Sun so far been produced in which the Moon at apogee has wholly covered the Sun, likewise at apogee. Neither can proposition 8 of Aristarchus be admitted, in whose exposition he says it is manifest that the Sun is wholly eclipsed, but with no delay in the eclipse, if he means this universally and at any distance of the luminaries.

SCHOLIA.

Scholia.

I. In Book 3, chapter 10, Scholium 2, I said that a change in the Sun's distance and apparent diameter cannot be evidently demonstrated from eclipses. For if someone supposes its semidiameter to be perpetually 16′ 10″, not only the

remaining eclipses but also, which seems more difficult, the total and annular eclipses of the Sun will stand, provided one retains the semidiameter of the Moon established by us, as will be clear to anyone contemplating the last column of the earlier small table set before number 18. For the lunar semidiameter will always be larger than the solar, as total eclipses of the Sun require, and smaller, as annular eclipses demand. And conversely, the apparent semidiameters of the Sun could indeed be so fitted that the Moon's semidiameter was always the same, and with the Sun's semidiameter varied, the eclipses would nonetheless be preserved. But this conflicts with the most evident difference of the lunar distances from the Earth, which has been detected from the difference of the notable parallaxes by us and by the others who practise this art faithfully. As for the difference of the solar distances, however, it is not so manifest from observations, because it could be put down to an error of observations, or of causes partly optical and partly physical.

[Margin: The variation of the apparent diameter is more certain in the Moon than in the Sun.]

Therefore, although it is almost beyond doubt for me that the distance of the Sun varies, and so its apparent diameter, yet if invariability of these were to be ascribed to one or other of the luminaries, we are certain that it cannot be attributed to the Moon, but only to the Sun.

II. It was said in chapter 15 that, because of refraction, the diameter of the Moon observed on the horizon is sometimes 38′ and nearly 40′, but the other 36′. From the varied constitution of the vapours of the horizon, therefore, the diameters can vary, and hence the measure of eclipses.

CAPVT XVII.

Chapter 17.

De Vera Magnitudine Lunæ ad Terram Solemq. collata, Portione Lunæ viſa à nobis, illuminata à Sole, deq. Portione & Diametro apparenti Terræ ex Luna ſpectatæ, ac tandem de Vmbra Lunari.

On the true size of the Moon compared with the Earth and the Sun; on the portion of the Moon seen by us and illuminated by the Sun; on the portion and apparent diameter of the Earth as seen from the Moon; and finally on the lunar shadow.

I. In this chapter we take up these problems, because they spring up for us from the distance of the Moon and its apparent semidiameter, now established. For the following propositions, then, let this be the figure, in which the Sun is EMF and the Moon ABCD; and let the extreme rays of the Sun, touching it and the Moon, be EAG and FCG, meeting at G and enclosing the lunar shadow AGC. Let the straight line KHG be drawn through the centres of the luminaries to the apex of this shadow, and from their centres let radii be drawn

to the points of contact of the rays, namely KE and KF in the Sun and HA, HC in the Moon. For there, with the tangent rays, by 16 of the third book of Euclid, they will make right angles at E, A, and so on. When the straight line HL is drawn from the centre of the Moon parallel to GAE, by 29 and 33 of the first book of Euclid it will make the angle AHI and the angles at L on either side right angles, and EL equal to AH, the Moon's semidiameter; and it will make the external angle LHK equal to the internal and opposite angle EGK, that is, to the half-angle of the cone of the lunar shadow, since KG falls on the parallels EG and LH. With these set down, let the following hold.

Diagram. A geometric figure of the Sun, the Moon and the Moon's shadow, set in the right column beside the text. The Sun stands at the top as a shallow bowl, an arc below a nearly straight upper line, with two lines running down from K to E and to F; two straight lines descend from E and F, touch a small circle (the Moon) at A and C, and converge at the apex G at the foot. A vertical line runs from K at the top through M, B, H and D down to G. Inside the Moon's circle, radii run from the centre H to A and C. A slanting line runs from L down to the Moon's circle by I, towards H.

- K (top centre, on the upper line): K
- E (upper left end of the Sun): E
- F (upper right end of the Sun): F
- L (within the Sun, left of the vertical): L
- M (on the Sun's lower arc, on the vertical): M
- I (just above the Moon's circle, left of the vertical): I
- B (top of the Moon's circle, on the vertical): B
- H (centre of the Moon's circle): H
- A (left of the Moon's circle): A
- C (right of the Moon's circle): C
- D (bottom of the Moon's circle, on the vertical): D
- G (apex of the shadow, at the foot): G

PROBLEMA I.

Problem 1.

Diametrum Veram Lunæ & inde Circumferentiam, Aream circuli in Luna maximi, & Soliditatem Lunaris globi inueſtigare.

To find the true diameter of the Moon, and from it the circumference, the area of the Moon's greatest circle, and the solidity of the lunar globe.

II. In the preceding figure let the base HG be given, the distance of the Moon from the centre of the Earth, and the angle AGH, subtending the apparent semidiameter of the Moon as seen from that same centre. [Margin: Summary of the rule.] For since the triangle AGH is right-angled at A, by what was said in number 1, if one makes as the radius to the distance of the Moon, so

the sine of the apparent semidiameter to another term, the true semidiameter of the Moon will be known in the parts in which the distance HG was given. Alternatively, the logarithm of the distance added to the logarithm of the apparent semidiameter will make up the logarithm of the true semidiameter, and by doubling this you will have the diameter. If one makes as 100 to 314, so the diameter to another term, the circumference will be known. A quarter of this, multiplied by the diameter, will give the area of the Moon's greatest circle in square measures, while the whole circumference multiplied by the diameter will give the convex surface in square measures. Finally, the tripled ratio of the terrestrial diameter to the true diameter of the Moon will give the solidity of the Moon, according to what was said in Book 1, chapters 4 and 6. Now, on the occasion offered in this Book 4, chapter 16, from the Moon's apparent semidiameter of 16′ 24″ at a distance of 54 58′ Earth semidiameters, we find the true semidiameter of the Moon to that of the Earth, and so the diameter of the one to the diameter of the other, to be as 26⅕ to 100. Nothing else remains, then, except by this method to deduce the size of the Moon according to the various astronomers, from the lunar distances given in chapter 14 and the apparent diameters given in chapter 16. Here, then, is the fruit of this treatise in the following table, which is to be compared with the magnitudes of the Sun given in Book 3, chapter 11. I have also added, from Aristarchus's little book *On the Sizes and Distances of the Three Bodies*, the measures from his proposition 18, in which, given certain premises, he concludes: *The Earth is to the Moon in a greater proportion than 1259712 to 79507, and in a lesser than 216000 to 6859.* For the rest, see the Scholia.

The true size of the Moon compared with the Earth and the Sun.

From the data, by the authors		True diameter of ☾ (the Earth's diameter taken as 100)	Of the Earth	Of ☼	Solidity of the Moon to that of the Earth: as	to	Solidity of the Moon to that of the Sun: as	to
Aristarchus	more than	31⅔	100	633	1	16	1	5832
	less than	39⅘	100	716	1	32	1	8000
Ptolemy		29⅙	100	550	1	40	1	6648
Copernicus		28 4/7	100	545	1	42⅞	1	6999 1/60
Mästlin		28⅓	100	546	1	44	1	7300
Tycho		29⅙	100	520	1	41 11/12	1	5848
Longomontanus		26⅚	100	580	1	51 1/49	1	10000
Kepler		25 7/10	100	1500	1	59	1	204671
Lansbergen		28	100	757	1	45 11/20	1	19770
Boulliau		27 11/12	100	700	1	45 29/30	1	15765⅓

From the data, by the authors	True diameter of ☾ (the Earth's diameter taken as 100)	Of the Earth	Of ☼	Solidity of the Moon to that of the Earth: as	to	Solidity of the Moon to that of the Sun: as	to
Wendelen	26⅝	100	6400	1	53	1	13833632
Athanasius Kircher	29 7/17	100	520	1	40	1	5600
Šírek	28 3/7	100	1000	1	42 19/20	1	42875
Ourselves	26⅕	100	3383	1	55	1	2123000

PROBLEMA II.

Problem 2.

Lunæ Magnitudinem in Stadijs, Milliaribus, aut Leucis exhibere.

To express the size of the Moon in stadia, miles or leagues.

III. This depends on the preceding problem and on what was said about the diameter of the Earth in Book 2, chapter 7, for by using the rule of proportions this problem is easily solved. [Margin: The opinion of Tycho.] Meanwhile Tycho, in volume 1 of the *Progymnasmata*, p. 474, with the terrestrial diameter set at 1720 German miles, gives the diameter of the Moon 495 German miles. [Margin: The opinion of Galileo and Biancani.] But Galileo in the *Sidereus Nuncius*, p. 13, and with him Biancani in Book 9 of the *Sphere*, chapter 9, taking from Copernicus the Moon's diameter to the terrestrial as 2 to 7, assign to the lunar diameter 2000 Italian miles, because evidently they reckon the diameter of the Earth to contain 7000 Italian miles. [Margin: The opinion of Athanasius Kircher.] But Athanasius Kircher, in his *Magnes*, p. 561, gives the lunar diameter 2281 Italian miles; and so others in other ways. [Margin: Our view.] For our part, since in Book 2, chapter 7, we found 8278 modern Bolognese miles in the Earth's diameter, and since for us the true diameter of the Moon is to the terrestrial as 26⅕ to 100, we certainly recognize in the Moon's diameter 2168 84/100 Bolognese miles. Again, therefore, as 100 to 314, so 2168 84/100 to 6750 16/100, the Bolognese miles contained in the circumference of the Moon. Now in chapter 8 of this book, in Scholium 2, we set out the height of certain lunar mountains on the Copernican hypothesis, which assumes the lunar diameter to be to the terrestrial as 2 to 7, because we had not yet established our own measure of this proportion. Now, however, we establish it as 26⅕ to 100. [Margin: The height of the lunar mountains corrected.] So the heights must be somewhat corrected of the mountain Piccolomini, which we there set at 9 Bolognese miles high, and of Saints Michael and Xavier, which we set at 12, with the diameter of the Moon set at 2265 Bolognese miles. But for us it is 2168⅖. Therefore the mountain Piccolomini is only 8 14/25 Bolognese miles high, and Saints Michael and Xavier 11½.

PROBLEMA III.

Problem 3.

Quota Pars Lunæ videatur à terreſtribus ſpectatoribus.

What part of the Moon is seen by terrestrial observers.

IV. In the following figure, the portion ADC of the lunar hemisphere seen from the eye at G falls short of a perfect hemisphere by an arc as great as the angle AGC, which subtends the Moon's apparent diameter, as we showed in a similar case for the Sun in Book 3, chapter 11, number 3. So, given the Moon's anomaly, and its apparent diameter found by means of it, when this is subtracted from the hemisphere's arc of 180°, there will remain the arc ADC seen from the eye at G. From the table of chapter 16, then, here for you is this portion as seen by us in the syzygies.

Diagram. The same kind of figure as on the preceding page, set in the right column beside the text: the Sun at the top as a shallow bowl under a nearly straight upper line, two straight lines descending from E and F past the small circle of the Moon at A and C to the apex G at the foot, and a vertical line from K through M, B, H and D down to G. Inside the Moon's circle, radii run from H to A and C. A slanting line runs from L down towards the Moon's circle by I.

- K (top centre, on the upper line): K
- E (upper left end of the Sun): E
- F (upper right end of the Sun): F
- L (within the Sun, left of the vertical): L
- M (on the Sun's lower arc, on the vertical): M
- I (just above the Moon's circle, left of the vertical): I
- B (top of the Moon's circle, on the vertical): B
- H (centre of the Moon's circle): H
- A (left of the Moon's circle): A
- C (right of the Moon's circle): C
- D (bottom of the Moon's circle, on the vertical): D
- G (apex, at the foot): G

[Margin: Arc of the Moon seen by us in the syzygies.]

Portion of the Moon seen by us in ☌ and ☍ is 179° and in addition:

Authors	☾ at apogee	☾ at perigee
Ptolemy	28′ 40″	24′ 40″
al-Battānī	30′ 30″	24′ 40″
Alfonso	31′ 0″	23′ 52″
Copernicus	30′ 0″	24′ 22″
Tycho, in ☍	28′ 0″	24′ 0″
in ☌	34′ 24″	31′ 12″
Longomontanus	28′ 0″	26′ 0″

Authors	☾ at apogee	☾ at perigee
Kepler	30′ 0″	25′ 50″
or	30′ 0″	27′ 16″
Lansbergen	30′ 0″	24′ 22″
Gassendi	33′ 24″	28′ 54″
Boulliau	28′ 50″	26′ 0″
Wendelen	31′ 20″	27′ 18″
Kircher	30′ 38″	25′ 20″
Ourselves	32′ 0″	26′ 35″

PROBLEMA IV.

Problem 4.

Quota pars Lunæ vltra hemiſphęrium illuminetur à Solis radijs irrefractis.

How much of the Moon beyond a hemisphere is illuminated by the Sun's unrefracted rays.

[Margin: Why the Problem is restricted to unrefracted rays.]

V. We limit the Problem to unrefracted rays because, if the Moon has around it an air or aether of its own that is denser, and of such density as to be fit for a sensible refraction of the Sun's rays, there is no doubt that much more of the Moon is lit by refracted rays than by unrefracted ones; and indeed more by as much as the horizontal refraction doubled, as we said in a similar case about the Earth illuminated by the Sun in Book 3, chapter 11, Problem 4, and shall show in Book 5, chapter 5, Problem 7. But this refraction at the Moon has not yet been established by any certain experiment or argument. Speaking, then, of the Sun's rays that touch the Moon unrefracted: it was shown in the preceding figure that AH equals EL and that triangle HLK is right-angled at L. So if the Moon's true semidiameter AH, which is supposed known from the first Problem, is subtracted from the Sun's true semidiameter EK, known from Book 3, chapter 11, and from the table of Problem 1, the remaining side LK will be known. And if from the Sun's distance, known from Book 3, chapter 7, the Moon's distance at new Moons, known from chapter 14 of this Book, is subtracted, or conversely if the Sun's distance is added to the Moon's at ☌, the distance HK between the luminaries will be known. Therefore, with base HK and side LK, you will find the angle LHK by means of trigonometry; this angle measures the arc IB. This arc, added to the quadrant AI (since in section 1 we showed that angle AHI is right, and therefore that arc AI is 90°), will make up the arc BA, whose double is the arc ABC of the Moon illuminated by the Sun's unrefracted rays.

[Margin: Short form of the rule.]

In brief, therefore: multiply side LK by the radius and divide the total by the base KH, for the quotient will be the sine of angle LHK; or, to the logarithm

of side LK add the residual of the logarithm of base KH, and the logarithm of angle LHK will result. But if you use the semidiameters of the table placed at the end of Problem 1, in which the Earth's semidiameter is supposed to be 100, you must first multiply the distance between the Sun and the Moon by 100.

[Margin: Authors on this Problem.]

Ibn al-Haytham in *De Crepusculis*, prop. 5, Witelo in Book 10, prop. 59, and Magini in the *Primum Mobile*, Book 11, prop. 28, set out this Problem in general or applying it to the Earth illuminated by the Sun. It is applied to the Moon, however, by Reinhold on Peuerbach's *Theoricae*, p. 165, and by Kepler in the *Astronomia Optica*, from p. 230. But Reinhold takes the diameters and distances of the luminaries from Ptolemy, deducing from them the arc ABC as 181° 45′, and so the angle KHL as 52′ 30″, which is too large, as will soon appear from Kepler's calculations and our own [Epilogiſmis]. Kepler therefore says that on the Ptolemaic hypothesis, when each luminary at apogee is seen under the same angle, the arc ABC ought to have come out at 180° 31′ 20″ in ☌ and 180° 28′ 12″ in ☍. We, however, taking the true distances and semidiameters from Book 3, chapters 7 and 11, and from Book 4, this chapter and chapter 14, have drawn up the following table.

[Margin: How this Problem is to be solved at the Moon's quadrature.]

Furthermore, at the quadratures the distance of the Moon from the Sun is very nearly as great as that of the Sun from the Earth. For in the figure placed in Book 3, chapter 7, Problem 3, triangle SKT is sensibly isosceles, and KS is as great as TS, because angle KTS differs little from the right angle TKS, and differs from it by only 30′, as we said there, and indeed by only 15′ if we believe Wendelen. He thinks that, because of the refraction of the Sun's rays in the lunar air, the Moon's apparent dichotomy is hastened in the first quadrature and delayed in the second by 15′ of the zodiac, that is, in time by about 30^{m}. But if that were so, the horizontal refraction of the Sun's rays in the lunar air would be very nearly 15′, as we shall demonstrate in the book on optics.

Arc beyond the quadrant of the lunar hemisphere illuminated by the Sun's unrefracted rays; when doubled, it gives the portion illuminated by them beyond a hemisphere.

From the data of the authors	In ☌: ☼ at apogee, ☾ at perigee	In ☌: ☼ at perigee, ☾ at apogee	In ☍: ☼ and ☾ at apogee	In ☍: ☼ and ☾ at perigee	In □: at the mean distance of the luminaries
Ptolemy	15′ 30″	16′ 50″	14′ 6″	15′ 12″	15′ 20″
Copernicus	14′ 30″	17′ 6″	14′ 18″	15′ 18″	15′ 33″
Tycho	15′ 0″	15′ 54″	13′ 40″	14′ 22″	14′ 40″
Longomontanus	14′ 50″	16′ 4″	13′ 42″	14′ 44″	14′ 46″
Kepler	14′ 49″	15′ 30″	13′ 24″	14′ 15″	15′ 0″
Lansbergen	16′ 46″	18′ 8″	15′ 30″	16′ 42″	16′ 44″
Boulliau	16′ 10″	16′ 49″	14′ 56″	15′ 30″	15′ 50″
Wendelen	14′ 48″	15′ 18″	14′ 40″	15′ 16″	14′ 58″
Ourselves	15′ 18″	16′ 39″	15′ 4″	16′ 22″	15′ 48″

From the data of the authors	In ♂: ☼ at apogee, ☾ at perigee	In ♂: ☼ at perigee, ☾ at apogee	In ☍: ☼ and ☾ at apogee	In ☍: ☼ and ☾ at perigee	In □: at the mean distance of the luminaries

PROBLEMA V.

Problem 5.

Semiangulum & Angulum Coni Vmbræ Lunaris à radijs Solis irrefractis comprehenſæ inquirere.

To find the half-angle and the angle of the cone of the lunar shadow comprehended by the Sun's unrefracted rays.

VI. In section 1 we showed that angle KHL is equal to angle AGH, which is the half-angle of the cone of the lunar shadow, and we have already taught how to find it in Problem 4. Indeed, for the syzygies and quadratures of the Moon we have already found it, at the extreme and mean distances. So consult Problem 4 and its table: the angle of the cone of the lunar shadow is thus as great as the portion of the ☾ beyond a hemisphere illuminated by the unrefracted rays of the ☼.

PROBLEMA VI.

Problem 6.

Axem Vmbræ Lunaris à Solis radijs irrefractis comprehenſæ metiri.

To measure the axis of the lunar shadow comprehended by the Sun's unrefracted rays.

VII. In the preceding figure triangle AGH is right-angled at A, and in it there is given from Problem 1 the side AH, the Moon's true semidiameter in parts of which the Earth's semidiameter is one; and by Problem 5 the angle AGH is given, and from it the angle AHG. Therefore, by analysis of this triangle, you will find the base HG, which is the very height or axis of the lunar shadow. So make: as the radius to side AHG, so the secant of the angle [recte: *AH, ita ſec. ang. AHG*, "to side AH, so the secant of the angle AHG", per the book's errata] to another, and you will have the base HG. Or join the logarithm of side AH to the residual of the logarithm of angle AGH, and the logarithm of the base GH will result. Or, since triangles LHK and EGK are proportional, because the angles at E and L are right and K is common, and LHK is equal to angle EGK: make as KL to KH, so KE to KG; and when KH is taken away, the axis HG of the lunar shadow will remain. Take, therefore, this small table for the Moon's syzygies; for the axis of this shadow at the quadratures is roughly midway between the extreme axes of the syzygies.

Axis or length of the lunar shadow, from unrefracted rays.

From the data of the authors	In ♂: ☼ at apogee, ☾ at perigee (Earth's semidiameters)	In ♂: ☼ at perigee, ☾ at apogee (Earth's semidiameters)	In ♂: ☼ and ☾ at apogee (Earth's semidiameters)	In ♂: ☼ and ☾ at perigee (Earth's semidiameters)	In ♌: ☼ and ☾ at apogee (Earth's semidiameters)	In ♌: ☼ and ☾ at perigee (Earth's semidiameters)
Ptolemy	64 9/25	59 6/52	53⅘	59 35/52	70 12/13	65 9/[?]3
Copernicus	69	58	63⅗	59	73⅔	65⅕
Tycho	66½	63	66 13/49	62 44/49	73⅛	69⅗
Longomontanus	59⅗	57⅘	62 48/55	64	70 0	63⅓
Kepler	61	59	62 0	57⅔	65 7/15	59 9/16
Lansbergen	57 4/7	57⅓	57 9/73	58 48/73	62 2/73	57 0
Boulliau	60	57	59⅔	57½	64 7/17	62⅙
Wendelen	63	61	62 0	59¼	64 13/32	61⅓
Ourselves	60	53	59⅓	53 26/33	59⅔	56 26/33

[Margin: Conflict of Tycho and Kepler with each one's own hypothesis.]

From the preceding table you will gather that the lunar shadow of Tycho and of Kepler does not cohere with the semidiameters of the luminaries at apogee; for then the lunar shadow is longer than the distance of the Moon from the centre of the Earth, as is clear from the distances given in chapter 14. Therefore the whole Sun ought then to be able to be darkened for us by the Moon, with duration, in an eclipse. But Tycho nowhere admits total eclipses of the Sun, while Kepler makes the semidiameters of the luminaries at apogee equal, and so admits a total eclipse there, without any duration.

[Margin: Usefulness of this Problem in eclipses of the Sun.]

From this same small table, and from the Problem on which it rests, and from the distances given in chapter 14, you will gather how far the Moon's shadow reaches at ecliptic new Moons: whether exactly to the Earth, so that the eclipse of the Sun is total without duration; or beyond the Earth, so that the eclipse of the Sun is total with duration; or whether its apex stops short of the Earth, so that a central eclipse of the Sun is nevertheless partial, a bright ring being left all around, free of the eclipse. But more fully on this in Book 5, chapter 9, where we also treat the tract of the Earth occupied by this shadow.

PROBLEMA VII.

Problem 7.

Quanta ſit Diameter Terræ apparens oculo in Luna conſtituto; & Quanta Terræ portio de hemiſphærio videatur ab eodem oculo.

How great the apparent diameter of the Earth is to an eye placed on the Moon; and how great a portion of the Earth's hemisphere is seen by the same eye.

VIII. In Book 2, chapter 6, Scholium 5, it was said that from a distance of 60 Earth semidiameters the Earth is seen under an angle of 1° 55′. But if you want to be more accurate, use the figure of section 1 here, and imagine the eye at G and the Earth at H. For since, in triangle AGH, right-angled at A, the Earth's semidiameter is supposed given as one, and the Moon's distance from the Earth GH in Earth semidiameters, the angle AGH will also be given. When this is doubled, the angle AGC will be known, under which the Earth H can be seen from the Moon G, and which measures the apparent diameter of the Earth seen from there. And since the angles at A and C are right, and in triangle AGH there are two right angles by Euclid I.32, if from the remaining right angle you subtract the angle AGH just found, the angle AHD will be known, which measures the arc AD. When this is doubled, the arc ADC will be known, which is the part of the terrestrial hemisphere seen by the eye at G. For me, from the distances of the Moon established in chapter 14, the angle AGC and the arc ADC stand, for the syzygies and quadratures, as you see in the following small table. Kepler, however, in the Rudolphine Tables, precept 149, and in the *Epitome of Copernican Astronomy*, p. 874, says that this is obtained if from the Moon's horizontal parallax you subtract one and a half times the parallax of the Sun; for this, he says, gives the Earth's semidiameter seen from the Moon.

☾	Diameter of the Earth seen from the Moon	Portion of the Earth's hemisphere seen from the Moon
In ☌ and ☍: Apogee	1° 47′ 0″	178° 13′ 0″
In ☌ and ☍: Perigee	2° 7′ 50″	177° 52′ 10″
In □: Apogee	1° 43′ 16″	178° 16′ 44″
In □: Perigee	2° 13′ 54″	177° 46′ 6″

PROBLEMA VIII.

Problem 8.

Quanta portio Hemiſphærij Terreſtris illuminetur à Lunæ Plenæ radijs irrefractis, aut etiam refractis.

How great a portion of the terrestrial hemisphere is illuminated by the unrefracted, or also the refracted, rays of the full Moon.

IX. With Witelo, Book 2, proposition 28, Pisanus [John Pecham] in the *Perspectiva communis*, prop. 24, Maurolico in the *Photismi*, theorem 17, and the other writers on optics, I suppose that the Moon, being a sphere smaller than the globe of the Earth, illuminates less than a whole hemisphere of the Earth, however full of light it may be. How great this portion is you will learn from the figure of section 1, by imagining the Moon at H and the Earth at K, and the portion of the Moon illuminated by the Sun through unrefracted rays to be ABC, from all of whose points rays are reflected to the Earth. So, when the Moon's true semidiameter AH, known by Problem 1, is taken from the Earth's true semidiameter EK, the side LK of triangle LHK, right-angled at L, will be left known; and in this triangle the distance KH of the Moon and Earth is supposed given. Therefore, as above in Problem 4, the angle LHK will become known, whose complement to a right angle, by Euclid I.32, is the angle LKB, that is, EKM, which measures the arc EM. And when the arc EM is doubled, the arc EMF of the Earth illuminated by the full Moon will be known, but imagining the rays to be unrefracted. But if you add to this arc the doubled horizontal refraction of the Moon, the arc illuminated by refracted rays will be known, in line with what is to be said in Book 5, chapter 5, Problem 7, on the refracted rays of the Sun, which can be applied to this problem without sensible dissimilarity. So, taking the Moon's horizontal refraction from Tycho as 33′, here is a sample of the Problem from our hypotheses. After this, as we did for the Sun in Book 3, chapter 11, we shall collect in one place the theorems worth knowing that can be gathered from the Problems already mentioned.

Portion of the Earth illuminated by the Moon in non-ecliptic ☌.

☾	With unrefracted rays	With refracted rays
☾ Apogee	178° 41′ 2″	179° 47′ 2″
☾ Perigee	178° 25′ 46″	179° 31′ 46″

THEOREMATA

Theorems.

Ex Lunæ Diſtantia, & vera Diametro ſequentia.

Following from the distance and true diameter of the Moon.

1. Theorem. When the Moon appears larger and nearer to us, we in fact see less of it; but when it appears smaller, more: from Problem 3 and its table.
2. Th. As great a portion of the lunar hemisphere is hidden from us as the Moon's diameter apparent to us: from Problem 3.
3. Th. The angle of the cone of the lunar shadow is as great as the portion of the Moon beyond a hemisphere illuminated by the Sun's unrefracted rays, and conversely: from Problem 4.

4. Th. When the Moon shines more for us, namely at full Moon, it is less illuminated by the Sun, both intensively and extensively. As to extension, this is clear from the table of Problem 4; as to intensity, from the greater distance of the Moon from the Sun in ☍ than in ☌, since rays further from their origin are weaker.
5. Th. The circle separating the illuminated part of the Moon from the dark part is hardly sensibly distant from a great circle of the Moon. For from Problem 4 it is gathered that the arc beyond the quadrant illuminated by the Sun on one side is about 16′ if unrefracted rays are used, or 32′ if refracted, that is, a quarter or a half of a degree, of which there are 180 in the arc of the lunar hemisphere. But Aristarchus, in the book on the distances and sizes of the Earth and the luminaries, prop. 4, says: "The circle in the Moon that divides the dark from the bright does not differ sensibly from the great circle in it", because from his hypotheses he concludes that the arc by which this circle is distant from the great circle is less than the 3960th part of a right angle, and so like a point.
6. Th. The portion of the Earth's hemisphere viewed from the Moon is as great as the complement to two right angles of the apparent diameter of the Earth viewed from the Moon, and this is as great as the doubled horizontal parallax of the Moon: from Problem 7.

SCHOLIA.

Scholia.

[Margin: The prudence of Aristarchus.]

I. From the diversity of opinions about the size of the Moon compared with the Sun and with the Earth, which we set out in the table of Problem 1, it appears how prudently Aristarchus, in his little book on their sizes and distances, did not dare to define them, but thought it enough to confine them between certain extremes, in order to refute the Pythagoreans, who, devoted to certain harmonic proportions, had wrongly established the intervals of the luminaries.

[Margin: The opinion of Thales on the bulk of the Moon.]

Before him, however, Thales of Miletus, as Laërtius reports, had said that the orb of the Moon is the 720th part of the size of the Sun. We, however, although we do not deny that God, Best and Greatest [D.O.M.], constructed the heavens above all with a certain supreme symmetry, nevertheless admit that this symmetry is still unknown to us. For although the distance and size of the Moon compared with the Earth are known more certainly from parallaxes and the apparent diameter, the distances and parallaxes of the Sun and the other planets are not so certainly established, and so we can form a conjecture about their size only within certain limits. So about these too we must press home that saying of Pliny the Elder, Book 2, chapter 23: "These things are undiscovered

and inextricable", and, as he says in the same place, "it is not right to hope that the measure of the heavens and the world should come at once to the fingers."

[Margin: What the true praise of God by astronomers is.]

And so the true commendation of Divine Wisdom by astronomers is at last the one that arises not from defining these measures and from proportions that smile on us, but from the candid confession of our ignorance, which lowers its fasces and its pride before God, who through his Ecclesiasticus, chapter 1, said, not ὑπερβολικῶς [Greek: hyperbolikōs, "hyperbolically"] but ἀληθῶς [Greek: alēthōs, "truly"]: "The height of heaven, and the breadth of the earth, and the depth of the abyss, who has measured?" Notwithstanding all this, the proportions in which Tycho, Kepler and certain others took excessive delight must be reviewed.

[Margin: Tycho's foundation and opinion on the size of the luminaries.]

II. So Tycho, in volume 1 of the *Progymnasmata*, p. 474, establishes that the Moon's diameter is contained 18 times in the Sun's diameter, because of the agreement between the diameter of the lunar heaven and the diameter of the solar heaven. For he thinks that the distance of the Moon at apogee is contained in the distance of the Sun at apogee very nearly 18 times, provided that the Sun's distance is taken as 1190 Earth semidiameters, and the Moon's not as 68 according to Copernicus or 64 according to Ptolemy but as a mean, limiting the opinions of both craftsmen, for 66 is contained in 1188 18 times. With these laid down, he adds: "A beautiful and elegant symmetry indeed, and not considered until now by anyone, so far as I know."

[Margin: Tycho's empty boast and the conflict of his hypotheses.]

But Tycho did not then remember that this symmetry conflicts with his own hypotheses; for on page 119 he had established the Moon's greatest distance from the Earth as 60⅗ Earth semidiameters, and the Sun's greatest distance as 1180, as I reported in Book 3, chapter 7. But 60⅗ is contained in 1180 19 7/12 times. So that proportion vanishes.

[Margin: Kepler's foundations on the same subject, and his inconstant opinion.]

III. Similarly, Kepler, in the foundation of the *Ephemerides*, p. 2, defined the distance of the Sun at apogee as 1800 Earth semidiameters, so that this round number would be about thirtyfold the distance of the Moon from the Earth, which is about 60 Earth semidiameters; and so the proportion of the intervals of the Sun and the Earth would be the same as that of the revolutions of the Earth and the Moon around the same centre, as it were, of the Earth. But in the *Epitome of Copernican Astronomy*, as I said in chapter 14, section 7, he wants the distance of the Moon at apogee to be 59 Earth semidiameters, so as to be the mean proportional between the Earth's semidiameter and the semidiameter of the great orb, that is, the distance of the Sun from the Earth; and in the same place, p. 479, he wants the Moon's apogee diameter to be the 720th part of its orb, so that it can cover the Sun without duration. He then says: "Therefore, as the body of the Earth is contained in the body of the

Sun as many times as the Earth's semidiameter is contained in the distance or semidiameter of the solar orb, so the body of the Moon is contained in the body of the Earth as many times as the Earth's semidiameter is contained in the distance or semidiameter of the lunar orb. Hence it comes about that the distance of the Moon at apogee from the Earth is a mean proportional between the distance of the Sun and the Earth and the Earth's semidiameter. Therefore the square root of the Sun's interval of 3469⅓ is about 59; when the cube of the Earth's semidiameter is divided by this, and the cube root of the quotient is taken, the true semidiameter of the Moon results, as 1 to 59 to the Earth; so the Sun," says Kepler, "contains the Moon more than 200000 times." And so Kepler sometimes uses a solar interval of 3469⅓ and sometimes 1180; on this inconstancy or variation I have already spoken in Book 3, chapter 8. What follows concerning their size from the intervals of the luminaries deduced by Father Bettini from harmonic proportions there is no need to weigh here, and I have sufficiently refuted it in chapter 14, in the Scholia.

CAPVT XVIII.

Chapter 18.

De Lunæ Motibus, ac de Diurni Motus Velocitate.

On the motions of the Moon, and on the speed of the diurnal motion.

[Margin: The 15 motions of the Moon are enumerated.]

I. There are fifteen names of motions, or respects, or motions of the Moon, although not all really distinct, nor certain among all authors, by which the multiform Providence of God has commanded it to be driven around the Earth: namely, the Diurnal, to the west, which is also that of the primum mobile; the Monthly motion in Longitude, to the east; the Elongation from the Sun; the motion in Latitude; the motion of the Nodes; the motion of the Apogee; the motion of the Deferents of the Aux; the motion of the simple, or free, or Periodic Anomaly; that of the compound, or monthly, or Synodic Anomaly; that of the Variation or Reflection; that of the Anomaly of Latitude and of the Nodes; that of the Centre of the Eccentric, or of the Eccentricity; that of Altitude; that of Rotation; and that of Libration. So that with every right Pliny the Elder, Book 2, chapter 9, although unaware of all these motions, said: "The last star surpasses all in wonder, the one most familiar to the Earth, and devised by Nature as a remedy for darkness: the Moon. With this multiform winding she has tortured the minds of those who contemplate her, and who resent that the nearest star should be the one most unknown." The motions listed, then, must be explained briefly, since they will often come up. Their measures must first be stated here approximately, and then the technique set out by which astronomers reached an exact knowledge of them and bound them by fixed laws of numbers and hypotheses.

[Margin: 1. Diurnal motion of the Moon.]

II. The Moon's *first* motion is the diurnal one. From east toward west, and from there back again to the east, it appears to revolve in 24 hours, or rather in nearly 25, on the hypothesis of a stationary Earth; or it is the motion by which, travelling westward, the Moon returns from a given meridian to the same meridian after nearly 25 hours. For in this motion the Moon is the slowest of all the stars and does not keep pace with the motion of the primum mobile in 24 hours. If, tonight, at the first hour after sunset, you observe the Moon on the meridian, crossing Mid-heaven together with Spica or some other fixed star, then on the following night you will see that, although Spica has already returned to the meridian, the Moon has not yet returned but stands about 13° 10′ 35″ east of it, to which 48^{m} of time correspond on the equator. The Moon therefore appears to strive against the motion of the primum mobile and, by another motion of its own, to advance about 13° 10′ 35″ each day, and so not to return to the same meridian or horizon until after about 24^{h} 48^{m}. This, certainly, is the reason why in many places the flow and ebb of the sea repeat their alternations each day about an hour later than the day before.

[Margin: 2 and 3. Motions of the Moon in the zodiac, and from the Sun.]

III. The Moon's *second* motion is its proper motion in longitude beneath the zodiac. It shows itself in the fact that each day, as I have just said, the Moon appears farther east by 13° 10′ 35″, a mean motion between the shortest and the slowest. By this motion it completes the zodiac in about 27^{d} 7^{h} 43^{m} 7^{s}. But since the Sun moves eastward each day along the ecliptic by 59′ 8″ 20‴, if you subtract this from the Moon's mean motion in longitude there remains the Moon's *third* motion, its mean motion from the Sun, which Ptolemy in his tables calls the Distance of the Moon from the Sun, and others the Elongation of the Moon from the Sun. There remains, I say, a daily motion of 12° 11′ 26″ 41‴.

[Margin: 4. Motion of the Moon's Nodes.]

IV. The Moon's *fourth* motion is that of the Nodes of the lunar orbit against the order of the signs [in antecedentia]. For the lunar orbit is inclined to the plane of the ecliptic at an angle of 5° at new and full Moons, or of 5° 18′ at the quadratures, as was said in chapter 12. This greatest latitude is called the Limit [Limes], or by the Arabs the Bellies of the Dragons [Ventres Draconum], from their likeness to the snakes which the circumference of the lunar ecliptic forms with the circumference of the lunar orbit; and one is northern, the other southern. The points at which there is no latitude and at which the lunar orbit cuts the ecliptic, diametrically opposite each other, are called the Nodes. One of these is called Northern and Ascending, because the Moon, crossing it by its proper motion, becomes northern; and as it heads toward it from the southern Limit and passes through it as far as the northern Limit, it grows higher each day for us who are northern, and ascends toward our pole and zenith. From this ascent Ptolemy calls this Node [Greek: συνδεσμός ἀναβιβάζων (syndesmos anabibazōn, "ascending node")] and the Arabs the Dragon's Head [Caput Draconis], and it is marked with this character: ☊. The other Node is called

Southern and Descending; for Ptolemy it is [Greek: συνδεσμός καταβιβάζων (syndesmos katabibazōn, "descending node")], for the Arabs the Dragon's Tail [Cauda Draconis], and its sign is ☋. In the ephemerides, moreover, both for the Moon and for the other planets, the letters SA indicate Northern [Septentrionalis] Ascending latitude, in which the planet is between the northern Node and the northern limit; SD, Northern Descending latitude, namely from the northern limit to the southern Node; MD, Southern [Meridionalis] Descending latitude, that is, from the southern Node to the southern limit; and MA, Southern Ascending latitude, namely from the southern limit as far as the northern Node. These lunar Nodes, near which eclipses occur, have been observed to retrogress against the order of the signs, as is shown by eclipses which, other things being equal, occur beneath places of the ecliptic farther west than before. The mean motion of the lunar Nodes is toward the preceding signs, its daily amount about 3′ 10″ 38‴, so that they complete their period beneath the ecliptic in about 18 Egyptian years $228^{d}\ 3^{h}\ 50^{m}$, as Kepler holds in the *Epitome of Copernican Astronomy*, p. 788; or, as Copernicus has it, in $6798^{d}\ 3^{h}\ 49^{m}$; or in 18 years $223^{d}\ 6^{h}\ 12^{m}$, as Magini has it in his *Theoricae Planetarum*; and Antonín Maria Šírek nearly the same, Book 4, chapter 2, Member 4. Others believed this motion to be uniform; but Tycho found it irregular outside the syzygies, as he himself attests in volume 1 of the *Progymnasmata*, p. 115. He is followed by Longomontanus, *Theorica*, Book 1, chapter 7; Kepler in the Ephemerides, number 6, in the *Epitome of Copernican Astronomy*, p. 820, and in the Rudolphine Tables, chapter 27; Mulerius in the *Tabulae Frisicae*, p. 415; Magini in the *Supplement to the Ephemerides*, Canon 7; Boulliau, Book 3 of his *Astronomy*, chapter 13; and in part Lansbergen in his Tables, precept 9.

[Margin: 5. Motion of the Moon's Anomaly of the Nodes.]

From this, therefore, arises the *fifth* motion, namely that of the Anomaly of the Nodes, which is monthly according to Tycho but semiannual according to Kepler. In the other planets, however, the motion of the Nodes is in the order of the signs.

[Margin: 6. Motion of the Moon in latitude.]

V. The *sixth* motion is the motion of latitude, or in latitude. It is made up of the Moon's motion in longitude along its own orbit [Greek: εἰσ τά ἐπόμενα (eis ta epomena, "toward what follows")], that is, in the order of the signs [in Consequentia], and of the motion of the Nodes [Greek: εἰς τά προηγούμενα (eis ta proēgoumena, "toward what precedes")], namely against the order of the signs [in Antecedentia]. On account of this, the Moon acquires its greatest latitude with respect to the ecliptic at ever different places of its orbit. Adding together, then, the Moon's mean daily motions in longitude, about 13° 10′ 35″, and of the Node ☊, about 3′ 10″ 38‴, the mean daily motion of latitude comes to about 13° 13′ 45″ 38‴, so that one period, which from the Dragons already mentioned Wendelen calls the Draconic Orbit [Orbis Draconticus], is completed in very nearly $27^{d}\ 5^{h}\ 6^{m}$. By this motion, then, the Moon each day

moves away from the Nodes in the order of the signs, even though Ptolemy and Copernicus preferred to count the motion of latitude from the northern limit rather than from the northern Node.

[Margin: 7. Motion of the Anomaly of latitude.]

Since, however, this motion is compounded of the motion in longitude toward the east, which for everyone is genuinely anomalous, and of the motion of the Nodes toward the west, which on Tycho's view is anomalous, there arises from this the *seventh* motion, that of the Anomaly of latitude.

[Margin: 8. Motion of the Moon's simple Anomaly.]

VI. The *eighth* motion is that of the Moon's simple Anomaly or Irregularity, which Ptolemy calls the First inequality, the Alphonsines and Prutenics the Periodic, and Kepler the Free [Soluta], that is, not tied to the Moon's phases. For it has been observed that, even leaving parallaxes out of account, the Moon does not return to new and full Moons at the same interval of time, and that true synodic months are now longer, amounting to nearly 30 days, and now shorter, as I shall say in the next chapter, number 1, where the synodic month is treated. So too it is established that the return to the same star, observed when the Moon is at the nonagesimal degree of the ecliptic reckoned from the ascending point of the ecliptic, that is, when it has no parallax of motion in longitude, is completed not in equal but in unequal intervals of time. It was further observed that when the Moon is at apogee and farthest from the Earth, as the parallaxes of altitude have shown, it is slowest, and at perigee fastest. Then, by observing the return from apogee to apogee, that is, from the point of greatest slowness to the point of greatest slowness, or from perigee to perigee, and by comparing many eclipses, Hipparchus and the other astronomers found that this period, taking the mean between the longest and the shortest, is completed in about $27^{d}\ 13^{h}\ 18^{m}\ 35^{s}$, and that this is the anomalistic month; so that each day the Moon by this motion, whether on Ptolemy's epicycle or on an arc of some other equivalent circle or quasi-circle, completes $13^{\circ}\ 3'\ 35''\ 56'''$. How, by the force of an epicycle or eccentric, a planet's motion which is regular in itself can appear unequal to us, and how by the same means the mean motion is reduced to the true and apparent one, is clear from what was said of the Sun's epicycle or eccentric in Book 3, from chapter 19 on, from where the diagrams may be repeated. Finally, it has been observed that in the syzygies, that is, in ☌ and ☍, the Moon undergoes this one simple inequality, and that nothing is needed but a single simple prosthaphaeresis or equation to make the Moon's mean motion agree with the apparent and true one. This equation Ptolemy calls Epicyclic, others the equation of the First and simple inequality, and Kepler the equation of the free Moon; at its greatest it does not exceed 5°.

[Margin: 9. Motion of the Moon's composite Anomaly.]

VII. The *ninth* motion is that of the composite Anomaly, which some call Eccentric or Synodic and Kepler the Monthly Temporary [Menstruam temporaneam], because it depends on the phases of the Moon, which make up the month, and on the varying elongation of the Moon from the Sun. For it

has been observed that the Moon's place outside the syzygies (derived from observation, and also outside the parallaxes of longitude, as when the Moon is observed at the nonagesimal, by taking its distance from fixed stars of known longitude) does not agree with the Moon's place deduced for that moment from the tables and, by the first equation, equated, but that a further, second equation is needed, which in the perigean quadratures is at times nearly 2° 40′ by itself, and 7° 40′ when compounded with the simple one. From this, therefore, astronomers recognized that outside the syzygies the Moon undergoes a double and composite Anomaly or inequality in its motions.

[Margin: 10. Motion of the Variation.]

VIII. The *tenth* motion is, as Tycho puts it on p. 126, the motion of the *Variation*; or, as Kepler puts it in the *Epitome*, the motion of the *perpetual monthly Anomaly*; while Boulliau, Book 3, chapter 13, calls it that of the *Reflection*. For Tycho states that the Moon, stripped of the first and second inequalities, with its place outside the syzygies equated not only by the simple but also by the composite equation, does not agree with its apparent and true place, even when this is observed beyond the risk of parallax in longitude, as when the Moon is observed at the nonagesimal. It therefore undergoes a third inequality or Variation and needs a third prosthaphaeresis, which taken by itself reaches, at its greatest, 41′ 30″, or, as Kepler says, 51′ or 49′. Longomontanus agrees with Tycho, *Theorica*, Book 1, chapter 6, that it is greatest at the places midway between the quadratures and the syzygies, that is, in the octants, namely at 45°, 135°, 225° and 315°, counted in the order of the signs from the preceding conjunction of the Moon with the Sun. But Kepler and Boulliau do not agree on the manner and measure of this Variation and Equation. Kepler, however, called this Equation the Perpetual Monthly, because it returns every month and is of the same quantity in any month; whereas the second Equation, discussed in the preceding number, according to the opinion he finally chose in the *Epitome* and in the Rudolphine Tables, does not return every month and is not the same, but decreases over about 6 or 7 months until it vanishes, and increases again over as many months until it reaches its greatest. For this reason he called that one the Temporary Monthly. Boulliau, for his part, saw that this very Variation is variable, now much less than 40′ and now much greater.

[Margin: 11. Motion in altitude.]

IX. The *eleventh* motion is the motion in altitude, by which the Moon is now higher and now lower, either because of the Eccentricity, or because of the Epicycle, or the eccentric of the Eccentric, or a simple elliptical eccentric, according to the different hypotheses of astronomers. For from the Moon's parallaxes in altitude, in accordance with what was said in chapter 14, it is evident that the Moon stands at different distances from the centre of the Earth at different times.

[Margin: Error of Girolamo Fracastoro and Amicus.]

This is not inferred only from its greater or lesser apparent diameter, or from a hypothesis that feigns epicycles or eccentrics in order to account for

the inequalities of the motions, as Girolamo Fracastoro, Amicus [Giovanni Battista Amico] and others thought, against whom we spoke in Book 3, chapter 7. They therefore tried in vain to explain this appearance by variety of vapours, or by different parts of the heavens that are more or less diaphanous, while keeping the same distance of the planets from the centre of the Earth. For the evident variety of the lunar parallaxes shows the variety of the lunar distances.

[Margin: 12. Motion concerning the apogee.]

X. The *twelfth* motion is the Motion of the lunar Apogee. For the true place on the ecliptic beneath which the Moon is found by observation when it is at apogee is ever different toward the east, and it has therefore been observed that its apogee advances in the order of the signs, slowly indeed but nonetheless evidently, by very nearly 6′ 41″ each day, so that, according to Kepler's *Epitome of Copernican Astronomy*, Wendelen in the 4th *Lampas*, and common opinion, it completes its period in about 8 Egyptian years $311^{d}\ 5½^{h}$.

[Margin: Christmann's error.]

Since Jacob Christmann thought that this time approached half the period of the lunar Nodes, he ventured in his *Theorica* of the Moon to make this motion commensurable with the motion of the Nodes. Longomontanus censures this error, *Theorica*, Book 1, chapter 7, p. 269, and rightly: for the doubled motion of the Apogee is 17 Egyptian years $277^{d}\ 12^{h}$, and falls short of the period of the Nodes by $315^{d}\ 15^{h}\ 50^{m}$; and that much time is required for the Moon's Node to return to the same place on the ecliptic to which the Moon's Apogee has already returned twice, after both Node and Apogee began their motion from the same point of the zodiac, whether the motion of the Apogee then arises from a motion of the Epicycle slower than the motion of the Eccentric, or from some other source. Kepler, however, with a great many others, in Book 6 of the *Epitome of Copernican Astronomy*, holds that the motion of the lunar Apogee is uniform and regular in itself. This motion in the Ptolemaic form they call the motion of the Apogee of the Epicycle, or of the true Apogee, and Kepler that of the Apogee of the free Moon.

[Margin: 13. Motion of the monthly Apogee.]

XI. The *thirteenth* motion is the motion of the Apogee of the Eccentric in the Ptolemaic form, or of the Deferents of the Aux of the Eccentric, or, as some have it, of the monthly Apogee, or, as others have it, the motion of the Centre of the Eccentric; and this motion is against the order of the signs. Not everyone gives it the same measure, however, because of the variety of hypotheses. Some, with Magini, say that its period is completed in 31 days $19^{h}\ 29^{m}\ 52^{s}$; others in 32 days $3^{h}\ 4^{m}\ 38^{s}\ 31‴$. For they derive it by subtracting the Sun's daily motion from the Moon's daily elongation from the Sun; others again, with Boulliau, Book 3, derive it by subtracting the daily motion of latitude from the doubled motion of the Moon from the Sun.

[Margin: 14. Motion of Rotation.]

XII. The *fourteenth* motion is the motion of the Rotation of the Moon about the centre of its own body. This is not the one invented by Berossus, so that the Moon should turn its self-luminous hemisphere toward us at full Moon and away from us at new Moon; for we rejected that one, because of the phases of the lunar spots, in chapter 9, number 2. So too we rejected the total Rotation introduced by Francesco Fontana, by which the southern spots become northern, on which see the same place, number 1. It is rather the one which, in the hypothesis that uses epicycles carrying the Moon, is necessary to keep the same face of the Moon turned toward us through the whole month, on which see the same place, number 2. To defend this phase, and to avoid harming his magnetic fibres and their inclination, Kepler, in the *Epitome of Copernican Astronomy*, p. 613, did not hesitate to suggest a certain motion of the Moon's inner globe, different from and really distinct from the outer crust of the lunar surface, like that of a kernel from its rind, so that the rind should preserve the position of the spots and the kernel follow the motion of the fibres. So fertile in daring is this man's ingenuity.

[Margin: 15. Motion of Libration.]

XIII. The *fifteenth* motion is the motion of Libration, whether to explain the motion of some Anomaly or, rather, to uphold the varying distance of the lunar spots from the limbs, on which see what was said in chapter 9, number 5.

De Velocitate motus Diurni Lunæ.

On the speed of the Moon's diurnal motion.

XIV. To express the speed of the Moon's diurnal motion toward the west, we suppose that each day, in 24 hours of the primum mobile, it runs through 346° 49′ 25″, since it falls short of 360° by 13° 10′ 35″, which it travels by its proper motion in longitude toward the east. So in one hour it runs through 14° 27′ 4″, and in one minute 14′ 27″ of its heaven, whose daily circuit varies with the varying distance of the Moon from the Earth. Secondly, therefore, we suppose from chapter 14 that the Moon's greatest distance, in apogean □ [quadrature], is 66 semidiameters of the Earth; its least, in perigean □, 52 semidiameters of the Earth and 10′; and its mean, 59 semidiameters of the Earth and 5′. Doubling this gives the varying diameter of the lunar heaven, which, by what was said in Book 1, chapter 4, is to the circumference as 100 to 314. From this, then, we shall infer how many diameters of the Earth the Moon runs through in one hour and how many in one minute of an hour. These diameters we shall resolve into Italian Bolognese miles, of which there are 8278 in the Earth's diameter, as we showed in Book 2, chapter 7. If you multiply the number of these miles by 90 and divide the sum by 72, you will have the Italian Roman miles in use under Augustus and Vespasian, for 90 of those equal 72 modern Bolognese, as we showed in the same chapter 7.

[Margin: Method of deriving the Moon's diurnal speed.]

So that beginners may have one example of the method, let the Moon's greatest distance from the centre of the Earth, in □, be 66 semidiameters of the Earth, and let the Moon be on the equator; the diameter of the lunar heaven is therefore 66 diameters of the Earth. Now as 100 is to 314, so 66 is to 207 24/100 diameters contained in the circumference of the lunar heaven, which circumference consists of 360°, or 1296000″. Therefore, if 360° contain 207 24/100 terrestrial diameters, then 346° 49′ 25″, which we said the Moon completes by its mean motion in 24 hours, contain 199 10/13 diameters. Dividing these by 24 gives to one hour 8⅓ diameters, which the Moon runs through in one equal hour. Multiplying the diameters already mentioned by 8278 Bolognese miles, then, we shall have the miles corresponding to the Moon's daily and hourly motion; dividing these by 60, we shall have the miles matching one minute; and dividing these further by 60, we shall have the miles corresponding to one second of an hour, which is nearly equal to one pulse-beat of a human artery. Let the following table therefore stand.

LVNÆ VELOCITAS IN DIVRNO VERSVS OCCIDENTEM MOTV.

The Moon's speed in its diurnal motion toward the west.

In □	Moon's distance from the centre of the Earth: semidiameters of the Earth	Circumference of the lunar heaven: diameters of the Earth	Moon's motion in one day: diameters of the Earth	Moon's motion in one day: Italian miles	Moon's motion in one hour: diameters of the Earth	Moon's motion in one hour: Italian miles	In 1 minute: Italian miles	In 1 second: Italian miles
Greatest	66 0	207 14	199 10/13	1652734	8 1/3	68864	1142 0	19 1/8
Mean	59 5	185 30	176 6/7	1460431	7 1/3	60851 1/2	1042 1/5	16 4/5
Least	52 10	164 36	160 0	1324480	6 2/3	55186 1/3	920 0	15 1/3

SCHOLIA.

Scholia.

[Margin: Biancani's method for the Moon's diurnal speed, less exact.]

I. Biancani, Book 9 of the *Sphaera*, chapter 2, multiplies the 21600 miles of the Earth's circumference by the Tychonic mean distance of the Moon from the Earth, 56 semidiameters of the Earth, and derives a circumference of the lunar heaven of 1200600 miles, the 24th part of which, namely 50400, he takes to be the Moon's hourly motion. But what the Moon takes away from the 360° by its proper motion should have been subtracted.

II. Kepler, in the book *De Stella Nova*, chapter 16, since he assigned the diurnal motion toward the west not to the Moon but to the Earth, could not cite the speed of this motion. Instead he took up the other daily motion, namely

that of longitude toward the east, which is much slower than the diurnal motion toward the west: for by the latter the Moon runs through 346° 49′ 25″ each day in mean motion, whereas by its proper motion toward the east it completes each day, in mean motion, only 13° 10′ 35″. He did this to show that according to the hypothesis of Copernicus any planet is the faster the nearer it is to the Sun.

[Margin: The speed of the planets in the hypothesis of Copernicus.]

For in that hypothesis, in one hour ♄ travels through nearly 300 German miles, ♃ 400, ♂ 600, the centre of the Earth 740, ♀ 800, ☿ 1000.

[Margin: The speed of the Earth in the same.]

But since the Earth rolls eastward not only annually around the Sun but also daily around its own centre, the parts of its surface lying beneath the equator complete 240 German miles in one hour by their lasting [diutino] motion. If you add these to the miles traversed from the centre in the meantime, the sum is 980, which a city located on the equator traverses in one hour of midnight; but if you subtract those from these, there remain 500 miles, which such a city completes in one hour of midday.

[Margin: The speed of the Moon in the hypothesis of Copernicus.]

The Moon, however, around the centre of the Earth, when it is 60 terrestrial semidiameters distant from it, passes through 476 German miles in one hour by its proper motion toward the east. If you add these to the miles of the Earth's centre, the journey of the full Moon through the aethereal air in one hour is 1216 German miles; but if you subtract them, there remains for the new Moon an hourly journey of 264 miles. For at full Moon it is like a planet direct around the Earth, tending in the same direction as the centre of the Earth; at new Moon it is carried in the opposite direction, like a retrograde one. So Kepler. Yet far greater, and nearly fifteenfold, is the Moon's speed in the common hypothesis, which assigns the diurnal motion to the Moon; for in it, in one hour, at its mean distance from the Earth, the Moon completes 60851½ of our Italian miles, that is, 15212 German.

[Margin: Hérigone's method for the speed of the Moon.]

III. Pierre Hérigone, in volume 5 of his *Cursus Mathematicus*, p. 583, investigates the Moon's speed differently. He says that, since the Earth's semidiameter is to the semidiameter of the lunar orbit as 1 to 56, and the Moon runs through its orbit once while the Earth completes about 28 daily revolutions around its centre, the ratio of the Earth's motion around its centre to the Moon's proper motion in the order of the signs is as 28 to 56, that is, as 1 to 2; and that it follows from this that in one minute of an hour the Moon completes 30 Italian miles around the Earth. Of these miles he holds that 60 are contained in one degree of a great circle of the Earth, and 21600 in the whole circle. Dividing these by 24 yields 900 for one hour, and subdividing these by 60 yields for one minute the 15 Italian miles which the Earth runs through. So if the Moon does double, it runs through 30 Italian miles in one minute around the Earth. Although it is always moved with the centre of the Earth in the order of the

signs, with respect to someone viewing from the Sun, yet since the Earth by its annual motion completes 47 Italian miles in each minute, if these are added to the Moon's motion around the Earth at full Moons they make 77; but if they are subtracted at new Moons, there remain 17 Italian miles, which, all things reckoned, the Moon completes in one minute. But Hérigone on p. 581 had supposed a distance of the Earth from the Sun of 1142 terrestrial semidiameters, which for me is much larger.

[Margin: Galileo's opinion on the cause of the tides.]

Opportunely, moreover, he relates that, according to Galileo, from the annual motion of the Earth around the Sun and its motion around its own centre, of which the former is to the latter as 47 to 15, there arises the flow and ebb of the sea, just as from the unequal motion of a vessel the water runs now to one end of the vessel, now to the other.

[Margin: The inequality of the Moon's motion, from Geminus.]

IV. The Moon's motion is so unequal that, according to Geminus, *Elements of Astronomy*, chapter 15, its least daily motion in the zodiac is 11° 6′ 11″ 35‴ and its greatest 15° 11′ 35″. For he follows the measure of the motions deduced from the Chaldean period, of which we shall speak in chapter 19. But see what we shall say about the variation of the synodic month in the next chapter, number 1, from the opinion of Tycho, following which Kepler, in the *Epitome of Copernican Astronomy*, p. 815, has this on the variety of the Moon's motion:

[Margin: The harmonic inequality of the Moon's motion, from Kepler.]

"The hourly motion of the Moon from the equinox," he says, "in the syzygies is made 29′ 41″ at apogee and 38′ 32″ at perigee; in the quadratures, 29′ 35″ at apogee and 35′ 11″ at perigee. But with the Anomaly increased by the Variation, in the syzygies it is 29′ 50″ at apogee and 38′ 45″ at perigee; in the quadratures, 29′ 26″ at apogee and 34′ 39″ at perigee. And when the Sun's mean hourly motion of 2′ 27″ is taken away, the motion of the Moon from the Sun at perigee in syzygy is 36′ 5″, and at apogee in quadrature 27′ 8″; between these there is precisely a diatessaron, that is, the ratio of 3 to 4. So the Moon's motion from the Sun at apogee in syzygy becomes 27′ 14″, and at perigee in quadrature 32′ 44″; between these there is precisely a minor third [tertia mollis], that is, the ratio of 5 to 6. So too between the purely syzygial motions there is a diatessaron, and between the purely quadrature motions a lesser ditone [Ditonus minor]. The difference of the syzygies from the quadratures gives a minor tone." You will understand these things better from our Book 9, of the Systems, where the Harmonic System is treated.

CAPVT XIX.

Chapter 19.

De Menſibus, Anniſq. Lunaribus, Epactis & Cyclis magnis Luniſolaribus, Primoq. Artificio inueſtigandi per hos Cyclos motus Lunæ.

On lunar months and years, epacts and the great lunisolar cycles, and on the first technique for investigating the Moon's motions by these cycles.

I. The month [Mensis] is in Greek [Greek: μήν μηνός (mēn, mēnos, "month")], and from this the Moon is called [Greek: μήνη (mēnē, "moon")], because, just as the Sun was the cause of distinguishing years, so the Moon was the cause of distinguishing months, by its revolution back to the Sun after traversing the zodiac. One kind of month is Political or Civil; another, Astronomical. The Astronomical is either Periodic, or Synodic, or of Illumination; to which could be added the Anomalistic, Apogistic and Draconic month.

[Margin: The Civil month.]

The *Civil* month is a period of 30 natural days or thereabouts: 30, indeed, among the Persians of old and the Egyptians and certain Greeks before Solon, but variable among the Romans under Romulus, Numa and Julius Caesar; and these months were rather midway between solar and lunar ones. For those who have used and still use purely lunar months, such as the Hebrews and most others, employed them alternately of 30 days, which we call [Greek: πλήζους [recte: πλήρους, per the book's errata] (plērous, "full")], that is, Full, and of 29, which we call [Greek: κοίλους (koilous, "hollow")], namely Hollow. But we say more on these in the Chronological Book, where we also treat of both Common and Intercalary months.

[Margin: The Periodic month.]

The *Periodic* month, or month of traversal, which Kepler calls the physicians' Critical month, is one by the Moon of the zodiac, or of its own orbit equivalent to the zodiac as it traverses it: a revolution from one point of the zodiac or orbit back to that same point. For in Greek [Greek: περίοδος (periodos, "a going round")] means a going round. If you consider the motions as they truly appear to us, this month is unequal; but if you consider the mean motion between the slowest and the fastest, it is supposed equal, and is called the Mean Periodic Month [Mensis Periodicus Medius]. The Mean month is about $27^{d}\ 7^{h}\ 43^{m}\ 7^{s}$.

[Margin: The Synodic month.]

The *Synodic* month, that is, the Conjunctive month, is one revolution of the Moon to conjunction with the Sun, or meeting with the Sun; for in Greek [Greek: σύνοδος (synodos, "meeting")] means conjunction and meeting, or coming together. This month is longer than the Periodic because the Moon, setting out from a new Moon and a conjunction with the Sun made, for example, at the beginning of Aries, and having traversed the zodiac by mean motion in $27^{d}\ 7^{h}\ 43^{m}$, does not find the Sun at the same point of the zodiac but advanced farther toward the east. So, to be conjoined with it again, the Moon must still traverse nearly one sign, which the Sun has meanwhile completed,

before it overtakes the Sun, which all this while is still advancing. This month is also called a Lunation [Lunatio]: True if you consider the true motions, Mean if the mean ones. [Margin: The mean, longest and shortest lunation.] The Mean Lunation is about $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}$. The longest, however, is for Ptolemy $30^{d}\ 3^{h}\ 14^{m}\ 3^{s}$, for Tycho $30^{d}\ 1^{h}\ 24^{m}\ 3^{s}$; the shortest for Ptolemy $28^{d}\ 22^{h}\ 14^{m}\ 3^{s}$, for Tycho $28^{d}\ 23^{h}\ 24^{m}\ 3^{s}$. For Kepler, in the *Epitome of Copernican Astronomy*, p. 790, with the Sun at perigee or fastest and the Moon at apogee or slowest, the month is longest, $29^{d}\ 19^{h}\ 47^{m}$; but with the Sun at apogee and the Moon at perigee the month is shortest, $29^{d}\ 6^{h}\ 42^{m}$.

The month of *Illumination* is the time from the Moon's first phase, or evening appearance after new Moon, to its last morning appearance before the following new Moon; for the remaining time, which slips by between these two phases, is called the interlunium. Between these phases there intervene interlunia sometimes of 2, 4 or 6 days, and so the month of illumination is sometimes 27½ days, or 25½ days, and very rarely 23½ days. But see what we said about these phases in chapter 3, from no. 4 to no. 10, and in chapter 4, no. 10.

[Margin: The Anomalistic month.]

The *Anomalistic* month is the time in which the Moon on the epicycle, according to Ptolemy, moves from its greatest slowness through mean speed to its greatest speed, and returns from the apogee of the epicycle to the same apogee. By the mean motions this time is about $27^{d}\ 13^{h}\ 18^{m}\ 34^{s}$.

[Margin: The Apogistic month.]

The *Apogistic* month, if one may so speak, is the time in which the orb carrying the apogee of the lunar eccentric completes its period against the order of the signs [in præcedentia Signorum]: namely, about $32^{d}\ 3^{h}\ 4^{m}\ 24^{s}$.

[Margin: The Draconic month.]

The *Draconic* month, or month of Latitude, is the time in which the Moon completes its period in the motion of latitude: namely, about $27^{d}\ 5^{h}\ 5^{m}\ 36^{s}$. I mean the motion compounded of the Moon's motion in the zodiac in the order of the signs and of the motion of the Nodes against the order of the signs.

[Margin: What the lunar year is, and of how many kinds.]

II. The *lunar year* is either Common, or Embolismaean, that is, Embolismic or intercalary. The former consists of 12 lunar months, the latter of 13. The civil Common year is of 354 days, not of 355 as once among the Greeks and the less advanced Romans; the Embolismic, of 384 days. Astronomically, however, the mean lunar year, if Common, is about $354^{d}\ 8^{h}\ 48^{m}\ 38^{s}\ 12'''$; if Embolismic, $383^{d}\ 21^{h}\ 41^{m}\ 18^{s}\ 24'''$.

[Margin: What an epact is.]

Epacts, in Greek [Greek: ἐπακταὶ ἡμέραι (epaktai hēmerai, "added days")], from the verb [Greek: ἐπάγω (epagō, "I bring in")], which means I introduce, I take on, I add, are the days to be added to the common lunar year so that it is made equal to the solar year. In civil reckoning these are usually 11 days and sometimes 12, rarely 10 or 13, as may be seen in the Calendar of Clavius,

chapter 12, no. 13. Astronomically, however, according to the mean motions, they vary with the varying opinion about each of the two years. For now it is enough to know that in the Rudolphine Tables they are $10^{d}\ 15^{h}\ 11^{m}\ 21^{s}\ 49'''\ 53''''$, but in the Prutenic Tables, as given by Clavius, $10^{d}\ 15^{h}\ 11^{m}\ 21^{s}\ 52'''\ 24''''$. When their accumulated sum after 3 years exceeds the quantity of the synodic lunar month, one month is thrown out and the remainder is retained as the epact of the following year, while the preceding year consists of 13 lunations. On this, however, there is more in the Ecclesiastical Computus.

III. The lunisolar cycle can be taken in two ways. *First*, for the period produced by multiplying the Cycle of the Moon by the Cycle of the Sun, in the sense used by the calendar computists. For them the Cycle of the Moon is a period of 19 years, serving the golden number and, through it, the finding of new Moons. The Cycle of the Sun is the cycle of weekdays [feriarum] recurring through the weeks of the Julian year. If the year were always Common, of 365 days, this cycle would be of 7 years; but because of the leap years [Biſſextiles] it is of 28 years, after which the weekdays return to the same day of the year, and the same Dominical letter returns. So when 28 is multiplied by 19, the result is 532 years, after which it was believed that new and full Moons return not only to the same day of the year but also to the same weekday. [Margin: The period of Victorinus and Dionysius, of 532 years.] Such was the Cycle devised by Victorinus of Capua and begun from the Baptism of Christ. It was restored, however, by Dionysius, surnamed Exiguus (the Small), but Great in wisdom and uprightness of character, as his contemporary Cassiodorus affirms; and it was begun from the Incarnation of the Lord, in order to abolish the custom of numbering years from the beginning of the reign of the most impious tyrant Diocletian. There is more on this period in Scaliger, *De emendatione temporum*, Book 2, p. 157, and Book 4, p. 300, and in Pétau, *De doctrina temporum*, Book 6. [Margin: The Julian period, of 7980 years.] Similar to it is the Julian Period, introduced by the same Scaliger (p. 2 of the Prolegomena to *De emendatione temporum*). It is produced by multiplying the Dionysian period of 532 years by the 15 indictions, and is of 7980 years. In the Chronological Book we show that it is not so very suitable for recording dates, much less necessary, as Scaliger asserts, or very useful, as Pétau affirms in the *Rationarium temporum*, Part 2, Book 1, chapter 4.

[Margin: The second meaning of the lunisolar cycle.]

Second, the lunisolar cycle is taken for a period of years at whose end the syzygies of the luminaries, namely new and full Moons, return to the same hour and moment of the hour of the same day, say the first day of January; granted that some require for this that they return to the same point of the zodiac, and a restitution [Greek: ἀποκατάστασιν (apokatastasin, "restitution")], that is, a revolution, not only of the luminaries but of all the celestial phenomena. This is by no means necessary; indeed it is perhaps not even possible in mathematical rigour, because of that incommensurability in the celestial motions which we suspect, together with Kepler at the end of the *Prodromus* or *Mysterium*

Cosmographicum, Boulliau, Book 2, chapter 3, and others. In this second sense, then, but restricted to the restitution of the syzygies to the same time of year, we here take lunisolar cycles, because astronomers and computists used them to investigate, as best they could, the measure of the solar and lunar year, and from that of the months and the other motions, as we shall shortly relate.

Cyclorum Lunisolarium aut Periodorum enarratio.

An account of the lunisolar cycles or periods.

[Margin: The cycle of Cleostratus, Harpalus and Eudoxus, of 8 years.]

IV. The first period was that of Cleostratus of Tenedos, and then of Harpalus, consisting of 8 solar years and therefore called the octaëteris [Greek: ὀγδοετήρις (ogdoetēris, "eight-year period")]. On it see Censorinus, *De Die Natali*, chapter 6; Macrobius, *Saturnalia*, Book 1, chapter 13; Scaliger, *De emendatione temporum*, Book 2, p. 64. For although there was previously in use the Trieteris, of 2 full years but 3 when the terms are counted, in which the year had alternately 12 and 13 months; and although, when the error had been recognised, a Tetraeteris was then made, of 4 whole years but 5 when the terms are counted, and therefore called the Penteteris; yet the Octaëteris seemed more correct, which, if you count the terms, was an enneateris, that is, of 9 years. So Cleostratus, thinking with many others that the lunar year is of 354 days and the solar of 365¼, judged that eight lunar years, with as many solar excesses, namely 2922 days, complete 99 lunations. He began this period, in civil reckoning, from the winter solstice. [Margin: The month of Cleostratus.] But since the lunar month is about 29 days and 12¾ hours, and Cleostratus had taken it as only 29½ days, as is plain to anyone who divides 2922 days by 99 lunations, it came about that Cleostratus's new Moons fell ahead after 8 years by 6 hours. And when at length they fell ahead by two days, while the true new Moons lagged by as many days, Harpalus, correcting Cleostratus's Octaëteris, made it of 2924 days. [Margin: The month of Harpalus.] When this is divided by 99, the lunar month comes out as 29 days and 12 84/99 hours, or $50^{m}\ 54^{s}\ 33'''$. But when 2924 is divided by 8, the solar year comes out too large: $365^{d}\ 12^{h}$, or, as the codex of Censorinus has it, chapter 7, $365^{d}\ 13^{h}$. Scaliger, however, makes the Harpalean syzygy

29 days and 12 28/33 hours, or about 51^{m}. [Margin: The month of Eudoxus.] This was later found to be too large by Eudoxus of Cnidus, who compared it with eclipses observed by the Egyptians. So, as Geminus reports in chapter 6, and Diogenes Laertius, he introduced another Octaëteris, of 2923½ days. Dividing this by 99 lunations gives one lunation of $29^{d}\ 12^{h}\ 43^{m}\ 38^{s}\ 11'''$; but dividing 2923½ by 8 gives a year of $365^{d}\ 13½^{h}$. But let us leave these Octaëterides aside, and also those of Nauselis, Doxitheus and Menesistratus, on which see Censorinus, chapter 6; for none of them was suited to restoring the new Moons to their original positions.

[Margin: The cycle of Meton, of 19 years, or of Philippus and Euctemon.]

V. The *second* period, of 19 years, was that of Meton. On it see Theophrastus in his book on Prognostics; Censorinus, chapter 6; Diodorus Siculus, Books 2 and 12; Tzetzes, Book 12 of the *Variae historiae*; Aelian, *De varia historia*, Books 10 and 13; Manutius, Question 6, by letter, which concerns Meton's Great Year; Giraldi, in his book on years, p. 20; Scaliger, *De emendatione temporum*, Book 2, p. 72, Book 4, p. 292, and Book 5, p. 410; Clavius, chapter 8 of the Calendar; and Pétau, *De doctrina temporum*, Book 2, from chapter 11 on, and in the *Uranologion*, Book 6, chapter 10. A trace of this cycle does seem to have existed long before in Magna Graecia, since Livy relates that Numa so adjusted lunar years to the Sun that, when the 19th year was complete and the twentieth beginning, *the days would coincide at the same turning-point of the Sun from which they had set out, the full spans of all the years being accomplished*. And in his *Elements of Astronomy*, chapter 6, Geminus attributes the discovery of this cycle to Euctemon and Philippus. Yet the others just cited, and Festus Auicenus [Rufius Festus Avienus], soon to be cited, ascribe it to Meton. So Meton of Athens, son of Pausanias and in astrology a pupil of Phaenus, himself most celebrated for his prognostics, for the drawing and levelling of waters, whose springs he conducted to Athens, and for the heliotrope, or sundial, which he dedicated there in the forum, devised or restored the cycle of 19 years. It was therefore called [Greek: δεκαιννεατήρις (dekainneatēris, "nineteen-year period")] or [Greek: ἐννεαδεκατήρις (enneadekatēris, "nineteen-year period")], that is, the Decennoval Cycle. In it, as Censorinus and Geminus attest, intercalation was made seven times, and it contained 6940 days, as the corrected codex of Censorinus has it in Manutius, Scaliger and Pétau; for the corrupt one has six thousand CCCCXL, when DCCCCXL should have been written. In these days Meton thought that 235 lunations, of which 110 are Hollow and 125 Full, and 19 years of the Sun, are completed with no minutes of an hour over or short. [Margin: How long Meton's year and month are.] So Geminus says that the year was fixed by Philippus and Euctemon at 365 5/19 days, and Censorinus, chapter 7, says it was fixed by Meton at *365 days with five nineteenth parts of a day*, which comes to the same thing, namely $365^{d}\ 6^{h}\ 18^{m}\ 56^{s}\ 51'''$. Dividing the 6940 days by the 235 lunations gives one lunation of $29^{d}\ 12^{h}\ 47^{m}\ 26^{s}\ 48'''\ 30''''\ 38'''''\ 18''''''$, whatever others may gather [Greek: ἀλογιστικῶς (alogistikōs, "without reckoning")] from the data given by Meton. But 19 Julian years contain only $6939^{d}\ 18^{h}$. In these, as Clavius computes in chapter 8 of the Calendar, there occur by the Prutenic reckoning 235 mean lunations, but these take up only $6939^{d}\ 16^{h}\ 32^{m}\ 27^{s}\ 18'''$. So the new Moons precede the end of the 19 Julian years by $1^{h}\ 27^{m}\ 32^{s}\ 41'''$. If instead you use equinoctial years, each of which for me is $365^{d}\ 5^{h}\ 48^{m}\ 40^{s}$, 19 years make $6939^{d}\ 14^{h}\ 24^{m}\ 40^{s}$, and so the mean new Moons follow later by $2^{h}\ 7^{m}\ 47^{s}\ 18'''$. [Margin: The defect of the Metonic cycle.] Either way, therefore, the cycle of Meton was defective, although, because of its civil use, it was held in such esteem that its number was called Golden. [Margin: The cycle of Rabbi Adda.] Rabbi Adda, however,

corrected it in the Hebrew Calendar. For he took a cycle of 6939 days, 16 hours and 595 parts [punctorum], that is, $33^m\ 3^s\ 20'''$. Divided by 19 years, this gives a year of $365^d\ 5^h\ 55^m\ 25^s\ 26'''\ 20''''$; but divided by 235 lunations, it gives one lunation of $29^d\ 12^h\ 44^m\ 3^s\ 20'''$. As for true lunations, let the following table serve as a sample of this falseness, drawn from observed lunar eclipses reduced to the meridian of Bologna according to the difference of longitude established in our Geographical Book.

CYCLORVM METONICORVM EX VERIS PLENILVNIIS EXAMEN.

Examination of the Metonic cycles from true full Moons.

[Margin: Eclipses of the Metonic cycle.]

Year of Christ	Month and day	Mid-point of the lunar eclipse, after noon	Time	But at Bologna	Interval between the eclipses in Julian years	Difference from the completed 19th year
1584	November 17	Tycho, at Uraniborg	$13^h\ 9^m$	$13^h\ 9^m$		
1603	November 18	Lansbergen, at Goes	$6^h\ 30^m$	$7^h\ 4^m$	19 yr $0^d\ 17^h\ 55^m$	$17^h\ 55^m$
1590	July 16	Mästlin, at Tübingen	$16^h\ 50^m$	$17^h\ 4^m$		
1609	July 16	Longomontanus, at Copenhagen	$12^h\ 9^m$	$12^h\ 10^m$	18 yr $364^d\ 19^h\ 6^m$	$4^h\ 54^m$
1594	October 28	Tycho, at Uraniborg	$19^h\ 16^m$	$19^h\ 16^m$		
1613	October 28	Longomontanus, at Copenhagen	$3^h\ 56^m$	$3^h\ 57^m$	18 yr $364^d\ 8^h\ 41^m$	$15^h\ 19^m$
1601	December 9	Kepler, at Prague	$6^h\ 59^m$	$6^h\ 48^m$		
1620	December 9	van Langren, at Brussels	$5^h\ 36^m$	$6^h\ 9^m$	18 yr $364^d\ 23^h\ 21^m$	$0^h\ 39^m$
1605	April 3	Lansbergen, at Goes	$8^h\ 35^m$	$9^h\ 9^m$		
1624	April 3	Ritelius [?], at Stuttgart	$6^h\ 8^m$	$6^h\ 22^m$	18 yr $364^d\ 21^h\ 13^m$	$2^h\ 47^m$
1609	January 19	Lansbergen, at Goes	$14^h\ 10^m$	$14^h\ 44^m$		
1628	January 20	Gassendi, at Aix	$9^h\ 36^m$	$10^h\ 5^m$	18 yr $364^d\ 19^h\ 11^m$	$4^h\ 49^m$
1612	November 8	Father Aleni, at Macau	$10^h\ 7^m$	$0^h\ 47^m$		
1631	November 8	Gassendi, at Paris	$11^h\ 14^m$	$11^h\ 58^m$	19 yr $0^d\ 12^h\ 45^m$	$12^h\ 45^m$
1613	October 28	Longomontanus, at Copenhagen	$3^h\ 56^m$	$3^h\ 57^m$		
1632	October 27	Hortensius, at Leiden	$11^h\ 32^m$	$12^h\ 4^m$	18 yr $364^d\ 8^h\ 7^m$	$15^h\ 53^m$

Year of Christ	Month and day	Mid-point of the lunar eclipse, after noon	Time	But at Bologna	Interval between the eclipses in Julian years	Difference from the completed 19th year
1623	April 14	Hortensius, at Leiden	$10^h\ 52^m$	$11^h\ 24^m$		
1642	April 14	We, at Bologna	$14^h\ 44^m$	$14^h\ 44^m$	19 yr $0^d\ 3^h\ 20^m$	$3^h\ 20^m$

[Margin: The beginning of the Metonic cycle.]

As for the beginning of this cycle: since Euctemon and Meton observed solstices, but [Greek: ὁλοσχερῶς (holoscherōs, "roughly")], that is, less exactly, as Hipparchus attests in Ptolemy, *Almagest*, Book 3, chapter 2, it is no wonder if Meton began the epoch and start of his cycle in year 4 of Olympiad 86, from the 13th day of the Athenian month Scirrhophorion, as Diodorus Siculus relates, Book 12. That is, in the year 432 before Christ, on June 27, when the mean solstice occurred, and on the 28th the true solstice followed, as the Emendator of Times, cited above, and Pétau in the *Uranologion* point out. Ptolemy, moreover, agrees wonderfully with this mark of time and with Diodorus, in Book 3, chapter 2, where he relates that a solstice was observed by Euctemon and Meton in year 316 of Nabonassar, on the 21st of the Egyptian month Phamenoth, in the morning. That is, as is clear from our Chronological Book, in the year 432 before Christ, on June 27, weekday 1. From this you will note in passing that the order of the golden number introduced by Meton was afterwards interrupted, and is not the same as that later used by St Cyril of Alexandria, Dionysius Exiguus, Bede and the Roman Church in its Calendar. For going back through the Golden Numbers of the Roman Calendar, the golden number in that year 432 before Christ would have been 7, whereas for Meton it was 1.

The last new Moon [Neomenia], moreover, then fell on June 15, and the first of Hecatombaeon on July 15. The Greeks generally began their year at the summer solstice, and so Meton began his cycle from there. Festus Avienus also touches on this, after first praising Harpalus, in these verses:

> For that [?] which holds that the Sun is rolled through the aether
> for nine winters, so that the Moon's former interval may return,
> Harpalus brings it back more swiftly to its seats and its ancient
> moments. To his numbers Meton, by Cecropian art, is said to have
> added long decades again, and he took root in men's minds. Skilful
> Greece grasped the matter at once, and sent the discovery on down
> the long years. But Meton took his first beginnings from the year
> when Phoebus was scorching Cancer with his ruddy star, when the
> sea carried far off the belt of Orion, and when Sirius was blazing
> with its dark-blue star.

[Margin: The Chaldaean period, of 12 and 18 years, but when tripled, 54.]

VI. The *third* period was the Chaldaean: not that [Greek: δωδεκαετήρις (dōdekaetēris, "twelve-year period")] of 12 years which, as Censorinus says in chapter 6, the Chaldaeans kept *adapted not to the courses of the Sun and Moon*

but to other observations, because they said that in it the weather and the yields of crops, and likewise barrenness and diseases, come round. It was rather the one of which Geminus treats, *Elements of Astronomy*, chapter 6, Ptolemy, *Magna Constructio*, Book 4, and Boulliau, *Astronomy*, Book 3, chapter 2. It consisted of 6585 days and 8 hours, that is, 18 Egyptian years $15^{d}\ 8^{h}$, but 18 Julian years $11^{d}\ 8^{h}$. For they thought that in it there were completed 223 lunations, 241 periodic months together with 10° 40′, 239 revolutions of anomaly and 242 periods of latitude. So that they might nevertheless have a period without a fraction of a day, they tripled this and made it of 19756 days, in which there are 54 Egyptian years 46 days, but 54 Julian years and 34 days; periodic months, as the Chaldaeans thought, 723, and 32° besides; synodic months 669; anomalistic 717; of latitude 726. Now if you divide the 19756 days by the 669 months, one month comes out as 29 days and 13 63/669 hours. [Margin: The defect of this period.] Hipparchus saw that this conflicts with all observations of eclipses, and so he turned to devising other periods, as will be said a little below. Certainly, as Boulliau too notes, Book 3, chapter 2, from the total lunar eclipse observed by Tycho at Uraniborg in 1577 September 26, at $13^{h}\ 3^{m}$ after noon, to the total eclipse observed by him at the same place in 1595 October 7, at $20^{h}\ 29^{m}$ after noon, there are not $6585^{d}\ 8^{h}$, as the first period of the Chaldaeans requires, but $6585^{d}\ 7^{h}\ 26^{m}$. Again, from the total eclipse observed by him at Uraniborg in 1588 March 12, new style, at $15^{h}\ 2^{m}$ after noon, to the total eclipse observed by us at Bologna, and correspondingly at Uraniborg, in 1642 April 14, at $14^{h}\ 44^{m}$ after noon, there are not 19756 days, as the Chaldaeans thought, but $19755^{d}\ 23^{h}\ 42^{m}$.

[Margin: The opinion of Geminus.]

Geminus, however, notwithstanding these things, in the *Elements of Astronomy*, chapter 15, relying on the Chaldaean period of 19756 days, with 669 lunations, 717 circles of anomaly, and 723 circles of longitude or of the zodiac and 32° completed in that interval, gathers the synodic month as about 29½ 1/33 days and the periodic as 27 1/18 days, and the Moon's mean daily motion as 13° 10′ 35″, its greatest as 15° 11′ 35″ and its least as 11° 6′ 12″, as is found in Pétau, *Uranologion*, p. 64. [Margin: The Chaldaean month and that of Geminus.] But Boulliau shows that the text of Geminus is erroneous and that the periodic month must be restored as $27^{d}\ 13^{h}\ 20^{m}$. The synodic month, on dividing 19766 days by 669 lunations, comes out as $29^{d}\ 13^{h}\ 5^{m}\ 39^{s}\ 24'''\ 11''''$. Yet it is surprising that Geminus, himself a Rhodian, who flourished in the time of Cicero and was thus later than Hipparchus of Rhodes, as Pétau shows, did not know of, or ignored, the correction Hipparchus applied to the Chaldaean period.

[Margin: The cycle of Philolaus and Oenopides, of 59 years.]

VII. The *fourth* period was that of Philolaus the Pythagorean, and of Oenopides of Chios, of 59 years. But Philolaus, as Censorinus relates, chapters 6 and 7, introduced 21 intercalary months and made the solar year of 364½ days. Therefore, says the Emendator of Times, Book 2, p. 166, he took the lunar

month, roughly in the civil manner, as of 29½ days, which doubled makes 59 days. So he assigned as many years to his period, and 21505½ days; for when these are divided by 59, there results a year of 364½ days. His lunar year was of 354 days, and there were 729 lunations in this period. But Oenopides of Chios, whose cycle, as Aelian affirms in Book 10 of the [Greek: ποικίλης (poikilēs, "varied")], that is, of the *De varia historia*, was publicly displayed at the Olympic Games, retained indeed the cycle of 59 years. But the solar year, as Censorinus records, chapter 7, he defined as 365 days and the fifty-ninth part of 22 days, that is, 8 56/59 hours. So this period exceeds 59 Julian years by an excess of 7 days and 6 hours. Now multiply 365 days by 59 and add 22 days to the sum: this period then comes to 21557 days, containing 730 lunations. [Margin: The month of Oenopides.] When those days are divided by these, one lunation comes out as $29^{d}\ 12^{h}\ 43^{m}\ 24^{s}$. This is that fifty-nine-year period which Plutarch touches on in *De placitis philosophorum*, Book 2, chapter 32.

[Margin: The period of Calippus, of 76 years.]

VIII. The *fifth* period, by far the most celebrated of those before it, was that of Callippus of Cyzicus, mentioned by Geminus, *Elements of Astronomy*, chapter 6; Ptolemy, *Almagest*, Book 4, chapter 9; Censorinus, *De Die Natali*, chapters 6 and 7; Regiomontanus, in the *Epitome*, Book 4, Proposition 3; Copernicus, Book 4, chapter 4; Scaliger, *De emendatione temporum*, Book 2, from p. 81 on; Clavius in the Calendar, chapter 8; Pétau, *De doctrina temporum*, Book 2, from chapter 16 on, and Book 10, chapter 37; and Boulliau, Book 2, chapter 2. For Callippus judged from eclipses that, when Meton's cycle is quadrupled, that is, taken as 76 years, the new Moons come one day ahead, because Meton's solar year was too large, being 27760 days in 76 years. So, one day being taken away and 27759 days divided by 76, he obtained a solar year of 365¼ days. Geminus and Ptolemy also attest this, and by them the text of Censorinus must be corrected, which in chapter 7 has 365 days, with the quarter omitted. And so in 76 years 19 intercalary days were required, as many as there were years in the simple cycle of Meton. Again, because in 76 years 235 lunations taken four times, that is 940, were completed exactly, as he thought, provided that the days of the period were only 27759, he divided these by 940 and found the synodic lunar month to be about $29^{d}\ 12^{h}\ 44^{m}\ 12^{s}\ 45'''\ 57''''\ 26'''''\ 49''''''$. [Margin: The month of Calippus.] Geminus and Censorinus add that of the 940 months 28 in that period were intercalary; on this intercalation Scaliger says more, and learnedly. This year of $365^{d}\ 6^{h}$, moreover, was favoured by Archimedes, Geminus, Sosigenes and nearly by Aristarchus, who, however, as Censorinus attests, added to it the 1532nd part of a day. But the excessive quantity of the year at length revealed a defect in the year and in the lunar month derived from it.

[Margin: The defect of this period, and its beginning.]

As for the epoch [Greek: ἐποχὴν (epochēn, "epoch")] and beginning of the Calippic period, it is established from Ptolemy, Book 3, chapter 2, Book 4, chapter 11, Book 6, chapter 5, and Book 7, chapter 3, that it was begun in

the year 330 before Christ, which followed the victory of Alexander the Great over Darius at Arbela; begun, I say, on June 28, the solstice having preceded on the 27th, this 28th day being the new Moon of Hecatombaeon among the Athenians, as Scaliger, Pétau and Boulliau agree. This period may then have spread among the people and passed into civil use, as Scaliger thinks, or it may have remained only with the learned, as Pétau contends. Certainly Diodorus Siculus, Book 22, writes that in his time the cycle of Meton was in use.

[Margin: The cycle of Democritus, of 82 years.]

IX. The *sixth* period was that of Democritus of Abdera, whose work is entitled in Laertius *The Great Year*, *Astronomy* and *Parapegma*. A parapegma, as Vitruvius indicates, Book 9, chapter 7, is a catalogue of predictions and prognostics compiled chiefly from the risings and settings of the stars. The period of Democritus, in Censorinus, chapter 6, was of 82 years with 28 intercalaries. From this Scaliger, *De emendatione temporum*, Book 2, p. 167, concludes that for him the lunar year was of 355 days, the solar of 365¼ days and the lunation $29^{d}\ 14^{h}$, both of which exceed the truth. [Margin: The month of Democritus.] Next to this was that Hebrew period of 84 years, which Guldin, in the Calendar, Book 5, chapter 13, gathers from Munsterus [Sebastian Münster] and from Pétau's notes on Epiphanius, heresy 51. It was far from the truth, since in 84 Julian years the epacts are about $28^{d}\ 5^{h}\ 52^{m}\ 53^{s}$, which lack more than 30 hours of a whole lunation.

[Margin: The period of Gamaliel, of 247 years.]

X. The *seventh* period, in Paulus of Fossombrone [Paul of Middelburg], Book 8 of the *Summa*, was that of Gamaliel, of 247 years, in which the mean new Moons are restored very nearly to their original place. For the epacts of 247 years, by the Rudolphine Tables, are only $1^{h}\ 1^{m}\ 2^{s}\ 53'''\ 39''''$. Further, in 247 years 3055 lunations are completed; and in 247 Julian years, which were already in use in Gamaliel's time, there are 90216 days and 18 hours. [Margin: The month of Gamaliel.] Dividing these by 3055 gives a lunar month of $29^{d}\ 12^{h}\ 44^{m}\ 4^{s}\ 20'''$, closer to the truth.

[Margin: The first period of Hipparchus, of 304 years.]

XI. The *eighth* period, now, was that of Hipparchus, but threefold; the first indeed more civil, the others astronomical. On the first, see Ptolemy, *Magna Syntaxis*, Book 4, chapter 2; Censori- nus, chapter 6; Regiomontanus in the *Epitome*, Book 4, Proposition 3; Copernicus, Book 4, chapter 4; Scaliger, *De emendatione temporum*, Book 2, p. 108; and Clavius in the Calendar, chapter 8. Hipparchus flourished about 200 years after Callippus, and in those years, from observations of eclipses, he noted that the Moon was already running ahead of the Calippic new Moons by 16 hours; and therefore by a whole day, or 24 hours, in 300 years, which contain very nearly four periods of Callippus, amounting to 304 years. He therefore thought that one day should be taken away from them, so as to have a juster measure of the year and a more exact restitution of the lunisolar cycle. Accordingly, a three-hundredth part of a day was to be removed from the Calippic year of 365¼ days, and he noticed

from observing the equinoxes that this did not conflict with their revolutions. Now the three-hundredth part of a day is $4^{m}\ 48^{s}$; taken from 365 days and 6 hours, it leaves the Hipparchan year received by Ptolemy, $365^{d}\ 5^{h}\ 55^{m}\ 12^{s}$. And so from four Calippic periods (which would have been 111036 days) he took away one day, so that there were 111035 days. Dividing these by 3760 lunations, he obtained one synodic lunar month of $29^{d}\ 12^{h}\ 44^{m}\ 41^{s}\ 6'''$. In this period, moreover, as Censorinus says, there was intercalation 112 times. Clavius, however, maintains that for the new Moons to come ahead by one whole day requires not 300 or 304 years but 312½, evidently because he uses a smaller year. But Copernicus had noticed that the revolution of the Moon with respect to the Sun occurs only when the year is of such a size that a three-hundredth particle of a day must be taken from the Calippic and Julian year; for he thought that the quantity of the year varied.

[Margin: Praise of Hipparchus.]

[Margin: And a period of about 345 or 441 years.]

Since, however, this period of 304 years did not restore the conjunctions of the luminaries very exactly, and much less the revolutions of anomaly and latitude, that towering man, to whom those two things never commended enough to any astronomer, Labour and Truth, were dear (for Ptolemy calls him [Greek: φιλόπονον καὶ φιλαλήθη (philoponon kai philalēthē, "labour-loving and truth-loving")] and Pliny the Elder a sharer in nature's counsels), compared with one another many eclipses formerly observed by the Chaldeans and Egyptians along with those he had observed himself. He noticed that in 344 Julian years, 363 days and 1 hour, that is, in 126007 days and 1 hour, 4267 synodic months are completed, 4612 periodic months less 7° 30′, and 4573 revolutions of simple anomaly; but that the restitution [apocatastasim] of latitude is not restored except in 5458 lunations, at which point 5923 revolutions of latitude are complete. Therefore, in the matter of the Moon, the revolutions of solar years must be set aside; and these four periods, namely of synodic and periodic months, of anomaly and of latitude, have not yet been made commensurable by so great an interval of time that all would be exactly restored in one and the same interval. Ptolemy, Book 4, chapter 2, judges this matter difficult, but Scaliger, Book 4, p. 288, [Greek: ἀδύνατον (adynaton, "impossible")]. [Margin: The month of Hipparchus.] Further, dividing 126007 days and 1 hour by 4267 gives a synodic month of $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 15'''\ 44''''\ 39'''''\ 4'''''''$.

[Margin: Viète's period of 3400 years.]

XII. The *ninth* period was that of François Viète, in Proposition 31 of his Calendar, which he falsely calls Gregorian, since it was not approved by Gregory. This period is of 3400 Julian years, or 1241850 days, containing 42053 synodic months, in which he affirms that the revolutions of both luminaries are exactly restored. Clavius, however, in the Calendar, chapter 24, denies that this can happen unless the synodic month is $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 10'''\ 43''''$; which Viète has not yet demonstrated, though he does admit that the lunar month is of 29 12313/42053 days. But Clavius too grants that this period comes very close

to the truth, and Kepler, in the Rudolphine Tables, Precept 135, thinks that it falls short of exact restitution by only $1^m\ 19^s$, and therefore strongly commends it. More than 200000 years would therefore be required before this period could be convicted from observations of a defect of one hour. But just as it cannot yet be demonstrated that this period is false, so, to speak with Boulliau, Book 2, chapter 3, who has ever demonstrated that the motions of the Sun and Moon are fully commensurable? [Margin: The month of Viète and of Kepler.] Now in 3400 years there are 42053 lunations. When 1241850 days are divided by these, according to Viète, Viète's lunar month comes to $29^d\ 12^h\ 44^m\ 3^s\ 11'''$; but by my reckoning to $29^d\ 12^h\ 44^m\ 3^s\ 10'''\ 43''''\ 50'''''\ 41''''''\ 25''''''\ 43''''''''$. If, however, you divide 1241850 days and $1^m\ 19^s$ by the same number, according to Kepler, the lunar month will be $29^d\ 12^h\ 44^m\ 3^s\ 10'''\ 43''''\ 57'''''\ 26''''''\ 40''''''$; and yet in the Rudolphine table of syzygies he gives $29^d\ 12^h\ 44^m\ 3^s\ 10'''\ 51''''$. Viète, however, adds in Proposition 35 that the greatest period of the lunar restitution is of 165580000 Gregorian years, which is less than the same number of Julian years by 3400 Julian years.

SCHOLION.

Scholium.

Do not look here for other cycles, as though they had been passed over unadvisedly or omitted through forgetfulness. For these are either certain Great Years for some other end than restoring the conjunctions of the Moon and the Sun to the same time, or else civil or rather ecclesiastical cycles, set up for feast days and festivals, or for restoring, after a fashion, new Moons and especially the Paschal ones, such as those of Anatolius, Theophilus, Cyril, Victorinus, Dionysius Exiguus, Bede and countless others, some of whom I indicated in Book 3, chapter 14, Scholia 3 and 4. It will be enough, then, to have recorded the nine already mentioned.

CAPVT XX.

Chapter 20.

De Secundo Artificio, quo Aſtronomi Reuolutiones & inde Motus Lunæ inueſtigarunt.

On the second technique by which astronomers have investigated the revolutions and, from them, the motions of the Moon.

I. SINCE it is clear from what was said in the preceding chapter that it is vain to investigate the restitution of the Moon's meetings with the Sun to the same moment of the year from cycles of the Moon and the Sun together, or from a commensuration of the lunar year and month together with the solar year, the care of the solar year must meanwhile be set aside, for we have already investigated its quantity better from the equinoxes, in Book 3, chapter

15. Just as there, then, from two observed equinoxes separated by a very long interval, but reduced to equal time and, from a moderate knowledge of the solar apogee, converted into mean equinoxes, we investigated the measure of the tropical year; so now, from two observed lunar syzygies, that is ☌ or ☍, separated by the longest possible stretch of time, but reduced to equal time and, from a moderate knowledge of the anomaly of the luminaries, brought back to mean syzygies, we must investigate the measure of the lunar month and of the other revolutions of the Moon. Ptolemy presents this technique in Book 4 of the *Magna Constructio*, from chapter 1; Regiomontanus there in the *Epitome*, Book 4, from Proposition 4; Biancani in the *Sphere*, Book 9, chapter 2; and Boulliau, Book 3 of his *Astronomy*, from chapter 2 to chapter 7. The remaining astronomers pass over it, except Copernicus, who deals with the third technique, of which I shall speak in chapter 22. No one, however, has set out this whole method clearly and in order, as we would wish, and so we shall present it distinctly in the following precepts.

[Margin: Precept 1.]

II. *First*, from observations of eclipses one must choose two eclipses of the Moon separated by the longest possible interval of time, so that an error, if one creeps in, spread over very many years, becomes insensible; and so that, on the contrary, an error arising in a small interval is not multiplied over a large interval and, from insensible, made enormous and manifest, as Ptolemy warns, Book 3, chapter 2, and Book 4, chapter 1. I said eclipses of the Moon because, since the Moon is then truly deprived of the light received from the Sun, eclipses of this kind occur in the same way on all horizons, as regards the duration and quantity of the eclipsed part, and toward the same regions of the world; nor is any other difference found in them than the count of time arising from the difference of geographical longitude, or from the distance of the meridians. Therefore, at the middle of a lunar eclipse, which can be known from its beginning and end without sensible error, the Moon is diametrically opposite the Sun and 180° from it in the zodiac, and the Sun's place at the stated time can be known quite exactly from the tables established in Book 3, and that either independently of parallax, which cannot be known without the Moon, or dependently on a parallax known in some fashion, though not exquisitely (for that is not required here). So, by the addition or subtraction of 180° from the Sun's place in the zodiac, the true place of the Moon in the zodiac can be known at once, and no parallax of the Moon intervenes to disturb this business.

But if eclipses of the Sun were chosen, or if two places of the Moon had to be gathered from its approach to fixed stars, or from its distance from two fixed stars, or from the Sun on one side and one of the fixed stars on the other, or from azimuths and from right ascension and declination observed and seen, it would be necessary to disentangle the Moon's parallaxes in longitude and latitude. These differ on different horizons at the same time; they make the same eclipse of the Sun greater and longer in some places, smaller and shorter in others, and nil elsewhere; they introduce into its beginning and end another

difference beyond the one that arises from the distance of the meridians; and finally they make the visible place differ greatly from the true one. So Ptolemy, Book 4, chapter 1, rightly preferred lunar eclipses in this practical procedure, because of their uniformity everywhere on Earth. For, as is said in the same place, [Greek: Τὸ τεταγμένον, καὶ τὸ ὅμιον [?], τῶν ἀτάκτων καὶ ἀνομίων ἀναγκαῖον ἂν εἴη πρωυποκεῖσθαι], that is: *What is ordered and alike must necessarily be put before what is disordered and unlike.* In addition, outside the syzygies the Moon is subject to two or three inequalities or anomalies, indicated in chapter 19, whereas at the syzygies it undergoes only a simple and single inequality.

[Margin: Precept 2.]

III. *Second*, as Ptolemy prescribes, Book 4, chapter 2, two eclipses of the Moon must be chosen in both of which the Moon is in the same degree of its simple anomaly and the Sun in the same degree, but of its own anomaly; this, however, he admits is very difficult to find. But at least the Moon should be in the same degree of anomaly, or equally distant in the order of the signs from the point of greatest or least slowness. This can be known from the durations of eclipses and the hourly motion, and from lunar tables constructed from them and adequately proved by their truth in predicting eclipses without enormous error. But the best eclipses, as Regiomontanus advises, Proposition 5, would be those in both of which the Moon was slowest, that is, at the apogee, or fastest, that is, at the perigee of the epicycle. And since the Moon in overtaking the Sun depends on the varying speed of the motion not only of itself but also of the Sun, I said accordingly that it was more prudent that the Sun too should occupy the same degree of its anomaly in both eclipses. Two other eclipses of the Moon can also be chosen for proof, which show the greatest inequality and yet do not contain whole revolutions of anomaly: for instance, if in the first interval it began from the apogee and did not end in the perigee, while in the other interval of eclipses it entered from the perigee and did not end in the apogee.

In this or in the preceding precept, however, there is no need for those two eclipses to have occurred in the same degree of the zodiac, provided the other conditions are present.

[Margin: Threefold Precept 3.]

IV. *Third*, the apparent time of the observed lunar eclipses must be converted into equal time, according to the precepts of Book 3, chapter 31. Next, from astronomical tables, which through smaller cycles and smaller intervals of time are supposed to have the motion of the Moon's anomaly and the equations of the first inequality known in some fashion and, as I said, to have been already proved by a tolerable prediction of eclipses, the apparent eclipse full Moons must be converted into mean ones, by adding the equation where otherwise it would have been subtracted and subtracting it where it would have been added. And finally, if the eclipses were not observed under the same meridian, their mean time must be reduced to the same meridian, according to the difference

of geographical longitude, known from another eclipse observed at the same time in both places, or from the distance of the places, or from some other source.

[Margin: Precept 4.]

V. *Fourth*, the interval of time, equated as above, must be resolved into days; and from this must be found the number of whole revolutions of the Moon completed in it, namely of lunar synodic and periodic months, and also of simple anomaly and of latitude. Hipparchus, and others following in his footsteps, have searched these out for us with such certainty, from ephemerides, from smaller cycles and from observations handed down, by comparing the times of greatest, least and middling speed and also of latitude, that there is now no danger of error in one whole monthly revolution.

For Hipparchus, as Ptolemy reports, Book 3, chapter 2, and Book 4, chapter 2, from the Chaldean period of 18 years and, tripled, of 54 years, from the Calippic period of 76 years and, this quadrupled, of 304 years, was certain that in 19 Julian years, that is, in 6939 days and 18 hours, 235 lunations and a very little more are completed; and in 54 years, or 19756 days, 669 lunations; and in 76 years, or 27759 days, only 940 whole lunations and somewhat more; and in 304 years, or 111035 days, 3760 lunations are completed, with almost no sensible remainder, as we also said in chapter 20, numbers 5, 6, 8 and 11. But from a deeper comparison of eclipses he saw that in the 54 years of the Chaldean period there are not exactly completed 669 lunations, nor 723 periodic months with 32°, nor 717 anomalistic circuits, nor 726 circuits of latitude, as the Chaldeans had thought; but that in 344 Julian years and 1 hour, or 126007 days and 1 hour, there are exactly completed 4267 lunations, 4612 zodiacs less 7½°, and 4573 revolutions of anomaly; but that the revolutions of latitude are not restored except in 5458 lunar months, or 441 Egyptian years $213^{d}\ 23^{h}\ 23^{m}$, at which point 5923 draconic circuits are completed. Having examined these things, I say, he left us this rule: we are to multiply the days elapsed in the interval between the eclipses by 4267, and divide the sum by 126007, to have the number of lunations. These, moreover, are to the revolutions of anomaly as 4267 to 4573, and to the revolutions of latitude as 5458 to 5923. And Hipparchus set the mean synodic month at very nearly $29^{d}\ 31'\ 50''\ 8'''\ 9''''\ 20'''''$.

[Margin: Hipparchus's slight error in the Moon's anomaly and latitude.]

Ptolemy, however, Book 4, chapters 2 and 3, affirms that the proportions already mentioned are correct as regards synodic months, without danger of sensible error; but that in the revolutions of anomaly and latitude Hipparchus erred by some sensible error; and that this became known to him by a simple method, of which we shall speak in chapter 22. But this method cannot be taught unless the mean motions of the Moon, investigated by the rules already mentioned, and then corrected or to be corrected in the manner still to be taught, are set out first. He adds that as 4267 is to 4573, so 251 is to 269, and that we can therefore use these smaller numbers.

[Margin: Wendelen's rule.]

Govaert Wendelen, moreover, an outstanding investigator of lunar secrets, after long reflections over very many years, affirms in the *Idea of the Atlantic Tables* that in precisely 79736 days, which are 2184 Egyptian years with 166 days, exactly 27000 lunations are completed; and that in 16831 synodic lunar months are completed 18038 eccentric circuits, that is, as Ptolemy would say, epicycles or circles of anomaly; but in 6890 synodic months, 7477 draconic circuits, or circuits of latitude. In this reckoning he agrees with Hipparchus, both as to whole anomalies, since by Hipparchus's rule 18038 anomalistic circuits are completed in 16831 months, and in the circuits of latitude, since according to Hipparchus 7477 draconic circuits correspond to 6890 months.

[Margin: Boulliau's rule.]

Ismaël Boulliau, an astronomer of deep inquiry, in Book 3 of his *Astronomy*, chapters 3 and 4, from several but smaller intervals of eclipses, limited on either side, shows that in 18 years etc., that is, in $6585^{d}\ 11^{h}\ 8^{m}$, 223 months are completed, 241 zodiacs and 239 circuits of anomaly, and that the Moon's apogee completes the zodiac twice in this time. Next, advancing to greater intervals of eclipses, he shows that in $652685^{d}\ 3^{h}\ 51^{m}$ there are completed 22102 months, 23885 zodiacs less $3^{\circ}\ 3'\ 58''$, and 23661 anomalies, while the Moon's apogee meanwhile completes the zodiac 198 times. Likewise, that in $652863^{d}\ 9^{h}\ 31^{m}$, 22108 months are completed. But in chapter 6, true eclipse full Moons having been reduced to mean ones, he teaches that in $736404^{d}\ 16^{h}\ 20^{m}\ 5^{s}$ there are completed 26725 anomalies and a further $114^{\circ}\ 35'\ 34''$; but in $76404^{d}\ 9^{h}\ 1^{m}\ 29^{s}$, 24937 months.

[Margin: On Mulerius's table of syzygies.]

Nicolaus Mulerius, in the *Tabulae Frisicae*, constructed for us a table for finding the number of lunations or syzygies for given Julian years. In it, I believe rather through the fault of copyists or printers, there are some errors; so in its place we shall substitute another, made from the Rudolphine epacts, and we shall give it at the end of this work as an aid for beginners, so that for given Julian years the mean lunations completed in them can be summed, together with the remaining epacts.

[Margin: Precept 5.]

VI. *Fifth*, then, divide the days of the interval between the lunar eclipses mentioned above by the lunations, or by the revolutions of anomaly or of latitude found by means of Precept 4, and the result will be the time corresponding to one mean lunation, or one anomalistic revolution, or one draconic circuit, or circuit of latitude. The remaining part of the precept, on the daily motions, will be given more conveniently below, from number XII.

It is a pleasure now, with three lunar eclipses from our His- tory of Eclipses, selected as examples, to illustrate the precepts already mentioned and, by the same effort, to lay the foundation of our hypothesis on the lunar motions. For no other such observed eclipses exist so far that are better suited to this purpose.

EXEMPLVM I.

Example 1.

[Margin: First eclipse.]

VII. The Chaldeans at Babylon, as Ptolemy reports, Book 4, chapter 11, observed a lunar eclipse in the year 336 of Nabonassar, which was the year 383 before Christ, on day 26 of Thoth, namely December 22 completed, at 19^h 20^m after noon; but at Alexandria at 18^h 30^m. Therefore, as I shall show in the Geographical Book, at Uraniborg and Bologna it was at 16^h 47^m, but in equated time at 16^h 37^m 52^s. For the Sun was in ♐ 26° 35′, its anomaly 203° 40′. The Moon's true place was ♊ 26° 35′, its anomaly 230°; and the mean motion of the Sun was ♐ 25° 49′, of the Moon ♊ 22° 45′, and the difference from the true opposition 3° 4′, to which, from the Moon's mean motion from the Sun, correspond 6^h 4^m. So the mean opposition preceded the true one, and it occurred at 10^h 34^m.

[Margin: Second eclipse.]

Next, in 1628, on January 20 in the New Style, but on day 10 in the Old, an eclipse of the Moon was observed, as I have it in the Letters of Gassendi and as Wendelen reports, at Aix [Aquis Sextijs] by Pierre Gassendi at 9^h $36½^m$ after noon, although Wendelen draws it to 9^h 26^m, because he thinks that Brussels and Aix are under the same meridian, while at Brussels he observed the end 10^m later than Gassendi. But by my reckoning, from my Geographical Book, Brussels is farther west than Aix by 4^m of time, and Aix farther west than Bologna by 29^m. Keeping therefore to the observation of Gassendi, whom I have found very accurate from other eclipses observed by him and by us, the middle of the eclipse at Bologna was at 10^h $5½^m$, but in equated time at 10^h $14½^m$. For the Sun was at the very beginning of ♒, at 36′ of it, and its anomaly 203° 30′. The Moon was in ♌ at minute 36′, and its equated anomaly was 231° 4′. The mean motion of the Sun was ♑ 29° 48′, and of the Moon ♋ 26° 52′. It therefore fell short of the place of opposition by 2° 56′, to which, from the Moon's mean elongation from the Sun, correspond 5^h 47^m. So the true opposition followed the mean one, and the mean one was at Bologna at 4^h $27½^m$ in equal time.

[Margin: The interval of time between these eclipses.]

Now, from the time of the first mean eclipse full Moon to the end of the year there are 8^d 13^h 26^m; from there to the beginning of the years of Christ there are 382 whole years; and from there to the time of the second, but mean, eclipse full Moon there are 1627 whole years 10^d 4^h $27½^m$. Added together, these make 2009 Julian years 18^d 17^h 53^m 30^s. Now 2009 multiplied by the 365 days of a common year makes 733285 days. Again, dividing 2009 by 4 gives the leap years in that number of years, and so 502 days. When these, along with 18^d 17^h $53½^m$, are added to the 733285 days, the whole interval of the eclipses comes to 733805^d 17^h 53^m 30^s. So, multiplying these, according to Hipparchus's rules,

of which see Precept 4, by 4267, and dividing the sum by 126007 days, we find 24849 mean lunations; and multiplying these by 269 and dividing the sum by 251, we find 26631 revolutions of anomaly, though these are to be brought back under the geometer's file below, in the next chapter.

[Margin: Finding the mean synodic month.]

Further, when 733805 days are divided by 24849 lunations, the result is 29^{d} 12^{h} 44^{m} 0^{s} $32'''$ $36''''$ $26'''''$. When the 17^{h} 53^{m} 30^{s} that remained are resolved into $3864600'''$ of time, and these are divided by 24849 lunations, the result is 2^{s} $36'''$ $21''''$ $43'''''$ $12''''''$. When all are added together, then, one mean lunation, or one synodic month, comes to 29^{d} 12^{h} 44^{m} 3^{s} $8'''$ $57''''$ $43'''''$ $12''''''$.

Likewise, when 733805 days and 17^{h} $53½^{m}$ are divided by 26631 anomalies, one revolution of simple anomaly comes to 27^{d} 13^{h} 18^{m} 34^{s} $25'''$ $21''''$ $43'''''$ $12''''''$. This comparison of eclipses is outstanding, because the Sun's anomaly was the same in both, the Moon's anomaly in both was very nearly in the same degree, and the interval is very large: conditions that I have not been able to find so exquisite in any other comparison of eclipses, as will be clear from the last table of the next chapter.

EXEMPLVM II.

Example 2.

[Margin: The first eclipse, Hipparchus's.]

VIII. In the year 548 of Nabonassar, as Ptolemy, Book 4, chapter 11, reports from Hipparchus, which was the year 200 before Christ, a total eclipse of the Moon was observed at Alexandria on day 9 of Mechir, that is, March 19, at 13^{h} 20^{m} p.m. At Bologna it was 11^{h} 37^{m}, but in equated time 11^{h} 38^{m}. For the Sun was in ♓ 25° 10′, its anomaly 289° 5′, and its mean place ♓ 23° 18′. The Moon's equated anomaly was 103° 42′, and its mean place ♎ 0° 31′. So the mean eclipse preceded the true one by 7° 13′, that is, by 14^{h} 14^{m}, and it occurred on day 18 at 21^{h} 24^{m}.

[Margin: The second eclipse, observed by us.]

In the year of Christ 1642, on April 14, but on the 4th in the Old Style, we, with many others at Bologna, observed a total eclipse of the Moon, from the altitudes of Spica and Aquila; the times of its duration were also counted by the oscillations of the pendulum, on which see Book 2, chapter 20; and the penumbra was separated, and the beginning of the spurious eclipse from the legitimate one, by means of a very clear telescope. The middle was exactly at 14^{h} 45^{m} p.m. There is no eclipse of which I have so many observations made in different places, for it was observed in 15 widely separated cities in Europe, and by distinguished men, for the most part very skilled in astronomy, as I shall relate in the History of Eclipses. The equated time was 14^{h} 38^{m}, for the Sun was in ♈ 25° 13′, its equated anomaly 288°, and its mean motion ♈ 23° 21′. The Moon's equated anomaly was 103° 47′, and its mean place ♏ 0° 4′.

So the mean opposition preceded the true one by 6° 43′, or 13^{h} 16^{m}; the mean full Moon therefore occurred at 1^{h} 22^{m}. And in both eclipses the anomaly of the Sun and that of the Moon were the same, so that this pairing too is very valuable.

The interval of time between the two mean oppositions was 1841 Julian years 16^{d} 3^{h} 58^{m}, or 672441^{d} 3^{h} 58^{m}, in which, by Precept 4, 22771 whole syzygies and 24404 anomalies were accomplished. When 672441^{d} 3^{h} 58^{m} are divided by 22771, then, the mean synodic month comes to 29^{d} 12^{h} 44^{m} 3^{s} 21‴ 23⁗ 30⁗′, and one anomaly to 27^{d} 13^{h} 18^{m} 32^{s} 35‴. So that you may see how much it matters that the Moon's anomaly should be the same and the interval of time as great as possible, do not be reluctant to bear with a third example.

EXEMPLVM III.

Example 3.

[Margin: The first and most ancient of all observations of an eclipse.]

IX. Ptolemy, Book 4, chapter 6, relates from Hipparchus that an eclipse of the Moon was observed by the Chaldeans at Babylon in the first year of Mardocempadus and the 27th of Nabonassar, which was 721 before Christ, on day 29 of the month of Thoth, that is, March 19, and that the Moon began to be eclipsed after its rising, a good hour having passed. From the duration he gathers that the middle of the eclipse was 2^{h} 30^{m} before midnight, but at Alexandria 3^{h} 20^{m}, that is, 8^{h} 40^{m} after noon; therefore at Bologna 6^{h} 57^{m}, but in equated time 7^{h}. For the true place of the Sun was ♓ 23° 30′, its mean place 21° 9′, and its anomaly 295°. The Moon's mean place was ♍ 27° 59′, and its equated anomaly 66°. So the mean opposition preceded the true one by 6° 20′, or 12^{h} 30^{m}, and it occurred on day 18 at 18^{h} 30^{m} p.m.

[Margin: The second eclipse, observed by Wendelen.]

In the year of Christ 1621, an eclipse of the Moon was observed by Govaert Wendelen at Bethasiis in Belgium on November 28 at 15^{h} p.m. So at Bologna it was 15^{h} 22^{m}, but in equated time 15^{h} 13^{m}; for the true place of the Sun was in ♐ 7° 9′, its mean place ♐ 8° 10′, and its anomaly 151° 10′. The Moon's mean place was ♊ 11° 51′, and its equated anomaly 67° 36′. So the mean opposition preceded the true one by 3° 41′, namely by 7^{h} 14^{m}, and it occurred at 8^{h} 8^{m} p.m.

The interval was 2341 Julian years 244^{d} 13^{h} 38^{m}, or 855294^{d} 13^{h} 38^{m}. By Precept 4, then, there are 28963 lunations and 31040 anomalies. When 855294^{d} 13^{h} 38^{m} are divided by 28963, one mean lunation comes to 29^{d} 12^{h} 44^{m} 3^{s} 25‴ 51⁗ 25⁗′ 18⁗′′, and one revolution of anomaly to 27^{d} 13^{h} 18^{m} 36^{s} 12‴.

CAPVT XXI.

Chapter 21.

De Tertio Artificio, quo Motus Lunæ in Longitudinem eiusque prima Inæqualitas demonstratur, eaque occasione Semidiameter Epicycli inueſtigatur.

On the third technique, by which the Moon's motion in longitude and its first inequality are demonstrated, and on that occasion the semidiameter of the epicycle is investigated.

I. THIS technique is presented by Ptolemy, Book 4, chapters 6 and 7; by Copernicus, Book 4, chapters 4, 5 and 6; and by Boulliau, Book 3 of his *Astronomy*, from chapter 3 to chapter 7. In it the tables of the Moon's mean motion are assumed to have been constructed first, from the synodic and anomalistic month already known beforehand, by the technique treated in chapter 20. Next, three ancient eclipses of the Moon are employed. These are arranged on a circle, and from the true and mean motions of the luminaries the eccentricity of the Moon, or the semidiameter of the epicycle, is investigated, and from that the anomaly, the equation, and the difference between the true and mean place of the Moon. The very same is done by means of another three, more recent, eclipses of the Moon. Finally, by comparing the ancient anomaly with the recent one, it is gathered whether the Moon's mean motion, as held in the tables, agrees with those equations, or instead falls short or exceeds; and by spreading the shortfall or excess over the time between the ancient and the recent eclipse, the tables of the motion of anomaly are corrected. In the same way, by comparing the true places of the Moon, the synodic month is corrected, and from it the mean motion of the Moon's elongation from the Sun and in the zodiac, if need be.

[Margin: Three earlier eclipses examined by Ptolemy.]

II. Now Ptolemy first takes three eclipses observed at Babylon by the Chaldeans. The *first* was in year 1 of Mardocempadus, with day 29 of Thoth completed, at $9^h\ 30^m$ p.m., but at Alexandria $8^h\ 40^m$, with the Sun in ♓ and the Moon in ♍ 24° 30′. The *second* was in year 2 of Mardocempadus, on day 18 of Thoth, at $11^h\ 10^m$ p.m. at Alexandria, with the Sun in ♓ and the Moon in ♍ 13° 45′. The *third* was in the same year 2 of Mardocempadus, on day 15 of Phamenoth, at $7^h\ 40^m$ p.m. at Alexandria, with the Sun in ♍ and the Moon in ♓ 3° 15′. With these laid down, he gathers from them that the semidiameter of the epicycle, or the eccentricity, is very nearly 5 13′ in parts of which the radius of the eccentric carrying the epicycle is 60; that in the second of these eclipses the Moon's mean place was at ♍ 14° 44′, and therefore the equation to be subtracted 59′, since the true place was 13° 45′; and that the anomaly, or distance from the apogee of the epicycle, was 12° 24′. The first of these eclipses was total, the second of 3 digits from the south, the third of more than 6 digits from the north.

[Margin: Three other eclipses examined by Ptolemy.]

III. Again, he selects three other eclipses, observed by himself at Alexandria with the greatest accuracy. The *first* was in year 17 of the Emperor Hadrian, with day 20 of Payni completed, at $11^h\ 15^m$ p.m., with the Sun in ♉ and the

Moon in ♏ 13° 15′; and it was eclipsed totally. The *second* was in year 19 of Hadrian, with day 2 of Choeac completed, at 11^{h} p.m., with the Sun in ♎ and the Moon in ♈ 25° 10′; and three-quarters of the lunar disc, or 9 digits, was eclipsed from the north. The *third* was in year 20 of Hadrian, with day 19 of Pharmuti completed, at 16^{h} p.m., with the Sun in ♓ and the Moon in ♍ 14° 12′; and half of the Moon was eclipsed from the north.

[Margin: How large the semidiameter of the Moon's epicycle is.]

From these he gathers that the eccentricity, or the semidiameter of the epicycle, is 5 14′ in parts of which the radius carrying the epicycle is 60, or 8722 parts of which that radius is 100000; that in the second of the recent eclipses the Moon's mean place was in ♈ 29° 30′, and so the equation to be subtracted 4° 20′, since the true place was 25° 10′, just as the Sun's true place at the middle of the second eclipse was ♎ 25° 10′; and that the anomaly was 64° 38′.

[Margin: Correction of Hipparchus's lunar motion, made by Ptolemy.]

IV. Having done this, Ptolemy, chapter 7, reasons as follows. At the moment of the second ancient eclipse full Moon, the Moon's mean place was ♍ 14° 44′ and its anomaly 12° 24′; at the moment of the second recent eclipse full Moon, its mean place was ♈ 29° 30′ and its anomaly 64° 38′. It is clear, then, that in the time between these two eclipses, whole circuits set aside, the Moon advanced by its mean motion in longitude 224° 46′, the distance from ♍ 14° 44′ to ♈ 29° 30′, but on the circle of anomaly 52° 14′, the distance from degree 12° 24′ of anomaly to degree 64° 38′. Now the time between the second of the ancient and the second of the recent eclipses, but equated and converted from apparent to equal, was 854 Egyptian years 73^{d} 23^{h} 20^{m}, or 311783^{d} 23^{h} 20^{m}. To this time, according to Hipparchus's tables compiled from the synodic month, by accumulating the daily motion, which in them is 13° 10′ 34″ 58‴ 53⁗ 30⁗′ 30⁗′′, there correspond 224° 46′, the whole zodiacs traversed by the Moon being set aside. He therefore concludes that the synodic month, and the Moon's mean motion in the zodiac and in elongation from the Sun, as found by Hipparchus and distributed by Ptolemy himself into tables, are correct. On the other hand, to the same time, according to Hipparchus's tables of the daily motion of the lunar anomaly, which was 13° 3′ 53″ 56‴ 29⁗ 38⁗′ 38⁗′′, there correspond, the whole circles of anomaly being discarded, 52° 31′; whereas, from what was demonstrated by the two trios of eclipses, only 52° 14′ should correspond. Hipparchus's anomalistic motion therefore exceeds by 17′ in 311783^{d} 23^{h} 20^{m}. In one day, then, it exceeds by 11⁗ 46⁗′ 39⁗′′. So the exact daily mean motion of anomaly is 13° 3′ 53″ 56‴ 17⁗ 51⁗′ 59⁗′′.

[Margin: Three other eclipses observed by Copernicus.]

V. Nicolaus Copernicus, for his part, Book 4, chapter 5, takes the three eclipses observed by Ptolemy that were discussed in number 3, and reduces their time to the meridian of Frombork and Kraków, which he judges to be one hour west of Alexandria. He then selects three other eclipses of the Moon observed by himself at Frombork. The *first* was in the year of Christ 1511,

with October 6 past, and its middle, or eclipse full Moon, was at $0^h\ 35^m$ after midnight, with the Sun in ♎ and the Moon in ♈ 22° 25′; and it was eclipsed totally. The *second* was in the year of Christ 1522, with 5 days of September elapsed, at $1^h\ 20^m$ after midnight, with the Sun in ♍ and the Moon in ♓ 22° 12′; and it was again eclipsed totally. The *third* was in the year 1523, with 25 days of August past, at $4^h\ 25^m$ after midnight, with the Sun in ♍ and the Moon in ♓ 11° 21′; and the Moon was eclipsed totally.

[Margin: How large the semidiameter of the Moon's epicycle is.]

From these, and from the mean motions of the luminaries, taken from tables made according to Ptolemy's motion of the Moon, he gathers that at the moment of the second of his eclipses the semidiameter of the epicycle was 8604 parts of which the semidiameter of the deferent carrying the epicycle is 100000; the Moon's mean place in ♓ 26° 50′; its subtractive equation 4° 38′, since the true place was in ♓ 22° 12′; its anomaly 74° 27′; and the Moon's mean elongation from the Sun 182° 51′. Copernicus's comparisons are, moreover, far better than Ptolemy's, because the Moon was in almost the same anomaly and the Sun in the same anomaly.

[Margin: Correction of Ptolemy's lunar motion, by Copernicus.]

VI. After this, Copernicus, chapter 6, continues as follows. In the second Ptolemaic eclipse, the mean distance of the Moon from the Sun was 182° 47′ and the anomaly 64° 38′; in the second of the recent eclipses, the Moon's mean elongation from the Sun was 182° 50′ and the anomaly 74° 27′. The time between these eclipses, reduced from apparent to equal, was 1388 Egyptian years $302^d\ 3^h\ 34^m$, or $506922^d\ 3^h\ 34^m$. To this time there correspond, according to Ptolemy's tables, beyond 17165 whole lunations, 358° 38′, and of anomaly, whole circles being discarded, 9° 11′. But in fact 17166 months and 0° 4′ were completed; and of anomaly, whole circles being discarded, 9° 49′ were completed. So Ptolemy's motion falls short on both counts in 506922 days: by 26′ as to the elongation from the Sun, and by 38′ as to the anomaly. Distributing these over $506922^d\ 3^h\ 34^m$, I find that to Ptolemy's daily motion of the Moon's mean elongation from the Sun, which is 12° 11′ 26″ 41‴ 20⁗ 17⁗′ 59⁗′′, there must be added 11⁗ 4⁗′ 43⁗′′, so that the daily elongation is 12° 11′ 26″ 41‴ 31⁗ 22⁗′ 42⁗′′; which, however, the Prutenic Tables put at 12° 11′ 26″ 41‴ 29⁗ 57⁗′ 50⁗′′. And to the daily mean motion of anomaly, which for Ptolemy is 13° 3′ 53″ 56‴ 17⁗ 51⁗′ 59⁗′′, there must be added 16⁗ 11⁗′ 30⁗′′, so that it is 13° 3′ 53″ 56‴ 34⁗ 3⁗′ 29⁗′′; which, however, the Prutenic Tables, by I know not what reckonings [Epilogiſmis], put at 13° 3′ 53″ 56‴ 23⁗ 57⁗′ 41⁗′′.

VII. It remains for us to give, by one example, the method by which Ptolemy investigated the semidiameter of the epicycle and the anomaly of the Moon by means of three lunar eclipses. For this we shall choose, from the 9 eclipses, the three in the middle, namely Ptolemy's own. In the first interval of these, as is clear from what was said in number 3, the Moon, having completed whole circles, traversed a further 161° 55′, the distance from ♏ 13° 15′ to ♈

25° 10′; in the second interval, after whole circles, it completed 139° 2′, the distance from ♈ 25° 10′ to ♍ 14° 12′. To the equated time of the former interval, which is $531^{d}\ 23^{h}\ 39^{m}$, there corresponds, from the earlier tables of Hipparchus and Ptolemy, after whole circles are discarded, a mean motion of the Moon in the zodiac of 169° 37′, and of anomaly 110° 21′. To the equated time of the latter interval, namely $502^{d}\ 5^{h}\ 30^{m}$, there corresponds a mean motion of the Moon in the zodiac of 137° 34′, and of anomaly 81° 36′, whole circles being discarded. Now compare the true places of the Moon, known from the true places of the Sun, with the mean ones, and you will see that to the anomaly of 110° 21′ there correspond 7° 42′, to be subtracted from the mean motion, but to the anomaly of 81° 36′ there correspond 1° 21′, to be added to the mean motion of the Moon in the zodiac. Keep these aside.

With these laid down, let ABC be the epicycle of the Moon, on which the first Ptolemaic eclipse is at point A, the second at B, the third at C, and let the Moon's passage in this order be understood as toward the preceding signs. Let arc AB be 110° 21′ and BC 81° 36′; for the complement to the circle, arc CEA, will be 168° 3′. To this is due an additive equation of 6° 21′, so that from this and the equation due to arc BC, which was 1° 21′, there comes a total additive equation of 7° 42′ due to arc BCA, as great as was the subtractive equation of arc AB in the former interval. Now since the apogee of the epicycle is not in BC or AEC, as these call for an additive prosthaphaeresis and are less than a semicircle, it must be in arc ALB (since in the lunar theory the body of the Moon is taken to move toward the preceding signs on the circumference of the epicycle, and so the equation of the first inequality is additive in the lower part of the epicycle but subtractive in the upper), and the perigee in arc AEB. So let D be the centre of the ecliptic, about which the centre K of the epicycle moves uniformly; from D let the straight lines DA, DB, DC be drawn to the three places of the eclipses, and let these be connected by the straight lines EB, EC and CB; and finally let EZ be drawn perpendicular to DB, EI perpendicular to DC, and CT perpendicular to EB.

For angle ADB, that is EDZ, will be 7° 42′, since it subtends the subtractive equation due to arc AB; and since the angle at Z is a right angle, the ratio of DE to EZ will be known in the right triangle EDZ.

Diagram. Geometric figure set into the left column, with the text running in a narrow measure beside it. A circle, the lunar epicycle, stands above a long narrow fan of straight lines that converge at the point D at the foot. One line rises from D through M and K to L at the top of the circle. Other lines rise from D to A (upper left on the circle), passing through E (lower left on the circle), and to B (upper right on the circle). A further line from D rises near C and I. Inside the circle, K is joined to A and to S (left on the circle) through N; E is joined to B through T and to C; T is joined to C; C is joined to B; a short line runs from E past I to Z, outside the circle at lower right. Labels read (13): L, A, B, S, N, K, T, E, M, C, I, Z, D. Labels not read: none. C is set small, above I.

- L (top of circle): apogee of the epicycle
- A (upper left on circle): place of the first eclipse
- B (upper right on circle): place of the second eclipse
- C (lower right on circle, small): place of the third eclipse
- E (lower left on circle): point where DA cuts the circle
- M (bottom of the circle, on DL): perigee of the epicycle
- K (centre of the circle): centre of the epicycle
- D (foot of the figure): centre of the ecliptic
- S (left on circle), N (on KS), T (on EB), I (on EZ), Z (outside the circle, lower right): construction points

So too in triangle BEZ, EBZ is known; for AEB, by proposition 20 of the third book of Euclid, is half the angle at the centre K subtending AB, of 110° 21′, and is therefore 55° 10½′; and AEB, by 32 of the first book, equals the two interior and opposite angles, namely angle EDB, which is 7° 42′; so EBD is 47° 28′ 30″. In triangle EZB, then, right-angled at Z, the ratio of BE to EZ will be known; but the ratio of DE to EZ was known, so the ratio of DE to EB will be known. Similarly, in triangle EDI, right-angled at I, because angle EDI is known, at 6° 21′, the ratio of DE to EI will be known. Now angle BEC is known, being, by 20 of the third book, half the angle at the centre K subtending arc CB, which was 81° 36′; so BEC is 40° 48′. Together with AEB this makes AEC 95° 58½′, to which, by 32 of the first book, EDC, of 6° 21′, and ECD are together equal; so ECD is 89° 37½′. In triangle CEI, therefore, right-angled at I, angle CEI is 6° 21′. So the ratio of EC to EI will become known. But the ratio of DE to EI was already known, so the ratio of DE to EC will be known. Again, in triangle ECT, right-angled at T, angle TEC is known, at 40° 48′, and so ECT is 49° 12′. So the ratio of ET and of TC to EC will be known, EC being already known in the parts in which DE was already known; and so the remaining TB, out of the whole EB now known, will become evident; and from TC and TB known, CB will be known. Now from arc CB, of 81° 36′, the chord CB is known in parts of which the radius of the epicycle is, for example, 100000; so in the same kind of parts the chord EC will be known. Hence arc EC will be known, and from this the whole arc ECB; and since ALB is known, the remaining arc AE will be known, which Ptolemy finds to be 95° 16′ 50″. From this it was clear that the centre K of the epicycle falls in the segment ABE; so, when the straight line DMKL is drawn, M comes out as the perigee and L as the apogee of the epicycle. Further, by means of the chord EC or BC, the ratio of DE to the chord EA will be known, and so the whole DEA in parts of the semidiameter of the epicycle, and DE in the same parts. But by Corollary 1 of 36 of the third book of Euclid, and 6 of the second, the rectangle formed from AD and DE, together with the square of the semidiameter of the epicycle, is equal to the square of KD; so the ratio of LK, the semidiameter of the epicycle, to KD, the semidiameter of the deferent carrying the epicycle, will be known. Ptolemy finds it to be 5 14′ in parts of which DK is 60.

VIII. As for the anomaly in the second eclipse, let KNS be drawn perpendicular to DEA, and the straight line AK. For since the ratio of DE to EA was known, and EA is bisected by KS, and so NE is half of AE, the ratio of ND to DK will be known; hence angle DKN will be known, and its arc MES, and so the whole of MSA, and therefore also AL, the remainder to the semicircle, which is the anomaly in the first eclipse. AB was already known, so arc LB will be known, namely the Moon's anomaly in the second eclipse, and LBC in the third. Ptolemy finds AL to be 45° 43′, LB 64° 38′ and LBC 146° 14′.

[Margin: Tycho's judgement on this technique of Ptolemy and Copernicus.]

IX. But Tycho, volume 1 of the *Progymnasmata*, page 114, affirms that eclipses derived from the Ptolemaic and Copernican motions sometimes depart from the truth by more than a whole hour, and that three eclipses are not enough to establish the Moon's motions; rather, more eclipses and many other phenomena of the Moon must be examined. Yet he brings out no examination of these. He only indicates the times of 21 eclipses of the Moon and 8 of the Sun observed by himself, and their magnitude; and then he adds the daily and annual motion of the Moon derived from them, keeping back the technique by which he investigated it. But that error of a whole hour can be attributed in part to the uncertain distance of the meridians and to the fallibility of observations; for before the use of the telescope, some wrongly took the penumbra and the spurious beginning of an eclipse for the shadow and the genuine beginning, and Tycho himself or the Tychonians erred in something similar, as Govaert Wendelen has shown in the eclipses listed below. In these, because of προσκοτίαν [Greek: proskotian, "adumbration"], or ἐπισκοτίαν [Greek: episkotian, "overshadowing"], that is, the sootiness and appendage of shade that usually hastens the beginning of lunar eclipses and delays their end, the Tychonians recorded the eclipse full Moons otherwise than a truer calculation requires.

Eclipsium Lunæ Tychonicarum Dissidium à Vendelini Calculo.

Disagreement of the Tychonic eclipses of the Moon with Wendelen's calculation.

Years of Christ	Month	Day	Apparent time recorded at Uraniborg by the Tychonians, hours p.m.	Time from Wendelen's calculation	Difference
1573	Dec.	8	$8^h\ 3^m$	$7^h\ 50^m$	13^m
1177	April	2	$8^h\ 50^m$	$8^h\ 25^m$	25^m
1577	Sept.	26	$13^h\ 3^m$	$13^h\ 17^m$	14^m
1580	Jan.	31	$10^h\ 10^m$	$10^h\ 30^m$	20^m
1581	Jan.	19	$9^h\ 57^m$	$10^h\ 6^m$	9^m
1581	July	15	$16^h\ 57^m$	$16^h\ 45^m$	12^m
1584	Nov.	17	$13^h\ 12^m$	$12^h\ 50^m$	22^m
1592	June	24	$10^h\ 16^m$	$10^h\ 31^m$	15^m
1592	Dec.	18	$7^h\ 41^m$	$7^h\ 21^m$	20^m
1595	April	23	$16^h\ 36^m$	$16^h\ 13^m$	23^m
1596	April	12	$9^h\ 29^m$	$8^h\ 46^m$	43^m

Moreover, the middle of the eclipse of the year 1581 July 15 Tycho puts in the *Epistles* at $13^h\ 6^m$, but in the Catalogue, volume 1 of the *Progymnasmata* at $13^h\ 12^m$.

[Margin: Tycho's sometimes doubtful reliability, and the error of the tables.]

And in the eclipse of the year 1596 April 12, Tycho, as Wendelen says, writes his own refutation. For he admits that the middle of the eclipse was not visible, and the middle seems to have been determined from the mere observed beginning and then from the Tychonic tables, rather than from observation. For that reason Pétau could find it from no tables in such a way as to come closer to the Tychonic determination. But Kepler too, in the *Ephemerides*, page 11, has this: *Simon Marius writes, in his Prognostication for the year 1617, that some notable defect remains in Tycho's lunar reckonings* [Lunaribus]*, not only in the time but also in the magnitude of eclipses. He proves it from the observation of the solar eclipse of 1614 October 3, which both occurred earlier and became larger. The same was observed by me at Linz, and the same at Venice and Munich.*

[Margin: Longomontanus's judgement on the technique of Ptolemy and Copernicus.]

X. Longomontanus, Book 1 of the *Theorica*, chapter 2, in the commentary on the Moon, and chapter 6, affirms that with Herculean labour he toiled alongside Tycho for a whole seven years on the theory of the ☾, and completed it in the year 1600, so that it holds good for all ages. He too, however, criticises Ptolemy and Copernicus, because in hunting down the eccentricity and the semidiameter of the epicycle, and from these the anomaly and the lunar motions, by means of trios of eclipses, they erred in connecting the motions of the Moon, the Sun and the fixed stars, and neglected the equant circle in the Moon. He brings forward three eclipses of the Assyrians, three of Hipparchus and three of Ptolemy, and then 23 eclipses of the Moon observed at Uraniborg or Copenhagen. But more than once he reports their observed time differently from Tycho, which leads Wendelen to be suspicious of his reliability. Computing their places from the mean equinox, using his own equations but omitting the equation of time that arises from the obliquity of the zodiac to the equator, he gathers the Moon's mean motions, but chiefly from Hipparchus's eclipse of the year 548 of Nabonassar. Finally, in chapter 6, he concludes from the examination of ancient and recent eclipses that the Prutenic Tables err by almost nothing sensible in the motion of longitude and of anomaly, and that the Prutenic Canon should therefore be used almost whole.

[Margin: Lansbergen's attempt.]

XI. Lansbergen, in the *Thesaurus observationum*, class 1 of the lunar observations, having examined 38 eclipses of the Moon, sees that their observations agree with his tables; and so he concludes that his tables hold good for all ages, and that the motions of the Moon derived from so many eclipses are correct. But in truth, in his eclipse 10, the middle of the eclipse at Alexandria, recorded by Ptolemy as $8^h\ 24^m$, comes out from Lansbergen's tables as $8^h\ 51^m$. But he thinks that Ptolemy's text is corrupt. And in eclipse 7 he relates from Ptolemy,

Book 4, chapter 9, that the middle of the eclipse observed at Babylon was one hour before midnight,

[Margin: The doubtful reliability of the same man.]

whereas Ptolemy says twice that it was observed at Babylon half an hour before midnight, and at Alexandria one hour and a third of an hour. And in eclipse 16 he puts the hour of the observation at $11^h\ 6^m$, whereas Regiomontanus puts it at $11^h\ 14^m$. Justly, then, Johannes Phocylides [Holwarda], in his πανσελήνω [Greek: panselēnō, "full Moon"], convicts him of doubtful reliability, because of another unreliable account similar to this one; on it see Boulliau, Book 3, chapter 7.

[Margin: Lansbergen's deceitful technique.]

But recognise the man's plainly artless technique, ἄτεχνον [Greek: atechnon, "without art"], and a cunning rather detected by us. Lansbergen saw that the observed times of seven eclipses disagreed with his tables; these are eclipses 18, 23, 25, 27, 35, 36 and 37 in his own order. So that this disagreement should not rebound into the fallibility either of the observations or of his tables, he devises a certain second equation in lunar eclipses, which rests on no theory at all and has no consistency. Not content with this, in the precepts, Precept 8, he makes this equation universal and decrees that it be applied to the mean time whenever eclipses have occurred in the same parts of the zodiac, as if that position by itself produced this variation. Here these words must be noted: *But the cause of the discrepancy is not any error in the equation of days, but comes from the Moon, and is still hidden. Until, therefore, it is brought to light, a second equation of time must be established from observations of eclipses. We have observed, moreover, that when either luminary is eclipsed at the beginning of Aries,* 30^m *must be added to the mean time*, etc. But if there were no deception in the observations or the tables, this equation could have been gathered from the agreement of other observations with Lansbergen's tables; so it should first have been demonstrated that there is no deception underlying either. Then why did he not use a similar equation in eclipses 7, 10 and 16, discussed above, but instead showed us either a corrupt text or his own good faith corrupted in reporting the time?

[Margin: Boulliau's attempt on this matter.]

XII. Ismaël Boulliau, Book 3 of the *Astronomia Philolaica*, from chapter 3 to chapter 7, investigates the synodic and the anomalistic month, partly by the method given in chapter 20 and partly by means of several eclipses arranged on a circle. He does not use the technique of Ptolemy or Copernicus described from number 1 to number 8, but a method of Viète's very like it, discussed under the eccentricity and apogee of the Sun, Book 3, chapter 24. He employs now 3, now 4 eclipses, and intervals of years now small, now large, but chiefly of 1788 Egyptian years $65^d\ 3^h\ 51^m$, between two eclipses recorded on the one side by Hipparchus and on the other by Tycho. Not resting on these, he adduces 19 ancient eclipses and 34 recent ones and, attending to the difference between their observed place and the place computed from tables, he uses a double supposition: one that assumes the motions of the Moon already investigated by

him in chapter 6, the other that subtracts 5′ from the Moon's longitude and 30′ from its anomaly. Yet he does not dare to attribute so much to his tables as to say that they hold good for all ages. He does suspect, however, that Ptolemy may have altered the times of the eclipses and drawn them to fit his hypotheses, or that the ancients sometimes recorded their times with too little accuracy.

[Margin: What Wendelen did on this matter!]

This perhaps was the reason why Wendelen used only recent eclipses in the *Idea of the Atlantic Tables*, since these were usually observed more accurately, at least after the use of the telescope; unless perhaps in the *Apotelesmata* of eclipses that he promises there he has also used ancient eclipses.

XIII. For my part, after a long suspension of judgement, arising from the suspicion of deception in observations of eclipses before the use of the telescope, and from the diversity of opinions on the difference of meridians, on the places of the fixed stars, and on the calculations by which the times of eclipses are derived, I have decided to bring all eclipses back to calculation according to the hypotheses of all the tables. This will be done, God willing, in volume 3 of the *Almagest*, so that from the comparison of these and of other phases of the Moon it may appear which are the best of all, and what more probable correction can be applied. Meanwhile, from the 86 eclipses that I shall list shortly, I have selected 40, that is, 20 pairs, which I have weighed by the method and technique given in chapter 21, and some also by the Ptolemaic and Copernican method. In these I applied the usual equation of time and the difference of meridians established in my Geographical Book, which I have judged should be set before the eclipses to be listed, assuming from the same book that the meridian of Bologna and of Uraniborg is the same, with no evidently sensible difference.

Differentia Temporis, qua loca infraſcripta ſunt Vraniburgo, & Bononia OR, vel OC. ideſt Orientaliora, vel Occidentaliora

Difference of time by which the places written below are OR or OC from Uraniborg and Bologna, that is, more eastern or more western.

Places where the eclipses were observed	Difference of time	Kind of difference
Alexandria Aegypti	$1^h\ 43^m\ 0^s$	Or.
Antiochia Syriæ	$2^h\ 38^m\ 40^s$	Or.
Aquæ Sextiæ	$0^h\ 29^m\ 0^s$	Oc.
Arachta	$2^h\ 23^m\ 0^s$	Or.
Babylon	$2^h\ 33^m\ 0^s$	Or.
Bethaſij	$0^h\ 28^m\ 35^s$	Oc.
Bruxellæ	$0^h\ 33^m\ 0^s$	Oc.
Cracouia	$0^h\ 33^m\ 40^s$	Or.
Dinia	$0^h\ 25^m\ 0^s$	Oc.
Fruemburgum	$0^h\ 32^m\ 0^s$	Or.
Hafnia	$0^h\ 0^m\ 48^s$	Oc.
Goeſa	$0^h\ 34^m\ 0^s$	Oc.
Leodium	$0^h\ 27^m\ 0^s$	Oc.
Leyda	$0^h\ 32^m\ 0^s$	Oc.

Places where the eclipses were observed	Difference of time	Kind of difference
Lugdunum	$0^h\ 30^m\ 16^s$	Oc.
Matritum	$1^h\ 10^m\ 0^s$	Oc.
Mellicum	$0^h\ 18^m\ 40^s$	Or.
Oenipons	$0^h\ 7^m\ 0^s$	Oc.
Patauium	$0^h\ 2^m\ 0^s$	Or.
Parifij	$0^h\ 44^m\ 0^s$	Oc.
Praga	$0^h\ 11^m\ 0^s$	Or.
Rho us	$1^h\ 35^m\ 0^s$	Or.
Roma	$0^h\ 12^m\ 0^s$	Or.
Tubinga	$0^h\ 14^m\ 0^s$	Oc.
Vienna Auftriæ	$0^h\ 22^m\ 0^s$	Or.
Viterbium	$0^h\ 10^m\ 0^s$	Or.

29

ECLIPSES 86. SELECTÆ ET REDVCTÆ AD TEMPVS APPARENS post Meridiem Bononiæ & Vraniburgi.

86 eclipses of the Moon, selected and reduced to apparent time after noon at Bologna and Uraniborg.

Order of the eclipses	Vertical label	Observers	Places of observation	Years elapsed: of Nabonassar	Years elapsed: before Christ	Egyptian month and day	Julian month and day	Time after noon	Sun's anomaly	Moon's equated anomaly	Digits eclipsed
1	*Chaldæi apud Ptolemęum*	*Ptolem.* lib.4. c.6.	Babylon.	27	721	Thoth 29	March 19	$6^h\ 57^m$	173°	66°	Total
2		*Ptolem.* lib.4. c.6.	Babylon.	28	720	Thoth 18	March 8	$9^h\ 27^m$	219°	13°	3 A. 0′
3		*Ptolem.* lib.4. c 6.	Babylon.	28	720	Phamenoth 15	Sept. 1	$5^h\ 57^m$	93°	160°	6 B. 0′
4		*Ptolem.* l.5. c.14.	Babylon.	127	621	Athyr 27	April 22		325°	341°	3 A. 0′
5		*Ptolem.* l.5. c.14.	Babylon.	2[?]5	523	Phamenoth 17	July 16		48°	29°	6 B. 0′
6		*Ptolem.* l.4. c.9.	Babylon.	246	502	Epiphi 28	Nov. 19	$9^h\ 3^m$	172°	4°	3 A. 0′

[29] place names in the table stand as printed. The identifications that follow are editorial and not on the page. Alexandria in Egypt; Antioch in Syria; Aix; Arachta and Bethafij, not identified here; Brussels; Cracow; Digne; Frombork; Copenhagen; Goes; Liège; Leiden; Lyon; Madrid; Melk; Innsbruck; Padua; Paris; Prague; Rhodes, the page setting a gap in the word; Rome; Tübingen; Vienna in Austria; Viterbo.

Order of the eclipses	Vertical label	Observers	Places of observation	Years elapsed: of Nabonassar	Years elapsed: before Christ	Egyptian month and day	Julian month and day	Time after noon	Sun's anomaly	Moon's equated anomaly	Digits eclipsed
7		*Ptolem.* l.4. c.9.	Babylon.	257	491	Tybi 3	April 25	$8^h\ 37^m$	237°	102°	2 A. 0′
8		*Ptolem.* l.4. c.11.	Babylon.	366	383	Thoth 26	Dec. 22	$16^h\ 47^m$	204°	230°	0 30′
9		*Ptolem.* l.4. c.11.	Babylon.	366	382	Phamenoth 24	June 18	$6^h\ 31^m$	333°	29°	☩ ☩
10		*Ptolem.* l.4. c.11.	Babylon.	367	382	Thoth 16	Dec. 12	$8^h\ 27^m$	256°	182°	Total
11	*Hipparch. alÿue*	*Ptolem.* l.4. c.11.	Alexandria	547	201	Mesorì 16	Sept. 22	$5^h\ 17^m$	110°	300°	7 A. 0′
12		*Ptolem.* l.4. c.11.	Alexandria	548	200	Mechir 9	March 19	$11^h\ 37^m$	288°	110°	Total
13		*Ptolem.* l.4. c.11.	Alexandria	548	200	Mesorì 5	Sept. 12	$12^h\ 32^m$	90°	250°	Total
14		*Ptolem.* l.6. c.5.	Alexandria	574	174	Phamenoth 27	April 30	$12^h\ 37^m$	331°	164°	7 B. 0′
15		*Ptolem.* l.6. c.5.	Rhodus	607	141	Tybi 2	Jan. 27	$8^h\ 20^m$	239°	180°	3 A. 0′
				Nabon.	*Christi*						
16		*Ptolemæus* l.4. c.9.	Alexandria	872	125	Pachon 17	April 5	$6^h\ 41^m$	306°	252°	2 A. 0′
17		*Ptolemæus* l.4. c.6.	Alexandria	880	133	Payni 20	May 6	$9^h\ 32^m$	337°	316°	Total
18		*Ptolemæus* l.4. c.6.	Alexandria	882	134	Choeac 2	Oct. 20	$9^h\ 17^m$	81°	65°	9 B. 0′
19		*Ptolemæus* l.4. c.6	Alexandria	883	136	Pharmuti 19	March 5	$14^h\ 17^m$	276°	147°	6 B. 0′
20		*Albategnius* c.30.	Arachta	1631	883		July 23	$5^h\ 37^m$	49°	115°	11 B. 0′
21		*Albategnius* c.30.	Antiochia	1649	901		Aug. 2	$13^h\ 12^m$	59°	112°	nearly total [T. ferè]

Order	Observers	Places of observation	Years of Christ	Month	Day	Time after noon	Sun's anomaly	Moon's anomaly	Digits eclipsed
22	*Purbach. & Regiomont.*	Mellicum	1457	Sept.	3	$10^h\ 48^m$	80°	192°	Total
23	*Regiomontanus* in obſ.	Vienna Auſtr.	1460	July	3	$8^h\ 13^m$	17°	103°	4 14′
24	*Regiomont.* in obſeruat.	Viterbium	1462	June	11	$14^h\ 38^m$	355°	273°	7 B. 0′
25	*Regiomontanus* ibidem	Vienna	1461	June	22	$13^h\ 0^m$	335°	39°	Total
26	*Regiomontanus* ibidem	Patauium	1464	April	12	$12^h\ 57^m$	296°	113°	Total
27	*Copernicus* l.4. c.14.	Roma	1500	Nov.	5	$13^h\ 48^m$	141°	295°	Total
28	*Copernicus* l.4. c.13.	Cracouia	1509	June	2	$11^h\ 14^m$	347°	162°	8 A. 0′
29	*Copernicus* l.4. c.5.	Fruemburgū	1511	Oct.	6	$12^h\ 3^m$	110°	185°	Total
30	*Copernicus* l.4. c.5.	Fruemburg.	1522	Sept.	5	$12^h\ 48^m$	80°	77°	Total
31	*Copernicus* l.4. c.5.	Fruemburg.	1523	Aug.	25	$15^h\ 53^m$	69°	23°	Total
32	*Tycho* t.1. Progymn.	Vraniburgum	1573	Dec.	8	$8^h\ 3^m$	141°	234°	Total
33	*Tycho* (*Pagina 114.* set vertically beside rows 33–37)	Vraniburgum	1576	Oct.	7	$11^h\ 32^m$	110°	66°	☩
34	*Tycho*	Vraniburgum	1577	April	2	$8^h\ 50^m$	295°	217°	☩
35	*Tycho*	Vraniburgum	1577	Sept.	26	$13^h\ 3^m$	100°	12°	Total
36	*Tycho*	Vraniburgum	1578	Sept.	15	$13^h\ 17^m$	61°	317°	2 30′
37	*Tycho*	Vraniburgum	1580	Jan.	31	$10^h\ 10^m$	224°	148°	Total
38	*Tycho* ibidem	Vraniburgum	1581	Jan.	19	$9^h\ 57^m$	210°	351°	Total
39	*Tycho* ibidem	Vraniburgum	1581	July	15	$16^h\ 57^m$	25°	148°	Total
40	*Tycho* ibidem	Vraniburgum	1584	Nov.	17	$13^h\ 12^m$	138°	128°	Total
41	*Tycho* ibidem	Vraniburgum	1587	Sept.	16	$9^h\ 16^m$	76°	301°	☩
42	*Tycho* ibidem	Vraniburgum	1588	March	12	$15^h\ 2^m$	251°	110°	Total
43	*Mæſtinus, Keplerus*	Tubinga	1590	July	16	$16^h\ 42^m$	15°	136°	2 50′
44	*Tycho* ſupra	Vraniburgum	1591	Jan.	9	$6^h\ 55^m$	189°	279°	☩ ☩
45	*Tycho* ſupra	Vraniburgum	1592	June	24	$10^h\ 16^m$	353°	35°	8 0′
46	*Tycho, Lansbergius*	Vraniburgum	1592	Dec.	18	$7^h\ 41^m$	168°	185°	5 0′
47	*Lansbergius* theſaur.	Goëſa	1594	Oct.	28	$19^h\ 26^m$	118°	64°	9 B. 30′
48	*Tycho* ſupra	Vraniburgum	1595	April	23	$16^h\ 36^m$	292°	213°	Total

Order	Observers	Places of observation	Years of Christ	Month	Day	Time after noon	Sun's anomaly	Moon's anomaly	Digits eclipsed
49	*Tycho* ſupra	Vraniburgum	1595	Oct.	27	$8^h\ 36^m$	110°	8°	☩ ☩
50	*Tycho* ſupra	Vraniburgum	1596	April	12	$9^h\ 29^m$	281°	168°	☩ ☩
51	*Keplerus,* Aſtron. Optica	Praga	1598	Feb.	20	$18^h\ 26^m$	231°	44°	11 30′
52	*Tycho* ſupra	Vraniburgum	1598	Aug.	16	$7^h\ 37^m$	45°	190°	Total
53	*Longomontanus*	Vraniburgum	1599	Feb.	9	$17^h\ 56^m$	215°	348°	Total
54	*Lansbergius*	Goëſa	1601	Dec.	9	$6^h\ 46^m$	159°	172°	10 37′
55	*Keplerus,* finem	Praga	1602	June	4	☩ ☩	☩	☩	☩ ☩
56	*Keplerus, Lansbergius*	Praga	1603	May	24	$12^h\ 7^m$	322°	270°	7 30′
57	*Keplerus, Lansbergius*	Praga	1603	Nov.	18	$7^h\ 8^m$	137°	77°	3 0′

[Table continues.][30]

Order of the eclipses	Observers	Places of observation	Years of Christ	Month	Day	Time after noon	Sun's anomaly	Moon's anomaly	Digits eclipsed
58	*Lazsbergius* [recte: Lansbergius]	Goëſa	1605	April	3	$9^h\ 9^m$	273°	156°	11 B. 30′
59	*Lansbergius*	Goëſa	1605	Sept.	26	$17^h\ 4^m$	86°	295°	8 A. 30′
60	*Lansbergius*	Goëſa	1609	Jan.	19	$15^h\ 14^m$	199°	277°	9 B. 30′
61	*Longomontanus*	Hafnia	1609	July	16	$12^h\ 11^m$	15°	83°	Total
62	*Longomont. Vendelinus*	Hafnia	1612	May	14	$10^h\ 23^m$	313°	256°	6 30′
63	*Longomontanus*	Hafnia	1613	Oct.	28	$3^h\ 57^m$	117°	4°	☩ ☩
64	*Griembergerus*	Roma	1616	Aug.	26	$15^h\ 8^m$	55°	187°	Total
65	*Vendelinus*	Machlinia	1619	Dec.	20	$15^h\ 30^m$	169°	170°	11 0′
66	*Gaſſendus, Galterius*	Aquæ Sextiæ	1620	June	14	$13^h\ 42^m$	343°	320°	Total
67	*Vendelinus*	Bethaſij	1620	Dec.	9	$18^h\ 10^m$	158°	124°	Total
68	*Keplerus, Gaſſendus*	Aquæ Sextiæ	1621	Nov.	28	$15^h\ 22^m$	147°	74°	4 0′
69	*Gaſſendus, Hortenſius*	Leyda	1623	April	14	$11^h\ 5^m$	282°	154°	11 0′

[30] observers and places stand as printed; the identifications that follow are editorial and not on the page. l. and lib. mean book, c. chapter, t. volume; ibidem "in the same place", ſupra "above". *Albategnius* is al-Battānī; *Purbach.* Georg Peuerbach; *Mæſtlinus* Michael Mästlin; *Lansbergius* Philips van Lansbergen. Mellicum is Melk; Vienna Auſtr., Vienna in Austria; Viterbium, Viterbo; Patauium, Padua; Roma, Rome; Cracouia, Cracow; Fruemburgū and Fruemburg., Frombork; Vraniburgum, Uraniborg; Tubinga, Tübingen; Goëſa, Goes; Praga, Prague; Rhodus, Rhodes; Antiochia, Antioch; Arachta is not identified here. The vertical label *Chaldæi apud Ptolemęum* means "Chaldaeans in Ptolemy". The vertical label beside rows 11–15 prints *Hipparch. alÿue* [recte: alijque?], "Hipparchus and others".

Order of the eclipses	Observers	Places of observation	Years of Christ	Month	Day	Time after noon	Sun's anomaly	Moon's anomaly	Digits eclipsed
70	*Michaël Flor. Langren.*	Bruxellæ	1624	Sept.	26	$8^h\ 43^m$	85°	243°	Total
71	*Gaſſendus, Mydorgius*	Pariſij	1625	March	23	$14^h\ 0^m$	266°	54°	1 0′
72	*PP. Soc. IESV* initium	Matritum	1627	July	27	✝ ✝	✝	✝	✝ ✝
73	*Gaſſendus*	Aquæ Sextiæ	1628	Jan.	20	$10^h\ 6^m$	204°	231°	Total
74	*Langrenus*	Bruxellæ	1630	Nov.	19	$11^h\ 30^m$	143°	53°	8 30′
75	*Hortenſius*	Leyda	1631	Nov.	8	$11^h\ 59^m$	132°	2°	Total
76	*P. Auxon* emerſionem	Lugdunum	1631	May	15	✝ ✝	✝	✝	Total
77	*P. Andreas Arzet*	Oenipons	1632	Oct.	27	$11^h\ 55^m$	121°	312°	5 22′
78	*Gaſſendus, Langrenus*	Dinia	1634	March	14	$9^h\ 36^m$	256°	38°	10 15′
79	*Gaſſendus, Langrenus*	Aquæ Sextiæ	1635	March	3	$9^h\ 13^m$	245°	346°	Total
80	*Gaſſendus, Bullialdus*	Pariſij	1635	Aug.	27	$15^h\ 36^m$	60°	136°	Total
81	*Gaſſendus*	Dinia	1636	Feb.	20	$11^h\ 5^m$	234°	293°	3 0′
82	*Vendelinus, Langrenus*	Bruxellæ	1638	Dec.	20	$14^h\ 39^m$	174°	118°	Total
83	*Nos,* & plurimi	Bononia	1642	April	14	$14^h\ 44^m$	285°	105°	Total
84	*Nos* & plures	Bononia	1642	Oct.	7	$16^h\ 46^m$	99°	242°	Total
85	*Nos* & plures	Bononia	1643	Sept.	27	$7^h\ 31^m$	89°	195°	6 B. 0′
86	*Gaſſendus, Bullialdus*	Pariſij	1645	Feb.	10	$7^h\ 50^m$	223°	275°	10 0′

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Among the eclipses already mentioned, the pairs [bigæ] set out below seemed to us the best and the principal ones for investigating the motion of longitude and of anomaly, because of their outstanding distance apart and the nearer equality of the Moon's anomaly and, for the most part, of the Sun's. God willing, we shall give an examination of these and of the rest according to the tables of all the astronomers in the third volume of the *Almagestum Novum*,

[31] observers and places stand as printed; the identifications that follow are editorial and not on the page. *Griembergerus* is Christoph Grienberger; *Vendelinus* Govaert Wendelen; *Gaſſendus* Pierre Gassendi; *Galterius* Joseph Gaultier de la Valette; *Hortenſius* Martinus Hortensius; *Michaël Flor. Langren.* and *Langrenus* Michael Florent van Langren; *Bullialdus* Ismaël Boulliau; *Mydorgius*, *P. Auxon* and *P. Andreas Arzet* are not identified here. *PP. Soc. IESV* means "the Fathers of the Society of Jesus", initium "the beginning", emerſionem "the emersion", *Nos, & plurimi* "we and very many others", *Nos & plures* "we and several others". Hafnia is Copenhagen; Machlinia, Mechelen; Aquæ Sextiæ, Aix; Bethaſij, not identified here; Leyda, Leiden; Bruxellæ, Brussels; Pariſij, Paris; Matritum, Madrid; Lugdunum, Lyon; Oenipons, Innsbruck; Dinia, Digne; Bononia, Bologna.

although we have already weighed very many of them in this way, so that the Moon's motions are almost established for us.

Further, from all of these, but chiefly from the first, I set the mean synodic month at $29^{d}\ 12^{h}\ 44^{m}\ 3^{s}\ 10'''$. This is almost midway between Tycho's on the one hand and Lansbergen's and Wendelen's on the other, and comes very close to the Prutenic and the Rudolphine.

[Margin: Mean synodic month.]

The anomalistic month, on the other hand, I set at $27^{d}\ 13^{h}\ 18^{m}\ 34^{s}\ 30'''$. For, having chiefly consulted three eclipses observed by us in the years 1642 and 1643, I found that at the equated time of the eclipse of 1643 September 27, which at Bologna was $7^{h}\ 29^{m}\ 25^{s}$ after noon, the Moon's anomaly was not 194° 51′, as I had gathered from the tables I had adopted, but 195° 10′. For this reason I judged that the motions of anomaly should be corrected from here.

[Margin: Mean anomalistic month.]

Order of the eclipses	Anomaly in degrees: ☉	Anomaly in degrees: ☾	Interval of years
8	204°	230°	
73	204°	231°	2009
12	288°	105°	
83	288°	110°	1842
1	295°	66°	
68	151°	67°	2342
3	93°	160°	
85	89°	195°	2362
5	48°	29°	
31	69°	22°	2045
6	172°	4°	
49	199°	7°	2096
11	110°	300°	
77	121°	311°	1833
13	90°	250°	
84	99°	242°	1842
16	306°	252°	
62	312°	256°	1487
13	90°	250°	
70	85°	242°	1824

Order of the eclipses	Anomaly in degrees: ☉	Anomaly in degrees: ☾	Interval of years
17	336°	316°	
66	343°	320°	1487
3	96°	163°	
28	346°	162°	2228
2	219°	12°	
49	199°	7°	2314
1	173°	66°	

Order of the eclipses	Anomaly in degrees: ☉	Anomaly in degrees: ☾	Interval of years
47	118°	64°	2314
4	325°	341°	
79	245°	346°	2255
7	327°	102°	
83	285°	105°	2132
14	331°	164°	
28	346°	162°	1682
15	239°	180°	
46	168°	184°	1732
18	81°	147°	
47	118°	149°	1460
20	48°	115°	
82	173°	116°	754

CAPVT XXII.

De Inueſtigando Motu Latitudinis Lunæ ac Nodorum.

Chapter 22. On investigating the motion of the Moon's latitude and of the nodes.

[Margin: Hipparchus's opinion on the restitution of the Moon's latitude.]

I. Hipparchus judged that the restitution [apocataſtaſis] of the Moon's latitude is completed only in 5458 synodic months, or 441 Egyptian years $213^{d}\ 23^{h}\ 23^{m}$, and that 5923 draconic orbits [orbes Dracontici] are then completed. Therefore, if any other number of lunations is multiplied by 5923 and the sum divided by 5458, one will have the number of whole draconic orbits, or revolutions of latitude, though not of the remainder of the motion of latitude accomplished beyond them. But Govaert Wendelen, in the *Idea of the Atlantic Tables*, affirms that 7477 draconic orbits are completed in 6890 synodic months. In this he follows Hipparchus's ratio as regards whole circuits: for as 5458 is to 5923, so is 6890 to 7477. But since Hipparchus relied on proportions that were not very exact, as Ptolemy teaches in Book 4, at the end of chapter 2 and the beginning of chapter 9, let us rather listen to Ptolemy.

[Margin: Ptolemy's method for the Moon's latitude, and his opinion.]

II. Accordingly Ptolemy, in Book 4, chapter 2, lays down that two lunar eclipses are to be chosen in each of which the Moon had either no latitude, while nonetheless standing at the same node, or an equal latitude of the same kind, that is, towards the same quarter of the world; that, if possible, she was ascending in both or descending in both, and at the same degree of anomaly, or at least with the luminaries at the same distance from the Earth; and that the size of the eclipsed part lay towards the same quarter. Accordingly, in chapter 9, he chose two eclipses. The first was observed at Babylon in the 31st year of Darius I, the 3rd day of Tybi having elapsed, half an hour before midnight, but at Alexandria at hour 1⅓, when 2 digits were eclipsed from the south: namely in the year 491 before Christ, April 25, the Sun's anomaly being 237° and the Moon's 102° or 100⅓°. The second was observed by Ptolemy himself

at Alexandria in the 9th year of Hadrian, the 17th day of Pachon having passed, $3^h\ 36^m$ before midnight at Alexandria, and again 2 digits were eclipsed from the south: namely in the year of Christ 125, April 5, when the Sun's anomaly was 306° but the Moon's 252°. So the Moon stood 102° from apogee in the first and 108° in the second, and thus at very nearly equal distance from the Earth and near the same node, although Ptolemy puts the distance in the first at 100° 19′. Therefore, according to the anomaly of the earlier eclipse, the Moon's true motion was less than its mean motion by 5°; according to the anomaly of the second, the true motion was greater than the mean by 4° 53′. Accordingly, in the elapsed time, which was 615 Egyptian years $133^d\ 21^h\ 50^m$, the Moon's true motion in latitude completed whole circles, but the mean motion fell short of completing the last revolution by 9° 53′. According to Hipparchus's mean motion of anomaly, however, it would fall short by 10° 2′, that is, with an excess of 9′. So, dividing 9′ by the elapsed days, which were 224609, gives 8‴ 39⁗ 18⁗⁗. Added to Hipparchus's mean daily motion of latitude, which was 13° 13′ 45″ 39‴ 40‴ 17⁗ 19⁗⁗, these make the exact motion 13° 13′ 45″ 39‴ 48‴ 56⁗ 37⁗⁗.

[Margin: Copernicus's method and opinion.]

III. Copernicus, in Book 4, chapter 13, having said that several circumstances are required for defining the motion of the Moon's latitude, namely two lunar eclipses alike and equal in every respect, and that it is therefore harder to find, and that two eclipses agreeing in these conditions are rarely found, adds: *To us, certainly, they have not occurred at present.* He is therefore forced to use another method, namely two eclipses that took place around opposite nodes but were alike as regards the other conditions. The first, as it stands in Claudius Ptolemy, Book 6, chapter 5, was in the 7th year of Ptolemy Philometor, which was year 150 of Alexander the Great, the 27th day of Phamenoth having passed: namely in the year 174 before Christ, April 30, at $14^h\ 20^m$ after noon at Alexandria, but at Kraków, as Copernicus judges, at $13^h\ 20^m$. At that time at most 7 digits were eclipsed from the north, around the descending node. The second was observed by Copernicus himself under the meridian of Kraków in the year of Christ 1509, June 2, at $11^h\ 36^m$ after noon, and very nearly 8 digits were eclipsed from the south, around the ascending node. Using equated time, Copernicus computes the Moon's mean anomaly in the first eclipse as 163° 33′ and the equation as 1° 23′, by which the Moon's true place was less than its mean place; in the second eclipse, the mean anomaly as 159° 55′ and the equation as 1° 44′, *by which the equal motion was less than the apparent*. Those are Copernicus's words, but the good old man certainly erred: for if the anomaly was 159° 55′, the equation is to be subtracted from the mean motion, and the mean is greater than the true. The Sun, moreover, stood 29° from apogee in the first and 13° in the second. And since the second eclipse was greater by one digit, that is by 2′ 30″, he concludes that the Moon in the second [secunda; recte: prima?] eclipse was farther from the north node by about 30′ than she was from the

south node in the second eclipse, and so that the true motion of latitude, after the completed revolutions, was 179° 30′. The difference of the equations was 21′, so the mean motion of latitude after whole orbits was 179° 51′. The time between the eclipses already mentioned was 1683 years 88^{d} 22^{h} 35^{m}, in which, after 2577 completed revolutions of latitude, 179° 51′ remain. This agrees with the numbers Copernicus set down towards the end of chapter 4, where he had said that the mean motion of latitude according to Hipparchus is, annually, after 13 whole revolutions, 148° 42′ 46″ 20‴ 3⁗, and daily 13° 13′ 45″ 39‴ 40⁗; but that the annual exceeds by 1″ 2‴ 42⁗, and so is 148° 42′ 45″ 17‴ 21⁗, although the Prutenic Tables have it somewhat differently.

[Margin: Longomontanus's opinion.]

IV. By what reasoning the Tychonians emended the Prutenic Tables at this point, however, is not yet clear. For Longomontanus, in Book 1 of the *Theorica*, chapter 6, speaks very obscurely on this matter, in these words: *We use the Prutenic Canon in full for the Moon's longitude and anomaly. As regards the argument, or motion, of latitude, I indeed see, from the comparison of the second of the three most ancient eclipses, which occurred at Babylon in the year 720 before Christ, as also from other particular ancient ones, that something must here be taken away from the Prutenic Tables, about 7′, so that that most ancient eclipse which I have just named may agree closely in magnitude with the one of Lord Tycho made in the year 1578 after Christ, on 15 September, provided that the size of the Earth's shadow and of the Moon in the following ones is duly proportioned. But in Tycho's earlier restitution, which was carried out at Prague, I remember that 12′ were added to the Prutenic motion of latitude. Nor can anyone doubt that everything concerning the Moon's latitude was then in sufficiently good order, since we very frequently tested it with new observations even outside eclipses: both for the variation of latitude found around the quadratures, and in investigating parallax from the Moon's meridian altitude in each Tropic, and finally in the sextile and trine aspect with the Sun, where the variation of the motion of the nodes is found greatest on either side. For this reason, if from the year before Christ, as I said, 720, up to the completed year 1584 after the birth of Christ, to which the earlier particular restitution of the Moon refers, I distribute the said combined emendation of the mean motion of latitude, that is 19′, over the 2305 equal years between, then 29‴ 39⁗ fall to each common year in correcting the Prutenic Tables.* I, however, doubt how 7′ to be taken away from the Prutenic motion of latitude and 12′ to be added can make a combined emendation of 19′. Nor does the comparison of those eclipses seem well suited to establishing the motion of latitude: for in the earlier, of the year 720 before Christ, March 8, the Sun's anomaly was 162°, the Moon's 12°, and 3 digits were eclipsed from the south; whereas in the later, of 1578 September 15, the Sun's anomaly was 61°, the Moon's 317°, and the digits 2½.

[Margin: What Tycho and Kepler hold on the motion of the nodes and of latitude.]

V. But not even Tycho himself, in volume 1 of the *Progymnasmata*, page 127, makes clear how he found the mean motion of latitude and of the nodes to vary

insensibly in the syzygies [Copulis] and quadratures [Quadris], yet to grow to as much as 1° 45′ in the intermediate places. He must be believed until his observations are published and we weigh our own, set up for this purpose, in volume 3 of the *Almagest*. Meanwhile Kepler, in the *Ephemerides*, pages 7 and 8, admits that he has kept the retrograde motion of the nodes uniform, with the ancients, lest with this new little observation he should fix a new cross for the astrologers, who hold that great power resides in these nodes. Thanks must nevertheless be given to Origanus, because in his *Ephemerides* this Tychonic libration of the nodes, by which their mean mo- tion is made anomalous, he has put to use. But neither of them has disclosed to us the foundations on which he defined the motion of latitude in his Tables, just as the Alphonsines did not in former times, nor Wendelen later.

[Margin: Boulliau's method and opinion.]

VI. Boulliau, for his part, in his *Astronomia Philolaica*, Book 3, having noted in chapter 2, from Ptolemy, that the revolutions of latitude do not return with the months unless there comes about an [Greek: ἀνταπόδοσις (antapodosis, "correspondence", "return in kind")] of eclipses — that is, unless a likeness of eclipses occurs from the first interval to the later one, or unless the eclipses are equal in quantity of obscuration and in duration, and have the same latitude, and of the same kind — later, in chapter 8, first uses two eclipses. One is of 1462 June 11, observed by Regiomontanus at Viterbo, of 7 digits, near the south node, with lunar anomaly 273°. The other was observed by Tycho in 1592 June 24, of 8 digits, near the south node, with lunar anomaly 35°. From these he gathers 7 revolutions of the nodes, lacking 4° 57′ 11″, in an interval of 130 Egyptian years $35^{d}\ 19^{h}\ 36^{m}$. Next he chooses two others with a greater interval. The *first* is the very one Longomontanus used, namely the second of the Babylonian eclipses in Ptolemy, Book 4, chapter 6. It was in 720 before Christ March 8, at 12^{h} p.m. at Babylon, or as Ptolemy has it [Greek: κατ' ἀυτὸ τὸ μεσονύκτιον (kat' auto to mesonyktion, "at midnight itself")], but at Uraniborg $9^{h}\ 58^{m}$, as Boulliau has it; of 3 digits from the south, when the Sun's anomaly was 219°, the Moon's 13°, and the distance from the north node 9° 30′ 34″. The *second* was in 1634 March 14, at $9^{h}\ 20^{m}$ at Uraniborg, of 11 digits from the south, when the Sun's anomaly was 257°, the Moon's 38°, and the distance from the north node 5° 30′ 48″. Gassendi, however, who observed this eclipse at Digne, puts it at 10¼ digits. Having related these things, Boulliau divides the 2354 Egyptian years $218^{d}\ 23^{h}\ 22^{m}$ of the interval between these eclipses by the interval of the two preceding ones, namely by 130 years $35^{d}\ 19^{h}\ 36^{m}$. From this he gathers 18 periods consisting of seven revolutions of the nodes each, that is 126 revolutions, less some degrees, and in addition 12 years 23 days $2^{h}\ 16^{m}\ 30^{s}$. And he at once concludes thus: *There are therefore 126 zodiacs 151° 36′ 31″, or 12 second sexagesimals and 38 first sexagesimals of degrees, together with 31° 36′ 31″. But of days there are 3 third sexagesimals, 58 second and 43 first, together with 48^{d} 58′ 25″. Therefore, dividing the sexagesimals of degrees by the sexagesimals of days, we*

shall gather the daily motion of the nodes toward the preceding signs as 0° 3′ 10″ 38‴ 25⁗ 3⁗′ 12⁗′′ 7⁗′′′ 26⁗′′′′ 35⁗′′′′′. And adding the Moon's daily motion from the equinox, we shall have the Moon's daily motion from the node as 13° 13′ 45″ 39‴ 38⁗ 5⁗′ 39⁗′′ 35⁗′′′ 17⁗′′′′ 3⁗′′′′′, and the year as 4ˢ 28° 42′ 46″ 6‴ 44⁗ 25⁗′ 49⁗′′ 38⁗′′′ 45⁗′′′′ 15⁗′′′′′. But neither was the comparison of these eclipses suitable enough, because of the inequality of the anomalies and of the digits. Granted that from this he has fished out rather the motion of the nodes, and from that the motion of latitude, and that his effort deserves praise. And from this one may recognise that 7 revolutions of the nodes toward the preceding signs are completed in about 47486 days, but in 859429 days, 126 revolutions and about 152° besides.

[Margin: Our method and opinion.]

VII. We, however, by the singular favour of God, Best and Greatest, have observed one eclipse entirely similar to a very ancient eclipse, and one to be preferred, for establishing the motion of latitude, to all those observed up to now, at least those that survive. The *first* of the ancient eclipses, then, in Ptolemy, Book 4, chapter 6, is the third observed at Babylon by the Chaldaeans in year 2 of Mardocempadus and 28 of Nabonassar, when day 15 of Phamenoth had been completed, or in the year 720 before Christ, when September 1 had been completed. The Moon began to be eclipsed after its rising, and half of the lunar disc was eclipsed from the north. So Ptolemy gathers from the duration of the eclipse that the middle of the eclipse was at Alexandria at $7^h\ 40^m$ after noon, granted that Longomontanus computes $7^h\ 50^m$ from the Prutenic Tables. It was therefore, according to what is said in my *Geographical Book*, at $5^h\ 57^m$ at Bologna, but in equated time $6^h\ 5^m$. For the Sun's true place was ♍ 0° 26′ and its mean place ♍ 2° 24′, because its anomaly was 93° 10′; the Moon's mean place was ♓ 2° 7′ and its simple anomaly 158° 20′. So the mean full Moon followed the true one by 34^m of time, and was at Bologna at $6^h\ 39^m$.

The *second* eclipse, observed by us and by many others, was at Bologna in the year of Christ 1643, September 27, but 17 in the old style, at $7^h\ 30^m\ 52^s$ at the height of the eclipse, which was likewise of 6 digits to the north; but the equated time was $7^h\ 29^m\ 25^s$. For the Sun's anomaly was 89° 5′ 40″, its true place in ♎ 4° 29′ 34″, and its mean place 6° 27′ 52″. The Moon's simple anomaly was 195° 10′ and its mean place ♈ 3° 9′ 4″. Therefore the mean full Moon followed the true one by $6^h\ 2^m\ 6^s$, and was at Bologna at $13^h\ 31^m\ 31^s$.

[Margin: Observers of this eclipse.]

This same eclipse of 6 digits was also observed by Wendelen at Antwerp, by Boulliau at Paris, by Gassendi at Succij [?], by Hevelius at Danzig, by the most illustrious D. Giulio Giustiniani at Venice, by D. Vicenç Mut in Majorca, and by Father Io. Bapt. Zuppo [Giovanni Battista Zupi] at Naples, as I shall relate in the History of Eclipses. In both eclipses, then, half of the lunar disc was eclipsed to the north near the south node; the estimate of 6 digits is by far the most certain of all partial eclipses, as Kepler too notes in the *Optics*; the

Sun was at the same degree of anomaly; and the interval is the greatest. This comparison therefore surpasses all others. And although in the second eclipse the Moon was a very little higher, and therefore a little nearer the south node, than in the earlier eclipse, it was nevertheless at very nearly the same distance from the Earth, since it was 160° from the apogee in the first and 165° in the second.

The interval of time between the mean full Moons was 2362 Julian years 16^{d} 6^{h} 42^{m} 31^{s}, or 862736^{d} 6^{h} 42^{m} 31^{s}. In this time, by Precept 5 of chapter 21, there came about 29215 equal lunations or syzygies. Therefore, according to the same precept, when 29215 is multiplied by 5923 and the product divided by 5458, there result 31704 whole draconic circuits [Orbes Dracontici], or revolutions of latitude. Dividing the interval between the eclipses, or 862736^{d} 6^{h} 42^{m} 31^{s}, by these, I find that one mean month of latitude comes to 27^{d} 5^{h} 5^{m} 36^{s} $0'''$ $55''''$. Nor is any other elaborate labour [periergia] needed, since we have obtained two such eclipses, alike in every respect, of the kind astronomers once wished for in vain, God reserving them for us, or us for them.

CAPVT XXIII.

De modo colligendi reliquos Lunæ Motus, ex Menſtruis ipſius reuolutionibus: cum Synopſi Opinionum de Lunæ Motibus.

Chapter 23. On the way of deriving the Moon's remaining motions from its monthly revolutions; with a synopsis of opinions on the Moon's motions.

[Margin: The lunar year and the epact.]

I. The common and mean lunar year is easily obtained from the mean synodic month multiplied by 12. If you add one month more to it, the result will be the embolismic lunar year [Annus lunaris Emboliſmicus], that is, one of 13 months. The remainder, however, by which the common lunar year falls short of completing the civil solar year of 365 days will give the epact of one common year. And in this way the epacts of several years are accumulated, taking account of leap years, yet so that the synodic month is cast out of the accumulated sum as often as it is contained in it, and the remainder is kept as the epact.

[Margin: The periodic month.]

II. You will derive the periodic month from the synodic month as follows. To the Sun's mean motion corresponding to the synodic month add the zodiac, or 360°; for this is the space that the Moon, having traversed the zodiac, runs through in a synodic month. Then divide by this sum the amount produced by multiplying the synodic month by the zodiac, that is by 360°, and you will have the time corresponding to the periodic month. But it is better first to resolve all these amounts into the smallest fractions of one kind, for example into thirds. The anomalistic and draconic months, however, have already been derived from what was said in chapters 21 and 22.

EXEMPLVM.

Example.

For me the synodic month is $29^d\ 12^h\ 44^m\ 3^s\ 10'''$; or, resolved into thirds of time, it is 153086590‴. In that time, by my mean motion of the Sun, on which see Book 3, chapter 17, the Sun's motion is 29° 6′ 24″ 17‴. To this I add 360°; the sum is 389° 6′ 24″ 17‴, which resolved into thirds is 84047057‴. And the zodiac, 360°, resolved into thirds is 77760000‴. When 153086590‴ is multiplied by these, the amount is 1190 401323840000‴. When this is divided by 84047057‴, the periodic month results as 141635098‴ 836/840, that is, $27^d\ 7^h\ 43^m\ 4^s\ 58'''\ 59''''\ 45'''''$.

[Margin: The Moon's daily motion in the zodiac.]

III. You will find the Moon's daily mean, or equal, motion in the zodiac as follows. To the Sun's mean motion corresponding to the synodic month add 360°. Multiply this sum, resolved into fractions, say thirds, by the fractions of time of one day resolved, say thirds; and divide the product by the fractions of the synodic month, likewise thirds. There will result the fractions that the Moon completes in the zodiac in one natural day by its mean motion, and these you will reduce to degrees by dividing by sexagesimals.

EXEMPLVM.

Example.

The sum of 360° and the motion the Sun makes in one synodic month, resolved into thirds, is, by what was said in precept 2, 84047057‴. And 1 day, resolved into thirds, is 5184000‴. When these numbers are multiplied together, the amount is 435699943488000‴. When these are divided by the thirds of one synodic month resolved, 153086590‴, the Moon's mean daily motion in the zodiac results as 2846101‴ 466/1531. When these are divided and subdivided by 60, the same motion comes out as 13° 10′ 35″ 1‴ 18'''' 16'''''.

[Margin: The Moon's daily motion from the Sun.]

IV. You will have the daily elongation, or distance, of the Moon from the Sun according to the mean motions of both bodies if you subtract the Sun's daily mean motion from the Moon's daily mean motion in the zodiac.

EXEMPLVM.

Example.

For me the Sun's daily mean motion is 59′ 8″ 19‴ 51'''' 46''''' 20'''''', and the Moon's daily motion in the zodiac, from what was just said, is 13° 10′ 35″ 1‴ 18'''' 16'''''. When the former is subtracted from the latter, there remains the Moon's daily mean motion from the Sun, 12° 11′ 26″ 41‴ 26'''' 29''''' 40''''''.*

[Margin: Daily motion of the anomaly.]

V. You will have the daily motion of the simple anomaly if you multiply the circle of 360°, first resolved into fractions, by the fractions of time of one day,

and divide the product by the anomalistic month, first resolved into fractions of the same kind as those of the day and of the resolved circle.

EXEMPLVM.

Example.

360° resolved into thirds are 77760000‴, and one day resolved into thirds of time is 5184000‴. When these are multiplied together, the amount is 403108440000000‴. And the anomalistic month for me, from what was said at the end of chapter 21, is 27^{d} 13^{h} 18^{m} 34^{s} 30‴, that is, 142842870‴. Dividing the above-mentioned amount by these, there results the daily mean motion of the anomaly, 13° 3′ 53″ 56‴ 30⁗.

[Margin: Daily motion of latitude.]

VI. You will get the daily mean motion of latitude if you divide the amount produced by multiplying the circle of 360° by the fractions of the day resolved, by the draconic month, first resolving all of these into fractions of the same kind.

EXEMPLVM.

Example.

The amount from multiplying the thirds of one day by the thirds of one circle, from what was just said, is 403108440000000‴. And the month of latitude, or draconic month, for me, from what was said in chapter 22, section 7, is 27^{d} 5^{h} 5^{m} 36^{s} 0‴ 55⁗, that is, 14068161‴ [recte: 14106816‴?]. Dividing the above-mentioned amount by these, there remains the Moon's daily mean motion in latitude, 13° 13′ 45″ 29‴ 28⁗.

[Margin: Daily motion of the node ☊.]

VII. You will get the daily motion of the lunar nodes toward the preceding signs if you subtract the Moon's daily mean motion in the zodiac from its mean motion in latitude.

EXEMPLVM.

Example.

From my daily motion of latitude, 13° 13′ 45″ 29‴ 28⁗, take away my daily motion of the Moon in the zodiac, 13° 10′ 35″ 1‴ 18⁗ 16⁗⁗, there remains the daily motion of the node ☊, as also of ♈, toward the preceding signs, 3′ 10″ 38‴ 9⁗ 44⁗⁗.

[Margin: Daily motion of the mean apogee.]

VIII. You will get the daily mean motion of the mean apogee from the line of the Sun's mean place, that is, toward the preceding signs — which is the same as the motion of the centre of the eccentric toward the preceding signs according to Ptolemy and his followers — by subtracting the Sun's daily mean motion from the daily mean elongation from the Sun; granted that Boulliau thinks it is obtained by subtracting the daily motion of latitude from the doubled motion of the Moon from the Sun.

EXEMPLVM.

Example.

From my daily lunar elongation from the Sun, which is 12° 11′ 22″ [recte: 26″?] 41‴ 26⁗ 35⁗′ 40⁗′′, subtract my daily motion of the Sun, 59′ 8″ 19‴ 51⁗ 46⁗′ 20⁗′′, and there remains the daily motion of the mean apogee, 11° 12′ 18″ 21‴ 34⁗ 49⁗′ 20⁗′′.

[Margin: Daily motion of the apogee under the zodiac.]

IX. You will have the daily motion of the apogee in the zodiac by subtracting the daily motion of the simple anomaly from the Moon's daily motion in the zodiac.

EXEMPLVM.

Example.

I take my daily motion of the anomaly, 13° 3′ 53″ 56‴ 30⁗, away from my daily motion of the Moon in the zodiac, which is 13° 10′ 35″ 1‴ 18⁗ 16⁗′, and there remains the daily motion of the apogee in the zodiac, 6′ 41″ 4‴ 48⁗ 16⁗′.

[Margin: Hourly and minute motion.]

X. You will have any hourly motion by dividing the daily motion by 24, and the motion of one minute by subdividing the hourly motion by 60, and so on. Yet you will have the hourly motion more easily if you add together double the daily motion and half the daily motion, but change the titles of degrees into minutes and of minutes into seconds. You will note this device in the Alphonsine and Prutenic Tables, in Arabic form.

EXEMPLVM.

Example.

For me the Moon's daily motion in the zodiac is 13° 10′ 35″ 1‴ 18⁗ 16⁗′. Its double is 26° 21′ 10″ 2‴ 36⁗ 32⁗′, and its half 6° 35′ 17″ 30‴ 39⁗ 8⁗′. The sum of the double and the half is 32° 56′ 27″ 33‴ 15⁗ 40⁗′. Therefore the Moon's hourly motion in the zodiac is 32′ 56″ 27‴ 33⁗ 15⁗′ 40⁗′.

[Margin: Investigation of the lunar months from the daily motion.]

XI. Finally, since in the tables of many astronomers the daily motion is given but the month is lacking, you will derive the synodic, or periodic, or anomalistic, or draconic month from it by dividing the circle of 360° by the daily motion; but to derive the synodic month you must first increase the circle by the Sun's daily motion. Remember, however, that the annual motion is not exactly contained in the hourly motion given in the tables written below, because of some further fractions omitted or reduced to the preceding ones; so an error not to be despised could be committed in the sum of many years. It is better, therefore, to multiply the daily motion by 365 days so as to have the annual motion. The lunar year can also be derived from the more accurate

epacts of certain authors, by subtracting them from 365 days; and when the lunar year is divided by 12, the synodic month can be derived.

XII. It remains for us to bring into tables the opinions of the more distinguished astronomers on the lunar motions. The foundation of these tables is either the synodic, periodic, anomalistic and draconic month in the authors treated from chapter 19 to chapter 21 inclusive, to whom, for the synodic month, add the Prutenic Tables, page 84½; Clavius, chapters 8 and 28 of the *Calendar*; Kepler in the Rudolphine Tables, page 99; Lansbergen in his Tables, page 36; Mulerius in the *Tabulae Frisicae*, page 344; Magini in the *Theoricae Planetarum*, Book 1, and in the *Canon of the Second Mobile*. Or their foundation is the lunar year, which is derived from the epact of one year subtracted from 365 days; these epacts, as far as fourths, are shown by Clavius, chapter 28 of the *Calendar*, and by Kepler in the Rudolphine Tables, page 99. Or finally the foundation is the Moon's daily motion, which is given by Ptolemy, Book 4 of the *Almagest*, chapter 3; the Alphonsine Tables, pages 60, 65, 92 and 100; Reinhold in the Prutenic Tables, the tables from page 45 to page 48; Tycho, *Progymnasmata*, volume 1, page 113; Longomontanus, *Theorica*, Book 1, chapters 2 and 6; Kepler in the Rudolphine Tables, pages 47, 88 and 94; Lansbergen in the *Treasury of Observations*, page 95, and in his tables from page 23 to page 25; Wendelen in the *Idea of the Atlantic Tables*, page 29; Boulliau, *Astronomia Philolaica*, Book 3, chapters 6 and 7; Vincenzo Renieri in the lunar *Tabulae Mediceae*; and Antonín Maria Šírek, or de Rheita, in his *Radius Sidereomysticus*, Book 3. From these let there be

☾ **TABVLA I. LVNÆ MOTVS MEDII.**

Table 1. The Moon's mean motions.

LVNÆ MOTVS (the Moon's motions)	Annual, or of 365 days							Daily						Hourly		
The Moon in the zodiac	Circles cast out, it is 129° and							It is 13° 10′ and besides						it is 32′ 56″ 27‴ and		
Authors	′	″	‴	⁗	⁗′	⁗″	⁗‴	″	‴	⁗	⁗′	⁗″	⁗‴	‴	⁗′	⁗″
Ptolemy	22	46	13	50	32	30	0	34	58	33	30	30	0	26	23	46
Alfonso	23	2	37	22	22	52	55	35	1	15	11	4	35	33	7	58
Copernicus	23	1	49	50	23	0	0	35	1	9	24	52	55	32	48	26
Tycho	23	3	17	58	0	0	0	35	1	21	52	7	27	33	24	40
Kepler	23	2	33	16	49	50	52	35	1	14	30	42	43	33	6	17
Longomontanus	23	2	36	54	59	50	0	35	1	15	45	52	0	33	9	25
Lansbergen	23	2	18	59	21	32	0	35	1	12	29	25	0	33	1	13

LVNÆ MOTVS (the Moon's motions)	Annual, or of 365 days							Daily						Hourly		
Wendelen	23	2	6	40	23	25	0	35	1	10	8	17	0	32	55	21
Boulliau	23	2	24	19	57	4	22	35	1	13	2	27	28	32	3	36
We, for the present	23	3	17	24	58	20	0	35	1	18	16	0	0	33	15	40
The Moon's elongation from the Sun	Circles cast out, it is 129° and							It is 12° 11′ and besides						it is 30′ 28″ 36‴ and		
	′	″	‴	⁗	⁗′	⁗″	⁗‴	″	‴	⁗	⁗′	⁗″	⁗‴	⁗	⁗′	⁗″
Ptolemy	37	21	28	29	23	55	0	26	41	20	17	59	0	43	20	45
Alfonso	37	23	15	20	23	7	15	26	41	37	51	50	39	44	4	40
Copernicus	37	22	27	16	47	0	0	26	41	29	57	49	37	43	44	55
Tycho	37	22	39	43	0	0	0	26	41	32	0	27	13	43	50	1
Kepler	37	22	25	58	37	16	51	26	41	29	44	58	44	43	44	22
Longomontanus	37	22	25	16	46	50	0	26	41	29	57	50	0	43	44	55
Lansbergen	37	22	12	18	30	50	0	26	41	27	30	10	0	43	36	53
Wendelen	37	22	18	43	59	40	0	26	41	28	33	22	0	43	41	23
Boulliau	37	22	23	39	41	11	41	26	41	31	0	46	13	43	47	32
We, for the present	37	22	26	46	58	20	0	26	41	26	29	40	0	43	15	40
The simple anomaly	Circles cast out, it is 88° and							It is 13° 3′ and besides						it is 32′ 39″ 44‴ and		
	′	″	‴	⁗	⁗′	⁗″	⁗‴	″	‴	⁗	⁗′	⁗″	⁗‴	⁗	⁗′	⁗″
Ptolemy	43	7	28	41	13	55	0	53	56	17	51	59	0	50	44	40
Alfonso	43	14	49	38	10	39	5	53	57	30	21	4	13	53	45	53
Copernicus	43	8	5	45	53	0	0	53	56	23	57	40	46	50	59	54
Tycho	43	7	45	54	0	0	0	53	56	20	41	41	51	50	51	44
Kepler	43	10	15	44	46	2	20	53	56	45	19	41	16	51	53	18
Longomontanus	43	8	5	45	53	0	0	53	56	23	57	41	0	50	59	54
Lansbergen	43	13	13	30	51	5	0	53	57	14	33	1	0	53	6	23
Wendelen	43	5	46	39	16	10	0	53	56	1	5	38	0	50	0	33
Boulliau	43	7	34	20	51	42	45	53	56	18	57	40	33	50	9	29

LVNÆ MOTVS (the Moon's motions)	Annual, or of 365 days							Daily						Hourly		
We, for the present	43	8	42	30	0	0	0	53	56	30	0 [?]	0	0	51	15	0
The Moon's latitude	Circles cast out, it is 148° and							It is 13° 13′ and besides						it is 33′ 4″ 24‴ and		
	′	″	‴	⁗	⁗′	⁗″	⁗‴	″	‴	⁗	⁗′	⁗″	⁗‴	⁗	⁗′	⁗″
Ptolemy	42	47	12	44	21	5	0	45	39	48	56	37	0	9	32	22
Alfonso	42	44	31	27	31	58	45	45	39	22	25	53	45	8	26	5
Copernicus	42	44	22	12	46	0	0	45	39	30	46	28	53	8	46	56
Tycho	42	45	26	56	59	50	0	45	39	31	33	1	58	8	44	42
Kepler	42	44	23	42	53	33	6	45	39	32	39	55	39	8	51	40
Longomontanus	42	45	51	50	55	55	0	45	39	34	38	47	0	8	59	7
Lansbergen	42	45	22	12	46	25	0	45	39	30	46	29	0	8	46	56
Wendelen	42	45	30	25	10	40	0	45	39	32	7	36	0	8	50	19
Boulliau	42	46	6	44	25	49	39	45	39	38	5	39	35	8	9	15
We, for the present	42	45	28	24	58	20	0	45	39	28	0	0	0	7	55	40
The node ☊, toward the preceding signs	It is only 19° and							It is 3′ of minutes and besides						it is 7″ 56‴ and		
	′	″	‴	⁗	⁗′	⁗″	⁗‴	″	‴	⁗	⁗′	⁗″	⁗‴	⁗	⁗′	⁗″
Ptolemy	20	0	58	53	48	35	0	10	41	15	26	7	0	43	8	36
Alfonso	19	41	54	5	9	5	50	10	38	7	14	49	10	35	18	7
Copernicus	19	43	32	22	23	0	0	10	38	21	21	35	58	35	58	30
Tycho	19	42	8	58	0	0	0	10	38	9	40	54	31	35	24	2
Kepler	19	42	50	26	3	42	24	10	38	18	9	12	56	35	45	23
Longomontanus	19	43	14	55	56	5	0	10	38	19	52	55	0	35	49	42
Lansbergen	19	43	3	13	24	53	0	10	38	18	17	4	0	35	45	43
Wendelen	19	43	23	44	47	15	0	10	38	21	59	19	0	35	54	58
Boulliau	19	43	42	24	28	45	17	10	38	25	3	12	7	36	5	39

We, for the present	19	42	11	0	0	0	0	10	38	9	44	0	0	34	40	0

RESIDVVM TABVLAE PRAECEDENTIS.

The remainder of the preceding table.

LVNÆ MOTVS (the Moon's motions)	Annual, or of 365 days							Daily						Hourly		
Apogee of the Moon	It is only 40° and							It is 6′ and besides						it is 16″ 42‴ and		
Authors	′	″	‴	⁗	⁗′	⁗″	⁗‴	″	‴	⁗	⁗′	⁗″	⁗‴	⁗	⁗′	⁗″
Ptolemy	39	35	45	9	18	35	0	41	2	15	38	31	0	35	39	6
Alfonso	39	57	47	44	12	13	50	41	3	44	50	0	22	29	22	5
Copernicus	39	53	44	4	30	0	0	41	4	45	27	12	9	41	48	32
Tycho	39	55	32	4	0	0	0	41	5	1	10	25	36	42	32	46
Kepler	39	52	17	32	3	48	32	41	4	29	11	1	27	41	12	59
Longomontanus	39	54	31	9	6	50	0	41	4	51	48	11	0	42	9	31
Lansbergen	39	49	5	28	30	27	0	41	4	57	56	24	0	35	54	50
Wendelen	39	56	20	1	7	15	0	41	5	9	2	39	0	42	54	48
Boulliau	39	54	48	59	5	21	37	41	4	54	4	46	55	41	54	7
We, for the present	39	54	34	54	58	20	0	41	4	48	16	0	0	42	0	40

☾ TABVLA II. ANNVS LVNARIS Medius, & Communis, eſt Dierum 354. &.

Table 2. The mean and common lunar year is 354 days and:

Authors	Hours	I	II	III	IV	V	VI	VII	VIII
Geminus, with the Chaldeans	13	7	52	50	12	0	0	0	0
Harpalus	10	10	54	36	0	0	0	0	0
Meton	9	29	21	42	7	43	36	0	0
Eudoxus	8	43	38	12	0	0	0	0	0
Oenopides	8	40	48	0	0	0	0	0	0
Callippus	8	50	33	11	29	21	48	0	0
Gamaliel	8	48	52	0	0	0	0	0	0
Hipparchus, Ptolemy	8	48	40	0	0	0	0	0	0
Rabbi Adda	8	48	40	0	0	0	0	0	0
Alfonso	8	36	35	46	16	24	0	0	0

Authors	Hours	I	II	III	IV	V	VI	VII	VIII
Copernicus, Reinhold	8	48	38	9	36	0	0	0	0
Clavius, from the Prutenic Tables	8	48	38	7	37	52	1	0	0
But he uses	8	48	38	9	36	0	0	0	0
François Viète	8	48	38	8	46	8	17	8	36
Tycho	8	37	43	57	21	36	0	0	0
Kepler	8	48	38	10	12	0	0	0	0
Lansbergen	8	48	38	24	0	0	0	0	0
Wendelen	8	48	38	24	0	0	0	0	0
Boulliau	8	48	37	55	25	59	51	3	0
We, for the present	8	38	0	0	0	0	0	0	0

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☾ TAB. III. MENSIS LVNARIS Medius, & Communis eſt Dier. 29. &.
Table 3. The mean and common lunar month is 29 days and:

Authors	Hours	I	II	III	IV	V	VI	VII	VIII
Geminus, with the Chaldeans	13	5	39	24	11	0	0	0	0
Harpalus	12	50	54	33	0	0	0	0	0
Meton	12	47	26	48	30	38	18	0	0
Eudoxus	12	43	38	11	0	0	0	0	0
Oenopides	12	43	24	0	0	0	0	0	0
Callippus	12	44	12	45	57	26	49	0	0
Gamaliel	12	44	4	20	0	0	0	0	0
Hipparchus, Ptolemy	12	44	3	20	0	0	0	0	0
Rabbi Adda	12	44	3	20	0	0	0	0	0
Alfonso	12	44	3	2	58	51	22	0	0
Copernicus, Reinhold	12	44	3	10	48	0	0	0	0
Clavius, from the Prutenic Tables	12	44	3	10	38	9	20	5	0
But he uses	12	44	3	10	48	0	0	0	0
François Viète	12	44	3	10	43	50	41	25	43
Tycho	12	44	3	8	39	46	48	0	0
Kepler	12	44	3	10	51	0	0	0	0
Lansbergen	12	44	3	12	0	0	0	0	0
Wendelen	12	44	3	12	0	0	0	0	0
Boulliau	12	44	3	9	37	9	59	15	15
We, for the present	12	44	3	10	0	0	0	0	0

CAPVT XXIV.

De Conſtituendis Epochis Lunarium Motuum.

Chapter 24. On establishing the epochs of the lunar motions.

[32] In the rows for Alfonso, Tycho and We, for the present, the page sets no 48 after the 8 hours, so that each row's figures stand one place to the left of their neighbours and the last places are filled with 0. Twelve times the month of the same row in Table 3 runs 8 hours, 48, then the figures printed here. The figures are given as printed.

[Margin: Authors who treat this subject.]

I. We must recall here what we said about the epochs and roots [Radicibus] of the celestial motions in general, on the occasion of the solar roots, in Book 3, chapter 33. Keeping to that, we shall set the lunar epochs for the meridian of Bologna, beginning the year from noon of the last day of December, and in these matters we shall use the device that Ptolemy teaches in Book 4, chapters 8 and 9, Copernicus in Book 4, chapters 7 and 14, and Boulliau in Book 3, chapter 9.

[Margin: Root of the Moon's longitude from Aries.]

II. *First*, we shall establish the root of the Moon's longitude from the equinox as follows. In chapter 22, section 7, we had the eclipse full Moon at Bologna in 1643, on September 17 Old Style but September 27 in the Gregorian style, at equal time $7^{h}\ 29^{m}\ 25^{s}$ p.m.; and at that moment the Moon's place, as regards mean motion in the zodiac, ♈ 2° 50′, and the mean motion of the anomaly 195° 10′, as we said there and also at the end of chapter 22. Therefore, according to the mean motion we established in chapter 23, at noon of the last day of December, on which the year 1643 was completed, the Moon's mean motion was $5^{s}\ 20°\ 28′\ 23″\ 53‴$. For from the equated time of the full Moon to the end of the year 1643 there were $94^{d}\ 16^{h}\ 30^{m}\ 35^{s}$. To these corresponds a mean motion of $5^{s}\ 17°\ 38′\ 23″\ 53‴$, whole circles being cast out; and this, added to the mean motion the Moon then had, namely 2° 50′, makes a motion of $5^{s}\ 20°\ 28′\ 23″\ 53‴$.

[Margin: Root of the Moon's anomaly.]

III. *Second*, by our tables a motion of the anomaly of $5^{s}\ 7°\ 5′\ 26″\ 18‴$ belongs to the already mentioned interval of $94^{d}\ 16^{h}\ 30^{m}\ 35^{s}$. If this motion is added to the motion the Moon had at the equal time of the full Moon, namely, as we said, 195° 10′, there results at the end of the year 1643 a motion of the anomaly of $11^{s}\ 22°\ 15′\ 26″\ 18‴$.

[Margin: Root of the motion of the node.]

VI. [recte: IV.] Finally, at the same equal time of the eclipse full Moon of 1643 September 27, the Sun's anomaly was 89° 5′ 40″, and the Moon's 195° 10′ but, equated, 196° 30′ 3″. The apparent semidiameter of the Earth's shadow was exactly 48′, according to the distances of the luminaries we have now established. And since the eclipse was of exactly 6 digits, the Moon's true latitude was therefore as great as the semidiameter of the Earth's shadow at mid-eclipse. The angle opposite this at the south node, that is, the Moon's greatest latitude, is 5°, according to what was said in chapter 12. Therefore the distance of the true centre of the shadow from the more westerly south node was, by the rules of trigonometry, 9° 13′ 6″, and accordingly the distance of the Sun's true place from the north node was as great. But the Sun's place, according to the true place, was ♎ 4° 29′ 34″; therefore the place of the north node ☊ was in ♍ 25° 16′ 28″. From then to the end of the year 1643 there are, as I said, $94^{d}\ 16^{h}\ 30^{m}\ 35^{s}$, to which, of the mean motion of the north node ☊ toward the preceding signs, 5° 0′ 50″ correspond. Therefore, when these are

subtracted from the place of the node, the root of the node ☊ at the end of the year 1643 was 5ˢ 20° 15′ 38″. Going back from here, then, to the beginning of the first year of the Christian era, and to other years backward and forward, we have put the roots of the lunar motions into the tables.

CAPVT XXV.

De Secunda Lunæ Inæqualitate, & de Hypothesi, qua Ptolemæus hanc & primam Inæqualitatem explicauit.

Chapter 25. On the Moon's second inequality, and on the hypothesis by which Ptolemy explained this and the first inequality.

[Margin: Ptolemy's progress in each inequality.]

I. In Book 4 of the *Almagest* Ptolemy had set out the Moon's first inequality, had noticed it alone at new and full Moons, and had found that a single equation was therefore then needed to convert the Moon's mean motion into the true, or the true into the mean. In chapter 10 he accordingly fixed the greatest epicyclic equation at 5° 1′, subtractive at 96° of anomaly and additive at 264°, using here, that is, a semidiameter of the epicycle of 5° 14′ of which the radius of the Moon's eccentric orb is 49° 41′, as he had gathered from sets of three eclipses in chapter 6.

[Margin: Hipparchus's errors in this matter.]

Next, in chapter 11, he relates that Hipparchus, using the epicycle hypothesis on sets of three eclipses, found the semidiameter of the epicycle to the semidiameter of the deferent of the epicycle to be as 4° 46′ to 60°, or as 247½ to 3122, with a greatest equation of 4° 34′; but that, using the eccentric hypothesis, he found the eccentricity to be 6° 15′ of which the radius of the eccentric is 60°, or 327⅚ parts of which the same radius is 3144, with a greatest equation of 5° 49′, although equal results ought to have been gathered from either hypothesis. But Ptolemy shows that Hipparchus went somewhat astray in computing and equating the interval of time between the eclipses, and that the discrepancy arose from this, not from any fallacy of the hypotheses.

[Margin: The way in which the Moon's second inequality was detected.]

II. With this done he passes to Book 5, and in chapter 1 describes the astrolabe made of armillary rings, which we too shall describe in volume 2, in the *Liber Organicus*. In chapters 2 and 3 he draws on the observations of Hipparchus and on his own, in which he observed with the armillary rings the distances of the Moon from the Sun, especially when the Moon stood at the nonagesimal degree of the ecliptic and so lacked all parallax of longitude. He relates that from the Sun's known place he often observed the Moon's true place in this way, both at the quadratures and outside them. And in gathering from the tables already constructed the Moon's true motion, equated, for the moment of the observation, he found that at the syzygies ☌ and ☍ the simple equation spoken of above was indeed enough to obtain a true place agreeing with the observed one; but that outside the syzygies another equation was needed besides this

one, and that this one was greatest at the quadratures, when the equation of the first inequality was greatest in them, and that it reached 7° 40′. So the second inequality grows from new Moon to the first quadrature, then shrinks as far as full Moon; and from full Moon it grows again as far as the second quadrature, then shrinks as far as the next new Moon. From this he concludes that the period of the second inequality is completed twice in each month.

To explain both inequalities, then, he assigned the first to the epicycle, on whose circumference the Moon is carried in its upper part toward the preceding signs and in its lower part toward the following signs; and the second to the eccentric that carries the epicycle, so that after the syzygies the centre of the epicycle approaches the centre of the world more and more, according to the proportions he determines in chapter 4.

[Margin: Authors following or expounding Ptolemy's hypothesis.]

A great many embraced this hypothesis, down to Copernicus. It must therefore be made clear with an accompanying figure, not least so that it may smooth our way to understanding the hypotheses of others. Those who treat of it are: Ptolemy in Book 5, from chapter 2 to 10; Proclus in the *Hypotyposes*; al-Battānī in chapter 30; Jābir ibn Aflaḥ in Book 4 of the *Astronomy*; Regiomontanus in the *Epitome of the Almagest*, Book 5, from proposition 4 to 13; Georg Peuerbach in the *Theorica Lunae*, and on it Giovanni Battista Capuano, Campanus of Novara, Erasmus Oswald Schreckenfuchs, Pedro Nunes, Erasmus Reinhold and Christian Wurstisen. Likewise Jean Fernel in Book 1 of the *Cosmotheoria*, chapter 4; Francesco Maurolico in Dialogue 3 of the *Cosmographia*, from page 79; Alessandro Piccolomini in the *Theoricae Planetarum*, Book 1, from chapter 11; Clavius at the end of the *Sphere*; Francesco Barozzi in Book 4 of the *Cosmographia*, from page 250; Biancani in Book 9 of the *Sphere*, chapter 2; Joseph Lange in the *Elementa Astron.*, part 2, chapter 10; Pierre Hérigone in volume 5 of the *Cursus Mathematicus*, from page 514; Andrea Argoli in the *Pandosion*, chapter 40, and in the *Ephemerides*, Book 1, chapter 9; Pierre Gassendi in Book 2 of the *Institutio Astronomica*, chapter 2; and Boulliau in Book 3 of his *Astronomia*, chapter 6. All of these, like the Alphonsines, follow the Ptolemaic hypothesis with a very few figures corrected, except Biancani, Lange, Hérigone, Gassendi and Boulliau, who merely explain it so as to lead beginners by the hand into astronomy. Let there be, then,

Ptolemaicæ Hypotheſis Schema cum Proportionibus Orbium.

The scheme of the Ptolemaic hypothesis, with the proportions of the orbs.

III. In chapter 14, section 2, we obtained the Moon's distances in terrestrial semidiameters. We shall now reduce them to parts of which the semidiameter, or radius, of the eccentric is 100000, and in this kind of parts we shall have the proportion of the Moon's orbs; for otherwise you will find them badly corrupted

in other authors, through a fault either of calculation or of the printing. Here they are, then, in a small table.

PTOLEMAICAE HYPOTHESIS SYNOPSIS

Synopsis of the Ptolemaic hypothesis.

Intervals	Lines	To a radius of the orb of 100000	Semidiameters of the Earth
Semidiameter of the eccentric	I D	100000	48 51′
Semidiameter of the epicycle	B D	10576	5 10′
Semidiameter of the small circle, and eccentricity	A I	20766	10 92′
Distance in ☌ ☍, greatest	A B	131352	64 10′
Distance in ☌ ☍, mean	A D	120766	59 0′
Distance in ☌ ☍, least	A F	110190	5[?] 50′
Distance in □, greatest	A p	89814	[?]3 53′
Distance in □, mean	A b	79234	38 43′
Distance in □, least	A K	68662	33 33′
Greatest equation in ☌ ☍	G A D	5° 1′	
Greatest equation in □	m A Z	7° 40′	

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With these things assumed, let A be the centre of the Earth, from which the Moon's ecliptic ♈ ♋ ♎ ♑ is to be described, which, carrying the other, lower orbs with it, moves the nodes of the Moon's path toward the preceding signs. Let BAC be the line of the mean syzygies, that is, of the mean new Moon and full Moon, and ♎ ♈ the line of the mean quadratures. This is set down by way of example, granted that the latter do not always occur under ♎ or ♈, nor the former under ♋ or ♑. Beneath the ecliptic let there be understood the orb that carries the aux [Augem] of the eccentric, in whose plane lie the planes of all the remaining orbs, these being inclined to the ecliptic at 5°; and let its semidiameter be AB. Next, from A, the centre of the world, let the small circle IXβY be described, carrying the centre of the eccentric toward the preceding signs, and let its semidiameter AI be the Moon's eccentricity.

First, then, with the centre taken on the line of the syzygies and on the circumference of the already mentioned small circle, that is, at the point I, and with the interval of the semidiameter ID, which is to be 48 56′ semidiameters of the Earth, let the eccentric D p f b D be described, with f as its perigee and D as its apogee. From D as a centre let the epicycle BGFH be described, with DB as its semidiameter. And from the centre of the world let two straight lines Aθ and Ax be drawn as far as the ecliptic, touching the epicycle at G and H.

Second, let the centre of the eccentric, I, be moved toward the preceding signs through a half-quadrant as far as M. With the centre made at M, and with

[33] The figure 92 in the row of the small circle stands as printed, although it cannot be a number of minutes.

the interval ML, equal to the semidiameter ID, let there be described another, not so much eccentric as position of the eccentric, namely L ε *e* N H L, with L as its apogee. From L let a straight line be drawn through M to O, the point opposite the centre M itself. Meanwhile the centre of the epicycle will have advanced toward the east, that is, toward the following signs, by as much as the centre of the eccentric, M, and the apogee, L, have receded toward the preceding signs from the line of the mean syzygies, so that the centre of the epicycle is at N. From N let the epicycle PR be described, equal to the first. Through its centre N let the line ONQ be drawn from the point O, for it will be the line of the mean apogee of the epicycle, namely of the point Q; and from A, the centre of the world, the line AP, the line of the true apogee of the epicycle, which, produced as far as the ecliptic, is the line of the Moon's mean place. And if the Moon is at R, having meanwhile completed about a half-quadrant of the epicycle, PR, let the straight line ARS be drawn through the centre of the Moon's body, for it will be the line of the Moon's true place.

Third, let the centre of the eccentric have reached X, the quadrant IX being completed; and from the centre X, with the interval as above, let the third position of the eccentric be described, namely a γ E V F a. Let the aux of the eccentric be at a, and the point opposite the aux at V; and to this V let the centre of the epicycle have meanwhile come, having completed about a quadrant of elongation from the line of the mean syzygies. And from the centre V let an epicycle be described equal to the earlier ones.

Fourth, let the centre of the eccentric have reached β, the semicircle I X β being completed, and let its apogee be at Φ and its perigee at u. From β let an eccentric be described equal to the earlier ones, namely Φ δ p u b γ Φ; and from the centre Φ let an epicycle be described equal to the earlier ones, for the centre of the epicycle will have come to Φ, having completed a semicircle of elongation from the line of the syzygies.

Fifth and last, let the centre of the eccentric have come from β to Y, and the apogee of the eccentric to π. From Y let an eccentric be described equal to the earlier ones, namely π P R m n ε δ π; and let its perigee be Z, to which the centre of the epicycle, Z, will have come. From Z let the epicycle λ m b n be described, whose tangents drawn from the centre of the world to the ecliptic are to be A m μ and A n ξ. These could indeed have been drawn also to the points of contact e h of the epicycle, as could several other positions of the eccentric; but to avoid a tangle of lines let us be content with those already mentioned.

Diagram. Ptolemaic scheme of the Moon's orbs, set in the left column. An outer circle, the Moon's ecliptic, is marked at its four quarters with ♋ at the top, ♈ at the right, ♎ at the bottom and ♎ at the left, this last set sideways. A vertical and a horizontal diameter cross at the centre A, around which is a small circle. Solid and dotted circles, the successive positions of the eccentric, surround the centre, and five small epicycles stand on them: at the top, at the

upper left, at the left, at the right and at the bottom. Straight lines run from the centre to the outer circle. Labels read:

- θ, x (top, on the outer circle, either side of ♋): ends of the tangents from A
- ♋ (top), ♈ (right), ♑ (bottom), ♎ (left, sideways): the signs at the quarters
- B, D, F (top epicycle, on the vertical line): its top, centre and foot
- G, H (either side of the top epicycle): points of tangency
- T, S (upper left, on the outer circle): ends of the lines AP and ARS produced
- P, R, Q, N (upper-left epicycle): true apogee, Moon, mean apogee, centre
- u (on the vertical line below the top epicycle)
- I (on the vertical line above A, on the small circle)
- L (upper right): apogee of the second position of the eccentric
- μ (right, on the outer circle): end of the tangent A m μ
- m, n (top and foot of the right epicycle)
- λ, Z (inside the right epicycle; the Z is a Z with a stroke)
- b (right side of the right epicycle)
- a (right, within the outer circle)
- M, X (right of the centre, on the small circle)
- Y, A (at the centre; Y left of A)
- o (small, lower left of the centre)
- β (below the centre, on the small circle's foot)
- f (on the vertical line below β)
- ξ (lower right, on the outer circle, sideways): end of the tangent A n ξ
- ε, γ (lower right, on the eccentrics)
- E (on the vertical line above the bottom epicycle)
- Φ, C (inside the bottom epicycle)
- h, V, K (left epicycle: above it, inside it and at its right side)
- π, p (left, on the dotted circle and the next circle)
- [?] (a sort like a 5 beneath p)
- e (below the left epicycle)
- δ (lower left, on a dotted circle, sideways)

De Quatuor Lunæ Orbibus, & totidem præcipuis Motibus per eos factis.

On the Moon's four orbs, and the same number of principal motions produced by them.

IV. There are, then, four orbs of the Moon's heaven according to Ptolemy, Alfonso, Peuerbach and the others mentioned in section 2: namely the ecliptic, the deferent or deferents of the aux of the eccentric, the eccentric, and the epicycle; and the same number of principal motions are produced by them.

[Margin: 1. The Moon's orb and the motion in it.]

The *first* orb is the Moon's ecliptic, which moves toward the preceding signs and carries westward the nodes of the Moon's path, that is, of the plane in which the remaining lower orbs lie. For Ptolemy it completes 3′ 10″ 41‴ 15⁗ 26⁗′ 7⁗′′ daily and 19° 20′ 0″ 58‴ 53⁗ 48⁗′ 35⁗′′ yearly, so that it finishes its whole period beneath the ecliptic of the highest heaven in about 18 Julian years 7 months 12 days, or $6798^{d}\ 3^{h}\ 49^{m}\ 40^{s}\ 16'''$. There are those who call this orb the Moon's equant.

[Margin: 2. The orb and motion of the apogee of the eccentric.]

The *second* orb is the deferent of the apogee of the eccentric. In place of it, it is enough to attend to the motion of the centre of the eccentric about the centre A, made on the circumference of the small circle Iβ, since that centre moves with the same kind and quantity of motion as the apogee of the eccentric, namely toward the preceding signs from I toward M. This motion is regular about the centre of the world. Its recession from the line of the syzygies BC, or from the line of the Sun's mean motion, toward the preceding signs is as great as the mean elongation of the centre of the epicycle from the same line toward the following signs. For that reason most authors gather the motion of the aux of the eccentric from the motion of the Moon from the Sun, by subtracting the Sun's mean motion from it: thus Wurstisen, Clavius, Barozzi and Gassendi, who accordingly put its daily motion at 11° 12′ and say that its period is completed in $32^{d}\ 3^{h}\ 5^{m}$. Lange, however, puts it at 11° 12′ 18″ 21‴ 52⁗ and the period at $32^{d}\ 3^{h}\ 4^{m}\ 38^{s}\ 51'''$. But subtracting the Sun's daily motion from the Moon's daily elongation from the Sun, there remains, according to Ptolemy, a daily mean motion of this apogee of 11° 12′ 18″ 24‴ 7⁗ 5⁗′ 28⁗′′. Boulliau, however, thinks it is gathered by subtracting the mean motion of latitude from the doubled motion of the Moon from the Sun, by which reckoning the daily motion of this apogee would be, for Ptolemy, 11° 9′ 7″ 42‴ 51⁗ 39⁗′ 21⁗′′.

[Margin: Motion of the mean apogee of the epicycle.]

Moreover, by force of this motion there is a shift in the point on the small circle Iβ opposite the centre of the eccentric, from which point a line drawn through the centre of the epicycle to the circumference of the epicycle determines the point of the mean apogee of the epicycle itself. Therefore this apogee also shifts: toward the preceding signs as long as the centre of the epicycle is in the first and fourth quadrants of the eccentric, and toward the following signs as long as it is in the 2nd and 3rd quadrants. Reinhold on page 73 and Wurstisen on page 97 make this clear with accompanying diagrams, which are by no means necessary here.

[Margin: 3. The orb and motion of the eccentric.]

The *third* orb is the eccentric, which carries the centre of the epicycle toward the following signs, receding from the line of the mean syzygies, or from the line of the Sun's mean motion, by as much as the apogee and centre of the eccentric recede from the same line toward the preceding signs. This motion is irregular over the centre of the eccentric, but its mean daily amount is 13° 3′ 52″, and it completes its course in $27^{d}\ 13^{h}\ 13^{m}$. Over the centre of the world, or

of the zodiac, A, however, it is regular, and its mean daily amount is 13° 10′ 34″ 58‴ 35⁗ 30′′′′′ 30′′′′′′, and it completes the periodic month in about 27^{d} 7^{h} 43^{m} 7^{s}. If you take the Sun's daily motion from this, there remains the elongation of the Moon, or rather of the centre of the epicycle, from the Sun, daily and mean, of 12° 11′ 26″ 41‴ 20⁗ 17′′′′′ 59′′′′′′, and it completes the synodic month in about 29^{d} 12^{h} 44^{m} 3^{s}. But if to the daily motion of the centre of the epicycle in the zodiac you add the daily motion of the node ☊ toward the preceding signs, the motion of this centre becomes the motion of latitude. Its daily amount is 13° 13′ 45″ 39‴ 48⁗ 56′′′′′ 37′′′′′′, and it completes its period, or draconic circuit [orbem Draconticum], in 27^{d} 5^{h} 5^{m} 36^{s}, according to the Ptolemaic figures. The nearer the centre of the epicycle is to the aux of the eccentric, the faster it moves; and the farther from it, the slower.

[Margin: 4. The orb and motion of the Moon on the epicycle.]

The *fourth* orb is the epicycle, whose centre is on the circumference of the eccentric and whose plane is in the plane of the eccentric. On the circumference of the epicycle is the centre of the Moon's body, which, by the motion of the epicycle, moves in the lower part, as HFG, toward the following signs, from H to F, and so on. The Moon therefore moves faster at that time, because this motion is joined to the motion of the centre of the epicycle going toward the east. But in the upper part of the epicycle, as GBH, it moves against the succession of the signs, and so the Moon's motion is then slower. This motion is counted from the mean apogee: for example, at the mean syzygies from B toward H. With respect to the true apogee and the centre of the epicycle it is irregular, but with respect to the mean apogee it is regular, and it is called the motion of the argument, or of the anomaly. Its daily amount is 13° 3′ 53″ 56‴ 17⁗ 51′′′′′ 59′′′′′′, and it completes its course in 27^{d} 13^{h} 18^{m} 34^{s}. Since, then, the Moon's motion is very swift, the Moon cannot be called stationary; and since the Moon's motion on the epicycle is less than the motion of the centre of the epicycle in the zodiac, it cannot be called retrograde, but always direct.

[Margin: Motion of the true apogee.]

Moreover, since the motion of the eccentric is swifter than the motion of the epicycle, the true apogee of the epicycle advances toward the following signs and completes its course in 8½ years. For its daily motion is 6′ 41″, and it is obtained by subtracting the motion of the anomaly, or of the epicycle, from the motion of the Moon in the zodiac.

COROLLARIA.

Corollaries.

Ex Lunæ Motibus prædictis & Orbium descriptione.

From the already mentioned motions of the Moon and the description of the orbs.

[Margin: 1st Corollary.]

V. From what was said in numbers 3 and 4 it follows, *first*, that twice in every month the centre of the epicycle is at the apogee of the eccentric, namely at every mean ☌ new Moon and ☍ full Moon: for example, once at D and again at Φ; and twice at the perigee of the eccentric, as about the first quadrature at V and about the second □ at Z. Indeed, it is from these distances that the motion of the apogee of the eccentric has been detected.

[Margin: 2nd Corollary.]

Second, at every mean ☌ and ☍ the line of the mean motion and that of the motion of the apogee of the eccentric coincide, and the same line is the line of the mean conjunction or opposition, namely BAC.

[Margin: 3rd Corollary.]

Third, at every mean ☌ and ☍, and also at every mean □ quadrature, the line of the apogees of the eccentric, or of the epicycle, both true and mean, is one and the same.

[Margin: 4th Corollary.]

Fourth, at mean new Moons and full Moons, no equation is needed if the Moon is at the apogee of the epicycle, as at B or at C, that is, if its mean argument or anomaly is nil, or if it is at the perigee of the epicycle, as at F or at E, that is, if its anomaly is exactly 6^{s} or 180°; instead, the mean motion and place coincide with the true. But if it is at the points of contact, as at H, that is, if it has an anomaly of 96° according to *Ptolemy*, the maximum equation of the first inequality is needed, to be subtracted from the mean motion; for *Ptolemy* this is 5° 1′. At the point G, however, that is, at an anomaly of 264°, it is to be added to the mean motion, so that the true motion may be had. Elsewhere this equation is smaller, but in the first semicircle of the anomaly, as in BHF, it is to be subtracted, and in the second, FGB, to be added to the mean motion, so that the true or apparent motion may be had.

[Margin: 5th Corollary, and the way of deriving the maximum equation and its place.]

Fifth, the maximum equation of the simple inequality, that is, the epicyclic equation, is derived in triangle AGD, right-angled at the contact G, with the following given or already found: the semidiameter of the epicycle GD, the semidiameter of the eccentric DI, and its eccentricity IA, and hence the whole of AD as base. For from these, by the rules of trigonometry, one derives the angle of the maximum equation, GAD, whose complement to a right angle is angle ADG, equal to angle ADH. This angle, then, subtracted from the 180° of the semicircle BHF, gives the place of the anomaly H, where the maximum subtractive equation occurs; added to the same semicircle, it gives the point G, of the maximum additive equation.

[Margin: 6th Corollary.]

Sixth, outside the mean syzygies a double equation is often needed, one of the epicycle and the other of the eccentric. Thus if the Moon is at R, the anomaly QR must be equated by adding the arc PQ, so that the true anomaly

may be had, that is, the anomaly computed from the true apogee P; therefore the angle PNQ will be needed.

[Margin: Maximum equation about the quadratures.]

In the quadratures, however, if the Moon were at the apogee or perigee of the epicycle, no equation would be needed, as if it were at ρ, b, K, λ. But if it is at the contacts of the tangents to the epicycle, as at *m* or at *n*, the equation is at its maximum, 7° 40′ for *Ptolemy*.

[Margin: 7th Corollary.]

Seventh, if a new Moon occurs at the apogee of the epicycle, as at B, the first quadrature will occur about the end of the first quadrant, as at *h*; the full Moon about the perigee of the epicycle, as at E; and the second quadrature about the end of the third quadrant of the epicycle, as at *m*. But if the new Moon occurs at the perigee of the epicycle, as at F, the first quadrature will occur at the end of the third quadrant, as at *e*; the full Moon at the apogee of the epicycle, as at C; and the other quadrature at the end of the first quadrant, as at *n*. Now if the new Moon occurs at the end of quadrant 1, at H, or of quadrant 3, at G, the first quadrature will occur in the one case at the perigee K, in the other at the apogee ρ; the full Moon in the one case at the end of quadrant 3, in the other at the end of quadrant 1; and the second quadrature in the one case at the apogee *b*, in the other at the perigee λ. From these it is easy to understand the positions of the other syzygies and quadratures on the circumference of the epicycle.

[Margin: 8th Corollary.]

Eighth, throughout the month both the centre of the epicycle and the Moon describe by their motion not a circle but a certain egg-shaped and almost elliptical line, as can be seen in the following figure, in which the line PCHDP represents the line described by the centre of the epicycle. Its greater semidiameter PT consists of the semidiameter of the eccentric and the eccentricity; its lesser, TC, of the remainder of the semidiameter of the eccentric after the eccentricity has first been taken from it. The former is therefore 59 0′ semidiameters of the Earth, the latter 38 43′ semidiameters of the Earth. Similar to this is the line described by the centre of the Moon's body, but now wider, now narrower, according as it is nearer to or farther from the apogee of the epicycle at the syzygies and quadratures. Reinhold set out this egg-shaped figure at the end of Peuerbach's *Theoricae*.

Diagram. At the head of the right column, standing inside the sentence above: a large outer circle (the eccentric's path) with a vertical diameter from A at the top to S at the bottom and a horizontal diameter from Z at the left to V at the right, their intersection near T. Four small epicycles stand on an inner egg-shaped line taller than it is wide: one at the top with centre P, one at the bottom with centre H, one at the left with centre C, one at the right with centre D. A straight line runs up from the area of T to M at the upper rim, passing near B.

- A (top, outside the circle)
- M (top, to the right of A, at the end of the slanted line)

- P (in the upper epicycle, right of the vertical)
- B (in the upper epicycle, at its right edge)
- ☍ (in the upper epicycle, to the left of P)
- L (left of the vertical, below the upper epicycle)
- □ (in the left epicycle, above the horizontal)
- Z (far left, outside the circle)
- E (at the left, the horizontal line running through it)
- C (in the left epicycle, below the horizontal)
- L (just to the right of the left epicycle, below the horizontal)
- T (above the horizontal, at the vertical)
- K (below the horizontal, right of the vertical)
- □ (in the right epicycle, above the horizontal)
- D (in the right epicycle, below the horizontal)
- F (at the right edge of the right epicycle)
- V (far right, outside the circle)
- R (above the lower epicycle, left of the vertical)
- ☌ (in the lower epicycle, left of H)
- H (in the lower epicycle, right of the vertical)
- S (bottom, outside the circle)

Summa Definitionum ac Regularum ad Lunæ Theoricam Ptolemaicam, & præcipuè ad Prosthaphæreses spectantium.

Summary of the definitions and rules pertaining to the Ptolemaic theory of the Moon, and especially to the prosthaphaereses.

[Margin: True place.]

[Margin: Line of the true place.]

VI. The Moon's *true place* [Locus Verus] in the zodiac is the point of the zodiac on which falls a straight line drawn from the centre of the zodiac, that is, of the Earth, through the centre of the Moon's body: as, in the earlier figure, the point S, on which ARS falls if the Moon is at R. And this line is called the line of the true, or apparent, place.

[Margin: Mean place.]

The Moon's *mean place* [Locus Medius] is the point of the zodiac on which falls a straight line drawn from the centre of the Earth through the centre of the epicycle: as the point T, on which A N T falls if the centre of the epicycle is at N. And that line is called the *line of the mean place or motion.*

[Margin: Mean longitude.]

Mean longitude [Longitudo Media] is sometimes taken for the mean or middling distance of the Moon from the Earth, and sometimes for the point of the epicycle or eccentric at which it stands a quadrant away from the apogee of the epicycle or eccentric, that is, half of the greatest and semicircular distance: as are 90° and 270° of anomaly.

[Margin: True apogee of the epicycle.]

The *true apogee of the epicycle* is the point on the circumference of the epicycle most distant from the centre of the Earth, as B or C. But by an extended use of the name, any point of the epicycle on which falls a straight line drawn from the centre of the Earth through the centre of the epicycle is also called a true apogee: as the point P, on which ANP falls; that line is accordingly called the *line of the true apogee*.

[Margin: Mean apogee of the epicycle.]

The *mean apogee of the epicycle* is the point on the circumference of the epicycle on which falls a straight line drawn through the centre of the epicycle from the point opposite the centre of the eccentric and as far from the centre of the Earth as the centre of the eccentric is from the same centre of the Earth: as is the point Q, on which the straight line ONQ falls, with the centre of the epicycle placed at N and that of the eccentric at M. And this is called the *line of the mean apogee*.

[Margin: True apogee of the eccentric.]

The *true apogee of the eccentric* is the point on the circumference of the eccentric on which falls the straight line drawn through the centre of the eccentric from the centre of the world.

[Margin: True anomaly.]

The Moon's *true argument* or *anomaly* is the Moon's distance from the true apogee, computed on the circumference of the epicycle to the right, that is, westward: thus if the Moon is at R, its true anomaly will be the arc PR.

[Margin: Mean anomaly.]

The *mean argument*, or *mean anomaly of the orb*, is the Moon's distance from the mean apogee of the epicycle, computed on the circumference of the epicycle westward: as is the arc QR.

[Margin: Centre of the Moon.]

The *centre of the Moon*, or *anomaly of the eccentric*, or, as Peuerbach puts it, *double longitude*, is the arc of the zodiac intercepted between the line of the apogee of the eccentric and the line of the mean place or of the true apogee of the epicycle, but counted from the line of the aux of the eccentric in the order of the signs: as is the arc TBL, with the aux of the eccentric placed at L and the epicycle at N. This arc is obtained by doubling the Moon's mean elongation from the Sun;

and therefore it is usually called the doubled longitude of ☾ from ☉; and it is called the centre because from it the place of the centre of the eccentric is derived.

[Margin: Epicyclic equation.]

The *equation of the argument* [Aequatio Argumenti], or epicyclic equation, or equation of the first inequality, is the arc of the zodiac between the line of mean motion and the line of true motion. It is nil if the Moon is at the apogee or perigee of the epicycle, as at B or F; it is at its maximum if the Moon is at the contacts of the lines tangent to the epicycle and drawn out from the centre of the Earth, as if it is at H or at G, at *m* or at *n*. At the syzygies it is 5° 1′, in □

7° 40′. It is obtained in the same way as the Sun's equation in the hypothesis of the epicycle, as in triangle DGA, right-angled at G, given AD and GD and the complement of the anomaly to, or its excess over, 180°.

[Margin: Eccentric equation and its method.]

The *equation of the centre* [Aequatio Centri], or eccentric equation, is the arc of the epicycle intercepted between its true and its mean apogee, as the arc PQ. It is so called because it is obtained from the centre of the Moon, that is, from the doubled mean elongation of the Moon from the Sun. For the arc PQ, for example, is as great as the angle N subtending it, which by 15 of Book 1 of *Euclid* equals the angle ONA. This angle becomes known in the same triangle, because in it are given the eccentricity AO and the side ON, which in this case is equal to the line drawn from M to N, that is, the semidiameter of the eccentric; and the angle OAN is the complement, to two right angles, of the angle NAL, which measures the doubled distance of the Moon from the Sun. The maximum equation of the centre, however, does not occur at the Moon's mean longitudes but a little below them, as *Peuerbach, Maurolico*, and the others above teach; that is, about the sextile and trine, or, as *Nunes* shows with *Ptolemy* at p. 224, at the point of the eccentric distant from its aux by 114° 3′ 55″; and the maximum equation itself is 13° 8′ 7″.

[Margin: Where the maximum equation of the centre occurs and how great it is.]

But because *Giovanni Battista Capuano* and certain others thought that the maximum equation occurs when the centre of the epicycle is on the line that meets the line of the apogee of the eccentric orthogonally at the opposite point, that is, when the line of the mean apogee makes right angles with the line of the apogee of the eccentric, as if NO were to fall orthogonally on OL, *Pedro Nunes* above, p. 213, and *Reinhold*, p. 74, therefore demonstrate that this is false and that this equation becomes greater below that point. Conversely, this equation is nil when the centre of the epicycle is at the apogee of the eccentric, D or Φ, or at its perigee, V or Z.

[Margin: Use of both equations.]

Now the epicyclic equation is added to the mean motion, so that the true motion may be had, if the anomaly is greater than 6^s; it is subtracted if less. But the eccentric equation is added to the mean anomaly if the centre is less than 6^s; it is subtracted if greater.

[Margin: Diversity of the diameter.]

The *diversity of the diameter* [Diuersitas Diametri] of the epicycle is the arc of the zodiac by which the perigean equation of the epicycle exceeds the apogean epicyclic equation. Thus in the earlier figure, with the centre of the epicycle placed at the perigee of the eccentric Z, the arc corresponding to the maximum equation is μ ♈ or ♈ ξ; with it placed at the apogee D, the arc is θ B or B *x*. Therefore the excess by which μ ♈ exceeds θ B is called the diversity of the diameter; for it is as great as the portion by which the arc μ ξ, subtending the diameter *m n*, exceeds the arc θ *x*, subtending the diameter GH.

[Margin: Proportional minutes.]

Proportional minutes [Scrupula proportionalia] are the sixtieth parts of the diversity of the diameter by which the true argument, or true anomaly, is equated. For these minutes are set against the signs and degrees of the centre, and by means of the true argument the diversity of the diameter is taken from the table. If the minutes are 60, the diversity is added whole to the equation of the argument; if they are fewer than 60, a part proportional to them is added, and thus the true equation of the argument is had for such a position of the epicycle. By means of this, the composite equation of the Moon is then drawn from its own table. For others, however, with whom the eccentricity varies, these minutes are the difference between the longest line of the apogee and the shortest line of the perigee, that is, the eccentricity divided into 60 parts.

[Margin: Method of calculation.]

The method of calculating longitude is this. From the tables one takes the Moon's mean longitude from ♈, its distance from the Sun, and the anomaly of the orb; and the distance from the Sun is doubled, so that the anomaly of the eccentric, or of the centre, may be had. With this, one looks up in the tables the equation of the centre, together with the proportional minutes. This equation is added to or subtracted from the anomaly of the orb, and the equated anomaly of the orb is had. With this, one looks up in the tables the equation of the orb with its excess, which would be taken whole if those minutes were 60; but if they are fewer, a part proportional to them is taken from the excess and is always added to the equation of the orb, so as to make the absolute equation. For this, added to or taken from the mean longitude, gives the Moon's true longitude from the equinox.

CAPVT XXVI.

Chapter 26.

De Lunari Hypothesi Copernici, Magini, Tychonis, & Longomontani.

On the lunar hypothesis of Copernicus, Magini, Tycho and Longomontanus.

[Margin: Absurdity of the Ptolemaic hypothesis.]

I. Copernicus, Book 4, chapter 2, rejects the Ptolemaic hypothesis about the Moon for three reasons. The first is that he holds it absurd that the celestial motions should be unequal in themselves and with respect to their own centres, as is the motion of the Moon on the epicycle, which according to *Ptolemy* is unequal about the centre of the epicycle, and that they should beg their equality from another's centre, namely from the centre of the Earth. The second is the enormous size of the Ptolemaic parallaxes at the quadratures, which by what was said in chapter 14 is 103′ at perigee, that is, almost twice as great as that of the Moon at apogee in the syzygies, although *Copernicus* affirms that these differ very little or not at all from one another. The third is the likewise

enormous apparent size of the Moon at perigee in ☐: at a distance, that is, of 33 33′ semidiameters of the Earth, by what was said in chapter 16, its diameter would appear to be 55′, nearly twice as great as in the apogean syzygies, where for *Ptolemy* it is 31′ 20″. *Copernicus* says that this is contrary to the evidence of the phenomena.

[Margin: Objections to Copernicus on behalf of Ptolemy.]

II. But someone could adduce on behalf of *Ptolemy*, against Copernicus, the change in the Moon's eccentricity, which would have grown smaller with time. For that enormity of the visible diameter and of the parallaxes of the Moon at perigee in ☐ arises from its excessive closeness to the Earth, which could be corrected so as to fit these centuries of ours if the eccentricity were reduced: as if, in the first figure of chapter 25, no. 3, the semidiameter A X of the small circle became shorter, and the eccentric were described from the point X with the interval XV; then certainly V, the centre of the Moon's epicycle, and K, the perigee of the epicycle, would be moved away from the centre of the Earth, A. That is, unless someone wanted to call in the contraction of the semidiameter of the epicycle as part of this correction. As for the Moon's moving equally about another's centre, those more recent astronomers who have admitted the equant circle, not only for the five minor planets but also for the Sun, as we said in Book 3, chapter 22, think this not at all absurd. Copernicus certainly seems hardly consistent with himself in this, since he invented a diminution of the eccentricity in the Sun, as was said in Book 3, chapter 26, and did not devise the like for the Moon. Nevertheless, let us meanwhile hear him, in chapters 3, 8 and 9; *Mästlin* in the Appendix to Kepler's *Prodromus*, p. 151; and *Magini* in the first *Theorica* of the Moon and in the *Canon of the Second Mobiles*, who with Copernicus substitute the hypothesis set out below for the Ptolemaic one.

Copernicæ Hypothesis Synopsis

Synopsis of the Copernican hypothesis.

For the distances see ch. 14	Lines or angles	To the radius of the concentric orb, 100000	Earth semidiameters, 1
Semidiameter of the concentric	A C	100000	60 19
Semidiameter of the epicycle, greater	C E	10970	6 36
Semidiameter of the epicycle, lesser	P E	2370	1 26
Distance at ♂ and ☍, max.	A N	108600	65 30
Distance at ♂ and ☍, mean	A C	100000	60 19
Distance at ♂ and ☍, min.	A O	91400	55 9
Distance at ☐, max.	A P	113340	68 20
Distance at ☐, mean	A C	100000	60 19
Distance at ☐, min.	A Q	86660	52 17
Equation at ♂ ☍	G A I	5° 0′	

For the distances see ch. 14	Lines or angles	To the radius of the concentric orb, 100000	Earth semidiameters, 1
Maximum at ☐	C A D	7° 40′	

III. With these things supposed, from A, the centre of the Earth, let a portion of an orb or circle concentric with the Earth, BCD, be described at an in- terval AC, of 100000 parts, and from the centre C the greater epicycle EHGF, with semidiameter CE of 10970 parts, carrying the lesser epicycle P S N, whose semidiameter PE is of 2370 parts. Let the epicycles I K L M, and OTQ, and R D, etc., be equal to this one, so that the lesser epicycle with its apogee P describes the circle P L Q D P, and with its perigee N the circle N I O. And let straight lines be drawn from the centre of the Earth A: one, A I, tangent to the epicycle O Q at T, or to the circle N O at I; the other, A D, tangent to the epicycle R D at D, or to P Q at D.

[Margin: Method of deriving the proportion of the epicycles.]

For it has been observed that when the Moon is half-lit, that is, placed at the quadratures and at the nonagesimal, where the parallax of longitude ceases, its observed or apparent place, when it differs most from the Moon's mean place as derived from the tables of equal motions, differs by 7° 40′; and that this maximum prosthaphaeresis occurs when in ☐ the Moon has an anomaly of about 3^{s} or 9^{s}; whereas in ☌ and ☍ the maximum prosthaphaeresis needed to reduce the mean place to the true and observed place is 5° 0′. With A I drawn, then, tangent to the circle N O at I, the triangle A I C is, by 16 of Book 3 of *Euclid*, right-angled at I. Therefore, given A C, 100000, and the angle C A I, 5°, trigonometry yields C I, of 860 [recte: 8600] parts, to which CN is equal. And with AD drawn, tangent to the circle PQ at D, in the triangle CAD, right-angled at D, from the base AC, 100000, and the angle CAD, 7° 40′, one derives CD, of 13340 parts, to which CP is equal. With CN taken from CP, therefore, NP remains, of 4740 parts, and half of it, that is, the semidiameter of the lesser epicycle, PE or NE, is of 2370 parts; and with NE added to NC, the semidiameter of the greater epicycle, CE, comes to 10970 parts.

[Margin: Form of the lunar motions.]

Now let the centre C of the greater epicycle move in the order of the signs toward B, with the Moon's equal motion in the zodiac; if you take from this the mean motion of the Sun, the mean motion of the Moon from the Sun remains. Meanwhile let the centre E of the lesser epicycle move against the order of the signs, the motion being counted from E toward F, with the motion of the mean anomaly, which is of course the distance of the centre of the lesser epicycle, computed from the apogee E of the greater epicycle. And finally let the body of the Moon move on the circumference of the lesser epicycle against the order of the signs from N through S to P, counting from N the double motion of the Moon from the Sun; that is, under this rule, that when the line AC is the line of the Sun's mean motion, the Moon at mean ☌ and ☍ is at the perigee N of the lesser epicycle, and in ☐ is at its apogee P. For in this way

it will come about that the Moon runs through the small epicycle twice in a month, in which time the centre C will have returned once to the Sun; and the new and full Moon will appear to describe the smallest circle N I O, but at the quadratures the largest, P L Q D, and therefore to undergo in the former case smaller, in the latter larger differences of the parallaxes, of the visual diameters, and of the prosthaphaereses. And with these things laid down, and with their motions supposed, on which see chapter 23, *Copernicus* holds that all the lunar appearances agree with this hypothesis.

Diagram. In the left column, with the narrowed text running down its left side: the greater epicycle's circle centred at C, carrying around it four small epicycles at top, left, bottom and right, enclosed by a larger circle through P at the top and Q at the bottom, with an inner circle about C through N and O. A long vertical line runs from P at the top through C to A at the foot, and two lines run down from the circles to meet at A. Point C stands near the centre; a slanted line passes through C from upper right to lower left.

- P (top, outside the circle)
- E (in the upper epicycle, right of the vertical)
- S (right edge of the upper epicycle)
- N (below the upper epicycle, on the inner circle)
- K (at the top of the left epicycle)
- R (at the top of the right epicycle)
- C (centre)
- B (far left, outside the circle)
- L (next to B, on the left edge of the left epicycle)
- H (in the left epicycle)
- I (inside, at the right edge of the left epicycle)
- F (in the right epicycle, a drawn line running through the sort)
- D (far right, outside the circle)
- M (below the left epicycle)
- O (on the vertical, above the lower epicycle)
- T (in the lower epicycle, left of the vertical)
- G (in the lower epicycle, right of the vertical)
- Q (below the lower epicycle)
- A (at the foot, where the lines meet)

[Margin: Investigation of the equations.]

IV. Further, in chapter 9 he investigates the equations, and the difference by which the Moon appears to move unequally from the apogee of the lesser epicycle, by this method. In this second figure let the greater epicycle be AB, and let the lesser, with centre E, have withdrawn from the highest apsis A against the order of the signs by any arc AE; and let the Moon be at M or L, the contacts of the lines C M or CL. For since the angles M and L, by 16 of Book 3 of *Euclid*, are right, and CE is of 10970 parts of which M E or LE is 2370, the angle ECL follows as 12° 28′, namely the maximum difference by which the Moon varies from the highest apsis F of the lesser epicycle. This, he

says, occurs when the Moon in its mean motion is distant from the mean place of the Sun by 38° 46′, and by as much on either side of the mean opposition.

Diagram. In the right column, with the narrowed text running down its left side: a circle centred at C, the greater epicycle, with A at the top and B at the bottom, joined by a vertical line that continues down to D. Three small circles stand on its circumference, at upper left, upper right and lower right. Lines radiate from C to points on the small circles, and two long lines run from the left and right small circles down to meet at D.

- G (top left, at the top of the upper-left small circle)
- A (top, outside the circle)
- M (left of the upper-right small circle's centre)
- E (centre of the upper-right small circle)
- F (top right, at the edge of the upper-right small circle)
- E (centre of the upper-left small circle)
- I (below the upper-left small circle, right of the line to D)
- L (lower edge of the upper-right small circle)
- C (centre)
- P (in the lower-right small circle)
- N (right edge of the lower-right small circle)
- K (below N, at the same edge)
- B (bottom of the circle, outside)
- D (at the foot, where the lines meet)

With this done, in chapter 10 he gives the way of demonstrating the apparent motion geometrically, from the mean motion given for a given time, together with the canon of the lunar prosthaphaereses and their foundations. For in the second figure let the Moon be at G, and let the arc IG, or angle C E G, be known from the doubled motion of the Moon from the Sun. Then in the triangle CEG, given the angle just mentioned and the sides CE, 10970, and EG, 2370, the angle ECG and the side GC will be found by the rules of triangles. Now let the anomaly APBE be given, and hence its complement E A, that is, the angle ECA, which, subtracted from the semicircle AEB, leaves the angle ECD. To this, ECG added will give the angle GCD, in the triangle GCD, in which GC has been found and DC, 100000, is given. Therefore by trigonometry the angle GDC will not remain hidden, namely the prosthaphaeresis to be added to the mean motion of the Moon from the Sun; just as, with the angle ECG taken from the angle ECA, there remains the true distance of the Moon from the apogee of the first epicycle, G C A, and thus the equated anomaly. But if the Moon were at K, with mean anomaly AP, and with the arc KN known from the double motion of the Moon from the Sun, that is, the angle KPN, and its complement to two right angles, C PK, then certainly in the triangle CPK, in which CP is of 10970 parts and PK of 2370, the side CK would become known, and the angle KCP, to be added to the mean anomaly so that it would be equated, DCK. With this, and with the sides CK, just found, and CD, of 100000 parts, the angle CDK would become known, which is the

prosthaphaeresis to be subtracted from the mean elongation of the Moon from the Sun, so that the true elongation may be known. I pass over the way of finding the Moon's motion by tables, on which see *Copernicus*, Book 4, chapter 12, *Reinhold* in the precepts of the Prutenic Tables, and ourselves in the practical part at the end of this work. From this he deduces the maximum equation of the first inequality as 4° 56′ 19″ at the 95th and 265th degree of anomaly, and the maximum equation of the second inequality as 7° 40′ at 98° and 262° of anomaly.

[Margin: Fallacy of the Copernican hypothesis.]

But since the Copernican parallaxes of the Moon are fallacious, as we said in chapter 14, no. 4, it is no wonder if, both for this reason and from mean motions not so exactly established, this hypothesis strays somewhat from the phenomena. Besides this, it follows from it that the lunar spots would appear to us at the perigee of the lesser epicycle and be hidden at the apogee, unless the epicycles mentioned are transformed into eccentrics; which *Copernicus*, Book 4, chapter 3, concedes can be done. *Kepler* adds, in the Rudolphine Tables, p. 57, and in the *Epitome of Astronomy*, p. 674, that from these two epicycles it follows that the planet wanders outward around the mean longitudes and is twisted into various bends, whereas observations attest that in those places it enters toward the sides and contracts its distances.

V. Accordingly *Giovanni Antonio Magini*, Book 1 of the *Theoricae*, from p. 70, having seen that it follows from Copernicus's epicycles that the lunar spots would be hidden from us at the apogee of the first epicycle, which *Averroes* had long ago objected to the Ptolemaic epicycle, transformed the Copernican hypothesis into eccentrics, one in particular being carried on the circumference of a hypocycle, that is, a small circle placed near the Earth, yet making the centre of the Earth and of the ecliptic one and the same; and he used solid orbs, while retaining almost the Copernican numbers and intervals. The synopsis of his hypothesis is set out below.

Maginicæ Hypotheseos Synopsis

Synopsis of Magini's Hypothesis

Interval	Lines	In parts of an orb radius of 100000	Semidiameters of the Earth, 1
Semidiameter of the eccentric	C G	100000	60 19
Semidiameter of the small circle	B C	2375	1 26
Eccentricity, greatest	A C	13330	8 2
Eccentricity, least	A D	8580	5 10
Distance at conjunction and opposition, greatest	A H	108580	65 29
Distance at conjunction and opposition, mean	A M	100000	60 19
Distance at conjunction and opposition, least	A F	91420	55 9
Distance at quadrature, greatest	A G	113330	68 21
Distance at quadrature, mean		100000	60 19

Interval	Lines	In parts of an orb radius of 100000	Semidiameters of the Earth, 1
Distance at quadrature, least	A E	86670	52 17

[Margin: Six orbs in Magini's lunar theory.] In the lunar heaven, then, six orbs must be conceived. One, the highest, lies in the plane of the ecliptic GPT, described from A, the centre of the Earth. It carries the nodes of the lunar orbit against the order of the signs by 3′ 10″ 47‴ daily and by 19° 20′ 33″ 44‴ yearly, and completes its circuit in 18 Egyptian years $223^{d}\ 6^{h}\ 12^{m}$. The others, carried round by the highest, must be conceived in a plane inclined to the ecliptic by 5°, which is the Moon's greatest latitude. Among these, the two immediately below the ecliptic carry the mean apogee of the eccentric in the order of the signs, from the first star of Aries, by 6′ 40″ 56‴ 28⁗ daily and by 40° 39′ 3″ 32‴ yearly, and they complete their course in 8 Egyptian years $312^{d}\ 9^{h}\ 27^{m}\ 46^{s}$.

[Margin: The mean apogee of the eccentric and its motion.] The mean apogee is the point in the highest lunar heaven determined by the line drawn from the centre of the world A through the centre B of the small circle carrying the eccentric: for example G, if the centre of the eccentric is at C; or the point X, if the centre of the eccentric is at V.

[Margin: The motion of the anomaly of the apogee and eccentricity.] Below these are the fourth and fifth orbs, namely those of the anomaly of the apogee and of the eccentricity. They move in the order of the signs with the doubled motion of the Moon from the Sun, 24° 22′ 53″ 23‴ daily, and complete their circuit in $24^{d}\ 18^{h}\ 22^{m}\ 1^{s}$. This motion takes place about the centre B of the small circle CD and is counted from its perigee D, or at any rate from the perigee F of the eccentric HMFZ. By the force of this double motion the centre of the eccentric is restored to the perigee D twice in one synodic month. Some call this motion the centre of the Moon; *Copernicus* and the *Alphonsines*, the doubled distance of the Moon from the Sun; *Magini*, the anomaly of the apogee and eccentricity. It is counted as far as the line drawn from the centre B through the centre of the eccentric, as is the line BVL in the following figure, if the centre of the eccentric is at V.

Finally the sixth orb, which carries the body of the Moon against the order of the signs, is the eccentric, whose mean daily motion is 11° 18′ 59″ 27‴, and it completes a circle in $31^{d}\ 19^{h}\ 29^{m}\ 12^{s}$. This motion takes place about a moving centre on the periphery of the small circle CVD, and is counted from the mean apogee of the orbs of the anomaly, which is defined by the straight line drawn from the centre B through the centre of the eccentric, as is the line BVL.

Diagram. Magini's lunar hypothesis, in the left column. A large outer circle carries G at the top and T at the bottom, joined by a vertical diameter through A. On that diameter from the top stand G, R, H, C, B, D, A, E, F, T. A small circle round B passes through C and D. Two eccentric circles stand inside the outer circle, one with its top at G and its foot at E, the other with its top at H

and its foot at F; a further arc runs from the region of X and K down past L to S. A horizontal line joins I, C and Y; another joins M and D and runs on to the letter beside L. A line runs from B through V to L. Lines radiate from D, C and A to the upper left, ending on the outer circle at the labels there, and a line runs from A through the region of D and V up through K to X.

- G (top of outer circle); T (foot of outer circle)
- X, K (upper right, on the line to the outer circle)
- R, H, C, B, D, E, F (on the vertical diameter, in that order downward, with A between D and E)
- A (on the vertical diameter)
- V (right of D, on the small circle)
- I (left end of the upper horizontal); Y (right end of the same)
- M (left end of the lower horizontal); L (right, on the outer region)
- Z (immediately left of L, with a slanting foot)
- S (lower right); Θ (left of the vertical, above F)
- on the outer circle at upper left, in order from the left: a glyph like a turned 9, read as Q [?]; P; N; O
- inside the outer circle at upper left: ε (below N O); δ (below P)

VI. Therefore, on the line of apsides TG, bisected at A, the centre of the world, let the ecliptic GT be described, and at the interval AB, of 10955 parts of which CG, the radius of the eccentric, is 100000, let the centre B be taken, and with the semidiameter BC of 2375 parts let the small circle CVD be described. On its periphery, for the mean conjunctions and oppositions, let the centre of the eccentric be at D, and let the eccentric be HMFZ, with its apogee at H and its perigee at F, and the points of mean longitude M, Z, determined by the straight line MDZ, perpendicular to GT. And let us suppose that at that time the Moon is not at H, where the true and mean motions agree, but at δ. For the line Dδ Q, drawn from the centre of the eccentric through the body of the Moon, will be the line of mean motion, and the mean place will be at Q; but the line Aδ P, drawn through the Moon's centre from the centre of the Earth, will be the line of true motion, the true place will be at P, and the equation to be subtracted will be the angle AδD.

Now for the mean quadratures let the centre of the eccentric be at C, and the eccentric be GIEY; and let the Moon be not on the line GT, where the equations cease, but at ε. For the line of mean motion will be CεN, and of true motion AεO, and the equation to be subtracted the angle AεC; and the points of mean longitude will be I and Y, determined by the line ICY. In the other semicircle of the eccentric, however, the equations would be additive.

The greatest equations occur a little below the points of mean longitude. The equation of the first inequality, 4° 56′ 19″, occurs at conjunction and opposition, a little below the points M and Z, that is, with the Moon distant from the apogee of the eccentric by the arc HM or HZ of 98°, and with the centre of the eccentric placed at D; and the base of the angle of the equation is the least eccentricity AD, of 8580 parts. But the greatest equation of the

composite or second inequality, 7° 40′, occurs at quadrature, with the Moon placed below the mean longitudes I or Y, distant from the apogee G by the arc GI or GY of 95°, and with the centre of the eccentric placed at C; and the base of this greatest equation is the greatest eccentricity AC, of 13330 parts. The rest will be understood from what has been said. This hypothesis, if the Copernican numbers on which it rests are corrected and it is freed from the entanglements of solid orbs, is not unfitting.

Tychonis & Longomontani Hypothesis.

The Hypothesis of Tycho and Longomontanus.

VII. *Tycho Brahe*, with *Longomontanus* assisting him, saw that the eccentricity and the semidiameter of the Ptolemaic epicycle were too great, that the semidiameters of the Copernican epicycles were too great, and likewise the Moon's distances arising from them. He therefore adopted an eccentric with Ptolemy and two epicycles with Copernicus, but reduced the measures of both in proportion to the lunar distances he had observed, as was said in chapter 14. And since, besides the first and second inequality of the lunar motion, he had observed a certain third variation from the syzygies to the quadratures, and indeed greatest in the octants, he added a small circle of libration and hammered out a new hypothesis. It is explained by *Tycho* himself, volume 1 of the *Progymnasmata*, p. 116; by *Longomontanus*, Book 1 of the *Theorica*, chapter 6; by *Kepler*, before the *Ephemerides*, pp. 3 and 5, and in the Rudolphine Tables, pp. 79 and 82; and by *Nicolaus Mulerius*, in the *Isagoge* to the *Tabulae Frisicae*, part 2, chapter 3. Its measures are as follows, with the semidiameters of the Earth added from our chapter 14.

Tychonicæ Hypotheseos Synopsis

Synopsis of Tycho's Hypothesis

Intervals	Lines	In parts of a radius of 100000	Semidiameters of the Earth, 1
Semidiameter of the eccentric	A I	100000	56 30
Greatest eccentricity	A D	4348	2 28
Semidiameter of the small circle	A C	2174	1 14
Semidiameter of the greater epicycle	I L	5800	3 17
Semidiameter of the lesser epicycle	M L	2900	1 38½
Distance at conjunction and opposition, with the centre of the eccentric at A: greatest	A M	102900	58 8
The same: mean	A I	100000	56 30
The same: least	A X	97100	54 52
Distance at quadrature			
Greatest, at the apogee of the eccentric and at the perigee of the lesser epicycle		107248	60 36
Mean, as above		100000	56 30
Least, at the perigee of the eccentric and the apogee of the lesser epicycle, with the centre of the eccentric at D		92752	52 24

Intervals	Lines	In parts of a radius of 100000	Semidiameters of the Earth, 1
Greatest equation at conjunction and opposition		4° 58′ 27″	
Greatest equation at quadrature		7° 28′ 0″	

[Margin: Tycho's device in this theory.] The reason for these proportions is as follows. *Ptolemy*, Book 4, chapter 6, investigating the semidiameter of the epicycle through sets of three eclipses, had found it to be 5° 14′, of which the radius of the orb, but of the orb concentric with the world, is 60; that is, 8696 parts, of which the radius of the orb is 100000. *Tycho* bisected this quantity and took its half, 4348, for the diameter of the small circle AD and for the greatest eccentricity. He also divided the Ptolemaic epicycle into two epicycles, of which the greater should have a semidiameter twice as great as the semidiameter of the lesser, the semidiameter of the former being 5797⅓ parts and of the latter 2898⅔. In these, however, he chose round numbers: 5800 for the one, 2900 for the other. Using the quantity 8700, not as the sine of the greatest equation, as happens in the eccentric hypothesis, but as the tangent, he found the greatest equation at the syzygies to be 4° 58′ 22″ at 90° of anomaly. This he chose to make equal to the greatest latitude at the syzygies, just as the ancients, equating longitude with latitude, had made each of them 5°; but at an anomaly of 92° he made it 4° 58′ 27″. Bisecting this, he made the greatest equation to be added to it at quadrature 2° 29′ 13″, so that the whole was 7° 27′ 40″, or 7° 28′. Such, evidently, is the device by which *Tycho* and *Longomontanus*, attending also to the parallaxes and distances discussed in chapter 14, built up the lunar theory for us.

VIII. Let this, then, be the scheme, at least for the syzygies. In it let A, the centre of the Earth and of the world, and indeed, at the true syzygies, at which there is no eccentricity, be the same as the centre of the circle ITBV, at other times eccentric, now concentric. Let its diameter IB be the line of the true syzygies and quadratures. Let it be extended to P; and with the centre taken at I, let the greater epicycle LFEG be described at the interval IL, of 5800 parts of which the radius AI is 100000. With the centre taken at L on its periphery, at the interval PL of 2900 parts, let the lesser epicycle PNMO be described. For L will be the apogee of the greater epicycle and E its perigee, while P will be the apogee of the lesser and M its perigee.

Diagram. Tycho's lunar scheme at the syzygies, set in the left column with the text running narrow beside it. A large circle has a vertical diameter running through it from a small circle at the top down to B at the foot. Across the top of the large circle, centred on its upper point, is the greater epicycle; above that, centred on the greater epicycle's top, the lesser epicycle. A very small circle with R to its left and S to its right surrounds the point I on the large circle's top. Inside the large circle, on the diameter above A, is a small circle with H at its left, K at its right and C within it.

- P (top of the lesser epicycle); L (its centre); N (its left); O (its right)
- M (foot of the lesser epicycle, inside the greater)
- R (left of the very small circle); I (its centre); S (right of it)
- F (left, where the greater epicycle meets the large circle); G (right, the same)
- E, D (on the diameter, in that order downward, below the greater epicycle)
- H (left of the inner small circle); C (its centre); K (right of it)
- A (on the diameter, at the foot of the inner small circle)
- T (left, inside the large circle); V (right, inside the large circle)
- B (foot of the large circle)

[Margin: Motion of the centre of the eccentric.] Finally, taking the semidiameter AC of 2174 parts, let the small circle of the eccentricity AKDH be described. On its periphery let the centre of the eccentric move in the order of the signs, its motion beginning from A towards K, with the doubled motion of the Moon from the true place of the Sun, but with the Moon's motion equated by the prosthaphaeresis of the prior inequality; so that at every true full Moon and new Moon the centre of the eccentric is at A, the centre of the world, and at every true quadrature it is at D; and twice in a month the centre of the eccentric runs through the circumference of this small circle.

[Margin: Line of the mean and true apogee.] At the syzygies let the one line BI be that of the mean and true apogee; outside them, let the line of the mean apogee be the one that passes through the centre of the eccentric parallel to the diameter AD of the small circle, and that of the true apogee the one through the centres of the Earth and of the eccentric. Therefore anyone who wants to draw the figure for himself for the quadratures must, with the centre taken at D and the semidiameter AB, describe the eccentric, on whose periphery the centre of the greater epicycle is towards L for the Moon at apogee and towards B for the Moon at perigee; and he must draw the two epicycles, as above, on either side.

[Margin: Motion of the epicycles.] Further, let the centre I of the greater epicycle move in the order of the signs towards T, its motion beginning from the mean apogee, such as the point I or another on which the line of the mean apogee falls; let it move, I say, with the mean motion of the simple anomaly, discussed in chapter 23. Meanwhile let the centre L of the lesser epicycle likewise move with the motion of the simple anomaly, but against the order of the signs, from L to G, this motion always beginning from the point on the periphery of the greater epicycle on which falls the straight line drawn from the centre of the eccentric through the centre of the greater epicycle; and let it complete its motion in $27^{d}\ 13^{h}\ 18^{m}\ 35^{s}$.

[Margin: Motion of the Moon.] Finally, let the body of the Moon on the circumference of the lesser epicycle, that is, by the motion of that epicycle, move in the order of the signs from M to O, with a motion twice as great as the motion of the epicycles singly, or with the doubled motion of the simple

anomaly, so that it completes its period in $13^{d}\ 18^{h}\ 39^{m}\ 17^{s}\ 30'''$. This motion always begins from the point of the periphery of the lesser epicycle nearest the centre of the greater epicycle; for example, in our scheme, from the point M. The straight line drawn from the centre of the greater through the centre of the lesser epicycle either coincides, as in our scheme, or, outside conjunction, opposition and quadrature, is parallel to the diameter AD. Hence it happens that when the centre L is midway between L and E, that is, at F or at G, the Moon is at P, the apogee of the lesser epicycle. Whenever the centre of the greater epicycle is at the apogee I, that is, when the motion of the anomaly is 0^{s}, the Moon is at M, the perigee of the lesser epicycle; but if the anomaly is 6^{s} and the centre of the greater epicycle is at the perigee of the eccentric, the Moon is at the apogee of the lesser epicycle, and in both places it lacks all prosthaphaeresis. When, however, the centre of the greater epicycle is towards T or V, that is, when the anomaly is 3^{s} or 9^{s}, the Moon is at the mean distances from the apogee of the lesser epicycle, as at N or O; and so at the points of contact, at which the equations become greatest.

IX. [Margin: Small circle of libration for the third variation.] To save the third inequality, or Variation, which is nil at conjunction, opposition and quadrature and greatest in the octants, let there be a small circle RS, though it is rather to be conceived in the surface of the sphere of the fixed stars, as *Kepler* notes in the *Epitome of Astronomy*, p. 812. On its diameter let the centre I of the greater epicycle librate to and fro with a motion analogous to circular motions, as we saw done in Copernicus in Book 3, chapter 29. This motion is to be commensurable with the doubled true distance of the Moon from the Sun, and it is to produce a prosthaphaeresis, to be added, from conjunction and opposition as far as quadrature, but subtracted, from quadrature to conjunction and opposition, to the Moon's mean longitude from the Sun, so that the true place of the centre I of the greater epicycle may be established. The greatest variation is to be $40'\ 30''$, added in the first and third octant after conjunction, subtracted in the second and fourth octant. And so at the syzygies the centre of the greater epicycle will be at I, and in the sextile and trine aspects at R and at S. See, however, another scheme in chapter 29, no. 2.

Kepler notes, in the *Epitome of Astronomy*, p. 812, that the portion of $40'\ 30''$, taken as the whole sine, that answers to the sine of the doubled elongation from the Sun, already equated, is the Variation that applies. Moreover, says *Longomontanus*, just as the anomaly of the eccentric, accounted for by the small circle DHAK, depends on the distance of the Sun and Moon from the Earth, through a power flowing out of the epicycle of the Sun which varies the Moon's motion under the greatest angle of vision, so the third Variation comes from the Sun, according to the Moon's varying disposition towards it and towards the Earth. For while the Moon is joined to the Sun or opposed to it, greater efficacy is assigned to the Earth, as the primary centre of the lunar orbit, because of the solar rays falling on the Earth and reflecting themselves, together with the motive force, from it jointly and directly to I, the common centre of the

Moon's epicyclic system, which for that reason moves more hastily in the order of the signs. Around the quadratures, however, it moves more slackly, because the power of the solar rays is dissipated in the greatest obliquity of aspect. So far *Longomontanus*.

[Margin: A method of deriving the Moon's motions from this hypothesis.] Since the form of lunar calculation in this hypothesis is somewhat intricate, although *Magini* made it easier through his tables in the *Supplement to the Ephemerides*, and *Renieri* after him in the *Tabulae Mediceae*, I should still like to take from Tycho himself the geometric method of deriving the lunar motions from it.

EXEMPLVM.

Example.

X. *Let there be given: the Sun's mean place, 5^{s} 5° 50′ 21″, and its true place, in Virgo 4° 5′ 12″; the Moon's mean longitude from the Sun, 9^{s} 25° 13′ 13″; and its mean anomaly, 45° 37′ 29″. Now, as in the following figure, let the small circle AKD be described, whose semidiameter AC is 2174 parts, of which the radius of the eccentric OF is 100000. And since the doubled longitude of the Moon from the Sun is about 230°, if it is counted from A through K, the centre of the eccentric will fall at O, from which describe the eccentric FIB. The straight line FB, drawn through O, will mark the mean apogee at F, just as the straight line drawn through the centres AO, that is, AOE, will mark the true apogee at E. So count from F, in the order of the signs, the arc FI of the mean anomaly, 45° 37′ 29″; and from I describe the first epicycle, whose semidiameter IL is 5800 parts, of which OF is 100000. Having drawn a straight line from O through I as far as G, count against the order of the signs from G towards L the arc GL, likewise of 45° 37′ 29″; and from L describe the second epicycle, whose semidiameter LM is 2900 parts, of which IG is 5800.*

Diagram. The worked example of Tycho's lunar hypothesis, in the left column. A large circle with F at the top and B at the foot is joined by a vertical line through O. In its lower right part a small circle carries D at its top, K at its right, A at its foot and H at its left, with C at its centre and O on its upper left. At the upper left, on the large circle, stands a very small circle round I, with S and R on it; round I is a larger circle carrying G at its far left; on that circle, above, is a further circle round L, with M at its top, T at its upper right and N at its foot. Straight lines run from T to O and from T to A; from G through I to O; from A through O to E on the large circle; and from C to O, to H and to A.

- F (top of large circle); E (on the large circle, left of F); B (foot of large circle)
- M (top of upper epicycle); L (its centre); T (its upper right); N (its foot)
- G (far left of the greater epicycle)
- I (centre of the very small circle); S (right of it); R (lower left of it)

- D (top of the small circle); O (its upper left); C (its centre); K (its right); H (its left); A (its foot)

Extend ILM and count from N, in the order of the signs, the arc NT of the doubled anomaly, namely 91° 14′ 58″; for the Moon will be at T. To it draw the semidiameter LT, and connect the centre I with the Moon T by the straight line IT. For in the triangle ILT the sides IL, 5800, and LT, 2900, are given, and the angle NLT, 91° 14′ 58″; therefore by trigonometry you will derive the angle TIL, 26° 18′ 43″, and the side IT, of 6541 parts. Next draw from the Moon T the straight lines to the centres of the eccentric and of the Earth, namely TO and TA; for in the triangle TIO are given OI, of 100000 parts, TI, found to be 6541, and the angle TIO, 108° 3′ 48″, since it is the complement of the angles LIT, found to be 26° 18′ 43″, and GIL, which measures the arc GL of 45° 37′ 29″. So from the laws of triangles the angle TOI becomes known, 3° 29′ 16″, which is the epicyclic equation, together with the side OT, of 102217 parts, of which OF is 100000.

Now add the Sun's mean place to the mean longitude from the Sun, and the Moon's mean longitude from the vernal equinox becomes 3^s 1° 3′ 34″. When the epicyclic equation is subtracted from this, the Moon's equated longitude from the equinox remains 2^s 27° 34′ 18″. And when the same equation is subtracted from the anomaly itself, the equated anomaly remains 42° 8′ 13″. After this, from the Moon's longitude from Aries, epicyclically equated, subtract the Sun's true place, and the Moon's true longitude from the Sun is left, 9^s 23° 29′ 6″. Its double, a whole circle being cast out, is 7^s 16° 58′ 12″. So, this arc being counted from A through K and D, and the 6^s of the semicircle AKD being cast out, the arc DO remains, 46° 58′ 12″. Therefore, the semidiameter of the small circle CO being drawn, the isosceles triangle OCA is formed, in which the angle OCA, the complement of the angle OCD, that is, of the arc DO, is 133° 1′ 48″; and the angle at the circumference, namely OAD, is by the 20th of the third book of Euclid *half the angle OCD, and is 23° 29′ 6″. This, evidently, is the angle of the second anomalistic equation, to which, by the 28th of the first book of* Euclid*, the angle EOF is equal; it is to be subtracted from the coequated anomaly TOF, which was 42° 8′ 13″, and the anomaly equated a second time remains 18° 39′ 7″. This, subtracted from the semicircle, leaves the angle TOA, 161° 20′ 53″.*

Further, the side of the eccentricity OA is investigated as follows. Since the arc OHA was found above to be 133° 1′ 48″, its half is the arc AH, 66° 30′ 54″, and the right sine of this is 9171634. So as the radius is to this sine, so is the semidiameter of the small circle AC, of 2174 parts, to 1994 parts, whose double gives the chord AO, of 3988 parts, of which OF is 100000, namely the eccentricity. Thus in the triangle TOA are given the sides OA, 3988, and OT, already found above, 102217, and the angle TOA was 161° 20′ 53″. Therefore by the analysis of triangles there will be given TA, the distance of the Moon from the Earth, and in addition the angle ATO, 41′ 22″, which is the equation of the eccentricity, to be subtracted from the coequated longitude of the Moon from Aries; and the Moon's nearly true place remains 2^s 26° 52′ 56″.

Finally, since the motion of libration, commensurable with the double distance of the luminaries and beginning to be counted from its own apogee in the contrary

direction, that is, in the order of the signs with respect to the first epicycle, is 7ˢ 16° 58′ 12″, and the sine of its excess beyond the semicircle is 7309965: therefore, as the whole sine 10000000 is to 7309965, so the greatest Variation, or the semidiameter of the small circle RS, 40′ 30″, is to the Variation 29′ 36″, to be subtracted from the Moon's last place, so that the Moon's exact place remains in Gemini 26° 23′ 20″ on its orbit. But from this hypothesis, as from the Copernican, it follows that the lunar spots are seen at perigee and hidden from us at apogee, unless those who use an epicycle give the Moon a motion of rotation about its centre, which, however, Tycho *denies, volume 1, p. 742.*

CAPVT XXVII.

Chapter 27.

De Lunari Hypothesi Ioannis Kepleri, P. Alberti Curtij, Fabricij, Bullialdi, Lansbergij, Vendelini, & Recentiorum.

On the Lunar Hypothesis of Johannes Kepler, Father Albert Curtz, Fabricius, Boulliau, Lansbergen, Wendelen, and More Recent Authors.

I. *Kepler* had indeed decided, in his *Commentaries on Mars*, published in 1608, and in the first part of the *Epitome of Copernican Astronomy*, published in 1616, that the causes of the inequality of the celestial motions are not to be sought a priori from mere geometry and the bare lines of circles, whether these were concentrics with epicycles or eccentrics; but that they are physical, and that either all or part of that inequality is real, and not merely optical, that is, apparent to one viewing the motions from a centre other than their own. This physical cause, he held, is the Sun with respect to Saturn, Jupiter, Mars, Venus, Mercury and the Earth, and with respect to the Moon the Earth itself, through a power received from the Sun. [Margin: Kepler's variations in constructing the lunar hypothesis.] By these the planet in motion would be moved according to the disposition of magnetic fibres and according to the ratios of the balance and the lever, from which the path of the moved planet would turn out elliptical, as I shall say more clearly in the Scholia of this chapter.

But because of the manifold anomaly of the Moon he still used a double eccentric, with the distances of the Moon at apogee at opposition and conjunction different from the distances at quadrature; and a double eccentricity from the centre of the Earth. One of these, depending on the magnetic fibres of the Moon, was free of the Moon's phases and independent. The other depended on the shape of the illumination and its comparison with the magnetic fibres; it would arise twice each year and die away again, and its apogee, if it began from new Moon, would die away at full Moon, but if it began at full Moon, would finally die away, after 6 months, at new Moon; and so it was tied to the phases. On this pattern he composed the *Ephemerides* for 1617 and 1619, as is clear from their short prefaces to the reader.

But as one day teaches another, when he came to the *Ephemeris* for 1618 he noticed that among the physical causes the illumination could be placed, not of the Moon, but of the Earth, and that the two real and physical eccentricities could, without great inconvenience, be fused into one geometric eccentricity, whose apogee would have an unequal motion and whose eccentricity would vary through the year. On this pattern he computed the single *Ephemeris* for 1618, as is clear from his letter to the reader prefixed to that *Ephemeris*.

Finally, in 1620 April, from a deeper study of the physical causes, he chose a hypothesis in which there is a single orbit, that of an eccentric ellipse, and a single real eccentricity; and so the Moon's distances from the Earth are equal both at the syzygies and at the quadratures, namely apogee distances to apogee distances and perigee to perigee, and so on, as is clear from the *Epitome of Copernican Astronomy*, in the preface to Book 5 and pp. 614 and 820, and from the Rudolphine Tables, p. 79. So anyone who wants to understand this last hypothesis must consult the second part of the *Epitome of Astronomy*, published in 1621, Book 4, from pp. 549 and 609, and Book 6, the whole of its part 4, and also chapters 20, 25 and 26 of the Rudolphine Tables, besides what he teaches in Book 5 of the *Epitome* as common to the Moon and all the planets. As for what concerns the physical causes of accelerated or retarded motion, namely the magnetic fibres, the lever and the balance, partly common to the other planets and partly proper to the Moon, we shall set these aside for the Scholia of this chapter and refute them in part; or, as regards the motion in latitude, for the Scholia of Chapter 29. The rest, which concerns the geometrical or arithmetical computation of the Moon's motion in longitude, we shall set out here not a little more clearly and distinctly than the author himself.

[Margin: The Moon's threefold anomaly.]

II. In Kepler's hypothesis there is a threefold inequality or anomaly of the Moon, and therefore a threefold equation.

[Margin: 1. Free.]

He calls the first *Free* [Soluta], that is, not tied to the variety of the phases and of the Moon's elongation from the Sun; others call it the first and simple anomaly, and the Ptolemaics the epicyclic. But in this theory it arises from the invariable and real eccentricity of the Moon's elliptical orbit from the centre of the Earth.

[Margin: 2. Temporary Monthly anomaly.]

The second is called *Temporary Monthly* [Menstrua Temporanea], because it is tied to the variety of the phases that return through the month and make it up, yet in such a way that it neither returns nor is of the same quantity in every month. It depends both on the line of syzygies, which leaves the Moon's apogee, and on a variable eccentricity: not indeed a real one, that is, of the centre of the eccentric orbit from the centre of the Earth, but that of a certain monthly point, and of the semicircle of the whole orbit, from the plane of the circle of illumination, by which the Earth is illuminated by the Sun and which separates the diurnal hemisphere from the nocturnal.

[Margin: Variable eccentricity.]

This eccentricity, with its anomaly, decreases from greatest to least, indeed to none, over 6 months, and over about another six increases from none to greatest.

[Margin: Period of this anomaly.]

For that reason it is not semi-monthly, nor does it return twice in any month, as in Tycho; it has a half-yearly growth and a half-yearly decline, and so it is annual, and twice a year it dies and rises again. For when the Moon's apogee is at the quadratures, the equations for the whole month are simple, such as they are in the Free anomaly; and there is no eccentricity of the monthly or equating [Æquatorij] point, since the line of the apogee lies in the plane of the Earth's circle of illumination.

[Margin: Empty month.]

Kepler calls that month *Empty* [Vacuus], that is, lacking the monthly equation. But in the following month, when the Sun's quadrature has left the apogee (which is slower than the Sun, since it completes in the order of the signs no more than 6′ 41″ 4‴ a day), some eccentricity of the monthly point begins, and some small perceptible monthly equation, even if the Moon were at the mean apogee, where by the laws of the Free anomaly it ought to lack all equation. As the months succeed one another, the eccentricity of the monthly or equating point grows, and with it the monthly equation, until the line of syzygies, namely when the seventh month from the extinction of the eccentricity has begun, has reached the apogee. In that month the equations become greatest and join themselves to the equations of the Free anomaly, so that at last, at one and the same moment, namely at the Moon's quadratures, both become greatest.

[Margin: Full month.]

And that month is called *Full* [Plenus]. From there they decrease again through as many degrees.

[Margin: Monthly apogee.]

Just as a second eccentricity, called monthly, is given for this anomaly, so also a second apogee and perigee, called monthly, which is always near the Sun's apogee or perigee and on a line parallel to the line of syzygies. This, moreover, is the equation that Ptolemy called the Nod [Annutum] or [Greek: πρόσνευσιν (prosneusin, "inclination")] of the epicycle; the Arabs, the parallax of the epicycle; Copernicus, the prosthaphaeresis of the eccentricity of the second epicycle; and Tycho, the prosthaphaeresis of the eccentricity, all supposing in their hypotheses something optical and geometrical. For Kepler, however, it is purely physical and comes from purely physical causes, as he himself notes on p. 803 of the *Epitome*.

[Margin: 3. Anomaly.]

The *third* inequality and anomaly, which the older authors included in the second and confused with it, is called by Tycho the *Variation*, but by Kepler the *Perpetual Monthly* [Menstrua perpetua], because it returns every month and

is of the same quantity at the same distances of the Moon from the Sun and from the Earth. So it is called monthly on two grounds: because it returns the same every month, and because it is tied to the phases that make up the month. For it depends on the line of syzygies, indeed, but not on the variable eccentricity. It is nil at the syzygies and quadratures, and greatest at the octants, that is, at degrees 45, 135, 225 and 315, counted in the order of the signs from the conjunction of the Moon with the Sun. Because of it the Moon becomes slow twice in the month, namely at each quadrature, and fast twice, namely at each syzygy, and it is the faster the nearer it is to the syzygies.

The place of the Moon equated by the first equation, that of the Free anomaly, is therefore called the true place of the Moon at the syzygies; but outside them it is called the feigned place, or the place first equated. If the Temporary Monthly equation is added to it, it is called the nearly true place, or the place second equated. If finally the Perpetual Monthly equation is added, it is called the true and absolute place of the Moon. But what has been said in summary will be better grasped in the figures that follow. Before them, however, we shall set out in our usual way the measures of the distances and of the greatest equations. For the distances, though, see what was said in chapter 14, number 7.

TRIVM KEPLERI HYPOTHESIVM SYNOPSIS.

Synopsis of Kepler's Three Hypotheses.

Source	Distances	Lines	Radius 100000	Earth's semidiameters, ′
From the *Ephemerides*	Eccentricity, greatest, at conjunction and opposition	☩	6543	3 52′
From the *Ephemerides*	Eccentricity, least, at quadrature	☩	4362	2 36′
From the *Ephemerides*	Distance from the centre of the Earth, at conjunction and opposition, greatest	☩	106543	62 52′
From the *Ephemerides*	Distance from the centre of the Earth, at conjunction and opposition, least	☩	93457	55 8′
From the *Ephemerides*	Distance from the centre of the Earth, at quadrature, greatest	☩	104362	61 36′
From the *Ephemerides*	Distance from the centre of the Earth, at quadrature, least	☩	95638	56 24′
From the *Ephemerides*	Distance from the centre of the Earth, mean	☩	100000	59 0′
In the *Epitome of Astronomy* and the Rudolphine Tables, chapters 25 and 26	Perpetual eccentricity	A B	4362	2 32′
In the *Epitome of Astronomy* and the Rudolphine Tables, chapters 25 and 26	Semidiameter of the ellipse, greater	B D	100000	56 28′
In the *Epitome of Astronomy* and the Rudolphine Tables, chapters 25 and 26	Semidiameter of the ellipse, lesser	B E	99810	
In the *Epitome of Astronomy* and the Rudolphine Tables, chapters 25 and 26	Distance from the centre of the Earth, both at conjunction and opposition and at quadrature, greatest	A P	104362	59 0′

Source	Distances	Lines	Radius 100000	Earth's semidiameters, ′
In the *Epitome of Astronomy* and the Rudolphine Tables, chapters 25 and 26	Distance from the centre of the Earth, both at conjunction and opposition and at quadrature, mean	A E	100000	56 28′
In the *Epitome of Astronomy* and the Rudolphine Tables, chapters 25 and 26	Distance from the centre of the Earth, both at conjunction and opposition and at quadrature, least	A R	95638	53 56′
From the Rudolphine Tables, p. 98	Eccentricity	A B	3486	1 57′
From the Rudolphine Tables, p. 98	Distance from the centre of the Earth, both at conjunction and opposition and at quadrature, greatest	A P	103486	57 54′
From the Rudolphine Tables, p. 98	Distance from the centre of the Earth, both at conjunction and opposition and at quadrature, mean	A E	100000	55 57′
From the Rudolphine Tables, p. 98	Distance from the centre of the Earth, both at conjunction and opposition and at quadrature, least	A R	96514	54 0′

Greatest equation of the Free anomaly, 5° 0′. Of the Temporary Monthly, taken separately from the Free, 2° 30′; compounded with it, 7° 30′. Greatest Variation, 51′ or 49′.

III. Let this now be the first scheme, in which the line of apsides is PR, that is, of the apogee P and the perigee R; and let the semidiameter BP be of 100000 parts, of which AB, the real eccentricity of the centre B from A, the centre of the Earth, is 4362. From B let the circle PDRC be described, and through its centre B let another diameter DC be drawn at right angles. Let A be one of the two points of comparison [ex comparatione], that is, one of the foci or umbilici of the ellipse to be inscribed in the circle already mentioned; the other equally distant from the centre B, be S. For by what Apollonius demonstrated in the *Conics*, if from A or S two points E, V are taken on the diameter DC, at a distance from A or S equal to the semidiameter BP of the circle, the elliptical section will pass through those points; and EV will be the minor diameter, or latus rectum, of the ellipse, and PR the transverse side, or major diameter, or axis. In numbers, you will have BE if from the square of the semidiameter BP, that is AE, you take away the square of the eccentricity AB, assumed as the distance of one focus from the centre of the ellipse: for the square root of the remainder will give the quantity EB, which in this theory is 99810 parts, of which DB is 100000 and DE is 190. To find other points through which the elliptical circumference must pass, let perpendiculars to PR, such as GF and NQ, be drawn from any number of points you please on the circular periphery, say from G and N; with respect to the circle these are called sines, but with respect to the ellipse ordinates [Ordinatim applicatę]. But let the arc which those points bound be given, such as the arc PG, or that of its complement to the semicircle, such as NR. For if you make, as the whole sine DB is to the

minor semidiameter of the ellipse EB, so the sine GF of the given arc GP to HF, or so the sine NQ of the arc NR to OQ, you will have, by what Apollonius demonstrated, the points H and O through which the ellipse passes; and so for the other points. In this way the ellipse PHEMORTKVP can be described.

[Margin: Error in the circle commonly used in place of the ellipse.]

For although Kepler, in chapter 25 of the Rudolphine Tables and on p. 800 of the *Epitome*, says that such great subtlety is not necessary, and therefore uses perfect circles there, asserting that no error greater than 17″ can be committed in the Moon's place, he nevertheless adds the equations he constructed from the elliptical form; and it is better to become used to this from the beginning, if you wish to know exactly the foundations of the Rudolphine calculation. Therefore, with the line of apsides laid down as PR, EV is the line of mean longitudes; and whenever

Diagram. Kepler's lunar ellipse inscribed in a circle, standing in the left column. An outer circle and, just inside it, an ellipse share the vertical line of apsides from P at the top to R at the bottom; the horizontal diameter DC runs through the circle's centre B. A small circle centred on A, the centre of the Earth, passes through B at its top. Two horizontal lines cross the figure below DC, one through A from M to T, and one through Q from N and O to Y. Further lines join A and B to the points G and H at upper left, A to D and E at left, and A to K and L at right; a dotted line runs from B through the small circle towards Y; a line runs from O through A towards K, and through the small circle at Z.

Labels read (all as printed, on the figure):

- P (top): apogee
- G, H (upper left): G on the circle, H on the ellipse
- F (on PR, level with H): foot of the perpendicular
- S (on PR, between F and B): the other focus
- B (centre of the circle)
- D, E (left end of the horizontal diameter): D on the circle, E on the ellipse
- V, C (right end of the horizontal diameter): V on the ellipse, C on the circle
- K, L (right, below C): K on the ellipse, L on the circle
- I (on PR, between B and A)
- A (centre of the Earth)
- M (left), T (right): ends of the line through A
- Z (on the small circle, lower left of A)
- O, N (lower left): O on the ellipse, N on the circle
- Q (on PR, foot of the small circle)
- Y (lower right)
- R (bottom): perigee

Labels not read: none noticed. Show-through from the facing leaf's table lies under the lower half of the figure and was not read as part of it.

the Moon is at apogee P or perigee R, there is no need of an equation; rather, the true place of the lunar apogee or perigee is also the Moon's true place in the zodiac. But whenever it is at the points E or V, the distance EA, etc., of the Moon from the centre of the Earth A is mean, and equal to the major semidiameter BP, and there the physical equation of the eccentric anomaly is greatest. Its measure is the area of the triangle EAB, based on the whole eccentricity AB and having its vertex at the centre of the planet placed at E, reduced to parts of which the area of the whole ellipse is 360°; or the area of the companion triangle DAB, because of its equipollence, of which shortly. In other positions, however, as if the Moon is at H, to which the point G of the circle corresponds, when these points are joined to the centres of the Earth A and of the eccentric B, two equating triangles [Triangula Aequatoria] arise: one, a segment of the elliptical plane, namely HAB, based on the eccentricity AB and having its vertex at the Moon H; the other, a segment of the circular plane, namely GAB, likewise based on AB but ending at G, which nevertheless is equipollent to the triangle HAB, as we shall say. So the physical equation is the area of the triangles already mentioned, and Kepler here uses it alone; the optical equation, however, would be the angle H or G, if it had any use. Moreover, the line EV, passing through the centre of the ellipse, that is its minor diameter, is called by Kepler [Greek: διάκεντρος (diakentros, "through the centre")], or Diacentra; but the line MT parallel to it, passing through the centre A, at which he places the Sun in the hypothesis of the other planets, is called [Greek: διήλιος (diēlios, "through the Sun")], or Dihelia; here, however, since A is the centre of the Earth, it can be called [Greek: διάγειος (diageios, "through the Earth")], or Diagea.

IV. Now let the centre of the eccentric B move in the order of the signs, from B to Z, on the circumference of the small circle BZQ described around the centre of the Earth A at the distance of the eccentricity AB, carrying the apogee P forward, likewise in the order of the signs, with an equal daily motion of 6′ 41″ 4‴, so that it completes its period in 8 Egyptian years $311^{d}\ 6^{h}$. This is granted even though, with respect to the Sun, which moves faster, it may be imagined to move retrograde, and the line of syzygies, in which the Sun lies, overtakes the Moon's apogee after approximately 13½ months. The Moon itself recedes from the apogee P in the order of the signs, and moves on the elliptical periphery PHEO, etc., unequally, by reason of its curvature and of its varying distance from the centre A; although it may be imagined to move equally on the periphery of the circle PGDN, and to advance daily with the simple motion of the eccentric anomaly by 13° 3′ 54″, so that it completes its period in $27^{d}\ 13^{h}\ 18^{m}\ 35^{s}$.

[Margin: What the eccentric anomaly is.]

So let the Moon be at H. The eccentric anomaly will be the arc PH, or by equivalence and denomination the arc PG, which is measured by the angle GBP, namely the one counted in the order of the signs from the apogee P, or line of apsides, up to the place of the Moon, so long as it does not exceed a

semicircle. For if it does exceed one, as if the Moon is at Y, its excess above the semicircle is applied, namely the arc RY; or if it is beyond V, its complement to the circle. The measure of this anomaly is potentially contained in the elliptical area BHP, or by equivalence in the circular area BGP, reduced to degrees, etc.; and it is the one that in the Rudolphine Tables, p. 80, stands in the first column in whole numbers of degrees. The area of the equating triangle HAB, or by equivalence the area GAB, reduced to degrees or minutes, of which the whole area of the ellipse or circle is 360°, is the one that in the same column of the Rudolphine Tables is set, with its minutes, beneath the eccentric anomaly. Added to the eccentric anomaly, it makes up the value of the whole area AHP or AGP, that is, the mean anomaly, which in this hypothesis is the sum of the eccentric anomaly and the physical equation.

[Margin: Coequated anomaly.]

The coequated anomaly, on the other hand, is the angle at the Earth (in the other planets, however, at the Sun) A, made by the line of apsides and by the line drawn through the body and apparent place of the planet in its own orbit, or its complement to 4 right angles. This angle subtends the arc of the zodiac to which the lines already mentioned are understood to be produced, and in which this anomaly is properly counted, beginning from the line of apsides: as in our example, with the Moon placed at H, it is the angle HAP; but with the Moon placed at K, it would be the angle KAP. It shows the planet's true distance from the apogee, and it is the one that in the Rudolphine Table, p. 80, stands in the third column. Its difference from the mean anomaly is the equation, to be subtracted from the mean anomaly itself in the first semicircle PDR and added to it in the second, RCP, so that the planet's true distance from the apogee may be established. So if the true place of the apogee in the zodiac is known, the planet's true place from the equinox in its own orbit will also be known from this; or if the planet's mean place from the equinox was already known, then by adding or subtracting the equation already mentioned, the planet's true place becomes known; and for the Moon this holds at the syzygies, for outside them it will be the feigned place, and only first equated.

V. Three elements therefore [printed ego; recte: ergo] are needed to investigate the equation, in this hypothesis common at least to the Moon's Free anomaly and to the other planets. The *first* is the eccentric anomaly, or arc PG, and this is obtained by subtracting the planet's mean motion from the motion of the apogee already set up in the tables; and it is always as great as the area BGP (for we have already placed the Moon at H) in parts of which the whole area of the circle is 360; so it becomes known with no further device. The *second* is the physical equation of the eccentric anomaly, whose measure is the area of the triangle HAB or GAB. For this we need the eccentricity AB, as given, and the sine of the eccentric anomaly, namely the sine GF of the arc PG; for this sine gives the height of the triangle GAB over the line PAR, and this height is less than the whole sine DB, that is, than the height of the greatest equating triangle. So since they stand on the same base, the triangle

GAB is smaller and is contained in the triangle DAB. Let there therefore be taken within DAB itself the right-angled triangle EAB over BA, equal to GAB; and let EAB be such, with EB assumed by fiction not as the semidiameter of the ellipse but as a line equal

[Margin: Method of finding the physical equation, and by it the mean anomaly.]

to the sine GF. For if the sine GF, that is EB, is multiplied by half the eccentricity AB, the area of the triangle EAB, that is of GAB, will be known. This is reduced to degrees, of which the area of the whole circle is 360, if it is multiplied by 360 and the result is divided by the area of the whole circle; with the radius BD set at 100000 parts, this area is for Kepler 31415926536. He himself, however, on p. 686 of the *Epitome* uses this shortcut. First he finds the area of the greatest triangle DAB by multiplying half the eccentricity by the whole sine; this half in the Moon is 2181, and so the result is 218100000, namely the area of the triangle DAB. This he multiplies by 360°, resolved into seconds, 1296000″, and divides the result by the area of the whole circle, that is by 31415926536; and he finds the greatest physical equation of the eccentric anomaly, that is, the value of the greatest triangle DAB, to be 2° 29′ 57″. Then this value, first resolved into seconds, 8997″, he multiplies by the sine GF of the given arc PG, and divides the result by the whole sine. For the quotient gives the value of the triangle GAH [recte: GAB], that is the physical equation, to be added to the eccentric anomaly in the first semicircle PDR and subtracted in the second, RCP, so as to have the value of the whole area, namely GAP, that is the mean anomaly. So the number of seconds, 8997″, will always be useful.

The *third* is the angle GAP, or HAP, for which we need either the sides of the right-angled triangle, namely GAF, or the base GA, that is the planet's distance from A, and one of the sides. But the distance itself is gathered in two ways. The first of these is by the sides themselves about the right angle: for in the triangle GFA, right-angled at F, there is given BF, the second sine of the given arc PG, which, joined to the eccentricity AB, makes the side AF; with this and with the first sine GF of the same arc PG you will find trigonometrically the distance GA, in parts of which the sides GF and FA are known. But if you wish to work more accurately, first make, as DB the whole sine is to EB the minor semidiameter of the ellipse, so GF the sine of the arc PG to HF; with this side and AF you will find the most exact distance HA. But if the planet were below the Diacentra EV, yet above the Dihelia or Diagea MAT, say at K, the distance KA is to be sought in the right-angled triangle KIA, in which the side AI is obtained by subtracting from the eccentricity AB the second sine BI of the arc RL, known from the eccentric anomaly; and the side IL is the first sine of the same arc, which, if it is made to KI as DB to EB, will give the other side IK. If finally the planet is below the Dihelia or Diagea, as at O, the distance OA will be investigated in the right-angled triangle OQA, in which, from the sine NQ of the arc NR, which completes the anomaly PDN, the side

AQ will become known as above, by subtracting the eccentricity AB from BQ, the second sine of the arc NR.

[Margin: What Libration is, and the method of finding the planet's distance.]

Now the difference between the mean distance AE and any other, say AH or AO, Kepler on p. 679 of the *Epitome* calls the Libration, because, as in the motion of scale-pans, it is slow at the beginning and at the end, when the planet is most distant from A or nearest to it, but fast in the middle. By means of it, on p. 682, he seeks the planet's distance; for he says that when the second sine BF of the given arc PG is multiplied by the eccentricity AB and the result is divided by the whole sine, the Libration comes out, to be added to the mean distance in the upper semicircle, which is above the Diacentra, but to be taken away in the lower, so as to have the planet's distance from A. With the planet's distance obtained, then, as the base of a right-angled triangle, namely GA of the triangle GAF, and with one of the sides, namely GF or FA, the angle at A, namely the coequated anomaly, is easily found by trigonometry. Kepler, however, on p. 690 of the *Epitome* uses the base GA and the side FA, found as above; this he multiplies by the whole sine, and when the result is divided by the distance GA, the quotient gives the sine of the angle FGA, whose complement to a right angle is the angle GAF, that is the planet's coequated anomaly. Its difference from the mean anomaly is the equation of the place or mean motion of the planet, computed from the equinox. Without this, however, if the true place of the apogee is known, the coequated anomaly itself, that is the planet's true distance from the apogee, will show the planet's true place, by adding it to the place of the apogee. And what has been said so far applies to the five minor planets and to the Moon as Free, or indeed as Monthly but enjoying the greatest eccentricity, that is, when the centre of the eccentric B is on the line of syzygies: for these the scheme of number III serves.

VI. But if the centre of the eccentric, because of the faster motion of the Sun than of the lunar apogee, has receded from the line of syzygies, then, in order that the Monthly eccentricity and its variation, and the monthly point, and the monthly apogee, and the manner of the monthly equation may be understood, let there be a second scheme, in which A is the centre of the Earth; B the centre of the eccentric PDRC; PR the line of apsides of the Moon as Free, on which let P be the apogee and R

Diagram. The second scheme of Kepler's lunar hypothesis, standing at the head of the right column. A large circle carries H and M at the top, P at upper right, D at upper left, O at left, Q at right, S and T at the ends of a horizontal line lower down, R at lower left, K and N at the bottom and C at lower right. A small circle at the centre, about A, has its lower half hatched. A middle circle about A passes through B at its upper right. Two near-vertical lines run down from H and from M; the horizontal line OQ passes through G and B, and the line ST through X, A and I. Oblique lines join P through B and A towards R, D through B towards C, and O and D to points near A and C. A short dotted vertical line drops from near L to X.

Labels read (as printed, on the figure):
- H, M (top)
- D (upper left), E and F (on the short line running right from D)
- P (upper right)
- V, B, L (on or near the middle circle, upper part)
- O (left), G (on the line from O), Q (right)
- S (left), β (between S and X), X, A (centre), I, T (right)
- an R-shaped sort (a small bowl at the top and a long leg sweeping down to the right), right of I [?]
- a looped sort (a closed loop at lower left rising to an open hook at the top), below right of I [?]
- C (lower right)
- a sort in the shape of the sign of Aries (♈) on the middle circle, lower left, and Y on the vertical line below A
- R (lower left), K and N (bottom)

Labels not read: none noticed; show-through from the facing leaf lies behind the figure and was not read as part of it. The two flagged sorts are given as seen; the prose of this page names only one Y.

the perigee; and let DBC, drawn at right angles to it through B, be the line of mean longitudes. Let the hemisphere of the Earth in which A stands be nocturnal, and the rest diurnal; and let XI be the diameter of the circle of illumination separating these hemispheres. Produced to the periphery of the eccentric, it becomes the line ST; and let it be called the line of terrestrial illumination.

[Margin: What the line of illumination is.]

To this let OBQ be drawn parallel through the centre of the eccentric, and let it be called the boundary of the new and variable eccentricity. The line HAK, perpendicular to both and drawn through the centre of the Earth, serves as the line of syzygies, in which the Sun is at H, where conjunction occurs, and its opposite K, at which the Moon's opposition occurs; and the same line is the line of the monthly apogee, on which are the monthly eccentricity and the centre of the new eccentricity, deputy of the centre of the Earth, or the monthly equating point.

[Margin: Which is the monthly eccentricity, and what it is.]

In this scheme, however, the temporary monthly eccentricity is AG, and its centre is G. This eccentricity is not that of the centre of the eccentric B, that is AB, always the same; it is that of the whole semicircle of the eccentric divided in two by the line OGQ, that is, by the parallel to the line of illumination drawn through the centre of the eccentric. For the distance of this semicircle from the plane of the circle of illumination is called the Monthly and variable eccentricity, according to the variation in quantity of the line GA.

[Margin: What the monthly apogee is, and which is its line.]

Just as the perpetual eccentricity AB, produced beyond the centre B to the periphery of the eccentric, marks on it the point P, the apogee of the Free

anomaly, whose opposite R is the perigee, so the monthly eccentricity AG, produced beyond G, the centre of the monthly eccentricity, to the periphery, marks at H the monthly apogee, whose opposite K is the monthly perigee; and the line AH is called the line of the monthly apogee. It is always in the greater part of the eccentric divided by the line of illumination; but the line AK is the line of the monthly perigee, and it is in the lesser part of the eccentric.

VII. Eight cases must be noted, however, for which Kepler gives eight schemes in chapter 25 of the Rudolphine Tables; but for us these two will suffice. Of these eight cases, four are more notable, because of the four cardinal points of this eccentricity and monthly anomaly.

[Margin: When and where the Full month and the greatest monthly equation occur.]

For if the centre of the eccentric B is on the line of syzygies, so that PR, which is the line of apsides of the Free anomaly, becomes the line of syzygies and coincides with HK, with the centre B situated either at V or at Y, then the line of mean longitudes is parallel to the line of illumination, as in the earlier scheme, and the greatest eccentricity is the line AV or AY, and therefore equal to the Free eccentricity AB, that is of 4362 parts, of which BP is 100000. And then the month is called Full, because then the physical equation is the greatest among the monthly ones, and the same point performs a double office, namely of the centre of the eccentric and of the monthly equating point; and AV or AY that of a double eccentricity. But of these two cases, either the centre B is at V, and then the new Moon occurs in both apogees at P, coinciding with H, and the full Moon in the perigee at R, coinciding with K, without any sensible equation; and the quadratures occur in the mean longitudes at D and C of the earlier scheme, coinciding with OQ as regards the mean quadratures, for the true ones occur on the line of illumination MT of the earlier scheme, or at ST of the later. At these quadratures occurs the greatest equation of both the Free and the monthly anomaly, and the value of the greatest equating triangle ADB of the earlier scheme is doubled. The equation of the Free anomaly, compounded of the physical and the optical, is 5°, and the physical by itself is 2° 30′; so that when this is compounded, the greatest equation then becomes 7° 30′. The same things happen if the centre B is at Y, but then the new Moon occurs in the perigee of both the Free and the monthly anomaly, and the full Moon in both apogees. These are the two cases of the Full month, because in that month twice, namely at the quadratures, at the mean longitudes, the greatest equations of all occur; and throughout that month the monthly equations, compared with the monthly equations of other months, are greatest, inasmuch as throughout that month the Moon's apogee does not move perceptibly away from the line of syzygies except after many days.

[Margin: When and where the Empty month and zero eccentricity occur.]

VIII. If, on the other hand, the centre B comes to Z or β, the points of the line of illumination, so that the line of illumination ST becomes the line of apsides RP of the Free anomaly and divides the eccentric in two, then the line that

ought to be the limit of the monthly eccentricity, such as OGQ drawn through the centre of the eccentric, is not merely parallel to the line of illumination ST but actually coincides with it. There is therefore no eccentricity, since the monthly point G is extinguished in the centre of the Earth A; and then, though only at the one moment at which this happens, the month is empty of all monthly equation, even at the quadratures, because these occur on the line of illumination. One of the quadratures falls in the apogee, the other in the perigee, while the syzygies occur at the mean longitudes.

[Margin: Where the Variation is greatest, where it is zero?]

But both in these two cases and in the two earlier ones, the third anomaly, which is called the Variation, is not extinguished unless the Moon is placed in the syzygies or in the quadratures; it is greatest in the octants. The remaining four cases occur when there is some monthly equation and eccentricity intermediate between none and the greatest: for instance if the centre of the eccentric is at B or at δ, where the apogee P of the Free anomaly precedes the monthly apogee H, or at L or γ, where it follows. Now although the Moon runs through the whole periphery of the eccentric in any month, the Sun, or the line of syzygies, does not move away from the apogee of the Free anomaly so as to return to the same place except in almost 13½ months. In this chart the line of syzygies HK is imagined as immobile, on which supposition the motion of the centre of the eccentric and of the apogee of the Free anomaly must be imagined as retrograde. But if the line of syzygies is supposed to move in the order of the signs with the Sun, it will spend 6¼ months going from V, the point of greatest eccentricity, to β, the point of no eccentricity, and then as many months to Y, the other point of greatest eccentricity; and so the anomaly from V to β is half-yearly, and from V to Y about annual.

Diagram. The lunar eccentric with its monthly eccentricity, set in the left column. A large outer circle (the eccentric) has its centre at B. The vertical line HK passes through H at the top, E, V, G, the Earth's centre A and Y to K at the foot; a parallel vertical MN passes through B and I. A horizontal OQ runs through B, and a horizontal ST through A, carrying β, X, A, I and Z. A middle circle about A carries V at its top, L upper left, B upper right, β at left, Z at right, γ lower left, δ lower right and Y at its foot. A small circle about A, half hatched, stands for the Earth. Straight lines join D to F (horizontally, crossing HK at E), P through B and A to R, and various points to C at lower right and to D; a dotted vertical drops from the line through L towards ST.

- H (top, left of centre) · M (top, right of H)
- E, F (on the horizontal from D, upper centre)
- D (outer circle, upper left) · P (outer circle, upper right)
- V, G (on the vertical through A) · B (centre of the outer circle) · L (middle circle, upper left)
- O, Q (ends of the horizontal through B) · S, T (ends of the horizontal through A)
- β, X, A, I, Z (on ST; the last is cut as ζ or an italic Z)

- γ, δ (middle circle, lower part; δ is a looped sort) · Y (foot of the vertical through A)
- C (outer circle, lower right) · R (outer circle, lower left) · K, N (foot of the outer circle)

Labels not read: none noticed. All letters the text of this page cites stand on the block.

IX. Let us now come to those monthly equations that are not the greatest, which are purely physical, for the greatest monthly equation is equal to the greatest physical equation of the Free anomaly, that is 2° 30′.

[Margin: Method of finding the monthly equation.]

Let it be, as in our scheme, that the centre of the eccentric is at B, and the rest as above, so that the eccentricity is AG, and the distance of the Sun, as of the monthly apogee H, from the apogee P of the Free anomaly, reckoned in the order of the signs on the ecliptic, is the angle HAP, since the ecliptic is understood to be described from the centre of the Earth. But so that it may be reckoned on the periphery of the lunar eccentric, let a straight line MN be drawn through its centre B, parallel to HK; for by Euclid I.29 the angle MBP will be equal to the angle HAP, and therefore the arc MP, measured by the angle MBP, will be the distance of the Sun from the apogee of the Free anomaly, but numbered on the Moon's eccentric. From this, moreover, there will arise BI, equal to the monthly eccentricity AG. Now let the Moon be at D, and AD the line of the Moon's first-equated place; for by the steps of the following precepts you will arrive at the monthly equation for that moment [Æquationem menstruam temporaneam], and at the Moon's nearly true place.

[Margin: 1st precept: on the anomaly of the eccentric and the monthly argument.]

First. Find the place of the apogee P of the Free Moon, the place of the Sun, set at H or at K, and the Moon's place at D, first-equated with the equated anomaly according to the precepts given in no. 5. In this way the anomaly of the eccentric, or arc DP, will be known, and also the distance of the Sun, or of the opposite place H, from the apogee of the Free anomaly, namely arc MP, which the angle MBP, equal to angle HAP, measures. When this distance is subtracted from the anomaly of the eccentric, the monthly argument remains, namely arc DM; for the monthly argument of longitude is the arc between the Moon's first-equated place and the parallel to the line of the monthly apogee, drawn out towards the same side on which the monthly apogee lies. *So let DMP, for example, be 81° 42′ 24″ and MP 36°; for when the latter is subtracted from the former, there will remain DM, the monthly argument, 45° 42′ 24″. The anomaly first equated, moreover, will be 77° 18′ 30″; and let the Moon's apogee be at 24° Aries and the Sun at the beginning of Gemini.*

[Margin: 2nd precept: on the monthly eccentricity.]

Second. In the right-angled triangle GAB, find the monthly eccentricity AG by means of trigonometry: for in it the base AB is given, namely the whole eccentricity, of 4362 parts, and the angle GAB, equal to the angle MBP, and as

large as the distance of the Sun from the apogee of the Free anomaly, already known by precept 1. *In our example this was 36°. With these given, AG comes out as 3529 parts, of which AB is 4362, or BP 100000; and BI is also that much.*

[Margin: 3rd precept: on the area of the greatest triangle of such a month.]

Third. When IQ or IO has been drawn, there arises the greatest equating triangle of that month and moment, BIQ or BIO. You will find its area if you multiply half the monthly eccentricity, taken on the line BI, by the whole sine BQ. *In our example, half of the monthly eccentricity BI, of 3529 parts, is 1764½, and this multiplied by 100000 makes the area of the greatest triangle in such a month 176450000 square parts.*

[Margin: 4th precept: on the area of the greatest triangle in the Full month.]

Fourth. Have to hand the area of the greatest triangle in the Full month, such as is the triangle DBA in the earlier scheme, for it will always be useful in the other cases. *It is of 218100000 parts; for half the whole eccentricity AB, which is 2181, multiplied into the whole sine 100000, amounts to such an area.* Let this area now be valued at 60 minutes.

[Margin: 5th precept: on the monthly minutes.]

Fifth. Multiply by 60 the area of the greatest triangle treated in precept 3, and divide the result by the area of the greatest triangle in the Full month, and the monthly minutes of longitude [Scrupula menstrua longitudinis] will come out. *So in our example the area 176450000 multiplied by 60 makes 10587000000; and when this is divided by the area 218100000, there remain monthly minutes 48′ 33″.*

[Margin: 6th precept: on the area of the monthly equating triangle and its value.]

Sixth. Since the Moon is at D, drop the perpendicular DF from it to the diameter of the eccentric; this will be the sine of the monthly argument treated in precept 1. Look up its sine in the tables of sines. Then join the Moon D by straight lines to the four points A, G, I, B; for there will arise the triangle of the physical monthly equation, containing that equation in its area, which will be DAG, being based on the monthly eccentricity AG. But because the area of this triangle cannot be found by means of the sine DF, and one would need to know its portion DE, the companion triangle DBI is taken, which here is somewhat larger. Find its area, and its value in minutes proportional to the area of the greatest triangle in this month, as follows. Multiply the sine DF of the monthly argument by the monthly minutes treated in precept 5, reduced to seconds, and divide the result by the whole sine, and there will remain the proportional minutes of the equating triangle DBI. Then multiply these by the greatest physical equation of the Full month, which is 2° 30′, and divide the result by the value of the greatest triangle in the Full month, reckoned as 60, first reducing both this and the 2° 30′ to seconds; for the quotient will give the area and value sought, namely the physical monthly equation, which must however be corrected, as below, by the exempt particle [particulam exortem]. *So in our example the monthly argument was 45° 42′ 24″, whose sine 71577, multiplied*

by the monthly minutes 48′ 33″, that is by 2913″, makes 20850380l. When this is divided by 100000, there remain minutes 2085″. These multiplied by 2° 30′, that is by 9000″, make 18765000″; and when this is divided by 60′, that is by 3600″, there remain 5212″, that is 1° 26′ 52″. This, then, is the monthly equation, but it needs a slight correction.

[Margin: 7th precept: on the correction of the equation by the exempt particle.]

Seventh. Correct the monthly equation found in precept 6 by the exempt particle and its value, which is nothing other than the difference between the companion triangle, here DBI, and the truly equating triangle DAG. This is to be subtracted when the latter is smaller than the companion triangle, that is, when the Moon is in the semicircle in which the perigee of the Free anomaly lies, as in our case, in which the Moon D is in the semicircle MDN, in which R, the perigee of the Free anomaly, lies. For the semicircle is here bounded by the line parallel to the line of syzygies HK. But the value of the exempt particle must be added when the companion triangle is smaller than the truly equating triangle, that is, when the Moon is in the monthly semicircle in which the apogee of the Free anomaly lies: as if the Moon were at C, the truly equating triangle would be CAG, greater than the companion triangle CBI, and in the semicircle MCN would lie the apogee P of the Free anomaly.

Now the exempt particle can be found in many ways, but for brevity it is convenient to take in its place the uncia, or twelfth part, of the Variation, of which I shall speak shortly, to which it is very nearly equal, as Kepler says on p. 805 of the *Epitome of Copernican Astronomy*; especially since it could also be omitted because of its smallness, as is noted in the same place. For if the greatest Variation does not, for Kepler, exceed 5l′ or 49′, then the exempt particle, a twelfth of it, certainly does not exceed 4′ 15″ or 4′ 5″, and will very often be less.

[Margin: 8th precept: on the anomaly second coequated, and the Moon's nearly true place.]

Eighth. Add the monthly equation treated in precept 7 to the anomaly first coequated in the semicircle HDK, reckoned from the monthly apogee in the order of the signs; but in the other semicircle KCH, if the Moon were at C, subtract it. The anomaly second coequated will come out, or the distance of the Moon D from the apogee of the Free anomaly; and from its place in the zodiac you will learn the Moon's second-equated place, or nearly true place, and also its nearly true elongation from the Sun H. *So in our example of precept 1 the anomaly first equated was 77° 18′ 30″; if you add to this the equation of 1° 26′ 52″, treated in precept 6, the anomaly second equated becomes 78° 45′ 22″. And the Moon's apogee was at 24° Aries and the Sun at the beginning of Gemini. Therefore the nearly true place of the Moon D is Cancer 12° 45′ 22″, and the nearly true elongation from the Sun 42° 45′ 22″.*

[Margin: 9th precept: on the Variation and its use.]

Ninth. Find the Variation, or perpetual monthly equation, by multiplying the sine of the doubled nearly true elongation of the Moon from the Sun by the

greatest Variation, either Tycho's 40′ 30″ or Kepler's 51′ or 49′, and dividing the result by the whole sine. *So since the elongation by precept 8 was 42° 45′ 22″, doubled it is 85° 30′ 44″, whose sine is 99693; this multiplied by 51 gives 5084343. When this is divided by 100000, there remain 50′ 84/100, or 50′ 45″, for the Variation.* This Variation is added when the Moon is in the first or in the third quadrant, numbered in the order of the signs from the preceding new Moon, but in the second and fourth it is subtracted from the Moon's nearly true place, and the Moon's absolute and third-equated place results; if, however, you have subtracted the twelfth part of the Variation, which here is 4′ 15″, as the exempt particle, from the monthly equation for that moment, according to what was said in precept 7. *So from the Moon's nearly true place, which was Cancer 12° 45′ 22″, with 4′ 15″ subtracted but 50′ 45″ added, that is with 46′ 30″ added, the Moon's absolute place becomes Cancer 13° 31′ 52″.*

[Margin: Properties of the ellipse.]

X. Moreover, since in the above precepts Kepler relies on certain properties of the elliptical plane, partly demonstrated by Apollonius in the *Conics* and partly by himself in Book 5 of the *Epitome of Copernican Astronomy*, and indicated in chapter 20 of the Rudolphine Tables, it will not be out of place here to indicate them summarily, repeating here the figure of no. 3 and the explanation given there. They are these.

[Margin: 1st property of the ellipse.]

1. *If an ellipse is inscribed in a circle, the elliptical periphery cuts in the same ratio the ordinates applied to the major diameter of the ellipse and drawn from the periphery of the circle;* therefore, as DB is to EB, so is GF to HF, and NQ to OQ, and LI to KI, etc.

[Margin: 2nd property.]

2. *The major semidiameter of an ellipse, drawn from the extremities of the ellipse's minor diameter to the major diameter, marks on it by comparison two points equally distant from the centre of the ellipse, which are now called the foci or umbilici of the ellipse.* So if from E or V you draw to A or to S a straight line EA or VA, equal to BP, A and S will be the foci of the ellipse PERVP, and AB will be equal to SB.

[Margin: 3rd property.]

3. *Any two straight lines drawn from one point of the elliptical circumference to the foci of the ellipse are, taken together, equal to the major diameter of the ellipse; and to the same are equal two lines drawn from one of the foci to points of the ellipse opposite through the centre.* Thus the straight lines drawn from H to S and to A, and likewise the straight lines drawn from A to H and to the opposite point Y, are equal to the diameter PR; for H and Y are opposite through the centre, since the straight line HY, drawn to those points, passes through the centre B.

[Margin: 4th property.]

4. *If the planet's eccentricity is equal to the distance of either focus from the centre of the ellipse, the planet's path on the circumference of the ellipse will agree*

with the observed intervals and motions. For if the eccentricity is equal to the distance AB, its greatest distance from the centre A will be the line AP, with the apogee or aphelion at P; its least AR, with the perigee or perihelion at R; their difference AB; and the mean distance AE, at the points a quadrant distant from the extremes P and R. It is established, moreover, that at these points a quadrant distant from the highest and lowest apsis the planets are at a mean distance from the centre A, namely from the Earth in the case of the Moon and from the Sun in the case of the other planets, and that there they take on the greatest equations of their motions.

[Margin: 5th property.]

5. *The eccentricity of the centre of the ellipse from the foci is the mean proportional between the major diameter and the excess of the major diameter of the ellipse over the minor.* Kepler, p. 662 of the *Epitome*. So here AB is the mean proportional between DC and DE doubled. But the minor semidiameter EB is more easily had in numbers, if the square of the given eccentricity AB is taken from the square of the major semidiameter AE; for there will remain a square whose root will be EB.

[Margin: 6th property.]

6. *Ordinates applied to the major diameter of the ellipse cut both the ellipse and the circle circumscribed about it into proportional arcs and areas.* Kepler, p. 663 of the *Epitome*. Thus as the arc PG is to the arc GD, so is PH to HE; and as PG is to PH, so is GD to HE; likewise as GH is to HF, so is the area of the segment PGF to the area of the segment PHF; and as BD is to EB, so is the area of the quadrant PBD to the area of the elliptical quadrant PBE, and the area of the semicircle PDRBP to the area of the semi-ellipse PERBP. From this he concludes that the most exact measure of a planet's delay in elliptical arcs lies in the area of the elliptical segment comprised by the arc of the ellipse traversed and by the straight lines drawn to the extremes of this arc from the centre that moves the planet, that is, from the Earth A, which moves the Moon, or from the Sun A, which moves the other planets; and so the planet's delay in the arc PH is measured by the area of the segment HAP.

Diagram. The ellipse inscribed in its circumscribed circle, set in the right column. The vertical major axis PR runs through P at the top, F, S, B, I, A and Q to R at the foot, with the centre of the circle at B. A horizontal through B runs from D on the circle through E on the ellipse to V on the ellipse and C on the circle. Horizontal ordinates cross the axis: G on the circle and H on the ellipse above F; N on the circle and O on the ellipse below, at Q; and on the right K on the ellipse and L on the circle, level with I. A small circle or oval surrounds A. Lines radiate from A, among them to H, to E, to L, to T on the right, to M and O on the left and to Y at lower right; a dotted line runs from B through to Y, and four lines run down from G and H towards B and A.

- P (top) · R (foot)

- G, H (upper left, on circle and ellipse) · F, S (on the axis, upper part)
- D, E (left end of the horizontal through B) · B (centre) · V, C (right end of the same)
- I (on the axis below B) · K, L (right, level with I)
- M (left, level with A) · A (axis) · T (right, just below L)
- a sort like Z or ζ, left of the axis below A · O, N (lower left) · Q (on the axis) · Y (lower right)

Labels not read: none noticed. M, T and the Z-like sort stand on the block and are not cited in the text of this page.

[Margin: 7th property.]

7. *The plane of the circle circumscribing the ellipse, together with its areas, is equivalent to the plane of the ellipse and its areas, in measuring the delays of the planet in the arcs completed.* Kepler, p. 671 of the *Epitome*. For by property 6, as the area PDR is to the area PER, so is the area PGA to the area PHA, or PDA to PEA; and by permutation, as PER is to PHA or to PEA, so is PDR to PGA or PDA. And as the semiperiodic time which the planet spends on the semicircumference PER is to the time it spends on PH or on PE, so is the area PER to the area PHA or PEA. Therefore, as the half-time of the arc PER is to the time of the arc PH or PE, so is the area PDR to the area PGA or PDA. In these segments of the circular plane, therefore, there is an exact measure of the planet's delay in the arcs of the ellipse; and those segments become known by means of sines, as we taught above in the precepts.

Indicatur Hypothesis P. Alberti Curtij Societatis Iesu.

The hypothesis of Father Albert Curtz of the Society of Jesus is indicated.

XI. Kepler himself indicates this hypothesis in chapter 25 of the Rudolphine Tables in these words: *There arises also a fifth hypothesis, that of Father Albert Curtz, S.J., my Uranian friend, one of the most beautiful ingenuity, which, while it leaves me the balance for forming the elliptical path, nevertheless snatches away the lever; that is, it transfers the area of the triangle to the Ptolemaic equating point, one of the foci of the ellipse, around which the mean anomaly is ordered by equal angles; but which in its effect on longitude, which we are considering here, coincides entirely with the Ptolemaic equant.* It therefore doubles the eccentricity AB, so that AS in the preceding scheme is of 8724 parts, of which AB is 4362 and BP 100000. You have a specimen of this equant in Book 3, chapters 22 and 23.

Indicatur Hypothesis Dauidis Fabricij.

The hypothesis of David Fabricius is indicated.

XII. The same Kepler touches on this one also in a few words in the *Epitome of Copernican Astronomy*, p. 673, and in chapter 20 of the Rudolphine Tables, where he admits that the elliptical orbit of a planet can be produced by a composition of circles, and that Fabricius attempted this using the Ptolemaic

apparatus, namely by the motion of a small circle, regular around a centre not its own, that is, in reality unequal; or rather by two small circles, which would make the centre of the eccentric librate on the transverse or shorter diameter of the ellipse, and with a single libration indeed, but one requiring contrary motions of two equal circles; and that at the same time he so made the apsides librate that, by a count begun from the highest apsis and carried to the body of the planet, he measured the time on the periphery of the eccentric. But Kepler adds that he obtained neither pure uniformity of the motions nor complete precision, nor any saving of labour, and that he renounced physical causes of the motions.

Ismaelis Bullialdi Hypothesis.

The hypothesis of Ismaël Boulliau.

XIII. The ingenuity of this author can be seen in this hypothesis of the Moon, which he attempts to explain in Book 3 of the *Astronomia Philolaica*, especially in chapters 1, 10, 11, 12 and 14. It has two parts: one deals with the suppositions of the lunar motions and their threefold inequality and measure, the other with the exposition of the scheme by which the Moon, carried in an ellipse by the Earth's annual motion, undergoes those three anomalies. The first and simple one he calls *Periodic*, depending on the line of apsides and the Moon's departure from it. This anomaly is completed in about $27^{d}\ 13^{h}\ 12^{m}$, and for Boulliau it is partly optical, arising from the real eccentricity of the centre of the ellipse from the centre of the Earth, and partly physical, because of the unequal and real delays or accelerations of the motion made in equal arcs of the elliptical periphery, a periphery which he had shown in Book 1, chapter 20, to be equivalent to infinitely many equating small circles. The second he calls *Synodic*, depending on the Moon's digression from the synodic line or line of syzygies, which of course passes through the centres of the Sun and the Earth. The third, Tycho's *Variation*, he himself calls the *Reflection of the Apogee*. The first, indeed, has its origin from the Moon's own periodic motion and the position of the apogee under the zodiac; the second and third he calls monthly, and he attributes their cause to the translation of the lunar system, carried out by the Earth as it moves with annual motion in its ellipse, by virtue of which the centre of the lunar ellipse acquires a new and real eccentricity; and for that reason the second and third inequality is immediately optical, but mediately from a physical cause. So, to set out this hypothesis more clearly, we shall first state the suppositions and measures, and then present the schemes and their function in these anomalies.

[Margin: Suppositions of the first inequality.]

XIV. As for the first. As observations establish, the Moon in every true syzygy needs no equation; but in every mean syzygy it needs a single and simple equation, unless it is in the apogee or perigee, or on the line of apsides: for then it lacks every inequality, first as well as second and third, and the mean

and true place coincide. In the month in which the new or full Moon occurs on the line of apsides, the quadratures occur in the mean longitudes, or at about degrees 90 and 270 of the Periodic anomaly, and in such quadratures occur the greatest equations of the first inequality; and they are, by themselves, 5° 0′. But if the new and full Moon occur in the mean longitudes, or near 90° and 270° of the periodic anomaly, the quadratures occur on the line of apsides, one in the apogee and the other in the perigee, or very nearly; and the Moon then lacks sensible periodic inequality indeed, but not all synodic inequality, and sometimes needs some monthly equation. The beginning of the Periodic anomaly is the Moon's apogee, and from there to the perigee, throughout the first semicircle, its prosthaphaereses are subtractive; but from the perigee to the apogee, in the second semicircle, they are additive, if you wish to derive the true and apparent motion from the mean.

[Margin: Suppositions of the 2nd inequality.]

As for the second inequality, as is evident from observations: when the new or full Moon occurs in the apogee or perigee, or at 0° or 180° of the Periodic anomaly, or when the synodic line agrees with the line of apsides, at that moment there is indeed no first or second inequality; yet in that month the quadratures occur at the mean longitudes, or at degrees 90 and 270 of the Periodic anomaly, and at these quadratures occur the greatest equations, both Periodic, 5°, and Synodic, 2° 30′, so that compounded they are 7° 30′. For the Moon's place then observed does not agree with the place derived from the tables, even when corrected by the equation of 5°, but differs from it by 2° 30′. Boulliau confirms this by an observation of the Moon in conjunction with Aldebaran near quadrature and at degree 261° 11′ of the anomaly, from which greatest equation he also derives the eccentricity of the lunar orbit as of 4362 parts, as we shall say later. On the other hand, when the synodic line falls in the mean longitudes, or when the new and full Moon occur one at 90° and the other at 270° of the Periodic anomaly, and thus the quadratures occur at the apogee and perigee, throughout that month there is almost no second inequality, but still some, and greater and greater in the succeeding months. Moreover, the beginning of the second inequality, from which it begins to grow, is the mean longitude with which the synodic line lies, or 270° of the Periodic anomaly: for as the synodic line recedes from there, the second inequality begins to grow, and it grows until the synodic line reaches the apogee; and from there it decreases to 90° of the Periodic anomaly, where it is extinguished. In this semicircle of the monthly anomaly, from conjunction to opposition, the monthly equation is subtractive. Then from degree 90 the second inequality begins to grow again up to the perigee of the Periodic anomaly, with which the synodic line is joined; and from there it decreases to 270°, where it vanishes again. In this semicircle of the monthly anomaly, from conjunction to opposition, the monthly equation is additive. Hence it comes about that in every true syzygy both equations are nil; in quadrature they are greatest; and in the first quadrant of the Periodic anomaly both are

subtractive, and in the 3rd both additive; but in the 2nd the first is subtractive and the second additive; in the 4th the first additive, the second subtractive. Now since the Moon completes a synodic month in about $29^{d}\ 12^{h}\ 44^{m}$, and daily is elongated from the Sun by mean motion 12° 11′ 26″ 41½‴, while it moves from its apogee daily by mean motion 13° 3′ 54″, and the Moon's motion exceeds the motion of the line of syzygies daily by 52′ 27″, it is elongated from that same line by just as much:

and therefore, when the synodic line falls on the first of the mean longitudes, there are almost no additive equations of the second inequality; but on the second of the mean longitudes, no subtractive ones.

[Margin: Suppositions of the 3rd inequality.]

As for the third inequality, although Boulliau in the end subscribes to Tycho, and for the time being, until a greater light shines from experiment, admits that the greatest Variation is 40′ 30″, and that it reaches that size in the octants, in such a way that from the new Moon to the first quadrature and from the full Moon to the second quadrature it is to be added to the Moon's mean motion, but from the first quadrature to opposition and from quadrature 2 to conjunction it is to be subtracted, he nevertheless calls it into doubt because of 24 observations made in the octants counted from conjunction, when the Moon stood 90° on either side of the Periodic apogee. For he found that it is neither always greatest precisely in the octants, nor always 40′ 30″, but sometimes nearly 57′ and sometimes only 23′; from this he conjectures that the Tychonists, being in doubt, chose the middle value, namely 40′ 30″. Whether this Variation itself is anomalous, however, and undergoes some fourth inequality, he does not determine. But he admits that he tried without success to refer this apparent anomaly of the Variation to the motion of libration, by which the lunar spots appear now nearer to, now farther from, the edge: say the *Caspian* to the western edge and the *Lake of Posidonius* [Lacus Possidonij] to the eastern. And he urges those who possess the journals of the Tychonic observations to compare a great many observations made on successive days, and so to see whether this inequality lasts through some months, and when it recurs. Finally, to Wendelen's assertion that *the Moon's equations in the octants are equal to one another both above and below the mean longitudes*, he says that some of his own observations agree, provided however that the equation of natural days is admitted, which Wendelen does not admit.

XV. Now let us come to the diagrams of Boulliau's theory, first setting out, from our chapter 14 and from what he posits in Book 3, the measures of the distances, as in the following small table.

BVLLIALDICÆ HYPOTHESEOS SYNOPSIS.

Synopsis of Boulliau's hypothesis.

Distances	Lines	Radius 100000	Semidiameters of the Earth
Semidiameter of the ellipse: major	AC	100000	59 5′

Distances	Lines	Radius 100000	Semidiameters of the Earth
Semidiameter of the ellipse: minor	BC	99810	58 55′
Eccentricity from the Earth: greatest, in fig. 2	TXC	8724	5 10′
Eccentricity from the Earth: least, in fig. 1	TC	4362	2 35′
Distance from the centre of the Earth at conjunction and opposition: greatest	TA	104362	61 40′
Distance from the centre of the Earth at conjunction and opposition: mean	TB	100000	59 5′
Distance from the centre of the Earth at conjunction and opposition: least	TP	95638	56 30′
Distance from the centre of the Earth at quadrature: greatest	TVA	108724	64 15′
Distance from the centre of the Earth at quadrature: mean		100000	59 5′
Distance from the centre of the Earth at quadrature: least	TP	91276	53 55′

Greatest equation: Periodic 5°; Synodic 2° 30′; compound 7° 30′. Variation or Reflection: mean 40′ 30″; greatest 57′; least 24′, in the limits.

Let there now be two figures, the former for the syzygies, the latter for the quadratures, for from these the rest can be understood. In them let AP be the line of apsides and axis, or major diameter of the ellipse, of 200000 parts, in which A is the apogee and P the perigee; and let BD be the Diacentra [Diacentros], or minor diameter of the ellipse, whose half BC is 99810 of the parts of which AC is 100000. Let C be the centre of the ellipse and V the umbilicus, and let their distance CV be of 4362 parts. But let T be the centre of the Earth, with which the umbilicus is united in the syzygies; and therefore let VC be the least eccentricity, 4362. In such a case let the line of the mean longitudes, or the parallel to the Diacentra drawn through the umbilicus, namely MVN, coincide with the Diagea FTG, which of course is parallel to the Diacentra and drawn through the centre of the Earth. For thus in the syzygies the line BT or DT will be the mean longitude, taken for the Moon's mean distance from the Earth, as being drawn from the umbilicus V, placed at T, to the ends of the Diacentra; and both BT and DT will be of 100000 parts. And then also the points B and D will be mean longitudes, taken for the Moon's mean distance from the apogee in the orbit; and the arc ALB will be 90°, and ABPD 270°. In the quadratures, however, as you see in the latter figure, the umbilicus V does indeed lie on the line of apsides, but most distant from T, the centre of the Earth; and let VT, the distance of the umbilicus from the centre of the Earth, be as great as VC, the distance of the umbilicus from the centre of the ellipse; and so let VT be of 4362 parts, but the whole CVT double, namely 8724.

Diagram. The first of two figures set side by side at the head of the right column, the one the text assigns to the syzygies: an ellipse, pointed at top and bottom as drawn, with the vertical axis AP and the horizontal minor diameter BD crossing at C. A small circle lies below C on the axis, with its horizontal diameter HK and its centre I; the horizontal FG runs through its lowest point. Straight lines join L to C and L to T, and B and D to T. Two dotted lines run from the axis at T toward the lower right edge of the ellipse.

- A (top end of the axis)
- L (on the upper left of the ellipse)
- B (left end of the minor diameter)
- C (centre, above the crossing)
- D (right end of the minor diameter)
- H, I, K (left point, centre and right point of the small circle)
- F, G (left and right ends of the lower horizontal)
- M, T, V, N (along the lower horizontal, inside the ellipse, in that order from the left)
- P (bottom end of the axis)
- Z (on the lower right of the ellipse)

Diagram. The second figure, the one the text assigns to the quadratures: the same ellipse, with the horizontal BD through C, a shorter horizontal MN through V, and the lower horizontal FG. The small circle now stands with its top at V, its centre at I, its horizontal diameter HK and its lowest point on FG at T. A solid vertical runs from A to P through C; a second solid near-vertical runs down the left of it from S to Y, and a dotted line runs between them from E at the top to Q at the bottom. A straight line runs from R on the upper right through the top of the small circle down to F.

- S, E, A (at the top of the ellipse, in that order from the left)
- R (on the upper right of the ellipse)
- B, D (left and right ends of the upper horizontal)
- C (centre, on the upper horizontal)
- V, X (side by side at the top of the small circle)
- M, N (left and right ends of the second horizontal)
- H, I, K (left point, centre and right point of the small circle)
- T (below the small circle)
- F, G (left and right ends of the lower horizontal)
- Y, P, Q (at the foot of the ellipse, in that order from the left; the Q carries a long tail)

Therefore if the Moon, when it is joined to or opposed to the Sun, is at A or at P, no equation is needed, and in that month the quadratures occur at B and D, where the equations are greatest. But if in the mean syzygies the Moon is off the line of apsides, as at L, only a simple equation is needed; and the line of mean motion will be CL, while that of the true or apparent motion, to an eye at T, will be TL; and the optical part of this equation will be the angle VLC, as arising from the eccentricity TC. It differs from the physical part, since the Moon in equal times does not traverse equal arcs of the elliptical periphery with respect to the centre C of the ellipse, although it may traverse equal arcs of the circle circumscribing the ellipse.

Now if the Earth did not move with an annual motion, says Boulliau, this simple ellipse would suffice, as in the first figure, and no other second inequality, let alone a third, would be seen in it. But because the Earth too moves in its own ellipse, or annual orb, toward the east, the centre of the lunar system also,

which is the very centre of the Earth, is carried away from its own proper and connatural seat; and it no longer has a simple relation to the centre of the World and to the Sun, as it does in the syzygies, in which it stands on the same straight line with them, say on AIP or on LTZ. Instead it acquires a varied position, and the evection of the apogee occurs. For the umbilicus V moves on the periphery of the small circle from T toward K, with double the Moon's motion from the Sun, so that twice a month the umbilicus is united with the centre of the Earth, namely at conjunction and opposition, because twice a month the greatest monthly equations occur, in the quadratures. From this arises a real change of the eccentricity, which cannot come about by an approach of the centre of the ellipse to the centre of the Earth, since otherwise at times, off the line of apsides, the whole first inequality would perish, which is false. Nor can it come about by a libration on the straight line TIC, on which the umbilicus would librate, because, as Boulliau demonstrates in Book 3, chapter 1, it would at times happen that the second equation in that arc of the anomaly would be additive, say in the last quadrant, in which nevertheless, as we supposed from the observations at no. 14, it must be subtractive. Nor, finally, does this libration occur on the transverse line HK; otherwise, when a quadrature occurred at the mean longitudes there would be no monthly inequality, but when it occurred on the line of apsides it would be greatest, whereas it is established from observations that it is greatest at the former and nil at the latter. It remains, therefore, that the umbilicus of the ellipse moves on the small circle already mentioned, with double the Moon's motion from the Sun, in such a way that from the syzygy, where the umbilicus was at T, to the first quadrature, V comes through K to X, the semicircle of the small circle being completed; from the quadrature, V then descends through H to the centre of the Earth T, where the other syzygy occurs, and so one period of the small circle has now been completed once; and again from T, through K, it comes to X, where the second quadrature occurs, and finally from X, through H, it descends to T, where a new lunation begins.

XVI. Boulliau continues, and says that the Earth by its annual translation imprints on the whole lunar system a motion by which the centre of the ellipse C, following the motion of the umbilicus V, and with these the apogee A, is carried outside the place that by its nature it would keep in a lunar system not translated; and the Earth throws it away from itself, moving the perigee P toward itself. And from this motion, by means of the small circle already mentioned, arises the second inequality; and the apogee in the quadratures is at its greatest evections, and remains longer in them, at almost the same rate, because then the body of the Moon is on the periphery of the annual orb, or very near it, and moves with the same motion about the centre of the universe H as the centre of the Earth does. But in the syzygies the evections of the apogee grow or shrink most quickly, because the body of the Moon recedes furthest from the annual orb, and after conjunction strives against the motion of the centre of the Earth, heading toward the opposite quarter of the World; but from the full Moon on it goes along with it.

[Margin: Cause of the third inequality.]

The third inequality, or Variation, arises from the same translation; for it too is monthly, and is completed twice in a month's revolution. Thus, while the whole line ACVP, which passes through the centre of the ellipse, the umbilici and the apogee, is carried from the parallel position in which it would naturally rest if the Earth stood still, and does not remain in the parallel position SY which it held in the syzygies, the apogee by a certain natural appetite turns back toward the line of its natural position; and there fight against each other, on one side the love of rest in that line, on the other the translation impressed from outside. Therefore the place of the apogee settles midway between those parallel lines AP and SY, namely at E; and because this turning of the apogee occurs about the umbilicus V, as the centre of this turning, the perigee too is reflected to the opposite point at Q. Since, then, the motion of the evection, from new Moon to first quadrature and from full Moon to second quadrature, carries the apogee in the order of the signs from A toward E while the umbilicus moves through the semicircle TKX, the Moon seems to be advanced; for as the apogee advances, to which it is bound as to the limit of its motion, it too must be advanced. But while the umbilicus runs through the other semicircle, namely from the quadratures to the syzygies, the apogee is reflected against the order of the signs, and so the Moon is drawn back against the order of the signs, and a retardation of its motion occurs. Finally, in the octants, when the umbilicus is at K or at H, the greatest evection or reflection occurs, or what for Tycho is the Variation [Varietas]. And although on this sheet the small circle is represented as unmoved, it must nevertheless be conceived as moved, since its centre I is carried along with the Earth T.

[Margin: A supposititious confirmation of the Copernican system.]

With these supposed as certain, Boulliau in Book 3, chapter 1 confirms Copernicus's hypothesis, because from it there follows that this motion is impressed on the lunar system by the Earth, as being translated by an annual motion, but that a like motion is not impressed on the planetary system by the Sun, as not being translated; whereas in Tycho's hypothesis the Earth, though at rest, displaces the lunar system from its place, and yet the translated Sun does not displace the planetary system: statements which he thinks are made less coherently.

Gotifredi Vendelini Hypothesis, & ferè Leandri Bandtij Abbatis.

The hypothesis of Govaert Wendelen, and nearly that of Abbot Leander Bandtius [?].

XVII. The *Apotelesmata* of this outstanding astronomer have not yet appeared, and he has not brought to light his whole theory of the Moon; he hesitates, though now close to seventy, while I and others urge him in vain to publish it. But the part that touches the syzygies he has indicated, for the sake of eclipses, in two small works which he himself most generously sent me as a gift.

One is titled *Lampas Arcanorum Caelestium* [ARCANORVM CÆLESTIVM LAMPAS] [Greek: ΤΕΤΡΑΛΥΧΝΟΣ (tetralychnos, "with four lamps")]; the other, *Lunar Eclipses of Govaert Wendelen, Luminarcanus* [GOTIFREDI VVENDELINI LVMINARCANI ECLIPSES LVNARES], observed from the year 1573 to 1643, on which the Atlantic Tables are built, and their Idea is set out. In these he has most elegantly wrapped four doctrines in as many verses:

1. *In the heavens tomorrow's sum is equal to today's.* [In Cœlis par est hodiernæ crastina summæ.]
2. *An equal Oscillation moves the Moon at its highest and its lowest.* [Par summam atq. imam ciet Oscillatio Lunam.]
3. *A sextantal ratio gives the law to the apsides.* [Apsidibus ratio dat Sextantaria legem.]
4. *The Moon will triple the Year's turns at the Sun's apsis.* [Luna vices Anni triplicabit ad Apsida Solis.]

The sense of the first is that all natural days are equal to one another, and so there is no need of an equation of them. This he strives to confirm from observations of eclipses, which mostly favour this position remarkably; on this subject we said not a little in Book 3, chapters 31 and 32. The sense of the second verse is that the Moon's prosthaphaereses are oscillatory. For just as a pendulum drawn out of the perpendicular line and released to itself swings to and fro with equal librations, or with vibrations isochronous to sense, as we showed in Book 2, chapter 20, so the Moon's equations at a number of degrees equidistant both from the apogee and from the perigee are equal. For example, let the Earth be G, and ABC the path of the centre of the lunar body, which is B, describing by its mean motion the circle eccentric to the Earth. But the lunar body does not describe about B the epicycle HF, as in the Ptolemaic theory; instead it oscillates, or librates, from E to D through F, and in turn from D to E through F; hence the equations are equal at an equal distance from the line GFB. They would not be so if the Moon ran round through the upper space DHE of the epicycle. So writes Leander Bandtius [?], Abbot *Disimbergensis* [?] in Flanders, to my friend Uranicus [?], who communicated these things to me; and with such a diagram he sets out his opinion, which he strengthens by Govaert's authority.

Diagram. Set into the right column, with the text running down its left side. A circle centred at B, its vertical diameter running from H through B to F and continuing down as a solid line to a small round body at the foot of the figure. A shallow arc crosses the circle through B from A on the left to C on the right. Short dotted lines join B to D and to E on the circle. Two longer dotted lines run from D and from E down to the small body, whose interior is hatched and which carries the letter G.

- H (top of the circle)
- B (centre of the circle)
- A (left end of the arc, outside the circle)
- D (on the circle, left, just below the arc)

- E (on the circle, right, just below the arc)
- C (right end of the arc, outside the circle; heavily inked)
- F (just below the circle, on the vertical)
- G (inside the small body at the foot)

[Margin: Boulliau's vain attempt against Wendelen.]

Wendelen continues, and says that it has hitherto been believed that the lunar equation at 45° and 315° of anomaly is 3° 20′, but at 135° and 225° of anomaly 3° 46′. He himself, however, affirms that at all these four points of the anomaly, since the former stand equally distant from the apogee and the latter from the perigee, it is the same and precisely 3° 32′, which he contends is confirmed by an epilogism [Epilogismo] of eclipses; while at 90° and 270° the greatest equation is 5°. He is speaking, moreover, of the Periodic equations; for on the monthly ones he has not yet published his opinion, although Boulliau in Book 3, chapter 15 transfers his view to the monthly ones as well, and indeed in the octants, and in the same place tries to overturn it, saying that in this hypothesis of Wendelen's there must necessarily be no eccentricity with respect to the Earth, and no epicycle substituted for the eccentricity. For if an eccentricity is posited, the equating angles subtending it, and equally distant, one from the apogee and the other from the perigee, will not be equal; but if an epicycle, then under the same equating angle there will not be contained equal arcs of the epicycle, intercepted by the line of the true place and of the apsides, but the arc in the upper part of the epicycle will be greater than in the lower. But if Wendelen does not use the upper portion of the epicycle, as Abbot Leander indicates, or uses no epicycle, but a pure libration in the eccentric, then the weapon hurled from the upper arc of the epicycle, or from the pure eccentric, is aimed in vain. For the monthly equation, however, Wendelen employs an eccentricity, which he certainly states expressly as of 654465 parts, of which the radius of the eccentric is 10000000, but invariable and perpetually constant; and he says that to it there corresponds the greatest compound equation, 7° 30′, and that it is to the Sun's eccentricity, 174500, as 15 to 4; and that the Moon's apogee, most manifestly of all, advances, completing a period in about 3231^{d} 5^{h} 30^{m}. But whether, with the eccentricity posited as invariable, he posits the Moon's apogeal distance from the Earth as equal both in the syzygies [Copulis] and in the quadratures [Quadris] is not yet clear; and so I adjourn the verdict [amplius pronuncio] and await the publication of this whole theory. It only remains that, beyond the rule of the lunar motions that I gave from Wendelen in chapter 20, you should survey, from what was said in chapter 14 and from Wendelen's small works, the measures set out below.

Vendelini Hypotheseos Semisynopsis.

Half-synopsis of Wendelen's hypothesis.

	Radius 10000000	Semidiameters of the Earth
Semidiameter of the eccentric	10000000	60 0′

	Radius 10000000	Semidiameters of the Earth
Perpetual eccentricity	654465	3 55⅗′
Distance from the centre of the Earth at conjunction and opposition: greatest	10654465	63 55′
Distance from the centre of the Earth at conjunction and opposition: mean	10000000	60 0′
Distance from the centre of the Earth at conjunction and opposition: least	9345535	56 4⅖′

Greatest Periodic equation 5°; Synodic, together with it, 7° 30′.

Philippi Lansbergij Hypothesis.

The hypothesis of Philips van Lansbergen.

[Margin: The device of Lansbergen's hypothesis.]

XVIII. Lansbergen set out his lunar hypothesis briefly in the *Theoricae motuum caelestium* [Theoricis motuum Cælestium], chapter 3, in Book 1 of the *Uranometria*, and in the *Treasury of Observations*, pp. 95, 98 and 107. If considered more attentively, it seems to be built out of the Copernican hypothesis as transformed by Magini, and out of Tycho's small circle of libration. For the semidiameters of the Copernican epicycles, 10970 and 2370, melted into one, make Lansbergen's greatest eccentricity, 13340; and the semidiameter of the smaller epicycle, 2370, makes for Lansbergen the semidiameter of the hypocycle carrying the eccentric, even though he corrected and contracted the distances from the Earth. But why he gives the librating small circle a semidiameter of 7000 he nowhere explains. Let us first set out, in our usual way, the measures of this hypothesis, for which recall the lunar distances given in chapter 14.

LANSBERGIANAE HYPOTHESEOS SYNOPSIS.

Synopsis of Lansbergen's hypothesis.

Distances	Lines	Parts of the radius 100000	Semidiameters of the Earth
Semidiameter of the eccentric	MK	100000	59 5′
Semidiameter of the small circle	NI	2370	1 24′
Diameter of the same	PN	4740	2 48′
Eccentricity: greatest	AP	13340	7 53′
Eccentricity: least	AN	8600	5 5′
Semidiameter of the libration	DG	7000	4 8′
Distance from the centre of the Earth at conjunction and opposition: greatest	AN, with MK	108600	64 10′
Distance from the centre of the Earth at conjunction and opposition: mean	MK	100000	59 5′
Distance from the centre of the Earth at conjunction and opposition: least	NA taken from ML	91400	54 0′
Distance, etc., at quadrature: greatest	AP, with MK	113340	66 58′

Distances	Lines	Parts of the radius 100000	Semidiameters of the Earth
Distance, etc., at quadrature: mean	MK	100000	59 5′
Distance, etc., at quadrature: least	PA taken from ML	86660	51 12′

Greatest equation at conjunction and opposition 4° 58′ 44″; at quadrature 7° 40′.

Let A, then, be the centre of the Earth, and the greatest eccentricity AP of 13340 parts of which the radius of the orbit, MK or MD, is 100000, and the least, AN, 8600, and their difference the diameter of the small circle, PN, 4740. When this is bisected at I, with the semidiameter PI, 2370, let the small circle of the eccentricity PMNV be described, on which let the centre of the eccentric move in the order of the signs, beginning from N toward V, with the doubled motion of the Moon from the Sun, which is called the motion of the anomaly of the centre, and is daily 24° 22′ 53″ 22‴ 55⁗ 0⁗′ 20⁗′′. In accord with this motion let the Moon's apogee be continually shifted, so that in the mean syzygies the centre is at N, but in the mean quadratures at P. Outside these, however, it is elsewhere, say at M.

Diagram. Set into the left column, with the text running down its left side. A large circle, crossed by a vertical CE and a horizontal running from H on the left to B on the right; a diagonal runs from K at the upper left to L at the lower right, crossing the others near M. On the right of centre a small circle stands on the horizontal, with a short vertical through its top P, its centre I and N below. On the left a second small circle straddles the large circle's edge, with the horizontal passing through it.

- K (upper left of the large circle, end of the diagonal)
- C (top of the vertical)
- G, H, R (top, left and bottom of the left-hand small circle)
- D (inside the left-hand small circle, where the large circle crosses the horizontal)
- F (right of the left-hand small circle, on the horizontal)
- M (near the crossing of the lines)
- P, I, N (top, centre and below the right-hand small circle)
- V (right of I, at the right edge of the right-hand small circle)
- B (right end of the horizontal)
- A (on the diagonal, below N)
- E (foot of the vertical)
- L (lower right, end of the diagonal)

Therefore from the centre M let the eccentric KDLB be described, on which the Moon is elongated from the Sun in the order of the signs daily by 12° 11′ 26″ 41‴ 27⁗ 30⁗′ 10⁗′′; and parallel to the line AIP let the line CME be drawn, passing through M, the centre of the eccentric. For C will be the line of the mean apogee, which advances in the order of the signs about the centre A, in the order of the signs, daily by 6′ 41″ 3‴ 57⁗ 56⁗′ 24⁗′′. But the line drawn

through A, the centre of the Earth, and M, the centre of the eccentric, namely KL, marks the true apogee K and the perigee L; this advances in the order of the signs in accord with the motion of the centre of the eccentric. Besides these motions, the Moon's eccentric librates with a double libration: one from west to east and back, on the diameter GDR of the small circle GHRF, by force of which the Moon, placed on the eccentric, is carried now to R, now to G; and the motion of this anomaly is fourfold of the Moon's mean motion from the Sun, and is counted on the small circle already mentioned from H to G, against the order of the signs. The semidiameter of this small circle, namely DG, is of 7000 parts of which MK is 100000; or it is 4° 1′ 10″ of which the Moon's orbit BCDE is 360°. The other libration occurs from south to north at the centre A, and differs in no way from the libration of the obliquity of the zodiac.

Given, therefore, the Moon's equal motions, their equations and apparent motions are gathered from this, unless the line of the mean apogee and that of the true coincide together with the line of the syzygies or quadratures; for then no equation is needed. For there are gathered, for the given time, but equated, the mean motions of the Moon and the Sun and of the Moon from the equinox, together with the mean anomaly of the orbit.

EXEMPLVM.

Example.

XIX. For the sake of example, let the Sun's mean motion be 5^s 5° 36′ 2″; the Moon's mean motion from the Sun 9^s 25° 3′ 1″, and from Aries 3^s 0° 39′ 3″; and the mean anomaly of its orbit 1^s 17° 4′ 4″. Therefore in the following figure, having described the small circle of the anomaly of the centre NVPM, whose semidiameter IP is to be of 2370 parts, and PN 4740, let this be produced as far as A, so that NA is of 8600 parts of which the radius of the eccentric, soon to be described, is 100000. Then let the doubled motion of the Moon from the Sun, which, a whole circle being cast away, is 7^s 20° 6′ 2″, or 230° 6′ 2″, be counted from N through V toward M; and so let MP be 50° 6′ 2″; and let the centre of the eccentric be at M, through which and through A let the line of the true apogee K be drawn, which is KL; and through M let CE be drawn, parallel to PA itself, so that the mean apogee is at C. From it the arc CD of the mean anomaly of the orbit, counted in the order of the signs, namely 47° 4′ 4″, marks the Moon's mean place at D: from which let the small circle HGR be described, with semidiameter HD of 7000 parts of which MC is 100000. On it let the doubled motion of the anomaly of the centre, that is, the quadrupled motion of the Moon from the Sun, which, whole circles being cast away, is 100° 12′ 4″, be counted from H toward G as far as F. Therefore, with the quadrant HG subtracted, GF remains, 10° 12′ 4″. Its second sine DQ is of 98420 parts of which HD is 100000; but in parts of which HD is 7000, DQ is 6889. To this corresponds the arc DQ, 3° 57′. Subtracted from the arc DC of the mean anomaly of the orbit, which was 47° 4′ 4″, it leaves the arc CQ of

the coequated anomaly, 43° 7′ 4″. Its sine QB is 68348 and its second sine BM 72996, in parts of which MQ is 100000.

Now the arc MP was 50° 6′ 2″. Its sine MT is 76716 and its second sine TI 64145, in parts of which the radius IP is 100000. Therefore in parts of which PI is 2370, TI will be 1520 and MT 1818, and the whole TNA 12490. Now add BS, equal to MT, of 1818 parts; add it, I say, to the sine QB, and the whole QS becomes 70166. Likewise add AT to the second sine MB, that is, to its equal ST, and the whole AS becomes 85486. Therefore in the triangle ASQ, right-angled at S, the sides QS and AS are given, and hence the angle QAS, 39° 22′ 42″. Its difference from the angle QMC of the coequated anomaly, 43° 7′ 4″, is 3° 44′ 22″, namely the equation to be subtracted from the Moon's mean motion, which was 3^{s} 0° 39′ 3″. And so the Moon's true place, reckoned from the mean equinox, is 2^{s} 26° 54′ 41″. If, following Lansbergen, the additive prosthaphaeresis of the equinoxes, 15′ 58″, is added to this, the Moon's true place counted from the true equinox becomes Gemini 27° 10′ 39″.

But if you want to know the true place of the true apogee, use the triangle MTA, right-angled at T, in which, as appeared above, AT 12490 and MT 1818 are given; therefore the angle MAT is 8° 17′. Its difference from the angle QAS, already found above to be 39° 22′ 42″, is 31° 5′ 42″. Subtract this from the Moon's true motion, and there will remain the true place of the apogee K, in Taurus 26° 4′ 57″.

This hypothesis is certainly the simplest of the rest, and it does not overturn the position of the lunar spots, as the Copernican and the Tychonic do.

Diagram. Set in the left column, between line and line. A large circle with a small circle on its periphery at the upper left and a second small circle inside it at the lower right. Straight lines run from K and from C down through M, reaching the foot of the large circle at L and E as drawn; lines run from the small circle at the upper left to M and to A. Dotted lines run from the upper-left small circle horizontally through B to S, and from S vertically down to T.

- C (top of the large circle)
- K (upper left of C, on the large circle)
- H (left point of the upper-left small circle)
- G (top of the upper-left small circle)
- Q (inside the upper-left small circle, above D; the letter carries a tail)
- D (inside the upper-left small circle)
- F (right side of the upper-left small circle)
- R (below the upper-left small circle, where it meets the large circle)
- B, S (on the dotted horizontal, B left of S)
- P (top of the inner small circle)
- M (left of the inner small circle)
- T, I (inside the inner small circle, T above I)
- V (right of the inner small circle)
- N (foot of the inner small circle)
- A (below N)

• E, L (at the foot of the large circle, E left of L)

Diagram. Set in the right column, between line and line. An outer circle, a second circle slightly displaced from it, and a small circle near the middle. The horizontal EC and the vertical DF cross in the small circle; a second horizontal IM and a second vertical KL cross at B. A line runs from R through N to the centre region, and a line from P through the small circle to Q. A crescent-and-circle sign stands at N. Zodiac signs stand beside the four ends of the outer circle's diameters.

• D (top of the outer circle), with the sign of Cancer
• C (right end of the outer circle's horizontal), with the sign of Aries
• F (foot of the outer circle), with the sign of Capricorn
• E (left end of the outer circle's horizontal), with the sign of Libra
• R (upper left, beyond the outer circle)
• N (upper left, on the second circle)
• K (top of the second vertical)
• L (foot of the second vertical)
• I, M (left and right ends of the second horizontal)
• P (upper right, on the second circle)
• Q (lower left, on the second circle; the letter carries a tail)
• H (upper right of the small circle)
• A (inside the small circle, above the second horizontal)
• B (left point of the small circle, on the second horizontal)
• G (below the small circle)

Antonij Mariæ Schyrlæi de Rheita Hypothesis.

The hypothesis of Antonín Maria Šírek of Rheita.

XX. This author indicates his lunar theory in the book entitled *Radius Sidereomysticus*, Book 3, from chapter 2, or p. 151. Let A be the centre of the Earth, from which let the upper ecliptic [Ecliptica suprema] CDEF be described, whose beginning from Aries is at C. Around the same centre A let the small circle BGH be described, whose semidiameter AB, that is, the constant and perpetual eccentricity of the Moon, is of 1000 parts of which BK, the semidiameter of the eccentric, is 10000. For thus AB will be as great as the semidiameter of the solar globe. Let the centre of the eccentric, B, move in the order of the signs from B toward G, H, etc., by about 6′ 30″ daily. From B, at the distance BK of 10000 parts, let the Moon's ecliptic, or its eccentric KILM, be described; let the line of apsides be drawn through the centres A and B, and let Q be the apogee and P the perigee, which are to move with the motion of the centre B. Again, let the straight line IBM be drawn through B, parallel to the diameter EAC of the upper ecliptic, which is to be the line of the equinoxes, and likewise KBL, parallel to the line DAF of the solstices; and let the Moon's body move in its own ecliptic in the order of the signs, with a mean daily motion of 13° 10′ 35″.

This motion is to be counted not from C, the beginning of the upper ecliptic, but from M, the moving and librating beginning of the lunar ecliptic, because of the motion of the centre B. Let the motion of the apogee also be counted from M, through N, toward Q, in the order of the signs. Now let the Moon be at N, distant from the beginning M by the arc MKN, known from the tables of mean motion in the zodiac. Through it let the line of mean motion BN be drawn from the centre of the eccentric B, and the line ANR of true motion as far as the upper ecliptic; and let the motion of the apogee Q from the beginning M be counted by the arc MNQ. For when the motion MN, measured by the angle NBM, is taken from the arc and motion MNQ, there will remain the known arc NIQ, measured by the angle NBQ, whose complement to two right angles will be the angle NBA in the triangle NBA, in which, besides the angle mentioned, the eccentricity AB, 1000, and the radius BN, 10000, are given. Therefore by solving this triangle the angle N becomes known, that is, the prosthaphaeresis to be added to the mean motion when the Moon is in the semicircle ascending from the perigee P to the apogee Q, as in our case; but in the other semicircle QMP it is to be subtracted, so that the Moon's true place becomes known in that part of the zodiac in which the plane of the lunar orbit lies.

Šírek adds that the semidiameter of the ecliptic is of 3389830 parts of which the radius of the lunar orbit is 10000, or that it is of 100000 parts of which the latter is 295; and that others need epicycles or equations because they have not determined the correct eccentricity. And in Book 4, chapter 2, section 4, he adds that the 29½ days of the lunar month give us the Moon's mean distance from the Earth, namely 29½ diameters of the Earth, that is, 59 terrestrial semidiameters. From this, however, it follows that when the triangle NAB is right-angled, the equation is greatest, 5° 44′ 30″, and that the apogeal distance is equal both in the quadratures and in conjunction and opposition. This runs against the diversity of the parallaxes observed by us. Nor does there appear any excess of the equation outside the syzygies over the Moon's equations in the syzygies; nor will it ever show in the syzygies an equation of 5° 44½′. And so this hypothesis does not satisfy the phenomena. Let a synopsis of its distances nevertheless stand in the accompanying table.

Antonij Mariæ Rheitensis Hypothesis.

The hypothesis of Antonín Maria Šírek.

Distances	Lines	Radius 10000	Semidiameters of the Earth
Eccentricity	AB	1000	5 54′
Semidiameter of the eccentric	BP	10000	59 0′
Moon's distance from the centre of the Earth, both in conjunction and opposition and in quadrature: greatest	AQ	11000	64 54′
Moon's distance from the centre of the Earth, both in conjunction and opposition and in quadrature: mean	AN	10000	59 0′

Distances	Lines	Radius 10000	Semidiameters of the Earth
Moon's distance from the centre of the Earth, both in conjunction and opposition and in quadrature: least	AP	9000	53 6′
Greatest equation	5° 44′ 30″		

SCHOLIA.

Scholia.

[Margin: Rotation of the Sun and of others about their own centre.]

I. Since Kepler holds that the Moon is moved in a circuit by the Earth in almost the same way as the primary planets by the Sun, and derives much from their theory into the lunar theory, his universal principles must be touched on here and applied to the Moon's hypothesis. First, then, in the *Ephemerides* and in the Rudolphine Tables, but especially in the *Commentaries on Mars* and in the *Epitome of Copernican Astronomy*, Book 4, from p. 513, he teaches that the Sun is at the centre of the World, and is moved by no other motion than a rotation of 25 or 26 days about its own axis and centre, in the plane of the ecliptic, toward the east. He confirms this rotation, as we do in Book 3, chapter 4, from the sunspots, which are faster and larger around the middle of the Sun's disc, but more contracted and slower around the edges. Hence he thinks it probable, on p. 549, that all the primary planets, or at least those that have secondaries around them, are moved by rotation about their own centre, and indeed toward the east: namely Saturn and Jupiter, which their satellites surround, and the Earth, around which the Moon runs as its attendant and secondary planet.

[Margin: Periods of the planets.]

Moreover, he teaches that the Sun is a source no less of motion than of light, heat and life, and that from the centre of its body, animated by a material soul and kept in perpetual rotation, it emits a certain image [speciem], or a power and quality very like the magnetic, and diffuses it in every direction through the ethereal air, thinned in a uniformly difform way; and that this power, together with the Sun, turns with an equal rotation, and in the same direction, that is, toward the east, seizes the planets and drags them around. For thus a physical cause is provided for why the planets more remote from the Sun complete their periods more slowly, namely: Saturn in 29 Egyptian years $174^{d}\ 4^{h}\ 58^{m}\ 25^{s}\ 30'''$; Jupiter in 11 years $317^{d}\ 14^{h}\ 49^{m}\ 31^{s}\ 56'''$; Mars in 1 year $321^{d}\ 23^{h}\ 31^{m}\ 56^{s}\ 49'''$; the Earth in $365^{d}\ 5^{h}\ 49^{m}\ 15^{s}\ 48'''$; Venus in $224^{d}\ 17^{h}\ 53^{m}\ 2^{s}\ 14'''$; and Mercury in $87^{d}\ 23^{h}\ 15^{m}\ 36^{s}$; and also why the same planet is carried faster the nearer it comes to the Sun, and slower the more remote it is, since the Sun's power of course prevails more where it is stronger than where, because of the distance, it is weaker and thinner. Yet he thinks that each planet's passive inertia toward the motion of translation also contributes in its own measure to this diversity of motion, and by force of it their period is longer than the Sun's rotation.

II. To explain, and as it were to give shape to, this magnetic motion: just as in our magnet there is a double face and force extending along the same length, one by which it draws and attracts to itself the friendly face of another magnet, the other by which it repels the hostile face, so in the planets moved by the Sun there is a double power or disposition extended along straight lines, which he calls magnetic fibres [fibras magneticas]. One is friendly to the Sun's centre, namely the one that at the beginning of the World was turned toward the Sun, and which the Sun would at length have united to itself by continued motion, if it did not itself rotate and if the resistance of those fibres did not stand in the way while they strive for parallelism. The other is akin to the surface of the Sun but hostile to its centre, namely the one that was then turned away from the Sun toward the fixed stars. Therefore the Sun's power, falling perpendicularly on the fibres already mentioned, that is, on a position of the planets neither friendly nor hostile to the Sun's centre, neither attracts them nor repels them, but holds them, and so seized carries them around.

Meanwhile those fibres, because of an innate appetite for facing the same quarter, toward the east or the west, in which they were once placed, strive to keep their parallelism while they are moved by the Sun. And so the one end, the Sun-seeking [Solipeta], comes nearer the Sun, and the hostile one farther away; and thus the Sun can more easily draw the planet to itself, on that side, from the circumference of a perfect circle, until it inclines the fibre to a position tending directly toward the Sun.

[Margin: Reinhold ahead of Kepler in scenting out the elliptical path of the planets.]

From there, however, with the fibre resisting, its position is at length restored to parallelism; and from this arises the elliptical circumference, or oval path of the planet, such as Reinhold had described much earlier in the last lunar hypothesis, to say nothing of the hypothesis of Mercury, at the end of Peuerbach's *Theoricae*.

III. For the clarity of what has been said, let the following diagram stand, somewhat elucidated, from Kepler's *Epitome of Copernican Astronomy*, pp. 587, 596 and 654. In it let Cβ be the diameter of the eccentric, A on it the centre of the Sun, and B the centre of the eccentric. Let some planet be at aphelion C, and let its magnetic fibre be represented by the little arrow GF in its natural position, which the fibre itself and the whole body of the planet, compacted of such fibres, loves; and on the fibre let F be the end friendly to the Sun, or Sun-seeking, and G the Sun-fleeing [Solifuga]. Let the Sun's power fall along the straight line AC, perpendicular to the fibre GF; for at the beginning of this motion it will neither draw nor repel the planet, because of the equal distance of the ends G and F from the Sun. Let the planet then be drawn toward the east to H. For there, if the planet's fibre did not strive for a position parallel to the straight line GF, it would be in the position LK; but because it does strive for it, it inclines of its own balancing into the line HO. But because the end K comes nearer the Sun than it was at F, the Sun inclines it still more, into the position MN, and at the same time it is drawn inward; and so the centre of the

planet H departs somewhat from the perfection of the circular path, and is bent into the elliptical one.

Diagram. Set in the right half of the right column, with the text running down its left. A vertical straight line from C at the top to β at the foot, with a curved path bulging to the left between them. Five small circles stand on the path, each crossed by a short arrowed line: at the top C, at the upper left H, at the left D, at the lower left T, and at the foot β. A small rayed Sun with a face and the letter A stands on the vertical, below B. Solid lines run from the circle at H to O and from the circle at D to B, and a line runs from the circle at T to Y. Dotted lines run from N to P and from N to B, from R to A, and from V to Y.

- C (inside the top circle, on the vertical)
- G, F (left and right ends of the arrow through the top circle)
- K (on the path, upper right of the circle at H)
- H (inside the circle at the upper left)
- M (left end of the upper arrow)
- L (at the lower left of the circle at H, set on a slant)
- N (at the point of the arrow through H)
- O, P, B (on the vertical, in that order downward)
- D (inside the circle at the left)
- Q (at the far left of the circle at D, set sideways)
- a sign resembling Ω, above and to the right of the circle at D [?]
- R (at the lower right of the circle at D)
- A (in the rayed Sun)
- X (left end of the arrow through the circle at T)
- T (inside the circle at the lower left)
- V (at the point of that arrow)
- Y (on the vertical, in the circle at the foot)
- β (inside the circle at the foot)

When it has come to the mean longitude at D, let the fibre QR tend directly toward the Sun, and there let it depart most from the circular periphery, so that DB is the transverse semidiameter of the ellipse, and AD is equal to the major semidiameter BC. Then let the fibre gradually begin to be restored to its parallelism; at T let it take up the position XV, and finally at the perigee β let it be perfectly restored to its position and be parallel to GF. And by a similar libration, arising from the power of the Sun, which moves but repels the hostile part, and from the strength of the fibre struggling for parallelism, let the other semicircle be completed.

[Margin: Physical cause of unequal motion in the planets.]

For in this way the elliptical path of the planet will come about with the properties of the ellipse preserved, and a physical cause can be provided for why the planet, in equal times, moves more slowly at the greater distances, and why the motion is accelerated in the semicircle of the orbit that contains the perigee; and the inequality of the planets' motion is not all optical, or apparent

to us who are placed outside the centre of the celestial motions, as Aristotle and Copernicus contend, but part of it is physical, and comes from a real acceleration and retardation, from the Sun's physical motion and the balancing of the fibres; if these did not strive for parallelism, the motion would be perfectly circular. And without the superfluous apparatus of eccentrics and epicycles, the inequality will be produced by one most simple elliptical periphery. And if these two powers, of the Sun and of the fibres, do not perfectly compensate each other, the parallelism is restored not exactly at the apogee C but beyond it; and so the planets' apogee will appear to advance gradually.

[Margin: The ratios of the lever and the balance are included in the motion of the planets.]

IV. Now since the libration by which the Sun moves the planets comes about, according to Kepler, by the ratios of the balance and the lever, and is measured from the ratios of the balance, let there be supposed a steelyard from whose shorter arm a heavier weight hangs, and from whose longer arm a lighter weight in equilibrium with it; and as the longer arm on that side is to the heavier weight on this side, so, by permutation, is the shorter arm on this side to the lighter weight on that side. Then let the heavier weight and its power be imagined to be at the yoke [iugo] of the steelyard; for there it will have greater force to raise a weight that is less distant on the other arm than the same weight at a greater distance, for the reasons already known in the lever. Let the Sun, then, says Kepler in the *Epitome of Copernican Astronomy*, p. 579, be placed at A, as at the yoke; for it will have more strength to move a nearer planet P from its place than a more remote one C.

Again, since the raising of one pan in a balance and the lowering of the other is measured by the second sine of the angle that the arms of the balance make with the perpendicular line of the handle passing through the point of the yoke, or the centre of the balance, as Kepler shows in the same place, p. 650, and as is easily evident: for the raising of the lighter pan at the end of the arm, or viewed on the diameter of the balance, is as great as the second sine of the angle already mentioned, as will be evident, in the motion of the pans, to anyone who describes the balance in a circle made by its arms. Since, I say, the sine already mentioned is the measure of the motion of the pans, so the measure of the libration, by which the planets' fibres are inclined from their parallelism, is the sine of the angle that the line of the fibre makes at the centre of the planet with the perpendicular drawn to the major diameter of the ellipse. For that reason he teaches that the ratios of the balance are included in this libration. And so in the preceding figure the fibre QR, like the arm of a balance, is inclined below the perpendicular DB by the arc δR, or by the angle ADB, which measures that arc, whose sine is the sine of the angle already mentioned. He shows moreover (Kepler, on p. 596), that as AD (which, with the Sun placed at A, one of the foci, in the ellipse, is equal to the mean distance or semidiameter BC) is to BA, the eccentricity or half the libration, which is the measure of the angle ADB, so is the planet's semidiameter DR, taken as the

whole sine, to the sine of the angle of greatest inclination δDR; and likewise, as the sine HO of the arc CH completed by the planet is to the whole sine DB, so is the angle NHO to the angle RDδ. Then, from p. 654, he shows that the sine of the arc of the orbit completed by the planet is the sine of the libration, and thus that the sine HO of the arc HC is the measure of the libration. And on p. 656, using Pappus's demonstration in Book 5 of the *Collection*, prop. 36, and citing his own attempt in *Mars*, chapter 57, he shows that the measure of the libration accomplished, begun from the apogee C, is the versed sine [Sinum Versum] of the arc completed on the orbit. For as the whole diameter Cβ is to the whole libration of the semicircle, or the semidiameter BC to half the libration reckoned by the eccentricity AB, so is the versed sine of any arc on the orbit, say OC of the arc CH, to the part of the libration accomplished. From this it follows that, just as the motion of the pans in a balance is fast in the middle and slow at the extremes, so the planet's motion undergoes smaller differences near the apogee and perigee, but larger ones at the mean longitude.

[Margin: Application of what has been said to the Moon's hypothesis.]

V. Now what the Sun is to the primary planets, the Earth is to the Moon. For the Earth, turned toward the east by a daily rotation about its own axis, propagates from itself the magnetic power received from the Sun; and this power inclines to itself the friendly part of the lunar fibres and repels the hostile part, while the fibres meanwhile strive to keep themselves in a parallel position toward the same parts of the World. From this arises an ellipse, in one of whose foci is the centre of the Earth. For this, on p. 610, he shows this figure, in which the centre of the Earth, A, A, etc., is moved around the Sun by an annual motion toward the east through the great orb [Orbem magnum]; and the body of the Earth is turned about its own centre A by a daily rotation, likewise toward the east, carrying the Moon with it in the same direction. The Moon's fibres, indicated by an arrow, have the part akin and friendly to the Earth at BCDEFGHIKL, and the opposite part, as at MNOP, is hostile to the Earth. Let the Moon at BL be at the middle place between apogee and perigee, but at EFI a little before apogee and at HK a little before perigee; and let the Moon appear to the Earth new at C, full at B, E, H, and dichotomised at D.

Diagram. Set in the left column, between line and line. A rayed Sun with a face stands in the middle. Around it a curved line passes through six positions of the Earth, each a circle lettered A, half hatched, with a larger circle around it; on or beside each larger circle stands a small Moon, half hatched, crossed by a short arrow. Two of the larger circles carry two Moons.

- A (six times, inside each Earth)
- G, F (at the heads of the arrows through the two Moons at the upper left; no letter read at the tail of the arrow at G)
- M, E (tail and head of the arrow through the Moon at the upper right)
- C (head of the arrow through the lower Moon of that circle; no letter read at its tail)
- R (above the Moon just left of the Sun)

- I (head of the arrow through that Moon)
- a letter at the tail of the arrow through the Moon at the far left, reading O
- D (head of that arrow)
- N, B (tail and head of the arrow through the Moon at the right)
- H (head of the arrow through the Moon at the lower left; no letter read at its tail)
- L (head of the arrow through the Moon at the lower right; no letter read at its tail)
- P, K (tail and head of the arrow through the Moon at the foot)

Therefore while the Moon goes around the Earth, its mover, in the circle MC, DI, etc., always keeping the same periodic eccentricity, it acquires the first, free anomaly [Anomaliam Solutam] by reason of this eccentricity, because of which it becomes now apogeal, now perigeal. And if its apogee falls in the syzygies, then the magnetic image or power of the Earth, diffused into a larger orb and thinned by a uniformly difform decrease, is weakened; at perigee, on the contrary, it is stronger; and there it retards the motion, here it accelerates it. But if the Moon's apogee falls in the quadratures, it behaves in a middle way, feeling neither loss nor gain of the Earth's moving power or the Sun's promoting power. The second and third monthly anomaly it acquires from the varied relation of the eccentric to the lunar phases, and from the varied eccentricity of the semicircle of the eccentric, with respect to the circle of illumination on the Earth that divides day from night; and because it becomes accidentally now nearer to the Sun, now farther from it, it is also advanced by the Sun in a varied way by means of the illumination. And the Moon's apogee stays in the same sign through almost the whole year, and the fibres PK, etc., remain almost parallel to themselves through the whole circuit. But the Earth, with the Moon's heaven, passes from sign to sign each month. Hence it happens that the fibres are applied differently at different times to the circles of illumination, both lunar and terrestrial, made by the Sun, as you see in the arc RI, by which the end I of the fibre declines from the circle of illumination R.

[Margin: Monthly anomaly of the Moon from the Sun's illumination.]

VI. To explain better the monthly anomaly arising from the Sun's illumination, Kepler in the *Epitome of Copernican Astronomy*, pp. 560, 616 and 791, uses the following figure, in which BA is the Earth; the full Moon is at DC, the new at GH; the first quadrature at KI, the second at EF; the first octant from the new Moon is N, where the Moon is 45° from the Sun and appears horned; the second octant O, in which it is 135° from the Sun and appears gibbous; the third octant L, in which it is 225° from the preceding conjunction with the Sun but 135° from the following one, and appears gibbous; and finally the fourth octant M, in which it is 315° from the preceding conjunction with the Sun but 45° from the following one, and appears horned.

Diagram. Set in the right column, between line and line. A large central circle, its upper half hatched, is the Earth, with a vertical line through it from

the top of the figure to the foot. Around it runs a ring on which stand eight small circles, each with its upper half hatched and a stroke through it: at the top, the upper left and upper right, the left and right, the lower left and lower right, and the foot.

- D, C (left and right of the circle at the top)
- L (in the circle at the upper left)
- O (in the circle at the upper right)
- E (above the circle at the left)
- Q (on its side, at the far left of that circle)
- T (inside that circle)
- P (to the right of that circle)
- F (below that circle)
- B, A (left and right ends of the Earth's horizontal)
- R (to the left of the circle at the right)
- K (above the circle at the right)
- V (inside that circle)
- S (to the right of that circle)
- I (below that circle)
- M (in the circle at the lower left)
- N (in the circle at the lower right)
- G, H (left and right of the circle at the foot)

Tycho further observed that the Moon's mean motion, stripped of that second anomaly which exists in all planets because of the eccentricity of the orbit, has another one, and is always faster in the syzygies DC or GH and slower in the quadratures EF, IK than the reasoning of the eccentricity allows, whether the Moon is situated at the apogee, at the perigee, or at any other place on its eccentric. This anomaly, therefore, cannot be ascribed to the eccentricity. But neither can it be ascribed to the image or power of the Earth, turned by the terrestrial rotation through the annual orb LMNO; for at an equal distance of the Moon from the Earth its speed is uniform, both in the syzygies and in the quadratures. This anomaly therefore comes from elsewhere, from a cause connected with the Moon's phases; and these are produced by the Sun. The Sun therefore assists the Moon's motion around the Earth; just as elsewhere it was said, according to Kepler's view, that the Sun, by illuminating the terrestrial globe, spurs on by its own rotation the daily turning of the Earth, to such a degree that the Earth, which of itself would make only the 360 days of the archetypal year, nevertheless produces a year of 365¼ days. Therefore, while the Sun by its whirling spurs on the whirling of the Earth that moves the Moon, it spurs on the Moon as well.

For although, both in the syzygies and in the quadratures, half of the terrestrial and of the lunar globe is illuminated by the Sun, yet the Moon's illumination, which is bounded at the surface of the lunar globe, namely the part of the circle of illumination, as CD, GH, etc., is applied in a different way to the image of the Earth, or the magnetic power, turned through the circle

LMNO. For in the syzygies DC and GH, the image of the light and the image of the Earth are applied by contact, so that the portion DC or GH coincides through the whole arc DC or GH with the circle of illumination DC or GH; and because of this conjunction the Earth's power is greatly strengthened by the light, and accelerates the Moon's motion. But in the quadratures, the circle of illumination PQ or RS cuts the arc of terrestrial power EF in the single point T, or KI in V, at right angles. It is no wonder, therefore, if the application of light to the terrestrial power, at one point and almost nil, in no way strengthens the Earth's faculty and in no way accelerates the Moon. In other places, where this application and section occurs at oblique angles and imperfectly, there is some acceleration, but not so great. Since, however, the Earth's image, turned in the order of the signs and spurred on in that direction by the Sun's rotation, always moves the Moon in the order of the signs, whereas the force of the light can spur it even against the order of the signs, with respect to the centre of the Sun at new Moons, the illumination therefore strengthens the image of the terrestrial power separately, and without regard to the quarter in which the terrestrial power is directed.

Kepler next teaches, in the same place, from pp. 564 and 616: if the Moon were not advanced by the illuminating Sun, the lunar year would coincide with the Sun's sidereal year, and the Moon in it would re- turn to the Sun exactly twelve times. But because it is accelerated, after 12 synodic months completed in the sidereal year it also runs through 132° 45′, which, distributed through the second and the monthly equations, produce the greatest Variation in the octants, 51′. But if that increment is reduced to 127° 10′, which the Moon makes in order to arrive after 12 lunations at its initial place among the fixed stars, from which the Sun is distant after 360 days, the greatest Variation will be 49′. You will note also that when the apogee is in the quadrature EF and the perigee in the quadrature IK, and that month lacks a synodic equation, the parts of the luminary acceleration are equal in both syzygies, because of the Moon's equal distance from the Earth. In the following months, however, because of the inequality of the distances in the syzygies — as if the apogee should be at L and the perigee at N — some monthly equation arises, and throughout that month it is greatest in the quadratures. [Margin: The greatest size of the Variation.]

VII.

[Margin: A refutation of the objection from the lunar spots against the position of the fibres.]

Two more of the countless fictions of the Keplerian hypothesis remain to be pointed out, from p. 613 of the *Epitome*, where he says that it is not necessary for the Moon's magnetic fibres to be directed straight toward the Earth at the two opposite times of the period. It is enough that at those moments they should at least be inclined toward the Earth on alternate sides, and that this position of the fibres should stay parallel throughout the Moon's circuit; for in this way it can happen that now one face of the fibre and now the opposite one nods closer to the Earth. This inclination, if it is small, cannot be observed

by us on the lunar disc, because of the slightness of the new spots at the edges of the Moon. If it is somewhat larger, this alternation can be attributed to an interior globe of the Moon, enclosed within the crust of the lunar surface like a kernel and having a rotation different from the rotation of the crust. So it can happen that the outer shell always turns the same spots toward the Earth, and yet the magnetic fibres of the inner globe nod toward the Earth with their sides turned to it alternately. This astronomer of most audacious ingenuity has dared to imagine a kernel of the same kind in the Earth too.

[Margin: A second eccentricity, and a difference of distances tied to the phases, is rejected.]

To establish that the second inequality does not come from any second eccentricity, he says on p. 614 that this second eccentricity conflicts with the observed parallaxes, when the study of eclipses is added, along with the reasoning, taken a priori, from the proportion of the bodies to their distances. Therefore, although he thought otherwise elsewhere, he has at last found that the lunar distances do not change by reason of the phases, and that the distance is not tied to the phases in such a way that the greatest in the syzygies is less than the greatest in the quadratures, or the reverse. On the contrary, the apogeal distance at conjunction and opposition [☌ ☍] is equal to the apogeal at quadrature [□], and the perigeal to the perigeal. He confirms this further from the fact that the astronomers who made this change tied to the phases nevertheless found it smaller and smaller all the time. For Ptolemy had made it enormous. When Regiomontanus had censured it, Copernicus corrected and halved it, and transposed it from the form of an eccentric into the form of a second epicycle. Tycho, reducing it again, assigned part of it to the equating small circle, with a motion double that of the Moon from the Sun. Kepler at last removed this change of distances tied to the phases, and transformed the change from apogee to perigee from a monthly into an annual one, and from the half-monthly change of a semicircle into a half-yearly one. But let this be enough and more than enough of Kepler's fictions; yet I have only pointed them out, so as not to be prolix to no purpose.

VIII.

[Margin: Kepler's hypothesis is disapproved.]

For on many grounds I do not approve of Kepler's hypothesis, however great the engines of ingenuity that prop it up. First, because it does not represent the lunar motions, as we have found in observations of more recent eclipses and in the Rudolphine calculation applied to some ancient eclipses. Second, because the equality of the parallaxes and of the lunar distances in the syzygies and in the quadratures alike goes against observations most evident to me, let alone those of Tycho, of Copernicus and of very many others, since the distance is sensibly greater and the parallax smaller at apogees in quadrature than at apogees in conjunction and opposition; and conversely, the distance is smaller and the parallax greater at perigees in quadrature than at perigees in conjunction and opposition. But the man, inclined to fictions and to daring, and impatient of

observing, misuses Tycho's observations, to the point that he does not fear to drag them by force to the ideas and mysteries not of the heavens but of his own ingenuity, believing that these could not be convicted of falsity by any observer in future.

[Margin: A contradiction in Kepler's hypothesis.]

And surely whom would that inconstancy in the lunar parallaxes, which I reported in chapter 14, number 7, and in the preceding chapter toward the end, number 2, not strike with a huge suspicion of violated astronomical good faith? In the Rudolphine work itself, chapter 25, does he not set the real eccentricity at 4362 parts, of which the radius is 100000? From this follows a distance of the Moon at apogee of 104362 parts and at perigee of 95638. And yet in the tables on p. 98 he sets the Moon's horizontal parallax at apogee at 58′ 22″ and at perigee at 63′ 41″. From these there follows a distance from the Earth, in semidiameters of the Earth, at apogee of 57 54′, at perigee of 54 0′, and a mean of 55 57′. So if for Kepler this mean distance equals the radius 100000, then certainly the greatest is 103486 and the least 96514, and thus the eccentricity is 3486; and yet he has gathered the lunar equations from 4362 parts of eccentricity.

Third, an inclination of the lunar fibres so great that at the mean longitudes they tend toward the Earth conflicts with the phenomena of the spots. And as for the remedy for this, discussed in Scholium 7 and begged from a kernel of the lunar globe, there is no one who does not see how forcibly, not to say how hot-headedly, it has been hunted up.

[Margin: The complications and troubles of calculation in the Keplerian hypothesis.]

Fourth, because while Kepler parades the simplicity of a single elliptical eccentric, with a small circle carrying its centre around, and sings a triumph over vanquished epicycles, equants and doubled eccentrics — as may be seen in the Rudolphine Tables, chapters 20 and 26, and in the Letter to the Reader prefixed to Book 5 of the *Epitome of Astronomy* — in reality he falls into more complications, of many triangles, of equants and of the exempt particle [particulæ exsortis]. So much so, that he himself says in the Rudolphine Tables, chapter 26: "It is rather the purpose of the present work, by explaining the diagrams, to lay the foundations of the calculation, lest, if these are not known, the labour of calculation should become less pleasant, tedious enough in itself because of the remarkable variety of cautions brought forth by this hypothesis, not newly laid down but offered freely by Nature." And at the foot of the same chapter, in precept 125: "But since the equation of light is not extracted exactly enough for the taste of the curious, let them therefore take on themselves the labour, surely not a little troublesome, of extracting from the table the monthly minutes [Scrupula menstrua] and the Exempt [Exsortem], and of forming the equation by means of them." What if everything had to be worked out by geometrical niceties [apices Geometricos], without the help of tables?

Fifth, the application of the light that strengthens the Earth's image is nil in the quadratures for this reason, that it takes place at a point. But it takes place at

a point no less in every other position in which the plane of the circle separating the illuminated part of the Moon from the dark part cuts the circumference of the lunar orbit; for granted that it cuts at oblique angles, it still cuts at no more than a point. Therefore it cannot aid the Earth's image outside the syzygies, contrary to what Kepler lays down. Re-read what was said in Scholium 6, and you will grasp the force of our objection. So much for the things proper to the Moon.

IX.

Against the general hypothesis, common to the Moon and the other planets, not a few things tell. First, the annual and diurnal motion of the Earth, and the Sun's rest at the centre of the world from every motion τῆς φορᾶς [Greek: τῆς φορᾶς (tēs phoras, "of conveyance")], that is, of translation; many things to be adduced in Book 9 forbid me to subscribe to this. And as regards the Moon, I think it absurd that the Moon is placed among the secondary planets, as Kepler places it so as not to disturb the order of his system. Second, that in seeking physical causes he begs them from the likeness of the magnet, the balance and the lever — things surely too material and gross. And he is wholly taken up with seeking the efficient cause, when he ought to have sought the final cause, if he could; a cause that Aristotle, in *Physics* 2, recognises as the most physical of all [Greek: φυσικοτάτην (physikotatēn, "most physical")]. For the planets do not move along such a path, at such distances from the Sun and from the Earth, and with such a difference of speed and slowness, for this reason: that the Earth or the Sun strives to carry them along with itself or even to unite them to itself, and that they, in order to preserve the position of their fibres, partly resist this seizure and union. Nor is this a cause sufficient to settle our intellect. Rather, Divine Providence without doubt had certain ends in these motions, and architectonic and lofty ones at that, far nobler than to show us the magnetic librations of bodies. Yet Kepler conceals these, while he so binds the planets to material necessity and to the blind striving of these motions, like millstones to a mill, as to banish from them every mind that orders and directs these motions, and with arguments surely frivolous to proscribe the Intelligences from the motion of the heavenly bodies, as I shall show in Book 9.

X.

But Kepler nevertheless admits, in Book 4 of the *Epitome*, p. 516, that the consideration of the celestial harmonies, treated in the last chapter of Book 5 of the *Harmonics*, suggests conjectures about a moving Intelligence of the heavenly bodies. Even so, all that Keplerian harmony rests on the musical scale, neatly fitted to our ears, or on the distances of the 5 regular bodies, or on certain round numbers, which he counts as ideal and archetypal for this reason, either because they are round, or because they contain certain ratios purely mathematical, or at most harmonic, but with a harmony tied to our hearing. So the year, if you listen to him, ought to have been of 360 days and the month of 30 days, and the distances of the planets to the proportions flattering our imagination or pleasing to our hearing ought to have been

adjusted. [Margin: Kepler's inconstancy once more.] Hence it came about that he fell into plainly conflicting opinions. In the *Ephemerides*, p. 2, he defined the distance of the Sun from the Earth as 1800 Earth semidiameters, so that it would be thirty times the lunar distance of nearly 60 semidiameters, just as the Moon's monthly revolution around the centre of the Earth is thirty times the Earth's daily revolution around that same centre of its own. But in the *Epitome of Astronomy*, Book 4, from p. 478, and Book 6, p. 776, he makes the distance of the Sun 3469 Earth semidiameters and that of the Moon 59, so that it may be the mean proportional between the Earth's semidiameter and the distance of the Sun. But more against this harmony in Book 9, on the Harmonic System. Meanwhile I say that the harmony of that divine Harmonizer [Harmostæ] and the geometry of that supreme Demiurge are far too sublime to need measuring by the scale of Guidi Aretini [Guido d'Arezzo], or by the compasses and rule of Archimedes or of Euclid; and that God, Best and Greatest [D.O.M.], had other ends. Such indeed He had when He distributed the fixed stars, not into regular small plots bounded by figures, say of a quadrangle, a pentagon or a hexagon (which would of course have pleased Kepler's ideas uniquely). And if the year, for example, for an end that pleased God, is of 31574925 seconds, as Tycho holds, this number, or another closer to it but more exact, is round and perfect to the divine mind; or else it is to be reduced to further and more exact minutiae, so that it may be rounder.

[Margin: Kepler censured by Boulliau.]

Moreover, lest anyone should think that Kepler is censured on this point by me alone, or undeservedly, let him hear Boulliau, who says this in Book 3, chapter 14, of his *Astronomy*: "Hence I cannot wonder enough at Kepler's intemperance [Greek: ἀκολασίαν (akolasian, "intemperance")], by which, driven as if by the violence of some stormy spirit, he boldly departs from the authority of observations and passes over to the archetypal ratios of a cosmic music that he devises. As though what he has imagined for himself could command the bodies of the world, so that they should keep to it. Cosmic music is to be understood from the motions and their properties; those motions are not to be ordered from our musical ratios. For the symphonies of those motions admit other compressions [Greek: καταπυκνώσεις (katapyknōseis, "compressions")], condensations, intensifications and relaxations, different from ours." He rightly said "is to be understood"; for it is not to be perceived by the senses, or judged by the laws of sense.

CAPVT XXVIII:

Chapter 28

Anacephaleosis Præcedentium Hypotheseon; Iudicium de illis, & Indicium de Nostra.

A recapitulation of the foregoing hypotheses; a judgment on them, and an indication of our own.

I.

[Margin: The hypothesis of Ptolemy.]

So the diversity of the lunar distances and motions has driven astronomers into so many and so varied hypotheses. Ptolemy explained the first inequality by the Moon's motion on an epicycle, and the second by the motion of the epicycle's centre on an eccentric and its approach to the Earth and recession from it. But he moved the centre of the eccentric on a circle described around the Earth, using always the same eccentricity; and the sum of the semidiameters of the epicycle and of the eccentric [recte: of the hypocycle, per the book's errata][34] was for him 31342 parts, of which the radius of the eccentric was 100000. This hypothesis was worked by the Arabs, and also by Gherardo Cremonensi [Gherardo da Cremona], Sacrobosco and Peuerbach, into the matter and solidity of several partial orbs. But since the semidiameters already mentioned were found to be enormous, and no observations ever showed the lunar diameter, parallax and distance twice as great at perigee as at apogee, as the Ptolemaic hypothesis nevertheless required, this hypothesis was rejected by Regiomontanus; but he put no other in its place.

[Margin: The hypothesis of Copernicus.]

Copernicus therefore, rejecting the eccentricity, chose a concentric with an epicycle on an epicycle [Epicepicyclo]. He distributed the first inequality to the first epicycle and the second to the second, and restricted the aggregate of both semidiameters to 13340 parts, which is nearly half of the Ptolemaic aggregate.

[Margin: The hypothesis of Tycho and Longomontanus.]

Tycho, with the help of Longomontanus, kept the two epicycles of Copernicus on a concentric for the syzygies, and on an eccentric outside them, moving its centre on an equating hypocycle, and assigning the greatest eccentricity to the quadratures and none to the syzygies. He added besides, for the third inequality, the Libration outside the lunar system. The aggregate of the semidiameters of the epicycles and of the greatest eccentricity is for him 13048 parts, although in his hypothesis these three quantities never concur to make up the difference of the greatest distance from the least.

[Margin: The hypothesis of Magini and Lansbergen.]

Magini and Lansbergen, rejecting the epicycles, used a mere eccentric; but in place of the epicycle carrying the Moon they substituted a hypocycle, or small circle carrying the eccentric. This small circle does not reach the centre of the Earth with its periphery, as in Tycho, but stands wholly distant from it; and so it varies the eccentricity from the least to the greatest, which for Magini is

[34] in this copy a hand has struck through the printed Eccētrici and written Hypocycl· in the margin beside it

13330 parts and for Lansbergen 13340, so that he may keep close to Copernicus. But Lansbergen, for the Tychonic third variation and for other reasons, adds a small circle on whose diameter, not on whose periphery, the Moon is librated.

[Margin: The hypothesis of Wendelen, Leander and Šírek.]

Wendelen, Leander and Antonín Maria Šírek, also using an eccentric without an epicycle, nevertheless keep the eccentricity unvaried, and so describe a small circle carrying the eccentric around the Earth. But the first two try to uphold the lunar inequalities by oscillating librations from east to west and back, and the last by a double ecliptic, of which the lunar one is carried by the force of that small circle. The eccentricity is for Wendelen 6545 parts and for Šírek 10000, of which the radius of the eccentric is 100000.

[Margin: The hypothesis of Kepler.]

Kepler, having rejected the eccenter of the eccentric, or the difference of real eccentricity, let alone the epicycles, moves the Moon, as he does the other planets, on a simple eccentric but of elliptical periphery, whose centre is carried on the periphery of a small circle described around the Earth with a semidiameter of constant eccentricity of 4362 parts. He does not acknowledge an apogeal distance in the syzygies different from the apogeal in the quadratures, as he does not acknowledge a perigeal different from the perigeal. In place of equant circles, however, he uses areas of triangles based on an eccentricity not real but fictitious. And acknowledging the Moon's second and third inequality as physical and real, he refers it to the differing illumination by the Sun and to the application of light with the Earth's magnetic force moving the Moon.

[Margin: The hypothesis of Boulliau.]

Boulliau, finally, himself also using an ellipse, nevertheless admits a change of the real eccentricity, and for this he uses a small circle passing with its periphery through the centre of the Earth and carrying not the centre of the elliptical orbit but the umbilicus or focus of the ellipse. For it the least eccentricity is 4362 parts and the greatest 8724, of which the radius, or greater semidiameter of the ellipse, is 100000. The second and third inequality he ascribes to the Earth, carried by an annual motion and impressing on the whole lunar system a motion by which it thrusts the apogee away from itself and draws the perigee to itself, and produces a varying real eccentricity, etc. The hypotheses of the others are not yet fully established.

II.

[Margin: Objections against epicycles.]

What displeases me in the hypotheses already mentioned, so that I acquiesce in none of them, I think should be briefly indicated. The Ptolemaic, Copernican and Tychonic epicycles do not preserve the same face of the Moon toward us, nor the same position of the spots. On them the Moon's body moves in such a way that the spot Caspia, for example, which appears near the western limb in the first phases, would have to appear on the eastern edge when the Moon is already full, and from full Moon to interlunium none of those spots that we see on it ought to appear. This false supposition, manifestly contrary

to the phenomena of the spots both old and new, we can indeed remedy by a monthly motion of rotation, by which the Moon's globe, turned the contrary way around the centre of its own body, would keep the same face of the lunar disc toward us. But if no necessity presses from elsewhere, motions are not to be multiplied. Moreover, the Copernican and Tychonic epicycles twist the planet into excessively winding bends, contrary to one another; and the planet, or its mover, in a free and fluid heaven without solid orbs, as Tycho holds, must look to the fictitious centres of several circles, and those perpetually shifting. For while the Moon moves itself, or its mover moves it on the last epicycle, it must meanwhile move it toward that part toward which the centre of the smaller epicycle is imagined to move, and at the same time direct this very motion to where the centre of the greater epicycle is imagined to turn. And in this very act it must attend, in the Tychonic hypothesis, to the fictitious motion on the hypocycle near the Earth, and in these things must make use of the fiction of one of these motions in the contra- ry direction. Not to repeat here what we said about the measure of the distances a little earlier against Ptolemy, and in chapter 14 against him and Tycho: for in the former the difference of the apogeal distances from the perigeal is too great, in the latter too small, and it goes against observations of parallaxes carried out by us with the greatest good faith and diligence.

[Margin: What is to be said against the hypocycles of some?]

But the hypocycles described around the Earth in place of epicycles by Kepler, by Šírek and perhaps by Wendelen, in order to keep the Moon's eccentricity always the same in imitation of Ptolemy, go against the evident difference of distances between apogees in the syzygies and apogees in the quadratures. For the latter are greater than the former by an excess of two semidiameters of the Earth, and the perigees less than the perigees. Therefore, since this difference is not compensated by Ptolemy's epicycle or by something else equivalent, such as a libration of the planet upward or downward, the fallacy of the hypothesis is not avoided.

[Margin: Objections against eccentrics.]

So the eccentric of Magini and Lansbergen, which varies the eccentricity and carries the body of the Moon directly, avoids all the inconveniences already mentioned. But since these men and all who use an eccentric set another centre different from the centre of the Earth, in order to uphold the real equality not only of the distances but also, and above all, of the motions, in itself and with respect to their own centre, they imagine some point or other in the universe, displaying no other prerogative, whose good or dignity should prevail, and to which the motion of the planets should rather be ordered; and with respect to which they are to be said to move in themselves and primarily with an equal motion, but accidentally to move unequally with respect to the centre of the Earth. For that point, as was said in chapters 25, 26 and 27, is distant from the centre of the Earth, in Earth semidiameters: for Ptolemy, 10⅙; for Magini and Lansbergen, sometimes 5⅙ and sometimes 8; for Tycho at quadrature and for

Kepler, always 2½ very nearly; for Boulliau, at least 2⅗ and at most 5⅙; for Wendelen, always 3 11/12; for Šírek, always 5 11/12; and among those who vary the eccentricity it has an inconstant distance. Where in the world, then, is this point, so noble that it demands to be regarded in itself and primarily by the planet's mover? In the elementary sphere? (For this must be said, if you deny that the heaven of the Moon extends beyond the perigeal distance of the lunar body.) Then either in the air or in the fire, since it is outside the terraqueous globe. So a reason must be given why the Moon should move in itself and equally in relation to the air or the fire and some point of it, but accidentally and unequally in relation to the earth and the water. No such reason, I believe, can be devised with sufficient probability. For if you say that the air and the fire have a greater kinship, both in place and in quality, with the Moon, which is on the border of celestial and elementary bodies, many will not agree, who assert rather that it is another Earth, or more like the terraqueous globe; on them see chapter 2. Nor indeed do any living creatures live, or any meteorological effects occur, in that part of the air, let alone of the elementary fire, in which the centre of the eccentric lies, on account of which the lunar motion should have been directed there. I confess that I tried to set one of these points on the surface of the Earth and another on the surface of the elementary sphere, and that I imagined many other similar things; but in vain, because the measure of the distances, perceived by me through observations of parallaxes, could not in good faith be adapted to these fictions. But you will say that this centre was set there for the reason that the motion of a celestial body had to be uniform and equal, and it cannot be defended as such unless the centre of this motion is far distant from that centre in which the same motion appears unequal. Yet here, from this very invented position of the centre, something is already assumed which is both disputed among astronomers and which I shall show below, with greater probability, to be false. This argument holds in its own measure against epicycles, devised so that the physical and real equality of planetary motion should be safe on every side, and against that part of the elliptical orbit with respect to which some physical equality of motion at least, if not all of it, is sought. For in so far as Kepler and Boulliau acknowledge a physical inequality in this motion, they do not fear the blow of this argument.

[Margin: Objections against the elliptical orbit.]

What then do we find lacking in their form? Namely, that they should give a reason why the planet had to move in an ellipse, and in its motions had to regard only one of the two foci, or umbilici, the other being neglected and empty, and why it should besides regard the centre of the ellipse, however far that may be from the Earth, and though it has no character of good in the universe for which it should have been regarded; a reason, moreover, that does not already suppose that the motion must take place in an ellipse, or assume other uncertain things as certain. Add to this that in Boulliau's form the Earth is often outside both umbilici of the ellipse, and so three points must frequently be regarded: namely the centre of the Earth, the centre of the ellipse, and one

of the umbilici. Nor is there any need for me to press again here either the motions of the Earth, on which those forms rest, or the equality of the distances in the syzygies and quadratures, assumed by Kepler falsely and against the good faith of the observations; or the mystical trifles of the magnetic fibres and of the terrestrial crust and kernel, invented by Kepler to prop up his hypothesis; for enough has been said of these in the Scholia of the preceding chapter.

III.

[Margin: The heavenly motions are physically unequal, and a moving Intelligence regulates them.]

Since, therefore, we cannot avoid the inequality of distances that is manifest in the planets' approach to the Earth and in their recession, and the senses clearly perceive that the motions of the planets are not equal in all their parts, and no reason a priori, whether efficient or final, stands against this inequality, we shall acknowledge a physical inequality in them. And, with very wise men, we shall attribute the immediate efficient cause of this motion to Intelligences rather than to blind nature, because it is the work of a Mind that knows the end and the idea of that inequality. But because this controversy about the inequality of the heavenly motions, about their hypothesis and their moving cause, is universal, it will be treated by us more fully and more opportunely in Book 9, which is on systems and hypotheses; there we shall also blunt the frigid arguments that Kepler devised against moving Intelligences.

IV.

Now, however, for custom and for I know not what necessity of founding the calculation of the planets on some geometrical figure, and until the observations of Tycho are published — whose edition our Father Albert Curtz directs by order of the Emperor — so that from these, and from those of others and our own, made at very many eccentric places of the Moon, a more accurate form may be set out in volume 3 of the *Almagest*, we propose the following very simple hypothesis. We propose it not as true and such as shines forth in the mind of the Intelligence moving the Moon, but such as we judge suitable, so that from the fiction of an equal motion with respect to the centre of the eccentric we may, for a given variation of the real eccentricity, try out the equations to be applied to the mean motion, so that the true motion, or one very nearly true, may result. But the measures must first be set out, for which one must recall what we established in chapter 14.

[Margin: Kepler too, in the *Ephemerides*, no. 20, admits that from the parallaxes observed by Tycho the greatest equation in the syzygies comes out as 5° 1′; although, not trusting this subtlety and serving his own novelties, he sets it at 5° 0′.]

NOSTRAE FICTAE HYPOTHESIS SYNOPSIS.

Synopsis of our fictitious hypothesis.

A, centre of the Earth; B, centre of the eccentric at ♂ ☍; G, centre of the eccentric at □	Letters of the lines	Distances, to radius 100000	Distances, in semidiameters of the Earth	Horizontal parallax
Semidiameter of the eccentric	B C	100000	59 0′	
Eccentricity, greatest, at □	A G	13053	7 42′	
Eccentricity, least, at ☍ & ♂	A B	8745	5 10′	
Diameter	B G	4308	2 33′	
Distance of the Moon from the centre of the Earth at ♂ & ☍, greatest	A C	108745	64 15′	53′ 30″
Same, mean	A O	100000	59 0′	58′ 16″
Same, least	A D	91255	53 45′	63′ 55″

RESIDVVM TABVLAE PRAECEDENTIS.

The remainder of the preceding table.

A, centre of the Earth; B, centre of the eccentric at ♂ ☍; G, centre of the eccentric at □	Letters of the lines	Distances, to radius 100000	Distances, in semidiameters of the Earth	Horizontal parallax
Distance of the Moon etc. at □, greatest	A H	113053	66 42′	51′ 32″
Same, mean	A Q	100000	59 0′	58′ 16″
Same, least	A K	86947	51 20′	66′ 56″

Greatest equation at ♂ & ☍: the angle AFB, 5° 1′, at the point F, and at the arc of anomaly CNF, of 95° 1′, to which corresponds the arc CFV, 264° 59′; since AB is 8745, the sine of the arc FN is 5° 1′.

Greatest equation at □: the angle APG, 7° 30′, at the point P, where the anomaly HNKP is 262° 30′, to which corresponds the arc 97° 30′, where the equation is equal; since AG is 13053, the sine of the arc MP is of 7° 30′.

[Margin: Explanation of the figure for the conjunctions and oppositions.] Let A, then, be the centre of the Earth and B the centre of the eccentric in the syzygies, so that BC is the semidiameter of the eccentric, of 100000 parts, and AB the least eccentricity, of 8745 such parts. Let the eccentric CFDE be described from B, and let C be the apogee, with AC the Moon's greatest distance from the centre of the Earth in the syzygies, and D the perigee, with AD the least distance in the syzygies. Now let a perpendicular be drawn to A from the periphery of the eccentric, namely FA, and let it be joined to B. For in the triangle ABF, right-angled at A, the angle F will be the greatest equation, since the angle F is the greatest of all those having their vertex on the periphery of this circle and subtending the eccentricity AB as their base. This greatest equation occurs in the syzygies when the Moon is at F, a point below the mean longitude, that is, at the end of the arc of anomaly CNF, 95° 1′. For since AB is 8745 of the parts of which BC, that is, the whole sine, is 100000, this same AB is the sine of the arc NF, of 5° 1′. The mean longitude taken as the planet's mean distance from the apogee C is the arc CN, of 90°. But the mean longitude taken as the planet's mean distance from the centre of the Earth, A,

is the line AO; for it the point O is sometimes taken, that is, the place at which the planet holds its mean distance from the Earth. This point is below N and above F, and it is the point from which the perpendicular OI is drawn from the periphery to the line of apsides, falling at half the eccentricity and ending at I. For when O, A and B are joined, two equal triangles arise, in which BO, drawn from the centre of the eccentric, is equal to OA itself; and hence OA is equal to the radius BO, or BC, which is the mean distance between the greatest, AC, and the least, AD. For it is false, and contrary to observations, that the greatest equation occurs at the points of the mean longitudes, except in the sense on which see Book 3, Chapter 20. Finally, if we imagine the Moon to be at F, BF will be the line of the mean place and AF that of the true place, and the anomaly will be the arc CNF.

Diagram. A geometric figure set within the left column: two large overlapping circles of equal size, one drawn around G and one around B, cut by a vertical line of apsides running from H at the top to D at the foot. A small circle lies on that line between G and B, with T inside it and R just to its right. From the left periphery, broken and dotted lines run to B and to A; a solid line runs from F through to B; a horizontal solid line runs from F through A to P; a short solid line runs from O to I on the vertical. On the right, a dashed line runs from G to M, a dotted line from G to Q, a solid line from G to P, and a dashed line from A up to Q.

- H (top of the vertical, on the upper circle): apogee of the eccentric drawn around G
- C (on the vertical, below H, on the lower circle): apogee of the eccentric drawn around B
- G (on the vertical, top of the small circle): centre of the eccentric at the quadratures
- T (inside the small circle): not explained in the text of this page
- R (right of the small circle): on the small circle BRG
- B (on the vertical, foot of the small circle): centre of the eccentric at the syzygies
- I (on the vertical, below B): foot of the perpendicular OI
- A (on the vertical, below I): centre of the Earth
- K (on the vertical, on the upper circle's foot): perigee of the eccentric drawn around G
- D (foot of the vertical, on the lower circle): perigee of the eccentric drawn around B
- M, Q, V (right periphery, descending); P (right, on the horizontal through A)
- N, O, F (left periphery, descending); S (lower left, right of F, on the horizontal)
- E (lower right periphery)

Labels read: H, C, G, M, Q, T, R, N, B, I, O, F, S, A, P, V, E, K, D. Labels not read: none.

Let the centre of the eccentric now be at G for the quadratures, so that AG, the greatest eccentricity, is 13053 of the parts of which the radius GH is 100000. [Margin: Explanation of the figure for the quadratures.] From G let the eccentric HSKM be described, and let H be the apogee, with AH the greatest distance from the centre of the Earth, and K the perigee, with AK the least distance. Then, when the perpendicular PA has been drawn to A and P has been joined to G, the right triangle GAP will result, in which the angle P, subtending the greatest eccentricity AG, 13053, will be the greatest equation, 7° 30′ — and not 7° 40′, as those would have it who, with Copernicus, set the greatest eccentricity in the syzygies, that is, the sum of the semidiameters of the epicycles, at about 13340. But the distances we have observed in the apogee and perigee quadratures by no means allow this. Here too the equations occur at the point P, or at the point corresponding to it on the left, where the arc MP is computed from the eccentricity AG and its sine as 7° 30′. But the mean longitude taken as the mean distance is AQ, and the point Q that ends it is below M by the arc MQ, which is 3° 44′ 30″, as its sine shows, that is, half the eccentricity AG, which is 6526 of the parts of which the radius GH is 100000.

Just as no equation is needed when the Moon is on the line of apsides, that is, in the apogee or perigee of the eccentric, and this whether in the syzygies or quadratures or outside them, so the greatest equations occur a little below the mean longitudes. Therefore, in the month in which the new Moon falls in the apogee C, the first quadrature will occur at the point S with the greatest subtractive equation, and the full Moon nearly in the perigee D, and the second quadrature at P with the greatest additive equation. But if the new Moon occurs in the perigee D, the first quadrature will occur at P with the greatest additive equation, the full Moon nearly in the apogee C, and the second quadrature at S with the greatest subtractive equation. Conversely, if the first quadrature occurs in the apogee H, the second will occur in the perigee K with almost no equation; but the full Moon will occur at F, with the greatest syzygial equation to be subtracted, and the new Moon at V, with the greatest equation to be added. In turn, if the first quadrature occurs in the perigee K, the second will be in the apogee H, the full Moon at V with the greatest equation to be added, and the new Moon at F, nearly with the greatest equation to be subtracted. For this is what the observations require; and from them we have also learned that when the doubled distance of the once-equated Moon from the Sun is 180°, the centre is at G and it is the time of quadrature, and that in the mean longitudes the greatest equations are as great as we have said.

V. [Margin: Explanation of the hypothesis outside the syzygies and quadratures.] For the hypothesis outside the syzygies we must recall what is certain so far, from our own observations and those of others, about the mixing of the second inequality with the first, and what was already sufficiently indicated, with Boulliau, in the preceding chapter, no. 14. We note, however, that it is not necessary to imagine the centre of the eccentric as moved on the small circle BRG; it is enough that it be carried up or down from the point B to the

point G — not, indeed, along the same straight line in the world, since the line of apsides DH must be conceived as moving continually eastward, and so the centre of the eccentric must change its position by an oblique ascent and descent. For while the Intelligence describing a spiral line with a slower motion than the motion of the primum mobile, seems to us to retard the planet toward the east, it at the same time, in measuring the distances, observes or is imagined to observe a centre of the eccentric that is always further and further to the east. Yet at any given moment we can employ this centre as though it were then unmoved and, letting fall a perpendicular from it to the centre of the Earth, measure the eccentricity. Nor indeed is it necessary for us to use the sines of the arcs of the small circle BRG in order to measure the appendix to be added to the least eccentricity AB, namely the appendix BT, even granting that for this we use the doubled motion of the Moon from the Sun. Instead we have so far used another, easier method, by which we have been accustomed to derive equations agreeing with the observations. We therefore propose it, until, from a sufficient number of observations — a number that an unseasonable sky has not rarely cut short — something can be perfected more exactly by the astronomical file. We suppose, moreover, that the Moon's monthly motion is tied to the true rather than to the mean motion of the Sun; that this too was observed by Tycho you will learn from the Appendix to volume I of the *Progymnasmata*.

Diagram. A geometric figure set within the left column: a large circle, the Sun's circle, drawn around A, with a smaller circle, the Moon's eccentric, inside it, drawn around B. A vertical line runs from T at the top to Z at the foot, through G, Q, B, C, A and P. Two further straight lines cross at A, one from H to K and one from F to N. A dotted horizontal line joins D to E through B. From L on the inner circle, and from M on it, straight lines run to A, to B and to C.

- T (top of the outer circle), Z (foot of the outer circle), on the vertical
- H (upper left), N (lower left), F (upper right), K (lower right), on the outer circle
- ♈ (on the outer circle at its rightmost point)
- ♎ (on the outer circle at its leftmost point, opposite ♈)
- G (top of the inner circle), P (foot of the inner circle), on the vertical
- Q, B, C (on the vertical, descending, between G and A); A (the crossing of the lines)
- L (upper left on the inner circle), M (lower right on the inner circle)
- S (where HK cuts the inner circle, upper left), I (where FN cuts it, upper right)
- R (where FN cuts it, lower left), V (where HK cuts it, lower right)
- D (left end of the dotted line), E (right end of the dotted line)

Labels read: T, G, H, L, S, Q, B, D, E, I, F, C, A, M, V, R, P, N, K, Z, ♈, ♎. Labels not read: none.

Let the line of apsides be GAP, movable toward the east, on which lie A, the centre of the Earth and of the world, and the least eccentricity AC, of 8745 parts, and the greatest, AQ, of 13053 of the parts of which the radius of the eccentric, BG, is 100000. Now, however, let the centre of the near-circular spiral be at B, from which the circle GDPE will be described; and so let the temporary or monthly eccentricity be AB and the appendix of the eccentricity be BC, always to be added to the least eccentricity AC if you wish to know the momentary eccentricity AB. Now let the Moon be at L or M, and let these places be joined by three straight lines each: to A, the centre of the Earth; to C, the centre of the other eccentric, used for the Moon in the syzygies; and to B, the centre of the present eccentric. For two triangles of equation will arise. One is LAC or MAC, which in this figure does not indeed contain the exact equation of the first inequality, because the centre of this eccentric is not at C; yet it indicates that equation, so that the imagination may be helped and so that the drawing of the other eccentric, on which the Moon in fact is not now, may not lead us into another, greater deception. The other triangle is LCB or MCB, in which the angle at L or M is the exact measure of the monthly equation considered separately from the first. The angle L in the triangle LBC therefore becomes known by trigonometry, given now the radius BL, 100000, the appendix of the eccentricity B[?][35], and the sum of the angles BLC and BCL. By Euclid 1.32 this sum is as great as the external angle LBG, that is, as great as the anomaly of the once-coequated Moon, counted from the apogee G in the order of the signs. Likewise when the Moon is at M, in the triangle MCB there must be given the radius BM, 100000, the appendix of the eccentricity BC, and the sum of the angles BCM and BMC; when the anomaly is greater than a semicircle, this sum is equal to the complement of the anomaly to the circle, namely the angle GBM. Next, the angle found at L or M, that is, the monthly equation, is added to or subtracted from the place of the once-equated Moon, so that the Moon's true place, or its twice-equated place, results. Here, however, we must repeat the rules of addition or subtraction, gathered from observations of the Moon's true place; and they are those written below.

VI. [Margin: Figure for the rules of monthly addition or subtraction.] Describe the Sun's circle THNZKF from the centre of the world A, and draw through A, in any way, the synodic lines HAK, or NAF, or TAZ, cutting the Moon's eccentric at the points S, V, R, I, G, P; if the Moon were at these points, there would be either a new Moon or a full Moon. Finally, when the line of the lunar apsides GAP has been cut in two and at right angles at B, let the points D and E be marked. For, as we said in the preceding chapter, no. 14, the periodic anomaly, or first inequality, begins from the apogee G, and from there it grows up to the first mean longitude near D, where it becomes greatest; from there it decreases as far as the perigee P, where it vanishes, and the equations are subtractive in the semicircle GDP. (For to avoid confusion we

[35] the second letter is blotted on the page, a C-shaped bowl with a dark mark inside it, unlike the G set two lines below; the parallel sentence for M reads BC

here employ this eccentric just as if it were the one that serves the Moon when set in the syzygies, and as if it should be described from the centre C.) From there it begins to grow as far as E, the second longitude, where it is greatest, and from there it decreases as far as G, where it vanishes; and in the semicircle PEG the equations of the first inequality are additive. But the second anomaly, or monthly inequality, begins from the mean longitudes of the periodic anomaly, namely from the points E and D, of which E is at the 270th degree counted from G through DP, and D at the 90th. Now if the synodic line is joined with the line of the lunar apsides and the Sun is at T or at Z, when the Moon, joined or opposed to it, is at G or at P, there is no first inequality; but in that month, as we have said, the quadratures will occur at the mean longitudes D and E. And therefore, when the synodic line falls on the periodic apogee G or perigee P, we have said that the monthly inequality becomes greatest in that month. The monthly inequality therefore begins to grow from the point E, on which the synodic line falls, and it grows as far as G and decreases again as far as D. And if the synodic line falls within this earlier semicircle of the monthly anomaly, namely the one containing the periodic apogee G — as if the Sun is at F and the synodic line AF falls on I, or if the Sun is at H and the synodic AH falls on S — the monthly equations from ♂ to ♌ are to be subtracted, but from ♌ to ♂ added, to the once-equated place of the Moon. Conversely, if the synodic line falls in the later semicircle of the monthly anomaly, the one containing the periodic perigee P — as if the Sun is at N and the synodic line AN falls on R, or the Sun is at K and the synodic line AK falls on V — then the monthly equations from ♂ to ♌ are to be added, and from ♌ to ♂ subtracted. For example, if the Moon is at L and the Sun at F, the Moon will be in the earlier semicircle of the month, that is, it will have departed from a new Moon made near F, and will be tending toward a full Moon to be made a little beyond N, and the synodic line will fall on I, a point of the earlier semicircle of the monthly anomaly. Therefore the monthly equation will have to be subtracted. With these things set down, let the whole progress of the calculation now follow.

VII. [Margin: Precept 1. Method of finding the true place of the Moon.] *First.* For the given time, gather from our tables the true place of the Sun, the once-equated place of the Moon, and the once-equated anomaly of the Moon. Subtract the true place of the Sun from the Moon's once-equated place, taking a whole circle if necessary, so as to have the elongation of the Moon from the Sun; and finally double this. If so doubled it does not exceed 180°, use it; if it does exceed, use its complement to 360°. In either case let it be called the anomaly of the eccentricity.

[Margin: Precept 2.] *Second.* Multiply the anomaly of the eccentricity by the whole difference of the eccentricities, CQ, namely by 4308, and divide the sum by 180. For the quotient will be the appendix of the eccentricity, as CB; added to the least eccentricity AC, it will make up the temporary or monthly eccentricity AB. And this same appendix added to the radius, 100000, will make

the sum of the sides, and subtracted from the same it will make the difference of the sides; these are the two data required for solving the equating triangle.

[Margin: Precept 3.] *Third.* Take the once-equated anomaly of the Moon if it does not exceed 180°, or its complement to 360° if it does exceed 180°, and divide it in two; for its half is the half-sum of the angles adjacent to the base of the equating triangle. With this, and with the sum and difference of the sides mentioned in Precept 2, find the angle opposite the appendix of the eccentricity; for that will be the monthly equation. Therefore, if you wish to proceed by sines and tangents, multiply the tangent of the half-sum of the angles by the difference of the sides; and the sum, divided by the sum of the sides, will give the tangent of the angle always to be subtracted from the half-sum of the angles; for the remainder will be the monthly equation. [Margin: A short way of solving the triangle of equation.] But if you use the short way of logarithms, add together the mesologarithm of the half-sum of the angles, the logarithm of the difference of the sides, and the residual of the logarithm of the sum of the sides. For the sum, with unity taken away from the left, will be the mesologarithm of the angle to be subtracted from the half-sum of the angles, and the remainder will be the monthly equation.

[Margin: Precept 4.] *Fourth.* Subtract the once-equated anomaly from the once-equated place of the Moon, taking a whole circle if this is necessary for the subtraction, and you will have the place of the lunar apogee. Subtract this from the true place of the Sun, and you will have the longitude of the Sun from the Moon's apogee. If this is less than 90° or greater than 270°, the synodic line will be in the first semicircle of the monthly anomaly; but if it is greater than 90°, yet less than 270, it will be in the second.

[Margin: Precept 5.] *Fifth.* Finally, if the synodic line is in the first semicircle of the monthly anomaly and the Moon is tending from new Moon to full Moon, subtract the monthly equation from the Moon's already once-equated place; but if it is tending from full Moon to new Moon, add it. Conversely, if the synodic line is in the second semicircle, add it from ♂ to ♌ and subtract it from ♌ to ♂; and you will have the true place of the Moon, twice equated. But if you wish to know the Moon's distance from the centre of the Earth, you must, in the triangle resting on the eccentricity, seek the base or third side, namely the side LA in the triangle ALB, given the radius LB, the monthly eccentricity BA, known from Precept 2, and the complement of the coequated anomaly to a semicircle, or its excess if it should exceed one. But an example will make the precept clear.

EXEMPLVM.

Example.

VIII. In the year 1641, at noon on February 13, the motions of the ☾ and the ☉ stand as follows from our tables.

	Value
☉ True place	10ˢ 24° 19′ 14″
☾ Place once equated	0ˢ 6° 19′ 6″
☾ Anomaly once equated	10ˢ 5° 12′ 44″
☾ elongation from ☉	1ˢ 11° 59′ 52″
Doubled	2ˢ 23° 59′ 44″
or	83° 59′ 44″

Since, then, the doubled elongation of the Moon from the Sun does not exceed 180°, when 83° 59′ 44″, or 84°, is multiplied by 4308, the sum is 361872. When this is divided by 180, there remains the appendix of the eccentricity, CB, of 2010 parts. When this is subtracted from the radius BM, 100000 (for we imagine the Moon to be at M), there remains the difference of the side CB from the side BM, 97990; and when it is added to the radius, the sum of these sides results, of 102010 parts. Now the complement of the once-equated anomaly to the circle was 54° 47′ 16″, namely the angle MBG, which is equal to the angles BCM and BMC taken together, and their half-sum is 27° 23′ 38″, whose tangent is 51822. This tangent, then, being multiplied by the difference of the sides, 97990, and the sum divided by the sum of the sides, 102010, there will remain 49786, the tangent of the angle of 26° 28′. Subtracted from the half-sum, 27° 23′ 38″, this leaves the monthly equation, 55′ 38″.

Now when the equated anomaly of the Moon is subtracted from the once-equated place of the Moon, there is left the place of the lunar apogee, 2ˢ 1° 0′ 22″. Subtracted from the true place of the Sun, K, this leaves the arc GLRV, the distance of the Sun from the Moon's apogee, 8ˢ 23° 18′ 52″, or 263° 18′ 52″. So the synodic line was in the later semicircle of the monthly anomaly, that is, within the arc DPE, and the Moon was tending from new Moon to full Moon, as is clear from its elongation from the Sun, and it was near the first octant. The monthly equation of the Moon, 55′ 38″, must therefore be added to the once-equated place of the Moon, 0ˢ 6° 19′ 6″, and the true place of the Moon becomes 0ˢ, that is ♈, 7° 14′ 44″.

But by Tycho's calculation at Bologna it was ♈ 7° 18′; for Argoli it is ♈ 6° 24′, for Lansbergen ♈ 6° 14′, for Copernicus ♈ 5° 52′. Nor is this surprising, for sometimes the difference in the twice-equated lunar place among these authors and others rises to a whole degree. So our calculation ought not to seem false to anyone because it sometimes differs by several minutes from the place given by others; rather, the sky is to be consulted, and from observations the hypotheses are to be corrected, whether those of others or our own. As for that Variation which Kepler (Book 6 of the *Epitome*, p. 816, and in the Rudolphine Tables, p. 83, col. 2) and Boulliau (Book 3, Chapter 14), the former from Tychonic observations, the latter from his own and those of others, teach to be an anomaly, and to be now 50′ ½, now 30′ ½, now midway between these: it is not yet so known to me that it can be bound to fixed laws.

CAPVT XXIX.

Chapter 29.

De Theorica Latitudinis Lunæ.

On the theory of the Moon's latitude.

I. In Chapter 12 it was said what the Moon's greatest latitude is and how great, both in the syzygies, 5°, and in the quadratures, 5° 18′; and the way of finding these by observations was given there. In Chapters 22 and 23, moreover, we taught the way of determining the mean or true motion of the lunar nodes against the order of the signs [in antecedentia], which, added to the Moon's mean motion in the zodiac in the order of the signs [in consequentia], constitutes the mean motion of latitude; we determined this too in the same place, and the period of this motion in Chapter 18. Nothing else remains, then, but to set out the theory of this motion. [Margin: The ancients' hypothesis on the Moon's latitude.] Now Ptolemy in Book 4, al-Battānī in Chapter 38, Peuerbach in the *Theorica* of the Moon and his commentators there, Copernicus in Book 4, Chapters 4, 23 and 24, Magini in the *Theoricae Planetarum*, Book 2, Chapter 2, and Barozzi in Book 4 of the *Cosmographia*, p. 243, were not much concerned about this part of the hypothesis, since they supposed with Ptolemy that the Moon's greatest latitude is perpetually and invariably 5°. So it was enough for them to suppose that the plane of the lunar orbit, together with the plane of the epicycle or epicycles, is inclined to the plane of the ecliptic at an angle of 5°, and that these planes cut one another in a diameter passing through the centre of the world. Those who posited solid orbs added that the axis of the lunar orb intersects the axis of the ecliptic at the centre of the world, with an angle of intersection of 5°, so that the poles of the lunar orb are 5° from the poles of the ecliptic, and that these poles are on the periphery of the circle passing through the limits, that is, on that circle of latitude whose arc intercepted by the Moon's orbit and the ecliptic is the greatest latitude. Copernicus and Lansbergen, however, chose to count latitude from these limits rather than from the nodes. In the hypothesis of these authors, moreover, the equations of the motion of latitude are not different from the equations of the motion of longitude, but are the very ones, of exactly the same quantity and character, by which the motion of longitude is equated, since this motion is included in the motion of latitude and the motion of the nodes in itself is, with them, always uniform.

II. [Margin: The anomaly of the motion of latitude and of the nodes, detected by Tycho.] Tycho, however, as he himself affirms in volume 1 of the *Progymnasmata*, p. 127, and after him Longomontanus (*Theorica*, Book 1, Chapter 7) and Mulerius (*Isagoge* to the *Tabulae Frisicae*, p. 415), Origanus (second part of the *Ephemerides*, Chapter 4) and Magini (*Supplement to the Ephemerides*, canon 7), affirms from his observations that in the syzygies, and nearly in the quadratures, the motion of the nodes does not need any equation, and that the

greatest latitude of the lunar orbit is perpetually 4° 58′ 30″; but that outside the syzygies the motion of the nodes is unequal and therefore needs an equation, which sometimes grows to 1° 46′; and hence that from this motion the latitude of the Moon, since it is further from the nodes than the motion of the ancients would allow, can turn out about 10′ greater than usual; and that the greatest latitude of the lunar orbit in the quadratures is 5° 17′ 30″. This is to be believed on his word in the meantime, until his observations come to light, whose edition, as I have said more than once, is in the charge of Father Albert Curtz of our Society, by order of the Emperor. On this assumption, Tycho and Longomontanus there account for this anomaly with the following figure. From B, the pole of the ecliptic CFDG, let the ecliptic be described, which the semicircle of latitude CBD cuts at right angles at C and D; on whose periphery, here along a straight line as represented, let A be the pole of the lunar orbit KFHG, distant from the pole B by 5° 8′ — that is, by as much as the mean inclination of this orbit to the ecliptic, or the Moon's mean latitude if it is at K, measured by the arc CK; and the nodes will then be at F and G. Again, from A let the small circle EMZI be described, whose semidiameter AE is 9′ 30″.

Diagram. In the left column: a large circle with concentric-looking eccentric arcs. The vertical diameter runs from C at the top through B and A down to D, with H just below D outside the circle. Near the centre, about A on the vertical line below B, stands a small circle with a horizontal diameter. Three arcs cross the upper part of the vertical at R–L–S, X–K–d and N–Y–O, and curve down to the rim on both sides. Labels read on the plate:

- C (top of the vertical): the point where the semicircle of latitude meets the ecliptic
- R, L, S (upper arc, from left to right across the vertical)
- X, d (next arc, left and right of the vertical)
- K (on the vertical, below L)
- N, Y, O (next arc, from left to right across the vertical)
- B (on the vertical): the pole of the ecliptic
- T (left) and V (right): rim points at the height of the small circle's upper part
- F and P (left rim, side by side)
- G and Q (right rim, G above Q)
- I (top of the small circle), E (its left end), A (its centre), Z (its right end), M (its foot, on the vertical)
- D (foot of the vertical), H (below it)

This is the small circle of the third anomaly, or Variation, and also of the anomaly of latitude and of the nodes. On its periphery the pole of the lunar orbit moves in the order of the signs [in consequentia] from I to E, with a motion double the Moon's elongation from the Sun, so that at new and full Moons this pole is at I. From I the lunar orbit TRSV is understood to be described, with the nodes at T and V, and with the smallest angle of inclination, CTR, or the smallest latitude, 4° 58′ 30″. At that time there is no Variation in the

motion of longitude, and no anomaly of the motion of latitude and of the nodes. Conversely, when the pole of the lunar orbit is at M, which happens at the quadratures, the orbit is PYQ, etc., with the nodes at P and Q, and the inclination CPY and the latitude CY at their greatest, 5° 17′ 30″. Then too there is no Variation and no inequality of the nodes, because the arc CBAMD, etc., is continuous. In octants 1 and 3, however, the pole is at E, and in octants 2 and 4 at Z; and there the greatest Variation occurs (which can, however, be accounted for by a libration along the diameter EZ alone), together with the greatest anomaly of the nodes. Now let arcs of circles of latitude, RBZ and SBE, be conceived, cutting the arc CD at the pole B: for the spherical angle ABZ will be the measure of the equation of the nodes, since it is equal to the angle XBK, just as ABE is equal to the angle dBK. In the triangles ABZ and ABE, moreover, the side BA, 5° 8′, is always given, and the side AZ or AE, 9′ 30″; and the angle BAZ or BAE that they comprehend is known from the doubled motion of the Moon from the Sun, for it is equal to this elongation if that is not greater than 180°, as with BAE; but if it is greater, the angle is equal to its complement to the circle, as is the angle BAZ. This anomaly is counted from I, where the pole is at conjunction [☌], towards E, etc.; and the prosthaphaeresis is subtractive from the motion of the nodes in the first semicircle of the doubled elongation, IEM, but additive in the second, MZI.

[Margin: Verdict on this hypothesis.] III. In this hypothesis, then, the Variation and the anomaly of latitude and of the nodes are monthly. But this hypothesis, as Tycho meant it, is indeed a fiction, made as a geometrical model for calculation on paper and a foundation for the imagination, as Kepler admits in the *Epitome of Copernican Astronomy*, p. 817; it is not, however, real. For Tycho himself did not later admit solid orbs, nor did he hold that the planets are moved by the motion of the whole machinery of their heaven, but rather that they move freely in a fluid aether, by the force of some physical cause; and so the motion already mentioned, on that small circle devised about the poles of the ecliptic, is not real. For us, however, all this is accomplished by an Intelligence that leads the Moon round in spirals, bending it from the ecliptic and intersecting the ecliptic in different ways and at different places, accelerating and retarding its motion as harmony requires — not the harmony of the ears, which Kepler pursued all too clumsily, by an analogy utterly [Greek: ἄλογον (alogon, "irrational, without ratio")] from the divine plan, but the harmony of minds, which Divine Providence has fixed in advance according to the alternation of effects, and whose idea it willed to shine forth from its archetype in the mind of the Intelligence that moves the lunar globe.

[Margin: Kepler's hypothesis on the Moon's latitude.] IV. Kepler, however, in the foundations of his *Ephemerides*, from no. 20 on, in the *Epitome of Copernican Astronomy*, Book 6, pp. 620, 788, 797 and 817, and in the Rudolphine Tables, Chapters 25 and 27, partly adheres to Tycho and partly departs widely from him. But later he relapses into Tycho's view, by an inconstancy or a craftiness that is certainly remarkable, though familiar to this mind, so that he

may not either bring forth nothing new, or have the offspring of his novelty found illegitimate through disagreement with the phenomena. Accordingly he grants Tycho that the Moon's greatest latitude at the quadratures is 5° 17′ 30″, or in round figures 5° 18′; that in the other syzygies, those without eclipses, the smallest latitude is 4° 58′ ½, or rather, in round figures, 5°; and that when the Moon's nodes are in the octants it has a small latitude. But in most respects he disagrees as regards the form of the hypothesis, even granted that as regards the effect of the calculation, as will be said later, he differs from Tycho imperceptibly. He says, then, that after Tycho's death he discovered from eclipses of the Sun and Moon that the angle of latitude is greatest, 5° 18′, throughout the whole month in which the Sun or the Earth's shadow passes the places of the nodes; that in the following months, he says, this angle diminishes until at the syzygies it comes to 5° 0′; and that this anomaly is annual, not monthly, and is restored twice in one year. Accordingly Tycho, from the eclipse syzygies, releases the Moon from a small angle of inclination to the greatest at the quadratures, so that this increase is nil at the end, greatest at the beginning but slow, and steep in the middle; and the nodes reciprocate twice in any month, turning from direct to retrograde. Kepler, by contrast, from the eclipse syzygies releases the Moon at a large angle of inclination, which it keeps throughout the eclipse month, so that when the node is at quadrature, the Moon runs out on a straight, not a bent, path to the syzygies at a small angle of 5°, and acquires a latitude equal to this at the syzygies; and the change is slow only when the increments of latitude are greatest, and fastest when they vanish; nor do the nodes ever become direct, but rather resemble stationary bodies twice in the year, and their inequality is semi-annual. Moreover he advanced the places of the nodes 25′ beyond what Tycho's calculation gives for these times. But in order that, with a large angle of inclination posited, constant from the syzygies to the quadratures, the latitude should not be greater than Tycho's observations allow when the Moon has its nodes in the octants, he introduced — after many perplexities and great errors, as he admits in Chapter 27 of the Rudolphine Tables — an annual libration and prosthaphaeresis of the nodes, so that this should take away as much as the large angle had added in the eclipse latitudes. And although in the *Ephemerides* he had set up a libration of the limits on an axis released from the Sun, that is, on the line connecting the nodes and the centre of the Earth, yet in the *Epitome of Copernican Astronomy*, pp. 794, 818 and 821, he set up a libration of the monthly limit on an axis and line tied to the line of the syzygies. And finally, forced to choose a middle way, he introduced a double prosthaphaeresis of the node, three-monthly rather than annual, which would be of very small quantity when the node is near the Sun; but when the node is in the octants, and 45° from the Sun on one side or the other, it would be equal to Tycho's monthly equation. Yet in that Chapter 27 he professes that these matters are not yet clear in every respect.

[Margin: The physical cause of this hypothesis.] V. As for the physical cause of the varying latitude, Kepler in Book 4 of the *Epitome of Copernican*

Astronomy, p. 620, assigns it to the light of the Sun and its application to the Moon's fibres, but to the fibres of latitude. For just as our stomach has three positions of fibres, for attracting, retaining and expelling, so it is no wonder if in the Moon there are two kinds of fibres, of which one kind strives to keep the same position, parallel to itself, as regards longitude in the universe, and the other the same position as regards latitude. When, therefore, the light, which has a certain breadth [latitudinem] since it is a surface, is fitted to the fibres of latitude, it increases the latitude; otherwise it does not increase it. [Margin: A fictitious scheme of the same.] As for the hypothesis devised to help the imagination, recall here the Keplerian Sun set out in Chapter 27, no. 6, and imagine a plane drawn through the points A and G, oblique at 5° to the plane of the ecliptic, on a line passing through the centre of the Earth, A, and the places of the nodes; in which plane let the line of the syzygies be HGAK, on which the Moon's eccentric PDRC is inclined to the plane I spoke of earlier — not always, however, but according to these laws. If S and T are the places of the quadratures among the fixed stars, and the limits of the Free anomaly are on the line HGAK, then the plane of the eccentric is united with the former plane, with no inclination between them, though their common inclination to the plane of the ecliptic is kept. But when the limit of the Free anomaly, the northern one for example, begins to digress from H, the place of opposition, towards the quadrature K, and has come, say, to D, then as the sine of the angle HAP is to the sine of the right angle HAT, so let the portion of the northward libration, by which the monthly limit K librates, be to the whole libration or inclination of the planes, which is 18′. This whole inclination occurs in the month in which the northern limit of the Free anomaly comes to T, the place of the quadrature. Hence the northern limit of the Free anomaly rises northward from the former plane until it falls on the quadrature K, and from there gradually inclines back towards the former plane; and finally, on coming to K, it recedes into that plane itself. But when the northern limit of the Free anomaly passes the point K, towards T, the semicircle HCK becomes southern with respect to the monthly latitude.

VI. The method of calculation by which one arrives at the monthly equation of latitude, moreover, is this. First, with the Sun's distance from the north node, the minutes of latitude are sought, that is, the sine of the complement or of the excess, to or beyond one or three quadrants, of the Sun's distance from the Moon's node ☊, reduced to astronomical minutes of which the whole sine is worth 60. Second, by the monthly argument, that is, by the distance of the Moon's true and finally equated place from the Sun's true place, the monthly latitude is taken out, namely the arc of a great circle under the ecliptic, whose plane, passing through the centres of the Earth and the Moon, stands erect at right angles on the plane inclined at a constant angle of 5° to the plane of the ecliptic; it is, I say, the arc intercepted between this plane, when extended, and the place under which the centre of the lunar body lies. Third, the monthly latitude is multiplied by the minutes of latitude, for from this the appropriate

portion of latitude is obtained. But see the precepts of the Rudolphine Tables, Chapter 27, on which we treat in the practical part. Whether, moreover, the nodes have as much power as the astrologers imagine is disputed by Kepler in the foundation of the *Ephemerides*, no. 25; consult him.

[Margin: Lansbergen's scheme.] VII. Lansbergen's theory for the latitude and the motion of the nodes, given by him in Chapter 5 of his *Theoricae*, is as follows. From A, the centre of the world, let a circle of latitude be understood to be described, of which, for brevity's sake, we here mark only the arcs QG and DR; let it be at right angles to the plane of the ecliptic, marked here by the straight line CAB. And let the limits of the smallest latitude be F and E, and those of the greatest G and D, joined by straight lines; for the angle CAF will be the smallest inclination of the lunar orbit to the plane of the ecliptic, and CF the smallest latitude, 5°, at the syzygies; but the angle CAG and the arc CG will be the greatest, at the quadratures, 5° 16′. Therefore the mean, that is the arc CI, will be 5° 8′, and the difference between them 16′, half of which is the arc FI, 8′. With this interval let the small circle HFKG be described, the circle of the anomaly of the obliquity of the lunar orbit, which is as great as the anomaly of the centre, that is, as great as the doubled distance of the Moon from the Sun, and is counted from F through K, G, H, in the order of the signs. The libration of the lunar orbit in latitude itself takes place along the diameter FG, reciprocally, and is restored twice in a month.

Diagram. Set into the left column, with the text running in a narrow measure beside it on the left. From A, low in the figure, straight lines rise to an upper arc and descend to a lower arc, the two arcs being opposite ends of a circle of latitude. At the upper right on the arc stands a small circle with a diameter through its centre. Lines rise from A to P and C on the left of the fan, and a dotted line rises from A through O towards the small circle; below A the lines descend, one of them dotted, to D, N, E and B on the lower arc. Labels read on the plate:

- Q (far left end of the upper arc), C (on the upper arc), F (left rim of the small circle)
- H and M (above the small circle), I (its centre), L (within it, to the right of I), G (its right rim), K (its foot)
- P (on the left of the fan, below the upper arc), O (on the right of the fan, level with P)
- A (the crossing point of the lines)
- D, N, E, B (along the lower arc from left to right), R (right end of the lower arc)

[Margin: Method of geometrical and logistic calculation for the latitude.] VIII. With these premises, let the Moon's southern latitude observed by Tycho be 5° 13′, in 1587 August 17, Old Style, at the hour p.m. $19^{h}\ 24^{m}$ at Uraniborg — but, as examined, $19^{h}\ 18^{m}$ — and at Goes $18^{h}\ 33^{m}$. At that time the Sun's mean motion from Lansbergen's tables was 155° 36′ 2″. The Moon's mean elongation from the Sun was 295° 3′ 1″, which, doubled, is 590° 6′ 2″. When a circle of

360° is discarded, then, there remains the anomaly of the centre, and also of the obliquity of the lunar orbit, 230° 6′ 2″. The mean motion of latitude was then 180° 35′ 9″, and the subtractive prosthaphaeresis of the orb 3° 44′ 18″; therefore the true motion of latitude was 176° 50′ 51″. So, drawing the intersecting line NM, which marks at L the latitude CL that we are seeking, let 230° 6′ 2″ be counted from F through K; for when the semicircle FKG of 180° is discarded, there remains the arc GM, 50° 6′ 2″, whose second sine, namely LI, is 64145 of the parts of which IF is 100000. But of the units of which IF is 8′, LI is 5′ 1/10, and LF 13′ 1/10. And when the smallest latitude is added, the southern latitude CL becomes 5° 13 1/10′, which is very nearly what Tycho observed. But it can be corrected as below.

For let the plane of the lunar orbit at that time be marked by the intersecting line NAOM, which we may use as if it were a semicircle; and let N be the northern limit of the temporary latitude and M the southern, with the south node at A; and let the motion of latitude, 176° 50′ 51″, be counted from N as far as O. For when 90°, that is the arc NA, are discarded, there remains the arc AO, that is, the Moon's distance from the south node, 86° 51′. So a perpendicular arc is dropped from O to P. [Margin: Finding the places of the nodes.] For since in the spherical triangle APO, right-angled at P, the angle at A is given, 5° 13 1/10′, and the base AO, 86° 51′, the side OP, that is the Moon's latitude, will be found to be very nearly 5° 13′. And since the motion of latitude counted from N, then the northern limit, was 176° 50′ 51″, if you add to it 90° against the order of the signs [in præcedentia], you will find the motion of latitude counted from A, but from the north node, 266° 50′ 51″. If you take this from the Moon's motion, which was then ♊ 27° 11′, there will remain the true place of the Dragon's Head in ♎ 0° 20′, and of the Dragon's Tail in ♈ 0° 20′. [Margin: The hypothesis of Pierre Hérigone.] Hérigone, moreover, in volume 5 of his *Cursus Mathematicus*, follows Lansbergen's hypothesis in this theory, as can be seen on p. 588.

IX. Ismaël Boulliau, in Book 3 of the *Astronomia Philolaica*, Chapter 13, pressing his own opinion — which should be recalled here from our Chapter 27, from no. 13 — affirms that if the Moon were not moved with an annual motion around the Earth, the nodes would advance in the order of the signs; but that if, carried along by the Earth with the annual motion alone, it had not been raised up month by month, there would be no reciprocation of the nodes such as Tycho observed. But since the Moon is carried off by the motion of the Earth with its whole system towards the east, and the Moon's apogee is advanced in the same direction, nature has provided that the lunar system should be drawn back towards the west by certain restraints, so to speak. Because of the evection, moreover, a reciprocal libration of the nodes follows each month: for when the Moon's ellipse is displaced, the northern part is raised further to the north, and the Moon's latitude increases from the syzygies to the quadratures, and decreases from the quadratures to the syzygies. Meanwhile, from ☌ to the first quadrature, and from ☍ to the second, the nodes are dislocated, and the Moon, ascending from the syzygies to the quadratures, is lifted towards the north; and

so the node moves faster against the order of the signs, and its greatest speed comes in the octants. From the quadratures to the syzygies, however, the north node passes to the south and the southern to the north, and over the same time the node's motion is faster in the order of the signs.

Now let the annual orb be ABC, with the Earth at B, carrying the Moon around the Sun; let the circle KAL be in the plane of the annual orb, and let the Moon's orbit be HOFZ, with the north node ☊ and the south node ☋, and the southern semicircle ☋F☊. At the syzygies the nodes are carried against the order of the signs with a simple and uniform motion from ☊ to F; but outside the syzygies a synodic motion supervenes on them, while each month the umbilicus B is carried out through the small circle BDEG; and to this motion the raising of the node ☊ and the lowering of the node ☋ are commensurate. At the syzygies, indeed, the umbilicus B is joined to the Earth; but when the umbilicus has moved away from B through the quadrant BD, that is, by the double motion of the Moon from the Sun, and so with the Moon placed at 45° from ☌, or in the first octant, the node ☊ recedes to R and ☋ to O; and at these points occurs the greatest equation of the nodes, additive to the retrograde motion, and the retrogradation is increased for those nodes; but from there to the first quadrature, with the umbilicus now brought to E, the node that was at R tends to the mean place ☊, and the other node returns from O to ☋; and at these points no equation occurs. From the first quadrature to the second octant the umbilicus is at G, and the node ☊ recedes from its mean place in the order of the signs towards Z, and the node ☋ to C, where the subtractive equation is greatest. When the umbilicus is again brought to B, at the other syzygy, the nodes return to their mean places, and all their equation ceases. From there a second revolution takes place through the circle BDEG, and this anomaly returns twice in the whole month. The inclination of the planes, however, librates by another law, since it increases from the syzygies to the quadratures but decreases from the quadratures to the syzygies. This libration of the nodes, says Boulliau, seems similar to the swinging of pendulums nodding this way and that while they are being transported.

Diagram. Set into the left column, with the text running in a narrow measure beside it on the left. A circle with a vertical diameter, inside it a second, displaced circle, and at the centre a small circle standing on an arc that crosses the figure from lower left to the right and runs out beyond the rim. Labels read on the plate:

- H (above the top, outside), L (below the foot, outside), F (on the vertical, near the foot)
- K (on the vertical, upper part), E (on the vertical, at the top of the small circle), B (on the vertical, at the foot of the small circle)
- G (left of the small circle), D (right of the small circle)
- O (upper left rim), Z (upper right rim)
- ☋ (left rim), ☊ (right rim)

- C (on the crossing arc, left), R (on the crossing arc, right), A (at the arc's right end, outside the rim)

[Margin: Tycho praised, and soon unpraised, by Boulliau.] He adds that from his own observations of the Moon applied to the fixed stars it is established to him that Tycho's latitude of the Moon agrees exactly with the heavens, and therefore that he has changed nothing as regards this measure; but that the form of the Tychonic hypothesis is not so consistent with itself, since the Tychonic Moon, though not transported by the Earth nor impelled by a Sun that moves the Earth, nevertheless suffers so many varieties of motion in longitude and latitude that have a relation to the Earth and the Sun; whereas the other planets, by contrast, suffer nothing of the kind from the Sun, although it is moved, neither in longitude nor in latitude. [Margin: A weak confirmation of the Earth's motion.] For it seems that one should rather say that the body that induces these varieties, namely the Earth, moves; and that the body that does not induce them, namely the Sun, does not move with a motion of translation. But do you not see, Boulliau, that it follows from this that, since the Moon undergoes these varieties with respect not only to the Earth but also to the Sun, and in them follows its elongation from the Sun, not only the Earth but also the Sun should be transported; or, if only one of these is transported, that the translation of the Sun and the rest of the Earth suffice? Secondly, it is assumed here that these varieties come from the Earth or from the Sun as from an external cause physically moving the Moon; but this has not yet been shown; and it suffices that the Earth and the Sun should be the terms of the relation that the Moon's mover, whatever that is, has in moving the Moon.

[Margin: Solid orbs denied.] Boulliau says in that Chapter 13 that it is indeed remarkable in the system of the heavenly bodies that, although it does not consist of solid orbs or material circles, but the planets move freely without these machines, they nevertheless move according to fixed and immutable laws just as if they were moved by the force of material circles, and describe ellipses around scalene cones, conceivable by the intellect.

[Margin: An indication of our hypothesis.] X. For my part, I have already said that the physical efficient cause of these motions is a moving Intelligence, in whose mind there are images divinely infused and formed from the divine archetype, which, according to the harmony of the effects to be produced in sublunary things, fixed in advance by Divine Providence, represent the admirable modes and alternations of these motions; and that to carry out and direct these motions the angels do not need our regular figures, but restore the planet to its due places according to rhythms and numbers in the most perfect order. If anyone nevertheless needs some scheme, let him by all means use the small circles or librations that Tycho, or Lansbergen, or Boulliau used. I, however, have no need to distribute this anomaly according to the sines of arcs in small circles assumed for the purpose. For example, in Lansbergen's scheme set out at no. VII, I use the diameter FG, as I did in the preceding chapter in investigating the eccentricity, and from it the monthly equation. Counting the

doubled motion of the Moon from the Sun from F through K, if it does not exceed 180°, I use those degrees; but if they exceed it, I use their complement to the whole circle; and I multiply those degrees, or this complement, by the difference of the extreme latitudes, namely by 18′, divide the sum by 180°, and add the quotient to the smallest latitude, so that the true latitude results. In this way too, I see, the librations of the nodes and the Variation can be dealt with.

EXEMPLVM.

Example.

Let the example be the same one that Lansbergen used; for in it the doubled elongation of the Moon from the Sun was 230° 6′ 2″. The complement of this to the circle is 129° 53′ 58″, that is, very nearly 130°; when these are multiplied by 18′, the sum is 2340; and when this is divided by 180, there remain 13′, namely the portion LF of the diameter, which, added to the arc CF, or the smallest latitude, 5°, makes the latitude CL 5° 13′, exactly what Tycho then observed.

CAPVT XXX.

Chapter 30.

De Reductione Lunæ ad Eclipticam.

On the reduction of the Moon to the ecliptic.

I. Since the Moon's orbit, or the path along which it moves in longitude, is inclined to the ecliptic, it follows that the place of the Moon, when it is outside the nodes or limits, is not the same in the ecliptic as it is in the orbit; nor are there as many degrees and minutes from the node or limit to the point of the ecliptic determined by the circle drawn through the poles of the ecliptic and through the centre of the lunar body, as there are in the lunar orbit, if they are counted from the same node or limit up to the point of the orbit through which the circle already mentioned passes, which is called the circle of latitude. From tables or geometrical hypotheses, however, the Moon's longitude in its own orbit is first collected. It must then be reduced to the ecliptic, so that we may have its place in the ecliptic or according to the ecliptic. This is commonly done by means of the spherical triangle ADC, in which let AED be a quadrant of the ecliptic, and ABC a quadrant of the lunar orbit[36]; and let the portion of the circle of latitude, drawn through the poles of the ecliptic orthogonally to the ecliptic, be CD, if the Moon is at D, one of the limits. For then the Moon's place in the ecliptic, namely the point C, is the same as that at D, the point of the orbit; that is, the arc AD is as great as AC. But let the Moon be at E, and let the portion of the circle of latitude be EB, making a right angle at B but an oblique one at E. For, given the base AE, that is, the Moon's distance in its

[36]the rest of the paragraph, and the figure, treat AED as the lunar orbit and ABC as the ecliptic; the two arcs appear to be interchanged in the print

orbit from the nearer node A, and given the angle A, the side AB, that is, the Moon's distance from the nearer node computed in the ecliptic, is easily found by trigonometry. And by this device the Table of Reduction is constructed.

Diagram. Set into the right column, with the text running in a narrow measure beside it on the left. A spherical triangle: an upper arc from D on the left through E to A on the right; a lower arc from A through B down to C; a side from D to C; and a short arc from E down to B. Labels read on the plate:

- D (left), E (top of the upper arc), A (right, in a larger sort), B (on the lower arc), C (foot)

Tycho, however, in volume 1 of the *Progymnasmata*, p. 118, used for the angle A the mean inclination, 5° 8′, because in this way an error beyond half a minute is never committed. It is better, however, to use the Moon's true latitude, in the way that Lansbergen, in the figure of no. 7, having drawn the arc OP at right angles at P, the point of the ecliptic, from O, the point of the lunar orbit, uses the latitude OP and the Moon's distance AO to investigate the arc of the ecliptic AP. But Boulliau, in Book 3, Chapter 16, though he knew this, nevertheless kept Tycho's little table of this reduction unvaried, as did Magini in the *Supplement to the Ephemerides*, Canon 9; and Lansbergen finally transcribed it into his own *Canones*, p. 30.

II. The reduction of the Moon to the ecliptic, moreover, is at most about 7′, and it is to be subtracted if the argument of latitude is signs 0, 1, 2, 6, 7, 8; in the others it is to be added to the Moon's place in the orbit; but the contrary must be done if the reduction is to be made from the ecliptic to the orbit. Nor is this equation to be neglected, since otherwise in applications to the fixed stars a notable error can be committed, when the Moon's place, observed by instruments in the ecliptic, is brought back to the place in the lunar orbit. Moreover, according to Kepler it is a little greater in the eclipse syzygies, because the angle of inclination of the Moon's path is then 5° 18′; the same author also notes this in the Rudolphine Tables, precept 130.

LIBRI QVARTI FINIS.

End of Book 4.

LIBER QVINTVS DE LVNÆ ET SOLIS ECLIPSIBVS.

Book Five. On the Eclipses of the Moon and the Sun.

CAPVT I.

Chapter 1.

De Nomine, & falſis Eclipſium cauſis ac ſpeciebus, in omnibus Sideribus: Deque in his detegendis, illiſq. prædicendis Aſtronomorum Præſtantia.

On the name, and on the false causes and the kinds of eclipses, in all the heavenly bodies; and on the excellence of astronomers in detecting and predicting them.

[Margin: The names of an eclipse.]

I. We now enter the darkness of eclipses, than which, nevertheless, nothing in the whole theatre of astronomy is more illustrious, and nothing more sublime than the contemplation of them. *Eclipsis* is a Greek word: from ἐκλείπω [Greek: ekleipō, "I fail"], that is, *deficio*, comes ἔκλειψις [Greek: ekleipsis, "eclipse"], namely a failing [deliquium] or defection [defectio]. So the sick who, deprived of the vital spirit, seem to suffer a swoon of the soul and a faintness very like the death agony are said to have fallen into ἔκλιψιν [recte: ἔκλειψιν] [Greek: ekleipsin, accusative of the same word]. Therefore the Moon, when at the full it falls into the shadow of the Earth and grows pale, deprived of the Sun's life-giving rays, and the Sun in its turn, when through the interposition of the Moon, failing not for itself but for us, it seems to be darkened, are both said to fail and to suffer eclipse, or a failing, deficiency and defection [deliquium, defectum, & defectionem]. For Pliny the Elder uses these names along with Cicero; and in Pliny the Elder and in Tacitus the shadow of the Earth darkening the Moon is called *hebetatrix* [the dimmer]. Long ago, however, Anaxagoras called eclipses simply σκιὰς [Greek: skias, "shadows"], that is, shadows; Plutarch and Ptolemy called them ἐπισκοτήσεις [Greek: episkotēseis, "darkenings"], that is, obscurations; and the Latin poets after Virgil called them the labours of the Moon and of the Sun. For what Lucretius had said in bk. 5:

> Likewise also the failings of the Sun, and the hidings of the Moon,

Virgil expressed more elegantly in *Georgics*, bk. 2, thus:

> But as for me, first of all may the Muses, sweet before all things,
> Whose sacred emblems I bear, struck by a mighty love, Receive me,
> and show me the paths of the sky and the stars, The Sun's various
> failings and the Moon's labours.

[Margin: Barozzi, in correcting Virgil, himself needs correcting.]

The last of these lines Barocius [Francesco Barozzi] (bk. 4 of the *Cosmographia*, p. 328) to no purpose brought out inverted, thus:

> Defectus Lunæ varios, Soliſq. labores. [The Moon's various failings,
> and the Sun's labours.]

His reason is that the same Virgil said in *Aeneid* 1:

> Hic canit errantem Lunam, Soliſq. labores, [Here he sings the
> wandering Moon and the Sun's labours,]

and that the Sun does not truly fail, or is not truly deprived of light, as the Moon is in an eclipse. But the Sun does not truly labour any more than it truly fails; and Barozzi has surely fallen into an eclipse of memory, since he ought to have remembered that in Pliny the Elder and Tacitus, not to mention the poets, the eclipse of the Moon is everywhere expressed by the name of labour, and its failing by the verb *laborare*, as we shall presently see.

[Margin: Why the name of labour was transferred to the eclipse of the Moon.]

[Margin: Fictions of Stesichorus, Pindar and other poets about the cause of eclipses of the Sun and Moon.]

II. For, as Pliny the Elder testifies (bk. 2, chap. 12), there was an ancient error in the wretched mind of men, which at the failings of the stars dreaded crimes or some death of the heavenly bodies. It is evident that the sublime mouths of the poets Stesichorus and Pindar were in this fear at an eclipse of the Sun; and that in the case of the Moon mortals laid the blame on sorcery and for that reason came to its aid with a discordant din. The idea, of course, was that the Moon failed because, by the magic incantations of witches, above all Thessalian ones, with a bronze rhombus wound round with threads and whirled in a circle like a top, it was compelled to come down to earth and to froth onto the herbs with some virulent dew or other, which the sorceresses needed as something of immediate effect for whatever they wished. The Moon, it was held, usually struggled against being called down from the sky, but was at last overcome by the chants, unless it were deafened to the witches' chants by the ringing of bronze and a disorderly din, and the noisy superstition of mortals came to the aid of the Moon labouring in that struggle; and this superstition persists to this very day among certain barbarians. It will not be out of place to confirm this by the testimony of poets and historians, as well as the other opinion, about the failing of the Sun on account of men's crimes. There is that well-turned line of Virgil, *Eclogue* 8:

> Songs can even draw the Moon down from the sky.

Ovid, *Amores* 1, elegy 8, on the old woman Dipsas:

> She knows well what a herb can do, what threads set whirling on the twisted rhombus, What the venom of a mare in heat avails. When she has wished, the clouds mass over the whole sky; When she has wished, again the day shines in the world.

And in the same place, bk. 2:

> Songs draw down the horns of the blood-red Moon.

And *Metamorphoses* 12:

> His mother was Mycale, who, it was agreed, had often by her chanting Drawn down the horns of the resisting Moon.

And in the *Epistle of Hypsipyle*:

> She strives to draw the resisting Moon down from her course,

and *Metamorphoses* 4:

> Or beneath a glowing whiteness, When the bronzes sound in vain in aid of the Moon.

Of this redness the same Ovid says in the *Amores*:

> As roses gleam when mixed among their lilies, Or when the Moon labours with her enchanted horses.

So in *Metamorphoses* 7 that sorceress boasts:

> You too, Moon, I draw down, although the Temesan Bronzes lessen your labours.

And another, in Petronius, in the *Satyrica*:

> Why should I speak of lighter things? The image of the Moon descends, Drawn down by my songs.

And Medea in Seneca, Act 4:

> I see the swift chariot of Trivia, Not the one she drives all night, bright with full face, But the one she drives, ghastly with mournful face, When, harried by Thessalian threats, She skirts the sky on a closer rein. So, pale as you are, pour your sad light With your torch through the air; Terrify the peoples with a new horror, And in your aid, Dictynna, Let the costly bronzes of Corinth sound.

Tibullus, bk. 1, elegy 8:

> Song also tries to draw the Moon down from her chariot, And will do it, if the struck bronzes do not sound.

Lucan, bk. 6:

> And, pressed down by chant, she suffers such great labours Until, closer now, she froths onto the herbs beneath.

And shortly after:

> —— she supplies lunar venom in plenty.

Martial too (bk. 9, epigram 22), with a shameful grief, mourns the death of Philaenis, the most ancient of bawds and witches:

> What woman now will know how to draw down the Moon with a Thessalian rhombus, What woman, etc.?

[Margin: A witty passage of Martial, and of Juvenal.]

But more wittily, of a certain eloquent man, one accustomed to report whatever went on in the world (bk. 12, epigram 46):

> He will say how many hands of the world are beating bronzes When the Moon, cut by the Colchian rhombus, takes a beating.

But more sharply Juvenal, *Satire* 6, on a garrulous little woman:

> —— Such a torrent of words falls That you would say so many basins and bells Were being struck at once. Now let no one tire out trumpets and bronzes: One woman alone will be able to come to the aid of the labouring Moon.

[Margin: Seneca: an ingenious fiction.]

But Seneca more ingeniously still brings on the Chorus in Act 2, applying a very recent failing of the Moon, whether real or invented, to the beauty of Hippolytus, by whose beauty Diana, turned to a blush as if by a magic incantation, checked her course:

> The star born after the ancient Arcadians Will not be able to turn her white chariot. And lately she blushed, though no dirtier cloud Stood in the way of her bright face; But we, anxious at the troubled light, Thinking her drawn by Thessalian songs, Gave out our ringings. You had been the labour, You the cause of the delay.

This fiction is surely far more generous than the one that Ludouicus Viues [Juan Luis Vives], hostile to a city of the Swiss and to some magistrate or other (rather, I do know him, but I do not want him named, since he was slandered unjustly), made up: namely, that the man was so stupid that when a donkey-driver blamed the failing of the Moon on his little donkey, which had

swallowed with a brisk draught the Moon shining back from the bottom of a trough, he went so far in believing him that he went out of the city wanting to look into the matter, had the wretched little donkey cut open with more than triumviral scourges and made to vomit up the Moon; and that, the Moon meanwhile having been restored to its former splendour, he returned to the city full of self-congratulation. It should then have been added, so that this splendid fiction might not lack its tail, that a law was passed forbidding beasts of burden in future to drink water under the open sky at full Moon. But enough now of trifles, and of the wrinkles with which I fear the brows of the sterner sort are furrowed; let us smooth them again with a little flower or two from the Christian poets.

Papinius Statius, *Thebaid* 6:

> ..... whenever from the stunned stars The Sun's darkened sister is torn away; far off the nations Clash their aiding bronzes and fear in vain; but the victorious Thessalian woman Laughs at the panting team when her song is heard.

And Petrus Apollonius, *On the Destruction of Jerusalem*, bk. 1:

> ..... A mingled Shout goes up to the stars, as great as when peoples striving to help the frightened Moon Redouble it with ringings of sharp bronze, And strive that the Thessalian song may not reach The goddess's deafened ears.

Now to other writers, leaving the poets for a while.

[Margin: The superstition of the ancients, mixed in with little tales.]

III. St Basil (*Hexaemeron*, homily 6) relates that ridiculous little stories of old women are going around, who say that the Moon, "by μαγγανείαις [Greek: manganeiais, "trickeries, sorceries"]", that is, by sorceries, is "moved from its own seat and drawn down to earth"; and Apuleius, bk. 1, reports that "by a magic whispering the Sun is checked, the Moon made to froth", etc. Caesarius too, in dialogue 3, ridiculing the superstition of the ancients, says: "The Moon, although she does not let herself down, nevertheless burns with grief, shudders, hides herself by putting on a cloud, and, following her brother's example, disdains to lavish her splendour on the unworthy, etc. How do some say that the Moon is pulled from the sky by incantations and conjuring tricks? Certainly the greatest part of the human race, persuaded and preoccupied by the report of this thing, think by beating iron and bronze to free her from her error by the din, as if she had gone out of her power." There survives by St Maximus, bishop of Turin, a homily entitled *On the Failing of the Moon*, which contains this: "When I asked what this shouting meant, they told me that your outcry was coming to the help of the labouring Moon, and was aiding her failing with its shouts." But this abuse was far older. For Livy relates in bk. 26: "The unwarlike multitude of the Campanians, stationed on the walls, raised a shout with such a clash of bronze as usually happens on a silent night at a failing of the Moon." There is a remarkable passage on this subject in Tacitus, bk. 1 of the *Annals*, where he relates that the Pannonian legions, on hearing of the death of

Augustus and the succession of Tiberius, turned to mutiny, and that Drusus, sent there, barely escaped the fury of the threatening men. He then adds:

[Margin: An eclipse of the Moon, opportune for quelling a mutiny.]

"Chance softened a night that was threatening and about to break out into crime: for the Moon, in a suddenly clear sky, was seen to grow faint. The soldier, ignorant of the reason, took this as an omen of the present state of things, likening the failing of the star to his own labours, and thinking that what they were carrying out would turn out well if the brightness and clarity of the goddess were restored. So they made a din with the sound of bronze and the blended noise of trumpets and horns; as she grew brighter or darker, they rejoiced or grieved. But after clouds rose and blocked the view, and she was believed buried in darkness, they lamented, as minds once struck are easily moved to superstition, that eternal labour was portended for them, and that the gods turned away from their crimes. Caesar, thinking that advantage must be taken of this inclination and that what chance had offered must be turned to wisdom, orders the tents to be visited." And a little later: "There lasted too the dread of heavenly anger, and that not in vain did the stars grow dim against the impious, and the storms rush down," etc.

[Margin: An eclipse of the Moon predicted by Sulpicius Gallus.]

Let Plutarch follow this, who reports the following in his *Life of Aemilius Paulus*: "That night, when after dinner the soldiers had given themselves over to sleep, suddenly the Moon, while it was shining with full light and high in the sky, was darkened, and having lost its light passed, covered over with various colours, into darkness. When the Roman soldiers, according to their custom, tried to call back its splendour by the ringing of struck bronze, and raised many lights with kindled torches towards the sky, nothing of the kind was done by the Macedonians; instead, horror and stupor pervaded the camp, and the common talk murmured that the king's destruction was portended by that prodigy. Aemilius, however, although he was not altogether ignorant of the ecliptic alternations by which the Moon, carried round, falls at fixed times into the shadow of the Earth and stays hidden until, having passed beyond the shadow, it is again lit by the Sun, nevertheless, being a religious man and an augur, as soon as he saw the Moon purged of darkness, sacrificed eleven calves to it."

Christophorus Borus [Cristoforo Borri] also relates, in his report of the year 1620 on the kingdom of Cochinchina, sent to Urban VIII, that on the night following December 8 the prince of Caciani rushed out into the open with a great part of the city and, with drums and trumpets ready, waited in vain for a lunar eclipse predicted by that ruler's astrologers, who erred by one day; for it occurred on the night of the 9th, as the mathematicians of our Society had determined. Then indeed, with a frantic din, the cry went up that they should bring help to the Moon. For some Arabs hold that the Moon then either slips into the sleeve of Muhammad or is swallowed by the celestial Dragon, the little tale taking its occasion from the nodes and limits of the Moon's path, which

they have called the heads and tail and bellies of the Dragon. Levinus Hulsius also reports (chap. 30 of the *Dutch Voyages*) that in the year 1599 an eclipse of the Moon on August 6 was seen by the inhabitants of the Moluccas, Tidore and Ternate, and by the Dutch there, at about 8^{h} in the evening, and was received with a huge din of cymbals and bells.

[Margin: The supposed cause of solar eclipses.]

IV. It was a very ancient opinion that the Sun suffers eclipses by day because, with its eyes turned elsewhere, it recoils in horror from the crimes of mortals and from some monstrous deed, either already committed or soon to be perpetrated; and this opinion was not rarely succeeded by the fear, arising from these eclipses, of the imminent end of the world. Well known is that passage of Seneca in Act 4, on Thyestes, as he feeds on the flesh of his own children at an unspeakable banquet and insults the Sun itself as it draws in its rays:

> O long-suffering Phoebus, though you have fled backward And
> removed the day, broken off in mid-sky, You have set too late, etc.

After this the Chorus, having lengthily lamented the failing of the Sun, ends in these complaints:

> Have we, out of so great a people, seemed Worthy to be crushed
> by the world With its hinge overturned? On us the last Age has
> come. O we, born to a hard Lot, whether, wretched, we have lost
> The Sun or driven it away: Let complaints depart, go away, fear:
> He is greedy of life who does not wish, When the world perishes
> with him, to die.

As if to say that, when the Sun is dying and threatening the destruction of the world, no one should take his own death hard. Pliny the Elder made the same point in passing about the failing of both luminaries (bk. 2, chap. 12), saying: "For who, seeing these things and the appointed labours of the stars (since it has pleased us to call them so), would not, being born mortal, pardon his own necessity?"

What Virgil sang in *Georgics* 2 about the Sun darkened after Caesar's death seems to point here:

> He too took pity on Rome when Caesar was extinguished, When
> he covered his shining head in dark rust, And the impious ages
> feared eternal night.

But most think that this was not a true eclipse of the Sun but a ball of clouds, seen beneath the Sun for that entire year, as Pliny the Elder relates (bk. 2, chap. 30): "There occur prodigious and longer failings of the Sun, such as when the dictator Caesar was killed and in the Antonian war, with a continuous pallor of almost the whole year." Yet that unforeseen darkening, of which Petronius speaks in the *Satyrica*, does seem to signify an eclipse:

> At once the slaughter of men and the losses to come
> Were revealed by the auspices of the gods: for with bloody mouth
> The disfigured Titan covered his face in gloom.

And Lucan, bk. 1:

> Titan himself, when he bore his head in mid-Olympus, Hid his burning chariot in black gloom, And wrapped the world in darkness, and forced the nations To despair of day.

But Heruartus [Johann Georg Herwart von Hohenburg] (chap. 145 of his *Chronology*) moves that darkening to the beginning of the year under the following consuls, Hirtius and Pansa. On prodigies of this kind, however, and on spurious eclipses, see the Scholia of the following chapter, and below, chap. 20, at the year 43.

[Margin: Opinions of the old philosophers on the cause of eclipses.]

V. Now let us not be reluctant to review, from Plutarch (*De placitis philosophorum*, bk. 2, chaps. 24 and 29), the babbling school of the ancients on the causes of eclipses. Anaximenes thought that the failing of the Moon happens when the mouth of its rim [absidis] is stopped; Berosus, when the extinguished part of it is turned towards us; Heraclitus, when the convexity of its bowl is turned over; certain of the Pythagoreans, by an obstruction of the light by our earth and a reflection from that of the Antichthones. Anaximander said that the failing of the Sun happens when the breathing-hole in the rim of the solar wheel is blocked; Heraclitus, by the turning of the solar bowl, which shines only on its hollow side and is then rolled round upward; Xenophanes, by a real extinction of the Sun, another meanwhile being reborn, for there are, he said, several Moons and Suns over different tracts of the earth. Aristarchus counted the Sun among the fixed stars and said that the Earth turns around the Sun and by its own inclinations and motions shades the Sun for itself.

[Margin: Truer opinions.]

But Plutarch also adds, in the same place, the authors of truer causes, saying: "Plato, Aristotle, the Stoics and the mathematicians agree that failings occur when the Moon runs into the shadow of the Earth, and by the interposition of the Earth between the two heavenly bodies." And in chap. 24 he had said: "Thales was the first to say that the Sun fails when the Moon passes perpendicularly beneath it, since she is by nature opaque like the Earth; and this is seen as in a mirror with a disc placed underneath." The same author, however, in the *Nicias*, gives first place, but among the Athenians, to Anaxagoras:

[Margin: Danger to learned men from the ignorant.]

"For the first who most openly and most freely dared to publish in writing an account of the illumination and shading of the Moon was Anaxagoras, and he no man of old; nor was this then discussed publicly, but secretly and cautiously among a few. For the Athenians did not then tolerate those who professed the nature of things and celestial and sublime matters, as men who tore the divine power apart into absurd causes and necessary affections. Thus Protagoras too was driven into exile, and Anaxagoras, thrown into prison, Pericles barely saved," etc.

VI. The true cause of eclipses, then, which Plutarch has already touched on, is for the Sun the opaque body of the Moon interposed between our sight and the Sun, and for a failing of the Moon the shadow of the Earth interposed

between the Sun and the Moon. "For," says Pliny the Elder (bk. 2, chap. 10), "it is evident that the Sun is hidden by the intervention of the Moon, and the Moon by the interposition of the Earth, and that they repay each other in turn: the Moon by its interposition takes the same rays of the Sun away from the Earth, and the Earth from the Moon. As the Moon passes beneath, sudden darkness is drawn over; and in turn the star grows dim by the other's shadow."

[Margin: Sulpicius Gallus's priority among the Romans in the science of eclipses.]

[Margin: And that of Thales among the Greeks.]

But Pliny the Elder discloses in the same bk. 2, chap. 12, from where the Romans learned these things, saying: "And the first of the Roman race to make public the explanation of both heavenly bodies was Sulpicius Gallus, who was consul with M. Marcellus, but was then a military tribune; he freed the army from anxiety on the day before King Perseus was overcome by Paulus, being brought forward before the assembly by the commander to foretell the eclipse, and soon afterwards also by composing a book. Among the Greeks, however, the first of all to investigate it was Thales of Miletus, in Olympiad 48, year 4, when he predicted the failing of the Sun that took place under King Astyages, in the year 170 of the founding of the City." Now this passage of Pliny the Elder, and Plutarch's account of the eclipse that arose before the defeat of Perseus, which we reported towards the end of no. 3, must be illustrated by the account of Livy (Decade 5, bk. 4), in which he says:

"When the camp had been fortified, C. Sulpicius Gallus, a military tribune of the second legion, who had been praetor the previous year, with the consul's permission called the soldiers to an assembly and announced that on the coming night, so that no one should take it for a portent, the Moon would fail from the second hour to the fourth hour of the night; since this happens in a natural order at fixed times, it can be both known beforehand and predicted. Thus, just as they did not wonder, since the risings and settings of the Sun and Moon are fixed, that the Moon shone now with a full orb, now, when waning, with a slender horn, so they ought not to take it as a prodigy either that she is darkened when hidden by the shadow of the Earth. On the night that was followed by September 4 [pridie nonas Septembris], when the Moon had failed at the stated hour, the wisdom of Gallus seemed almost divine to the Roman soldiers. It moved the Macedonians as a grim prodigy portending the fall of the kingdom and the ruin of the kingdom, and the seers took it no differently; there was shouting and howling in the Macedonian camp until the Moon emerged into her own light."

[Margin: Sulpicius Gallus's prediction of the eclipse was held almost divine.]

VII. And it was truly something divine to bring the so abstruse principles of eclipses, which leave no trace of themselves in the sky and happen so far from our regions, back to laws so fixed that astronomers can foretell all those that have been or will be, and how great their magnitude and duration are, and the moments of their beginning and end. And not undeservedly, says Reinhold

in the *Theoricae* (p. 209), do such things arouse admiration: for the keenness of human ingenuity could never have penetrated to the knowledge of such great matters without God going before, as Plato most weightily says in the *Epinomis*. Nor was Dionysius extravagant in once paying Helicon of Cyzicus a talent for each eclipse properly predicted; nor is it unfitting that even now the ruler of Cochinchina awards a mathematician an entire estate for the same thing. If this generosity resided with other princes, astronomers would not be compelled to degenerate into astrologers, and to recook the very stars into gold for themselves with alchemical industry, or to hunt their living among the beasts of the zodiac with invented predictions. With sounder counsel they would imitate Hipparchus, of whom Pliny the Elder says (bk. 2, chap. 12), after the praises of Thales and Sulpicius Gallus:

[Margin: Encomium of Hipparchus and of astronomers, from eclipses predicted long in advance.]

"After them Hipparchus foretold the course of both heavenly bodies for six hundred years, embracing the months of the nations, and the days and hours, and the sites of places and the villages of peoples, with time as his witness; in no other way than as one sharing in the counsels of nature." Having recalled these things he exclaims: "Mighty men, above the nature of mortals, who discovered the law of such great divinities, and set free the wretched mind of men, etc. Hail to your genius, you interpreters of the sky, you who comprehend the nature of things, by which you have overcome gods and men."

[Margin: St Augustine's most wholesome warning to astronomers.]

But unless astronomers refer the divination and glory of these alternations to the Father of lights, with whom there is no change nor shadow of alteration, there is a danger that they themselves may run into deeper darkness and shadows. Let them therefore listen to Augustine, the summit of theologians, who wholesomely warns them (*Confessions*, bk. 5, from chap. 3, and bk. 7, chap. 6): "Those who know these things exult and are lifted up; and, through impious pride falling back and failing from their own light, they foresee the future failing of the Sun so far ahead, and do not see their own in the present; for they do not ask religiously from where they have the ingenuity with which they ask these things."

SCHOLIA.

Scholia.

[Margin: Do other heavenly bodies suffer eclipses?]

I. This book treats of the failings of the Sun and Moon, as phenomena to be wondered at among those that are manifest. For the eclipses of the other heavenly bodies are either non-existent, or almost daily, or hidden from the common sight of mortals and observable only by the most skilled astronomers with the Belgian spyglass [Specillo Belgico]. To one who asks, then, whether other heavenly bodies have their eclipses, there respond, some one way and

some another: Pliny the Elder (bk. 2, chap. 10); Ptolemy (*Almagest*, bk. 9, chap. 1), and on that passage Regiomontanus in the *Epitome*, prop. 1; Clavius in the *Sphere*, p. 70; Barozzi in the *Cosmographia*, bk. 4, p. 326; Wurstisen on Peuerbach's *Theoricae*, p. 357; and Kepler in the *Optics*, p. 304, and in the *Epitome of Copernican Astronomy*, bk. 6, p. 825. But a distinction is needed, and we must first speak of eclipses made by a shadow, of which kind is the eclipse of the Moon.

[Margin: First reply, on the first kind of eclipses, those from shadows.]

Therefore that statement of Pliny the Elder, that no other heavenly body is darkened in the same way as the Moon, that is, by the shadow of the Earth, since the shadow does not extend beyond 282 terrestrial semidiameters (from what was said in bk. 3, chap. 11, Problem 8), is called into doubt by Kepler on account of the parallaxes of Mars, because of which it does not seem impossible that Mars, opposite the Sun, is grazed by the Earth's shadow. It never, however, reaches Saturn and Jupiter; nor does the Earth ever come between the Sun and Venus or Mercury. So even if the Earth's shadow extended to their heaven, they would still never fall into it, because of their small digressions from the Sun, which do not exceed 50°. The same Kepler adds that it seems fitting that, just as the point of the Moon's shadow ends at the Earth, so the point of Venus's shadow should end at the Moon when near the Sun, and Mercury's at Venus, and the Earth's at Mars, and that of Mars at Jupiter, and that of Jupiter at Saturn. On this matter we cannot decide before we have established their distances, and whether they have their own light; for that is necessary if one is to undergo an eclipse from the shadow of another. But it is now certain that the Satellites of Jupiter frequently suffer eclipses from Jupiter's shadow, and this will be shown below, in bk. 7, when we treat of Jupiter.

[Margin: Second reply, on eclipses from the interposition of a heavenly body.]

II. As for the second kind of eclipses, those that occur when a heavenly body comes between our sight and another body, it is certain that all the fixed stars that lie below the zodiac [infra Zodiacum], and all the planets, can be covered by the Moon, just as the upper planets can be covered by the lower. For astronomers everywhere record conjunctions of this kind that are at the same time occultations. Thus in the year 241 before Christ, September 4, Jupiter as a morning star [Eous] occulted the southern Ass [Aſellum Auſtrinum], according to Ptolemy, bk. 11, chap. 3.

[Margin: Fixed stars occulted by planets.]

And in the year 272 before Christ, January 18, Mars in the morning covered the northern star in the forehead of Scorpius; and on October 12 Venus in the morning covered Vindemiatrix, according to Ptolemy, bk. 10, chaps. 4 and 9. And in the year of Christ 1574, September 16, Mästlin saw Regulus covered by Venus in the morning, and Kepler saw the same in the year 1598, September 25, at 3^h in the morning.

[Margin: Planets occulted by planets.]

Now Jupiter occults Saturn only very rarely, within many centuries; yet Kepler thinks this happened in the year 1464. Proclus, for his part, reports some comet or other covered by Jupiter; on which, Tycho, *Progymnasmata*, vol. 1, p. 619. Furthermore, Mästlin and Kepler saw Jupiter covered by Mars at Tübingen, 1591 January 9: the fiery colour of Mars showed it to be the lower. Proclus reports that Venus was observed running beneath Mars, and Mercury beneath Venus; for Venus and Mercury can occult each other, now the one and now the other being the upper. Mästlin also saw Mars occulted by the whiter Venus in the year 1590, October 3, at 5^{h} in the morning.

[Margin: What of Venus and Mercury with respect to the Sun?]

There remains the controversy over Mercury and Venus: whether they can occult, not indeed the whole of the Sun, but a part of it. Certain more recent authors mentioned in Ptolemy, bk. 9, chap. 1, set these two above the Sun for that reason, as al-Biṭrūjī also did in the case of Venus: because they had never been seen to occult the Sun at their conjunctions. But Ptolemy rejects those authors, because these planets can be conjoined with the Sun and yet, on account of latitude (just as the Moon, on account of latitude, usually does not cover the Sun), or else on account of the smallness of their visible diameter, as Regiomontanus there and Clavius above add, either not cover the Sun or cover it imperceptibly. For if, as al-Battānī and Thābit ibn Qurra hold, the visual diameter of Venus is ten times smaller than the Sun's diameter, that is, of 3′, then the visual circle or disc of Venus will be to the Sun's disc as 1 to 100; since by prop. 20 of the Sixth and prop. 2 of the Twelfth of Euclid, circles are to one another as the squares of their diameters, and the squares of the diameters are in the duplicate ratio of the diameters. Or, as Barozzi says, if Venus and Mercury were to pass beneath the Sun in the same plane with the Sun and our sight, they would still either not appear, on account of their diaphaneity, or would appear as a black point on the Sun. But we have already shown, in the Scholia to bk. 3, chap. 3, that Mercury has been seen under the Sun and has more than once occulted a small part of it. Venus, says Kepler, cannot cover the Sun in this century; yet it could almost 200 years ago and will be able to some day, and the smallness of its diameter will be no obstacle if it is observed with a telescope, since it was no obstacle in the case of Mercury, which is smaller than Venus. From this you see with what limitation we must understand the dissertation of Martinus Hortensius on Mercury seen under the Sun and Venus unseen.

[Margin: Spurious or extraordinary eclipses, especially of the Sun.]

III. Besides the regular or ordinary eclipses of the luminaries, there occur certain spurious, or extraordinary, or prodigious ones, with which astronomy does not concern itself: whether from snows [niuibus] or clouds, or from flocks of birds, or from misty globes accompanying the luminaries beneath them, or from the shadows of comets, or from ashes and smoke belched into the heavens out of volcanoes like Vesuvius [ex Veſuuijs], or finally from a coming-together of spots. Kepler, in the *Optics*, p. 304, surmises a shadow of comets on account of the darkness that lay over the lands in the year 47.

[Margin: Eclipses from ashes.]

As for ashes, Cornelius Gemma relates that in Portugal in the year 1562 the Sun was covered by them. And Dio [Cassius Dio] records such an eclipse of the Sun too, seen at Rome under Domitian, when Vesuvius belched ashes as far as Palestine; and under Titus, for the same cause, the Sun was hidden just as if it had been eclipsed, says Dio. So too Marcellinus [Marcellinus Comes], in the *Chronicle*, relates a night-time darkness by day on account of the ashes and murk of Vesuvius. And because these fires usually come with earthquakes, while the prodigious eclipse of the Sun at the death of Christ came with an earthquake, Kepler, in the *Epitome of Astronomy*, p. 896, does not hesitate to attribute it to ashes of this kind, against the manifest authority of St Dionysius the Areopagite; but on this see below, chap. 19, no. 3, opinion 4.

[Margin: And from misty globes.]

Of dense globes running beneath the luminaries, and as it were clinging to them, several examples survive. Such was that clouding-over for the whole year after the murder of Caesar, which was discussed in the previous chapter, no. 4, even though Ovid, *Metamorphoses* 15, places this failing before Caesar's death. On these eclipses see what I said in bk. 3, chap. 3, Scholium 1; and add to them the darkness lasting many days, of which Trebellius Pollio writes in the *Gallieni*; and the darkness of 18 days in the year 798, so great that ships at sea went astray; and that passage of Tertullian in *Ad Scapulam*: "For that Sun too, in the assembly at Utica, its light almost extinguished, was so great a portent that it could not have suffered this from an ordinary eclipse, being placed in its own exaltation [hypſomate] and domicile." And this of St Jerome to Pammachius: "We split the Church, we who a few months ago, around the days of Pentecost, when the Sun was darkened and the whole world feared that the judge was coming at any moment," etc. Baronius understands this of a prodigious eclipse, under the year 393, and Scaliger agrees with him in the *Prolegomena*, p. 51, since from the year 391 to 395 the Moon's latitude allows no eclipse of the Sun around the days of Pentecost.

CAPVT II.

Chapter II.

De Admiranda DEI Prouidentia in Eclipſibus; Deq. illarum multiplici vſu, Effectibus, & Portentis.

On the wonderful Providence of God in eclipses, and on their manifold use, effects and portents.

I. I do not deny that, for the foolish and for those prone to slander the works of God, the failings of the Moon and Sun could be turned into failures of a Providence not understood: as if eclipses defiled or infested the heavens, or had been established rashly, from uncorrected errors of motion, for the sake of no good. For this is what created, for Anaxagoras when he asserted that the

Moon is shaded by the Earth, that danger among the Athenians which we have already reported from Plutarch's *Nicias*. For they would not tolerate that the Moon, as if it were some divinity, should of necessity suffer the deprivation of its light from our Earth, for absurd causes that had no definite end in view; and Plutarch pointedly notes this there. But in truth they themselves were wrong by the whole width of heaven, while Providence erred not at all. For the supreme Maker of all things had many ends, and excellent ones, in eclipses, and commanded the light of his divine wisdom to shine out of this darkness, as will be clear from their manifold use, shortly to be shown.

[Margin: 2 Corinthians 4.]

[Margin: First end and use of eclipses, theological or moral.]

II. The first end, then, unless I am mistaken, which concerns religion and morals, was that failings of this kind should be proof to us that the Moon and Sun are not divinities, since their light can be either wholly taken away or impeded in its propagation; and that true divinity is to be acknowledged in him alone with whom there is no change, nor shadow of alteration: for God is light, and in him there is no darkness at all.

[Margin: 1 John 1.]

Next, and closest to this, that whoever contemplated these alternations of the luminaries should be reminded again and again of their own mortality, and of the destruction that will at last come to all things, however beautiful. To this purpose is that verse of Ecclesiasticus 17: "What is brighter than the Sun? And this too will fail." Or, if it is permitted to repeat here the words of Pliny the Elder from bk. 2, chap. 12: "Let anyone who sees these things and the appointed labours of the stars (since it has pleased us so to call them), being born a mortal, pardon his own necessity." What if eclipses were also destined for this: that no one should be so bent towards the ground and forever gazing at earthly things, so careless and contemptuous of the heavens, as not to be forced at some time, by the very horror of this unexpected darkness, to raise his eyes; whether, thunderstruck, he merely looked up at the heavens, or also began to surmise and seek out the causes of these darkenings [Greek: ἐπισκοτήσεων (episkotēseōn, "darkenings")] and of the unlooked-for gloom? And those who grasped these causes, and were now able to divine and foretell other similar portents to come, drawn by this particle, or rather participation, of divinity, might be joined by closer reverence and love to God, who foreknows all things to come, and might then be invited to cultivate astronomy further by the good fortune of so great a success. Or at least that from portents of this kind they might dread some huge calamity, the avenger of an offended Deity; and, some day, whether the last day of the World or that night, as soon to fall upon them, or might dread in these shadows its lesson and its shadow.

[Margin: Second end and use of eclipses, astronomical.]

III. After these or other supreme causes and ends, eclipses are more useful to no art than to astronomy, just as nothing demands greater skill than the exact prediction of eclipses. Kepler, in the *Epitome of Astronomy*, p. 900, not inaptly

called eclipses the tutors of astronomers for thoroughly learning the celestial motions. Pliny the Elder, bk. 2, chap. 11, puts it aptly: "This reckoning draws mortal minds up into the heavens and, to those contemplating as if from there, discloses the size of the three greatest things in nature. For the whole Sun could not be taken from the lands by the Moon's coming between, if the Earth were larger than the Moon. From both, the Sun's vastness is disclosed more surely, so that there is no need to search out its extent by the evidence of the eyes and the conjecture of the mind."

[Margin: Pliny the Elder's lapse.]

Here Pliny the Elder must be pardoned the error by which he thought that eclipses of the Sun could be universal for the whole hemisphere of the Earth, and therefore that the entire globe of the Earth could be enclosed within the conical shadow of the Moon's body. If that happened, Pliny the Elder's argument would without doubt establish that the Moon is larger than the Earth. Nor is there any reason for Iacobus Milichius [Jacob Milich], on this passage, to invert the text in order to vindicate Pliny the Elder; for Pliny the Elder continues in the same opinion, saying of the Sun: "The failing of the Moon declares its size by a reckoning beyond doubt, just as, failing itself, it declares the smallness of the Earth." Yet rightly, in chap. 10, after the mention of the Moon made in chap. 9, he adds: "that its own failings and the Sun's, a thing most wonderful in all contemplation and like a portent, are the indicators of their sizes and of the shadow." And indeed it is incredible to say how many things most worth knowing astronomers have attained through the shadows of eclipses, climbing as by ladders into heaven, more happily than by any giant's effort.

> Happy the souls whose care it was first to know these things, and to climb to the homes above. They brought the distant stars close to our eyes, and set the aether beneath their own genius. So is heaven sought: not that Olympus should bear Ossa, and the peak of Pelion touch the stars!

That is how it is. From eclipses of the Moon we have learned the Moon's simple motion and its first inequality, and also the shape of the Earth's shadow; from the Moon's motion, its parallaxes and distances from the Earth; from the Moon's distances, the distances and parallaxes of the Sun; from these, the thickness and height of the Earth's shadow. From eclipses of the Sun, moreover, we have confirmed the Moon's parallaxes and have obtained the measure of the Moon's shadow; further, from the Sun's intervals we have searched out the intervals and parallaxes of the remaining planets; and at last, from the distances of all the planets together with their apparent diameters, we have hunted down by geometrical ingenuity their true and threefold magnitude. For that reason we owe a great part of astronomy, and the most delightful part, to eclipses, or rather to immortal God, their architect. And yet, even if nothing else were learned from it, the theory and practice of foretelling eclipses was to be sought for its own sake, as a most splendid proof of the divine genius in ordering them, and of human genius, not without God, in searching them out.

[Margin: Third end and use of eclipses, geographical.]

IV. Who is so uncultured [Greek: ἄμουσος (amousos, "without the Muses", uncultured)] or so ignorant of geography [Greek: ἀγεόγραφος (ageographos, "ignorant of geography")] as not to know that geography would be maimed and obscure if there were no eclipses? For there was a time when geography, as to the distances of places, teemed with foul errors, because, as Ptolemy says in bk. 1, chap. 3, "mathematical considerations were not yet exactly grasped, nor had many lunar eclipses yet been observed at the same time in different places." For the difference of longitude, or distance between the meridians of widely separated places, can be measured with no other ten-foot rod and no other device than the difference of time at which the same phase of the same eclipse, say the beginning or end of the failing, is observed by two spectators in those separated places; for that difference, converted into degrees of the equator, at once yields the difference of the meridians. Thus, for example, we have recognised that Paris lies further west than our Italian Bologna by exactly 11° reckoned on the equator, because the several phases of the lunar eclipse of the year 1642, April 14, were observed at Paris by Father Georges Fournier, S.J., together with the eminent astronomers Pierre Gassendi and Ismaël Boulliau, 44^{m} of time later than by us at Bologna. So too it has been established that Macau, on the borders of China, lies 15° further west than Nagasaki, the chief city of Japan, from the eclipse of the Moon of the year 1612, November 8. Its beginning was observed at Nagasaki at 9^{h} 30^{m} after noon by that most illustrious and unconquered martyr of Christ, Father Carlo Spinola, and at Macau at 8^{h} 30^{m} by Father Giulio Aleni, S.J., once a pupil of Giovanni Antonio Magini, to whom this observation was written; we found it here at Bologna among his papers. Now 1 hour corresponds to 15°, and 44^{m} of time to 11° on the equator. Therefore by paired observations of eclipses and their not difficult calculation, as with an enormous measuring-rope, or by throwing a very great bridge, we join places far apart from one another and set them in their proper seats on the geographical globe [Glogo] [recte: Globo], the altitude of the pole being known.

Further, whether total eclipses of the Moon are better suited to this business, as Ptolemy, Tycho, Magini and most others have believed; or those of the Sun, as Kepler prefers in the Rudolphine Tables, chaps. 16 and 32, precept 176; or the very frequent eclipses of the satellites of Jupiter, observed with suitable telescopes, as Galileo, Don Vincenzo Renieri and Pierre Hérigone have thought; or finally the illuminations of small spots on the Moon's disc, both in an eclipse and outside one, or observations made suddenly and noted in different places with excellent spyglasses together with the exact moment of time, as that man so dear to me, my friend in things celestial [amicus Vranicus], Michael Florent van Langren, Royal Cosmographer, hopes and is working to do: this controversy we have discussed in our *Geographical Book*, which will be the fourth in the second volume of our *New Almagest*. Meanwhile, all you who take pleasure in geographical maps of the world or in globes, "whom it delights

[viuat] [recte: iuuat] to run over the world with unmoving feet": credit to eclipses, and to the Deity who dispenses them, this most beautiful and most pleasant opportunity. Those maps of yours shine out of this illustrious darkness; with the brushes of such great shadows are terrestrial hemispheres or globes painted. Finally, so that in this point of the universe the points of places may be distinguished for you, the lamps of the world are extinguished again and again. Have we, and our learning, been so close to the heart of God?

[Margin: Fourth end and use of eclipses, chronological.]

V. Now that most ancient witness of the times, bond of the ages, lamp of history and high priest of truth, chronology, would never have escaped from the utterly obscure hiding-places of years otherwise uncertain, had not the luminaries darkened on known days, and the eclipses noted by historians, carried a torch before it. How true this is, and how frequent and immediate a remedy it is for setting the dislocated limbs of chronicles, I have discussed abundantly in the *Chronological Book*, which is the fifth of volume 2 of the *New Almagest*. There too we have exposed the false reasonings of many, and above all of Kepler, who, from eclipses noted around the birth or death of Christ, try to undermine this era of the times, in very common use throughout the Christian world for a thousand years and more. As if the epoch of Christ, the Son of God, and of the sect of Christians so dear to him, had been less a concern to God than that of Iphitus or Nabonassar, in his providence for the years of the Olympiads or of the Egyptians!

[Margin: Note the remarkable argument for divine Providence in preserving and restoring the celestial motions and the series of the times.]

Yet the order of the times mattered so much to God that he willed nothing ever to be taken from their intervals, not even through that prodigy by which, when Christ was about to die, the Moon, retreating from the place of the full Moon, darkened the Sun for three whole hours by passing under its disc. For having performed this obedience, the Moon returned not to its former place but to a place about 1½° further east than the place of the full Moon, as far as it would have advanced in the zodiac during that delay of three hours. Otherwise we could not, by calculating backwards, have recognised the eclipses of 700 years and more before Christ that were observed by the Chaldaeans and recorded in Ptolemy, nor have confirmed them by the moments of the hours handed down; we should have to apply a prosthaphaeresis of three hours in order to reproduce them exactly. From this I likewise gather that the customary series of the times was not interrupted either as a favour to Hezekiah, in whose time the Sun went back on the dial of Ahaz, or at the command of Joshua, "who made the Sun stand still in his days"; rather, whatever had been taken away by that prodigy was restored and made good from heaven. This was also the opinion of St Thomas Aquinas, part 3, question 44, art. 2.

[Margin: Fifth end and use of eclipses, physical.]

VI. I now pass to the physical uses, of which there are doubtless many, though unknown to us. For just as the alternations of winter and of night have

their advantages in nature, so the withdrawal of the Moon's light at that time when, full, it would otherwise have lit the lands, and the blocking of the Sun's rays, appear to serve for tempering or interrupting influences. Nor, if eclipses are harmful to some things, should we believe that they benefit no sublunary nature at all. As for the calamities that most astrologers appear to foretell from the failings of the luminaries, we shall say something in the Scholia.

[Margin: Sixth end and use of eclipses, political.]

VII. There remains the last use of eclipses, the civil use, which has shown itself on various occasions. On these, different people have used or abused them in different ways, through skill or lack of skill, to the happiness or the ruin of the commonwealth. How well Sulpicius Gallus used a lunar eclipse to steady the Roman soldiery, and Drusus to quell a mutiny of the soldiers, we have already related in chap. 1, no. 3, from Tacitus and Plutarch; to these we shall here add similar select cases.

[Margin: An eclipse of the Sun predicted by Thales.]

VIII. The first eclipse of the Sun, then, predicted to the Ionians by Thales of Miletus, was useful not only to him but also to the Medes and Lydians. Of this Pliny the Elder says, bk. 2, chap. 12: "Among the Greeks, moreover, Thales of Miletus was the first of all to investigate it, in the fourth year of the forty-eighth Olympiad, predicting the eclipse of the Sun that occurred under King Alyattes, in the year 170 from the founding of the City." In the life of Thales, Diogenes Laertius has this: "Most think that he was the first to search out the secrets of astrology, and that he foretold eclipses and solstices of the Sun, as Eudemus says in his history of astrology; for which reason he was greatly admired by Xenophanes and Herodotus." And shortly after: "He was also the first to find the Sun's course from solstice to solstice; and the first, as some hold, to say that the Moon's orb, compared with the Sun's size, is a 720th part of it." But to our purpose, Herodotus, bk. 1, relates as follows: "In the sixth year of the war between Cyaxares, king of the Medes, and Alyattes, king of the Lydians, when they were fighting a pitched battle on equal terms, it happened while the battle-lines stood that day was suddenly turned to night. Thales of Miletus had predicted this change of the day to the Ionians, fixing in advance the year in which it occurred. When the Lydians and Medes saw day turning to night, they broke off the fight, and for that reason hurried all the more readily to establish peace between them, the authors of this agreement being Syennesis the Cilician and Labynetus the Babylonian."

[Margin: Peace entered upon between the Medes and Lydians by force of this eclipse.]

We shall say more on this in chap. 19, at the year 581 before Christ.

[Margin: An eclipse of the Sun terrifies Cleombrotus; and another terrifies the ship-captain of Pericles, not Pericles.]

IX. In the year 478 before Christ an eclipse of the Sun, October 2, at 2^{h} after noon at Athens, or another nearer to this one, greatly terrified Cleombrotus while he was fortifying the Isthmus, as Herodotus relates, bk. 9. But another,

in the year 431 before Christ, August 3, at about 6^h after noon at Athens, did not terrify Pericles, because he had learned the causes of eclipses from Anaxagoras. Of this one Plutarch relates, in the *Pericles*, that while Pericles was setting sail for the Peloponnese night was drawn over the sky, and at once the helmsman of the commander's ship was terrified; but Pericles ran up and freed him from his panic terror by wrapping the man's head in the covering of a cloak and asking him whether he feared anything from that cloak. When the man said no, Pericles added that the Sun was covered by the Moon in just this way, and would soon be present again; and that there was no other difference except that the Moon was of course larger than the cloak of Pericles. Of this same eclipse Thucydides, who flourished at that time, speaks in bk. 2, saying: "In the same summer, on the first day of the month, at the new Moon [in Neomenia], the only time at which it is evident that the Sun can fail, the Sun was so darkened after noon that several stars were seen in the sky; and shortly after, when it had taken the shape of a half Moon, it returned to its former shape."

[Margin: An eclipse of the Sun ominous for Pelopidas.]

X. It did not go so well with Pelopidas, who, as Diodorus Siculus relates, bk. 15, in Olympiad 104, that is, in the year 364 or 365 before Christ, fought against Alexander of Pherae. And Plutarch says in the *Pelopidas*: "And now, as the commander was about to set out, the Sun failed, and darkness seized Thebes in the daytime. Then Pelopidas, seeing the crowd disturbed by that portent, was unwilling to compel men who were trembling and had no good hope, or to lead into danger the seven thousand citizens who had offered themselves to him. He therefore gave himself to the Thessalians, with three hundred horsemen following him of their own accord and with mercenary soldiers. Neither the augurs nor his friends approved of that expedition, thinking that this so great a sign had been sent from heaven to so great a man. And indeed the Thessalians did win, but Pelopidas, fighting with unconquered spirit, died pierced by wounds after a very great slaughter of the enemy, and was buried with great honour on a pyre built of soldiers' shields, with the bridles and trappings taken from the horses, so that they too seemed to mourn the death of so outstanding a commander."

[Margin: The dexterity of Agathocles in the use of an eclipse of the Sun.]

XI. I should now like to bring forward the acumen and dexterity of a Sicilian king, namely Agathocles, when he was taking his fleet across to Africa, in the consulship of Gaius Junius and Quintus Aemilius. Justin says, then, in bk. 22: "But the sight of a portent frightened them, because the Sun had failed as they were sailing. The king gave an account of this with no less care than of the war: if it had happened before they set out, it would be believed to be a prodigy against those about to set out; but as it was, because it had happened after they had departed, it portended against those to whom they were going. Further, the failings of natural stars always change the present state of affairs, and it was certain that a change was signified for the flourishing wealth of the Carthaginians and the adverse fortunes of the Sicilians." Of this failing

Diodorus Siculus says: "On the next day there was so great a darkening of the Sun that you would have said it was plainly night, the stars appearing on all sides." Salianus conjectures it to have been in the year of the world 3744, or 310 before Christ, August 15, at $8\frac{1}{4}^{h}$ after midnight at Syracuse, and says it was almost total.

[Margin: The skill of the Emperor Claudius in the use of an eclipse of the Sun.]

XII. Let us now hear Dio, bk. 6, on the Emperor Claudius, in the consulship of Marcus Vinicius and Statilius Corvinus: "Since an eclipse of the Sun was going to occur on his birthday, he feared that some disturbance might arise from it, as certain other prodigies had also happened; so he publicly announced not only that this failing would occur, and both its time and its extent, but also the causes for which it would necessarily come about."

[Margin: The inexperience and timidity of Nicias on account of an eclipse of the Moon.]

XIII. But so that we do not pass over lunar eclipses untouched, we must recall the timidity of Nicias arising from the eclipse of the Moon of the year 413 before Christ, August 27, about midnight at Syracuse. On account of it, says Thucydides, bk. 7, he feared to lead his fleet out of the harbour, created ruin for himself and his men, and exposed himself with his whole army to be killed by the Syracusans. Polybius, bk. 9, Diodorus Siculus, bk. 13, and Plutarch in the *Nicias* concur; Plutarch says: "When everything was ready for departure, and none of the enemy was watching, the Moon failed by night. This cast great terror into Nicias and others who, deceived either by inexperience or by superstition, fear portents of that kind. For it was already then, after a fashion, commonly understood that the Sun's light is sometimes darkened at the time of new Moon, and that the Moon brings this about; but what the matter was, that the Moon, suddenly, when it is shining in full light, should lose it and show various colours, was not easily understood, and was construed as a prodigy by which some great calamity was being foreshown by divine agency." After recounting the inexperience of the Athenians, and the danger of Anaxagoras, who was the first among them to profess openly the true causes of lunar eclipses, he adds:

[Margin: The judgement of Philochorus on the portent of an eclipse.]

"Nicias at that time by chance had no skilled seer with him; for Stilbides, who used to remove much of the superstition, had died shortly before. For, as Philochorus asserts, that sign is by no means bad for men in flight, but very advantageous; for things done in fear require darkness and hiding-places, and light is against them. And in any case the portents that occur about the Sun or Moon are ordered to be guarded against for only three days, as Autoclides relates in his *Exegetics*; but Nicias ordered them to wait for another circuit of the Moon, as though he would not at once see it restored to its brightness after it had passed the place of its opposition to the Sun."

[Margin: The act of Dion, contrary to Nicias.]

Dion, King of Sicily, by contrast, about to sail from Zacynthus against Dionysius, when the Moon had failed and others were turning this into a bad omen, was not ignorant of the cause himself: he pursued the undertaking he had begun, drove out the tyrant, and took possession of Syracuse; for he had been taught by Plato. So Plutarch.

[Margin: Christophorus Columbus used an eclipse of the Moon to good effect.]

XIV. Christophorus Columbus [Cristoforo Colombo] profited not a little from his knowledge of eclipses. For when in the year 1493 he had landed on the island of Jamaica, and the Spanish force there was suffering from want after a mutiny, and no food could be obtained from the Jamaicans, Colombo threatened them with a plague and, as a sign of it, with an eclipse of October 22, shortly to appear. When it occurred, those barbarians vied with one another to bring supplies and threw themselves at his knees to obtain pardon. On this matter one may also see Io. de Roias [Juan de Rojas] in the preface of the *Astrolabe*, addressed to Charles V.

[Margin: The prediction of an eclipse opens a way for the Gospel.]

XV. Let the line be closed by the lunar eclipse of the year 1620, December 9, which the mathematicians of our Society had foretold to the Prince of Caciani [?] in the Kingdom of Cochinchina as going to occur on the night following that day, when his other astrologers maintained that it would occur on the night following the 8th. So when they had waited for it in vain that night, with a great display of drums and trumpets, and it came on the night of the 9th, it is incredible how much those wretched astrologers blushed at this, and how great a splendour of reputation accrued to our men. So much so that (if what Christophorus Borus [Cristoforo Borri] relates in the Relation sent to Urban VIII is true) that ruler, together with many men of letters, embraced our religion. Surely no eclipse of the Moon was ever more fortunate: out of its darkness God willed so much light to shine forth for that people walking in darkness.

SCHOLIA

Scholia.

[Margin: The opinions of astrologers on the significations of eclipses set out.]

I. Astrologers derive not a few things from eclipses for their predictions. For Hermes says in the *Centiloquium*, no. 53: "Many misfortunes come about in the world when there is an eclipse of both luminaries in one month, and especially in those places over which their signification is particular." And Messahallach [Māshāʾallāh], ch. 5, teaches that the signification of a bad or good outcome is to be sought from the degree ascending at the moment of mid-eclipse and from the lord of the ascendant, but that the Part of Fortune is not to be neglected. Ptolemy, however, says in the *Tetrabiblos*, Book 2, ch. 7: "There is no prior or

more powerful cause of such occurrences than the one drawn from the ecliptic syzygies of the Moon and Sun, and from the transits of the stars at their time." He first gives guidance for the regions in which eclipses signify some outcome. He then asserts that the effects of a solar eclipse last as many years as the eclipse lasted hours, but as many months as a lunar eclipse lasted hours. Finally he adds many things on the kind of these outcomes, which Porphyry elucidates, and also Cardano, who says in Book 2, text 21, that the eclipse of the year 1539, which took place in Taurus, a part of his horoscope, brought nothing bad to himself or to Milan, because it did not accord with the city in which he was then living. Yet still more rules on these matters are handed down by Proclus and, following him, by Giuntini, from p. 307 to p. 314. Giuntini brings as an example the birthday of the city of Venice, which was 421 March 25, with the 25th degree of Cancer ascending, and the solar eclipse of the year 1591, which took place in the 26th degree of Cancer. He therefore says, with Luca Gaurico etc., that it portends for the Venetians seditions and many evils for senate and people, betrayals and naval disasters etc., but that their liberty will last until the year 1880.

[Margin: The duration of the Venetian Republic.]

Garsæus [Johannes Garcaeus] too, p. 288, contends that Bohemia is subject to Cancer rather than to Leo, because of the evils that befell it from eclipses taking place in Cancer: namely from the solar eclipses of the years 1424, 1433, 1469 and 1536. He also contends that Nuremberg is subject to Gemini or to Virgo, because it suffered many calamities from solar eclipses occurring in those signs: on 1448 August 29, and in 1551 in the 17th degree of Virgo, and in 1560 in the 8th degree of Virgo, with a plague following. But he had said at p. 57 that an ecliptic place, if it is referred to a nativity chart, always signifies something bad, in whatever part of the sky it may be. Finally Origanus, in vol. 1 of the *Ephemerides*, part 3, ch. 2, having reviewed Ptolemy's rules from Book 2 of his *De Iudicijs*, from ch. 7 on, foretells many good things for mankind from the lunar eclipse of 1607 August 26, with Jupiter as its ruler and Venus as its companion.

But I, though I do not deny that eclipses have great force in either direction, nevertheless judge that much among the astrologers is empty. To show this by at least one example: Ptolemy lays down, Book 2, text 28, that if an eclipse, especially of the Sun, falls on the western horizon, its effects will begin around the last 4 months and will show themselves most in the last third of the whole duration. Yet the plague that in 1630 followed or accompanied the solar eclipse of June 10, which we observed at Forum Naevii [?] [Foro Næuij], had begun to show itself in the month of April, and it raged in June, July and August.

[Margin: The deceptiveness of astrological rules.]

Kepler too, in the *Ephemerides*, p. 7, no. 25, judges that there is no force in the nodes of the lunar orbit; granted that in the *Optics*, p. 407, he agrees with Ptolemy that a greater signification can be sought from the inclinations of eclipses.

II. It is nevertheless a pleasure, as a courtesy to the learned reader, to compile a short chronicle of some eclipses, with the more notable events more closely connected with them, although I am well aware that some eclipses occur in every year, as I shall say in ch. 15, and that there is no year in which some more notable event does not happen.

CHRONICON ALIQVOT ECLIPSIVM

A chronicle of some eclipses.

Ex Aliorum magis quàm Noſtra opinione Aſtrologiæ deſeruiens.

Serving astrology in the opinion of others rather than in our own.

Years before Christ	Date	Entry
754	July 5	But according to the old calendar on April 21: an eclipse of the Sun, on which day the foundations of Rome were laid, if Taruntius Firmanus is to be believed, about whom Plutarch in *Romulus*.
721	March 19	A total eclipse of the Moon, on which Ptolemy, Book 4 *Almagest*, ch. 6. End of the empire of the Assyrians and beginning of the Babylonian, now established.
585	May 28	An eclipse of the Sun predicted by Thales, from which came peace between the Medes and Lydians. Herodotus, Book 1, and Pliny the Elder, Book 2, ch. 12.
523	July 16	An eclipse of the Moon, on which Ptolemy, Book 5, ch. 14. The killing of Cambyses followed. Salianus, Year of the World 3531.
502	November 19	An eclipse of the Moon in Ptolemy, Book 4, ch. 9. Defeat of the Sabines, death of Valerius Poplicola. Salianus, Year of the World 3552.
491	April 25	An eclipse of the Moon, on which Ptolemy, Book 4, ch. 9. A severe famine at Rome, beginning of the Peloponnesian War. Salianus, Year of the World 3563.
463	April 30	An eclipse of the Sun. The Persian war, and the revolt of the Egyptians from the Persians. Eusebius in the *Chronicle*.
431	August 3	A total eclipse of the Sun, on which Thucydides, Book 2, and Plutarch in *Pericles*. A comet, and plague at Athens.
413	August 27	A total eclipse of the Moon. Nicias cut down with his fleet by the Syracusans. Polybius, Book 9; Thucydides, Book 7; Diodorus Siculus, Book 13; Plutarch in *Nicias*.
394	August 14	An eclipse of the Sun. The Persians routed by Conon in a naval battle. Xenophon, Book 4.
364	or 365	A great eclipse of the Sun; Pelopidas killed in battle. Plutarch in *Pelopidas*; Diodorus Siculus, Book 15.
331	September 20	A total eclipse of the Moon, 11 days before Darius was defeated by Alexander the Great. Diodorus Siculus, Book 17; Q. Curtius, Book 4; Arrian, Book 3; Pliny the Elder, Book 2, ch. 70; Plutarch in *Alexander the Great*.
310	August 15	An eclipse of the Sun. Agathocles conquers the Carthaginians, but with ill fortune. Justin, Book 22.
217	February 11	A failure of the Sun. The Romans defeated by Hannibal at Trasimene. Livy, Book 22, and Julius Obsequens in his little book on prodigies.

Years before Christ	Date	Entry
190	March 14	A great eclipse of the Sun. Antiochus routed by Scipio. Livy, Book 37.
168	June 20 [?]	A total eclipse of the Moon. Perseus, King of Macedon, defeated by Paulus Aemilius on the following day. Livy, Book 35; Plutarch in *Paulus Aemilius*.
50	or 49	An eclipse of the Sun. Caesar crosses the Rubicon: beginning of the civil wars. Dio, Book 41.
44	or 43	An eclipse of the Sun after the killing of C. Julius Caesar: the second civil war. Aurelius Victor; Josephus the Hebrew, Book 14 *Antiquities*, ch. 21; and many others.
4	or 1	An eclipse of the Moon before the death of Herod the child-slayer. Josephus, Book 17 *Antiquities*, ch. 8.
Years of Christ		
14	March 28	An eclipse of the Sun. Augustus dies on August 19. A severe famine followed. Dio, Year of the City, ch. 757; Nicephorus, Book 1, ch. 17; St Jerome in the *Chronicle*.
14	September 26	An eclipse of the Moon, by which Drusus quells the mutiny of the Pannonian legions: Tacitus, Book 1 of the *Annals*.

RESIDVVM CHRONICI.

The rest of the chronicle.

Years of Christ	Date	Entry
49	✝	A total eclipse of the Sun, after which, from Lucan and Dio, Book 4, Caesar, having crossed the Rubicon, casts the die and the torch of the civil wars.
59	April 30	An eclipse of the Sun. Pliny the Elder, Book 2, ch. 70, and Tacitus, who places it among the prodigies on account of the murder of Agrippina by Nero.
69	October 18	An eclipse of the Moon. Nero, on account of the killing of the Holy Apostles Peter and Paul, takes his own life. Dio, Book 65.
72	February 9	An eclipse of the Sun, which was followed on day 22 by an eclipse of the Moon. Jerusalem captured by Titus: more than eleven hundred thousand [supra vndecies] Jews slain, 97000 taken captive. Pliny the Elder, Book 2, ch. 13, and Josephus the Hebrew.
95	✝	An eclipse of the Sun. Domitian killed. Philostratus in the *Life of Apollonius*.
97	January 27	An eclipse of the Sun on the day of Nerva's death. Aurelius Victor, Eutropius, and perhaps Plutarch on this one, in the opuscule *On the Face of the Moon*.
192	✝	An eclipse of the Sun before the death of the Emperor Commodus, from Herodian.
237	April 12	A total eclipse of the Sun. A sign that Gordian would not rule long, as Capitolinus says in the *Gordians*. The sixth persecution in the Church.
301	September 2	An eclipse of the Moon. Diocletian's persecution against the Church. See the Acts of St Felix the Martyr in Baronio, year 302, and Scaliger, p. 18, in the Prolegomena to *De emendatione temporum*.
306	July 27	An eclipse of the Sun. Stars seen in the sky. The Emperor Constantius dies. From the ancient *Fasti*.

Years of Christ	Date	Entry
337	✝	An eclipse of the Sun, shortly before the death of Constantine, who died on May 22, the feast of Pentecost. From the *Life of Constantine*.
450	April 27	An eclipse of the Sun, on which St Gregory of Tours, Book 2, ch. 3, speaking of the coming of Attila and of the heresies troubling the Church, where he adds: "Then too the Sun appeared hideous, so that scarcely a third part of it shone out; I believe, indeed, for such great crimes and for the shedding of innocent blood." [Margin: Even saints turn eclipses into a portent.]
605	May 5	An eclipse of the Sun, and after it a plague of 3 months raging most fiercely throughout all Italy. Regino [Regino of Prüm], Book 1 of the *Chronicle*.
813	May 4	An eclipse of the Sun of 10½ digits. Michael Curopalates dies; Leo the Armenian succeeds. Theophanes in the *Miscellanea*.
840	May 2	A dreadful eclipse of the Sun: a month and a half later Louis the Pious dies. *Annales Franc.*
944	✝	An eclipse of the Sun on the sixth weekday. The King of the Saracens defeated by Ramiro [Rhadamiro]. Sigebert [Sigebertus] in the *Chronicle*.
1009	✝	An eclipse of the Sun. Jerusalem captured by the Saracens. Lycosthenes in the *Chronicle*.
1039	May 14	An eclipse of the Sun. The Emperor Henry dies. Ubbo Emmius.
1133	August 2	An eclipse of the Sun with stars appearing. Schism of three antipopes. Dodechinus and Sigebert.
1181	July 15	An eclipse of the Sun. Alexander III and the Emperor Louis Crassus die. Sigebert.
1187	August 29	An eclipse of the Sun. Baldwin, on September 28, by Saladin, and Jerusalem captured: Lycosthenes in the *Chronicle*. Pope Urban dies at Ferrara on October 20 [13. Kal.Nouembr.]. Spondanus [Henri de Sponde] in the *Annals*.
1197	September 4	An eclipse of the Sun with stars visible. The Emperor Henry dies on September 28, excommunicated by Celestine. Gregorius Tholosas, Book 21 *De Rep.*, ch. 7, and Spondanus in the *Annals*.
1239	June 2	An eclipse of the Sun in Gassendi, in the *Life of Peiresc*. War between Gregory and the Emperor Frederick, who is excommunicated.
1307	February 11	An eclipse of the Sun and then of the Moon on February 26. Our people driven from Rhodes by the Saracens. Lycosthenes in the *Chronicle*.
1419	April 5	An eclipse of the Sun. Bohemian disturbances and calamities from Jan Žižka [Io. Zisca]. Origanus in the preface to vol. 3 of the *Ephemerides*.
1448	August 29	An eclipse of the Sun. Wars between Albrecht of Brandenburg and the Emperor: Origanus above. The Venetians defeated: Lycosthenes.
1518	June 17	An eclipse of the Sun, from which the astrologers conjectured the death of the Emperor Ferdinand. Spondanus in the *Epitome* of the *Annals*.
1520	November 20	An eclipse of the Sun. With Luther excommunicated, turmoil in the Church. Selim dies of plague; Suleiman bursts into Hungary: Spondanus and others in the *Annals*. Origanus above.
1540	April 7	An eclipse of the Sun. A great drought: forests burned up by the Sun; many fires. Origanus above.
1560	August 11	A total eclipse of the Sun observed by Clavius at Coimbra. King Francis of France dies of an abscess on December 5. Gustavus, King of Sweden, dies on September 29. The Spanish fleet battered, with 18000 men perishing, and Andrea Doria [Andreas Auria] dies.

Years of Christ	Date	Entry
1598	March 7; August 16; and 31	An eclipse of the Sun, and on August 31, and of the Moon on the 16th. Rains and floods; was it because it took place in Pisces? From this, crops ruined, dearth of corn, buildings submerged with their people. Origanus above and at this year. Ferrara recovered by Clement VIII on the death of Duke Alfonso; Philip, King of Spain, dies on September 13. Theodore, Duke of Muscovy, died. Eclipses had preceded.
		I pass over the eclipses of our own age, for just reasons; for I am now tired of playing too long a game in the uncertain dust of the astrologers.

CAPVT III.

Chapter III.

Definitio, Cauſæ & Species ac Varietates Lunarium Eclipſium.

The definition, causes, species and varieties of lunar eclipses.

[Margin: Definition of an eclipse of the Moon.]

I. An eclipse of the Moon is the privation, in the Moon, of the primary light it has previously received from the Sun, because of the interposition of the terrestrial globe between it and the Sun. So, more or less, Aristotle, 2 *Poster.*, ch. 1; Pliny the Elder, Book 2, ch. 10; Plutarch in *Nicias*; Livy, Book 4, Decade 5; and others, on whose opinions the same Plutarch, ch. 29, and we ourselves in Book 4, ch. 5, where see what is said against Berossus. I have added the word "primary" [Primarij] in the definition, because the Moon in eclipse is not always deprived of secondary light, that is, of rays refracted in the air around the Earth, as we shall show later. To explain the cause of a lunar eclipse, it is usual in *Theoricae* of the planets to set out first a threefold figure of the shadow, from Cleomedes, Book 2; Pliny the Elder, Book 2, ch. 11; Witelo, props. 26, 27, 28; and from the common *Perspectiva*, props. 22, 23, 24.

[Margin: Cylindrical shadow.] [recte: Vmbra Cylindrica; the note prints Vmbra lindrica]

For either the luminous sphere, as in the figure below, AC, is equal to the illuminated sphere GE, and then the shadow that the illuminated sphere casts has a columnar or cylindrical shape; and for this reason it is called in Greek [Greek: κυλινδροειδὴς (kylindroeidēs, "cylinder-shaped")]. Such is the shadow GKEM, but it extends to infinity.

[Margin: Top-shaped or basket-shaped shadow.]

Or the illuminating sphere is smaller than the illuminated sphere, as in the second figure is sphere EG compared with sphere ABC; and then the shadow of the larger sphere imitates the form of a straight top, or of a truncated inverted pyramid, or of a basket, from which it is also called [Greek: καλαθοειδὴς (kalathoeidēs, "basket-shaped")]. Such is the shadow ANPC; but this too is of infinite length. Or finally the illuminating

Diagram. The first figure of the shadow, set at the foot of the right column. Two equal circles stand side by side on one horizontal line, joined by straight tangents above and below. The right half of the right-hand circle is solid black, and a hatched rectangle runs from it to the right.

- A (above the top of the left circle): point on the luminous sphere
- C (below the foot of the left circle): point on the luminous sphere
- D (inside the left circle, left of its centre cross): no text gloss
- B (right edge of the left circle, on the axis): no text gloss
- F (on the axis between the circles): no text gloss
- H (inside the right circle, at its vertical diameter): no text gloss
- G (above the top of the right circle): point on the illuminated sphere and corner of the shadow
- E (below the foot of the right circle): point on the illuminated sphere and corner of the shadow
- K (top right corner of the hatched rectangle): corner of the shadow
- M (bottom right corner of the hatched rectangle): corner of the shadow sphere is larger than the sphere to be illuminated, and the shadow of this sphere has the figure of a turning-post or cone, one indeed finite and ending in the sharpness of a point. Such, in the adjoining scheme, is the shadow EMG of the sphere EG, illuminated by the larger sphere ABC. Hence that shadow is called [Greek: κωνοειδὴς (kōnoeidēs, "cone-shaped")].

[Margin: Conical shadow.]

Diagram. The second and third figures of the shadow, drawn as one block in the upper left column. A large circle stands in the middle. Above it a hatched inverted truncated cone widens upward to a flat top. Below it two straight lines converge from the sides of the circle to a small circle, half black, and under that small circle a hatched cone narrows to a point.

- N (top left corner of the upper hatched band): corner of the shadow
- O (top centre, on the axis): no text gloss
- P (top right corner of the upper hatched band): corner of the shadow
- A (left of the large circle): point on the sphere ABC
- C (right of the large circle): point on the sphere ABC
- R (inside the large circle, left of its centre): no text gloss
- D (inside the large circle, right of its centre): no text gloss
- B (at the foot of the large circle): point on the sphere ABC
- F (on the axis above the small circle): no text gloss
- H (inside the small circle): no text gloss
- E (left of the small circle): point on the sphere EG and corner of the shadow
- G (right of the small circle): point on the sphere EG and corner of the shadow
- M (apex of the lower cone): apex of the shadow

II. It is evident, moreover, from the Sun's distances from the Earth established in Book 3, ch. 7, no. 15, and from the apparent size of the Sun, on which see the same book, ch. 10, that the true size of the Sun is greater than the whole terraqueous globe. So, from what was said in the same place, ch. 11, according to Tycho, who attributes the least to it, the body of the Sun contains the globe of the Earth 140 times; but according to Wendelen, who attributes the most, 262144 times; according to us, for the present, 38600 times; and according to others otherwise.

[Margin: That the Earth's shadow is conical.]

Therefore the shadow of the Earth illuminated by the Sun must be conical, and so finite. This is confirmed even by the fact that Saturn or Jupiter, let alone any of the fixed stars that are in the Zodiac, never suffer an eclipse from the Earth's shadow, however much the Earth is interposed between these stars and the Sun, provided that one holds that they receive their light from the Sun; otherwise let this confirmation be void. In its place let there follow the demonstration given in Book 3, ch. 11, Problem 8, on the axis or length of the Earth's shadow. With the Sun at perigee, no one puts this at less than 213 terrestrial semidiameters and no one at more than 268 semidiameters; with the Sun at apogee, no one puts it at less than 231 semidiameters and no one at more than 282 semidiameters.

[Margin: The increase and decrease of the Earth's shadow with the Sun's recession or approach.]

For it is certain that the shadow of the Earth, and of any smaller globe illuminated by the Sun or by any larger globe, becomes blunter, narrower and shorter as the Sun approaches the Earth; but as the Sun recedes, it becomes sharper and comes out more extended in length and thicker or ampler in breadth. Erasmus Reinhold demonstrates this in Peuerbach's *Theoricae*, p. 208; but it will be readily evident from the mere drawing of the figures, provided that the same size of the Sun and the Earth is kept at any distance.

[Margin: Argument 1 for the cause of a lunar eclipse from the Earth's shadow.]

III. With these foundations laid, we must now demonstrate that the Moon suffers eclipse from the Earth's shadow. First, from the time. For this eclipse never occurs except at full Moon, when the Moon, distant from the Sun in the Zodiac by a whole semicircle, stands diametrically opposite to it, and so is in that tract of the sky into which the shadow of the Earth illuminated by the Sun is cast. So it is in the present diagram, in which the Sun is AB, the Earth CD, the Earth's shadow CFD. For unless the Moon is in that region of its sky which CFD takes in, it is never naturally eclipsed.

Diagram. The Sun, drawn as a rayed disc with a human face, stands at the top. Two straight lines converge from its sides past the Earth to a point at the foot. The Earth is a circle with its lower half solid black, and below it a hatched cone narrows to the point. Four small circles mark places of the Moon: a white one at the left edge of the cone and a dark one inside it on the same level, a

smaller dark one lower down inside it, and a white one outside the cone beside its lower part.

- A (left of the Sun): one side of the Sun
- B (right of the Sun): other side of the Sun
- C (left of the Earth): one side of the Earth
- D (right of the Earth): other side of the Earth
- H (left of the small white circle at the edge of the cone): the Moon on entering the shadow
- G (right of the dark circle inside the cone): the Moon in the shadow
- I (right of the small dark circle lower in the cone): the Moon in the shadow, more remote from the Earth
- K (right of the small white circle outside the cone): the Moon clear of the shadow
- F (below the apex): apex of the Earth's shadow

[Margin: 2nd argument.]

Second, from the distance of the Moon compared with the height of the Earth's shadow. For this, as I said in no. 2, extends to 213 terrestrial semidiameters and more when it is least; whereas the Moon's greatest distance from the Earth does not exceed 70 terrestrial semidiameters, from what was demonstrated in Book 4, ch. 14. So when the Moon at full Moon is on the ecliptic or near one of the nodes, so that the distance of the Moon's centre from the axis of the Earth's shadow, that is, the Moon's latitude, is less than the sum of the semidiameters of the shadow and the Moon, either the whole of it or a part must be darkened by the Earth's shadow, as if it is at G or I. For the axis of this shadow, cast towards the part opposite the Sun, always lies in the ecliptic, as the Sun does; and the Moon has its primary light from the Sun, from what was said in Book 4, ch. 5. If on the contrary the distance already mentioned is equal to that sum or greater than it, as if the Moon is at K, the Moon at full Moon avoids the Earth's shadow and so is not darkened at all. And it is established from observations of nearly 2400 years that the Moon is always eclipsed when that distance is less than that sum, but free of eclipse when it is equal or greater.

[Margin: 3rd argument.]

Third, from the beginning and end of a lunar eclipse. For it always begins to be eclipsed on its eastern side, because with that side it runs into the Earth's shadow, and it begins to emerge from the shadow likewise on its eastern side; so that the last part to be freed from the shadow is the western. For the Sun moves from B to A, and the Earth's shadow from C to D, and the Moon from H to G, all in the order of the signs; but because the Moon is swifter than the Sun and the shadow in apparent motion in longitude, it at length overtakes the shadow. So when the Moon is placed at H, its western part will appear bright and its eastern part darkened; and when it is placed at G, its eastern part will begin to appear.

[Margin: 4th argument.]

Fourth, from the duration of the eclipse, which, other things being equal, is greater when the Moon is nearer the Earth, as if it is in the tract HG, because there it must traverse a larger space of thicker shadow before it escapes the darkening by its motion of elongation from the Sun. On the contrary, it is less when the Moon is more remote from the Earth, as at I, for the opposite reason.

[Margin: 5th argument.]

Fifth, from the shape of the eclipsed part of the Moon. For the darkened part appears as a segment of a black circle, as may be seen in the first of the following schemes and in others to be presented in ch. 6, exactly as the shape of a shadowy cone requires. For this reason the portion of the Moon free from the shadow is sickle-shaped or horned towards the shadow.

Sixth, from the places of the Zodiac in which lunar eclipses occur. For since it is established from the motion of lunar latitude that the places of the nodes, near which eclipses occur because of the small latitude, move continually against the order of the signs, it has also been observed that the places of eclipses follow this motion and retreat against the order of the signs; so that if the eclipse of one year took place at the beginning of Taurus, the eclipse of the following year takes place in Aries, and so on for the rest.

Seventh, from the part and quarter of the Moon in a partial eclipse. For it is always the northern part when the Moon has southern latitude; but it fails in the southern part when it has northern latitude that is less than the sum of the semidiameters of the Moon and the Earth's shadow.

Eighth and finally, from the infallible prediction of lunar eclipses, which, resting on these suppositions and on the comparison of the sum of the semidiameters of the Moon and the shadow with the latitude, has always been found true. It is evident, therefore, physically and astronomically, that no other more fitting cause can be assigned for the Moon's undergoing eclipse than its falling into the Earth's shadow. Cleomedes, Book 2, ch. 5, presents some of these arguments in learned fashion.

[Margin: The species of lunar eclipses.]

IV. Now there are four species of lunar eclipse, indicated by the same number of schemes added here. In these the ecliptic is the line EL; the Moon's path, inclined to the ecliptic and measuring the span of the duration, is BP; the large black circle is the one whose plane cuts the cone of the Earth's shadow at right angles, at the place through which the Moon passes; the other, smaller ones represent the Moon's disc at the beginning, middle and end of the eclip-

Diagram. Caption: *Partial eclipse of the Moon.* A large black circle is crossed by the horizontal line EL. A slanting line runs from B at upper left to P at right, passing through three small white-outlined circles, the middle one overlapping the top of the black circle.

- B (inside the small circle at upper left): beginning of the Moon's path
- G (above the middle small circle): no text gloss
- P (inside the small circle at right): end of the Moon's path
- E (left end of the horizontal line): end of the ecliptic

• L (right end of the horizontal line): end of the ecliptic

Diagram. Caption: *Total eclipse of the Moon without duration.* A large black circle is crossed by the horizontal line EL. A slanting line runs from B to P through three small circles; the middle one lies wholly inside the black circle, and the outer two touch its edge from outside.

• B (inside the small circle at left): beginning of the Moon's path
• P (inside the small circle at right): end of the Moon's path
• E (left end of the horizontal line): end of the ecliptic
• L (right end of the horizontal line): end of the ecliptic se. The first species is the Partial Eclipse [Eclipſis Partialis], as at G, when at the height of the darkening not the whole Moon but only a part of it is eclipsed. This happens when the Moon's true latitude is less than the sum of the semidiameters of the shadow and the Moon, but greater than the excess of the shadow's semidiameter over the Moon's semidiameter.

[Margin: Partial eclipse of the Moon: how great, and when?]

The second species is the Total Eclipse without duration in the shadow [Totalis ſine mora], as in the second scheme. It occurs when the Moon's apparent semidiameter together with its true latitude exactly equals the shadow's semidiameter, or when the Moon's latitude equals the excess of the shadow's semidiameter over the Moon's semidiameter. For then, at the height or middle of the eclipse, the whole Moon is indeed eclipsed, but at the same moment it begins to emerge from the shadow.

[Margin: Total eclipse without duration.]

Diagram. Caption: *Total eclipse of the Moon with duration.* A large black circle stands in the left column beside the narrow text. A line runs from B to P through a small circle lying wholly inside the black circle, nearer the middle; two further small circles, holding B and P, cross its edge, their centres lying just outside it. A second line, EL, crosses the black circle below the first.

• B (inside the small circle at left): beginning of the Moon's path
• P (inside the small circle at right): end of the Moon's path
• E (left end of the lower line): end of the ecliptic
• L (right end of the lower line): end of the ecliptic

Diagram. Caption: *The greatest of total eclipses, or central.* A large black circle is crossed by the horizontal line EL. A slanting line runs from B at lower left to P at upper right through a small circle at the centre of the black circle; two further small circles, holding B and P, touch its edge from outside.

• B (inside the small circle at lower left): beginning of the Moon's path
• P (inside the small circle at upper right): end of the Moon's path
• E (left end of the horizontal line): end of the ecliptic
• L (right end of the horizontal line): end of the ecliptic

The third species is the Total Eclipse with duration [Totalis cum mora], as in the 3rd scheme. It occurs when the Moon's true latitude is less than the excess of the shadow's semidiameter over the Moon's semidiameter.

[Margin: Total eclipse with duration.]

The fourth species is the Central Eclipse [Eclipſis Centralis], which is nevertheless reduced to the Total with duration. It occurs when, at the middle of the eclipse, the Moon has no true latitude but occupies one of the Nodes with its centre, as in the 4th scheme; and this, other things being equal, is the greatest of lunar eclipses.

[Margin: Central eclipse.]

V. The causes, then, that come together to produce the species of lunar eclipses and to vary their magnitude and duration are 4. The first is the height and thickness of the Earth's shadow, which is greater at a greater and less at a lesser distance of the Sun from the Earth; and the greater it is, the greater the quantity and duration of the eclipse it produces. The second is the Moon's apparent semidiameter, which is greater at a lesser and less at a greater distance of the Moon from the Earth; and the greater it is, other things being equal, the smaller the eclipse. The third is the Moon's true latitude: the less it is, the greater the eclipse it produces, and the greater it is, the lesser. The fourth is the Moon's hourly motion from the Sun, which varies because of the varying slowness of the motion of the luminaries; and the slower this motion of the Moon is, the greater, other things being equal, the duration of the lunar eclipse. Now the causes that increase or diminish these four, whether of themselves or incidentally, also increase or diminish the duration and magnitude of eclipses, as we shall indicate in the Scholia.

[Margin: Causes varying the measure of eclipses.]

SCHOLIA.

Scholia.

I. The first cause varying eclipses of the Moon, discussed in no. 5, is itself varied, first, by the varying eccentricity of the Sun, if that is admitted, on which I have said enough in Book 3, Chapter 26: for the greater the eccentricity, the greater the Sun's distance, the thicker the shadow and the greater the eclipses of the Moon.

[Margin: The Sun's eccentricity varies eclipses of the Moon.]

It is varied, second, by the varying shape and size of the terraqueous globe, not so much because of the varying height of mountains as because of the atmosphere, or the denser air surrounding the Earth. The height of this air is usually estimated from twilight, as we said in Book 1, Chapter 31, and it is not thought to exceed 70 Italian miles; indeed, if its measure is derived from the Sun's refracted rays, it contracts to about 24 miles, as Wendelen holds in the preface to the *Idea of the Atlantic Tables*, and very nearly Kepler in the *Optics*, p. 287. The Earth, therefore, being larger than itself because of this crust of thick air, produces a thicker shadow and increases the shadow's apparent semidiameter, so that Wendelen thinks an eclipse of the Moon begins 17^s or 20^s of time sooner and ends later. But if this air has swollen with clouds or vapours, he thinks this increase can be felt up to a full half of a minute of time. He adds

that our air is lowest under the Equator and the Torrid Zone and higher under the Poles, from which the Earth's shape turns out oval, as Clavius too noted in the *Sphere*; and that account should be taken of this at least in total eclipses without duration.

[Margin: The air around the Earth increases the shadow.]

However, since the shadow's semidiameter, as I shall say shortly, is determined by the Sun's unrefracted rays, and so by rays touching not so much the Earth as the thick air, whatever increase arises from that is understood to be already computed in the semidiameter of the shadow that the astronomical tables give. Conversely, if the Sun's apparent diameter is enlarged because of refractions, the shadow contracts, as Longomontanus notes in Book 1 of the *Theorica*, pp. 292 and 293.

II. The second cause, of which no. 5 speaks, namely the Moon's apparent semidiameter, appears to be varied by the denser air or aether around the Moon, of which we spoke in Book 4, Chapter 2. For David Fabricius, in Kepler's *Ephemerides*, pp. 9 and 22, holds that because of this air the Moon's semidiameter is enlarged, the beginning of the eclipse is therefore hastened and its end delayed, and its duration increased, expanding the Moon's diameter on this account to 4′ or 5′. Wendelen, in the preface to the *Eclipses*, p. 5, supposes that from this the duration can be increased by a whole minute and the beginning hastened by half a minute of time. Nor would Galileo deny it, in the *Sidereus Nuncius*, p. 13, since he admits that the orb of this denser substance, when lit by the Sun, presents us with a larger Moon.

[Margin: Whether the air around the Moon increases eclipses.]

But Kepler, in the *Optics*, p. 302, and in the two passages of the *Ephemerides* noted above, denies that the Moon's diameter or the duration of the eclipse is sensibly increased by this. That enlargement, he says, is a deception of sight, to which white and luminous things next to black or dark ones appear larger, because of the confused image at the back of the eye, the bright part prevailing and acting more strongly. A sign of this, he says, is that the Moon seen through the telescope appears either without that air or with air so thin that the measure of eclipses is in no way varied by it; or even, on the testimony of Fabricius himself, who says of the eclipse of 1613 October 28: "Near the end of the eclipse of the Moon, when to the naked eye a digit was judged to be missing, through the telescope the whole Moon appeared illuminated and round." From this he infers the conclusion that Kepler rejects: therefore the limb that seemed to the naked eye to be lacking was the limb of the lunar air above the Moon's body. But in truth, if that air were something real and objectively increased the diameter and duration, the same should have been seen with the telescope, and indeed more clearly and larger, since that is the telescope's proper effect. That phase, therefore, is not in the sky, but in the eye, overwhelmed by the brightness of our air.

III. Although an eclipse of the Moon is said to be Universal, that is, of the same quantity and duration in all the parts of the Earth where it appears, because

it consists in a true privation of the direct light usually received from the Sun, which occurs in the same way everywhere on Earth; and therefore, if an eclipse of the Moon is partial to one spectator, it should be thought partial to any spectator as well; nevertheless Wendelen acutely noted, in the preface to the *Eclipses*, p. 7, as did Father Paolo Casati by his own wit, that in strictness this is not true. For the same eclipse can be observed as Partial by one spectator and as Total by another, since the Moon's limb can be outside the shadow and the angle of contact, towards the north, and yet be hidden from a southern observer far removed from us; and so an eclipse that is Partial to us can appear Total to our Antoeci, and conversely. Wendelen admits, however, that this difference is so insensible that it deserves to be neglected.

[Margin: A paradox about an eclipse of the Moon, Partial to one and Total to another.]

CAPVT IV.

Chapter IV.

De Triplici menſura Vmbræ Terræ à radijs Solis Irrefractis comprehenſæ.

On the threefold measure of the Earth's shadow as bounded by the Sun's unrefracted rays.

I. The threefold measure of this shadow or penumbra consists in the angle of the shadowy cone, in the height or axis of this cone, and in the breadth, or its apparent semidiameter at the place through which the Moon passes at full Moon. And on the angle and semiangle of the cone of the Earth's shadow, and also on its height, with the Sun's unrefracted rays in view, enough has already been said in Book 3, Chapter 11, Problems 5 and 8, where we also reviewed the opinions of others. Our own opinion is that the already mentioned semiangle is 14′ 50″ with the Sun at apogee and 16′ 8″ at perigee; and there is no one from whose hypothesis there follows a value greater than 16′ 8″ or less than 12′ 10″. Here I add only what I forgot there: that Kepler, in the *Epitome of Copernican Astronomy*, p. 861, derives this semiangle by subtracting the Sun's horizontal parallax from the Sun's visual semidiameter, which is very nearly true. In the same place we fixed the axis of the Earth's shadow at 231½ terrestrial semidiameters with the Sun at apogee, but 213 at perigee. Nor is there anyone who makes it greater than 282 semidiameters of the Earth or less than 213.

Here, however, the terrestrial semidiameters are to be understood as those not of the bare Earth, but of the Earth surrounded by that denser air, which the extreme rays of the Sun touch as they go on beyond without refraction, and not those rays that are commonly imagined to touch the Earth itself; for those are neither extreme rays nor unrefracted. Kepler, however, in the passage cited, derives this axis by multiplying the Sun's distance by the true semidiameter of the Earth, and dividing the total by the excess of the Sun's true semidiameter

over the true semidiameter of the Earth. It remains, then, for us now to deliver what we promised there, in no. 10, and to determine the apparent and the true semidiameter of the Earth's shadow at the place of the Moon's transit; and after that we shall also complete the dimensions and properties of the penumbra.

PROBLEMA I.

Problem 1.

Semidiametrum Vmbræ terreſtris ad datam Lunæ per Vmbram tranſeuntis diſtantiam à Terra, inuenire.

To find the semidiameter of the Earth's shadow at a given distance from the Earth of the Moon passing through the shadow.

II. The first method is by a lunar eclipse of exactly six digits, that is, when exactly half of the lunar disc suffers eclipse at the height of the darkening. Ptolemy gives it in Book 5 of the *Almagest*, Chapter 14; Regiomontanus in the same place, prop. 18; Reinhold in Peuerbach's *Theoricae*, p. 209; and Biancani, Book 9 of the *Sphere*, Chapter 8. For this method, in the adjoined figure

[Margin: 1st method, by an eclipse of 6 digits.]

Diagram. An uncaptioned figure in the left column. A straight line runs from A at the left to C at the right, carrying the points G, B, E and I. A large circle is centred on B. A second straight line runs from C down to the left to D, through three small circles with centres P, M and F; the small circle at M, on the edge of the large circle, has its part within the large circle hatched. Straight lines join B to F, to M and to P, and G to F, E to M and I to P.

- A (left end of the upper line): the ecliptic ABC, as the text gives it
- G, I (on the upper line): feet of the latitudes FG and PI
- B (on the upper line): centre of the shadow
- E (on the upper line): foot of the latitude ME
- C (right end of the upper line): the nearer Node
- D (left end of the lower line): the path of the lunar orbit CD
- O, L (where the lines from B meet the large circle, near F and near P): points on the circle of the shadow OML
- M (centre of the middle small circle): the Moon's centre at the height of the eclipse
- P (centre of the right small circle): the Moon's centre at the beginning of the eclipse
- F (centre of the left small circle): the Moon's centre at the end of the eclipse

let the ecliptic be ABC, on which the centre of the shadow is B, from which the circle of the shadow OML is described; and let the path of the lunar orbit be CD. At the height of the eclipse let the Moon's centre M touch the periphery of the shadow, so that exactly half of the lunar diameter is darkened. For when the centres B and M are joined, BM will be the apparent semidiameter of

the shadow, which Ptolemy holds to be sensibly equal to the Moon's latitude. Therefore, for the moment of such a greatest darkening, derive from the tables the Moon's true latitude and the lunar anomaly, to which the Moon's distance from the Earth corresponds; and the shadow's semidiameter will become known for the given anomaly of the Moon and distance from the Earth, for it will be as great as the latitude.

Thus Ptolemy, above, selects an eclipse of half the Moon towards the north, which took place in the year 225 of Nabonassar, or 523 before Christ, on the 17th of Phamenoth going into the 18th, or July 16, and was observed at Babylon 1^h before midnight. For that equated moment he derives the Moon's anomaly as 28° 5′, and its latitude, and so the shadow's semidiameter, as 40′ 40″.

[Margin: An example from Ptolemy.]

Here, however, we are using arcs as if they were straight lines, or chords of the arcs. In strictness, when the Moon's centre touches the periphery of the shadow, the radius MB makes with the chord MC a right angle at M, by 16 of Book 3 of Euclid. So in triangle CBM the angle B cannot be a right angle; nor is BM the arc of the circle of latitude or its chord, since that must make a right angle with the ecliptic; instead it is called the arc through the centres. The arc of latitude, or its chord, is the line ME, which makes a right angle with the ecliptic at E.

III. Our second method, then, by an eclipse of 6 digits, is this: for the moment of the greatest eclipse, let the Moon's true latitude ME be derived from the tables, and the distance of the lunar centre M from the nearer Node C, namely the arc MC. For from these data, in the spherical triangle right-angled at E, the angle C is easily derived; and with it and the arc MC, in the triangle CBM, right-angled at M, you will derive by trigonometry the arc BM, namely the semidiameter of the Earth's shadow.

[Margin: Our 2nd method.]

IV. Our third method works from these data, at the beginning or end of any lunar eclipse whatever: the Moon's visual semidiameter, its latitude, and the distance both of the shadow's centre and of the Moon's centre from the nearer Node, all of which I suppose can be known from tables already corrected. For in the preceding figure let the Moon's centre be at P at the beginning of the eclipse, or at F at the end; let these centres be joined to B, the shadow's centre, by the lines BP and BF; and from the Moon's centres let there be drawn lines orthogonal to the ecliptic AC, representing the Moon's true latitude, and let them be PI and FG.

[Margin: Our 3rd method.]

Therefore, if you use the beginning: in triangle CIP, right-angled at I, in which are given the base PC, namely the distance of the Moon's centre from the neighbouring Node, and PI, the true latitude, derive angle C. With it, in triangle CBP, and with PC, known as above, and BC, the distance of the shadow's centre from Node C, derive the side BP. From it subtract the Moon's semidiameter LP, and the shadow's semidiameter BL will remain. But

if you use the end: in triangle CGF, right-angled at G, from the base FC, namely the Moon's distance from the nearer Node, and the latitude FG, derive angle C. With it, in triangle CBF, and with FC and BC, known as above, derive the side BF. From it take away the Moon's semidiameter FO, and the shadow's semidiameter BO will be known. Martinus Hortensius, however, in the dissertation with Gassendi on Mercury seen in the Sun, uses an eclipse greater than 6 digits and the duration between the two moments at which the Moon's centre touches the periphery of the shadow, and so appears eclipsed by half of itself.

V. The fourth method is by observation, receiving the Moon's image on a white tablet, especially at the height of a partial eclipse, provided the tablet is equidistant from the lunar disc, and marking three points on the periphery of the lunar image and also three points on the periphery of the shadow, as in the following scheme, in which the image of the shadow is ABC and that of the Moon AFC.

[Margin: 4th method, by reflected observation.]

Diagram. An uncaptioned figure at the foot of the right column, beside the narrow text. Two overlapping circles stand one above the other, the upper larger than the lower, on a vertical line running from B at the top to F at the bottom. The arcs of the two circles cross at A on the left and C on the right, and the straight line AC crosses the vertical line. Straight lines join A and C to E on the vertical line, and A and C to I on the vertical line. The upper circle is inscribed with the words VMBRA VEL SOL (shadow or Sun), the lower with the word LVNA (Moon).

- B (top of the vertical line): on the periphery of the shadow's image ABC
- E (on the vertical line, just below the inscription): centre of the shadow's image
- G (on the vertical line, in the common part of the circles, above K): not explained on this page
- K (on the vertical line, at its crossing with AC): not explained on this page
- D (on the vertical line, below K): on the periphery of the shadowy image
- I (on the vertical line, beside LVNA): centre of the Moon's image
- F (foot of the vertical line): on the periphery of the Moon's image AFC
- A (left crossing of the circles): common point of both peripheries
- C (right crossing of the circles): common point of both peripheries

For when the three points A, F, C have been marked on the periphery of the lunar image, and the points A, D, C on the periphery of the shadowy image, the centre of each, E and I, and the semidiameter are easily found with compasses, by the scholium to Book 4, prop. 5 of Euclid; and from that the proportion of the semidiameters, of the shadow AE to the Moon's semidiameter AI. So, once the Moon's semidiameter is known, the shadow's semidiameter will at once be known.

In this way Father Andreas Arzet of the Society of Jesus, in the eclipse of 1630, on the night following November 19, observed that the diameter of the Moon is to the diameter of the shadow as 100 to 266; and in the eclipse of 1632 October 27, that it is to the diameter of the spurious shadow as 100 to 303, but to that of the legitimate shadow as 100 to 261. And Father Georges Fournier, in the eclipse of 1642 April 14, writes that the semidiameter of the Moon to that of the shadow, or 17 to 40, was observed by him, Gassendi and Boulliau. For it is useful to divide the Moon's diameter into as many and as minute particles as possible; and therefore its image is usefully received through the telescope, by which it is amplified with its clarity and distinctness preserved; and before observing the shadow, the diameter and image of the full Moon are usefully received on the tablet, at a distance determined from the telescope, to be kept from then on.

VI. Our fifth method is this: in a lunar eclipse of 6 digits, at the moment when exactly six digits are darkened, the distance between the lunar horns is taken, and shortly before or after the Moon's diameter is observed in the same kind of parts. For let the Moon's disc be DFKG, its centre A, its diameter FAG and its semidiameter FA; and taking the shadow's centre B, let the arc of the shadow CAE be described with the interval AB, passing through the centre A of the lunar disc. The Moon's horns will be C and E, their distance CE and their half-distance CM. Let one of the horns, as C, be joined by straight lines to the centres A and B, by drawing CA and CB. The shadow's semidiameter will be both BA and BC, and so triangle ABC is isosceles, in which the angles at the base, namely CAB and ACB, are equal by 5 of Book 1.

[Margin: Our 5th method, by observation.]

Diagram. An uncaptioned figure in the left column, beside the narrow text. A tall triangle has its apex at B, its two long sides running down to C at the left and E at the right, and a vertical line from B through D and M to A, the centre of a circle with K at its foot. An arc from C through A to E crosses the circle, the straight lines CE and FAG cross it near each other, and straight lines join C and E to A.

- B (apex): centre of the shadow
- D (top of the circle, on the vertical line): on the Moon's disc DFKG
- M (on the vertical line, on CE): midpoint of the distance between the horns
- C (left end of the arc): one horn of the Moon
- E (right end of the arc): the other horn of the Moon
- F (left end of the diameter, just below C): on the Moon's disc
- G (right end of the diameter, just below E): on the Moon's disc
- A (on the vertical line, below M): centre of the lunar disc
- K (foot of the circle): on the Moon's disc

So make: as the observed FA is to the observed CM, so the radius FA, 100000, to CM, the sine of the arc CD, which measures the angle CAD. Since, therefore, in triangle ABC all the angles will be known individually (for CBA

is the complement to two right angles of the equal angles at A and C), and the base AC is known because it equals the semidiameter AF, neither leg, AB or CB, will escape the trigonometer, in parts of the kind in which AF was observed. Accordingly, if AF is known from elsewhere in minutes and seconds, then from the already mentioned proportion the shadow's semidiameter AB will also become known in minutes and seconds. Thus in 1643 September 27, in an eclipse that at its height was of 6 digits, we observed the shadow's semidiameter AB as 48′. By what means small intervals are taken more exactly, not with the astronomical radius, which is deceptive, but with another instrument, we shall show in the *Liber Organicus*.

It is not necessary, however, for the eclipse to be of 6 digits at its height; it is enough if it is of 6 digits at some moment. We must now come to the methods that are connected with the search for the true semidiameter, or that rest on figures serving the investigation of both.

PROBLEMA II.

Problem 2.

Veram ſimul & Apparentem Semidiametrum Vmbræ Terreſtris inueſtigare.

To investigate at once the true and the apparent semidiameter of the Earth's shadow.

VII. The sixth method is used by Ptolemy, Book 5, Chapter 14; Regiomontanus there, prop. 22; al-Battānī, Chapter 30; Copernicus, Book 4, Chapter 19; and Magini, Book 2 of the *Theoricae*, Chapter 24, although they do not use exactly the same method or figure. It is of this kind. Let the Sun be ABC, its centre D and its diameter AC; and the Earth KLM, its centre N; and let the extreme rays of the Sun touching the Earth at K and M be CKS and AMS. So the Earth's shadow is KSM, its axis NS, and the distance of the Sun from the Earth DN, in the same straight line; and on this line let there be the Moon at apogee, EFG, at distance HN from the Earth. Here Ptolemy supposes that the angle subtending the Sun's visual semidiameter, namely CND, is equal to the angle subtending the Moon's apparent semidiameter, namely angle ENH. Now let the line NP, equal to the distance NH, be taken on the axis NS. P will then be the place of the Moon at apogee in the Earth's shadow, and IQ, orthogonal to NS, will be the diameter of the circle of the shadow at the place of transit of the Moon at apogee.

[Margin: 6th method.]

[Margin: Ptolemy's method for investigating the semidiameters and distances of the shadow, the Sun and the Moon.]

First, Ptolemy, from the observation of two eclipses, about which we spoke in Book 4, Chapter 16, derives the visual semidiameter of the Moon at apogee as 15′ 40″. From this and its distance HN, in triangle EHN, right-angled at E, he derives EH, the true semidiameter of the Moon, as 0 17′ 33″ of terrestrial

semidiameters. And from the observation of the same eclipses he had derived the semidiameter of the Earth's shadow at the place of the Moon nearly at apogee as 40′ 40″, by the first method given in Problem 1; and he adds that from many other observations the Moon's semidiameter was found to be to the shadow's semidiameter as 1 to 2⅗, or as 5 to 13. Let it be made, therefore, as 5 to 13, so the Moon's true semidiameter, 17′ 33″, that is 1053″, to the true semidiameter of the shadow, 2737⅘″, or 45′ 37″ 48‴, or as Ptolemy has it, 45′ 38″, of which the Earth's semidiameter is 60′. Next let it be made, as 5 to 13, so 15′ 40″, the Moon's apparent semidiameter, to 40′ 44″, the shadow's apparent semidiameter.

[Margin: The true semidiameter of the Earth's shadow.]

Diagram. A tall uncaptioned figure in the right column, with the narrow text at its left. A vertical axis runs from the apex S down through P, L, N, F, H and B to D and on to the foot of a large circle. From S two long lines descend to C and A, the ends of the horizontal diameter of the large circle at the bottom. Three chords cross between these lines: IQ through P; KM through N, the diameter of a circle with L at its top; and a line from E through H and G to T, with a small circle above it whose top is F. From N two further lines descend to C and A, passing through E and G.

- S (apex): apex of the Earth's shadow
- I, Q (ends of the upper chord): diameter of the circle of the shadow at the place of transit of the Moon at apogee
- P (on the axis, on IQ): place of the Moon at apogee in the shadow
- L (top of the middle circle): on the Earth KLM
- K, M (ends of the chord through N): points where the Sun's extreme rays touch the Earth
- N (on the axis, centre of the middle circle): centre of the Earth
- F (top of the small circle): on the Moon EFG
- E, G (either side of H): on the Moon EFG
- H (on the axis): centre of the Moon
- T (right end of the line through H, on the long line): end of HT
- B (top of the large circle): on the Sun ABC
- C, A (left and right ends of the large circle's horizontal line): on the Sun ABC
- D (on the axis, on CA): centre of the Sun

Ptolemy, however, supposes that the chords AC, EG and KM differ insensibly from the diameters of these circles, because of the great distance of these three bodies from one another. Second, since HT and PQ, parallel to it, are equally distant from NM, therefore HT and PQ together are double NM, which Regiomontanus says is easily evident; and so they are together equal to two NM, semidiameters of the Earth, or 120′. Therefore, since HG is 17′ 33″ and PQ 45′ 38″, that is together 63′ 11″, when these are taken from 120′ there remains GT as 56′ 49″, of which NM is 60′. Now by the second of Book 6 of Euclid, in triangle ANM, the straight line NM is to GT as AN to AG; and NG to AG as

NH to DH. So NM to GT is as ND to DH; and therefore, if ND is 60′, DH will be 56′ 49″ and HN 3′ 11″. And so the proportion is known of HN, namely the distance of the Moon at apogee from the Earth, to the Sun's distance ND. Now HN is 64 10′, of which NM is one; therefore ND, the distance of the Sun at apogee, is almost 1210, of which NM is one. Further, NH is to HG as ND to DA, but NH is to HG as 64 10′ to 0 17′ 33″, and ND is 1210; therefore DA is 5 30′. Finally, NM to PQ is as NS to PS; therefore the proportion is known of NP, the distance of the Moon at apogee, to NS, the axis of the shadow, which he finds to be 268 semidiameters of the Earth. I wanted to set out the whole of Ptolemy's method here from the man himself, because we presented it otherwise, with Lansbergen, in Book 3, Chapter 7, Problem 2.

[Margin: The Sun's distance.]

[Margin: The axis of the shadow.]

Al-Battānī, however, in Chapter 30, although he assumes the same distance of the Moon at apogee and the same visual semidiameter of the Sun as Ptolemy, nevertheless sets the Moon's semidiameter at 14′ 45″ and the shadow's at 38′ 20″. From these he derives the horizontal parallax of the Sun at apogee as 3′, its distance from the Earth as 1146 terrestrial semidiameters, and the shadow's axis as 254 terrestrial semidiameters. But from his data Lansbergen, in the Prolegomena to the *Uranometria*, derives the true semidiameter of the shadow at the place of the Moon at apogee as 42′ 9317350/10000000, of which one semidiameter of the Earth is 60′; and the axis of the shadow as 225 33′ terrestrial semidiameters, and

[Margin: Al-Battānī's opinion, and its inconsistency.]

the Sun's distance 7936½ semidiameters of the Earth, and its horizontal parallax 0′ 26″.

Copernicus, for his part (Book 4, chapters 19 and 23), puts the Moon's distance at apogee at 65½ semidiameters of the Earth, and the apparent semidiameters of the Moon at 15′ 0″, of the shadow at 40′ 18″, and of the Sun at apogee at 15′ 50″. From these he thinks it follows that the axis of the shadow is 265 semidiameters of the Earth, the Sun's distance 1179, and its horizontal parallax 2′ 55″. But Lansbergen, in the same place, derives from Copernicus's earlier data a true semidiameter of the shadow of 46′ 721760/10000000, of which one semidiameter of the Earth is 60′; an axis of the shadow of 282⅛ [?] semidiameters of the Earth; a distance of the Sun of 941 53′; and a horizontal parallax of 3′ 39″. As for the greatest shadow, Copernicus derives 47′ 52″.

[Margin: Copernicus's opinion and its inconsistency.]

Tycho, however, in volume 1 of the *Progymnasmata*, puts the Moon's distance at apogee at 58 8′ semidiameters of the Earth, and the visual semidiameters of the Moon at apogee at 16′ 0″, of the shadow at 43′ 0″, and of the Sun at apogee at 15′ 0″. These do not cohere. For, as Lansbergen shows in the place cited, there follows from them a true semidiameter of the shadow of 43′ 6306944/10000000, of which one semidiameter of the Earth is 60′, and a semi-angle of the cone of the shadow, namely K S N, of 16′ 8″. That is greater than the angle C N D,

which subtends the Sun's apparent semidiameter, set at 15′, whereas by Euclid, Book I, proposition 32, it ought to be smaller.

[Margin: Tycho's opinion and its inconsistency.]

Longomontanus too (Book 1, *Theorica*, chapter 9) puts the Moon's distance at apogee at 57 38′ semidiameters of the Earth, and at apogee the apparent semidiameters of the Sun at 15′, of the Moon at 16′, and of the shadow at 43′. But from these, on Lansbergen's reckoning, there follows a true semidiameter of the shadow of 43′ 2554304/10000000, of which one semidiameter of the Earth is 60′, and a semi-angle of the cone of the shadow of 16′ 37″. That is certainly greater than the apparent semidiameter of the Sun set by him, whereas it ought to be smaller. But here one should recall what we said about this incongruity and conflict of the hypotheses in Book 3, chapter 7, Problem 6, together with Problem 5, where we dealt fully with the semi-angle of the cone of the shadow, and Problem 7, where we dealt fully with the axis of the shadow, the Sun's rays being taken as unrefracted.

[Margin: Longomontanus's opinion and its inconsistency.]

VIII. The seventh method, however, that of Longomontanus (Book 1, *Theorica*, chapter 9), is as follows. In the accompanying diagram let A be the centre of the Earth and B the centre of the Sun, let C M E and D N E be the rays touching both globes, and so let M E N be the shadow of the Earth. With B A E drawn through the centres, let the Sun's distance A B be a mean one, of 1288 semidiameters of the Earth; and with A D drawn, let the Sun's apparent semidiameter, or its measure the angle B A D, be 15′ 30″. First, then, from triangle A B D, right-angled at B, the Sun's true semidiameter is derived as 5 807/1000 terrestrial semidiameters, and C B is of that size. So, with M R drawn parallel to B A, the angles at R will be right, and R B will equal one semidiameter of the Earth, A M. When this is subtracted from B C, there remains C R, 4 807/1000 semidiameters of the Earth. Further, by Euclid, Book VI, proposition 2, as C R, 4 807/1000, is to the Sun's distance R M, 1288, so B C, 5 807/1000, is to B E, 1556. With A B subtracted, therefore, there remains the axis of the shadow A E, 268 terrestrial semidiameters. Next, let A G be the Moon's mean distance, 56 semidiameters of the Earth. When this is subtracted from A E, there remains G E, 212 semidiameters of the Earth. And with P G drawn parallel to M K, it will be that as E A, 268, is to A M, 1, so E G, 212, is to G P, the true semidiameter of the shadow, 47′ 28″, of which one semidiameter of the Earth is 60′. Finally, with A P drawn, there are given in triangle A G P, right-angled at G, the side A G, 56 semidiameters of the Earth, and G P, 0 47′ 28″. Therefore the angle P A G, subtending the shadow's apparent semidiameter, is 48′ 34″.

[Margin: 7th method: Longomontanus's.]

[Margin: Axis of the shadow at the mean distances of the luminaries, according to Longomontanus.]

[Margin: True and apparent semidiameter of the shadow at the mean distances of the luminaries.]

Diagram. A tall narrow figure inset in the left column, beside the narrowed text. At the top a large circle, the Sun, is cut by the horizontal diameter C–D, on which R and B stand. Two long tangents run down from C and D, through M and N on a middle circle, and meet at the apex E. Between the Sun and the middle circle stands a small circle holding F. The middle circle holds T, A and K on the line M–N. Below it a small circle holds G on the line P–L, inside the cone. Straight lines run down from R and B and to the lower points.

- C (left end of the top diameter): point where the ray touches the Sun
- R (on the top diameter, left of centre): foot of M R
- B (on the top diameter, centre): centre of the Sun
- D (right end of the top diameter): point where the ray touches the Sun
- F (inside the small upper circle): no gloss on this page
- M (left of the middle circle): where the ray C M E touches the Earth
- T (inside the middle circle, left): no gloss on this page
- A (inside the middle circle, centre): centre of the Earth
- K (inside the middle circle, right): point on M N used in the text
- N (right of the middle circle): where the ray D N E touches the Earth
- P (left of the lower circle): end of G P
- G (inside the lower circle): the Moon's place on the axis
- L (right of the lower circle): end of G L
- E (apex): end of the shadow

But Longomontanus adds that at a pole-height of 56° this angle, from many observations of lunar eclipses, has been found no greater than 44′ 10″. From this he confirms that the thickness of the air around the horizon, by refracting the Sun's rays, thins the semidiameter of the Earth's shadow, though only moderately; but that perhaps in farthest Norway, Ireland and regions nearer the Pole, because of thicker vapours, it is contracted somewhat more.

[Margin: Breadth of the penumbra.]

By a wholly similar method, varying the distances of the luminaries, he derives a varying axis and a varying semidiameter of the Earth's shadow, as can be seen in the following small table; although afterwards, in the Table of semidiameters, he limits it and sets it differently, as we shall see below in the Synopsis of opinions.

☉ Distance in semidiameters of the Earth	☾ Distance in semidiameters of the Earth	Apparent semidiameter of the shadow
	Apogee 57 38′	46′ 49″
Mean 1288	Mean 56 0	48′ 34″
	Perigee 54 23′	50′ 24″
Apogee 1334	Mean 56 0	49′ 0″
Perigee 1242	Mean 56 0	48′ 4″

But he first sets out the apparent semidiameter of the shadow at the mean distance of the luminaries, from these authors: Ptolemy 44′ 48″, Alfonso 41′, Copernicus 43′ ½, Reinhold 43′ ¾, Tycho 44′ ⅛ [?]. And he adds that the

ancients, measuring the shadow from the Moon's transit, fell short, because they thought the Moon's motion slower. But since the basis of a varied semidiameter of the shadow is uncertain and has not been demonstrated by this author, and since it contains that conflict spoken of above at the end of number 7, we cannot safely follow him.

IX. The eighth *motus* [recte: modus, "method"] is that of Lansbergen, *Uranometria*, Book 2, elements 5, 6 and 7. First, with Hipparchus, Ptolemy, al-Battānī and, as he himself says, Copernicus, he supposes that the Moon's apparent semidiameter is to the apparent semidiameter of the shadow as 5 to 13. He confirms this by observation of the eclipse of 1601 December 9, of 10 digits 56′ according to Kepler but according to himself 10 digits 40′, with the Moon and Sun nearly at perigee, when the Moon's apparent semidiameter was 17′ 49″ and its northern latitude 32′ 32″. So the sum of the two was 50′ 21″, which is how large the shadow's semidiameter would have been if the eclipse had been of 12 digits. But one digit and a third remained unharmed, to which 4′ 2″ of the Moon's diameter correspond. When these are subtracted from 50′ 21″, a semidiameter of the shadow of 46′ 19″ remains. And as 17′ 49″ is to 46′ 19″, so is 5 to 13. From this he derives that the apparent semidiameter of the shadow, with the luminaries at apogee, is 39′. From these, moreover, he derives the axis and the true semidiameter of the Earth's shadow, and the semi-angle of the cone, by the following method.

[Margin: 8th method: Lansbergen's.]

[Margin: Apparent semidiameter of the shadow.]

In the figure immediately above, let A G be the Moon's distance and B A the Sun's, A E the axis of the shadow, A N the semidiameter of the Earth and G L that of the shadow; and let K L be drawn parallel to A G, and A L, so that G A L is the apparent semidiameter of the shadow. First, then, in triangle G A L, right-angled at G, the Moon's distance A G from the centre of the Earth is taken as given, and the angle G A L, the apparent semidiameter of the shadow; therefore G L, the true semidiameter of the shadow, can be found trigonometrically. Second, with G L subtracted from A N, there remains K N, known, in triangle K L N, in which K L too is known, being equal to the lunar distance; therefore the angle K L N can be found, which by Euclid, Book I, proposition 29, equals the interior angle A E N, namely half the angle of the cone of the shadow. Third, as A N, the sine of the angle A E N, is to A E, the second sine of the same angle, so A N, one semidiameter of the Earth, is to A E, the semidiameters of the Earth contained in the axis of the shadow. So at the greatest distance of the luminaries he derives an apparent semidiameter of the shadow of 39′ and a true one of 43′ 67825/100000, of which one semidiameter of the Earth is 60′; a semi-angle of the cone of the shadow of 14′ 34″; and an axis of the shadow of 236 semidiameters of the Earth. We, however, deduced the semi-angle of the cone and the axis of the shadow somewhat differently from the data he gives, in Book 3, chapter 7, numbers 6 and 9. Moreover,

Lansbergen relies on an estimate of the eclipsed digits, which is very deceptive when they are not precisely six.

[Margin: True semidiameter and axis of the shadow.]

[Margin: Lansbergen's flaw.]

X. The ninth method, Boulliau's (*Astronomia Philolaica*, Book 4, chapter 1), relies on the lunar eclipse of 1638 December 21. He puts the time of its incidence [incidentię], accurately observed, at $1^h\ 1^m\ 30^s$, the anomaly at 118° 29′ 31″, and the Moon's hourly motion at 33′, and so the Moon's semidiameter at 16′ 54″; and from the duration of the eclipse he derives an apparent semidiameter of the shadow of 44′ 9″. From another observation taken at that time, of the Moon's altitude compared with the altitude of the leading star in the foot of Gemini, he derives the Moon's parallax, and from it the Moon's distance, 56⅔ terrestrial semidiameters. And yet, without any better-known foundation, feeling his way, he assumes in this eclipse a lunar distance A G of 57 51′ semidiameters and a semidiameter of the shadow G A L of 45′ 4″. From these he derives the true semidiameter of the shadow G L, 45′ 30″, of which A N is 60′. With G L, or A K, subtracted from A N, there remains K N, 14′ 30″. Then in triangle K L N, right-angled at K, from K N just found and K L, equal to the lunar distance, he derives the angle K L N, that is, the semi-angle of the cone equal to it, B E D, 14′ 21″. Next, from the Sun's apparent semidiameter at perigee, B N D, 16′ 45″, which by Euclid, Book I, proposition 32, equals the two angles B E D and E B N, he derives the Sun's parallax at perigee, 2′ 24″, namely by subtracting the angle B E D, 14′ 30″[37], from the angle B N D, 16′ 45″; and the Sun's distance at perigee, 1433 52′ semidiameters of the Earth. But from the Sun's eccentricity, 1784, of which the half of the transverse axis in the ellipse is 100000, he derives the Sun's distance at apogee, 1485 56′ semidiameters of the Earth, its horizontal parallax 2′ 19″ and its apparent semidiameter 16′ 9″; and from these the Sun's mean distance, 1459 54′. The Moon's distance at apogee in the syzygies he derives as 61′ 40″[38] semidiameters of the Earth, and at perigee as 56 30′. Finally he derives the axis of the shadow with the Sun at apogee as 249 52′ semidiameters of the Earth, and at perigee as 237 37′; and the apparent semidiameter of the shadow with the Sun at apogee and the Moon at perigee as 47′ 5″, but with Sun and Moon at apogee as 41′ 51″; and the total variation from the Sun's apogee to its perigee as 35″; and accordingly with the Sun at perigee and the Moon at apogee 41′ 16″, but with Sun and Moon at perigee 46′ 30″. For the rest, this whole machine relies on a semidiameter of the shadow and a lunar distance in that eclipse that are assumed rather than demonstrated, as will be clear to anyone reading the author himself. And I have already said, in Book 4, chapter 16, number 10, that the time of incidence in that eclipse was observed differently by others.

[Margin: 9th method: Boulliau's.]

[Margin: Boulliau's flaw.]

[37]so printed; the same angle stands as 14′ 21″ just above

[38]so printed, with minute and second marks

[Margin: Axis and apparent semidiameter of the shadow.]

XI. The tenth method is ours. It relies on the semi-angle of the cone of the shadow and on the axis of the shadow, already demonstrated in Book 3, chapter 11, Problems 5 and 7, from two very simple elements, namely the Sun's distance and its semidiameter, first the apparent and then the true; from these we derived the following quantities trigonometrically.

[Margin: 10th method: ours.]

☉	Axis of the shadow, in semidiameters of the Earth	Semi-angle of the cone, that is, A E N
At apogee	231½	14′ 50″
At perigee	213 0	16′ 8″

With these set down, and with the lunar distances in the syzygies taken from Book 4, chapter 14, let the Moon's distance A G, in the preceding figure set out at number 8, be at apogee 64 15′ semidiameters of the Earth, and at perigee 53 45′. When these are subtracted from the axis of the shadow A E, the side G E remains known in triangle G E L. With it and the semi-angle of the cone E, the true semidiameter of the shadow G L is derived; and with this and the Moon's distance A G, the angle G A L in the same triangle G A L, which measures the apparent semidiameter of the shadow. Here, then, is the sum of the calculation.

☉	☾	Semidiameters of the Earth: ☾ distance A G	Semidiameters of the Earth: axis of the shadow A E	Semidiameters of the Earth: remainder G E	True semidiameter of the shadow G L, of which A N is 100000	True semidiameter of the shadow G L, of which A N is 60′ 0″	Apparent semidiameter of the shadow G A L
Apog.	Apog.	64 15′	231 30′	167 15′	72085	43′ 15″	38′ 32″
Apog.	Perig.	53 45′	231 30′	177 45′	76610	45′ 58″	49′ 2″
Perig.	Apog.	64 15′	213 0	148 45′	69764	41′ 52″	37′ 30″
Perig.	Perig.	53 45′	213 0	159 15′	74688	44′ 50″	47′ 45″

XII. The eleventh method, without geometry, is by the ratio of the Moon's apparent semidiameter to the apparent semidiameter of the Earth's shadow at the place of the Moon's transit. This ratio is as 1 to 2⅗, or as 5 to 13, with the Sun at apogee, as Ptolemy asserts was observed from various eclipses (Book 5, chapter 14), and from him Reinhold on Peuerbach's *Theoricae*, p. 207, Wurstisen in the same place, p. 348, and Barozzi, *Cosmographia*, Book 4, p. 345. But Lansbergen (*Uranometria*, Book 2, element 6) uses this ratio of 5 to 13 also with the Sun nearly at perigee, as we saw at number 9. Copernicus, for his part (Book 4, chapters 19, 21 and 23), says that with the Sun at apogee it is as 150 to 403, but at perigee more than 5 to 13, and that therefore a mean can be chosen. But from the observations of Fathers Arzet and Fournier reported at number 5, the Moon's semidiameter would be to the shadow's semidiameter as 5 to 11 13/17, or as 5 to 13 3/10. For my part, since the Moon's semidiameter for me

is 14′ at apogee and 16′ 45″ at perigee, if I wished to follow Ptolemy's rule it would be: as 5 to 13, so 14′ to the shadow's semidiameter, 36′ 24″, at apogee, and 43′ 33″ at perigee. This differs too much from the semidiameter found by the preceding method at number 11. So if we wish to know the semidiameter of this shadow without geometrical inquiry, we shall rather have recourse to the rule of Kepler and Wendelen, of which presently.

[Margin: 11th method: from Ptolemy, by the ratio of the Moon's semidiameter to the shadow's semidiameter.]

XIII. The twelfth method, then, is Kepler's, in the Rudolphine Tables, precept 148, and in the *Epitome of Copernican Astronomy*, p. 862, where he says: "Gather the horizontal parallaxes of the Sun and Moon into one sum, and from that sum cast off the Sun's apparent semidiameter, and the correct semidiameter of the Earth's shadow will be left." Wendelen approved this precept in the *Idea of the Atlantic Tables*, p. 32, and elegantly enclosed it in the following couplet:

> *Iunge Parallaxes Solis Lunæq. deinde Subtrahe ſemiſſem Solis, ſic noſcitur Vmbra.*
>
> Join the parallaxes of Sun and Moon, then subtract half the Sun: so the shadow is known.

[Margin: 12th method: Kepler's and Wendelen's.]

But since neither of them demonstrated this truly most beautiful rule, the demonstration falls to us, and we easily accomplish it with the help of the following scheme. In the accompanying figure, then, let A B C be the Sun, with its centre at S on the same straight line S V on which lies the centre of the Earth T; and let C H V be the extreme ray of the Sun, touching the Earth at H and serving as the physical horizon with respect to the eye at H. From T let two straight lines be drawn: T R, to the point R, imperceptibly distant from the point C, and T K, to the centre of the Moon, to be conceived at K. For the angle T K H will be the horizontal parallax of the Moon, and T C H that of the Sun: for the parallax of the upper limb at C is as great as that of the centre S when it is at C. Afterwards let R T be produced towards N, and, with K I drawn, let it be the semidiameter of the Earth's shadow at the place of the Moon in its transit,

[Margin: The rule of Kepler and Wendelen is demonstrated.]

Diagram. A tall narrow figure in the right column, beside the narrowed text. At the top S stands at the head of a vertical line running down to V at the foot; straight lines run from S out to A at the left and C at the right, and a further line from S to R, which stands just inside C. A semicircular arc hangs from A to C, its lowest point at B on the vertical. Two long lines run down from A and C and meet at V. Below the arc a small circle holds L, with P on its left and G on its right. Below it a larger circle holds F and T, with H on its right edge. Below the larger circle N stands to the left of the left-hand line, and a short horizontal joins I on the vertical to K on the right-hand line. Further lines run from C and R down through T.

- S (top of the vertical): centre of the Sun

- A (left end of the Sun's arc): the Sun, A B C
- B (lowest point of the arc, on the vertical): the Sun, A B C
- C (right end of the arc): the Sun's limb
- R (just inside C): end of T R, imperceptibly distant from C
- L (inside the small circle): no gloss on this page
- P (left of the small circle): no gloss on this page
- G (right of the small circle): no gloss on this page
- F (inside the larger circle, upper): no gloss on this page
- T (inside the larger circle, on the vertical): centre of the Earth
- H (right edge of the larger circle): where the ray C H V touches the Earth
- N (left, below the larger circle): R T produced runs towards N
- I (on the vertical, low): end of K I
- K (on the right-hand line, low): centre of the Moon
- V (foot, apex): end of the line S V and of the ray C H V

I K, and its measure the angle I T K. For the external angle N T K, by Euclid, Book I, proposition 32, equals the two interior angles of triangle T C K opposite it, at C and K, that is, the horizontal parallaxes of the luminaries taken together. So when the angle N T I is subtracted from N T K, that is, from their sum (N T I being equal, by proposition 15 of Book I, to the angle S T R, which measures the Sun's apparent semidiameter), there remains the known angle I T K, which measures the apparent semidiameter of the Earth's shadow.

That moderate difference, however, between the semidiameter of the shadow derived by this method and the one deduced at number 11, an almost negligible difference, we attribute to that operation, as being more composite than this one; and it matters little which you prefer to follow in tables and in practice. I admit, moreover, that the semidiameter I reported as deduced from observation, in Book 4, chapter 12, number 4, and here at number 6, also differs somewhat from this one. And no wonder: for the horns of the shadow in a partial lunar eclipse, whether observed by direct view or by reflection on a small board, are blunted and show a smaller arc, because the bright part of the Moon prevails, and white and luminous things next to black or shadowy ones appear larger. But at that time I was pressed by the need to investigate the semidiameter of the shadow, so as to find from it the Moon's latitude independently of parallax; now, however, we already take the parallax as known.

Kepler adds, in the place cited above, that this is the demonstrative semidiameter of the shadow, which is often found to be greater than the one Tycho presents in the *Progymnasmata*, shaped from observations of lunar eclipses; but that those observations are deceptive, because of a concurrence of physical causes, which are inconstant. In the Foundations of the Ephemerides, number 29, he teaches that the semidiameter of the shadow follows necessarily from the parallaxes of the luminaries and the semidiameter of the Sun; and that the Tychonic semidiameter of the shadow in the table of the *Progymnasmata* therefore does not cohere by a geometrical bond with the semidiameter of the

☉ and the parallaxes of the luminaries, because Tycho followed observations instead. For Kepler, moreover, the greatest semidiameter of the shadow, with the ☉ at apogee and the ☾ at perigee, is 49′ 41″, and the least, with the ☉ at perigee and the ☾ at apogee, 43′ 52″. For Wendelen the former is 46′ 39″ and the latter 38′ 43″.

[Margin: Tycho's inconsistency.]

[Margin: Semidiameter of the shadow according to Kepler and Wendelen.]

XIV. It is now pleasing to set out in the following table the apparent semidiameter of the shadow, or rather of the penumbra, as comprehended by the Sun's unrefracted rays; and such rays are not those that touch the Earth, for those are refracted, but those that touch the thicker atmosphere around the Earth. The table will have two parts: the first as it stands in the authors; the second contains the semidiameter that follows from the data given by the same authors for the semidiameter of the ☉ (Book 3, chapter 10) and the horizontal parallaxes of the ☉ (Book 3, chapter 8) and the ☾ (Book 4, chapter 14). From these the conflict will appear, great in some cases, and in others none or almost none.

First part of the table: semidiameter of the Earth's shadow at the place of the ☾'s transit, comprehended by the Sun's unrefracted rays, as it stands in the authors. Second part: as it follows from the data given by the authors and the method of number 13.

Authors	First part: ☉ apogee, ☾ apog.	First part: ☉ apogee, ☾ perig.	First part: ☉ perigee, ☾ apog.	First part: ☉ perigee, ☾ perig.	Second part: ☉ apogee, ☾ apog.	Second part: ☉ apogee, ☾ perig.	Second part: ☉ perigee, ☾ apog.	Second part: ☉ perigee, ☾ perig.
Ptolemy	40′ 44″	51′ 3″	39′ 58″	50′ 15″	40′ 45″	51′ 2″	39′ 58″	50′ 15″
al-Battānī	38′ 20″	51′ 11″	39′ 0″	50′ 15″	✢	✢	✢	✢
Alfonso and Peuerbach	37′ 42″	46′ 57″	36′ 46″	46′ 1″	✢	✢	✢	✢
Copernicus	40′ 18″	47′ 52″	39′ 21″	46′ 55″	40′ 7″	50′ 5″	39′ 5″	49′ 13″
Reinhold, at the ☉'s least eccentricity	39′ 48″	49′ 46″	38′ 21″	46′ 55″	40′ 5″	40′ 3″	39′ 2″	49′ 0″
Reinhold, at the ☉'s mean eccentricity	39′ 59″	49′ 56″	39′ 53″	48′ 51″	40′ 10″	40′ 8″	38′ 57″	48′ 55″
Reinhold, at the ☉'s greatest eccentricity	40′ 7″	50′ 5″	39′ 4″	49′ 1″	40′ 14″	40′ 12″	38′ 52″	48′ 50″
Tycho and Mulerius	43′ 0″	47′ 0″	42′ 4″	46′ 4″	46′ 53″	49′ 33″	46′ 16″	48′ 46″
Longomontanus	43′ 0″	47′ 0″	42′ 4″	46′ 4″	46′ 44″	50′ 15″	45′ 50″	49′ 21″
Magini in the *Supplement to the Ephemerides*, and from him Argoli	43′ 0″	47′ 0″	42′ 32″	46′ 32″	47′ 11″	49′ 25″	46′ 54″	50′ 8″
Kepler	44′ 22″	49′ 41″	43′ 52″	49′ 11″	44′ 22″	49′ 41″	43′ 52″	49′ 11″
Lansbergen	39′ 0″	46′ 19″	38′ 2″	45′ 21″	38′ 59″	49′ 5″	37′ 57″	48′ 3″
Wendelen	39′ 1″	46′ 33″	38′ 30″	46′ 2″	39′ 1″	46′ 33″	38′ 30″	46′ 2″

Authors	First part: ☉ apogee, ☾ apog.	First part: ☉ apogee, ☾ perig.	First part: ☉ perigee, ☾ apog.	First part: ☉ perigee, ☾ perig.	Second part: ☉ apogee, ☾ apog.	Second part: ☉ apogee, ☾ perig.	Second part: ☉ perigee, ☾ apog.	Second part: ☉ perigee, ☾ perig.
Boulliau	41′ 51″	47′ 5″	41′ 16″	46′ 30″	41′ 54″	47′ 1″	41′ 23″	46′ 30″
We, for the present, from numbers 11 and 13	38′ 32″	49′ 2″	37′ 30″	47′ 45″	38′ 42″	48′ 55″	36′ 56″	47′ 38″

[Margin: Conflict in the tables and hypotheses of the astronomers.]

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I. The semidiameter of the Earth's shadow, by the method discussed above at numbers 11 and 13, is the one derived from the Sun's unrefracted rays, as is clear from the demonstrations given there. But the Sun's unrefracted rays are not those that touch the Earth; for those that touch the Earth's globe are refracted on entering the denser air around the Earth. They are rather those that touch the atmosphere, that is, that air so dense that the remaining rays are sensibly refracted on entering and leaving it. The semidiameter of this atmosphere, beyond the semidiameter of the Earth, contains for Kepler as much as the height of the greatest mountains, which in this matter scarcely bears a sensible proportion to the semidiameter of the Earth; and for Kepler it is 2 Italian miles. For Wendelen, in the *Idea of the Atlantic Tables*, p. 6, it is less than 24 miles, since the height of the twilight atmosphere, the whole of which is not suited to sensible refraction, is in his opinion 24 Italian miles above the Earth. Now the Earth's shadow, which is contained by unrefracted rays, is not pure shadow but penumbra, of which I shall speak in the following chapter. Therefore the semidiameter derived here by the preceding method, whether ours or that of Kepler and Wendelen, is not that of the pure terrestrial shadow but of the atmospheric penumbra. And it is this that truly serves for producing and measuring eclipses; and if it had been rightly derived from observations by other astronomers, no other semidiameter than this would have been arrived at, granted that it takes its name from the Earth, which has the largest part in the sphere of the atmosphere. Hence the axis too is that of the penumbra, not of pure shadow, and its measure is to be understood not in semidiameters of the bare Earth, but of the Earth clothed with the air already mentioned. Meanwhile it is worth noting that this semidiameter of the shadow must not be increased by 17″ to obtain the semidiameter of the penumbra, as Wendelen teaches above, p. 5; since the semidiameter of the shadow, as deduced by him from the parallaxes of the luminaries and the Sun's semidiameter, presupposes unrefracted rays, and so is the semidiameter not of the shadow but of the penumbra.

[Margin: It is rather the semidiameter of the penumbra than of the shadow that serves for eclipses.]

[Margin: Wendelen corrected.]

II. In his preface to the *Idea of the Atlantic Tables*, p. 6, Wendelen notes that astronomers, in the quantity of lunar eclipses, treat the Moon as a disc, although it is a globular body and its boss is nearer to us than the rest of the body; and so the shadow there is greater than in the place through which the Moon's great circle passes. Moreover, since over each distance measured in terrestrial semidiameters the shadow's semidiameter varies by a whole minute, as for him it subtends 39′ at apogee and about 47′ at perigee, the shadow is therefore increased at the boss by a quarter of a minute, or 15″, over what it is at the centre of the lunar disc, because that boss is nearer to us than the Moon's centre by a quarter of a terrestrial semidiameter:

[Margin: Wendelen's acute observation on the place of the Moon's transit through the shadow.]

as a result, every eclipse appears there one-tenth of a digit greater than the calculation directed at the Moon's centre allows, and in place of 6 digits, 6 digits 10′ are seen. Moreover, those for whom the Moon stands overhead see the same boss of the Moon that would be seen by people looking from the Earth's centre; but the rest see one boss or another, which Wendelen calls parallactic [Parallacticos]. From this he thinks that a difference of 15″ can arise, so that an eclipse that appears from the vertical as one of 6 digits can appear to northerners as one of 6 digits 10′ and to southerners as one of 5 digits 50′. Combining both differences, therefore, it can rise to a third of a digit, or to 20′, of which one digit contains 60. But it is false that that boss is so much nearer to us, and that so great a difference can arise from it.

For let A be the centre of the Earth EC, and CDFE the frustum of the Earth's shadow, enclosed by the Sun's rays CD and EF, which touch the Moon at G on its entry into the shadow and at M on its exit. From these points let the radii, or semidiameters, GK and ML be drawn to the Moon's centre. From A, through the centres of the Moon, let the arc KL be described, along which the Moon's centre travels, so that NI is the arc of the shadow's diameter that astronomy assumes the Moon to pass through; and through the points G and M, from A, let the arc GM be described, which the Moon traverses from the beginning to the end of the eclipse. Then let the centres A and K be joined by the straight line AK, which is the distance of the Moon's centre, and finally from the point C let the straight line CK be drawn.

Diagram. Set into the left column, with the text running down its left side. A solid black frustum of the Earth's shadow narrows upward from a circle (the Earth) at its base; a vertical line runs through its middle. Two small circles (the Moon) straddle the frustum's sides at left and right, linked by a pair of arcs across the shadow, and a line runs from the base up to the right-hand circle.

- F, B, D (top edge of the frustum, left, centre, right)
- L (centre of the left-hand circle)
- N, I (upper arc inside the shadow, left and right)
- K (centre of the right-hand circle)
- P, H (outside the right-hand circle, right, upper and lower)

- M, G (lower arc inside the shadow, left and right)
- O (small, on the line just below the right-hand circle)
- E, A, C (base: left, centre, right)

The Moon's boss of which Wendelen speaks, therefore, will be the segment GOH; but this does not enter the shadow at the point O but at the point G, which is distant from the point I, assumed by the old astronomy, only by the arc GI. This arc is as large, very nearly, as the angle GKI, namely as large as the angle GCK, which subtends the Moon's apparent semidiameter, and this at the Moon's perigee does not exceed 17′. Therefore the arc GH lies insensibly distant from the arc IKP, since GO is 89° 43′ and GI only 17′. So although the Moon's true semidiameter KO is almost a quarter of the Earth's semidiameter, from what was said in Book 4, chapter 17, and the point O is nearer the centre A than the point K by a quarter of the Earth's semidiameter, nevertheless the point G, at which contact and the beginning of the eclipse occur, is insensibly nearer than the point K, by scarcely a two-hundredth part of the Earth's semidiameter. For the sine of the arc GI, 17′, is 495, which to the whole sine KO, 100000, is less than 1 to 200. Therefore the arc MG too is insensibly greater than the arc NI.

[Margin: Wendelen's subtlety, here too scrupulous, is corrected.]

III. There have been some who believed that if the Earth's shadow were cylindrical or calathoid, the Moon would suffer an eclipse at every full Moon. So teach Cleomedes, Book 2, chapter 5, and Wurstisen on Peuerbach's *Theoricae*, p. 337, and Erasmus Reinhold's teacher Jacob Milich on Book 2 of Pliny the Elder, chapter 11. But they are wrong. For if it were cylindrical, as in the first figure of Chapter 3, its apparent diameter KM would not be greater than twice the horizontal parallax of the Moon at perigee, and this does not exceed 140′. But if it were calathoid, as in the second figure, even if the Sun were a single point M, in which case the breadth of the shadow NOP would be greater, still at the distance DO of the Moon at perigee the apparent semidiameter of the shadow OP, under the angle ODP, would not exceed 1° 58′. For from the distance MD of the Sun at perigee (this is what increases a calathoid shadow), the least among the authors being 1000 of the units of which DC, the Earth's semidiameter, is one, there follows the angle DMC, 3′ 26″. Now add to this 1000 the distance DO of the Moon then at perigee, so that it is only 30 semidiameters of the Earth, and the whole MO is 1030. From this and from the angle OMP, 3′ 26″, you will gather OP as 1 3/100 semidiameters of the Earth; and with this side and the side OD, taken as 30, the angle subtending OP is gathered as 1° 58′. But the Moon at full can have so great a latitude that it exceeds the sum of this semidiameter and the Moon's semidiameter, since it sometimes has a latitude of 5°. Therefore, granted even a shadow as great as this, it would still avoid eclipse; how much more if the shadow were cylindrical, since its semidiameter would not exceed 70′.

[Margin: The error of Cleomedes, Wurstisen, Milich and others concerning a necessary eclipse of the Moon if the shadow were cylindrical or calathoid.]

CAPVT V.

Chapter 5.

De Penumbra Terræ, & varijs Lunæ in Eclipſi, aut prope Eclipſim, Phaſibus aut Coloribus à Refractione radiorum ortis.

On the Earth's penumbra, and on the various phases or colours of the Moon in or near eclipse that arise from the refraction of rays.

I. Since that shadow of the Earth which dulls the Moon is itself dulled by the Sun's refracted rays, and enlarged by the same rays when they are obstructed, or is also diluted by the rays of the Moon, and since from this the various phases and colours of eclipses appear, whatever pertains to this subtlety must be dealt with in this one chapter. And first, the manifold notion of the penumbra as a whole must be explained. For sheer shadow [Mera Vmbra] is the privation of the whole of the primary light, direct as well as reflected and refracted, or blunted and weakened. Penumbra, on the other hand, is the privation sometimes of part of the primary light, sometimes of the whole of the direct, unblunted or unweakened light; yet with some partial primary light, or at least with refracted and therefore blunted and weakened light. Let us be allowed, then, to call the first species of penumbra the image of the shadow [Vmbræ imaginem] or, in contracted form, the *Umbrago* [Vmbraginem]; and the second, the *Penumbra*: although many, with Kepler, call the first the penumbra, in the *Astronomia Optica*, p. 239, or [Greek: λιπαύγειαν (lipaugeian, "failure of light")].

[Margin: The manifold notion of the penumbra.]

II. The umbrago in the Moon, then, is the privation of some primary and direct light otherwise propagated from the Sun, along with some primary and direct light. For let DRP be the Sun, and DFC and PMC its outermost rays touching the Earth, and let VOS be the sheer shadow of the Earth B, and LQZ the Moon's orbit, with the Moon at T before its eastern limb is impaired by any shadow. For although its half-disc IT is lit by all the Sun's rays contained within DR, that is, enclosed within the two ray-cones DFR and TFI, it nevertheless lacks the other rays arising within RP, on the part where it sees less of the Sun, rays it nonetheless enjoyed before it approached the Earth's shadow. It is no wonder, then, that before the genuine and sheer shadow and before the legitimate beginning of the eclipse, the Moon should seem to the unwary to be darkened and suffused with a sort of smoke in its more easterly part IT, especially since that part too, as can be seen in the *Selenographia*, Book 4, Chapter 7, is darker in itself on account of larger and ancient spots, namely the Philippic Ocean [Oceano Philippico], and becomes darker still through the privation of these rays. The more recent and more careful astronomers therefore usually distinguish, at the beginnings of lunar eclipses, this umbrago from the shadow, and the smoke and spurious eclipse from the legitimate one.

Those who do otherwise overshoot enormously, by a quarter of an hour and more, in noting the beginning and end of the eclipse, and lengthen the duration on both sides beyond half an hour, counting that pallor and virtual agony as part of the eclipse and, so to speak, of its death. Indeed, Wendelen, in the preface to his Eclipses, that is, in the *Idea of the Atlantic Tables*, p. 3, judges that this optical enchantment sometimes introduces an error of 30′ on this side and as many on that, so that the duration appears an hour longer to weaker eyes,

[Margin: What the umbrago is, and what its effect is in an eclipse of the Moon.]

[Margin: The beginning of the spurious eclipse, and its twofold cause.]

[Margin: The fallacy of the inexperienced in observing the beginning of a lunar eclipse.]

Diagram. Set into the right column, with the text running down its left side. From a semicircle (the Sun) at the foot, labelled P, R, D, lines rise through a large circle (the Moon's orbit) to the apex C of a narrow black cone (the sheer shadow), with dotted lines marking the rays bounding the lighter zones on either side. Several small circles (the Moon in successive positions) lie along the cone and the orbit, one of them, at X, stippled in part.

- C (apex at the top)
- O (small, inside the cone near the apex, on its axis)
- K, E (small circles against the right side of the cone, upper and lower)
- Y, X (a further small circle to the right, with Y on its left edge and X inside it)
- L (small circle on the left side of the cone, on the orbit)
- I, T, G (on the small circle on the right side of the cone, on the orbit)
- Q, Z (left and right ends of the large circle)
- M, V, B, S, F (along the band at the base of the cone, left to right)
- N (below the band, under the small semicircle at the base of the cone)
- R, P, A, D (the semicircle at the foot: top, left, base centre, right) and bleary ones. For the rest, keen eyes, whether naked or equipped with the telescope, clearly distinguish in the Moon this pallor and first beginning of the shadow from the true shadow. For, as in the Moon X, the true and sheer shadow blackens conspicuously, and like the edge of a horse's hoof begins with a small circular segment to impair the Moon's eastern limb, occupying a very small part of it from the Philippic Shore [Littore Philippico] onward, as you see at Y; whereas that umbrago occupies a large part, and displays no blackness but only pallor and a languid light.

III. The penumbra more strictly taken is the privation of direct and primary light, with some light, but refracted or weakened. For this reason it is subdivided into two species, as it were. The first would hold alone if the Sun's rays passed through the atmosphere, that is, through the thick vapours surrounding the Earth's globe: as in the figure given above, if around the Earth VBS there is a hemisphere of vapours MNF, through which rays contained on this side within DO and DC, and on that side within PO and PC, pass unrefracted as regards

their figure, yet blunted or weakened and enfeebled in their degree of intensity by the denser and more opaque medium, as I assume from my Optical Book, and generate a penumbra enclosed within the rays already mentioned, whose base is MF and whose apex is C, and which clothes the sheer shadow VOS like a crust. If that were to happen, there is no doubt that the Moon would begin to be dimmed by this penumbra, as can be seen at E, before being impaired on its eastern limb by the shadow simply so called, and would emerge from the penumbra later, as at L, than from the sheer shadow. This, however, does not in fact happen, because the Sun's rays in the vapours and in our denser air are not only weakened in intensity and vigour of light but are also refracted, and change the figure of the penumbra, as will be set out in the following figures. Therefore this penumbra in fact, not disjunctively but conjunctively, *is the privation of direct light, with some light both weakened and refracted alike*. Meanwhile, however, three beginnings and three ends of a lunar eclipse, and so three durations, ought to be distinguished here: the first from the umbrago, as at T; the second from the penumbra, as at E or L; the third from the shadow, as at K. This would hold both in observations and in predictions, if the inconstancy of the physical causes, which vary the density and opacity of the atmosphere in height and in degree of intensity, allowed these limits to be defined, and to be determined to these times according to the greater or lesser removal of the luminaries from the horizon and according to the different temper of the air; I see that this has been attempted in vain. Since this is not yet allowed, therefore, we shall be content with that moderate knowledge of the penumbra and more exact knowledge of the shadow which God has granted us. For as to Wendelen's statement above, p. 5, that the semidiameter of the shadow is increased by the penumbra by about 17″, we have rejected it in Scholium 1 of chapter 4.

[Margin: The penumbra: its two species and causes.]

[Margin: A truer definition of the penumbra.]

[Margin: The threefold beginning, end and duration of a lunar eclipse.]

IV. Further, the penumbra produced by the Sun's rays refracted in the air around the Earth produces a redness or other colours in the eclipsed Moon. In order that the penumbra may be distinguished from the sheer shadow, and the measures of each be made known separately, some points must be explained, which Kepler sets out beforehand for this purpose in the *Astronomia Optica*, from p. 269 to 279. With centre A describe the Sun's hemisphere EF, and with centre B the Earth HPQK and the atmosphere RTVS. Let ERD and FSD be the Sun's outermost unrefracted rays, touching the sphere of our air at R and S and bounding the aggregate of penumbra and shadow RDS. Next, let there be the rays ET and FV, which, if the density of the air did not prevent them, would touch the Earth at H and K, go straight on and meet at G, producing the straight shadow HGK of the Earth alone and separating it from the penumbra of the thick air, which would be RDGH on one side and SDGK on the other. But in reality they are refracted at T and at V, and strike the points P and Q

of the Earth. So let there be two other, higher rays, which by the force of refraction can touch the Earth's globe, namely the rays EI and FC; the former at I, the latter at C, are so refracted toward the perpendicular that, touching the Earth at L and M, they make for X and Z, which are points on the air's convex surface. They would meet straight on at N if the second refraction, on leaving the air, did not prevent it, and would produce the sheer shadow of the Earth LNM. But because on exit into the rarer air, according to the laws of refractions treated in the Optical Book, they are refracted away from the perpendicular line that would be drawn from the centre B through the points X and Z, they are therefore bent toward the point O, at which they bound the sheer shadow LXOZM; but the remaining rays, higher than the rays EI and FC yet below the unrefracted rays ER and FS, are similarly refracted by a double refraction, one on entering the denser air and the other on leaving it, there converging toward the perpendicular, here diverging from the perpendicular, but not so as to touch the Earth's globe; and they meet at other points, partly below and partly above O. Therefore, if no refracted rays reached below O, the height of the Earth's sheer shadow would be BO.

[Margin: An account of the penumbra produced by refracted rays.]

Diagram. Heads the right column, interrupting the sentence that runs from the left column's foot. At the top a broad semicircle (the Sun's hemisphere) opens upward; lines from its corners converge downward past a double circle (the Earth inside the atmosphere) and on to a long narrow apex at the foot. Solid lines from X and Z, at the foot of the circles, meet at the point O on the central line; two pairs of dotted lines run on past it and meet lower down, at N and at G.

- A (top centre), E, F (upper corners of the semicircle)
- I, T (upper left of the outer circle)
- V, C (upper right of the outer circle)
- P, B, Q (inside the inner circle: left, centre, right)
- R, H (left edge of the circles, outer and inner)
- L, M (inside the inner circle, lower left and lower right)
- K, S (right edge of the circles, inner and outer)
- X, Z (foot of the outer circle, left and right)
- O, N, G, D (down the central line below the circles, top to bottom; D at the apex)

Lemma ad ſequentia Problemata.

Lemma for the following problems.

V. The lemma is as follows. *The angle formed by a ray of the Sun taken as unrefracted and touching the Earth, and by a ray of the Sun radiating from the same point of the Sun but refracted in the air and touching the Earth, is sensibly equal to the angle of horizontal refraction.* From the centre B let the hemisphere of the Earth MHP be described, and that of the refracting atmosphere AIK, and let the

straight line through the centre dividing the hemispheres be AG, on which let BG be the axis of the shadow, made, or in the manner of the ancients imagined, by the unrefracted ray EG touching the Earth at H, at which point a right angle is made, by III.18, by the tangent and the semidiameter BH. Next let there be another ray FI, which on entering at I, if it were not refracted, would go straight toward R; but let it be refracted at I so that it eventually touches the Earth, as at L, and let IL be the refracted ray, produced as far as N. To the point of contact L let the semidiameter BL be drawn, making with the tangent IL a right angle by Euclid III.18. With these laid down, since the rays EHG and FIR are assumed to issue from the same point of the Sun's limb (although the Sun is not drawn here, as it was in the preceding figure), and their distance SI, because of the slight height of the air suited to the refraction of rays, is small, as I assume from what was said in Book 1, chapter 31, and from what is to be said in the Optical Book, therefore, by what was demonstrated in Book 3, chapter 5, number 6, they are sensibly parallel. For this reason the refracted ray ILN will meet the unrefracted ray EHG at some point, say C, and will make with it angles at the vertex C that are equal, by Euclid I.15. And since IC falls on lines sensibly parallel, the angle ICE will be sensibly equal to the angle of refraction RIC; and to this same angle is equal the angle GCN, which is vertical to the angle ICE: which was to be shown. Further, the refraction of the first ray, among those refracted, that touches the Earth and issues from the Sun's upper limb is as great as the Sun's horizontal refraction, for this refracted ray ILN is the physical horizon of an eye at L, along which the Sun is first seen; and this refraction, ordinarily, from Tycho's observations in Denmark, is 34′. Therefore in the same place the angle GCN is 34′; and the angle of refraction NXO at the exit X is as great as the angle RIC at the entry, by Witelo X.9, so that this too is 34′ by Tycho's reckoning.

Diagram. Set into the left column, with the text running down its left side. A long vertical line on the right carries the letters of the central axis, from A at the top to G at the foot. Two arcs (the atmosphere and the Earth) curve to the left of this axis, and straight and dotted lines, the rays, cross between them near the left limb and converge down on the axis.

- A, M (head of the vertical line, top to bottom)
- F, E (top left, the upper ends of two rays)
- B (on the vertical line, at the middle)
- P, K, O, N, G (down the vertical line below B; G at the foot)
- I (on the outer arc, left)
- S (on the inner arc, near the top of the crossing lines)
- R (on the outer arc, below I)
- H, C, L (at the crossing of the rays near the inner arc)
- T (on the outer arc, below R)
- X (on the outer arc, lower, on a ray running down to the axis)

PROBLEMA I.

Problem 1.

Terræ ſolius meram Vmbram comprehenſam à radijs Solis refractis terramq. tangentibus metiri quoad altitudinem.

To measure, as to its height, the sheer shadow of the Earth alone, enclosed by the Sun's rays refracted and touching the Earth.

VI. In the figure of the preceding lemma, the height of the sheer shadow that we are seeking is the axis BO. Now in the triangle CGN the angle GCN is given by the same lemma, because it is equal to the Sun's horizontal refraction, and according to Tycho it is 34′. The angle CGN is also given, from what was said in Book 3, chapter 11, number 6, because it is the semiangle of the cone of the Earth's shadow made by unrefracted rays, and for Ptolemy, in Kepler, it is 12′ 48″; together with the angle GCN it makes a sum of 46′ 48″. And the external angle LNB, or XNO, is as great as this, by Euclid I.32. In the small triangle XNO, therefore, there is given the angle of refraction at the exit, NXO, 34′, and XNO, 46′ 48″, whose sum is 1° 20′ 48″; to this the external angle XOB is equal, by Euclid I.32. Now let the refracted ray OX be continued toward the atmosphere until at T it makes a right angle with the perpendicular BT drawn to it from the centre of the Earth. For the triangle OTB will be right-angled at T, and in it, besides the right angle T, there is given the angle TOB, 1° 20′ 48″, and the semidiameter TB, taken as one, which is slightly greater than the semidiameter BL of the Earth alone. Therefore the height BO will be 43 of the units of which BT is one. But if you take our semiangle of the shadow's cone CGB, 14′ 50″, with the Sun at apogee, according to what was said in Book 3, chapter 11, number 6, and retain the Tychonic refraction of 34′, BO will come out as 41½ of the units of which BT is one. And if the portion LT of the refracting air is given, then by multiplying it by 41½ and adding the product to the BO that was found, there will remain a more exact measure of BO itself. Now the air suited to refraction is for us at most 25 Italian miles high, but for Kepler much lower. And 41½ multiplied by 25 makes 1037, which is almost a third of the Earth's semidiameter. Therefore OB remains for us 41 5/6 semidiameters of the Earth. We are speaking here, however, of rays not prevented by mountains from passing out, of rays touching the Earth before their exit, and of ordinary refraction. For rays that have passed out but do not touch the Earth fall below O, and so the height of the sheer shadow is less than BO; and it is still less if, as the refraction or the angle of the shadow's cone grows, the angle TOB turns out larger.

[Margin: The height of the sheer terrestrial shadow.]

PROBLEMA II.

Problem 2.

Quantus ſit ſemiangulus coni Vmbræ meræ terreſtris à radijs Solis refractis, & Terram tangentibus comprehenſæ.

How great is the semiangle of the cone of the Earth's sheer shadow, enclosed by the Sun's rays refracted and touching the Earth.

VII. We shall know the angle XOB from what was said in Problem 1. For this angle, as was proved in the course of the demonstration of Problem 1, is composed of the Sun's refraction taken twice and of the semiangle of the shadow made, or imagined, by the Sun's unrefracted rays; and from what Kepler lays down it is 1° 20′ 48″, while from our foundations it is 1° 22′ 50″. Its double therefore gives the Keplerian angle of the sheer shadow as 2° 41′ 36″, and ours as 2° 45′ 40″. In this way, very easily, from the semiangles of the cone of the unrefracted shadow set out in Book 3, chapter 11, number 6, by adding to them the Sun's horizontal refraction doubled, you will gather, in the opinion of others, the semiangle of the sheer shadow; and by doubling it you will have the whole angle.

[Margin: How great is the angle of this cone of sheer shadow?]

PROBLEMA III.

Problem 3.

Quantò plus de Terra illuſtret Sol radijs in aere refractis, quàm radijs, vt veteres putabant non refractis.

How much more of the Earth the Sun lights with rays refracted in the air than with rays not refracted, as the ancients supposed.

VIII. As regards unrefracted rays, we have already completed this problem in Book 3, chapter 11, Problem 4. Using now the figure set out here at number 5, we showed that the part of the hemisphere MHP lit by the unrefracted ray EHG, touching the Earth at H, and by the others contained between this and the central ray AM, is greater than the quadrantal arc MS by as much as the arc SH, measured by the angle SBH; and that to this angle is equal the semiangle of the cone of the shadow made by unrefracted rays, namely the angle HGB. But since the Sun's rays are in reality refracted in our air, if we take the first one that is higher than the ray EG and, once refracted, touches the Earth as soon as may be, say the ray IL, it will certainly touch below H, say at the point L, and still more of the arc MLP is lit, more by as much as the arc HL, measured by the angle HBL. The task now is to find the measure of this angle. I say that it is as great as the horizontal refraction of the Sun's ray, namely, in Tycho's opinion, 34′ in Denmark; which I also asserted in Book 3, chapter 11, Problem 4, with Kepler and very nearly with Bettini, although in the *Apiarium* 8, Progymnasma 2, Proposition 4, having assumed an elevation of the ray that is not at all necessary, he adds a single minute and makes the arc HBL 35′.

Now that assertion is easily demonstrated as follows. The angle ICH is sensibly equal to the angle of horizontal refraction RIC, by our lemma at number 5. But this is equal to the angle HBL. For when a straight line is drawn from B to C, the point at which the rays touching the Earth intersect, namely the unrefracted EHG and the refracted ILN, the trapezium CHBL is divided into two triangles CHB and CLB, right-angled at the contacts H and L. Therefore the four angles of this trapezium taken together are, by Euclid I.32, equal together to four right angles; and to these likewise are equal the four angles at the point C made by the said rays, by Euclid I.13. Taking away, then, the common obtuse angle HCL, there remain the three angles of the trapezium, H and B and L, together equal to the three angles ICG, the obtuse one, and the two acute ones ICH and GCL together. But the two angles ICG and GCL are equal to two right angles by I.13, that is, to the two angles at the contacts H and L. Taking those away on the one side and these on the other, there remains the angle ICH, which is the measure of the Sun's horizontal refraction, equal to the angle HBL: which was to be shown. Therefore, by adding the Sun's horizontal refraction to the arc HS of the ancients, on which see Book 3, chapter 11, Problem 4, the whole arc SL will be produced, which the Sun lights beyond the quadrant. Moreover, Bettini adds above, in a corollary, that the limits of the illuminated Earth do indeed grow from refraction, but the shadow does not decrease, because he thinks that the refracted rays die away in the valleys and strike against the mountains in such a way that none passes out; I shall speak of this at number 11.

PROBLEMA IV.

Problem 4.

Vmbræ & Penumbræ terreſtris ſegmenta varia diſtinguere; eorumq. altitudines.

To distinguish the various segments of the Earth's shadow and penumbra, and their heights.

IX. Let the second figure be this one, in which the straight line from the centre of the Sun through the centre of the Earth to the apex of the penumbra is ABD; the centre of the Earth and of the atmosphere is B; the Earth's circle is HK and the atmosphere's AMG. Let the Sun's extreme rays, which touch the atmosphere and bound the penumbra, be RD and SD. Let the rays next to them, δ[?]M and δ[?]G, fall on the convex surface of the atmosphere and, with the greatest refraction at the points M and G, be refracted to L, so that they are the outermost of the refracted rays touching or nearly touching the atmosphere. From the points L, on emerging, let them be refracted again to O and, intersecting each other there, make for PP, and so enclose the Earth's sheer shadow LOL, whose height or axis is BO, although, to avoid confusion, not all of it is blackened here, but only its portion XOZ. Now let the straight lines BM

and BL be drawn. Next, let there be two other outermost rays of the refracted ones, but touching the Earth, namely EI and FC, refracted on entry at I and C towards the perpendicular and touching the Earth at H, K. Having passed on to X and Z, let them be refracted there on exit away from the perpendicular, making for the point N, and, intersecting each other there, let them continue, X to V and Z to T, so as to enclose the Earth's shadow XNZ, were it not cut short by the rays LOL. Further, let both these and all the other refracted rays contained within the rays EXN and RPD on one side, and within FZN and SPD on the other, refracted by a double refraction, one on entry through the arcs MI and GC, the other on exit from the arcs LX and LZ, all of them, I say, intersect at the nearest points m, m. From there, diverging, let them proceed towards ω ω, between the segments VmP and TmP. Finally, from B to one P let the straight line BP be drawn, and two other straight lines, PP and ωNω.

Diagram. Set into the left column, with the text running down its left side and the figure reaching to the foot of the column. A double circle stands at the top: the outer circle (the atmosphere) carries A at its head and M and G at its left and right limbs; the inner circle (the Earth) has its centre at B. Below the circles a long cone tapers downward to its apex D. Its upper part is solid black as far as O; below that a lattice of fine lines, the refracted rays, crosses and recrosses around the central axis; the lower part is solid black again from T and V down to D. Rays rise from the circle's top to the letters outside it.

- A (head of the outer circle, on the axis)
- R, E (upper left, outside the circle), with a small flag-like sign beside R, read as δ [?]
- F, S (upper right, outside the circle), with the same sign beside S, read as δ [?]
- Φ (on the axis, inside the outer circle below A)
- I, C (on the outer circle, upper left and upper right)
- B (centre, on the axis)
- M, G (left and right limbs of the outer circle)
- H, K (inner circle, left and right)
- L, L (lower arc of the outer circle, left and right)
- Ψ (on the axis, foot of the inner circle)
- X, Z (lower arc, left and right of the axis)
- m, m (boxed in white in the black of the shadow, left and right)
- O (on the axis, foot of the upper black part)
- two small italic signs just below O, left and right of the axis, perhaps Q, Q [?]
- P, P (outer edges of the cone, left and right)
- ω, ω (outer edges, lower, left and right)
- N (on the axis, boxed in white)
- Y, Y (either side of the axis, below N)
- a small open triangle (on the axis, below Y, Y)

- T, V (edges of the cone, left and right, at the head of the lower black part)
- D (apex)

These 9 or 10 segments of the penumbra and shadow then arise, which are solid frusta of the cone MDG. The *first* is XOZ, which however ought to be HXOZK, all black, and this is the Earth's shadow. The *second*, MPQm, and the *third*, GPQm, are sheer shadows, but of the atmosphere, because within them there are no rays either direct or refracted. The *fourth*, QmO on the left, and the *fifth*, QmO on the right, are half-lit segments of the atmospheric penumbra, lit by refracted rays, on the one side by those from the left alone, on the other by those from the right. The *sixth* is OQNQ, a kernel of refracted light and a rhomboid of sorts, in which the refracted rays unite and those from the left grow stronger together with those from the right; this, however, could be subdivided into two segments. The *seventh* is TNQP and the *eighth* VNQP, segments of the atmospheric penumbra that, after passing out of the biconical frustum NQOQ, are lit by refracted rays, on this side those from the left and on that side those from the right, separately. The *ninth*, finally, deprived by these of every ray direct and refracted, is the biconical segment NTDV, sheer shadow, but of the whole atmosphere, consisting of the cones TDV and TNV.

[Margin: The segments of the penumbra and shadow, 9 or 10.]

So the cone that darkens the Moon has nine segments: one the sheer shadow of the Earth; three the sheer shadow of the atmosphere; five the penumbra of the atmosphere, lit by the Sun's refracted rays, which raise the Sun above the horizon when it is really below it, and make day, and which light up towers and trees in such a way that these cast a distinct shadow. What wonder, then, if they can illuminate the Moon, even when it is immersed in the penumbra?

X. As regards the measures of these segments useful for our purpose, Kepler's method, *Astronomia Optica* p. 280, is as follows. Taking the horizontal refraction at the entry points I and C as 34′, it follows by the rules he gives in the same place, chapter 4, number 6, proposition 9, that the refraction DML of the ray δ[?]M of greatest inclination, almost touching the air, is 61′ 30″. Therefore the angle δ[?]ML is 178° 58′ 30″, and so the arc ML, or the angle MBL, is 2° 3′. Kepler confirms this rather obscurely from Euclid III.2, but it is shown more clearly by III.32: for the angle in the alternate segment MAGXL equals the angle that the tangent DM makes with LM, which cuts the circle. But by III.20 the angle at the centre MBL is double the angle at the circumference standing on the arc ML; therefore it is also double the angle DML, to which the angle in the alternate segment has just been shown equal. The semiangle MDB, moreover, is on Ptolemy's value, according to Kepler, with the Sun at apogee, 12′ 48″; and BMD is right, because δ[?]M differs insensibly from the tangent RM and it is just as if it touched, since it is physically next to the tangent. Therefore MBD is 89° 47′ 12″; and when MBL, 2° 3′, is subtracted, LBD will be 87° 44′ 12″, and LBO the same. Now because δ[?]M is taken to touch the circle as far as the senses can tell, the ray LO will also touch the circle

at L, by Witelo X.9; so OLB will be right. But LBO was 87° 44′ 12″; therefore LOB is 2° 15′ 48″, by Euclid I.32. So, all the angles in the triangle LBO being given, and the side LB taken as one semidiameter of the atmosphere, it is found by the rules of triangles that the axis BO is no longer than 25⅔ atmospheric semidiameters; and so great, or no greater, is the height of the sheer terrestrial shadow, if the horizontal refraction of the ray all but touching the atmosphere is 34′. Kepler was therefore right, at p. 269, although he had gathered from the refracted rays touching the Earth that the height of this shadow was 43 semidiameters, to add that it is made shorter by other refracted rays, namely those touching the air. For us, however, BO is 24 semidiameters of the Earth, because the angle MDB is 14′ 50″; indeed it is made shorter by the refracted rays that travel between HX and ML.

[Margin: The height of the sheer shadow, BO.]

Second, the angle DOP, as being equal by I.15 to the angle LOB, was found to be 2° 15′ 48″; and ODP on Ptolemy's reckoning is 12′ 48″. Therefore by Euclid I.32 OPG, the exterior angle equal to those two, is 2° 28′ 36″. But the circumference MLG was 179° 34′ 24″. For, from what we said in Book 3, chapter 11, theorem 4, the arc of the smaller sphere illuminated by the Sun beyond the hemisphere is as great as the angle of the cone of shadow; so when the angle MDG, which for Ptolemy is 25′ 36″, is added to 180°, the arc MAG becomes 180° 25′ 36″, and so MLG 179° 34′ 24″. And when ML, which was 2° 3′, is taken away, LXG remains, 177° 31′ 24″. And since the lines drawn from P, the right-hand one, to G, and to L, the left-hand one, are tan- gents, the arc of the angle GBP will be half the arc LXG, and therefore of 88° 45′ 42″, and BPG will be 1° 14′ 18″. From this, in the right-angled triangle BGP, it is gathered by trigonometry that PG is 46⅓ of the parts of which BG is one. For us, however, for whom the angle BDG is 14′ 50″, GP comes out at only 44½ atmospheric semidiameters.

[Margin: The height of the lateral shadow, GP and MP.]

Third, HΦK had to be 180° 25′ 36″ by what was set down above and from what Kepler established, chapter 6, number 3, if the Sun's rays had arrived without refraction; but an observer at H and K sees the Sun higher than is correct by 34′. The arc HΦK is therefore increased by 1° 8′ and becomes 181° 33′ 36″, and so HΨK is 178° 26′ 24″, half of which, HΨ, that is the angle HBN, is 89° 13′ 12″. When HX∇ is continued, the angle H∇B is concluded to be 46′ 48″; and XNB, the exterior angle, equals that together with the complement of ∇XN, or 34′. So XNB is 1° 20′ 48″. And since NX, continued, cuts BM between M and H, very close to H, it makes little difference whether we take BH as the measure or something a little greater, though much less than BM. Since, then, HBN is 89° 13′ 12″, and BNX, which is now BNH, is 1° 20′ 48″, BHN will be 89° 26′. Therefore, of parts of which BH, or a little more, is one, BN is about 43, on the assumptions Kepler makes in the *Optics*, chapter 6, numbers 3 and 8.

[Margin: The height BN, of the limit N.]

Granted that, forgetting himself, or with the figures wrongly written through a slip of the imagination or of the pen (which happens to great men at times, but is worthy of pardon), he gathers BN as 70 semidiameters in the *Optics*, p. 281. Hence it comes about that on his account, with the horizontal refraction of the refracted ray touching the Earth set at no more than 34′, the Moon, which at full Moon never rises as far as 70 semidiameters of the Earth, is never, not even in the middle of an eclipse, darkened by the sheer shadow of the atmosphere, DN, but only by the penumbra, passing between PP and ω ω. But when it is darkened by the sheer shadow DN, the refraction becomes greater, so that the height BN becomes less than the Moon's distance. So Kepler, p. 282; and from this, as we shall say at number 17, and from what was said in Book 3, chapter 10, last scholium, he gathers that the Sun's horizontal refractions are sometimes and in some places 51′, indeed 60′, indeed more than one degree.

[Margin: Kepler's slip.]

[Margin: A notable corollary.]

But with Kepler's figures and calculation corrected, the Moon in the middle of a total eclipse does in fact enter the sheer shadow of the atmosphere, and the following figure must be substituted for the preceding one. In it the height of the sheer terrestrial shadow is BO, very nearly 26 terrestrial semidiameters; the height of the segments of sheer atmospheric shadow MPI and GPL, namely MP and GP, is about 46 semidiameters of the Earth; the distance of the tip N from the Earth's centre B, that is BN, is about 43 semidiameters of the Earth; and the base TV of the inverted cone TNV is distant from the Earth's centre very nearly 73 semidiameters of the Earth, as I have calculated, using half the angle TNV, given above as 1° 20′ 48″, and half the angle D, 12′ 48″, and ND of 228 semidiameters of the Earth, which is obtained by subtracting BN, 43, from the axis BD, which on Ptolemy's values is 271 with the Sun at apogee, according to what was said in Book 3, chapter 11, number 9; and by solving the triangle DTN. For I found the side TN to be 31 semidiameters of the Earth; so in the triangle TNV the perpendicular from N to the base TV comes out at about 30, and these, added to BN, 43, make 73. Further, the arc AF marks the path of the Moon at apogee, and RS at perigee. So the cone of sheer atmospheric shadow that darkens the Moon in the middle of a total eclipse is TNV, unless the variation in the refractions becomes so enormous that BN exceeds the Moon's distance from the Earth.

Diagram. Set into the left column, with the text running down its left side and the figure reaching to the foot of the column. A ring (the atmosphere) stands at the top, its lower half filled black by the top of a long black cone that tapers down to its apex D. A little below the ring, rays from the ring's two sides cross in a light diamond on the axis; below the diamond the rays spread to the cone's sides, where two arcs cross the cone. Below them the cone is solid black again from T and V to the apex.

- M, G (left and right, on the ring)

- H, K (inside the ring, on the black, left and right)
- B (centre, on the axis)
- O (head of the light diamond, on the axis)
- I, L (inside the diamond, left and right of the axis)
- N (foot of the diamond, on the axis)
- P, P (cone's edges, left and right)
- R, S (the upper arc, left and right)
- A, F (the lower arc, left and right)
- T, V (cone's edges, left and right, at the head of the lower black part)
- D (apex)

Applicatio Doctrinæ de Vmbra & Penumbra ad Eclipſes Lunæ.

The application of the doctrine of the shadow and penumbra to eclipses of the Moon.

PROPOSITIO I.

Proposition 1.

Luna non patitur Eclipſim ab Vmbra mera ſolius Terræ.

The Moon does not suffer eclipse from the sheer shadow of the Earth alone.

XI. This is Kepler's paradox, in the *Astronomia Optica* pp. 268 and 270, and it is proved as follows. At full Moon the Moon is never less distant from the Earth than 52 semidiameters of the Earth, as is clear from what was said in Book 4, chapter 14, and when it is at apogee it is much further; but the height of the sheer shadow of the Earth alone does not exceed about 26 semidiameters of the Earth. Indeed if the Sun's refracted rays between those touching the Earth and those touching the refractive atmosphere combine to cut it short, as they do, it is far smaller, and smaller still if the horizontal refraction is greater than 34′, as is clear from what was said partly in Problem 1, partly in Problem 4, number 10.

[Margin: Kepler's paradox.]

Nor does the authority of the ancient astronomers or cosmographers stand in the way, who attribute the eclipse of the Moon to the terrestrial shadow, as Bettini objects to Kepler, though without naming him, in *Apiarium* 8, Progymnasma 2, Proposition 5. Chief among them are Cleomedes, *Cyclical Theory* Book 2, chapter 5; Pliny the Elder, Book 2, chapters 10, 11, 12; Ptolemy, Book 5, chapter 14, and Theon there; and Plutarch in the little work *On the Face of the Moon* and in the *Nicias*. For they did not distinguish the sheer shadow of the Earth from the penumbra, but by the name of shadow meant the deprivation, caused by the Earth, of the direct and primary sunlight by which the Moon usually shines outside full Moon, or at a full Moon without eclipse. They did not exclude the dense matter around the Earth, which could deprive the Moon

of such light, or else they named that deprivation after the stronger and greater mass of this globe.

[Margin: The first objection of Father Bettini against Kepler is dissolved.]

Nor, likewise, does it stand in the way that the same author objects against him in the same place, Proposition 6, namely that the air capable of refracting the rays of the stars is very low: by his own conjecture in Proposition 3 about 151 paces, and according to Kepler in the *Optics*, pp. 117, 129, 135 and 271, no more than two Italian miles; on this matter see what was said in Book 2, chapter 19. For from this, Bettini judges, it comes about that the rays refracted on entering this air do not pass out of it, but strike against the flanks of mountains or are buried in valleys. Granted that in Proposition 1, number 6, and Proposition 2, number 2, Bettini may have conceded that the Earth's shadow is cut short by them, not absolutely but on condition that the Earth were not rough with mountains and valleys, or that the height of the refractive air were much greater; this is certainly a sharp objection and not to be despised. But it is answered: first, because in our opinion the height of the refractive air is greater, as we indicated in Book 2, chapter 19, and shall teach in the Optical book; next, because many great tracts of the Earth are level expanses of plains and seas, and many are rough with very low hills, so that the greatest part of the rays once refracted can pass out and, refracted again, cut the shadow short. Finally, if none of the refracted rays passes out of the atmosphere, and they neither dilute nor diminish the shadow of this globe, then this shadow is indeed sheer, without any admixture of penumbra, but it is nevertheless the shadow of the whole atmosphere enclosing the Earth, not of the Earth alone.

[Margin: The second objection of the same author is dissolved.]

PROPOSITIO II.

Proposition 2.

Luna Eclipſim patitur ſub initium finemq. Eclipſis à Penūbra Atmoſphæræ; circa medium autem partim ab eadem Penumbra, partim verò ab Atmoſphæra mera Vmbra.

Towards the beginning and end of an eclipse the Moon suffers eclipse from the penumbra of the atmosphere; around the middle, however, partly from the same penumbra, partly from the atmosphere's sheer shadow.

XII. By the name of atmosphere I here understand the terraqueous globe surrounded by air, and distinguished from the aether (whether the aether is pure elemental fire or a heavenly and most fluid substance coming down as far as the air), and I suppose that all this air is denser and more opaque than the aether, but uniformly difformly [vniformiter difformiter], more and more so towards the Earth. And so, although exhalations predominate in its upper part and vapours in its lower, the whole is fit for some refraction, in places scarcely sensible, in others more and more sensible. With this laid down, the

proposition is proved from the history of eclipses and from our figure. For after the umbrago and smoke, on which see number 2, the Moon never begins or ceases to be eclipsed by the eclipsing elementary globe without some light and some colour appearing on the eclipsed segment; and there happens what Galileo, as quoted by Kepler in the *Ephemerides* p. 22, reports of the eclipse of 1610 December 30, observed with the telescope: *The limit of the shadow appeared indistinct, confused and as if clouded over.*

[Margin: The notion of the atmosphere.]

But this, whatever it is, of grudging light, or of colour usually reddish, does not come from the Sun's rays reflected from the Earth on to the Moon, since at full Moon the reflected rays of the Sun make for the opposite part, towards the Sun; nor from rays reflected from any other assignable body. For, granted that, as Barozzi would have it (*Cosmographia* p. 318), they could be reflected from the remaining part of the Moon not yet darkened, yet when it is now wholly darkened they cannot. Nor does it always come from the direct illumination of the planets, which are often in a position and place unsuited to this; nor from the fixed stars, since otherwise they would also illuminate the Moon in the middle of an eclipse, and at an even rate, and on a moonless night we should see the Earth lit by them with a brighter and direct light than we see the Moon by their light reflected from the Moon to us. Nor does it come from sunlight drunk in, as Liceti would have it, and preserved in much the way that the light-drinking [lucibibus] Bologna stone preserves it for some time; for if that were so, a trace of this light would always appear even in the middle of a total eclipse, or it would gradually weaken towards the eclipse's end, whereas rather it grows brighter towards the end. Nor does it come from its own light, which we determined in Book 4, chapter 6, that it does not have; nor, finally, from any other probable cause.

And on the other side it seems not only probable but certain that very many of the Sun's rays pass out of the atmosphere refracted towards the axis of the shadow, and so diminish the shadow and illuminate the region in which the Moon is, already or still suffering eclipse. Now this compound, of the deprivation of primary light arising from the whole eclipsing globe and of refracted and weakened light sent through the vapours and exhalations of the air, is a penumbra, not of the air alone, nor of the terraqueous globe alone, but of the whole atmosphere. For since a penumbra consists of the deprivation of direct and primary light and of positive indirect light, which in our case is refracted, not reflected, both concur in the deprivation of direct light: the terraqueous globe, which by its opacity blocks the passage of all light, and the density and some opacity of the air, which block the direct propagation of light. So, when all the parts of this argument are gathered together, towards the beginning and end of an eclipse the Moon is darkened by the atmosphere's penumbra. Nor, around the horizon, at the entry or exit of the Sun's rays from the atmosphere, are clouds ever so foul as to block all passage entirely to the Sun's rays, since with the Sun above the horizon it never becomes sheer night.

So if clouds do not snatch the sight of the eclipse from us, that shadow which dulls the Moon at the beginning and end, however dark it appears, always, with some small particle of light mixed in, shows us some colour, not purely black, on the eclipsed part of the Moon. And, as Scheiner says: *That rust colour which the Moon shows when it moves around the edges of the terrestrial shadow is nothing other than the Sun's light, with an admixture of terrestrial shadow, that has glided on to it and been sent back to us.* And this is confirmed, because from what was said at the end of number 10 the Moon is always within PT and PV.

Kepler adds, in the *Epitome of Copernican Astronomy* p. 870: *Often too a luminous substance inflamed around the Sun irradiates the outermost limits of the shadow, and so the Moon, although wholly immersed in the shadow, does not seem immersed; see* Astronomia Optica *fol. 301. And Galileo too thinks so; but that appendage of inflamed substance around the Sun not seldom paints itself on paper, when we catch the Sun's image through a hole.*

XIII. The second part of the proposition is shown from the same history of eclipses. For sometimes the Moon, in the middle of a total eclipse, watched with the sky clear around it, so disappears that no trace of it in the sky, no colour of a ruddy, or livid, or pale Moon glimmers through. Sometimes, however, it appears ruddy or pale even in the middle of a total eclipse, and *failing, and yet conspicuous in its failure*, as Pliny the Elder says, Book 2, chapter 9. Sometimes, finally, it disappears entirely for some and not entirely for others, keeping a trace of a faint glow, as I shall shortly relate.

For so that there may be some example of this uncertainty from our own observations: on 1642 April 14, at Bologna, the Moon, already immersed in the penumbra and reddening, shortly after seemed to die and to be withdrawn entirely from our eyes, and for a long time we searched in vain for the place in the sky where it was; then it reddened again, and not very long after emerged gradually from the penumbra. And yet for the whole time of the eclipse the sky seemed clear to us, with the stars glowing all around.

[Margin: The Moon extinguished in the middle of an eclipse, for some.]

The same, as regards the Moon's extinction in the middle, was observed at Prague by Fathers Theodorus Moretus [Theodoor Moretus] and Vuolfangus [?], of the Society of Jesus, and at Paris by Father Georges Fournier, S.J., with D. Pierre Gassendi and Ismaël Boulliau, as I have it from their letters delivered into my hands. Indeed Scheiner says in the *Disquisitiones Mathematicae* p. 59: *It is established that the Moon in the very middle of the terrestrial shadow is entirely deprived of all light and colour.* To others, however, it did not seem entirely extinguished. For the most illustrious D. Giulio Giustiniani writes to D. Fortunio Liceti, and Liceti to me, that it was seen with a tiny portion of reddish glow, like Mars or the nebula of Praesepe; and he adds that the same was seen by Bonuicino [?] and Pieronio [?] at Vienna. At Glacij [Kłodzko] too Father Balthasar Conrad, S.J., and at Nyſſæ [Nysa] Father Scheiner saw a faint glow always clinging to the Moon like a star, but of the rest of the Moon they saw nothing at all.

[Margin: The same not entirely extinguished for others.]

At Pisa, D. Vincenzo Renieri writes to me about this eclipse as follows: *What seems worthy of consideration in a phenomenon of this kind is the total loss, so much so that the Moon was not only entirely stripped of that redness with which it shines in such a case, but the limb of the disc itself did not appear at all, and only a tiny whiteness was seen, from that part of the disc believed to face the south-west* [Notolybicum]. *Let those who think the Moon shines with its own light see here how they can save a phenomenon of this kind.* Wendelen tells it a little differently in the preface to the *Eclipses*: *It sometimes happens that the Moon, once immersed in the shadow, so vanishes that unless carefully sought it cannot be found; which Tycho left in writing about the eclipse of 1588, which very thing happened in 1620, and finally in 1642, in April, we all noted it.* And in Kepler's *Astronomia Optica* p. 277: *On 1588 March 3, at Mulpronnæ [Maulbronn] in Württemberg, I could scarcely make out the Moon with my eyes, through its ashen colour, when it was in the middle of the shadow.* Now the Moon was then at mean distance from the Earth, at 25° Libra; Venus at 20° Aries; Saturn at 20° Pisces and Jupiter at 8° Pisces; that is, in a place most suited to imparting some light to the Moon, which stood almost opposite.

But in the same year, on October 7, in the middle of a total eclipse, we saw the Moon grow pale after it had been red, but not wholly extinguished. It was nearer perigee than apogee, and at 15° Aries; and it could then have been illuminated by Venus, placed at the end of Scorpio. Gassendi saw the same in the eclipse of 1623, in the month of April. And Wendelen adds, in the place above: *When it is now set in the midst of the darkness, it reddens like a glowing coal*; which we too have seen at other times. Nor is what Scheiner pronounced universally above true.

So although, when the Moon shines in the middle of a total eclipse and no planet stands opposite it in a position suited to illuminate it, it seems certain that it is with part of itself in the penumbra and receives secondary light from the Sun's refracted rays, especially if it does not shine in the middle as it does at the edges, and so that it is with part of itself outside, and with the middle of itself inside, the cone TNV; yet when planets suited to illuminate it are present, whether the Moon in the middle of the eclipse is visible to all observers, or only to some, or to none, it is uncertain whether it is in the sheer shadow of the atmosphere and its cone TNV, or within ON, since because of extraordinary refractions it is not impossible that its limit N rises above the Moon's distance from the Earth, to 70 semidiameters, and so it appears glimmering, or, if planets illuminating it are present, it appears to some and not to others, because of a different temper of the air or attention not kept up in observing; or whether, as I said, it is in the lucid segment NQOQ of the penumbra, but does not appear because of small clouds interposed between the Moon and us, or thick vapours around the horizon, at about the same time spread around such a position of the atmosphere that, by an enormous refraction, they at once refract and weaken either the Sun's rays on entering or leaving the atmosphere, or the Moon's rays on entering it, so that the Moon can scarcely, or not even scarcely, be

seen. And I say the same when no planets suited to illuminating the Moon are present and it does not appear; for that total failure need not necessarily be attributed to their absence rather than to other causes. Finally, we have very few observations in which the appearances of the middle of a total eclipse have been accurately noted and committed to record. I do hope that posterity will be more exact in these matters; yet astronomy will always waver here, because of the inconstancy of physical causes.

PROPOSITIO III.

Proposition 3.

Luna in medio Eclipſi totalis poteſt ſed rarò plus ſublucere, quàm ante aut poſt medium; quando autem tota in medio obſcuratur, craſſiorem Vmbram permeat in Apogeo quàm in Perigeo, & meræ Vmbræ conus Lunam obſcurans inuerſo mucrone Terram reſpicit.

In the middle of a total eclipse the Moon can, though rarely, glimmer more than before or after the middle; but when it is wholly darkened in the middle, it passes through a thicker shadow at apogee than at perigee, and the cone of sheer shadow that darkens the Moon faces the Earth with its point inverted.

XIV. Father Bettini touched ingeniously on the foregoing paradoxes, in other words, but from a hypothesis he himself did not absolutely admit, in *Apiarium* 8, Prop. 2, Prop. 2[39], and he deduces them from Kepler's figure and from what was said at numbers 9 and 10. Namely, if the Moon is in the segment NQOQ, whose middle lies beyond 46 and below 70 semidiameters of the Earth, and the Sun's refracted rays from left and right unite in it and illuminate the Moon with doubled light; and if these same rays, at a smaller distance from the Earth, say within OQ, are less spread apart and more united, and because of their smaller distance from their origin more intense, than at a greater distance, say within QN. Conversely, if the Moon is in the atmosphere's sheer shadow TNV, then the nearer it is to the Earth, the nearer it will be to the limit N, and it will enter a smaller part of the shadow at YY than if, being further from the Earth, it comes nearer to the shadow's greater breadth TV.

[Margin: Father Bettini's ingenious paradoxes.]

The former part, however, is deduced from the hypothesis of Kepler's measure, which is nevertheless erroneous, from what was said at the end of number 10. Yet that part can be verified, if the horizontal refraction varies so much that the height BN is greater than the Moon's distance; but that happens very rarely, and almost always, as in our second figure, substituted at the end of number 10, the Moon in the middle of an eclipse is above ON and within the cone TNV.

[39] *Prop.2.* is printed twice; the same work is cited further on this page as *Apiario 8. Prog.2. prop.9.*

The third part of the proposition, on the inverted point, is clear from our figure at the end of number 10.

PROPOSITIO IV.

Proposition 4.

Eclipſium Lunarium Quantitas & Duratio eadem eſt quoad ſenſum, ſiue æſtimetur ex Penumbra Atmoſphæræ, ſiue ex putata à Priſcis mera Vmbra Terræ.

The magnitude and duration of lunar eclipses is the same as far as the senses go, whether it is reckoned from the penumbra of the atmosphere or from what the ancients took to be the sheer shadow of the Earth.

XV. This, of course, is what Kepler meant when he said in the foundations of the *Ephemerides*, number 29: *The business of refractions does indeed cause the redness in the failing Moon, but it does not at all disturb the geometrical figure of the terrestrial shadow*; and in the *Astronomia Optica* p. 268: *That neither eclipses should be cavilled at from refractions, nor refractions from eclipses*; and p. 271, where he teaches that the astronomical calculation will not be disturbed if, in place of the ancients' terrestrial shadow, the penumbra from the air is used. See the reason for this statement in Scholium 1 of chapter 4.

PROPOSITIO V.

Proposition 5.

Poteſt eſſe & videri Eclipſis Lunæ, etiam in medio, quamuis vtrumq. Luminare ſupra Horizontem appareat, & tamen interim alijs apparere infra Horizontem.

An eclipse of the Moon can exist and be seen, even at its middle, although both luminaries appear above the horizon; and yet it can meanwhile appear to others below the horizon.

XVI. Goclenius denied this in *Urania* p. 128, of both luminaries above the horizon; but it seemed not ἀδύνατον [Greek: ἀδύνατον (adynaton, "impossible")] to Pliny the Elder but παράδοξον [Greek: παράδοξον (paradoxon, "paradoxical")], when he says in Book 2,

[Margin: Goclenius's error.]

chapter 13: *And what is most wonderful in this marvel: although it is agreed that the Moon is dulled by the Earth's shadow, this happens to it now from the side of the setting, now from that of the rising. And in what way, since at sunrise that dulling shadow must be under the Earth, did it once happen that the Moon failed at its setting, with both bodies visible above the Earth?*

[Margin: A history of such eclipses.]

Mästlin testifies that something of the kind was seen by him in 1590 July 17, in the *Theses on Eclipses*, thesis 55, and Kepler after him in the *Astronomia Optica* p. 136. Mästlin's words are: *Here at Tübingen, with the Sun's centre emerging above the horizon, we saw the Moon, already failing from the south by several digits, raised almost 2°; and conversely, with the Moon's centre descending to its setting, we noted the Sun's height above its rising as 2°. But the Moon set before it had come to its greatest darkening.* And the Romans will see a similar eclipse in 1668 May 25; on this and similar ones, see what I said against Clavius in Book 2, chapter 2, Scholium 3.

Cleomedes, moreover, in Book 2, chapter 5, which concerns the Moon's eclipse, appears to deny such eclipses absolutely. For he had said: *Some say that eclipses of the Moon have been seen with both luminaries visible above the horizon, which would plainly show that the Moon does not fail because it runs into the Earth's shadow*; and he had refuted the ancient mathematicians who untied this knot by saying that it can happen to an eye raised above the globular Earth; and he had granted them that it can happen if the sight were very high and raised above the Earth's globosity, but by no means if it rests closer to the Earth, as it usually does. Having related this, I say, he adds:

[Margin: Cleomedes's ambiguous judgement on this phenomenon.]

But first those who say this must be met with the point that this story has been made up by certain philosophers and astrologers: for there are extant many eclipses of the Moon, both total and partial, all of which are set down in writing, and no one down to our own times has reported such an eclipse, neither a Chaldean, nor an Egyptian, nor any other mathematician or philosopher; but what is said is invented.

[Margin: Cleomedes's first response.]

Now Kepler in the *Optics* p. 145, and Mästlin with him, judge that it is Pliny the Elder who is here rebuked by Cleomedes without being named. Our Bettini, on the contrary, in *Apiarium* 8, Progymnasma 2, Proposition 9, thinks that Pliny the Elder took this from Cleomedes, and so he wonders that some are so ignorant of chronology as to have thought that Cleomedes took it from Pliny the Elder. But there were two men named Cleomedes: one who flourished under Theodosius the Younger, about the year of the Lord 390, and so later than Pliny the Elder by three centuries; the other who flourished in the century before the birth of CHRIST, shortly after Posidonius, and wrote two books κυκλικῆς θεωρίας μεθεωρῶν [Greek: κυκλικῆς θεωρίας μεθεωρῶν (kyklikēs theōrias meteōrōn, "of the cyclical theory of the heavenly bodies")] [recte: μετεώρων], as well as an *Arithmetic* and a *Harmonics*, as Robertus Balforeus [Robert Balfour] testifies, who translated the first work and elucidated it with scholia; and Biancani after him in the *Chronologia Mathematicorum*. Thus the parachronism [parachronologismus] of Kepler and Mästlin was rightly noted by Father Bettini. And indeed, how could Cleomedes deny that such an eclipse had ever been committed to writing by anyone, if he had known that this one observation was extant in Pliny the Elder? Yet I do not agree with Bettini when he asserts that Pliny the Elder copied from Cleomedes those words *and*

in what way … once; for Cleomedes does not say that it once happened, and indeed he denies that any such thing is read anywhere in the history of eclipses.

[Margin: The parachronism of Kepler and Mästlin, rightly noted by Bettini, though not without a blemish of his own.]

Cleomedes, however, goes on, also on the hypothesis of such an eclipse having been seen or being yet to be seen. He responds, secondly, that it can happen either because of an image of the Sun reflected from a thicker cloud to our sight, or because of our visual rays in moist and more humid air, which nevertheless present to us above the horizon a Sun placed below the horizon. Here, in the second part of his response, as if stammering, he touches on the true cause; for his own words are: *Further, the ray poured out from the eyes, having met moist and more humid air, can also* κατακλᾶθαι [Greek: κατακλᾶθαι (kataklasthai, "to be broken" — the text's own gloss follows)] [recte: κατακλᾶσθαι]*, that is, be broken, and reach the Sun, already hidden under the horizon: for we have observed something similar in a gold ring put into an empty vessel, which from a moderate distance is not seen, but from the same distance appears if the vessel is filled with water, etc.* In this, however, Cleomedes errs: he both defends vision by the emission of rays, and thinks it necessary that the air at the horizon be sprinkled with abundant moisture, whereas the greater density of our air alone is enough. Yet his response is approved by Milich on Book 2 of Pliny the Elder, chapter 13; by Io. Pisanus [John Peckham], *Perspective* Book 3, propositions 5 and 7; and by Stöffler on Proclus, p. 63½.

[Margin: The second response of Cleomedes, of Milich, of Pisanus, of Stöffler.]

XVII. The true cause of this phenomenon, therefore, especially as regards the beginning of a morning lunar eclipse or the end of an evening one, can in part be the one which the ancients in Cleomedes, the mathematicians, pointed out: namely the raising of the eye above the ground, whether on a hill or in a tower or some other lookout; for from there it can see the Sun, even though the Sun is placed below the horizon, as is gathered from what was said in Book 2, chapter 10. But the principal cause, and one that always intervenes, is the refraction of the rays, solar on the one side and lunar on the other, which visibly lifts the luminaries above the horizon, although one of them, or even both, is really situated below it, as Kepler teaches above. He adds that the refraction by whose force Mästlin saw such an eclipse in Swabia was greater than 1° in each of the luminaries, since it raised the luminaries into his view in spite of the mountains besetting the horizon of Tübingen.

[Margin: The second cause of this phenomenon.]

In that Proposition 9, Bettini also ingeniously adds that the same lunar eclipse can be seen by some above the horizon, and by others below the horizon of the former but above their own (add: and by others on the horizon), and this at the same moment of time, namely by the force of a refraction that raises the luminaries for each of them towards his own zenith. So in the following

figure, in which let the Earth's centre be M, and the astronomical horizon AMB, serving both the inhabitant HI and his antipode KL.

[Margin: Bettini's ingenious observation.]

Diagram. Set in the left column. At the left, three circles stand one above another, touching: the upper labelled C, the middle A, the lower D; a curved line runs through C and D, passing the junctions with A. In the middle is the Earth, with a centre M, its left half showing land and dotted water, its right half hatched. A straight line runs from A through M to a point B at the far right, and two tangent lines run from the region of A's contact points past the top and bottom of the Earth, converging on B to form the hatched cone of the shadow. Around that cone stand, on a second curved line, a circle E above and a circle F below, with a smaller circle G on the axis between them, overlapping both.

- Upper left circle: C
- Middle left circle: A
- Lower left circle: D
- Top of the Earth: H I
- Centre of the Earth: M
- Bottom of the Earth: K L
- Upper right circle: E
- Circle on the axis: G
- Lower right circle: F
- Far right, apex: B

For although the Sun be at A, for example, and the Moon at G, already eclipsed by the shadow or penumbra IBL, yet by the benefit of refraction the eye H or I will be able to see the Sun at C and the eclipsed Moon at E, at the same moment at which the eye K or L sees the Sun at D and the eclipsed Moon at F. Indeed, even while the eye H sees the luminaries on the horizon, the other eye I will be able to see one of them above it. Indeed, what is more wonderful, all those whose horizons intersect at the same point where the failing Moon stands will be able to see their own distinct image of the luminaries, and one real eclipse can be multiplied into almost infinite apparent ones, in diverse images, by the force of refraction, because horizons of this kind are infinite, or finite without limit.

[Margin: Infinite eclipses of the Moon from one.]

XVIII. The assertion of Father Bettini, however, which is found on this occasion in the corollary of that Proposition 9, must be corrected. For he affirms that at the time such an eclipse appears (the one discussed above with Pliny the Elder) the luminaries are on the horizon, and that therefore, once their apparent altitude has been observed by instrument and the horizontal parallax added to it, the measure of the horizontal refraction is at once obtained. This would indeed be true if it were known that at that moment the true place of the luminaries is on the astronomical horizon; but this is not always certain independently of refraction, and indeed it is mostly certain that the luminaries

are already either below or above the horizon. For when the Sun appears on the physical horizon, it is below the astronomical one, since these horizons are apart only by the arc of the horizontal parallax, which in the Sun is not 4′, whereas refraction raises it about 34′. Hence, for it to be on the astronomical horizon when it appears above the physical one, it must be seen raised about 30′ above the physical one; and this can be concluded with determinacy only if its horizontal refraction is known and presupposed beforehand. The Moon, because of its greater parallax, is subject to a greater variation in this; but more on these matters in the Optical Book.

[Margin: Bettini's fallacious way of measuring refraction from such eclipses.]

SCHOLIA.

Scholia.

I. The matters concerning the Moon's own light, asserted by some on account of the light and redness it has in its eclipses, have already been sufficiently discussed in Book 3, chapter 6, from number 11, although in this chapter, at number 12, we have touched on most of the arguments on this subject. On the colours of the eclipsed Moon, however, we said some things incidentally in Book 3, chapter 6, near the end, where we reported the false causes of these colours advanced by certain people, and conjectured the true one to lie in the diverse vapours of the atmosphere, more or less dense and opaque, and in a threefold refraction in them: namely of the Sun's rays once on entering the atmosphere and again on leaving it, and a third time of those same rays reflected from the Moon to us and refracted on their return through the atmosphere.

[Margin: Whether the Moon has its own light.]

[Margin: The cause of the diversity of colours in an eclipse of the Moon.]

Sometimes the variety of these colours is so great that Kepler, in the *Astronomia Optica* pp. 274 and 282, thinks they resemble the colours of the rainbow. For Cornelius Gemma, in Book 2 of the *Cosmocritice* p. 64, relates of the eclipse of 1569 March 3: *In the morning at the third hour Phoebe suffered a horrible eclipse, marked by dreadful colours. First it shone dusky, then blood-red, soon also crimson, and green, and livid, and finally misshapen with an incredible variety.* And Apollonides in Plutarch, in the little work *On the Face in the Orb of the Moon*: *Since therefore we see the Moon take on so many colours in the shadow, those people are not right to seize on the ember-colour.* And in the *Aemilius Paulus* Plutarch says that the Moon, overlaid with various colours, went off into darkness.

[Margin: The variety of these colours.]

Mostly, however, either the pale or the red predominates in it. Hence Frodoard says in his *Chronicle* for the year 926: *On April 1 [Cal.April.] the Moon suffered eclipse and was turned to pallor, with a small part of its light left, as though it were a second-day Moon; and so, with dawn already beginning, it was changed entirely into a blood colour.* There is also extant in the *Acts of the Holy Martyrs* this passage about St Felix, an African bishop who with his companions gained

martyrdom in Campania because he refused to hand over the divine books, that is, sacred Scripture, and to obey Diocletian's edict in this: *And he was led to the place of his passion, when the Moon itself also was turned to blood, on August 30 [die tertio Kal.Sept.].* Scaliger, however, in the prolegomena to the *De emendatione temporum* p. 18, because he assigns this eclipse to the year of CHRIST 301, September 3 [3.Nonas Septembris], thinks that August 30 [3. Kal. Septembris] was the day on which Felix was led to the place of his passion, but that the eclipse that followed is described soon after.

[Margin: The red colour is the more frequent in these eclipses.]

So Cardano, Book 3 of *De subtilitate*: *It is agreed that in the greatest eclipses the Moon reddens like a sort of ember of fire.* Scheiner, in the *Disquisitiones* number 26, calls this colour rust-red. The poets chime in here, speaking of the colours of the failing Moon. Ovid, *Amores* 2:

Songs draw down the horns of the blood-red Moon.

Seneca in the *Hippolytus*, Act 2, Chorus: *And lately it reddened.* For the rest see chapter 1, number 2. Lucan, Book 6:

and serene Phoebe, Not otherwise beset by the dreadful poisons of words, Grew pale, and burned with black and earthly fires.

That is, from the Earth's penumbra.

II. Besides the phases discussed so far, there are at times notable *Hapsi*, that is, small clouds or spots hanging under the Moon, arising when it is already darkened, and suspended beneath it. Mästlin saw one of this kind on the evening of 1605 April 3, in the eclipse of that year. For he says in the *Theses* on eclipses, thesis 152, that he saw on the Moon's body, towards the north, a blackening spot darker than the rest of the body, and of the colour of glowing iron; perhaps it was the lake of Panciroli, on which I spoke in the *Selenography*, Book 4, chapter 7. Wendelen, for his part, in the preface to the *Eclipses* p. 3, says that he has often seen small clouds hanging under the Moon while it was still set in darkness, as if heavy with much water, which at times, because of their shape arched in the manner of the shadow, can imitate the beginning and end of an eclipse produced by the true penumbra. Thus in the year 1638, although he was certain from the better tables and the remaining phases that the whole eclipse had ended at $4^{h}\ 8^{m}$ after midnight, yet because of an arched small cloud clinging to or accompanying the Moon, it was seen to last until $4^{h}\ 40^{m}$. Therefore Tycho and the ancients must be pardoned if, in noting the beginnings and ends of eclipses, they sometimes go enormously astray, deceived either by the umbrago, or by these Hapsi, or by suffusions of the eyes, especially since the telescope had not yet been discovered.

[Margin: A false and spurious penumbra from Hapsi and arched small clouds.]

III. As for the astrological judgements to be made from the colours of eclipses, they rest rather on conjectures and on a certain remote analogy. So Ptolemy, in Book 2 of the *Quadripartitum*, chapter 9, t. 51, 52, 53, and after him Origanus in the *Introduction to the Ephemerides*, part 3, chapter 2, teach that a black and

somewhat livid colour signifies Saturnine effects; white, Jovial; ruddy, Martial; yellow, those of Venus; variegated, Mercurial. But neither are the astronomical judgements on the future colours of eclipses any more certain, although some have handed down rules of some kind or other, among them Apollonides in Plutarch, in the little work *On the Face in the Orb of the Moon*: *We see the Moon in eclipse take on now one colour, now another, which the mathematicians distinguish by the reckoning of the time as follows: if towards eve-*

[Margin: The fallacious prejudgements, astrological on the effects of colours, and astronomical on the future colours of eclipses.]

ning the Moon fails, it is horribly foul; if at midnight, it becomes crimson and red; towards dawn it puts on a fierce and dark blue face. But the Alphonsines, following Io. de Linerijs [Jean de Lignières] or following the Arabs, Apian in the *Astronomicum Caesareum* statement 27, Magini in Canons 73 and 89 of the *Secunda Mobilia*, and Renieri in the *Tabulae Mediceae*, second edition, p. 355, set out small tables that do not answer to experience.

IV. Just as in a Moon that is not full the section between the part lit for us and the dark part is winding, because of the roughness of the Moon's valleys and mountains, so in partial lunar eclipses the borders of the penumbra and of the bright part appear roughened and broken, whether you look through the telescope or receive its image on paper. Kepler shows something of the kind in the *Optics* p. 247 and in the *Dissertatio cum Nuncio Sidereo* p. 6½. He thinks the cause of this phase is that, at the border of the eclipse, some parts of the Moon take up the weakened ray of the Sun more strongly, others more weakly, and send back the Sun's direct ray, and indeed also the ray refracted in the penumbra; and so in the darkened part that redness appears blacker in some places than in others, as can be seen in the figure of the eclipse of 1602 December 31, 6^{h}, observed on paper, which Kepler has in the *Optics* p. 247.

[Margin: The roughness and inequality of the penumbra.]

Wendelen, however, in the preface to the *Eclipses* p. 6, attributes this jagged edge and fringed border of partial eclipses to vapours that are lower under the Torrid Zone and higher under the Poles. So it comes about that the convexity of the atmosphere is itself also unequal and has its own valleys and mountains, so to speak, and so the rays, refracted at its surface by a double or triple refraction, produce this inequality at the border of the penumbra. For my part I judge that both causes, each in its own measure, contribute to this. Now follows the table mentioned at the end of number III.

[Margin: The twofold cause of this.]

COLORES ECLIPSIVM.

The colours of eclipses.

In an eclipse of the Sun		In an eclipse of the Moon				
Distance of the Sun from the Dragon's Head or Tail	**Colours**	**Moon's latitude**	**Colours**	**Argument of latitude**	**Colours**	**Argument of latitude**
1°	Deep black [Niger preſſus]	from 0′ to 10′	Blackest [Nigerrima]	0ˢ	Ruddy [Ruffa]	12ˢ
2°	Dark black [Niger obſcurus]			1ˢ	Whitish ash [Albicinerea]	11ˢ
3°	Dusky black [Niger fuſcus]	from 10′ to 20′	Blackish green [Nigrouiridis]			
4°	Dusky saffron [Fuſcicroceus]			2ˢ	Ash-grey [Cinerea]	10ˢ
5°	Clear dusky [Fuſciclarus]	from 20′ to 30′	Blackish red [Nigrorubra]			
6°	Dusky red [Fuſcirubeus]			3ˢ	Blackish pale [Nigropallens]	9ˢ
7°	Ruddy [Ruffus]	from 30′ to 40′	Blackish pale [Nigropallens]			
8°	Ruddy [Ruffus]			4ˢ	Blackish reddening [Nigrorubens]	8ˢ
9°	Ruddy grey-blue [Ruffiglaucus]	from 40′ to 50′	Ashen [Cineritia]			
10°	Red grey-blue [Rubeoglaucus]			5ˢ	Blackish greening [Nigrouirens]	7ˢ
11°	Saffron [Croceus]	from 50′ to 60′	Whitish ash [Albicinerea]			
12°	Whitish saffron [Albicroceus]			6ˢ	Blackest [Nigerrima]	6ˢ

CAPVT VI.

Chapter 6.

De Lunarium Eclipſium Terminis, Magnitudine, Duratione, & Medio; vbi & de Horario motu Luminarium.

On the limits, magnitude, duration and middle of lunar eclipses; where also on the hourly motion of the luminaries.

PROBLEMA I.

Problem 1.

Terminos Eclipſis Lunę determinare.

To determine the limits of a lunar eclipse.

I. The limit [Terminus] of a lunar eclipse is the distance of the Moon's centre from the nearer of the nodes, beyond which an eclipse of the Moon is naturally impossible, and within which it is either necessary or at least possible. And so these limits are subdivided into possible and necessary. Their determination depends on three elements: the apparent semidiameter of the Moon, the apparent semidiameter of the shadow of the Earth or rather of the atmosphere, and the true latitude of the Moon. For if the latitude is not less than the sum of the apparent semidiameters of the Moon and of the shadow or penumbra, an eclipse is not possible. Therefore, to have the limits of a possible eclipse, we must take the greatest semidiameter of each, namely of the Moon at perigee and of the shadow or penumbra with the Sun at apogee. For their sum will give the side EB in the spherical triangle ABE, right-angled at E, in which the angle at the node A is supposed given, measuring the Moon's greatest latitude in the syzygies. By the rules of triangles the base BA will then be found, namely the distance from the nearer node A within which an eclipse of the Moon will be possible, and beyond which it will not.

[Margin: How the possible limits are found.]

To have the limits of a necessary eclipse, on the other hand, the least apparent semidiameter of the obscuring and the obscurable body must be taken, namely of the Moon at apogee and of the shadow or penumbra with the Sun at perigee. For their sum gives the side EB, and this with the angle A gives the base BA, which is the distance from the node A within which the Moon, once placed there, necessarily suffers eclipse.

[Margin: How the necessary limits are found.]

Diagram. Set in the right column with the text running round it. A spherical figure with its apex at the right: an upper arc runs from D through E to A, a lower arc from C through B to A, a line joins E to B, and a line joins D to C.

- Left end of the upper arc: D
- On the upper arc: E
- Apex at the right: A
- On the lower arc: B

• Left end of the lower arc: C

So, once the Moon's greatest latitude in the syzygies has been taken from Book 4, chapter 12, the Moon's apparent semidiameter from Book 4, chapter 16, and the semidiameter of the shadow, or rather of the penumbra, from Book 5, chapter 4, number 14, the limits already mentioned can easily be defined according to the various hypotheses of the astronomers. They can, however, also be gathered from the small tables of semidiameters and from the motion of latitude, or its argument, already arranged in tables, by seeking in the body of these tables the Moon's visible latitude equal to the sum of the semidiameters of the Moon and of the shadow or penumbra, and in the margin the degrees of the argument of latitude under sign 0 or above sign 11, and using, if need be, the proportional part.

In these ways, then, the ecliptic limits were established by Ptolemy, Book 6, chapter 5; al-Battānī, chapter 43; Regiomontanus in the *Epitome of the Almagest*, Book 6, proposition 8; the Alphonsines in the tables, proposition 32; Copernicus, Book 4, chapter 30; Reinhold in the Prutenic Tables, precept 50, and on Peuerbach's *Theoricae* p. 198; Wurstisen in the same place, p. 347; Neander in the *Elements* p. 129; Mulerius in the *Tabulae Frisicae* pp. 77 and 428; Magini in the *Theoricae*, Book 2, chapter 26, and Canon 67 of the *Secunda Mobilia*; Tycho, *Progymnasmata* vol. 1, p. 126; Biancani, Book 9 of the *Sphaera*, chapter 5; Langius in the *Theoricae*, chapter 19; Kepler in the *Epitome of Copernican Astronomy* p. 863 and in the Rudolphine Tables p. 98. From these, and from the tables of these and of others, let the ecliptic limits be trigonometrically corrected and indicated in two ways: one by the Moon's distance from the node; the other, and better, by the Moon's true latitude itself, and that in true full Moons, where the words *In mean oppositions* [In Medijs ☍] will not be added.

Limits of an eclipse of the Moon: authors	Possible: distance from the node	Possible: true latitude of the Moon	Necessary: distance from the node	Necessary: true latitude of the Moon
Ptolemy, in mean oppositions	15° 38′ 0″			
in true oppositions	12° 12′ 0″	1° 8′ 43″	10° 50′ 0″	0° 55′ 38″
al-Battānī, in mean ones	14° 45′ 0″	1° 8′ 51″	10° 17′ 20″	0° 53′ 45″
Alfonso and Peuerbach	12° 31′ 17″	1° 5′ 1″	9° 52′ 0″	0° 51′ 16″
Copernicus	12° 39′ 30″	1° 5′ 41″	10° 26′ 55″	0° 54′ 21″
Reinhold, with the Sun's eccentricity greatest	13° 7′ 0″	1° 7′ 52″	10° 24′ 10″	0° 54′ 12″
Reinhold, with the Sun's eccentricity least	13° 1′ 0″[40]	1° 7′ 35″	10° 23′ 55″	0° 53′ 53″
Tycho, Mulerius	12° 36′ 0″	1° 5′ 0″	11° 12′ 0″	0° 58′ 4″
Magini, in the *Supplement to the Ephemerides*	12° 28′ 0″	1° 4′ 20″	11° 11′ 55″	0° 57′ 47″

[40]the degree figure is written over by hand in this copy; it reads 13 as it stands

Limits of an eclipse of the Moon: authors	Possible: distance from the node	Possible: true latitude of the Moon	Necessary: distance from the node	Necessary: true latitude of the Moon
Longomontanus	12° 24′ 0″	1° 4′ 0″	11° 12′ 0″	0° 58′ 4″
Kepler	12° 0′ 0″	1° 6′ 3″	10° 40′ 0″	0° 58′ 52″
Lansbergen	12° 3′ 0″	1° 4′ 8″	10° 11′ 18″	0° 53′ 2″
Boulliau	12° 25′ 0″	1° 4′ 5″	10° 58′ 40″	0° 56′ 51″
Wendelen	12° 8′ 13″	1° 3′ 0″	10° 11′ 20″	0° 53′ 3″
Ourselves, taking the penumbra into account	12° 50′ 0″	1° 6′ 0″	10° 0′ 0″	0° 52′ 6″

To these Kepler, in the Rudolphine Tables p. 98, adds limits of 11° 54′ with the Moon and Sun at perigee, but of 10° 46′ with the Moon and Sun at apogee. Therefore, in our opinion, taking the semidiameter of the penumbra into account, if the Moon in true full Moons is more than 13° from either of the nodes, an eclipse will not be possible; but if less than 10°, there will necessarily be an eclipse of the Moon. This shows how carelessly Argoli said in his tables, chapter 17: *If the Moon's nodes are not more than 15° distant from the Moon, the Moon will suffer an eclipse.*

PROBLEMA II.

Problem 2.

Magnitudinem Eclipſis Lunæ determinare.

To determine the magnitude of an eclipse of the Moon.

[Margin: How the magnitude of an eclipse is estimated.]

II. The magnitude of a lunar eclipse is the portion of the Moon that is obscured by the penumbra of the atmosphere or, in the opinion of the ancients, by the shadow of the Earth. It is usually expressed either in inches [vncias] or in digits [digitos], of which the Moon's visual diameter contains twelve; as in the figure added here, in which, of the Moon's diameter AF, roughly a third part DF is shown, that is, 4 inches or digits, eclipsed by the shadow or penumbra towards the south.

Diagram. A circle representing the Moon's disc, set into the left column, with a vertical diameter from A at the top to F at the bottom. The diameter carries a series of short cross-ticks. The lowest part of the disc is a solid dark segment with a convex upper edge, the part in shadow; the upper end of the shaded part meets the diameter just below D.

- A (top, outside the circle)
- G (left, outside the circle)
- K (right, outside the circle)
- C (on the right of the diameter, inside the circle)
- D (just below C, at the upper edge of the dark segment)
- F (bottom, outside the circle)

But when the whole Moon is eclipsed with some stay [mora] in darkness, then beyond the 12 digits as many digits are said to be eclipsed as there are between the circumference of the eclipsed disc and that of the eclipsing shadow or penumbra. For if the Moon's true latitude at mid-eclipse is greater than the sum of the visual semidiameters of the Moon and the shadow, there is no eclipse; if equal to it, the periphery of the Moon touches the periphery of the shadow; if less, some part of the Moon is obscured. If the latitude equals exactly the semidiameter of the shadow, exactly half, or 6 digits, are obscured. If the latitude is less than the semidiameter of the shadow, and equal to the remainder of the shadow's semidiameter when the Moon's semidiameter has been subtracted from it, the eclipse is total without stay, and of 12 digits; if less than this remainder, it is total with stay, and of more than 12 digits. For example, if the Moon enters the shadow so deeply that, if its diameter were of 15 parts of the kind of which it now has 12, it would still be wholly obscured, 15 digits are said to be eclipsed. Now to ordinary people either luminary appears a foot across, and so, like a foot, its diameter is divided into 12 inches, which the Greeks call δακτύλους [Greek: daktylous, "fingers", that is, digits], and some indeed call points. The eclipsed digits are determined in two ways and are set out in tables; these are given by Ptolemy, Book 6, chapter 7; Regiomontanus in the *Epitome*, Book 6, proposition 16; Copernicus, Book 4, chapters 31 and 32; Mulerius in the later part of the *Isagoge* to the *Tabulae Frisicae*, chapter 9; Magini in the *Theoricae*, Book 2, chapter 27; Kepler, precept 152 of the Rudolphine Tables.

[Margin: Rule 1, for the digits to be eclipsed.]

The first rule is this. Subtract the Moon's true latitude from the sum of the apparent semidiameters of the Moon and the shadow or penumbra. Multiply the remainder by 6 digits and divide the total so obtained by the Moon's semidiameter; or multiply the remainder by 12 and divide the total by the Moon's apparent diameter. Either way, there will result the digits, and the minutes or sixtieths, of the digits to be eclipsed. But if you wish to work more precisely, use, in place of the latitude, the arc between the centres at mid-eclipse, which I shall discuss in Problem 4 and in Scholia 1 and 2.

[Margin: Rule 2, for the portion of the diameter, and indeed for the area of the eclipsed disc.]

The second rule comes from Copernicus and runs as follows. In this second diagram let ABC be the circle of the shadow or penumbra, whose apparent semidiameter is AE; and let AFC be the Moon's disc, whose apparent semidiameter is AI; both are supposed given. And let the Moon's true latitude EI at the moment of greatest obscuration be given. For triangle AEI will be one of known sides, and so its angles will become known through the analysis of triangles; and when the straight lines EC and IC are drawn to the point of intersection C, triangle ECI will be not only similar but also equal to triangle EAI.

Diagram. Two intersecting circles set into the right column, the upper larger and the lower smaller, with a common vertical line through both centres from B at the top to F at the bottom. The upper circle is lettered in large capitals VMBRA VEL SOL [SHADOW OR SUN], split by the vertical line; the lower LVNA [MOON]. The circles cross at A (left) and C (right), joined by a horizontal chord through K. Straight lines run from E to A and C, and from I to A and C, forming a rhombus with the vertical line as its other diagonal.

- B (top of the upper circle, on the vertical)
- E (on the vertical, centre of the upper circle)
- G (on the vertical, above K, on the upper edge of the overlap)
- K (on the vertical, at the chord AC)
- D (on the vertical, below K, on the lower edge of the overlap)
- I (on the vertical, centre of the lower circle)
- F (bottom of the lower circle, on the vertical)
- A (left point of intersection)
- C (right point of intersection)

Therefore from angle AIC the arc AGC will be known, and from angle AEC the arc ADC, in units of which the whole periphery in each disc is 360 degrees. But since, from what was said in Book 1, chapter 4, the diameter of a circle is to the circumference as 100 to 314, and the semidiameter as 50 to 314, let it be made: as 50 to 314, so the Moon's given visual semidiameter AI to another quantity; and likewise the shadow's semidiameter AE to another. There will result, in the one case AGC, in the other ADC, in the parts in which the apparent semidiameters were known. In this way there will also result what is contained under EA, AD, and under IA, AG, which are the halves of the sectors AEC and AIC, the one of the one and the other of the other.

Moreover, in the isosceles triangles AEC and AIC, from the angle already found at E and at I, and from the semidiameters that form the equal legs, the common base AKC will be known by trigonometry. And if it is made: as the radius to the semidiameter AE or AI, so the second sine of the angle AIG or AED to another quantity, then KE on the one side and KI on the other will be known. Therefore, in the rectangle of AKE, when AK and KE are multiplied together, the parallelogram or area will be known, to which the double of triangle AKE is equal, that is, triangle AEC. Similarly, when AK and KI are multiplied together, the area of triangle AIC will be known. So [recte: ergo; printed ego], when each of these triangles is taken from its own sector, namely triangle AEC from sector AECDA and triangle AIC from sector AICGA, the eclipsed segment AGCDA will remain. If anyone is so curious as to want to know how large a part of the surface of the luminary is eclipsed. For Copernicus says, Book 4, chapter 32: *Many hold that the judgement should be made not by the diameters but by the surfaces, for it is not lines but surfaces that fail*; and before him Ptolemy, Book 6, chapter 7: *Most of those who observe eclipse indications measure the magnitudes of the observations not by the diameters of the circles but by their whole surfaces.* Finally, with the angles AIG and AED known

as above, if it is made: as the radius to the semidiameters of the obscuring and the obscured body, so the versed sine to another quantity, then KG and KD will be known, which taken together make up GD, the eclipsed portion of the diameter.

PROBLEMA III.

Problem 3.

De Duratione Eclipſis Lunæ eiuſq. partibus.

On the duration of an eclipse of the Moon and its parts.

III. The duration of an eclipse is the time from the beginning to the end of the eclipse. To know it, one must know beforehand the minutes of the zodiac that the Moon runs through from beginning to end, and the Moon's hourly motion from the Sun according to the Moon's equated anomaly: for those minutes, divided by the hourly motion, will give the time sought. Further, the astronomers, with Ptolemy, Book 6, chapter 7; Regiomontanus, Book 6 of the *Epitome*, propositions 17 and 18; Copernicus, Book 4, chapter 32; Reinhold on Peuerbach's *Theoricae* p. 199; Wurstisen in the same place, p. 353; Tycho, vol. 1, p. 130; Mulerius in the *Frisicae* p. 430; and Kepler in the Rudolphine Tables, precept 151, divide the minutes of the whole duration and the time corresponding to them: they divide them, I say, into two parts if the eclipse is not total with stay, but into four if it is total with stay; and these are the parts listed below.

[Margin: What are the minutes of incidence?]

The minutes of incidence [Incidentiæ Scrupula], or minutes of falling [minuta Casûs], in Ptolemy ἐξήκοστα τῆς ἐμπτώσεος [recte: ἐμπτώσεως] [Greek: exēkosta tēs emptōseōs, "sixtieths of the incidence"], are the minutes of the zodiac that the Moon, in overtaking the motion of the Sun, passes through from the beginning of the eclipse to mid-eclipse, if the eclipse is not total with stay, or from the beginning of the eclipse to the beginning of total obscuration, if it is total with stay.

[Margin: What are the minutes of repletion?]

The minutes of repletion [Repletionis ſcrupula], or minutes of evasion or of emersion [minuta Euaſionis, ſeu Emerſionis], in Ptolemy ἐξήκοστα τῆς ἀναπληρώσεως [Greek: exēkosta tēs anaplērōseōs, "sixtieths of the refilling"], are the minutes of the zodiac that the Moon, in overtaking the motion of the Sun, runs through from mid-eclipse to the end, if the eclipse is not total with stay; or from the beginning of its appearance out of the shadow, if it is total with stay.

[Margin: What are the minutes of half-stay?]

The minutes of half-stay [Moræ dimidiæ ſcrupula], in Ptolemy ἐξήκοστα τῆς ἡμίσεως μονῆς [Greek: exēkosta tēs hēmiseōs monēs, "sixtieths of the half stay"], are the minutes of the zodiac that the Moon, overtaking the Sun in motion,

runs through from the beginning of total obscuration to the middle or height of the eclipse; and equal to them are the minutes that it passes through from the middle to the beginning of its new appearance. The time corresponding to these is called the time of incidence, of repletion, and of half-stay.

For greater clarity, let the shadow BCESD be described about the centre A, with the ecliptic PR; and for a partial eclipse let the Moon's path be FGH, on which the centre of the Moon at the beginning of the eclipse is at F, and the contact with the shadow at C; at mid-eclipse it is at G, and at the end at H, with the contact with the shadow at B. Through these points let the straight lines ACF and ABH be drawn, though they are equivalent to arcs; and let AG be perpendicular to the Moon's orbit, which is called the arc between the centres. For the minutes of incidence will be the arc FG, and the minutes of repletion GH. But for a total eclipse with stay, let the Moon's path be KLQ, and the centre of the Moon at the beginning of the eclipse at K, with contact at I; at the beginning of total obscuration let it be at N, with internal contact at E; at mid-eclipse let it be at L; at the beginning of emersion at V, with interior contact at D; and finally at the end of the eclipse let it be at M [recte: Q], with external contact at O. And let the centres of the Moon K, N, L, V, Q be joined by straight lines to the centre of the shadow A, and let the arc between the centres be AL, perpendicular to the Moon's path. For the minutes of incidence will be the arc KN; those of half-stay NL for the earlier and LV for the later; and those of repletion VQ; which we shall presently teach how to find.

Diagram. A large solid dark disc, the shadow, stands in the lower part of the left column, with straight lines radiating from its centre A. Eight small discs of the Moon, each engraved with a human face and lettered at its centre, lie on two paths across it. Along the upper path, one at the top within the shadow's upper edge and two outside the shadow to left and right. Along the lower path, five in a row, overlapping one another and the shadow's lower half. A horizontal line through A carries P on the left and R on the right; a vertical line through A carries M at the top and S at the bottom.

- M (top, outside the shadow, above the upper central Moon)
- G (at the centre of the upper central Moon)
- H (at the centre of the upper left Moon, outside the shadow)
- F (at the centre of the upper right Moon, outside the shadow)
- B (on the shadow's edge, at the upper left Moon)
- C (on the shadow's edge, at the upper right Moon)
- A (centre of the shadow)
- P (left end of the horizontal line)
- R (right end of the horizontal line)
- Q (at the centre of the lower left Moon)
- O (on the shadow's edge, at the lower left Moon)
- D (on the shadow's edge, below O)
- V (at the centre of the second lower Moon)
- L (at the centre of the middle lower Moon)

- N (at the centre of the fourth lower Moon)
- K (at the centre of the lower right Moon)
- I (on the shadow's edge, at the lower right Moon)
- E (on the shadow's edge, below I)
- S (bottom, below the shadow)

PROBLEMA IV.

Problem 4.

Arcum inter Vmbræ terreſtris & Lunæ centra in ſummo ſeu medio Eclipſis inuenire.

To find the arc between the centres of the Earth's shadow and the Moon at the height or middle of the eclipse.

IV. In the diagram attached, let HAB be the ecliptic, in which A is the centre of the Earth's shadow KPN, and B the nearer node; and let BG be the Moon's path, on which the Moon's centre at the beginning of the eclipse is at C, at the end at G, and at mid-eclipse at I.

Diagram. A circle, the shadow, set in the right column, with centre A. The straight line HAB runs from the left edge through the centre and out to B far to the right. A second straight line, the Moon's path, runs from B down to the left across the lower part of the circle to G. Two small circles, the Moon, are centred on this path: one at C on the right near the shadow's edge, and one at G on the lower left. Lines run from A to C and from A to G, and from A down to L and to I on the path.

- P (upper part of the circle)
- A (centre, lettered just above the line HAB)
- H (left end of the line, outside the circle)
- B (right end of the line)
- N (on the right edge of the circle, at the line AC)
- C (in the small right circle)
- K (on the lower left edge of the circle, at the line AG)
- G (in the small left circle)
- L (on the path, below A)
- I (on the path, beside L)

Let the points of the lunar centres be joined to the centre of the shadow by the lines AC and AG, which are the sum of the semidiameters. Let AI be perpendicular to the Moon's orbit: this is called the arc between the centres [Arcus inter centra], and is the shortest distance of the lunar centre from the centre of the shadow. And finally let LA be perpendicular to the ecliptic, or the Moon's latitude given for the moment of full Moon, which occurs when the Moon is at L. For LI will be the arc by which BL, the distance of the Moon from the node, differs from AB, the distance of the centre of the shadow from the same node B. Therefore, as Kepler observes in the *Epitome of Astronomy* p.

864, and assumes in the Rudolphine Tables, chapter 31, precept 146, the arc LI will be equal to the arc of the reduction of the Moon to the ecliptic, which is known from the Moon's argument of latitude, either from its own small table based on the obliquity of the lunar orbit, which for most writers is 5° in eclipses, for Tycho 4° 58½′, and for Kepler 5° 18′; or from the solution of the triangle, discussed in Book 4, last chapter. With this laid down, there are three ways of finding the arc AI.

[Margin: First way of finding the arc between the centres: Kepler's.]

The first is Kepler's, in the *Epitome of Astronomy* p. 864. Subtract the arc LI of the reduction from the Moon's argument of latitude itself, or from its distance from the nearer node, if the Moon's latitude is northern ascending or southern descending; or add it, if the latitude is southern ascending or northern descending. Then IB will be known, a side of the triangle ABI, right-angled at I, in which the base AB is likewise supposed known for the moment of full Moon, namely the distance of the centre of the shadow from the same node B, or the angle B at which the Moon's orbit is inclined to the ecliptic. So the arc AI cannot remain hidden.

[Margin: Way 2: ours.]

The second way is ours, through the triangle ALI, right-angled at I, in which are given the base AL, which is the Moon's true latitude at opposition, and LI, the reduction of the Moon to the ecliptic; so the arc AI will easily be found.

[Margin: Shortcut for both ways.]

Briefly, in either way, let it be made: as the radius to the second sine of the given side, namely either IB or, in my way, LI, so the secant of the base AB or, in my way, AL, to the secant of the arc sought, AI. Or to the second logarithm of the given side add the residual of the second logarithm of the given base, and the result will be the second logarithm of the arc AI.

[Margin: Way 3: Kepler's and ours.]

The third way comes from Kepler in the Rudolphine Tables, chapter 31, precept 146, and his tables p. 98. Subtract the place of the nearer node from the place of the centre of the shadow, or the latter from the former if it comes first. With the degrees of the remainder enter the margin of the already mentioned table, under the minutes belonging to those degrees, and in the body of the table you will find, in place of the latitude, the arc between the centres. For Kepler holds that this differs insensibly from the Moon's latitude at the height of the eclipse, as I shall explain in Scholium 2. Or consult our table of the Moon's latitude for ecliptic conjunctions and oppositions; for there you have the arc between the centres.

PROBLEMA V.

Problem 5.

Inueſtigare ſcrupula Incidentiæ, Repletionis, Moræ dimidiæ, & Differentiæ inter momentum Plenilunij, & momentum ſummæ Eclipſis.

To investigate the minutes of incidence, of repletion and of half-stay, and those of the difference between the moment of full Moon and the moment of the height of the eclipse.

V. Small tables by practitioners do exist for this purpose. They are given by Ptolemy, Book 6, chapter 8; Alfonso, folio 116; Peuerbach, the penultimate table of eclipses; Reinhold in the Prutenic Tables p. 125; Magini in the *Secunda Mobilia* p. 687; and Lansbergen p. 40 and Renieri p. 349 of their tables. But they rest on the Moon's latitude at the moment of full Moon, used in place of the arc between the centres, and on the solution of a rectilinear triangle in place of a spherical one.

[Margin: On incidence in an eclipse of the Moon without stay.]

To solve the problem more exactly, then, repeat here the diagram of Problem 4, and in the triangle AIC, right-angled at I, in which are given the arc AI, found by Problem 4, and the base AC, consisting of the apparent semidiameter AN of the shadow and NC of the Moon, find the side CI, namely the minutes of incidence in an eclipse without stay. Equal to these are the minutes of repletion IG, because the triangle GAC, from the equality of the sums of the semidiameters AC and AG, is isosceles, and the perpendicular arc AI bisects the base GC, just as the chord of the arc AI bisects the chord of the arc GC, by 10 of the first book of Euclid.

[Margin: On incidence in an eclipse of the Moon with stay.]

But if the eclipse is with stay, repeat here the diagram of Problem 3, and in the triangle ALK, right-angled at L, from the arc AL, found by Problem 4, and the sum of the semidiameters AK, find the arc LK, namely the minutes of half-duration, to which the minutes LQ are equal. Then in the triangle ALN, right-angled at L, with the arc AL and with the base AN, which is the excess of the shadow's semidiameter over the Moon's semidiameter, find the minutes of half-stay LN, to which the minutes LV are equal; and when these are subtracted from LK, there will remain, in an eclipse with stay, the minutes of incidence alone, NK, which are equal to the repletion VQ.

Since, then, in all these triangles the base and one side are given, proceed as I said of the triangles of Problem 4. Or, in the manner of the old astronomers, treat them as rectilinear and, by 47 of the first book of Euclid, take the square number of the given side from the square of the given base; for a square will be left, whose square root will be the other side sought.

[Margin: Our quickest way.]

Or if you want to work more quickly, use the small tables, in the manner of the old astronomy, listed at the start of this Problem; but to the minutes of incidence found through them add the reduction of the Moon to the ecliptic, or subtract it, when it is to be added to or subtracted from the Moon. In a

total eclipse with stay, add or subtract that same reduction both to or from the minutes of half-duration and to or from the minutes of half-stay. The reduction itself is the difference between the opposition and mid-eclipse. Finally, multiply the already mentioned minutes by 3600″ of time, and divide the total by the Moon's hourly motion from the Sun, first reduced to seconds, and you will have the time corresponding to them.

PROBLEMA VI.

Problem 6.

De Horario Motu Lunæ à Sole.

On the hourly motion of the Moon from the Sun.

[Margin: On the hourly motion of the Moon from the Sun.]

VI. The Moon's hourly motion from the Sun becomes known by subtracting the Sun's true hourly motion from the Moon's true hourly motion in the zodiac. Both depend on the equation of the argument or anomaly, for the motion is neither perpetually equal nor always the same. For us, from what was said in Book 3, chapter 17, the Sun's mean hourly motion is 2′ 27″ 50‴ 50⁗, and the greatest equation of the solar motion, from Book 3, chapters 21 and 24, is 1° 59′. The Moon's mean hourly motion, from Book 4, chapter 23, is 32′ 56″ 27‴ 33⁗ 15ᵛ 40ᵛⁱ; and so the Moon's mean hourly motion from the Sun is 30′ 28″ 36‴ 43⁗ 15ᵛ 40ᵛⁱ. For others it is slightly different, from what was said in the same place; but there is no need of such subtlety, and it is enough to stop at seconds or thirds. There are, then, three ways in which the hourly motion of each heavenly body is obtained.

[Margin: Way 1 of finding the hourly motion.]

The first way is through the true motion of the Sun and Moon for the given day and for the day immediately before or after. For the difference of the true daily motion, whether it is collected directly from astronomical tables or from an accurate ephemeris, if divided by 24, will give the hourly motion sought. In place of division you can use the table that Reinhold made in the Prutenic Tables, p. 95, the use of which he teaches in precepts 19 and 31.

[Margin: Way 2 of finding the hourly motion.]

The second way, by which tables of the hourly motion are constructed, is this. Find the Sun's equation for the given time, and the equation belonging to a solar anomaly differing by one degree next to it, and take their difference. Of this take the proportional part according to the ratio of the mean motion 2′ 28″ to 60′. Subtract this proportional part from the Sun's mean hourly motion 2′ 28″ if the Sun's anomaly is less than 92° or 93°, at which the equation is greatest; but add it if the anomaly is greater than 93°, up to 183°; and you will have the Sun's true hourly motion. In this way you will construct a table of the solar hourly motion, such as Kepler has in the Rudolphine Tables p. 92. For the Moon, take the equation of the simple anomaly for the given time,

and the equation belonging to an anomaly next greater by one degree; and of their difference take the proportional part according to the ratio 41′ 49″ to 60′. Subtract this from the Moon's mean hourly motion 32′ 56″ if the anomaly is less than 95°; add it if greater, up to 180°; and thus you will construct a table of the Moon's hourly motion. When the Sun's hourly motion is subtracted from this, you will have the Moon's hourly motion from the Sun at syzygies. But if you use the difference of the equation composed from the first and second inequality, you will have the hourly motion outside the time of syzygies, the method by which tables of the Moon's hourly motion from the Sun are usually made for half a day before or after syzygies, so that it may serve for reducing mean syzygies to true or quasi-true ones.

[Margin: What gave the Tychonics the occasion of the Moon's Variation.]

Kepler notes in the Rudolphine Tables, chapter 31, precept 147, from Longomontanus's *Astronomia Danica* p. 173, that the occasion for detecting the lunar Variation was the acceleration observed at syzygies, and that the variation was cut off from the Moon's stay in the shadow. Further, the reason for the precept handed down by Regiomontanus is that the Sun's anomaly increases in one hour by 2′ 28″, and up to 93° of anomaly the equations grow, and from there they decrease; in the Moon's case, however, it is that one hour before or after mean syzygy the true anomaly differs from the mean by the hourly motion of the mean anomaly and by the equation of the centre, to which 41′ 49″ correspond.

[Margin: Way 3 of finding the hourly motion of the Moon and Sun.]

The third way is given by al-Battānī, chapters 31 and 43; Regiomontanus, Book 5 of the *Epitome*, proposition 21; Reinhold on Peuerbach p. 206; Magini, Book 2 of the *Theoricae*, chapter 25; and Barozzi, Book 4 of the *Cosmographia*, p. 341. For they say that the true hourly motion of the Sun is to the Sun's apparent diameter as 5 to 66, that is, in the subtredecuple sesquififth ratio [ratione ſubtredecupla ſeſquiquinta]. The Moon's hourly motion, moreover, is to the Moon's visual diameter as 48 to 47; or, on Copernicus's hypotheses, at apogee as roughly 29 to 30, but at perigee as roughly 37 to 35, or more accurately as 2255 to 2138.

[Margin: How to obtain the Moon's greatest and least hourly motion.]

Further, to have the least hourly motion of the Moon from the Sun, subtract the hourly motion of the Sun at perigee from the hourly motion of the Moon at apogee; but for the greatest, subtract the Sun at apogee from the Moon at perigee.

[Margin: Kepler's variation.]

Yet Kepler, in the *Epitome of Astronomy* p. 868, reports both his own and Tycho's opinion on the Moon's true hourly motion somewhat differently from the Rudolphine Tables, and differently from Tycho himself. For he says that the Moon's least hourly motion at apogee is for Tycho 29′ 46″, for himself 29′ 56″; the greatest at perigee, for Tycho 38′ 30″, for himself 38′ 43″. When the Sun's mean hourly motion is taken from these, there remains the hourly

motion of the Moon from the Sun: the least, for Tycho 27′ 18″, for himself 27′ 28″; but the greatest, for Tycho 36′ 2″, for himself 36′ 15″, because of the Variation, which is greater for him than for Tycho. Yet he concedes that it is better to use the hourly motion which holds at any given time. See also what the same Kepler teaches in our Book 4, chapter 19, Scholium 4. But Tycho, vol. 1, and following him Longomontanus, Mulerius and Magini, put the Moon's hourly motion at apogee as 27′ 43″, at perigee as 33′ 24″, at syzygies; but with the Sun placed at its mean distance, and its hourly motion as 2′ 28″. When this is added to the Moon's motion, the Moon's true hourly motion is obtained: in the former case 30′ 11″, in the latter 35′ 52″, as we have set down in the small table that follows.

[Margin: Authors of tables of the hourly motion of the Sun and Moon.]

If anyone wants tables of the hourly motion for mean and true syzygies, they are given, for Ptolemy, Copernicus and Tycho, by Mulerius in the *Tabulae Frisicae* pp. 128, 176, 306, 432; by Peuerbach in the Tables of Eclipses; Reinhold in the Canon, pp. 95 and 97; Tycho, vol. 1, p. 128; Longomontanus, Book 1 of the *Theorica*, chapter 9, p. 301; Magini in the *Secundum Mobile*, vol. 2, p. 673, and in the *Supplement to the Ephemerides* p. 290; Ke- pler in the *Epitome*, p. 868, and in the Rudolphine Tables, pp. 92 and 98; Lansbergen in the Tables, p. 36; Wendelen in the *Idea of the Atlantic Tables*, pp. 33 and 34; and Boulliau in the *Philolaic Tables*, p. 138. Among these, Peuerbach, Reinhold and Magini in his old Canon set out the true motion for a number of hours and days, the last adding the time for verifying the syzygies that corresponds to the hourly motion of the Moon from the Sun. Now let the true hourly motion of the luminaries, least at apogee and greatest at perigee, be taken from the authors just cited.

The true or apparent hourly motion of the luminaries; but of the Moon in conjunction and opposition.

Authors	Sun at apogee ′	″	‴	Sun at perigee ′	″	‴	Moon at apogee ′	″	‴	Moon at perigee ′	″	‴	Moon at apogee from Sun at perigee ′	″	‴	Moon at perigee from Sun at apogee ′	″	‴
Ptolemy	2	22	28	2	33	50	30	50	30	37	21	10	28	16	40	34	58	42
Alfonso and Peuerbach	2	23	0	2	34	0	30	0	0	36	0	0	27	26	0	33	17	0
Copernicus and Reinhold, with the Sun's eccentricity greatest	2	21	55	2	34	18	29	6	5	37	29	32	26	31	47	35	7	37
mean	2	22	32	2	33	5	29	6	5	37	29	32	26	33	0	35	7	0

Authors	Sun at apogee ′	″	‴	Sun at perigee ′	″	‴	Moon at apogee ′	″	‴	Moon at perigee ′	″	‴	Moon at apogee from Sun at perigee ′	″	‴	Moon at perigee from Sun at apogee ′	″	‴
least	2	23	15	2	32	45	29	6	5	37	29	32	26	32	35	35	6	17
Magini, formerly	2	22	20	2	33	30	29	2	0	37	35	0	26	28	30	35	12	40
Tycho and Longomontanus	2	23	0	2	33	0	30	11	0	35	52	0	27	38	0	33	29	0
Kepler in the Rudolphine Tables	2	23	0	2	33	0	29	44	0	38	28	0	27	11	0	36	5	0
Lansbergen	2	23	0	2	33	0	29	48	0	36	42	0	27	15	0	34	18	0
Wendelen	2	23	0	2	33	0	29	30	0	38	8	0	27	38	0	33	30	0
Boulliau	2	23	0	2	33	0	29	40	0	38	10	0	27	7	0	35	47	0
Ourselves	2	23	0	2	34	0	29	20	0	35	30	0	26	46	0	33	7	0

PROBLEMA VII.

Problem 7.

De Maxima Duratione Lunaris Eclipſis & Maxima Mora in Totali Eclipſi.

On the greatest duration of an eclipse of the Moon and the greatest stay in a total eclipse.

[Margin: Method of finding the greatest duration and the stay of an eclipse of the Moon.]

VII. Once the hourly motion of the Moon from the Sun is known, it is easy to determine the longest duration of an eclipse of the Moon, and also the longest stay in the shadow or penumbra. For, as Kepler teaches in the *Epitome of Astronomy*, p. 869, the sum of the apparent diameters of the Moon and of the shadow or penumbra, divided by the true hourly motion of the Moon from the Sun, will give the hours and minutes of the greatest duration. Again, the difference of those same diameters, divided by the true hourly motion of the Moon from the Sun, will give the hours and minutes of the greatest stay in the shadow or penumbra. These occur when the Moon at mid-eclipse is at the node, for then the Moon's path differs imperceptibly from the arc of the ecliptic. And so, in the figure placed at no. 3, the eclipse begins at R and ends at P, and from the beginning of the total eclipse to its end the Moon traverses the diameter RAP and, besides that, as much space as the Moon's own diameter.

[Margin: The longest duration of an eclipse of the Moon, and the stay in darkness.]

It is enough for us at this point to gather into the following small table the view of Kepler on this matter, and from him that of Tycho, from p. 869 of Kepler's own *Epitome of Astronomy*, so that it may be clear that the duration is greatest when both luminaries are at apogee. For, as Kepler notes in the *Epitome of Astronomy*, p. 868, although the Earth's shadow is thicker at the place of the Moon at perigee, the Moon is faster there, and the hourly motion stands in a greater ratio than the traverse at perigee does to the traverse at apogee. Later we shall add a second small table, constructed by ourselves.

The greatest duration of an eclipse of the Moon, from Kepler's Epitome.

Luminaries: Sun	Moon	By Tycho's values	By Kepler's values
at apogee	at apogee	$4^h\ 20^m\ 4^s$	$4^h\ 18^m\ 18^s$
at apogee	at perigee	$3^h\ 39^m\ 22^s$	$3^h\ 38^m\ 46^s$
at perigee	at apogee	$4^h\ 19^m\ 40^s$	$4^h\ 18^m\ 6^s$
at perigee	at perigee	$3^h\ 38^m\ 40^s$	$3^h\ 37^m\ 26^s$

The longest stay of an eclipse of the Moon in the Earth's penumbra.

Luminaries: Sun	Moon	By Tycho's values	By Kepler's values
at apogee	at apogee	$2^h\ 8^m\ 36^s$	$2^h\ 7^m\ 58^s$
at apogee	at perigee	$1^h\ 50^m\ 38^s$	$1^h\ 50^m\ 46^s$
at perigee	at apogee	$2^h\ 7^m\ 20^s$	$2^h\ 6^m\ 38^s$
at perigee	at perigee	$1^h\ 49^m\ 24^s$	$1^h\ 48^m\ 48^s$

The greatest duration of an eclipse of the Moon, with the Sun and Moon at apogee, by our own computation and from the data of the authors named.

From the data of	Duration
Ptolemy	$4^h\ 6^m\ 42^s$
Copernicus	$4^h\ 8^m\ 8^s$
Tycho	$4^h\ 17^m\ 37^s$
Longomontanus	$4^h\ 15^m\ 0^s$
Kepler	$4^h\ 22^m\ 30^s$
Lansbergen	$3^h\ 57^m\ 40^s$
Boulliau	$4^h\ 7^m\ 45^s$
Wendelen	$4^h\ 13^m\ 0^s$
Ourselves	$3^h\ 57^m\ 6^s$

For physical reasons, however, a varying refraction of the Sun's rays can make the thickness of the penumbra vary, and with it the duration and the stay of an eclipse of the Moon.

SCHOLIA.

Scholia.

[Margin: The common error about the midpoint of an eclipse and the parts of its duration.]

I. It is commonly believed that the moment of full Moon and the moment of greatest obscuration are the same, and therefore that the Moon's latitude is the same at both moments, and that there is as much space from the beginning of the eclipse to the moment of full Moon as from the full Moon to the end of the eclipse. But if one is to proceed by strict astronomical right, these three things differ from one another, as is affirmed by Ptolemy, Book 6, chapter 17; Regiomontanus in the *Epitome*, Book 6, Prop. 15; Kepler in the Rudolphine Tables, chapter 31, Precept 145; Wendelen in the *Idea of the Atlantic Tables*, p. 38; and Boulliau, Book 4 of the *Astronomia Philolaica*, chapter 7, and in the Tables, p. 138. For the greatest obscuration occurs at the moment at which a perpendicular to the Moon's path can be drawn from the centre of the shadow or penumbra to the Moon's centre: this is the shortest line, by which the Moon's centre comes nearest to the centre of the shadow, unless the full Moon takes place at the nodes. But the moment of full Moon is commonly taken to be that at which a perpendicular can be drawn from the Moon's centre to the place on the ecliptic diametrically opposite the Sun, just as the moment of new Moon is when a perpendicular can be drawn from the Moon's centre to the Sun's centre on the ecliptic. Thus in the diagram of Problem 4 set out there, if two perpendiculars are drawn, one from the centre A to the Moon's centre at I, the other from the Moon's centre placed at L to the point A on the ecliptic, AI will certainly be shorter than AL, because in triangle ALI, right-angled at I, the right angle LIA subtends a greater side, AL, than the side AI, which is subtended by the acute angle ALI, by 19 of Book 1 of Euclid. Indeed, when the straight lines ANC and AKG are drawn, at the time when the Moon is there at the beginning and here at the end of the eclipse, no line shorter than AI can be drawn from A to GG [recte: GC] in triangle GAC. In this diagram, therefore, the moment of greatest obscuration, which occurs at I, precedes the moment of full Moon, which occurs at L, by the whole arc or side LI; namely, because the Moon has passed the node B.

Diagram. A geometric figure at the head of the left column. A large circle has its centre at A, with a straight line running from H on its left edge through A and out beyond the circle to B on the right. A second straight line runs from B down to the left, crossing the right edge of the large circle and its lower part. Two small circles, one at each end of this chord, lie on the edge of the large circle: the right-hand one with centre C, the lower-left one with centre G. Straight lines run from A to the chord through N towards C, through K towards G, and two short lines from A down to the chord at L and at I.

- P (upper part of the large circle, above A): label
- A (centre of the large circle): label

- H (left end of the horizontal line, on the circle): label
- B (far right, where the two straight lines meet): label
- N (on the line from A towards C, at the right edge of the large circle): label
- C (inside the right-hand small circle): label
- K (on the line from A towards G, at the lower-left edge of the large circle): label
- G (inside the lower-left small circle): label
- L, I (side by side on the chord below A, L to the left): labels

But if the Moon is approaching the node, the moment of greatest obscuration follows the moment of full Moon. Hence it comes about that, although the space IC, from the beginning of the eclipse to the height of the eclipse, is equal to the space IG, from the height of the eclipse to the end, because the perpendicular AI bisects the base GC of the isosceles triangle GAC, by 10 of Book 1 of Euclid, yet the space CL, from the beginning of the eclipse to a full Moon occurring outside the nodes, is longer than the space LG, from the full Moon to the end of the eclipse, when the Moon is receding from the nodes, that is, has a northern ascending or southern descending latitude; but conversely the former is shorter than the latter when the Moon is approaching the node.

[Margin: Whether the spaces between the beginning and end of an eclipse are equal?]

Finally, the Moon's latitude is less at the moment of greatest obscuration than at the moment of full Moon, and the arc between the centres, that is, the perpendicular determining the greatest obscuration, is greater than the latitude at the height of the eclipse but less than the latitude at full Moon. In the figure above, the arc between the centres is AI. But in the figure placed and explained already in chapter 4, problem 1, the arc between the centres is BM, the latitude belonging to the greatest obscuration is ME, and the latitude at the beginning of the eclipse is PI.

[Margin: What is the arc between the centres?]

[Margin: Whether the differences just mentioned should be passed over.]

II. Now Ptolemy, and Regiomontanus subscribing to him, say that it is indeed shameful for an astronomer not to know these differences and subtleties, but that they deliberately omitted them in the calculation of eclipses, because they are differences in some measure imperceptible to the appearance of the senses; and that if anyone seeks exact knowledge of this matter, the difficulty will distress him far more than the usefulness will delight him. But the more recent astronomers, Kepler, Wendelen and Boulliau, thought this difference not to be despised, since it can reach about 6^{m} of time. So in triangle ABL of the diagram above, right-angled at A, the latitude AL at the moment of full Moon is supposed given, together with the true distance of the centre of the shadow A from the node B; therefore the angle ALB, that is ALI, can be found. With this angle and with the latitude AL, in the small triangle ALI, right-angled at I, the side LI is found. Divided by the Moon's hourly motion from the Sun,

LI will give the time to be subtracted from the moment of full Moon if the Moon is receding from the nearer node, and to be added if it is approaching, so as to have the moment of greatest obscuration, which bisects the duration of the eclipse and the stay in the shadow. For that moment compute the Moon's place and reduce it to the ecliptic: for the difference between the place M in the orbit and the place E on the ecliptic will be BE, in the diagram of chapter 4, no. 2. With that difference and with the arc between the centres, BM, you will compute in triangle BME, right-angled at E, the Moon's true latitude belonging to the moment of greatest obscuration.

We do not need this latitude, however; it is enough to compute the arc between the centres, namely AI, in the former diagram, by means of the latitude AL and the arc LI, which is equal to the reduction of the Moon to the ecliptic; or with AB, the distance of the centre of the shadow from the node, and IB, the argument of latitude, which differs from the distance AB by the arc IL of the reduction of the Moon to the ecliptic, as Kepler notes in the *Epitome*, p. 863. Astronomers, moreover, use straight lines and chords of arcs in place of arcs, because in these, given their smallness, there is no danger of sensible error, as Ptolemy, Book 6, chapter 17, Copernicus, Book 4, chapter 32, and Kepler in the *Optics*, p. 418, point out; though we could also use arcs.

[Margin: Kepler's and Boulliau's tables for these things.]

For the arc between the centres and the latitude at the greatest obscuration, however, you can use Kepler's table, of which he speaks in the Rudolphine Tables, Precept 45, or our own table made for the purpose; and for the difference of the moment of full Moon from the moment of greatest obscuration, use Boulliau's small table, which he has on p. 138 of his Tables, or another like it in our own Tables, or the reduction of the Moon to the ecliptic converted into the time of the Moon from the Sun.

III. Nor is this the end of the subtleties. For Tycho, in a most learned letter to Rothmann, dated 1587 January 20, p. 72, relates that in 1584 on November 7, Old Style, he observed the beginning of an eclipse of the Moon, by the equatorial distance of stars from the meridian, at $11^h\ 12^m$ after noon at Uraniborg; the middle at $13^h\ 8^m$ and the end at $15^h\ 0^m$, so that the earlier part of the duration was longer than the later by 4^m of time. And he adds in parenthesis: *For these times are not always equal, as has been believed until now.*

[Margin: Whether the parts of the duration of an eclipse, computed from the moment of greatest obscuration, are equal.]

Yet Kepler, Wendelen and Boulliau passed over this inequality. Indeed Kepler says in the Rudolphine Tables, chapter 31, Precept 151: *There will remain the time of incidence or of emersion, without distinction: for they are equal, because we are working by the moment of greatest obscuration.* So, reckoning not from the moment of true full Moon but from the instant [punctiſtitio] of greatest obscuration, he counts the time of incidence equal to the time of repletion.

[Margin: Suspicion about Tycho's credit.]

Now as for Tycho's observation, Wendelen in the *Eclipses*, p. 48, charges it with being of slight credit. And why? Because the same Tycho, drawing it to his own tables in the catalogue, puts the middle at $13^h\ 12^m$; and because Wendelen, making the two times on either side of the middle equal, says that the middle was at $13^h\ 6^m$, since the duration was $3^h\ 48^m$.

But whatever may now be said of Tycho's credit, it must first be admitted that the moment of greatest obscuration cannot be observed exactly, because at the height of a partial eclipse the Moon lingers for some time; and in a total eclipse, such as that one of Tycho's was, the middle is not observed at all. In both, therefore, we are forced to infer the middle: in a partial eclipse from its beginning and end, by bisecting the time elapsed; but when the eclipse is total, the time of the stay is usually divided in two, and half of it shows the middle of the eclipse. We can, however, observe the time of incidence and of emersion, and it is not seldom found to be unequal. And in truth the times of these are not always equal, nor is the time from the beginning of total obscuration to the greatest obscuration equal to the time from there to the beginning of emersion. For neither the motion of the Earth's shadow, whose axis does not stay in the same place, nor the motion of the Moon is equal throughout the time of the eclipse (granted that the space of the ecliptic or of the Moon's orbit from the beginning to the height of the eclipse is equal to the space from the height to the end); rather, the motion becomes faster as the Moon tends towards perigee, and slower towards apogee. And although we use the hourly motion of the Moon from the Sun sought for the full Moon or for the point of greatest obscuration, and it seems to be the mean between the faster and the slower motion that belongs to the Moon from the beginning or at the end, it is nevertheless not always so. So, if every subtlety is to be taken into account, it is better to compute for the beginning and end of the eclipse, determined in the usual way, the most exact anomaly, and from it the hourly motion and the difference of motion, and by that difference to correct the moment of mid-eclipse and to separate exactly the time of incidence from the time of emersion. But because the difference is very small, and because, if it seems greater in observations, the excess is put down to the fallibility of observations, astronomers everywhere usually disregard it.

[Margin: On the Moon's stay in the sheer shadow.]

IV. From what was said in chapter 5, at the end of no. 10, the Moon's stay in the sheer shadow of the atmosphere could have been separated out, apart from the penumbra that makes the beginning and end of the eclipse on either side, and the axis of the cone TNV described there could have been brought to a measure. But because that measure varies with the inconstancy of refractions, and the moment at which the Moon begins to enter that inverted cone or to leave it is not easily observable, we have not wished to add this new labour to the calculation of eclipses or to draw up new small tables for that part of the eclipse.

V. Whatever was said in chapter 5 and in its Scholia 2 and 3 about the umbrago, about the arched clouds, and about the variation of the penumbra from various refractions should be recalled here, because these things can vary the duration and the beginning and end of the eclipse for physical and optical causes; as can any causes that vary the Moon's apparent diameter by accident.

[Margin: Whether the duration of an eclipse of the Moon is varied by a lunar air or by the Moon's refractions.]

Among these causes David Fabricius, as reported by Kepler in the *Ephemerides*, pp. 9 and 22, and Wendelen in the *Eclipses*, p. 5, posit a circumlunar air, on account of which the Moon's diameter is dilated up to 4′ or 5′, and the Moon appears larger, and is therefore touched by the shadow sooner and emerges from it later. But this does not please Kepler, there and in the *Optics*, p. 286; nor is that air so evident, or so dense or thick for this effect, as the increase of the atmospheric penumbra above the shadow computed in the old way is evident. It is to this increase, then, rather that the acceleration of the beginning and the greater duration of the eclipse are attributed. The refraction of the lunar rays, by contrast, on account of which the Moon looks larger near the horizon, does not change the beginning or end of the eclipse, though it may visibly raise the visible points of the lunar disc and make the uneclipsed digits larger; on this matter see what I reported from Longomontanus in Book 4, chapter 16, no. 8. Finally, remember the small equation to be applied in equinoctial eclipses, if indeed what

Kepler says in the Rudolphine Tables, Precept 171, is true: that the Moon goes under the Sun or the shadow later around the point of the vernal equinox, and earlier around that of the autumnal one. On this equation consult him, and ourselves in the precepts of the Tables.

[Margin: The optical enlargement of the Moon in sight has no laws.]

He also rightly points out, in the *Astronomia Optica*, p. 285, that in ecliptic full Moons the Moon appears larger next to the Earth's shadow, because bright things next to dark ones appear larger, and that thus the Moon's edges are not thinned but enlarged by an optical deception; yet the astronomical tables are not to be changed on that account, since this cause is inconstant and varies with the variety of eyes.

CAPVT VII.

Chapter 7.

De Typo Eclipſis Lunaris graphicè oculis ſubijciendo; eiuſq. plagis ac flexu tenebrarum, ſeu Inclinationibus.

On setting the delineation [Typus] of an eclipse of the Moon graphically before the eyes; and on its quarters and the bending of the darkness, that is, its inclinations.

[Margin: Authors who treat this technique.]

I. This technique too was handed down to us by Ptolemy, Book 6, chapter 13; al-Battānī, chapter 43; Peuerbach, proposition 17 of *On Eclipses*; Peter Apian in the *Astronomicum Caesareum*, statement 27; Reinhold in Peuerbach's *Theoricae*, p. 203, and in the Prutenic Tables, Precept 65; Magini in the *Secunda Mobilia*, Canon 74; Tycho, vol. 1, p. 131; Longomontanus in the *Astronomia Danica*, p. 309; and Origanus, vol. 1, p. 267. Most of them set out such delineations on their pages so that the north is in the upper part and the south in the lower; Regiomontanus, however, has the opposite in the diagram and notes to al-Battānī, chapter 43, as you see in the following diagram, in which the east is at I and the west at D, and the ecliptic is IFD, which the line AC cuts at right angles at F.

Diagram. A geometric figure in the left column. A large circle AICD has its centre at F; a vertical line runs from A at the top through F to C at the foot, and a nearly horizontal line from I on the left through F to D on the right. Inside it and concentric with it stands a second circle, on which G and S lie. A small circle stands on the vertical line above F, with L at its top, E at its middle and H at its foot. Two further small circles straddle the large circle: one upper left with centre M, one right with centre K. Straight lines run from M to L; from M through E to K; from H to K; from M through F to T on the large circle at lower right; and from K through F to Q on the large circle at lower left.

- Meridies (above A, top): "South"
- A (top of the large circle): label
- Septent. (below C, foot): "North"
- C (foot of the large circle): label
- I (left of the large circle): label
- D (right, on the large circle): label
- F (centre, below the crossing of the lines): label
- M (inside the upper-left small circle): label
- G (on the inner circle, below the upper-left small circle): label
- K (inside the right-hand small circle): label
- S (on the inner circle, left of the right-hand small circle): label
- L, E, H (on the vertical, in the small circle above F, top to foot): labels
- Q (lower left, outside the large circle): label
- T (lower right, outside the large circle): label

The apparent semidiameters of the Moon and of the shadow or penumbra must be known beforehand, as well as the Moon's true latitude at the beginning and end and at the middle of the eclipse, with the kind or quarter of the latitude. With these set down, from the point F, the centre of the shadow, at the distance FK, which should equal the sum of the semidiameters of the Moon and the shadow, describe the circle AICD; and from the same centre F, at the distance of the semidiameter FS of the shadow alone, describe the shadow's circuit LGS. Now let the Moon's latitude at full Moon be FE, southern, as in this delineation, and the Moon's centre at E; and on the line FA let FH be taken, equal to the

Moon's latitude belonging to the beginning of the eclipse. For if from the point H you draw to the outer circumference a straight line HK perpendicular to AF, you will have the point K, at which the Moon is at the beginning of the eclipse. From there, then, at the distance of the lunar semidiameter KS, describe the Moon's disc touching the shadow at S. Again on AF take the latitude FL belonging to the end of the eclipse, since in our delineation the latitude is increasing; and when ML is drawn perpendicular to AF, you will have at M the centre of the Moon touching the shadow at G, for the end of the eclipse. When the centres K, E, M are joined, you will mark out the Moon's path in the lunar orbit. A different procedure is needed if the Moon's latitude decreases, and a different one again if the latitude changes its kind in the course of the eclipse, as when, from K, having passed the node F, the Moon comes to Q by ascending, or from T to M by descending. For the proportion of those lines it is useful to have ready a line divided into 60 or more equal small parts representing minutes, from which lines in proportion can be carried over into the delineation of the eclipse. Recall also, from the diagram set out in chapter 4, no. 2, the delineation of an eclipse of exactly six digits, with the Moon receding from the ascending node; and, from chapter 6, no. 3, the delineation of a partial and of a total eclipse.

II. As for the quarters or regions towards which eclipses, that is, the portion of the eclipsed luminary, bend or incline, let us hear Ptolemy, Book 6 of the *Almagest*, chapter 11, who says: *It follows now that we should also consider the inclinations of the obscurations, in Greek* [Greek: ἐπισκοτήσεων προσνεύσεις (episkotēseōn prosneuseis, "inclinations of the obscurations")]*; the understanding of these depends on knowledge of the inclination that both the eclipses themselves have to the ecliptic and the ecliptic itself to the horizon, each of which, at the individual times of an eclipse, produces a very great and incomprehensible change in the transits, if anyone should wish with useless care to investigate the inclinations to come throughout the whole time of the eclipse, since this minute prediction is neither necessary nor useful.*

[Margin: Whether the quarters and inclinations of eclipses are useful to know.]

From this I see that both the theory and the practice of these inclinations have been neglected by almost all astronomers: only Regiomontanus, in Problem 50 of the *Primum Mobile* and in the *Epitome of the Almagest*, Book 6, Propositions 28, 29 and 30, and Kepler in the *Astronomia Optica*, from p. 407 on, have tried this ford. To astrologers, however, the diversity of these inclinations and quarters seems somewhat more useful, so that they may be able to predict which eclipses or phases of eclipses will do most harm or most good to which regions, as may be seen in Ptolemy in the *Quadripartitum*, Book 2, chapter 7, and his interpreters.

[Margin: What is the quarter or aspect of an eclipse?]

So Ptolemy, not to seem to have passed over this topic altogether, gave certain easier methods for the beginning and end of eclipses, and for the beginning

of total obscuration and of repletion. We too shall imitate him, after setting down two definitions first. Let the first be this: the quarter, or rather the aspect [respectus], of an eclipse is the point of the horizon towards which the part of the luminary eclipsed, or placed on the border of the eclipse, points straight; that is, the point on which falls the line or arc drawn from the centre of the obscured body through the centre of the obscuring one.

[Margin: What is the inclination of an eclipse?]

The inclination of an eclipse, or the bending of the darkness, is the angle between the ecliptic and the arc drawn through the centres of the obscuring and the obscured luminary. Thus, in the preceding figure, if the Moon at the beginning of the eclipse is at K, and we conceive the circle AICD to be the horizon, the line or arc KFQ drawn from the centre K through F, the centre of the shadow, will mark at Q the quarter on which the obscured part, or the part soon to be obscured, looks straight; and the inclination of the eclipse will be the angle KFD, equal to the angle QFI that the arc KFQ makes with the ecliptic IFD. But if at the end of the eclipse the Moon's centre is at M, when the line MFT is drawn, the quarter on which the eclipse looks will be T, and the inclination the angle MFI. Conversely, if the Moon were at T, the quarter would be M; and if at Q, the quarter would be K. Hence it comes about that at the beginning of an eclipse of the Moon or of total obscuration the eclipsed part usually inclines towards the eastern part of the horizon, and looks on the rising, now equinoctial, now summer, now winter, or on other quarters of the winds; while at the end of an eclipse of the Moon, or at the beginning of emersion, it inclines towards the western part of the horizon and towards the setting, equinoctial or summer or winter, or another region of the wind; and in the middle it usually inclines towards the north if the latitude is southern, and towards the south if northern. By contrast, in an eclipse of the Sun the eclipsed part of the Sun ordinarily inclines towards the setting at the beginning and towards the rising at the end, and in the middle, if the Moon's visible latitude is southern, towards the south, and if northern, towards the north. With these things explained, let there be.

PROBLEMA I.

Problem 1.

Inclinationem Eclipſis inueſtigare.

To investigate the inclination of an eclipse.

[Margin: Our method is better than Ptolemy's.]

III. Ptolemy (Book 6, chapter 11) and Regiomontanus (Book 6, Proposition 28) use straight lines for curved ones, and treat the Moon's path as parallel to the ecliptic, because the arcs are so small. But this parallelism is not necessary here, nor indeed are straight lines; and Regiomontanus himself warns us of this in the same place, saying: *But if you wish to rest on complete precision, do the work by the*

science of spherical triangles. In the preceding figure, then, let FH be given, equal to the Moon's latitude at the beginning of the eclipse, and FSK, the sum of the semidiameters of the obscuring and the obscurable body. For in the triangle FHK, right-angled at H, the angle HFK can be found, and its complement to a right angle will be the inclination KFD. Likewise at the end of the eclipse, FL, equal to the latitude, and FGM, the sum of the semidiameters, will in the right-angled triangle MLF reveal the angle MFL, which is the complement of the inclination MFI. But for the beginning of total obscuration, or of emersion, one must use for the sides, taken on the line AC, the Moon's latitude belonging to those phases, and for the base FK or FM the remainder of the shadow's semidiameter after the Moon's semidiameter has been subtracted from it. In an eclipse of the Sun, use the visible latitude. Ptolemy, however, used the arc between the centres at the middle of the eclipse, found by means of the ecliptic digits, and the right angle made by that arc both with the ecliptic and with the Moon's path. From that arc and the sum or remainder of the semidiameters he sought, in a right-angled rectilinear triangle, the angle opposite the arc through the centres, which, because of the sensible parallelism, is equal to sense to the angle of the inclination sought. And he composed the following small table, supposing the Moon to be at a mean distance from the Earth.

[Margin: The Ptolemaic small table of the inclinations of eclipses.]

The inclination of eclipses to the ecliptic.

Dig-its	Sun, at beginning and end of eclipse	Moon, at beginning and end of eclipse	Moon, at beginning of total obscuration and of emersion
0	90° 0′	90° 0′	0° 0′
1	66° 50′	72° 30′	
2	56° 59′	65° 10′	
3	49° 16′	59° 27′	
4	42° 36′	54° 27′	
5	36° 35′	50° 14′	
6	31° 1′	46° 15′	
7	25° 46′	42° 31′	
8	20° 44′	39° 2′	
9	15° 51′	35° 42′	
10	11° 6′	32° 29′	
11	6° 25′	29° 23′	
12	1° 42′	26° 23′	90° 0′
13		23° 28′	63° 27′
14		20° 36′	52° 24′
15		17° 48′	43° 26′
16		15° 1′	35° 41′
17		12° 18′	28° 38′
18		9° 36′	22° 1′
19		6° 55′	15° 43′
20		4° 15′	9° 36′
21		1° 36′	3° 35′

PROBLEMA II.

Problem 2.

Punctum Horizontis, quod flexus Eclipſium reſpicit, determinare.

To determine the point of the horizon towards which the bending of eclipses looks.

IV. Regiomontanus sets out this problem in Problem 50 of the *Primum Mobile* and in Book 6 of the *Epitome*, Proposition 30, by solving three triangles. Consult him, if you please, for exercise in these matters, which are not at all necessary. Or else, at any rate, take a globe and set it to the given elevation of the pole, turning it until the rising point of the ecliptic that belongs to the time of the phase of the eclipse stands on the eastern horizon. Then, according to the Moon's latitude given at the time of the eclipse and the inclination taken from the preceding table, draw an arc of a great circle through the centres of the obscuring and the obscured body; for that arc will mark on the horizon the point sought. But if the Moon has no latitude, the ortive amplitude of the ascending point of the ecliptic at the beginning of the eclipse and of the stay, or at the end, will indicate the point of the horizon sought. What pertains here specifically to an eclipse of the Sun will be stated below, in chapter 13, from Kepler's *Astronomia Optica*, p. 411, and the astronomical *Epitome*, from p. 887 on.

SCHOLION.

Scholion.

[Margin: Eclipse of the lunar spots.]

Someone might wish that, in the delineation of eclipses, the lunar spots were distinguished: those obscured first and those obscured last, or those across which the section of the Earth's shadow passes at a given time within the duration of the eclipse. For this our selenography, on which see Book 4, chapter 7, is not useless. But we reserve this for the Geographical Book, where the difference of longitudes is to be corrected by means of eclipses of the spots.

CAPVT VIII.

Chapter 8.

Quid & Quotuplex ſit Eclipſis Solis; & in quo differat ab Eclipſi Lunæ.

What an eclipse of the Sun is, and of how many kinds; and in what it differs from an eclipse of the Moon.

[Margin: The causal definition of an eclipse of the Sun, and its cause.]

I. An eclipse of the Sun, if you look for the definition drawn from its cause, following Aristotle in Book 2 of the *Posterior* [*Analytics*], chapter 1, *is the interposition of the Moon*, that is, between the Sun and our sight. *For it is evident that the Sun is hidden by the Moon coming between, and the Moon by the Earth's being set against it; and that they return the turn, the Moon by its interposition taking the same rays of the Sun away from the Earth, and the Earth from the Moon*, says Pliny the Elder, Book 2, chapter 10. It is evident, he says, but to the wiser. For although, according to Plutarch, Book 2 of *De placitis philosophorum*, chapters 24 and 29, a few held otherwise, and Anaximander said that the failing of the Sun comes about *when the breathing-hole in the rim of the solar wheel is blocked*; or at any rate, as Heraclitus says in the same author, *by the turning of the solar bowl, bright only on its hollow side, when it is then rolled over upwards*; or, if Xenophanes were listened to, who thought that there are, or appear to be, several Moons and Suns in different regions, *by a real extinction of the Sun, while another is meanwhile reborn*: nevertheless, as the same Plutarch reports in the same place, Plato, Aristotle and the mathematicians agree with Thales, *who first showed that the Sun is eclipsed when the Moon passes under it on the perpendicular, since the Moon is opaque by nature, just like the Earth.* Pericles convinced a frightened ship's captain of this by drawing his own cloak over the man's eyes, as we related in chapter 2, number 9, from Plutarch, who adds in the *Nicias* that it was somewhat better known to the Athenians that the Sun is darkened by the Moon coming between than that the Moon is darkened by the Earth's interposition. We have also already reported from Dio, in chapter 2, number 10, that by order of the Emperor Claudius the true cause of an eclipse of the Sun that threatened on his birthday was to be explained to the Roman people as well, lest some disturbance should arise from it.

[Margin: The essential definition of an eclipse of the Sun.]

II. Defined formally, as they say, or quidditatively, an eclipse of the Sun is nothing other than *the privation of solar light in the sublunary region, arising from the Moon's interposition between the Sun and that region.* For the Sun itself does not truly fail in itself, or lose its own light. It is rather that tract of aether, air and earth between which and the Sun the opaque body of the Moon intervenes, and hence also our sight, when we are in that tract. That is why Cleomedes, Book 2 of the *Cyclical Theory*, said: *It is surely not the Sun's condition but our sight's that is called a defect in the Sun.* For granted that the body of the Moon is of far smaller bulk than the body of the Sun, as we showed in Book 4, chapter 17, and is contained in the Sun not only 5848 times, as Tycho thinks, but 2123000 times, as we deduced there from our own principles, and indeed 13833632 times, as Wendelen's tenets require. Yet because of the Sun's immense distance it appears a foot across, and no larger than the Moon, indeed sometimes smaller.

[Margin: Pliny the Elder's slip.]

Yet Pliny the Elder erred in Book 2, chapter 11, in saying: *For the whole Sun could not, of course, be taken away from the lands by the Moon coming between, if the Earth were larger than the Moon.* For he thought that an eclipse of the Sun

occurs over the whole terrestrial hemisphere; if that were granted, Pliny the Elder's inference would be true. But it is not so. And Cleomedes, Book 2 of the *Cyclical Theory*, chapter 3, holds that *the Moon's own shadow cannot extend on the Earth beyond four thousand stadia.* Hence it comes about that at the same moment at which, for some, the whole Sun AB is covered by the Moon DE, as for a spectator at K, whom the outermost rays of the Sun, AK and BK, reach after touching the Moon at D and E, for others meanwhile the whole Sun appears, as for a spectator at F, whose visual rays FA and FB take in the whole Sun. For others again half of the Sun shines, as for a spectator at G, to whom the portion AC of the Sun appears while the portion CB is hidden from him; and for others it is otherwise.

[Margin: Why an eclipse of the Sun is not universal.]

For an eclipse of the Sun is not naturally universal over the whole diurnal terrestrial hemisphere, as an eclipse of the Moon is over the whole nocturnal hemisphere, because the Sun itself is not deprived of its own light, as the Moon is deprived of light that is another's, that is, received from the Sun, and because the Earth FGK is larger than the Moon DE.

Diagram. Set into the left column, beside the passage above. At the top, a large disc drawn as a rayed Sun with a human face. Two straight lines run from the left and right edges of the disc, crossing at the small Moon below, and a third runs down the centre from the foot of the disc. The small Moon, about two-thirds of the way down, has its lower half black. At the bottom a globe with land masses, its upper part drawn with coastlines, and a small standing human figure on top of it. Two further outer lines run from the sides of the Sun down to the sides of the globe.

- A (left of the Sun): the Sun's left edge
- B (right of the Sun): the Sun's right edge
- C (foot of the Sun, on the centre line): point on the Sun
- D (left of the small Moon): the Moon's left edge
- E (right of the small Moon): the Moon's right edge
- G (beside the standing figure on the globe): spectator on the Earth
- F (left of the globe): spectator on the Earth
- K (right of the globe): spectator on the Earth

[Margin: The cause and definition of an eclipse of the Sun are proved by 9 arguments.]

III. That an eclipse of the Sun comes from the Moon's intervention the astronomers show, with Ptolemy, Book 6. First, because it never occurs except at conjunctions of the luminaries, that is, only at apparent new Moons, which is of course the only time at which the Moon, shown in Book 4, chapter 14, to be lower than the Sun, can slip between the Sun and our sight.

[Margin: Second argument.]

Second, because it does not occur at all new Moons, but only at those at which the Moon's apparent latitude is less than the sum of the visual semidiameters of the Sun and the Moon.

[Margin: Third argument.]

Third, because it does not occur at the moment of the true new Moon unless that coincides with the visible one; when it does occur, it occurs at the visible conjunction, that is, when, along the longitude of the zodiac, the Moon is seen conjoined beneath the Sun by us who watch the luminaries from the surface of the Earth, even if, because of the parallaxes, it is not then always conjoined with the Sun with respect to a straight line drawn from the centre of the Earth to the luminaries.

[Margin: Fourth argument.]

Fourth, because an eclipse of the Sun is not total for all spectators in the hemisphere where it is otherwise day, but only for those from whose eyes a straight line can be drawn through the centres of both luminaries, provided that the Moon's apparent diameter is not less than the Sun's visual diameter. Nor is as much covered for all those for whom a part of the Sun is covered, but more for some and less for others, according to the difference in the position of the places, at which the Moon is seen so conjoined with the Sun that it cuts off for them a greater or lesser portion of the solar disc, as the lunar parallax in latitude requires.

[Margin: Fifth argument.]

Fifth, because an eclipse of the Sun never lasts longer than the time that the Moon's motion from the Sun requires for the Moon to be able to traverse, apparently, as much space as the Sun's disc or apparent diameter occupies; and, other things being equal, eclipses of the Sun are shorter when the Moon is faster and longer when it is slower.

[Margin: Sixth argument.]

Sixth, because an eclipse of the Sun begins at the Sun's western part, when the Moon, carried eastward by its apparent motion, which is faster than the Sun's, overtakes the Sun from that side; and for the same reason it ends at the Sun's eastern part.

[Margin: Seventh argument.]

Seventh, because ordinarily, when the Moon's apparent latitude is northern, the eclipsed part of the Sun, for those to whom it fails partially, is northern; but when the Moon's visible latitude is southern, the covered part of the Sun is southern.

[Margin: Eighth argument.]

Eighth, because the shape of a solar eclipse, when the whole Sun is not hidden, is such as the Moon's apparent circular figure requires when compared and conjoined with the Sun's circular figure, namely horned. So at the beginning and end of the eclipse, and also at the middle of a partial eclipse, the Sun appears horned like the Moon, or curved into horns, with the horns turned towards the west at the beginning and towards the east at the end of the eclipse, and towards the south if the Moon's visible latitude at the middle of the eclipse is southern, towards the north if northern; while the portion covered by the Moon appears as a segment of a black circle.

[Margin: Ninth argument.]

Ninth, because for those from whose eyes a straight line can be drawn through the centres of the luminaries, when the Sun's visible diameter is less than the Moon's apparent diameter, the whole Sun is eclipsed, with some stay in the eclipse and with dreadful darkness, and night is so turned into day that stars appear in the daytime. But when it is greater than the Moon's, a shining small circle of the Sun's body appears all round the Moon. And when, according to the anomaly of the luminaries and their distances already known from elsewhere, the apparent diameter of each is of the same size, the whole Sun is covered with no stay in the eclipse. Moreover, arguments 2, 3, 4, 5, 7 and 9 together show that among the causes of a solar eclipse are the parallaxes, or variations of view, in breadth and in length.

[Margin: The kinds of eclipse of the Sun.]

IV. Further, the kinds of eclipse of the Sun are as many as those of the Moon, indeed more.

[Margin: Total with a stay.]

For some are *total with duration*, in which the whole Sun is covered, so that for some stay the Moon remains beneath a Sun covered on all sides; and then more than twelve digits, or twelfths [vnciæ], of the Sun are said to be eclipsed, because if the Sun had more than 12 digits, it would still be obscured.

[Margin: Total without a stay.]

Some are *total without duration*.

[Margin: Partial, annular.]

Some are *partial and central*, or, so to speak, *annular*, in which a bright ring of the Sun, or crown, appears around the Moon. These three kinds occur when, and for those to whom, the Moon's apparent latitude is nil or almost nil, that is, when a straight line can be drawn from the eyes through the centres of the luminaries. But total eclipses with duration do not occur unless the Moon is nearer perigee and the Sun nearer apogee; and central ones do not occur unless the Moon is nearer its apogee and the Sun nearer perigee.

[Margin: Partial, horned.]

Others, finally, are *partial and horned*, in which the part of the Sun covered by the Moon appears crescent-shaped, that is, like a Moon curved into horns.

The diagrams of a partial non-annular eclipse, and of a total one with or without duration, are like the lunar diagrams set out in chapter 3, if in place of the shadow's circle the Moon is understood, and in place of the Moon there the Sun, but almost equal to the Moon in apparent size.

[Margin: How far the tip of the Moon's shadow extends in eclipses of the Sun.]

Meanwhile, it should be noted that in a central and annular eclipse the Moon's sheer shadow does not reach the Earth with its tip, as you see in the adjoining diagram, in which the sheer shadow HKI of the Moon HFI does not, with its tip K, reach the Earth P. And so of the Sun A only the portion enclosed by the arc CSD is hidden, while the small portion of it lying between BC and

DE is seen. You will understand this better from the following diagram, in which the eye at B sees the portion of the Sun MG and SK around

Diagram. Set into the right column, beside the passage above. At the top, a large circle, the Sun, with its centre at A; radii run from A out to points on the circumference at the left and right, and a vertical line runs down from A through the foot of the circle along the whole figure. From the two sides of the circle two long lines converge downward to a point at the foot of the figure. Partway down, a small circle, upper part blank and lower part solid black, with a horizontally hatched cone running down from its black part to a point. Below that, a second small circle, likewise part blank and part black. The long lines run on past it and meet at the bottom.

- A (centre of the large circle): the Sun's centre
- B (left, above the circle's side): point on the circumference
- E (right, above the circle's side): point on the circumference
- C (left, inside the circumference, lower than B): point on the circumference
- D (right, inside the circumference, lower than E): point on the circumference
- e (lower-case letter left of C, outside the circle): not cited in the text on this page
- S (just below the circle, on the vertical): foot of the arc CSD
- F (above the upper small circle, on the vertical): the upper Moon's top
- G (inside the upper small circle, on the vertical): not cited in the text on this page
- H (left of the upper small circle): the upper Moon's left
- I (right of the upper small circle): the upper Moon's right
- K (tip of the hatched cone): tip of the sheer shadow
- P (above the lower small circle, on the vertical): the Earth, as cited in the text
- L (inside the lower small circle, on the vertical): not cited in the text on this page
- N (left of the lower small circle): not cited in the text on this page
- M (right of the lower small circle): not cited in the text on this page
- R (below the point where the long lines meet): not cited in the text on this page the Moon, which lies between the rays ML and GEB on one side and KL and SDB on the other. And then in the tract of the Earth from which the rest of the Sun is hidden there is not sheer shadow but penumbra or umbrago, because, although it lacks many of the Sun's direct rays, it is still lit by some.

Diagram. Set into the left column, under its caption, *Figure 2*. At the top, the lower part of a large circle, the Sun, closed by a straight line across its top, with a second arc inside it leaving a crescent between the two. Dotted lines run from the two sides of the Sun down past the sides of a small Moon, part blank

and part black, with a short horizontally hatched cone below it; the dotted lines meet at the summit of a dome below, the Earth, closed by a straight base.

- S (top left): end of the Sun's top line
- K (top, right of S): point on the Sun's top line
- M (top, left of G): point on the Sun's top line
- G (top right): end of the Sun's top line
- D (left of the small Moon): the Moon's left
- E (right of the small Moon): the Moon's right
- L (tip of the hatched cone): tip of the shadow
- B (summit of the dome): point on the Earth's surface
- A (left end of the dome's base): the Earth
- C (right end of the dome's base): the Earth

But when an eclipse of the Sun is total, yet without any stay, the Moon's simple shadow then reaches the Earth exactly with its tip, as in Figure 3, in which the Sun is SFG, the Earth ABC and the Moon DE, and the outermost rays of the Sun, SDB and GEB, touch the Moon at D and E and meet at B, a point of the Earth's surface, and so bound the Moon's shadow DBE, ending at the point B.

Diagram. Set into the left column, under its caption, *Figure 3*. At the top, the lower part of a large circle, the Sun, closed by a straight line across its top. Two straight lines run from the ends of that line, touching the sides of a small Moon, part blank and part black with a hatched cone below it, and meeting at the summit of a dome, the Earth, closed by a straight base.

- S (top left): end of the Sun's top line
- G (top right): end of the Sun's top line
- F (below the Sun's arc, on the centre): the Sun's lowest point
- D (left of the small Moon): the Moon's left
- E (right of the small Moon): the Moon's right
- B (summit of the dome): the point where the rays meet
- A (left end of the dome's base): the Earth
- C (right end of the dome's base): the Earth

Finally, when an eclipse of the Sun is total and with duration, the Moon's shadow extends with its tip beyond the surface of the Earth, if the Sun's rays are imagined to penetrate the Earth. And just as the Moon in its eclipse transit cuts the Earth's conical shadow straight across, as if, to sense, with a flat disc, so that that conic section is, to sense, circular, so now the Earth cuts the Moon's shadow in such a way that on the surface of the Earth itself the shadowed tract is circular. So it is in Figure 4, in which the Moon DE is nearer the Earth ABCP, its shadow is DTE, and the part of the Earth obscured by it is BC.

Diagram. Set into the left column, under its caption, *Figure 4*. At the top, the lower part of a large circle, the Sun, closed by a straight line across its top. Two straight lines run from the ends of that line past the sides of a Moon, drawn larger than in the figure above, part blank and part black, and on to a point at

the base of a dome, the Earth. A hatched cone runs from the Moon's black part down through the dome to that point on its base.

- S (top left): end of the Sun's top line
- G (top right): end of the Sun's top line
- F (below the Sun's arc, on the centre): the Sun's lowest point
- D (left of the Moon): the Moon's left
- E (right of the Moon): the Moon's right
- B (left, where the cone enters the dome): the Earth
- C (right, where the cone enters the dome): the Earth
- A (left end of the dome's base): the Earth
- P (right end of the dome's base): the Earth
- T (below the base, at the cone's tip): the centre of the Earth, as cited in the text

Although in this diagram the tip of the lunar shadow is set as ending exactly at the centre of the Earth, T, that is not necessary in every total eclipse with duration. Indeed, when the Moon is at perigee, that tip is to be conceived far beyond the centre of the Earth, as we shall say in the next chapter. But this diagram was set down for the sake of an example.

V. From what has been said so far and what is yet to be said, it is easy to draw the comparison of lunar eclipses with solar ones and to discern the differences between them, which are very many; but we shall set them out more suitably in chapter 17.

SCHOLIA.

Scholia.

[Margin: Spurious eclipses of the Sun.]

I. This passage concerns an eclipse of the Sun taken astronomically, and its natural causes, of which there can be certain foreknowledge. Otherwise, that there are other eclipses from accidental causes, not falling under astronomical science, such as clouds, mists and the ashes of Vesuviuses, we have already related in chapter 1, Scholion 3; and in number 4 of the same chapter we showed that the extraordinary crimes of men are commonly counted among the moral causes of eclipses. These are nothing to us here.

II. The total and annular eclipses of the Sun that we have supposed here will be confirmed below, in chapters 19 and 20, by clear authorities and examples; although we have already argued for annular ones in Book 4, chapter 6, Scholion 1. Of the portents of evening and morning eclipses I shall speak in chapter 20, from number 7.

CAPVT IX.

Chapter 9.

De Vmbra & Penumbra Lunæ varijſq. Eclipſium Solarium Phaſibus inde ortis.

On the shadow and penumbra of the Moon, and the various phases of solar eclipses arising from them.

[Margin: What is the shadow of the Moon?]

I. Sheer shadow, which in this place is also simply so called, is the privation of all solar light, direct as much as refracted, in the sublunary region, because of the Moon's interposition. For the reflection of some rays from the Earth into air otherwise shadowed by the Moon, owing to the unevenness of valleys and mountains, does not fall under a definite measure, nor does it disturb the measures of eclipses.

[Margin: What is the penumbra or umbrago of the Moon?]

The Moon's penumbra, taken broadly, is rather an umbrago, that is, the privation not of all the Sun's direct light but only of some of it, because of the Moon's interposition, in the sublunary region. For those who see partial eclipses of the Sun are lit by some of the Sun's direct rays, but not by all, as those are who see the whole Sun. In this way the name penumbra is taken by Kepler in the *Epitome of Copernican Astronomy*, p. 875, and in the Rudolphine Tables, chapter 31, in these words: *The lunar penumbra is the space of the Earth's disc from which not the whole Sun but a part is covered by the Moon; the shadow, the space from which the whole Sun is covered.* For he restricts its subject to the Earth, because it is on the Earth that we observe eclipses.

[Margin: What is the penumbra of the Moon, taken strictly?]

The penumbra taken more strictly, as I said of a like case in chapter 5, is the privation of direct and unrefracted light, but with refracted light present, in the sublunary region, because of the interposition of the Lunatmosphere [Lunatmoſphęræ]: for so let us be allowed to call the sphere of the Moon clothed in an air or aether that is thicker and fit for refracting the Sun's rays, if indeed there is such a mantle around the Moon, on which see below.

II. It must be recalled, furthermore, what I said in the preceding chapter, number 4: namely, that the Moon's sheer shadow does not reach the Earth in annular eclipses; but that in total ones without duration it reaches the surface of the Earth exactly with its tip; and that in total ones with duration it extends beyond the surface of the Earth, the Sun's rays being prolonged by a fiction, and that by this fiction its tip ends now short of the centre of the Earth, now beyond it, now at the centre itself. First, therefore, we must discuss the Moon's shadow and its penumbra or umbrago, and their measures, disregarding refraction.

PROBLEMA I.

Problem 1.

Semiangulum & inde Angulum Coni Vmbræ Lunaris à radijs Solis irrefractis comprehenſæ in Nouilunijs dimetiri.

To measure, at new Moons, the semiangle, and hence the angle, of the cone of the Moon's shadow enclosed by the Sun's unrefracted rays.

III. We have already dealt with this problem in Book 4, chapter 17, Problem 4, by means of the diagram added here, in which the Sun is EMF, the Moon ABCD, and their centres K and H, on the straight line KG; and the Moon's shadow is the cone AGC, enclosed by the rays EG and FG, which touch the Sun at E and F and the Moon at A and C, and which therefore, by 18 of Book 3 of Euclid, make right angles at E and A with the semidiameters of the luminaries, EK and AH.

Diagram. Set into the right column. At the top, the lower part of a large circle, the Sun, its top closed by a shallow line through K and by lines running from K to E and F. Two long straight lines run from E and F, touching the sides of a small full circle, the Moon, and meeting at G at the foot. A vertical line runs from K through M and the centre H of the small circle down to G. A further slanted line runs from L, in the Sun, down to the centre H of the small circle.

- K (top, on the vertical): the Sun's centre
- E (left of the Sun): point of tangency on the Sun
- F (right of the Sun): point of tangency on the Sun
- L (in the Sun, on the slanted line): foot of the parallel HL
- M (below the Sun's arc, on the vertical): the Sun's lowest point
- I (top left of the small circle): point on the Moon
- B (top of the small circle, on the vertical): the Moon's upper point
- H (centre of the small circle): the Moon's centre
- A (left of the small circle): point of tangency on the Moon
- C (right of the small circle): point of tangency on the Moon
- D (bottom of the small circle, on the vertical): the Moon's lower point
- G (foot of the figure): tip of the cone

When HL is drawn, therefore, parallel to EAG, there arises, by 29 and 34 of Book 1 of Euclid, a right angle on each side at L, and a right angle AHI; EL is equal to AH, and the angle LHK is equal to the interior angle EGK; and the triangle LHK, right-angled at L, is, by 2 of Book 6 of Euclid, similar to the triangle EGK and has proportional sides. With these things set down, if the Moon's distance, known from Book 4, chapter 17, is subtracted from the Sun's distance, known from Book 3, chapter 7, or otherwise from tables showing these distances at each of the anomaly's degrees, the distance of the luminaries at new Moon will be known; this is KH, namely the base of the triangle LHK. And if the true semidiameter of the Moon, AH, that is, EL, is subtracted from the true semidiameter of the Sun, EK (on these see Book 4, chapter 17, Problem 1), the side LK will be known. So by trigonometry you will find the angle LHK, that is, AGH, which when doubled will give the angle

of the cone, AGC. It is useful, however, to repeat that angle here from the table of Book 4, chapter 17, Problem 4, leaving aside full Moons and quadratures, for the two extreme cases: with the Sun at apogee and the Moon at perigee, when this angle is at its smallest and the cone of the shadow at its most acute; and with the Sun at perigee and the Moon at apogee, when the angle is at its largest and the cone at its most obtuse, so that from these you may judge of the other, intermediate cases. See it, then, in the table of Problem 2, to be given shortly. Note, however, that if the semidiameters of the luminaries are taken in parts of which one semidiameter of the Earth is 100, as we ourselves did in Book 4, chapter 17, Problem 1, their distance too must first be multiplied by 100. Here and below I take the rays as unrefracted, because I do not suppose that there is air around the Moon sufficient for a sensible refraction.

PROBLEMA II.

Problem 2.

Lunaris Vmbræ Axem ſeu Longitudinem definire in Nouilunijs, poſitis radijs Solis irrefractis.

To determine the axis, or length, of the Moon's shadow at new Moon, assuming the Sun's rays to be unrefracted.

IV. In the preceding figure this is done in two ways. First, in the triangle AGH, right-angled at A, in which the angle AGH is given from Problem 1, along with the true semidiameter of the Moon, AH; for from these one finds the base HG, which is the axis of the cone of this shadow. Second, by the proportional triangles EGK and LHK, once these are known: KH, the interval of the luminaries at new Moon (which is had by subtracting the Moon's distance from the Sun's distance), and LK, the difference between the true semidiameter of the Sun, EK, and the true semidiameter of the Moon, AH, that is, EL. Take these semidiameters from Book 4, chapter 17, Problem 1. For let it be: as LK to EK, so KH to KG; for when KH is subtracted from KG, thus found, the axis of this shadow, HG, will be known. We have moreover dealt with this in Book 4, chapter 17, Problem 6. So let the following table serve for new Moons, when this axis is at its longest, namely with the Sun at apogee and the Moon at perigee, or at its shortest, with the Sun at perigee and the Moon at apogee.

[Margin: Axis and angle of the cone of the Moon's shadow.]

Axis of the Moon's shadow and semiangle of the cone, with the Sun's rays unrefracted.

From the data of the authors	Sun at apogee and Moon at perigee: axis of the shadow (Earth semidiameters)	Sun at apogee and Moon at perigee: semiangle of the cone	Sun at perigee and Moon at apogee: axis of the shadow (Earth semidiameters)	Sun at perigee and Moon at apogee: semiangle of the cone
Ptolemy	64 9/25	15′ 30″	59 6/52	16′ 50″

From the data of the authors	Sun at apogee and Moon at perigee: axis of the shadow (Earth semidiameters)	Sun at apogee and Moon at perigee: semiangle of the cone	Sun at perigee and Moon at apogee: axis of the shadow (Earth semidiameters)	Sun at perigee and Moon at apogee: semiangle of the cone
Copernicus	69 1/20	14′ 30″	58 0	17′ 6″
Tycho	66 1/2	15′ 0″	63 0	15′ 54″
Longomontanus	59 3/5	14′ 50″	57 4/5	16′ 4″
Kepler	61 0	14′ 49″	59 0	15′ 30″
Lansbergen	57 4/7	16′ 46″	57 0	18′ 8″
Boulliau	59 2/3	16′ 10″	57 0	16′ 49″
Wendelen	63 0	14′ 48″	61 0	15′ 18″
Ourselves	60 2/3	15′ 18″	52 35/60	16′ 39″

[Margin: Mean axis of the Moon's shadow according to Biancani.]

[Margin: Greatest axis of this shadow according to Boulliau.]

Among other authors, however, you may perhaps find the measures already mentioned somewhat different, but most of them neglect these measures. Thus Biancani, Book 9 of the *Sphere*, chapter 6, sets the Moon's shadow at mean distance at 60 Earth semidiameters in conjunction and 67 in opposition. But Boulliau, Book 4, chapter 3, with the Sun at apogee and the Moon at perigee, deduces the axis CE in the following figure as 58 55′ Earth semidiameters. For he draws the straight line ACE perpendicular to the semidiameters of the Sun, AB, and of the Moon, CD, just as if the Sun's ray BE touched the luminaries at B and D. He then draws FD parallel to AC, the interval of the luminaries, which for him is 1429 26′ Earth semidiameters. He sets BF, the difference of the true semidiameters, to BA, the Sun's semidiameter, as 0 16′ 45″ to 7 parts. And he makes it: as BF to BA, so FD, to which AC is equal, to AE, 1488 21′. Therefore, when AC, 1429 26′, is taken away, there remains for him CE, 58 55′. From his data, however, we have deduced that the true semidiameter of the Moon, CD or AF, is to the Sun's semidiameter, BA, as 28 to 700, and so BF is 672; and therefore we have deduced the axis CE as 59 2/3.

Diagram. Set into the right half of the right column, beside the passage above. A straight horizontal line at the top, with a shallow semicircular arc hanging below it for the Sun. Two straight lines run from the ends of the top line and converge downward to a point at the foot. A vertical line runs from the top line down the centre to that point, and a second vertical line, to the right of it, runs from the top line to the small circle. The small circle stands about two-thirds of the way down, its lower half black. Below it a short horizontal line crosses the cone, and the part of the cone beneath that line is hatched with horizontal strokes.

- A (top line, at the head of the centre line): end of the Sun's semidiameter on the line of centres
- F (top line, at the head of the second vertical): foot of the line parallel to AC
- B (top line, right end): where the Sun's ray leaves the Sun
- C (centre of the small circle): the Moon's centre
- D (right edge of the small circle): where the ray touches the Moon
- L (left of the short horizontal line): uncited in this passage
- N (at the centre line, on the short horizontal line): uncited in this passage
- M (right of the short horizontal line): uncited in this passage
- T (inside the hatched area, just right of the centre line): uncited in this passage
- E (foot, the point of the cone): the tip of the shadow

PROBLEMA III.

Problem 3.

Tabulam Axis & Semianguli Vmbræ Vniuerſalis vtilem Eclipſibus Solis condere: poſitis radijs Solis irrefractis.

To construct a table of the axis and semiangle of the universal shadow, useful for eclipses of the Sun, assuming the Sun's rays to be unrefracted.

V. This table is made by the method already given in Problems 1 and 2. But, as is usually done for the Earth's shadow, it is enough to find the Moon's shadow with the Sun at apogee, for each degree, or at least for every fifth degree, of the Moon's anomaly, and to add to it, in a separate column, the difference to be subtracted from the axis and added to the semiangle, according to each degree, or every fifth degree, of the Sun's distance from the solar apogee. In this way I thought the tables of eclipses should be enriched; they have neglected this part, though it is by no means to be neglected, as we shall soon see from its use. Let this now be a specimen of the table, for each whole sign of the anomaly; for you will have the full table, God granting, at the end of the work. Let the Moon's equated anomaly, then, be 3^{s}, and the Sun's likewise 3^{s}. Under the title *Axis* you will have 60 0′, and under the next title the difference 3 40′. Subtract this from the axis, and the axis of the Moon's shadow that remains is 56 20′. To the semiangle, 15′ 24″, add 41″, so that it is 16′ 5″.

Axis of the Moon's shadow with the Sun at apogee, and semiangle of the cone of the shadow in conjunction, with the difference up to the Sun at perigee.

Whole signs of the anomaly	Axis of the Moon's shadow (Earth semidiameters)	Difference to be subtracted from the axis (Earth semidiameters)	Semiangle of the cone of the Moon's shadow	Difference to be added to the semiangle	Whole signs of the anomaly
0^{s}	59 20′	0 0′	15′ 30″	0′ 0″	12^{s}

Whole signs of the anomaly	Axis of the Moon's shadow (Earth semidiameters)	Difference to be subtracted from the axis (Earth semidiameters)	Semiangle of the cone of the Moon's shadow	Difference to be added to the semiangle	Whole signs of the anomaly
1^{s}	59 30′	1 0′	15′ 28″	0′ 12″	11^{s}
2^{s}	59 43′	2 10′	15′ 26″	0′ 25″	10^{s}
3^{s}	60 0′	3 40′	15′ 24″	0′ 41″	9^{s}
4^{s}	60 17′	5 10′	15′ 22″	0′ 57″	8^{s}
5^{s}	60 30′	6 20′	15′ 20″	1′ 10″	7^{s}
6^{s}	60 40′	6 55′	15′ 18″	1′ 22″	6^{s}

PROBLEMA IV.

Problem 4.

Inueſtigare an ſpectatori Centralis Eclipſis Solis, ea ſit Annularis, an Totalis ſine mora, an Totalis cum mora, atq. adeò an apex Vmbræ Lunaris citra Terram, an ad Terram, an vltra ſuperficiem Terræ extendatur: poſitis radijs Solis irrefractis.

To investigate whether, for an observer of a central eclipse of the Sun, it is annular, or total without duration, or total with duration; and hence whether the apex of the Moon's shadow reaches short of the Earth, or to the Earth, or beyond the Earth's surface, assuming the Sun's rays to be unrefracted.

[Margin: First method of the Problem.]

VI. From the table of apparent semidiameters, of which you have a specimen for the Sun in Book 3, chapter 10, and for the Moon in Book 4, chapter 16, take the semidiameters of the luminaries, but as seen from the surface of the Earth. For if they are equal, the eclipse of the Sun will be total without duration for an observer placed on the straight line passing through the centres of the luminaries; and the apex of the Moon's shadow will end precisely at the Earth's surface. But if the Moon's semidiameter is smaller than the Sun's, the eclipse will be annular for the observer already mentioned, and the apex of the shadow will not reach the Earth. And if it is greater than the Sun's, the eclipse will be total with duration for the same observer, and the apex of the Moon's shadow will extend beyond the Earth's surface. Moreover, you will derive the semidiameters seen from the Earth's surface from the usual tables of semidiameters, if the distances from the Earth are set beside them, and if you look them up for distances reduced by one Earth semidiameter.

[Margin: Second method of the Problem.]

This Problem can also be solved by the following one, but more laboriously.

PROBLEMA V.

Problem 5.

Diſtantiam Apicis Vmbræ Lunaris à Terræ tum ſuperficie tum centro, citra aut vltra metiri.

To measure the distance of the apex of the Moon's shadow from both the surface and the centre of the Earth, whether short of it or beyond.

[Margin: Five cases of this Problem.]

VII. Find the Moon's distance from the centre of the Earth from its own tables, or by the Problems given in Book 4, chapter 14, and reduce it by subtracting one semidiameter of the Earth. Then find the axis of the Moon's shadow by Problem 2, and compare it with the Moon's distance, both whole and reduced. For if the axis is less than the reduced distance, the apex of the shadow will not reach the Earth's surface. If it is equal to the reduced distance, it will reach precisely to the Earth's surface. If it is greater than the reduced distance but less than the whole distance, it will penetrate the Earth, but short of the Earth's centre. If it is equal to the whole distance, the apex of the shadow will stop at the Earth's centre. And finally, if the axis is greater than the whole distance, it will stretch beyond the Earth's centre. The difference between the axis and the distance will show the distance of the shadow from the surface or the centre of the Earth, either short of it, toward the Moon, or beyond it. These things hold, however, for a place where there is no parallax of latitude; elsewhere some difference intervenes, but not so great that it is worth labouring over.

PROBLEMA VI.

Problem 6.

Inueſtigare Arcum ſuperficiei Terreſtris occupatum ab Vmbrâ Lunæ mera: Quando Eclipſis Solis eſt Totalis cum mora; poſitis radijs Solis irrefractis.

To investigate the arc of the Earth's surface occupied by the Moon's sheer shadow, when an eclipse of the Sun is total with duration, assuming the Sun's rays to be unrefracted.

VIII. Plutarch, Cleomedes, Kepler and Boulliau touch on this Problem. First, then, Plutarch, in the little work *On the Face in the Orb of the Moon*, or rather Empedocles in Plutarch, sang as follows of the Moon covering the Sun:

And it deprives of light as wide a space of the Earth As the orb of the grey-eyed Moon takes in.

But this conflicts with geometric rigour. For since the Moon is smaller than the Sun, and the Moon's shadow is therefore conical, and the Moon stands many Earth semidiameters away from the Earth, the circle in which this shadow cuts the Earth must be smaller than the base of this cone, whose diameter is very nearly equal to the Moon's diameter; and, for example, in Figure 4 of chapter 8, BC must be smaller than DE.

[Margin: Empedocles must be corrected, together with Kepler, his corrector.]

Nor did Kepler, reporting Empedocles' verses in the *Astronomia Optica*, p. 288, sufficiently qualify his opinion when he said: *Understand a somewhat smaller space.* For it is not somewhat but much smaller, since the Moon's true diameter is to the diameter of the Earth as 26 1/5 to 100, and the Earth's diameter takes up 8278 Bolognese miles, by what was said in Book 2, chapter 7. Now as 100 is to 26 1/5, so 8278 is to 2168 4/5, the number of miles that the diameter of the Moon's shadow would occupy if it were equal to the Moon's diameter. But when it is at its greatest, it does not occupy 400 miles, as we shall show below, in number 9. It is therefore at least five times smaller, and not merely somewhat smaller.

[Margin: Cleomedes' opinion on the Moon's shadow darkening the Earth.]

Cleomedes, for his part, in Book 2 of the *Cyclical Theory*, chapter 3, reasons in this way: *When therefore the Moon is joined to the Sun, and at the conjunction is in the middle of the zodiac, a conoid shadow must be sent down from it onto the Earth. This shadow itself is said to advance more than four thousand stadia; for every place in which the Sun is not then seen, with the Moon running beneath the Sun, is the Moon's shadow.* Cleomedes' basis is that, as he goes on to explain more fully in the same place, an eclipse of the Sun seen as total at the Hellespont was nevertheless observed at Alexandria (which he places on the same meridian, and 10000 stadia from the Hellespont) as partial, with 2 digits of the Sun, or a sixth part, standing clear; while at Rhodes, which lies midway between Alexandria and the Hellespont and is 5000 stadia from either, 1 digit was seen. From this he infers that the tract of Earth in which the Sun is not seen (understand: the whole Sun) is greater than 4000 stadia, or 500 Italian miles. But Cleomedes takes in not only the sheer shadow but also the umbrago, or penumbra; and, as Kepler holds in the *Optics*, p. 293, he relies on Eratosthenes, who assigns the Earth's circumference 250000 stadia.

[Margin: Kepler's teachings on the Moon's shadow.]

Kepler, in the *Astronomia Optica*, p. 287, shows that if nocturnal darkness falls by day because of an eclipse of the Sun, the Moon must be seen at a greater angle than the Sun, in such a way that the Sun's rays reflected from the Earth do not illuminate the air occupied by the Moon's shadow, and the material of twilight is thus extinguished. From this he infers that if the material of twilight were about 12 German miles high, as Tycho judged with the ancients, the Moon's shadow would have to occupy on the Earth's surface 300 German miles, of which the Earth's semidiameter holds 860; and that this would require a diameter of the Moon 20′ greater than the Sun's. Since these things are absurd, the material of twilight is therefore much lower than 12 German miles. Moreover, in the *Epitome of Astronomy*, p. 874, and in the Rudolphine Tables, precept 160, he says: *Take the minutes of latitude, or of the arc between the centres, from the semidiameter of the shadow, if there is one; and add that difference, and convert this sum into two arcs of a great circle of the Earth; for the difference of these will show the space of the Earth, extended in width, that the Moon's shadow envelops.*

[Margin: Boulliau's method.]

Finally, Boulliau, in Book 4 of his *Astronomy*, chapter 3, in the figure we have already set out towards the end of Problem 2, deduces the axis CE of the Moon's shadow, which, with the Sun at apogee and the Moon at perigee, he says extends beyond the Earth's centre N by 2 25′ Earth semidiameters. For when NM is drawn parallel to CD, the Moon's true semidiameter, two triangles arise, CED and NEM, with proportional sides, and to these the triangles AEB and FDB are also proportional. So let it be: as CE, the axis of the shadow, to NE, the excess of the shadow beyond the Earth's centre, so CD, the Moon's true semidiameter, to NM, in parts of which the Earth's semidiameter is known. Then let it be: as the Earth's semidiameter to the whole sine, so NM to the sine of the arc occupied by the Moon in the middle of the Earth's disc. In this way, from the data set out at the end of our Problem 2, he deduces that NM, at its greatest, is the 86th part of the semidiameter of the Earth's disc, and a sine of 39′, and that to these correspond 10 German miles; but that at the very edge of the limb the versed sine corresponding to NM is 1° 7′, or 16 3/4 German miles, and that this is the greatest semidiameter of the Moon's shadow, whose double is 33 1/2 German miles, that is, 134 Italian.

[Margin: First case of the Problem.]

IX. It seems to us that this procedure should be worked out differently, with the help of the following figure, in which the Moon is EK, the Earth LF, the centre of the Moon A, that of the Earth B, and the axis of the shadow in a total eclipse with duration is either AC, if it passes the Earth's centre, or AB, if it ends at the Earth's centre, or AI, if the apex of the shadow lies between the surface G and the centre B. These are already known from Problem 5. So if it ends at the Earth's centre, the apparent semidiameter of the Moon, seen from the Earth's centre, is the measure of the angle ABK. Let this now be imagined as GBF, which measures the arc GF, whose double is the arc of a great circle of the Earth occupied by the Moon's shadow.

[Margin: Second case.]

But if the apex of the shadow is beyond the Earth's centre, as at C, then when BF is drawn to the point F, where the extreme ray cuts the Earth's circumference, the triangle CBF arises. So in it, once BC is known, the excess of the axis of the shadow over the Moon's distance, by Problem 5, and the semiangle of the shadow, that is, ACK, by Problem 1, and BF, one semidiameter of the Earth, find the angle BFC. For the external angle GBF is equal to this angle and the angle BCF taken together, by proposition 32 of the first book of Euclid; and GBF measures the arc GF, namely half the arc occupied by the shadow on the Earth's surface.

[Margin: Third case.]

Lastly, if the apex of the shadow is short of the Earth's centre, as at I, and the Sun's ray is ELI, cutting the Earth at L, join L with B by the straight line

LB, and the point I with B; for this gives the triangle ILB, in which LB is known, one semidiameter of the Earth, in parts of which BI is known, the distance of the apex I from the Earth's centre, known by Problem 5; and the

angle BIL is known, which is the complement to two right angles of the semiangle of the shadow, that is, of the angle EIA, known by Problem 1. So find the angle LBI, that is, LBG, which measures the arc LG, half the arc occupied by the Moon's shadow on the Earth's surface. And by what was said in Book 2, chapter 7, it is easy to find the miles that correspond to these arcs. This Problem too holds for a place where there is no parallax of the Moon in latitude; elsewhere it undergoes some variation, but not so great as to be worth investigating.

Diagram. Set into the right half of the left column, beside the passage above. At the top, a circle for the Moon, its upper half white and its lower half black, cut by a horizontal diameter. From the ends of that diameter two lines converge downward to a point at the foot, enclosing a hatched cone. A vertical line runs down the axis of the cone. Lower down, a larger circle for the Earth, hatched round its rim, with its centre on the axis; the cone enters it at the top and leaves it at the bottom, the tip standing below the circle.

- E (left end of the Moon's diameter): edge of the Moon
- A (on the diameter, centre): the Moon's centre
- K (right end of the Moon's diameter): edge of the Moon
- L (top of the Earth's circle, left, inside the hatching): where the ray meets the Earth when the apex falls short of the centre
- G (top of the Earth's circle, centre): the surface point on the axis
- F (top of the Earth's circle, right): where the extreme ray cuts the Earth's circumference
- I (on the axis, inside the Earth): apex of the shadow in the third case
- B (on the axis, lower within the Earth): the Earth's centre
- C (below the Earth): apex of the shadow in the second case

[Margin: How great is the largest portion of the Earth occupied by the Moon's shadow.]

By this method we have found the greatest arc of the Earth that can be occupied on the Earth's surface by the Moon's sheer shadow, which occurs with the Sun at apogee and the Moon at perigee, to be 3° 31′ 30″, or 255 Italian miles of Bologna, or 317 ancient Roman miles. Here we take from Problem 1 the angle C as 15′ 18″; from Problem 2 the axis AC as 60 40′ Earth semidiameters; and from Book 4, chapter 14, the distance of the Moon at perigee, AB, as 53 45′ semidiameters. For from these BC comes out as 6 55′ Earth semidiameters, and the arc GF as 1° 45′ 45″, whose double is therefore 3° 31′ 30″. But when the apex of the shadow ends at the Earth's centre, which occurs at an anomaly of the Sun of 30° and of the Moon of 95°, the semidiameters of the luminaries are then equal, and are 15′ 20″. So the arc occupied by the shadow is 30′ 40″, or 36 55′ Bolognese miles and 49 12′ ancient Roman miles. Granted, Scheiner, in the *Rosa Ursina*, p. 612, says the Moon's shadow occupies at most only 350 Italian miles, and Origanus, in volume 1 of the *Ephemerides*, p. 266, thinks it occupies 64 German miles, that is, 256 Italian.

PROBLEMA VII.

Problem 7.

In Eclipſi Solis Annulari, Inquirere tum Latitudinem annuli lucidi de Sole viſibilis aut partis lunatæ eiuſdem, tum Tractum Terræ, intra quem integer hic annulus ſpectabilis eſt, poſitis radijs Solis irrefractis.

In an annular eclipse of the Sun, to find both the width of the bright ring visible of the Sun, or of its crescent part, and the tract of Earth within which this whole ring can be seen, assuming the Sun's rays to be unrefracted.

X. Consider the following diagram, in which the Sun is DRE, the Moon FG and the Earth NP; and the centres are A for the Sun, B for the Moon and C for the Earth, on the same straight line AC, on which the observer of the central eclipse is at S. When the Sun's rays touching the Moon are drawn, namely DFI and EGI, let the Moon's sheer shadow be FIG, whose apex stands at the interval IS from the Earth; and let the same rays, produced to the Earth, end at O and T. Then from the Earth's centre draw two rays touching the Moon and reaching the Sun, namely CFH and CGM, so that the angle HCM subtends and measures the apparent semidiameter of the Moon, FG; and two others touching the Sun, namely CD and CE, so that the angle DCE measures the visual semidiameter of the Sun, DAE. Finally, from Y draw the ray YFN, touching the Moon at about F and the Sun at Y; and ZGP, touching the Moon near G and the Sun at Z. For on the Earth's surface the arc OST will be the one within which, and not beyond, the bright ring of the Sun can be seen on either side, encircling the Moon. Outside it, however, if the observer is within NO, he will see the part of the Sun DH, but not the whole of ME; conversely, one who is within TP will see the part of the Sun ME, but not the whole of DH; while observers at the points O and T will see the Sun crescent-shaped, with its horns meeting at E or D; and an observer at S will see a ring of the Sun of equal width around the Moon. Further, the arc NSP will be the one occupied by the penumbra, or umbrago, of the Moon, lit by some direct rays of the Sun but not by all, that is, the arc within which some part of the Sun will be seen covered by the Moon; but outside it, as from N to Q and from P to V, the whole Sun will be seen.

[Margin: If the Sun and Moon were represented here as much farther from the Earth, as is proper, the lines DF and EG would fall beyond T and O, and would take in a larger arc than OT, indeed larger than the angle DCE subtends.]

Diagram. Occupying the full width of the right column. At the top a shallow arc, the Sun's upper limb, carrying D, H, A, M and E, with a large semicircle below it running from D to E through R. From the points on the arc, straight lines run down and cross one another. A small circle, its lower half black, stands below the crossing K, with a hatched cone beneath it ending at I.

Lower still a large arc, the Earth's surface, carries O, S and T near its top and N and P on its flanks; a straight line across the base runs from Q through C to V, with lines from C up to N and to P, and up through the Moon to the Sun's arc.

- D (left end of the upper arc): the Sun's limb
- H (on the upper arc, left): end of the ray CFH
- A (on the upper arc, centre): labelled as the Sun's centre on the axis
- M (on the upper arc, right): end of the ray CGM
- E (right end of the upper arc): the Sun's limb
- Z (left, on the Sun's lower semicircle): point of contact of ZGP
- Y (right, on the Sun's lower semicircle): point of contact of YFN
- R (foot of the Sun's lower semicircle): the Sun's limb on the axis
- K (crossing of the rays, above the Moon): uncited in this passage
- F (left of the small circle): edge of the Moon
- B (in the upper half of the small circle): the Moon's centre
- G (right of the small circle): edge of the Moon
- I (below the hatched cone, on the axis): apex of the sheer shadow
- O (on the Earth's arc, left of centre): end of the ray DFI
- S (on the Earth's arc, centre): the observer of the central eclipse
- T (on the Earth's arc, right of centre): end of the ray EGI
- N (left flank of the Earth's arc): end of the ray YFN
- P (right flank of the Earth's arc): end of the ray ZGP
- Q (left end of the base line): limit on the Earth
- C (centre of the base line): the Earth's centre
- V (right end of the base line): limit on the Earth

[Margin: First part of the Problem.]

With these things set down, the *first* part of the Problem is correctly solved, following Kepler in the Rudolphine Tables, precept 151, if you subtract the apparent semidiameter of the Moon from the apparent semidiameter of the Sun, that is, the angle BCG, or ACM, from the angle ACE. For this leaves the angle MCE, which measures the width ME of the solar ring as it appears to an eye at C or S; this is equal to the width DH, and double this will be the width of the crescent Sun as it appears to observers at O and T.

[Margin: Greatest width of the solar ring and of the solar crescent.]

It is at its greatest when the Sun is at perigee and the Moon at apogee, in which case the apparent semidiameter of the Sun is, by my reckoning, 16′ 34″ and that of the Moon 14′. So the greatest width of this ring is 2′ 34″, and that of the crescent Sun 5′ 8″.

The *second* part of the Problem, in which the arc OST is sought, is solved by means of the triangle ICT, in which there are taken as given the angle CIT, equal to the semiangle FIB of the Moon's shadow, known by Problem 1; the side CT, which is one semidiameter of the Earth; and the side CI, which is the distance of the apex of the Moon's shadow from the Earth's centre, known by subtracting the axis BI, known by Problem 2, from the Moon's distance BC. So with these three given, trigonometry will reveal the angle ICT, which

measures the arc ST, whose double is the arc OT, within which the bright ring of the Sun can be seen. This arc is at its greatest when the Sun is at perigee and the Moon at apogee, in which case the angle CIT is for us 16′ 39″, the axis BI 52 35′, and the Moon's distance BC 64 10′. So IC is 11 35′, in units of which CT is one; therefore the angle ICT, and the arc ST, is 2° 32′, and OT is 5° 4′, that is, 360 Bolognese miles or 480 ancient Roman miles. But Cleomedes, as Kepler reasons in the *Optics*, p. 292, relying on the hypotheses of Eratosthenes and Hipparchus, assigns to the arc OT 6°, and so 4000 stadia, or 500 miles. This Problem too holds for a place where the Moon has no parallax in latitude; outside this place it holds, but with some small difference, though not so large that further labour need be imposed on astronomers. As for the greatest arcs, however, which we have investigated both in this Problem and in the preceding one, it holds exactly, because they are not greatest except with respect to the already mentioned place on the Earth, and in other cases they are always smaller. See the notes in the margin at the diagram

PROBLEMA VIII.

Problem 8.

Inueſtigare Arcum Terræ occupatum à Penumbra vel Vmbragine Lunæ, ſeu intra quem non totus Sol videtur; ſi recta linea per centrum Luminarium ducta cadat in centrum Terræ.

To find the arc of the Earth occupied by the penumbra or umbrago of the Moon, that is, the arc within which the whole Sun is not seen; if a straight line drawn through the centre of the luminaries falls on the centre of the Earth.

XI. We seek the arc NSP, according to the preceding figure; but on this more conveniently below, chapter 12, Problem 5, and at the end of Problem 6.

XII. Gathering, then, into one synopsis the conclusions drawn from our hypotheses by Problems 6, 7 and 8, let the following be the theses.

Th. 1. The greatest Total Eclipse of the Sun with stay occurs with the Sun at apogee and the Moon at perigee, and with the centres of the Earth and of the luminaries placed on the same straight line.

Th. 2. The greatest arc of the Earth occupied by the Moon's shadow occurs when the greatest of the Total Eclipses of the Sun with stay occurs.

Th. 3. The greatest arc of the Earth occupied by the Moon's shadow is 3° 31′ 30″[41], or 255 Bolognese Italian miles, 317 ancient Roman miles.

Th. 4. In annular eclipses of the Sun, the greatest breadth of the bright ring, or the greatest crescent-shaped part of the Sun, occurs with the Sun at perigee, the Moon at apogee, and the centres of the Earth and of the luminaries lying on the same straight line.

[41]the page sets the last value as 30′, with a single mark; seconds are meant

Th. 5. The greatest and uniform breadth of the bright ring of the Sun that can be seen is 2′ 34″; that of the solar crescent, however, with its horns joined in one point, is 5′ 8″.

Th. 6. The greatest arc of the Earth within which the greatest bright ring of the Sun can be seen, and outside which the Sun is seen like the Moon but with its horns separated, or else round, is 5° 4′, or 360 Bolognese miles and 480 ancient Roman miles.

Th. 7. The penumbra or umbrago of the Moon is greatest when an annular eclipse of the perigee Sun is made by the apogee Moon, and the centres of the Earth and of the luminaries are on the same straight line.

SCHOLIA.

Scholia.

I. The Problems of the preceding chapter from 4 to 8, with their figures, are suited above all to a spectator who has the luminaries at the zenith with no parallax at all, and accordingly with the Moon having no latitude and the luminaries placed on a straight line in which the centre of the Earth lies. Outside this case the Moon's parallax in latitude disturbs those problems somewhat and varies them. As regards the measures of the arcs occupied by the shadow or penumbra, however, the difference does not come to so much that multiple diagrams and solutions of problems on its account would be worth the astronomers' concern; and, as regards these arcs, we can use those problems without inconvenience, just as if the centres of the luminaries were on a straight line drawn out from the centre of the Earth. For in an eclipse of the Sun there is always some spectator who is on the straight line drawn through the centres of the luminaries, by virtue of parallax in latitude. I am speaking, however, of cases in which the cone of the shadow cuts the Earth on both sides.

[Margin: The method of correcting the preceding Problems in cases of parallax.]

But if anyone wishes to be shown the method he ought to follow in other cases, I shall give at least one example in the following diagram, in which A is the centre of the Moon and B that of the Earth, and let the cone of the lunar shadow be ECK, whose axis AC is given by Problem 2, passing through the eye at I. Draw therefore through I, from B, the straight line BD, up to the zenith of the lunar heaven DAK, and let the Moon's distance AB be given, and let either semiangle of the cone at C be known by Problem 1. Then to the points G and F, at which the sides of the cone of the shadow cut the circumference, draw from the centre of the Earth, B, the straight lines BG and BF; and let the apparent distance of the Moon from the zenith be given, namely the angle DIA. For the remaining angles at I will then be given, namely CIR, vertically opposite to it, and BIA and CIM, its complement to two right angles.

First, then, in triangle BIA, in which the angle BIA is given, and the line BA, in parts of which BI is one, find the side AI. When this is subtracted from the

axis AC, there remains CI, in triangle CIR, in which the angles CIR, as I said, and RCI, the semiangle of the cone, are known. In this triangle, therefore, find the side IR. When this is subtracted from the Earth's semidiameter BI, the side of triangle BRF will be known, in parts of which BF, the Earth's semidiameter, is one; and the angle BRF is known, since it is the complement to two right angles of the sum of the known angles CIR and RCI. Find therefore the angle RBF, which measures the arc IF.

Second, in triangle CIM the angles CIM, as I said, and MCI, the semiangle of the cone, and the line CI are known. Find therefore the side IM, and join it to the Earth's semidiameter BI, so that the side BM becomes known, in the parts of which the side BG is the Earth's semidiameter, in triangle GBM, in which the angle GMB is likewise known, being the complement to two right angles of the sum of the angles CIM and MCI. Find therefore the angle GBM, which measures the arc GI; when this is joined to the arc IF, the arc of the Earth occupied by the Moon's shadow will be known. Likewise in other cases you must be diligent and resourceful, provided, however, that one side of the shadow, say AGC, does not fall outside the Earth.

Diagram. Let into the right half of the left column at its foot. A circle, the Earth, with its centre marked below the middle. At the upper right a small circle stands on an arc that runs from the upper left to the small circle and on past it. Two straight lines run from the point at the lower left, one touching the small circle's upper side and one its lower side, and cross the large circle. A line runs from the lower centre of the large circle up through the circle to the small circle and on to the arc; further lines run from the centre to points on the circle's upper edge.

- D (upper left, at the start of the arc): the zenith end of the lunar heaven DAK
- E (on the arc, upper side of the small circle): end of the cone's base, ECK
- A (centre of the small circle): the centre of the Moon
- K (right of the small circle, on the line beyond it): end of the cone's base, ECK
- M (on the cone's upper side, above the large circle): point of triangle CIM
- G (on the large circle's upper left): where a side of the cone cuts the circumference
- I (at the top of the large circle): the eye
- F (on the large circle, right of I): where a side of the cone cuts the circumference
- R (just below I): point of triangles CIR and BRF
- B (centre of the large circle): the centre of the Earth
- C (at the lower left, the apex of the cone): the apex of the shadow

Labels read: D, E, A, K, M, G, I, F, R, B, C. Labels not read: none known.

II. So far the discussion has been of the Sun's unrefracted rays. You will now ask what happens if they are refracted in air or in a denser ether around

the Moon. I reply that such air or ether is still unknown to me, and that with telescopes however excellent, which I have at last obtained, I have not been able to detect it around the Moon. And if there is such a thing, I do not think that its height or its refractions can be determined by anyone; rather, once it is admitted, it must only be asserted that the Moon's sheer shadow is shortened proportionally, as we said of the Earth's shadow in chapter 5; and that in this case the measures of the arcs of the Earth treated in Problems 4 to 8 nevertheless hold, but that the shadow placed there is the penumbra of the lunar atmosphere [Lunatmoſphæræ]; and that those arcs are not to be increased, except by a scarcely sensible little portion, because the height of the lunar atmosphere is very small, if I am not mistaken.

[Margin: Whether refractions vary the measure of eclipses.]

Because of the refractions of the Sun's rays that occur in the air around the Earth, the Sun is indeed visibly raised. But when this happens, and the sky is clear enough not to cut off the view of it, the Moon too is visible by the Sun's rays reflected from the Earth onto it and reflected again from the Moon to us; and these are likewise refracted on entering our atmosphere, by a refraction sensibly equal to the solar refraction. And so this refraction does not sensibly change the size of eclipses.

Longomontanus, however, teaches the contrary in *Theorica*, Book 1, chapter 9, p. 293, asserting that because of this refraction the Moon's diameter is not indeed diminished, but rather the Sun's apparent diameter is enlarged, so that in thicker air and under a more elevated pole a total eclipse of the Sun never occurs for that reason. To set this before the eyes, he uses the diagram added here, in which A is the Earth, B the Sun, and H and L the Moon. For just as at full Moons the Earth's shadow, which would be RPQ, becomes more contracted because of the refractions in our air, so that it comes out as RMQ, so because of the same refractions he thinks that the Moon's shadow, HV, is contracted. He adds, moreover,

Diagram. Let into the right half of the right column and running down it from the middle of the page to the foot. At the top a large circle, the Sun, with a horizontal diameter, inside a ring of hatched triangles. Two pairs of long lines descend from it and converge. About a third of the way down, a small circle, half shaded, stands between two curved arcs; below it a hatched cone narrows to a point. Further down a circle with its lower part shaded and hatched stands on the long lines, with a horizontal line through its centre. Below it a second small circle, half shaded, with a hatched cone beneath, and the long lines converge at the foot.

- E (far left of the top circle, outside the ring)
- C (left end of the diameter, inside the ring)
- B (centre of the top circle): the Sun
- D (right end of the diameter, inside the ring)
- F (far right of the top circle, outside the ring)
- H (in the upper small circle): the Moon

- G (on the second arc, left of the cone below H)
- K (on the second arc, right of that cone)
- V (the point of that cone, just above the third circle): the end of the Moon's shadow, HV
- R (left end of the line through the third circle)
- A (centre of the third circle): the Earth
- Q (right end of the line through the third circle)
- L (in the lower small circle): the Moon
- M (near the foot, on the axis): the end of the contracted shadow, RMQ
- P (at the foot, the point where the long lines meet): the end of the Earth's shadow, RPQ

Labels read: E, C, B, D, F, H, G, K, V, R, A, Q, L, M, P. Labels not read: none known.

that the Moon's shadow is thinned and lessened by the Sun's rays reflected from the Earth, so that the angle V, under which the Moon is seen, becomes narrower, and therefore some part of the Sun that would otherwise be hidden can be seen. But he errs on both counts. He does not attend to the refractions of the lunar rays, of which I have already spoken, and which compensate for whatever the refraction of the solar rays would increase; and the Moon's shadow can indeed be diluted by the Sun's reflected rays as regards degree of intensity, but cannot be lessened as regards extensive quantity. For even if, with the Moon otherwise covering the whole Sun, innumerable other lights were kindled between the Moon and us, even those of a second Sun wondrously shining through, we would not, by the force of these lights illuminating the Moon's shadow, see the ordinary Sun covered by the Moon. Read again here the other points I brought against Longomontanus in Book 4, chapter 16, number 8.

[Margin: Longomontanus's double optical delusion in eclipses of the Sun.]

III. It is customary for Lansbergen in his *Treasury of Observations*, Class 6 of the lunar observations, when the Sun's diameter from his own tables is greater or smaller than the observations of some eclipses of the Sun examined by him would require, to have recourse in the one case to an amplification, or optical dilation, of the Sun's rays, and in the other to a contraction. But how he contracts the rays he has not explained to us, and it seems a plainly arbitrary fiction. It would have been better to invent thick round clouds that amplified the Moon optically and covered more of the Sun than she could have done on her own. That amplification, moreover, although it holds when some part of the Sun is otherwise seen, because bright things placed next to dark ones appear larger, as we said of the Moon's bright ring in Book 4, chapter 6, yet when the whole Sun is covered it cannot be amplified. Kepler noted this in *Astronomia Optica*, p. 297, in these words: "Since these are bright, they are dilated, and they must cast the first ray to that place; but those which do not radiate, being impeded by the interposition of opaque bodies, as the Sun

by the interposition of the Moon, will not be able to be dilated either. This phenomenon is therefore not to be ascribed to vision."

[Margin: Lansbergen's double fiction in eclipses of the Sun.]

[Margin: Kepler's sounder teaching on this.]

He is speaking there of the cause for which, in a central conjunction of the luminaries, something of the Sun appears, as happens in an annular eclipse; and he assigns a threefold cause. The first is the visual diameter of the Moon, smaller because of its great distance from the Earth than the diameter of the Sun when nearer the Earth. The second, when the first does not hold, is a substance of air, or rather of denser ether around the Sun, set on fire by contact with the Sun, which seems to us to be the Sun's limb although it is not in fact. The third, when neither the first nor the second is present or acceptable, is air or denser ether around the Moon, in which the refracted rays of the Sun do not thin the Moon's diameter, as Tycho puts it, but amplify the Sun's diameter and make visible parts of the solar limb that would otherwise be hidden, just as the same rays refracted in our air bring it about that the Sun, otherwise hidden below the horizon, nevertheless appears to us. But on this third cause, hear what he says on p. 302: "I prefer, therefore, to attribute to the Moon an air, not of great depth, so that at times, but very rarely, it can advance as far as this effect. But I much prefer that the cause adduced in the next-to-last place should hold, and that this one should not be in use but stand in reserve."

[Margin: The threefold cause of an annular eclipse, from Kepler.]

Supposing, however, that the Moon does not of itself cover the whole Sun, he concedes, on p. 286, that especially in weaker sight the edges of the Sun around the Moon are amplified, the image of bright things placed next to dark ones predominating. But because of these accidents of vision, which are too various, he rightly denies that the astronomical tables are to be changed, or that one should rest on them in estimating eclipses: for it is the astronomer's part to bind by laws the things that occur of themselves, and to leave sight to the physicians for correction. This we also said of the amplification of the Moon, from the same chapter 6, at the end of Scholium 5. Granted, then, that the little table of Longomontanus adduced in Book 4, chapter 16, number 8, held for eclipses of the Moon, it does not hold for eclipses of the Sun. Rather, one must either have recourse to Kepler's causes, or better, the diameters of the luminaries must be corrected, if that can be done, so that they satisfy the observations without these evasions; or, if not even that can be done, human weakness must be forgiven.

IV. As for what is wanted at this place on the Moon's secondary light, by which the Moon is seen in eclipses of the Sun, enough has already been said of it in Book 4, chapter 6, from number 5. For we ascribed it to the Sun's rays reflected from the Earth to the Moon, and from there to us.

[Margin: The Moon's light in an eclipse of the Sun.]

As for the colours of eclipses of the Sun, apart from what was said in chapter 5, Scholium 3, note that the cause of their variety, in the part of the Sun not

covered by the Moon or in the surrounding air, is, as Kepler notes in the *Optics*, p. 303, both the preceding clearness of the air and diverse vapours. For if great clearness has preceded, as with a very serene sky and in summertime, when the Sun's rays, reflected more directly, illuminate the air more, then an eclipse suddenly bearing down seems more dreadful to those strongly steeped in the image of light, because it is more unusual; but if clouds or rain have preceded, then although the eclipse increases the darkness, it is nevertheless not seen as dreadful. And so it happened in the eclipse of the year 1590, July 31, in Swabia, on whose harvesters it cast a yellow colour everywhere, and in Styria a reddish one; whereas the eclipse of the year 1598, March 4, with a stormy and rainy sky and a low Sun, excited no complaint about colours. For what Cardano asserts in Book 4 of *De subtilitate*, p. 164, does not always happen: that at the time of a solar eclipse all things appear saffron, and headache comes to many whose heads are weak. Rather, just as at dawn the colours appear now saffron, now crimson, now blue-black, because of the diversity of vapours, so also in an eclipse of the Sun.

[Margin: The colours of the Sun or of the air in an eclipse of the Sun.]

CAPVT X.

Chapter 10.

De Parallaxibus Luminarium, earumq. effectibus, & vſibus in Eclipſibus Solis.

On the parallaxes of the luminaries, and on their effects and uses in eclipses of the Sun.

I. We must recall what we said on parallaxes in general in Book 1, chapter 35, on the parallaxes of the Sun in Book 3, chapter 8, and on the parallaxes of the Moon in Book 4, chapters 13 and 14, where we taught the methods of constructing a table of parallaxes in the circle of altitude, or in the vertical circle, and indeed a twofold table: one for a given true altitude, the other for the apparent altitude of a star above the horizon; and also the method of using either of these tables in both ways, or of transforming one into the other. With these things set down, we must explain the four principal kinds that arise from the parallax of altitude and always presuppose it as known: namely, parallax of longitude, of latitude, of right ascension, and of declination.

[Margin: On parallaxes of altitude.]

Yet only the parallax of altitude, longitude and latitude has usually been treated in eclipses, by these: Ptolemy, Book 5, chapter 19; al-Battānī, chapter 39; al-Farghānī, Differentia 27; Regiomontanus in the *Epitome of the Almagest*, Book 5, from Proposition 26 to 32; the Alphonsine astronomers, in Proposition 32 and the Canon on p. 112; Peuerbach in *De Eclipsibus*, from Proposition 8 to 12; Copernicus, Book 4, chapters 26 and 27; Reinhold in the Prutenic Tables, from Precept 54 to 64 and the Canon on p. 99, and on Peuerbach's *Theoricae*,

pp. 192 to 199; Wurstisen on the same *Theoricae*, from p. 320; Erasmus Oswald Schreckenfuchs on the *Passiones Planetarum*, and Nonius, annotation 7; Magini in the *Primum Mobile*, Book 12, Problems 12 and 13, and in the *Secunda Mobilia*, Canon 75, and their tables, p. 681 and from 691 to 703, and in the *Supplement to the Ephemerides*, from p. 279, or Canons 8 and 9 of the compendium of eclipses; Tycho, Volume 1, *Progymnasmata*, p. 132; Origanus, Volume 1, Part 2, chapter 10; Mulerius in the *Tabulae Frisicae*, pp. 434, 446 and 463; Longomontanus in *Astronomia Danica*, Book 2 of the *Spherica*, chapter 10, and Book 1 of the *Theorica*, chapter 8; Lansbergen in the Precepts of his Tables, 28 and 29, with the Canon on p. 51; Kepler in *Astronomia Optica*, chapter 9, in *Epitome of Copernican Astronomy*, p. 883, and in the Rudolphine Tables, chapter 28; Boulliau, Book 4, chapter 4, and Book 12, Precept 22, with the table on p. 173; Vincenzo Renieri in the *Tabulae Mediceae*, in their second edition, Part 2, chapter 3; and Langius [?] in the *Elementale*, from p. 406.

[Margin: Authors on the parallaxes of the luminaries.]

Among these, Magini in the *Primum Mobile*, Problem 14, also treats of the parallax of right ascension and of declination. From these authors, then, and from our own reflections we shall derive the whole doctrine of these parallaxes. For these, in the diagram added here, let there be the meridian HVP, etc., the horizon HAC, etc., and the quadrant of the circle of altitude VA, in which is the vertical point V. Let the pole of the world, or of the equator, be P, and from it let a small circle be described, which the pole of the ecliptic is imagined to trace by its motion. Let this pole now be at B, and from this pole let the ecliptic EGC, etc., be described. From the same pole of the ecliptic let arcs of the circle of latitude be drawn through the centre of the star as far as the ecliptic, or beyond it, as the northern or southern position of the star requires; so that if the star is at L, let the arc B

Diagram. Let into the right half of the right column near its foot. A semicircle stands on a straight base line. At the upper right a small circle touches the semicircle's outer edge. Several arcs cross the semicircle from upper right to lower left and from upper left to lower right; a line descends from the top of the semicircle to the middle of the base.

- V (top of the semicircle): the vertical point
- B (upper right, on the small circle): the pole of the ecliptic, at present
- P (centre of the small circle): the pole of the world, or of the equator
- E (upper left, on the semicircle): on the ecliptic EGC
- L (below V, right of the vertical line): the star, in the case set out
- D (below E, on an arc): the end of the arc from B through L
- F (below D, on the crossing lines)
- K (below F)
- M (below K, near the base)
- G (right of F, at the crossing of the arcs): on the ecliptic EGC
- H (left end of the base): on the meridian and the horizon
- A (middle of the base): the foot of the quadrant VA

• C (right end of the base): on the horizon and the ecliptic EGC

Labels read: V, B, P, E, L, D, F, G, K, M, H, A, C. A short upright stroke in the crossing of the lines between F and G may be a further letter, I, but it is merged with the lines and is not settled. The letters F, K, M stand close together below the crossing.

[Margin: Explanation of the figure for parallaxes.]

LD be drawn; but if it is at F, the arc BF; and if at K, the arc BK. These arcs, as is taught in the definitions and properties of these circles in the doctrine of the primum mobile, always make right angles on both sides with the ecliptic, which they cut or on which they fall, just as arcs of declination make right angles with the equator. With these things set down.

Definitiones Parallaxium, vel ad Parallaxes ſpectantium.

Definitions of parallaxes, or of things pertaining to parallaxes.

II. 1. The true place of a star, or of any phenomenon, is that point in the world on which a straight line drawn from the centre of the Earth and of the universe falls, according to the more common opinion, which holds these two centres to be the same. For what must be said according to the opinion of the Pythagoreans and Copernicans will be said in the Optical Book and in the Book of Systems.

[Margin: What is the true place?]

2. The apparent or seeming place of a star is that on which a straight line falls, drawn from the eye of a spectator positioned elsewhere than at the centre of the Earth and of the universe, or, as commonly happens, placed on the surface of the Earth.

[Margin: What is the apparent place?]

3. The true altitude of a star is the arc of the vertical circle, or circle of altitude, intercepted between the star's true place and the rational or astronomical horizon, as in the following diagram: if the star is at L, its true altitude will be the arc AL.

[Margin: What is the true altitude?]

4. The true distance of a star from the zenith is the arc of the vertical circle intercepted between the zenith, or vertical point, and the star's true place; or it is the complement of the true altitude to a quadrant. Thus if the star is at L, the true distance from the zenith V will be the arc VL, which with the true altitude AL makes up the whole quadrant ALV of the vertical circle.

[Margin: True distance from the zenith.]

Diagram. Let into the right half of the left column beside definition 4. A semicircle stands on a straight base line. At the upper right a small circle touches the semicircle's outer edge. Arcs cross the semicircle from the upper right to the lower left and from the upper left to the lower right; a line descends from the top of the semicircle to the base.

- V (top of the semicircle): the zenith
- B (upper right, on the small circle): the pole of the ecliptic
- P (centre of the small circle): the pole of the world
- E (upper left, on the semicircle): on the arc CE
- L (below V, on the vertical line): the star's true place
- D (below E): where the circle of latitude BD falls
- F (below D): the apparent place of the star at L
- K (below F): a further true place
- M (below K, near the base): the apparent place of the star at K
- G (right of F): on the arc MG
- H (left end of the base)
- A (middle of the base): the foot of the quadrant ALV
- C (right end of the base): the beginning of Aries and of the ecliptic

Labels read: V, B, P, E, L, D, F, K, M, G, H, A, C. Labels not read: I, which the text cites in the arcs KI and IG; a short upright stroke in the crossing of the lines between F and G may be this letter, but it is merged with the lines and is not settled.

5. The apparent or seeming altitude of a star is the arc of the vertical circle intercepted between the horizon and the star's apparent place. Thus if the star is at L, but appears at F, the apparent altitude will be the arc AF.

[Margin: What is the apparent altitude?]

6. The apparent distance of a star from the zenith is the arc of the vertical circle intercepted between the zenith and the star's apparent place; or it is the complement of the apparent altitude to a quadrant. Thus if the star is at L, but is seen at F, the apparent distance will be the arc VF.

[Margin: What is the apparent distance from the zenith?]

7. The true latitude of a star is the arc of the circle of latitude, that is, of the great circle drawn through the poles of the ecliptic, the ecliptic, and the centre of the star, intercepted between the ecliptic and the star's true place. Thus if the star's true place is at L, and the northern pole of the ecliptic is B, the true latitude of the star L will be the arc DL.

[Margin: What is the true latitude?]

8. The apparent latitude is the arc of the circle of latitude drawn through the star's apparent place, intercepted between the ecliptic and the star's apparent place. Thus if the star is at K, but appears at M, the true latitude will be the arc KI, but the apparent latitude GM.

[Margin: What is the apparent latitude?]

9. The true longitude of a star is the arc of the ecliptic counted from the beginning of Aries, following the succession of the signs, up to the point of the ecliptic on which falls the circle of latitude drawn through the centre of the star placed in its true place. Thus if the beginning of Aries and of the ecliptic is C, and CE is a western arc of the ecliptic, and the star's true place is at L, its true longitude will be the arc CD, because the circle of latitude BD, drawn through L, falls on the point D.

[Margin: What is the true longitude?]

10. The apparent or seeming longitude is the arc of the ecliptic counted from the beginning of Aries in the order of the signs, up to the point of the ecliptic on which the circle of apparent latitude falls. Thus if the star is at L, but appears at F, and EC is a western portion of the ecliptic, the apparent longitude will be the arc CF. But if you suppose the arc CE to represent a portion of the ecliptic that is eastern on one side and western on the other, and the star is in the eastern part, the true longitude of the star L would be the arc counted from the beginning C as far as E, and from E as far as D; whereas the apparent longitude would be the arc made up of CE, western, and EF, eastern.

[Margin: What is the apparent longitude?]

11. The true declination of a star is the arc of the circle of declination, that is, of the circle drawn through the poles of the equator, the equator, and the centre of the star, intercepted between the equator and the star's true place.

[Margin: What is the true declination?]

12. The apparent declination of a star is the arc of the circle of declination drawn through the star's apparent place, intercepted between the equator and the star's apparent place.

[Margin: What is the apparent declination?]

13. The true right ascension of a star is the arc of the equator counted from the beginning of Aries in the order of the signs, up to the point of the equator on which falls the circle of declination drawn through the star's true place.

[Margin: What is the true right ascension?]

14. The apparent or seeming right ascension is the arc of the equator counted from the beginning of Aries in the order of the signs, up to the point of the equator on which the circle of apparent declination falls.

[Margin: What is the apparent right ascension?]

There is no need to illustrate the last four definitions with examples or a special diagram; it is enough if the circles of declination are understood to be drawn through the pole of the world, P, and CE represents the equator.

15. The parallax of altitude is the difference between the true and the apparent altitude, and it coincides with the parallax of distance from the zenith. Thus if the star is at L, but appears at F, the arc LF is the parallax of altitude with respect to the horizon A, and of distance from the zenith with respect to the zenith V. The parallax of altitude is called by Ptolemy [Greek: παράλλαξις μηκοπλατῆς (parallaxis mēkoplatēs, "parallax in length and breadth")], because at times it is the same as that of latitude or of longitude, or causes both.

[Margin: What the parallaxes of altitude, latitude, longitude, right ascension and declination are.]

16. The parallax of latitude, for Ptolemy [Greek: παράλλαξις κατὰ πλάτος (parallaxis kata platos, "parallax in breadth")], is the difference between the true and the apparent latitude of a star; and it is called northern if it depresses the star towards the north, southern if towards the south.
17. The parallax of longitude, for Ptolemy [Greek: παράλλαξις κατὰ μῆκος (parallaxis kata mēkos, "parallax in length")], is the difference between the true and the apparent longitude; the parallax of right ascension, that between the true and the apparent right ascension; and finally the parallax of declination, the difference between the true and the apparent declination. Thus if the star is at L and appears at F, the parallax of latitude is the arc DL, and that of longitude the arc DF. But if the star is at K, but appears at M, the parallax of longitude will be the arc IG, while that of latitude will be the arc by which the apparent latitude MG exceeds the true latitude KI.
18. The angle of longitude is the one opposite the parallax of longitude, as in the figure given above the angle DLF.
19. The angle of latitude is the one opposite the parallax of latitude, or the one made by the ecliptic and the vertical, as in the figure given above the angle LFD.

[Margin: What the angle of longitude and of latitude is.]

20. The parallax of the Moon from the Sun is the difference by which the Moon's parallax exceeds the Sun's parallax, since, as we showed in Book 1, chapter 35, other things being equal, the parallax of a star nearer to the Earth, such as the Moon, is greater than that of a more remote one, such as the Sun.

[Margin: Parallax of the Moon from the Sun.]

The kinds already mentioned are, moreover, subdivided into other kinds, as genera are: thus the parallax of altitude, of longitude and of right ascension into an eastern and a western parallax, according as the star is in the eastern or the western quadrant of the vertical circle, or of the ecliptic, or of the equator; indeed, into a northern and a southern parallax, if the circle of altitude turns out to be either the meridian or the circle of latitude. The parallaxes of latitude and of declination, however, are subdivided into northern, southern, or mixed of both.

[Margin: The subdivision of parallaxes.]

That this may be understood, for the kinds of latitude and their various cases let there be a second diagram, in which all things are set out as in the preceding one, but for five cases let five arcs of latitude be drawn. The first is when the apparent as well as the true latitude remain of the same kind and of some quantity, but the true latitude is greater than the apparent, as if the star is at D and appears at F, and so the true northern latitude is the arc DM, but the apparent northern latitude is FN. The second is when the true latitude is

[Margin: Five cases of parallax in latitude.]

Diagram. Set across the right column near its foot. A semicircle stands on a straight base line, with a small circle at the upper right that crosses the semicircle's edge. Several arcs fan out from a point on the small circle, toward the left side of the semicircle. Straight lines descend from the top of the semicircle and cross the arcs, and meet the base.

- V (top of the semicircle)
- E [?] (upper left, on the semicircle; the letter could be read as F)
- D (right of E, below the top, on the straight line): the star in the first case
- M (left, below E): the end of the arc of true latitude, DM
- F (right of M): the star's apparent place in the first case
- N (below M): the end of the arc of apparent latitude, FN
- G (right of N)
- O (right of G)
- I (below N)
- K (below I)
- Q (right of K)
- B (upper right, on the small circle): the pole of the ecliptic
- P (in the small circle): the pole of the world
- H (left end of the base)
- A (on the base, below K)
- C (on the base, right of A)
- R (right end of the base)

Labels read: V, D, M, F, N, G, O, I, K, Q, B, P, H, A, C, R; with E [?]. Labels not read: none known.

of some quantity, but the apparent latitude is nil; for example, if the star is at F but appears at G, and the true latitude is northern, FN, while the apparent latitude, at G, is nil. The *third* case is when the parallax is mixed, from a true latitude on one side and an apparent latitude on the other; for example, if the star is at F and appears at I, and the parallax of latitude is composed of the true northern latitude FN, which it cancels, and the apparent southern latitude IO. The *fourth* case is when there is no true latitude, but the apparent latitude is of some quantity; for example, if the star is at G but appears at I, with the apparent southern latitude IO. The *fifth* case is when the apparent and true latitudes are of the same kind, but the apparent is greater than the true, because both are reckoned toward the other pole; for example, if the star is at I, with the true southern latitude IO, but appears at K, with the apparent southern latitude QK.

[Margin: What the nonagesimal degree is.]

21. The *nonagesimal* degree is the one 90° distant from the ascending degree: reckoned on the ecliptic if it is the nonagesimal of the ecliptic, or on the equator if it is the nonagesimal of the equator. In the following figure, for instance, if the arc NC of the ecliptic EC, standing above the horizon HCR, is 90°, the point or degree N is called, and is, the nonagesimal. Understand always the nonagesimal of the ecliptic, if nothing else is added.

[Margin: What the eastern and western quadrants are.]

22. The *eastern quadrant* of the ecliptic or equator is the one between the nonagesimal and the ascending degree; the western is the one that lies between the nonagesimal and the descending degree of the ecliptic or equator.

[Margin: What the horoscope or ascendant is.]

23. The *horoscope* or *ascendant* is the rising point of the ecliptic, that is, the one placed on the eastern horizon, as the point C in the following figure.

Theoremata Selecta de Parallaxibus prædictis cum Corollarijs ad Eclipſes Solis ſpectantibus.

Selected theorems on the parallaxes already mentioned, with corollaries bearing on eclipses of the Sun.

[Margin: Theorem 1.]

III. Theorem 1. A star at the nonagesimal degree of the ecliptic, in itself or in respect of its longitude, undergoes no parallax of longitude. This comes from Ptolemy, Book 5, Ch. 19, and from Copernicus, Book 4, Ch. 26, and it is common among the astronomers cited in Section 1. Regiomontanus demonstrates it as well, in Book 5 of the *Epitome*, prop. 28, and in *De Cometis*, Problem 5, as does Nonius on Peuerbach, p. 296. The reason is that the circle of latitude determining the star's longitude then passes at once through the nonagesimal of the ecliptic and through the vertex, and so also becomes a circle of altitude, cutting the ecliptic at right angles; and therefore it passes through both the true place and the apparent place. So it is in the adjoined figure, in which let the horizon be HCR, the meridian HVR, the vertex V, the ecliptic EC, and on it the nonagesimal N, distant from the ascendant C by the quadrantal arc NC.

Diagram. Let into the right half of the left column. A semicircle stands on a straight base; a heavy double-ruled line, slightly curved, runs from the upper left of the arc down to the base right of centre. Two small horned crescents, the Moon, lie along the upper part of the figure near that line. At the upper right a small circle stands on the arc.

- H (left end of the base, outside): horizon, per HCR, and meridian, per HVR
- A (on the base, just inside H): not cited on this page
- C (on the base, right of centre): the ascendant
- R (right end of the base): horizon and meridian, per HCR and HVR
- V (top of the arc): the vertex
- E (upper left, at the head of the heavy line): ecliptic, per EC
- R (beside a disc, upper left): the star's place in the text's "at R"
- N (beside a disc, below that R): the nonagesimal
- K (below that, beside the heavy line): a place of the star
- M (below K): not cited on this page

- B (at the top of the small circle): not cited on this page
- P (at the centre of the small circle): not cited on this page

Labels read: H, A, C, R (twice), V, E, N, K, M, B, P. Labels not read: none known.

So whether the star, the Moon for instance, is at R, at N, or at K, its place of longitude, both true and apparent, will still be the nonagesimal N itself; yet in that case there can be parallax in altitude and in latitude. This happens once a day; and the Moon at the nonagesimal, if it is horned, holds its horns upright, as I said in Book 4, Ch. 3, no. 3; see the figure there.

[Margin: Theorem 2.]

Theorem 2. For a star placed at the nonagesimal degree of the ecliptic, in itself or in respect of its longitude, the parallax of latitude is not merely equal to the parallax of altitude but the same. For, as I said, the circle of latitude and the circle of altitude then become one and the same. Thus in the figure just given, if the Moon is at R and appears at N or at K, the same arc RN or RK that is the parallax of altitude will also be the parallax of latitude.

[Margin: Theorem 3.]

Theorem 3. If the true conjunction of the luminaries occurs at the nonagesimal degree of the ecliptic, it will coincide with the visible or apparent conjunction; that is, the true and the apparent new Moon will be the same. This follows from Theorem 1, but the Moon is taken as reduced to the ecliptic.

[Margin: Theorem 4.]

Theorem 4. A star placed at the nonagesimal degree of the equator has no parallax of right ascension; but its parallax of declination is the same as its parallax of altitude, and if a true conjunction of the luminaries then occurs, it will occur at noon. The reason is that the same circle of altitude becomes a circle of declination, and indeed the meridian too.

[Margin: Theorem 5.]

Theorem 5. If the ecliptic passes through the spectator's vertex, as happens for many within the Torrid Zone, and the star is on the ecliptic itself, though away from the vertex, there is no parallax of latitude, while the parallax of altitude and that of longitude are the same and equal. For the ecliptic then becomes a circle of altitude, and on it lie the star's true and apparent places. You will easily understand this if, turning a globe or an armillary sphere, where the altitude of the pole does not exceed 23° 30′, you set some degree of the ecliptic under the zenith.

[Margin: Theorem 6.]

Theorem 6. For a star placed away from the nonagesimal of the ecliptic and in the eastern quadrant, the true longitude is less than the apparent, and the apparent conjunction of the luminaries precedes the true by an arc as great as the parallax of longitude, and by a time equal to the time produced by dividing the parallax of longitude by the Moon's hourly motion from the Sun, as then proper to the Moon: as in the first figure of Section II. If the arc EC of the ecliptic is eastern, and the place of the true longitude of the star L is D, while

that of its apparent longitude is F, the star will be seen at F, further east and further advanced in longitude than it really is, by an arc as great as the parallax DF; and if the true conjunction is going to occur at F, it will already be seen to have occurred, although the star is still short of it, at D.

[Margin: Theorem 7.]

Theorem 7. For a star placed away from the nonagesimal of the ecliptic in the western quadrant, the true longitude is greater than the apparent, and the apparent conjunction of the luminaries follows the true by an arc as great as the parallax of longitude, and by as much time as is produced by dividing this parallax by the Moon's hourly motion from the Sun. For the theorems already mentioned, see Reinhold on Peuerbach, p. 195.

[Margin: Theorem 8.]

Theorem 8. In regions on this side of the Tropic of Cancer the parallax of latitude diminishes a northern latitude; if it equals the true latitude, it cancels it; if it exceeds it, it carries it over into an apparent southern latitude as great as that excess. A southern latitude, however, it always increases. In the same way, in regions beyond the Tropic of Capricorn it either diminishes a southern latitude, or cancels it, or turns it into a northern one; but a northern latitude it always increases.

[Margin: Theorem 9.]

Theorem 9. Within the Torrid Zone the parallax of latitude can, for a spectator whether northern or southern, both increase and diminish either kind of latitude, according as the star on a vertical circle is depressed for them by the parallax of altitude toward the ecliptic or further away from the ecliptic. You will learn this more easily from the use of a globe or an armillary sphere than through long and multiple rules.

[Margin: Theorem 10.]

Theorem 10. If the culminating point of the ecliptic, that is, the one crossing mid-heaven, is further south than the inhabitant's vertical point, the parallax of latitude is southern, that is, it depresses the star toward the south; but if further north, it is northern, that is, it depresses the star toward the north. Ptolemy, Book 5, Ch. 19. I am not saying here that the apparent latitude is southern or northern, but the parallax itself; for the apparent latitude can remain northern and yet the parallax be southern, because it casts the star down toward the south, etc. But Ptolemy's statement must be limited to a star that is on the ecliptic; otherwise it would not be universally true.

[Margin: Theorem 11.]

Theorem 11. If the angle of the ecliptic and the horizon that faces north is greater than a right angle, and the parallax of latitude depresses the star to the south, the parallax of longitude is in the order of the signs [ad Succeſſionem Signorum]; but if the parallax of latitude depresses it to the north, it is against the order of the signs [in præceſſionem Signorum]. Conversely, if that angle is less than a right angle and the parallax of latitude is to the south, the parallax of longitude is against the order of the signs [in præcedentia Signorum]; but if the

parallax of latitude is to the north, it is in the order of the signs. Ptolemy, Book 5, Ch. 19; and recall here that it is one thing for the parallax to be to the south or north, and another for the apparent latitude itself to be northern or southern.

[Margin: Theorem 12.]

Theorem 12. If the star's distance from the meridian reckoned on the horizon, that is, the eastern azimuth, equals the rising amplitude of the rising degree of the ecliptic, or the western azimuth equals the setting amplitude, the star has no parallax in longitude; for it is then at the nonagesimal, and the azimuth of the nonagesimal equals the amplitude already mentioned. Nonius demonstrated this on Peuerbach, part 2, note 7, and Regiomontanus touched on it in the book *De Cometis*, Problem 5.

[Margin: Theorem 13.]

Theorem 13. All the parallaxes of latitude, in whatever degree of the ecliptic the star may be, are equal, provided that the same point of the ecliptic rises at the same altitude of the pole, and the star is equally distant from the eye. Kepler, in the *Astronomia Optica*, Ch. 9, no. 4, prop. 1, where he demonstrates this.

[Margin: Theorem 14.]

Theorem 14. All the parallaxes of latitude, in whatever degree of the ecliptic the star may be, at the same altitude of the pole and with the same point of the ecliptic rising, even if the star is unequally distant from the eye, provided it is equally distant from the centre of the Earth, are as nearly as possible equal. Kepler, above, prop. 2, where he also shows that the greatest inequality among them does not exceed 37″, and that this occurs only if the spectator's position lies midway between the one under the ecliptic and the one under the pole of the ecliptic, and when the parallax of altitude is at its greatest; otherwise that difference is much less than 37″.

[Margin: Theorem 15.]

Theorem 15. All the parallaxes of latitude are in fact equal, in whatever degree of the ecliptic the star may be, provided that it is seen on the ecliptic, that the star's distance from the centre of the Earth and the altitude of the pole are the same, and that the same point of the ecliptic is rising. Kepler, above, prop. 3.

[Margin: Theorem 16.]

Theorem 16. The parallax of altitude is rarely equal to the parallax of longitude or of latitude; usually it is greater than either, and never less. Mulerius, p. 434. To demonstrate these theorems here, along with others that bear on comets rather than on eclipses, would have been long and out of place, since they belong to the common doctrine of the primum mobile, or to our optical book, Section 1.

[Margin: Theorem 17.]

Theorem 17. The nonagesimal degree of the ecliptic is in the eastern quadrant when northern signs are ascending, but in the western when southern signs are ascending, for inhabitants of the north; the opposite happens for those of the south. This is evident to anyone who turns a sphere.

[Margin: Theorem 18.]

Theorem 18. The greatest parallax of longitude is, in the southern signs, below the horizon; at the equinoctial points, on the horizon; in the northern signs, above the horizon, to us in the north. Kepler, in the *Epitome of Copernican Astronomy*, p. 884.

PROBLEMATA.

Problems.

Parallaxium Lunæ & Solis, Eclipſibus Solis deſeruientia, aut ad has Parallaxes pertinentia.

Problems of the parallaxes of the Moon and the Sun, serving eclipses of the Sun or pertaining to these parallaxes.

[Margin: Three kinds of problems prerequisite for the parallaxes of the luminaries.]

IV. There are three kinds of these problems. The first consists of general problems that bear remotely on these parallaxes, such as finding the *right ascension* for a given point of the ecliptic, and the *angle of the ecliptic and the meridian* for a given point of the ecliptic, and, for a given time, seeking the *right ascension of mid-heaven* as well as the *culminating point of the ecliptic*, and other things of the kind. It is certainly not the business of this place to set these out; they are to be assumed here and sought from the doctrine of the primum mobile, to be set out in Book 10. The second kind consists of problems that are indeed general and to be set out under the primum mobile, but are required more nearly or most nearly for the parallaxes of the luminaries; and these we must touch on here, though as briefly as possible, with their fuller doctrine referred to Book 10. Such are those that deal with the *rising point of the ecliptic*, the *nonagesimal of the ecliptic*, the *altitude of the nonagesimal* or the *distance of that nonagesimal from the vertex*, and also the *altitude of the luminaries*; and finally the *angle of the ecliptic and the vertical*, which is called the *parallactic angle* [Angulus Parallacticus], whether that vertical is drawn through the true place of the luminaries, when they are on the ecliptic, or through the Moon, when it is off the ecliptic. The third kind includes the problems proper of the parallaxes of altitude, latitude and longitude, to be deduced directly. Let us now begin with the second kind.

PROBLEMA I.

Problem 1.

Dato Tempore apparente veri Nouilunij ante vel poſt meridiem, & Loco Luminarium in Zodiaco, Inuenire punctum Eclipticæ Aſcendens ſeu exoriens, atq. inde Nonageſimum Gradum, ſeu punctum Eclipticæ, ſitumq. Luminarium reſpectu Nonageſimi.

Given the apparent time of the true new Moon before or after noon, and the place of the luminaries in the zodiac, to find the ascending or rising point of the ecliptic, and from it the nonagesimal degree or point of the ecliptic, and the position of the luminaries with respect to the nonagesimal.

V. First, from tables or problems of the primum mobile look up the right ascension of the point of the ecliptic at which the luminaries are, and add the apparent time after noon, converted into degrees of the equator, to the right ascension, discarding 360° if the sum reaches or exceeds them. The remainder or the sum will be the right ascension of mid-heaven. Add 90° to it, and you will have the oblique ascension in an oblique sphere, or the right ascension in a right sphere, of the horoscope or ascending point of the ecliptic; this is to be used if it does not exceed 180°, or its complement to 360° if it exceeds them.

Second, if the place you are working for is in a right sphere, look up in the body of the table of right ascensions the right ascension of the rising point: opposite it on the left you will find the degree or point, and at the head of the table the sign of the ascending degree or point. But if you are working for an oblique sphere, look up the rising point or ascending degree in the same way, but in the tables of oblique ascensions suited to the given altitude of the pole, looking up in the body of the table the oblique ascension, or its complement, discussed here in instruction 1.

Third, subtract 90° from the ascending point of the ecliptic, and the nonagesimal degree of the ecliptic will remain known. If this point coincides with the true place of the luminaries on the ecliptic, they will be at the nonagesimal, which also holds in its own way of the Moon, if it has been reduced to the ecliptic. But if they are on this side of it, against the order of the signs, they will be in the western quadrant of the ecliptic; if beyond it, in the order of the signs, they will be in the eastern quadrant of the ecliptic. To these you will apply Theorems 1, 2, 3, 4, 6 and 7, discussed above.

Idem Aliter.

The same by another method.

VI. First, add together into one the given time, but the apparent time, from noon and the semidiurnal quantity, and convert the sum into degrees of the equator, discarding 24^h if it should exceed them. Then, with the given place of the Sun, look up under the appropriate altitude of the pole its oblique ascension; add this to the degrees of the equator already mentioned, discarding 360° if it should exceed them, and you will have the oblique ascension of the ascending

point. Second, therefore, in the body of the table of oblique ascensions look up, as above, the ascending point corresponding to the oblique ascension, and do the rest as above. Another way, by tables of houses (if you lack tables of oblique ascensions): find the Sun's place for the given time from noon; with it, in a table of houses under the appropriate altitude of the pole, in the column of the 10th house, look up the Sun's place, for opposite it in the left column stands the corresponding time from noon. Add to this the time from noon for which you calculated the Sun's place, and look up the combined time in the same table of houses: for opposite it, under the title *first house* or *ascendant*, you will find the point of the horoscope. Such tables are given by Magini in the Introduction to the Ephemerides, and by Argoli and Origanus in the same place.

PROBLEMA II.

Problem 2.

Altitudinem Nonageſimi ab Horizonte, atq. adeò eius diſtãtiam à Vertice inueſtigare.

To find the altitude of the nonagesimal above the horizon, and hence its distance from the vertex.

VII. In the adjoined figure let the horizon be HCR, the meridian HVR, on which are the vertex V, the pole of the world F, and the pole of the zodiac B, from which let there be described a portion of the ecliptic ENC, and on it the nonagesimal N, through which and the vertex V let there pass the circle BVNA, both vertical and of latitude.

[Margin: Finding the culminating point.]

First, then, from tables or problems of the primum mobile, with the Sun's place and the given time, look up the right ascension of mid-heaven, as I said in Problem 1, instruction 1; and with this, looked up in the body of the table of right ascensions, find the culminating point of the ecliptic, that is, the one then passing through the meridian, namely point E.

[Margin: Finding the angle of the ecliptic and the meridian.]

Second, from tables or problems of the primum mobile look up the angle HEC of the ecliptic and the meridian, corresponding to the culminating point of the ecliptic, and find the difference of longitude between the culminating point E and the point C, the ascending point of the ecliptic, known by Problem 1, by subtracting the longitude of the culminating point from the longitude of the ascendant. Call that difference, namely EC, the base of the spherical triangle HEC, right-angled at H, to which is adjacent the angle HEC of the ecliptic and the meridian.

Diagram. Set across the left column, interrupting the sentence above after "the point C,". A semicircle stands on a straight base. From the top, V, dotted and solid lines run down to the base. A curved line and a straight line run from

the upper left of the arc down to the right of the base; another arc runs from the left of the figure up to the right, to a small circle at the upper right.

- H (left end of the base): horizon and meridian, per HCR, HVR
- A (on the base, right of H): the foot of the circle BVNA
- [?] sign (on the base, right of A): a closed loop over two out-curving feet, not identified; not cited on this page
- T (on the base, right of that): not cited on this page
- C (on the base, right of centre): the ascending point
- R (right end of the base): horizon and meridian, per HCR, HVR
- V (top of the arc): the vertex
- E (upper left): the culminating point
- D (upper middle): a place of the star in Case 2
- N (below and to the left of D): the nonagesimal
- G (left, below N): a place of the star in Case 2
- O (between D and K, smaller than the other letters): not cited on this page
- L (below G): not cited on this page
- I (right of L): a place of the star in Case 3
- M (below I): not cited on this page
- K (right of I): not cited on this page
- B (upper right, on the arc): the pole of the zodiac
- F (in the small circle): the pole of the world

Labels read: H, A, S, T, C, R, V, E, D, N, G, O, L, I, M, K, B, F; one sign on the base, between A and S, not identified [?]. Labels not read: none known.

Third, by analysis of the triangle already mentioned, look up the other angle adjacent to the base, namely HCE. For it will be the angle of the east [Angulus Orientis], which measures the arc AN, because the angles at N and A are right angles, and NC, and hence CA, are quadrants. And AN is the altitude of the nonagesimal above the horizon, and the complement of this altitude to 90° will give the distance NV of that nonagesimal from the vertex.

Idem aliter per Tabulas.

The same by another method, using tables.

[Margin: Ptolemy's table of the angle of the east.]

VIII. First, in Ptolemy's *Almagest*, at the end of Book 2, you have tables of the angle of the east and of the west and of a luminary's distance from the vertex, entered with the equal hours by which the luminary is distant from the meridian. But they serve only seven climates, that is, from polar altitude 16° 2′ as far as altitude 48° 31′, and only the whole signs in which the luminaries are.

[Margin: And Copernicus's table.]

Second, Copernicus gives a table of the same angle of the east, Book 2, Ch. 10, p. 42, which is entered, under polar altitudes of 39°, 42°, 45°, 48°, 51°, 54° and 57°, with the ascending point of the ecliptic known by Problem 1. But it

has only every sixth degree of the ascending signs, and it is constructed for an obliquity of the ecliptic of 23° 28′.

[Margin: The table of the angle of the east in Magini.]

Third, in the second volume of the *Primum Mobile* of Giovanni Antonio Magini and in his tables, which they call tables of directions, from p. 200, there is a better table of the angle of the ecliptic and the horizon, for each altitude of the pole from 30° to 54° and for each degree of the ascending signs, entered from the sides with the ascending degree known by Problem 1. But this table, like his preceding table of the angle of the ecliptic and the meridian, is constructed for the Tychonic obliquity of the ecliptic, 23° 31′ 30″.

[Margin: The table of the nonagesimal's distance from the vertex and from the meridian in Mulerius.]

Fourth, there is in Nicolaus Mulerius, in the *Tabulae Frisicae*, p. 448, a table containing both the distance of the nonagesimal from the vertex and its distance from the meridian reckoned on the ecliptic, entered with the degree of the ecliptic that is culminating or at mid-heaven, known by instruction 1 of Problem 2. He constructed this table for each culminating degree of the signs, and for polar altitudes of 42°, 45°, 48°, 51°, 52°, 53⅙° and 55° 55′, assuming the Tychonic obliquity of the ecliptic.

[Margin: The best table of the angle of the east, in Kepler,]

[Margin: and in Renieri.]

Fifth and last, Kepler, in the Rudolphine Tables, from page 26, has given us a table of the angle of the east, or, what is equivalent, of the altitude of the nonagesimal, and that for absolutely every altitude of the pole from 1° to 90° and for every third degree of the rising signs, expressing the minutes of the angle of the east in parts of a unit [per partes affis]. He teaches its use in the instructions, Ch. 14, or instruction 39. Don Vincenzo Renieri, in his *Tabulae Mediceae*, printed a second time, from p. 324, abridged this table to every third degree of polar altitude up to 30°, then to every degree but only up to 53°, then again to every third up to 62°, and to every second degree of the ascending signs; and the same table, but exactly as it stands in Kepler, Boulliau copied into his *Philolaic Tables*, p. 158. Kepler admits, in Ch. 12 of the Rudolphine Tables, that he constructed these tables of the primum mobile for the Tychonic obliquity of the ecliptic, 23° 31′ 30″, as if constant, although he says that from his own solar parallax, which does not exceed 1′, an obliquity of the ecliptic of 23° 30′ 30″ follows.

[Margin: The blemish of Renieri and Boulliau.]

But since this table does not give the altitude of the nonagesimal considered on the Moon's orbit, Renieri, in the instructions for eclipses, Ch. 3, instruction 3, and Boulliau, Book 12, instruction 22, say that the true place of the node ☊ is to be subtracted from the true place of the nonagesimal on the ecliptic; that with the remainder, as an argument of latitude, the latitude is to be sought as if it were the Moon's; and that this is to be added to the altitude of the nonagesimal if the latitude is northern, and subtracted if southern, so as to obtain the altitude

of the nonagesimal considered not on the ecliptic but reduced to the orbit of the Moon.

[Margin: Praise of Don Vicenç Mut.]

Don Vicenç Mut, a man most learned in astronomy, disapproves this method, as I gather from various letters he has sent me, on this subject too. First, because the same vertical cannot cut both the ecliptic and the Moon's orbit at right angles, except when the node is in the east. Next, because when the node is at the nonagesimal of the ecliptic, the node is then not a quadrant of the lunar orbit away from the rising. Finally, because at the same meridian altitude of the lunar orbit, the parallaxes of lunar latitude would be equal in any part of the orbit according to Kepler's doctrine as applied here, on which see our Theorem 13; and that, however, is false. So although it is better to use at least this method than no correction, still we do not attain complete precision by it, and in some places the error exceeds a whole 1′.

PROBLEMA III.

Problem 3.

Datis Nonageſimo Eclipticæ, eiuſq. Diſtantia à vertice, & ſideris ad idem tempus Longitudine ac Latitud. Inquirere ſideris in Ecliptica aut extra poſiti Altitudinem ab Horizonte Diſtantiamq à vertice, necnon Angulum in centro ſideris aut loco eius in Ecliptica factum à Verticali & Ecliptica aut circulo latitudinis.

Given the nonagesimal of the ecliptic, its distance from the vertex, and the longitude and latitude of a star at the same time, to find the altitude above the horizon and the distance from the vertex of a star placed on the ecliptic or off it, and also the angle at the centre of the star, or at its place on the ecliptic, made by the vertical and the ecliptic or circle of latitude.

[Margin: Four cases of this problem.]

IX. This problem has four cases, according as the star is either at the nonagesimal itself without latitude, or at the nonagesimal as to longitude but with latitude; or away from the nonagesimal without latitude; or away from the nonagesimal with latitude. The altitude AN of the nonagesimal and the distance NV are assumed as known by Problem 2.

[Margin: Case 1.]

First, then, in the figure given before no. 8: if the star is at the nonagesimal N, its altitude is already the same as the altitude of the nonagesimal.

[Margin: Case 2.]

Second, if in its place of longitude it is at the nonagesimal N, but has northern latitude ND or southern latitude NG, because the star is at D or G, add the northern latitude ND to the altitude AN of the nonagesimal, or subtract the

southern NG. In the first case AD will result; in the second AG will remain, the altitude of the star. In this case we do not need the angle of the vertical made at the centre of the star, because there is no parallax of longitude.

[Margin: Case 3.]

Third, if the star as to longitude is away from the nonagesimal, but on the ecliptic itself without latitude, as at I, let the vertical VIS be drawn through it, so as to form the triangle VNI, which is right-angled at N. In it are given NV, the distance of the nonagesimal from the vertex, by Problem 2, and the arc NI of the ecliptic, which is the difference between the nonagesimal known by Problem 1 and the place of the star's given longitude. Therefore by trigonometry the base IV, which is the star's distance from the vertex, will not remain hidden; its complement to 90°, IS, will be the star's true altitude. Nor will the angle HIV [recte: NIV], the one sought, remain hidden.

[Margin: A shortcut for the Problem.]

In brief, let it be: as the radius is to the second sine of arc NV, so is the second sine of arc NI to the second sine of the base IV. Then let it be: as the radius is to the tangent of arc NV, so is the second secant of arc NI to the tangent of the angle sought, NIV. Or join together the second logarithms of the arcs NV and NI, and their sum will be the second logarithm of the base IV. Then the mesologarithm of arc NV, added to the residual of the logarithm of arc NI, will make up the mesologarithm of the angle sought, NIV.

[Margin: Case 4.]

Fourth, if the star is away from the nonagesimal and has latitude, as when it is at K or at L, with its place in longitude at I, let there be drawn through it both the vertical, such as VKT or VLQ, and a third, VIS; and the circle of latitude, such as BKI or BIL.

Diagram. Set across the left column, interrupting the word that follows the sentence above. A semicircle stands on a straight base. From the top, V, solid and dotted lines run down to the base. A curve runs from the upper left of the arc down to the right of the base; another arc runs from the left of the figure up to the right, to a small circle at the upper right, and a line runs from B down toward the centre of the figure.

- H (left end of the base): the altitude HE, in Section X
- A (on the base, right of H): not cited on this page
- Q (on the base, right of A, a swash letter set close against the next): the foot of the vertical VLQ
- S (on the base, joined to that sort): the foot of the vertical VIS
- T (on the base, right of that): the foot of the vertical VKT
- C (on the base, right of centre): not cited with this figure
- R (right end of the base): not cited with this figure
- V (top of the arc): the vertex
- E (upper left): the culminating point, in Section X
- D (upper middle): not cited on this page

- O (below D, a small round letter): not cited in the fourth case; the foot of the perpendicular KO in the passage under Section X
- N (below and to the left of D): the nonagesimal
- G (left, below N): not cited on this page
- L and I (centre, side by side, L on the left, I on the right at the junction of the lines): a place of the star, and the place in longitude
- M (below them): the foot of the perpendicular LM
- K (right of centre): a place of the star
- B (upper right, on the arc): the end of the circles of latitude BKI, BIL
- F (in the small circle): not cited on this page

First, as in the third case, find the base VI in the triangle NVI, and also the angle NIV. Then, if the star is at K, consider the triangle IKV, in which the northern latitude IK is given, and the side IV has been found, as has the angle VIK, which is the complement to a right angle of the angle NIV already found (for the circle of latitude always makes right angles with the ecliptic). By trigonometry, therefore, you can find the third side or base KV, namely the star's distance from the vertex; and its complement to a quadrant, which is the altitude TK; and the angle IKV, whose complement to two right angles is the angle VKB. But if the star is at L, look at the triangle LIV, in which the southern latitude LI is given, and the side IV has been found, as has the angle LIV contained by them, which is made up of NIV, already found, and the right angle LIN. You can therefore find the base LV, namely the star's distance from the vertex, whose complement to a quadrant is the altitude QL. Then you will seek the angle VLI, that is, the angle sought, VLB.

[Margin: A shortcut for the Problem in Case 4.]

I am of course aware that at ecliptic new Moons, because of the Moon's slight latitude, astronomers treat the Moon just as if it were on the ecliptic. I do not, however, think that this latitude should be neglected. And if you shrink from the tedium of further trigonometry, you ought to subtract the northern latitude IK from the base IV already found, so as to have KV, or to add the southern latitude LI, so as to have LV; for in this way you will come closer to the truth.

Idem Problema Aliter iuxta Ptolemæum Regiomontanum & Copernicum, datis Longitudine, ac Latitudine ſideris, & puncto Eclipticæ Culminante.

The same Problem otherwise, according to Ptolemy, Regiomontanus and Copernicus, given the longitude and latitude of the star and the culminating point of the ecliptic.

[Margin: Case 1.]

X. Ptolemy (*Almagest*, Book 2, Ch. 12, and Book 5, Ch. 19), Regiomontanus (*Epitome*, Book 2, prop. 45, and Book 5, prop. 30) and Copernicus (Book 2, Ch. 12, and Book 4, Ch. 26), when these passages are taken together, complete both parts of the Problem as follows. Let everything be set out as in the preceding

figure for the second and this third Problem, and let the star, or the star's place, be given at I, together with the point of the ecliptic at E. For the arc EI will be given, and, from the Tables or Problems of the primum mobile, the angle IEV of the ecliptic and the meridian in the triangle IEV; and from the tables of declination and the polar altitude there will be given, for the culminating point E, both the altitude HE and the distance EV from the vertex. From these three data in the triangle EIV you will therefore find both the distance IV of the point I from the vertex and the angle EIV.

[Margin: Cases 2 and 3.]

Next, let the star be at K or at L. For once the arc IV and the angle EIV have been found in the first case, and that angle has been taken from the right angle EIK or joined to the right angle LIE, there will be evident the angle VIK in the one instance and LIV in the other. So, as we did above in the third and fourth cases, Copernicus seeks from the given latitudes IK or LI the distance KV and the angle VKI in the triangle IKV, or the distance LV and the angle VLI in the triangle LIV. Ptolemy and Regiomontanus, by contrast, draw from K to IV, already found, the perpendicular KO; and in the triangle IKO, right-angled at O, from the angle OIK, found as above, and the latitude IK, they seek IO and OK. Then, with IO taken from IV, the arc OV becomes known to them in the triangle VOK, right-angled at O; and so, with OK already found, they seek the distance KV and the angle OKV, which, joined to the angle IKO, makes the desired angle IKV, whose complement to two right angles is the angle VKB.

Similarly, when the star is at L, with the perpendicular LM drawn to SV, they seek the arcs LM and MI in the triangle LIM, right-angled at M, from the angle LIM, which equals the angle OIK, and the latitude LI. With IV and MI joined, the arcs MV and ML become known in the triangle LVM, right-angled at M; and so in that triangle they seek the distance LV, with which, in the triangle LIV, together with the latitude LI and the distance IV, they seek the angle VLI.

[Margin: Reason why Ptolemy multiplies so many operations.]

In the small triangles LMI and KOI, moreover, they use curved lines in place of straight ones, because of the imperceptible difference arising from the nearness and smallness of these arcs; and they turn the more willingly, when they can, to right-angled triangles, for they think that this ease offsets the labour of the many-fold operation. Regiomontanus, though, in Book 5, prop. 31, gives another method using spherical triangles alone.

[Margin: A saving of labour according to Ptolemy, Regiomontanus and Copernicus.]

Yet when all this has been done, Ptolemy (Book 5, Ch. 19), Regiomontanus (Book 5, prop. 30) and Copernicus (Book 4, Ch. 26) admit that it is neither expedient nor necessary, in hunting down the Moon's parallaxes, to use her latitude, and that we can safely use the altitude of the Moon's place on the ecliptic and the angle made there by the vertical and the ecliptic. For the greatest error that can arise from this in the parallax of altitude is 10′, as Ptolemy

and Regiomontanus say, or 7′, as Copernicus says, and that only when the Moon's latitude at perigee is 5° and the parallax is horizontal. In eclipses of the Sun, moreover, an error greater than a minute and a half, or 1′ 30″, cannot be committed in the parallaxes, because in them the latitude cannot be greater than 1° 30′.

Idem Aliter ex Magino aut D. Vincentio Muto per ſola triangula rectangula, Datis Longitud. & Latitud. Sideris vnà cum Pūcto Eclipticæ Oriente, & Angulo Orientis.

The same otherwise, from Magini or D. Vicenç Mut, by right-angled triangles alone, given the longitude and latitude of the star together with the rising point of the ecliptic and the angle of the rising point.

[Margin: Problem of D. Vicenç Mut.]

XI. In the letters of D. Vicenç Mut, of which I spoke at the end of number 8, a part of this Problem is supplied to me, and it certainly deserved to be made whole by me and inserted here. Let the horizon be HR, the meridian HVR, the ecliptic EC, and let there be three vertical quadrants VQ, VS, VT. Let the Moon be now at I, without latitude, now at K, with the southern latitude KI, now at O, with the northern latitude OI; through these three places let the circle of latitude BOIK pass, and from these three places let three arcs OC, IC, KC be drawn to the rising point C.

Diagram. Set across the right column, interrupting the sentence above after "VQ, VS,". A semicircle stands on a straight base. From the top, V, straight lines run down to the base. A curve runs from the upper left of the arc, at E, down to C on the base; another curve rises from the lower left through the centre of the figure to B and beyond the arc at the upper right; further lines converge on C.

- H (left end of the base): horizon and meridian, per HR, HVR
- Q (on the base, a swash sort, right of H): the foot of the vertical quadrant VQ
- S (on the base, right of Q): the foot of the vertical quadrant VS
- T (on the base, right of S): the foot of the vertical quadrant VT
- C (on the base, right of centre): the rising point
- R (right end of the base): horizon and meridian, per HR, HVR
- V (top of the arc): the vertex
- E (upper left of the arc): the ecliptic, per EC
- B (upper right, outside the arc): the circle of latitude BOIK
- O (right of centre, upper): a place of the Moon with northern latitude
- N (left of centre, upper): cited in the second case, angle NCQ
- I (centre): a place of the Moon without latitude
- L (right of I): not cited on this page
- K (left of I, lower): a place of the Moon with southern latitude

[Margin: Case 1.]

First, if the Moon is at I, then in the triangle ISC, right-angled at S, the angle of the rising point ICS is given, and from the Moon's place I compared with the rising point C the arc IC of the ecliptic is given. Therefore the Moon's altitude SI will not remain hidden, nor will the parallactic angle SIC; for it is so called because it is useful for deriving parallaxes in longitude and latitude.

[Margin: Case 2.]

Second, let the Moon be at K, which is the case set out by D. Vicenç. In that event, in the triangle KIC, IC is given as above, and the southern latitude KI; therefore KC will not remain hidden, nor the angle KCI, since the triangle is right-angled at I by virtue of the arc of latitude. Then in the triangle KQC, right-angled at Q, with the base KC now found, and the angle KCI, already found, subtracted from the given angle of the rising point ECQ, so that KCQ is known, you will seek the Moon's true altitude QK and the arc QC. Finally, in the triangle NQC, right-angled at Q, with the arc QC found, as I said, and the angle of the rising point NCQ given, you will seek the parallactic angle QNC.

[Margin: Case 3.]

Third, let the Moon be at O. For then in the triangle IOC, right-angled at I, with the arc IC given as above, and the northern latitude IO, you will seek the base OC and the angle ICO, which you will add to the angle of the rising point ECH, so that the angle TCO is known. Then in the triangle OTC, right-angled at T, with OC and TCO now known, you will seek both the Moon's true altitude TO and the arc TC. Lastly, in the triangle LTC, right-angled at T, with the arc TC now found and the angle of the rising point TCL given, you will seek the parallactic angle TLC.

[Margin: Otherwise, from Magini.]

Otherwise, from Magini's General Table of the primum mobile you will find the altitude of the Sun by entering it with the angle of the ecliptic and the horizon and with the Sun's distance from the rising point. And with the same data, in the table of the third analogy, which is found in the Tables of Directions of the same Magini, seek the angle, marking at the foot of the table the Sun's distance from its rising or setting and on the left side the angle of the rising point; for the angle found and subtracted from 90° will give the angle of the ecliptic and the vertical.

Idem Problema aliter Per Tabulas aut vſum Globi vel Sphæræ, quoad altitudinem Luminarium, Dato eorum loco in Eclip-tica ad datum tempus, & Data Poli Altitudine.

The same Problem otherwise, by tables or the use of a globe or sphere, as regards the altitude of the luminaries, given their place on the ecliptic at a given time and given the polar altitude.

XII. In the Prutenic Canons, p. 99, there are tables constructed by Erasmus Reinhold, which Magini, in the tables of the *Secunda Mobilia*, p. 691, Renieri,

in the first edition of the *Tabulae Mediceae*, p. 296, and Argoli, in his own tables, p. 165, copied out in mutilated form, but Lansbergen, on p. 51 of his tables, in full. These tables are entered with whole hours after or before noon, and under the beginning of the signs in which the luminaries stand, at certain polar altitudes. Their second column, opposite the given hour, shows the distance of the luminaries from the vertex, whose complement to 90° is their altitude above the horizon. But those tables are only for polar altitudes of 16°, 24°, 31°, 36°, 41°, 45°, 49°, 52°, 54°, 57°, 60°, 63°, 66° and 70°. So when the polar altitude differs from these, or the luminaries are not at the very beginning of the signs, or the hours are not whole, or all these differences come together, one has to hunt for the proportional part by repeated calculation, to the point of tedium.

[Margin: A blemish in the Prutenic Tables corrected.]

Moreover, these tables assume the star to be on the ecliptic itself, without any latitude, on account of the assurance which, as we saw at the end of the preceding number 10, Ptolemy, Regiomontanus and Copernicus urged; yet we should be safer if the northern latitude were subtracted from the distance found in these tables, or the southern added.

[Margin: Kepler's slip.]

Meanwhile one must be very much on guard against a slip of imagination or memory by Kepler, in the *Astronomia Optica*, p. 320, where he says that the already mentioned tables of Reinhold show the distance of the nonagesimal from the vertex, when what they show is the distance not of the nonagesimal itself but of the star. This is clear from Reinhold's precept 54 and Lansbergen's precept 29, and even from the table itself. For in it, for example, under the polar altitude 45°, with the Sun at the beginning of Aries, at 6 hours before noon, you will find a distance of 90° from the vertex, which is certainly the distance of the Sun, since the nonagesimal, which is the beginning of Cancer, is then on the meridian, and on it stands only 21° 30′ from the vertex.

[Margin: The use of a globe or sphere substituted for the tables.]

But without so much trouble over the proportional part, the wiser course for anyone who dreads trigonometrical calculation is to use a globe or armillary sphere, with the meridian raised to the given polar altitude, and with the sphere or globe turned as far as the time of the distance from noon requires, when converted into parts of the equator. For a pair of compasses from the horizon to the degree of the ecliptic where the luminary is, or the arc of a quadrant drawn through that place and divided into degrees, will show the luminary's altitude just as well as the already mentioned tables, indeed better.

PROBLEMA IV.

Problem 4.

Data Luminarium Altitudine ab Horizonte, aut Diſtantia à Vertice per Problema 3. Inquirere Parallaxim Altitudinis vtriuſq. & inde Parallaxim Lunæ à Sole.

Given the altitude of the luminaries above the horizon, or their distance from the vertex, by Problem 3, to find the parallax of altitude of each, and from that the parallax of the Moon from the Sun.

[Margin: Authors of parallactic tables.]

XIII. This Problem is completed by the methods already given in Book 3, Ch. 8, as regards the Sun, and in Book 4, Ch. 13, as regards the Moon; or by tables constructed for the purpose, though constructed for the true altitude of the luminaries, which are presented by Ptolemy, Book 5, Ch. 18; Copernicus, Book 4, Ch. 24; and Reinhold in the Prutenic Canons, p. 120, but for new Moons only, a table that Magini copied in the *Secunda Mobilia*, p. 681. Tycho also gives his own, in Vol. 1 of the *Progymnasmata*, pp. 80 and 120, and from there Longomontanus, Book 1 of the *Theorica*, Ch. 8, p. 279; Magini in the *Supplement to the Ephemerides*, pp. 59 and 293; Mulerius in the *Tabulae Frisicae*, p. 421; Argoli in his tables, p. 154; Renieri in the first edition of the *Tabulae Mediceae*, p. 185; and Lansbergen likewise gives his own, pp. 46 and 47. For horizontal parallaxes alone, one may consult Kepler in the Rudolphine Tables, p. 98, and at the head of the parallactic table, p. 320 of the *Astronomia Optica*; Renieri in the later Medicean Tables, p. 312, and outside the syzygies, p. 364; Boulliau in the *Philolaic Tables*, p. 138, and outside the syzygies, p. 139; and Wendelen in the *Eclipses*, p. 34.

[Margin: Parallax of the Moon from the Sun.]

Now if the Sun's parallax is subtracted from the Moon's parallax, what remains is the parallax of the Moon from the Sun.

PROBLEMA V.

Problem 5.

Datis Parallaxi Altitudinis Luminarium per Problema 4. & Angulo Verticalis & Eclipticæ, ſi careant latitudine per Problematis 1. partem 1. vel Angulo Verticalis & Circuli latitudinis, ſi latitudinem habeant, per Problematis 5. partem 2. Inquirere Parallaxim Latitudinis & Longitudinis.

Given the parallax of altitude of the luminaries by Problem 4, and the angle of the vertical and the ecliptic, if they lack latitude, by Problem 1, part 1, or the angle of the vertical and the circle of latitude, if they have latitude, by Problem 5, part 2, to find the parallax of latitude and of longitude.

[Margin: Authors of this Problem.]

XIV. This Problem is more difficult when the Moon has a true latitude than when she has no latitude; and therefore Ptolemy laboured over it in Book 5, Ch. 19; Regiomontanus in the *Epitome*, Book 5, props. 26 to 32; Copernicus, Book 4, Ch. 26; Tycho, Vol. 1, p. 132; Longomontanus, but very obscurely, in Book 2 of the *Spherics*, Ch. 10, and Book 1 of the *Theorica*, Ch. 8, p. 282; Magini in the *Primum Mobile*, Book 12, problems 12 and 13; and Mulerius in the *Tabulae Frisicae*, p. 439. But no one did so more expeditiously than Magini, although he too uses several figures, or superfluous lines, and yet does not cover all the cases. Ptolemy indeed, Copernicus and Mulerius in the places above, and Regiomontanus in prop. 32, judge that no perceptible error is committed if, even when the Moon has latitude, we treat her as having no latitude, and use the angle of the vertical and the ecliptic made at the Moon's place on the ecliptic in place of the angle of the vertical and the circle of latitude made at the Moon's own centre. They judge that there is no need for such precision, and that more labour than profit is gained from greater subtlety. Finally, Ptolemy and Copernicus use rectilinear triangles in place of spherical ones, thinking that because of their apparent smallness no perceptible error is made. We, however, to remove every scruple from inquisitive astronomers, shall first cover all the cases with a single figure, and shall use true angles and right-angled spherical triangles, because with the help of logarithms they are now no harder than rectilinear ones. Then we shall add a second figure, by which the same problem can be solved more exactly and more generally.

[Margin: Case 1 of the Problem.]

Let the horizon be HAR; the meridian HVR, on which are the zenith or vertex V and the pole of the world P, around which is the Arctic small circle, on whose circumference let B be, the pole of the ecliptic ESC; and let the vertical or circle of altitude be VIA, on which, *first*, let the Moon be at I, without any true latitude, but because of the parallax of altitude IL let her appear at L. For when the circle of latitude BL has been drawn from B through the Moon's apparent place on the vertical, Q will be her apparent place on the ecliptic, and so IQ will be the parallax of longitude and LQ that of latitude, which in this case will be the apparent latitude. And there will arise the triangle ILQ, right-angled at Q, in which are given the base IL, the parallax of altitude, and the angle LIQ of the ecliptic and the vertical. The parallaxes of longitude IQ and of latitude LQ will therefore not remain hidden. For let it be: as the radius or whole sine is to the sine of the angle LIQ, so is the sine of the base LI to the sine of the arc LQ. Or join the logarithm of the angle LIQ to the logarithm of the base LI, and the logarithm of the arc LQ, that is, of the parallax of latitude, will result. Again, let it be: as the radius is to the secant of the angle LIQ, so is the second tangent of the base LI to the second tangent of the arc IQ. Or add the residual of the second logarithm of the angle LIQ to the second mesologarithm of the base LI, and the second mesologarithm of the arc IQ will result, that is, of the parallax of longitude. In this and the following cases, however, one must use the parallax of altitude of the Moon from the Sun.

[Margin: Case 2 of the Problem.]

Second, let the Moon be at F, but appear at K. For when the quadrants of latitude have been drawn from B as far as the ecliptic, BFD through the Moon's centre and BKG through the place of the apparent altitude, D will be her true place on the ecliptic and G her apparent place; and so DG will be the parallax of longitude, the true latitude FD and the apparent latitude GK, though of the same side or kind. Therefore, with the arc KO drawn from K parallel to DG, and so perpendicular to FD, the parallax of latitude will be the arc OF.

Diagram. Set across the left column, interrupting the sentence above after the word `atque`. A semicircle stands on a straight base. From the top, V, straight lines run down to the base. From B, at the upper right, a sheaf of curved lines runs down to the left across the figure; a small circle at the upper right has P at its centre, with a line from P up to B. Longer curves run from the left side of the figure across to R at the right end of the base. The centre of the figure is a crowded cluster of small letters and sorts, not all of which are separated.

- H (left end of the base): the meridian, by the earlier statement HVR, and the horizon HAR
- A (on the base, right of H): the foot of the vertical VIA
- λ (on the base, right of A): the foot of the vertical Vλ
- ε (on the base, right of λ): the end of the quadrant Vε
- C (on the base, right of centre): the foot of the vertical VC
- R (right end of the base): the common point of the arcs RΦX and RθY
- V (top of the arc): the vertex
- E (upper left of the arc): the ecliptic, by the earlier statement ESC
- O (right of E): the foot of the arc KO
- F (right of O, below V): a place of the Moon
- B (upper right): the pole of the ecliptic
- P (centre of the small circle): the pole of the world
- D (left, below E): the Moon's true place in Case 2
- K (right of D): a place of the Moon
- G (below D): the Moon's apparent place in Case 2
- T (left, below G): cited in Case 5
- I (right of G): a place of the Moon
- L (below T): a place of the Moon
- Q (a swash letter right of L): cited in Cases 1, 4, 5
- S (below Q): cited in Case 4
- two sorts [?] stacked on the vertical line just left of X, the lower resembling δ (the text's true place of the Moon in Case 9); neither settled
- X (centre, upper right): the Moon's true place in Cases 8 and 9
- Z and ϒ (side by side below S, ϒ against the vertical line): cited in Cases 4 and 9
- Y (right of the central cluster): the Moon's apparent place in Cases 8 and 9
- N (lower left of the cluster): cited in Case 4

- M (right of N, lower): cited in Case 4
- Φ (right, upper): a place of the Moon in Case 8
- Ψ (below Φ): cited in Case 8
- θ (right of Ψ): the Moon's apparent place in Case 8

The arc OK, although strictly it is not a portion of a great circle, will nevertheless, because of the nearness, be such to the senses, and equal to the parallax of longitude DG. And there will arise the triangle KFO, right-angled at O, in which are given the base FK, which is the parallax of altitude, and the angle KFO of the vertical and the circle of true latitude. Let it be, therefore: as the radius is to the secant of the angle F, so is the second tangent of the base FK to the second tangent of the arc OF. Or add the residual of the second logarithm of the angle E [recte: F] to the second mesologarithm of the base FK, and the second mesologarithm of the arc OF will result, namely of the parallax of latitude. Again, let it be: as the radius is to the sine of the angle F, so is the sine of the base FK to the sine of the arc KO. Or add the logarithm of the angle F to the logarithm of the base FK, and the logarithm of the arc KO will result, which is equivalently the parallax of longitude.

[Margin: Case 3 of the Problem.]

Third, let the Moon be at K, but appear at I, a point of the ecliptic, so that the parallax cancels the whole true latitude. For with the single quadrant of latitude BKG drawn through the Moon's centre, the triangle GKI will be formed, right-angled at G, in which are given the base IK, which is the parallax of altitude, and the angle at K made by the vertical and the circle of latitude. Therefore, by exactly the same rules as in the second case, the arc GK will become known, which is the parallax of latitude, and GI, which is that of longitude.

[Margin: Case 4 of the Problem.]

Fourth, let the Moon be at L, but appear at M. For with two arcs of latitude drawn from the pole B across the ecliptic, one to the place of the apparent altitude M, which is to be BSM, the other through the Moon's centre as far as the point N, on which there falls from M the arc MN, parallel to the ecliptic or perpendicular to LN, Q will be the Moon's true place on the ecliptic and S her apparent place. And so the parallax of longitude, taken formally, will be the arc QS, while the true latitude, but toward the other pole, will be the arc LQ, and the apparent latitude MS. Therefore, with the arc LZ drawn from L, the Moon's centre, parallel to QS or perpendicular to SM, the parallax of latitude will be the arc MZ, and that of longitude, taken equivalently, the arc LZ. But because of the need to solve the problem, the triangle LMN is employed, right-angled at N, in which, because of the nearness and the already mentioned parallelism, the arc NL is perceptibly equal to MZ, the parallax of latitude, and NM to LZ and QS, the parallax of longitude. And in it are given the base LM, which is the parallax of altitude, and the angle NLM, made at the Moon's centre by the vertical and the circle of true latitude. Therefore, by altogether the same rules as in the second case, the arc LN will become known, which is equivalently the parallax of latitude, and NM, which is equivalently the parallax of longitude.

[Margin: Case 5 of the Problem.]

Fifth, let the Moon be at K, on this side of the ecliptic, but appear at L, beyond the ecliptic, so that the parallax changes her latitude from northern to southern, if the pole B is northern, or from southern to northern, if the pole B is southern. For draw two arcs of latitude from the pole B beyond the ecliptic: one to the place of the apparent altitude L, which is to be BL; the other, BGT, through the Moon's centre K as far as the point T, to which from L the arc LT can be drawn, parallel to the ecliptic or perpendicular to BGT. For the Moon's true place on the ecliptic will be the point G, and her apparent place Q; and so the formal parallax of longitude will be the arc GQ, and the true latitude toward one pole will be the arc KG, and the apparent latitude toward the other pole the arc QL. And so the parallax of latitude will be the arc KGT, because GT, on account of the nearness and the parallelism of the arc LT, is equal to LQ, just as TL is to GQ. There therefore arises the triangle KTL, right-angled at T, in which are given the base KL, which is the parallax of altitude, and the angle TKL, made at the Moon's centre by the vertical and the circle of true latitude. Therefore, by altogether the same rules as in the second case, the arc TL will become known to you, which is equivalently the parallax of longitude, and the arc KGT, which is equivalently the parallax of latitude. If you subtract from it the true latitude KG, there will remain GT, that is LQ, the Moon's apparent latitude.

[Margin: Case 6 of the Problem.]

The *sixth* case is when the vertical circle in which the Moon stands and the circle of latitude are the same; but then the star is at the nonagesimal, either with no latitude or with some, and then the parallax of altitude equals the parallax of latitude, as I said in Problem 3, Cases 1 and 2, and there is then no parallax of longitude.

[Margin: Case 7 of the Problem.]

The *seventh* case is when the vertical circle in which the Moon stands and the ecliptic are the same, a case that occurs only within the Torrid Zone; and then the parallax of altitude is the same as the parallax of longitude. Thus, if the quadrant Vε is at once the vertical and the ecliptic, and the Moon is at X, but appears at Y, the arc XY will be the parallax of altitude and of longitude, and there will be no parallax of latitude.

[Margin: Case 8 of the Problem.]

The *eighth* case is when the ecliptic is one of the verticals, but the Moon, because of her latitude toward your pole, is on another vertical; as if the ecliptic is Vε, but the Moon's vertical is VC, and the Moon is at Φ, but appears at θ. In this case, with the arcs of latitude drawn through the true and apparent altitudes as far as the ecliptic, namely RΦX and RθY (for the pole of the ecliptic will then be either on the horizon itself or not far from it), the Moon's true place on the ecliptic will be the point X and her apparent place Y, and so the parallax of longitude will be the arc XY, the true latitude ΦX and the apparent latitude θY. In the triangle RΦθ, therefore, seek the arc Rθ; for in it are given the angle

at Φ, made by the vertical and the circle of true latitude; RΦ, the complement of the true latitude to a quadrant; and Φθ, the parallax of altitude. For when θR is taken from 90°, there will remain θY, the apparent latitude. And with the arc ΦΨ drawn parallel to the ecliptic or perpendicular to RY, the arc Ψθ will be the parallax of latitude, which will be known when the true latitude XΦ, equal to YΨ, is subtracted from the whole Yθ. And so in the triangle ΦΨθ, right-angled at Ψ, are given the base Φθ, the parallax of altitude, and the side Ψθ, the parallax of latitude. The side ΦΨ will therefore not remain hidden, which differs imperceptibly from the parallax of longitude XY.

[Margin: Case 9 of the Problem.]

The *ninth* case is when the ecliptic is one of the verticals, such as Vε, but the Moon, because of her latitude toward the other pole, is on another vertical, such as Vλ. For let the Moon be at δ, but appear at ϒ; and with the circles of latitude produced as far as there, let the true latitude be δX and the apparent latitude ϒY, and let the Moon's true place on the ecliptic be X and her apparent place Y, and so the parallax of longitude XY. Draw, then, the arc ϒX, and in the triangle ϒδX seek both the arc ϒX and the angle ϒXδ, since in it are given the angle at δ, made by the vertical and the circle of true latitude; ϒδ, the parallax of altitude; and δX, the true latitude. Then in the triangle ϒYX, right-angled at Y, seek both ϒY, the apparent latitude, and XY, the parallax of longitude, since in it are given the base ϒX, already found, and the angle ϒXY, from the subtraction of ϒXδ, already found, from the right angle YXδ.

But for Cases 8 and 9 the Moon's altitude and the angle of the vertical and of latitude are found otherwise than by the method given above in Problem 3. For here, from the Moon's true place on the ecliptic, X, and the culminating point V, the difference between them, that is the arc VX, is obtained; and in the triangle VXΦ, right-angled at X, with the true latitude XΦ, both VΦ, the Moon's distance from the vertex, and the angle VΦX are obtained; or in the triangle VXδ, right-angled at X, with the true latitude Xδ, both Vδ, the Moon's distance from the vertex, and the angle VδX are obtained, whose complement to two right angles is the angle ϒδX. In this way I believe I have covered all the cases of these parallaxes when a star, as the Moon does at ecliptic or near-ecliptic new Moons, has a modest latitude.

Idem Problema exactiùs abſoluere Datis Latitudine vera Sideris, Parallaxi Altitudinis, & dato vel inuento Angulo Parallactico, ideſt facto ab Ecliptica & Verticali per centrum ſideris ducto.

To complete the same Problem more exactly, given the star's true latitude, the parallax of altitude, and the parallactic angle, whether given or found, that is, the angle made by the ecliptic and the vertical drawn through the star's centre.

XV. This Problem does not need arcs of circles that are not great circles, which, because of their nearness and parallelism, would be assumed to be

sensibly equal to arcs of great circles. It rests instead on arcs of great circles, whether remote from the ecliptic or near it, and it is therefore more exact, and indeed also more universal, and it holds for any star, however far from the ecliptic. So in the adjoined figure, placed at the beginning of the chapter at number 2, let everything be as set out there.

[Margin: Case 1.]

And *first*, let the star be at G, a point of the ecliptic EC, without any latitude, but let it appear at I, a point of the vertical VA.

Diagram. Set across the left column, interrupting the sentence above within the word `pun- / cto`. A semicircle stands on a straight base, with H at its left end, A and C below the base, and R at its right end. V stands at the top of the arc; a straight line runs from V down to the left, through D, to the base at A, and another short line runs from V to the right, to P. A small circle at the upper right has B near its centre, and a line runs from B down to P, at the circle's lower edge. From P a sheaf of curved lines runs down to the left across the figure, through the crowded cluster of letters. A longer curve runs from the upper left of the arc, through G, down to C on the base.

- Labels read: V (top of the arc); B (inside the small circle at upper right); P (lower edge of the small circle); D (on the line from V); M, N, G, O (the central cluster, N set small and close against G); I; Q (with a short flourish beneath it); K; H (left end of the base); A (below the base); C (below the base); R (right end of the base).
- Labels not separated: the letter at the upper left of the arc, which reads as E or F `[?]`, and the letter just below D, which reads as F or E `[?]`. The text calls for both an E and an F in this figure, but which stands where is not settled here.

For when the arc of latitude PI is drawn to the apparent place, the star's apparent place on the ecliptic will be the point O, and the parallax of longitude GO, and that of latitude, or rather the apparent latitude itself, IO, in the triangle GIO, right-angled at O. In this triangle GI, the parallax of altitude, is given, and the parallactic angle IGO, or at any rate let it be found by the method to be given at the end of the Problem. So IO, the parallax of latitude, and GO, that of longitude, will not remain hidden.

[Margin: Case 2.]

Second, let the star be at D, but appear at F. For when the arcs of latitude are drawn, PDM through the place of the true altitude and PFN through that of the apparent altitude, the true place on the ecliptic will be M and the apparent place N, and the true northern latitude DM, but the apparent latitude FN. So in the triangle MGD, right-angled at M, the latitude DM is given, and the parallactic angle MGD. Seek, then, the base GD and the side MG; next subtract the parallax of altitude DF from the base DG, and there will remain, known, the base FG in the small triangle NGF, right-angled at N, in which the parallactic angle NGF is given. Seek, then, the arc NF, which is the apparent latitude, and which, subtracted from the true latitude MD, will give the parallax of latitude.

Then find the arc NG, which, subtracted from the arc MG, will leave MN, the parallax of longitude.

[Margin: Case 3.]

Third, let the star be at F, but appear on the ecliptic itself at G. For in the triangle NGF, right-angled at N, the true latitude NF is given, the whole of which is cancelled by the parallax, and so the parallax of latitude is as great as the true latitude. The angle NGF is also given in the same triangle. Seek, then, the arc NG, namely the parallax of longitude.

[Margin: Case 4.]

Fourth, let the star be at I, but appear at K. For when the arcs of latitude are drawn, POI through the place of the true altitude I, and PQK through that of the apparent altitude, the true place on the ecliptic will be the point O, but the apparent place the point Q, and the true southern latitude OI, but the apparent latitude QK. Now in the triangle IGO, right-angled at O, the true latitude OI is given, and the parallactic angle IGO; therefore find the arcs GO and GI separately. Then in the triangle KGQ, right-angled at Q, by joining GI, the arc found, and IK, the parallax of altitude, the base GK will be given; with this and with the parallactic angle KGQ, search out the arc GQ, and KQ. For if from the arc GQ you subtract GO, found above, there will remain the parallax of longitude OQ; and KQ will be the apparent latitude, greater than the true latitude. So when the true latitude is subtracted from it, the parallax of latitude will be known.

[Margin: Case 5.]

Fifth, let the star be at F, but let it be thrown down by parallax across the ecliptic to I. For first, in the triangle NGF, right-angled at N, with NF, the northern latitude, given, and the parallactic angle at G, you will find the arcs NG and FG. Then, when FG is subtracted from the parallax of altitude FI, there will remain, known, the base GI of the triangle GIO, right-angled at O, with which and the angle G you will find GO and IO. So when the arcs of the ecliptic NG and GO are put together, the parallax of longitude NO will be known. The parallax of latitude, moreover, will consist of the true northern latitude NF, which it cancels, and of the apparent southern latitude IO.

[Margin: Method of finding the parallactic angle.]

This method is much quicker than the one indicated by Copernicus, Book 4, Ch. 26, or by Longomontanus, Book 1 of the *Theorica*, Ch. 8, as will be plain to anyone who compares it with them. For finding the parallactic angle is not difficult, since it is the angle opposite the true latitude, which is already given in a right-angled spherical triangle, in which there is also given, by Problem 3, the angle made at the star's centre by the vertical and by the circle of latitude. For example, in the triangle MDG, given the angle at D and the latitude MD, MGD is easily found; and so for the others.

Idem Problema per Tabulas Parallacticas aut illis æquiualentes abſoluere, Datis Longitudine Luminarium, & Tempore ante vel poſt meridiem, cum Altitudine Poli, vel Angulo Orientis, & Angulo Parallactico.

To complete the same Problem by parallactic tables or tables equivalent to them, given the longitude of the luminaries and the time before or after noon, with the polar altitude, or the angle of the rising point, and the parallactic angle.

[Margin: Method 1, by the Alphonsine, Prutenic and Lansbergian tables.]

XVI. The first method is by the tables of Alfonso, p. 112, which run from a polar altitude of 16° 39′ to 48° 40′; or rather by those of Reinhold in the Prutenic Tables, p. 99, and of Lansbergen in his tables, p. 51, from 16° to 70°. These are entered under the whole sign in which the luminary stands, under the appropriate polar altitude, and with the whole hours by which the luminaries are distant from noon, and one seeks the side of longitude [latus longitudinis], the side of latitude [latus latitudinis], and the luminary's distance from the vertex. With that distance one seeks, in the tables of parallaxes of altitude, the parallax of the Moon and of the Sun, and from these the parallax of the Moon from the Sun. But since in the right-angled triangle on which the tables already mentioned rest, the parallax of altitude is always assumed as 60′, therefore, if the parallax of altitude is not 60′, one must find the side of latitude and the side of longitude proportional to it, by making: as 60′ is to the corresponding parallax of altitude, so the side of latitude found in the table is to the parallax of latitude; and again, as 60′ is to the corresponding parallax of altitude, so the side of longitude found in the table is to the parallax of longitude. This would indeed be no great labour if the luminaries were precisely at the beginning of the signs, and the hours before or after noon were whole, without minutes, and the polar altitude set at the head of the tables were the very one being worked for. But since for the most part these things do not coincide, beyond the two operations already mentioned one must hunt again and again for the proportional part, for the difference between polar altitudes, between whole signs and between whole hours, as can be seen in the precepts that are given on this practice by the Alphonsines, prop. 32, from number 123; Reinhold, precept 54; Magini in the *Secunda Mobilia*, Canons 75 and 76; and Lansbergen, precept 29. This trouble becomes much greater when all this has to be repeated for the beginning and the end of a solar eclipse, so that Kepler, in the *Astronomia Optica*, p. 316, rightly said of Reinhold and his followers: *the ardour of the ingenious was crushed by the tedium of seeking the proportional part.*

[Margin: Method 2, by the tables of Peuerbach.]

The *second* method is by the tables of eclipses of Peuerbach, but these were computed only for Climates 6 and 7. In them, given the hours before or after noon, the whole signs in which the luminaries stand, and the whole signs of the lunar anomaly, one finds both the distance of the apparent conjunction

from the true conjunction and the parallax in latitude. But when these are not whole, a manifold search for proportional parts is needed.

The *third* and quickest method is by the parallactic table that Kepler constructed and presented in the *Astronomia Optica*, p. 320, based on Theorems 13, 14 and 15, brought forward above at number 3; from him Vincenzo Renieri copied it out at the end of the *Tabulae Mediceae* in their second edition, p. 332, and Boulliau in the *Philolaic Tables*, p. 173.

[Margin: Method 3, by the tables of Kepler, Renieri and Boulliau.]

In this table, under the luminary's distance from the centre of the Earth, or under the horizontal parallax of the Moon from the Sun, placed at the head of the table, which is supposed known from the anomaly and the tables of semidiameters, etc., and opposite the distance of the nonagesimal from the vertex, considered on the Moon's orbit, known by Problem 2, and indeed most easily by the precept and use of the table described there at number 8, precept 5, but corrected by the advice of Renieri in the same place: with these two elements, I say, the parallax of latitude is found in the fields of the table. Then by the altitude of the nonagesimal, under the greatest and horizontal parallax, the greatest parallax of longitude is sought; and when this has been sought again at the head of the table, opposite the apparent distance of the luminaries from the nonagesimal degree, the parallax of longitude is sought. Both are obtained accurately at the time of solar eclipses, because the Moon is almost on the ecliptic as far as appearance goes. The apparent distance of the Moon from the nonagesimal is obtained if the nonagesimal, by Problem 1, and the Moon's place on the ecliptic are known, for their difference will give their true distance. With this seek, rather roughly, the parallax of longitude, and add it to the true distance if the Moon is beyond the nonagesimal, subtract it if she is on this side of it, and the result will be the apparent distance from the nonagesimal. With this seek again, as above, a more exact parallax of longitude. This precept is to be observed at least when the Moon's apparent place is known from observation, as Kepler advises in the *Optics*, p. 320.

[Margin: Method 4, by logarithms.]

The *fourth* method is by logarithmic tables, either Kepler's in the Rudolphine Tables or ours in the Trigonometric Book, which will be the first of the second volume of the *Almagestum Novum*. So, following Kepler in the *Epitome of Copernican Astronomy*, p. 884, add into one sum the logarithm of the horizontal parallax of altitude and the antilogarithm, or second logarithm, of the angle of the rising point; and the sum will be the logarithm of the parallax of latitude. Then add into one sum three logarithms, namely those of the horizontal parallax of latitude, of the angle of the rising point, and of the star's distance on the ecliptic from the nonagesimal; and the result will be the logarithm of the parallax in longitude.

[Margin: Method 5, by the General Table of Magini.]

The *fifth* method is by Magini's General Table of the primum mobile, on whose right side the parallactic angle is sought, and at the head the parallax of

altitude of the Moon from the Sun, or of a star on the ecliptic or placed very near it; for in the field you will find the parallax of longitude. And in the same table, by seeking the parallax of altitude of the Moon from the Sun at the head, and the parallactic angle on the left side, the field gives the parallax of latitude of the Moon from the Sun. So Magini in the *Supplement to the Ephemerides*, p. 280, admitting that he used a rectilinear triangle, because its base never exceeds 66′.

SCHOLIA.

Scholia.

I. The definitions of parallax placed in the preceding chapter at number 2 were given according to the common view of those who hold that the centre of the world and of the Earth are the same; but in the view of some Pythagoreans and of the Copernicans these centres are different, and the centre of the world is the centre of the Sun itself. Whether, then, from parallaxes that suppose the same centre and yet display the phenomena quite accurately, any argument can be drawn against the Copernican system, I shall examine below in Book 9, where this system is treated. Meanwhile I am well aware that Kepler, in the *Optics*, p. 321, denies that from the appearances this Optical system rather than the Tychonic can be chosen.

[Margin: Comparison of the old astronomy with the recent in the hunt for these parallaxes.]

II. Although Ptolemy presents us, in Book 5, Ch. 19, with a lucid outline [hypotypoſim] of the parallaxes and of their variation, not so well understood by those who follow the Latin version, which is corrupt at this point, yet in use and practice he confidently judged that the subtlety he professed in theory could be neglected, because of the slight error that arises from this neglect, as I already said toward the end of number 10. Hence he knowingly admits that he neglected the parallax of the Sun, chose the analysis of rectilinear triangles in place of curvilinear ones, and took the angle of the vertical and the ecliptic in place of the angle of the vertical made on the Moon's orbit, neglecting the Moon's latitude. With him agrees Regiomontanus, as we have already said at the end of number 10, and Copernicus, Book 4, Ch. 26, who says that in the hunt for these parallaxes the Moon's latitude may safely be disregarded: "This computation, which is spent on the smallest matters, has more labour than profit." Moreover, in Ptolemy, in the same place, Hipparchus, seeing astronomers misuse the altitude of the Moon's place on the ecliptic and the angle made there by the vertical in place of the altitude of the Moon herself and the angle made at her, tried, though unsuccessfully, to remove this misuse. But neither did Reinhold, Lansbergen, Argoli and whoever else have used, in a table of the parallax of altitude, the altitude of the Moon's place on the ecliptic, just as if she had no latitude, remove it. Finally Kepler, who removed the tedium of solving triangles, or of repeatedly seeking the proportional part in the Prutenic

Tables, by his new and most highly praised parallactic table in the *Astronomia Optica*, p. 320, nevertheless slipped in two ways in the same place.

[Margin: Praise of Kepler, but a double blemish.]

First, he teaches there that one should use the distance of the nonagesimal from the vertex, with the nonagesimal itself considered on the ecliptic, whereas it is more correct to use its distance when considered on the Moon's orbit.

[Margin: Praise of Renieri, Boulliau and Vicenç Mut.]

This was corrected by Renieri and Boulliau, but more exactly by Vicenç Mut, as I said at number 8, precept 5. Second, he said that the distance of the nonagesimal from the vertex was to be looked up in Reinhold's table, p. 99, whereas that table contains the distance from the vertex not of the nonagesimal itself but of the luminary, as we showed at number 11. For he had not yet composed the Rudolphine Tables; but even when they were composed he did not recognise this error [Greek: ἀμάρτημα (hamartēma, "error")]. On the contrary, in Ch. 28, precept 132, he sent the reader back to his *Optics*, granted that he taught there and in the *Epitome of Copernican Astronomy* another method of seeking the parallaxes of longitude and latitude through his logarithms and antilogarithms, based on the angle of the ascending point and the Moon's distance from the nonagesimal.

[Margin: Disparity of the parallaxes because of the change of the solar apogee.]

III. One scruple remains, arising from the change of the apogee and of the eccentricity. As to the first, Reinhold, on Peuerbach's *Theoricae*, p. 194½, observes that points of the ecliptic equally distant from the solstitial points have the same variation of parallaxes, but in different quadrants of the ecliptic standing above the horizon. For example, because the beginning of [?] and of Pisces are equally distant from the solstitial points, in Climate 7 the true new Moon differs by 1¾ hours because of the parallax of longitude, if the true new Moon occurs 4 hours before noon with the Sun placed at the beginning of [?], or if it occurs 4 hours after noon with the Sun at the beginning of Pisces. This, he observes, is exactly true when the Sun's apogee stands near the solstitial points, and he indicates that it is not so exactly true when the apogee is farther from them. And so I did not list this property among the Theorems given at number 3.

[Margin: A small equation of the parallaxes because of the Moon's eccentricity.]

IV. As to the eccentricity, but of the Moon, the Alphonsines in their tables, p. 116, and in the precepts, prop. 32, marginal number 112, and Renieri in the later *Tabulae Mediceae*, pp. 286 and 313, but setting out the Alphonsine form of calculation, teach that the horizontal parallax given in the tables holds when the Moon, at the very moments of the syzygies, is at the apogee of the eccentric; but that outside those moments, even if only a few hours before or after conjunction and opposition, the Moon approaches the Earth because of the eccentricity and so increases the parallax, and that a correction is needed,

which is at most 51″. They therefore give a small table of these sexagesimal parts [horum ſcrupulorū], which, multiplied by the horizontal parallax, with the sum divided by 60, give the part to be added to the parallax, etc.

CAPVT XI.

Chapter 11.

De Terminis, Magnitudine, Duratione, Initio, Medio, Fine, alijſq. momentis Eclipſium Solarium.

On the limits, magnitude, duration, beginning, middle, end and other moments of solar eclipses.

THEOREMA I.

Theorem 1.

Cum PROBLEMATIBVS de Terminis Eclipſium Solis.

With Problems on the limits of solar eclipses.

[Margin: What the limits of a solar eclipse are, and how many.]

I. Some limits [Termini] of solar eclipses are possible, others necessary. They are the distances of the luminaries from the nodes of the lunar orbit with the ecliptic, within which a solar eclipse is possible somewhere, or even necessary, and outside which it is not possible, or is not necessary. Their determination depends on the apparent semidiameters of the Moon and the Sun, and on the Moon's visible, or apparent, latitude. For if the apparent latitude is equal to or greater than the sum of these semidiameters, there will be no eclipse; if it is less, there will be a solar eclipse somewhere. But the apparent latitude itself depends on the parallax of latitude, and this on the horizontal parallax of altitude, which is not the same among all astronomers, just as neither is the true latitude the same at the same distance from the nodes, nor the sum of the semidiameters the same; and from this a manifold variety of opinions on this subject arises. Besides, some fix these limits for mean new Moons, others for true new Moons; and again, some express them by the distance of the luminaries from the Moon's nodes, others by the motion of latitude counted from the northern limit, though from this the distance from the node is derived; others by the apparent latitude; and this last, some for the moment of the true synod, others for that of the apparent or visible synod. Finally, nearly all assign these limits for inhabitants on this side of the Torrid Zone, but not beyond the seventh Climate, with Ptolemy, that is, not beyond 48° 31′ of north polar altitude, and scarcely anyone determines them universally. We shall therefore divide these authors into various classes, and afterwards bring forward our own view, supplying what is lacking on this point in Copernicus, Tycho, Longomontanus, Boulliau and Wendelen, who have set down no limits.

[Margin: Opinion of Ptolemy and his followers.]

II. First, then, Ptolemy, Book 6, Ch. 5, and with him Regiomontanus, Book 6 of the *Epitome*, prop. 7; Reinhold on Peuerbach, p. 198, and in the Prutenic Tables, precept 61; Wurstisen on Peuerbach, p. 349; Neander in the *Elementa*, p. 130; Magini in Canon 84 of the *Secunda Mobilia*; Origanus in part 2 of Vol. 1 of the *Ephemerides*, Ch. 10; Mulerius in the *Tabulae Frisicae*, p. 77; and Argoli in his tables, Ch. 31, fix the possible limits: if at mean new Moons the Moon has not been distant from either node by more than 20° 41′ toward the north, or in round figures 20° 40′, and by no more than 11° 22′ toward the south, or, as Regiomontanus has it, 10° 55′. Understand this with respect to those who are on this side of the Torrid Zone and live in the North Temperate Zone, not beyond 49° of polar altitude. But with respect to those who live beyond the Torrid Zone, in the South Temperate Zone as far as 49°, if a solar eclipse is to be possible, the Moon must not be distant from either of the nodes by more than 20° 41′ toward the south, or 11° 22′ toward the north, as the parallax of latitude requires. For if she is distant beyond 11° 22′ toward the other pole, the parallax will increase the latitude, which at that distance is equal to the sum of the semidiameters; but if she is distant by more than 20° 41′ toward the pole of the place for which the calculation is undertaken, the latitude will be so great that, whatever parallax is computed as diminishing it, it will still never be less than the sum of the semidiameters of the Sun and the Moon. But if she is distant from the nearer node by less than 11° 22′ in the one case, and by less than 20° 41′ in the other, an eclipse will still not follow at once. For it can be that, either because of the semidiameters decreasing toward the apogee, or because of the parallax not diminishing the latitude sufficiently, or increasing it toward the other pole, there follows no eclipse, which could have occurred with another proportion [ſymmetria] of semidiameters and parallaxes.

[Margin: Argoli's threefold error in his way of speaking.]

Caution is therefore needed with Argoli, who does not speak with precision when he pronounces all too carelessly: *If, at the middle of the true conjunction of the luminaries, the equal motion of the Moon's latitude is not distant from the northern node ☊ by more than 20° 40′, or from the southern ☋ by 11° 22′, that synod will be ecliptic.* And giving an example he says that, since in it the Moon was not distant from the southern node by 11° 22′, *for that reason that conjunction had to be with an obscuration of the Sun.* For he errs in three ways. First, because he confuses the limits of the mean new Moon with the limits of the true new Moon. Next, because he confuses the limits of a possible eclipse with the limits of a necessary eclipse: for even if an eclipse necessarily occurs somewhere within the Torrid Zone if the Moon is not distant from the nodes by more than 15° 40′, let alone 11° 22′, still outside the Torrid Zone this is not true, and much less so if she is distant by less than 20° 41′ but by more than 15° 40′. Lastly, he confuses distance toward the north with distance from the northern node. For the Moon can be distant from the northern node and yet have a southern latitude, and so, even if she is distant from it by less than 20° 41′, a solar eclipse can be

impossible; and conversely she can be distant from the southern node and yet have a northern latitude, and so an eclipse can be possible, even if she is distant from it by more than 11° 22′. I do indeed believe that he understood it correctly, but his unconsidered way of speaking could harm beginners who have not been instructed from elsewhere. Ptolemy distinguished those distances from the nodes of the ecliptic most correctly by these formulas, [Greek: πρὸς αρκτους (pros arktous, "toward the north"); the mark over the alpha [?]] and [Greek: πρὸς μεσ[?]βρίαν (pros mes[?]brian, "toward the south")], that is, toward the north and toward the south; and Wurstisen most distinctly, saying that an eclipse is possible if it is less distant from the northern node than 11° 22′ before it or 20° 41′ after it, and from the southern node, before it, than 20° 41′, and after it, than 11° 22′. Mulerius, for his part, from Ptolemy's hypothesis added both the limits of an eclipse necessary somewhere, if at true new Moons the Moon's distance from the nodes is less than 15° 38′, and the limits of a possible eclipse for true new Moons, 17° 41′ toward the north and 8° 22′ toward the south.

[Margin: Opinion of al-Battānī and the Alphonsines.]

III. Yet some departed somewhat from Ptolemy: first al-Battānī, Ch. 43, because of a different opinion on the diameters of the luminaries, on account of which he fixes the possible limits at mean ☌ at 20° 12′ toward the north and 10° 40′ toward the south; second, the Alphonsines, prop. 32, who, in broad terms [Greek: ὁλοσχερῶς (holoscherōs, "in general, broadly")], defined these limits at 12° on each side.

[Margin: And of Langius, from Copernicus.]

Our own Langius, in the *Theorica* of his *Elementale*, decrees a solar eclipse impossible if the apparent latitude at the apparent synod of the luminaries exceeds 34′ 51″, possible if it is less than 34′ 51″, but necessary if it is less than 30′ 40″. In this he follows Copernicus, Book 4, Ch. 30, where, although he does not set down these limits, he nevertheless teaches that this criterion [Greek: κριτήριον (kritērion, "criterion")] is to be taken from the Moon's visible latitude at the moment of the visible conjunction. This requires the labour of investigating the parallax in longitude and latitude. So that such great labour may not be undertaken in vain, the others brought forward at number 2, with Ptolemy, seem to me to advise wisely that we should first form a conjecture about the risk of an eclipse for the time of the mean, or at most of the true, new Moon, from the Moon's bare distance from the nodes. So Renieri, in the precepts of eclipses, Ch. 1, number 4, considering the latitude, parallax and semidiameters of the Alphonsines, fixes these limits as in the following small table of the opinions already mentioned.

Limits of solar eclipses. Authors	Possible, North Temperate Zone to 49°: distance of ☾ from either node, toward the north	Possible, North Temperate Zone to 49°: distance of ☾ from either node, toward the south	Possible, South Temperate Zone to 49°: distance of ☾ from either node, toward the north	Possible, South Temperate Zone to 49°: distance of ☾ from either node, toward the south
Ptolemy, Reinhold, Neander, Wurstisen, Magini, Origanus, Mulerius, in mean ☌	20° 41′	11° 22′	11° 22′	20° 41′
Mulerius, in true ☌, from Ptolemy	17° 41′	8° 22′	8° 22′	17° 41′
Al-Battānī, in mean ☌	20° 12′	10° 40′	10° 40′	20° 12′
The Alphonsines, more or less	12° 0′	12° 0′	12° 0′	12° 0′
Regiomontanus, in mean ☌	20° 30′	10° 55′	10° 55′	20° 30′
Renieri, from the Alphonsines, in true ☌	17° 0′	✢	✢	17° 0′

Necessary limits in true ☌ are gathered from Ptolemy by Mulerius at 15° 38′. Necessary, I say, at least somewhere, that is, in the Torrid Zone. Langius, however, sets necessary limits in apparent ☌ if the apparent latitude is less than 30′ 40″, and possible limits if the apparent latitude is not greater than 34′ 51″.

[Margin: Kepler's method and opinion.]

IV. Kepler differs from these. In the *Epitome of Copernican Astronomy*, p. 880, and in the Rudolphine Tables, p. 98, he argues about these limits as follows. The semidiameter of the disc of the Earth and of the lunar shadow equal the sum arising from the parallax of the Moon from the Sun and the semidiameters of the luminaries; with those at apogee, this sum is 87′ 23″, and it is called the latitudinal arc [arcus latitudinarius]. But as the Moon's parallax 58′ 22″ is to the parallax of the Moon from the Sun 57′ 23″, so the latitudinal arc 87′ 23″ is to the latitudinal arc of the Moon's centre, 86′ 5″, to which there corresponds a distance from the Node of 15° 43′, as the limit of partial eclipses. With the Sun at perigee and the Moon at apogee, however, it is 15° 49′. The limit of total solar eclipses in this configuration is 10°, because the arc of the Earth's disc is 56′ 54″ and the latitudinal arc of the Moon is 56′. With the Moon at perigee and the Sun at apogee, on the other hand, the sum of the parallax of the Moon from the Sun and the semidiameters, which equals the sum of the semidiameter of the disc of the Earth and of the penumbra, is 92′ 37″, to which there corresponds a distance from the Node of 16° 57′ for the limit of partial eclipses; but with the Sun at perigee, 17° 3′. And since in this configuration the semidiameter of the Earth's disc is 62′ 12″, to which the latitudinal arc of the

Moon of 61′ 14″ corresponds, the limit of total eclipses is therefore a distance from the Nodes of 11° 7′; and it is not much different with the ☉ at perigee. In the Rudolphine Tables, p. 98, however, he sets these limits slightly differently, according to the Moon's motion first equated and nearly true, assuming an equal motion of the Nodes and a demonstrative semidiameter of the Sun and Moon, not one enlarged from physical causes. Here, then, is the following small table for his opinion.

Limits of possible ☉ eclipses according to Kepler (distance of the ☾ from the Node on either side)	☉ Apog., ☾ Apog.	☉ Apog., ☾ Perig.	☉ Perig., ☾ Apog.	☉ Perig., ☾ Perig.
In partial eclipses	15° 43′	16° 57′	15° 49′	17° 3′
In total eclipses	10° 0′	11° 7′	10° 0′	11° 7′
But from the Rudolphine Tables, at a nearly true ☌	15° 58′	16° 4′	17° 12′	17° 19′

V. For our part, in order to try these limits universally at true new Moons, we shall take from the astronomers' hypotheses the sum of the apparent semidiameters of the Moon and Sun, together with the horizontal parallax of the Moon from the Sun in the circle of altitude, when they are at perigee; for the parallax of latitude cannot be greater than this, but at most equal to it. Treating this aggregate as a latitude, sought in the body of the table of latitude for the time of new Moon, we shall look for the corresponding degree of the Moon's distance from the nearer Node, bearing in mind, however, Tycho's semidiameter of the Moon, contracted by him at new Moons, and Kepler's hypothesis, which at eclipse syzygies puts the angle at the Nodes at 5° 18′. And so we shall have the limits of a possible solar eclipse in the Temperate Zones, indeed even in the Frigid, if the Moon is in the semicircle of the ecliptic facing them, and with respect to those for whom the Moon's latitude tends toward their own pole. Next, with the aggregate of the semidiameters, treated as a latitude, we shall likewise seek in the body of the tables of latitude the distance from the nearer of the Nodes, or argument of latitude, which we shall set down as the limit of a possible eclipse in the same Zones when the Moon's latitude is toward the other pole. The necessary limits, at least for the Torrid Zone, we shall gather from the sum of the semidiameters of the Moon and Sun, but when they are at apogee, namely by seeking the Moon's distance from the nearer Node for this sum, looked for in the body of the table of latitude. Let this, then, be our small table.

TERMINI ECLIPSIVM SOLIS In Veris Nouilunijs ex Datis — Limits of Solar Eclipses at True New Moons, from the Data

From the authors	Possible in the North Temperate Zone and, if the Moon is in the northern signs, in the Frigid: toward the north	Possible in the North Temperate Zone and, if the Moon is in the northern signs, in the Frigid: toward the south	Possible in the South Temperate Zone and, if the Moon is in the southern signs, in the Frigid: toward the north	Possible in the South Temperate Zone and, if the Moon is in the southern signs, in the Frigid: toward the south	Necessary, at least in the Torrid Zone
Copernicus	18° 36′	6° 45′	6° 45′	18° 36′	5° 53′
Tycho	17° 21′	5° 22′	5° 22′	17° 21′	6° 3′
Longomontanus	18° 12′	6° 20′	6° 20′	18° 12′	5° 38′
Kepler	17° 19′	5° 46′	5° 46′	17° 19′	5° 25′
Lansbergen	18° 54′	6° 52′	6° 52′	18° 54′	6° 5′
Boulliau	18° 2′	6° 30′	6° 30′	18° 2′	6° 4′
Wendelen	18° 1′	6° 4′	6° 4′	18° 5′	5° 36′
Ourselves	18° 40′	6° 25′	6° 24′	18° 40′	5° 40′

All columns give the distance of the Moon from one or other of the Nodes within which they are possible or necessary.

[Margin: Our method, the quickest and the newest.]

But if you wish to know without labour whether any eclipse will occur, turn to Problem 4 of Chapter 12; for if there is to be no eclipse of the Earth, there will be none of the Sun either; but if there is some eclipse of the Earth, there will also somewhere be some eclipse of the Sun. And in the same place, Problem 4, no. 6, we shall set out these limits by another, more certain method, used by no one until now.

THEOREMA II.

Theorem 2.

Et PROBLEMA de Magnitudine Solaris Eclipſis.

And the Problem of the Magnitude of a Solar Eclipse.

[Margin: What the greatest solar eclipse is.]

VI. The magnitude of a solar eclipse is usually expressed in parts of an as, or unciae, or digits, of which the Sun's apparent diameter contains 12. But if a solar eclipse is total, and with stay, more than 12 digits are said to be eclipsed, because if the Sun had more than 12 they could still be covered by the Moon. So, when the excess of the Moon's apparent diameter over the Sun's has been multiplied by 12, and the sum divided by the Sun's apparent diameter, the result will be the digits, or fractions of digits, to be added to the twelve digits in a

total eclipse with stay. We have already shown that this is possible, in Book 4, Chapter 16, no. 15, and in this Book, Chapter 8, Scholium 3. For us, therefore, for whom the diameter of the Sun at apogee is 30′ 30″ and that of the Moon at perigee 33′ 30″, a solar eclipse cannot be greater than 13 digits 11′; and in the hypothesis of no celebrated astronomer is it greater than 13 digits 45′, or at the very most 14 digits. Magini, however, in his *Theoricae Planetarum*, Book 2, Chapter 31, states that beyond 12 digits hardly 17′ can be counted, and that the stay is therefore not sensible. This does not conform to Copernicus, whom he was otherwise following there. For in Copernicus the diameter of the Moon at perigee is 35′ 40″, and that of the Sun at apogee 31′ 24″; the excess is therefore 4′ 16″. Multiplied by 12 this makes 49′, and when these are divided by 31′ 24″ the result is 1 digit 34″ of excess over the Sun's diameter; and 4′ 16″ requires a time of about 8^{m}, which is certainly a sensible time. But Magini may then have followed another opinion about the diameters.

Further, you will find the digits of the Sun to be eclipsed, or to be covered by the Moon, by subtracting from the sum of the apparent semidiameters of the Sun and Moon the arc between the centres, or the distance of the centres of the Moon and Sun, multiplying the remainder by 6, and dividing the sum arising from this by the Sun's apparent semidiameter. How the arc between the centres is found from the apparent latitude at the moment of visible new Moon you will learn from solving a similar triangle, on which see Chapter 6, Problem 4. For what I said there about the true latitude and the true distance of the centre of the shadow from the Node applies here to the apparent latitude and distance. Or use the reduction of the Moon to the ecliptic, compiled for the moment of visible ☌, and the apparent latitude, so that in the same figure, by solving the triangle A L I, you may find the arc A I; or use precept 5 of Chapter 4 of Renieri, on which see below among the precepts of the tables. The ancient practitioners, however, used instead of the arc between the centres the Moon's apparent latitude at the moment of visible ☌, subtracted from the sum of the semidiameters, multiplying the remainder by 6 and dividing the sum by the Sun's apparent semidiameter. By this device was constructed the small table of digits given by Ptolemy, Book 6, Chapter 8, by Alfonso, fol. 117, and by Peuerbach in the penultimate table of Eclipses;

Reinhold, in the Prutenic Canons, p. 124; Magini, in the *Secunda Mobilia*, p. 683; Lansbergen, p. 38; and Renieri, p. 345 of the *Tabulae Mediceae*.

THEOREMA III.

Theorem 3.

Cum PROBLEMATIB. de Durationе & Partibus Eclipſis Solis.

With Problems on the Duration and Parts of a Solar Eclipse.

[Margin: Parts of the duration of a solar eclipse.]

VII. The duration of a solar eclipse is divided into two parts if it is not with stay, and into four if it is total with stay. In a partial eclipse, or a total one without

stay, the incidence, in Greek [Greek: ἐμπτώσις (emptōsis, "incidence")], is the interval from the beginning to the middle or maximum of the eclipse; the repletion, in Greek [Greek: ἀναπληρώσις (anaplērōsis, "filling up")], is the interval from the middle to the end. In a total eclipse with stay, however, the incidence is the interval from the beginning to total obscuration, and the repletion the interval from the beginning of emersion, or the point at which a small part of the Sun begins to appear again, up to the end, when the whole disc of the Sun appears to us full of light. The stay in total obscuration is divided into an earlier and a later part. Now the common run of astronomers think these parts equal. And they are indeed sensibly equal as regards the arcs of the zodiac and the diameters of the luminaries, as Ptolemy says in Book 6, Chapter 10, and Regiomontanus in Book 6 of the *Epitome*, prop. 26. But as regards time they are not always equal, whether incidence to repletion or earlier stay to later, as Ptolemy and Regiomontanus note in the places cited above, and Reinhold in Prutenic precept 63. These authors, however, except for Ptolemy, do not here distinguish the quadrants of the equator from the quadrants of the ecliptic, and they do not set out distinctly the causes of this variation; nor is Ptolemy's teaching complete, nor the translation clear enough.

[Margin: How great Ptolemy's diligence is here.]

Take both, therefore, from Ptolemy as elucidated by us. When the apparent synod of the luminaries occurs at the nonagesimal of both the ecliptic and the equator (which happens only when the cardinal points, that is, the equinoctial and solstitial points, are culminating on the meridian), then the time of incidence and the time of repletion, and the earlier and the later stay, are sensibly equal. For by as much as the parallaxes decrease in the eastern incidence, owing to the Moon's ascent toward the meridian, by so much do they increase in the western repletion, owing to its descent; and a compensation of the apparent slowness and speed results. For as the Moon gradually ascends the parallaxes of longitude decrease, on account of which the ☾ appears, in the eastern quadrant of the ecliptic, farther east than it really is, and so appears faster in its apparent motion in longitude. Therefore, as these decrease in this part, it appears in its ascent to the meridian to move more and more slowly, as regards apparent motion. Conversely, as the Moon descends toward the horizon the parallaxes of longitude increase; and on account of them it appears, in the western quadrant of the ecliptic, farther west than it actually is, and therefore slower in its apparent motion than in its true motion. But within this apparent slowness itself it is slower nearer the meridian, because of the smaller parallaxes, and faster farther from the meridian. Ptolemy therefore rightly concludes, on this account, that the times of a solar eclipse always come out greater because of the apparent motion, and that its times are slower in the motions nearer the meridian. But when the whole time of incidence passes in the eastern quadrant of the equator, whether the repletion is wholly completed in the same quadrant or part of it occurs in the eastern and part beyond the meridian in the western quadrant, the time of incidence is shorter than the time of repletion, because the Moon's

apparent motion is faster there and slower here, owing to the nearness of the meridian. Conversely, when the time of incidence passes either wholly or in part in the western quadrant, and the whole time of repletion in the western, the time of incidence is longer than the time of repletion. Nor is this contradicted by the fact that it may meanwhile be in the eastern quadrant of the ecliptic while, with respect to the meridian and the equator, it is in the western quadrant, or vice versa. For it is one thing for the apparent motion to be slower or faster than the apparent motion, and for the parallaxes of longitude and the time arising from them to have to be added to or subtracted from the true motion; it is another for the true motion to be slower or faster than the apparent motion. Kepler therefore rightly said [recte: dixit], in the *Epitome of Copernican Astronomy*, p. 886, that the visible hourly motion of the ☾ from the ☉ is almost always shorter than the true hourly motion, namely as regards the arc which it makes in one hour, and most of all near the nonagesimal, but that near the horizon they are almost equal.

[Margin: Why the method of calculating a solar eclipse is indicated here in advance.]

We cannot, however, determine in this place either the maximum duration or the times and conditions of the other phases belonging to solar eclipses unless we indicate here, by what is certainly a timely anticipation, the order and method of the calculation, which otherwise seem to belong to the second part of this volume. The examples are deferred to that part, but certain things that will have to be passed over there, so that the calculation is not disturbed, are not passed over here; especially since not all authors use the same completeness or the same clarity in this matter.

[Margin: Authors of the calculation of solar eclipses.]

Those to whom we nevertheless owe most in this are: Ptolemy, Book 6 of the *Almagest*, Chapter 10; al-Battānī, Chapter 44; the Alphonsines, prop. 32; Peuerbach, in the tables of Eclipses, from prop. 8 to 13; Regiomontanus, Book 6 of the *Epitome of the Almagest*, from prop. 19 to 26; Copernicus, Book 4, from Chapter 30; Erasmus Oswald Schreckenfuchs, on Peuerbach's *Theoricae*, from p. 341; Erasmus Reinhold, in the Prutenic Tables, from precept 54 to 65; Tycho, vol. 1, p. 136; Mulerius, in the *Tabulae Frisicae*, pp. 434, 443 and 463; Longomontanus, in the *Astronomia Danica*, Book 1 of the *Theorica*, Chapters 8 and 9, especially from p. 302; Magini, in the *Secunda Mobilia*, from Canon 75 to 91, and in the *Supplement to the Ephemerides*, from p. 279, or from Canon 9 of the Eclipses to 15; Lansbergen, from Precept 26 to 37; Kepler, in the *Epitome of Copernican Astronomy*, from p. 880, and in the Rudolphine Tables, from Chapter 29 to 32; Boulliau, in the *Astronomia Philolaica*, Book 4, Chapters 4 and 5, and Book 12, Chapters 22 and 24; Andrea Argoli, in his tables; Balthaſſar Capra [Baldassarre Capra], in the *Tyrocinium Astronomicum*; and Vincenzo Renieri, in the second part of the second edition of the *Tabulae Mediceae*, from Chapter 3 to 7.

[Margin: Comparison of these authors.]

Of these, Copernicus, Tycho and Longomontanus are too sparing; Kepler, Boulliau and Renieri are the most diligent; the rest stand in a middle position.

[Margin: Method of solar eclipses and first step.]

VIII. First, then, having found the true new Moon and, for that time, the true place of the luminaries, the equated anomalies, the true place of the Node ☊, and from these the argument of latitude, or the distance of the Sun from the Node ☊: if the ecliptic limits given in no. 5 give you reason to suspect a coming solar eclipse, convert the equal time of the new Moon into apparent time. And for that time, if the luminaries are not at the nonagesimal (for then the true and the apparent new Moon are the same), find by Problem 1 of Chapter 10 the rising point of the ecliptic and the nonagesimal point. When you have prepared, by Problems 2, 3 and 4 of the same chapter, what is necessary for Problem 5 of the same chapter, find by that Problem 5 the parallax of the Moon from the Sun in longitude; keep this, and call it the parallax of the new Moon, or the synodic parallax.

[Margin: Step 2, and the quickest way of collecting the Moon's apparent hourly motion and, by means of it, the apparent new Moon.]

Secondly, if the luminaries and their true synod are in the eastern quadrant of the ecliptic, add the synodic parallax of longitude to the Moon's true place found at the synod, and you will have the first apparent place of the Moon. Likewise add one hour to the time of the true new Moon, and for this aggregate time collect both the true place of the Moon and the parallax of longitude; add the latter to this second true place, and you will have the second apparent place of the Moon. Its difference from the first apparent place will be the Moon's apparent hourly motion; and when the Sun's hourly motion has been subtracted from this, there remains the apparent hourly motion of the Moon from the Sun. Multiply, therefore, by one hour, or rather by 60^{m}, the synodic parallax of longitude, and divide the sum by the Moon's apparent hourly motion; and you will have the apparent time, which, added in this eastern quadrant to the moment of the true new Moon, will give the exact moment of the apparent new Moon. You could, however, with the apparent hourly motion, in the tables of verification which Magini gives at p. 676 of the *Secunda Mobilia*, and we after him, look up the apparent hourly motion of the ☾ from the ☉ at the head and the synodic parallax of longitude at the side; for in the body you will find the time to be added to the true new Moon.

But if the true synod of the luminaries occurs in the western quadrant of the ecliptic, subtract the synodic parallax of longitude from the Moon's true place, and you will have the first apparent place of the Moon. Likewise subtract a whole hour from the moment of the true new Moon, and for the remaining time collect both the true place of the Moon and the parallax of longitude; subtract the latter from the second true place, and you will have the second apparent place. Its difference from the first apparent place of the Moon will be the Moon's apparent hourly motion; and when the Sun's hourly motion has been subtracted from this, there remains the apparent hourly motion of the

Moon from the Sun. Therefore, with this find, as above, the time corresponding to the synodic parallax of longitude, and subtract it from the moment of the true new Moon, and there will remain the exact moment of the apparent new Moon, or of the visible synod. So Regiomontanus, Book 6, prop. 21, and Magini, Canon 10 of the Compendium of computing eclipses. This way of deriving the Moon's apparent hourly motion has seemed to me far quicker than the one suggested by the same Regiomontanus in the second place above, by Reinhold, precept 55, by Magini in the *Secunda Mobilia*, Canon 77, by Lansbergen, precept 30, by Mulerius, p. 445 of the *Tabulae Frisicae*, by Renieri in the precepts on eclipses, Chapter 3, nos. 6 and 7, and by Kepler in the Rudolphine Tables, Chapter 32, precept 155.

[Margin: A still more exact moment.]

But if further meticulousness [Greek: περιεργίας (periergias, "over-elaborate care")] does not weary you, collect for the moment of apparent new Moon the true places of the luminaries, and from these their true distance from each other, and the parallax of the Moon in longitude. If this is equal to the distance of the luminaries in the zodiac, that was the most exact moment of apparent new Moon. But if they disagree, the difference must be converted into time, according to the proportion of the hourly motion of the Moon from the Sun already investigated above. You will add this time to, or subtract it from, the moment of apparent new Moon first found, according to the following small table, and the result will be the most exact moment of apparent new Moon.

If the true distance of the luminaries, compared with the parallax of longitude of the ☾ from the ☉, is	In the quadrant	Instruction (at the visible ☌)
GREATER	Eastern	Add the time
GREATER	Western	Subtract the time
LESS	Eastern	Subtract the time
LESS	Western	Add the time

For in a visible conjunction the distance of the true places of the Moon and Sun must be equal to the parallax of longitude, as Regiomontanus shows, Book 6, prop. 22, and as Reinhold takes from there in precept 58.

[Margin: Step 3, and the determination of the ecliptic new Moon.]

Thirdly, for the most exact moment of apparent new Moon, collect the Moon's true latitude and also the parallax of latitude, according to Problem 5 of Chapter 10. In the Temperate and Frigid Zones this parallax is always toward the opposite pole; in the Torrid Zone, however, it is in some places and at some times toward the visible pole, at other times toward the opposite one. If, therefore, the latitude is toward the same region toward which the parallax of latitude tends, add the parallax to the true latitude, and the result will be the apparent latitude, of the same region or denomination. But if the latitude is toward one pole of the ecliptic and the parallax of latitude toward the other, and they are equal to each other in quantity, the apparent latitude will be nil, and the centre of the Moon will be visibly on the ecliptic with respect

to the place on Earth for which you began the calculation. If the parallax is less than the true latitude, their difference will be the apparent latitude, of the same region or character as the latitude. But if the parallax is greater than the true latitude, the apparent latitude will be toward the opposite region, or of the contrary character. Compare, therefore, the apparent latitude with the sum of the visual semidiameters of the Moon and Sun, collected earlier or now, by means of the anomaly, from their own tables; for if the apparent latitude is less than that sum, there will be a solar eclipse at that place on Earth; if it is not less, there will not.

[Margin: Step 4, on the magnitude of a solar eclipse.]

Fourthly, as I said in no. 6, you will collect the ecliptic digits, using the arc between the centres rather than the apparent latitude.

[Margin: Step 5, for the minutes of incidence, etc.]

Fifthly, so that the whole duration of the eclipse and its parts may be ascertained: if the eclipse is without stay in darkness, then with the sum of the visual semidiameters of the Sun and Moon at the head of the tables, and with the apparent latitude corresponding to the apparent new Moon at the side of the tables, the minutes of incidence, or minutes of fall, are usually sought.

[Margin: Authors of tables of incidence, but imperfect ones.]

These tables are given by Ptolemy, Book 6, Chapter 8; Alfonso, fol. 116½; Peuerbach, penultimate table of Eclipses; the Prutenic Canons, p. 125; Magini in the *Secunda Mobilia*, p. 687; Lansbergen, p. 40; Argoli, p. 160; and Renieri, p. 349.

[Margin: Another way, by analysis of triangles.]

But in these tables the minutes of incidence and half-stay together, and then of half-stay alone, are not set out separately, even granting that these authors admit total solar eclipses with some stay. Therefore, to collect the minutes of each interval, solve in the old manner the rectilinear triangle treated by Ptolemy, Book 6, Chapter 11; al-Battānī, Chapter 44; Regiomontanus in the *Epitome of the Almagest*, prop. 24; and Mulerius in the *Tabulae Frisicae*, p. 443; and indicated by Tycho, p. 136. In the following diagram, then, let the ecliptic be EQ, on which is the centre of the Sun S and its semidiameter SG, from which the disc of the Sun is to be described; and let the sum of the semidiameters of the luminaries be S C on one side and S D on the other. At the beginning of the eclipse let the centre of the Moon be at C, at the middle at B, and at the end at D, so that the Moon's apparent path is CBD. To this path, from the centre S, let there be drawn the perpendicular S B, which is to be called the arc between the centres, or the apparent latitude of the Moon at the middle of the eclipse. The remaining lines belong to another problem, on which see below.

Diagram. A figure set into the right column, with the text narrowed to its left. Three equal circles stand on a line slanting from upper left to lower right. The middle circle is a disc black at the centre, with a pale flame-like band round its rim, carrying S at its centre and B within the black part. The upper and lower circles are each engraved with a human face and a hatched lower

rim. Straight lines pass through the three circles along the slant, a horizontal line runs through S, and a long straight line runs from near the left end of that horizontal line down to the lower right, outside the circles.

- N (top of the figure, above the upper circle): label only
- Q (left of the upper circle): label only; the prose names the ecliptic EQ
- V (left end of the horizontal line): label only
- G (on the long lower-left line, at the rim of the middle circle): the prose names the Sun's semidiameter SG
- S (centre of the middle circle): centre of the Sun
- I (on the limb of the middle circle, within the flame-like band, below S): label only
- B (within the black part of the middle circle): point on the Moon's path at the middle of the eclipse
- R (right of the middle circle): label only
- M (right of the lower circle): label only
- C (in the lower circle): centre of the Moon at the beginning of the eclipse
- H (in the lower circle, left of C): label only
- A (foot of the long lower-left line): label only
- E (foot of the lower circle): the prose names the ecliptic EQ
- D (in the upper circle, between the eyes of the face): centre of the Moon at the end of the eclipse
- F (in the upper circle, below the face's mouth, near the rim): label only

With these things laid down, because of the nearness and ease of calculation, and the security from sensible error, as the authors already mentioned suppose, they use, in place of the arcs of which the ecliptic, the Moon's path and the circle of latitude consist, their chords or semichords; and they use the Moon's path from beginning to end as if made through the same plane. In the triangle SBC, then, taken as rectilinear and right-angled at B, from the given side S B, which is the apparent latitude at the middle of the eclipse, and the base SC, which is the sum of the semidiameters, they investigate the side BC. This is the interval of incidence, and to it is equal the straight line BD, which contains the minutes of repletion, since the triangle DSC is isosceles and the perpendicular SB cuts its base CD in two at B, by 10 of Book 1 of Euclid. So, in solving the triangle already mentioned by 47 of Book 1 of Euclid, the square number of the side S B, subtracted from the square of the base SC, leaves the square of the side BC, whose square root gives the minutes of incidence.

[Margin: Inquiry into the apparent arc between the centres, and from it a more accurate incidence.]

But because of what is to be said in Scholium 1, you will work more accurately if you use the apparent arc between the centres, which is sought proportionally, as in a lunar eclipse, by the method already given in Chapter 6, Problem 4. For collect for the moment of apparent new Moon the apparent latitude and apparent longitude of the Moon, with the reduction of the Moon to the ecliptic, and make: as the radius to the second sine of the reduction, so the secant of

the apparent latitude to the secant of the arc SB. Or to the second logarithm of the reduction add the residual of the second logarithm of the apparent latitude, and the result will be the second logarithm of the arc SB, the apparent arc between the centres. When this arc has been obtained, let: as the radius to the second sine of the apparent arc between the centres, so the secant of the sum of the semidiameters to the secant of the incidence. Or to the 2nd logarithm of the apparent arc between the centres add the residual of the 2nd logarithm of the sum of the semidiameters, and the result will be the 2nd logarithm of the incidence. But if, in place of the sum of the semidiameters, you use the excess of the lunar semidiameter over the solar, and complete the rest as above, you will have the minutes of half-stay.

[Margin: The imperfection of this problem indicated.]

In this treatise, however, what error is committed by assuming the Moon's apparent path to be made, from the beginning to the middle or end, in the same plane and inclined at the same angle to the ecliptic, and by taking the apparent latitude at the moment of apparent new Moon for the arc between the centres, just as if that moment coincided with the moment of greatest obscuration and the middle of the eclipse, must be told below, more suitably, in the Scholia.

[Margin: Step 6, for the time of the duration of a solar eclipse and of its parts.]

Sixthly, so that the minutes of incidence and repletion, or if need be of each half-stay, may be converted into time, and finally the duration of the eclipse and its parts determined, the Moon's apparent hourly motion is needed, both before and after the middle of the eclipse. This is collected for the incidence one hour before the moment of apparent synod, and likewise one hour after for the repletion, by the method already given above in precept or step 2; then the minutes of incidence by

60 minutes of time, that is, by one hour, are multiplied, and the sum divided by the apparent hourly motion of incidence will give the time of incidence; the same sum divided by the hourly motion of repletion will give the time of repletion. But if the eclipse is total with stay, multiply the minutes of the half-stay by 60 and divide the sum by the hourly motion of incidence, for you will then have the time of the first half-stay; divide the same sum by the hourly motion of repletion, and you will have the time of the second half-stay. The aggregate of the times of incidence and repletion will be the duration of the eclipse, and the aggregate of both stays will be the time of the whole stay.

[Margin: Method of determining the greatest duration of a solar eclipse.]

IX. The greatest duration of a solar eclipse, moreover, occurs where and when these 2 elements come together: namely, the greatest sum of the semidiameters, and the least hourly motion of the Moon from the Sun, both true and apparent. Now the sum of the semidiameters is greatest when both luminaries are at perigee; but the Moon is then swiftest in true motion, and its true hourly motion is greatest. We must therefore try which of the four comparisons is better suited to this purpose, so that it may be clear which of them yields

the longer time. To that end we shall take the semidiameters and the hourly motion of the Moon from the Sun from our own hypothesis. Look at this in the following small table, so that you may take from it a similar specimen for the hypothesis of others.

From our hypothesis	☉ & ☾ at apogee	☉ & ☾ at perigee	☉ at apog., ☾ at perig.	☉ at perig., ☾ at apog.
Hourly motion of ☾ from ☉	26′ 57″	32′ 56″	24′ 12″	30′ 44″
Sum of the semidiameters	29′ 15″	33′ 20″	32′ 0″	30′ 34″
Time corresponding to the sum	$65^m\ 44^s$	$60^m\ 46^s$	$79^m\ 23^s$	$59^m\ 48^s$

It is clear, then, that the longest time for traversing a space as great as the sum of the semidiameters occurs with the Sun at apogee and the Moon at perigee, and that the time from the beginning to the middle of the eclipse, if it were accomplished by the Moon's true motion, is then $1^h\ 19^m$. To this, as Kepler teaches in the *Epitome of Copernican Astronomy*, p. 890, a quarter of an hour at most can be added on account of the apparent motion in central and greatest eclipses, so that the greatest half-duration from the beginning to the middle is $1^h\ 34^m$. Therefore, if the middle of this eclipse falls at noon and in the nonagesimal, where the time of incidence is equal to the time of repletion, the greatest duration of an eclipse by my hypotheses will be $3^h\ 8^m$, although for Kepler, in the same place, it does not exceed 2½ hours, because from the true hourly motion, taken separately, he had determined from his hypotheses a greatest duration of 2 hours. I, however, from his hypotheses, gather a greatest duration, considering the true hourly motion, of $2^h\ 23^m$; adding half an hour, then, as he himself teaches, it will come to $2^h\ 53^m$.

[Margin: Greatest duration of a solar eclipse.]

And indeed, in their calculations of solar eclipses, computers not rarely extend eclipses, even those that are not greatest, beyond 2½ hours: so Tycho, p. 136, who gives his own ☉ eclipse $2^h\ 52^m\ 56^s$; and Reinhold in the Prutenic Tables, precept 63, who gives about 2^h and 52^m; and Magini in canon 86 of the *Secunda Mobilia*, where he gathers the time of incidence as $1^h\ 21^m\ 17^s$ and of repletion as $1^h\ 16^m\ 8^s$, and hence of the duration as $2^h\ 37^m\ 25^s$. Now let there be a small table for the greatest duration of a solar eclipse.

Solaris Eclipſeos Duratio Maxima nempe ☉ Apogæo & ☾ Perigæa — Greatest duration of a solar eclipse, namely with the ☉ at apogee and the ☾ at perigee

From the data of	Duration
Ptolemy	$3^h\ 8^m$
Copernicus	$3^h\ 18^m$
Tycho	$2^h\ 56^m$
Longomontanus	$3^h\ 3^m$

From the data of	Duration
Kepler	$2^h\ 53^m$
Or as he himself would have it	$2^h\ 30^m$
Lansbergen	$3^h\ 18^m$
Boulliau	$3^h\ 12^m$
Wendelen	$3^h\ 0^m$
Ourselves	$3^h\ 8^m$

SCHOLIA.

Scholia.

I. What was said in chap. 6, Problem 4 and Scholium 1 against the ancient astronomers, about their confusing the middle of an eclipse with the moment of full Moon, and the arc between the centres, which determines the greatest eclipse, with the true latitude of the full Moon, holds here far more strongly for a solar eclipse. For the moment of the apparent new Moon is not the same as the moment of the middle or greatest eclipse; nor, to seek the minutes of incidence exactly, should one use that apparent latitude of the Moon which was found for the moment of the visible new Moon, but rather the arc between the centres of the Sun and Moon, drawn perpendicular to the Moon's path — at least when the eclipse is very large, or the Moon's apparent path is parallel to the ecliptic as far as the senses can tell. In this case, following Kepler's precept (*Epitome Astron.*, p. 890), look at the figure set out in the preceding chapter, no. 8, precept 5, in which let the Moon's path be CBD, parallel to the ecliptic EQ. For then, for a certain chosen interval of time before or after the apparent new Moon, the Moon's apparent latitude is to be gathered. Its antilogarithm, or second logarithm, subtracted from the antilogarithm of the sum of the visual semidiameters of the ☾ and ☉, leaves the antilogarithm of the difference, very nearly, between the places of longitude that the beginning and end of the eclipse require. To this difference the Moon's visible motion, gathered over that chosen interval of time, must be equal; or, if it is not, the interval must be corrected until they are equal. Therefore in the diagram already mentioned, and in triangle SHC, right-angled at H, having found for the beginning of the eclipse the apparent latitude CH, with this and the base SC, which is the sum of the semidiameters, you will find the minutes of arc SH, which are equal to the minutes of incidence CB. So too in triangle SDF, right-angled at F, having found for the end of the eclipse the Moon's apparent latitude DF, from this and from the sum of the semidiameters you will seek FS, that is, BD, the minutes of repletion.

[Margin: A more accurate inquiry into incidence, but in large eclipses.]

But surely, if the Moon's path is parallel to the ecliptic as far as the senses can tell, what is the point of so much [Greek: ὑπεριεργία (hyperiergia, "excess of labour")]? For then the arc between the centres SB and the arcs of latitude CH and DF are parallel as far as the senses can tell; it is enough, therefore, to use the

arc between the centres, and to work according to the rules of the preceding chapter, no. 8, precept 5. But if the eclipse is central for some place, the very sum of the visual semidiameters of the ☾ and ☉ is equal, as nearly as may be, to the minutes of incidence.

[Margin: In small solar eclipses it would be greater.]

II. I said "when eclipses are very large, or the Moon's visible path is nearly parallel to the ecliptic", because if they are small, or that path is very oblique, then, as Kepler points out in the *Astronomia Optica*, pp. 406 and 418, that is, chap. 11, Problems 30 and 32, and in the *Epitome of Copernican Astronomy*, p. 890, the angle of inclination of the Moon's visible path to the ecliptic is not 5°, as astronomers before Kepler have hitherto supposed, nor 5° 18½′, as Tycho did. It varies instead, so that at times it turns out twice, three times or even four times as great; as when the Moon is near the Node ☊ and the parts near the beginning of ♋ are rising, or near the Node ☋ and the parts near the end of ♐ and the beginning of ♑ are rising. The Moon's path is also sometimes inclined contrary to the true path, which happens when the Moon, near ☊, is rising toward the north and the signs near the beginning of ♑ are rising. From this it follows that the approach of the centres of the Sun and Moon must not rarely be considered in a circle drawn through the centre of the Sun and inclined toward the visible Node. In this, moreover, there is so great a variety that it cannot be embraced in a few rules, just as the duration of an eclipse in any given place cannot either, because of the great variations of the parallaxes which the Moon undergoes from beginning to end, as Ptolemy indicates in Book 6, in the last chapters, where he also declares that he does not pursue the highest precision in this matter.

[Margin: A flaw of the Prutenic Tables, of Magini and of Lansbergen.]

The reader therefore had to be warned not to rely on precepts 63, 64, 65 of the Prutenic Tables, and canons 86, 87, 88 of Magini's *Secunda Mobilia*, or precepts 35, 36, 37 of Lansbergen, and the like, where, assuming a constant angle of inclination of the Moon's path to the ecliptic of 5°, they teach universally how to find the minutes of incidence and the apparent latitude for the beginning and end of an eclipse, and from this to form the delineation of a solar eclipse. For that latitude is not apparent but nearly apparent, and the diagram of the eclipse built by those precepts is not true but nearly true. And since the Moon's visible path is twice, three times, four times as oblique to the ecliptic, the moment of greatest eclipse also differs more widely from the moment of the apparent new Moon, and the times of incidence and emersion vary more widely.

[Margin: Inclination of the Moon's visible path to the ecliptic of almost 20°.]

Outside eclipses, moreover, the same variation sometimes occurs, as Kepler confirms by the example of an observation of Bernhard Walther, with the Moon at its last quadrature and 37° from the Node ☊. Taking that observation as given, he shows that the Moon's visible path was inclined to the ecliptic at 19° 21′.

III. Besides what was said in Scholium 2, Kepler, in *Astron. Opt.*, chap. 11, Problem 32, does indeed grant that the space which the Moon covers in a solar eclipse in three hours at most may fairly be likened to a straight line, although it is an arc of a great circle; but nevertheless he rightly denies

[Margin: The Moon's visible path winding and compound.]

that the centre of the Moon proceeds through the same plane of a great circle from the beginning to the end of the eclipse. For this, he says, the parallaxes of longitude and latitude do not allow, since they vary unequally, and the faster the nearer they are to the nonagesimal; and therefore the Moon's path is winding and compounded of various curves. He concludes, however, that this tiny difference does not appear sensible except around the nonagesimal and the meridian.

CAPVT XII.

Chapter 12.

De Terræ Eclipſi ab Vmbra vel Penumbra Lunari obſcuratæ, ex Luna ipſa ſpectata.

On the eclipse of the Earth, darkened by the lunar shadow or penumbra, as viewed from the Moon itself.

I. Not every property of a solar eclipse viewed from the Earth has yet been sufficiently examined, when Kepler carries us, still unlearned, from the Earth to the Moon, so as to instruct us from there. In the *Epitome of Copernican Astronomy*, from p. 874, and in the Rudolphine Tables, chap. 32, precept 158, he introduces us to the true eclipse of the Earth itself, as viewed or viewable from the Moon.

[Margin: The intellect of Kepler and Boulliau praised.]

With him lighting the way, Ismaël Boulliau, a man of the keenest intellect, in Book 4 of the *Astronomia Philolaica*, chap. 4, considers that what is called an eclipse of the Sun is in reality an eclipse of the Earth itself, since the Earth is deprived of the Sun's light by the intervention of the Moon, just as an eclipse of the Moon is by the intervention of the Earth. Pliny the Elder, with his acumen, had touched on this in Book 2, chap. 10, in these words: *For it is plain that the Sun is hidden by the intervention of the Moon, and the Moon by the interposition of the Earth, and that they repay each other in turn, the Moon by its interposition taking the same rays of the Sun from the Earth, and the Earth from the Moon. As the Moon passes beneath, sudden darkness is drawn over; and again by the Earth's shadow the star is dulled.*

Boulliau had seen that the inquiry into the lunar parallax must be repeated so many times in calculating a solar eclipse, to the point of wearying anyone's astronomical patience, and that in the parallactic tables of the Prutenics, of Magini and of Lansbergen it is as unreliable as it is troublesome. He therefore considered that a solar eclipse could be calculated without parallaxes if in its

place one considered the eclipse of the Earth, darkened by the shadow or penumbra of the Moon, as viewable from the Moon. For just as we calculate a lunar eclipse without parallax, because the Moon is truly deprived of the Sun's light, wholly or in part, and the place of the Earth's shadow through which the Moon passes and the Moon's body are equally distant from the eye of the viewer, whereas someone who was then on the Moon would nevertheless judge the eclipse, which was really the Moon's, to be an eclipse of the Sun, brought about by the [Greek: ἐπιπροσθέσει (epiprosthesei, "interposition")] of the Earth, and would have to seek the parallax according to the proportion of the Earth's semidiameter: so we, while viewing from the Earth an eclipse of the Sun through the interposition of the Moon, must take account of the lunar parallax. But if we were on the Moon, we should see the Earth really deprived of the Sun's light by the intervention of the Moon, in the same place where the shadow or penumbra of the Moon is, without any parallax between the Earth and the place of the lunar shadow or penumbra. And just as in a lunar eclipse the Moon, with respect to two viewers of whom one is at the centre and the other on the surface of the Earth, undergoes parallax, but the place of the Earth's shadow through which the Moon's centre passes undergoes exactly the same parallax, so an eclipse of the Earth viewed by two people, of whom one is on the surface and the other at the centre of the lunar globe, has parallax, but the same as, and equal to, that which the place of the lunar shadow or penumbra through which the Earth's centre passes has. So, more or less, Boulliau, but not so explicitly.

[Margin: The author's judgement of this method.]

II. Indeed we admit that this inquiry and [Greek: ψηφοφορίαν (psēphophorian, "reckoning")] is ingenious, and useful, and easier than the customary method of computing solar eclipses, and deserving further cultivation, as I shall presently say, because of the fertility of new theorems and problems which it holds in its lap. Yet by this method alone we do not succeed in being able either to weigh solar eclipses already observed, or to predict future ones, with respect to viewers living on the Earth. That cannot be done without an inquiry into parallax, as is clear in itself and from the example of the solar eclipse of the year 1639 June 1, observed by Boulliau at Paris and brought to this new form of calculation in Book 4, chap. 5. For when he wants to reduce its phase to the meridian and horizon of Paris, and to determine the times of the phases, he investigates the parallax of altitude, of longitude and of latitude, and uses a calculation no less prolix and troublesome than others do, once a method of deducing parallaxes has been supposed; which method, however, he teaches in Book 12, precepts 22 and 24. So by this new method the astronomical Hercules is assigned to a new labour, though not unwillingly, yet is not wholly released from the old one.

[Margin: The method of the eclipse of the Earth advanced by the author.]

Meanwhile we must develop the method already mentioned, adding some demonstrations omitted by Kepler and Boulliau, and also by Renieri in his latest precepts for eclipses.

PROBLEMA I.

Problem 1.

Definire Semidiametrum Apparentem Diſci Terræ à Sole illuſtrati ex Luna viſam.

To determine the apparent semidiameter of the disc of the Earth illuminated by the Sun, as seen from the Moon.

III. Just as the luminaries viewed from the Earth, although they are spherical, look flat, because owing to their excessive distance the eye cannot distinguish the greater distance of points on a spherical surface from the lesser, but judges them to be equally distant and so all in the same plane, so the globe of the Earth viewed from the Moon would appear as a flat circle or disc, and viewed at new Moon it would appear, like a full Moon, illuminated by the Sun. Which of the three circles of the Earth that seem to bear on this matter is the disc we are speaking of must be determined from the adjoining diagram. On its perpendicular straight line AB, let A be the centre of the Moon in conjunction with the Sun, and C the centre of the Earth, from which let the terraqueous globe BFTE be described, whose diameter is DCE. From the extremities of this diameter, straight lines being drawn to the centre of the Moon, let an eye A be imagined to see the Earth under the angle DAE; for then the apparent semidiameter of the disc will be CE or CD, and its measure the angle CAE.

Diagram. Set into the right column, with the text running in a narrow measure on its left. A large circle stands above a small circle whose upper half is solid black; four long straight lines, in two close pairs, converge downward from the large circle to the small circle, with the vertical between them. A vertical line runs through the large circle from the top of the figure to the small circle. Three nearly parallel horizontal chords cross the large circle at about its middle, with several crossing lines between them near the right limb. Short lines stand above the upper chord inside the large circle. A long straight line, tangent at the right limb, descends from upper right to lower right.

- O (upper left of the large circle, at the end of a short horizontal line, beside the point where it meets the limb): O
- S (top of the large circle, to the left of B): S
- B (top of the vertical line): B
- P (inside the large circle, right of the vertical line, below B): P
- I (inside the large circle, left of the vertical line, below S): I
- N, V, M (standing on the upper chord, left to right): N, V, M
- H, D, F (left limb, top to bottom): H, D, F
- K, E, G (right limb, top to bottom): K, E, G

- C (centre, just above the lowest chord): C
- L (below C, under the lowest chord): L
- T (bottom of the large circle, on the vertical): T
- R (upper end of the tangent, upper right): R
- Q (lower end of the tangent, lower right): Q
- A (in the white lower half of the small circle): A

Now let the rays AF and AG be drawn from A, touching the Earth at F and G, where, by Euclid III.18, a right angle will be formed with the radius GC; and let the points of contact be joined by the straight line FG. For to the viewer at A, FG will be the diameter of the disc and LF its semidiameter, and its apparent measure the angle LAF. This angle is a very little greater than angle CAD, just as angle CAG is slightly greater than angle CAE, namely by about 15″, as will be clear to anyone who gathers these angles first in triangle CGA, right-angled at G, taking side CG as one semidiameter of the Earth and CA, the lunar distance, as base; then in triangle ACE, right-angled at C, taking as one side one semidiameter of the Earth, CE, and as the other side the lunar distance CA.

[Margin: 1. Method of determining the semidiameter of the Earth's disc.]

[Margin: 2. Renieri's method.]

In this second way, then, Renieri, in the new *Tabulae Mediceae*, Part 2, chap. 5, no. 1, takes the apparent semidiameter of the Earth's disc, when he instructs us to seek the Moon's horizontal parallax and to use it as its measure.

Thirdly, let QR be a ray of the ☉, touching the Earth at K, and by other rays let the arc EK beyond the quadrant TE be illuminated, to which let the arc DH be equal. When the straight line HK is drawn, then, it will be the diameter of a circle that is not a great circle, separating the segment HTK, illuminated by the Sun, from the unilluminated HBK; and therefore it will be the diameter of the terrestrial disc illuminated by the Sun, and its semidiameter MK, whose measure is angle MAK.

[Margin: 3. The method of Kepler and of Boulliau.]

In this 3rd way, then, Kepler, in the *Epitome of Copernican Astronomy*, p. 874, and in the Rudolphine Tables, chap. 31, precept 149, takes the semidiameter of the Earth's disc apparent to a lunar viewer at A, and so does Boulliau in Book 4, chap. 5. How much, however, the angle MAK is smaller than the angle of lunar parallax CAG, Kepler defines from his hypothesis, saying: *If from the Moon's horizontal parallax you take away one and a half times the Sun's horizontal parallax, there will remain the semidiameter of the Earth's disc.* This is very nearly true, however, in any hypothesis, as will be clear if you investigate the angle CAK (which is the same as MAK) in triangle CAK, in which AC, the distance of the Moon from the Earth, is given, and CK, one semidiameter of the Earth, and the angle they enclose. For that angle consists of the right angle ACE and the acute angle KCE, which measures the arc EK, illuminated beyond the quadrant by the unrefracted rays of the Sun, which it is sufficient to use here; and this arc becomes known through Problem 4 of chapter 11 of Book 3. It

will be clear if you then compare the angle thus gathered with the Moon's horizontal parallax. Yet the difference will be slight if in many problems we use CE itself for the semidiameter of the disc, because the angle CAE is nearly midway between the parallactic angle CAG and Kepler's angle CAK.

PROBLEMA II.

Problem 2.

Inueſtigare Semidiametrum Apparentem Vmbræ Lunaris, Porrectæ vltra Diſcum, aut centrum Terræ.

To investigate the apparent semidiameter of the lunar shadow, extended beyond the disc or the centre of the Earth.

IV. Here the Moon's shadow is supposed to extend beyond the centre of the Earth; how this is determined I have already shown in chap. 9, Problem 4. For if the apparent semidiameters of both luminaries are equal, the apex of the lunar shadow ends at the centre of the Earth, as in figure 4 of chapter 8. So for it to extend further, the Sun's apparent semidiameter must be less than that of the Moon. Then, therefore, says Kepler in the *Epitome of Copernican Astronomy*, p. 874, and in the Rudolphine Tables, chap. 31, precept 150: *if the Sun's apparent semidiameter be taken from the Moon's apparent semidiameter, there remains approximately the semidiameter of the lunar shadow as viewed from the centre of the Moon.*

To demonstrate this, since Kepler's *Hipparchus* has never, so far as I know, come to light, in the following figure let there be on the straight line AB the centres: A of the Sun, whose quadrant is SV; C of the Moon, whose hemisphere is DEF; and T of the Earth, whose hemisphere is GHK. Let TH be a semidiameter of the Earth, perpendicular to AB, and let H be joined to C by the straight line CH; for the angle TCH will measure the apparent semidiameter of the Earth's disc TH, viewed from the Moon. Next draw the straight line PB, tangent to the Sun at P and to the Moon at E, until it meets the straight line AB at B. For CBE will be the semicone of the lunar shadow, whose apex B is beyond the centre of the Earth T. And therefore, since the Moon is smaller than the Sun and much smaller than the Earth, PB will cut the Earth's semidiameter at M; let this point be joined to the Moon's centre C by the straight line CM. For the angle TCM will be the measure of the shadow's semidiameter TM, apparent to a viewer placed at C; and CME will be the angle subtending the Moon's apparent semidiameter, viewed from M, that is, very nearly from the centre of the Earth. Finally, from the centre of the Earth let the straight line TP be drawn, tangent to the Sun at a point that differs insensibly from the point P, because of the Sun's enormous distance. For the angle ATP will measure the Sun's apparent semidiameter, viewed from the centre of the Earth.

Diagram. Set into the left column, on the right of a narrow measure of text. A long vertical line runs from A at the top to B at the bottom. A short

horizontal line runs from A to V, and a second line from A down to P, just below V. A quadrant arc runs from S on the vertical up through P to V. A pair of close, nearly parallel lines descends from P to the lower left, one reaching T and one reaching B. Lower down, a small semicircle D-E-F stands against the vertical, and below it a larger semicircle G-H-K, with a horizontal line from T to H and several lines crossing between C, E, F, T, M and H.

- A (top left): A
- V (top right): V
- P (just below V): P
- S (on the vertical, at the foot of the quadrant arc): S
- D, C, F (on the vertical, against the small semicircle, top to bottom): D, C, F
- E (on the right of the small semicircle): E
- G, T, K (on the vertical, against the large semicircle, top to bottom): G, T, K
- M (just right of T, below the horizontal line): M
- H (right end of the horizontal line): H
- B (foot of the vertical): B

[Margin: Lemma 1.]

With these laid down, I say first that the two straight lines PT and PB are parallel as far as the senses can tell, as I have already shown in Book 3, chap. 5, from no. 6, granted that TB has no sensible ratio to the Sun's distance ATB. And it has none, because between TM, which is less than the Earth's semidiameter, and AT, the Sun's distance, which in my reckoning is 7500 semidiameters of the Earth, there is no ratio perceptible by us through any observations.

[Margin: Lemma 2.]

I say therefore secondly that the external angle ATP, that is, the Sun's apparent semidiameter, because of the sensible parallelism of the lines PT and PB, on which the straight line AB falls, is by Euclid I.29 equal, as far as the senses can tell, to the internal and opposite angle TBM; and so TBM is, without sensible difference, equal to the angle measuring the Sun's visual semidiameter.

[Margin: The solution of the problem and its proof.]

With these premised, since by Euclid I.32 the external angle CME, that is, the Moon's apparent semidiameter, is equal to the two internal and opposite angles CBM, that is, TBM, which is equal to the Sun's semidiameter, and BCM, that is, TCM, which measures the semidiameter of the lunar shadow TM, taken together: therefore, when the angle of the Sun's semidiameter CBM is taken from the angle CME of the Moon's semidiameter, there will remain TCM, the measure of the apparent semidiameter TM.

PROBLEMA III.

Problem 3.

Semidiametrum Penumbræ Lunaris apparentem ſpectatori in Luna collocato, & tranſeuntem per Terræ centrum Inquirere.

To find the semidiameter of the lunar penumbra apparent to a viewer placed on the Moon, and passing through the Earth's centre.

V. Kepler, in the *Epitome of Copernican Astronomy*, p. 875, and in the Rudolphine Tables, chap. 31, precept 150, says: *if you gather into one sum the apparent semidiameters of the Sun and Moon, and about half of the Sun's horizontal parallax, the semidiameter of the penumbra will be accumulated*; and he affirms that this was demonstrated in his *Hipparchus*. I demonstrate the same as follows.

In the diagram added at this place, let there be on the straight line ACB the centres: A of the Sun, which is DEF; C of the Moon; and B of the Earth, whose disc and diameter is HM. To this diameter let the extreme rays descend from the Sun, touching the Moon on opposite sides, namely DPL and FTG; and from very nearly the same points D and F to the points G and L let the Sun's rays DG and FL be drawn, so that from G and L the whole Sun begins to be seen, under the angles DGF and FLD. For the space GL will be that which the Moon's penumbra darkens, because within it there is no point that is illuminated by all the Sun's rays, but only by some; therefore GL will be the diameter of the penumbra, and GB its semidiameter.

Diagram. Set into the right column, with the text running in a narrow measure on its left. At the top, a shallow arc D–E–F hangs from a nearly straight line D–A–F, the arc bulging downward to E. A vertical line runs from A through E, C and S to B at the foot. Two heavy near-vertical lines run from D and from F down to the base line. Two pairs of slanting lines cross: from D toward the lower right and from F toward the lower left, crossing the vertical above a small circle centred at C; further lines from A slant down to G. Below, a large semicircle stands on the horizontal base H–G–B–L–M. A dashed line runs from N on the left limb of the semicircle to B.

- D (top left): D
- A (top centre): A
- F (top right): F
- E (below the arc, on the vertical): E
- T (above K, at the upper left of the small circle, where a ray touches the circle): T
- K (left of the small circle): K
- C (centre of the small circle): C
- P (right of the small circle): P
- T (small sort, on the upper left limb of the semicircle): T
- O (right of T, on the same limb): O
- S (top of the semicircle, on the vertical): S
- R (upper right limb of the semicircle): R
- N (left limb of the semicircle, at the head of the dashed line): N
- H, G, B, L, M (on the base, left to right): H, G, B, L, M

When the straight line GC is drawn from G to the Moon's centre C, then, the angle GCB will subtend and measure the semidiameter of the penumbra GB, apparent to the viewer at C. Now from the Sun's centre A let the straight line AG be drawn to G, so that the angle AGF subtends and measures the Sun's apparent semidiameter AF; and finally from A, the Sun's centre, let a ray be drawn tangent to the Earth at N, so that the angle NAB is the Sun's horizontal parallax. With these laid down, I say that the angle GCB, that is, the measure of the semidiameter of the lunar penumbra viewed from the Moon's centre, is equal to the angles AGF, which measures the Sun's apparent semidiameter, and TGC, which measures the Moon's apparent semidiameter (as the angle KBC also measures it, since the ray GIF touches the Moon at T), and the angle GAB, which is smaller than the Sun's horizontal parallax HAB [recte: NAB], all taken together. For by Euclid I.32 GCB is equal to the angle GAB and the angle AGC taken together; but these two are equal to the three already mentioned, since AGC consists of the angles AGF and TGC.

[Margin: Part 2 of the problem, concerning half of the Sun's parallax.]

Now the parallactic angle NAB is the angle under which an eye at A would see the arc NS of the Earth, which is almost a quadrant, since BNA is a right angle and NAB, the Sun's horizontal parallax, is not greater than 3′. The arc OS, as I shall say at the end of Problem 5, is about 35°; and OI is about as great as the angle IGO, that is AGF, of the Sun's apparent semidiameter, which is about 15′. The arc SI, therefore, never much exceeds 35° 15′, and will be about 36°. So if NS, of almost 90°, is seen under the angle NAB of the whole solar parallax, the arc IS, which is about 36°, will be seen under the angle GAB, about half of the horizontal solar parallax: and this Kepler asserts but does not demonstrate. You will therefore work more exactly if, to the sum of the semidiameters of the luminaries, you add not half but 4/9 of the Sun's horizontal parallax. Now, to have a specimen of the three preceding Problems from our hypothesis, look at the following small table.

Semidiametri Apparentes Oculo in ☾ fito. — Apparent semidiameters for an eye placed on the ☾.

Apparent semidiameter	Value
Of the Earth's disc: ☾ at apogee	53′ 20″
Of the Earth's disc: ☾ at perigee	63′ 45″
Of the lunar penumbra: ☾ & ☉ at apog.	29′ 27″
Of the lunar penumbra: ☾ & ☉ at perig.	33′ 30″
Of the lunar penumbra: ☾ at apog., ☉ at perig.	30′ 46″
Of the lunar penumbra: ☾ at perig., ☉ at apog.	32′ 11″
Of the sheer shadow of the ☾, greatest, with ☾ & ☉ at perigee	1′ 30″

PROBLEMA IV.

Problem 4.

Inquirere in vero Nouilunio Latitudinem veram Centri Penumbræ Lunaris, ſeu potiùs Arcum inter centrum eius, & centrum Diſci Terræ inuenire, Terminos Poſſibilis, aut Neceſſariæ Eclipſis Terræ ac Solis, & ſpeciem futuræ Eclipſis determinare.

To investigate, at the true new Moon, the true latitude of the centre of the lunar penumbra, or rather to find the arc between its centre and the centre of the Earth's disc; and to determine the limits of a possible or a necessary eclipse of the Earth and of the Sun, and the kind of the coming eclipse.

[Margin: Precept 1.]

VI. First, the true latitude is investigated here in the same way as the latitude of the Moon itself; for the axis and centre of the lunar penumbra recede from the ecliptic marked on the Earth's disc by just as much, and toward the same quarters of the world, as the Moon itself recedes in the sky. The arc between the centre of the lunar penumbra and the centre of the Earth's disc is likewise investigated in the same way as the arc between the centres of the Moon and of the Earth's shadow, on which see chapter 6, Problem 4. Next, the apparent semidiameter of the Earth's disc is to be sought by Problem 1, and that of the lunar penumbra by Problem 3. With these prepared, let there be a diagram in which the Earth's disc at true new Moon is AKI, with its centre C and semidiameters CI, CK, CL, and the terrestrial ecliptic is ACB, on which the nodes of the penumbra's path are at B or D.

Diagram. Set into the right column across its full measure, between the words prǽpa- and ratis. A large circle has its centre at C. A heavy straight line runs from A on its left limb through C and D, out past the right limb, to B. Lines run from C up to M and K at the top of the circle, and from C down through G and I. Four small stippled circles stand around it: the circle centred at L straddles the limb at the upper right; the circle centred at H lies inside the limb at the upper left of L; the circle centred at G lies inside the limb below the line; the circle centred at E lies below the limb and touches the circle G at I. Each small circle carries radii from its centre. Lines run from B to L and from B to E; lines run from D to H and from D to G.

- A (left limb of the large circle, on the line): A
- K (top of the large circle, outside the limb): K
- M (just inside the limb, below K): M
- L (centre of the upper right small circle): L
- H (centre of the upper left small circle): H
- C (centre of the large circle): C
- D (on the line, inside the right limb): D
- B (far right, the end of the line): B
- G (centre of the lower small circle inside the limb): G
- I (on the lower limb, where the two lower small circles meet): I

• E (centre of the lowest small circle): E

[Margin: Case 1, in which there is no eclipse.]

Second, let the centre of the penumbra be at E, the node of the penumbra's path EB with the terrestrial ecliptic at B, and the straight line CE from the disc's centre perpendicular to the penumbra's path; and let the penumbra's limb I, at the closest approach of the centre E to C, remain outside the limb of the disc, or merely touch it. For then there will be no eclipse of the Earth, and none of the Sun either. The test for this will be if the arc CE between the centres is not smaller than the sum of the apparent semidiameter CI of the disc and IE of the penumbra.

[Margin: Notable corollary on the limits of eclipses of the Earth and of the Sun.]

Now the greatest apparent semidiameter of the terrestrial disc can be known, which of course occurs when it is viewed from the Moon at perigee; and the greatest semidiameter of the lunar penumbra can also be known, which of course occurs with the Sun and Moon at perigee; and it can be known at the moment of true new Moon as well. The greatest of the arcs CE between the centres is therefore given. The angle of inclination of the penumbra's path to the ecliptic is supposed from the hypothesis of each of the more celebrated astronomers, since it is as great as the Moon's greatest latitude at ecliptic syzygies. In the triangle CBE, then, right-angled at E, given CE, the greatest arc, and the angle CBE, one can know EB, the distance of the centre of the lunar penumbra from the nearer node, and the distance of the Moon itself from the same node, at which or beyond which an eclipse of the Earth and of the Sun is not possible. If, however, the least arc is used, which occurs with the luminaries at apogee, the distance from the node for the limits necessary somewhere on Earth will become known; and the arc between these two distances will be for eclipses, but not necessary ones. I am glad, therefore, to bring these limits into one view in the following small table, according to the hypotheses of the more celebrated astronomers. The foundations of its synopsis are the parallaxes of the luminaries, on which see Book 3, chapter 8, and Book 4, chapter 13; their apparent semidiameters, on which see Book 3, chapter 10, and Book 4, chapter 16; the greatest inclination of the lunar path to the ecliptic, on which see Book 4, chapter 12; and finally Problems 1 and 3 of this chapter.

TABVLA PRO TERMINIS ECLIPSIVM TERRÆ & SOLIS. — Table for the limits of eclipses of the Earth and of the Sun.

From the data of the authors	Apparent semidiameter of the Earth's disc: greatest, ☾ at perig.	Same: least, ☾ at apog.	Apparent semidiameter of the lunar penumbra: greatest, ☾ & ☉ at perig.	Same: least, ☾ & ☉ at apog.	Arc between the centres at ecliptic ☌: greatest, ☾ & ☉ at perig.	Same: least, ☾ & ☉ at apog.	Inclination of the ☾'s path to the ecliptic, at ecliptic ☌	Limits of eclipses of the ☉ and of the Earth, and distance from the nearer node: possible	Same: necessary somewhere
Ptolemy	63′ 40″	53′ 24″	36′ 10″	32′ 50″	99′ 50″	86′ 14″	5° 0′ 0″	19° 25′	16° 42′
Copernicus	62′ 44″	52′ 46″	36′ 12″	32′ 20″	98′ 56″	85′ 6″	5° 0′ 0″	19° 12′	16° 25′
Tycho	62′ 30″	59′ 0″	31′ 54″	29′ 15″	94′ 24″	88′ 15″	4° 58′ 30″	18° 25′	17° 9′
Longomontanus	63′ 4″	59′ 27″	34′ 22″	32′ 18″	97′ 24″	91′ 45″	4° 58′ 30″	19° 0′	17° 50′
Kepler	62′ 11″	56′ 50″	32′ 25″	30′ 30″	94′ 36″	87′ 20″	5° 18′ 0″	17° 16′	15° 55′
Lansbergen	63′ 30″	53′ 24″	37′ 0″	32′ 53″	100′ 30″	86′ 17″	5° 0′ 0″	19° 32′	16° 43′
Boulliau	59′ 22″	55′ 46″	34′ 55″	32′ 54″	94′ 17″	88′ 40″	4° 58′ 30″	18° 22′	17° 13′
Wendelen	61′ 8″	53′ 36″	31′ 50″	29′ 12″	92′ 58″	82′ 48″	5° 0′ 0″	18° 1′	15° 58′
Ourselves	63′ 45″	53′ 20″	33′ 30″	29′ 27″	97′ 0″	82′ 47″	5° 0′ 0″	18° 49′	15° 58′

[Margin: Case 2, of a half-shadow.]

Third, if the arc between the centres, as CL in the diagram, where the centre of the penumbra is at L, is equal to the apparent semidiameter of the Earth's disc, only half of the penumbra, and half of the shadow if there is any shadow, will occupy the Earth. That eclipse will concern only the Frigid and Temperate Zones; the remaining half of the penumbra will fall outside the globe of the Earth, and there will scarcely anywhere be a central eclipse.

[Margin: Case 3, of the whole penumbra occupying the Earth without stay.]

Fourth, if the arc between the centres is smaller than the apparent semidiameter of the Earth's disc by as much as the apparent semidiameter of the penumbra, the whole penumbra will occupy the Earth, but without any stay of the whole penumbra. So it is in the diagram with the arc CG, the centre of the penumbra being placed at G; this arc differs from the semidiameter CI by GI, the semidiameter of the penumbra.

[Margin: Case 4, of the whole penumbra staying on the face of the Earth.]

Fifth, if the sum of the arc between the centres and the apparent semidiameter of the penumbra is smaller than the apparent semidiameter of the Earth's disc, the whole penumbra will occupy the Earth, with some stay of its whole self. So

it is in the diagram, the centre of the penumbra being placed at H, if the sum of the arc CH and the semidiameter HM is smaller than the semidiameter CK.

[Margin: Corollary for the discoveries of the new astronomy.]

In case 1, therefore, there will be no eclipse either of the Earth or of the Sun; in cases 2, 3 and 4 there will be some eclipse, and somewhere on Earth, both of the Earth and of the Sun. Astronomy may rejoice now for the first time that this can now be known without the tangles of the parallaxes. Here it is not worth the trouble of worrying over the sheer shadow, since it rarely occupies the Earth perceptibly, and then with a semidiameter of no more than 1′ 30″.

PROBLEMA V.

Problem 5.

Magnitudinem Eclipſis Terræ determinare & ſcrupula Diſci Terræ conuertere in arcum circuli magni Terræ.

To determine the magnitude of an eclipse of the Earth, and to convert minutes of the Earth's disc into an arc of a great circle of the Earth.

[Margin: Why there is no total eclipse of the Earth or universal eclipse of the Sun.]

VII. The Moon's horizontal parallax, by which we said the semidiameter of the Earth's disc viewed from the Moon is estimated, is always greater than the semidiameter of the lunar penumbra, that is, than the sum of the apparent semidiameters of the ☾ and ☉ and the semi-parallax of the ☉. For that reason there is no total eclipse of the Earth; and from this it also follows that there is no universal eclipse of the Sun over one entire terrestrial hemisphere. Speaking, then, of a partial eclipse of the Earth, it is as great as the diameter of the lunar penumbra, or the part of it occupying the Earth, compared with the diameter of the Earth's disc. Now to someone viewing from the ☾ the Earth illuminated by the Sun, the Earth would seem to the senses two feet across, since it is seen under nearly double the angle under which we see the luminaries, which we estimate at one foot. Since, then, the magnitude of an eclipse of the ☾ and of the ☉ is usually expressed in digits or inches, of which one foot has 12, it will not be unfitting to set out an eclipse of the Earth in inches or digits, but in such units as the apparent diameter of the Earth's disc has 24 of.

[Margin: Method 1 of determining the magnitude of an eclipse of the Earth.]

Let it be, then, as the apparent diameter of the Earth's disc is to 24, so the diameter of the penumbra, if the whole of it occupies the Earth, or the part of the diameter of the penumbra occupying the Earth, to the digits or inches of the Earth's disc obscured.

[Margin: Method 2 of the magnitude of an eclipse of the Earth.]

The same magnitude, however, is expressed another way by seeking the great arc of the Earth occupied by the penumbra, with the help of the figure

to be repeated here from number 3 of this chapter[42], in which we shall use the part of the Earth DBE as if it were DTE, for the solution of the problem will come to the same thing. Let the centre of the penumbra, then, be at B, at no distance from the place standing perpendicularly over the centre of the disc, and let the given semidiameter of the penumbra be VM, equal to IP, for example 32′, of which the disc's semidiameter HM, very nearly as great as DC, is 60′. Say, therefore: as HM or DC, 60′, is to the radius 100000, so 32′ is to IP, 53333, the sine of the arc SB. From the tables of sines, then, the arc SB is 32° 16′, and in this example the whole diameter of the penumbra occupies 64° 32′ of a great circle of the Earth. Next, let the centre of the penumbra be at V, its given semidiameter NV 32′, and the arc between the centres VM 10′. The whole NM is then 42′, of which, as above, HM, or nearly DC, is 60′. Say, therefore: as HM, 60′, is to the radius 100000, so NM, 42′, is to 70000, the sine OP of the arc OB, of 44° 26′. Then say again, or make it: as 60 is to 100000, so VM, 10′, is to IP, 16666, the sine of the arc SB, of 9° 36′. When these are taken from the arc OB, 44° 26′, there remains the arc OS, 34° 50′, occupied by the semidiameter of the penumbra; and this doubled will give the arc occupied by the whole penumbra, 69° 40′. Work in the same way, therefore, in other cases. As for how an arc of this kind is to be converted into miles, according to one's opinion of the size of the Earth, see Book 2, chapter 7.

[Margin: Kepler's rule is defective.]

Kepler's rule in the Rudolphine Tables, chapter 32, precept 159, comes to almost the same thing, as far as a portion of the disc beginning from the centre is concerned. But we have made it both easier and more universal, and have fortified it with a demonstrative diagram. For he does not teach how to find this arc except as reckoned from the centre of the disc, and he says that from its logistic logarithm the logarithm of the disc's semidiameter must be subtracted if this is less than 60, or[43] added if it is greater, so as to produce a logarithm which, sought among the logarithms of the semicircle or in the logarithms of the Heptacosias, will give the required arc in the column of arcs.

At the end of the following Problem I shall give in a small table the greatest and least arc of the Earth that the whole penumbra can occupy.

PROBLEMA VI.

Problem 6.

[42]no figure is printed at this place

[43]printed twice, vel at the line end and again at the head of the next line

Definire Durationem Eclipſis Terræ atque adeò Solis in Terra ſpectatam; Moramq eius, & Vmbræ ſi qua eſt, & Initium ac Finem Eclipſis huius.

To define the duration of the eclipse of the Earth, and so also of the Sun, as seen on the Earth; its stay, and that of the shadow if there is one; and the beginning and end of this eclipse.

VIII. In the following diagram let AB be the trace of the ecliptic on the Earth, and on it C the centre of the Earth's disc, which is AKF. Let the path of the lunar penumbra be BH, and the nearer node B. On this path let the centre of the penumbra first be at D, which is joined by the straight line CD, and let its limb touch the Earth at I. Next, if the whole penumbra is to occupy the Earth, let its centre be at G and the contact of the limbs at F, and let the centre G be joined to C by the straight line GC. Further, at the end of the shading, when the whole penumbra has now gone out, let its centre be at H and the contact of the limbs at L; and let H be joined to C by the straight line HC, for it will pass through the contact L, just as DC passes through I and CG through F, by Euclid III.12. Finally, from C let the arc CP be drawn, perpendicular to the path of the shadow, which at the moment of the middle of the eclipse of the Earth, that is, the moment next to the true new Moon, is known by Problem 4, number 6.

Diagram. Set into the right column across its full measure. A large circle has its centre at C. A heavy straight line runs from A on the left limb through C, out past the right limb, to B. A second straight line runs from B, slanting down to the left, through the small circle D and the small circle G and past P, to the centre H of a third small circle. Three small stippled circles stand on this line: D outside the right limb, touching the large circle from without at I; G inside the lower right limb, touching it from within at F; H outside the lower left limb, touching it from without at L. Straight lines run from C through I to D, from C through G to F, and from C through L to H, and a short line from C down to P. Each small circle carries radii from its centre.

- K (top, inside the large circle): K
- A (left limb, on the line): A
- C (centre of the large circle): C
- B (far right, the end of the line): B
- I (on the right limb, where the circle D touches): I
- D (centre of the right small circle): D
- G (centre of the small circle inside the limb): G
- F (lower right limb, below the circle G): F
- P (foot of the short line from C, on the slanting line): P
- L (lower left limb, above the circle H): L
- O (small sort, on the lower left limb, at the circle H): O
- H (centre of the lower left small circle): H

[Margin: Method 1, ours, of solving the problem.]

First, then, in the triangle CPD, right-angled at P, from the given CP, the arc between the centres, and the base CD, the sum of the semidiameters of the disc, CI, by Problem 1, and of the penumbra, ID, by Problem 3, find the arc PD, namely the minutes of the half-duration of the eclipse, to which PH, the minutes of the remaining half-duration, is equal.

[Margin: Minutes of the half-duration.]

Then in the triangle CPG, right-angled at P, from the given side CP, as above, and the base CG, which becomes known by subtracting the penumbra's semidiameter GF from the semidiameter CF of the Earth's disc, find PG, the minutes of the half-stay of the whole penumbra on the Earth; doubled, these will give the whole stay.

[Margin: Minutes of the half-stay of the whole.]

These minutes, converted into time according to the true hourly motion of the Moon from the Sun, will give the time of the whole stay and of the whole duration. The time of the half-duration subtracted from the middle of the eclipse, or from the ecliptic new Moon, will give the beginning, at which the Moon's penumbra starts to shade the face of the Earth; added, it will give the end, when the whole penumbra leaves the Earth.

[Margin: Minutes of the half-stay of the centre.]

But if you want the half-stay of the centre, resolve the triangle CPO, and let the base CO be the semidiameter of the disc, etc. Briefly: let it be, as the radius is to the second sine of the arc CP, so the secant of the base CD or CG to the secant of the minutes PD or PG. Or to the residual logarithm of the arc CP add the second logarithm of the base CD or CG, and the second logarithm of the arc PD or PG will result. But if the centre of the penumbra were to traverse the eclip- tic of the Earth, the sum of the semidiameters of the disc and penumbra would make up the minutes of the half-duration, while the difference of these semidiameters would give the minutes of half the stay. Further, if there is ever a sheer shadow of the Moon occupying the Earth, and someone wishes to know its duration and stay, and the beginning of the shading and the end, do everything as above, but using the semidiameter of the sheer shadow.

[Margin: Method 2, Kepler's.]

Second, Kepler proceeds differently in the Rudolphine Tables, chapter 32, precept 158. He says that the sum of the semidiameters of the disc and penumbra is to be found, and that from its antilogarithm the antilogarithm of the arc between the centres is to be subtracted, so that the antilogarithm of the minutes of the half-duration remains. But if from the antilogarithm of the disc's semidiameter the antilogarithm of the arc between the centres is taken away, he says, there remains the antilogarithm of the minutes of half the stay, for which the whole shadow stays on the face of the Earth; and these minutes are to be converted into time by the true hourly motion of the Moon from the Sun. But he confirms these things with no demonstration.

[Margin: Method 3, partly Renieri's, partly ours.]

Third, Renieri gives a special table for this in the second edition of the *Tabulae Mediceae*, p. 356, and its precept in Part 2, chapter 6, number 2. We could also use the usual table of incidence and half-stay, seeking on its side the minutes of the arc between the centres and at its head the minutes of the base CD, if that table were extended beyond 68 minutes. Let there now come forth from the foregoing Problems the greatest of the eclipses of the Earth, and the least of those in which the whole penumbra of the Moon falls on the face of the Earth, with the greatest and least duration and stay, according to the principles of our hypothesis.

Magnitudo Eclipſis Terræ. — Magnitude of an eclipse of the Earth.

Magnitude	☾ at apogee, ☉ at perig.	☾ at perigee, ☉ at apog.
Digits, of 24	13 3/5	12 2/3
Arc of a great circle of the Earth	71° 22′	57° 0′
Italian Bolognese miles	5136	1065
German miles	1065	900

Tempora Eclipſis Terræ. — Times of an eclipse of the Earth.

Time	Greatest, ☾ & ☉ at perigee	Least, ☾ & ☉ at apog.
Duration of the lunar penumbra on the Earth	$5^h\ 52^m$	$4^h\ 30^m$
Total stay of the penumbra on the Earth	$1^h\ 52^m$	$1^h\ 45^m$

PROBLEMA VII.

Problem 7.

Determinare locum in Terra, vbi eſt centrum Penumbræ Lunaris, atq. adeò vbi in Nonageſimo Eclipticæ gradu fit vera ſimul ac viſa Coniunctio, & Centralis Eclipſis Terræ Soliſq. & Horam qua ibi fiet.

To determine the place on the Earth where the centre of the lunar penumbra is, and so where the true and also apparent conjunction occurs at the nonagesimal degree of the ecliptic, and a central eclipse of the Earth and Sun; and the hour at which it will occur there.

IX. We shall teach how to determine this place on the Earth without a diagram, which can hardly be represented on a plane; the reader will be better helped by using and turning a terrestrial globe.

[Margin: Precept 1.]

First, find the altitude of the nonagesimal in the region where the Sun is centrally eclipsed in the nonagesimal itself, and so where the true and apparent synod coincide. So, according to Kepler in the Rudolphine Tables, precept 161, convert the arc between the centres of the disc and penumbra, or at least the

latitude of the Moon itself found at the moment of true new Moon, into an arc of the quadrant, following what was said at the end of Problem 5; for the complement of this arc to the quadrant is the altitude of the nonagesimal at the middle of the duration of the eclipse, which it was proposed to find. The nonagesimal will incline to the south if the ☾'s latitude is northern; to the north, if southern. Or, as Renieri teaches in Part 2 of the *Tabulae Mediceae*, chapter 5, number 2, look for the latitude belonging to the moment of true synod in the body of Kepler's parallactic table, placed in the *Optics*, p. 312, under the semidiameter of the Earth's apparent disc; for opposite the latitude, on the left side, you will find the altitude of the nonagesimal you want.

[Margin: Precept 2.]

Second, to the degree of the ecliptic in which the true synod takes place add 90°, and you will have the degree ascending in the region sought. For if you look for this degree on the side of the table showing the altitude of the nonagesimal, and go across from it until you find the altitude of the nonagesimal found by precept 1, it will show at the head of the table the altitude of the pole of the region in which the central eclipse occurs, etc. But if you do not find the altitude of the nonagesimal in it, the defect [Defectus] will be either central nowhere, or only in polar regions. In this case, subtract the sum of the semidiameters of the Sun and Moon from the true latitude of the Moon, which will then be greater, and with the remainder enter the parallactic canon, at whose head is the horizontal parallax of the luminaries representing the semidiameter of the terrestrial disc. Under this look for the remainder already mentioned, and opposite it you will have the altitude of the nonagesimal. With this, and with the ascending point of the ecliptic, as above, you will find the required elevation of the pole for a region outside the polar circles, in which the least eclipse of all will occur, and the Moon will only graze the Sun's margin. Therefore, if you subtract from the Moon's latitude something less than the sum of the semidiameters of the luminaries, and repeat the same operation as above with the remainder, you will find a region outside the polar circles at whose elevation some part of the Sun will be eclipsed. See Renieri above, chapter 5, number 3, and Kepler in the Rudolphine Tables, precept 162.

[Margin: Precept 3.]

Third, for the moment of true new Moon, find the right ascension of the degree of the ecliptic in which the luminaries are joined; and to that right ascension add the apparent hours at which the new Moon happens, converted into degrees of the equator, and you will make up the right ascension of mid-heaven for the meridian for which the astronomical tables are constructed. Next, under the altitude of the pole found by precept 2, find the oblique ascension of that ascending point spoken of at the beginning of precept 2, and from its oblique ascension subtract 90°; the result is the right ascension of mid-heaven of the region sought. Subtract this from the right ascension of mid-heaven of the tables, and there will emerge the difference between the meridian of the tables and the meridian of the region sought. Since, then, the longitude of

this region is known from this, and the altitude of the pole from what was said above, the place will also be known in which the central eclipse occurs, and with respect to which the true and also apparent synod of the luminaries occurs at the nonagesimal of the ecliptic. The difference of the meridians, converted into time, will give the time at which such an eclipse of the Earth and Sun will occur in that same place.

[Margin: Renieri's timely warning.]

But when the Moon's southern latitude is very great, or so great that, with the parallax of latitude toward the south added, the eclipse will occur only in a southern region, everything is to be done as above; but one must use not the place where the true and apparent conjunction occur together, but the opposite place, as Renieri rightly warns above, number 8, and Boulliau, Book 4, chapter 5, and the Rudolphine Tables, precepts 161, 162, 166, 167.

PROBLEMA VIII.

Problem 8.

Determinare an in loco Tabularum poſſibilis ſit Eclipſis Terræ, quæ alibi futura eſt centralis.

To determine whether an eclipse of the Earth that will be central elsewhere is possible at the place of the tables.

X. Look, by Problem 11, to be shown a little later, for the tract of regions within which the eclipse is completed. For if the place of the tables is within that tract, the eclipse can be seen there; if not, it cannot be seen. The place of the tables (which I want understood in other problems too) is the place for whose meridian the roots of the motions of the Sun and Moon are constructed, which anyone may choose to use.

PROBLEMA IX.

Problem 9.

Determinare locum, vbi Oriente Sole Penumbra Lunæ Diſcum Terræ incipit tangere, atque adeò Luna ſtringere limbum occiduum orientis Solis; qui proinde locus viſurus eſt Defectum ab initio ad finem. Et locum cui Occidente Sole, Penumbra Lunæ Terram deſerit; atq. adeò Luna ortiuum Solis marginem deſerit, finiturq. cum fine diei Labor, qui proinde locus vidit iam deliquium ab initio ad finem.

To determine the place where, at sunrise, the Moon's penumbra begins to touch the Earth's disc, and so where the Moon begins to graze the western limb of the rising

Sun; this place will therefore see the defect from beginning to end. And to determine the place for which, at sunset, the Moon's penumbra leaves the Earth, and so the Moon leaves the Sun's eastern margin, and the labour ends with the end of the day; this place has therefore already seen the eclipse from beginning to end.

[Margin: Precept 1.]

XI. First, take the minutes of the half-duration found by Problem 6, not yet converted into time, and subtract them from the true argument of the Moon's latitude; there will remain the true argument for the moment at which the eclipse first begins on the Earth. Or add them, and you will have the argument for the moment at which the eclipse ends. With the argument, moreover, work out the Moon's true latitude, and divide it by the sum of the three apparent semidiameters of the Sun, the Moon and the Earth's disc. For the remainder [Reſiduum], in the small column of the parallactic table spoken of above, the column standing under 60′, will show opposite it, on the left side, the altitude of the nonagesimal of the region for which the rising Sun begins to be eclipsed by the Moon, or, in the second case, for which the setting Sun ceases from the eclipse.

[Margin: Precept 2.]

Second, with the minutes of the half-duration found by Problem 6, but now converted into time, look up the Sun's mean motion, and subtract it from the Sun's place at true new Moon for the first phase; add it for the last. Next, look for the remainder or the sum on the side of the table showing the altitude of the nonagesimal, running across its cells to the right or left. For the cell in which you find the altitude of the nonagesimal found by precept 1 will show at the head of the table the altitude of the pole sought. If you do not find it, know that the region you seek is southern and that the Antarctic Pole is elevated in it, and that therefore the operation already mentioned is to be carried out with the place of the ecliptic opposite the one in which the true new Moon occurs.

[Margin: Precept 3.]

Third, with the altitude of the pole found for the first phase of the eclipse, under that altitude, and with the Sun's place at its moment, found as above by subtracting the ☉'s mean motion, look up the oblique ascension. Subtract 90° from it, so as to make the right ascension of mid-heaven for the meridian of that region. Next, from the time of true new Moon subtract the time produced from the half-duration of Problem 6, added to the moment of true new Moon, and turn the remainder into degrees of the equator. Add these to the right ascension of that degree of the Sun whose oblique ascension you have just found, and you will have the right ascension of the meridian of the tables at the moment of the first phase. By subtracting from this the right ascension of the former meridian, you will find the difference of longitude between the place of the tables and the place of the region sought, whose altitude of the pole you have already found. So the place of the first phase will be known.

[Margin: Precept 4.]

Fourth, for the last phase, with the Sun's place opposite to the one you found by precept 2 in order to obtain the altitude of the pole, and with the altitude of the pole found there for this phase, look up the oblique ascension and subtract 90° from it, so that the right ascension of mid-heaven remains for the meridian of the place sought. Next, add the time of the half-duration of Problem 6 to the apparent time of true new Moon, and add the sum, converted into degrees of the equator, to the right ascension of the Sun's place found for the moment of the last phase; for this will make the right ascension of mid-heaven for the meridian of the tables. By subtracting from this the right ascension of mid-heaven of the former meridian, the difference of longitude will be known, and consequently the longitude of the place sought, whose altitude of the pole is already known.

[Margin: Renieri's salutary warning.]

But if, in either of the phases, the altitude of the pole found is that not of the Arctic but of the Antarctic Pole, then add 180° to the right ascension of mid-heaven of the tables, and from the total subtract the right ascension of the former meridian, so that the difference of the meridians becomes known.

See, if you like, for this Problem the Rudolphine Tables, precepts 164, 166, 167, and Renieri above, chapter 6, numbers 3, 4 and 5, with examples there and in Boulliau, Book 4, chapter 5.

PROBLEMA X.

Problem 10.

Determinare locum Terræ, cui Sol Oriens aut Occidens centraliter deficit; ſiue Eclipſis ſit totalis, ſiue annularis.

To determine the place on the Earth for which the rising or setting Sun is centrally eclipsed, whether the eclipse is total or annular.

[Margin: Precept 1.]

XII. First, look up the minutes of the half-stay of the centre of the lunar penumbra and convert them into time, as stated in Problem 6, precept 1. Subtract this from the moment of true new Moon, and you will have the time of this first phase, that is, the time at which the central eclipse first appears on the Earth, in a place not yet known. Add that time again, and you will have the time of the second phase, at which the central eclipse will end.

[Margin: Precept 2.]

Second, take the minutes of the half-stay already mentioned, but not converted into time, and take them away from the true argument of latitude found for the time of true ☌, for the first central phase; add them for the second. With that argument look up the Moon's true latitude, and search for this latitude in the small columns of the parallactic table, under the minutes placed at the head that represent the semidiameter of the disc, so that on the left side you obtain the altitude of the nonagesimal for each phase.

[Margin: Precept 3.]

Third, take the Sun's mean motion corresponding to the interval of time between the true new Moon and the phases already mentioned. Subtract it for the first phase, and add it for the second, to the Sun's place found at true new Moon. With the remaining place or the total, looked up on the side of the table of the altitude of the nonagesimal, look in its cells for the altitude of the nonagesimal dealt with in precept 2, and at the head you will have the desired altitude of the pole.

[Margin: Precept 4.]

Fourth, with the altitude of the pole found, look up the oblique ascension of the Sun's place belonging to the first or second phase, and do all the rest as in precepts 3 and 4 of Problem 9, to obtain the difference of longitude from the meridian of the tables. When this is known, together with the altitude of the pole, the place on the Earth will be known in which the rising Sun first, or the setting Sun last, is centrally eclipsed. On these see Renieri above, chapter 6, numbers 6, 7, 8, 9; Kepler in the Rudolphine Tables, precepts 163, 166, 167; and Boulliau, Book 4, chapter 5.

PROBLEMA XI.

Problem 11.

Determinare Regiones in Terra, quibus toto tempore Durationis Eclipſis Penumbra Lunæ ſuperfertur.

To determine the regions on the Earth over which the Moon's penumbra is carried during the whole time of the eclipse's duration.

[Margin: Precept 1.]

XIII. First, having found by Problem 9 the places in which, at sunrise, the Moon's penumbra begins, or at sunset ceases, to touch the Earth: if you conceive an arc of a circle through them, passing through the place in which the central eclipse occurs at the nonagesimal, on which see Problem 7, you will have the tract of the Earth within which, during the whole time of the duration, the Moon's penumbra darkens the Earth with some part of itself.

[Margin: Precept 2.]

Second, having found by Problem 10 the places in which the rising Sun begins, or the setting Sun ceases, to be centrally eclipsed, that is, in which the centre of the lunar penumbra begins or ceases to be on the Earth: if an arc through these passes through the place of the central eclipse occurring at the nonagesimal, on which see Problem 7, you will have the tract of the Earth within which the phases of a total or central eclipse are observed. See Renieri above, chapter 6, number 11.

So far I have chosen to point out how many Problems concerning the eclipse, not only of the Earth but also of the Sun, we can settle without the circuitous

routes of parallaxes. The rest, with their examples, will be made clear in Part 2 of this volume.

[Margin: On the two phases omitted.]

But if anyone wishes to know two other phases, namely the place for which the rising Sun ceases to be eclipsed on its lower or eastern part, or the setting Sun begins to be eclipsed on its western and lower part, let him consult Kepler in the Rudolphine Tables, precepts 165, 166, 167.

CAPVT XIII.

Chapter 13.

De Formando Typo Eclipſis Terræ ac Solis & de Inclination-ibus Phaſium.

On forming the delineation of an eclipse of the Earth and Sun, and on the inclinations of the phases.

I. What I said in chapter 7 about the delineation of eclipses of the Moon will indeed help in understanding the present chapter, but it is not sufficient. So one should indeed consult Ptolemy, Book 6, chapter 13; al-Battānī, chapter 44; Peuerbach, toward the end of his precepts on eclipses; Regiomontanus, Problem 50 of the *Primum Mobile*, and in the *Epitome of the Almagest*, Book 6, propositions 28, 29 and 30; Reinhold in the Prutenic Tables, precept 65; Tycho, volume 1 of the *Progymnasmata*, p. 136; Kepler in the *Ephemerides*, pp. 41 and 44; Magini, Canon 90 of the *Secunda Mobilia*; Lansbergen, precept 37; and Renieri, Part 2, chapter 3, number 14. But they are to be corrected from Kepler's warnings in the *Astronomia Optica*, chapter 11, Problems 30, 31, 32, and in the *Epitome of Copernican Astronomy*, p. 890. Leaving aside the delineations that commonly circulate without art, the ones made with art are to be set out.

Diagram. In the left column, between the first and second sections, headed `Meridies.` (South) above and `Septentrio.` (North) below. A large circle has a vertical diameter from B at the top to D at the bottom, and a horizontal diameter from C on the left to A on the right, crossing at the centre E. A smaller concentric circle stands within it, and a further circle of the same size as two others is centred about H, above E. Two more circles of that size are centred on the circumference of the large circle: one upper right at Q, one on the left at G, the latter passing outside the large circle. A chord runs from G through H to Q, and P stands on the vertical above H. Lines run from Q through E to R at the lower left, and from G through E to S at the lower right. M stands on the vertical just below H, and K on the line from Q to E.

- Top, outside the figure: `Meridies.` (South)
- Bottom, outside the figure: `Septentrio.` (North)
- Top of the vertical: B
- Bottom of the vertical: D
- Left of the horizontal: C

- Right of the horizontal: A
- Centre: E
- On the vertical above the centre, downward: P, H, M
- Upper right: Q, with K on the line below it
- Left: G
- Lower right: S
- Lower left: R

II. The old way of forming this delineation is as follows. Given the apparent semidiameters of the luminaries, let their sum be AE. At that interval, from the centre E, let the circle ABCD be described, whose diameter AC is to represent the ecliptic, and BD the arc of latitude. And from E let the Sun's disc be described at the interval of the Sun's semidiameter EK. Next, let the apparent latitude of the Moon be given for the beginning of the eclipse; let EP be taken equal to it, and from P let a perpendicular to BD be drawn out as far as the circumference of the circle BD, to the point Q, where the centre of the ☾ is at the beginning of the eclipse. From this point, at the interval of the lunar semidiameter KQ, let the Moon's disc be described. Again, for the end of the eclipse let the apparent latitude of the Moon be given, and let EM be made equal to it. Then MG, drawn perpendicular to BD, will mark at G the place of the lunar centre at the end of the eclipse, from which a disc of the Moon equal to the former is to be described. With this done, let the Moon's centres G and Q be joined by the straight line GHQ, for this will be the Moon's apparent path. Where this path cuts at H the arc of latitude, or the arc through the centres of the luminaries at the middle of the eclipse, there, with that point as centre, let a disc of the Moon be described equal to the former ones. Finally, the straight lines QER and GES, drawn from the centres Q and G through the Sun's centre E, will mark at R and S the quarters of the horizon, but those opposite to the ones toward which the darkness tends: for at the beginning of a solar eclipse the darkness bends toward the western hemisphere, and at the end toward the eastern. These quarters and the bending of the darkness are themselves investigated in the way I indicated in chapter 7, Problems 1 and 2, where I also set out the small Ptolemaic table; but using the apparent latitude, and applying to the Sun what was said there about the Earth's shadow.

[Margin: Faults of the method already mentioned.]

III. But in fact the method already mentioned for forming the delineation of solar eclipses rests on a constant angle of 5°, at which the Moon's apparent path is supposed to be inclined to the ecliptic during the whole time of the eclipse. Indeed, in investigating the angle formed by the ecliptic and the sum of the semidiameters, or the semidiameter of the shadow there and of the Sun here, Ptolemy, Book 6, chapter 11, supposes straight lines in place of curved ones. He also supposes the Moon's latitude at the moment of true ☍ or apparent ☌ in place of the arc passing between the centres of the darkening and darkened bodies at the moment of the middle and greatest eclipse; and finally he supposes the Moon's path parallel to the ecliptic, because of the smallness and nearness

of the arc, and lying in the same plane; although he did sometimes correct the quantity of the arc between the centres from the ecliptic digits.

[Margin: Regiomontanus's warning.]

But as Regiomontanus warns, Book 6 of the *Epitome*, proposition 28, it would have been better to use spherical triangles, and the latitude of the Moon belonging to the beginning of the eclipse or of the stay.

[Margin: Kepler's notable warning.]

In an eclipse of the Sun, however, Kepler's warning and observation, in both the passages cited from the *Optica* and the *Epitome*, is of greater weight. For the form already mentioned can be tolerated when the Moon's visible path is parallel to the ecliptic, and in the greatest eclipses, as I said in Scholium 1 of chapter 11, where I also gave, from Kepler, a way of determining more accurately the minutes of incidence and repletion in large eclipses of the Sun. But when the eclipses are small, and the lunar visible path deviates noticeably from parallelism to the ecliptic, because of the great variation of the parallaxes in latitude, the Moon's visible path turns out winding and does not lie in one plane, and its angle with the ecliptic takes on an inclination greater than 5°, so that it sometimes reaches 19° or 20°, as I said in Scholium 2 of chapter 11. In this case, then, a delineation of the eclipse built to the old standard is not true. Moreover, the approach of the centres of the Moon and Sun is to be considered in a circle inclined from the centre of the Sun to the visible node; and in this matter, because of the difference of the parallaxes, the variation is so great that, even on Kepler's testimony, it cannot be comprehended by small rules. Lastly, the Moon's visible path is sometimes inclined contrary to the true path, as I said in the same Scholium. Read it again here, and repeat from it the following diagram for some delineation of an eclipse of the ☉, in which DC, the visible path of the ☾, is shown inclined contrary to the true path NR.

Diagram. In the right column, inset at the right with the text of the fourth section running down its left side. Three discs of equal size lie along an oblique line from upper left to lower right. The middle disc is the Sun, with centre S and a dark central disc marked B, rimmed by a stippled band. The upper and lower discs are the Moon, each drawn with a face, overlapping the Sun at upper left and lower right. A straight line runs obliquely along the lower left of the three discs, and another along their upper right. A line through S, the vertical, runs from V at the left to R at the right.

- Upper left, outside the upper disc: Q and N
- Inside the upper disc: D at the centre, F at its lower edge
- Right of the Sun's disc: R
- Left of the Sun's disc: V
- Below the Sun's disc, on the lower straight line: G; in the rayed band just above it: I
- Sun's centre: S, with B above it
- Inside the lower disc: C at the centre, H at its lower edge
- Right of the lower disc: M

• Below the lower disc: A and E

[Margin: On the quarters of eclipses of the Sun, from Kepler.]

IV. As regards the quarter of the world on which the defect stands at the Sun's margin, it is found as follows, from Kepler in the *Epitome of Copernican Astronomy*, p. 891, and the Rudolphine Tables, precept 177. Let the ecliptic be ESQ, the Moon's true path NR but its apparent path DC, and the Sun's centre at S. Let the Moon's centre be at B at the middle of the eclipse, at C at the beginning and at D at the end, and let the arc through the centres be BS. Let the vertical VR be drawn through the centre of the Sun, and the arcs through the centres of the luminaries: at the beginning the arc SC, at the middle the arc SB, at the end the arc SD. First, then, the angle RSE is to be investigated, which the vertical already mentioned makes with the ecliptic, and next the angles CSE, BSE and DSQ. The angle RSE becomes known if from the mesologarithm of the distance of the nonagesimal from the zenith you subtract the logarithm of the Sun's distance from the nonagesimal; for there will remain the mesologarithm of the angle sought. The other angles become known if you subtract the logistic logarithm of the distance of the centres, SC or SB or SD, from the logistic logarithm of the apparent latitude at these three moments, which is CH, BS or DF; for there remains the logarithm of the angle CSE or BSE or DSF. Thus, with CSE subtracted from the angle RSE, the angle RSC becomes known, from which the azimuth for the beginning of the eclipse can be gathered. Again, with RSE subtracted from the angle BSE, there remains the angle BSR, for the azimuth at the middle of the eclipse. And with the angles DSQ and RSE subtracted from two right angles, that is, from 180°, there remains the angle DSR, for the azimuth at the end of the eclipse. Or, from precept 177 of the Rudolphine Tables, use three points of the ecliptic that are supposed known from the calculation of eclipses: namely, the point in which the centre of the Sun is, the culminating point, and the nonagesimal degree. These, connected with the vertical point by means of arcs, make up two triangles, in which the angle at the nonagesimal is a common right angle. With their help the angle at the zenith is investigated, which measures the arc of the horizon that the Arabs call the azimuth. From the apparent time given for the moment of the eclipse, moreover, and from the Sun's place, the nonagesimal is supposed known, and its distance from the zenith, from the culminating point and from the Sun. The rest, concerning the quarters of the horizon toward which the darkened part of the Sun tends, you will complete more readily (says Kepler) by imagination or by inspecting a sphere.

SCHOLIA.

Scholia.

[Margin: Paradox 1, on the quarters in an eclipse of the Moon.]

I. Although an eclipse of the Moon ordinarily begins on the eastern part of the Moon and ends on the western, yet it sometimes happens that it begins on

the western and ends on the eastern. Pliny the Elder indicated this, Book 2, chapter 13, in these words:

[Margin: An obscure passage of Pliny the Elder explained.]

"And what is most wonderful in this marvel: although it is agreed that the Moon is dimmed by the shadow of the Earth, this happens to it now on the side of the setting, now on that of the rising." Kepler, in the *Astronomia Optica*, chapter 11, Problem 31, shows that this can happen when at true full Moon the Moon's true latitude is equal to the sum of the semidiameters of the Moon and the shadow, and the Moon's latitude is either southern decreasing or northern increasing. For then the arc between the centres turns out shorter than the latitude, and some eclipse of the Moon occurs, though hardly a hundredth part of the diameter is darkened. If the latitude is southern decreasing, the mark of the eclipse tends toward the west; if northern increasing, it will tend toward the east. He adds that such an eclipse was seen in the year 1603 November 18, at $6^h\ 21^m$, and that a similar one was seen by Tycho in the year 1594 October 19, in the morning, at $5^h\ 56^m$.

[Margin: Paradox 2, on the quarters in an eclipse of the Sun.]

II. An eclipse of the Sun, too, ordinarily begins on the western part of the Sun and ends on the eastern; but, as Kepler shows in the *Astronomia Optica*, chapter 11, Problem 31, and in the *Epitome of Copernican Astronomy*, p. 895, it sometimes begins on the eastern and ends on the western; and indeed more evidently because of the parallaxes in latitude, which sometimes produce an inclination of the Moon's visible path of nearly 20°. For then, if at the moment of visible synod the visible latitude equals the sum of the semidiameters of the ☾ and ☉, the arc between the centres, or the distance of the centres, being smaller than the apparent latitude, will be able to produce an eclipse of one digit and more. For an eclipse of the Sun in such a case to be able to begin from the east, however, the Moon's true latitude must be either northern decreasing, with the eclipse occurring at the rising, or southern decreasing, with the eclipse at the setting. But if it must end from the west, the latitude required is either northern increasing, with the eclipse at the setting, or southern increasing, with the eclipse at the rising. But the parallax of latitude contributes chiefly to this. Now, the angles of the ecliptic and horizon begin to increase with ♈ rising, as far as ♎, and they increase most with ♋ rising, and the parallaxes of latitude decrease most. Therefore the Moon above the horizon is carried visibly toward the north more than it is by true motion toward the south, especially if by true motion it is ascending. So then a small southern eclipse of the Sun placed toward the west will be able to begin from the east, and a small northern eclipse to end from the west. Conversely, with ♑ rising, from ♎ to ♈, the angle of the horizon and ecliptic diminishes and the parallax of latitude increases, and the Moon is carried by visible motion toward the south. Therefore a small northern eclipse of the Sun placed in the eastern quadrant will be able to begin from the east, and a small southern one to end from the west. What Kepler has in the *Epitome* comes to almost the same thing: namely, near the eastern

horizon, in signs of right ascension, with the Moon slipping down to the node ☋, the Sun is seen to undergo eclipse either at its vertex or toward the left, and soon to be refilled from the same quarter; or near the western horizon, in signs of right descension, with the Moon fleeing from the node ☊ toward the north, the Sun is seen to undergo eclipse at its lowest margin, or a little above the left, and the eclipse is seen to end from the same quarter, higher up. Because also, with the Sun placed near the nonagesimal, with ♊ or ♋ rising and the parallax of latitude therefore very rapidly diminished, but with ♐ or ♑ rising and the parallax of latitude very rapidly increased, if the Moon in the former case ascends toward the north near ☊, and in the latter descends toward the south near ☋, the true and apparent variation accumulate, and the angle of the visible path becomes very great, and an eclipse of quite noticeable size can begin and end from the same quarter.

[Margin: Kepler's diligence.]

All of this we owe to Kepler's diligence and subtlety.

[Margin: Examples of such eclipses.]

III. To confirm this doctrine, Kepler relates that similar eclipses, varying from the usual quarters, occurred in the year 1588 February 16, the hour of the beginning at Uraniborg being $1^h\ 32^m$ after noon; and in 1595 October 3, a little after noon at Tübingen, with Mästlin observing and asserting in thesis 53 that the edge of the Sun that failed at the beginning inclined not to the west but to the east, by 9°, and that 2½ digits were eclipsed; and that the diagrams of the same eclipse observed at Strasbourg, found in Tycho, display the eclipse at the left. And he relates that in the year 1598 a great eclipse of the ☉ in ♓ near ☊ began at Graz in Styria from the right, precisely as if in the nonagesimal, and ended from the left, but near the vertex of the Sun. Thus in the year 1599 July 22, in the morning, the rising Sun was seen by Tycho to fail at its vertex, from which he deduces that in some southern climate the defect stood on the west from beginning to end. Again, in the year 1593 May 20, at Zerbst [Serueſtæ], the Sun failed to 2 digits, at the left, toward the beginning and the end. Lastly, in the year 1614, an eclipse of the ☉ in ♎ near ☋ began at Munich about noon from the right, and ended on the lower part of the Sun.

CAPVT XIV.

Chapter 14.

De Interuallis, Numero, ac Reuolutione Eclipſium Lunæ Solis ac Terræ.

On the intervals, number and revolution of eclipses of the Moon, Sun and Earth.

I. We have now come to the contemplation, as pleasant as it is noble, of divine Providence in dispensing eclipses. For it was fitting that there should be neither no eclipses, lest we be deprived of those opportunities we listed in chapters 1 and 2, nor too frequent ones, lest either so frequent a withdrawal

of the life-giving lights should do too much harm to the natures of sublunary things, or some, stupefied by excessive terror, and others, on the contrary, with their wonder growing stale, should turn out too contemptuous of heaven and of celestial spectacles. I have decided, then, to propose in separate theorems first the intervals of eclipses, then their number, and finally their restitution, that is, their revolution to the same day of the year or part of the zodiac, after first giving a single diagram and two small tables, which greatly help in discerning these things more easily.

Diagram. In the right column, inset at the right with the text running down its left side. Two circles of nearly equal size cross at A on the left and B on the right, the ecliptic ARBP and the Moon's orbit AMBS. R stands on the top of one circle and M just inside it below; S stands on the bottom of the other and P just inside it above. Short strokes cross the band between the circles on either side of A and of B, marking the limits.

- Top, outside: R
- Top, inside: M
- Bottom, inside: P
- Bottom, outside: S
- Right, at the crossing: B, followed by the sign ☊
- Left, at the crossing: A, preceded by the sign ☋
- Upper right, on the band, from the top: L, F
- Lower right, outside the band, from the top: C, G
- Upper left, on the band, from the top: K, E
- Lower left, outside the band, from the top: D, H

Let the ecliptic, then, be ARBP, and the circle of the Moon's orbit AMBS, whose southern semicircle is AMB and northern BSA, the southern node A and the northern B. Let the limits of a possible eclipse near B be F and C for lunar eclipses, but L and G for solar; and near the node A let them be D and E for lunar, H and K for solar. And let us take from chapter 7, number 1, the greatest arc BF and those like it as 15° 38′ in mean ☍, but 13° in true ☍, and the least as 10°. From chapter 11, numbers 3 and 5, and chapter 12, number 6, let us take the arc BL and those like it, the greatest as 20° 41′ in mean ☌, but 19° in true, and the least as 5° 25′, looking to the authors who hold the extreme opinions; for what holds of these will hold much more of the intermediate ones. Let us also take the likewise extreme opinion on lunar months from Book 4, chapter 19, and the motion of latitude from Book 4, chapter 23, taking into account, however, the equation, which at its greatest can befall this motion after one month or more. And so let the first table be the table of the arcs comparable in the foregoing figure, so that, looking at the motion in latitude of the ☾ in the second table, you may distinguish the intervals of months suited to eclipses from those not suited, according as the motion in latitude puts the Moon within or beyond the limits of eclipse.

I. TABVLA. Arcus Comparabiles cum motu Latitudinis ☾ pro Eclipſibus.

Table 1. Arcs comparable with the motion in latitude of the ☾, for eclipses.

Arcs	Greatest in ☍: in mean	Greatest in ☍: in true	Least: in true
For an eclipse of the ☾			
BF. BC. AD. AE	15° 38′	13° 0′	10° 0′
FC. DE	31° 16′	26° 0′	20° 0′
BSD. AMF	170° 0′	170° 0′	164° 22′
BSE. AMC	195° 38′	193° 0′	190° 0′
DMC. FSE	211° 16′	206° 0′	200° 0′
CSD. EMF	160° 0′	160° 0′	149° 36′
FSD. CSE. DMF. EMC	185° 38′	183° 0′	174° 22′
For an eclipse of the ☉			
BL. BG. AH. AK	20° 41′	19° 0′	5° 25′
LG. HK	41° 22′	38° 0′	10° 50′
BSH. AML	174° 35′	174° 35′	159° 19′
BSK. AMG	200° 41′	199° 0′	185° 25′
HMG. LSK	221° 22′	218° 0′	190° 50′
GSH. KML	169° 10′	169° 10′	138° 38′
LSH. GSK. HML. KMG	195° 16′	193° 35′	164° 44′

[Margin: Table of some lunar months and of the motion in latitude in them.]
II. TABVLA.
Table 2.

Number of lunar months	[Kind]	Time	Motion in latitude
1	Mean	$29^{d}\ 12^{h}\ 44^{m}$	30° 40′
1	Greatest	$30^{d}\ 3^{h}\ 14^{m}$	32° nearly
1	Least	$28^{d}\ 22^{h}\ 14^{m}$	28° nearly
5	Mean	$147^{d}\ 15^{h}\ 40^{m}$	153° 21′
5	Greatest	$150^{d}\ 16^{h}\ 10^{m}$	159° 5′
5	Least	$144^{d}\ 15^{h}\ 10^{m}$	147° 37′
6	Mean	$177^{d}\ 4^{h}\ 24^{m}$	184° 1′
6	Greatest	$180^{d}\ 19^{h}\ 24^{m}$	189° 51′
6	Least	$173^{d}\ 13^{h}\ 24^{m}$	178° 21′
7	Mean	$206^{d}\ 17^{h}\ 8^{m}$	214° 42′
7	Least	$202^{d}\ 12^{h}\ 38^{m}$	208° 47′
7	Greatest	$210^{d}\ 22^{h}\ 38^{m}$	220° 37′

THEOREMA I.

Theorem 1.

Non poſſunt eſſe ſingulis menſibus Eclipſes Solis aut Lunæ.

There cannot be eclipses of the Sun or Moon in every month.

II. For Pliny the Elder is right, Book 2, chapter 13: "But the eclipses of both are neither fixed nor monthly, because of the obliquity of the Zodiac and the

Moon's many-wandering windings, as has been said, the motion of the stars not always agreeing in the minutes of the degrees." That is, as Milichius [Jacob Milich] explains there, because of the obliquity of the ecliptic to the Moon's orbit, and of the orbit to the ecliptic, and because of the Moon's latitude, which prevents diametrical oppositions of the luminaries, or conjunctions, from always or for the most part occurring within the degrees and minutes required for eclipses. For let the earlier eclipse of the ☾ have occurred within arc FC, or of the ☉ within LG, and double, triple or quadruple the motion in latitude given in Table 2. You will always find the Moon, when the second, third and fourth lunar month is complete, beyond C and G but short of D and H, toward S; that is, in arc CSD, in which no eclipse of the ☾ occurs, or in GSH, in which none of the ☉ occurs. And the same happens if an eclipse of the ☾ occurs within DE, and of the ☉ within HK: for whether after the 2nd, after the 3rd or after the 4th month, the Moon will be within arc EMF, in which none of the ☾ occurs, or within KML, in which no eclipse of the ☉ occurs.

THEOREMA II.

Theorem 2.

Poſſunt Sol & Luna in Sex Menſibus Lunaribus bis Eclipſim pati.

The Sun and Moon can undergo eclipse twice within six lunar months.

III. So say Ptolemy, Book 6, chapter 6; Regiomontanus, Book 6 of the *Epitome*, proposition 9; Milich on Book 2 of Pliny the Elder, chapter 13; Magini, Book 2 of the *Theoricae*, chapter 26; and Mulerius [Nicolaus Mulerius] in the *Tabulae Frisicae*, p. 78. It is plain to anyone who compares the arcs of the first table with the motion in latitude of the second table, that is, with the motion that belongs to the months, whether mean, greatest or least, as the need may be. Ptolemy and Regiomontanus, however, use mean months, whose motion in latitude is 184° 1′. Therefore, if the earlier eclipse occurs within FBC for the ☾, or within LBG for the Sun, the Moon can certainly be, after six months, within DAE, the limits of an eclipse of the ☾, or HAK, the limits of an eclipse of the ☉. We have very many examples of eclipses in every sixth month below, in chapter 21. If, however, the earlier eclipse occurs very close to F, the Moon can be, after 6 lesser months, short of D, within arc DSC; or if it occurs very close to C, it can be beyond E after 6 greater months, and in this way a second eclipse does not occur.

Diagram. Set into the right column, with the text of the third section running in a short measure to its left. Two circles of equal size cross at two points, B on the right and A on the left. Near the top the upper arc of one lies outside the other, with R on the outer arc and M on the inner; near the foot the order is reversed, with P on the inner arc and S on the outer. Short cross-strokes mark off the limits on either side of each crossing.

- Top, outer arc: R
- Top, inner arc: M
- Upper right, nearer the rim: L; just below it: F
- Right crossing: B, with the sign ☊ outside it
- Lower right, on the outside: C; below it: G
- Bottom, outer arc: S
- Bottom, inner arc: P
- Lower left, on the outside: D; below it: H
- Left crossing: A, with the sign ☋ outside it
- Upper left, inside the rim: K; just below it: E

Labels read: R, M, L, F, B, C, G, S, P, D, H, A, K, E, ☊, ☋. Labels not read: none.

[Margin: The error of Martianus Capella.]

From this you may refute the error of Martianus Capella in Book 8 of *De Nuptiis Philologiae*, in the chapter on the eclipse of the Moon, where he says: "An eclipse cannot recur within the sixth month."

THEOREMA III.

Theorem 3.

Poſſunt Sol & Luna in quinque Menſibus bis Eclipſim pati, ſed rarò & paruam, nec niſi verſus eumdem Polum.

The Sun and Moon can undergo eclipse twice within five months, but rarely, and a small one, and only toward the same pole.

[Margin: The sagacity of Hipparchus.]

IV. So say Ptolemy, Book 6, chapter 26; Regiomontanus, Book 6 of the *Epitome*, propositions 10 and 12; Magini and Mulerius in the places above; and in part Pliny the Elder, Book 2, chapter 13, with Milich on that place. For [Pliny the Elder] says: "Within two hundred years it was discovered by the sagacity of Hipparchus that an eclipse of the Moon also sometimes occurs in the fifth month after the previous one, but one of the Sun in the seventh." For the motion in latitude of five greatest months is 159° 5′, while the least arc CSD, or EMF, is 149° 36′. Therefore, if the earlier eclipse occurs not far from C, between B and C, the Moon can be, in the fifth month, beyond D, within DA, and a second eclipse can occur, although nearer to D itself than to A; and so neither eclipse is large. And because in five months the Moon cannot pass from C, through S, beyond A, nor from E, through M, beyond B, both eclipses are therefore toward the same pole. It is easier, however, as far as the arcs themselves are concerned, for two eclipses of the Sun to occur after five greater months, because arc GSH, which the motion in latitude has to surmount, is smaller than arc CSD, just as KML is smaller than arc EMF. Regiomontanus adds that if the earlier eclipse of the ☉ was near sunset, the other will fall 6^h before sunset; or if the earlier was at 3^h after noon, the other

will fall at 3^h before noon; and if the earlier was in 15° ♎, the other will be in 15° ♓, or thereabouts. Now we have more frequent examples of solar eclipses of the fifth month than of lunar ones. For, as will be clear from chapter 21, from the year 1490 to 1700 we have only two pairs of lunar ones: namely between 1658 June 14 and November 9, and again between 1668 November 18 and 1669 April 15. Of solar ones, however, there are more, as you see in the following list.

If, then, an eclipse of the Moon recurs twice within one century in the fifth month, there is no need to wait two hundred years, as Pliny the Elder thought, with Hipparchus.

Solis Eclipſes interuallo Menſium Quinque.

Eclipses of the Sun at an interval of five months.

Years of Christ	Month	Day	Years	Month	Day
1601	December	24	1624	April	17
1602	May	21	1624	September	26
1602	June	11	1627	August	11
1602	November	13	1628	January	6
1605	October	12	1639	January	4
1606	March	8	1639	June	1
1609	July	30	1652	October	2
1609	December	26	1653	February	27
1613	May	19	1660	November	2
1613	October	13	1661	March	29
1616	September	10	1676	June	25
1617	February	5	1677	May	31
1620	January	4	1690	September	3
1620	May	31	1691	February	27

THEOREMA IV.

Theorem 4.

Non poteſt in menſibus 7. licet minoribus fieri bis Eclipſis Lunæ poteſt autem Solis, ſed verſus eumdem Polum.

An eclipse of the Moon cannot occur twice within 7 months, even lesser ones; but one of the Sun can, though toward the same pole.

V. So say Ptolemy, Book 6, chapter 6; Regiomontanus, Book 6 of the *Epitome*, propositions 11 and 13; and in part Pliny the Elder, whose words I reported in Theorem 3. For let the Sun be slow and moving in the upper semicircle of the eccentric, but the Moon fast and in the lower part of the epicycle, and so let the 7 months be lesser, and the least motion in latitude 208° 47′; and let the arc FSE, or DMC, in mean ☍ be at its greatest 211° 16′. Yet in true ☍, which are what is under discussion when the subject is months that are not mean but lesser, it is only 203° according to Ptolemy, though according to

others 206° at most. And so the Moon, having undergone eclipse at the limit F, is found after seven lesser months beyond E, by way of S. Conversely, however, if an eclipse of the Sun occurs at L, then, because arc LSK in true ☌ is nearly 218° and greater than the 208° 47′ of latitude, there can occur within seven lesser months one eclipse around L and the other around K, or one around H and the other around G, and so both toward the same pole. Yet this rarely happens, and once or twice within one and the same century. Such was the interval between the eclipse of the Sun of 1623 October 23 and that of 1624 May 18.

THEOREMA V.

Theorem 5.

Non poteſt Solis Eclipſis in vno menſe bis fieri verſus eumdem Polum; poteſt verſus alterum; non tamen vtraq. valde magna.

An eclipse of the Sun cannot occur twice within one month toward the same pole; it can toward the other pole, yet not both of them very large.

[Margin: Corrupt manuscripts have eamdem.]

VI. So say, as to the first two parts, Ptolemy, Book 6, chapter 6; Regiomontanus, Book 6 of the *Epitome*, proposition 14; and before them Hipparchus in Pliny the Elder, Book 2, chapter 13, where, with Milich, one must read thus: "Within two hundred years it was discovered by the sagacity of Hipparchus that an eclipse of the Moon also sometimes occurs in the fifth month after the previous one, but one of the Sun in the seventh; that the same Sun is hidden twice within thirty days above the lands, but that this is seen by different peoples." For if it can occur within one month, this is surely when the earlier occurs near L and the other near G, with the parallaxes of latitude helping; or the earlier near H and the later near K. Therefore, if one occurs toward the north pole in semicircle BSA, the other will occur with a noticeable true southern latitude, which will be increased by parallax, and thus there will be no eclipse with respect to those in the north. For the same reason, if the earlier occurs in the southern semicircle AMB, the later will be no eclipse visible to those in the south, since parallax increases the northern latitude. Moreover, that two eclipses of the ☉ can occur within one month, at least a lesser one, is clear, because the mean motion in latitude of one mean month is 30° 40′, and therefore the least, of a least month, is less than 30°, while arc LG, or HK, in true ☌ can be nearly 38°. And so, from one new Moon to the next, the luminaries can be within the ecliptic limits. Yet both eclipses cannot be very large: for if LB as well as GB is 19°, then even if the least motion in latitude of one least month is taken as only 26°, in one of the eclipses the Moon will still be farther from node B than from the limit L or G. But if LB is taken as 19° and BG as 5° 25′, it is clear that there will be no second eclipse.

[Margin: A rather obscure passage of Pliny the Elder explained.]

On this occasion Milich emends those words of Pliny the Elder, Book 2, chapter 13, "For that both heavenly bodies should be looked for within twelve days also happened in our own age, when the Emperors Vespasian were consuls, the father for the third time and the son for the second," by substituting "fifteen". But it is not extraordinary for two eclipses to occur within 15 days, one of the ☉, the other of the ☾. Pliny the Elder, however, is reporting something extraordinary and rare; therefore he is speaking of 12 whole days elapsing in a lesser month. On these, see the history of eclipses below, chapters 19 and 21.

THEOREMA VI.

Theorem 6.

Poſſunt in vno Menſe tres Eclipſes fieri, vna Lunæ & duæ Solis; Quo caſu Lunæ Eclipſis eſt neceſſaria & plerumq. Totalis; & Solis Eclipſes vna ad vnum, altera ad alterum Polum ſpectat, neq. vtraq. valde magna eſt.

Three eclipses can occur in one month, one of the Moon and two of the Sun. In this case the eclipse of the Moon is necessary and usually total; and of the Sun's eclipses, one looks toward one pole and the other toward the other pole, and they are not both very large.

VII. A large part of the theorem has already been proved in Theorem 5, namely the part on the two eclipses of the ☉, but what I said about the lunar one follows from it. For if two eclipses of the Sun happen, the Moon, in coming from the one to the other, must cross the node and travel from L, through B, to G, or [vel vel] from H, through A, to K. In addition, at the full Moon between them it must be distant from the new Moons on either side by at least $14^{d}\ 11^{h}\ 7^{m}$, which is the least semilunation; and so it must be distant from the node in motion in latitude by far fewer degrees than 10. And therefore an eclipse of the Moon is necessary, and usually total because of the nearness of the node, from which in this case it is usually less than 5° distant, and it has a latitude smaller than 26′, and so smaller than the difference of the semidiameters of the shadow and the Moon.

THEOREMA VII.

Theorem 7.

Quando Plenilunium fit in Nodo aut quamproximè, ideòq. Totalis Eclipſis Lunæ; rarò ac fortè nunquam vtrumque Nouilunium circumſtans caret Eclipſi Solis, ſed neutrius Eclipſis valde magna eſſe poteſt.

When full Moon occurs at the node or as close to it as can be, and therefore a total eclipse of the Moon, rarely and perhaps never are both new Moons on either side without an eclipse of the Sun; but the eclipse of neither can be very large.

VIII. So says Kepler, as to the first part, in the *Epitome of Copernican Astronomy*, p. 881. This can easily be proved from my figure and small tables set out in number 1. For if, for example, an ecliptic full Moon occurs at node B ☊, then the new Moon is no more than $15^{d}\ 1^{h}\ 37^{m}$ distant from it, this being of course the greatest semilunation. But in that time the motion in latitude will not exceed 19° or 20°; and if it comes to that much in one half-month, it will not come to that much in the other. Therefore in one or the other of the new Moons the Moon will be within arc BL or BG, of 19°, and so there will be some eclipse of the ☉ somewhere, yet very small, whether both or one, because of the great distance from the node.

THEOREMA VIII.

Theorem 8.

Omnis Partialis Lunæ Eclipſis vicinum habet Nouilunium Eclipticum; & omne Nouilunium Eclipticum paruæ Eclipſis, vicinum habet Plenilunium Eclipticum.

Every partial eclipse of the Moon has an ecliptic new Moon nearby; and every ecliptic new Moon with a small eclipse has an ecliptic full Moon nearby.

IX. The first part is Kepler's, above; the second is mine. For if an eclipse of the ☾ is partial, the Moon's latitude is greater than the difference of the semidiameters of the shadow and the Moon, and so greater than 30′, and therefore the distance from the nearer node is greater than 5°. Let the Moon, then, be partially eclipsed at F, and arc FB be at least 5°. For after $15^{d}\ 1^{h}\ 37^{m}$, which is the greatest semilunation, the motion in latitude will not be greater than 19° or 20°. So when 5° is subtracted, arc BG will be left, not greater than 14° or 15°, and thus within the ecliptic limits. And very often it will be smaller, because neither will the motion in latitude be so great, since usually the month is not the greatest, nor will arc FB be only 5°. Therefore, if a partial eclipse of the Moon occurs near F, with southern latitude decreasing, the following new Moon will be ecliptic, at least for those in the north, in arc AH; but if with southern latitude increasing, as at E, the preceding new Moon was ecliptic, at least for those in the north, in arc BG. By a similar argument the second part of the theorem will be clear. For let a partial and small eclipse of the ☉ occur in

arc BL, at L. The Moon at L, then, will be distant from node B, in true ☌, by at most 19°, as far as H will be distant from node A. Now let the semilunation be the least, $14^{d}\ 11^{h}\ 7^{m}$, and the motion in latitude the least, 13°, or indeed nearly 14°. For when these are subtracted from 19°, arc FB will be left, at least 5° or 6°, to which arc DA is equal.

THEOREMA IX.

Theorem 9.

Quando Nouilunium fit in Nodo aut proximè, ideòq. Eclipſis Solis alicubi centralis reſpectu centri Diſci Terræ, Plenilunia circumſtantia tranſeunt ſine Eclipſi; & viciſſim ſi Plenilunia Nodum circumſtantia tranſeunt ſine Eclipſi, Nouilunium intercedēs eſt cum Eclipſi Solis centrali in Nodo, aut quam-proximè.

When a new Moon occurs at the node or very near it, and therefore an eclipse of the Sun that is somewhere central with respect to the centre of the Earth's disc, the full Moons on either side pass without eclipse; and conversely, if the full Moons on either side of the node pass without eclipse, the new Moon between them comes with a central eclipse of the Sun at the node, or as close to it as can be.

X. Both parts are Kepler's, above. And the reason is that the least motion in latitude of the least half-month, running between ☌ and ☍, is about 14°, and so greater than the arc contained within the limits of a possible eclipse, which in true ☍ does not exceed 13°.

THEOREMA X.

Theorem 10.

Poſſunt interſtitio ſemeſtri fieri duæ Eclipſes Solis Centrales aut quaſi Centrales. quo caſu toto anno nullæ ſunt Lunæ Eclipſes.

Two eclipses of the Sun, central or nearly central, can occur at an interval of six months. In this case there are no eclipses of the Moon in the whole year.

XI. So says Kepler, above. The second part of the theorem is indeed clear from Theorem 9. The first is shown as follows: if an eclipse of the ☉ occurs at one node, say ☊, the six months that follow can be slightly greater than six lesser months and slightly less than six mean months, and so the motion in latitude can be 180° or very near it; and when semicircle BSA is completed, a new Moon can occur at the other node ☋, or very near it, since the motion in latitude of 6 mean months is 184° 1′, and of least months 178° 21′. Further,

within this century, from the year 1600 to 1700, the years already mentioned, with only two eclipses of the Sun and no lunar ones at all, are 1604, 1611, 1615, 1622, 1633, 1640, 1644, 1651, 1655, 1662, 1666, 1680, 1691, 1698, as will be clear from chapter 21.

THEOREMA XI.

Theorem 11.

Si in vno anni ſemiſſe contingat Eclipſis Lunæ Centralis aut quaſi, ſeu proxima Nodo, non poterit oppoſitus anni ſemiſſis carere ſua Lunari Eclipſi.

If in one half of the year there occurs an eclipse of the Moon that is central or nearly so, that is, very close to the node, the opposite half of the year cannot be without its lunar eclipse.

XII. This too is Kepler's, above. For if, for example, an eclipse of the ☾ occurs at node ☊, then after six mean months the Moon will be no farther than 4° 1′ from node ☋; after six least months, no farther than 1° 39′; and after six greatest months, no farther than 9° 51′, as is clear in the second small table of number 1. Therefore it will always be within the limits of a necessary eclipse.

DE NVMERO ECLIPSIVM.

On the number of eclipses.

THEOREMA XII.

Theorem 12.

Nullus Annus vacat ab omni Eclipſi, nullus pauciores quàm Duas, aut plures quàm Octo Eclipſes habet.

No year is free of every eclipse; none has fewer than two or more than eight eclipses.

[Margin: The number of eclipses.]

XIII. For in the course of one lunar year, let alone a solar one, the Moon celebrates the syzygies ☌ and ☍ at least twice near the nodes. Now if on the first day of the year an ecliptic new Moon occurs, say in L, and the ecliptic full Moon following it around B, and the succeeding ecliptic new Moon at G; and if after five greater months further eclipses occur, of the ☉ in H, of the ☾ at A, and again of the ☉ in K; then the remaining months that succeed will not be lesser, but mean or greater. So toward the end of the year, when 5 greater months and 7 mean have been completed, the motion in latitude will be 362° 8′. Therefore the Moon, by way of SM, will return to L and will be 17° 52′ from node B; and so a seventh eclipse of the year, but the fifth of the ☉, can occur, and after about 15 days an eighth at B, but the third of the ☾. But by

then the lunar year will be complete, and indeed, before the following new Moon, the solar year as well. So if an eclipse of the ☉ returns at G, it will not belong to the preceding year. The same follows if 7 mean months come first and 5 greater follow; and nearly the same if six months are least and the other six middling, for they amount to a motion in latitude of 362° 22′. Nor indeed can all of them be either least, or mean, or greatest.

THEOREMA XIII.

Theorem 13.

Poteſt vnus Annus habere duas tantùm Solis abſque vlla Lunæ, vel duas Lunæ Eclipſes cum vnica Solis.

One year can have only two eclipses of the Sun without any of the Moon, or two eclipses of the Moon with only one of the Sun.

XIV. This is clear from Theorems 7, 8 and 9 taken together and carefully weighed.

THEOREMA XIV.

Theorem 14.

Anniuerſarius Eclipſium numerus, plurium ſæculorum Eclipſibus numerandis conuenientior, eſt potiùs medius inter Ternarium & Quaternarium, quàm Quaternarius, & quàm Quinarius.

The annual number of eclipses better suited to counting the eclipses of many centuries is a mean between three and four, rather than four, and rather than five.

[Margin: What is the annual number of eclipses?]

[Margin: 1. The opinion of Longomontanus.]

XV. Longomontanus stands for the number four, in the *Astronomia Danica*, Book 1 of the *Theorica*, toward the end of chapter 9. There he supposes that at the mean distance of the luminaries the greatest ecliptic limits are 12° for the ☾ and 6⅙° for the ☉, but with the aid of parallax 17½°. These he doubles around one of the nodes and quadruples around both, so that for the Moon they come to 48 and for the Sun to 70. He also supposes that the Moon, after 12 mean lunations, before the end of the solar year, completes 132° 35′; and, adding the annual precession of the nodes of 19° 20′, he collects the Moon's whole annual excess over the Sun as 151° 55′, after whole revolutions. Then he makes: as 12 semirevolutions of the Moon are to the excess, so 48 to 1⅔, the number of lunar eclipses; and as 12 semirevolutions are to the excess, so 70 to 2 5/12, the number of eclipses. So when the two are combined the number of eclipses comes to 4, if mean motion is considered. For if true motion is considered, he admits that

there can be more or fewer, yet in such a way that over a period of about eight or nine years, in which the Moon's apogee completes its circuit beneath the zodiac, that number of four eclipses comes round again, and in every 8 or 9 years 32 eclipses occur, which he confirms from the *Ephemerides* of Origanus,

[Margin: 2. The opinion of Kepler.]

Kepler, on the other hand, fights for the number five, in the *Epitome of Copernican Astronomy*, p. 882. For he says that in nearly all half-years eclipses of the Sun occur, more often two in succession, very rarely and almost never none; and that in all lunar years there occur either two or three eclipses of the Sun, and one or two of the Moon, and so 5 in all to which a sixth eclipse is sometimes added around the beginning or end of the solar year. And indeed, since the extremes of the possible number are 2 and 7, the mean between these is 4½, that is, in whole numbers, either 4 or 5.

[Margin: 3. Tycho's opinion and its kindly interpretation.]

[Margin: Ignorance of eclipse doctrine in some interpreters of the sacred oracles.]

But Tycho's statement in volume 1, p. 328, that *six eclipsations occur every year in the Sun and Moon, by reason of different horizons*, holds at most under the Ptolemaic hypothesis and the limits of possible eclipses taken by the hundred [centenos], as Longomontanus kindly interprets him above. Nor does this stand in the way of Tycho's chief purpose, which is to refute those who interpreted the natural eclipses that occur every year as the signs of the impending end of the world given by the Prophets and by CHRIST. To refute them it is enough that about six, or nearly six, eclipses can occur every year by the necessity of the course of the luminaries; Longomontanus also refutes them in the same place. In actual fact, moreover, there are some years made up of six eclipses, such as the year 1620; on these see Kepler in his ephemeris for that year, at the beginning, and our chapter 21 below.

[Margin: How many eclipses from the beginning of the world.]

But since from the year of Christ 1600 to 1700, as will be clear from chapter 21, there are 350 eclipses in all, if 100 gives 350, then one year gives 3½ rather than 4 or 5; and in the 57 centuries elapsed from the beginning of the world, up to the year of Christ 1650, about 20,000 eclipses have occurred. See the rest in Scholium 3.

THEOREMA XV.

Theorem 15.

In eadem Terræ Regione Plures fiunt Eclipſes Lunæ quàm Solis; In toto autem Orbe reſpectu diuerſarum Regionum Plures Solis quàm Lunæ. Vniuerſim autem plures ſunt Partiales Solis quàm Lunæ qualibet; Totales verò Solis pauciores quàm Lunæ qualibet ſeu omniuariæ; Totales verò Solis plures quàm Totales Lunæ; denique Totalium Solis, & omnimodarum Lunæ idem penè numerus.

In the same region of the Earth more eclipses occur of the Moon than of the Sun; but in the whole world, taking account of different regions, more of the Sun than of the Moon. In general, moreover, partial eclipses of the Sun are more numerous than eclipses of the Moon of any one kind; total eclipses of the Sun, however, are fewer than eclipses of the Moon of any one kind or of all kinds; but total eclipses of the Sun are more numerous than total eclipses of the Moon; and finally the number of total eclipses of the Sun and of eclipses of the Moon of every kind is almost the same.

XVI. Mulerius holds this as to the first two parts, in the *Tabulae Frisicae*, p. 78; as to the others, Kepler in the *Epitome of Copernican Astronomy*, p. 881. It can be confirmed from the preceding theorems, from the motion in latitude of the ☾ compared with the arcs spoken of in number 1, and from the ephemerides and the history of eclipses. See, then, if you please, chapters 19 and 21 below.

THEOREMA XVI.

Theorem 16.

Diuina Prouidentia in Oeconomia Eclipſium Admiranda imprimis & Amanda eſt Mortalibus.

Divine providence in the management of eclipses is above all to be admired and loved by mortals.

[Margin: Arguments of divine providence in the dispensing of eclipses.]

XVII. For since through eclipses he has so provided for us that they should harm us as little as possible, GOD is to be loved by us and honoured in the highest degree with a grateful heart. He is to be admired, moreover, because he has so distributed the eclipses of the Sun, which do more harm to the nature of sublunary things, that they should be of shorter duration than lunar ones; that they should not be universal to a whole hemisphere at the same time; that they should not be large in the month in which the greatest and central or nearly central eclipses of the Moon occur; or that, when eclipses of the Sun are central, there should be no lunar ones in that month; and finally that two of the Sun should not occur in the same month toward the same quarter of the world. In this way compensation would be made by the smallness, or the fewness, or the dissociation of solar eclipses. These and similar things may be contemplated in the preceding theorems.

DE APOCATASTASI ECLIPSIVM.

On the restitution of eclipses.

[Margin: On the revolutions of eclipses.]

XVIII. Let us begin with Pliny the Elder, who in Book 2, chapter 13, states it thus: *It is certain that eclipses return to their own circuits in two hundred and twenty-two months.* I believe indeed that he has this from the opinion of Hipparchus, for a little later he adds: *Within two hundred years it was discovered by the sagacity of Hipparchus that an eclipse of the Moon, too, sometimes occurs in the fifth month from the previous one, but one of the Sun in the seventh.*

[Margin: Opinion on the revolution of the node and of eclipses: the opinion of Pliny the Elder; of Ptolemy; of Copernicus; of Milich; of Magini; of Martianus Capella.]

He is speaking of the revolution to the same place of the zodiac, but with respect to the same node, not with respect to the luminaries. For when 222 lunar months are complete, the node ☊, moving against the order of the signs, has completed the zodiac; or in 18 Egyptian solar years $223^{d}\ 6^{h}\ 12^{m}$, as Magini, with Copernicus, judges in Book 2 of the *Theoricae*, chapters 26 and 30; or in 18 Egyptian years $228^{d}\ 3^{h}\ 50^{m}$, as Kepler holds in the *Epitome*, p. 788; or in 18 years and 9 months, as Milich gathers from the Ptolemaic motion of the nodes, on Pliny the Elder, Book 2, chapter 13. But Martianus Capella, in *De nuptiis Philologiae*, Book 8, in the chapter on the parts of the latitude of the zodiac, where he treats of the Moon, says: *Yet it could not return to the same point, that is, just as it was placed, in the same month, in the same part, to the Sun in this way, at the same portion of latitude, except in the 235th month, that is, in the 19th year.* For in 19 solar years there are 235 lunar months, and this is the Metonic period. But this revolution of eclipses is not one of the node to the same degree of the zodiac; it is one of ecliptic new and full Moons to the same day of the year or the next to it, and to the same degree of the zodiac or the next to it. On the truth of this revolution, see the examples of lunar eclipses set out by us for nine Metonic cycles in Book 4, chapter 19, number 5.

[Margin: Kepler's opinion on these matters.]

But Kepler, in the *Epitome of Copernican Astronomy*, p. 895, affirms that because of the many principles that concur in eclipses, especially of the Sun, no exact order of revolution is known to us. Speaking, however, of the lunar ones, which are simpler, he says that there is a notable period of 223 months, or 18 lunar years, so that one simple lunar year remains to make up the nineteen-year cycle; for in this period eclipses mostly return not much different, though each time preceding by 10° of the zodiac. He adds that sometimes two eclipses of the Sun recur, on completion of the cycle of 19 years, at the same place of the zodiac, as happened in the years 1579 and 1598, for each was very large, and each fell on February 25, old style, at 16° ♓; but that this was a singular concurrence of causes, and that a similar eclipse was expected in vain in the following cycle, in the year 1617.

[Margin: The opinion of Cicero and of Macrobius.]

But if a perfect restitution of eclipses had to occur at the same moment of the year, in the same minute of the zodiac, with the same magnitude and duration, and at the same distance from the same node toward the same quarter of the world, the motions and periods of the Moon and the Sun would have to be commensurable, and the Greatest Year would have to be completed, to say nothing of that Great Year of which Cicero speaks in Book 6 of the *Republic*, in Macrobius on Book 2 of the *Dream of Scipio*, chapter 11, saying: *For just as the Sun once seemed to men to fail and be extinguished when the soul of Romulus entered these very precincts, so whenever the Sun shall again have failed in the same part and at the same time, then, with all the signs and stars called back to the same starting point, consider the year completed.* This year, on the opinion of some natural philosophers or other, Macrobius there defines as 15,000 years.

SCHOLIA.

Scholia.

[Margin: 1. The error of certain people about the end of the world, drawn from natural eclipses.]

I. Because of ignorance of the science of eclipses, a double error above all has gained ground among some interpreters of the sacred oracles, or others, in this and the preceding century. The first was that of those who twisted natural failures of the luminaries to those extraordinary and preternatural ones by which CHRIST the Lord, or other prophets, foretold the end of the world. Their error is refuted by Tycho, volume 1 of the *Progymnasmata*, p. 328, and by Longomontanus, Book 1 of the *Theorica*, p. 314, on the ground that every year from the beginning of the world there have occurred, if not 6, at least 4 eclipses, when the years in which there were more are reckoned with the years in which there were fewer. Let preachers beware, then, lest, by stirring the people to terror and tears with empty arguments, they themselves provoke laughter at their own expense among the experts.

[Margin: 2. The error about there being more eclipses now than formerly.]

II. The second error was that of those who saw future eclipses noted even below the horizon in the *Ephemerides* of Origanus, or of Kepler, or of similar practitioners, and therefore more of them than the older writers of ephemerides used to note, since these noted only those that were visible above the horizon of their tables; and who suspected that the number of eclipses is greater in these centuries of ours than formerly. Origanus refutes them at length in the preface to volume 2 of his *Ephemerides*, showing that an equal number of eclipses existed formerly, from the beginning, in ancient centuries, as the number in recent centuries; and that the number of these could not have grown greater now through any variation, whether of the Sun's apogee, of the eccentricity, of the obliquity of the ecliptic, or of the lunar latitude.

[Margin: The number of eclipses and of revolutions from the beginning of the world.]

III. Since from the beginning of the world up to the year of CHRIST 1650 there are, according to our chronological book, not more than 5703 years and not fewer than 5694, it is clear, at any rate from the syzygies that Mulerius set in order in the *Tabulae Frisicae*, that there were not more than 70,538 lunations and not fewer than 70,426. Again, eclipses of the ☉ and ☾ together, from what was said in the preceding chapter, number 15, were not more than 28,515, by multiplying 5703 by 5, and not fewer than 22,776, by multiplying 5694 by 4. And ecliptic revolutions of 19 years were not more than 317 and not fewer than 316; but of 18 years, not more than 300 and not fewer than 299. And so anyone can try out his own number according to the opinion he has chosen on the age of the world. But from what was said above, at the end of number 15, the more probable number is 20,000 eclipses.

[Margin: The inequality of ecliptic intervals.]

IV. Proclus, in the *Commentary* on Book 1 of Euclid's *Elements*, proposition 8, uses proposition 7 of the same book to show that three eclipses cannot occur at equal intervals from one another. Father Mario Bettini, in volume 1 of the *Aerarium Philosophiae Mathematicae*, p. 250, testifies that an explanation of this was sent to him by Scipione Chiaramonti. But that demonstration supposes an anomalous motion of the luminaries and an irregular motion in latitude; otherwise, if those motions were equal, and completed equal arcs in the ecliptic or the Moon's orbit in equal times, the intervals in months between several eclipses, to say nothing of three, could be equal.

[Margin: The Sun and the world created in a great eclipse, in the opinion of Kepler.]

V. Kepler, throughout the Rudolphine Tables, but especially at the ends of the epochs, and in chapter 18, p. 52, and chapter 29, p. 97, judges that the world was created in the year 3993 before CHRIST, on July 24. And in the same chapter 29, precept 138, from his own table containing the cycle of the meetings of the ☉ and of the nodes of the Moon ☊ and ☋, he gathers that on that day there was a great eclipse of the Sun, the luminaries being joined at the node ☊, at noon of the 24th, at the solstitial point itself, that is, at the beginning of ♋, but by the true motions at 28° ♊, with a southern latitude of the ☾ of about 11′; and that a total eclipse of the Sun was seen in Ethiopia, midway between those living under the Tropic of Cancer and those living under the equator. From this, as Kepler hints in the margin, some astrologer might infer that the Ethiopians had the cause of their blackness from this first eclipse. He adds, moreover, in accordance with our Theorem 9, that neither full Moon, whether that of June, if there had been one, or that of August, was ecliptic; and that on the preceding January 29, if the stars had existed then, there would have been a great eclipse of the Sun at ☋, and perhaps a small one of the Moon on February 12.

[Margin: But an improbable one.]

But I cannot be persuaded that the world or the Sun was made by God in an eclipse, or that the first phenomenon of the Sun was its failure. On the contrary, in this the beginning of the innocent world was the opposite of the end of a world defiled by the filth of sins: the former began from the light and gladness of the divine countenance, represented by the Sun as by a symbol; the latter is to end in the foulest eclipses of the luminaries, foreshadowing the dreadful judgement and wrath of God.

CAPVT XV.

Chapter 15.

De Modo Obſeruandi Eclipſes Luminarium & de Cauſis Diſſidij Obſeruationes inter ac Tabulas Aſtronomicas.

On the method of observing eclipses of the luminaries, and on the causes of the disagreement between observations and astronomical tables.

[Margin: Authors on this subject.]

I. Few indeed have handed this down, and very little; and not even all of them together have handed down everything that this practice requires. But in a matter of such moment, in which above all the crucial test of astronomy and the trial of astronomical reliability consist, I will not allow my own diligence to be found wanting. What it has achieved will be judged if this chapter is compared with what has been indicated on this subject by Gemma Frisius, *De Radio Astronomico*, chapter 18; Reinhold, on Peuerbach's *Theoricae*, p. 198; Maurolico, Dialogue 3 of the *Cosmographia*, p. 87; Daniel Santbech, *De Observatione* [Greek: τῶν φαινομένων (tōn phainomenōn, "of the phenomena")], Proposition 13; Barozzi, Book 4 of the *Cosmographia*, p. 325; Longomontanus, Book 1 of the *Theorica*, chapter 9; Kepler, in the *Astronomia Optica*, chapter 11, from p. 349; Tycho, in the letter to Rothmann of 1587 January 20, old style; Biancani, in the *Sphaera*, Book 10, chapter 19, and on the mathematical passages of Aristotle, at Problem 10 of Section 15; Bettini, *Apiarium* 6, Progymnasma 2, chapter 6; and Wendelen, in the preface to the eclipses of his own age. First, however, the things to be observed in eclipses of the luminaries must be listed; then this method of observing them must be taught in order and point by point, free of all deception, so far as is permitted to man.

[Margin: Things to be observed in eclipses.]

II. The things to be observed, then, are these. First, the quantity or magnitude of the eclipse: whether it is total, or partial, or none, especially where it is established that the astronomical tables disagree on this, some promising a partial eclipse, others a total one, others none or an imperceptible one; and if it is partial, how many digits or inches or points it is at the height of the failure. Second, the ratio of the semidiameters of the obscuring and of the obscured body. Third, the apparent semidiameters of the obscuring and of the obscured body. Fourth, the arc or distance between the centres of the obscuring and of

the obscured body. Fifth, the shining or the dark arc of the circumference of the obscured luminary at the middle or height of the eclipse. Sixth, the Moon's path and the position of the horns, so that it may be established when they are parallel to the Moon's path; and thus, in a partial eclipse, the moment of the height of the eclipse. Seventh, the inclinations and quarters toward which the failure tends. Eighth, the interval of time from beginning to end, both of the spurious eclipse arising from smoke or umbrago, and especially of the true and legitimate eclipse. Ninth, the stay in darkness, and the incidence and repletion of a total eclipse. Tenth, the moment of time at which the eclipse begins and ends; and if it is total, at which the obscuration becomes total and at which the luminary begins to emerge from the obscuring body. Eleventh, the moments of time of other principal phases, namely when 3 digits, or 6, etc., appear obscured or already recovered; and when the Moon changes colour, or in some total eclipse loses those colours along with all its secondary light, and recovers them again. Twelfth, the spots of the Moon, or islands, or mountains, or small areas through which the section of the shadow or penumbra passes; and the moments of time at which that section either grazes or leaves parts of the Moon. Thirteenth, the colours of the eclipsed luminary, or of the surrounding air, and the state of the air around the times of the eclipse. Fourteenth, the stars that appear in an eclipse of the Sun, and whether they are planets or not. Fifteenth, the position of the luminaries in the principal phases of the failure, for example the transit through the meridian, the azimuths and verticals in which they stand, the distance of the centres or edges from the more notable fixed stars, the line drawn through the centres of the luminaries or through the horns, and through two fixed stars; and finally the altitude of the luminaries. As for the method of observing the diameters of the luminaries, I have taught it sufficiently in Book 3, chapter 10, and Book 4, chapter 16.

PROBLEMA I.

Problem 1.

De Magnitudine Eclipſis Obſeruanda, & Arcu circumferentiæ obſcurato.

On observing the magnitude of an eclipse, and the arc of the circumference obscured.

[Margin: 1. Method of observing the magnitude.]

III. The first method is estimation by eye. One first conceives beforehand, by the power of the imagination, the apparent size of the luminary as if it were a foot long, with its diameter divided into twelve digits or inches; then one inspects the luminary with a direct gaze, but with the eye protected against the injury of the Sun's glare by coloured glasses, or by the interposition of translucent paper or of a rather fine veil. If you use a telescope, those are better here that show the whole disc of the luminary at once; of this kind are small ones, or large ones indeed, but with a second, or even three, convex lenses

set near the eye at a certain interval (which experiment will determine), and a fourth and principal convex lens far from the eye, turned toward the object.

[Margin: The deceptiveness of this method.]

But except for the case of exactly six digits obscured, in which the deception is usually smaller, as Kepler points out in the *Optics*, p. 349, this direct estimation by the eyes is very deceptive. Thus in the eclipse of the ☾ of 1603 May 24, Lansbergen gave 7½ digits, Wendelen 8; and on November 18 Lansbergen 3 digits, Kepler less than 3, others more than 3; and in the eclipse of the ☾ of 1612 May 14, Longomontanus and Wendelen gave 6½ digits, but Scheiner 8; again, in the eclipse of the ☾ of 1621 November 28, Kepler gave 4 digits

Gassendi 3⅔, Wendelen 3; and in the eclipse of the ☾ of 1643 September 27, Wendelen gave more than 6 digits, Father Theodorus Moretus fewer, or hardly 6. We estimated exactly 6. And you will see the same throughout in the history of eclipses, chapter 19. Judgement is also slippery when an eclipse is on the border of total without stay, or with a very small stay, or nearly total but not wholly, because of that doubtful boundary of the penumbra or of the edge; or when the eclipse is so small that it escapes duller eyes and is barely evident to sharper ones. We have an example of the first case below in chapter 19, in the eclipse of the ☾ of 1616 August 26, when some asserted it was partial and others total; and in the eclipse of the year 1580. Of the second case, in the eclipse of the ☾ of the year 1625, and perhaps in the eclipse of the ☾ of the year 1626 and of 1619 June 26.

[Margin: 2. Method.]

[Margin: Its deceptiveness.]

The second method, formerly used by Cornelius Gemma and Tycho, is by the astronomical radius, with which, outside an eclipse, one takes the distance between the edges of the Sun or Moon, that is, the apparent diameter, and at the height of an eclipse the greatest distance of the periphery of the obscuring body from the periphery of the obscured one. But Kepler in the *Optics*, p. 350, rightly rejects this method as crude, and Tycho finally repudiated it because of the imperfection of this instrument, on which see what was said in Book 3, chapter 10, number 3.

[Margin: 3. Method.]

IV. The third method, either used or praised by Gemma Frisius, Reinhold, Santbech, Mästlin, Barozzi, Maurolico, Biancani and Bettini, and not displeasing to Kepler in the *Astronomia Optica*, chapter 11, Problem 7, is by the image or ray of the luminaries, let in through a slit, or a hole, or a small window, into a dark room and received on a wall, a tablet, or white paper. But it must be done in such a way that the plane both of the hole and of the tablet or paper receiving the bright base of the radiant cone is illuminated perpendicularly by the central ray of the luminaries, and that their planes are parallel to each other, so that the image is circular, not oval; and the tablet receiving the image must be kept at a fixed and constant distance from the hole during the whole time of the eclipse.

Therefore, if the eclipse is solar you can, on the preceding day, or if lunar, on the same night, with the tablet fixed to the instrument, namely a small beam at one end of which is the plate with the hole, observe the diameter of the circle of the bright base. Half of this will give the semidiameter for describing a circle from a suitable centre, that is, from the point on which the ray passing through the centre of the hole falls perpendicularly. Although the periphery of this circle is usually subdivided into 360°, or at least its semiperiphery into 180°, and its diameter not only into 12 equal parts but into more small particles, it is better, nevertheless, to describe on another tablet, separate from the instrument, a circle equal to it, and to divide its semicircumference into 180° and its diameter into as many and as small equal particles as possible; for this division is done more conveniently and more exactly than on a tablet already fixed to the beam. With this done, once the eclipse has begun, the small beam must be moved so that the bright image follows the motion of the Sun or Moon from the opposite side and the bright circle coincides with the circle described.

Then, whether at the height of the eclipse or at any other moment you please, take with the compasses the greatest interval, such as GB, which lies between point G of the periphery GCHD of the obscuring body and point B of the periphery BCEDB of the obscured luminary. For when the compasses are transferred to the diameter of the equal circle, already divided into particles, the portion then free of the eclipse will be known, and its complement to the whole diameter, namely GE, will be the eclipsed portion. Let it be made, then: as the particles of the whole diameter BE are to 12 digits, so is that complement GE to the required amount, namely to the digits then obscured.

Diagram. Set into the foot of the left column, with the text wrapped to its left. Two concentric circles form a narrow ring at the top, the outer marked B and the inner N, with centre A on a vertical line that runs from B down to H; the outer circle continues as a white arc through the disc, crossing the vertical at E. A large solid dark disc, the obscuring body, rises from below into the lower part of the circles, its top at G and its foot at H; a white vertical line continues through it. Two arcs cross the upper circles above the disc. The vertical line carries, from the top, B, N, A, P, G, I, E, F, H.

- B (top of outer circle)
- N (top of inner circle)
- A (centre of the upper circles, on the vertical)
- P (on the vertical, on the lower of the two arcs, MPR)
- G (on the vertical, top of the dark disc)
- K (left end of the arc, outside the circle)
- M (on the arc, left)
- R (on the arc, right)
- L (right end of the arc, outside the circle)
- C (left, where the circle meets the dark disc)
- D (right, where the circle meets the dark disc)
- I, E, F (on the white vertical within the dark disc, in that order downward)

• H (foot of the dark disc)

Others, however, usually keep ready several circles, larger and smaller, on the same tablet or on different tablets, so that at the time of the eclipse they may choose the one with which the bright base of the radiant cone let in through the hole agrees most closely, as Kepler reports that Mästlin did, in the *Optics*, p. 352; or, as Kepler teaches in the *Optics*, chapter II, Problem I, they use various small crescents.

[Margin: The deceptions of this method.]

This method, besides the deception to be exposed under the fifth method, has this inconvenience: if the distance of the passive tablet from the hole is small, the diameter of the circle can be divided into only a few particles, and small ones too, and from their excess or defect a greater error results in measuring the magnitude of the failure. But if the distance is increased, as when the hole is in the roof of a lofty church, the way Kepler relates above that Mästlin used it, the image becomes fainter and weaker, the borders of light and shadow are harder to discern, and the motion of the bright image is more fleeting, so that the points of the compasses can hardly be applied at the right moment to the desired extremes; and while one hesitates and the hand trembles, the opportunity slips away. For that reason it is advantageous to have two people, each of whom with a very sharp needle quickly pricks the tablet at one of the points B and G, to which the compasses may then be fitted in a calm operation; and, moving the instrument before the failure perceptibly grows or shrinks, to test whether the section of the shadowing body passes through the marked points.

[Margin: The absent or shining arc of the circumference.]

The absent or obscured arc of the circumference, such as arc CED in the diagram, will become known either by opening the compasses to the interval of the chord CD, or by two observers pricking the circumference BCD at C and D with two needles, then taking the interval CD with the help of the compasses and applying it to the degrees of an equal circle or semicircle already divided. But if you want to work more exactly, let the semidiameter of the hole be known to you; subtract it from the semidiameter AB, so that AN remains; with this interval describe circle NMR; add the same to the portion EG, so that EP results; and with the interval FP describe the circle or arc MPR. For points M and R will bound the arc MNR shining on one side and the arc MIR obscured on the other. The reason for this correction will be indicated presently, in numbers 6 and 9.

[Margin: 4. Kepler's method.]

V. The fourth method is Kepler's, in the *Astronomia Optica*, p. 350, who says: *To rely on somewhat firmer supports, I was accustomed to estimate the absent arc of the Moon's circumference.* Let the figure set out in chapter 6, number 5, be repeated here, in which the absent or obscured arc of the Moon, AGC, is known; for when that is given, and the ratio of the diameters of the Moon and the shadow, the quantity of the defect is given.

Diagram. Set into the right column, with the text wrapped to its left. Two overlapping circles on one vertical line: the upper and larger is inscribed VMBRA VEL SOL (SHADOW OR SUN), with centre E; the lower and smaller is inscribed LVNA (MOON), with centre I. They cross at A on the left and C on the right. Straight lines join A and C to E and to I, and a horizontal chord joins A to C, crossing the vertical at K.

- B (top of the upper circle)
- VMBRA VEL SOL (legend across the upper circle)
- E (on the vertical, apex of the lines from A and C)
- G (on the vertical, top of the lower circle)
- K (on the vertical, on the chord AC)
- A (left intersection)
- C (right intersection)
- D (on the vertical, foot of the upper circle)
- I (on the vertical, centre of the lower circle)
- LVNA (legend across the lower circle)
- F (foot of the lower circle)

For let AGC be 60°, and AG 30°, namely the angle AIK; and therefore, by Euclid I.32, AKI is 90°, and AI, the secant, is 200,000 parts of which AK is 100,000. And let the ratio of the lunar semidiameter AI to the semidiameter of the shadow AE be as 1 to 3; for AE will be 600,000, the secant of the angle EAK, 80° 24′. The tangents of the same angles will be IK, 173,205, and EK, 591,236. These, subtracted from IG, of 200,000 parts, and from ED, of 600,000, leave GK, 26,795, and KD, 8,764, so that the whole deficient part DG is 35,559 of the parts of which AI, the apparent semidiameter of the Moon, is 200,000. Now let that semidiameter be 16′; for if one makes: as 200,000 to 16′, so 35,559 to another, GD, the obscured part, will be 2′ 51″. After handing this down in nearly these terms, Kepler adds: *Here, although sight and the estimate of sight can go somewhat astray, yet only a tiny little portion of that error must be reckoned to fall upon the Moon's diameter. I wish astronomers would take pains that some more certain methods of this observation be established. For on this one thing depends what is commonly made most of in astronomy, the measurement of the altitude and of the body of the Sun: for if the greatest parallax of the Sun is 3′, and one errs by a third part of a digit in estimating the lunar defect, it is all over with 600 semidiameters of the Earth. But if the greatest solar parallax is 2′, we shall err by 1700 semidiameters of the Earth, by the omission of one minute in the lunar defect, as can be seen on the front of our parallactic [?]* [in fronte noſtræ parallacticæ].

[Margin: Kepler corrected.]

In these words Kepler does not explain how he estimated the absent arc: whether by looking directly at the Moon, or in the bright image, with the points C and D marked as we taught above. Next, how can the error that spills over from this into the obscured part of the diameter be small, if an error of even one minute produces an error of 1700 terrestrial semidiameters in the Sun's distance? It is false, moreover, that the measurement of the Sun's distance and

magnitude depends on *this one thing*; for there are other methods, and indeed more certain than this one, on which see Book 3, chapter 7.

[Margin: 5. Kepler's method.]

VI. The fifth method, proposed by Kepler in the *Astronomia Optica*, chapter 11, Problems 8 and 9, supposes the bright image of the Sun received through a hole on a tablet set perpendicularly opposite the radiation, as we said in the third method; but in such a way that the semidiameter of the hole is known in parts of which the semidiameter of the bright base of the radiant cone is known. Therefore subtract the particles of the semidiameter of the hole or small window from the semidiameter of the bright base, AE, so that AI remains; and add the same particles to the remainder IG, so that the obscured part IP results; and complete the rest as above in method 3, or in the figure of the *fourth* method. For from this will come a more correct quantity of the eclipse and a more correct ratio of the semidiameters. The reasons for the addition and subtraction just mentioned are shown by Kepler in the *Astronomia Optica*, chapter 2, propositions 10, 11 and 12, and by us in the Optical Book [in libro Optico]. To these Kepler adds, in Problem 10 of chapter 11, a way of marking the points of the sections C and D in the earlier figure by means of a transverse rule [Regulæ tranſuerſalis]; but at p. 357 he shows it to be so deceptive that in the year 1600, on July 10, when 3 digits were failing in the bright image, the rule cut off only 1⅔ digits. For that reason, in Problem 13, he at last settles on various small paper crescents made beforehand from portions of different circles, so that when they are applied to the segment of the shadowing body, the one that best fits it may be chosen.

[Margin: 6. Method.]

VII. The sixth method is carried out with the help either of a whole telescope or of a single lens, whether convex or concave, by receiving the image of the Sun or Moon on a tablet or on very white paper. It is made very white if it is covered not with one but with several sheets of the cleanest paper, joined with a paste not of the ordinary kind but made from starch. Here too this tablet must be parallel to the plane of the glasses and directed straight, that is perpendicularly, onto the axis of the radiant cone. And before the eclipse the distance must be tested at which the image grows large, yet in such a way that the circumference of the bright base can still be clearly distinguished from the surrounding dark. At that distance the diameter of the bright base must be taken, either with compasses or by two points marked on the circumference with very sharp needles, so that a circle equal to it can be described on another tablet, its periphery divided into 360° and its diameter into 100 or 200 or more particles; and the rest is to be done as we said in the third method.

[Margin: Use of the telescope and choice of the glasses.]

As regards the glasses of the telescope, if you use it whole, the telescope must be such that, either by its shortness, or by the breadth of its lenses, or with the help of other lenses, it takes in the whole luminary and something more; otherwise it will be unfit to show you that obscured part which it does

not take in. For that reason some use a single convex glass, as we shall say in chapter 19 that Albert Linemann [Albertum Linnemanum] did at the eclipse of the Sun of the year 1639; others use a concave glass only, which we shall say in the same place that Gassendi used at the eclipse of the Sun of the year 1621. This is certainly better, because, other things being equal, it paints a cleaner image on the paper and a very fine periphery of the bright circle, with a very evident distinction between it and the surrounding dark field.

[Margin: 7. Method.]

VIII. The seventh method, better suited to an eclipse of the Sun, is by the image of the obscured luminary received in a mirror, either of water, in a basin full of water placed beneath, or of glass, and by estimating the deficient portion with the eyes.

[Margin: 8. Method, Langren's and ours.]

The *eighth* method, in a lunar eclipse, is to observe with an excellent and very bright telescope the parts of the Moon through which the section of the shadow passes, as Michael Florent van Langren pointed out to us by his example at the eclipse of 1634 March 14, of which I shall speak in chapter 19. For since the longitude and latitude of the lunar particles, and their distance from the edges, are already established from the selenography, on which see Book 4, chapter 7, it will also be easily established what fraction is deficient at that moment. But it will be useful to note at least three distinct particles or points through which that section passes, and to dictate them to an amanuensis or keep them in memory, so that the periphery of the shadow passing through them can be described.

PROBLEMA II.

Problem 2.

Proportionem Semidiametrorum Obſcurantis & Obſcurati, & data vnius Semidiametro, alterius Semidiametrum obſeruatione inquirere.

To find by observation the ratio of the semidiameters of the obscuring and of the obscured body, and, given the semidiameter of one, the semidiameter of the other.

IX. In the first figure, set in number 4, let three points C, G, D be marked at the height of a partial eclipse, or at any other more notable moment of a total eclipse, for example when 6 digits are obscured, or when the section of the periphery of the shadowing body passes through the centre A of the shadowed body; let them be marked, I say, at the same moment by two or three observers with very sharp needles; or, if the section were passing through the centre A, the points K, A, L. And let the interval between the extremes CD or KL be taken with compasses, as also their intervals from the third, intermediate point, namely the intervals CG and DG, or KA and LA. Then on another tablet or paper, having drawn a straight line equal to the interval CD or KL,

and having taken from its ends a point outside it in which the intervals of the points C and D from the point G, or the intervals of the points K and L from the point A, meet, describe, by the Scholium to proposition 5 of Book 4 of Euclid, a circle whose circumference passes through the three points C, G, D or K, A, L. For that circle will be equal to the image of the shadowing circle, that is, of the shadow in an eclipse of the Moon, or of the Moon in an eclipse of the Sun. Therefore measure its semidiameter GF in the particles into which the semidiameter AB has already been divided and which are known beforehand, and you will have the required ratio of the semidiameters. If, then, the apparent semidiameter of one is known in minutes and seconds, whether from observation or from astronomical tables, the semidiameter of the other will not remain hidden. For example, if AB is found to be 100 parts and GF 300, and it is established from elsewhere that the semidiameter of the Moon, AB, is 15′, it will follow: as 100 to 15′, so 300 to 45′, the semidiameter of the shadow. But if you use a bare hole rather than a whole telescope or a single glass, you must subtract the semidiameter of the hole from the semidiameter AB of the shining body, so that AN remains; and conversely add the semidiameter of the hole to the visible eclipsing portion GE, so that, for example, EP results. For AN will be the correct semidiameter of the shadowed body, and FP that of the shadowing body, as Kepler teaches in the *Optics*, chapter 11, Problem 9. Otherwise the semidiameter of the obscured body would be gathered as larger, and that of the obscuring body as smaller, than is correct, and the digits as fewer than in the sky, as he likewise teaches in chapter 2, propositions 10, 11, 12, and chapter 11, Problem 7. If, however, only the semidiameter of the luminary is sought, it is enough to subtract the semidiameter of the hole from the semidiameter of the bright base, as he teaches in the same chapter 11, Problem 2. Again, if you wish to continue the circumference CGD itself by proposition 5 of Book 4 of Euclid, it will suffice to note any three points on it quickly, etc.

PROBLEMA III.

Problem 3.

Diſtantiam Centrorum in ſummo Eclipſis, ſeu arcum inter centra Obſcurantis & Obſcurati obſeruatione deprehendere.

To determine by observation the distance of the centres at the height of an eclipse, that is, the arc between the centres of the obscuring and of the obscured body.

X. Find, by method 5 or 6 of Problem 1, the just and corrected quantity of the defect, namely IP in the figure of number 4, and by Problem 2 the apparent semidiameter of the shadowing body and of the shadowed body in minutes and seconds; and reduce the eclipsed part to the proportion of either of them. Subtract it from the sum of the semidiameters, and AF will remain known, namely the distance of the centre A of the shadowed luminary from the centre F of the shadowing shadow or Moon. For example, let the semidiameter AI

be 100 particles, but 15′ in minutes, and FP 300, but 45′ in minutes, and let the eclipsed portion PI be 30 particles. For if one makes: as 100 to 15′, so 30 to another, PI will be found to be 4½′. These, therefore, being subtracted from the sum of the semidiameters, which is now 60, AF, the distance of the centres, will remain, 55½′. So, nearly, Kepler in the *Optics*, p. 433, where he calls the uncorrected diameter of the luminary fringed [fimbriatam], but the diameter corrected by subtracting the diameter of the hole he calls enucleated [enucleatam].

PROBLEMA IV.

Problem 4.

Inclinationes Eclipſium obſeruare.

To observe the inclinations of eclipses.

XI. I assume here that the tablet on which the image of the deficient luminary is received is fixed immovably and at right angles to a small beam, and is parallel to the plane of the hole or of the glasses through which the ray is transmitted; and that a circle has already been described equal to the bright base of the radiant cone, as I said in method 3 of Problem 1, namely, in the following figure, the circle BCD, with its diameter marked there as EB, in such a way that it is inclined to neither side; and that, in whatever direction the small beam has been moved following the motion of the luminaries, EB remains in the plane of the vertical circle. With these things laid down, if the horns of the bright segment, say CD, are parallel to the horizon, and the straight line CD, drawn through them with the help of a stretched thread or a rule, cuts the diameter BE at right angles, the vertical circle will coincide with the arc drawn through the centres. Otherwise the arc through the centres will be inclined to the vertical circle. In that case, therefore, with the thread or rule passing through the centre A, draw a straight line parallel to the straight line drawn through the horns: for example, if one horn is B and the other M, let KL be taken parallel to the straight line BM, drawn through these in imagination. For the arc GK will be the arc measuring the angle GAK, the inclination sought; and this arc will be known with the help of compasses applied to the circumference of an equal circle already divided into degrees, as was said in Problem 1. At that time, moreover, the altitude of the luminary must be taken, without which these inclinations are of no use or little, as Kepler warns in the *Optics*, chapter 11, Problems 14 and 15; to which he adds, in Problems 16 and 17, a way of marking the inclinations on the floor.

Diagram. A geometric figure set into the left column, beside the text of number XI. A circle with centre A. A vertical diameter runs from B at the top down through A, and on through P and E to F at the foot. A horizontal line runs from G at the left through A to the right edge at M and H. A straight line runs obliquely from upper left, where R and K stand together, through A to

lower right, past D and L, ending outside the circle at Q. A second oblique line runs from C at lower left through A to T at upper right. A dotted line joins B to M at the upper right, and beside it a solid arc curves from B down to M inside the circle. A dotted line crosses the lower part of the circle from C to D. An arc bulging upward passes from C to D across the vertical above P; beyond C and D it continues outside the circle, down on each side to a straight base line through F. The circle's own lower edge crosses the vertical between P and E.

- B (top of the circle): point on the circle, the highest point
- R (upper left, outside or on the circle): end of the rule RAQ
- K (upper left, beside R): end of the line KL
- T (upper right, on the circle): end of the transverse CAT
- G (left, on the circle): end of the horizontal line
- M (right, on the circle): second horn in the example
- H (right, directly below M): on the circle
- A (centre): centre of the circle
- C (lower left, on the circle): horn of the bright segment
- D (lower right, on the circle): horn of the bright segment
- L (lower right, beside D): end of the line KL
- P (on the vertical, below A): between the upward arc and the lower edge of the circle
- E (on the vertical, below the circle)
- F (below the base line, at the foot of the vertical)
- Q (lower right, outside the circle): end of the rule RAQ

[Margin: Other methods for the Problem.]

But the image of the Moon on paper is not very clear, and the operations just mentioned cannot be carried out evidently enough in the darkness of the night, while the glare of a lighted lamp or candle is a hindrance to some. Kepler therefore, in the *Optics*, chapter 11, Problems 18 and 19, suggests two or three other methods. The *first* is by mere estimation with the eye, looking to see whether the horns of the eclipse are turned exactly upward on their backs or downward face-down; for then they lie parallel to the horizon, and there is no inclination from the vertical. It is the contrary if each horn stands perpendicularly upright; for then the arc through the centres will be parallel to the horizon and inclined to the vertical by 90°. The *second* is with the help of a circle, suspended or fixed on a tripod, which is held up to the Moon in such a way that it receives the ray or bright image of the Moon perpendicularly. Let its periphery BCD be divided into 360°, the division beginning from B, the highest point; and let there be a rule RAQ, longer than the diameter of the circle and turning about its centre A, with another transverse rule orthogonal to it, such as CAT. Then, with the eye transferred, let the rule be moved until it stands evenly under the horns B, M, as you see in the figure; for the transverse CAT will at once indicate the arc of inclination BT. The *third* is to observe the moment at which the horns of the eclipsed Moon are in the same straight line

with some fixed star or planet, or are turned perpendicularly toward it. For from the situation of the sphere, from the time and the place of that star, and taking account of the lunar parallaxes, you will find the inclination by means of trigonometry. In all these methods, moreover, Kepler at p. 367 supposes, as something known of itself or nearly known of itself, that the straight line joining the horns of the eclipse, that is, the points at which the circles of the Moon and of the shadow intersect, is orthogonal to the line drawn through the centre of each. Our *fourth*, for lunar eclipses, is to observe with a telescope two or more small spots of the Moon whose centres are in the same perpendicular line, and two or more through which the section of the terrestrial shadow passes; for from these, together with the time and the altitude of the Moon, the inclinations of a lunar eclipse will easily be clear. Note, however, as has been warned repeatedly in advance, that the telescope must be such that you can take in the whole disc of the Moon in a single look, and can determine more surely, by estimation with the eye, the vertical line of those of the already mentioned spots that are most distant from one another.

PROBLEMA V.

Problem 5.

Plagas Eclipſium obſeruare.

To observe the quarters of eclipses.

XII. How the quarters toward which eclipses tend are gathered from inclinations observed by Problem 4, and from azimuths and altitudes, was said sufficiently in chapter 7, number 2. When, however, in the image received through a hole or glasses on paper or a tablet, of which Problem 1, method 3 speaks, the horns are in a straight line parallel to the horizon, the azimuth alone, taken at that time, indicates the quarters.

PROBLEMA VI.

Problem 6.

Interualla temporum ad Eclipſes ſpectantia dimetiri.

To measure the intervals of time that pertain to eclipses.

XIII. These intervals are not only those that lie between the beginning and the end, or between the beginning and the total obscuration, and from there to the emersion, and from there to the end, or between, for example, six digits obscured and six digits recovered again; but also those counted from the transit of some fixed star through the meridian, or from its altitude or azimuth when taken, up to the beginning of the eclipse, or conversely from the beginning or end of the eclipse up to the transit of the star, etc. These intervals can be measured most exactly by the vibrations of that pendulum about which enough

was said in Book 2, chapter 20. If on that day you have tested sand-clepsydras or striking clocks against that pendulum, you can also use their times. Striking clocks, however, are also tested against a sciatheric solar clock, astronomical and horizontal [Horologium ſciatericum Solare Aſtronomicum & Horizontale]; either by stopping them so that the 12th hour after midnight does not strike until the shadow of the gnomon coincides exactly with the meridian line; or, what is better, against the transits through mid-heaven of two fixed stars of known right ascension, namely if the times of the striking clock are equal to the difference of right ascension converted into times of the equator. The other methods, by counted pulses of the artery, or by a certain stretch of a journey completed, or by the recitation of some verses, which Wendelen hints at in the preface to his *Eclipses*, seem to most people not so certain.

PROBLEMA VII.

Problem 7.

Principium ac Finem legitimæ Eclipſis, aut Totalis obſcurationis exactè diſcernere.

To discern exactly the beginning and end of a legitimate eclipse, or of total obscuration.

XIV. I speak of the legitimate eclipse, not of the smoke or umbrago, on which see chapter 5, especially number 2. The end is indeed more easily observed than the beginning, because the observer is already certain of the eclipse as practically finished, and does not wait in vain for the end of the phenomenon; but before it begins he has no certainty other than from an ephemeris or a calculation, which can go astray and keep the observer long in suspense, to the point of weariness; and unless he is patient and circumspect, the opportunity of the true beginning can escape him. It is certain, further, that the visible beginning of the eclipse etc. comes as soon as the disc of the Moon or Sun, viewed whether with the naked eye or with telescopes, and whether by direct looking or in the image received through a bare hole or glass on paper or a tablet, of which Problem 1, method 3 speaks, has begun to lose its perfect roundness, and its limb to be grazed or tasted by a small black lentil-shaped spot, as if by the overlay [recte: paring, per the book's errata] of a blackish little fingernail. Just so, it is established that the visible end of the eclipse is when the apparent disc of the luminaries has first returned to its perfect roundness, and has been so filled with primary light that no blemish in it, or trace of the former shadowing, appears. But one must beware of small arched clouds, which may perhaps imitate the disc of the shadowing circle, and also of solar spots, which might perhaps rise or set in the part of the limb first or last deficient.

[Margin: The difference between the true and the visible beginning, etc.]

I said above "of the visible beginning and end" because that blackish lentil-shaped spot, which first touches or last leaves the limb of the luminaries, and is the indicator of the beginning and end of the failing, is not an indivisible

point but some divisible part. However keen an eye or however excellent a telescope you use, it cannot become visible unless it has reached a certain quantity, as much as, and no smaller than, the least visual angle requires. Below that quantity it may be present when the eclipse has already begun, or remain when the eclipse is not yet completed, and yet not be seen by us. Hence it comes about that the true beginning of the eclipse, which occurs immediately upon the mutual contact of the Moon and the penumbra, or of the line drawn from the Earth and touching the western edge of the Sun and the eastern edge of the Moon, is earlier than the visible beginning, while the true end is later than the visible end. For the same reason, that lentil-shaped spot or small crescent of the lunar disc which is last immersed in the penumbra, or which first emerges from it, is not the indicator of the true immersion or emersion, since it is not visible to us except under a certain quantity; and the true immersion is later, but the emersion earlier, than the visible one. Indeed, it usually happens that although the spot just mentioned, whether bright or dark, could be seen by the eye or telescope, perfect and most attentive, it is nevertheless not seen, and about two or three minutes of time slip by between the true instant [verum punctiſtitium], at which the beginnings and ends of the phases just mentioned occur, and the instant visible to us or seen by us. For when I compared that smallest lentil-shaped spot with the small spots or small areas nearer the eastern limb of the Moon and marked on the Moon with a telescope, of which I have already spoken in Book 4, chapter 7, it appeared no smaller than those spots whose breadth is 20″; indeed, it usually appeared equal to those whose breadth is 25″ or 30″, of which seconds there are 1920, or at least 1900, in the diameter of the Moon, taken in that selenography as 32′. Now the mean hourly motion of the Moon from the Sun is 30′, or 1800″. Therefore, as 1800″ to one hour or 60^{m}, so 20″ to $0\frac{2}{3}^{m}$. And if one makes: as 1800″ to 60^{m}, so 30″ to another, the result will be one minute of time.

[Margin: Correction of the observed beginning and end of eclipses.]

We shall therefore come closer to the true moment of these phases if we subtract at least one minute of time from the visible beginning of the eclipse and of the emersion, and add one minute each to the end and to the immersion. But if that spot is somewhat larger, nearly two minutes will have to be taken from the time from noon in the former cases, and added in the latter.

PROBLEMA VIII.

Problem 8.

Tempus à meridie aut ante meridiem, quo Eclipſium Phaſes contingunt, obſeruare.

To observe the time from noon or before noon at which the phases of eclipses occur.

XV. This problem is common to the observing of the times of other phenomena, and so is to be treated in Book 10 among the problems of the primum mobile. Here, however, some of the chief methods must be indicated.

[Margin: 1. Method.]

The *first* method is by sundials, solar by day and lunar by night. It is indeed very useful at the time when an eclipse of the Sun and its phase occurs on the meridian or very close to it; for then the altitudes of the Sun that are taken differ insensibly from the meridian altitude, and so can, with a slight error of their own, produce a great error in the time. Therefore it is better then to use the inclination of the solar ray, or of the shadow, to the meridian line.

[Margin: 2. Method.]

The *second* is by the azimuths of the Sun, or of the Moon, or of the fixed stars. For from the complement of the azimuth to a semicircle, as the angle opposite one of the sides, and from the complement of the declination of the stars to a quadrant, as one side, and from the distance of the zenith from the pole of the equator, as the other side, one obtains, by the analysis of a spherical triangle, the angle contained by those sides. This angle measures the equatorial distance of the star from the meridian; added to the right ascension of a western star, or subtracted if the star is in the eastern quadrant, it leaves the right ascension of mid-heaven, that is, the point of the equator then culminating. From this at last, and from the right ascension of the Sun at that moment, the time from noon is found. But how slippery the practice of azimuths is, although Kepler trusts it too much in the *Astronomia Optica*, p. 375, Tycho had already warned in his *Mechanica Astronomica* and in the first volume of the *Progymnasmata*. We, so as to err here as little as possible, are accustomed to wait for the centre of the star until it passes through the plane of a vertical circle already determined and passing through the limits of whole degrees; to mark that plane with our thread triangle, that is, one consisting of threads, of which more below; and from the azimuth taken to count the times of the pendulum up to the desired phase of the phenomenon.

[Margin: 3. Method.]

The *third* method is by the equatorial distance from the meridian of a fixed star by night, or of the Sun by day, taken with equatorial armillaries [Armillis Æquatorijs]. Tycho considered this method most certain, in the *Epistolae Astronomicae*, p. 72; but the complicated construction and use of the armillaries do not satisfy as well as we should wish, as we shall teach in the *Liber Organicus*.

[Margin: 4. Method.]

The *fourth* method is by the altitude of the Sun by day, or of some fixed star by night, or, if clouds prevent this, by the altitude of the Moon. These apparent altitudes must be reduced to true ones, by adding the parallax and subtracting the refraction, if they have any that is sensible. It is well, however, to avoid the altitude of a star when it is nearer the horizon or the meridian: there, lest it be exposed to refractions that may be extraordinary; here, lest a difference from the meridian altitude that is insensible or small over much time produce a

great error in the time. The instruments with which the altitude of a star, or its distance from the zenith, is taken must be divided most exactly, and show single minutes of degrees, or even aliquot parts of minutes; the pinnules must be fixed exquisitely in their position; the plane of the instrument must first be set most perfectly to the perpendicular of the horizon; and the whole instrument must turn at the slightest touch, so that as soon as one of the observers has given the signal for taking the altitude, its plane can be directed at the centre of the star, and it or the alidade [dioptra] raised; and yet, if it has been tilted from the perpendicular position by that motion, it can at once be restored to the perpendicular position. All of this cannot be managed with Tychonic quadrants, as I shall teach in the *Liber Organicus*, where I shall also teach a more certain way of setting up these instruments quickly and fitting them at once in the required position. How the culminating point of the equator is obtained from the altitude of a star and its declination and right ascension, and from that and the right ascension of the Sun the time from noon, belongs to the problems of the primum mobile. Here someone might well wonder with what unconcern, not to say carelessness, Kepler, otherwise an outstanding weigher of minutiae [apicum trutinator], used in this business quadrants or astrolabes of paper, of the size of a three-quarter foot or a span, as is established from his *Optics*, pp. 367 and 368. But there he admits that he took the altitudes [Greek: ὁλοσχερῶς (holoscherōs, "roughly, in the gross")]; otherwise it is established that Tycho, at p. 72 of the *Epistolae Astronomicae*, condemned the observation of an eclipse made both by Heinrich Brucaeus [Henrico Brucæo] with a small brass quadrant, and by Paul Wittich [Vuitichio] with the large Landgrave's quadrant, but not duly set up.

[Margin: 5. Method.]

The *fifth* method is by the culminations of the stars, that is, their transits through mid-heaven or through the meridian, if you have previously obtained a most exact meridian line, and the plane of some instrument has been set on it, inclined to neither side with respect to the horizon but standing on it most exquisitely to the perpendicular, or at right angles. For since the right ascension of the star, as also of the Sun, is supposed to be known beforehand, the difference of the right ascensions, converted into times, gives the time from noon without any analysis of triangles. Therefore, if the beginning or another phase of the eclipse happens to take place at that moment, the matter is finished and you have got what you wanted; otherwise the interval of time between such a transit and the phase preceding or about to follow must be counted by the methods indicated under Problem 6, but above all with the help of the pendulum and its oscillations. This is not, however, the place to teach the way of obtaining a most exact meridian line; I presuppose it, partly from what was said in Book 1, chapter 10, partly from what will be said in Book 10. The greater difficulty lies in finding and using a plane that exactly represents the plane of the meridian and is in no way inclined from it. For the use of an ordinary plumb-line with a cord parallel to the very plane of a quadrant or astronomical

square, or to the plane of an astrolabe, does not attain so great a fineness and evi- dence as we require here. Not undeservedly does Wendelen, in his preface to the *Eclipses*, all but laugh at Kepler for saying: *We used large quadrants to note the arrivals of the stars at the meridian.* As if the size of quadrants were enough for this business! We therefore, after trying and rejecting many methods over a long time, have settled on this one alone, which we shall briefly describe in the following figure.

[Margin: Our method of marking out the vertical plane.]

XVI. First let a plumb-line AB be made, whose thread is twisted as little as possible, so that it does not wobble, turned about too much by its own rotation. Let the thread be passed through a fine sewing needle, and let the needle be passed through a metal cone CBD, exquisitely turned on a lathe, whose circular base CD is pierced at its very centre as far as the apex of the cone B. The point of the needle is to reach this apex and indeed to project beyond it; for in this way the parts of the cone will balance all round about the needle, as about the axis of the cone, and the tip of the needle B will lie on the same perpendicular straight line as A when the plumb-line hung from A has come to rest.

Diagram. Woodcut set in the left column, interrupting the sentence just rendered. At left a plumb-line hangs from A, with the cone at its foot: base CD above, apex B below. At upper right a plate with a pivot and two screw-clasps stands at the head of the window; from the loop beneath it a solid line runs obliquely down to the left to E, and a second solid line runs straight down through N to F, so that the threads form the triangle of the text. Along the bottom lies the horizontal rule, with its slotted plate at E on the left and its plate at F, and running on to the right past G to the eye-point O, where an eye is drawn. A dotted line runs from the star S at upper left, crossing the oblique thread, down to the eye at O. A chequered graduated arc is struck from F, from its upper end near the left of N round to its lower end at V; solid lines join N to the upper end of the arc and F to V.

- A (upper left): head of the plumb-line
- S (upper left, with a star sign): the star
- C, D (left, either side of the base of the cone): the base of the cone
- B (left, foot of the cone): apex of the cone and tip of the needle
- L (top right, head of the plate): screw-clasp
- K (top right, on the plate): the pin about which the plate turns
- M (top right, foot of the plate): screw-clasp
- I (right of centre, at the apex of the triangle): the loop
- N (right of centre, above F): notch of the needle-piece
- P, R, E (lower left, set close together as PR and E on the rule): end of the rule, slit and plate
- F (lower right of centre): the point on the meridian line
- G (lower right, on the rule): on the meridian line
- O (lower right, before the eye): place of the eye
- Q (top of the arc): upper end of the arc

• V (foot of the arc): lower end of the arc

Next let there be a very high window or door-leaf in the observatory, say of six, eight, ten or more cubits, and beneath its top a flat marble table, exquisitely levelled so as to be parallel to the plane of the horizon. On this table let there be a most exact meridian line GE, long tested. At the top of the window let a plate be fixed above the meridian line, movable about a small pin K and having a small loop I, from which the plumb-line AB is to be hung, of such a length that its point B almost touches the table already mentioned. Let an assistant be on hand to move the plate ML about K until the plumb-line, at rest from all oscillation and rotation (and to this rest it helps for the place to be closed off all round and protected from every breath of air, even the slightest), stands with its tip on the meridian line itself, say at F. When this has been done, let your assistant or companion clamp the plate to the small beam of the window with the screw-clasps L and M, so that it can no longer be moved from that position. You, for your part, mark the point F on the meridian line, over which the tip of the plumb-line came to rest, and there bore through the stone at right angles with a very small hole. Into this hole insert a piece of sewing needle FN, having its notch N, and solder it to the stone with molten lead. Then let a metal rule OP be made, with projecting plates pierced at F and E by the smallest possible hole, and let the small plate E have an open slit RE running up to the hole E, so that the thread can be put in through it directly. Afterwards fit the plate onto the small cylinder or needle FN. Take a fine thread, as little fibrous as possible, passed through the notch N and tied with a knot, and hang it from the loop I, so that, led through the loop and stretched downward, and passed through the slit R and the hole E, it reaches the small cylinder FN, to which, or to another hole set in the same straight line, it is to be tied. For the thread, stretched in this way, will form the triangle EIF, right-angled at F, whose plane, contained within the threads as its sides, will exactly coincide with the plane of a vertical circle, without any sensible inclination to the horizon; indeed, when the thread EF coincides with the meridian line RG, it will mark out for you the plane of the meridian circle.

So, when you are going to observe a star's transit through the meridian, unless the star is the Sun or the Moon, place two lamps near the threads, with their glare blocked by some obstacle so that it does not strike your eyes. With your head resting on or set into some support, and your eye still at O, where you see the thread EI covered by the thread FI, the visual ray leaving the centre of your eye (let me be allowed for now to speak with the old opticians) will run through the plane of the meridian, or of any vertical you like, at which you choose to await the star's arrival. Therefore, as soon as you see the star optically cut in two by that plane, or by the threads, now acting as one single thread, know that the centre of the star is crossing the meridian, or any other vertical, and give a signal to your companion, who must either take the star's altitude or count the times of another pendulum. But it is advisable to have warned him beforehand that the star is about to reach that position, so that he

may check the position of the quadrant and be ready for a sudden and precise capture of the altitude. In this way we have been accustomed to wait for the stars at a fixed degree of the circle VQ etc., described from the centre F, and to take azimuths much more accurately than, as far as we know, has been done so far by earlier astronomers. Nor can there be any doubt that the stretched thread EF marks out a perfectly straight line more evidently than the edge of any ruler.

[Margin: A reliable way of taking azimuths.]

While these pages were about to go to press, Father Iacobus Viua [?], S.J., wrote to me from Rome that Johannes Hevelius has published his *Selenographia*, cut and engraved with his own hands in a most graphic manner, in which he teaches a way of observing lunar eclipses by the Moon's spots and by the passage of the section of the shadow across them, in almost the same manner that I indicated above and had already told that same Father about. He adds that this same author directs that the vertical point of the Moon's upper limb be observed at every notable phase of the eclipse. This is done by noting two or more small spots that at that moment lie on the same perpendicular line, since that line, produced upward, will lead to the point already mentioned. He also wants the centre of the lunar disc as it appears to us to be observed, which he says varies every month, and indeed every day, because of the motion of the Moon's libration. This motion, which I have so far thought could be attributed to the variation of latitude, I shall not unwillingly accept, once I have obtained and read through this author's whole work, if observations persuade me of it; for I do not aim at [Greek: φιλοδοξίαν (philodoxian, "love of reputation")], but at [Greek: φιλαλήθιαν (philalēthian, "love of truth")].

[Margin: An addition from Johannes Hevelius.]

[Margin: On the motion of the Moon's libration.]

SCHOLIA.

Scholia.

I. To observe the quarters of eclipses, and to determine from the position of the horns their parallelism to the Moon's path, it helps, besides the doctrine of the primum mobile, to inspect an armillary sphere, or zodiacal armillaries duly set so as to correspond to the position of the celestial sphere. It is also useful to take the distances of the Moon's edges from fixed stars on either side, which Kepler attests in his *Optics*, p. 370, was done by Tycho; and to draw an arc through the centres of the Moon and the shadow and through some fixed star, with imagination and the eye as guide, or with a thread stretched between both hands; and to mark out the verticals with the azimuths in which the luminaries stand, which you will do very well with our thread triangle; and, if greater [Greek: ἀκρίβεια (akribeia, "precision")] is desired, to take at the same moment the altitude of each horn, or their distance from two fixed stars.

II. The great and frequent disagreement between observations and astronomical tables, at which you will marvel again and again in the history of eclipse observations to be given in chapter 19, and of which we have already given a sample in Book 4, chapter 21, no. 9, sometimes arises from the observation itself, because not everything discussed in the preceding chapter was exactly kept: as if smoke and umbrago were taken for the true eclipse, or the moments of the phases were not corrected, or some small cloud deceived the observer, or the instruments were not exactly made or placed, or were not used with care, and at the very instants of the phases, etc. Sometimes, however, even though the observation has been diligent and accurate in every respect, the error lies in the calculation by which observations are examined through trigonometry, either through the fault of tables swarming with mistakes, or through a slip of the pen, or through a disturbed and less attentive imagination. This happens also in the calculation by which eclipses, especially solar ones, are predicted, at times even to practised men, and Homerically

[Margin: Cause of the disagreement between observations and astronomical tables.]

nodding. Sometimes, finally, it arises from the very hypothesis on which the astronomical tables rest, when it is false and in places in conflict with itself; of this conflict we have already given examples in Book 3, chapter 11, no. 8, and in this very book, chapter 4, no. 14. And indeed so many elements sometimes come together for a single practical conclusion, or for completing one problem (such as the position of the meridian line, the altitude of the pole, the declination and right ascension of the stars, the obliquity of the ecliptic or of the Moon's path, the semidiameters of the luminaries and of the shadow, the latitude, the arc through the centres, the intricacies of parallaxes, and things like them), that one would have to be an Argus, not to say an angel, for no sensible error to creep in from all of these together. It is therefore very hard to judge to which side the cause of the disagreement should be referred, especially when the calculation is transferred from one meridian to another. And there is hardly any observer who does not commend his own diligence in observing. When, however, the calculation promises an eclipse that is either very large, or total, or with a sensible stay, and yet by the agreement of the observations there was none, or a slight one, or not a total one, or one without a sensible stay, the presumption stands against the tables. I hope, however, that from what we have said, and shall say in chapter 19, posterity will be roused to observe eclipses most accurately, and that astronomy will thereby be advanced to a wonderful degree.

[Margin: Why the image of the Sun in its eclipse is crescent-shaped, but the Moon's is not in a lunar eclipse.]

III. Aristotle asks (section 15, problem 10) why, in an eclipse of the Sun, rays passed through a sieve, or broad leaves, or crossed fingers onto the pavement appear as little crescents. He answers that this arises because the Sun's rays are forced into two cones, one having its base on the Sun, the other on the

pavement, and each its vertex in the hole; that this base would be circular if the interposed Moon did not intercept part of the rays, according to the shape of the segment occupying the Sun; that for this reason what is lacking from the circle of the bright base appears crescent-shaped on the ground, as does the remainder of the bright base; but that in the bright image of the Moon, whether failing or not yet full, this does not appear, because the brightnesses of its extremities are not manifest, or do not evidently move the sense. See, if you like, our own Biancani in his exposition of that passage, marginal number 350.

CAPVT XVI.

Chapter 16.

In quo Comparatio ſeu Antitheſis Eclipſium Lunæ, Solis, ac Terræ.

In which a comparison or antithesis of eclipses of the Moon, of the Sun, and of the Earth.

I. I am pleased to gather, by way of epilogue, the more notable marks and properties by which eclipses of the Moon and of the Sun or Earth differ from one another. We shall do this more briefly under one and the same heading, setting apart eclipses of the Moon on the left and of the Sun or Earth on the right, and yet we shall point out far more than do Peuerbach in the *Theoricae*, and Reinhold there, p. 204, and Wurstisen, p. 340; Magini, *Theoricae Planetarum*, Book 2, chapter 31; Barozzi, *Cosmographia*, p. 328; Biancani, *Sphaera*, Book 10, chapter 19.

[Margin: Essence. Cause. When visible. Occasion of the eclipse. Universality. Quarter toward which they incline. Greatest duration. Frequency. Affinity. Intervals.]

☾ Eclipse of the Moon.	☉ Eclipse of the Sun and of the Earth.
1 It is a deprivation of the Sun's primary light in the Moon, and so a true defect of the Moon.	1 It is a deprivation of the Sun's light not in ☉ but in the sublunary region; and so a true defect, not of ☉, but of the Earth and of the sublunary region.
2 It is caused by the shadow or penumbra of the Earth interposed between the Sun and the Moon.	2 It is caused by the shadow or penumbra of the Moon interposed between the Sun and the Earth.
3 It can be seen only at night, or at the extremes of the night.	3 It can be seen only by day, or on the boundary of day and night.
4 It occurs only at a true ☍, at full Moon.	4 It occurs only at an apparent ☌, at new Moon.
5 It is universal to the entire and whole nocturnal hemisphere of the Earth, and of equal magnitude and duration for all who watch it, except for a very few to whom an eclipse appears total that is not total for most.	5 It is not universal to the whole diurnal hemisphere: to some it appears greater and longer, to some smaller and shorter, or not at all; to some, on the contrary, total, or central and annular, and to others corniculate.
6 It ordinarily begins at the eastern edge of the Moon and ends at the western.	6 It ordinarily begins at the western edge of the Sun and ends at the eastern.

☾ Eclipse of the Moon.	☉ Eclipse of the Sun and of the Earth.
7 In the greatest partial obscuration, the obscured part inclines to the north if the Moon's true latitude is southern; but to the south if the latitude is northern.	7 In the greatest obscuration, the obscured part inclines to the south, if the apparent latitude is southern, for those in the north; to the north, if it is northern.
8 It has its greatest duration, of about 4 hours, with ☾ and ☉ at apogee.	8 It has its greatest duration with ☉ at apogee and ☾ at perigee: about 3 hours in the same region; about 5 in different ones.
9 It occurs more often than a solar eclipse in the same region of the Earth.	9 It occurs more often than a lunar eclipse in different regions of the Earth.
10 When it occurs at the node or very near it, the new Moon on either side has had or will have a small eclipse of the Sun.	10 When it occurs at the node or very near it, the full Moons on either side are entirely free of any eclipse.
11 When it occurs far from the node, it has a neighbouring ecliptic new Moon.	11 When it occurs far from the node, it has a neighbouring ecliptic full Moon.
12 It can occur twice in a year: in the sixth month after the former; rarely, and small, in the fifth; never in the seventh.	12 It can occur twice in a year: in the sixth month after the former; rarely, and small, and toward the same pole, in the fifth and seventh.
13 It cannot occur twice in the same month, that is, at two successive full Moons.	13 It can occur twice in the same month, but not toward the same pole.
14 In the same solar year it cannot occur more than three times.	14 In the same solar year it cannot occur more than five times.

CAPVT XVII.

Chapter 17.

De Vſu Crucis Noſtræ Geographicæ, pro non fruſtra admonendis futuræ Eclipſis obſeruatoribus, & aut pro tranſlatione calculi Ecliptici non fruſtra ſuſcepta ad alia atq. alia Terræ loca.

On the use of our Geographical Cross, so that the observers of a future eclipse may not be alerted in vain, and or [?] for a transfer of the eclipse calculation, not undertaken in vain, to one place on Earth after another.

I. Nineteen years ago I was thinking about transferring the calculation of eclipses to other places on Earth, so that a single labour might serve all for whom it could be done. I saw that what was needed for this above all was to distinguish day from night at one horizon after another, lest we should labour in vain, in a lunar eclipse for regions in which it will be day, or in a solar eclipse for those in which it will be night, for the whole time of the eclipse. And finally I saw that it was very tiresome to look up the altitudes of the pole one by one, and the semidiurnal arcs according to them, and that these could not be represented at once, at a single glance, on geographical globes. So I set my mind to devising a way by which, for any given moment of a given day, there might easily be represented on some geographical map all the places of the Earth, at least within the Torrid and Temperate Zones, in which the Sun would then be rising on this side and setting on that, and in which it would

be noon on one side and midnight on the other. From this one might see at a single glance where at that same moment it was day and where night; and what hour it was at any place whatever, reckoned from noon or before noon, from midnight or before midnight, from sunset or sunrise, or before sunset and sunrise; and finally in what proportion northern places had a greater or a smaller semidiurnal or diurnal arc than southern ones. This at length, with God, Best and Greatest, favouring our efforts, we found could be done in less time than it takes to say so, with the help of an instrument to which we gave the name of GEOGRAPHICAL CROSS; for it consists of two fine rulers, of which one represents the meridian and the other the equator, bisected at right angles by the meridian. The equator of this cross carries at its ends two arms imitating the shape of a large, Latin sigma, whose position and suitable inclination toward the meridian resolve all the problems already mentioned. But since, as early as the year 1643, with friends wresting it from us, we published at Bologna a little work in which the construction and use of this instrument are explained, together with a geographical map adapted to it, and since the same is to be inserted in our *Geographical Book*, which will be the fourth of volume 2 of the *New Almagest*, we refer the reader there.

[Margin: Application of the Geographical Cross to eclipses.]

II. Now therefore, if you duly place the Geographical Cross on our geographical map for the time of the beginning, middle and end of a lunar eclipse, you will at once be able to view on it, at a single glance, all the regions of the three principal zones in which it will be night, and in which, on the contrary, day; and so which places are suited to observing the whole eclipse, or its beginning, or middle, or end. For eclipses of the Sun, however, one must find the beginning and end of the eclipse of the Earth, that is, the moment at which the Moon's shadow or penumbra begins or ceases to be on the face of the Earth, according to Problems 9, 10 and 11 of chapter 12, and at those moments apply the Geographical Cross to the map, so that with its arms it embraces the tracts of land in which it will be day for the whole time of the eclipse, and for which, as far as this part is concerned, the labour of calculation will not be in vain.

CAPVT XVIII.

Chapter 18.

De Miraculo Eclipſis Solis in Morte CHRISTI Domini, & de Eclipſibus Præternaturalibus Extremam Mundi Diemne an Noctem? portendentibus.

On the miracle of the eclipse of the Sun at the death of Christ the Lord, and on preternatural eclipses portending the last day of the world, or its night?

I. The properties of natural eclipses of the Moon and Sun explained so far, and their antithesis briefly set out in chapter 16, remind us that before the history of all other eclipses we should recall the sacred horrors of that eclipse, and the

wondrous darkness from which the immortal divinity of Christ, dying on the cross, flashed out like lightning through the clouds of the world. That darkness, ending with its shadows the shadows of the old Law, offered itself as a most brilliant dawn to the Gospel. And to the great Dionysius, accustomed long ago to judge most justly in the darkness of the Areopagus, it brought, by its own, so to speak, sacrosanct darkness, such a splendour of almost prophetic light that, most justly and most wisely alike, he burst out from the miracle into that oracle: *Either the God of Nature is suffering, or the machine of the world will be dissolved.* For this reason we take up in this same chapter two kinds of eclipses divinely ordained above the order of nature: one, which was a witness of the divinity and innocence of God in Christ, most unjustly judged by men; the other, of those which will go before the machine of the world, now on the very point of dissolution, and before God in the same Christ, who will most justly judge the world.

[Margin: History of the eclipse of the Sun at the death of Christ, from the Gospel.]

II. First, then, concerning the former, we read thus in the sacrosanct Gospel: in Matthew, chapter 27: *From the sixth hour darkness fell over the whole earth until the ninth hour.* Then in Mark 15: *And when the sixth hour had come, darkness fell through the whole earth until the ninth hour.* But St Luke, chapter 23, is more expressive still: *It was about the sixth hour, and darkness fell over the whole earth until the ninth hour, and the Sun was darkened, and the veil of the temple was torn in the middle.*

[Margin: Oracles concerning this eclipse.]

This same eclipse had been foretold long before by Jeremiah 15, in these words: *Her Sun set while it was still day*; or, as St Jerome reads on Matthew 27: *The Sun went down while it was still midday.* And by Amos, chapter 8, in these words: *The Sun will set at noon, and I will make the earth grow dark on a day of light.* For St Jerome refers both prophecies to this event in the passage just cited; and indeed also that of Joel, chapter 2: *The Sun will be turned into darkness and the Moon into blood, before the great and horrible day of the Lord comes. And it shall be: Everyone who calls on the name of the Lord will be saved; for on Mount Zion and in Jerusalem there will be salvation.*

[Margin: St Jerome's interpretation.]

Since St Peter (Acts 2) and St Paul (Romans 2) cited these words to confirm this time of Christ's first manifestation, St Jerome therefore says in his commentary on Joel 2: *The Sun too was turned into darkness, when it did not dare to look on its Lord hanging; and the Moon into blood. This we may either believe happened according to the history, and these things were passed over in silence by the Evangelists (for not everything that Jesus did is reported in writing, etc.); or certainly, just as the Sun was turned into darkness, not because it was itself changed into darkness, but because it brought darkness over the world, so too ☾ was not turned into blood, but condemned the Jews, covered with the horror of blasphemies and of denial against Christ, by the eternal blood of their own testimony, when they said: His blood be upon*

us and upon our children. All these things he describes as going to happen before the great and horrible day of the Lord. The great and horrible day of the Lord, moreover, must be believed to be either that of the resurrection, or certainly, many ages later, the day of judgement, which is truly great and horrible. But since there follows: And it shall be that everyone who calls on the name of the Lord will be saved, and the Apostle Paul refers this to the time of the Lord's passion, it is rather to be understood of the day of the resurrection. And further down in the same place: *This passage is most difficult, and admits a threefold explanation: so that, under tropology, we may refer everything that has been said to those times to which the Apostles Peter and Paul referred it, that is, when he suffered and rose again.* Salmeron [Alfonso Salmerón] subscribes to this opinion, vol. 10, tractate 42, which is entitled *On the Darkness from the Sixth Hour.*

[Margin: Oracles of the Sibyls.]

To these is added the Sibylline oracle from Book 1 of the *Sibylline Oracles*:

But when, with hands spread out, having measured all things, he has borne a crown of thorns, and a reed, driven by hand, has pierced his side, for his sake, for three hours, a dark and monstrous night will fall at midday.

And that of another Sibyl, in Salmeron, above: *And the veil of the temple will be torn, and at midday there will be night for three hours.*

[Margin: Phlegon's testimony on this eclipse.]

Moreover, this enormous failing was also entered in his annals by Phlegon Trallianus [Phlegon of Tralles], Hadrian's freedman, on whom Suidas says more, and an outstanding reckoner of the Olympiads, as Origen relates, tractate 35 on Matthew, saying: *And Phlegon indeed wrote in his Chronicles that this happened in the principate of Tiberius Caesar, but he did not indicate that it happened at full Moon.* And in *Against Celsus*, Book 2, where he says: *Of the Sun darkened in the time of Tiberius Caesar, in whose reign Jesus is believed to have been fixed to the cross, and of the great earthquakes that then occurred, Phlegon wrote in the thirteenth, or as I judge the fourteenth, book On Times.* Eusebius Cæsariensis [Eusebius of Caesarea] also, in his *Chronicle* for the year of Christ 33, seems to recite the testimony not only of Phlegon but also of other pagans on these matters, when he speaks thus: *Jesus Christ, the Son of God, etc., according to the prophecies that had been uttered about him, came to his passion in the 18th year of Tiberius; at which time we find also in other commentaries of the pagans these things written word for word: An eclipse of the Sun took place, and darkness over the whole earth; Bithynia was shaken by an earthquake, and in the city of Nicaea very many houses collapsed. — All of which* (says Eusebius) *agree with what happened in the Saviour's passion.* Then he cites Phlegon as a distinct witness, saying: *Phlegon too, who is an outstanding reckoner of the Olympiads, wrote about these things, saying thus in his fourteenth book: — In the fourth year of the 202nd Olympiad a great eclipse of the Sun took place, one excelling all that had happened before it: the day at the sixth hour was turned into dark night, so that the stars of the sky were seen; and an earthquake in Bithynia overthrew many houses of the city of Nicaea.*

This testimony of Phlegon is then adduced on this matter above all by Cedrenus [Georgios Kedrenos] in the *Annals*, p. 155; Scaliger, Book 6, *De emendatione temporum*, p. 561; Baronio at the year of Christ 34, p. 176; Salmeron, vol. 10, tractate 42; Salianus in the *Annals* at the year of the world 4086, of Christ 33, no. 54; Cornelius à lapide [Cornelius a Lapide] on Matthew 27, verse 45.

[Margin: Testimony of St Lucian the Martyr.]

But long before, for the testimonies of the Gentiles already mentioned, the pagans themselves are cited, both by St Lucian in the acts of his martyrdom, in Eusebius, *Ecclesiastical History*, Book 9, chapter 9, who says: *I bring forward the Sun itself as a witness of these things for us, which, when it saw these things being done on earth by the impious,*

hid its light at midday in the sky. Search your annals, and you will find that in the time of Pilate the Sun was put to flight and the day broken off by darkness. And by Tertullian in the *Apologeticum*, where he has this to say about Jesus:

[Margin: And that of Tertullian.]

Yet, once nailed up, he showed many marks proper to that death of his; for he gave up his spirit of his own accord, with a word, forestalling the executioner's office. At that same moment the day was overcast, while the Sun was marking the middle of its circuit. Those who did not know that this too had been foretold concerning CHRIST certainly took it for a failing; and yet you have that disaster of the world recorded in your archives.

That stars were seen at that time is affirmed not by Phlegon alone, but also by Paulus Orosius, Book 7, chapter 4, whose words are these:

[Margin: Testimony of Paulus Orosius.]

On that same day too, at the 6th hour of the day, the Sun was wholly darkened, and a black night was suddenly drawn over the lands, and the impious ages feared eternal night. It is so plain that neither the Moon nor clouds stood in the way of the Sun's light that the Moon, on that day in its fourteenth day, is reported to have been very far from the sight of the Sun, with the whole region of the heavens lying between them, and that the stars then shone over the whole sky during the hours of daylight, or rather in that dreadful night. This is attested not only by the faith of the holy Gospels, but also by a fair number of books of the Greeks.

St Augustine too speaks as follows:

[Margin: And of St Augustine, Letter 80, to Hesychius.]

That failure of the Sun when Christ was crucified was not such as the other failures of the ☉, but truly marvellous and prodigious. But through the jealousy of the pagan Romans, lest heaven be thought to have mourned the death of Christ as of a divine man, they had this written down in their annals, yet forbade it to be made public.

Sedulius also chimes in, in Book 4 of the *Paschal Work*, as follows:

[Margin: Verses of Sedulius on the same eclipse.]

Meanwhile dreadful darkness suddenly came on and held the whole pole, and with mournful funeral rites wove the blackening day; the Sun, hiding his flashing rays in a cloud, veiled in a foul mantle, lay concealed, and filled the sad world with lamentation.

Because Delia, full, had her brother opposite her, when she was beneath the earth and he above at the height of heaven, the impious ages feared perpetual night.

III. But among so many ancient assertors of this eclipse none, the Evangelists excepted, is older or more trustworthy than the most clear-sighted eyewitness himself, St Dionysius the Areopagite. Having taught that the order and motion of the heavens can be changed by the power of God alone, of which kind was the Sun's retreat through 10 lines, he says:

[Margin: Testimonies of St Dionysius the Areopagite on this eclipse. Letter 7, to Polycarp.]

This rightly and deservedly terrified the Babylonians, and without a battle subjected them to Hezekiah, as though he were equal and like to God and surpassed men. I do not speak of the great works performed in Egypt, or of certain other signs and portents of God that happened elsewhere, but of those common and heavenly, which are celebrated both throughout the universe and on the lips of all. But Apollophanes flatly denies that these things are true. The priests among the Persians especially relate this by report, and the Magi moreover perform rites in memory of the threefold Sun; but let him indeed be allowed, through ignorance or inexperience, not to believe these things. Ask him, however, what he thinks about the failure of the Sun that occurred when the Saviour was driven onto the cross. For we were together, both of us, and were standing at Heliopolis, and we saw the Moon unexpectedly falling upon the Sun, for it was not the time of conjunction; and again the same Moon, from the ninth hour until evening, setting itself against the middle line of the Sun, contrary to the order of nature. Recall something else to his memory as well. For he knows too that the incidence itself was seen by us to begin from the rising of the Sun, to reach the Sun's edge, and then to vanish; and again that the incidence and the withdrawal did not occur from the same part of the Sun, but from the part that was, so to speak, diametrically opposite. These are the marvels of that time, which CHRIST, author of all things, alone can bring about, who does great and marvellous things without number. Refute these things, Apollophanes, if it is lawful for you and if you can, even against me, who was then, as you know, with you, and was present, and saw, and examined everything with the greatest wonder. Then indeed Apollophanes began even to prophesy, I do not know from what source, and, as if comparing with me what was happening, said this: These, Dionysius, are changes of things divine. Let this much be said by us, as far as the nature of a letter required. But you can both supply what is lacking and perfectly offer and consecrate to GOD a man who is endowed with great wisdom, and who perhaps will not be unwilling to learn calmly the most wise truth of our religion.

Again, writing to Apollophanes himself, who had by then surrendered to the truth, he writes as follows in Letter 11:

[Margin: A second testimony of St Dionysius.]

But as soon as the heavenly light of the Father's glory, by its own will, disposed its splendour to shine into the darkness of your mind, it straightway poured into the inmost chambers of my heart that I should set before your eyes a most pious recollection and memorial of what was done. Namely: at Heliopolis, you nearly my contemporary and I passing a span of about twenty-five years, we had settled and were staying

together, when, on a certain sixth day of the week and also at nearly the sixth hour, the Sun was suddenly covered with horrible darkness, with the Moon passing across it, because, being not God but a creature of God, it could not shine at the setting of that true light. For I was then asking you, who seemed to me truly very wise, what you yourself thought of it; and you with wonderful prudence added something that stuck so deep in the inmost parts of my mind that no forgetfulness can erase it, no image of death ever wear it away. For we ourselves were covered in darkness, the gloom occupying the Sun's disc on every side and evenly; and after the cleansing and restoration had been made, the Moon, turned to the Sun's diameter, fled away. And then too we took up the canon of Philip Arrhidaeus; and when we had found, what was also very well known, that the Sun could by no means have suffered an eclipse at that time, and that nonetheless the Moon had covered the Sun's radiance from the east with dark, blackening veils, though she usually comes under the Sun from the western side. Then, having wandered as far as the western bounds of the Sun, as we ourselves thought, she hid the light-making treasures with the darkness she herself had begotten, though the Moon lacked that time too, and the time of the synod was known not to be near. Then I said to you (for I was not yet aware of the mystery of so great a thing): O storehouse of immense prudence, what, I say, O mirror of learning, Apollophanes, do you ascribe to these secrets? To this you, by I know not what divine inspiration and not by human sense, added: These, good Dionysius, are surely changes of things divine. At last, having found by experience that the day of the week and the year, carefully noted, agreed and accorded with the announcement that holy Paul poured into our attentive ears, the signs consenting, I submitted my hands to the truth and was released from the bonds of falsehood. This truth I extol marvellously and share with you, the truth that is the way and the life and the true light, which enlightens every man coming into this world; and to it you at last, as a truly prudent man, have come.

O splendid gloom, O eclipse to be kissed, which joined these two wisest lights of Greece, Dionysius and Apollophanes, made subject to CHRIST, the eternal light!

Although some suspect these Letters of St Dionysius to be spurious, they are nevertheless accepted as genuine by St Thomas Aquinas, part 3, question 44, article 2; by Baronio, Cornelius à Lapide and Salmerón, with most of those in the places listed earlier; and by Neander, Maurolycus, Clavius, Barozzi and Biancani, who are to be brought forward below. This can be confirmed both from Michael Syngelus [Michael Syncellus] in the *Encomium of St Dionysius* and from Suidas in two places, where they add that, having heard Apollophanes, he cried out:

[Margin: What the exclamation of St Dionysius was.]

[Greek: ἄγνωστος πάσχει θεὸς δι' ὃν τὸ πᾶν ἐζόφωται καὶ σεσάλευται (agnōstos paschei theos di' hon to pan ezophōtai kai sesaleutai)], that is, *The unknown God is suffering in the flesh, and therefore the universe is shaken and darkened by this darkness*; or at any rate [Greek: Ἢ τὸ θεῖον πάσχει, ἢ τῷ πάσχοντι συμπάσχει (ē to theion paschei, ē tōi paschonti sympaschei)], namely *Either the Divinity is suffering, or it suffers with one who suffers*. Granted that, by a certain

tradition, that saying now goes around: *Either the God of nature is suffering, or the machine of the world is being dissolved*, as Sacrobosco reports at the end of the *Sphere*; Barozzi at the end of the *Cosmographia*; Biancani, Book 10, chapter 19; Salmerón above; Fonseca and Cornelius à Lapide. But the earlier words are reported by Neander in the *Elements of the Doctrine of the Sphere*, p. 130, and by Maurolycus in dialogue 3 of the *Cosmographia*, p. 86, from the *Onomasticon* of Suidas.

IV. That is now enough, and more than enough, of witnesses for the truth of the eclipse already mentioned. I now pass on to discussing its efficient cause, intending to speak later about the final cause; on the efficient cause I find eight opinions.

[Margin: 1. Opinion: on the withdrawal of the Sun's rays.]

The first is that of those who attribute it to the Sun itself, withdrawing its rays and as it were reabsorbing them inward. St Cyprian favours them in the sermon *On the Good of Patience*, saying: *Since the Sun, lest it be forced to look on the crime of the Jews, withdraws both its rays and its eyes.* St Chrysostom also favours them, as is contained in the *Catena*, in the sermon on the Lord's Passion: *The creature could not bear the injury done to the Creator; hence the Sun drew back its rays, so as not to see the crimes of the impious.* With these St Jerome agrees, on Matthew chapter 27, saying: *It seems to me that the brightest light of the world, that is, the greater luminary, drew back its rays, so that it should neither see the Lord hanging, nor the blaspheming impious enjoy its light.* Although Saints Cyprian and Chrysostom seem to have said these things metaphorically and in the manner of preachers, or almost poetically, as Virgil had sung, in Book 1 of the *Georgics*, about the Sun darkened after the slaying of Gaius Julius Caesar:

He too, with Caesar dead, took pity on Rome,
when he covered his shining head with dusky rust, and the impious ages feared eternal night.

St Jerome, however, seems to have adduced it as the more probable cause. For he had said: *Those who wrote against the Gospels suspect that the disciples of Christ, through inexperience, interpreted a failing of the Sun, which usually happens at fixed and appointed times, with reference to the Lord's resurrection; though a failure of the Sun never usually occurs except at the rising of the Moon. And no one doubts that at the time of Passover the Moon was at its fullest. And lest perhaps the shadow of the earth, or the disc of the Moon set opposite the Sun, should seem to have made a brief and rust-coloured darkness, a span of three hours is stated, so that every occasion for cavil might be removed.* And, after a few words in between, he adds: *And it seems to me that the brightest light of the world, etc.*, as already reported. To the same opinion Salmerón adds Cajetan and Paulus Orosius; but Orosius only denies that that eclipse was produced by the Moon or by interposed clouds, as is plain from what was said in no. 2.

[Margin: 2. Opinion, of Euthymius.]

The second opinion seems to be that of Euthymius, on Matthew chapter 27: that the cause is God, angered and denying his concurrence to the Sun's rays.

For he says: *This happened after they had had their fill of insolence and jeering, so that, with their fury now subsiding, something could be done by the miracle. For it was more wonderful for the one hanging on the cross to do these things than to come down from the cross. But if they did not suppose that he was doing these things, but that they came about with God working them, they surely ought to have been afraid, and to have understood that God had been moved to anger on this man's account. For they could not say that it was an eclipse of the Sun according to nature, since such an eclipse both occurs in one moment of time, and in one part of the earth, and not on the fourteenth day of the Moon; whereas this darkness both lasted up to three hours, and was over the whole earth, and fell on the fourteenth day of the Moon, when it is impossible according to nature for an eclipse of the Sun to occur.*

[Margin: 3. Opinion, of Origen.]

The third opinion was that of Origen, tractate 35 on Matthew. There he recounts at length the slanders of some against the Gospel, on the ground that it narrates an eclipse of the Sun at an unsuitable time, namely that of the full Moon; and who do not accept the answer of the faithful, who say that this happened not naturally but by a divine prodigy, on the ground that no one among the Greeks or barbarians records it in their chronicles, nor does Phlegon, who mentions it, say that it occurred at full Moon. So, in order to cut away the handle for slandering the Gospels, he says: *We say therefore that Matthew and Mark did not say that a failure of the Sun occurred then, nor did Luke, according to most copies, which have it thus: And darkness came over the whole earth, and the Sun was darkened; but in certain copies it does not stand as: Darkness came and the Sun was darkened; but thus: Darkness came over all the earth, the Sun failing.* He attributes this change of phrase either to one of the faithful who thought that that darkness could not have come about otherwise than through a failure of the Sun, or to one of the unbelievers, so that an occasion for slandering the Evangelists might arise from it. Finally he concludes that that darkness came over the whole earth, but that of Judea only, in which alone the rocks and the veil were split, and that it came from clouds. For he says: *We answer first of all that the Evangelists did not name the Sun at all in that place, but only that darkness came over all the earth. For if darkness came over all the earth with the Sun not named, it follows without doubt that one should understand certain very dark clouds, and perhaps not one but many, and larger ones, to have gathered over the land of Judea and Jerusalem to cover the Sun's rays.*

[Margin: The opinion of Origen rejected.]

But Origen is not consistent with himself, since he admitted that that particle is found in most copies of St Luke: *and the Sun was darkened*; and we are in any case obliged to follow it in the Vulgate version by the precept of the Holy Council of Trent and of Clement VIII. Next, the authority of Phlegon stands in the way, which was not unknown to Origen: he, like Paulus Orosius too, says that stars were then seen in the sky, and those thick clouds would surely have blocked them. Finally, the Evangelists did not say over the whole of that land, or over the whole land of Judah, but absolutely over the whole earth: [Greek:

εἰς ὅλην γῆν (eis holēn gēn, "over the whole earth")], or [Greek: εἰς πᾶσαν τὴν γῆν (eis pasan tēn gēn, "over all the earth")]. It agrees less with the letter, therefore, to confine that darkness, with Origen, Euthymius and Maldonato [Juan Maldonado], to Judea alone, than, with Chrysostom, Theophylact and others, whom Salmerón and Cornelius à Lapide follow, to extend it to the whole earth; and that better befits the power of God in this miracle, as we shall say below.

[Margin: 4. Opinion, of Kepler.]

The fourth opinion is Kepler's, in the *Epitome of Copernican Astronomy*, p. 896. There he deals with spurious eclipses, some of which are produced by globes of ash belched out by Vesuviuses, of which kind, he says, was the one that Cassius Dio reports seen under Domitian, when Vesuvius had spewed ash as far as Palestine. Because an earthquake usually accompanies or precedes these, and the eclipse at our Saviour's death came with an earthquake, he inclines to the view that it arose from ash spewed upward, and he compares it with the eclipse of Dio. For he says: *Compare also that darkness which veiled the Sun at the time of the Passion of our Lord; for it came with an earthquake, although Dionysius the Areopagite, in Suidas, ascribes this darkness to the motion of the Moon.* And he had held the same in the *Astronomia Optica*, taking no account of the authority of Dionysius.

[Margin: It is refuted.]

But this is to detract from the miracle; there would then have been neither darkness over the whole earth, nor stars seen in the sky, against the authority of Phlegon and Orosius.

[Margin: Kepler's bold cast of mind, here and on the Star of the Magi.]

But I am not surprised that Kepler delighted in detracting from this miracle concerning the luminaries at the death of CHRIST, since he had already detracted from the miracle performed at his Nativity concerning the new star that appeared to the Magi. For he says that it was lit by God, but at the conjunction of Saturn, Jupiter and Mars in the fiery triplicity, which took place in Julian Year 39; that the Magi, from the Chaldaean rules on great conjunctions and their effects, expected that conjunction to rouse some new star naturally; and that God, accommodating himself to them, lit it divinely at that time, and perhaps also placed it in the very location to which the eyes of the Magi were most of all being directed on account of the threefold gathering of the planets. So he says in the book *De Stella Nova* in Serpentarius, near the end of chapter 26, or from p. 134, subjecting to the natural faculty of the stars what the Fathers commonly recognize as a wholly sacred phenomenon and a spectacle born of pure miracle, to the point that they show from it that the Nativity of CHRIST did not depend on the stars or on the rules of the Chaldaeans.

But Julian Year 39 was too far from Julian Year 44 or 45, in which most assert that CHRIST was born. So, in order to be able to assert that he was born only a year or two after the great conjunction of the three superior planets, with only a two-year interval between the appearance of the new star and

the Magi's arrival in Judea, he undertook to defend the Theorems of the Pole Laurentius Suslyga [Wawrzyniec Suślyga], in which that author contends that our commonly used Christian Epoch falls short by four or five years, and that our Saviour was in truth born four years before it is commonly thought. To this end, therefore, Kepler wrote a little work which he calls the *Silva Chronologica* [Chronological Forest], as he openly testifies in its dedicatory letter and at the very beginning of the little work. And so that neither the Gospel of St Luke nor the authority of Phlegon on the year of the beginning of the preaching, or of the Passion, should stand in the way, he says at the end of the *Silva* already mentioned that the Saviour began preaching and came to Baptism at 32 years of age or a little more; and in the Rudolphine Tables, chapter 27, precept 62, he has this: *Phlegon of Tralles, as Eusebius reports, noted in the fourth year of Olympiad 202 a very great eclipse of the Sun at the 6th hour of the day. Eusebius understands this of the eclipse that occurred while the Saviour was hanging on the cross at the feast of Passover.* This year, from his own rules, he gathers to be Year 32 of the common Christian Epoch. Then, however, he adds: *But because the Passion of CHRIST certainly falls in the preceding 31, we understand from this that a corruption befell Phlegon's text, and that the numeral [Greek: γ (gamma, 3)] was changed to [Greek: δ (delta, 4)].* For what does this man, so very eager for innovation, not dare, lest the offspring of his most hot-headed intellect should fail to live?

But I long ago laid the axe to that chronological *Forest*, tangled as it is with the most tortuous knots and thickets of thorns, in my *Chronological Book*, which with God's favour will come out and will be the fifth of the second volume of the *New Almagest*. Tycho, by contrast, was far wiser in recognizing the Star of the Magi as wholly supernatural, volume 1 of the *Progymnasmata*, p. 325, condemning Cardano of impiety for having counted it among the comets.

[Margin: 5. Opinion, of Cornelius à Lapide.]

The fifth opinion is that of Father Cornelius à Lapide on Matthew chapter 27, verse 45. He holds that this eclipse was caused partly by the Moon, supernaturally interposed before the Sun, and partly by the Sun, which at God's action withdrew its rays from the Christ-killing earth. For he thinks that the Moon, being always smaller than the Sun, could not have covered the whole Sun, and that in any eclipse of the Sun much light is diffused from it over the earth; and therefore that, in the part where the Sun stood out, this eclipse was completed by another miracle, through the withdrawal of rays. But he does not

[Margin: Who is excused.]

remember, good father that he is, the dreadful eclipses in which stars were seen in the sky. He is nevertheless to be pardoned, not being a professor of astronomy, for what astronomers otherwise not to be disdained held with Tycho, namely that no total eclipses of the Sun can occur naturally; on these see what was said in Book 4, chapter 16, no. 15, and in this Book, chapter 20.

[Margin: 6. Opinion, of Scheiner.]

The sixth opinion is that of Scheiner in the *Rosa Ursina*, p. 612. Since the Moon's shadow occupies only 350 Italian miles, he thinks the eclipse was

produced partly by the interposition of the Moon and partly by an extraordinary mass of spots on the Sun.

[Margin: 7. Opinion, of the Jews.]

The seventh opinion was that of the Jews themselves, if credence were to be given to the apocryphal Gospel of Nicodemus, which Salmerón adduces in volume 10, tractate 42. For it recounts that the Centurion came to the Governor reporting all the signs that had occurred, and that, when the Jews had been summoned, he said: *Have you seen the signs that occurred in the Sun, and all the others that occurred while Jesus was dying?* And that the Jews replied: *An eclipse of the Sun occurred according to the old custom.* By the old custom they meant either the natural manner of a solar eclipse or, as Salmerón interprets it, the force of magical and demonic art, with which, as was their habit, they had out of envy more than once besmirched Christ.

[Margin: 8. Our own view, and that of very many.]

The eighth and true view is that this eclipse was produced by an interposition of the Moon between the Earth and the Sun that was natural indeed as regards substance (to speak in theological fashion) but supernatural as regards mode, or as to accidents. So hold Petrus Comestor; Vincent of Lérins, Book 8, chapter 45; Syngelus in the *Encomium of St Dionysius*; St Thomas Aquinas, part 3, question 44, article 2; Salmerón, volume 10, tractate 42; Baronio at the year of Christ 34; Salianus in the *Annals* at the year of the world 4086; Fonseca, *On the Life of Christ*, p. 594; Gretserus [Jacob Gretser], *On the Holy Cross*, Book 1, chapter 31; Barradius [Sebastião Barradas] on the Gospels, Book 7, chapter 20; and Iansenius [Cornelius Jansen] in the *Concordia*, chapter 143. Among the mathematicians: Johannes de Sacrobosco at the end of the *Sphere*, and Clavius in that place; Michael Neander in the *Elements of the Doctrine of the Sphere*, p. 130; Maurolycus in dialogue 3 of the *Cosmographia*, p. 86; Barozzi in the *Cosmographia*, p. 348; Biancani, Book 10 of the *Sphere*, chapter 19; and, as regards the principal cause, Cornelius à Lapide on Matthew chapter 27, verse 45, and Scheiner in the *Rosa Ursina*, p. 612. That it was miraculous, though not produced by the Moon or by clouds, we have already seen asserted by St Jerome, Orosius and Euthymius. St Augustine may stand for them, in Letter 80 to Hesychius, in Book 3 of the *City of God*, chapter 15, and in the book of *Questions on the New Testament*, question 105; in these places he teaches that this failure of the Sun was marvellous and prodigious because it happened at full Moon. And to Salmerón he seems to have held the same view, namely that it was produced by the interposition of the Moon.

[Margin: Enumeration of the miracles within the miracle of this eclipse.]

V. With the miracle and the cause of this eclipse established, the miracles contained in it as parts are now to be enumerated. Euthymius, Syngelus and Fonseca, in the places already listed, think there were three: namely that it was at full Moon, that it was universal, and that it came with duration of three hours, or that it began from the eastern part of the Sun and ended in the same part. Barozzi, however, counts five: namely the three already mentioned, and in

addition that the whole Sun was covered by the Moon, and that the Moon had a greater latitude than the ecliptic limits require. Biancani likewise counts five, but differently: namely the time of full Moon; 2. its totality and universality over the whole earth; 3. the beginning from the eastern part of the Sun and the end from the western; 4. the Moon's progress to conjunction with the Sun against the order of its own motion, that is, with a motion towards the west; 5. an unusual approach of the Moon to the earth, so that it could cover the whole Sun for so long for the whole earth. But St Thomas sets down five differently: 1. the time of full Moon; 2. that within an interval of three hours the Moon was joined to the Sun and then opposed to it; 3. that the eastern part of the Sun began to be hidden; 4. that the western part of the Sun began to emerge from the eclipse, although otherwise the same part that begins to be uncovered is the one that had begun to be covered; 5. from St Chrysostom, that it lasted for 3 hours. Cornelius à Lapide, however, lists seven: that is, 1. the time of full Moon; 2. a duration of three hours; 3. the beginning from the eastern part of the Sun; 4. the reversal of the Moon towards the east to opposition with the Sun, for so he interprets St Dionysius, in a manner contrary to Biancani; 5. the total obscuration of the Sun, even though the Moon was smaller than the Sun; 6. stars seen in the sky by day, and an earthquake, though these have been seen at other times; 7. the shadowing of the whole earth.

We, however, contemplate 10 things in this eclipse that are altogether marvellous and above the order of the natural course of the luminaries. For some of them, however, I assume for the present, from my *Chronological Book*, that CHRIST suffered in Year 33 of our common Christian Epoch, on April 3, as is also the view of Tonstatus [?], Roger Bacon, Io. de Muris [Johannes de Muris], Buntingus [Heinrich Bünting], Paul of Burgos, Johannes Stadius, Origanus, Scaliger, Io. Lucidus [Giovanni Lucido], Pererius [Benet Perera], Zarlinus [Gioseffo Zarlino], and the old Calendar of the Dominican Fathers.

[Margin: 1. Miracle.]

The first miracle was an eclipse of the Sun produced by the Moon at the time of full Moon. For it is certain that the paschal lamb was eaten by Christ in the evening of the fourteenth day of the Moon, and that he suffered on the fifteenth day of the Moon, that is, in the opposition of the Moon, at least the mean opposition, with the Sun; but it is also certain that an eclipse of the Sun cannot occur naturally by the Moon's interposition except at new Moon.

[Margin: 2. Miracle.]

The second was the beginning of the eclipse from the eastern part of the Sun, as St Dionysius testifies, although naturally, in an eclipse of the Sun, the Moon usually comes under it from the western part.

[Margin: 3. Miracle.]

The third was the beginning of the emersion in the part of the Sun different from and opposite to the one that had first been darkened, that is, in the western part, as St Dionysius likewise testifies; whereas naturally the same part that is

first to fail in the Sun is also the first to begin to be refilled and to emerge from the eclipse.

[Margin: 4. Miracle.]

The fourth was the Sun's stay in darkness for three equinoctial hours, whereas naturally the Sun in a total eclipse does not stay beyond 8^m or 10^m of time, from what was said in chapter 11, Theorem 2. I think this was done through a slowness in the Moon's hourly motion, which otherwise usually overtakes the Sun.

[Margin: 5. Miracle.]

The fifth was the total obscuration of the Sun by the Moon, occurring at a time when the Moon's apparent diameter was smaller than the Sun's diameter. For from my tables, in that Year 33, at noon on April 3, the anomaly of the ☉ was $10^s\ 1^\circ\ 13'\ 32''$, and therefore its apparent semidiameter $15'\ 34''$; while the Moon's equated anomaly was $1^s\ 6^\circ\ 17'\ 21''$, and therefore its visual semidiameter $14'\ 14''$. So if this miracle lay only in the motion of the luminaries, the Moon was made nearer the earth than it naturally should be, or the Sun was made more remote from the earth, upward, so that it could be wholly covered by the Moon.

[Margin: 6. Miracle.]

The sixth was the extent of the Moon's shadow, which occupied the whole hemisphere above the horizon of Palestine, since the Evangelists say that darkness came over the whole earth, or over all the earth. For St Matthew 27 has [Greek: εἰς ὅλην τὴν γῆν (eis holēn tēn gēn, "over the whole earth")], and St Mark [Greek: εἰς πᾶσαν τὴν γῆν (eis pasan tēn gēn, "over all the earth")]; and the testimony of Phlegon supports this, concerning this eclipse observed far from Palestine, to say nothing of Dionysius, its observer at Heliopolis. And there is the reason: if the miracle of the Sun's retreat through 10 lines in the time of Hezekiah extended as far as the Babylonians and beyond, how much more this one, especially since the Saviour's Passion was to be preached to the whole world? How this miracle was done, however, we cannot define with certainty; and the views of Cornelius à Lapide on the withdrawal of the Sun's rays, or of Scheiner on an extraordinary gloom owing to the gathering of several spots of the Sun around its edges, are not improbable. Otherwise the sheer shadow of the Moon, even at its greatest, that is, with the ☉ at apogee and the Moon at perigee, occupies on the surface of the earth only 255 Italian miles equal to the recent Bolognese ones, or 317 ancient Italian miles, or, as Scheiner holds, 350, as we said in chapter 9, no. 9; nor would it occupy the whole hemisphere even if the Moon descended all the way to the Earth, because it is smaller than the Sun and than the Earth.

[Margin: 7. Miracle.]

The seventh was a motion of the Moon contrary to its proper natural motion, by which it apparently tends towards the east. For St Dionysius saw it carried from the east towards the meridian, come under the Sun, and fall upon it from the eastern part of the Sun; granted that afterwards, going back through

the same semicircle, it returned to opposition with the Sun according to the succession of the signs.

[Margin: 8. Miracle.]

The eighth was the wonderful speed of the Moon in completing a semicircle of the zodiac twice within the briefest time, almost in a moment. For St Dionysius says that that eclipse was sudden and abrupt, that it occupied the whole Sun not gradually but *suddenly*, and that after the Sun was restored the Moon *fled away* to diametral opposition with the Sun; whereas otherwise it does not complete this semicircle except in about 14½ days, and the circle in 29½.

[Margin: 9. Miracle.]

The ninth was such an adjustment of the Moon's motion that the order of the celestial motions was not disturbed outside this time, but the Moon returned that evening to the place in the zodiac where it had to be, just as if no change had been made. St Thomas Aquinas noted this in the First Part, question 44, article 2. It is also gathered from the calculation of eclipses which, by astronomical tables, we find to have been observed by the Chaldeans in the years 700 before the birth of Christ, exactly at the time they recorded: for through this miracle the series of motions was not interrupted except for a short time, and that very lapse [intermorium] was corrected by a compensation made by God.

[Margin: 10. Miracle.]

The tenth was the changing of the lunar eclipse into a solar one outside the solar ecliptic limits. For on that day, 33 April 3, the true full Moon occurred at Jerusalem at 7^{h} 10^{m} after noon, with the Sun placed at 11° 25′ of Aries and the Moon at 11° 25′ of ♎; the anomaly of the Sun was 302°, and that of the Moon 38° 18′. The Moon was distant from the nearer node, namely the southern one, by 7° 10′, with a southern latitude of 37′. The corrected semidiameter of the Earth's penumbra was 41′ 36″, and that of the Moon 14′ 42″. The eclipse of the Moon was therefore of 7 digits 52′, by which the northern part of the Moon was obscured.

[Margin: Note the eclipse of the Moon on the evening on which Christ was buried.]

But at that distance from the node and with a southern latitude, an eclipse of the Sun was impossible for northerners on this side of the Tropic, because of the parallax, which increases the latitude and depresses the Moon towards the south; and this even if it had then been a new Moon. There was therefore an eclipse of the Moon on that day towards evening. Salianus teaches this, from unnamed mathematicians, at the year of the world 4086, no. 53, and before him Scaliger, book 6 of *De emendatione temporum*, page 561, saying: *That dreadful darkness of the Sun was therefore out of the ordinary, at the very full Moon itself; at which the astronomical reckoning* [Epilogismus] *shows that something of the Moon also failed around sunset.* This agrees very well with St Jerome too, who understands of an eclipse of the Moon on the day of the Lord's Passion that prophecy of Joel,

chapter 2: *The Sun shall be turned into darkness, and the Moon into blood*, as I have already reported in no. 2. Granted that the Evangelists did not report it, since it happened naturally and was to be slighted in comparison with the supernatural eclipse of the Sun. The blood-red or crimson colour of the failing Moon can already be recognised from what was said in chapter 5, in the scholia.

[Margin: Final cause of the eclipse that occurred at the Passion of Christ.]

VI. It is not my business, however, to digress into the mysteries or tropologies which the holy Fathers or the interpreters of the Gospels have derived from the eclipse of the Sun just mentioned. See, as standing for them all, Salmerón, vol. 10, tractate 42, who deduces eight mysteries from it, and Cornelius à Lapide on Matthew 27, verse 45; from these you will gather the end God intended in this eclipse.

[Margin: On the eclipses at the end of the age.]

VII. There remain the last eclipses of the luminaries, reserved for the last time of the world. On these, those Sibylline verses stand in St Augustine, *City of God* 18, chapter 23:

The Sun's radiance is snatched away, and the chorus of the stars perishes; The heavens will be dissolved, the Moon's splendour will pass away.

Most interpreters, moreover, also understand of these that passage of Isaiah, chapter 13: *For the stars of heaven and their splendour shall not spread forth their light*; and of Ezekiel 32: *And when you are extinguished I will cover the heavens, and I will make their stars grow black; I will cover the Sun with a cloud, and the Moon shall not give her light. I will make all the luminaries of heaven mourn over you, and I will set darkness upon your land.* And of Joel 2: *The Sun shall be turned into darkness and the Moon into blood, before the great and terrible day of the Lord comes.* And of chapter 3: *Peoples, peoples in the valley of slaughter: for the day of the Lord is near in the valley of slaughter; the Sun and Moon are darkened, and the stars have withdrawn their splendour.* But without doubt, whatever Alcazarius [Luis de Alcázar] may say on chapter 6, verse 12, drawing Christ's prophecy to the destruction not of the world but of the city of Jerusalem, it is of these that the passage of Matthew 24 is to be understood: *Immediately after the tribulation of those days the Sun shall be darkened, and the Moon shall not give her light, and the stars shall fall from heaven, and the powers of the heavens shall be moved.* Or as it is said in Mark 13: *But in those days, after that tribulation, the Sun shall be darkened, and the Moon shall not give her splendour, and the stars of heaven shall be falling, and the powers that are in the heavens shall be moved.* And in Luke 21: *And there shall be signs in the Sun and Moon and stars.*

[Margin: "Their light", that is, their accustomed light.]

These are not to be understood only comparatively, with respect to the splendour of Christ, beside which the stars would be seen as dark, as Saints Hilary, Chrysostom and Jerome, and Theophylact, at times held; nor only in a mystical or symbolic sense, of men otherwise illustrious in faith and grace like the stars, but who will fall from faith and grace because of the persecution of Antichrist, as St Augustine held in Letter 80 to Hesychius. They are to be

understood in the literal sense as well, as is learnedly taught above all by José de Acosta, book 4 of *De novissimis temporibus*, chapter 7, and by Cornelius à Lapide on Matthew 24, verse 29, and on that passage of Apocalypse 6, verse 12: *The Sun became black as sackcloth, and the whole Moon became as blood.*

By what means those eclipses will come about is not established, although Cornelius collects much on this matter. Yet it seems certain that they will be supernatural as to their mode; and among the astronomers Tycho, vol. 1, p. 328, and Longomontanus in *Astronomia Danica*, p. 314, teach the same, inveighing severely against those who interpret the prophecies of Christ of the eclipses that happen every year according to the laws of nature and of the celestial motions. On the falling of the stars, let the interpreters consulted there, and Tycho in the place above, be seen; for these matters are nothing to us now, lest everything be put into everything.

CAPVT XIX.

Chapter 19.

Eclipſium Solis ac ☾ ab Anno DCCLXXII. ante CHRISTVM ad Annum vſque CHRISTI MDCXLVII. obſeruatarū Hiſtoria: Indicatis ſubinde Quantitate & Duratione earum ex Tabulis Aſtronomicis; Quæ ſanè ingēs eſt pro ASTRONOMIA, GEOGRAPHIA, & CHRONOLOGIA THESAVRVS.

A history of the eclipses of the Sun and ☾ observed from the year 772 before Christ up to the year of Christ 1647, with their magnitude and duration indicated from time to time from astronomical tables; which is truly an immense treasury for astronomy, geography and chronology.

No one, as far as I know, has so far collected all the observations of eclipses that survive in the writers. Nor indeed do all those who treat of them, taken together, possess all observations of this kind, especially those which, observed in recent years by more recent astronomers, among whom are many professors of mathematics from our Society, have been communicated by letter to me or to Father Athanasius Kircher (to whose kindness and generosity I owe a very great deal).

[Margin: Authors from whom the history of eclipses comes.]

Yet either very many, or at least some, are reported by Ptolemy in books 4, 5 and 6 of the *Magna Constructio*; his own by al-Battānī in chapter 30; by Regiomontanus in the *Epitome of the Almagest*, books 4, 5, 6, and in his own and Peuerbach's observations, to which are added the observations of Bernhard Walther, published anew and revised by Willebrord Snel; his own and some of Ptolemy's by Nicolaus Copernicus in book 4; his own by Tycho in book 1 of the *Progymnasmata*, p. 113, or in the letters; a few of the ancients' and his own by Longomontanus in *Astronomia Danica*, book 1 of the *Theorica*, chapter 2, in the

Commentary on the Moon; some of the older ones and his own by Kepler in *Astronomia Optica*, chapters 7, 8 and 11; Ptolemy's by Nicolaus Mulerius in the *Tabulae Frisicae*, p. 79, and Tycho's lunar ones at p. 296; several, ancient and more recent, by Philips van Lansbergen in the *Treasury of Lunar Observations*, classes 1 and 6, and in the *Uranometria*, book 2, from p. 53 to 115. Very many are reported by Dionysius Petavius [Denis Pétau] of the Society of Jesus, book 8 of *De doctrina temporum*, chapter 13, and by Dom Vincentius Reinerius [Vincenzo Renieri], the Olivetan, pp. 1 and 52 of the *Tabulae Mediceae* in the second edition; some, ancient and recent, by Ismaël Boulliau in *Astronomia Philolaica*, book 3, from chapter 3 to 8, and book 4, chapters 5 and 7, and p. 465. The eclipses of his own age are reported by Govaert Wendelen in his *Eclipses*, or in the *Idea of the Atlantic Tables*, from the year 1573 to the year 1643.

[Margin: Historians who report eclipses.]

Besides these, not a few are given by Joseph Justus Scaliger in the books of *De emendatione temporum*; Johann Georg Herwart von Hohenburg in his *Chronology Established by Eclipses*; Salianus in the *Annals*; Bünting in his *Chronology*; Lycosthenes in the *Chronicle of Prodigies*; Theophanes and Idatius; and others to be named below.

As for the calculation, we have decided largely to omit here the places of the luminaries, over which Longomontanus, Lansbergen and Boulliau took pains in some cases, reserving a fuller calculation according to the various tables for volume 3 of our *Almagest*. We have therefore chosen only to indicate, where we judged it more useful, the magnitude and duration of the eclipse and the anomaly of the luminaries; for it is rather from these that the reliability of the tables is to be examined. Matters that concern the difference of meridians will be treated at length in the Geographical Book, which is the fourth of volume 2 of the *New Almagest*, and those that concern chronology in the Chronological Book, which is the fifth of the same volume.

[Margin: Year 772. ⊙ Total, by calculation.]

The eclipse of the Sun at the conception of Romulus, if not observed, at least asserted by backward calculation by Lucius Taruntius Firmanus, a very celebrated mathematician, whom Marcus Varro urged to this calculation: Dionysius of Halicarnassus, book 2, where he says that at the conception of Romulus *the whole Sun was obscured, and the sky covered in darkness as at night.* I am not reluctant to copy here the whole passage from Plutarch's *Life of Romulus*: *In the age of Varro the Roman philosopher, a man very well versed in histories, there was a certain Taruntius, his friend, a philosopher and mathematician by profession, who had also touched on astronomy through a study of contemplation, and seemed to excel in it. To him Varro proposed that he should gather the birthday and hour of Romulus from the things that had happened to him, in the way the resolution of geometrical questions is found from ends to beginnings. For it belongs to the same science, given the time of some man's birth, to foretell his life, and conversely, his life being set out, to investigate the time at which he was born. Taruntius complied, and having looked into the deeds of Romulus, what had happened to him, how long he had lived, by what*

death he had died, and other things of this kind, and having compared them all with one another, he boldly and fearlessly declared that Romulus was conceived in year 1 of the second Olympiad, on day 23 of the month which the Egyptians call Choeac, at the third hour, when the Sun suffered a total failing; that he was born on day 21 of the month Thoth, around sunrise; and that Rome was founded by him on the 9th day of the month Pharmuti, between the 2nd and 3rd hour.

[Margin: Herwart's opinion, and a chronological knot.]

✢ Nevertheless Georgius Heruartus [Johann Georg Herwart von Hohenburg], chapters 107 and 108 of his *Chronology*, conjectures this eclipse into the year 765 before Christ, on February 10, at $3^{h}\ 29^{m}\ 26^{s}$ before noon at Rome, and says that it was of 10¾ digits. Nor is the disagreement of 7 years to be wondered at, since Censorinus, *De Die Natali*, chapter 8, says of the time from the first Olympiad to his own age: *Concerning the third period, however, there was some dissension among the authors, of only six or seven years. But whatever darkness there was here, Varro dispelled, etc.* But this is not the eclipse Taruntius speaks of, since it did not fall on the 23rd of the month Choeac, that is, June 23 of year 1 of Olympiad 2, the year in which, according to Varro's opinion, we shall show in our Chronological Book that Romulus was conceived. Taruntius, however, did not say that that failing was seen in Italy or in Europe. ✢ For me, therefore, it was a total failing at about 21^{h} after noon at Rome (if Rome had existed then); it fell on the civil day June 24, and was seen beyond the Tropic of Cancer. Now since from Choeac 23 to Thoth 21 there are 291 days, in which are contained 9 lunar years [recte: months] and 25 days besides, the bearing of Romulus lasted ten months; and for that reason, perhaps, Romulus chose a lunar year of ten months. Ovid hints at this in *Fasti* book 1, in this couplet:

As much as suffices for an infant to come forth from its mother's womb, So much time he established to be in his year.

[Margin: Herwart's acute chronological observation.]

From this, Herwart thinks, it came about that through misuse of a twelve-month year in place of a ten-month one, great errors crept into the Consular Fasti, up to the year 404 from the founding of the City, from which the year of 12 months first came into use at Rome. But we treat this elsewhere.

[Margin: Year 754. ☉]

The eclipse of the Sun in year 1 from the founding of the City, in Plutarch's *Romulus*, who says: *It is agreed that the City was founded on the eleventh before the Kalends of May* [April 21]*, and the Romans call this day the birthday of their fatherland and keep it as a feast.* And shortly after: *But it is recorded that the rising of Rome fell on the thirtieth day of the month, and that there was then a conjunction of the Moon and Sun and a failing of the latter, which the poet Antimachus of Teos also observed, and which happened in the third year of the sixth Olympiad.* He speaks of the 30th day in the Greek manner, for he had said beforehand: *In our own time the Roman Kalends differ altogether from the beginnings of the Greek months.* And yet it is strange that Taruntius omitted this eclipse, since he knew that the City was founded in the yoke of the Moon with the Sun. For Cicero says,

On Divination book 2: *Lucius Taruntius Firmanus, our close friend, learned above all in Chaldean reckonings, also traced back the birthday of our city from the Palilia, on which he accepted that it was founded by Romulus; and he said that Rome was born when the Moon was in the Yoke; and he did not hesitate to sing its fates.* With Manilius, Libra can also be understood by the name Yoke, in which sign he says the City's horoscope stood; granted that Solinus writes that Rome was founded with the Sun placed in Taurus and the Moon in Libra. Now Scaliger, *De emendatione temporum* book 5, p. 388, denies that this new Moon was ecliptic, because of the eclipse of the Sun which happened exactly 38 years later, in the following month, the interval of eclipses being preserved. ✢ Pétau, book 8, chapter 13, computes for this year, on July 6, an eclipse of 3 digits at 6^h 51^m after noon at Rome, and in the following year another, but one which could not be seen in Asia by Antimachus; granted that Mulerius accepts it in the Appendix to the *Tabulae Frisicae*. ✢ Herwart, however, in chapter 109, throws this one back to the year 744 before Christ and to December 9, at 3^h before noon, of 7⅕ digits. ✢ But in the year 754, on July 5 according to the Julian year, but April 21 of the old year, there was an eclipse of ☉ of nearly 4 digits, at $4½^h$ after noon at Rome.

[Margin: Year 753. ☉]

✢ In the year 753, however, there was an eclipse on May 26 at 6^h 27^m after noon, but unobservable by Europeans because of the ☾'s southern latitude, greater than 46′. See what we said of the City's birthday in the Chronological Book.

[Margin: Year 721. ☾ Total.]

The total eclipse of the Moon observed by the Chaldeans at Babylon, an hour or more after moonrise at the start of the eclipse, in year 1 of Mardocempadus, but 27 of Nabonassar, on day 29 of the month Thoth; from Ptolemy, *Almagest* book 4, chapter 6, who says that the beginning was there 4^h 30^m before midnight and the middle 2^h 30^m, but at Alexandria 3^h 20^m before midnight. It was March 19 in the Julian manner, feria 2, in the 3rd year of the lunar cycle and the 17th of the solar, as Scaliger has it, *De emendatione temporum* book 5, p. 394. Lansbergen in the *Treasury of Observations* computes 17 digits 52′ and a duration of 3^h 50^m; Pétau, book 8, chapter 13, a duration of 4^h 6^m and 18 digits 30′; but Boulliau, book 3, chapter 7, 19 digits 10′. On this one, see what was said in book 4, chapter 20, no. 9.

[Margin: Year 720. ☾]

The eclipse of the Moon in year 2 of Mardocempadus, or 28 of Nabonassar, observed at Babylon on the night between the 18th and 19th of the month Thoth, at midnight itself, of 3 digits in the southern part of the Moon; from Ptolemy, book 4, chapter 6. Lansbergen in the *Treasury*, p. 55, corrupts his text, putting 3′ [ſcrupula 3] in place of 3 digits, so as to twist the observation to his tables and to 1¼ digits, whereas Ptolemy expressly says [Greek: ἀπὸ νότου δακτύλους τρεῖς αὐτοῦ τοῦ μεσονυκτίου (apo notou daktylous treis autou tou

mesonyktiou, "from the south, three digits, at midnight itself")]. It was the night between March 8 and 9, the following day being feria 6, as Scaliger has it, book 4, p. 323. Pétau, book 8, chapter 13, gathers 3 digits 20′.

[Margin: The unreliable good faith of Lansbergen.]

[Margin: Year 720. ☾]

A second eclipse of the Moon in the same year, observed at Babylon on the night between the 15th and 16th of Phamenoth. It began to fail after its rising, and failed on the northern side by 6 digits at most; from Ptolemy, book 4, chapter 6, who computes a duration of almost 3^h. But Lansbergen in the place above has 2^h 30^m and 5 digits; Pétau 2^h 42^m and 5 digits 20′; but Boulliau 6 digits 17′.

[Margin: Year 715. ☉]

The eclipse of the Sun at the death of Romulus, in year 2 of Olympiad 16, on the 8th of Choeac and the Nones of Quintilis in the old year; from Dionysius of Halicarnassus, book 2, Plutarch in *Romulus*, Seneca book 19 to Lucilius, St Augustine, *City of God* book 3, chapter 15, but above all Cicero in the *Dream of Scipio*, where this stands: *For just as the Sun once appeared to men to fail and be extinguished, when the soul of Romulus entered these very precincts, so when at some time the Sun shall have failed again in the same part and at the same time, then, all the constellations and stars being called back to the same starting point, consider the year completed.* Florus too says, of the departure of Romulus at the Marsh of the Goat: *But a storm arose, and the failing of the Sun gave the appearance of a consecration.* To this Ovid alludes in *Fasti* 2:

There is a place the ancients called the Marsh of the Goat; Perhaps you were giving laws there, Romulus. The Sun flees, and the clouds coming up take away the sky.

But Ovid refers this death to the Quirinalia in the month of February, while Lampridius in *Commodus* and Plutarch refer it to the Nones of Quintilis; since in our manner it was May 26, feria 7, on which, as Scaliger has it, book 5, p. 395, and Pétau, book 8, chapter 13, there fell an eclipse of 3 digits 45′, which lasted $1¾^h$.

[Margin: Year 709. ☉]

The eclipse of the Sun on July 18 indicated by Homer in the *Odyssey*, in Herwart's *Chronology*.

[Margin: Year 621. ☾]

The eclipse of the Moon in year 5 of Nabopolassar, but 27 of

Nabonassar, observed at Babylon on the night between the 27th and 28th of Athyr. It began to fail towards the end of the 11th hour of the night, and around the 6th hour after midnight at most a quarter of the Moon's diameter failed; from Ptolemy, book 5, chapter 14. It was April 22, feria 7. Lansbergen computes 2 digits 16′, but Pétau in the place above 3 digits 45′.

[Margin: Year 607. ☉]

The eclipse of the Sun on July 30, seen at Sardis at 1^h 43^m before noon, of 5 digits 40′, in Pétau, book 8, chapter 13.

[Margin: Year 603. ☉]

The eclipse of the Sun on May 28, of 7 digits 20′, in Pétau, book 8, chapter 13.

[Margin: Year 597. ☉]

The eclipse of the Sun on July 9, of 9 digits 22′, in Pétau, book 8, chapter 13.

[Margin: Year 585. ☉ Total, the eclipse of Thales.]

The eclipse of the Sun that Thales first predicted to the Ionians. It occurred in year 6 of the war between Cyaxares the Mede and Alyattes the Lydian, as Herodotus relates, book 1; Eudemus in Clement of Alexandria, *Stromata* book 1; and Pliny the Elder, book 2, chapter 12, who adds that it was year 4 of Olympiad 48 and year 170 of the City. I reported their words in chapter 2, no. 8. That it was total on the Hellespont, but of 10 digits at Alexandria, is attested by Cleomedes, book 2 of the *Cyclical Theory*, chapter 3, and by Theon in his Commentary on the *Almagest*, book 6, chapter 11, who says of Hipparchus: *He takes an eclipse in the places around the Hellespont, which occurred exactly in the whole Sun, so that nothing of it appeared in the surrounding places.* ✢ Calvisius wrongly thinks this was the eclipse of the year 607, departing from Pliny the Elder by a full 18 years. But Scaliger with Bünting, and Salianus at the year of the world 3469, recognise it in year 4 of Olympiad 48, but in year 168 from the founding of the City; for Pliny the Elder, they say, confused this one with the eclipse of the year 581 before Christ, March 16, of 2½ digits at Sardis.

[Margin: Year 581. ☉]

✢ Lansbergen computes it in year 3 of Olympiad 48, year 163 of Nabonassar, on day 13 of Tybi, that is, May 28, of 12 digits 20′ on the Hellespont, but of 10 digits 12′ at Alexandria. These years, however, are reconciled, because in the same year 585 year 4 of Olympiad 48 began, if not from this ♂, at least shortly after. ✢ Further, for me that eclipse, in that year 585, May 28, comes out around the Hellespont, in the place where it appeared central, as of 12 digits 56′, and at Sardis around the 6th hour after noon, and so in the very heat of the battle which this eclipse broke off, as was said in chapter 2, no. 8. ✢ Pétau attributes to it from his Parisian tables 11 digits 20′; and yet in book 10, chapter 1, he prefers for himself the eclipse of the year 597, of 9 digits 22′, July 9, which occurred in the earliest morning. This suits neither the time of a longer battle nor the magnitude of the eclipse seen in Pontus and Asia Minor, as D. Io. Antonius Rocca [Giovanni Antonio Rocca], a man of high judgement in mathematics, also observed against Pétau in a letter sent to me. Nor are Bünting's 11 digits 30′ from the Prutenic Tables sufficient. ✢ Kepler too accepts this one in the *Optics*, p. 290, and has it occurring in that summer from the middle of which year 4 of Olympiad 48 began. Since the Sun was almost at apogee itself, and the Moon very near perigee, distant from it by only 12°, Lansbergen judges that Hipparchus used this eclipse to demonstrate the distances and sizes of the Sun, Moon and Earth. But Kepler in the *Optics*, p. 292, thinks that it was observed by Timocharis or by Hipparchus, and so is different from the eclipse of Thales, though agreeing with it in magnitude.

[Margin: Year 523. ☾]

The eclipse of the Moon in year 7 of Cambyses, king of the Persians, or 225 of Nabonassar, observed at Babylon on the night between the 17th and 18th of Phamenoth, 1^h before midnight, at which 6 digits failed from the north; from Ptolemy, book 5, chapter 14. It was July 16, feria 4, according to Scaliger, book 4, p. 402. Mercator places it in year 2 of Olympiad 64. But Scaliger puts year 8 of Cambyses in place of 7; and Pétau, book 8, chapter 13, has 7 digits 25′; but Lansbergen, p. 59 of the *Treasury*, 6 digits 2′; and Boulliau in the place above 5 digits 15′.

[Margin: Year 502. ☾]

The eclipse of the Moon in year 20 of Darius son of Hystaspes, who succeeded Cambyses, or 246 of Nabonassar, observed at Babylon on the night between the 28th and 29th of the month Epiphi, 6^h 20^m of equal time after sunset and 24^m before midnight. Three digits failed from the south, or a quarter of the Moon's diameter: Ptolemy, book 4, chapter 9. It was November 19, the following day, the 20th, being feria 2. Lansbergen, p. 62 of the *Treasury*, computes 2 digits 25′, but Pétau 4, and Boulliau 3 digits 3′.

[Margin: Year 491. ☾]

The eclipse of the Moon in year 31 of Darius I son of Hystaspes, or 257 of Nabonassar, at Babylon, on the night following day 3 of the month Tybi, [Greek: ὥρας ς μέσης (hōras 6 mesēs, "at the middle of the 6th hour")], that is, in the middle of the sixth hour of the night. Two digits failed: Ptolemy, book 4, chapter 9. Or, as Trapezuntius [George of Trebizond] renders it, *half an hour before midnight*; for Ptolemy adds: *For the middle time preceded midnight by half an hour at Babylon, but at Alexandria by one hour and a third part of one equal hour.* There Trapezuntius or the printer omitted the one hour, content with a third part of an hour. Mulerius in the *Tabulae Frisicae*, p. 80, better restores 10^h 40^m after noon at Alexandria. Lansbergen, p. 61 of the *Treasury*, put it very badly: *one hour before midnight at Babylon*, and computes 1 digit 33′. ✢ But Pétau has a duration of 1^h 48^m and 2 digits 20′, and Boulliau 2 digits 40′. It was April 25, feria 4.

[Margin: Lansbergen's false report.]

[Margin: Year 480. ☉ Approximately.]

The eclipse of the Sun seen when Xerxes was setting out from Sardis for Abydos. It is much disputed, so we must be the more diligent in clearing up its date. Herodotus relates it thus in book 7: *Xerxes set out from Sardis, where he had wintered, towards Abydos at the beginning of spring. As he moved off, the Sun, leaving its seat in the sky, ceased to appear, although there were no clouds and the air was perfectly clear; and night took the place of day. Noticing this, he was seized by anxiety to ask the Magi what that portent foretold. They answered that the god was signifying to the Greeks the falling away of their cities, because he said that the Sun was the guardian of the Greeks and the Moon of themselves. On hearing this Xerxes went on his way exceedingly glad.* Then, speaking of the preparations of the Greeks against Xerxes, he adds: *The remaining allies too were spirited to action, for the time of the Olympiad had fallen on these affairs.* And not long after:

When the King, having taken Thermopylae, was going further, Arcadian deserters, asked what the Greeks were doing, answered that they were celebrating the Olympia and watching the gymnastic contest. Then in book 8: *From the crossing of the Hellespont, from which the barbarians began their march, they spent one month in crossing into Europe, three others until they reached Attica, while Calliades held the supreme magistracy at Athens, and they took the deserted city.* ✢ Agreeing on this time is Dionysius of Halicarnassus, book 9, near the beginning, saying: *Caeso Fabius, for the second time, was designated consul by the Senate, he who had prosecuted Cassius on a charge of tyranny; by the plebs Spurius Furius, in Olympiad 75, with Calliades chief at Athens, at which time Xerxes made his expedition into Greece.* ✢ And with him Diodorus Siculus, book 11, where he writes in these words: *Calliades therefore being in charge of the Athenians, the consuls created among the Romans were Spurius Cassius and Proclus Virginius, the thirtieth* [trigeſimus]*, in the seventy-fifth Olympiad, in which Asylus of Syracuse won the stadium race; at this time, therefore, King Xerxes took up arms against the Greeks.* ✢ Besides these, Pindar is said to have been born in Olympiad 65 and to have been 40 years old at the crossing of Xerxes, as Scaliger agrees, book 5, p. 407. Therefore the first expedition of Xerxes into Greece fell at the end of year 4 of Olympiad 74, and was completed with Olympiad 75 begun shortly before. On this time agree Scaliger in the place above, Bünting, and Kepler in the *Optics*, p. 291. ✢ Nor does Eratosthenes dissent in the *Canon of Times*, or another old anonymous chronologer in Clement of Alexandria, who count from the first Olympiad to the crossing of Xerxes 297 years, which make 74 whole Olympiads with one year, namely the first of Olympiad 75. For they had regard to the very arrival of Xerxes, and counted those years not as whole but with the last one only begun; granted that Scaliger, understanding them as whole, says that they erred. ✢ But Polybius, book 3, says that the treaty made between the Romans and the Carthaginians under the first consuls preceded Xerxes' departure into Greece by twenty-eight years, that is, by a full 7 Olympiads; and the first consuls were created at Rome in the year 244 from the founding of the City. Adding 28 years, therefore, Xerxes' departure falls in the year of the City 272 completed and 273 begun, the year 481 before Christ according to Varro, but according to Dionysius's computation 480. ✢ Thucydides, however, book 1, says that Xerxes came to Greece in the tenth year after the battle of Marathon; so too Gellius, book 17, chapter 21: *In the two hundred and sixtieth year after the founding of Rome, or not much later, it is recorded that the Persians were conquered by the Athenians, in that celebrated battle of Marathon under the leadership of Miltiades.* And Sulpicius Severus in his *Chronicle* records the same battle in the year 260 from the founding of the City. The City was founded in the year 753 before Christ according to Varro, or 752 according to Dionysius of Halicarnassus; so the battle of Marathon was either in the year 492 or 491 before Christ, and in year 1 of Olympiad 72, or in year 4 of Olympiad 71. Therefore Xerxes' arrival would have been in year 4 of Olympiad 74.

But when the eclipses around these years are examined, no eclipse of the Sun is found in the year 480 before Christ visible in Europe and western Asia.

[Margin: Year 481. ⊙]

But neither is that one to the point which Scaliger affirms in the year 481, on April 19, feria 4, and Bünting too; for either there was none at all, if we believe Pétau, book 8, chapter 13, and book 10, chapter 24, or it was of scarcely 3 digits, as Kepler says in the *Optics*, p. 291, and was therefore unfit to turn day into night.

[Margin: Year 478. ⊙]

As for the one that occurred two years later, that is, in year 3 of Olympiad 75, on the testimony of Herodotus, book 9, and that terrified Cleombrotus, it does not concern this time; it fell on October 2 and was of scarcely 7 digits. Finally, with the sky otherwise clear, a little ball of cloud beneath the Sun could not have made night out of day. And so, concerning this darkness reported by Herodotus, we still remain in a great darkness of uncertainty.

[Margin: Year 463. ⊙]

The eclipse of the Sun in year 1 of Olympiad 79, at Athens, on April 30, in Eusebius, in the *Chronicle*, at the year of Abraham 1556, which Pétau computes as of nearly 11 digits and of a duration of $2^h\ 8^m$. ✢ In its place Scaliger offers another, which occurred in the following year on October 13; but it is rejected by Pétau, because at Athens it fell at $21^h\ 21^m$ after midnight, and was thus nocturnal and invisible there.

[Margin: and Year 462. ⊙ Not observed.]

✢ But the one Eusebius reports under the consuls L. Ebusius and P. Servilius, Renieri wrongly assigns to the year 298 from the founding of the City; or else there is a transposition of figures, for it should have said the year 289 from the founding of the City.

[Margin: Year 431. ⊙ Nearly total.]

The eclipse of the Sun in the first year of the Peloponnesian War, while Pericles was setting sail for the Peloponnese, which terrified his ship's captain but not Pericles, as we said in chapter 2, number 9. Plutarch in the *Pericles* says that *night was drawn over the sky*. ✢ But Thucydides, who was flourishing at that time, says in book 2: *That same summer, on the first day of the month, at the new Moon, the only time at which it is evident that the Sun can fail, the Sun was so obscured after noon that some stars were seen in the sky; and shortly afterwards, when it had taken the form of a halved Moon, it returned to its former shape.* Moreover, the beginning of the Peloponnesian War was in the first year of Olympiad 87, according to Eusebius and Suidas; but the whole first year of the war took up, with the greater part of itself, the second year, as Diodorus Siculus relates, and as Scaliger teaches, book 5, p. 410, of *De emendatione temporum*, Pétau, book 8, chapter 13, Salianus at the year of the world 3623, and Kepler in the *Optics*, p. 291. They also assign this eclipse to August 3, feria 4; but Pétau reckons only 10 digits 25′ and a duration of $1^h\ 54^m$. Kepler asserts that it was total.

[Margin: Year 424. ⊙]

The eclipse of the Sun in year 8 of the Peloponnesian War, observed and reported by Thucydides, book 4, where he says that some part of the Sun failed. ✢ Mästlin, in Kepler's *Optics*, p. 291, computed 7 digits from the Prutenic Tables. ✢ Pétau, however, book 8, chapter 13, places it on March 22 [recte: March 21, per the book's errata] at 8^h 17^m after midnight at Athens, and of 9 digits with a duration of 2^h 34^m. ✢ And Salianus refers it to the year of the world 3630, year 1 of Olympiad 89.

[Margin: Year 413. ☾ Total.]

The eclipse of the Moon in year 19 of the Peloponnesian War, and in year 3 of Olympiad 93, reported by Polybius, book 9, Diodorus Siculus, book 13, Thucydides, book 7, and Plutarch in the *Nicias*. Plutarch says that Nicias, deterred by superstition on this account, was afraid to withdraw, and so offered himself and his army to the Syracusans to be slaughtered, as I related in chapter 2, number 13. It fell on August 27, as Scaliger says, book 1, p. 56, and Pétau, who adds that it was of 13 digits, at Syracuse at 10^h 11^m after noon, and that it lasted 3^h 28^m,

[Margin: A chronological knot.]

but with duration in total darkness of 41^m. ✢ But Apian, in the *Astronomicum Caesareum*, refers this eclipse to the year 414, September 8, when three digits were obscured after sunset; see him.

[Margin: Year 406. ☾ Total.]

The eclipse of the Moon in year 3 of Olympiad 93, in which the old temple of Minerva at Athens burned down, reported by Xenophon, book 2 of *On the Affairs of the Greeks*. ✢ Pétau assigns it to the year of the Julian Period 4308, April 15; he computes 15 digits, a duration of 3^h 45^m, a stay in the shadow of 1^h 14^m, and the middle at 8^h 49^m after noon at Athens.

[Margin: Year 404. ☉]

The eclipse of the Sun in the first year of Olympiad 94 and the last of the Peloponnesian War, in which Dionysius, tyrant of Syracuse, was defeated by the Carthaginians. On this eclipse, Xenophon, book 2 of the *Hellenica*. It fell on September 3, feria 6, in the sixth month after the Peloponnesian War was finished, as Scaliger has it, book 5 of *De emendatione temporum*, p. 411. ✢ Pétau, however, says that it was at 9^h 12^m after midnight, and that 8 digits 40′ failed at Athens.

[Margin: Year 394. ☉]

The eclipse of the Sun in Xenophon, book 4, who says that while Conon was defeating the Persians in a naval battle, the Sun appeared sickle-shaped like the Moon. Salianus refers it to year 2 of Olympiad 96. ✢ Pétau assigns it to the year of the Julian Period 4320 and to August 14, with 11 digits, and at Cnidus in Cyprus at 10^h 35^m after midnight.

[Margin: Year 383. ☾]

The eclipse of the Moon, in the archonship of Phanostratus at Athens, in the month of Posideon, in year 366 of Nabonassar, observed at Babylon on the

night between the 26th and 27th of Thoth, at 5^h 30^m temporal after midnight, from Ptolemy, book 4, chapter 11, who says that a small part of the Moon failed from the summer rising, when half an hour of the night remained, and that it set still in eclipse. ✢ It fell in the morning at the onset of December 23, feria 3, as Scaliger has it, and Mulerius in the *Tabulae Frisicae*, p. 81. Pétau computes 3 digits, Boulliau 2 digits 20′.

[Margin: Year 382. ☾]

The eclipse of the Moon, with Phanostratus still archon at Athens, in the month of Scirrophorion, but according to the Egyptians in year 366 of Nabonassar, on the night between the 24th and 25th of Phamenoth, at a temporal hour of at most 5½ before midnight at Babylon; and it failed from the summer rising. So Ptolemy, from Hipparchus, book 4, chapter 11, where he does not give the digits. ✢ But Pétau computes them, at June 18, as 7 digits 30′, whereas Boulliau has 6 digits 40′.

[Margin: Year 381. ☾ Total.]

The eclipse of the Moon, with Evander archon at Athens, in the first month of Posideon, but according to the Egyptians in year 367 of Nabonassar, on the night between the 16th and 17th of Thoth. It failed, says Hipparchus in Ptolemy, book 4, chapter 11, beginning from the summer rising when 4 hours of the night had elapsed, and it failed totally. And yet shortly afterwards he says that it was 2½ hours before midnight, and so read the codices, both the Basel one and two manuscripts of the Royal Library, as Boulliau affirms, p. 144. But Ptolemy contradicts himself: for after 4 temporal hours of the night, two hours remained until midnight, and therefore not two and a half. ✢ Boulliau therefore restores the letter γ, which signifies 3 hours elapsed, in place of the letter δ, which signifies 4, and holds that Ptolemy was speaking loosely, not strictly. ✢ This eclipse fell on December 12, and was of 17 digits according to Pétau, with a duration of 3 hours and a stay of 1^h 24^m. But Boulliau, book 3, chapter 7, has 19 digits 42′.

[Margin: Year 364 or 365. ☉]

The eclipse of the Sun in Olympiad 104, in the year in which Pelopidas fought against Alexander of Pherae, from Diodorus Siculus, book 15, and Plutarch in the *Pelopidas*, where he describes the omen of this eclipse clearly. I do not repeat that here, because this is the very eclipse of the Sun after which Pelopidas was slain, of which I spoke in chapter 2, number 10, when I was treating of eclipses that portend some omen.

[Margin: Year 343. ☉]

The eclipse of the Sun of which Herwart, chapters 80 and 81 of his *Chronology*, thinks Livy was speaking when he said that on July 1 [Calendis Iulij], in year 109 from the founding of the City, after the temple of Juno Moneta had been dedicated, *night seemed to come on in the daytime*; there he adds many things that bear on chronological knots.

[Margin: Year 331. ☾ Total.]

The eclipse of the Moon in year 2 of Olympiad 112, and 11 before the victory that Alexander the Great won over Darius at Arbela, from Diodorus Siculus, book 17, Q. Curtius, book 4, Arrian, book 3, Plutarch in the *Alexander*, and Pliny the Elder, book 2, chapter 70, where he adds: *At the famous victory of Alexander the Great at Arbela the Moon is recorded to have failed at the second hour of the night, and the same Moon while rising in Sicily.* ✢ It fell on feria 2, on the night between September 20 and 21, as Scaliger has it, book 5 of *De emendatione temporum*, p. 420. There he says that Plutarch is mistaken in saying that it was [Greek: τῇ εἰκάδι Βοηδρομιῶνος (tē eikadi Boēdromiōnos, on the twentieth of Boedromion)], namely on the 20th of Boedromion, and he contends that it was on the 20th day of [Greek: Μεταγειτνιῶνος (Metageitniōnos, of Metageitnion)]; and that Arrian is therefore still more mistaken, who has it in the month Panepsion, that is, in our November, with Aristophanes archon at Athens.

[Margin: Error of Arrian, Plutarch, Cicero.]

He also charges Plutarch with contradicting himself, since Plutarch admits that the battle was fought not indeed at Arbela but at the nearby place Gaugamela, [Greek: ἐνδεκάτη ἀπὸ τῆς ἐκλείψεως νυκτὶ (endekatē apo tēs ekleipseōs nykti, on the eleventh night after the eclipse)], on the eleventh night after the eclipse, and yet says shortly afterwards that it was [Greek: πέμπτη Βοηδρομιῶνος φθίνοντος (pemptē Boēdromiōnos phthinontos, on the fifth of waning Boedromion)], the 25th day of Boedromion; for how, from day 20 to 11, are there 5 days? Moreover, he convicts of the same error Cicero, *De Divinatione* 1, and the author of the little book on the rising and setting of the stars falsely attributed to Ptolemy. For Cicero says: *If the Moon should have failed shortly before sunrise in the sign of Leo, Darius and the Persians would be defeated by force of arms in battle by Alexander and the Macedonians, and Darius would die.* And the other author notes at August 10 [IV. EID. Augusti]: *An eclipse of the Moon usually falls on this day*; for both wrongly thought that this eclipse occurred with the Sun placed in Leo, whereas it was at 24° of Virgo, nor before sunrise, but at the second hour of the night, as the calculation shows and as Pliny the Elder rightly affirms; nor are eclipses tied to the day on which they once occurred. ✢ Pétau, book 8 of *De doctrina temporum*, chapter 13, computes 14 digits 17′, and says that the ☾ began to be obscured at $8^h\ 20^m$ after noon, was wholly obscured at $9^h\ 38^m$, emerged at $10^h\ 40^m$, and wholly escaped at $11^h\ 58^m$. Pliny the Elder's passage must therefore be understood of the beginning of the eclipse. ✢ But Eusebius places this victory and eclipse in the year of the world 4871, which for him is 328 before Christ.

[Margin: A chronological knot.]

✢ Apian, however, in the *Astronomicum Caesareum*, Enunciatum 29, contends that this eclipse was in the year 326 before Christ, on July 11, at $11^h\ 51^m$ after noon at Gaugamela, of 7 digits 15′; read him here on the chronology.

[Margin: Year 310. ☉ Total.]

The eclipse of the Sun in the consulship of C. Junius and Q. Aemilius, when Agathocles, king of Sicily, was taking his fleet across to Africa against Carthage;

he used this eclipse most skilfully to steady his soldiers, as I said in chapter 2, number 11. It was in year 3 of Olympiad 117, from Justin, book 22, and Diodorus Siculus, book 20, who adds: *On the following day so great an obscuration of the Sun took place that you would have called it plainly night, with stars appearing everywhere.* ✝ Salianus, following a series of consuls different from that of Diodorus Siculus, assigns this eclipse to the consulship of M. Rutilius and Q. Fabius Maximus, though he agrees on year 3 of Olympiad 117. He says, I believe from Bünting, that it fell on August 15, at Syracuse at $8^h\ 15^m$ after midnight, and of more than 11 digits; for the Prutenic calculation does not reach a total eclipse, as it ought to; nor does that of Pétau, who has 11 digits 10′ and puts the beginning at $6^h\ 57^m$ and the end at $9^h\ 5^m$. My tables give the diameter of the Sun as 31′ 22″ and of the Moon as 33′ 10″, the corrected anomaly of the Sun as 74°, and that of the Moon as 201°.

[Margin: Year 256. ☾]

The eclipse of the Moon in year 4 of Olympiad 130, before the death of Carneades, from Diogenes Laertius in his life.

[Margin: Year 220. ☉]

[Margin: Year 217. ☉]

Two eclipses of the Sun, commonly confused by writers because the years are close together, and placed in the year from the founding of the City either 533 or 536. Of these Livy, book 22, and Julius Obsequens say, under the consuls Gn. Servilius and C. Flaminius, *that the orb of the Sun was seen to diminish, and to be struggling with the Moon.* Silius Italicus adds that there was dense and immense darkness in Calabria, and that the light of day was taken away. ✝ Georgius Heruartus, in his *Chronology*, chapter 82, draws it back to the year 209, saying that it was in Sardinia at $2^h\ 13^m$ after noon and of 9 digits 50′ 27″. ✝ But Pétau says that this very eclipse began on February 11 at Cagliari in Sardinia after noon, at $2^h\ 3^m$ after midnight, ended at $4^h\ 20^m$, and was of 8 digits.

[Margin: Year 219. ☾ Total.]

The eclipse of the Moon in the year from the founding of the City 535, in the consulship of C. Livius Salinator and Aemilius Paulus, seen in Mysnia, of which Polybius, book 4. It fell on the night between March 19 and 20, of 12 digits 40′; it began there 18^m after midnight, was total at $1^h\ 45^m$, emersion at $2^h\ 25^m$, end at $3^h\ 52^m$, as our Pétau has it. ✝ But Herwart, chapter 83, refers it to the year 211 before Christ, and to the night between October 12 and 13.

[Margin: Year 203. ☉]

The eclipse of the Sun in the consulship of Gn. Cornelius Caepio and C. Servilius Geminus, seen at Frutinum [recte: Frusino, per the book's errata] in Latium, of which Livy, book 30, speaks rather obscurely. Others refer it to the preceding year, when the consuls were Cethegus and Tuditanus; ✝ Salianus to this year, with Bünting, and Pétau; for him it fell on May 6 around noon, of 5 digits and 40′.

[Margin: Year 202. ☉]

The eclipse of the Sun in the consulship of Tiberius Claudius Nero and M. Servilius Geminus, seen at Cumae in Campania, from Livy, book 30, or book 10 of decade 13 [recte: 3], in the year from the founding of the City 552. It fell on October 19, of 1 digit, and at $10^h\ 24^m$ after midnight, from Pétau.

[Margin: Year 201. ☾]

The eclipse of the Moon in year 54 of the Calippic Period and 547 of Nabonassar, observed at Alexandria and reported by Hipparchus in Ptolemy, book 4, chapter 11, where he relates that on the night between the 16th and 17th of Mesori the Moon began to be obscured half an hour before its rising, that it was replenished in the middle of the third hour, and that the middle time was at the beginning of equal hour 2 before midnight. It fell on September 22, feria 6, of 10 digits 17′ according to Pétau, but of 9 digits 8′ according to Boulliau, from the south, although he admits that it was observed as of 7 digits.

[Margin: Year 200. ☾ Total.]

The eclipse of the Moon in year 55 of the second Calippic revolution and 548 of Nabonassar, observed at Alexandria on the night between the 9th and 10th of the month Mechir; it failed totally, and began at $11^h\ 20^m$ after noon. So Ptolemy, from Hipparchus, book 4, chapter 11. ✢ This one, by a slip of the pen I believe, Renieri in his Catalogue refers to the year of Nabonassar 550, and to the day from the 8th into the 9th. Boulliau gives it, from his own calculation, 16 digits 14′. ✢ And it fell in March on the night between the 19th and 20th, feria 3, as Mulerius has it in the *Tabulae Frisicae*, p. 82.

[Margin: Year 200. ☾ Total.]

Another eclipse of the Moon, likewise in year 55 of the second period of Callippus and 548 of Nabonassar, observed at Alexandria on the night between the 5th and 6th of Mesori; the Moon began to be obscured at $6^h\ 40^m$ of the night and failed totally, as Ptolemy relates from Hipparchus, book 4, chapter 11, adding from his own calculation that the middle time was $14^h\ 15^m$ after noon. ✢ For Mulerius, in the place above, it fell on September 12, feria 4; for Pétau it was of nearly 18 digits and a duration of $3^h\ 16^m$, but for Boulliau of 19 digits 45′.

[Margin: Year 190. ☉]

The eclipse of the Sun in the year from the founding of the City 564, in the consulship of L. Cornelius Scipio Asiaticus and C. Laelius Nepos, on Quintilis 11 [V. Idus Quintiles] of the old year, of which Livy speaks, book 7 of decade 4: *With the sky clear the light was darkened by day, when the Moon had passed beneath the orb of the Sun.*

[Margin: Renieri's slip.]

✢ It fell not on August 9, as Renieri has it, p. 54, but on the day before the Ides, that is, March 14 of the Julian year, as Scaliger has it, book 5 of *De emendatione temporum*, p. 387, and Pétau, book 8, chapter 13, who adds 11 digits 36′, the beginning at $5^h\ 31^m$ after midnight, and the end at $7^h\ 20^m$. ✢ But Herwart, chapter 84, abusing the years, refers it to October, with the Sun at 22° ♎, at $1^h\ 16^m$ before noon, of 8 digits 4′.

[Margin: Year 188. ☉]

The eclipse of the Sun in the year from the founding of the City 565, in the consulship of C. Livius Salinator and M. Valerius Messala. The Sun failed at Rome between the third and fourth hour of the day: from Livy, book 8 of decade 3. ✢ For Pétau, book 8, chapter 13, it fell on July 17, of 11 digits 12′; it began at 5^h 5^m and ended at 7^h 7^m after midnight. ✢ Herwart, chapter 85, while confessing that it occurred in the year from the founding of the City 565, nevertheless says that it was in the year 180 before Christ, departing from the truth by a whole 8 years for the sake of his chronological mysteries, and says that it was at 2^h 12^m before noon, of 7 digits 48′.

[Margin: Year 174. ☾]

The eclipse of the Moon in year 7 of Philometor and 574 of Nabonassar, observed at Alexandria on the night between the 27th and 28th of Phamenoth at 14^h 20^m after noon, of nearly 7 digits, from the north, as Ptolemy reports, book 6, chapter 5, where he adds that it began to fail at 8^h and was replenished at 10^h. ✢ It fell on May 1 [Calendis Maij], feria 7, as Mulerius has it in the *Tabulae Frisicae*, p. 82; for Pétau of 7 digits 20′, but for Boulliau of 7 digits 13′.

[Margin: Year 168. ☾ Total.]

The eclipse of the Moon that Sulpicius Gallus was the first among the Romans to predict, as I related in chapter 1, number 6, in the year from the founding of the City 586, in the consulship of L. Aemilius Paulus and C. Licinius Crassus, *on the day before King Perseus was defeated by Paulus*, as Pliny the Elder says, book 2, chapter 12, and Plutarch in the *Aemilius Paulus*. It failed in Macedonia from the 2nd to the 4th hour on the night preceding September 4, as Livy has it, book 4 of decade 5; but the Pompilian year was dislocated by faulty intercalation, as Scaliger notes with several examples, book 2 of *De emendatione temporum*, pp. 186 and 584; ✢ he adds that it was on June 21 of the Julian year, feria 3, with feria 4 following. But he wrongly gives the Varronian year of the City as 584, although he admits that it was year 4546 of the Julian Period, namely 168 before Christ, in which year the Varronian year of the City 586 began and 585 ended. ✢ Pétau, book 8, chapter 13, computes nearly 16 digits, and says that the eclipse began in Macedonia after noon on June 21 at 5^h 59^m, total obscuration at 7^h 42^m, emersion at 8^h 22^m, end at 10^h 5^m. ✢ Herwart, chapter 86, following the *Fasti Capitolini*, places it in the year of the City 585, but in his usual way in the year 160 before Christ.

[Margin: Apian's slip.]

✢ But Apian, in the *Astronomicum Caesareum*, Enunciatum 30, clinging to the day, but September 2, and forgetting the dislocated year, contends that this eclipse was in the year 172 before Christ at 12^h 41^m.

[Margin: Year 159. ☉]

Herwart, chapter 127 of his *Chronology*, thinks that that phenomenon was an eclipse of the Sun of which Julius Obsequens says: *At Capua the Sun was seen at night; at night an appearance of the Sun shone forth at Pisaurum*; because in

fact on January 1 the Sun, failing by 11 digits at its setting, was seen by the aid of refraction.

[Margin: Year 141. ☾]

The eclipse of the Moon in year 37 of the third period of Callippus and 607 of Nabonassar, observed on the island of Rhodes on the night between the 2nd and 3rd of Tybi; it began at the start of the fifth hour of the night, and at most three digits failed, from the south. Ptolemy, book 6, chapter 5. ✢ It fell on January 27, feria 3, according to Mulerius; but Lansbergen gathers 2 digits 56′, while Pétau has 3 digits 50′ and a duration of 2^{h} 8^{m}.

[Margin: Year 127. ☉]

Herwart, chapter 128, posits an eclipse of the Sun on account of those words of Julius Obsequens, *The Sun seen at night*: because at the beginning of February a morning eclipse of the Sun of 9 digits 57′ was taken for a night of the Sun already risen.

[Margin: Year 113. ☉ False.]

The eclipse of the Sun indicated by Pliny the Elder, book 2, chapter 33, where he says: *A light from the sky was seen by night in the consulship of C. Caecilius and Cn. Papirius and often at other times, so that an appearance of day shone by night.* But, as Milich rightly explains there, what is meant is a chasm [Chaſma] and a meteorological burning, and not an eclipse of the Sun. ✢ Granted that Herwart, chapter 86 of his *Chronology*, drags Pliny the Elder to an eclipse so as to drag those consuls into the year 105 before Christ,

[Margin: Year 105. ☉ From calculation.]

in which year he finds an eclipse of the Sun at 12° of Aquarius, at 4^{h} after noon at Rome, of 11 digits 2′; and from this, by a remarkable chronological paralogism, he concludes that the common era, which assigns those consuls to the year 113, is in error by 8 years.

[Margin: Year 104. ☉]

The eclipse of the Sun in Varronian year 650 from the founding of the City, but according to the *Fasti Capitolini* 649, in the consulship of C. Marius, for the second time, and C. Flavius Fimbria. Of it Julius Obsequens writes as follows in his little book on prodigies: *The Moon appeared by day with a star, from the 3rd hour until the 7th. The regions among the Thurini were laid waste by fugitives and deserters. The Cimbri, having crossed the Alps, joined themselves to the Teutons through ravaged Spain. At the third hour of the day an eclipse of the Sun darkened the light. In Picenum three Suns were seen.* ✢ Herwart, however, obtained this eclipse by astronomical calculation in the year 104 before the common era of Christ, with the Sun at 22° of Cancer, at 4^{h} 4^{m} before noon at Rome, which was the 3rd temporal hour, of 11 digits 20′. Since he sees that the common fasti are confirmed by this eclipse, it is remarkable by how many tunnels and circuits he tries to undermine it through the following chapters. ✢ But Kepler too, from a corrupt codex, in *Astronomia Optica*, p. 293, and from him Renieri, p. 55, place it in the consulship of C. Marius and C. Flaccus, who were consuls in the year 100 before Christ.

[Margin: Year 102. ⊙ Doubtful, or occurred in the year 94.]

The eclipse of the Sun in the consulship of C. Marius and Q. Lutatius, and so according to the common fasti in the year 102 before Christ, foisted by Herwart, chapter 86 of his *Chronology*, on Julius Obsequens on account of those words: *In Gaul in the camp a light shone by night*, although this can be understood of a nocturnal chasm. But Herwart does this, in his usual way, so as to contend that our era runs over by eight years. For his argument, reduced to dialectical form, stands thus: Under the consuls already mentioned there was an eclipse of the Sun; but this was not in the year 102, to which the common fasti assign them, but in the year 94, at 2° of Cancer, of 8 digits 58′, at 4^h 20^m after noon, around the Fossae Marianae; therefore those consuls are to be recorded at the year 94, not 102.

[Margin: Year 49, perhaps. ⊙ Total.]

The eclipse of the Sun in the year from the founding of the City 704, in the consulship of L. Aemilius Paulus and Claudius Marcellus: the Sun failed wholly, namely at the time when C. Julius Caesar crossed the Rubicon. For Dio Cassius says in book 41: [Greek: ὅ, τε ἥλιος σύμπαν ἐξέλιπε (ho te hēlios sympan exelipe, and the Sun failed wholly)]. And it seems to be indicated by Lucan, who writes thus of that year in which Pompey fled:

When the Titan himself bore his head in mid Olympus, He hid his burning chariot in black darkness, And wrapped the day in shadows, and compelled the nations To despair of day: such a night as Thyestean Mycenae Drew on when the Sun fled through the east.

It is doubted, however, whether this is that eclipse of the Sun which Herwart, chapter 130, and Pétau, book 8, chapter 13, find in the year 51 before Christ, at 14° 21′ ♓, at Rome at 2^h 12^m after noon on March 7, since they say that it was of only 9 digits 2′ or 10 digits 34′. For that reason Herwart dares to say that this is the very one which in the year 38 before Christ preceded the civil war of Octavius Augustus and M. Antony, and that the prodigies of those civil wars were confused by writers, because in that year 38 there was a total eclipse, as we shall soon say. But it is certain for us that Caesar crossed the Rubicon in the year 49 before Christ, namely the year preceding his second consulship.

[Margin: Year 44 or 43. ⊙]

An eclipse of the Sun after the murder of C. Julius Caesar seems to be indicated both by Aurelius Victor, who says: *When the body of the murdered C. Julius Caesar had been placed before the rostra, the Sun is said to have hidden its orb*; and by Dio Cassius in these words: *The light of the Sun too was seen to diminish and be extinguished, and at times to shine out in three circles.* But Appian says nothing else to the point about the same year, except that many grim prodigies occurred around the Sun. Virgil sang of it more splendidly in book 4 of the *Georgics*, in verses already cited in chapter 1, number 4, where you also have the verses of Petronius Arbiter and the passage of Pliny the Elder. I now add, from Josephus the Jew, book 14 of the *Antiquities*, chapter 21, the words of M. Antony from a letter written to the high priest Hyrcanus: *We take*

vengeance on those who have committed injustices against men and wickedness against the gods. Indeed, on account of those crimes of theirs, we believe that the Sun too, abhorring them, turned its face away from them, as one that had itself looked on the crime committed against Caesar not willingly but with horror and disgust.

[Margin: Annular?]

Yet Georgius Heruartus, in chapter 145 of his *Chronology*, judges that this eclipse was the same as that appearance [Epiphaſi] of the Sun which occurred when C. Octavius Augustus was entering the City after Caesar's murder, in the consulship of Hirtius and Pansa; namely an annular eclipse, as we usually call it, with a bright small circle of the Sun left around the Moon. And he tries to drag here those words of Suetonius in the *Augustus*, chapter 95: *After Caesar's murder, when Augustus had returned from Apollonia and was entering the City, suddenly, in a clear and pure sky, a circle in the likeness of a rainbow surrounded the orb of the Sun.* And also those words of Julius Obsequens in his little book on prodigies: *When C. Octavius was entering Rome at the third hour of the day, with a huge crowd pouring round him, the Sun, enclosed in a moderate orb of a pure and clear sky, surrounded him with a circle of an outermost line, such as a bow usually stretches in the clouds*; and soon after: *Three Suns shone, and around the lowest Sun a crown like a spike of grain flashed out in a ring, and afterwards, the Sun being reduced to one circle, the light was feeble for many months.* This is confirmed by Orosius, book 6, chapter 20. But Seneca, book 1 of the *Natural Questions*, chapter 2, openly refers this to a meteoric appearance [Phaſma], saying: *It is recorded that on the day when the deified Augustus entered the City on returning from Apollonia, a circle of varied colour was seen around the Sun, such as there usually is in a rainbow; this the Greeks call* [Greek: ἅλως (halōs, halo)]*, and which we usually call a crown.* At other times, however, these crowns come from an eclipse. Thus Philostratus in the *Life of Apollonius*: *In the sky such a prodigy appeared: a certain crown like a rainbow surrounded the orb of the Sun, and obscured the rays and light of the Sun.* And after he has added that the prefect of Greece feared that all things would be turned to night, he adds that Apollonius said to him: *Take heart: from this night a light will rise, that is, the slaughter of the tyrant*, namely of Domitian. Soon after, recounting the killing of Domitian carried out by his chamberlain Stephanus, he says that this very thing was foreshadowed by the form of that prodigy, namely by that crown which obscured the Sun, since a crown in Greek is [Greek: στέφανος (stephanos, crown)]. The words of Pliny the Elder, book 2, chapter 28, also favour this: *Stars too are seen with the Sun, generally for whole days, and around the orb of the Sun crowns like spikes of grain and circles of many colours, as when Augustus Caesar, in his first youth, was entering the City after his father's death to take up his great name.* For if stars were seen, then the Sun seems to have been obscured in a great part of itself; and those words of Julius Obsequens (*the Sun enclosed in a moderate orb*) seem to indicate an annular eclipse.

But since it is certain for us, from the series of consuls and of Julian years, that C. Julius Caesar was killed in the year 44 before the era of Christ, on March 15, and that Hirtius and Pansa were consuls in the year 43, in which Augustus

also entered the City, then if that display [Emphaſis] of a bright crown and rainbow accompanied an eclipse, it cannot be the same as the eclipse of which Aurelius Victor spoke above, who affirms that it was seen when Caesar's body had been placed before the rostra. The new Moons of the year 44 after March 15 must therefore be examined. Since Herwart found no eclipse at them, he assigns this Caesarian eclipse, which he thinks was the same as the annular one of Octavian, to the year 38 before Christ,

[Margin: Year 38. ☉ From calculation, not from observation.]

in which on January 14 the Sun, at 22° 8′ of Aquarius, at Rome 3^h 6^m before noon, failed by 11 digits according to the Prutenic Tables, as Herwart has it; but I believe there is an error in the text, for how on January 14 could the Sun have been at 22° of Aquarius? Perhaps the day should be read as 24 and the degree as 2. He adds, however, from two different calculations, 11 digits 13′ or 11 digits 59′. From this he infers that Hirtius and Pansa were consuls in that year 38, and that there is an error of 5 years in the published fasti. But we have already refuted the violence of this chronologism in our Chronological Book.

[Margin: Year 4. ☾]

The eclipse of the Moon: in Julian year 42, on the night between March 12 and 13, which by the calculation of Pétau, book 8 of *De doctrina temporum*, chapter 13, and in the *Rationarium temporum*, book 4, chapter 22, was of 6 digits, and at Jerusalem began at 1^h 17^m after midnight and ended at 4^h 13^m. ✝ But by the calculation of Kepler in the Rudolphine Tables, p. 103, it was of 5 digits 34′, and began in Judaea at 3^h 20^m and ended at 5^h 15^m after midnight. They judge, moreover, that this is the eclipse which Josephus the historian, book 17 of the *Jewish Antiquities*, chapter 8, and book 2 of the *Jewish War*, chapter 21, relates was seen before the death of Herod the child-killer. That Herod, it is certain from the Gospel, was still alive when Christ the Lord was born. Kepler therefore contends, in his *Silva Chronologica*, that Christ was born in Julian year 41 or 40, and thus that to our common era, which places his birth in the year 45, at least 4 and perhaps 5 years are lacking, so that the year 1606, in which Kepler was writing that *Silva*, was in truth the year of Christ 1610 or 1611. But I have treated this matter at length throughout the whole of book 3 of my *Chronology*. Meanwhile I point out that there were other eclipses of the Moon before the Saviour's birth, which could have preceded Herod's death, if the history of Josephus, Dio, Tacitus and others did not, on other grounds, compel us to choose this eclipse.

[Margin: Year 1. ☾ ☾ Twin eclipse.]

For as Scaliger notes in *De emendatione temporum*, pp. 16 and 463, and as Salianus with many others notes at the year of the world 4053, number 23, an eclipse of the ☾ occurred in Julian year 45, on the night following January 8; from which Salianus concludes, and Scaliger does not deny it, that the Saviour was born in Julian year 44. But in that same year too, on December 29, there was an eclipse of the ☾ visible in Judaea, and it was total with stay.

SEQVVNTVR ECLIPSES POST NATVM CHRISTVM

Annis Epochæ vulgaris Christianorum consignatæ.

The eclipses after the birth of Christ now follow, marked by the years of the common era of the Christians.

[Margin: Years of Christ.]

[Margin: Year 5. ☉]

The eclipse of the Sun under the consuls L. Valerius Messala and Cn. Cornelius Cinna, concerning which Dio, book 55, says: *Some part of the Sun failed.* ✢ Pétau, book 8, chapter 13, places it on March 28, of 4 digits 45′, and has it begin at Rome at 3^h 21^m after noon and end at 5^h 2^m. ✢ But Herwart, chapter 131 of his *Chronology*, places its middle at 4^h 51^m, in ♓ 29° 57′, and gives it 5 digits 10′.

[Margin: Year 14. ☉ Total, or Year 17.]

The eclipse of the Sun shortly before the death of Augustus, of which Dio Cassius says: *For the Sun too had failed entirely.* And the Chronicle of St Jerome agrees, saying: *An eclipse of the Sun occurred, and Augustus dies at the age of 76.* Nicephorus, book 1, chapter 17, adds that a severe famine also followed. ✢ But because astronomical calculation shows no eclipse of the Sun in this year, Herwart, chapter 147 of his *Chronology*, supposes it to have been the one that Paulus, bishop of Fossombrone, reports as follows: *In the year of the Lord and Saviour 17 by the customary reckoning, the whole Sun was seen to be obscured at Rome and in very many places in Italy on February 15, at about the noon hour, by 12 points, in 16° of Aquarius, with the Sun holding the royal house; therefore, according to the opinions of the Chaldeans, it portended the death of the supreme King.* And because Herwart thinks that Augustus died in the year of Christ 19 of the common era, he therefore says that this eclipse occurred in the third year before Augustus's death.

[Margin: Year 14. ☾ Total.]

The eclipse of the Moon under the two Sextuses as consuls, Pompeius and Apuleius, in the year of the City 767 and Julian year 57, after the death of Augustus, who died on the 19th of the month of August. By the portent of this eclipse Drusus quelled the mutiny of the Pannonian legions, as we said in chapter 2, from Tacitus, *Annals* book 1, with whom Dio, book 56, agrees. ✢ On the authority of Scaliger, book 5, p. 465, this failing fell on September 27, feria 5. ✢ For Pétau, however, book 8, chapter 13, it was of 19 digits 30′, and began in Pannonia at 3^h 19^m after midnight; it was total at 4^h 22^m; the Moon began to emerge at 6^h 7^m and was wholly refilled at 7^h 7^m, the Sun having already risen. ✢ But because this eclipse condemns Herwart's chronology, and because Tacitus mentions clouds, on whose rising the mass of the soldiers believed that the Moon had again been hidden in darkness, the good Herwart affirms, in chapter 257 of his *Chronology*, that it was not a true eclipse but came

from clouds, and that from the death of Tiberius[44] to September 27 there was not enough time for the son of Blaesus to travel from Pannonia to Rome, and for Drusus to travel from there to Pannonia with the praetorian cohorts. But Herwart labours in vain. For unless it had been certain to the soldiers that the Moon, the sky around it having been clear beforehand, had undergone a true eclipse, Drusus would not have exposed himself to greater danger, as though he wished to frighten them with an empty portent and to turn into a prodigy what clouds very often do; yet once they had been terrified, it was easy for them to think the eclipse had been repeated. Nor would Tacitus have added the words: *The soldier, ignorant of its cause*; since even boys know that the Moon is often hidden by clouds. Nor would Dio Cassius have said outright: *Dismayed by the Moon's failing, they were calmed.* Herwart does admit that in this year 14 of the common era, on September 27, there was an eclipse of the Moon; but he denies that it fell in the consulship of Sextus Pompeius and Sextus Apuleius. ☩ And Renieri errs in his *Catalogue of Eclipses* when he says this one took place at the beginning of the night.

[Margin: Year 33. ☉ ☩]

The eclipse of the Sun at the death of Christ, which we discussed sufficiently in chapter 18.

[Margin: Year 45. ☉]

The eclipse of the Sun under the consuls M. Vinicius II and M. Statilius Corvinus, in the year of the founding of the City 798, of which Dio, book 60, says of the Emperor Claudius: *Since an eclipse of the Sun was going to occur on his birthday, he feared that some disturbance might arise from it, as certain other prodigies too had occurred; he therefore published not only that this failing would occur, and its time and magnitude, but also the causes by which it must necessarily come about.* Now Claudius died in the year of the City 800, on October 13 [3. Idus Octobris], and had been born on August 1 [Kalendis Augusti], as Suetonius attests, chapter 2. ☩ For that day, at this year 45, Pétau, *De doctrina temporum* book 8, chapter 13, computes an eclipse of 3 digits, with its beginning at $7^{h}\ 43^{m}$ and its end at $9^{h}\ 1^{m}$ after midnight. ☩ Scaliger, *De emendatione temporum* book 5, p. 466, says it was feria 1.

[Margin: Year 59. ☉]

The eclipse of the Sun under the consuls M. Vipsanius Apronianus and L. Fonteius, in Tacitus, *Annals* book 14, who counts it among the prodigies occasioned by Nero's killing of his mother Agrippina. Pliny the Elder, book 2, chapter 70, says this of it: *The eclipse of the Sun that took place under the consuls Vipsanius and Fonteius, who held office a few years ago, on April 30* [pridie calendas Maias]*, Campania noticed between the seventh and eighth hours of the day; the general Corbulo, in Armenia, between the tenth and eleventh hours of the day.* This passage serves to correct Martianus Capella, book 6, who gives April 21 [11. Calendas Maij], and hour 7 in Campania and hour 11 in Armenia.

[44] the page prints `Tiberij`; the death named earlier in this entry is that of Augustus.

[Margin: Year 69. ☾]

An eclipse of the Moon indicated by Dio, book 65, on October 18, which it hardly matters to examine.

[Margin: Year 72. ☉ and ☾.]

An eclipse of the Sun and, after twelve full days, a horizontal eclipse of the Moon are indicated by Pliny the Elder, book 2, chapter 13, in these words: *By what reason, since at the Sun's rising that dulling shadow ought to be below the Earth, it has once already happened that the Moon failed at its setting, with both bodies visible above the Earth; for that both bodies should be looked for within twelve days happened also in our own age, under the Emperors Vespasian.* We discussed this passage sufficiently, as regards the force of refraction, in chapter 5, number 16. But whether these are the eclipses, the solar one on the 8th and the lunar one on the 22nd of February, which the astronomical tables gather, the former shortly after sunrise and the latter shortly after sunset, I shall examine elsewhere.

[Margin: Year 95. ☉ Annular.]

The annular eclipse of the Sun recorded by Philostratus in the *Life of Apollonius*, as foretelling the murder of Domitian. For he says: *In the sky a prodigy of this kind appeared. A kind of crown resembling a rainbow surrounded the orb of the Sun, and obscured the Sun's rays and light.* Hence, when the Prefect of Greece feared [Greek: μὴ ἐς νύκτα μεταστῆ πάντα (mē es nykta metastē panta, "lest everything turn to night")], that all things might be turned into night, Apollonius replied: *Take heart: from this night light will arise*, signifying the slaughter of Domitian soon to be carried out by Stephanus his chamberlain; whose name, says Philostratus, the figure of that prodigy signified. Yet Ptolemy's series of the Emperors places this killing in the year 96, on September 18.

[Margin: Year 97 or 98. ☉ Perhaps total.]

The eclipse of the Sun on January 27, shortly after noon, the day on which the Emperor Nerva died, in the year of Christ 97 or 98; on this eclipse, Aurelius Victor and Eutropius. And perhaps it is the one that Plutarch, in his little work on the spots in the face of the Moon, designates as total in the words with which he reminds his interlocutors of that synod of the Sun with the Moon which had recently occurred, *in which, as it began right around noon, stars shone out in many places throughout the sky, and the temper of the air was such as it is in the uncertain light of twilight.* Then, in giving the reasons why *the darkness is not so deep in eclipses, nor the air taken over by them just as it is at night*, he does not resort to a part of the Sun exempt from the eclipse but to other things; he therefore shows that it was total, as Kepler rightly argues in the *Astronomia Optica*, p. 293.

[Margin: Year 113. ☉ Total.]

✝ Yet Kepler, in the Rudolphine Tables, precept 150, or p. 104, says that having examined many eclipses around the year of Christ 100, none presents itself as more likely, or as agreeing better with Plutarch's words, than the one that took place in the year 113 on June 1; but he does not add there the place where it was total.

[Margin: Year 125. ☾]

The eclipse of the Moon observed at Alexandria in Egypt by Ptolemy, in year 8 [recte: 9, per the book's errata] of Hadrian and year 872 of Nabonassar, the 17th of the month of Pachon having passed and the 18th following, 3^h 36^m before midnight, when a sixth part of the Moon's diameter failed from the south: Ptolemy, book 4, chapter 9. ✢ For Pétau this was on April 5, of 2 digits 18′, beginning at 7^h 31^m and ending at 9^h 20^m after noon. ✢ Boulliau gives year 8 of Hadrian and 2 digits 12′. Lansbergen, p. 64 of the *Treasury*, gives year 9 of Hadrian; 1 digit 48′. ✢ Renieri, p. 55, gives year 8 of Hadrian and the month Payni, and adds: *Lansbergen, from a corrupt text, states that it failed at hour 8 51′.* ✢ But in truth Lansbergen does not corrupt the text: for he admits that it has hour 8 24′ [Horam 8. 24′], but says that it should have been 8^h 51^m, given both his own calculation and the completed circles of latitude, as Ptolemy says, from the eclipse of year 31 of Darius son of Hystaspes to this one.

[Margin: Lansbergen vindicated from Renieri's accusation.]

The Greek codex of Basel has [Greek: τῷ ῆ ἔτει Ἀδριανοῦ (tō ē etei Hadrianou, "in the 8th year of Hadrian")], that is, in year 8 of Hadrian, as Herwart too points out, chapter 259 of his *Chronology*, although the Latin and the Arabic have year 9.

[Margin: Year 133. ☾ Total.]

[Margin: Renieri's error,]

The eclipse of the Moon observed with the greatest exactness by Ptolemy at Alexandria, in year 17 of Hadrian and year 880 of Nabonassar, the 20th of the month of Payni having passed, at 11^h 15^m after noon at the middle, and it failed entirely: Ptolemy, book 4, chapter 6. ✢ Renieri wrongly gives the 17th of Payni. Lansbergen computes 12 digits 26′, Pétau nearly 16 digits, Boulliau 13 digits. ✢ According to Mulerius in the *Tabulae Frisicae*, p. 80, it was on the night after May 6, feria 3. Copernicus uses this eclipse, book 4, chapter 5.

[Margin: Year 134. ☾]

The eclipse of the Moon observed at Alexandria by Ptolemy in year 19 of Hadrian and year 882 of Nabonassar, the 2nd of Chocac or Choeac having passed, at the 11th hour from noon; 10 digits failed toward the north: Ptolemy, book 4, chapter 6. ✢ For Mulerius, in the place above, it was on October 20, feria 3. Lansbergen inferred 10 digits 31′, Pétau 10 digits 20′, Boulliau 11 digits 6′, while setting the observed value at 9. Copernicus uses this eclipse, book 4, chapter 5.

[Margin: Year 136. ☾]

The eclipse of the Moon observed at Alexandria by Ptolemy, in year 20 of Hadrian and year 883 of Nabonassar, the 19th of Pharmuthi having passed, at the 16th hour after noon; and half of the Moon was darkened from the north: Ptolemy, book 4, chapter 6. ✢ It was on the 5th of March, with the 6th following. Lansbergen computes, p. 67 of the *Treasury*, 5 digits 47′, but Pétau 5 digits 24′, Boulliau 6 digits 27′. ✢ Further, in Copernicus, who uses this eclipse in book 4, chapter 5, the year of Christ is 135 [CXXXV], March 6 [Martij VI.]

having passed; but 136 [CXXXVI] should be read, because the years of Hadrian are to be reckoned from August 11 of the year 115, on which he entered on his rule. Year 20 therefore began on the same day in the year 135, in which on July 20 the year 883 of Nabonassar had already begun; and so the 19th of Pharmuthi falls in the leap year 136 of Christ and on March 5, and the 20th on March 6. ✝ Mulerius therefore rightly, on p. 80 of the *Tabulae Frisicae*, gave the day as March 6, feria 2, but with 4^h from midnight of the 20th of Pharmuthi, and 16^h after noon of the 19th of Pharmuthi, that is, March 5; ✝ and Pétau gives March 6, but with the hours counted from the preceding midnight.

[Margin: Year 192. ☉ Total.]

The eclipse of the Sun indicated by Herodian in the life of the Emperor Commodus, when he says that before Commodus's death stars were seen in the sky by day; Renieri, p. 56, takes it as total. Commodus was killed, moreover, on December 31 by conspirators.

[Margin: Year 237. ☉ Total.]

The eclipse of the Sun in the year in which Gordian the younger began to rule, of which Julius Capitolinus speaks in these words: *So great was the eclipse of the Sun that it was believed to be night, and nothing could be done without lamps lit.* ✝ Kepler, in the *Astronomia Optica*, p. 293, affirms that it was necessarily total, although he admits that on April 12 the Prutenic calculation shows less than 11 digits. ✝ Lansbergen, in the *Treasury*, p. 124, infers it at Rome at 5^h 2^m after noon and of 11 digits 27′, and adds that it was seen as total, because a particle of the Sun exempt from the eclipse was near the horizon.

[Margin: Lansbergen's bold error against Censorinus.]

But he errs badly when he says that in this year the consuls were Ulpius and Crinitus, and that Censorinus erred, who puts them in Julian year 283, that is, the year of Christ 238, because Lansbergen thinks this eclipse an index of that consulship. But where does Lansbergen get this fancy, since Censorinus says nothing of this eclipse? Or is it because Capitolinus said: *This was a sign that Gordian would not rule for long, that so great an eclipse of the Sun, that it was believed to be night, etc.*, that this was an index of that most famous year in which Censorinus fixed the roots of chronology, so splendid and so clear to himself that all good chronologers draw many axioms of chronology from them? ✝ Herwart, surely more prudently, chapter 256, places this eclipse in this year 237, but under the consuls P. Titius Perpetuus and L. Ovinius Rusticus Cornelianus, at 4^h 56^m after noon at Rome, and of 10 digits 46′. ✝ Pétau, book 8, chapter 13, and Scaliger, p. 492, agree with Censorinus, although Pétau infers 10 digits from his Parisian tables. My tables give then a semidiameter of the Sun of 15′ 32″ and of the Moon of 16′ 30″; for the anomaly of the ☉ was 316° and that of the ☾ 156°.

[Margin: Year 238. ☉]

The eclipse of the Sun in Julian year 238, under the consuls Ulpius Crinitus and Proculus Pontianus, beginning on the morning of April 2, by Pétau's calculation, at Rome at 6^h 29^m after midnight, of 8 digits 15′. Although this

could be taken for the one Capitolinus spoke of above, and could have seemed larger because it was in the morning near the vapours of the horizon, the one of the previous year is nevertheless more probable, as Pétau and Kepler hold. ✢ But Mulerius, in the *Tabulae Frisicae*, p. 236, says that it was this one of the year 238, and adds an Alphonsine calculation, because he does not find it from the Prutenic Tables.

[Margin: Year 291. ☉]

The eclipse of the Sun in year 7 of Diocletian, under the consuls Tiberianus and Dio, the year in which the Caesars Maximinus Armentarius and Constantius Chlorus were created, was seen at Carthage, as Idatius reports in the *Fasti*: namely, by Pétau's calculation, on May 15, of 11 digits 20′; it began there at 2^h 2^m after noon and ended at 4^h 32^m.

[Margin: Year 301. ☾]

The eclipse of the Moon mentioned in the acts of St Felix, martyr and African bishop, and of his companions, in Baronius, but at the year of the Lord 302. For in those acts it is said: *And he was led to the place of his passion, when the Moon itself too was turned to blood, on August 30* [die tertio Kal. Sept.]. Scaliger, in the Prolegomena to *De emendatione temporum*, p. 18, understands these words of a failing of the Moon that the astronomical tables show in the year 301 on September 3 [tertio Nonas Septēbris], and on the fifth day after St Felix was led with his companions into Campania to the place of his passion, because they had refused to hand over the divine books, that is, sacred scripture. Hence Scaliger convicts Baronius of the anachronism [Greek: ἀναχρονισμοὶ (anachronismoi, "anachronisms")], by which he assigns this martyrdom to the year 302.

[Margin: Year 316. ☉]

The eclipse of the Sun seen at Constantinople on July 6, from the Constantinopolitan *Fasti*; for Pétau it was of 4 digits, and began there at 4^h 11^m and ended at 5^h 42^m after midnight.

[Margin: Year 317. ☉]

The eclipse of the Sun in the year in which Crispus Constantinus, son of the Emperor Constantine, and Licinianus, son of Licinius, were named Caesars; on it, Aurelius Victor. ✢ For Pétau this was on December 20, of 7 digits 20′; it began at Paris at 11^h 52^m and ended at 14^h 28^m after midnight.

[Margin: Error in Renieri.]

✢ But Vincenzo Renieri must be corrected, who places it in the year 291, p. 57 of the *Tabulae Mediceae*.

[Margin: Year 334. ☉ Annular.]

The eclipse of the Sun under the consuls Optatus and Paulinus, of which Julius Firmicus, book 1, chapter 2, says: *The Sun at the time of midday, hindered by the rays of the Moon as if by certain obstacles, denied to mortals the brilliant light of its splendour*; and he adds that it was seen at Byzantium. ✢ For Pétau, book 8, chapter 13, it was on July 17 and of 10 digits 24′; it began at Rome at 10^h 19^m

and ended at $13^h\ 14^m$ after midnight. But Lansbergen, in the *Treasury*, p. 126, infers from his tables that it was annular, the whole orb of the Moon being seen within the orb of the Sun. For this is what those words of Firmicus, *the rays of the Moon* etc., signify; for the Moon appeared to radiate all around. My tables represent it as annular: for the anomaly of the ☉ was 46° and that of the Moon 296°, and therefore the apparent diameter of the Sun 31′ 8″ and that of the Moon 29′ 16″.

[Margin: Year 346. ☉]

The eclipse of the Sun that St Jerome assigns to the year of Eusebius 2364, but Theophanes in the *Miscella* to year 10 of Constantius; Kedrenos also speaks of it. ✢ For Pétau this was on June 6, of 7 digits 36′; it began at Constantinople at $4^h\ 44^m$ and ended at $6^h\ 18^m$ after midnight. Since therefore it was in the morning, and the Sun rose in eclipse, it is no wonder if Theophanes says that stars were seen in the sky. ✢ Kedrenos says that it was on the 6th of the month Daesius, that is, June 6; but Scaliger, p. 510, denies that this synod was ecliptic, for a reason to be given shortly.

[Margin: Year 347. ☉]

The eclipse of the Sun in year 11 of Constantius, as St Jerome notes in Eusebius's *Chronicle*. ✢ By Scaliger's calculation, book 5 of *De emendatione temporum*, p. 510, it was on October 20, feria 3. ✢ But for Pétau it was of nearly 7 digits; it began at $3^h\ 25^m$ after noon at Rome and ended at $5^h\ 15^m$, and the Sun set at $5^h\ 20^m$. But since between this ecliptic new Moon and the new Moon of June 6 of the year 346 there are only 16 lunations, and Scaliger does not consider this interval ecliptic, he therefore accuses Kedrenos of ignorance for assigning an eclipse of the Sun to year 10 of Constantius on June 6 [Greek: Λαισίου ς (Laisiou 6, "on the 6th of Laisios"; the Latin above gives the month as Desius)]. For although eclipses can return at the fifth and also at the sixth month, they cannot do so again straight away at the fifth month, and thus at 16 months, as we taught in chapter 14; Kedrenos therefore shows himself ignorant of the ecliptic intervals.

[Margin: Error of Marianus Scotus.]

✢ You may rightly also blame or correct Marianus Scotus, who moves this eclipse to December 15 [18. Kalend. Ianuarij], on which day there was certainly no new Moon, except in the year 350, in which he affirms there was no eclipse of the Sun.

[Margin: Year 348. ☉]

The eclipse of the Sun seen at Byzantium at hour 3 of the Lord's day: from Theophanes in the *Miscella*. For Pétau this was on October 9, of

8 digits; it began at $6^h\ 28^m$ and ended at $8^h\ 31^m$ after midnight.

[Margin: Year 360. ☉]

The eclipse of the Sun under the consuls Constantius X and Julian III, recorded by Ammianus Marcellinus, book 20, in these words: *Throughout the eastern regions the sky was seen dark with gloom, and from the first rising of dawn until noon stars glittered continuously. To these terrors was added the fact that, when*

the heavenly light was covered and light had been taken wholly from the world's sight, the frightened minds of men reckoned that the Sun had failed for a long while, first thinned to the shape of a horned Moon, then grown to the appearance of a month-and-a-half [ſeſquimeſtrem] *one, and afterwards restored whole.* ✢ This eclipse was on August 28, and was seen in Mesopotamia by the Tigris, on the same author's authority, and by the Prutenic calculation it was annular in some place. ✢ For Pétau it was of 11 digits; but because it began there at 4^h 29^m after midnight and ended at 6^h 33^m, stars were therefore seen in the sky; and in the eastern and more easterly regions it began in some places after the Sun had already risen, and lasted almost until noon. Ammianus, therefore, confusing the places of the observations on the strength of differing reports, exaggerated it as though it had lasted in Mesopotamia from dawn to noon.

[Margin: Kepler's error, and his contradiction of himself.]

✢ But Kepler contends in vain, in the *Optics*, p. 294, that it was total, while admitting that the Moon was at apogee but concealing that the Sun was then nearly at perigee. For the anomaly of the Sun was 157° and that of the Moon 358°, and therefore the diameter of the Sun by the Rudolphine Tables is 31′ and that of the Moon 30′; but by my reckoning that of the Sun is 33′ 2″ and that of the Moon 28′ 2″. He therefore contradicts himself.

[Margin: Year 364. ☾]

The eclipse of the Moon of which Theon speaks in his Commentary on Ptolemy, book 6, in the year of Nabonassar 1112, on the 6th of Phamenoth, or November 25, feria 5, as Scaliger has it, book 5, p. 496.

[Margin: Year 365. ☉]

The eclipse of the Sun in the year of Nabonassar 1112, on the 22nd of the month of Payni, whose beginning Theon, father of Hypatia, observed at Alexandria in Egypt at 2^h 50^m after noon, its middle at 3^h 45^m, and its end at about hour 4½, as Theon reports in his Commentary on book 6 of Ptolemy's *Syntaxis*. It was on March 10.

[Margin: Year 393. ☉]

The eclipse of the Sun under the consuls Theodosius III and Abundantius, at hour 3 of the day, of which Marcellinus and Prosper speak in the *Chronicle*, seen at Byzantium and at Rome. ✢ Pétau, therefore, fitting the calculation to both places, says that on November 20 it was at Rome of 9 digits 31′, from 8^h 31^m to 10^h 50^m after midnight, but at Byzantium of 10 digits 44′, from 11^h 16^m to 13^h 30^m. If the intervals of eclipses are therefore to be preserved, there could not have been in this same year in Europe an eclipse of the Sun at hour 4 of the day near Pentecost, as Baronius has it; and in this year Pentecost was on June 5, and the preceding new Moon on May 27.

[Margin: Year 402. ☉]

The eclipse of the Sun on November 11 [3. Idus Nouembris], under the consuls Honorius V and Arcadius V, in Idatius in the *Fasti*. ✢ For Pétau it was of 10 digits 30′, from 7^h 36^m after midnight to 9^h 42^m.

[Margin: Year 410. ☉]

The eclipse of the Sun in which stars were seen in the sky, in the year in which Alaric took Rome, from Nicephorus, book 13, chapter 36.

[Margin: Year 418. ☉]

The eclipse of the Sun on the 14th before the Kalends of August [XIV. Kalend.Auguſt.], or July 16[45], under the consuls Honorius XII and Theodosius VIII, which is asserted by Liechtembergius [?], at the time of St Martin; by Iornandus [Jordanes] in the *Chronicle*, and Marcellinus at Indiction 1; by Idatius in the *Fasti*; and by the *Chronicon Alexandrinum*. ✢ For Pétau it was of 11 digits 30′, from $11^h\ 32^m$ at Rome to $13^h\ 20^m$ after midnight, although Kepler, in the *Optics*, p. 294, considers it to have been total, because he is wholly taken up there with asserting total failings of the Sun.

[Margin: Year 421. ☉]

The eclipse of the Sun of which Prosper speaks in the short *Chronicle* edited by P. Pithou, saying: *Jerome dies at Bethlehem in the 98th year of his age; and in the following year, in this year, an eclipse of the Sun took place; Pharamond reigns.* It is established from the consular *Chronicle* of Prosper that St Jerome died under the consuls Theodosius IX and Constantius III, that is, as Marcellinus has it, in Indiction 3, or the year of Christ 420. The eclipse was therefore in the year 421, namely, as Scaliger says, book 6 of *De emendatione temporum*, p. 611, on May 17, feria 3; and thus the beginning of Pharamond belongs to the year 421 and not to 419, as Sigebert wrongly has it.

[Margin: Year 450. ☉]

The eclipse of the Sun of which St Gregory of Tours speaks, book 2, chapter 3, on the coming of the Huns: *Then the Sun too appeared foul, so that scarcely even a third part of it shone. For I believe that for such great crimes and the shedding of innocent blood* etc. It was in year 1 of the Emperor Marianus, under the consuls Valentinian VII and Avienus, Indiction 3, as Gregory notes, and according to Scaliger on April 27, feria 5; and in the following year, namely the year 451, Attila burst into the Gauls.

[Margin: Year 451. ☾]

The eclipse of the Moon at its rising, on September 26 [6. Kal. Octobr.], of which Idatius speaks in the *Fasti*, p. 28, seen at Compostela, where he lived, from hour 15 55′ to 19 25′, of 10 digits, as Pétau's calculation shows.

[Margin: Year 458. ☉]

The eclipse of the Sun on May 28 [V. Kalend. Iunias], in Idatius in the *Fasti*, of 11 digits, at Compostela at $9^h\ 9^m$ after midnight, as Pétau computes.

[Margin: Year 462. ☾]

The eclipse of the Moon on March 2 [VI. Nonas Martij], feria 6, as Idatius says in the *Fasti*, p. 40, seen at Rome, ✢ although Pétau directs the calculation to Compostela, and says that there it began on March 1 at $22^h\ 33^m$ after midnight and ended at $1^h\ 41^m$ of the following day, and finally that it was of 9 digits.

[45]the page prints 16; the 14th before the Kalends of August is July 19 by inclusive count

[Margin: Year 464. ☉]

The eclipse of the Sun on July 20 [13.Kal.Auguſti], seen at Constantinople, of which Idatius speaks in the *Fasti*, adding, p. 42, that it was seen at Rome from hour 3 to hour 6. ✢ Pétau, by his calculation, states 8 digits, and that it began at Byzantium at 7^h 28^m after midnight and ended at 9^h 55^m.

[Margin: Year 465. ☉ ☉ by calculation.]

The eclipse of the Sun computed, on the occasion of the preceding one, by Francesco Montebruni, who consulted me about it for I know not what purpose, and found from Lansbergen's tables on July 9, 21^m before noon, at Athens, together with another ecliptic new Moon on January 12, at 17^h 53^m after noon, at Constantinople.

[Margin: Year 484. ☉ Total.]

The eclipse of the Sun on January 13, *by which day was turned into night, for deep darkness came on and stars were seen*, as is found in Renieri's *Catalogue*, from the life of Proclus written by Marinus and from the Greek text, which is kept in the library of the Grand Duke of Tuscany. The corrected anomaly of the ☉ was 218° and that of the ☾ 169°, and therefore, for me, the diameter of the Sun was 32′ 50″ and that of the Moon 33′ 24″.

[Margin: Year 486. ☉]

The eclipse of the Sun on May 18, from the life of Proclus, of which above, in Renieri.

[Margin: Year 574. ☉]

The eclipse of the Sun in the year before the death of Justinian, which Paul the Deacon, book 2, chapter 31, says was so great that not even a fourth part of the Sun remained shining, but that it appeared foul and discoloured, as if a sack. But because he says that it was on October 1 [Kal. Octobris], ✢ and Scaliger, book 6, p. 617, finds it by calculation on September 1 [Kal. Septembris] rather than in October, Scaliger therefore corrects Paul the Deacon.

[Margin: Year 581. ☾]

The eclipse of the Moon on April 4. The Moon failed in year 6 of Childebert the Younger, as St Gregory of Tours writes, book 6, chapter 25. ✢ For Pétau it was of 7 digits; it began at 23^h 53^m after midnight and ended on the following day at 3^h 13^m.

[Margin: Year 582. ☾ Total.]

The eclipse of the Moon in year 7 of Childebert the Younger, on March 24, from St Gregory of Tours, book 5, chapter 21. ✢ By Pétau's calculation it was of 22 digits 7′; it began at Paris at 1^h 47^m after midnight; total immersion at 2^h 57^m; emersion at 4^h 55^m; end at 6^h 5^m.

[Margin: Year 582. ☾ Total.]

Another eclipse of the Moon, on September 17, in the same St Gregory of Tours, book 5, chapter 21. ✢ For Pétau it was of 21 digits; beginning at Paris at 22^h 45^m; total immersion at 23^h 55^m; emersion after midnight of the following day at 1^h 27^m; end at 2^h 37^m.

[Margin: Year 590. ☉]

The eclipse of the Sun of which St Gregory of Tours speaks, book 5, chapter 21, in year 15 of Childebert, where he says that at Tours in the month of October the Sun failed so that it scarcely had, for shining, as much as the horns of a fifth-day Moon have. ✢ By Pétau's calculation it was on October 4, of about 7 digits; it began at Paris at $9^h\ 7^m$ and ended at $11^h\ 3^m$ after midnight. ✢ Scaliger, book 5, p. 620, confuses this one with the eclipse of the year 597, of which we shall speak shortly.

[Margin: Year 592. ☉]

The eclipse of the Sun in year 9 of Maurice and year 32 of Guntherhamnus, in the spring, as the Constantinopolitan *Annals*, Theophanes and Fredegar in the appendix to Gregory of Tours have it, and the year there is 32. ✢ For Pétau it was on March 19, from $8^h\ 54^m$ to $11^h\ 23^m$, at Constantinople, and of 10 digits. ✢ Granted that Kepler, in the *Optics*, p. 294, places it among the total eclipses on a surely weak conjecture, namely because he thinks that eclipses of the Sun are rarely found in writers of those centuries, since they noted only total ones; but from the year 360 to 418 he had read of no eclipses of the Sun being noted.

[Margin: Year 597. ☉]

The eclipse of the Sun on November 15, of which Scaliger, book 6, p. 620, holds that Fredegar spoke when he said: *The Sun suffered an eclipse in the middle of the eighth month, and its light diminished so that it scarcely had, for shining, as much as the horns of a fifth-day Moon retain.* But the eighth month is October, so Fredegar or St Gregory speak of the eclipse of the year 590. ✢ Sigebert wrongly places this one in the year 596.

[Margin: Year 603. ☉]

The eclipse of the Sun in year 8 of Theodoric and Theodebert, kings of the Franks, of which Fredegar speaks in the appendix to St Gregory of Tours. By Pétau's reckoning it occurred on August 12, at Paris at 3^h after noon, of 11 digits 20′.

[Margin: Year 605. ☉]

The eclipse of the Sun on May 5, after which came a plague of 3 months, in which a good Angel together with a bad one was seen going round the cities and marking the houses, as Regino has it in his *Chronicle*, book 1, and Gregorius Tholosas [Pierre Grégoire of Toulouse], book 2 of *De Republica*, p. 128.

[Margin: Year 664. ☉]

The eclipse of the Sun on May 1 [Calendis Maij], granted that Bede, by mistake, in book 3 of the *History of the English*, chapter 27, puts the 3rd of the month; it was seen at London at about the third hour. ✢ Pétau says that it was of 10 digits 20′, that it began there at $3^h\ 7^m$ after noon and ended at $5^h\ 14^m$. Yet Kepler, in the *Optics*, p. 294, judges it to have been total, on the strength of that slight conjecture of which we spoke at the year 592.

[Margin: Year 682. ☉]

The eclipse of the Sun on May 3 [V. Nonas Maij], which an eclipse of the Moon had preceded, in Lycosthenes, in the *Chronicon prodigiorum*, an author, however, tainted by excessive spite against the authority of the Roman Pontiff.

[Margin: Year 719. ☉]

The eclipse of the Sun, total in Spain, from certain annals that Scheiner adduces in the *Rosa Ursina*.

[Margin: Year 760. ☉]

The eclipse of the Sun in year 20 of Constantine Copronymus, on August 15, at Constantinople, from Theophanes. For Pétau it occurred at 5^h after noon, of 10 digits.

[Margin: Year 787. ☉]

The eclipse of the Sun in year 7 of Constantine, son of Irene, on September 16, at hour 5 of the day, of which Kedrenos speaks under that year 7, and Theophanes. The Prutenic Tables give 11½ digits. ✝ Pétau gives 9 digits 47′, and the hour as 8^h 47^m after noon at Constantinople. But Kepler, in the *Optics*, p. 294, would have us believe that it was total.

[Margin: Year 806. ☾ Total.]

The eclipse of the Moon on September 2 [4. Nonas Septembris], in Aimoin, book 4, chapter 95, and Regino, book 2 of the *Chronicle*. ✝ For Pétau it was of 16½ digits; it began at Paris at 19^h 57^m after midnight, the total obscuration was at 21^h 11^m, the emersion at 22^h 47^m, and the end at 0^h 1^m of the following day.

[Margin: Year 807. ☉]

The eclipse of the Sun on February 11 [3. Idus Februarij], of which speaks an anonymous writer on the life of Charlemagne, published by Pierre Pithou, who is believed to be a monk of Angoulême. For Pétau it occurred there at 9^h after midnight, of 9 digits 42′.

[Margin: Year 807. ☾ Total.]

The eclipse of the Moon on February 26 [4. Kalend. Martij], of which Aimoin speaks, book 4, chapter 95, and Regino, at Paris near midnight; of 12 digits 20′ for Pétau.

[Margin: Year 807. ☾ Total.]

The other eclipse of the Moon in the same writers, which began on August 21 at 20^h 26^m after midnight; the complete immersion was at 22^h 8^m, the emersion at 22^h 32^m, the end on the following day at 0^h 14^m; 12 digits 12′ for Pétau.

[Margin: Year 808. ☉]

The eclipse of the Sun in year 8 of Charlemagne, on May 14, in Theophanes and Lycosthenes.

[Margin: Year 809. ☉ ☾]

An eclipse of the Sun and then of the Moon in December, in Regino, book 2.

[Margin: Year 810. ☉]

The eclipse of the Sun on November 30 [pridie Kal. Decembris], of which Regino speaks, book 2; at 11^h 48^m after noon at Königsberg in Prussia [Regiomontis Prussiae], as Pétau computes, and of 11 digits 10′. But the one that the same Regino says occurred on June 9 [V. Idus Iunij] is false; for there was no new Moon on that day, nor an ecliptic one in that month, as Pétau points out.

[Margin: Year 811. ☉]

The eclipse of the Sun in year 44 of the life of Charlemagne, on May 15 [Idib. Maij], in the *Annals of the Franks*.

[Margin: Year 812. ☉]

The eclipse of the Sun of which the anonymous writer on the life of Charlemagne speaks, and Regino, seen at Constantinople on the day before the Ides, not of June, as the corrupt codex has it, but of May, as Pétau would have it. At 14^h 13^m after midnight, of 9 digits.

[Margin: Year 813. ☉]

The eclipse of the Sun in year 2 of Michael Curopalates, of which Theophanes speaks at the year of the world 6305, and the *Miscella*, book 24, seen in Cappadocia on May 4 at sunrise; of 10 digits 35′ for Pétau. This year was the last of the Emperor Michael and the first of Leo the Armenian, at which the history of Theophanes ends.

[Margin: Year 840. ☉ Total.]

The eclipse of the Sun from the *Annals of the Franks*, in the year 840, on May 5, before the death of Louis the Pious, king of the Franks, of which Aimoin speaks, book 5, chapter 19, the anonymous writer published by Pithou, mentioned above, and the author of the life of Louis the Pious, who add that it was on the Vigil of the Ascension, and *very great, so that stars were even seen because of the darkness of the Sun, and the colour of things on earth changed*. Indeed, as the author of the life of Louis says: *So that it seemed to differ in nothing from true night; for the sparse order of the stars was so discerned that no star suffered the dulling of the Sun's light. Rather the Moon, which had presented herself opposite it, gradually making for the east, first restored to it a horned light from the western side, after her own manner when she is seen one or two days old; and so by increments the whole wheel of the Sun recovered all its beauty.* From this it appears that this author was skilled in astronomy and in distinguishing the stars; this is also confirmed by the conversation he adds, held with Louis the Pious, about the comet [stella Crinita] of the preceding year, as Kepler points out in the *Optics*, p. 295. ✢ It was therefore total, with the Moon set nearly at perigee; and therefore not of only 9 digits 20′, as Pétau computes for 11^h after midnight.

[Margin: Year 842. ☾]

The eclipse of the Moon on March 30 [3. Kal. Aprilis], feria 5 of Holy Week [Maioris Hebdomadae], at hour 10 of the night, of which the already mentioned *Annals of the Franks* speak. ✢ Pétau computes 14 digits 30′, and at

Paris the beginning at 0^h 48^m, totality at 2^h 2^m, the emersion at 3^h 14^m, and the end at 4^h 28^m from midnight.

[Margin: Year 878. ☾ ☉ Total.]

The eclipse of the Sun on October 29, at hour 9 of the day, which an eclipse of the Moon had preceded on the 14th, in year 29 of Louis the Stammerer, from the *Annals of the Franks*, in Kepler's *Optics*, p. 295. The author of the life of Louis says that *after hour 9 the Sun was so obscured for half an hour that stars appeared in the sky, and all thought that night was upon them*. Kepler, with Gemma Frisius, supposes that it was total, since it came from a Moon near perigee. Moreover, Louis died in the following year, on April 10 [4. Idus Aprilis]. For me it was total with no perceptible difference; for the anomaly of the Sun was 136° and that of the ☾ 220°, and the diameter of the ☉ 32′ 42″, but that of the ☾ 32′ 40″.

[Margin: Year 883. ☾]

The eclipse of the Moon in year 1631 of Nabonassar, 1206 from the death of Alexander the Great, and 1194 of the Terik era (that is, the era of Dhil-karnain), on the 23rd of the month Kenir, observed at Arachta in Syria at 8 equal hours or a little more after noon; al-Battānī, *De scientia stellarum*, chapter 30, where he says that it failed toward the north by more than 10 digits. Lansbergen concludes 11 digits 35′, Bünting and Pétau 11 digits.

[Margin: Year 888. ☾]

A great eclipse of the Moon on October 16, in Regino, in the *Chronicle*, book 2.

[Margin: Year 891. ☉]

The eclipse of the Sun in the year of Dhil-karnain 1202, 1214 from the death of Alexander the Great, on the 8th of Ab, that is August, feria 1, observed at Arachta by al-Battānī; there two and more parts of three of the Sun failed, and the apparent conjunction preceded the true by about an eighth part of an hour. So al-Battānī, chapter 30. See Scaliger, book 5, p. 428; Regiomontanus, book 5 of the *Epitome of the Almagest*, proposition 21; Lansbergen, who computes 9 digits 22′; and Pétau, who computes 9 digits 20′.

[Margin: Year 894. ☉ Or rather Year 901.]

The eclipse of the Sun observed by al-Battānī at Antioch in Syria on the 23rd of the month Huni the second, that is January, in the year of Dhil-karnain 1205, 1217 from the death of Alexander, at about 3^h 40^m before noon, and a half and a little more of the Sun was obscured. So al-Battānī, chapter 30. There, however, Lansbergen reads the year 1224 from the death of Alexander, that is, of Christ 901, on January 23; al-Battānī adds that it was observed at Arachta at about hour 3½ before noon, and that less than two thirds of the Sun was obscured there.

[Margin: Year 901. ☾]

The eclipse of the Moon observed by al-Battānī at Antioch in the year 1224 from the death of Alexander the Great, by night between the 2nd and 3rd of the month Ab, or August, at 15^h 20^m, but at Arachta at 15^h 35^m after noon. So

al-Battānī, chapter 30, affirming that the Moon was eclipsed almost wholly. Regiomontanus adds, in book 5 of the *Epitome of the Almagest*, proposition 21, that according to the Ptolemaic tables 10 digits ought to have been obscured, and the eclipse ought to have preceded by 50^{m}. Lansbergen, p. 69 of the *Treasury*, computes 12 digits 51′, but holds that al-Battānī did not see it wholly obscured, because it was not deeply plunged in the shadow. Pétau too concludes 12 digits 30′. ✝ Scaliger, p. 420, says that it was feria 1 going into the following feria 2. From my tables come fewer than 12 digits, as al-Battānī's observation requires.

[Margin: Year 926. ☾]

The eclipse of the Moon on April 1 [Calendis Aprilis], with a certain particle of light left, as though she were two days old; and so, with dawn now beginning, she changed wholly to a blood colour. Flodoard, in the *Chronicle*; on which see Kepler, p. 274 of the *Optics*.

[Margin: Year 944. ☉]

A terrible eclipse of the Sun on feria 6, at hour 3 of the day, on the day when Addaram [?], king of the Saracens, was defeated by Rhadamiro [Ramiro], king of Galicia. Sigebert, in the *Chronicle*.

[Margin: Year 95[?]. ☉ Total.]

The eclipse of the Sun on December 18 [XV. Kal. Ianuarij], in year 3 of Lothair, so great *that stars appeared from the first hour up to the third hour*, from the life of Lothair, and from Kepler in the *Astronomia Optica*, p. 295. For me the anomaly of the Sun at that time was 185° and that of the Moon 206°, and therefore the diameter of both Sun and Moon was 33′ 8″, and the eclipse was precisely total.

[Margin: Year 969. ☉]

The eclipse of the Sun on December 22, at hour 3 of the day, shortly after the death of Nicephorus Phocas, from Kedrenos and Curopalates.

[Margin: Year 1009.]

And another in year 7 of the Emperor Henry, of Christ 1009, in which Jerusalem was captured by the Saracens.

[Margin: Year 103[?]. ☉ ☉]

And another in the year 1032, on the 3rd before the Kalends

[Margin: Year 1039.]

[Margin: Year 1093. ☉ ☉]

of July [Iulij], at hour 6, in Lycosthenes, in the *Chronicle*. And another in the year 1039, on May 14 [prid. Idus Maij], in Vbo Emmius [Ubbo Emmius]. And lastly another in the year 1093, almost total, on October 1 [Calendis Octobris], at hour 3, in Meyer and Dodechinus, in the appendix to Marianus Scotus.

[Margin: Year 1113. ☉ Total.]

The eclipse of the Sun on August 2 [4. Nonas Augusti], at hour 7 of the day, on feria 6, very great, in Dodechinus, Sigebert, and Kepler in the *Astronomia Optica*, p. 295, who says that it was total; and rightly, for the anomaly of the

Sun at that time was 79° and that of the Moon 163°, and the diameter of the Sun 31′ 40″ and that of the Moon 33′ 18″.

[Margin: Year 1147. ☉]

The eclipse of the Sun at the end of October, in Sigebert, in the *Chronicle*.

[Margin: Year 1149. ☾]

[Margin: Year 1155. ☾ ☾]

Three eclipses of the Moon in Ubbo Emmius: one in the year 1149, on March 22 [XI. Kal. April.], the day before Palm Sunday, at about hour 9 of the night; the second in the year 1155, on May 22 [XI. Kal. Iun.]; and the third in the same year, on the Sunday before Christmas.

[Margin: Year 1177.]

[Margin: Year 1179. ☉ ☉ ☾]

Eclipses of the Sun in the year 1177, on September 13 [Idib. Sept.], at hour 6, in Lycosthenes; and in the year 1179, in September, in Emmius, with an eclipse of the Moon on August 18 [15. Kal. Septembr.], at hour 8½ of the night, in Lycosthenes.

[Margin: Year 1181. ☉]

The eclipse of the Sun on July 15 [Idib. Iulij], in Sigebert. Alexander III and the Emperor Louis the Fat [Ludouicus Crassus Imper.] died.

[Margin: Year 1187. ☉ Total.]

[Margin: A false note of Lycosthenes.]

The eclipse of the Sun on September 4, so great that stars are related to have been seen by day as if at night; Baldwin was captured by Saladin, and our people lost Jerusalem. Lycosthenes in the *Chronicle*, and Kepler in the *Astronomia Optica*, p. 295, who says that it was made total by the Moon's being at perigee. In that year the golden number was 10 and the Epact 19; therefore the ☌ could not have been on August 29 [4. Kal. Sept.], as Lycosthenes has it, but on September 4 [4. Sept.].

[Margin: Year 1191. ☉]

The eclipse of the Sun that Alberic affirms to have been total; but Kepler, in the *Astronomia Optica*, p. 298, denies it, because the Moon was at apogee.

[Margin: Year 1197.]

[Margin: Year 1239.]

[Margin: Year 1241. ☉ ☉ ☉ Total.]

[Margin: Year 1307. ☉ ☾ ☾]

Four eclipses of the Sun and two of the Moon. The first in the year 1197, on September 4 [prid. Nonas Septembr.], so great that stars appeared, of which Gregorius Tholosas speaks, book 21 of *De Republica*, chapter 27. The second in the year 1239, on June 3 [3. Non. Iunij], of which Gassendi speaks in the life of Claudius Fabricius Peireskius [Nicolas-Claude Fabri de Peiresc]. The third in the morning, on October 6, in the year 1241, and total, as Gemma Frisius has it, of which Kepler speaks in the *Optics*, p. 295. The fourth in the year 1307, on February 11 [3. Idus Febr.], which was followed by an eclipse of the Moon on February 26 [4. Cal. Mart.] and again on August 21 [12. Cal. Septembr.],

and our people were routed by the Saracens from Rhodes. Lycosthenes in the *Chronicle*.

[Margin: Year 1346.]

[Margin: Year 1347. ☾ ☾ ☾]

Three eclipses of the Moon from notes to Ptolemy in a Greek manuscript of the Most Serene Grand Duke of Tuscany, in Vincenzo Renieri. The first in year 1665 of Alexander the Great, on the 13th of Athyr going into the 14th, at $2^h\ 24^m$ after midnight at Thessalonica. The second in year 1666 of Alexander, on the 2nd of Athyr going into the 3rd, at $3^h\ 5^m$ after midnight at Thessalonica. The third in the same year, on the 30th of Pharmuthi, on the following night, at $1^h\ 47^m$ after midnight in the same place.

[Margin: Year 1387. ☉]

A great eclipse of the Sun on December 9, in Bonfinius [Antonio Bonfini], book 9 of Decade 2 of the *Hungarian History*.

[Margin: Year 1415. ☉ Total.]

The eclipse of the Sun on June 7, feria 6 after the feast of Corpus Christi, dreadful in the middle of Bohemia, so that birds, terrified by the sudden darkness, fell down. Of it speak, from the Polish historian, Lvovický and Lycosthenes; Reinhold in the *Theoricae* of Peuerbach, saying that it was seen at Constance at hour 6 of the morning, that is, as Lansbergen computes in the *Treasury*, p. 129, at $6^h\ 5^m$ after midnight; Gassendi, relating that it was observed at Aix at 7^h after midnight; and Kepler in the *Astronomia Optica*, p. 295, saying that it was total because of the Moon at perigee, which he confirms by calculation in the Rudolphine Tables, precept 169, adding $6^h\ 43^m$ after midnight at Prague. See also Origanus in the preface to volume 2, who confirms it from Mathias Mechovius [Maciej Miechowita], the Polish historian. My tables give the diameter of the Sun at that time as 30′ 50″ and that of the Moon as 33′ 28″, with the anomaly of the Sun at 351° and that of the Moon at 176°.

[Margin: Year 1419.]

[Margin: Year 1424.]

[Margin: Year 1448.]

[Margin: Year 1449. ☉ ☉ ☉ ☉]

Four eclipses of the Sun. The first in the year 1419, in ♈ 15°, in Origanus, preface to volume 3. The second in the year 1424, of which we spoke in the scholia to chapter 2. The third in the year 1448, on August 29, in Garcaeus, p. 288, and in Origanus, cited above. The fourth in the year 1449, on August 29 [4. Kal. Septemb.], at hour 6 of the day, in Lycosthenes, who adds that the Venetians were defeated at that time.

[Margin: Year 1457. ☾ Total.]

The eclipse of the Moon on September 3, at Melk in Austria, which is 11 German leagues west of Vienna, observed by Regiomontanus and his teacher Peuerbach. The beginning of total obscuration was when the last-but-one of the Pleiades stood 22° high in the east, from which they derive $10^h\ 24^m$ after

noon. The beginning of the emersion was when the same star stood 36° high, from which $11^h\ 49^m$. Snel, in the *Observations*, and, following him, Renieri, p. 59. Lansbergen, p. 71 of the *Treasury*, adds that it was of 14 digits 27′.

[Margin: Year 1460. ☾ ☾]

[Margin: Total.]

Two eclipses of the Moon observed by Peuerbach and Regiomontanus, as is found in their observations and in Snel, p. 14. The first was observed on July 3, at Vienna in Austria, of 4 digits 12′, at $8^h\ 13^m$, from the apparent eastern altitude of the Moon at the end, 15° 18′, whereas the Alphonsine Tables gave $9^h\ 10^m$ and 2 digits 56′. The second was observed in the same place on December 27. The beginning was at $11^h\ 42^m$ after noon, as they judged from the eastern altitude of Arcturus, 7°; the beginning of the stay at $12^h\ 47^m$, from the altitude of the same star, 17°; the end of the stay at $13^h\ 55^m$, from the altitude of the same star, 28°. But they assumed for Arcturus a declination of 22° and a right ascension of 297° 53′, which are to be corrected either from the Tychonic observations or from our own observations of the fixed stars, of which we shall speak in the next book. They add that at the beginning of the eclipse the Moon was seen on a great circle passing through the head of the leading Twin and the brighter star of Canis Minor, but at the end on one circle passing through the head of the following ♊ and Canis Minor. To this record Tycho attached this note: *If these things were exact, the places of the stars could be verified from here*, as Kepler reports in the *Astronomia Optica*, p. 371.

[Margin: Year 1461. ☾ ☾]

[Margin: Total.]

Two eclipses of the Moon observed by Regiomontanus, as is found in his observations and in Renieri, p. 59. The first was observed at Vienna in Austria on June 22: at the beginning of the stay Aquila stood 26° high, and at the end of the whole eclipse 47° 30′, in the east, from which he derived the middle at $13^h\ 21^m$ after noon. The second on December 17, at Vienna, of more than 8 digits; at the end Capella stood 38° 30′ high and Aldebaran 29°, from which he derived $0^h\ 54^m$ after sunset; whereas from the Alphonsine Tables, says Regiomontanus, it ought to have ended at $1^h\ 56^m$ after sunset. So indeed my codex; but Renieri adds that the Moon rose at Rome obscured by 10 digits.

[Margin: Year 1462. ☾]

The eclipse of the Moon observed at Viterbo by Regiomontanus on June 11, of 7 digits to the north, at $14^h\ 48^m$ after noon, at the middle. Lansbergen, p. 72 of the *Treasury*, concludes 7 digits 9′, and the beginning there at $14^h\ 45^m$.

[Margin: Year 1462. ☉]

The eclipse of the Sun on November 21, about noon, observed at Viterbo by Regiomontanus; it failed on the south by 2 digits. At the beginning the Sun stood 26° 30′ high in the east, and at the end 24° 36′ in the west. Regiomontanus in his observations. But Lansbergen, in the *Treasury*, p. 130, concludes 2 digits 48′, and at Viterbo the hour $0^h\ 15^m$ after noon.

[Margin: Year 1464. ☾ Total.]

[Margin: Renieri's slip.]

The eclipse of the Moon on April 21, observed at Padua by Regiomontanus on his way to Rome, from his observations, although Renieri, p. 59, puts it as observed at Rome. At the beginning Cor Scorpii stood 12° 45′ high in the east, from which, assuming the altitude of the pole at Padua as 45° 44′, and the rest from the table of Alfonso, he derived $11^h\ 15^m$; and total obscuration at $12^h\ 33^m$, the middle at $12^h\ 59^m$, the end of the stay at $13^h\ 25^m$, the end at $14^h\ 43^m$, and 13 digits 27′.

[Margin: Year 1469. ☉]

The eclipse of the Sun, on which see the evidence above, chapter 2, in the scholium.

[Margin: Year 1471. ☾]

The eclipse of the Moon observed by Regiomontanus on June 2, at Nuremberg. The Moon began to be eclipsed with Cor Scorpii standing 14° 15′ high in the west; and when 4 digits had been obscured, the muzzle of Pegasus [muscida Pegasi] stood 22° 30′ high in the east.

[Margin: Year 1473. ☉]

The eclipse of the Sun on April 27, observed at Nuremberg by Regiomontanus, when at the end the Sun stood 25° high from the west; from the same man's observations.

[Margin: Year 1478. ☉]

The eclipse of the Sun on July 29, observed at Nuremberg by Bernhard Walther. At the beginning the Sun's altitude in the west was 54° 30′, and at the end 41° 30′, *from the observations of Walther*. Lycosthenes is therefore to be corrected, who puts this one in the year 1479.

[Margin: Year 1485. ☉ Total.]

[Margin: Kepler's petty slander against Walther.]

The eclipse of the Sun on March 16, observed by Walther at Nuremberg, of nearly 11 digits from the south. The beginning at $3^h\ 26^m$ after noon, the middle at $4^h\ 27^m$, the end at $5^h\ 28^m$. The Alphonsine calculation gives 12 digits; Lansbergen, p. 131 of the *Treasury*, computes 10 digits 50′. Yet Kepler, in the *Astronomia Optica*, p. 295, rightly, and with my tables agreeing, contends that it was total, because the Moon was at perigee, and on account of these words of Lycosthenes: *There was sheer darkness, so that there was need of lighted candles; in the cities the hens, and out of doors animals of various kinds, betook themselves to their usual night-time places of rest*. And he adds that Walther was deceived by a fallacy of sight; as though indeed it had to be total everywhere, or at Nuremberg, because it was total elsewhere. The fallacy here is rather one of memory, on the part of Kepler, who forgot the parallaxes.

[Margin: Year 1487. ☾ Total.]

The eclipse of the Moon on February 8, observed by Walther at Nuremberg, total; the beginning at about 2^h after midnight, the end at $5^h\ 20^m$.

[Margin: Year 1491. ☉]

The eclipse of the Sun on May 8, observed at Nuremberg by Bernhard Walther. At the beginning of the eclipse ♊ 19° was in mid-heaven, and at the end ♋ 26°, in Walther's observations, fol. 38.

[Margin: Year 1493. ☾]

The eclipse of the Moon on October 22, predicted and then seen by Cristoforo Colombo, by the portent of which he terrified the barbarians and obtained provisions from them, according to what we said in chapter 2, number 14; recall it.

[Margin: Year 1494. ☉]

The eclipse of the Sun on October 10, observed by Walther, fol. 39, at Nuremberg. The beginning between hours 1 and 2 after noon, the end, precisely, at $4^h\ 24^m$.

[Margin: Year 1497. ☾]

[Margin: A fallacy of observations.]

The eclipse of the Moon on January 18, of which Johannes Werner speaks in the commentary on Ptolemy's *Geography*, book 1, chapter 4, where he says that the beginning was observed at Rome at $5^h\ 24^m$ but at Nuremberg at $4^h\ 52^m$ after noon. If that had been so, the difference of longitude between Rome and Nuremberg would be 32^m of time, or 8°, which neither any geographical maps nor other observations bear out; they give scarcely 5½°. It was therefore easy to err by 12^m in time, especially with each observer contributing his share of the error.

[Margin: Year 1497. ☉]

The eclipse of the Sun on July 29, whose end Walther observed at Nuremberg at $3^h\ 24^m$ after noon, fol. 39.

[Margin: Year 1500. ☾]

The eclipse of the Moon on November 5 going into the 6th, observed at Rome by Copernicus, as he himself relates in book 4 of *De revolutionibus*, chapter 14. He says that the middle was at 14^h after noon, and that 10 digits failed from the north. But Lansbergen, p. 73 of the *Treasury*, derives from his own tables 10 digits 35′.

[Margin: Year 1504. ☾]

The eclipse of the Moon on February 29, observed through an astrolabe by Johannes Stöffler at Ulm in Swabia, who says that its beginning was at about $11^h\ 49^m$ after noon. ✢ But Bernhard Walther at Nuremberg, with the equatorial armillaries, observed the end at $3^h\ 7^m$ after midnight; clouds hindered the rest.

[Margin: Year 1506. ☉ Nearly total.]

The eclipse of the Sun on January 13, feria 4, from hour 1 to hour 2½ after noon, so great that *stars were seen in broad daylight*, says João de Barros, and, following him, our people in the account of the observations of the comets of the year 1618. And yet for me the Sun did not fail wholly, for the diameter of the Sun was 32′ 58″ and that of the Moon 32′ 34″; it came close, however, to total eclipses.

[Margin: Year 1509. ☾]

The eclipse of the Sun[46] on June 2, observed at Kraków by Copernicus, book 4, chapter 13, of about 8 digits from the south, at 11^h 46^m after noon. But Lansbergen, p. 74 of the *Treasury*, concludes 7 digits 31′.

[Margin: Year 1511. ☾ Total.]

The total eclipse of the Moon on October 6, observed at Frombork in Prussia by Copernicus, from his book 4, chapter 5. The beginning at 10^h 52^m, the end at 14^h 20^m after noon, from which he derives the middle at 12^h 35^m. Lansbergen, p. 75 of the *Treasury*, concludes 12 digits 15′ and a duration of 3^h 22^m.

[Margin: Year 1516. ☉]

The eclipse of the Sun at the beginning of ♌, in Origanus, volume 3 of the *Ephemerides*, in the preface.

[Margin: Year 1518. ☉]

The eclipse of the Sun in June, in ♊ 26°, from which astrologers conjectured the death of the Emperor Ferdinand; from Henri de Sponde, in the *Epitome of the Annals*.

[Margin: Year 1520. ☉]

The eclipse of the Sun in [?] 10°[47], in Origanus, in the preface to volume 3 of the *Ephemerides*.

[Margin: Year 1522. ☾ Total.]

The eclipse of the Moon on September 5, observed at Frombork by Copernicus, book 4, chapter 5. The beginning at 11^h 36^m, and it failed wholly. Lansbergen concludes 12 digits 54′.

[Margin: Year 1523. ☾ Total.]

The eclipse of the Moon on August 25, observed at Frombork by Copernicus, book 4, chapter 5. The beginning at 14^h 48^m, and it failed wholly. Lansbergen, p. 77 of the *Treasury*, gives 14 digits 19′. ✝ Cardano says that at Milan it was at 15^h 15^m after noon, of 13 digits 50′. A huge plague followed.

[Margin: Year 1530. ☉ Total.]

A great eclipse of the Sun, and total for me, very early in the morning of March 29, shortly after sunrise, seen by Kepler's paternal uncle, as is said in the *Astronomia Optica*, p. 295, who reported that when day had scarcely dawned it was suddenly extinguished and turned into night.

[Margin: Year 1539.]

[Margin: Year 1540.]

[Margin: Year 1541. ☉ ☉ ☉ and ☾]

Three eclipses of the Sun. The first on April 18, at 4^h 19^m after noon, at Milan, of nearly 9 digits, from Cardano. The second in the year 1540, in ♈ 27°.

[46] the page heads this entry `Solis Eclipsis`, while the sign beside it in the margin, without rays, is of the kind this page uses for the Moon.

[47] the sign is a small pictorial sort, a many-legged creature with curled ends, unlike the Cancer sign printed in the Year 1491 entry

The third in the year 1541, in ♍ 7°, in Origanus, volume 3 of the *Ephemerides*, in the preface. And one of the Moon at the beginning of ♈, in the same place.

[Margin: Year 1544. ☉]

The eclipse of the Sun on January 14, observed at Leuven by Gemma Frisius through the hole of a window, of 10 digits, at $8^h\ 53^m$, as he himself has it in chapter 18 of the *Radius Astronomicus*, and Daniel Santbech, *De observatione*, proposition 13. ✢ But Functius, in Kepler's *Optics*, p. 296, says that the day began to grow dark, as if into evening twilight, and that the birds that had been merry at first light fell silent; and he estimated an eclipse of 11 digits, the figure Lansbergen gives. Yet Kepler maintains that it was total somewhere, as Reinhold had predicted it from calculation. For me it was partial, nor does the history prove otherwise, for morning eclipses appear larger because of the night that precedes and, as it were, continues.

[Margin: Year 1545. ☉]

The eclipse of the Sun on June 9 [5. Idus Iunii], at 4^h after noon, of 4⅔ digits, observed at Leuven by Gemma Frisius, chapter 15 of the *Radius Astronomicus*, and, from there, Daniel Santbech, *De observatione*, proposition 13.

[Margin: Year 1551. ☉]

The eclipse of the Sun in ♍ 17°, in Johannes Garcaeus, *De genituris*, p. 288.

[Margin: Year 1555. ☾]

The eclipse of the Moon on June 4, at 3^h after midnight, at Wittenberg, in Reinhold, in the *Theoricae* of Peuerbach.

[Margin: Year 1560. ☉ Total.]

[Margin: Tycho's error.]

The eclipse of the Sun on August 21, observed at Coimbra by our Clavius, total about noon and with stay, as he himself reports in chapter 4 of the *Sphere* of Sacrobosco, where he says: *The Sun lay hidden, covered for no short time; the darkness was in a way greater than that of night, for one could not see where to put one's foot, and stars appeared very brightly in the sky; birds too, marvellous to tell, fell from the air to the ground in horror at so foul a darkness.* Father Emmanuel Vega was present, who says that it lasted for nearly three hours, with women howling and crying aloud that the last day of the world had come; that the stars had never been seen so clearly, that men under their roofs scarcely recognized one another, and that lamps were needed; so Scheiner reports from his mouth in the *Rosa Ursina*, book 3, chapter 7. ✢ Egnazio Danti observed the same eclipse at Rome at $1^h\ 30^m$ after noon, as he has it in the *Astrolabe* [Astrolobio]. ✢ Tilemann Stella and Paul Fabricius observed it at Vienna in Austria, of 5 digits, and the end at $2^h\ 55^m$ after noon, as Gerard Mercator relates in the *Chronicle*. ✢ Giuntini, as he reports in the preface to his *Tables*, observed its beginning at Venice at $11^h\ 50^m$ after midnight. ✢ Lansbergen, p. 110 of the *Treasury*, says that it was observed at Brussels, of 6 digits 31′, and that the end there was at $1^h\ 48^m$ after noon, as Stadius says in his ephemeris for this year; but from calculation Lansbergen concludes 12 digits 4′, yet says that a longer stay was seen, because

the Sun contracted itself, so that an eclipse of 12 digits 38′ was seen; this is a subterfuge devised to prop up his hypothesis. But since Tycho, admitting no total eclipses of the Sun, did not believe this, he wrote in wonder to Clavius in the year 1600, as Kepler relates in the *Astronomia Optica*, p. 285. But the falsity of the Tychonic hypothesis on this point is refuted by so many total eclipses of the Sun, of which above and below, chapter 20. My tables then give the diameter of the Sun as 31′ 0″ and that of the Moon as 32′ 12″, with the anomaly of the Sun at 34° and that of the Moon at 130°.

[Margin: Year 1560. ☾]

The eclipse of the Moon on March 11, observed at Leuven by Cornelius Gemma, as he relates in his *Cosmocritice*, book 2. The beginning at $15^h\ 40^m$, the end at $17^h\ 26^m$ after noon. Lansbergen, in the *Treasury*, p. 78, gives 2 digits 58′.

[Margin: Year 1567. ☉ Annular.]

[Margin: Kepler's violence against observations.]

The eclipse of the Sun on April 9, observed by our Clavius at Rome, who affirms in chapter 4 on the *Sphere* of Sacrobosco: *At Rome the Sun did not fail wholly, but a certain thin shining circle was left all around.* ✢ This one, as Kepler reports in the *Astronomia Optica*, p. 297, Tycho says in a letter to Clavius that he observed as a youth at Rostock on the shore of the Baltic Sea, at almost exactly noon, of not quite 7 digits; and in the *Progymnasmata* he has 12^h after midnight, 6 digits 20′; and on another sheet 11^h, 9 digits. ✢ Mästlin, however, observed at Tübingen nearly 10 digits from the south. ✢ Gerard Mercator, at Duisburg in Cleves, the beginning at $10^h\ 25^m$ and the end at 1^h. ✢ Cornelius Gemma, at Leuven, the beginning at $10^h\ 12^m$, the greatest phase [vigorem] at $11^h\ 40^m$, the end shortly after $12½^h$, nearly 9 digits; the light very pale, the time as if evening, but no stars. Lansbergen shows from his tables that Clavius's observation was true. ✢ Nevertheless Kepler, in the *Optics*, from p. 297 to 302, calls it in doubt, because at such an anomaly his tables do not admit it. He therefore judges that that bright small circle was either a denser part of the aether set alight by contact with the Sun, or the Sun's rim optically enlarged by its own rays refracted in the air around the Moon. But our hypothesis needs no such subterfuges, and, with the credit of other observations unimpaired, upholds also this annular eclipse seen by Clavius; for it gives the diameter of the Sun as 31′ 28″ and of the Moon 30′ 54″, at an anomaly of the Sun of 69° and of the Moon of 100°.

[Margin: Year 1569. ☾ Total.]

The eclipse of the Moon on March 3, observed at Leuven by Cornelius Gemma, from book 2 of the *Cosmocritice*: after midnight, with dreadful colours, blood-red, crimson, green, livid, etc. I have seen Kepler on it, p. 274 of the *Astronomia Optica*.

[Margin: Year 1572. ☾]

The eclipse of the Moon on June 25, observed by Mästlin, of 6 digits, although Gemma, in book 2 of the *Cosmocritice*, affirms that about 8 digits were seen by him. See Kepler in the *Optics*, p. 349.

[Margin: Year 1573. ☾ Total.]

The eclipse of the Moon on December 8 at 8^h 3^m after noon, total, observed by Tycho at Uraniborg, from volume 1 of the *Progymnasmata*, p. 114, and by Longomontanus in the *Astronomia Danica*, book 1 of the *Theorica*, chapter 2. Wendelen, p. 43, computes it at 7^h 50^m at Uraniborg, and says that the sooty darkness of the penumbra drew out a spurious end.

[Margin: Year 1576. ☾]

The eclipse of the Moon on October 7, observed by Tycho at Uraniborg at 11^h 32^m after noon; the rest did not show. ✢ Egnazio Danti, in the *Astrolabe*, says that it was observed by him at Rome at 11^h 15^m, although it is established from elsewhere that Rome is more easterly than Uraniborg.

[Margin: Year 1577. ☾ ☾ Total.]

The 2 total eclipses of the Moon observed at Uraniborg by Tycho, from volume 1 of the *Progymnasmata*, p. 114. The first was on April 2 at 8^h 50^m, unless the penumbra deceived him; for Wendelen says in the *Eclipses* that it ought to have been at 9^h 15^m. ✢ But Egnazio Danti says that at Rome it was at 8^h 40^m, from his own observation, in the *Astrolabe*. The second was on September 26 at 13^h 3^m after noon; Wendelen's calculation, p. 44, requires the hour 13^h 17^m, from which he infers that Tycho's observation or calculation is spurious. ✢ [Margin: A note on the difference of meridians.] He adds that it was observed at Madrid by Lupo Velasco [Juan López de Velasco], at its end, at 2^h 16^m after midnight; at Toledo by Io. Alcantara [?] at 2^h 12^m; at Valladolid [Valle-Oletana] by Doctor Sobrino at 2^h 8^m; and at Seville by Roderico Zamorano [Rodrigo Zamorano] at 2^h 4^m. In Mexico, moreover, at S. Ioannem de Luâ [San Juan de Ulúa ?], the same end was observed at 7^h 50^m after noon, and at Angelopolis in Mexico at 7^h 36^m. From this Andreas Garcia Cespedius [Andrés García de Céspedes], because Mexico is 1° more westerly than Angelopolis, pronounced Mexico to be 100° more westerly than Toledo. These observations were undertaken both to correct the map of Spain and to settle the dispute between the Castilians and the Portuguese over the meridian dividing the western lands and the Moluccas from the eastern ones.

[Margin: Year 1578. ☾]

The eclipse of the Moon of 2½ digits, observed by Tycho at Uraniborg, as above, on September 15 at 13^h 17^m after noon. ✢ Wendelen adds, p. 45, that its end was observed at Toledo by Lupo Velasco at 13^h 20^m, and by others at Angelopolis at 6^h 46^m.

[Margin: Year 1579. ☉]

The eclipse of the Sun on February 25, observed by Tycho at Uraniborg at 5^h 50^m after noon, of 5 digits 50′, whose exeligmus [exeligmum] the eclipse of the year 1633 will reproduce, as Wendelen says, p. 46.

[Margin: Year 1580. ☾ Total.]

The eclipse of the Moon on January 31, total, observed at Uraniborg by Tycho, from volume 1 of the *Progymnasmata*, p. 114, at 10^h 9^m after noon, or at 10^h 10^m as Longomontanus has it. ✢ And Mästlin says in the *Isagoge* that it was observed by him as total, *but that it lay wholly hidden in the shadow for a very short time.* ✢ And as is said in the *Astronomia Optica*, Kepler at Vittemberga [Vittembergæ] at 10^h, *while still a boy, with his father pointing it out, saw it whole and red on every side*; and my hypothesis favours these things. [Margin: Disagreement of the calculation from observation.] Wendelen's calculation, p. 46 of the *Eclipses*, gives it as total with duration of about 4^m, but computes the middle at Uraniborg at 10^h 30^m. He therefore thinks that the penumbra deceived Tycho; [Margin: Wendelen's birth.] and he adds that he himself was born in this year, on June 6. But Lansbergen's calculation, p. 79 of the *Treasury of Observations*, gathers 11 digits 51′; and so he adds: *The Moon failed very nearly wholly, so that the remaining particle of light could hardly be distinguished from the darkened part.* On this night, moreover, Henry, King of Portugal, died, according to Henri de Sponde in the *Epitome Annalium*.

[Margin: Year 1581. ☾ ☾ Total.]

The 2 eclipses of the Moon observed at Uraniborg by Tycho. The first, on January 19, total, at 9^h 59^m after noon, or, as Longomontanus has it, at 10^h. Wendelen, by his own calculation, p. 47, gathers 10^h 6^m. The second, on June 15, total, at 16^h 57^m after noon; but Longomontanus has 17^h. Wendelen's calculation, p. 47, shows 16^h 45^m at Uraniborg. [Margin: A blemish in Tycho's observation.] He adds that beyond the beginning the remaining phases did not show to Tycho, since elsewhere, in the letters [in Epiſtolis], he writes: *This eclipse was observed at least so far that the Moon had entered half of the shadow before it set*; and therefore the rest were added from conjecture or calculation.

[Margin: Year 1584. ☉]

The eclipse of the Sun on April 30, in the old style of the year, observed by Tycho, as above, at Uraniborg at 5^h 39^m after midnight, of 3 digits 0′. Wendelen, p. 49, by his own calculation gives 5^h 36^m.

[Margin: Year 1584. ☾ Total.]

The total eclipse of the Moon on November 7, old style, observed at Uraniborg by Tycho at 13^h 12^m after noon; but in the *Epistolae Astronomicae*, p. 72, he has 13^h 8^m, and Longomontanus 13^h 9^m. ✢ But since Tycho says in the letters that the beginning was at 11^h 12^m and the end at 15^h 0^m, and the duration accordingly is 3^h 48^m, it seems to Wendelen, p. 48, that the middle ought to have been put at 13^h 6^m; and so he adds: *From this he makes it plain that he is no longer reciting his observations faithfully, but presenting the calculation of his*

tables. But it is rather Wendelen himself who (not in bad faith, indeed, but from forgetfulness, I would believe) omits Tycho's words in the letter of January 20 of the year 87, in which he says, on p. 72: *I found that the very first beginning was at 11^h 12^m from noon of November 7; the middle at 13^h 8^m; the very last end at 15^h 0^m. (For these times are not always equal, as has been believed up to now.)* On this matter, however, see what we have said in chapter 6, in the Scholia. [Margin: Wendelen's excessive censure of Tycho.] Wendelen nevertheless goes on and says: *Which is surely unworthy of that man, and an excessive unconcern about censure; and so we rightly understand that all credit in him, in other matters too, is weak.* But go more sparingly with such words toward a man who has deserved so well of Astronomy. For if later, rereading his calculations, he judged, with everything weighed, that two or three minutes should be added, did he on that account deserve so harsh and so sweeping a censure? Let Wendelen remember that saying of Horace:

We know it, and this indulgence we both seek and grant in turn.

Yet I would rather that Tycho had indicated the reasons for the correction, so that it might be clear that he had not departed from the observation when better considered. Tycho, moreover, adds in the same letter: *As for Wittich's noting the middle of this failing at Kassel at 13^h 3^m, as you wrote to me, it lacks exact precision. For your meridian is at least a fifth part of an hour more westerly than ours, as I also said above.* ✢ Further, Wittich, as Rothmann writes to Tycho, p. 88, using the Landgrave of Hesse's pair of clocks, corrected by the azimuths of the right shoulder of Orion, found at Kassel the beginning at 11^h 20^m, the immersion at 12^h 20^m, the emersion at 13^h 45^m, the repletion at 14^h 45^m; from which he concluded the middle at 13^h 3^m. But Rothmann admits that he had not attained the required precision. ✢ Tycho further adds this: *Heinrich Brucaeus, the distinguished mathematician at Rostock, a singular friend of ours, observed the same eclipse there and found its middle at 1^h 4^m after midnight [post M. N.], a time slightly earlier than ours, when rather it ought to have been almost the same, since the difference of the meridians is imperceptible. But he, by means of a quadrant, of brass indeed, but very small, granted to him by me some time ago, from altitudes of stars taken only in degrees, was examining the times in a broader manner; and this, especially in stars moving toward the meridian, is exposed to evident error.* [Margin: Wendelen's correction of Tycho, itself needing correction.] But Wendelen, passing over this remark of Tycho's, and relying rather on Brucaeus's observation, goes on to criticise Tycho's, and finally concludes from his own calculation that the duration of this eclipse was 3^h 36^m, not the 3^h 48^m of Tycho or the 3^h 25^m of Wittich. But of course Wendelen omits the equation of natural days: he is pleased to have omitted it when observations favour his calculation, but when they differ markedly, he carps at the observations of others.

[Margin: Year 1587. ☾]

The eclipse of the Moon on September 6, old style, observed at Uraniborg by Tycho, of 9 digits 45′, at 9^h 16^m. But Longomontanus has 9^h 30^m, certainly

a great error or disagreement. Wendelen, p. 49, by his own calculation shows 9^h 16^m, in remarkable agreement here.

[Margin: Year 1588. ☉]

The eclipse of the Sun on February 16. Tycho at Uraniborg observed the beginning at 1^h 32^m and the end at 2^h 51^m. At the start the failing is drawn, in Tycho's papers [Schedis], as inclining to the right by 36°, and at the end by 12° or 15°, again to the right, that is, the West, as Kepler reports in the *Astronomia Optica*, p. 414, where he weighs these inclinations in a rather lengthy calculation.

[Margin: Year 1588. ☾ Total.]

The total eclipse of the Moon observed at Uraniborg by Tycho on March 2, old style, at 15^h 2^m. And in the letters he says: *The clearness of the sky was moderate, and it was found that the middle exceeded the Prutenic calculation by half an hour*; which calculation, however, would give 14^h 18^m, says Wendelen on p. 50, adding from Kepler these words noted in Tycho's papers at the hour 2^h 58^m: *Now the Moon appeared brighter toward the East: therefore it has passed the middle.* Granted that from other phases he set the hour, in the *Progymnasmata*, p. 114, at 3^h 2^m after midnight; this was on account of the deceptiveness of the penumbra, says Wendelen; ✢ who adds that the beginning was observed by him at Herk about the middle of hour 1 after midnight, the immersion after the middle of hour 2, the emersion immediately after the third, and the end a little after hour 4, his father, the prefect of the Watch, regulating the clock. ✢ At Lisbon, however, Doctor Sobrino observed the beginning at 11^h 56^m. [Margin: A note on the difference of meridians.] But Wendelen adds: *Unless perhaps he meant to write 11^h 36^m; for then the agreement with the beginning at Herk would be stupendous.* ✢ Finally a Florentine nobleman, at the Cape of Currents [promontorium Currentium] beyond the Cape of Good Hope, noted the beginning at 4^h 15^m.

[Margin: Year 1588. ☾]

The eclipse of the Moon on the 26th coming into the 27th of August Julian, but the 6th into the 7th Gregorian, observed at Lisbon by the most learned Sobrino, began at 1^h 49^m after midnight. ✢ Its beginning was observed at Portus Vetus [Puerto Viejo] in Peru, under the Equator, at 8^h 54^m after noon, and was so reported in writing to the King, as Wendelen says, p. 51, [Margin: A note on the difference of meridians.] who gathers from this a difference of longitude of 4^h 55^m, or 73° 45′.

[Margin: Year 1589. ☾]

The eclipse of the Moon on August 25 Gregorian: Wendelen's father and others noted the end of the eclipse at about a quarter before the eighth hour; Wendelen's calculation, p. 51, requires 7^h 47^m from noon.

[Margin: Year 1590. ☾]

The eclipse of the Moon on July 17, in the morning. Mästlin at Tübingen saw the Moon failing by several digits from the South, and still almost two degrees

high above the horizon, although the centre of the Sun was already emerging above the horizon; and as the Moon's centre went down below the horizon, he saw the Sun raised by about 2°. Kepler attributes this to extraordinary refraction, in the *Astronomia Optica*, p. 137; but Wendelen, in the *Eclipses*, p. 51, says that a single degree is to be ascribed to the refraction of both luminaries, the rest to the parallaxes, since the Moon was far to the North. Mästlin adds that the Moon set before the greatest obscuration. Wendelen's calculation grants the beginning at Tübingen at $3^h\ 38^m$ after midnight, and the middle at $4^h\ 28^m$, 13^m after sunrise.

[Margin: Year 1590. ☉]

The eclipse of the Sun on July 31, new style, observed at Uraniborg by Tycho, of 5 digits, at $7^h\ 54^m$ after midnight. But on this I find, in Magini's *Tables of Directions*, p. 81, what was written in a letter of Tycho's to Magini from Uraniborg, of the year 590[48], December 1 [Cal. Decembr.], in these words: *The last eclipse of the Sun occurred here, as to its very first beginning, when $6^h\ 51^m$ had elapsed from midnight; the very last end came at exactly the ninth hour. And no more than four digits of the Sun were covered, that is, a third part of it, although the canons promised, in our climate, eight, and an interposition of the Moon twice as great.* He is speaking of the Prutenic canons. [Margin: Tycho's account is suspect.] But why in the *Progymnasmata*, p. 114, does he assign 5 digits to this eclipse? Was it to draw the observation toward his own tables? ✢ Mästlin observed it at Tübingen at $7^h\ 36^m$, as Kepler relates in the *Optics*, pp. 406 and 4[?]1; and on p. 396 he says that the diameter of the ☉ was observed to be to that of the ☾ as 24 to 23. Lansbergen, in the *Treasury of Observations*, p. 133, gathers 9 digits at Tübingen; and in the *Uranometria*, p. 111, he says, wrongly, that the middle was observed by Mästlin at $7^h\ 44^m$.

[Margin: Year 1590. ☾ In Gregorian reckoning 1591.]

The eclipse of the Moon on December 30, old style, but in Gregorian reckoning in the year 1591, on January 9, observed at Uraniborg by Tycho at $6^h\ 55^m$, or, as Longomontanus has it, at 7^h. But Tycho admits that he could not have all the phases. Wendelen, p. 52, computes $6^h\ 50^m$, omitting, as usual, the equation of natural days.

[Margin: Year 1591. ☉]

The eclipse of the Sun on July 10, old year, observed at Uraniborg by Tycho, of 2½ digits, at $3^h\ 33^m$ after noon.

[Margin: Year 1592. ☾]

The eclipse of the Moon on June 14, old style, observed at Uraniborg by Tycho, of 8 digits, at $10^h\ 16^m$ from noon, as Tycho has it in volume 1 of the *Progymnasmata*, p. 114, or at $10^h\ 20^m$, as Longomontanus has it in book 2 of the *Theorica*, chapter 2. But let us hear Kepler on it, in the *Astronomia Optica*, p. 350: *Tycho Brahe*, he says, *in the year 1592, on June 14, at the start observed by the radius the diameter of the Moon, still whole, as 32′. Then about the middle of*

[48]that is, 1590

the eclipse he found 14′ remaining, so that 18′ of these had failed. But this quantity departs much from his own calculation, which claims 26¼′ of the Moon's body for the shadow. And so I attribute this to the difficulty of observing. For even if someone should call into doubt the Tychonic calculation, or the diameter of the Moon, he will still never bring it about that this eclipse with this quantity squares with the others to one standard. Hence Wendelen, in his *Eclipses*, p. 54, concludes that there is nothing sound in this observation except the beginning of the failing, and the middle, which from this is reckoned to have occurred later.

[Margin: Year 1592. ☾]

The eclipse of the Moon on December 8, old style, or the 18th Gregorian, observed at Uraniborg by Tycho at $7^h\ 41^m$, to which is added *it did not show*. ✢ But Lansbergen, p. 83 of the *Treasury of Observations*, says that it was observed by him at Goes, of almost 5 digits, with the end at 8^h and the middle at $6^h\ 51^m$; but from calculation he gathers 4 digits 46′. Wendelen's calculation gives the middle at Goes at $6^h\ 49^m$.

[Margin: Year 1593. ☉]

The eclipse of the Sun on May 20 at Zerbst [Seruestæ], where the Sun was observed to fail, to the extent of two digits, from the lower part, the failing inclining at the start a little to the left, and at the end more. The hour was about $2^h\ 40^m$: Kepler in the *Optics*, p. 414.

[Margin: Year 1594. ☾]

The eclipse of the Moon on October 19, old style, observed at Uraniborg by Tycho, at the hour after noon $19^h\ 26^m$, with the addition *it did not show*. But Longomontanus has $19^h\ 16^m$. ✢ In the *Optics*, moreover, p. 413, Kepler adds, from Tycho's papers, that in the morning at $5^h\ 56^m$ it began to be darkened at Uraniborg at the top of the limb or a little to the right. ✢ Lansbergen, p. 84 of the *Treasury of Observations*, relates that the beginning was observed by him at Goes at $5^h\ 12^m$ from midnight, and 9½ digits; but from his own calculation 9 digits 21′, and in the middle $6^h\ 48^m$. Wendelen, p. 55, says: *My calculation grants the beginning on Hven at $5^h\ 56^m$ and the middle at $7^h\ 28^m$, which is A STUPENDOUS AGREEMENT, and by itself alone commends all my hypotheses.* All of them? Indeed, if in all of them there were such agreement. But why here is Tycho's observation not condemned, and no recourse had to the penumbra, while elsewhere it is otherwise? Is it not from an already presumed esteem for the Atlantic Tables?

[Margin: Year 1595. ☾ Total.]

The eclipse of the Moon on April 13, old style, observed at Uraniborg by Tycho at $16^h\ 36^m$, total. ✢ Lansbergen, p. 85 of the *Treasury of Observations*, says that the beginning was observed by him at Goes at $13^h\ 40^m$, and in his calculation sets 19 digits 40′. ✢ Kepler, in the *Astronomia Optica*, p. 367, says that the beginning was observed by him at Graz in Styria [Greek: ὁλοσχερῶς (holoscherōs, "roughly", "in general terms")] *with a paper astrolabe of three-quarters of a foot, suspended*; but he admits that the observation is not altogether

most certain, and that on Hven the beginning should in Tycho's reckoning have been at $14^h\ 51^m$. ✢ Wendelen, p. 55, relates that the beginning was observed at Herk when hour 2 from midnight had scarcely been heard, the immersion at the very stroke of 3, the emersion at about the hour $4^h\ 38^m$; and he contends that at Goes the middle ought to have been at $3^h\ 40^m$.

[Margin: Year 1595. ☉]

The eclipse of the Sun on September 23, old style, observed at Uraniborg by Tycho, of 3 digits 50′, at $1^h\ 8^m$ after noon. ✢ But Mästlin, in the disputation on eclipses published in the year 96, thesis 53, says: *At the beginning, a little after noon, the failing edge of the Sun inclined from the vertical not to the West but to the East by 9 degrees, and it failed by 2 digits and a half, and nothing beyond that, as careful observation at Tübingen attests.* ✢ Kepler adds in the *Optics*, p. 416, that at Strasbourg the beginning was observed before hour 12, and the end at exactly the first hour, and that all the diagrams show the shadow always inclining from the vertex to the left. ✢ For himself, he says, at Graz the Sun began to be darkened at the vertex when the Sun was 51⅙° distant from the vertex, by a small wooden quadrant; and when three digits had failed in the image on the tablet, the Sun stood 55° from the vertex; and a little later it began to incline from the vertex by 15°; and the apparent diameter of the Moon was less than that of the Sun. At Uraniborg, moreover, the end was observed at $2^h\ 5^m$, and the digits darkened there were 4.

[Margin: Year 1595. ☾ Total.]

The eclipse of the Moon on October 7, old style, observed at Uraniborg by Tycho, total, at $20^h\ 29^m$ from noon, which Longomontanus also has, and not $20^h\ 36^m$, as Wendelen, p. 57, reports; but from his own calculation he required $20^h\ 44^m$. The middle, moreover, was set by Tycho from the observation of the beginning and from conjecture or calculation.

[Margin: Year 1596. ☾]

The eclipse of the Moon on April 2, old style, observed at Uraniborg by Tycho at $9^h\ 29^m$; the rest did not show. Longomontanus has $9^h\ 25^m$. [Margin: A huge disagreement of Tycho's observation from calculation.] Here Pétau, in book 8 of *De doctrina temporum*, chapter 13, is at a loss, seeing that Tycho's observation differs greatly from the calculations of all the tables: the reason being, as Wendelen says on p. 58, that Tycho saw only the beginning, and the middle did not show. From his own Atlantic Tables he says that the middle at Uraniborg ought to have been at $8^h\ 45^m$, especially since at Tournai [Tornaci] ✢ the beginning was observed at 7^h and the end at $9^h\ 15^m$. If these things are true, Tycho's note of the observation differs from the truth by a full 44^m.

[Margin: Year 1598. ☾ Almost total.]

The eclipse of the Moon on February 9, old style, in Tycho's catalogue, p. 114, of 11½ digits, at $18^h\ 7^m$ after noon, in any case either from calculation or from Tycho's pupils, for Tycho had already left Denmark; or at $18^h\ 5^m$, as Kepler has it in the *Optics*, p. 368, adding *(the observation of the beginning aided*

by calculation, for the end was under the Earth). ✢ Kepler had, moreover, reported his own observation at Graz in Styria with a wooden quadrant of a span, and with clouds in the way; yet from it he conjectures the beginning at just before hour 5. ✢ At Leuven it began a little after hour 4 in the morning, or rather at $4^h\ 10^m$, as Wendelen says, p. 59. ✢ Lansbergen at Goes noted the beginning at $15^h\ 40^m$ and 11⅖ digits. ✢ At Aachen Io. Rotarius [?], on Lansbergen's testimony, observed the beginning at $4^h\ 30^m$, but gathered the middle at $5^h\ 30^m$ from midnight. In these observations Wendelen suspects deceit or carelessness, and trusts Kepler's rather, because it conspires with his own calculation, which gives the beginning at Graz at $4^h\ 54^m$. Origanus by his calculation makes it total, although all those above observed it as not wholly total; ✢ and Kepler says that it was observed at Wandsbek [Vandesburgi] as not quite total. My tables show it not total, but very nearly total.

[Margin: Year 1598. ⊙ Annular.]

The eclipse of the Sun on February 25, old style, of 9 digits 20′, at $0^h\ 44^m$ before noon, in Tycho's catalogue, p. 114. But Tycho himself, at Wandsbek [VVandesburgi], observed the beginning by the armillary spheres at $9^h\ 12^m$ after midnight, as Kepler says in the *Optics*, p. 383, adding, ✢ that on Hven a pupil of Tycho's observed the beginning at $10^h\ 3^m$, the end at $12^h\ 32^m$, and 9 digits 40′. ✢ At Graz, indeed, Kepler himself, lacking suitable instruments and under a cloudy sky, still somehow obtained the end at $12^h\ 55^m$, and conjectured the beginning from calculation at $10^h\ 19^m$. ✢ Lansbergen, from Kepler, in the *Treasury of Observations*, p. 114, relates that at Torgau in Meissen [in Miſnia] the eclipse was observed by Doctor Iessenio [Jan Jessenius] as almost central, with the Moon placed within the Sun and the entire margin of the Sun standing out like a bright circle. But Kepler, in the *Optics*, p. 299, in *De Stella Nova*, chapter 23, and in the *Epitome of Copernican Astronomy*, p. 895, contends that it was the brightness of the air, inflamed by contact with the Sun, because his hypothesis gives nothing else; further, he hunts out the times of this eclipse by a rather lengthy calculation in the Rudolphine Tables, p. 111, and its inclinations in the *Optics*, p. 419. ✢ Origanus, as he testifies in the ephemeris of this year, observed at Frankfurt on the Oder 9 digits at $11^h\ 28^m$ and again at $11^h\ 46^m$, and the beginning at $10^h\ 20^m$ and the end at $12^h\ 32^m$ from midnight; and he says that the Prutenic calculation errs, which shows it almost total. But I return to Lansbergen, who adds: *There was then such great darkness that it seemed to be a dusky night; yet the exact size of the failing could not be taken by us because of the cloudy sky*; that is, at Goes; and from calculation he gives 11 digits 15′. But Wendelen, p. 60, ascribes to him the saying *The Sun failed wholly*, which I find nowhere said by him. Finally Magini, in the *Tables of Directions*, p. 90, sets out the calculation of this eclipse under an elevation of the pole of 42° and a longitude of 36° 45′, as you see in this small table, and in hours after midnight of February 25.

[Margin: Disagreement of the tables.]

From the Tables of the Authors	Beginning	Middle	End	Duration	Digits
Alfonso	$10^h\ 46^m$	$11^h\ 44^m$	$12^h\ 42^m$	$2^h\ 56^m$	8 20′
Copernicus	$8^h\ 32^m$	$9^h\ 50^m$	$10^h\ 50^m$	$2^h\ 18^m$	11 26′
Tycho	$9^h\ 58^m$	$11^h\ 7^m$	$12^h\ 16^m$	$2^h\ 18^m$	10 6′

My Tables acknowledge this eclipse as annular, since they show the diameter of the Sun as 32′ 16″ and that of the Moon as 32′ 12″, at an anomaly of the Sun of 112° and of the Moon of 130°.

[Margin: Year 1599. ☾ Total.]

The eclipse of the Moon on January 30, old style, total, at $17^h\ 50^m$, as noted in Tycho's observations, p. 114 of volume 1 of the *Progymnasmata*. But Longomontanus has $17^h\ 56^m$, to whom Wendelen, p. 62, ascribes the observation, and he rejoices in a stupendous agreement with his own calculation, which shows $17^h\ 56^m$ on Hven. ✢ Kepler, p. 249 of the *Optics*, conjectured the beginning from the striking clocks [ex Horiſonijs] at $3^h\ 45^m$ from midnight, at Graz.

[Margin: Year 1599. ☉]

The eclipse of the Sun on July 12, old style, the 22nd Gregorian, at $4^h\ 8^m$ after midnight, of 3 digits, reported by Tycho, p. 114 of volume 1 of the *Progymnasmata*, and referred to Uraniborg, though from an observation at Prague; for Kepler says in the *Astronomia Optica*, p. 414, that at Prague a slight failing was observed by Tycho at the rising of the Sun, almost at the very vertex of the Sun's body.

[Margin: Year 1599. ☾]

The eclipse of the Moon on August 6, seen by the Dutch at Tidore and Ternate about hour 8 of the night, as Levinus Hulsius relates in chapter 30 of the Dutch voyages [nauigat. Holland.].

[Margin: Year 1600. ☾]

The eclipse of the Moon on January 30 in the morning, observed at Calidianum [?] in Provence, a town six minutes farther east than Aix, by Govaert Wendelen. The Moon began to fail at $5^h\ 45^m$ after midnight, and 2 digits failed toward the North; and it set, still in eclipse, as hour 7 was striking, the failing having lasted more than an hour; and after a quarter-hour the Sun rose, at the hour there of $7^h\ 10^m$, which corrected the striking clock, says Wendelen in the *Eclipses*, p. 65. For him the Atlantic calculation, assuming equal natural days and the Moon's oscillatory prosthaphaereses, as I explained in book 4, chapter 27, no. 17, gives the beginning at Calidianum at $5^h\ 46^m$, but the Tychonic at $5^h\ 49^m$.

[Margin: Year 1600. ☉]

The eclipse of the Sun on June 30 Julian, but July 10 Gregorian. In volume 1 of Tycho's *Progymnasmata*, p. 114, it is set at 5 digits 0′ and at $1^h\ 44^m$ after noon at Uraniborg. ✢ But Kepler in the *Optics*, from p. 422 to p. 430, setting out at length his own observation and those of others together with the calculation, says that Tycho's people at the castle of Benatek [Benátky], which lies 5 German

miles from Prague toward the north-east [verſus Euroaquilonem], observed the middle at 1^h $46½^m$ from noon, and about 5 digits; that Mästlin at Tübingen observed nothing other than that the Sun failed notably beyond the middle; and that he himself at Graz observed the beginning at 12^h $37½^m$ from midnight, the middle at 13^h $47½^m$, the end at 14^h $57½^m$, and 7½ digits. And he says that he later disputed about the beginning with Tycho, who feared that this observation would destroy his own, having obtained the beginning at 12^h 46^m. But in vain, because, as Kepler says on p. 427, he was farther south than Tycho by 3° and almost as many minutes, and saw the Sun more covered; and therefore he saw the beginning earlier and the end later, although he was farther east than Tycho. Magini, in the *Directions*, p. 91, gathers for the castle of Benátky, from the Tychonic tables, 4 digits 50′, and the beginning there at 0^h 47^m and the end at 2^h 47^m.

[Margin: Year 1601. ☾]

The eclipse of the Moon on December 9, on which Kepler says, in the *Optics*, p. 349: *A good part of the Moon's body indeed remained, but how much could not be discerned by anyone with confidence. Ambrosius Rhodius, at one time Tycho's calculator, estimated the failing at 10 digits at Wittenberg.* But on p. 371 he says that he himself observed the beginning by the Tychonic clock at 5^h 35^m, but from the culminations of the stars at 5^h 22^m; and the middle by the automaton at 7^h 7^m, but from the stars at 6^h 53^m. [Margin: A note on the difference of meridians.] ✢ Now Lansbergen, from p. 88 of the *Treasury of Observations*, observed at Goes the beginning at 4^h 35^m, the middle at 6^h 12^m, the end at 7^h 49^m, and 10 digits 37′ from the South. ✢ Adrianus Metius [Adriaan Metius], in the book *De usu Globi*, writes that he observed at Franeker in Frisia the beginning at 4^h 42^m, the middle at 6^h 30^m, the end at 8^h 18^m. ✢ Wendelen, in the *Eclipses*, p. 66, says that at Valensole [Valentiolæ] the Cluniacs observed the beginning a quarter of an hour after sunset, or at 4^h 38^m after noon, and the duration of exactly 3^h; but at Digne he and his people observed the beginning at 4^h 40^m, the end at about 7^h 45^m, and the middle at 6^h 11^m or 12^m. Magini, in the *Directions*, p. 92, by calculation gathers the digits: by the Alphonsine 11 digits 2′, by the Copernican 11 digits 17′, by the Tychonic 10 digits 55′.

[Margin: Year 1601. ☉ Annular.]

The eclipse of the Sun on December 24, observed at Goes by Lansbergen at 1^h 51^m after noon, in such a way that the whole Moon was within the Sun, and the northern margin of the Moon coincided with the northern margin of the Sun, while the southern margin stood 6¾′ from the Sun's margin. And he adds, but from the tables, that the diameter of the Moon at apogee was 29′ 15″, and that of the Sun, almost at perigee, 35′ 18″, from p. 103 of the *Treasury of Observations*. ✢ Kepler, in the *Astronomia Optica*, pp. 359 and 430, says that he observed it at Prague at about 2^h 53^m from noon, of 8 digits and more, and that the diameter of the Moon by the radius on the tablet was to the diameter of the

Sun as 30½ to 31. [Margin: A note on the diameters of the luminaries.] And in the Appendix to Tycho's *Progymnasmata* he says that this eclipse, seen by Tycho's people shortly after Tycho's death, did not agree with the diameters of the luminaries established by Tycho, even if it did not differ in other respects; and that this disagreement is removed if the Sun's diameter is assumed as 31 and the Moon's as 30½. This he says from his own hypothesis, as Lansbergen also does. But these hypotheses do not satisfy certain other eclipses, as was said in book 4, chapter 16.

☩ Longomontanus, in book 1 of the *Theorica*, chapter 9, reports that he observed this eclipse 18 miles beyond Trondheim [Nidrosiam], under an elevation of the pole of 64° 40′, at hour 1 after noon, on the horizon, with the Sun, because of the lunar parallaxes and the marked refraction, very conspicuous as to its upper limb. And when he had come to Bergen in Norway, he was shown by M. Andrea Fossio [Anders Foss], a distinguished theologian, botanist, historian and mathematician, a delineation of this failing, such as he had drawn from the report of fishermen; in it the Sun so surrounded the Moon on every side that the bright circle of the Sun stood out by about a digit and a half. [Margin: Note 2 on the diameters of the luminaries.] From this he strives to confirm that the visible diameter of the Moon in an ecliptic new Moon is diminished, and that of the Sun enlarged, especially where the vapours are thick. But although the image of bright things appears larger next to black ones, owing to its predominance in our eye, and although horizontal refraction optically enlarges the diameters of the stars, as we showed in book 4, chapter 15, and will teach more fully in the book on optics when we solve the objections, nevertheless this second enlargement is common to both luminaries and to the edges of the Moon lit by the Sun, and the first is no greater than what appears in a new Moon with its ring standing out, which is scarcely one or two minutes. So the whole of the excess already mentioned, of a digit and a half on this side and a digit and a half on that, that is, of 3 digits, by which the Sun's diameter was seen to exceed the Moon's, cannot be attributed to that optical enlargement, but at most two or three minutes of it. The rest must therefore be attributed to the ordinary excess of the Sun's apparent diameter over the Moon's. Now the Sun's anomaly was then 176° and the Moon's 7°, at which, for us, the Sun's diameter is ordinarily 33′ 8″ and the Moon's 28′ 4″. Let it be, therefore: as 28′ to 12 digits, so 33′ to 14⅐. For two of the three digits of excess will be owed to the Sun's ordinary excess, and the remaining one digit to the optical enlargement, that is, three minutes. And so we aligned our hypothesis for reconciling the diameters of the luminaries, in book 4, chapter 16, with this eclipse too, but with others along with it.

Moreover Scheiner reports in the *Rosa Ursina* that at Villeregia [?] in Spain some stars were seen in this eclipse. Finally Magini, in the *Tables of Directions*, p. 92, derives for Prague, from the Alphonsine calculation, 7 digits 36′; from the Copernican 9 digits 19′; from the Tychonic 9 digits 33′, with the beginning at $1^h\ 7^m$ and the end at $3^h\ 38^m$ after noon.

[Margin: Year 1602. ☾]

The eclipse of the Moon on June 4, in the evening, observed at Prague by Kepler, as he has it in the *Astronomia Optica*, p. 369: he observed the end with the help of a Tychonic clock showing minutes and seconds, regulated by the Sun's altitude. He observed it at $9^h\ 3^m$, and yet he admits that the Tychonic calculation requires $9^h\ 19^m$, taking the equation of time, the diameters and the difference of meridians from Tycho. *These*, he says, *can be changed without harm to the hypothesis that represents the very moments of the oppositions.* Wendelen's calculation, p. 69, gives, without the equation of time, the end at Prague at $9^h\ 5^m$.

[Margin: Year 1602. ☾]

The eclipse of the Moon on the morning of November 29, seen, contrary to the expectation the ephemerides had raised, by Wendelen, from the *Eclipses*, p. 70, who says that it began at Aix at $7^h\ 10^m$ after midnight. Origanus, however, by the Tychonic calculation, gives $7^h\ 30^m$ there, but with the equation assumed that Wendelen neglects.

[Margin: Year 1603. ☾]

The eclipse of the Moon on May 24, observed at Prague by Kepler, as is found in his *Optics*, at $12^h\ 32^m$ after noon. But because he adds that at $12^h\ 44^m$ *the horns were in a straight line with Jupiter, and the more westerly part was now being raised, which indicated the middle*, Lansbergen, on p. 90 of the *Treasury of Observations*, therefore gathers that the middle at Prague was at $12^h\ 40^m$ and not at $12^h\ 32^m$, which Kepler, he says, gathers from a faulty end and beginning. ✢ Lansbergen himself, however, says that he observed the middle at Goes at $11^h\ 56^m$, and 7½ digits from the South. But Wendelen, in the *Eclipses*, p. 72, because Kepler had said that at $12^h\ 25^m$ *the Moon stood on its back with its horns equally lowered*, and afterwards at $12^h\ 44^m$ *the horns with Jupiter*, etc., gathers from the places of Jupiter and the Moon and from the parallaxes that the horns were not parallel to the Moon's path at that time, but about half an hour before, and that the middle of the eclipse fell at $12^h\ 18^m$ at Prague. [Margin: Disagreement of observations.] ✢ He himself, moreover, at Valensole, observed the beginning at $10^h\ 14^m$, the middle at $11^h\ 38^m$, the end at $13^h\ 2^m$, the duration $2^h\ 48^m$, and 8 digits. Further, he says that Lansbergen could not have had the middle at Goes except at $11^h\ 33^m$, and that he was deceived by the penumbra or by his striking clocks, or else is deceiving us in order to foist his own tables on us. Finally he adds that by the Tychonic calculation the middle comes out at $12^h\ 40^m$ at Prague, but by his own at $11^h\ 38^m$ at Valensole and $12^h\ 18^m$ at Prague.

[Margin: Year 1603. ☾]

The eclipse of the Moon on November 18, observed by Wendelen, from p. 73: at Digne the beginning at about $5^h\ 52^m$, the end exactly at $7^h\ 4^m$, 3 digits. From this he sets the middle at $6^h\ 47^m$, as also from the Atlantic calculation. ✢ At Prague Kepler, as is found in his *Optics*, p. 417, observed the same eclipse.

He says that it began when it inclined 65° from the vertex to the left, and that it ended with the shadow not reaching quite to the vertex but still inclining to the left toward the east, since the Moon was descending southward and the failing was southern, in the eastern quadrant. Again he says that it began ten minutes after the right shoulder of Aquarius had culminated, with the Sun in ♏, 25° 55′, from which he infers the hour $6^h\ 21^m$; and that it ended three minutes after the head of Andromeda had culminated, and therefore at $8^h\ 17^m$, and that it lasted $1^h\ 56^m$, although the Tychonic calculation gives $2^h\ 10^m$. He adds that at the middle it seemed to him to be darkened by less than a quarter, and to others by more than a quarter, and that of the circumference less than ⅓ and more than ¼ was missing. He concludes: *The middle was at $7^h\ 19^m$, the very minute that Tycho's calculation shows on the meridian of Hven for the true conjunction in longitude, which differs somewhat from the middle of the eclipse.* [Margin: A note for geography.] From these, considering the end of the eclipse, Wendelen derives a difference of meridians between Digne and Prague of 35^m; but from the eclipse of the year 1601, of 41^m. He therefore chooses the mean, 38^m, the same number he had derived from the spring eclipse of this year.

[Margin: Year 1605. ☾]

The eclipse of the Moon on April 3, observed at Tübingen by Mästlin, who published a little book on it, and says in Thesis 152: *In the lunar eclipse on the evening of Palm Sunday in the year 1605, there was seen in the body of the Moon, toward the north, a certain blackish spot, darker than all the rest of the body, which presented the shape of red-hot iron. You would have said they were clouds extended over a wide region, heavy with rain and stormy showers, of the kind one not rarely happens to see from the ridges of high mountains, in the lower places of the valleys.* ✢ Lansbergen observed the same eclipse at Goes, from p. 92 of the *Treasury of Observations*, at $8^h\ 35^m$. He says that it lasted $3^h\ 18^m$ and that 11½ digits failed from the North, but from calculation 11 digits 25′. ✢ But Wendelen, in the *Eclipses*, p. 75, says that when he observed at Forcalquier, which is about midway between Digne and Aix, the beginning was at $6^h\ 37^m$ or 38^m and the greatest failing at $8^h\ 12^m$, of no fewer than 11 digits 40′, and that the horns were then parallel to the Moon's path, an argument for the middle of the eclipse, and that the remainder was not visible. And he adds that Lansbergen prolonged the end of the failing, and hence the middle, because of a spurious end due to the penumbra.

[Margin: Year 1605. ☾]

The eclipse of the Moon on September 26, whose middle Lansbergen observed at Goes at $16^h\ 30^m$ from noon, of 8 digits 30′, but from calculation he puts it at 8 digits 22′, p. 93 of the *Treasury of Observations*. ✢ Wendelen, p. 75, says that at Forcalquier he, with his pupil Scipio Arnaudus [?], observed the beginning at $2^h\ 42^m$ from noon [?], by night, fairly accurately on a sundial; that the digits did not much exceed 8; that it ended at $5^h\ 40^m$, the middle being at $4^h\ 11^m$. [Margin: A note on the difference of meridians.] He adds that Lansbergen,

while foisting on us a more prolonged calculation, cannot have seen the end at Goes, and is writing a refutation of himself. [Margin: Wendelen's untimely censure of Lansbergen.] But Lansbergen did not gather the middle from the end, but from the moment of the middle itself, compared with the observations. For the eclipse was partial, and from the earlier half-duration he gathered the later one, and only from this the end.

[Margin: Year 1605. Sun. Total.]

The eclipse of the Sun on October 12, Gregorian style, observed at Middelburg by Io. Rotarius [?] at $1^h\ 15^m$ after noon, of more than 10 digits and less than 11, from the South, as Lansbergen reports, p. 118 of the *Treasury of Observations*. ✢ But at Naples it was observed to be exactly total, or with a very brief stay, although the air all around appeared fiery, as Kepler reports in *De Stella Nova*, chapter 23, and in the *Epitome of Copernican Astronomy*, book 6, p. 893, adding that it was total, but that at Antwerp the whole Sun was not covered. ✢ So too Wendelen, from p. 77 of the *Eclipses*, saw the Sun at Forcalquier enter eclipse immediately after noon and leave it at $2^h\ 40^m$, the whole Sun being obscured except for a very thin thread of light toward the north, which he attributes not to the Sun but to the atmosphere shining through, either through our air or through the lunar air. ✢ He adds that at Marseille and on the nearby sea a total eclipse and dense darkness were observed. For me this eclipse was total, with the Sun's diameter 32′ 6″ and the Moon's 32′ 22″.

[Margin: Year 1607. Sun.]

The eclipse of the Sun, on the indication of which see the Scholia to chapter 2.

[Margin: Year 1608. Sun.]

The eclipse of the Sun on August 10, observed at Goes by Lansbergen at $4^h\ 8^m$ after noon, of 2 digits 30′, from the South, as he himself reports in the *Treasury of Observations*, p. 120. ✢ But Longomontanus, in the *Astronomia Danica*, book 2, chapter 9, says that it was observed at Wittenberg by Melchior I[?]östelius, of almost 2 digits; but that by him and by five other men of very sharp sight no trace of the eclipse could be noted at Copenhagen, although there ought to have been some; he puts the cause down to the thicker air of that place.

[Margin: Year 1609. ☾]

The eclipse of the Moon on January 19, observed at Goes by Lansbergen, as he reports on p. 94 of the *Treasury of Observations*: the beginning at $13^h\ 12^m$, the middle at $14^h\ 40^m$ after noon, 9 digits 30′ from the North, but from calculation 9 digits 36′. ✢ Adriaan Metius, in *De usu Globi*, says that he observed the middle at Franeker at $14^h\ 42^m$. ✢ Wendelen, from p. 77, observed at Forcalquier with Arnaudus the beginning at $1^h\ 10^m$ after midnight and the end at $4^h\ 16^m$.

[Margin: Year 1609. ☾ Total.]

The eclipse of the Moon on July 16, observed at Copenhagen by Longomontanus, who, reducing it to Uraniborg, says that it was at $12^h\ 10^m$.

☩ At Forcalquier, for Wendelen, from p. 78, it began to fail at 9^h 53^m and was immersed before the eleventh hour; soon clouds, thunder and lightning followed.

[Margin: Year 1610. ☾ ☾]

Two eclipses of the Moon. The first on July 6, observed at Forcalquier by Wendelen in the morning, 5^m before the third hour, and it set still failing. The second on December 30, about which Galileo writes in the letter prefixed to Kepler's *Dioptrice*; ☩ and Wendelen, p. 79, saying that at Forcalquier he observed the beginning a little before 2^h 40^m after midnight and the end at 4^h 50^m.

[Margin: Year 1612. ☾]

The eclipse of the Moon on the evening of May 14, which, as Wendelen says, Io. Remus [Johannes Remus Quietanus], physician and mathematician, observed at Rome but did not publish. ☩ At Liège Wendelen, as he relates in the *Eclipses*, p. 79, observed the beginning at about 8^h 37^m, the end exactly at 11^h 15^m, the duration 2^h 36^m, and 6 digits 30′. ☩ At Munich Father Christoph Scheiner noted the beginning half a quarter of an hour, as it were, before the ninth evening hour. He says that it ended at the twelfth hour itself, and that it exceeded the calculation in duration and magnitude, for it was of 8 digits. But Wendelen thinks that Scheiner counted the penumbra with the shadow. ☩ Longomontanus, in the *Astronomia Danica*, book 1 of the *Theorica*, chapter 9, writes that he and his people observed at Copenhagen the beginning with Regulus at an altitude of 35° 52′ in the west, from which he derived 9^h 14^m; but he says that it more probably happened a little earlier. The middle, with Spica at an altitude of 23° 57′ in the west, or at 10⅗ hours, and 6½ digits; although from his own calculation he gathers the beginning, in apparent time, at 9^h $13\frac{2}{3}^m$, the middle at 10^h $38\frac{2}{3}^m$, and 6 digits 35′.

[Margin: Year 1612. Sun.]

The eclipse of the Sun on May 29, observed by Father Christoph Scheiner at Munich, of 8 digits: the beginning at 10^h, the end at 12^h 45^m after midnight, as he relates in the *Apelles post tabulam*, p. 49. ☩ Longomontanus, in the *Astronomia Danica*, book 1 of the *Theorica*, chapter 9, says that he observed it at Copenhagen with seven others, and that the beginning was at 10^h $14\frac{2}{3}^m$, with the Sun at an altitude a little beyond 51°, and the end a little later than the calculation allows, which requires 12^h $23\frac{3}{4}^m$, because of the uncertainty arising from the Sun's almost meridian altitude; and 8 digits, although the Copernican calculation gives 9½.

[Margin: Year 1612. ☾]

The eclipse of the Moon on November 8, whose beginning was observed at Nagasaki at 9^h 30^m after noon by the most illustrious martyr Father Carlo Spinola of the Society of Jesus; ☩ but at Macau, on the borders of China, by Father Giulio Aleni, S.J., at 8^h 30^m, with the end at 11^h 45^m, from the Moon's altitude and azimuths. He adds that at Macau the altitude of the pole was

observed to be 22° 13′ to the north, and that Beijing is more easterly than Macau by 5°. Father Aleni wrote all these observations to Giovanni Antonio Magini, whose pupil he had once been at Bologna, and this letter, found among Magini's papers, was communicated to me by Magini's successor in the principal chair of mathematics and my very good friend, Father Bonaventura Cavalieri. This agrees with the observation of Father Vremannus [?] at Macau, sent to Belgium, which Wendelen reports, p. 80, where the beginning is given at $8^h\ 30^m$ and the end at $11^h\ 45^m$. ✢ Scheiner, at Ingolstadt, at about $4^h\ 53^m$, saw *the whole globe of the Moon, but tinged with a bloody and black colour; and its upper parts were still wrapped in darkness, so that the Moon shone through very grudgingly. As the fifth hour was striking, these wrappings began to slip away, but this struggle of light and darkness lasted for almost three quarters of an hour, until a little after the middle of the sixth evening hour the Moon smiled out serene; these remnants of the failing lasted almost* 40^m. From this he gathers the end at 5^h, as also from his tables.

[Margin: Year 1613. ☾ Total.]

The eclipse of the Moon on October 28, observed by Longomontanus at Copenhagen and reduced to Uraniborg, at $3^h\ 57^m$. ✢ David Fabricius, at Emden in Frisia, as Wendelen relates, p. 80, and Kepler in the *Ephemerides*, p. 21, thought that with the telescope he observed the end earlier than calculation or the naked eye would show, but he devised an extraneous cause, on which see Kepler in that place.

[Margin: Year 1614. ☾]

The eclipse of the Moon on the morning of April 24, whose beginning was seen at Leuven and at Mechelen by the night watchmen as the third hour after midnight was striking. So Wendelen, p. 81, whose calculation gives $3^h\ 1^m$.

[Margin: Year 1614. Sun.]

The eclipse of the Sun on October 3, in Frisia near Emden. Fabricius, and Kepler at Linz, and others at Venice and at Munich, namely Simon Marius and Scheiner, observed that the eclipse came earlier and was greater than Tycho's hypothesis allows, as Kepler relates in the *Ephemerides*, p. 11, and Wendelen, p. 81. For Tycho, in ecliptic new Moons, makes the Moon's diameter too small. ✢ Kepler adds, in the *Epitome of Copernican Astronomy*, p. 895, that at Munich the failing began about noon on the right and ended below.

[Margin: Year 1616. ☾]

The eclipse of the Moon on March 3. The end of the eclipse was observed at Goa at $7^h\ 53^m$ after noon, as Father Athanasius Kircher reports in his *Magnes*, which at Frankfurt on the Oder, from Origanus's calculation, should have been at $3^h\ 51^m$ after noon; but Wendelen's calculation, p. 82, requires $3^h\ 58^m$. From this he conjectures that Goa is further east than Frankfurt by $4^h\ 11^m$.

[Margin: Year 1616. ☾ Total.]

The eclipse of the Moon on August 26. At Rome our Grienberger observed the beginning at $1^h\ 43^m\ 30^s$ after the following midnight and the end at $4^h\ 56^m$

24ˢ; therefore the duration was almost 3^h 13^m and the middle at 3^h 20^m. ✢ At Rome likewise others, separately, using a telescope, noted the immersion at 3^h $6½^m$ and the emersion at 3^h $33¾^m$; therefore the middle at 3^h 20^m, in admirable agreement, as Kepler relates in the *Ephemerides*, pp. 11 and 12. ✢ But Wendelen, p. 83, says that at Liège he observed the beginning at 1^h 23^m, the immersion at 2^h 42^m, the emersion at 3^h 13^m, the end at 4^h 31^m, the duration 3^h 8^m, but with the deception of the umbrago 3^h 13^m. ✢ At Tübingen Mästlin, from Kepler above, observed the start at 1^h 33^m, by the altitude of 9° of the right shoulder of Orion, and the end at 4^h 43^m, by the altitude of Sirius of 11° 20′; therefore the middle at 3^h 8^m. ✢ At Linz Kepler, from the Moon's western azimuth of 31° 20′, observed the beginning at 1^h 48^m and the end at 5^h 16^m. But because of the faults and deceptions of sight, which he admits, he shortens the duration and puts the middle at 3^h 30^m. From this he gathers 5½° between Tübingen and Linz, agreeing very nearly with the distance by road, and 2½° between Linz and Rome, or 10^m of time. [Margin: A note on the difference of meridians.] In the Rudolphine Tables, however, p. 103, he says: *Of the observers, some assert that it was total, others partial.* But Wendelen thinks that the penumbra was on both sides the reason why the duration seemed to Kepler to be 3^h 32^m, when in reality it was 3^h 8^m; and he puts the middle at Linz at 3^h 34^m. ✢ At Ingolstadt, as Father Iacobus Viva writes to me, the total immersion was observed at 15^h 15^m after noon and the emersion at 15^h 45^m. In that case Rome would turn out to be further west than Ingolstadt, which is altogether false.

[Margin: Year 1617. ☾]

The eclipse of the Moon on August 16. Kepler, in a circle of nobles, observed the middle of the eclipse at Linz at 8^h 10^m after noon, as Wendelen gathers from his discussion, p. 85, who says ✢ that at Herk he observed the emersion at 8^h 15^m and the repletion at 9^h 17^m.

[Margin: Year 1619. ☾]

The eclipse of the Moon on June 26. At Herk, when midnight had passed, the Moon's limb was seen to be dulled very faintly, yet the quantity of 3 digits that the ephemerides promised nowhere appeared. Hence Wendelen, who reports this on p. 85, suspected either that the Moon's latitude in the syzygies is wider, as Kepler holds, or that the distance from the node was greater than the tables allow, or that the Earth's shadow was thinned by refractions. But this also confirms our hypothesis on the semidiameters, whose sum was 52′ 53″, while the Moon's latitude was 51′ 10″, and therefore the failing was not of a whole digit.

[Margin: Year 1619. ☾]

The eclipse of the Moon on December 20, observed at Mechelen by Wendelen, from his p. 86. The Moon began to fail at 1^h 27^m after the following midnight, when Canis Major had not yet passed the meridian by one degree. The end was at most at 4^h 28^m, the middle at 2^h $57½^m$, and almost 11 digits. ✢

At Herk, Wendelen's brothers and apprentices noted the beginning at $1^h\ 30^m$, when Canis Major had passed the meridian by a degree and a half and the hour $1^h\ 30^m$ was just striking. It ended at $4^h\ 30^m$, and they too judged the digits to be 11. ✢ At Ingolstadt, as Father Iacobus Viva writes to me, the smoke began at 2^h after midnight; the shadow was already noticeable at $2^h\ 15^m$; the Moon was leaving the shadow and the eclipse ending at $5^h\ 15^m$, with the Moon at an altitude of 32° 40′. [Margin: A note on the difference of meridians.] ✢ He adds that at Lisbon the altitude of the pole was taken as 39° 38′, and that the eclipse began with Sirius at an altitude of 35° 20′, that 11 digits were noted, and that it emerged completely with Sirius at an altitude of 18° 40′, namely at $15^h\ 48^m$; and that this was written from there to Ingolstadt.

[Margin: Year 1620. ☾ Total.]

The eclipse of the Moon on June 14, which Gassendi observed at Aix with the Lord Joseph Gaultier de la Valette, as is found in his letters and in Wendelen, p. 87: the beginning at $11^h\ 15^m$ after noon, from a western altitude of Spica of 16° 58′, its southern declination being taken as 9° 8′ and its right ascension as 196° 20′, and the altitude of the pole observed there as 43° 33′. The total obscuration at $12^h\ 27^m$, with Arcturus at an altitude of 35° 34′ in the west, its declination taken as 21° 13′ and its right ascension as 209° 37′. The emersion at 14^h, as far as the clouds allowed, with the Crown at an altitude of 37° 15′, and its declination 28° 2′ and right ascension 229° 39′. ✢ At Verona, Wendelen says, it lasted with the penumbra until $3^h\ 50^m$. Therefore, he says, without the penumbra the end would have been at $3^h\ 20^m$. ✢ At Herk, so far as the gaps in the clouds allowed, Wendelen conjectured the beginning at $11^h\ 12^m$, the immersion at $12^h\ 17^m$, and the emersion a little before 14^h, because at 2^h after midnight some small part of it was already shining.

[Margin: Year 1620. ☾ Total.]

The eclipse of the Moon on December 9. It was predicted beforehand by the Fathers of our Society to the Prince of Caciani [?] in the Kingdom of Cochinchina, as Cristoforo Borri writes to Urban VIII in his *Relation* on that kingdom. That ruler's astrologers had ordered it to be watched for on the night following the 8th, and the Prince and his people waited in vain, with a great array of drums, to help the labouring Moon. It was therefore observed to begin on the following night, at 11^h after noon; and this won great esteem for our men, and that Prince, with many men of letters, at length embraced the Catholic faith. ✢ At Brussels Michael Florent van Langren, Royal Cosmographer, noted the emersion at $6^h\ 25½^m$, with the bright star of Aries at an altitude of 51° 15′, and the end at $7^h\ 24^m$, with the same star at an altitude of 57° 34′, as Wendelen reports in the *Eclipses*, p. 88. He adds ✢ that at Bethasii [?] he observed the beginning at about $3^h\ 52^m$, the immersion at $4^h\ 51^m$, with degree 329° 45′ of the equator culminating, the emersion at $6^h\ 29^m$, with 354° 30′ culminating, and the end at about $7^h\ 28^m$; and from this he gathers the middle at $5^h\ 40^m$.

[Margin: A note on the difference of meridians.] He too reports the observation in Cochinchina, which he gathers to be further east than Bethasii by $7^h\ 8^m$.

[Margin: Year 1621. Sun.]

The eclipse of the Sun on May 21, observed at Middelburg by Lansbergen at $8^h\ 16^m$ after midnight, of 11⅙ digits to the north; and at Dordrecht by Martinus Hortensius at $8^h\ 16^m$, of 10 digits 7′, through a telescope, as Lansbergen reports in the *Treasury of Observations*, p. 120. ✢ And at Aix by Gassendi and Gaultier, with the Sun's image passed through the concave lens of a most excellent telescope onto a board covered with very white paper, in a darkened room. On the board the diameter of a circle equal to the circle of the Sun was divided into 12 digits, each digit being subdivided into 15 parts, and the circumference was divided into 360°. Gaultier, in the next room, took altitudes of the Sun with a quadrant of a radius greater than 2 feet. A companion was present to give Gaultier a signal, at Gassendi's order, by a knock on the wall. In this way the beginning was observed at $4^h\ 54^m\ 28^s$ before noon, with the Sun at an altitude of 25° 30′, the Sun being taken to be in ♊ 0° 15′. The end was at $2^h\ 29^m$, with the Sun at an altitude of 51° 17′; the duration $2^h\ 25^m\ 14^s$; and 9 digits 23′. And because 77° 30′ of the circumference were failing on each side, the diameters of the luminaries seemed equal. The Moon was further north than the Sun. So Gassendi writes in his letters to a friend at Rome, who communicated this to the Lord Antonio Rocca of Reggio, and he to me. The Sun's anomaly, moreover, was 322° and the Moon's 90½°, and so for me the Sun's apparent diameter was 31′ and the Moon's 30½′, which comes very close to Gassendi's observation.

[Margin: Year 1621. ☾]

The eclipse of the Moon on November 28, observed at Aix by Gassendi, as I have it from his letters sent to Italy, and it is confirmed from Wendelen's *Eclipses*, p. 88. The sky was somewhat cloudy at the start, with Aldebaran at an altitude of 52° 10′ from the west. The beginning was estimated at $13^h\ 54^m$ from noon, the end at $15^h\ 52^m$, with the same star at an altitude of 32° 30′ and the right shoulder of Orion at 39° 30′ in the west. The middle was at $2^h\ 53^m$, at which, so far as he could, he estimated 3 digits 38′. But he adds here: *But do not trust it too much.* ✢ Kepler at Linz noted, somehow or other through clouds, the beginning at $14^h\ 33^m$, the end at $16^h\ 34^m$, and 4 digits. ✢ At Bethasii Wendelen noted the beginning of the umbrago at $13^h\ 59^m$, shown by the sundial; but the true beginning at $14^h\ 2^m$, the true end at $3^h\ 57^m$, and the spurious end at $4^h\ 3^m$ and beyond, and 3 digits 1′. [Margin: A note on the difference of meridians.] From this he works out a difference of meridians between Linz and Aix of 42^m of time, and between Bethasii and Linz of 37^m.

[Margin: Year 1623. ☾]

The eclipse of the Moon on April 14, observed at Digne by Gassendi, where he had observed the altitude of the pole as 44° 6′. He observed the beginning at $10^h\ 5^m$ [recte: $15^h\ 9^m$, per the book's errata] after noon, with the tail of Leo at an altitude of 20° 6′ in the west; nearly 11 digits, and the remainder of the

Moon, seen a little before, equal to the diameter of Jupiter. The end could not be noted because the Moon was setting. Indeed he adds that the observation of the beginning was difficult, because it began from the left and lower part, where the spots are blacker. ☩ At Linz Kepler noted, under a cloudy sky, the middle at 4^h 50^m after the following midnight, whereas from calculation he had fixed it at 5^h 15^m; and therefore he looked for an equation of 19^m to be subtracted from the time. ☩ At Leiden Hortensius observed the beginning at 9^h 22^m and the end at 12^h 23^m.

[Margin: Year 1624. ☾]

The eclipse of the Moon on April 3. Wendelen, as he has it on p. 91, observed the end already over at Bethasii at the very hour 8^h after noon. ☩ At Stuttgart Fridericus Ritelius [?] noted the emersion at 6^h 58^m and the repletion at 7^h 58^m, looking at the striking clock. But looking at the altitude of Arcturus, of 22° 45′ when 3 digits had already emerged, and the hour from this was 7^h 48^m, and of 24° 18′ when 6 digits were shining, that is, at 7^h 57^m, it follows that the emersion was at 7^h 27^m by celestial time and the end at 8^h 27^m. But Wendelen suspects a displacement in Ritelius's quadrant, because he had already fixed 14^m of time between Stuttgart and Bethasii.

[Margin: Year 1624. ☾ Total.]

The eclipse of the Moon on September 26. Kepler at Linz, with a skilled companion in another house, observed the duration as 3^h 38^m, the stay as 1^h 45^m and the middle at 8^h 55^m from noon: for the beginning was at 7^h 6^m, the immersion at 8^h 3^m, the emersion at 9^h 47^m and the end at 10^h 44^m. But because all the tables promised a longer duration and stay, he suspected that the Earth's shadow was oval, as also the atmosphere of vapours producing the penumbra; Wendelen, p. 92, judges this an escape born of despair. Lansbergen, however, says in precept 24 of his tables that, given the difficulty of observing the exact time of entry into and exit from the shadow, it is no wonder if Kepler noted a duration 8^m shorter and a stay 7^m longer than his tables require. ☩ At Bethasii Wendelen noted the first threats of the failing at 6^h 25^m, and already at 6^h 28^m the failing was noticeable; the immersion at about 7^h 23^m, but until 7^h 29^m a secondary light clinging to the immersed limb; [Margin: A note on the secondary light of the Moon in an eclipse.] the emersion at 9^h 7^m and the end at 10^h 4^m, although the [Greek: ἐπισκοτία (episkotia, "darkening")] of the umbrago lasted until 10^h 12^m. The duration therefore was almost 3^h 39^m, the stay 1^h 44^m, throughout which the Moon glowed red with a pure brightness, a light little cloud arising toward the middle. The middle therefore at 8^h 15^m. ☩ At Brussels van Langren noted 6 digits obscured at 6^h 48^m, with degree 285° 30′ of the equator culminating; the immersion at 7^h $18\frac{1}{3}^m$, with 293° 27′ culminating; the emersion at 9^h 2^m, with 319° 12′ culminating; the repletion at 9^h 59^m 40^s, with 333° 40′ culminating; and therefore the stay as 1^h 44^m and the middle at 8^h 10^m. From this is gathered a difference of longitude between

Brussels and Bethasii of 5^m of time, even if Wendelen puts Bethasii further east by 4^m 20^s, and Linz further east than Brussels by 44^m. [Margin: And for the difference of meridians.]

[Margin: Year 1625. ☾]

The eclipse of the Moon on March 24, observed at Paris by Gassendi, with a steel quadrant of one foot by Mydorge, but most accurately divided and very nimble in motion. The start was at 12^h 54^m, with Regulus at an altitude of 36° 20′ in the west. The end at 13^h 37^m, with the northern Scale at an altitude of 31° 30′ in the east. The size of the eclipse was seen scarcely to exceed 1 digit, and was certainly less than two. So from his letters, ✝ and from Wendelen, p. 93, who says that he expected at Bethasii an eclipse of 3 digits and a duration of two hours, but that it turned out far otherwise, for 2 digits and a duration of 1^h 17^m, the middle at 1^h 30^m after midnight. Wendelen adds these words: *These last two eclipses at length warned me of not a few things to be corrected in my Tables: the Moon's anomaly; its distance from the Earth (which until now I had set at only 57½ semidiameters). They also dictated to me a far different eccentricity, and showed altogether eight semidiameters of the Earth from apogee to perigee. Finally they also pointed out the Nodes of the Dragons; and it is incredible with what labours, with how often repeated Tables, in fitting them both to this last eclipse and to other eclipses, both backward and forward, I have scarcely at last attained my wish, so as to gladden you, Reader.* [Margin: The hazard for astronomy in small eclipses.] For there is a greater hazard in these small eclipses, which vanish with a modest error in the Tables, than in large ones, which leave a wider field to hypotheses; and this will also be clear from the next one. For me, moreover, the sum of the semidiameters of the shadow and the Moon was 53′ 55″ and the latitude of the ☾ 52′ 0″, and therefore the eclipse was of about one digit.

[Margin: Year 1626. ☾ Promised in vain, and imperceptible.]

The eclipse of the Moon on August 7, promised by the ephemerides as a half-digit eclipse with a duration of 44^m, whose middle should have been at Linz at 7^h 45^m. But Kepler, attending carefully there from sunset until 8^h 30^m, found nothing lacking in the Moon's roundness. Wendelen, p. 94, adduces this in support of his own Tables, which give no eclipse. My tables, however, give an imperceptible one: namely, the sum of the semidiameters of the shadow and the Moon as 62′ 5″, and the latitude as 62′.

[Margin: Year 1627. ☾]

The eclipse of the Moon on July 27, observed at Madrid by Fathers of our Society. Their observation had been sent to Ingolstadt, and Father Iacobus Viua [?] forwarded it to me. At 16^h[49] after noon the smoke began; at 16^h 12^m the true beginning; at 16^h 43^m the Moon reached the horizon, already failing by 9 digits; it set at 16^h 45^m. Shortly afterwards the Sun rose, and earlier because of the refractions, since it should have risen at 16^h 52^m, with the altitude of the pole at Madrid taken as 40° 26′. So the observation sheet has it.

[49] the page sets **Hor.16′.**, a minute mark with no minute figure beside it

[Margin: Year 1628. ☾ Total.]

The eclipse of the Moon on January 20. As we have it from his letters, communicated to me by D. Antonio Rocca, Gassendi observed at Aix the beginning at 7^h 49^m from noon, with Sirius at an altitude of 21° 9′ in the east; the immersion at 8^h 48^m, with the heart of Hydra at an altitude of 14° 10′ in the east; the emersion at 10^h 25^m, with the same star at 28° 33′; the end at 11^h 24^m, with Rigel at an altitude of 26° 37′ in the west. Wendelen too has these numbers, p. 95. [Margin: Colours of the Moon in eclipse.] With the help of the telescope, moreover, Gassendi distinguished the true eclipse from the smoke of the umbrago; and he distinguished a honey colour, then purple, and finally rust-red, and the same colours at the emersion in reverse order. ✢ At Paris Mydorge [printed Midorpius] observed the beginning at 7^h 35^m, with Procyon at an altitude of 28° 3′, and the immersion at 8^h 34^m, with the same star at 36° 20′. So Gassendi in the same letter, and Ismaël Boulliau in the *Astronomia Philolaica*, p. 467, who adds ✢ that at Oxford Bainbridge [John Bainbridge] observed the total immersion at 8^h 23^m and the emersion at 10^h 0^m. [Margin: Difference of meridians.] ✢ Pétau adds, in a letter to Wendelen, that at Paris the beginning was seen without the telescope at 7^h 27^m and with the telescope at 7^h 35^m; that the end of the stay or of the emergence [euasionis] could not be observed because clouds passed across; but that a little after the end, from Arcturus at an altitude of 9° 30′, namely at 11^h 7^m, it was certain that the duration had not exceeded 3^h 37^m. Wendelen, p. 94, relates that at Brussels, because of snow and fog, nothing certain was apparent to him except the end at 11^h 15^m, because at 11^h 3 digits were lacking and at 11^h 15^m nothing but the pallor of the penumbra could be made out. [Margin: Colours of the Moon.] As the clouds parted again and again, the Moon was seen to glow red like a burning coal, and all but to shine. ✢ He adds that at Bethasiis his brothers, Nicolaus above all and Io. Acanthius [?], observed the emersion at precisely 10^h 20^m on the sundial, and the end clearly at 11^h 19^m; ✢ and that at Prague Kepler, with learned men, had the beginning from the striking clock at 8^h 30^m, the immersion at 9^h 30^m, the emersion at 11^h 10^m, the end at 12^h 10^m, and the duration 3^h 40^m. But from the altitudes of stars Kepler found that 11^m were to be subtracted from the striking clock, and that the middle was therefore at 10^h 9^m. [Margin: † Note on the contraction of the shadow.] But since he had computed from the tables a stay of 1^h 50^m, and found by observing that it was 10^m less, he attributed the cause to a shortening of the shadow.

[Margin: Year 1628. ⊙ Annular.]

The eclipse of the Sun on December 26, observed at Barcelona by Father Io. Baptista Cyssatus [Johann Baptist Cysat] at 4^h 1^m after noon, of 8 digits 45′, with the Sun at an altitude of 4° 30′ and the altitude of the pole already observed there as 41° 30′. ✢ Kepler observed it in Austria with a bright and vaporous ring, as I find in my papers.

[Margin: Year 1629. ☉]

The eclipse of the Sun on December 14, in West America and the province of Macha, at a southern pole altitude of 20°. The beginning was observed at 1^h 45^m before noon and the end at 1^h 45^m after noon, as was written to me.

[Margin: Year 1630. ☉]

The eclipse of the Sun on June 10. Lansbergen, in the *Treasury of Observations*, p. 123, relates that it was observed at Dordrecht by Martinus Hortensius, under a dark roof, through an optical tube, at 7^h 15^m, and of 10 digits, a little more; ✢ and by himself at Middelburg at 7^h 10^m after noon, of 10 digits 42′, whereas the true conjunction there was at 5^h 57^m. ✢ Gassendi says in his letters that he observed at Paris the beginning at 6^h 16^m, the middle at 7^h 12^m, and then 11 digits 32′; and that the end could not be observed, since the Sun set while still in an eclipse of 2 digits. ✢ Father Andreas Arzet [?] of the Society of Jesus, together with Father Cysat at Ingolstadt, writes to me that he observed the beginning at 6^h 40^m, the middle at 7^h $54½^m$, and then 11 digits 57′, with the Sun visibly at an altitude of 5° 18′, and the diameters of the luminaries scarcely sensibly different. [Margin: Note on the diameters of the luminaries.] This agrees excellently with my hypothesis, which gives the Sun, at an anomaly of 351°, a diameter of 30′ 50″, and the Moon, at the anomaly of 101° it then had, a diameter of 30′ 58″. ✢ Boulliau, in the *Astronomia Philolaica*, p. 467, relates that at Oxford Bainbridge observed the beginning at 5^h 58^m and the end at 7^h 48^m from noon. ✢ We ourselves, at Foro-Nævii [?], 14 miles from Parma (where we were on account of the plague, which raged there and throughout Lombardy from the beginning of spring until the following autumn), having no instruments, noted the beginning on the sundial at about 6^h 45^m after noon.

[Margin: Year 1630. ☾]

The eclipse of the Moon on November 19, omitted by Magini and Argoli but set down by Origanus in his *Ephemerides*. Though I did not know this at the time, I was for some reason or other practising calculation, and found it, and confidently predicted it to the Most Serene Odoardo Farnese. It happened, though with a huge quantity of rain, yet so that it could be seen through gaps in the clouds, but its phases could by no means be noted. ✢ But Father Arzet observed it at Ingolstadt, of 9 digits 27′, with the beginning at 9^h 50^m and the end at 12^h 58^m, and the diameter of the shadow to the diameter of the Moon as 266 to 100. So I have it from his letters, very generously sent to me. My hypothesis, however, requires, at the Sun's anomaly of 143° and the Moon's of 53°, which they then had, the semidiameter of the shadow to that of the Moon as 271½ to 100. ✢ Hortensius noted at Leiden the beginning at 9^h 22^m and the end at 12^h 23^m; his duration is therefore 8^m less than Arzet's. ✢ Van Langren, at Brussels, noted the middle at 10^h 17^m, and 9 digits 30′ optical but 8 true, as Wendelen gathers from his observation, p. 97, where he reports Hortensius's observation.

[Margin: Year 1631. ☾]

The eclipse of the Moon on May 15, observed at Ingolstadt by Father Andreas Arzet, S.J., about which, in a letter sent to me, as follows: *When the Moon had risen above the vapours it was already wholly in the shadow of the Earth; but it began to recover its light when the bright star of Lyra stood at an altitude of 27° 30′ from the east, that is, at 9^h 15^m after noon; it emerged wholly when Lyra stood at an altitude of 36° 35′.* At Lyon on the Rhône it emerged a little before the eighth hour, as Father d'Auxon, mathematician of our Society, told Wendelen: so Wendelen, p. 97.

[Margin: Year 1631. ☾ Total.]

The eclipse of the Moon on November 8. At Lugdunum Batavorum, or Leiden, Martinus Hortensius observed the beginning at 9^h 30^m from noon, with the Moon at an altitude of 43° 9′. But with Aldebaran at an altitude of 36° 40′ from the east, that is, at 10^h 4^m, 6 digits were failing. The immersion was at 10^h 36^m, with Aldebaran at an altitude of 40° 48′; the emersion at 12^h 17^m, with the right shoulder of Orion at an altitude of 36° 30′; and with the same star now at 39° 42′, namely at 12^h 50^m, 6 digits returned. The end was at 13^h 23^m, with the Moon at an altitude of 51° 5′ in the west. So Hortensius has it in his dissertation on Mercury seen in the Sun, with Gassendi. [Margin: Note on the difference of meridians.] ✢ Gassendi, however, writes that all those things happened at Paris 12^m later, as I read in his letter. ✢ Wendelen, however, p. 98, says that at Paris the beginning was not apparent, and that the immersion was noted from the altitude of Jupiter at 10^h 17^m 30^s, but from the altitude of the Moon at 10^h 22^m 30^s. ✢ At Tübingen Schickard noted the immersion at 10^h 55^m or 52^m, and the emersion at 12^h 35^m. ✢ At Bethasiis Wendelen observed the beginning at 9^h 33^m, the emersion at 12^h 23^m by the sundial, and the end clearly at 13^h 28^m, but the penumbra lasted until 13^h 36^m. ✢ At Madrid van Langren noted the immersion at 10^h $4\frac{2}{3}^m$, with degree 14° 58′ of the equator culminating, though the observation was difficult because the phase was long doubtful; the emersion at 11^h 50^m; the end at 12^h 54^m 40^s.

[Margin: Year 1632. ☾]

The eclipse of the Moon on October 27, observed at Innsbruck by Father Andreas Arzet of the Society of Jesus. The beginning at 10^h 37^m from noon, with Aldebaran at an altitude of 38° 20′ from the east; the end at 12^h 59^m, with Aldebaran at an altitude of 56° 17′; 5 digits 22′, but with the smoke 6; to others, however, it appeared as 6 digits 6′. Smoke had preceded the beginning for a quarter of an hour. So he says in his letters to me. [Margin: Difference of meridians.] ✢ At Lyon on the Rhône Gassendi obtained the beginning at 10^h 3^m and the end at 12^h 30^m from the altitudes of stars, with the altitude of the pole taken as 45° 36′, as Wendelen writes, p. 99. ✢ At Leiden Martinus Hortensius noted the pallor of the umbrago a little after the tenth hour was heard to strike, but the true beginning at 10^h 20^m, with Jupiter at an altitude of

41° 36′ in the east, and the end at 12^h 44^m, with Aldebaran at an altitude of 50° 13′; and scarcely more than 4 digits 40′. So he says. ✢ At Liège Otgerus [?], a skilled mathematician, obtained the beginning at 10^h 18^m and the end at 12^h 40^m.

[Margin: Year 1633. ☉]

The eclipse of the Sun on April 8, observed at Regensburg by Father Andreas Arzet, S.J., of 6 digits 44′, at 5^h 3^m. He says that he then found the diameter of the Sun to the diameter of the Moon, in the bright circle, as 100 to 97, which comes close to my hypothesis: for the Sun's anomaly was 281° and the Moon's 277°, and the apparent semidiameter of the ☉ was 15′ 49″ and of the ☾ 14′ 54″, that is, about as 100 to 95. ✢ At Avignon, as Father Theophilus Raynaudus [Théophile Raynaud] wrote to me, the beginning was observed at 3^h 29^m, and at Paris at 3^h 18^m. ✢ We at Bologna noted 5 digits on the sundial at 5^h 15^m.

[Margin: Year 1634. ☾]

The eclipse of the Moon on March 14. At Aix, Nicolaus Fabricius Peireskius [Nicolas-Claude Fabri de Peiresc] observed the beginning at about 7^h 33^m, and the end at 10^h 49^m. ✢ Gassendi at Digne had the beginning at 7^h 41^m, 10 digits at 9^h 2^m, and the end at 11^h 2^m, as Wendelen writes, p. 110. ✢ At Madrid van Langren observed, at 8^h $13½^m$, 9 digits 33′ failing, and the greatest failing, of 10 digits 15′, from which Wendelen gathers the middle at 8^h 42^m; the end was at 10^h 19^m at most. Van Langren himself writes to me the selenographic places through which the section of the Earth's shadow was passing, with the degrees of the equator then culminating: consult, therefore, book 4, chapter 7, and the following small table.

Parts of the Moon through which the section of the Earth's shadow passed as it left them [Lunæ Partes per quas tranſibat ſectio Vmbræ Terræ illas deferentis]	Degrees of the equator then culminating [Aequatoris Gradus tunc culminantes]
Port of St Vincent, Mount Moretus [Portus S.Vincentij Mons Moreti]	118° 7′ 30″
Port of St Vincent, Mons Torri [Portus S.Vincentij Mons Torri]	123° 42′ 30″
Lake of Posidonius [Lacus Poſſidonij]	132° 50′ 30″
Mount Philip [Mons Philippi]	132° 54′ 0″
Middle of the Adriatic Port [Portûs Adriatici medium]	137° 23′ 0″
Mons Torri	144° 43′ 30″
Mount Puteanus and the Port of St Louis uncovered [Mons Puteani & Portus S.Ludouici detectus]	146° 2′ 20″
Mount Wendelen appeared [Mons Vendelini apparuit]	149° 22′ 0″
Mount van Langren appeared [Mons Langreni apparuit]	✢ ✢ ✢
End of the eclipse [Finis Eclipſis]	152° 29′ 0″

[Margin: Year 1635. ☾ Total.]

The eclipse of the Moon on March 3. Gassendi, in Peiresc's house at Marseille, observed the beginning of the stay at 7^h 45^m. ✢ But at Aix it was observed at 7^h 55^m. ✢ At Danzig, by Crüger [Peter Crüger], at 8^h 47^m. ✢ At Stettin, by Laurentius Eichstadius [Lorenz Eichstaedt], at 8^h 27^m, with Regulus at an

altitude of 41° 13′. ✢ At Königsberg in Prussia at 8^h 54^m, with Procyon at an altitude of 4° 1′. [Margin: Difference of meridians.] ✢ At Amsterdam, by Hortensius, at 7^h 51^m, with Regulus at an altitude of 52° 46′, and the middle at 8^h 40^m, as Boulliau has it, pp. 137 and 466 of the *Astronomia Philolaica*. ✢ But at Paris van Langren's father noted the emersion at 9^h 23^m or 24^m. ✢ At Brussels van Langren had the beginning at 6^h 49^m and the end at 10^h $39½^m$. ✢ At Leiden, with Golius [Jacob Golius] observing, it began to emerge at 9^h 39^m, with Regulus at an altitude of 41° 3′. ✢ At Herk Wendelen, from the sundial and culminations, obtained the beginning at 6^h 53^m, the immersion at 7^h 59^m, the emersion at 9^h 40^m, and the end at 10^h $45½^m$. But he adds ✢ that at Aix the beginning was observed at about 6^h 41^m, with Regulus at an altitude of 30° 25′; the immersion at 7^h $55½^m$, with the same star at 42° 10′; the emersion at 9^h $32½^m$, with the same star at 55° 50′; and the end at 10^h 38^m, with Arcturus at an altitude of 31° 50′. ✢ Boulliau himself, however, in book 3 of the *Astronomia Philolaica*, chapter 6, says that he observed the middle at Paris at 8^h 21^m, ✢ and that Gassendi observed it at Aix at 8^h 42^m 45^s.

[Margin: Year 1635. ☾ Total.]

The eclipse of the Moon on August 28, carefully observed at Paris by Boulliau, as he attests in book 3 of the *Astronomia Philolaica*, chapter 6. He obtained the middle at 2^h 52^m after the preceding midnight, and adds ✢ that the middle was observed by Hortensius at Amsterdam at 3^h 11^m. ✢ But Wendelen, p. 102, relates that Boulliau observed the immersion at 2^h 5^m, the emersion at 3^h 40^m, and six digits restored at 4^h 12^m, all from altitudes of Lyra. ✢ At Aix Peiresc had the beginning without the telescope at 1^h 21^m, from Aldebaran's altitude of 27° 5′, but with the telescope applied at 1^h 30^m or 31^m, from the altitude of Capella and the Moon; the immersion at 2^h 28^m; and the end, with the telescope, at 5^h 1^m. [Margin: Difference of meridians.] But Wendelen, from other phases, more truly draws the beginning back to 1^h 25^m, the immersion to 2^h 24^m, and the end to 4^h 56^m. ✢ At Digne Gassendi, though struggling with clouds, nevertheless observed various phases, from which Wendelen deduces the beginning at 1^h 29^m and the immersion at 2^h 28^m. At Leiden Golius noted the immersion at 2^h 25^m. ✢ At Herk Wendelen noted the beginning at 1^h 29^m, the immersion at 2^h 28^m, and the emersion at 4^h 1^m. He adds that at Rome the beginning was apparent, after a fashion, at 1^h 54^m, and the middle, gathered from the phases, at 3^h 39^m. ✢ And that at Cairo in Egypt the beginning was at 5^h $14½^m$ and the immersion at 6^h $16½^m$, Father Agathangelus [?], a Capuchin, and his companion Ioannes Molinus [?] observing through altitudes of the Moon. ✢ But that at Aleppo in Syria Father Cælestinus a S. Liduina [?], a Discalced Carmelite, and Father Michaël Angelus [?], a Capuchin and skilled mathematician, obtained from altitudes of the Moon the beginning at 5^h 28^m and the immersion at 6^h 27^m, as Wendelen deduces from those altitudes; but Boulliau, book 2, chapter 8, deduces from them 6^h 30^m. Gassendi too, in the

Life of Peiresc, after reporting the already mentioned observations at Cairo and Aleppo, says that those places are about 15 degrees closer to Marseille than most geographical maps show.

[Margin: Year 1636. ☾]

The eclipse of the Moon on February 20. On the testimony of Wendelen, p. 104, and of Boulliau, book 3, chapter 6, Pierre Gassendi observed its middle at Digne at $10^h\ 40^m$ after noon, gathered from the following phases. At $9^h\ 26^m$, a thin shading; at $9^h\ 42^m$, the limb could still, though with difficulty, be made out with the telescope; at $9^h\ 44½^m$, the limb was growing dark in the telescope, from which he gathered the beginning; already at $9^h\ 52^m$, half a digit at most seemed to be failing; at $11^h\ 26^m$, it returned to half a digit; at $11^h\ 33½^m$, it was almost the end; at $11^h\ 34½^m$, the end as judged from the beginning; at $11^h\ 36^m$, the limb restored in the telescope; up to $11^h\ 51^m$, a very thin shading remaining; digits 3 exactly. So Gassendi. Because of this care, Wendelen and with him Sirmanus [?] called it the Queen of Eclipses, and material for a proper commentary. ✢ At Brussels van Langren obtained the beginning at $9^h\ 38^m$. ✢ At Aix the end was apparent in the telescope at $11^h\ 33^m$. ✢ At Amsterdam Hortensius obtained the end at $11^h\ 32^m$. ✢ At Königsberg in Prussia Albert Linemann observed from 2½ to 3 digits, the beginning with Arcturus at an altitude of 26° in the east, the middle roughly with the same star at 30° 20′, and the end with Jupiter at an altitude of 48° 20′, as Boulliau relates from his letter to Hevelius, book 12 of the *Astronomia Philolaica*, p. 466. My tables give an eclipse of 2 digits 20′.

[Margin: Year 1637. ☾]

The eclipse of the Sun on December 31, in the morning, observed among the Hurons in New France by Father Castellanus [?] of the Society of Jesus, at its middle, at $6^h\ 15^m$ after midnight. From this Wendelen, p. 105, gathers from his own hypothesis the longitude of the Hurons from the Atlantic meridian as 303° 30′, adding: *whether this is true, time will tell.*

[Margin: Year 1638. ☾ Total.]

The eclipse of the Moon on the night between December 20 and 21, whose observations are reported by Boulliau, book 3 of the *Astronomia Philolaica*, chapter 6, and by Wendelen in the *Eclipses*, p. 105. ✢ At Digne Gassendi obtained the beginning at $0^h\ 36^m$ after midnight, half a digit at $0^h\ 38^m$, the immersion at $1^h\ 37^m$ or 38^m, the emersion at $3^h\ 13^m$, the end at $4^h\ 14^m$, and therefore the middle at $2^h\ 25^m\ 45^s$. ✢ At Paris Boulliau obtained the middle at $2^h\ 10^m\ 30^s$ and the immersion at $1^h\ 23^m$. ✢ At Amsterdam Hortensius had the immersion at $1^h\ 32½^m$, with Aldebaran at an altitude of 37° 6′ in the west. [Margin: Difference of meridians.] ✢ At Brussels van Langren obtained the beginning at $0^h\ 29^m\ 8^s$, with degree 96° 33′ of the equator culminating; the immersion at $1^h\ 26^m\ 49^s$, with degree 111° 1′ culminating; the emersion at $3^h\ 5^m$, with degree 135° 38′ culminating; clouds stole the end. The middle

was therefore at 2^h 16^m, and the stay 1^h 38^m 10^s. ✢ At Leiden Golius saw the immersion at 1^h 31^m, the emersion at 3^h 4^m 40^s, and 6 digits restored at 3^h 34^m 40^s, from altitudes of Aldebaran, of the bright star of the foot of ♊, and of Arcturus; from which Wendelen gathers a stay of 1^h 34^m and the middle at 2^h 18^m. ✢ At Herk Wendelen himself had the sheer beginning at 0^h 34^m, the immersion at 1^h 32^m, the emersion at 3^h 10^m, and the end at 4^h, although the penumbra lasted until 40^m; the duration therefore 3^h 34^m, the stay 1^h 38^m, the middle at 2^h 21^m. ✢ At Franeker Johannes Phocylides Holwarda obtained the beginning at 0^h $33½^m$, a failing of 6 digits at 1^h 5^m, the immersion at 1^h 37^m, the emersion at 3^h 12^m, and 6 digits recovered at 3^h 44^m; therefore the middle at 2^h 24^m 30^s.

[Margin: Year 1639. ☉]

The eclipse of the Sun on June 1, observed at Königsberg in Prussia by Albert Linemann in a darkened chamber through a single convex lens, with the altitudes of the Sun taken with an Electoral astrolabe. The beginning very nearly at 5^h 25^m 30^s; 6 digits obscured at 5^h 56^m 30^s, and then the Sun appeared with very blunt and thick horns. The greatest obscuration, of 11½ digits, was at 6^h 24^m 30^s, and then ☿ was splendidly seen, which vanished after 10½ digits of the decreasing failing. [Margin: Disagreement of the tables from observ-[50].] At the moment of greatest obscuration the bright periphery of the circle occupied 212 degrees out of 360. The end was accurately at 7^h 20^m 30^s after noon. Earlier, at 1^h after noon, a solar halo had been seen, of a diameter of 43 degrees. So Boulliau reports, p. 465 of his *Astronomia*. ✢ But in book 4, chapter 5, he says that he observed at Paris the beginning at 4^h 21^m, the middle at 5^h 27^m, the end at 6^h 25^m, and the duration 2^h 4^m, which by his own calculation should have been 2^h 15^m 48^s. He judges, moreover, that the motion of the Moon, as yet unknown to us, was the cause of this disagreement. He adds that the image of the Sun, taken with the telescope on paper, whose semidiameter was one third of a Paris royal foot, was one in which the Moon's semidiameter was slightly smaller than the Sun's, even though, he says, it cannot be judged smaller from lunar eclipses; and he thinks that the cause was both that a dark thing within a bright one appears less, and the refraction of the Sun's rays in the air around the Moon. But our hypothesis gives a smaller one.

[Margin: Year 1641. ☾]

The eclipse of the Moon on October 18. Wendelen, p. 106, says that he observed the beginning at Herk at 6^h 8^m from noon; for at 6^h 12^m half a digit had failed, and at 6^h 15^m one digit. Then at 7^h 29^m the arc through the centres of the Moon and the shadow was passing through the breast of Cassiopeia, which for him was an argument for the height of the eclipse. The end at 8^h 50^m; digits altogether 6 45′. ✢ At Brussels van Langren obtained the middle at 7^h 24^m, and then 6 digits 47′. ✢ At Leuven Erycius Puteanus [Erik van der

[50] the note's last word is cut off at the binding on the scan

Putte] found the beginning at 5^h 59^m and the end at 8^h 53^m, and the middle therefore at 7^h 26^m. So Wendelen relates of both. But Boulliau, book 12, p. 465, relates from a letter of Hevelius of Danzig to Gassendi ✢ that at Danzig Hevelius observed the beginning at 7^h 1^m 15^s and the middle at 8^h 26^m 15^s. ✢ At Rome Father Franciscus Niceronus [Jean-François Niceron], a Minim [Paulinus], obtained the beginning at 7^h 13^m, with Lyra at an altitude of 67° 38′, as he wrote to Father Cavalieri at Bologna. [Margin: Difference of meridians.] But if that observation and Hevelius's were true, Rome would lie further east than Danzig: with all geographical maps protesting, or going astray?

[Margin: Year 1642. ☾ Total.]

The eclipse of the Moon on April 14, than which no eclipse has so far had more observers or surviving observations. Some of these are reported by Boulliau, p. 465 of his *Astronomia*, and by Wendelen in his *Eclipses*, p. 107; but many more were sent by the observers themselves either to me or to Father Athanasius Kircher. I will report them in order.

[Margin: Italian observations.]

At Pisa Dom Vincenzo Renieri, an Olivetan and first professor of mathematics, observed at my request with his excellent telescope, though with a middling quadrant, and sent me the following phases reduced to apparent time, in which the particle *or.* indicates east and *occ.* west. He adds that a somewhat doubtful, secondary little light clung to the Moon's disc, already immersed in the shadow, in the part that faces south-west; but that the usual redness of the Moon did not appear, and that the Moon was altogether lost; from which he concludes that the Moon lacks a light of its own, even a secondary one. He did not note the end, because neighbouring buildings blocked the view of the Moon, but he gathered it from the other phases.

Phases [Phaſes.]	Stars [Stellæ.]	Altitude of the stars [Altitudo Stellarum]	Time from noon [Tempus à Meridie]
Beginning [Initium.]	Spica ♍	35° 6′ west	12^h 33^m 6^s
3 digits obscured [Dig.obſc.3.]	Arcturus	66° 52′ west	✢
6 digits obscured [Dig.obſc.6.]	Spica	32° 10′ west	13^h 18^m 42^s
11 digits obscured [Dig.ob.11.]	Head of Ophiuchus [Cap.Ophiuc.]	49° 7′ east	13^h 39^m 52^s
Total obscuration [Total.obſc.]	Head of Ophiuchus [Cap.Ophiuc.]	50° 24′ east	13^h 49^m 18^s
Emersion [Emerſio.]	Bright star of Aquila [Aquilæ luc.]	40° 12′ east	15^h 21^m 23^s
1 digit lit [Dig.luc.1.]	Aquila	41° 20′ east	15^h 30^m 40^s
3 digits lit [Dig.luc.3.]	Aquila	41° 50′ east	✢
6 digits lit [Dig.luc.6.]	Aquila	43° 54′ east	15^h 47^m 14^s
Therefore the whole duration [Duratio ergo tota]			4^h 6^m 0^s

Phases [Phaſes.]	Stars [Stellæ.]	Altitude of the stars [Altitudo Stellarum]	Time from noon [Tempus à Meridie]
Stay in darkness, from the 5th and 6th phases [Mora in tenebris ex 5.& 6.phaſi]			$1^h\ 32^m\ 5^s$
Middle, from the 5th and 6th phases [Medium ex 5.& 6.phaſi]			$14^h\ 35^m\ 20^s$
But from the 3rd and 9th phases [Sed ex 3.& 9.phaſi]			$14^h\ 32^m\ 58^s$
And from the 4th and 7th phases [Et ex 4.& 7.phaſi]			$14^h\ 35^m\ 16^s$

At Venice a noble patrician, who ordered his name to be suppressed, as I have it in his letter written to Fortunio Liceti, observed the beginning at $12^h\ 46^m\ 9^s$ and the end of the total failing at $15^h\ 38^m\ 41^s$. I believe, however, that he means the emersion, or that by a slip of the copyist's pen 15 hours were written for 16. He attests, moreover, that at Venice and at Vienna in Austria the Moon was seen throughout the eclipse, glowing red like a coal, and he puts the reason why it was not seen in some places down to vapours raised from those regions or carried into them by winds.

At Mantua Fathers Wilhelmus Weilhamer [?] and Francesco Maria Grimaldi, S.J., used a larger astrolabe, and Father Grimaldi deduced the times from the altitudes of Regulus, Spica and Arcturus: the beginning at $12^h\ 38^m\ 27^s$, the immersion at $13^h\ 45^m\ 7^s$, the emersion at $15^h\ 15^m$, the end at $16^h\ 25^m$.

At Trent Father Michael Staudacher, S.J., noted the immersion at $13^h\ 32^m$.

At Bologna we observed this eclipse both with the naked eye and with a very bright telescope, with several of our companions as witnesses and helpers. Its beginning Father Bonaventura Cavalieri, proposing an example of calculation in his *Trigonometria*, Problem 8, deduced at $12^h\ 46^m\ 50^s$; but he used Tycho's declination and right ascension of Spica, though my altitude of the pole. We judged that our own should be used; and we considered that the times should be counted with the pendulum treated in book 2, chapter 20, proposition 15, by companions highly practised beforehand. Not long after the total immersion, however, we so lost the Moon that we searched in vain for its place in the sky, although the sky, as it seemed, was clear and cloudless all around. So the first visible beginning of the true eclipse was with Spica ♍ at an altitude of $33^\circ\ 44'\ 15''$ in the west, that is, at $12^h\ 47^m\ 27^s$; the visible immersion at $13^h\ 54^m\ 30^s$, with the same star at $26^\circ\ 25'\ 20''$. The emersion was not observed through altitudes of stars, because, the Moon being now lost, I was wholly intent on its first reappearance, nor had my companion moved the quadrant in time; but the signal was given at once to those counting the times of the pendulum, and we relied on these. The visible end was at $16^h\ 38^m$, with the bright star of Aquila at an altitude of $50^\circ\ 0'\ 50''$ in the east.

But from the transit of Arcturus through the meridian, which from its right ascension and the Sun's was at $12^h\ 26^m\ 56^s$, up to the visible beginning of

the eclipse, there were counted with the greatest correctness $22^m\ 10^s$ of time; from there to the immersion $62^m\ 4^s$; from there to the emersion $106^m\ 12^s$; and from there to the visible end $62^m\ 8^s$. I trust this second observation far more, because it is not made up of the small deceptions that can arise from the use of instruments and from an analysis of triangles that presupposes many elements, but rests on the simplest counting of the times of a pendulum examined long and often beforehand and afterwards, in which the error of the individual vibrations involves only single seconds of an hour; and the time of Arcturus's transit through the midheaven [M.C.] is gathered from nothing but the right ascension of the Sun and of Arcturus. Following this observation, then, I obtain the beginning of the eclipse at $12^h\ 49^m\ 6^s$, the immersion at $13^h\ 51^m\ 10^s$, the emersion at $15^h\ 37^m\ 22^s$, the end at $16^h\ 39^m\ 30^s$, the half-duration $1^h\ 55^m\ 12^s$, and the half-stay $53^m\ 6^s$; and I noted it so in my notebooks.

Later, when the hypothesis of the Moon and Sun had been established on other grounds, from the most certain celestial marks, and the calculation had been carried out for apparent time, I obtained a half-duration of $1^h\ 56^m\ 3^s$, a time of incidence of $1^h\ 4^m\ 4^s$, and a half-stay of $51^m\ 59^s$. Nor was I greatly surprised at so small a disagreement of the hypothesis with observation; but at last I noticed that this very disagreement almost wholly vanishes when a correction is applied, not voluntary but necessary, about which I have said enough in chapter 15, number 14. In accordance with it, by subtracting one minute each from the beginning of the eclipse and of the emersion, and on the other hand adding one each to the immersion and the end, I obtain a corrected beginning very nearly in agreement with the calculation, as you see in the following small table. The full Moon, moreover, followed the middle of the eclipse by 3^s, according to the rule given in chapter 6, scholium 1.

Phases of the eclipse [Phaſes Eclipſeos]	Corrected time of the observation [Tēpus obſeruationis correctum]	Apparent time from the calculation of our tables [Tempus Apparens ex Calculo Noſtrar.Tab.]
Beginning [Initium]	$12^h\ 48^m\ 6^s$	$12^h\ 48^m\ 4^s$
Immersion [Immerſio]	$13^h\ 52^m\ 10^s$	$13^h\ 52^m\ 8^s$
Emersion [Emerſio]	$15^h\ 36^m\ 22^s$	$15^h\ 36^m\ 6^s$
End [Finis]	$16^h\ 40^m\ 30^s$	$16^h\ 40^m\ 0^s$
Half-duration [Semiduratio]	$1^h\ 56^m\ 24^s$	$1^h\ 56^m\ 3^s$
Half-stay [Semimora]	$0^h\ 52^m\ 6^s$	$0^h\ 51^m\ 59^s$
Incidence [Incidentia]	$1^h\ 4^m\ 18^s$	$1^h\ 4^m\ 4^s$
Duration [Duratio]	$3^h\ 52^m\ 24^s$	$3^h\ 52^m\ 6^s$
Stay [Mora]	$1^h\ 44^m\ 12^s$	$1^h\ 43^m\ 58^s$
Middle of the eclipse [Medium Eclipſ.]	$14^h\ 44^m\ 30^s$	$14^h\ 44^m\ 7^s$
or [vel]	$14^h\ 44^m\ 16^s$	

Phases of the eclipse [Phaſes Eclipſeos]	Corrected time of the observation [Tēpus obſeruationis correctum]	Apparent time from the calculation of our tables [Tempus Apparens ex Calculo Noſtrar.Tab.]
Full Moon [Plenilunium]	$14^h\ 45^m\ 9^s$	$14^h\ 45^m\ 0^s$

[Margin: French observations.]

At Paris Father Georges Fournier, Pierre Gassendi and Ismaël Boulliau observed this eclipse, and Father Fournier, with his usual kindness, gave me the observation in a letter written to me, although I later obtained it also in Boulliau's *Astronomia*, p. 212. Father Fournier affirms, moreover, that of the Moon, already immersed in the shadow, nothing at all, or no colour, let alone light, had appeared, and that the diameter of the shadow to that of the Moon was observed as 40 to 17, and that the altitude of the pole at Paris, often taken by him, was 48° 50′. [Margin: Altitude of the pole at Paris.] But that ratio of the diameters is suspect to me, and it certainly does not agree with the semidiameter of the Moon established by Gassendi, nor with the semidiameters posited for this eclipse by Boulliau, p. 212, who sets that of the corrected shadow at 44′ 21″ and that of the ☾ at 16′ 24″, whose ratio is as 17 to 41 8/11. The rest of the observation went as follows.

Phases of the eclipse [Phaſes Eclipſeos]	Western altitude of the stars [Altitudo Stellarum Occidua]	Apparent time [Tēpus Apparens]
1 Beginning of the penumbra [Penumbræ initiū]	Regulus 31° 30′	$12^h\ 6^m$
2 Beginning of the eclipse [Initium Eclipſ.]	Regulus 30° 53′	$12^h\ 9^m$
3 6 digits obscured [Digit.6. obſcur.]	Regulus 26° 52′	$12^h\ 34^m$
4 Total obscuration [Totalis obſcur.]	Regulus 20° 53′	$13^h\ 11^m$
5 Emersion [Emerſio]	Arcturus 50° 22′	$14^h\ 47^m$
6 6 digits recovered [Digit.6. recuper.]	Arcturus 45° 28′	$15^h\ 20^m$
7 End of the eclipse [Finis Eclipſeos]	☾ lower limb 12° 30′	$15^h\ 51^m$
Therefore the duration of the eclipse [Ergo Duratio Eclipſis]		$3^h\ 42^m$
Stay in the shadow [Mora in vmbra]		$1^h\ 36^m$
Middle from phases 2 and 7 [Medium ex 2. & 7. Phaſi]		$14^h\ 1^m$
But from phases 3 and 6 [Sed ex 3. & 6. phaſi]		$13^h\ 57^m$
And from phases 4 and 5 [Et ex 4. & 5. phaſi]		$13^h\ 59^m$

But Boulliau says that the beginning was perhaps *two minutes earlier*, and that the whole duration was *not true but nearly true,* $3^h\ 43^m$. For from his own calculation he had deduced a stay of $1^h\ 40^m\ 32^s$ and a duration of $3^h\ 47^m\ 34^s$. Wendelen, moreover, p. 108, relates that Gassendi had *clouds and later times*.

[Margin: Altitude of the pole at Avignon.] At Avignon Antonius Franciscus Payen [Antoine-François Payen], having observed the altitude of the pole at

43° 53′, says that the beginning of the eclipse was at 11^h 59^m, with Arcturus at an altitude of 66° 32′ in the east, with an azimuth of 6° 50′; but the total obscuration at 13^h 1^m, with Spica ♍ at an altitude of 33° 15′ in the west, with an azimuth of 21° 58′; that the colour of the Moon was then ash-grey; and that clouds buried the rest. But how slippery the observation of azimuths is, unless it is carried out in the way I hinted at in chapter 15, towards the end of no. 16, and shall set out more fully in the *Liber Organicus*, I shall explain in that same place; and Tycho has already, in the end, noted it, in volume 1 of the *Progymnasmata*, p. 604.

[Margin: Observations of Belgium and Holland.] At Leuven Gerardus Gutiscovius [Gerard van Gutschoven], physician and professor of mathematics, with his father-in-law Guilielmus Regius [?], most skilled in mathematics, observed the beginning of the failing at the very point of midnight, with the Moon just one degree past the meridian, and 3 digits obscured at 0^h 15^m after midnight. Puteanus, for his part, in the citadel noted that at the beginning of the failing the Moon's shadow had scarcely passed the meridian.

At Herk Wendelen, having prepared a telescope, a sundial, two water-clocks and a compound striking clock [Horiſonio cōpoſito], and having engaged several young men of sharp sight and of good wit, above all the brothers Io. and Philippus Vendelarius [?], observed the following phases.

Order of phases [Ordo Phaſium]	Observed phases of the Moon and culminations of stars [Phaſes Lunæ obſeruatæ & Culminationes Stellarum]
1	The Moon was grazing the meridian with its western limb [Luna ſtringebat Meridianum limbo occid.]
2	Nadir on the meridian, therefore midnight [Nadir in Meridiano ergo Media nox.]
3	Beginning of the failing clear [Initium Deliquij manifeſtum.]
4	A sextant of the periphery, a reddening digit [Sextans peripheriæ digitus rubens.]
5	4 digits, strongly reddening [Digiti 4. valde rubentes.]
6	7½ digits, with more than a faint semicircle [Digiti 7 ½. plus ſemicirculo languido.]
7	10 digits, now dark [Digiti 10. iam tenebroſi.]
8	10½ digits obscured; the Southern Scale culminating [Digiti 10 ½. obſcur. culmin. Lanx Auſtr.]
9	It was ceasing to cast a shadow, although a digit was still shining, in fact immersed [Deſinebat vmbram fundere, etſi lucebat adhuc digitus, reuerà immerſus.]
10	A weak limb of half a digit was still seen [Adhuc debilis limbus ſemidigiti videbatur.]
11	Now immersed, it had completely disappeared [Iam immerſa planè diſparuerat.]
12	The Crown was culminating, the Moon completely extinguished [Culminabat Corona, Lunâ planè extinctâ.]
13	The hand of Ophiuchus passed the meridian, 1° 15′ [Ophiuchi manus transijt Merid. Gr.1. 15′.]
14	It was emerging, and shedding light [Emergebat, fundebatq. lumen.]
15	At least 3 digits were shining [Digiti 3. ad minimùm lucebant.]
16	6 digits, quite precisely [Digiti 6. ſatis præciſè.]
17	9 digits; the Head of Hercules culminating [Digiti 9. Culmin. Caput Herculis.]
18	A very small amount was lacking [Deerat aliquid perexiguum.]
19	The end, clear by the votes of all [Finis manifeſtus omnium ſuffragijs.]
20	The Head of Ophiuchus was culminating [Culminabat Caput Ophiuchi.]

Order of phases [Ordo Phaſium]	Observed phases of the Moon and culminations of stars [Phaſes Lunæ obſeruatæ & Culminationes Stellarum]

[Margin: Moon wholly extinguished.]

Order of phases [Ordo Phaſium]	Striking clock [Horiſonij]	Celestial hour [Hora Cęleſtis]
1	$12^h\ 0^m$	$11^h\ 55^m$
2	$12^h\ 5^m$	$12^h\ 0^m$
3	$12^h\ 8^m$	$12^h\ 3^m$
4	$12^h\ 15^m$	$12^h\ 9^m$
5	$12^h\ 30^m$	$12^h\ 23^m$
6	$12^h\ 45^m$	$12^h\ 37^m$
7	$1^h\ 0^m$	$12^h\ 51^m$
8	$1^h\ 7^m$	$12^h\ 58^m$
9	$1^h\ 12^m$	$13^h\ 2^m$
10	$1^h\ 15^m$	$1^h\ 5^m$
11	$1^h\ 30^m$	$1^h\ 19^m$
12	$2^h\ 0^m$	$1^h\ 47^m$
13	$2^h\ 45^m$	$2^h\ 29^m$
14	$3^h\ 0^m$	$2^h\ 43^m$
15	$3^h\ 15^m$	$2^h\ 57^m$
16	$3^h\ 30^m$	$3^h\ 31^m$
17	$3^h\ 45^m$	$3^h\ 25^m$
18	$4^h\ 0^m$	$3^h\ 39^m$
19	$4^h\ 2^m$	$3^h\ 41^m$
20	$4^h\ 6^m$	$3^h\ 45^m$

Wendelen goes on and says: *The whole duration was $3^h\ 38^m$, as much as for Gassendi too, who at Paris had some clouds, and later times, through the fault either of his instrument or of his assistant. The earlier struggle* [Lucta] *lasted 1 hour for Gassendi, as also for van Langren; for me $1^h\ 16^m$, spurious; the later one only 59^m. As for those observations at Leuven and at Herk, they are certain and beyond doubt to me; and the middle was at $1^h\ 52^m$. For certainly at $2^h\ 10^m$ by the striking clock, that is, at $1^h\ 57^m$ celestial time, the eastern limb of the Moon, sought with difficulty and found with trouble, was already somewhat more evident than the rest of the Moon's body, which was completely extinguished. So in truth the middle had already passed. My Atlantic calculation for Herk gives the beginning at $0^h\ 3^m$, the immersion at $1^h\ 2^m$, the middle at $1^h\ 52^m$, the emersion at $2^h\ 42^m$, the end at $3^h\ 41^m$.*

At Leiden, as Father Athanasius Kircher writes, from altitudes of Regulus and Arcturus the beginning was noted at $12^h\ 0^m\ 12^s$, six digits obscured at $12^h\ 37^m\ 34^s$, the immersion at $13^h\ 14^m\ 36^s$, 6 digits recovered at $15^h\ 25^m\ 18^s$, the end at $16^h\ 2^m\ 30^s$; the emersion was not seen because of clouds. The Moon disappeared completely, although stars were visible nearby.

[Margin: German observations.] At Cologne Fathers Lubertus Middendorpius [?] and Henricus Sturnius [?] of the Society of Jesus observed the

beginning at $12^h\ 26^m\ 30^s$ and the total obscuration at $13^h\ 41^m\ 30^s$, from altitudes of Lyra. At Paderborn Father Henricus Modersohu [?], professor of mathematics, with Father Gerardus Lansintkius [?] S.J., noted the beginning with Spica at an altitude of 28° 18′, or $12^h\ 15^m$, perhaps of a spurious eclipse; the total immersion with Spica at an altitude of 23° 5′, or $13^h\ 41^m$; the emersion at $15^h\ 21^m$, with Spica at an altitude of 12°; and the end at $16^h\ 12^m$, with Arcturus at an altitude of 36° 58′, the altitude of the pole there being taken as 51° 50′.

At Würzburg Father Henricus Marcellus S.J., with Father Rector and four other companions, obtained the beginning at $12^h\ 33^m$, the total immersion at $13^h\ 58^m$, the emersion at $15^h\ 18^m$, and the end at $16^h\ 32^m$. From this the Bishop of Würzburg discoursed publicly on a problem, and deduced 19 digits 53′.

At Ingolstadt Father Iacobus Viua [?] S.J. with others, using excellent telescopes, yet not agreeing exactly, especially at the beginning of the total obscuration, because of a certain little light that deceived some, and that lasted through the whole time of the eclipse near the Moon like a nebulous star: nevertheless he sent me the more probable moments as follows. The beginning of the legitimate eclipse at $12^h\ 46^m$, with Regulus at an altitude of 25° 15′; the immersion at $13^h\ 50^m$, with Regulus at an altitude of 13° 40′; the emersion at $15^h\ 28^m$, with Aquila at an altitude of 44° 40′; the end at $16^h\ 30^m$, with Aquila at an altitude of 44° 40′; of these altitudes of Aquila, the first was in the east, the second in the west.

At Eichstätt in Franconia Father Andreas Arzet [?] with Father Albert Curtz of the Society of Jesus observed the beginning at $12^h\ 44^m$, the immersion at $13^h\ 53^m$, and the emersion at $15^h\ 23\frac{1}{3}^m$; therefore the middle at $14^h\ 38^m$. From this they gathered the end at $16^h\ 32^m$, the duration $3^h\ 48^m$, and the stay $1^h\ 30^m$. He adds that for about a quarter-hour a certain imperfect obscuration, like smoke or fog, preceded the true beginning.

At Prague Fathers Theodorus Moretus and Vuolfangus, with Do- ctor Marcus, using a six-foot Tychonic quadrant granted by the Emperor, observed the beginning of a spurious darkness with Spica at an altitude of 29°; but the true beginning, as the telescope showed it, with Spica at an altitude of 28° 23′ in the west; the total obscuration at $14^h\ 10^m$, with Spica at an altitude of 22° 20′; and the emersion at $15^h\ 28^m$, with Spica at an altitude of 12° 15′. They say that no colours at all, no trace of its own light, could be seen in the Moon when it was totally immersed.

At Kłodzko Father Balthasar Conradus [?] of the Society of Jesus, using a quadrant of nearly six feet, observed the beginning at $14^h\ 26^m$, with Arcturus at an altitude of 52° 52′ 30″; but because of a little light that always clung to the Moon like a star, which Scheiner too noted at Neisse, this was doubtful, whereas the emersion was certain, at $15^h\ 48^m$, with the last star of the tail of Ursa Major at an altitude of 55° 7′ 38″.

At Stettin in Pomerania Lorenz Eichstaedt noted the beginning at $12^h\ 58^m$, with the tail of ♌ at an altitude of 39° 12′ in the west; the immersion at $13^h\ 56^m$,

with Spica at an altitude of 20° 18′ in the west; and the emersion at about 15^h 40^m, with Arcturus at an altitude of 42° 9′ in the west; from which he deduces the end at 16^h 38^m.

At Gedanum, or Danzig, Hevelius observed the beginning at 13^h 14^m, the immersion at 14^h 22^m, the emersion at 15^h 53^m, and the end at 16^h 56^m.

In Canada, at an altitude of the pole of 44° 20′, the eclipse was observed on day 13, at about 16^h 36^m, but not very accurately, as Father Bressani writes to Father Kircher.

Although the umbrago has misled some, I should still like to gather into one synopsis the intervals of this eclipse, both observed and deduced from the tables, so that at least from this the disagreement may be evident, both between the observations and the tables, and among the observations themselves, or among the tables themselves.

From observations [Ex Obſeruationibus]	Incidence [Incidentia]	Repletion [Repletio]	Duration [Duratio]	Stay in the shadow [Mora in Vmbra]
Bologna [Bononienſi]	1^h 4^m 18^s	1^h 4^m 8^s	3^h 52^m 24^s	1^h 44^m 12^s
Danzig [Dantiſcana]	1^h 8^m 0^s	1^h 3^m 0^s	3^h 42^m 0^s	1^h 31^m 0^s
Eichstätt [Eycſtadienſi]	1^h 9^m 0^s	1^h 8^m 38^s	3^h 48^m 0^s	1^h 30^m 22^s
Würzburg [Herbipolitana]	1^h 5^m 0^s	1^h 14^m 0^s	3^h 59^m 0^s	1^h 40^m 0^s
Herk [Hercana]	0^h 59^m 0^s	0^h 57^m 0^s	3^h 38^m 0^s	1^h 44^m 0^s
Ingolstadt [Ingolſtadienſi]	1^h 4^m 0^s	1^h 2^m 0^s	3^h 44^m 0^s	1^h 38^m 0^s
Leiden [Leydenſi]	1^h 14^m 24^s	1^h 4^m 14^s	4^h 2^m 18^s	1^h 43^m 40^s
Mantua [Mantuana]	1^h 6^m 40^s	1^h 10^m 0^s	3^h 46^m 33^s	1^h 29^m 53^s
Paderborn [Paderbonenſi]	1^h 26^m 0^s	0^h 53^m 0^s	3^h 57^m 0^s	1^h 40^m 0^s
Paris [Pariſina]	1^h 2^m 0^s	1^h 4^m 0^s	3^h 42^m 0^s	1^h 36^m 0^s
Pisa [Piſana]	1^h 16^m 12^s	1^h 17^m 49^s	4^h 6^m 0^s	1^h 32^m 5^s
Stettin [Stetinenſi]	0^h 58^m 0^s	0^h 58^m 0^s	3^h 40^m 0^s	1^h 44^m 0^s
Avignon [Auenionenſi]	1^h 2^m 0^s			
Cologne [Colonienſi]	1^h 15^m 0^s			
Prague [Pragenſi]				1^h 18^m 0^s
From the tables [Ex Tabulis]				
Prutenic [Prutenicis]	1^h 6^m 0^s	1^h 6^m 0^s	3^h 50^m 0^s	1^h 38^m 0^s
Tychonic [Tychonicis]	1^h 5^m 0^s	1^h 5^m 0^s	4^h 0^m 0^s	1^h 50^m 0^s
Magini's Tychonic [Magini Tychonicis]	1^h 4^m 33^s	1^h 4^m 33^s	3^h 49^m 48^s	1^h 40^m 42^s
Danish [Danicis]	1^h 5^m 15^s	1^h 5^m 15^s	3^h 54^m 12^s	1^h 43^m 42^s
Lansbergen's [Lansbergianis]	1^h 5^m 20^s	1^h 5^m 20^s	3^h 46^m 42^s	1^h 36^m 2^s
Rudolphine [Rudolphinis]	0^h 54^m 0^s	0^h 53^m 40^s	3^h 44^m 4^s	1^h 46^m 24^s
Philolaic [Philolaicis]	1^h 3^m 31^s	1^h 3^m 31^s	3^h 47^m 34^s	1^h 40^m 32^s
Atlantic [Atlanticis]	0^h 59^m 0^s	0^h 59^m 0^s	3^h 38^m 0^s	1^h 40^m 0^s
Ours [Noſtris]	1^h 4^m 4^s	1^h 3^m 54^s	3^h 52^m 6^s	1^h 43^m 58^s

[Margin: Year 1642. ☾ Total.]
The eclipse of the Moon on October 7, observed in eight places.

At Rome Father Jean-François Niceron, a Minim [Paulinus], says that at the beginning of the eclipse the Moon's ray fell, on a catoptric dial [horologio catoptrico], on the line of the 15th astronomical hour from noon, and that a quarter of an hour later Rigel stood at an altitude of 37° 10′ in the east; from which the beginning is gathered at $15^{h}\ 1^{m}$. But the total obscuration, he says, was with Rigel at an altitude of 39° 25′ and Sirius at 27° 37′, or at $16^{h}\ 5^{m}\ 28^{s}$.

At Catacium, or Catanzaro, Don Francesco Zupi, very skilled in astronomy, as Father Giovanni Battista Zupi S.J. writes to me, noted the beginning of the total obscuration at $16^{h}\ 16^{m}\ 32^{s}$, with Regulus at an altitude of 24° 8′ in the east.

At Milan Don Curzolanus [?], at my request conveyed through Father Aloysius Confalonerius [?], observed the beginning at $14^{h}\ 44^{m}$ from noon, with Procyon at an altitude of 28° 10′ in the east, and the total obscuration at $15^{h}\ 47^{m}$, with Procyon at an altitude of 39°; clouds took the rest away.

At Březnice in Bohemia Father Theodorus Moretus says that the beginning was with the Moon at an altitude of 29° 51′ 20″ and its azimuth 57° 46′ in the west, that is, at $14^{h}\ 57^{m}$; and that clouds begrudged the rest.

At Lleida Antonius Garretus [?], at one time an amanuensis of Gassendi, observed the beginning at $14^{h}\ 9^{m}\ 30^{s}$, one and a half digits at $14^{h}\ 17^{m}$, two digits at $14^{h}\ 19^{m}\ 30^{s}$, 7 digits at $14^{h}\ 47^{m}$, 10½ digits at $15^{h}\ 3^{m}$, and eleven digits or more at $15^{h}\ 11^{m}$, which in truth was the total immersion, but a spurious light lasted until $15^{h}\ 17^{m}$; and the emersion at $16^{h}\ 50^{m}\ 30^{s}$, with the Moon still at an altitude of 11°. So Wendelen reports, p. 109.

On Majorca Don Vicenç Mut, most skilled in astronomy, as is clear from his own letter to me, observed the beginning of this eclipse at $14^{h}\ 21^{m}\ 8^{s}$ after noon, with Procyon at an altitude of 24° 54′ in the east; the total obscuration at $15^{h}\ 24^{m}\ 16^{s}$, with the same star at 36° 43′; the emersion at $17^{h}\ 2^{m}\ 20^{s}$, with Rigel at an altitude of 40° 10′ in the west; and the end at $17^{h}\ 59^{m}\ 8^{s}$, with Rigel at 35° 42′; at an altitude of the pole of 39° 35′.

At Madrid Don Andreas à Brinsuela [?], and Fathers of the Society of Jesus, observed this eclipse with Sirius at an altitude of 12° 36′, as the same Don Vicenç Mut writes.

At Bologna we observed this eclipse in both ways, namely through the altitudes of stars and through culminations, applying the times of that pendulum discussed in book 2, chapter 20, proposition 15; counting, that is, from the beginning of the legitimate eclipse as ascertained through the telescope, the times up to the transit of Aldebaran across the meridian, from there on to the total immersion, and from there to the emersion. Hills and the setting of the Moon below the horizon denied us the end, but from other phases and from the moment of time at which 6 digits had been recovered we obtained the end. We did, however, add to the immersion and to the end one minute each, which we subtracted from the beginning and the emersion, for the same reason discussed in the preceding eclipse and in chapter 15, no. 14. Here, then, are the corrected times: the true beginning at $14^{h}\ 55^{m}\ 50^{s}$; the total obscuration

at $15^h\ 57^m\ 10^s$; the emersion at $17^h\ 46^m\ 40^s$; the end at $18^h\ 44^m$. Therefore the whole duration was $3^h\ 48^m\ 50^s$, the interval of incidence $1^h\ 2^m$, the stay in the shadow $1^h\ 49^m\ 30^s$, and the middle at $16^h\ 49^m\ 35^s$. The calculation from our tables, however, gives the beginning at $14^h\ 56^m\ 15^s$, the emersion at $17^h\ 46^m\ 31^s$, the end at $18^h\ 42^m\ 45^s$, the duration $3^h\ 51^m$, the stay $1^h\ 49^m\ 32^s$, the incidence $1^h\ 0^m\ 44^s$, and the middle at $16^h\ 51^m\ 45^s$; and the moment of the true full Moon at $16^h\ 54^m$ in apparent time.

[Margin: Year 1643. ☾]

The eclipse of the Moon on September 27, which Wendelen observed at Antwerp with Gutiscovius and others. At $6^h\ 36^m$ the Moon, struggling out of the clouds, was seen failing by half of itself: the elevation of the western horn and the depression of the eastern showed that the middle had not yet passed, for they were not yet parallel to the Moon's path; the arc through the centres could not be marked out because of clouds. At $6^h\ 54½^m$ the arc through the centres was passing through the head of Andromeda, from which he gathered that the middle of the eclipse was at $6^h\ 52^m$. Already at $7^h\ 6^m$ 6 digits were failing, and the arc through the centres was passing through the extremity of the wing of Pegasus; from which it was confirmed that the middle of the eclipse was at $6^h\ 52^m$. But at $7^h\ 51^m$ 2 digits were still lacking. The shadow, frayed with unequal fringes, appeared piecemeal. The end at last was at $8^h\ 5^m\ 30^s$. ✢ At Paris Ismaël Boulliau observed the end of this eclipse at $7^h\ 59^m\ 36^s$. [Margin: Difference of meridians.] ✢ But at Succium [?] Gassendi, at $8^h\ 1^m\ 38^s$ (Succium, moreover, is 3 miles from Paris). ✢ And at Danzig Johannes Hevelius, at $8^h\ 59^m$, as Boulliau reports. ✢ But at Naples Father Giovanni Battista Zupi, as is gathered from corrected altitudes of the end of the tail of Ursa Major, observed the same end at $9^h\ 5^m\ 40^s$. ✢ At Venice the end was observed at $8^h\ 44^m$ by a nobleman who does not allow himself to be named. ✢ On Majorca Don Vicenç Mut, where he says the altitude of the pole certain to him is 39° 35′, observed the end with the bright star of Aquila at an altitude of 56° 36′ in the west; clouds prevented the rest: so he wrote to me. Further, from our former difference of meridians, the end should have been seen there at $8^h\ 1^m\ 10^s$. ✢ At Bologna we, using a very clear telescope, counted from the beginning of the eclipse very carefully corrected times of our pendulum, $61^m\ 45^s$, up to a western altitude of Aquila of 53° 29′, that is, up to $7^h\ 20^m$. Therefore the beginning was at $6^h\ 18^m\ 15^s$ after noon. At the end, moreover, the western altitude of Aquila was 47° 22′, from which the end was at $8^h\ 43^m\ 30^s$; from which the middle at $7^h\ 30^m\ 52^s$, and the duration $2^h\ 25^m\ 15^s$. But the pendulum times gave a duration of only $2^h\ 25^m\ 10^s$. The digits eclipsed at the greatest were, by the agreement of all, precisely 6, or 6¼. ✢ At Březnice in Bohemia Father Theodorus Moretus, at an altitude of the pole of 50° 15′, observed the end with the Moon at an altitude of 25° 32′, and the bright star of Aquila was at an altitude of 45° 16′; and he affirms to me in his letter that the greatest eclipse barely reached 6 digits. The end, therefore,

was at 8^h 34^m. ☩ At Pisa Don Vincenzo Renieri observed the end with Aquila at an altitude of 50° 40′.

[Margin: Year 1645. ☾]

The eclipse of the Moon on February 10. At Frankfurt on the Main an unnamed observer, as Father Kircher wrote to me, from the stroke of hour 6, at which the eclipse began, up to the end, counted 8305 oscillations of a pendulum, of which 2820 make one hour; therefore 3^h 3^m 13^s; and so the end was at 9^h 3^m 13^s, the middle at 7^h 31^m 36^s, and the amount 10 digits 15′. ☩ At Paris Boulliau noted the beginning of the penumbra at 5^h 28^m, the beginning of the eclipse at 5^h 32^m, and the end at 8^h 40^m, from altitudes of ☾ and of Regulus. ☩ But Gassendi, separately in the same place, noted the beginning at 5^h 33^m, with Procyon at an altitude of 22° 53′ in the east; the end at 8^h 40^m, with Regulus at an altitude of 34° 35′; and 9 digits 30′. So Boulliau writes, together with Gassendi, to Father Mersenne, a letter I have had in my hands. They add, moreover, that from the Rudolphine Tables the beginning there should have been at 5^h 31^m and the end at 8^h 59^m; but from the Philolaic Tables, by Boulliau's own admission, the beginning at 5^h 12^m and the end at 8^h 28^m. The duration of this eclipse from the Rudolphine Tables is 3^h 28^m; from the Prutenic, 3^h 13^m; from the Tychonic, 3^h 21^m; from Duret's Lansbergian, 3^h 25^m; from the Lansbergian of Montebrunus [?], 3^h 18^m; from the Philolaic, 3^h 16^m; whereas from observation it was 3^h 8^m, or at most 3^h 12^m, if we suppose that the true beginning preceded the visible one by two minutes, and that the true end followed by as many. Boulliau thinks that this variation comes from the libration of the Moon, an indication of which was the Caspian spot, then round and very far from the western limb. ☩ At Pisa Don Vincenzo Renieri observed the beginning of this failing with Procyon at an altitude of 29° 30′ in the east, and the end with Aldebaran at an altitude of 46° 10′ in the west, as he relates before the first page of the reprinted *Tabulae Mediceae*. On Majorca Don Vicenç Mut observed the end with Procyon at an altitude of 53° 45′ in the east. At the middle of the eclipse the illuminated part of the Moon nearly equalled the distance between the two western stars in the quadrilateral of the Pleiades. ☩ At Naples Father Giovanni Battista Zupi observed the end with Sirius at an altitude of 31° 20′, but the end of the penumbra with the same star at an altitude of 32° 6½′.

[Margin: Year 1645. ☉]

The eclipse of the Sun on August 21. Its beginning on Majorca, as Don Vicenç Mut, its observer, writes to me, was with the Sun at an altitude of 57° 38′ in the east; its end, with the Sun at an altitude of 62° 20′ in the west. ☩ At Rome Father Daniel Bartolus [Daniello Bartoli] S.J. affirms that the beginning was observed at 16^h 25^m of the Italian solar hour, or 52^m before noon. ☩ At Prague Father Theodorus Moretus noted the end by the altitude of the Sun seen on a gnomon, whose right shadow was to the height of the gnomon as 3119 to 3102; and he says that the digits were a little more than 6. ☩ Renieri at

Lucca in Tuscany observed the Sun at an altitude of 57° 30′ when 3⅔ digits were failing; it was at an altitude of 58° when 4 digits were, before noon; but after noon it was at an altitude of 55° 40′ when 1 digit 30′ was failing, and at an altitude of 54° 30′ when the Sun was freed. ✢ At Paris Gassendi observed the beginning at 5^h 32^m after midnight, and the end at 8^h 40^m. ✢ But in the same place Father Boudinus [?] S.J. observed the beginning at 5^h 28^m and the end at 8^h 37^m. ✢ At La Flèche, for his part, Father Ioannes Deoauer [?] S.J. observed the beginning at 5^h 20^m and the end at 8^h 35^m. ✢ Father Theodorus Moretus writes, moreover, that by a certain friend of his at Aldersbach, in the western region of the Danube, one league from Vilshofen and four from Passau, the beginning was observed at 11^h 0^m 9^s after midnight, the end at 1^h 20^m 18^s after noon, and the middle at 12^h 10^m 13^s, with 5 unmixed digits; but that if the largest arc, of saffron colour, is counted in, it was 5 digits 45′, and if the inner purple arc, 5 digits 20′. The observer suspects that these arcs come from the air around the Moon, but Father Moretus from the colour of the glass through which the observation was made. ✢ Father Moretus adds that the end at Prague, observed by him, was with the Sun past the meridian by 33° 40′, and holding that altitude which is given by the height of a suspended plumb line casting its shadow on a plane exactly tested to the level of the horizon, taken to its shadow as 3102 to 3119, the height of this plumb line being 90 Roman palms; from which the hour is gathered as very nearly 1^h 59^m. He adds that the Sun was eclipsed a little beyond half. ✢ We at Bologna observed the beginning at 44^m 50^s before noon, the end at 1^h 47^m 10^s after noon, and 4½ digits, counting the pendulum times up to the moment of noon, noted by a very fine meridian shadow, and from there further on up to the end of the eclipse, whose moment was certainly observed most exactly with telescopes, for about the beginning there was not such agreement among my companions.

[Margin: Year 1646. ☾]

The eclipse of the Moon which, as Wendelen writes to me, Gassendi observed at Paris on the morning of January 31, at 6^h 1^m after midnight. ✢ But in Canada, at an altitude of the pole of 44° 20′, Father Bressani observed it with great care and wrote the observation to Father Athanasius Kircher, from whom I received it. He says, then, that it was January 30 there; that the beginning of a very thin shading was at 10^h 25^m after noon; that the true beginning, seen with the telescope, was at 10^h 44^m; nearly 6 digits were already obscured at 11^h 12^m; and the total immersion was at 11^h 41^m; the rest was hindered by clouds. ✢ At Pisa, with Don Vincenzo Renieri observing, the observed altitude of the Moon's centre was 24° when it began to be obscured; afterwards clouds arose.

[Margin: Year 1646. ☾]

The eclipse of the Moon on July 27, observed at Smyrna in Ionia by Io. Faber [?] of Marseille, who noted its end at 8^h 30^m after noon; so Renieri relates, from a letter of Boulliau's, at the beginning of the reprinted *Tabulae Mediceae*.

[Margin: Year 1647. ☾]

The eclipse of the Moon on January 20, observed at Paris by Gassendi, as to the middle, at $9^h\ 12^m$ after noon, as Wendelen wrote in a letter sent to me on November 14 of the year 1647. ✢ At Danzig Johannes Hevelius observed the same eclipse, as he himself reports in his *Selenographia*, where he sets out this observation above the others as an example of observing lunar eclipses by telescope on the spots of the Moon. Its summary is written below.

Phases of the eclipse [Phaſes Eclipſis]	Altitudes [Altitudines]	Computed time [Tēpus ſupputatum]	Time of the clock corrected by altitudes [Tēpus Horologij correcti per altitudines]
Nothing of the penumbra [Pe umbræ nihil]	Procyon 33° 42′	$8^h\ 40^m\ 40^s$	
Trace of the penumbra [Penumb veſtigiū]	Jupiter 41° 9′	$9^h\ 7^m\ 27^s$	
Thicker penumbra [Penumb craſſior]	Jupiter 41° 46′	$9^h\ 12^m\ 12^s$	
Noticeable penumbra [Penūb. notabilis]	Jupiter 42° 24′	$9^h\ 17^m\ 40^s$	
Beginning [Initi m]	Jupiter 42° 40′	$9^h\ 19^m\ 12^s$	
Digits eclipsed [Digit. eclipſ. /.]			
0 digits 52′	Procyon 37° 22′	$9^h\ 24^m\ 36^s$	
1 digit 35′	Procyon 37° 44′	$9^h\ 29^m\ 17^s$	
2 digits 15′	Jupiter 44° 55′	$9^h\ 37^m\ 23^s$	
3 digits 0′	Procyon 38° 48′	$9^h\ 44^m\ 7^s$	
3 digits 22′			$9^h\ 47^m\ 23^s$
3 digits 45′	Procyon 39° 19′	$9^h\ 52^m\ 11^s$	
4 digits nearly [4 ferè]	Procyon 39° 40′	$9^h\ 58^m\ 7^s$	
4 digits 15′	Jupiter 48° 9′	$10^h\ 8^m\ 11^s$	
4 digits 22′	Regulus 32° 55′	$10^h\ 10^m\ 31^s$	
4 digits 32′	Jupiter 49° 4′	$10^h\ 17^m\ 0^s$	
4 digits 45′	Jupiter 49° 24′	$10^h\ 20^m\ 22^s$	
5: greatest eclipse [5 Sūma Eclip.]	Jupiter 50° 14′	$10^h\ 29^m\ 5^s$	
5 digits nearly [5 ferè]	Jupiter 50° 41′	$10^h\ 34^m\ 2^s$	
4 digits 30′			$10^h\ 45^m\ 0^s$
4 digits 18′			$10^h\ 49^m\ 30^s$
3 digits 44′			$10^h\ 57^m\ 0^s$

Phases of the eclipse [Phaſes Eclipſis]	Altitudes [Altitudines]	Computed time [Tēpus ſupputatum]	Time of the clock corrected by altitudes [Tēpus Horologij correcti per altitudines]
3 digits 18′	Jupiter 52° 58′	$11^h\ 2^m\ 59^s$	
2 digits 45′			$11^h\ 10^m\ 2^s$
2 digits 30′			$11^h\ 13^m\ 40^s$
2 digits 0′			$11^h\ 16^m\ 0^s$
1 digit 32′			$11^h\ 19^m\ 15^s$
1 digit 22′	Regulus 41° 22′	$11^h\ 20^m\ 53^s$	
1 digit 7′			$11^h\ 22^m\ 20^s$
0 digits 38′			$11^h\ 24^m\ 45^s$
End [Finis]	Regulus 42° 5′	$11^h\ 27^m\ 44^s$	
Dense penumbra [Penumbra denſa]	Regulus 42° 19′	$11^h\ 30^m\ 3^s$	
Thin penumbra [Penūbra diluta]			$11^h\ 33^m\ 16^s$
Very thin penumbra [Penūbra dilutiſſ.]			$11^h\ 35^m\ 48^s$
End of the penumbra [Penumbræ finis]			$11^h\ 38^m\ 40^s$

He adds other things worth noting: namely, that shortly before the beginning of the eclipse one of the four satellites of Jupiter stood off from the centre of Jupiter's disc to the right, by 2⅓ Jovial diameters.

Of the remaining three, however, to the left, that is, toward the east, the nearest stood off by 1 diameter of Jupiter, the next by 1 ⅔, and the farthest by 3 ⅓, as is clear from his drawing. Likewise, he says that the beginning of the eclipse started on the lower right side, toward degree 5 from the nadir point, and 38° from the line perpendicular to the nonagesimal, toward the east. At the greatest phase of the eclipse, of 5 digits, the section of the shadow passed through the Island of Lemnos [Inſulam Lemnam], Mount Sypilus [M. Sypilū], *Didymi*, Mount Moschus [M. Moſchum], etc. For that is how he names those parts of the Moon; but comparing them with our Selenography, I see that it passed through the outermost edge of *Parisiorum* and the middle of *Innocentij* and *Ferdinandi Francisci*. And after other remarks he concludes that the end was 45° from the nadir point toward the right, and about 38° from the line perpendicular to the nonagesimal. The rest can be seen in his own account; I omit it here because of the ambiguity of the names by which he designates the spots of the Moon. Perhaps, however, in the Appendix to this Book I shall interpret the names of the more notable parts and compare them with the names we set down in our Selenography, Book 4, Chapter 7.

✢ But Don Vicenç Mut wrote to me that he observed the beginning of the eclipse at Majorca with Procyon 39° 10′ high in the east, namely at $8^h\ 16^m$, and

the end with Capella 68° 28′ high in the west, namely at 10^{h} 31^{m}. [Margin: Note for the difference of meridians.]

✢ And Ismaël Boulliau at Smyrna, as is related from his letter sent to Io. Rhodius [?] and by him to Father Kircher, observed the beginning of the penumbra with Regulus 31° 5′ high in the east, the true obscuration with the same star 35° 10′ high, the end with it 58° 40′ high, and the end of the penumbra with the same star 59° 40′ high. He observed the altitude of the pole there as 38° 15′ from the Sun's meridian altitude, but as 38° 22′ from the stars of Orion, a difference he attributes to a fault of the instrument. I afterwards found part of this observation in Renieri's *Tabulae Mediceae*, page 1.

CAPVT XX.

Chapter 20.

Eclipſes Solis Totales & Annulares ex præcedenti Hiſtoria & Auctoritate Aſtronomorum Confirmatæ, deque Affinitate quam cum illis habent Eclipſes Solis Matutinæ ac Veſpertinæ.

Total and annular eclipses of the Sun confirmed from the preceding history and from the authority of astronomers; and on the affinity that morning and evening eclipses of the Sun have with them.

I. Who would ever have believed that Tycho, commonly called the Phoenix of astronomers, would be so ignorant or so contemptuous of ancient observations as to deny that the whole Sun can be covered by the Moon? Yet he did deny it, and he clung to his opinion so far as to think that not even Clavius, a very sharp-eyed witness against him, should be believed. For as Kepler reports in the *Astronomia Optica*, Chapter 8, page 285: *When he wrote to Clavius in the year 1600, he said he was surprised that Clavius had seen a total eclipse in Portugal in the year 1560, and nocturnal darkness in the daytime. For his own observations, he said, do not allow the Sun to be wholly covered by the Moon, however low she may be.* [Margin: Total eclipses of the ☉: who has denied them and why.] And so in Volume 1 of the *Progymnasmata*, page 134, Tycho, holding to his opinion, says: *Since it has been noticed in an ecliptic ☌ of the luminaries that the Moon does not keep the same visible diameter as she has at other times, but that her margins are thinned by the force of the Sun's light, some optical reason suggesting this, so that about a fifth part of the ☾ is in this way lost and does not present itself to sight, we therefore found it necessary to vary the Moon's apparent diameter somewhat in ecliptic ☌.* Accordingly he gives a table in the same place, with the Moon's semidiameter at full Moons 16′ at apogee and 18′ at perigee, but at new Moons the former 12′ 48″ and the latter 14′ 24″. Mulerius in the *Tabulae Frisicae*, page 426, and a few Tychonizers in Germany follow him in this opinion and table. Longomontanus too, following Tycho in part, teaches in the *Astronomia Danica*, Book 1 of the *Theorica*, Chapter 9, that in eclipses of the Sun the Moon's diameter is diminished because of refractions and other causes, so much that

from a pole altitude of 53° to one of 74° it decreases by 2′, 3′, 4′, 5′ and 6′. The cause or occasion of Tycho's error was this: eclipses of the Sun, whether viewed directly or by reflection on a board on which the Sun's ray is received through a hole, show the Moon smaller than at other times. But if you then look directly at the Sun, the Moon appears smaller compared with the Sun (as long as the Sun is not wholly covered) for this reason: bright things set beside dark, and white beside black, appear larger, because light and whiteness predominate in moving the sight and affecting the organ of sense, and in overwhelming, or at least blunting, the image of black and dark things; as Kepler shows from the structure of the eye and the account of vision in the *Optics*, Chapter 5, toward the end. If, on the other hand, you let the Sun's rays through a hole or small window onto a board, and from the distance of the boards and the semidiameter of the ray, or bright base, you seek the Sun's apparent semidiameter, then unless you subtract the semidiameter of the hole from the semidiameter of the base, you will find the Sun's semidiameter larger than it should be. And if you do the same at the time of an eclipse of the Sun, you will get a diameter of the Sun larger, and a diameter of the Moon smaller, than they should be, as Kepler has demonstrated in the *Optics*, Chapter 2, from Proposition 10 to 12, and Chapter 11, Problems 2, 7, 8 and 9. Now Tycho and the Tychonists, in measuring the Sun's diameter, and from it the proportion of the Moon's, and the Moon's diameter itself in ecliptic ☌, used a channel or tube of 32 feet, as Tycho has it in Volume 1 of the *Progymnasmata*, page 471, but divided into 1000 parts, of which the hole, or rather the small square window, was 10, as Kepler has it in the *Optics*, page 341. But he did not subtract the window's semidiameter from the bright base, nor add the same, as he should have done, to the portion of the Moon obscuring the Sun; and from this came his occasion for slipping and for diminishing the Moon's diameter and the obscured digits of the Sun in ecliptic ☌, as Kepler acutely recognized in the *Optics*, page 286.

II. But against Tycho and Mulerius the whole of astronomy cries out, and has from its very cradle. For all other astronomers have so constructed their tables that the Sun's apparent diameter at new Moon is sometimes smaller than the lunar diameter, or not greater, and that the Sun can therefore be wholly covered by the Moon. [Margin: Total eclipses of the ☉ are confirmed by authority.] Their standard-bearer, Aristarchus, in his book *On the Distances and Magnitudes of the Three Bodies*, Proposition 8, says: *It is evident from observations that the whole Sun is eclipsed without stay.* After him, as Ptolemy reports in *Almagest* Book 5, Chapters 11 and 14, and Theon in his commentary on the same place, Hipparchus, in his book *On Magnitudes and Distances*, *assumes an eclipse, in the places around the Hellespont, that occurred precisely over the whole Sun, so that nothing of it appeared.* On this eclipse there is also Cleomedes, Book 2 of the *Cyclical Theory*, Chapter 4, saying: *The Sun, wholly eclipsed at the Hellespont, was observed at Alexandria with a fifth part of its diameter intact and the remainder eclipsed.* He asks in the same place how the Moon, though in fact smaller than the Sun, nevertheless sometimes covers it wholly. And Lucius

Taruntius Firmanus, in Plutarch's *Romulus*, assumed the same when he boldly declared to Varro that Romulus was conceived *when the whole Sun suffered a failing.* Ptolemy also assumes above the very thing Hipparchus does, namely that the whole Sun can be obscured by the Moon; and so does Pliny the Elder, Book 2, Chapter 11, in these words: *It cannot indeed be that the whole Sun is taken away from the lands by the intervening Moon, etc.*[51] Plutarch, in the little work *On the Face in the Orb of the Moon*, and al-Battānī, Chapter 30, hold this same thing as established, and so does Proclus in the *Hypotyposes*, q. 5, saying: *The whole Sun is at times seen to be covered from our view.* And Martianus Capella, in the book *De Nuptiis Philologiae* and of Mercury, says: *Often in the climate* [Greek: διὰ Μερόης (dia Meroēs, "through Meroe")] *the defect of the Sun shades the entire disc on every side.* Taking this truth as given, Vitellio [Witelo], as I said in Book 4, Chapter 6, no. 8, inquires into the reason why the Moon, in a total failing of the Sun, can nevertheless sometimes be seen by us. Lastly, Origanus too, in Volume 1 of the *Ephemerides*, Part 2, Chapter 10, and Magini in the *Supplement to the Ephemerides*, page 273, following Kepler's authority in the *Optics*, Chapter 8, departed from Tycho on this point; granted that Origanus, who professes to recognise total eclipses of the Sun, nevertheless in the Tychonic calculation of his *Ephemerides* presents them according to Tycho's mind. And the Braheans themselves, who, Tycho having died shortly before, attended in the year 1601 the observation of the solar eclipse of December 24 made at Prague by Kepler, acknowledged that the Moon's diameter is greater than Tycho states, as Kepler himself asserts in the *Optics*, page 359. And this is what is indicated in the Appendix of the *Progymnasmata*: namely, that by the eclipse of the Sun of the year 1601, shortly after Tycho's death (for he had died on October 24), his opinion on the diameter of the ☾ in ☌ was condemned.

III. [Margin: Examples of total eclipses of the Sun.] Kepler heaps up many examples of total eclipses of the Sun in the *Astronomia Optica*, Chapter 8; but he has neither brought forward all that exist in the writers, nor were all those he piles up necessarily total. Although these can be found from the preceding chapter, they must nevertheless be indicated briefly here, so that they may be ready to hand for anyone who wishes to pursue this controversy.

[Margin: 1. Eclipse of the ☉, total.] The first total eclipse of the Sun among those observed, then, was the one predicted by Thales, which was total at the Hellespont according to tradition; on it see what was said in Chapter 19, at the year 585 before Christ, May 28, and in Chapter 2, no. 8. The Sun's anomaly was then almost nil, the Sun being very near apogee, and the Moon's anomaly was 5^{s} 8°; and so for me the Sun's diameter was 30′ 50″ and the Moon's 33′ 10″.

[Margin: 2. Eclipse of the ☉, very nearly total.] The second, either total or very nearly total, was in the year 431 before Christ, August 3. For Thucydides, as I reported at that year, says that several stars were seen in the sky. The Sun's anomaly was then 32° and the Moon's about 90°, and for me the diameter

[51] the page prints *Non poſſe*, "it cannot be", a denial that seems to run against what the sentence is cited for; it is rendered as printed.

of the ☉ was 30′ 58″, but of the ☾ 30′ 23″. Now in an eclipse not quite total some, if not all, of the larger fixed stars or planets can be seen, as I shall say below, with Kepler's agreement. Still, it must have fallen as little as possible short of total; otherwise, if at its height the Sun had been visibly like the Moon, there was no reason for Thucydides to note that it afterwards became like a half Moon.

[Margin: 3. Eclipse of the ☉, total.] The third, in the year 310 before Christ, August 15, was plainly total, since Diodorus Siculus says: *the darkening was such that you would simply have called it night, with stars appearing everywhere.* And yet Tycho, who was not unaware of this eclipse, as is clear from Volume 1 of the *Progymnasmata*, page 745, does not present it as such. For us, however, the Sun's anomaly was then 2^{s} 14° and the Moon's 6^{s} 21°, and the Sun's diameter 31′ 22″, the Moon's 33′ 10″.

[Margin: 4. Eclipse of the ☉, total.] The fourth was in the year 49 before Christ, in which Gaius Julius Caesar crossed the Rubicon. For of the eclipse of the ☉ of that year Dio said [Greek: ὅτε ἥλιος σύμπαν ἐξέλιπε (hote hēlios sympan exelipe, "when the whole Sun was eclipsed")], *the whole Sun was eclipsed*; and because of it the nations despaired of the day, as Lucan sings, whose verses we have already brought forward at that year.

[Margin: 5. Eclipse of the ☉, total.] The fifth eclipse was before the death of Augustus, in the year of Christ 14 or the one before. For Cassius Dio says: *For the whole Sun too had been eclipsed.* This cannot be the one which Paulus, bishop of Fossombrone, affirms from calculation to have been total in the year of Christ 17; for that one was not quite total either, but very close to total, since the Sun's diameter was 32′ 8″ and the Moon's 31′ 20″; nor, according to the truer chronology, did Augustus die so late.

[Margin: 6. Eclipse of the ☉, total.] The sixth was the one Plutarch speaks of, referred in Chapter 19 to the year 97 or 98, although Kepler refers it to the year 113. For Plutarch says that stars shone out here and there in many places; and he gives another reason why the darkness was not altogether nocturnal but like twilight, and does not have recourse to a part of the Sun surviving and free from the eclipse. Moreover, in the years 97 and 98, on January 27, the day on which Nerva died shortly after the eclipse of the Sun, the Sun's anomaly was 216° and its diameter 32′ 26″, whereas the Moon's anomaly in the year 97 was 22°, with a diameter of 28′ 10″, but in the year 98 was 110°, with a diameter of 31′ 20″. So it was not total.

[Margin: 7. Eclipse of the ☉, nearly total.] The seventh was in the year of Christ 192, as Renieri judges, because Herodian in the life of the Emperor Commodus says that before his death stars were seen in the sky in the daytime; but it is enough that it was nearly total, unless something else is added.

[Margin: 8. Eclipse of the ☉, total.] [Margin: Lansbergen's tables erroneous.] [Margin: Kepler's error.] The eighth was in the year of Christ 237, April 12. For Julius Capitolinus says in the *Gordian* that it was so great *that it was not believed, and that nothing could be done without lights lit.* Lansbergen's

tables show this one as partial; but others, and ours, show it as total, for the Sun's anomaly was 10ˢ 16°, the Moon's 5ˢ 6°, and the Sun's diameter 31′ 4″, the Moon's 33′. I leave aside here the eclipse of the year 360, August 28, which Kepler shamefully and against his own tables makes total, as I said in Chapter 19 at that year; granted that, being a morning eclipse, it may have been like total ones: for see what I reported there from Ammianus Marcellinus.

[Margin: 9. Eclipse of the ☉, total.] [Margin: Lansbergen's tables erroneous.] The ninth was in the year of Christ 484, January 13. For Marinus says in the *Life of Proclus*: *The day was turned into night: for deep darkness came on, and stars were seen.* But Lansbergen's tables do not represent this one as total either; for me, however, the Sun's diameter was 32′ 50″, the Moon's 33′ 24″, the Sun's anomaly being 218° and the Moon's 169°.

[Margin: 10. Eclipse of the ☉, total.] The tenth was in the year of Christ 719, as Scheiner asserts in the *Rosa Ursina*, from the annals.

[Margin: 11. Eclipse of the ☉, total.] [Margin: Error of Petavius's, or the Parisian, tables.] The eleventh was in the year of Christ 840, May 5, since Aimoin, Book 5, Chapter 19, says it was very great, *so that stars were seen and the colour of things on Earth changed*, etc., and the author of the life of Louis the Pious says that *it differed in nothing from night, and all the fixed stars of the hemisphere were seen in their order, and the whole Sun was obscured by the Moon and then restored.* The Sun's anomaly was 10ˢ 25°, the Moon's 4ˢ 26°; and so for me the Sun's diameter was 31′ and the Moon's 32′ 38″. But Pétau, in his Parisian tables, computes this one at 9 digits 20′.

[Margin: 12. Eclipse of the ☉, total.] The twelfth was in the year of Christ 957, December 17, of which the author of the life of Lothair says: *There was an eclipse of the Sun, so that stars appeared from the first hour to the third.* The Sun's anomaly was then 185° and the Moon's 206°, and so for me the Sun's diameter was 33′ 8″ and the Moon's 33′ 6″, a difference that cannot be established; and so it must be counted exactly among total eclipses.

[Margin: 13. Eclipse of the ☉, total.] The thirteenth was, for Kepler, total, in the year of Christ 1113, August 2, because Dodechinus and Sigebert call it very great. The Sun's anomaly was 79° and the Moon's 163°, and so for me the Sun's diameter was 31′ 40″ and the Moon's 33′ 18″. It was therefore total somewhere.

[Margin: 14. Eclipse of the ☉, total.] The fourteenth was in the year of Christ 1187, September 4. For stars were seen as if at night, as Lycosthenes narrates. The Sun's anomaly was 2ˢ 18° and the Moon's 6ˢ 17°, and so for me the diameter of the ☉ was 31′ 38″ and of the ☾ 33′ 16″.

[Margin: 15. Eclipse of the ☉, total.] The fifteenth was in the year of Christ 1241, October 6, as Gemma Frisius affirms from tradition, and as all tables except Tycho's deliver. The Sun's anomaly was 3ˢ 20° and the Moon's 6ˢ 9°, and for me the Sun's diameter was 32′ 14″ and the Moon's 33′ 24″.

[Margin: 16. Eclipse of the ☉, total.] [Margin: Lansbergen's tables erroneous.] The sixteenth was in the year of Christ 1415, June 7, *so terrible that birds*

suddenly fell down, according to Polish history. And yet Lansbergen's tables make it partial. For us the Sun's diameter was 30′ 50″ and the Moon's 33′ 28″, with an anomaly of the ☉ of 11ˢ 21° and of the Moon of 5ˢ 26°.

[Margin: 17. Eclipse of the ☉, total.] The seventeenth was in the year of Christ 1485, March 16. For Lycosthenes says that there was *sheer darkness, and need of lit candles*, etc. The anomaly of the ☉ was 10ˢ and of the ☾ 5ˢ 16°, and for me the Sun's diameter was 31′ 20″ and the Moon's 33′ 18″.

[Margin: 18. Eclipse of the ☉, nearly total.] The eighteenth was that of the year 1506, January 14, in which Io. de Barres [João de Barros] says *that stars were seen on a clear day.* For it was as near as could be to total: the Sun's anomaly was 6ˢ 29° and the Moon's 7ˢ 12°, and the Sun's diameter 32′ 58″, the Moon's 32′ 34″.

[Margin: 19. Eclipse of the ☉, total.] The nineteenth, in the year 1530, March 29: for me the Sun's anomaly was 282° and the Moon's 150°, and the Sun's diameter 31′ 38″, the Moon's 32′ 58″.

[Margin: 20. Eclipse of the ☉, not quite total.] The twentieth, of the year 1544, January 14, was a morning eclipse; and so the birds that had been singing merrily at sunrise afterwards fell silent. Morning eclipses, though not quite total, nevertheless seem total if the eastern part of the Sun, free of the eclipse, is below the horizon or covered with thick vapours. For me it was not quite total, but close to total; for the Sun's diameter was 33′ and the Moon's 30′ 54″, with an anomaly of the ☉ of 208° and of the Moon of 102°.

[Margin: 21. Eclipse of the ☉, total.] The twenty-first was in the year 1560, August 21, seen at Coimbra by our Fathers Clavius and Emmanuel Vega, with Tycho opposing in vain; the Sun's anomaly was 34° and that of the ☾ 130°, and the Sun's diameter 31′, the Moon's 32′ 12″.

[Margin: 22. Eclipse of the ☉, total.] [Margin: Lansbergen's tables erroneous.] The twenty-second was in the year 1605, October 12; by the testimony of Wendelen the whole Sun was precisely covered, with dense darkness around Marseille. Granted that Lansbergen's canons make it partial. But for me the Sun's anomaly was 102° and the Moon's 134°, and the Sun's diameter 32′ 6″, the Moon's 32′ 22″.

One could add that total eclipse of the Sun which Zonaras, Volume 2, reports to have occurred when Scipio was fighting against Hannibal in Africa, if he had added other marks of the time.

De Annularibus Solis Eclipſibus.

On annular eclipses of the Sun.

IV. Annular eclipses of the Sun, in which, that is, a small circle or shining armlet of the Sun's disc appears around the Moon, like a golden ring of light, we have already asserted in Book 4, Chapter 6, Scholium 1, and we assert them again now. For all astronomers accept them, except Ptolemy and, nearly, Longomontanus, because in some position of the anomaly they grant that the

Moon's apparent diameter is smaller than the Sun's diameter; which Ptolemy denies, and Longomontanus too, of itself, granted that by the force of refractions he concedes in climates [Margin: Authors who deny annular eclipses of the Sun.]

nearer the pole of the World that the Sun's diameter can thus be enlarged, and the Moon's optically diminished, so that it appears smaller than the solar. But on Tycho's view all eclipses of the Sun would be annular in the places of the Earth where they appear central, or from which a straight line at the time of the eclipse passes through the centres of both luminaries, since at every ecliptic new Moon he makes the Moon's diameter smaller than the Sun's. These eclipses are accepted by Maurolico in Dialogue 3 of the *Cosmographia*, page 87; by Egnazio Danti in the *Astrolabium*, page 294; by Kepler in the *Astronomia Optica*, Chapter 8, no. 3; and by Bettini in *Apiarium* 8, Progymnasma 2, Proposition 10; if not of themselves, at least by benefit of refraction occurring in the air around the Moon. For otherwise, says Bettini, *the Moon's apparent diameter, in whatever position, always shows itself greater than the Sun's apparent diameter*; in which, however, he errs, but in company with Longomontanus and Ptolemy.

Let us now see the opinion of the more ancient astronomers. [Margin: The true opinion of Cleomedes.] Cleomedes, *Cyclical Theory*, Book 2, Chapter 4, says: *Some of the ancients held that in complete defects the centres of the luminaries are joined in a straight line, and that the orbit of the Sun appears all around on every side. This, however, is not among uncertain things. For all the projecting parts, being very bright and having no small projection, have been clearly seen by us, since the Sun is greater than the Moon.* So reads my codex, faithfully translated from the Greek. [Margin: Kepler must be corrected in reporting Cleomedes.] Granted that Kepler, in the *Astronomia Optica*, page 289, following some version or other, says that Cleomedes there speaks against Sosigenes' opinion, and says: *This has not been detected by observation, for that projecting limb would have been seen by us, being very bright.* The words of Cleomedes are these: [Greek: τοῦτο δὲ οὐ τῶν εὑρημένων ἐστίν. ἑωρᾶτο γὰρ ἂν ἡμῖν τὰ ἐξέχοντα λαμβρότατα γε ὄντα, καὶ μὴ βραχεῖαν ἐκφαίνοντα τὴν ἐξοχήν. (touto de ou tōn heurēmenōn estin. heōrato gar an hēmin ta exechonta lambrotata ge onta, kai mē bracheian ekphainonta tēn exochēn; "But this is not among the things found. For the projecting parts would have been seen by us, being very bright and showing no small projection.")] Nor would Cleomedes have argued rightly against the ancients by saying that he had never seen this; for the reply could be that he was not in the place of a central eclipse, or that no such eclipses occurred while he was observing. But neither was Sosigenes older than Cleomedes, so that he could be numbered among those ancients whom Cleomedes adduces. For Sosigenes flourished under the Dictator Gaius Julius Caesar and in the early days of Augustus, under whom he directed the correction of the calendar; whereas Cleomedes, author of the work [Greek: περὶ τῆς κυκλικῆς θεορίας (peri tēs kyklikēs theorias, "On the Cyclical Theory")], flourished a little earlier, as I have already said in Chapter 5, no. 16.

[Margin: The opinion of Sosigenes.] Moreover, Proclus Diadochus reports Sosigenes' opinion in question 5 of the *Astronomical Hypotyposes*. For having said, on Ptolemy's view, that the Sun's diameter never appears greater than the Moon's, he adds: *If this is true, what Sosigenes the Peripatetic relates in the work he entitled On Revolutions is not true: that the Sun, in eclipses occurring at perigee, is seen not wholly running forward, but at the extremes of its circumference escapes the Moon's circle and gives light by no means obstructed.* [Margin: Proclus an upholder of eclipses of the ☉ both total and annular.] Proclus himself certainly supports this opinion of Sosigenes when he says: *The whole Sun is at times seen to be covered from our view; at times, at the very moment when the two centres and the eye fall on the same line, the Moon is seen within the Sun's circuit.* [Margin: Plutarch's vote for annular eclipses.] Plutarch also favours it, in the little work on the spots in the face of the Moon, in these words: *Even if the Moon at times hides the whole Sun, that defect nevertheless lacks breadth and time; indeed, at times a brightness shines out around the orbit, not allowing the shadow to become deep and excessive.*

Eclipſes Solis Annulares.

Annular eclipses of the Sun.

V. It now remains to take up again from Chapter 19 the examples of annular eclipses of the Sun, certain or disputed.

[Margin: 1. Eclipse of the ☉ believed annular.] The first was, or seems to have been, the one believed to have been seen when Augustus entered the City; on it see what was said in Chapter 19, at the year 44 before Christ.

[Margin: 2. Eclipse, annular.] The second was in the year of Christ 95 or 96; on it see what was said in Chapter 19 at this year.

[Margin: 3. Eclipse, annular.] [Margin: Kepler's and Wendelen's tables in error.] The third was in the year of Christ 334, July 17; on it see what was said in Chapter 19 at this year. Kepler's and Wendelen's tables, however, show it as total, whereas ours leave it annular; for the Sun's anomaly was 46° and the Moon's 296°, and so for me the Sun's diameter was 31′ 8″ and the Moon's 30′ 16″.

[Margin: 4. Eclipse, annular.] The fourth was in the year of Christ 1567, April 9, which Clavius asserts he saw at Rome, saying in Chapter 4 of the *Sphere*: *at Rome the Sun was not wholly eclipsed, but there was left a certain thin shining circle all around.* And yet Kepler, in the *Astronomia Optica*, Chapter 8, who had condemned Tycho for not believing Clavius about the total eclipse of the Sun, calls into doubt in the same place, pages 297 and 302, Clavius's report or observation; evidently because, from the Keplerian hypothesis and tables, that eclipse cannot be computed as annular, and for him the Moon's diameter is then greater than the Sun's. [Margin: Tables of Kepler and of others erroneous.] In this the Alphonsine, Prutenic and Philolaic Tables also err, along with the Rudolphine and the Atlantic. And so Kepler, begging for escapes, says that that bright small circle appeared either because of the refraction of the Sun's rays

in the air that surrounds the Moon, or rather from the air or aether inflamed by contact with the nearby Sun; as if kindled air could have produced a bright ring, and not rather a broad space all around, suffused as with the light of dawn.

[Margin: 5. Eclipse, annular.] [Margin: Error of the astronomical tables.] The fifth was in the year 1598, February 25; on it see what was said in Chapter 19 at this year. But the Prutenic, Rudolphine, Philolaic and Atlantic Tables do not grant this one as such either; others, however, do, and mine above all, in which the Sun's diameter is 32′ 16″ and the Moon's 32′ 12″, with the Sun's anomaly 112° and the Moon's 130°.

[Margin: 6. Eclipse, annular.] The sixth was in the year 1601, December 24; on it see what was said in Chapter 19 at this year. The anomaly of the ☉ was 176° and the Moon's 7°, and for me the Sun's diameter was 33′ 8″, the Moon's 28′ 4″.

[Margin: 7. Eclipse, annular.] The seventh was in the year 1639, June 1, if what Boulliau says in his *Astronomia*, Book 4, Chapter 5, is true: that he observed the Moon's diameter at that time to be smaller than the Sun's diameter; for it must have been annular somewhere. Granted that, since his tables do not put the Moon's diameter smaller than the solar at that time, he too has recourse either to the optical enlargement of bright parts beside dark ones, or to the refraction of the Sun's rays produced by the thicker air around the Moon. But for me, without these detours, the Sun's diameter was then 30′ 56″, at an anomaly of 333°, and the Moon's diameter 29′ 54″, at an anomaly of 80°.

De Eclipſibus Matutinis ac Veſpertinis.

On morning and evening eclipses.

VI. [Margin: Morning and evening eclipses of the Sun similar to total ones.] Among morning eclipses of the Sun there are some in which the Sun rises astronomically covered in all that part of itself which otherwise stands above the horizon, so that the night seems to be prolonged. Therefore, even if they are not total at their height, they can nevertheless seem so somewhere. And since at dawn planets at times, and fixed stars that have not yet set heliacally, appear, they can appear much more easily when the Sun is then obscured. But because this does not seem unusual and new to common people, since stars were ordinarily seen earlier at the end of the night, the writers do not seem to be speaking of these eclipses when they narrate stars seen in the sky as something unusual and like a portent. The same must be judged of evening eclipses, in which the Sun, having suffered a failing a little before setting, sets in eclipse. For then night seems to fall more quickly, and stars which otherwise usually appear soon after sunset can be seen more easily, and that eclipse can seem total at its height, although in fact it is not, because the part of the Sun free of the eclipse has nevertheless sunk below the horizon.

VII. [Margin: Why morning eclipses of the Sun have been reported among portents.] But among morning eclipses of the ☉, those are commonly either less

known or more wonderful which begin a little before or a little after sunrise; so that the Sun rises shining either wholly or in some part of itself, but as the eclipse soon comes on or grows toward the east, the day seems to be extinguished and the night, which had been interrupted, to be called back. So much so that the ignorant crowd, not of birds only but of men, supposes that not the ordinary Sun, but some light like the Sun, has appeared unexpectedly in the night-time, and at times complains that it has been roused untimely to song or to work; while historians list phenomena of this kind, which are truly natural, among preternatural portents. And these too can seem to others total eclipses, although they are not. To this, not improbably, those words of Pliny the Elder, Book 2, Chapter 33, refer: *A light from the sky was seen by night when Gaius Caecilius and Gnaeus Papirius were consuls, and often at other times, so that a semblance of day shone by night.* Granted that Milich interprets this there of a meteorological chasm. To this also refers that passage of Julius Obsequens in the little book on prodigies: *The light shone so that, as if day had dawned, to work*

people rose; and again: *At Amiternum the Sun was seen by night, and its light was seen for some time*; and once more: *in Gaul, in the camp, a light shone at night*. Johann Georg Herwart von Hohenburg indeed refers these, not badly, to eclipses of this kind, in Chapters 86, 128 and 145 of his *Chronologia*, though he misuses them to establish dates, as I said in the chronological book, which will be the fifth of Volume 2 of our *Almagest*. Finally, to this class can be assigned that failing of the Sun which occurred in the Year of our Salvation 1544 January 14, since historians say that birds which had been cheerful at first light fell silent, etc. This eclipse was not indeed total, but it was very close to total ones, as I have already said at Eclipse 19. Such too was the one around the Year of Christ 957, on December 18 [15. Calendas Ianuarij], or in the 3rd year of Lothair, for me precisely total, of which the author of the life of Louis the Pious and Lothair says: *An eclipse of the Sun occurred, so that stars appeared from the first hour to the third hour.* Such was the one treated in Chapter 19 at the Year of Christ 360, from Ammianus Marcellinus, which began from the rising of dawn. And such without doubt was the one seen in the Year of Christ 1530 March 29 by Kepler's paternal uncle, who relates that, with the sky bitterly cold, *it had scarcely grown light when suddenly the light of day was extinguished and turned into night*. On this occasion Kepler in the *Optics*, page 295, prudently warns that morning eclipses of the Sun are gloomier, since the Moon's shadow from above the horizon strikes as if with a kind of blow and extinguishes the dawn. This one, however, was total for me.

VIII. It is pleasing now to imitate Kepler, who in the *Optics*, page 303, sets out certain notable corollaries on eclipses of the Sun; to these I will add some drawn from what has been said, thoroughly worth knowing for telling eclipses apart.

Theoremata de Solis Eclipſibus.

Theorems on eclipses of the Sun.

Theorem 1. *There are total eclipses of the Sun.*

This runs against Tycho and Mulerius, but it has been sufficiently shown above, nos. 1 and 2.

Theorem 2. *Whenever in an eclipse of the Sun the Moon's apparent diameter is greater than the Sun's, or not smaller, and the lunar shadow at the height of the eclipse falls wholly within the Earth's surface, that eclipse is total somewhere on Earth.*

This is clear from the definition of a total eclipse and from the causes that concur in it.

Theorem 3. *Whenever a total eclipse of the Sun occurs and the sky is clear, stars can be seen in the sky.*

So Kepler above, Corollaries 2 and 4. For if a sky filled with thick vapours glows around the Sun and is set on fire by the Sun, that meteorological light can hinder the appearance of the stars.

Theorem 4. *Whenever in an eclipse of the Sun, far from rising and from setting, stars appear as they do at night, that is evidence of a total rather than a non-total eclipse.*

Theorem 5. *The appearance of a few stars in eclipses of the Sun, especially evening or morning ones, is not sufficient evidence of a total eclipse, but it is evidence of one nevertheless very nearly total.*

So Kepler prudently above, Corollary 3; and it is clear from what was said above, nos. 6 and 7. See above Eclipses 2 and 17, among the nearly total ones. A concurrence of sunspots also contributes to the darkness of the air.

Theorem 6. *Morning and evening eclipses can seem total although they are not.*

This is clear from what was said in nos. 6 and 7, and from Corollary 5 of Kepler, who says that morning eclipses are more prone to darkness before their middle, namely in the descending node and semicircle, and evening eclipses after their middle, in the ascending node and semicircle; because, that is, they extinguish that part of the air lit by the Sun which, above our heads, brings us the most light.

Theorem 7. *Some eclipses that are not quite total can bring a greater darkness on the Earth than total ones.*

Kepler above, Corollary 7. It is clear in morning and evening non-total eclipses, if they are compared with those total ones in which the richer and thicker air is set on fire all round by the Sun.

Theorem 8. *There are annular eclipses, not only incidentally, from air or aether set on fire around the Sun, or from a refraction of the solar rays occurring in our air or in the air around the Moon, but of themselves, because of the excess of the Sun's apparent diameter over the lunar.*

This runs against Ptolemy, Bettini and, nearly, Longomontanus; but it is firmly enough established by what was said in nos. 3 and 4.

Theorem 9. *Whenever in an eclipse of the Sun the Sun's apparent diameter is greater than the lunar diameter, and around the middle of the eclipse the line drawn through the centres of the luminaries falls within the Earth, that eclipse is annular somewhere on Earth.*

This is clear from the definition of an annular eclipse and from the causes that concur in it.

Theorem 10. *Because of an eclipse of the Sun the Sun can seem to have risen at night, or to have risen twice on the same day.*

This is clear from certain morning eclipses, about which enough was said in no. 7.

CAPVT XXI.

Chapter 21.

Catalogus Eclipſium ex Ephemeridibus ab Anno CHRISTI 1485. vſq. ad Annum 1700.

A catalogue of eclipses from ephemerides, from the Year of Christ 1485 to the Year 1700.

I. It greatly helps, in comparing tables with the observations (19) set out above, and for the exeligmi and restitutions of eclipses, to have ready to hand, without the labour of calculation, the eclipses of many years computed continuously by the authors of ephemerides, even if not from tables perfect in every respect. So I should like here too to collect them in summary for candidates in astronomy, noting only the time, the duration and the digits. The authors are those listed below. [Margin: Authors of the ephemerides.] An unnamed author of an old ephemeris, from the year 1484 to 1506 inclusive; Iacobus Ptlaumen [Jacob Pflaum] [?] and Johannes Stöffler [Io. Stœflerinus], for the meridian of Ulm, from the year 1507 to 1534; Luca Gaurico [Lucas Gauricus], for a latitude of 45°, from the year 1534 to 1551; Peter Apian [Petrus Appianus], for the meridian of Leisnig [Leyſingæ], from the year 1538 to 1578; Stöffler [Io. Stœflerus], for the meridian of Tübingen, from the year 1543 to 1554; Pietro Pitati [Petrus Pitatus], for the Venetian meridian, from the year 1544 to 1556; Georg Joachim Rheticus [Georgius Ioachimus Rheticus], in the ephemeris of the year 1551; Nicolò Simi [Nicolaus Simus], for the meridian of Bologna, from the year 1552 to 1568; Michael Mästlin, for the meridian of Tübingen, from the year 1577 to 1590; Stadius [Io. Stadius], for the meridian of Antwerp, from the year 1554 to 1574; Giovanni Antonio Magini [Io. Antonius Maginus], for the Venetian meridian, from the year 1581 to 1630; David Origanus, for the meridian of Frankfurt on the Oder, from the year 1595 to 1654; Andrea Argoli [Andreas Argolus], for the meridian of Rome, from the year 1630 to 1700; Francesco Montebruni [Franciscus Montebrunus], for the meridian of Bologna, from the year 1641 to 1660. Among these, Stadius, Mästlin and Magini followed the Prutenic Tables;

Origanus the Prutenic and Tychonic; Argoli the Tychonic; Montebruni, like Duret too, the Lansbergian; the rest, for the most part, the Alphonsine.

II. So that the places for which, and the tables from which, the following eclipses were computed may be ready to hand, consult the following canon, in which the years are taken inclusively at both ends.

Ephemeridum Interualla, & Loca.

The intervals and places of the ephemerides.

Years	Place and tables
From 1485 to 1506	Place and author unknown
From 1507 to 1553	Ulm in Swabia [Vlma Sueu.], from the Alphonsine
From 1554 to 1576	Antwerp, from the Prutenic
From 1577 to 1585	Tübingen, from the Prutenic
From 1586 to 1594	Venice, from the Prutenic
From 1595 to 1600	Frankfurt on the Oder, from the Prutenic
From 1601 to 1640	Frankfurt on the Oder, from the Tychonic
From 1641 to 1660	Bologna, from the Lansbergian
From 1661 to 1700	Rome, from the Tychonic

For although Origanus computed up to the year 1654 in the Prutenic manner as well, and Argoli from the year 1630 to 1700 in the Tychonic, I nevertheless judged it enough, for trial and comparison, to give Prutenic eclipses of 24 years, Tychonic of 80 years, and Lansbergian of 20 years. The time noted, moreover, is apparent time, and from the year 1582 the days are Gregorian.

ECLIPSES EX EPHEMERIDIBVS Ab Anno Chriſti 1485. Ad Annum Chriſti 1700.

Eclipses from the ephemerides, from the Year of Christ 1485 to the Year of Christ 1700.

Year	Eclipse	Month	Day	From noon	Duration	Digits
1485	☉	March	16	$4^h\ 35^m$	$2^h\ 1^m$	Total
1486	☾	February	18	$5^h\ 41^m$	$3^h\ 34^m$	Total
1486	☉	March	5	$17^h\ 43^m$	$2^h\ 2^m$	9 0′
1487	☾	February	7	$15^h\ 49^m$	$3^h\ 32^m$	Total
1487	☉	July	20	$2^h\ 6^m$	$1^h\ 42^m$	7 0′
1488	☉	July	8	$17^h\ 30^m$	$1^h\ 22^m$	4 0′
1489	☾	December	7	$17^h\ 41^m$	$3^h\ 30^m$	Total
1490	☾	June	2	$10^h\ 6^m$	$3^h\ 50^m$	Total
1490	☾	November	26	$18^h\ 25^m$	$3^h\ 34^m$	Total
1491	☉	May	8	$2^h\ 19^m$	$2^h\ 10^m$	9 0′
1493	☾	April	1	$14^h\ 0^m$	$3^h\ 38^m$	Total
1493	☉	October	10	$2^h\ 40^m$	$2^h\ 8^m$	8 0′

Year	Eclipse	Month	Day	From noon	Duration	Digits
1494	☉	March	7	$4^h\ 12^m$	$1^h\ 28^m$	4 0′
2494	☾	March	21	$14^h\ 38^m$	$3^h\ 32^m$	Total
1494	☾	September	14	$19^h\ 45^m$	$3^h\ 36^m$	Total
1497	☾	January	18	$6^h\ 38^m$	$3^h\ 32^m$	Total
1497	☉	July	29	$3^h\ 2^m$	$1^h\ 12^m$	3 0′
1500	☉	March	27	by night		
1500	☾	October	5	$14^h\ 2^m$	$3^h\ 14^m$	10 0′
1500	☾	April	11	by day		
1501	☾	May	2	$17^h\ 49^m$	$3^h\ 44^m$	Total
1502	☉	September	30	$19^h\ 45^m$	$2^h\ 14^m$	10 0′
1502	☾	October	15	$12^h\ 20^m$	$2^h\ 2^m$	3 0′
1504	☾	February	29	$13^h\ 36^m$	$3^h\ 32^m$	Total
1505	☾	August	14	$8^h\ 18^m$	$3^h\ 26^m$	Total
1506	☉	July	20	$3^h\ 11^m$	$0^h\ 58^m$	2 0′
1508	☾	June	12	$17^h\ 40^m$	$3^h\ 46^m$	20 Tot.
1509	☾	June	2	$11^h\ 11^m$	$2^h\ 52^m$	7 0′
1511	☾	October	6	$11^h\ 50^m$	$3^h\ 26^m$	12½ Tot.
1512	☾	September	25	$3^h\ 56^m$	$3^h\ 30^m$	Total
1513	☉	March	7	$1^h\ 30^m$	$1^h\ 50^m$	6 0′
1515	☾	January	29	$15^h\ 18^m$	$3^h\ 34^m$	15 Tot.
1516	☾	January	19	$6^h\ 0^m$	$3^h\ 28^m$	13 T. 18′
1516	☾	July	13	$11^h\ 37^m$	$3^h\ 24^m$	12 T. 8′
1516	☉	December	23	$3^h\ 47^m$	$1^h\ 24^m$	3 0′
1518	☾	May	24	$11^h\ 19^m$	$3^h\ 8^m$	9 11′
1518	☉	June	7	$17^h\ 56^m$	$2^h\ 12^m$	11 0′
1519	☉	October	23	$4^h\ 33^m$	$1^h\ 58^m$	6 0′
1519	☾	November	6	$6^h\ 24^m$	$3^h\ 38^m$	17 Tot.
1520	☉	October	11	$5^h\ 22^m$	$1^h\ 32^m$	3 0′
1522	☾	September	5	$12^h\ 17^m$	$3^h\ 28^m$	12 T. 30′
1523	☾	March	1	$8^h\ 26^m$	$3^h\ 36^m$	15 Tot.
1523	☾	August	25	$15^h\ 14^m$	$3^h\ 34^m$	13 T. 50′
1525	☾	July	4	$10^h\ 10^m$	$1^h\ 46^m$	2 30′
1525	☾	December	29	$10^h\ 46^m$	$3^h\ 28^m$	13 Tot.
1526	☾	December	18	$10^h\ 30^m$	$3^h\ 36^m$	14 T. 10′
1529	☾	October	16	$20^h\ 23^m$	$3^h\ 14^m$	11 55′
1530	☉	March	28	$18^h\ 23^m$	$1^h\ 54^m$	8 24′
1530	☾	October	6	$12^h\ 11^m$	$3^h\ 40^m$	16 T. 30′
1532	☉	August	30	$0^h\ 49^m$	$1^h\ 24^m$	3 35′
1533	☾	August	4	$11^h\ 50^m$	$3^h\ 36^m$	13 T. 0′
1534	☉	January	14	$1^h\ 42^m$	$1^h\ 54^m$	5 45′
1534	☾	January	29	$14^h\ 25^m$	$3^h\ 28^m$	13 T. 40′
1536	☉	June	18	$2^h\ 3^m$	$2^h\ 6^m$	8 0′
1536	☾	November	27	$6^h\ 21^m$	$3^h\ 14^m$	10 15′
1537	☾	May	24	$8^h\ 3^m$	$3^h\ 48^m$	20 T. 13′
1537	☾	November	16	$14^h\ 56^m$	$3^h\ 38^m$	17 T. 20′
1538	☾	May	13	$14^h\ 24^m$	$1^h\ 50^m$	3 0′
1538	☾	November	6	$5^h\ 31^m$	$2^h\ 6^m$	3 37′
1539	☉	April	18	$4^h\ 33^m$	$1^h\ 54^m$	9 0′
1540	☉	April	6	$17^h\ 15^m$	$2^h\ 4^m$	12 T. 0′

Year	Eclipse	Month	Day	From noon	Duration	Digits
1541	☾	March	11	$16^h\ 34^m$	$3^h\ 38^m$	16 T. 0′
1541	☉	August	21	$0^h\ 56^m$	$1^h\ 18^m$	3 0′
1542	☾	March	1	$8^h\ 46^m$	$1^h\ 28^m$	1 38′
1544	☾	January	9	$18^h\ 13^m$	$3^h\ 28^m$	12 T. 46′
1544	☉	January	23	$21^h\ 16^m$	$2^h\ 6^m$	11 17′
1544	☾	July	4	$8^h\ 31^m$	$3^h\ 42^m$	17 T. 25′
1544	☾	December	28	$18^h\ 27^m$	$3^h\ 36^m$	14 T. 20′
1545	☉	June	8	$20^h\ 48^m$	$1^h\ 36^m$	3 45′
1547	☾	October	28	$4^h\ 56^m$	$3^h\ 20^m$	11 34′
1547	☉	November	12	$2^h\ 9^m$	$1^h\ 16^m$	9 30′
1547	☾	May	4	$10^h\ 27^m$	$3^h\ 0^m$	8 0′
1548	☾	April	22	$11^h\ 24^m$	$3^h\ 46^m$	18 T. 0′
1549	☾	April	11	$15^h\ 19^m$	$1^h\ 34^m$	2 0′
1551	☾	February	20	$8^h\ 21^m$	$3^h\ 32^m$	14 T. 10′
1551	☉	August	31	$2^h\ 0^m$	$1^h\ 52^m$	8 0′
1553	☉	January	12	$22^h\ 54^m$	$0^h\ 52^m$	1 22′
1553	☾	July	24	$16^h\ 0^m$	$0^h\ 36^m$	0 31′
1554	☾	December	8	$13^h\ 7^m$	$2^h\ 16^m$	10 12′
1555	☾	June	4	$15^h\ 0^m$	$3^h\ 50^m$	21 T. 48′
1556	☾	November	16	$12^h\ 44^m$	$2^h\ 14^m$	6 55′
1556	☉	November	1	$18^h\ 0^m$	$2^h\ 58^m$	9 41′
1558	☾	April	2	$11^h\ 0^m$	$3^h\ 18^m$	9 50′
1559	☾	September	16	$4^h\ 50^m$	$4^h\ 6^m$	18 T. 0′
1560	☾	March	11	$15^h\ 40^m$	$1^h\ 44^m$	4 13′
1560	☉	August	21	$1^h\ 0^m$	$1^h\ 48^m$	6 22′
1562	☾	July	15	$15^h\ 50^m$	$3^h\ 32^m$	16 T. 56′
1563	☾	July	5	$8^h\ 4^m$	$3^h\ 24^m$	11 34′
1563	☉	June	20	$4^h\ 50^m$	$2^h\ 26^m$	8 38′
1565	☉	March	7	$12^h\ 53^m$	✢	✢ ✢
1565	☾	November	7	$12^h\ 46^m$	$3^h\ 28^m$	11 46′
1566	☾	October	28	$5^h\ 38^m$	$3^h\ 46^m$	18 T. 0′
1567	☉	April	8	$23^h\ 4^m$	$2^h\ 52^m$	9 34′
1567	☾	October	17	$13^h\ 43^m$	$2^h\ 10^m$	2 40′
1569	☾	March	2	$15^h\ 18^m$	$3^h\ 24^m$	13 T. 32′
1570	☾	February	20	$5^h\ 46^m$	$3^h\ 28^m$	16 T. 48′
1570	☾	August	15	$9^h\ 17^m$	$3^h\ 44^m$	16 T. 49′
1572	☾	June	25	$9^h\ 0^m$	$2^h\ 24^m$	5 26′
1573	☾	December	8	$6^h\ 51^m$	$3^h\ 52^m$	19 T. 30′
1574	☉	November	13	$3^h\ 50^m$	$2^h\ 32^m$	5 21′
1576	☾	October	7	$9^h\ 45^m$	✢ ✢	✢ ✢
1577	☾	April	2	$8^h\ 33^m$	$3^h\ 48^m$	18 T. 20′
1577	☾	September	26	$13^h\ 4^m$	$3^h\ 52^m$	16 T. 28′
1578	☾	September	15	$13^h\ 4^m$	$1^h\ 54^m$	2 20′
1579	☉	February	15	$5^h\ 41^m$	$2^h\ 0^m$	8 36′
1580	☾	January	31	$10^h\ 7^m$	$3^h\ 34^m$	12 T. 29′
1581	☾	January	19	$9^h\ 22^m$	$3^h\ 48^m$	15 T. 14′
1581	☾	July	15	$17^h\ 51^m$	$3^h\ 30^m$	13 T. 2′
1582	☾	January	8	$10^h\ 29^m$	$1^h\ 4^m$	0 53′
1582	☉	June	19	$17^h\ 5^m$	$1^h\ 54^m$	7 5′

Year	Eclipse	Month	Day	From noon	Duration	Digits
1583	☾	November	28	$21^h\ 51^m$	$3^h\ 20^m$	12 T. 7′
1584	☾	May	9	$18^h\ 20^m$	$1^h\ 6^m$	3 36′
1584	☾	November	17	$14^h\ 15^m$	$3^h\ 43^m$	17 T. 2′
1585	☉	April	29	$7^h\ 53^m$	$2^h\ 8^m$	11 7′
1585	☾	May	13	$5^h\ 2^m$	$3^h\ 4^m$	6 54′
1587	☾	September	16	$9^h\ 28^m$	$3^h\ 22^m$	10 2′
1588	☉	February	26	$1^h\ 23^m$	$0^h\ 54^m$	1 3′
1588	☾	March	12	$14^h\ 14^m$	$3^h\ 46^m$	17 T. 19′
1588	☾	September	4	$17^h\ 30^m$	$3^h\ 50^m$	18 T. 21′
1589	☾	August	25	$8^h\ 1^m$	$2^h\ 9^m$	3 54′
1590	☾	July	16	$17^h\ 4^m$	$2^h\ 10^m$	3 54′
1590	☉	July	30	$19^h\ 57^m$	$2^h\ 13^m$	10 27′
1591	☾	January	9	$6^h\ 21^m$	$3^h\ 17^m$	9 40′
1591	☾	July	6	$5^h\ 8^m$	$3^h\ 55^m$	18 T. 46′

Year	Eclipse	Month	Day	From noon	Duration	Digits
1591	☉	July	20	$4^h\ 2^m$	$0^h\ 46^m$	1 0′
1591	☾	December	29	$16^h\ 11^m$	$3^h\ 48^m$	19 T. 37′
1592	☾	June	24	$10^h\ 13^m$	$3^h\ 19^m$	8 58′
1592	☾	December	18	$7^h\ 24^m$	$2^h\ 34^m$	5 54′
1593	☉	May	30	$2^h\ 30^m$	$1^h\ 15^m$	2 38′
1594	☉	May	19	$14^h\ 58^m$	$2^h\ 4^m$	10 23′
1594	☾	October	28	$19^h\ 15^m$	$3^h\ 19^m$	9 40′
1595	☉	April	9	In Tierra del Fuego		3 0′
1595	☾	April	24	$4^h\ 12^m$	$3^h\ 51^m$	20 T. 55′
1595	☉	October	3	$2^h\ 4^m$	$2^h\ 4^m$	5 18′
1595	☾	October	18	$20^h\ 47^m$	$4^h\ 2^m$	18 T. 37′
1596	☉	March	28	In the Kingdom of Chile		
1596	☾	April	12	$8^h\ 52^m$	$2^h\ 36^m$	6 4′
1596	☉	September	21	In the Kingdom of China		11 0′
1596	☾	October	6	$21^h\ 15^m$	$2^h\ 11^m$	3 33′
1597	☉	March	17	On St Peter's Island		8 30′
1597	☉	September	11	In Picora [?], etc.		9 49′
1598	☾	February	20	$18^h\ 12^m$	$3^h\ 32^m$	10 55′
1598	☉	March	6	$22^h\ 12^m$	$2^h\ 44^m$	11 57′
1598	☾	August	16	$8^h\ 15^m$	$3^h\ 35^m$	14 T. 35′
1598	☉	August	31	In Magellanica		8 34′
1599	☾	February	10	$17^h\ 21^m$	$3^h\ 59^m$	17 T. 13′
1599	☉	July	22	$4^h\ 31^m$	$1^h\ 3^m$	4 18′
1599	☾	August	6	Among the Antipodes		15 26′
1600	☉	January	15	In Java Major		11 48′
1600	☾	January	30	$6^h\ 40^m$	$2^h\ 1^m$	2 58′
1600	☉	July	10	$2^h\ 10^m$	$2^h\ 8^m$	5 39′
1601	☉	January	4	In Ethiopia, etc.		9 40′
1601	☾	June	15	$6^h\ 18^m$	$2^h\ 42^m$	4 52′
1601	☉	June	29	In southern China		4 29′
1601	☾	December	9	$7^h\ 6^m$	$3^h\ 9^m$	10 53′

Year	Eclipse	Month	Day	From noon	Duration	Digits
1601	☉	December	24	2^h 46^m	2^h 34^m	9 52′
1602	☉	May	21	In Greenland		2 41′
1602	☾	June	4	7^h 18^m	4^h 15^m	20 T. 31′
1602	☉	June	19	Toward New Granada		5 43′
1602	☉	November	13	In Magellanica		3 0′
1602	☾	November	28	10^h 2^m	3^h 44^m	17 T. 58′
1603	☉	May	10	In northern China		11 21′
1603	☾	May	24	11^h 41^m	3^h 5^m	7 59′
1603	☉	November	3	In Insul. Romeros [?]		11 17′
1603	☾	November	18	7^h 31^m	1^h 48^m	3 26′
1604	☉	April	29	In India and Arabia		9 32′
1604	☉	October	22	In Peru		6 49′
1605	☾	April	3	9^h 19^m	3^h 15^m	11 49′
1605	☉	April	18	Around Madagascar		5 31′
1605	☾	September	27	4^h 47^m	3^h 23^m	9 26′
1605	☉	October	12	2^h 32^m	2^h 10^m	9 24′
1606	☉	March	8	In northern Mexico		6 0′
1606	☾	March	24	11^h 17^m	3^h 47^m	16 T. 28′
1606	☉	September	2	In the Magellanic		6 40′
1606	☾	September	16	16^h 1^m	3^h 50^m	18 T. 40′
1607	☉	February	25	21^h 48^m	1^h 0^m	1 13′
1607	☾	March	13	6^h 36^m	1^h 27^m	1 22′
1607	☉	August	22	In Peru		9 48′
1607	☾	September	5	15^h 40^m	2^h 9^m	4 7′
1608	☉	February	15	Among the Antipodes		11 0′
1608	☾	July	27	0^h 30^m	1^h 31^m	1 53′
1608	☉	August	9	4^h 39^m	0^h 49^m	0 40′
1609	☾	January	19	15^h 21^m	3^h 26^m	10 32′
1609	☉	February	4	In Tierra del Fuego		5 52′
1609	☾	July	16	12^h 8^m	3^h 54^m	16 T. 53′
1609	☉	July	30	Toward Canada		4 10′
1609	☉	December	26	At the mouths of the Ob		5 50′
1610	☾	January	9	1^h 31^m	3^h 43^m	18 T. 6′
1610	☉	June	20	In Java Minor		10 46′
1610	☾	July	5	16^h 58^m	3^h 44^m	11 13′
1610	☉	December	15	In Cyprus, etc.		4 50′
1610	☾	December	29	16^h 47^m	2^h 22^m	4 23′
1611	☉	June	10	In the Sea of California		11 30′
1611	☉	December	4	Toward Madagascar		10 17′
1612	☾	May	14	10^h 38^m	2^h 55^m	7 22′
1612	☉	May	29	23^h 38^m	2^h 21^m	7 14′
1612	☾	November	8	3^h 22^m	2^h 14^m	9 49′
1612	☉	November	22	In Magellanica		9 0′
				Around [Cica] southern latitude 61°		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1613	☉	April	20	In northern Magellanica		
1613	☾	May	4	$0^h\ 35^m$	$3^h\ 42^m$	20 T. 21′
1613	☉	May	19	In eastern Tartary		
1613	☉	October	13	In western America		
1613	☾	October	28	$4^h\ 19^m$	$4^h\ 18^m$	1 T. 20′
1614	☉	April	8	In New Guinea		8 44′
1614	☾	April	23	$17^h\ 36^m$	$2^h\ 25^m$	5 25′
1614	☉	October	3	$0^h\ 57^m$	$1^h\ 56^m$	5 2′
1614	☾	October	17	$4^h\ 38^m$	$2^h\ 39^m$	4 56′
1615	☉	March	29	At Goa and Calicut		10 38′
1615	☉	September	22	In the Solomon Islands, etc.		
1616	☾	March	3	$1^h\ 58^m$	$3^h\ 45^m$	12 T. 2′
1616	☉	March	17	In Mexico		6 47′
1616	☾	August	26	$15^h\ 33^m$	$3^h\ 17^m$	12 T. 19′
1616	☉	September	10	In Magellanica		10 33′
1617	☉	February	5	In more southerly Magellanica		
1617	☾	February	20	$1^h\ 49^m$	$4^h\ 12^m$	17 T. 5′
1617	☉	August	1	In Biarmia, etc.		5 0′
1617	☾	August	16	$8^h\ 22^m$	$3^h\ 34^m$	15 47′
1618	☉	January	26	In Magellanica		
1618	☾	February	9	$3^h\ 29^m$	$2^h\ 4^m$	2 57′
1618	☉	July	21	In Mexico, Florida, etc.		
1619	☉	January	15	Toward California, etc.		
1619	☾	June	26	$12^h\ 40^m$	$2^h\ 14^m$	3 10′
1619	☉	July	11	In Africa	$2^h\ 57^m$	11 39′
1619	☾	December	20	$15^h\ 53^m$	$3^h\ 8^m$	10 47′
1620	☉	January	4	Around Mexico		9 0′
1620	☉	May	31	Beyond the Arctic		
1620	☾	June	14	$13^h\ 47^m$	$4^h\ 8^m$	18 T. 49′
1620	☉	June	29	In Magellanica		7 20′
1620	☾	December	9	$6^h\ 39^m$	$3^h\ 42^m$	18 T. 5′
1620	☉	November	23	In Magellanica		
1621	☉	May	20	$14^h\ 54^m$	$2^h\ 10^m$	10 44′
1621	☾	June	3	$19^h\ 42^m$	$3^h\ 15^m$	9 33′
1621	☉	November	13	In the Strait of Magellan		
1621	☾	November	28	$15^h\ 43^m$	$2^h\ 19^m$	3 38′
1622	☉	May	10	At the Cape Verde Islands		11 52′
1622	☉	November	2	At Narsinga, Malacca, etc.		
1623	☾	April	14	$17^h\ 19^m$	$3^h\ 11^m$	10 54′
1623	☉	April	29	Opposite the Cape of Good Hope		
1623	☾	October	8	$0^h\ 22^m$	$3^h\ 9^m$	8 35′
1623	☉	October	23	Of California, etc.		10 46′
1624	☉	May	18	In Novaya Zemlya		6 0′
1624	☾	April	3	$7^h\ 9^m$	$3^h\ 47^m$	17 T. 23′
1624	☉	April	17	Near the Antarctic		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1624	☉	September	12	In southern Magellanica		
1624	☾	September	26	$8^h\ 55^m$	$3^h\ 47^m$	19 T. 35′
1625	☉	March	8	Toward Mexico and Florida		
1625	☾	March	23	$14^h\ 11^m$	$1^h\ 49^m$	2 11′
1625	☉	September	1	On St Peter's Island		
1625	☾	September	16	$11^h\ 41^m$	$2^h\ 21^m$	5 6′
1626	☉	February	25	In Madagascar		8 27′
1626	☾	August	7	$7^h\ 48^m$	$1^h\ 1^m$	0 25′
1626	☉	August	21	In Mexico, Tingues [?], etc.		
1627	☾	January	30	$11^h\ 38^m$	$3^h\ 22^m$	10 21′

Year	Eclipse	Month	Day	From noon	Duration	Digits
1627	☉	February	15	In inner Magellanica		
1627	☾	July	27	$19^h\ 4^m$	$3^h\ 52^m$	15 T. 18′
1627	☉	August	11	In Tenduc and in		10 0′
1628	☉	January	6	Corte Reali		5 40′
1628	☾	January	20	$10^h\ 11^m$	$3^h\ 40^m$	18 T. 13′
1628	☉	July	1	Around the Cape of Good Hope		11 44′
1628	☾	July	16	$11^h\ 26^m$	$3^h\ 56^m$	12 T. 57′
1628	☉	December	25	In France, England, etc.		
1629	☾	January	9	$1^h\ 36^m$	$2^h\ 13^m$	4 27′
1629	☉	June	21	In the Gangetic Gulf		11 25′
1629	☉	December	14	In Peru		10 14′
1630	☾	May	25	$17^h\ 56^m$	$2^h\ 40^m$	6 0′
1630	☉	June	10	$7^h\ 47^m$	$1^h\ 38^m$	9 8′
1630	☾	November	19	$11^h\ 24^m$	$3^h\ 25^m$	9 27′
1630	☉	December	3	Toward New Guinea		10 10′
1631	☉	April	30	Around the Antarctic		
1631	☾	May	15	$8^h\ 15^m$	$3^h\ 40^m$	21 T. 32′
1631	☉	October	24	Toward the Antarctic		
1631	☾	November	8	$12^h\ 0^m$	$4^h\ 17^m$	19 T. 49′
1632	☉	April	19	Toward the Cape of Good Hope		9 2′
1632	☾	May	4	$1^h\ 24^m$	$2^h\ 37^m$	6 35′
1632	☉	October	13	In Mexico		8 37′
1632	☾	October	27	$12^h\ 23^m$	$2^h\ 46^m$	5 31′
1633	☉	April	8	$5^h\ 14^m$	$1^h\ 36^m$	4 30′
1633	☉	October	3	Toward the Maldives		12 T. 4′
1634	☾	March	14	$9^h\ 35^m$	$3^h\ 43^m$	11 18′
1634	☉	March	28	In Japan		10 19′
1634	☾	September	7	$5^h\ 0^m$	$3^h\ 11^m$	12 T. 10′
1634	☉	September	22	Toward the Cape of Good Hope		9 54′
1635	☉	February	17	Toward the Antarctic		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1635	☾	March	3	9^h 26^m	4^h 13^m	17 T. 41′
1635	☉	March	18	In northern Mexico		0 16′
1635	☉	August	12	In Iceland		5 0′
1635	☾	August	27	16^h 4^m	3^h 36^m	16 T. 59′
1636	☉	February	6	In Peru and New Guinea		
1636	☾	February	20	11^h 34^m	3^h 21^m	3 23′
1636	☉	August	1	In Tartary, etc.		11 20′
1636	☾	August	16	4^h 34^m	1^h 24^m	1 25′
1637	☉	January	26	Cambodia, Malacca, etc.		
1637	☉	July	21	In Yucatán, etc.		
1637	☾	December	31	0^h 44^m	3^h 8^m	10 45′
1638	☉	January	14	In Persia and India		9 45′
1638	☾	June	25	20^h 17^m	3^h 4^m	17 T. 7′
1638	☉	July	11	In the Strait of Magellan		9 5′
1638	☉	December	5	In southern Magellanica		2 10′
1638	☾	December	20	15^h 16^m	3^h 43^m	18 T. 8′
1639	☉	January	4	In Tartary		0 30′
1639	☉	June	1	5^h 59^m	1^h 54^m	10 40′
1639	☾	June	15	2^h 41^m	3^h 40^m	11 9′
1639	☉	November	24	In Magellanica		11 0′
1639	☾	December	9	11^h 57^m	2^h 19^m	3 46′
1640	☉	May	20	In New Spain		10 30′
1640	☉	November	13	In Peru		10 36′
1641	☾	April	25	1^h 2^m	3^h 3^m	9 49′
1641	☉	May	9	10^h 16^m	Peru	10 16′
1641	☾	October	18	8^h 19^m	2^h 47^m	6 31′
1641	☉	November	2	18^h 46^m		
1642	☉	March	30	In Estotilant		4 0′
1642	☾	April	14	14^h 31^m	3^h 47^m	18 T. 36′
1642	☉	September	25	In southern Magellanica		
1642	☾	October	7	16^h 45^m	3^h 46^m	20 T. 30′
1643	☉	March	19	13^h 53^m		
1643	☾	April	3	21^h 10^m	2^h 8^m [?]	3 9′
1643	☉	September	12	17^h 0^m		
1643	☾	September	27	7^h 38^m	2^h 35^m	6 0′
1644	☉	March	8	6^h 20^m		
1644	☉	August	31	18^h 10^m		
1645	☾	February	10	7^h 45^m	3^h 18^m	8 52′
1645	☉	February	26	In Insul. Romerij [?]		10 46′
1645	☾	August	7	2^h 4^m	3^h 46^m	13 T. 47′
1645	☉	August	21	0^h 35^m	2^h 30^m	4 40′
1646	☉	January	16	In the Strait of Anián		
1646	☾	January	30	18^h 11^m	3^h 42^m	18 T. 43′
1646	☉	July	12	6^h 57^m		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1646	☾	July	27	$6^h\ 2^m$	$3^h\ 41^m$	13 T. 48′
1647	☉	January	5	$12^h\ 10^m$		
1647	☾	January	20	$9^h\ 43^m$	$2^h\ 20^m$	4 47′
1647	☉	July	2	$0^h\ 9^m$	Southern	
1647	☉	December	25	$13^h\ 38^m$		
1648	☾	June	5	$0^h\ 55^m$	$2^h\ 23^m$	4 28′
1648	☉	June	20	$13^h\ 28^m$		
1648	☾	November	29	$19^h\ 17^m$	$2^h\ 58^m$	7 40′
1648	☉	December	13	$21^h\ 48^m$	Southern	
1649	☾	May	25	$15^h\ 20^m$	$3^h\ 45^m$	19 T. 49′
1649	☉	June	9	Within the Arctic		4 0′
1649	☉	November	4	$2^h\ 20^m$	$2^h\ 17^m$	5 19′
1649	☾	November	18	$19^h\ 56^m$	$3^h\ 54^m$	19 T. 58′
1650	☉	April	30	$5^h\ 54^m$	Southern	
1650	☾	May	15	$8^h\ 37^m$	$2^h\ 54^m$	7 57′
1650	☉	October	24	$17^h\ 17^m$	Eastern	
1650	☾	November	7	$20^h\ 29^m$	$2^h\ 30^m$	5 3′
1651	☉	April	19	$10^h\ 52^m$	Tuberonibus [?]	
1651	☉	October	14	$2^h\ 25^m$	Southern	
1652	☾	March	24	$16^h\ 52^m$	$3^h\ 10^m$	8 50′
1652	☉	April	7	$22^h\ 40^m$	$2^h\ 51^m$	9 59′
1652	☾	September	17	$7^h\ 27^m$	$3^h\ 7^m$	9 49′
1652	☉	October	2	$5^h\ 2^m$	Western	
1653	☉	February	27	In Magellanica		
1653	☾	March	13	$17^h\ 9^m$	$3^h\ 54^m$	18 T. 8′
1653	☉	August	22	More northern		
1653	☾	September	6	$23^h\ 45^m$	$3^h\ 50^m$	19 T. 0′
1654	☉	February	16	$19^h\ 10^m$	Eastern	
1654	☾	March	2	$19^h\ 25^m$	$2^h\ 2^m$	3 14′
1654	☉	August	11	$22^h\ 24^m$	$3^h\ 0^m$	9 28′
1654	☾	August	27	$11^h\ 49^m$	$1^h\ 35^m$	1 53′
1655	☉	February	6	$2^h\ 37^m$	$2^h\ 26^m$	4 20′
1655	☉	August	1	$14^h\ 19^m$	Below the horizon	
1656	☾	January	11	$9^h\ 4^m$	$3^h\ 8^m$	10 0′
1656	☉	January	26	$3^h\ 8^m$	$2^h\ 33^m$	5 0′
1656	☾	July	6	$3^h\ 17^m$	$3^h\ 43^m$	14 T. 26′
1656	☉	July	21	$11^h\ 48^m$	Below the horizon	
1656	☾	December	30	$23^h\ 30^m$	$3^h\ 43^m$	18 T. 26′
1657	☉	June	11	$11^h\ 20^m$	Below the horizon	
1657	☾	June	25	$9^h\ 35^m$	$3^h\ 29^m$	12 T. 25′
1657	☉	December	4	$20^h\ 0^m$	Eastern	
1657	☾	December	20	$7^h\ 47^m$	$2^h\ 1^m$	3 9′
1658	☉	May	31	$16^h\ 0^m$	Eastern	
1658	☾	June	14	$22^h\ 58^m$		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1658	☾	November	9	$13^h\ 56^m$		0 10′
1658	☉	November	24	$11^h\ 36^m$	Below the horizon	
1659	☾	May	6	$8^h\ 34^m$	$2^h\ 6^m$	8 5′
1659	☉	May	20	$17^h\ 4^m$	Eastern	
1659	☾	October	29	$16^h\ 16^m$	$2^h\ 39^m$	5 52′
1659	☉	November	14	$4^h\ 25^m$	$2^h\ 32^m$	9 51′
1660	☾	April	24	$21^h\ 58^m$	$3^h\ 49^m$	19 T. 47′
1660	☉	October	3	$22^h\ 34^m$		
1660	☾	October	18	$0^h\ 32^m$	$3^h\ 46^m$	21 T. 12′
1660	☉	November	2	$13^h\ 48^m$		
1661	☉	March	29	$22^h\ 32^m$		
1661	☾	April	14	$4^h\ 28^m$		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1661	☉	September	23	$1^h\ 36^m$	$1^h\ 43^m$	11 19′
1661	☾	October	7	$14^h\ 51^m$	$2^h\ 48^m$	7 4′
1662	☉	March	19	$15^h\ 8^m$		
1662	☉	September	12	$1^h\ 8^m$	Southern	
1663	☾	February	21	$16^h\ 11^m$	$2^h\ 3^m$	3 14′
1663	☉	March	9	$5^h\ 47^m$	Southern	
1663	☾	August	18	$8^h\ 45^m$	$3^h\ 34^m$	12 T. 22′
1663	☉	September	1	$8^h\ 8^m$		
1664	☉	January	27	$20^h\ 40^m$	Eastern	
1664	☾	February	11	$3^h\ 16^m$		
1664	☉	July	22	$14^h\ 48^m$		
1664	☾	August	6	$23^h\ 18^m$		
1664	☉	August	20	$22^h\ 10^m$		
1665	☉	January	15	$20^h\ 7^m$	Southern	
1665	☾	January	30	$18^h\ 47^m$	$2^h\ 3^m$	4 34′
1665	☉	July	12	$7^h\ 48^m$	Western	
1665	☾	July	26	$13^h\ 31^m$		0 10′
1666	☉	January	4	$21^h\ 33^m$		
1666	☉	July	1	$19^h\ 0^m$	$2^h\ 36^m$	11 10′
1667	☾	June	5	Near noon		
1667	☉	July	21	$2^h\ 32^m$		
1667	☉	November	15	$11^h\ 30^m$	At night	
1667	☾	November	30	$3^h\ 23^m$		
1668	☉	May	10	Below the horizon		
1668	☾	May	25	$16^h\ 26^m$	$3^h\ 8^m$	9 32′
1668	☉	November	4	$2^h\ 53^m$	$1^h\ 21^m$	5 50′
1668	☾	November	18	$3^h\ 54^m$	$2^h\ 54^m$	6 45′
1669	☾	April	15	$19^h\ 47^m$		
1669	☉	April	29	$18^h\ 18^m$	Southern	

Year	Eclipse	Month	Day	From noon	Duration	Digits
1669	⊙	October	24	$10^h\ 13^m$		
1670	⊙	April	19	$7^h\ 0^m$		
1670	⊙	September	10	$19^h\ 0^m$		
1670	☾	September	28	$15^h\ 43^m$	$2^h\ 56^m$	9 7′
1670	⊙	October	13	$12^h\ 5^m$		
1671	⊙	April	8	$23^h\ 29^m$		
1671	⊙	September	2	$21^h\ 25^m$		
1671	☾	September	18	$7^h\ 44^m$	$3^h\ 51^m$	19 49′
1672	⊙	February	28	$3^h\ 38^m$	Southern	
1672	☾	March	13	$3^h\ 17^m$		
1672	⊙	August	22	$6^h\ 43^m$		
1672	☾	September	6	$18^h\ 54^m$		
1673	⊙	February	16	$7^h\ 29^m$	Western	
1673	⊙	August	11	$21^h\ 44^m$	Southern	
1674	☾	January	21	$18^h\ 22^m$	$3^h\ 20^m$	11 21′
1674	⊙	February	5	$9^h\ 4^m$		
1674	☾	July	17	$9^h\ 40^m$	$3^h\ 49^m$	12 T. 17′
1675	☾	January	11	$8^h\ 29^m$	$3^h\ 36^m$	18 T. 53′
1675	⊙	January	25	$10^h\ 36^m$		
1675	☾	July	6	$16^h\ 31^m$	$3^h\ 47^m$	20 0′
1676	⊙	June	10	$21^h\ 26^m$	$2^h\ 34^m$	4 34′
1676	☾	June	25	$6^h\ 26^m$		
1676	⊙	December	4	$20^h\ 52^m$	Southern	
1677	⊙	May	31	$0^h\ 5^m$	Southern	
1677	⊙	November	24	$12^h\ 5^m$		
1677	☾	May	16	$16^h\ 25^m$	$3^h\ 26^m$	8 15′
1678	☾	May	6	$5^h\ 30^m$		
1678	☾	October	29	$9^h\ 17^m$	$3^h\ 48^m$	22 T. 18′
1679	⊙	April	10	$21^h\ 0^m$		
1679	☾	April	25	$11^h\ 53^m$	$2^h\ 43^m$	5 47′
1680	⊙	March	29	$23^h\ 22^m$		
1680	⊙	September	22	$7^h\ 57^m$		
1681	☾	March	4	Noon		
1681	⊙	March	19	$13^h\ 43^m$		
1681	☾	August	28	$15^h\ 22^m$	$3^h\ 24^m$	10 35′
1681	⊙	September	11	$15^h\ 43^m$		
1682	☾	February	21	$12^h\ 28^m$	$3^h\ 50^m$	19 T. 48′
1682	☾	August	17	$18^h\ 56^m$	$4^h\ 8^m$	18 T. 50′
1683	⊙	January	27	$1^h\ 35^m$	$2^h\ 11^m$	10 30′
1683	☾	February	9	$3^h\ 39^m$		
1683	☾	August	6	$20^h\ 36^m$		
1684	⊙	January	16	$6^h\ 34^m$		
1684	☾	June	26	$15^h\ 18^m$	$1^h\ 21^m$	1 35′
1684	⊙	July	12	$3^h\ 26^m$	$2^h\ 15^m$	12 0′
1684	☾	December	21	$11^h\ 18^m$	$3^h\ 22^m$	9 45′
1685	⊙	January	4	$16^h\ 0^m$		
1685	☾	June	16	$6^h\ 0^m$		
1685	☾	December	10	$11^h\ 26^m$	$4^h\ 11^m$	21 T. 45′
1686	⊙	May	21	$17^h\ 9^m$		

Year	Eclipse	Month	Day	From noon	Duration	Digits
1686	☾	June	6	At noon		
1686	☾	November	29	$12^h\ 22^m$	$2^h\ 58^m$	7 35′
1688	☾	April	15	$7^h\ 4^m$	$3^h\ 17^m$	6 49′
1688	☉	April	29	$16^h\ 27^m$		
1688	☾	October	9	At noon		
1688	☉	October	25	$19^h\ 40^m$		
1689	☾	April	4	$7^h\ 42^m$	$4^h\ 10^m$	21 T. 19′
1689	☾	September	28	$15^h\ 46^m$	$3^h\ 57^m$	20 T. 39′
1690	☉	March	10	At night		
1690	☾	March	24	$11^h\ 14^m$	$2^h\ 43^m$	5 53′
1690	☉	September	3	At night		
1690	☾	September	18	$2^h\ 42^m$		
1691	☉	February	27	$17^h\ 30^m$	At night	
1691	☉	August	23	$5^h\ 51^m$	Southern	
1692	☾	February	2	$3^h\ 20^m$		
1692	☉	February	16	$17^h\ 31^m$		
1692	☾	July	27	$16^h\ 9^m$	$3^h\ 41^m$	12 T. 28′
1693	☾	January	21	$17^h\ 25^m$	$3^h\ 53^m$	19 T. 32′
1693	☾	July	17	At noon		
1694	☾	January	11	Around noon		
1694	☉	June	22	$4^h\ 22^m$	$1^h\ 52^m$	6 22′
1694	☾	July	6	$13^h\ 51^m$		0 47′
1695	☾	May	28	Around noon		
1695	☉	May	11	$6^h\ 3^m$	Southern	
1695	☾	November	20	$8^h\ 0^m$	$3^h\ 0^m$	6 55′
1695	☉	December	5	$17^h\ 7^m$	Eastern	
1696	☾	May	16	$12^h\ 45^m$	$3^h\ 58^m$	21 T. 48′
1696	☉	May	30	$12^h\ 56^m$	At night	
1696	☾	November	8	$17^h\ 30^m$	$4^h\ 4^m$	21 45′
1696	☉	November	23	$17^h\ 32^m$	Eastern	
1697	☉	April	20	$14^h\ 32^m$	Eastern	
1697	☾	May	5	$18^h\ 27^m$		
1697	☾	October	29	$8^h\ 44^m$	$2^h\ 46^m$	8 54′
1698	☉	April	10	$9^h\ 13^m$	At night, western	
1698	☉	October	3	$15^h\ 29^m$	Eastern	
1699	☾	March	15	$8^h\ 14^m$	$3^h\ 13^m$	9 7′
1699	☉	March	30	$22^h\ 0^m$	Southern	
1699	☾	September	8	$23^h\ 22^m$	Antipodes	
1699	☉	September	22 [?]	$22^h\ 38^m$	$1^h\ 50^m$	9 58′
1700	☾	March	4	$20^h\ 11^m$	Western	
1700	☾	August	29	$1^h\ 42^m$	Eastern	

Hoc Anno omittitur Biſſextum in Kalend. Gregoriano
In this year the leap day is omitted in the Gregorian calendar.
LIBRI QVINTI FINIS.
End of Book Five.

LIBER SEXTVS DE STELLIS FIXIS.

Book Six. On the Fixed Stars.

CAPVT I.

Chapter 1.

De Nomine & Excellentia Siderum Inerrantium.

On the Name and Excellence of the Non-Wandering Stars.

[Margin: Difference between a star and an astrum.]

I. A star [stella], in Greek ἀστὴρ [Greek: astēr, "star"] and in Hebrew *Cocaû*, differs from a sidus, which in Greek is called ἄστρον [Greek: astron, "star, group of stars"] and in Hebrew *Nghas* or *Nghais*, if these names are used in their stricter sense. For, as Achilles Tatius points out in his *Isagoge* to Aratus's *Phaenomena*, ch. 14, and also Suidas in his lexicon, Galen on Book 1 of the *Epidemics*, and St Basil on Isaiah ch. 13, an ἀστὴρ or star is a single, separate, shining celestial body, whether it is a planet or not: such are Mercury, Saturn, or the bright star of Lyra. But ἄστρον δὲ τὸ ἐκ πολλῶν ἀστέρων σύστημα [Greek: "an astron is the system of many stars"], that is, *an astrum is a system composed of several stars*, and those non-wandering ones. Some think that the Latin name *sidus* suits this, as if it derived from the Greek σὺν εἴδειν [Greek: syn eidein, "to see together"], that is, from the fact that several stars are seen together in it; whereas this name is rather Latin, from *sedendo* or *subsidendo*, sitting or settling down, in setting. But the Hebrew word *Nghas* certainly suits it, from the verb *Nghus*, which means to be gathered together.

In a looser sense, however, you will find that not only the poets but also Plato among the Greeks and Cicero among the Latins call single stars, even wandering ones, *astra* and *sidera*. Conversely, Callimachus called the asterism of Coma Berenices ἄστερα [Greek: astera, "star"]. But you will more rarely find the celestial images [imagines], configured into one astrum, called a star by a singular name; to this bears that saying of Tatius above: *For every star is an astrum, yet not every astrum is a star.*

[Margin: Difference between the Astra of the Zodiac and the celestial Houses.]

The celestial images, however, especially those in the Zodiac or more closely connected with it, are called ἠστερισμένα ζώδια [Greek: ēsterismena zōdia, "starred animals"] by Hipparchus of Bithynia, that is, star-bearing or starred animals. They differ from those twelve segments of the Zodiac that the Greeks call δωδεκατεμόρια [recte: δωδεκατημόρια] [Greek: dōdekatēmoria, "twelfth-parts"] and the Latins the celestial Houses. For these segments are equal to one another, take in an equal space of the sky, and are in themselves invisible; but the figures of the celestial images, after which those houses are named, occupy unequal spaces of the sky and are visible in themselves. Sometimes they belong with one part of themselves to one house, or to one segment of the Zodiac, and

with another part to another. Some are larger than the dodecatemory named after them, some smaller. Finally, some belong to two domiciles: thus the image of Scorpio marks out the house of Libra with its Claws or arms, and the dodecatemory of Scorpio with the rest of its body.

[Margin: What are the celestial Signs?]

But because both these asterisms and the dodecatemories are called by the Greeks σήμα or σῆματα [Greek: sēma, sēmata, "sign, signs"], that is, Signs, you will find that some writers have confused the Astra of the Zodiac with the dodecatemories, and have attributed to the one certain things that suit only the other, as our Pétau learnedly shows in the dissertations of his *Uranologion*, Book 2, ch. 3 and following.

[Margin: Why are they called Fixed or non-wandering stars?]

II. Now those seven stars that with varied and anomalous motion repeatedly change their distance from one another and from the ecliptic are called wandering and erratic, or wanderers [errones], by the Latins, and πλανῆται [Greek: planētai, "wanderers"] by the Greeks. The rest, with which the sky above these glows on every side, are called *Fixed* or *Non-wandering* [Inerrantes], because they move with a regular motion, always keeping the same distance from one another and from the ecliptic, just as if they had been fastened into the sky like golden nails or light-spewing fire-gems [pyropi luciuomi], as Empedocles and Anaximenes held. As Plutarch says in Book 2 of *De placitis philosophorum*, ch. 13 and 14, they said that *the non-wandering stars are fixed or tied to crystal like nails*.

Hence the eighth sphere, filled with non-wandering stars, was called by the Greeks ἀπλάνη [Greek: aplanē], that is, non-wandering, or free from wandering; for πλάνη [Greek: planē] means wandering [error]. This is perhaps why the multitude of these stars, like a drawn-up battle-line of a camp, advancing in order at fixed intervals from one another, is more than once called in Holy Scripture *the host of heaven* [militia cæli]. Hence that line in the song of Deborah and Barak, who ascribe their victory to God: *From heaven the battle was fought against them: the stars, remaining in their order and course, fought against Sisera* (Judges ch. 5); and Ecclesiasticus 47: *A vessel of the camps on high*, etc., as below at the end of no. 3.

[Margin: Praises of the Fixed Stars.]

III. For a different reason, however, the fixed stars belong to the host of heaven. For they serve as soldiers for God, either when with their rays they hurl on the earth influences harmful to some, harmless and wholesome to others, yet always harmlessly, and fight with secret lightnings while they flash with open gleams; or when, like sentries armed with adamantine splendours, they seem to keep the night watches by the fixed turns of their evening risings. To this belong those words of the sacred canticle, Baruch 3, v. 34: *And the stars gave light in their watches and rejoiced; they were called, and they said: Here we are; and with gladness they shone for him who made them*; and the passage of Ecclesiasticus 47, of which below.

And yet it is God, God I say, who not only with a hundred starry eyes like Argus, but as wholly mind and wholly eye, either keeps watch over the good or keeps watch against the wicked: with one eye indeed, the one Jeremiah saw (ch. 1) and that we read of in the hieroglyphics as darting out of a rod and as it were blossoming from it, yet dispensing a manifold providence on account of its innumerable effects. Those innumerable stars are accordingly the symbol of this. You might call them the shining footprints of divine providence as it goes round all things, or the eyes of the upper World and, as it were, of God: metaphorically indeed, but more magnificently than if you called them the flowerets of the celestial field, or the gems of Olympus, or the torches of the ethereal ceiling, or the lamps of our life, or the funeral torches of the setting Sun, or the shining chorus of the Moon or Diana, or by other such ἄλογαις [Greek: alogais, "irrational"] analogies, by which, with a comparison by far the lowest, you would cast the stars down rather than raise them up.

I for my part would certainly prefer to draw their praise from the sacred word, and to call them the fiery and glowing tongues of heaven, which, by the perpetual wonder of their lights and the never-interrupted harmony of their most constant motions, tell the glory of their Maker while they call mortals to the praise of God; or which instruct the minds of those who contemplate more deeply in the more hidden mysteries of divine Providence toward us. So much so that for this reason they have become the symbol of sacred learning and of the learned, by the testimony of an angel, or rather of God, in Daniel ch. 12, where it is said: *But those who have been learned will shine like the splendour of the firmament, and those who instruct many to justice, like stars for perpetual eternities.* Or they are the signs of that heavenly glory to which they invite us, and as it were a kind of specimen of that inner and eternal tabernacle in which, without a veil, the Maker of the Stars will make his face shine upon the Elect. Or, finally, with a certain sacred dread of their splendour they terrify the wicked, who lie in wait amid the darkness of night and the hiding places of crime, as witnesses who will stand with the Saints against them at the Last Judgement; according to that passage of Ecclesiasticus 47, which gathers up much in their praise: *A vessel of the camps on high, shining gloriously in the firmament of Heaven. The beauty of heaven is the glory of the stars; the Lord illuminating the world on high. At the holy words they will stand for judgement, and will not fail in their watches.*

[Margin: Why the Astra contain irregular figures.]

IV. Yet to the foolish the stars could seem scattered and dispersed through the tracts of the sky by chance rather than by design, since they are not arranged at equal intervals from one another and are not enclosed in a regular figure such as we aim for in flower-beds, a square, say, a rhombus, a hexagon, a circle, an ellipse, but sprinkled ὁλοσχερῶς [Greek: holoscherōs, "wholesale, at large"] and with incredible variety, of irregular and dissimilar figures

distributed here sparsely, there thickly and in heaps. But this very thing is evidence of a providence that is indeed higher, yet more solid. First, that position and form were required by the variety of natural affections, which

had to flow down from the sky into the sublunary region in fixed alternations. Accordingly, figures that are irregular to our mere and bare geometry are regular to Theophysics, that is, to the divine science with which He governs Nature, and are commensurate with a heavenly plan and purpose. Next, had all the astra obtained a mathematically regular form, they would not be told apart by so evident a distinction, nor would their varied positions have marked out for us the different changes of the seasons. For suppose that the astra we now call the Wagons or Bears had only the form of a square or a rhombus: then, with the pole or tail turned over, they could by no means divide morning time from evening. In addition, it was in this way that the efforts of mortals were to be sharpened: only after long contemplation of the sky, devising by their own industry the images and figures of the astra and designating them with names fitted to their understanding, would they, as it were, win the sky for themselves and for all posterity. Finally, that variety, as the experience of the centuries has established, is of the greatest service not only to medicine and agriculture but above all to navigation and astronomy. Ship-masters recognise more clearly from the diversity of the astra that they are in different regions; and astronomers, observing the places of the planets or the moments of time from the altitude or the intervals of the stars, tell star from star by the surest marks, and the likeness of figures does not lead them into error.

V. Who, moreover, were the first to reduce the images of the astra to definite forms of animate or inanimate things — the Egyptians, the Chaldeans or the Greeks? And did they merely smear those images with the honey of fables, so that the young would commit them to memory more gladly, or did they intend, under the covering of those forms as under hieroglyphics, to indicate allegories and various tropologies, or a more secret power of influence upon these lower things? This is not the place to discuss it. I give only this notice about the order of my books. Because the places of the five minor planets cannot be properly observed without the places of the fixed stars, I have decided to place this book before the book on those five Wanderers.

[Margin: Authors who treat the fixed stars before the five planets.]

So too did Ptolemy, in Book 7 of the *Magna Constructio*; Geber [Jābir ibn Aflaḥ], Book 6 of his *Astronomy*; Regiomontanus in the *Epitome of Astronomy*; Copernicus, Book 2 of *De revolutionibus*, from ch. 14; Fracastoro, *Homocentrica*, section 2, from ch. 9; Johannes de Sacrobosco, ch. 1 of the *Sphere*, and Clavius there, from p. 148; Apian, proposition 4 of the *Astronomicum Caesareum*; Tycho, vol. 1 of the *Progymnasmata*, ch. 2 and 3; Longomontanus in the *Astronomia Danica*, Book 2 of the *Sphaerica*, ch. 4 and 5, and Book 1 of the *Theorica*, ch. 2, in the commentary on the fixed stars, and ch. 4, in the catalogue; Kepler, Book 3 of the *Epitome*, *of Astronomy*, and Book 4, part 1; Boulliau, Book 5 of the *Astronomia Philolaica*.

[Margin: Authors who treat the five planets before the fixed stars.]

Granted, some, following the order of the spheres, have delivered their treatment of the fixed stars, or of the motion of the eighth sphere, after the

doctrine of the five minor planets: namely, al-Battānī, ch. 51; al-Farghānī, from *differentia* 18; Peuerbach in the *Theoricae*; Io. Stadius [Johannes Stadius] in the *Canones Bergenses*, in the commentary on the fixed stars; and Biancani in the *Sphaera*, Books 17 and 18. The others who have dealt with the motion of the fixed stars without the minor planets, or have compiled various catalogues, calendars or canons of them, will be listed below in their proper places.

CAPVT II.

Chapter 2.

De Substantia & Qualitatibus Fixarum præsertim de Figura, Lumine ac Scintillatione.

On the Substance and Qualities of the Fixed Stars, Especially on Their Shape, Light and Twinkling.

I. Some things that these share with other stars I touched on in Book 3, ch. 1 and 2. More will have to be said in Book 9, on the System of the World: namely, whether the celestial bodies consist of sublunary matter, of a different matter, or of none; whether the stars are animate, or are moved and directed only by Intelligences that assist them, standing in one place or going around with them, or are moved by their own form though inanimate; again, whether they have been or will be receptacles of souls, when they were created by God, and the like.

[Margin: Opinions of the ancients on the nature of the stars.]

Speaking now of what seems to belong particularly to the fixed stars: if we believe Plutarch, *De placitis philosophorum*, Book 2, ch. 13, Thales held that the stars are ignited earth; Empedocles, ethereal fires fastened to crystal; Anaxagoras, stones snatched from the Earth by the ether, scorched by the vehemence of its revolution and turned into stars; Diogenes, exhalations of the world through holes, as in pumice; Plato, that they are for the greater part of themselves fiery, with an admixture of the other elements; Orpheus, Heraclides and the Pythagoreans, that each star is a world with its own elements; Xenophanes, coals kindled from inflamed clouds.

[Margin: And of the more recent.]

Among the more recent, Iordanus Brunus [Giordano Bruno], in his book *De Infinito & innumerabilibus*, holds that the stars are infinite worlds and as many Suns; and Io. Caramuel [Juan Caramuel y Lobkowitz], Suffragan of Mainz, that the fixed stars are an image of the Sun, replicated by the force of a refraction taking place in the Aplanes. Antonín Maria Šírek reports this in his *Radius Sidereomysticus*, p. 177, and refutes it, on the ground that the Sun, because of the enormous distance of the fixed stars, is a mere bright point relative to them, and insufficient to propagate its image in the eighth sphere. In the same place, however, he thinks it not improbable that the stars are parts of the Empyrean heaven, shining and flaming on all sides, which shine for us through holes

in a lower heaven, or through its more diaphanous parts. Mastri the Scotist, however, in disputation 2 *De caelo*, q. 2, art. 3, thinks this contradicts holy Genesis, which relates that the stars were made on the fourth day as something distinct from the Heaven and were placed in the firmament. But I am weary of these and similar opinions, since they are highly uncertain and have a very tiny spark of probability.

[Margin: Whether stars differ in species.]

Nor can we define with certainty what the Apostle says in 1 Corinthians ch. 15, *Star differs from star in brightness*. Is it to be understood of a merely accidental difference, in the intensity and brightness of their light, since we see in one and the same magnet the power of attracting iron on one face here and of repelling it on another there? Or, as most would have it, of a specific, substantial difference under the same genus? (For Aristotle, *De caelo* Book 2, t. 41, said that they are of the same genus, not of the same species.) This is what the different influences and the majesty and wisdom of the Maker seem to require: that He did not create so many and such vast bodies to no purpose all within one and the same lowest species, but several of them different in species as in power. We might perhaps discern that difference of species more clearly from other accidents, if we could look at them from closer by. As it is, because of the likeness of their light and shape and the uniformity of their motion, we judge them just as if we saw from a very long distance a forest full of various trees, or a ship carrying different living creatures, and, the rays and images being confused, reckoned all of them to be of one kind.

[Margin: Whether there is gravity, levity, penetrability in the fixed stars.]

II. As to qualities: just as you would falsely or rashly attribute gravity or levity to the non-wandering stars, so you would attribute to them still less probably penetrability, which the natural philosophers deny to every body, while the theologians grant it only to the bodies of the blessed, or by force of a miracle — even granted that Petrus Io. Prouenzalis [?] conceded it to the stars in *Quodlibet* 3, q. 7. Nor is there any danger, such as someone might perhaps suspect in the case of the planets, that they strike one another or are worn away by the ether in their very rapid motion, since they are seen to move at an always constant interval, together with the whole eighth sphere. On the fluidity and solidity of the celestial bodies, as on their sound, I shall have to speak in Book 9, on the System of the World.

[Margin: Density, rarity, opacity, diaphaneity.]

Now some density or rarity certainly belongs to the stars, since this is nothing other than a large or small number of parts of substance in the same bulk. There is also some opacity and diaphaneity, since they arrest and bound our sight in such a way that they nonetheless emit light and other influences not from their surface alone but from within themselves.

[Margin: Colour.]

From this comes some colour, at least an intentional one, especially in the nebulous stars or in those reddening like a live coal; many are of this kind, especially Cor Scorpii, Oculus Tauri, Arcturus and Capella.

[Margin: Heat.]

But whether these stars are hot in act, as some say with Telesius [Bernardino Telesio], *De natura rerum*, Book 4, ch. 16, and Cardano, *De subtilitate*, Book 4, or only in power, as the Peripatetics judge from *De caelo* 2, t. 42, is very difficult to establish. What I touched on regarding their fiery nature in Book 3, ch. 1, is however relevant here.

Granted, several sayings of the Church and the Fathers can be understood of a calorific and inflaming force, with no necessity of a heat that is, as they say, formal.

[Margin: The shape of the stars.]

III. The shape of the fixed stars, taken one by one, seems round to those looking from afar. According to Plutarch, *De placitis philosophorum*, Book 2, ch. 14, the Stoics affirmed that they are globular; Cleanthes, that they taper like a turning-post; Anaximenes, that they are like nails fixed in crystal; others, that they are as it were bright foils and plates of pictures, of no great depth. Scheiner, however, in *Apelles post tabulam*, and Antonín Maria Šírek, p. 198 of his *Radius Sidereomysticus*, hold that they are of different shapes, bounded by several faces and angles, with larger telescopes supporting this; yet because of their enormous distance they appear round, with better right than the flames of candles and lamps seen from afar. Granted, the Peripatetics assign them a spherical form on account of the perfection, on one side, of the spherical figure and, on the other, of the celestial bodies, as can be seen in the Coimbra commentators on *De caelo* 2, ch. 5, q. 1; in Rubeus [Antonio Rubio] in the same place, tr. 3, q. 1; and in other interpreters of Aristotle. But if you call them spherical at least physically, though mathematically many-angled and many-sided, like icosahedra or hecatehedra, you will walk a middle path and will not expose yourself to the refutations of the larger telescopes, to which, as Kepler says in the *Epitome of Copernican Astronomy*, p. 497, they appear not round but like bright points darting out rays.

DE LVMINE FIXARVM.

On the Light of the Fixed Stars.

[Margin: Whether the fixed stars have light from themselves or from the Sun.]

IV. Who is so blind as to deny light to so many lights of the sky? But is it received from the Sun, or implanted by nature and God? And is it greater than the light of the planets, or less?

[Margin: The opinion of Metrodorus; of al-Battānī and Witelo.]

All the fixed stars are illuminated by the Sun and receive their light from it, said Metrodorus, as Plutarch reports in *De placitis philosophorum*, Book 2, ch. 17. Al-Battānī, ch. 30, and Witelo, Book 4, prop. 77, thought the same, not only

of the Moon and all the planets but also of the non-wandering stars. Witelo therefore thinks that these change the configuration of their illuminated part from time to time with the motion of the Sun, because of the intersection of the bases of the visual pyramid and the pyramid of illumination, and he says: *And this indeed is more evident in the Moon because of its nearness to our sight; but in all the other stars, which receive their light and the actuality of their light from the Sun or from other stars, the same figures must arise from the three preceding theorems, and according to this the aspects and modes of the celestial influences are diversified. This does not appear visibly, however, in stars other than the Moon, because of their great remoteness from sight, by reason of which error befalls the sight, as is evident by 16 of this book. And so all the other stars besides the Moon are seen as always round, because of their remoteness from the eyes, for which reason a fire remote from the eyes is also seen as round. The same stars, however, when they are at their fullest, are seen sometimes larger, sometimes smaller, which we believe, from the premises, must be ascribed to the smallness of their illumination or to its abundance.*

[Margin: The opinion of Reinhold, Biancani and Scheiner.]

Reinhold subscribes to this in the *Theoricae*, p. 163. Biancani held the same in the *Sphaera*, Book 17, ch. 12, as did Scheiner in the *Disquisitiones Mathematicae*, no. 35, asserting: *If we admit that light flows to the Wanderers from the Sun, there is no reason why we should deny the same to the fixed stars*; and the Coimbra commentators on *De caelo* 2, ch. 7, q. 4.

[Margin: The view of Macrobius.]

Macrobius taught the opposite view, in Book 1 on the *Dream of Scipio*, ch. 19. Placing all the stars except the Moon above the Sun, he says: *A more perceptive observation, however, discovered the better order; and, besides the investigation of sight, this reasoning too commends it: the Moon, which lacks light of its own and borrows from the Sun, must necessarily lie below the source of its light. For this reasoning is what makes the Moon have no light of its own while all the other stars shine with their own, namely that those, being placed above the Sun, are in the very purest ether, in which whatever exists is natural light and its own.*

[Margin: Kepler's earlier opinion.]

Kepler had defended this view in thesis 25 and in the *Optics*, p. 261. There he had rejected those who said that the light of the stars comes from the Sun but passes through diaphanous bodies and is darted out by refraction: for if this were so, that light would pass only along a line diametrical from the Sun, as happens in the tails of comets, and they would not send their very bright rays to us, and would be seen only at a great distance from the Sun, contrary to experience and to Ptolemy's rules on the emersions of the stars from under the Sun's rays. With these arguments, I say, he had taught that all the planets, and especially Venus, have a light of their own and do not shine only with a light communicated by the Sun; I shall report his arguments about Venus in Book 7, ch. 5, no. 1. And he had confirmed this, because among sublunary things too many animate and inanimate things have some small light implanted in them,

and also because brilliance and twinkling argue a light of one's own, while nebulosity and sluggishness argue another's.

Afterwards, however, he read these words in Galileo's *Sidereus Nuncius*, p. 16: *The difference between the appearances of the planets and of the fixed stars also seems worth noting. For the planets present their globes exactly round and circular and, like so many little Moons flooded with light on all sides, appear orbicular; but the fixed stars are by no means seen bounded by a circular periphery, but rather as flashes of a sort, darting rays all around and twinkling a great deal. In the end they appear, with the spyglass [Perspicillum], endowed with a shape just like that which they have when viewed with natural sight, but so much larger that a little star of the fifth or sixth magnitude seems to equal the greatest Dog.* When he had read this, I say, he changed his opinion as regards the planets and speaks as follows in the *Dissertatio cum Nuncio Sidereo*, p. 9: *Another most delightful observation of yours is that of the rayed shape of the fixed stars, differing from the circular shapes of the planets. What else, Galileo, shall we gather from this, than that the fixed stars emit their light from within, while the opaque planets are painted from outside: that is, to use Bruno's words, that the former are Suns, the latter Moons or Earths?*

[Margin: The opinion of Giordano Bruno.]

For Giordano Bruno, in his book *De Infinito & innumerabilibus*, reckoned that the fixed stars are infinite in number, and as many Suns shining of themselves.

[Margin: And of Galileo.]

Galileo himself too, in Dialogue 3 on the World or Cosmic System, says in the person of Salviati: *See further how neatly that most headlong 24-hour motion is removed from the universe, and how the fixed stars, which are so many Suns conforming to our Sun, enjoy perpetual rest.*

[Margin: And of Descartes.]

Add to these Descartes, who in the *Principles* of the new philosophy, part 3, nos. 9, 10, 20 and 23, teaches that all the planets have their light from the Sun, because they are near it, and shine with a duller light; but that the fixed stars shine of themselves and are not smaller than the Sun, because they dart out a lively light, and that between them and Saturn, let alone the Sun, there is a very great interval, and around each one of them a very vast space. Our own Aguilonius [François d'Aguilon] too, in Book 5 of the *Optics*, prop. 81, and before these Cardano, *De subtilitate*, ch. 3, p. 138, affirmed that the fixed stars shine with their own light; indeed Cardano said that all do, even the Moon as regards its secondary light.

[Margin: And the opinion of Antonín Maria Šírek of Rheita.]

Finally Antonín Maria Šírek, p. 177 of his *Radius Sidereomysticus*, says that the Sun is so far from the fixed stars that it is like a point relative to them, and therefore insufficient to illuminate them, or to generate them by the refraction of its image.

The opinion of these men seems far more probable to me too, both because of the reasons brought by Galileo, Kepler and Descartes, and because it better suits the majesty of the divine Craftsman that He should have kindled not

one single star shining of itself, but several, like the Sun, and that they should acknowledge no other source of their light than GOD, the Father of all lights, according to that verse of Baruch 3: *The stars gave light in their watches; they were called, and they said, Here we are; and they shone with gladness for Him who made them.*

I do not know whether from forgetfulness of himself or for some other reason Kepler, as if doubting what he had already affirmed, says in the *Epitome of Copernican Astronomy*, p. 824, that the distance of the fixed stars is so great that the Sun appears to them at most 6‴, adding from this: *It is therefore doubtful to Galileo and to me whether from so great a distance they can receive from the Sun so much light that they reflect it to us; nor therefore do I affirm anything, but they can have it, just as certain animals and gems receive a constant light by a chemical force.*

[Margin: The bold opinion of Leucippus.]

There is no need, moreover, for us to refute Leucippus, who dared to assert that the very light of the Sun was kindled in it by the fixed stars, as Diogenes Laertius testifies in Book 9. But the discussion of the light of the planets will come up again in Book 7, ch. 5.

[Margin: The light of the fixed stars is compared with the light of the planets.]

V. The next question is whether the light of the fixed stars is greater and stronger than the light of the Sun, let alone of the other planets. Scheiner denies it in the *Disquisitiones Mathematicae*, no. 35, stating that the light of the fixed stars is, to us, much weaker than the brilliance of the Wanderers: otherwise these would tremble more than those, because they would be more easily overcome by vapours. He states too that those are mistaken who think that the light of the Dog is livelier than the light of the planets,

for it is whiter, and therefore more effective at moving the sight, but not on that account stronger. He then concludes: *The splendours of all the fixed stars are certainly as if most thinned with water; those of the planets, by contrast, are full of vigour. Who has ever read by the Dog, while one can by Venus?* Comparing the fixed stars with the Sun alone, Kepler says in the *Dissertatio cum Nuncio Sidereo*, p. 9, that *the body of this our Sun is by an inestimable measure brighter than all the fixed stars*, even taken together. For the single Sun, shining through a tiny hole into a closed room, lets you read clearly by that light; whereas all the fixed stars of the whole hemisphere pour so dim a light into the most open places that by its force you cannot see where you are putting your foot. Nor, he says, can this difference be put down to their excessive distance, *because the more distant they are, the greater in diameter than the Sun they are, even singly*. So he says; yet he had related in the *Optics*, p. 263, that in 1601 December he saw the distinct lights of Jupiter and of Spica through the same window on the same white wall. By contrast, Simon Marius in his *Mundus Iovialis* affirms that the fixed stars shine for us more strongly than the planets.

To me, as I said in Book 3, ch. 2, it seems not improbable that, if the fixed stars were as near to us as the Sun, while the Sun either, keeping the distance

it has, were divided into particles no larger in appearance than 2′ or 3′, or were removed as far from the Earth as the fixed stars are in the Copernican or Keplerian hypothesis, very many of the fixed stars would shine as much as the Sun, and conversely the Sun would then shine no more than the fixed stars do now. What moves me is, first, their very ruddy and very lively brilliance, striking our eyes in spite of so great an interval; and second, that the tiny rim of the Sun left immune from eclipse, though greater in length than any fixed star in apparent size, nevertheless does not overcome the darkness, and in spite of it fixed stars have sometimes been seen in the sky in an eclipse of the Sun that was not quite total, as I showed with Kepler in Book 5, ch. 19 and 20. A power united, moreover, is far stronger than the same power dispersed. And granted that they had a true diameter greater than the diameter of the Sun, the excessive distance of the fixed stars nevertheless makes their apparent diameter far too small, and prevents you from reading by the light of the Dog or of Lyra, as you can by the light of nearer Venus, with the light of twilight usually helping. The cause, however, for which the fixed stars, though more resplendent, seem to tremble more than the planets, is neither common to all of them nor drawn from their lesser light in respect of intensity, but is sought elsewhere.

DE SCINTILLATIONE STELLARVM.

On the Twinkling of the Stars.

VI. The twinkling of the stars, which belongs to all the fixed stars but is disputed by some in the case of the planets, has its cause either in the eye itself, or in the medium through which the rays pass, or in the object itself, or in all of these, or in two of them. The opinions of the authors who have written on this matter are therefore reduced to six classes.

[Margin: Opinion 1, attributing the twinkling of the stars to the eye.]

First, then, Aristotle, *De caelo* 2, t. 48 or t. 158, and there many Peripatetics, especially Philaltæus [?] and Amicus [Bartolomeo Amico], tr. 5, q. 7, d. 3, and with them Cardano, *De subtilitate*, attribute it to our sight, trembling and quivering towards distant things. For the Philosopher says, speaking against the rotation of the Sun asserted by some: *Of the stars the Sun alone seems to do this when rising and setting; and it does so not of itself but because of the extending of our sight: for sight stretched out far wavers because of its weakness. This perhaps is the reason why the fixed stars seem to twinkle and the planets not to twinkle. For the planets are near, so that the sight can reach them by its vigour; but towards the fixed stars it trembles because of the distance; and its trembling makes the star seem to move, since it makes no difference whether the sight moves or the thing looked at.*

[Margin: Argument for distance and place among the fixed stars, from twinkling.]

This opinion about the distinction between wandering and non-wandering stars has so prevailed that Copernicus, in Book 1, ch. 10, said: *That there is still a very great distance from Saturn, the highest of the wanderers, to the sphere of the fixed stars is shown by their twinkling lights, the sign by which they are chiefly*

distinguished from the planets. Indeed, Tycho too, vol. 1 of the *Progymnasmata*, ch. 6, p. 401, confirms from its continuous and great twinkling that the new star which appeared in Cassiopeia in the year 1572 was in the sphere of the fixed stars, saying: *Its outstanding twinkling, moreover, flashing beyond the other lights of the sky, showed most openly of all that it had chosen its place within the supreme and most vast region of the fixed stars, and had surmounted the revolutions of all the planets, let alone the lowest, elementary World.* And on p. 401: *It will have to be admitted, rightly and without any evasion, that this New star, since by a continuous twinkling, persisting as long as it could be seen, it not only imitated the fixed stars but also, especially at the start, surpassed them in this vibration of light, was not within those limits of the sky in which the revolutions of the seven planets are contained, but stood among the remotest non-wandering stars.* By a similar sign Kepler, in his book on the new star seen in the foot of Serpentarius in the year 1604, ch. 17, counts it among the fixed stars, but arguing *ad hominem* against the Aristotelians.

[Margin: Opinion 2, attributing twinkling to the medium and to restless vapours.]

Second, however, most writers on optics ascribe this to the medium through which the rays and images of the stars pass: especially Witelo, Book 10, prop. 55; Scheiner in the *Disquisitiones Mathematicae*, nos. 34 and 35; Io. Baptista Benedictus [Giovanni Battista Benedetti] in his disputation against Aristotle, ch. 21; and with them Biancani in the *Sphaera*, Book 17, ch. 12, and Bettini in *Apiarium* 8, Prog. and prop. 4. For the images of the stars formed on the convex surface of our atmosphere, they say, are spread apart and pulled asunder when the corpuscles of air and fire are moved up and down by force of levity and gravity, and around in a circle with the motion of the sky, because their continuous passage is interrupted, the rays of the more remote stars being unable to overcome and penetrate the density and opacity of the vapours. So all the fixed stars twinkle, and always; but the planets, whose rays, stronger because of their nearness, penetrate the vapours, do not twinkle except when thicker and watery vapours run between.

They confirm this, first, because the twinkling of the stars is greater near the horizon, where the vapours are more numerous and thicker, and greater in humid or windy weather, because of the greater thickness or agitation of those vapours. Second, they confirm it from the likeness of the image of the stars on the surface of running water, which seems to twinkle because of that rippling and tremulous quivering, as Witelo says; or of pebbles on the bottom of a brook, in which the fleeing water labours to tremble over the rough places, for their image too trembles from this, says Biancani; or of the bright image of the Sun, received on a small board, through the smoke of a chimney or dust raised upwards; or reflected from the gilded balls of towers when they are wet with dew, or with vapours and small clouds running between: for in the former case the image appears to tremble, in the latter those balls seem to leap this way and that, or up and down, says Scheiner. His conclusion in the end,

at no. 35, is this: *The twinkling of the stars is the one and only interruption of the images that have fallen into the eye, made by the restless running-between of vapours variously affected.* He explains this, however, in two ways: either by alternating refraction, by which denser but diaphanous vapours, refracting the rays of the stars, make the images more brilliant; or by the alternating obstruction which they interpose by their opacity to the transmission of the rays, especially from smaller stars or those radiating from a greater distance. From this, he thinks, the reason is given why all stars twinkle, but more so the fixed stars, the more northerly, those nearer the horizon, and in more humid weather; why they flicker with an irregular and disordered motion; and why Canis Major, viewed through a convex glass, seems to be almost wholly extinguished or stifled, and suddenly kindled whole. Finally he adds: *And the planets indeed all twinkle: with an agitation, though a gentle one, Saturn most, Jupiter little; with flashing, Jupiter much, Saturn virtually not at all; Mars most and violently, especially at apogee; Venus at apogee and perigee, less at mean distance; Mercury always; the Moon more rarely, as also the Sun, so far as it can be noted by the senses without a telescope.*

[Margin: Opinion 3, embracing more or less the two preceding.]

Third, the Coimbra commentators, on *De caelo* 2, ch. 8, q. 1, art. 1, ascribe this effect both to the weakness of the visual power towards distant things, because the images of the more remote stars at length give out and scarcely move the sense, and to vapours scattering the images. And they admit, with Simplicius, that some planets twinkle, especially Mercury. Yet the fixed stars move our sight far more strongly than many sublunary objects, even though those are nearer.

[Margin: Opinion 4, attributing twinkling to the stars themselves.]

Fourth, Giordano Bruno, in *De maximo & immenso*, Book 4, ch. 8, refers twinkling to the stars themselves, which, as Plato held, move about their own centre, so that their image seems to hop. Yet he says that Mars twinkles moderately because it shines with another's light; and that the Sun, if

it stood far off, would appear to twinkle because of its rotation. François d'Aguilon too, in *Optics*, Book 5, prop. 81, attributes twinkling to a very swift whirling [raptui], greater in some stars than in others, by which the stars, moved in their own cavities, turn towards us, in a reciprocal disparity, parts now brighter and now darker. In the same place, however, he rejects those who say that it comes from the speed of the diurnal motion, which very rapidly shifts the broken rays. For if this were the cause, the planets too would twinkle, and the fixed stars nearest the Pole of the World would not sparkle, because that motion is slow there. Raffaele Aversa approves Aguilon's opinion, *De caelo*, question 34, sect. 3, but he adds another probable cause, namely a mixture of opacity with diaphaneity in the very body of the stars, because of which the light is propagated irregularly to us from the innermost parts of the stars through the outermost, so that they appear to glow red.

[Margin: Opinion 5, attributing twinkling both to the stars themselves and to vapours.]

Fifth, twinkling is ascribed both to the stars themselves and to intervening vapours by Tycho, Vol. 1 of the *Progymnasmata*, ch. 6, from p. 402; by Kepler in *Astronomia Optica*, ch. 6, no. 12, in the book *De Stella Nova*, ch. 18, and in the *Epitome of Copernican Astronomy*, p. 825; and by Bartolomeo Mastri, Disp. 2 of *De caelo*, q. 5, from no. 163. Tycho grants that the planets do sometimes twinkle, but rarely and incidentally: Mercury and Venus most of all, Mars hardly at all, and Saturn least. Yet if Witelo's reasoning held, Saturn, being more remote, would have weaker rays and would twinkle more. Tycho adds that Sirius strikes our eyes with a stronger light than Mars, and so overcomes the vapours, and yet twinkles the most of all the other fixed stars. He further adds that the twinkling of both fixed stars and planets is assisted by the perpetual but varying agitation of the interposed air; but that the essential cause why the fixed stars twinkle, with a perpetual and continuous vibration of their light that is yet greater or smaller, is probably their revolution about their own centre, faster or slower, which the planets lack, and because of which the images of the fixed stars appear to spread apart.

Mastri, though he grants with Simon Marius that Mercury twinkles more than Saturn, and therefore rejects the view of Aristotle and Witelo, nevertheless recognizes the cause of twinkling as being incidentally from vapours and essentially from revolution about the star's own centre, subscribing to Aguilon and Aversa.

Finally, Kepler says in the *Epitome* that this happens either from a trembling due to intervening vapours, or from an internal alteration, as if from a paroxysm, or from the revolution of stars that display different angles, or colours or spots, one after another on their external surface. In the *Optics*, however, p. 262, he affirms that Sirius sparkles longer and more sharply than Arcturus, the Heart of the Scorpion more quickly than the Eye of the Bull, and that Venus, even in the clearest sky and the coldest air, twinkles and undulates most excitedly with a real twinkling, that is, either from a perpetual internal alteration of the body, as if in a paroxysm, or from revolution, with one part displayed after another. Lastly, in the book *De Stella Nova*, ch. 18, examining Scaliger, although he appears either to approve all Scaliger's reasons or to draw them to his own opinion, he nevertheless declares that he remains in the opinion given in the *Optics*: namely, that it arises from a motion either of the light in the body of the star or of the body of the star itself as it revolves. That is, it arises either from a reciprocal production of light through the diaphanous body of the star, from the centre sparkling like a heart, or from a turning such as there would be in a diamond, which, rolled about its own centre, would flash as it displayed different faces and angles.

He adds that all the fixed stars probably move about their own centre, and indeed all the planets as well, however remote, so that they may share with their whole surface in the life-giving rays of the Sun. The Moon is the exception, yet it is wholly illuminated by the Sun, part after part, through its own monthly turning about the Earth. He teaches moreover that twinkling belongs to Venus

and Mercury, obscurely to Mars, and not at all to Jupiter and Saturn; and that incidentally these planets and the fixed stars sparkle more when winds blow, or in moist air, or near the horizon, because of vapours or fumes disturbing and obstructing the continuous propagation of the rays. But distance alone, in itself, contributes nothing to twinkling, since the nearer planets twinkle; nor does the motion of the diurnal carrying-round, for otherwise Saturn and Jupiter would twinkle, and indeed more than Venus and Mercury, because they move faster in the diurnal motion.

[Margin: Opinion 6, coalescing from nearly all the others.]

Sixth and last, Scaliger, in Exercise 63, touches on and heaps together nearly all the causes already mentioned, but chiefly the motion of the air, the motion of the light in the star, and the motion of the star in itself, adding the size and brightness of the luminous body. For this reason, he says, larger and brighter stars twinkle more, other things being equal; and Kepler testifies that this happened in the new star when it appeared larger.

[Margin: The author's opinion.]

7. Although almost nothing is left for me to add to the opinions and modes already mentioned, and although the fifth and sixth seem more probable than the others, I add this: not only the vapours and exhalations of our atmosphere assist twinkling, but also the tiny particles of dust and fibres that fly about perpetually through the air, because of the continual fluctuation of the air. For when the air is moved in one part by winds in some region, it necessarily moves in to take its place in one part after another. In the Church of San Petronio in Bologna, when I was receiving the image of the Sun on a board, transmitted through an opening more than 170 feet from the board (on which see Book 3, ch. 14, no. 3), I observed that the image flitted about with a very rapid motion but with an immense and most manifest tremor, because of the restless fluctuation of those atoms, evidently boiling up in wonderful vortices in the Sun's ray. Fathers Francesco Maria Grimaldi, Zenus and others observed the same with me.

I say moreover that a complete rotation of the stars does not seem necessary; a libration or oscillation of them is enough. Further, if it seemed to Tycho that twinkling can be attributed to the rotation of the stars, then it is not an evident sign for placing a star among the fixed stars; for that rotation could coexist with a new star, even one placed in the region of the planets or in the sublunary region. Finally, if anyone were to admit the alternating nutation of discrete motion, of which I shall speak in ch. 18, no. 16, he would have a not improbable cause of twinkling.

SCHOLIA.

Scholia.

[Margin: On the density of the fixed stars.]

1. In Dialogue 1 of the *Cosmic System*, Galileo thinks that the fixed stars are nothing other than a denser part of their orb or heaven, which Suárez, in *De Opere sex dierum*, and many others also thought. His inference from this, that the density of the fixed stars exceeds almost infinitely the density of all the rest of the heaven, is indeed like Kepler's assertion about the Sun containing more matter than the whole aethereal air contains, of which we spoke in Book 3, ch. 2, no. 4. But I am not satisfied with Galileo's reason, which is, he says, that the heaven is supremely transparent and the stars supremely opaque. For it is uncertain whether the stars are supremely opaque, since not a few grant that they are partly diaphanous, as was said above in reporting the 4th and 5th opinions. And an increase of opacity does not necessarily bring with it an increase of density, since smoke or clouds or writing ink are more opaque than crystal, and yet not denser.

[Margin: On fixed stars viewed through glasses.]

2. Scheiner teaches in the *Disquisitiones Mathematicae*, no. 35, that if the images of the stars are received through a convex glass, the eye sees both a successive variety of colours and certain gaps and black openings between the bright parts. These effects, he says, come from the glasses, as Simon Marius said, when they too rotate and change position with the rotation of the glass; otherwise they come from the humours of the eye, or from droplets illuminated in the eye.

[Margin: On the colours of the stars.]

3. As to the colours of the fixed stars and planets, Kepler in *Astronomia Optica*, ch. 6, p. 261, says that if a black surface is strongly illuminated, it gives the colour of Mars; if a clear red one, of Jupiter; if a leaden or white one, of Saturn; if a yellow one, of Venus; if a blue one, of Mercury. In the Dog and Arcturus the colours of the rainbow are seen, in Lyra none, in the Heart of the Scorpion and the Eye of the Bull red. But these do not appear so evidently to every eye or at every time.

CAPVT III.

Chapter 3.

De Aſtrorum quæ ſunt in Zodiaco Forma, Numero Stellarum illa componentium, cum ſummula Fabellarum, & varia eruditione.

On the Form of the Constellations in the Zodiac and the Number of Stars Composing Them, with a Short Summary of the Fables and Various Learning.

[Margin: Authors from whom this chapter and the two following are drawn.]

1. The forms or images of the constellations are handed down, or expounded, or both, by Aratus in the *Phaenomena*, and there by Hyginus, Cicero, Germanicus Bassus and Rufus Festus

Avienus. After these come Vitruvius, Book 9, ch. 6 and 7; Marcus Manilius, Book 1 of the *Astronomica* [Aſtronomicōn], ch. 4; Julius Firmicus, Book 8,

from ch. 5 to 32, who adds the apotelesmata and the meanings of what they portend; Ptolemy, *Quadripartitum*, Book 1, ch. 9; Martianus Capella, *De Nuptiis Philologiae*, Book 8; St Isidore, *Origins*, Book 3, at the end; Bede, at the end of Vol. 1, in the opuscule *De Stellis & Signis*; Johannes Stöffler on the *Sphere* of Proclus, from p. 80, together with Proclus; the Arab Azophi [al-Ṣūfī] in his *Catalogue of Stars*; Peter Apian in the *Astronomicum Caesareum*, enunciation 4; Alessandro Piccolomini, after his *Sphere*, in the book *De Stellis Fixis*; Francesco Giuntini in the compendium *De Stellarum Fixarum observationibus*; David Origanus, Vol. 1 of the *Ephemerides*, part 3, ch. 2; Johannes Stadius in his commentary *De Stellis Fixis*; Johann Bayer Rhenanus in his *Uranometria*, made illustrious by its most beautiful engravings of the constellations; Christoph Grienberger in the catalogue of fixed stars with their diagrams; and the Persian Tables of al-Ṣūfī or Ebennensophi, in Boulliau, Book 5, ch. 3, and at the end of the *Philolaic Tables*.

✢ To these add, for the fables or histories, the *Mythologies* of Palaephatus, Fulgentius and Natalis Comes [Natale Conti], the *Dictionarium Poeticum* of Carolus Stephanus [Charles Estienne], and the interpreters of Ovid's *Fasti*. ✢ For the images, add the star globes made according to the mind of Tycho, or of the more distinguished navigators, published by Petrus Theodori [Pieter Dirkszoon Keyser], Jodocus Hondius, Willem Blaeu [Gulielmo Ianſonio], Jacob Floris van Langren, Jakob Bartsch [Iacobo Barſthio] and Frederick de Houtman [Friderico Houthmanno], to whom we owe very many southern stars observed on the island of Sumatra. The others, who compiled bare catalogues of the longitudes, latitudes, declinations, right ascensions, mediations of the heaven, and risings and settings of the fixed stars, do not belong here, but will be reviewed below in their own places. ✢ The astrological properties are reported by Ptolemy, Firmicus, Giuntini and Origanus in the places already named; but they are for the most part empty and utterly deceptive.

2. I have decided, therefore, to put the constellations of the zodiac first, which Manilius, Book 1, ch. 4, has gathered very elegantly in these verses:

> *Aries, the leader, shining in his gilded fleece, looks back in wonder at Taurus rising opposite, who with lowered face and brow calls to the Gemini. Cancer follows them, Leo Cancer, Virgo Leo. Then Libra, when day is equal to the time of night, draws on Scorpio, shining with his burning star, at whose tail he that is mixed with a horse aims his bent bow, now about to let fly a winged arrow. Then comes Capricorn, bent in his narrow constellation; after him Aquarius pours out his tilted urn into the accustomed waters, which Pisces eagerly enter; and these, closing the last of the signs, Aries touches.*

No less elegantly Plutarch, *De placitis philosophorum*, Book 1, ch. 7, has enclosed the zodiac in these verses, beginning from Cancer:

> *Cancer is in it, after this Leo, Virgo, then the Claws, Scorpio, the Archer, Capricorn; the pourer of water after these, the two bright constellations the Fishes; next Aries; Taurus follows, and the twin brothers.*

But Ausonius does it more briefly in the *Eclogarium*:

> *Here Aries, Taurus, Gemini, Cancer, Leo, Virgo, and Libra, Scorpio, the Archer, Capricorn, and he who holds the urn, and Pisces.*

The same Manilius, following the view of the Pythagoreans, on which see Ficino, *Platonic Theology*, Book 4, assigns to the twelve constellations or signs of the zodiac their own deities, singing in Book 2, ch. 11:

> *Pallas protects the wool-bearer; Cytherea protects Taurus; Phoebus the handsome Gemini, the Cyllenian Cancer; Jupiter himself, with the mother of the gods, rules Leo; Virgo is corn-bearing Ceres', and Libra, once fashioned, Vulcan's; warlike Scorpio clings to Mars; Diana cherishes the hunter, a man, yet of equine part; and Vesta fosters the narrow constellation of Capricorn; but Aquarius, opposite Jupiter, is Juno's star, and Neptune acknowledges his own Pisces in the sea.*

Ptolemy reports, *Quadripartitum*, Book 1, ch. 9, which planets' nature they share. The same Manilius, Book 2, ch. 13, teaches over which members these, or the twelve signs, preside: Aries over the head, Taurus the neck, Gemini the arms and shoulders, Cancer the chest, Leo the flanks and shoulder-blades, Virgo the loins, Libra the buttocks, Scorpio the groin, Sagittarius the thighs, Capricorn the knees, Aquarius the shins, Pisces the feet. As for the astrological fictions, on which see Ptolemy, Firmicus, Manilius, Giuntini and Origanus above, I am more ashamed than reluctant to report them.

ASTRA ZODIACI.

The Constellations of the Zodiac.

[Margin: The order of the constellations of the zodiac.]

[Margin: 1. Aries.]

3. Aries, the first of the constellations, is called [Greek: Κριος (krios, "ram")] Chrysomallus, Vervex [the Wether], Jupiter Ammon and Arietinum caput [Ram's Head]; in Arabic, Hammel, or with the article Elhammel. According to Ptolemy it consists of 13 stars, with 4 unformed; but according to Bayer of 19, among which three are of the third magnitude, two in the western horn and one in the eastern, called in Hebrew Sartai, Mesarthim. The star in the eastern horn, above the crown of Aries, however, seems to be of the second rather than the third magnitude, or midway between them. This star Tycho calls the principal and bright star above the head of Aries, and he preferred to begin from it the distances of the others in longitude, as from a more certain starting point, as is evident from Vol. 1 of the *Progymnasmata*, pp. 194 and 226. Copernicus, granted, in Book 2, ch. 14, reckoned the longitudes of the fixed stars not from the vernal section, which he thought irregularly movable, but from the first star, the one in the head of Aries, that is, from the westernmost in the horn.

The position of the first star of Aries in Ptolemy is not so certain, however, since he puts 170° from it to Spica and 115° 50′ to Regulus, whereas Tycho found

170° 39′ to Spica and 116° 40′ to Regulus. Now the bright star in the northern horn of Aries, or above its head, in the year 1600 stood 32° 6′ from the point of the vernal equinox, as Kepler gathers from Tycho in the Rudolphine Tables; but the first star, in the southern and western horn, stood, by the same source, 27° 37′. So Kepler rightly warns in the same place that anyone wishing to compare Tycho's longitudes of the fixed stars with Copernicus's must compare them with the southern and western star, from which Copernicus began.

[Margin: When the first star of Aries was at the beginning of the zodiac.]

Now if the first star of Aries was, in the year of Christ 1600, 27° 37′ east of the vernal section, this star would have been in the vernal section, or in the equinoctial colure (for it has a northern latitude of 7° 8′), in the year 349 before Christ by Tycho's reckoning, but by mine in 388. Govaert Wendelen, however, in Petavius [apud Petauium], Book 2 of the dissertations of the *Uranologion*, ch. 4, judges that in the year 1263 before Christ the first star of Aries preceded the vernal section by 15°; on this see Pétau's verdict in the same place.

[Margin: The fable of Aries.]

The reason for placing Aries in the heaven is sung to have been this. Ino, daughter of Cadmus and second wife of Athamas the Theban in place of the deceased Nephele, pursued with a stepmother's hatred Phryxus and Helle, the children of Athamas and Nephele. When she had given the farmers roasted seed so that it would not sprout, a great dearth of grain followed. A messenger sent for this reason to the Delphic oracle, bribed by Ino, reported that Phryxus and Helle must be sacrificed. As they stood at the altar with veiled heads, on the very point of being slaughtered, Nephele appeared over them, wrapped them in a cloud and snatched them away, and set them on a ram with a golden fleece so that they might escape. Helle fell from it into the Pontus and gave that sea the name of Hellespont. Phryxus, unharmed, reached Aeetes, king of Pontus, and after sacrificing that ram to Jupiter the Preserver, fastened the golden hide to the temple. For this reason Jupiter raised that ram to heaven. No wonder if it shines with few stars, since it left its golden fleece on earth; so Rufus Festus playfully jests, singing of it:

> *But though you may follow it in vain with your eyes through the aether,*
> *for it always lacks fires of its own, etc.*

Ovid writes elegantly of this fable in *Fasti* 3, but Hyginus tells it differently, where he treats of Jupiter Ammon; consult him.

[Margin: 2. Taurus.]

4. Taurus, the second constellation of the zodiac, is called [Greek: ταῦρος (tauros, "bull")], Bos [the Ox], Io, Isis, and in Arabic Ataur. Because of its bent knees Aratus calls it [Greek: πεπτηκῶτα (peptēkōta, "crouching")]. According to Ptolemy it consists of 33 stars with 11 unformed, that is, 44; but according to Bayer of 48, and according to Kepler, with the semi-Tychonic stars, of 52. Among these there is a single star of the first magnitude, in the southern eye of Taurus, called by the Greeks [Greek: λαμπαδίας (lampadias, "torch-like")], by Ptolemy [Greek: ὑπόκιρρος

(hypokirrhos, "somewhat tawny")], by the Arabs Aldebaran, that is, the ruling star, and also Atin Elthaur, that is, the eye of Taurus; by the Egyptians [Greek: πιώριων (piōriōn)], that is, the station or dominion of Horus, which the Sun, of course, exercises when in conjunction with this star; and by the Romans Palilicium, because it once rose at Rome on April 21 [XI.Kal.Maij], the day on which the Palilia, that is, the festival of Pales, the holiday of the shepherds and the birthday of the City, were celebrated, according to Varro, Book 5; Ovid, *Fasti* 4; and Pliny the Elder, Book 18, ch. 26. After this one the notable star is the one in the northern horn, of the second magnitude.

[Margin: The fable of Taurus.]

Taurus, moreover, is feigned to have been raised to heaven by Jupiter because, sailing to Sidon in a ship marked with the form of a bull, he carried off Europa, daughter of Agenor, as she played near the temple of Aesculapius, and took her away to Crete. From this comes the fable of Jupiter turned into a bull, on which see Ovid, *Fasti* 5, and, somewhat differently, Hyginus in the *Poetic Astronomy*. For some say that this constellation is Io, or Isis, turned into a cow by Juno and raised to heaven by Jupiter; and for this reason, or because its hinder parts are lacking, Ovid sang:

Whether it is a cow or a bull is not easy to discern: the front part shows, the hinder parts lie hidden. But whether it is a bull, or a female in this sign, against Juno's will it has this as love's reward.

[Margin: The Hyades and their little fable.]

The Hyades, of which Palilicium is one, are seven stars on the head of Taurus, two at the outgrowth of the horns and 5 on the face. They are called Hyades and Succulae, from [Greek: ἀπὸ τοῦ ὕειν (apo tou hyein, "from raining")], which means to rain, because at their cosmical rising they bring on rains and moisture; or because they imitate the gaping mouth of a pig, for in Greek pigs are [Greek: ὗες (hyes, "pigs")], and hence Suculae; or, as Bassus prefers, from the Greek letter [Greek: υ (hy, upsilon)], which they imitate; or because, born of Atlas and Hya, daughter of Oceanus, and weeping for their brother Hyas, killed by a lion in the hunt, they were raised to heaven for the sake of piety and solace and placed partly on the face of Taurus, partly at the beginnings of the horns. As Hyginus and Cicero, *De Natura Deorum* 2, report, the sisters of Hyas were Ambrosia, Eudora, Phesile, Coronis, Polixo, Phileto or Phaeo, and Thyene. Of them Ovid writes, *Fasti*, Book 5:

The face of Taurus gleams, radiant with seven flames, which the Greek sailor calls Hyades from the rain.

And in Book 6:

The next day calls forth the Hyades, the horns of the bull's brow, and the earth grows wet with much water.

Job too mentions them, ch. 9, v. 9, saying: *Who makes Arcturus and Orion, and the Hyades, and the inner parts of the South.* In Hebrew the Hyades are here called Kimah; and because among certain Hebraists that name means a

constellation made up of seven stars, as Pineda [Juan de Pineda] teaches on this passage, some refer it to the wagons of the Bears, others to the Pleiades. But the Vulgate better has Hyades. For when Job had said of God: *Who commands the Sun and it does not rise, and seals up the stars as under a seal; who alone stretches out the heavens and walks upon the waves of the sea*, describing his power in stirring up and calming storms and tempests, he adds: *Who makes Arcturus and Orion and the Hyades*, namely three constellations that are very stormy and the most tempestuous of all, rainy, gloomy and unclear. So Pliny the Elder, Book 2, ch. 39, calls the name of the Suculae rainy, and Book 18, ch. 26, stormy. For the tropological meaning of that passage, see Saints Gregory and Thomas Aquinas on that place in Job. Stöffler has more on these stars, and learnedly, on Proclus, p. 96.

[Margin: Whether the Pleiades are 7, and their little fable.]

The Pleiades too, on the back of Taurus, called Septistellium and the Huckster's Garment [Veſtis inſtitoris], are seven stars. They are called Vergiliae from the end of spring, when they once rose; in Arabic not Athoraie or Atarage but Altorieh or Benat Elnaásch, that is, the daughters of the assembly. The Egyptians call them [Greek: ἑξάστρον (hexastron)], that is, the constellation of six stars; the Hebrews call them Succoth Benoth, that is, the daughters of the tabernacle. In German they are Bruthean, in English Butrio, that is, a hen brooding her chicks, popularly the hen's brood [maſſa gallinæ]; to the Italians la Gallinella; to the Greeks [Greek: πλειάδες ἀπὸ τοῦ πλεῖν (pleiades apo tou plein, "Pleiades, from sailing")], because at their rising they show the time for sailing; or from their mother Plione, who bore them to Atlas. There are seven in Aratus and Hipparchus, but six in Attalus and Geminus, because one of them is rather nebulous and is hard to distinguish from the others. Hence Ovid's line, *Fasti* 3:

Which are usually called seven, yet are six.

For it is said in the fables that they were placed in the heaven because they had been the nurses of Jupiter or of Bacchus, and that their names were: Maia, the mother of Mercury, the brightest in the quadrilateral and of third rank; Sterope, Taygeta and Celeno, who form the quadrilateral; then Electra, Merope and Alcinoe. Among these, it is said, Electra after the burning of Troy refused to lead the dances with the others any longer, and therefore hides and scarcely appears. Others in Hyginus say that the one that hides is Merope, because she married a mortal man, Sisyphus, whereas the rest married immortal gods: Electra, Maia and Taygete married Jupiter, Sterope Mars, Alcinoe and Celeno Neptune.

But Michael Florent van Langren observed their exact figure and sent it to me; in it are added two stars unnamed by others, which he himself calls Atlas and Pleione. I do not know whether they are the ones that Wendelen says are observed as new because they appear at one time and hide at another. Viewed by telescope, however, more than 40 appeared to Galileo, as is told in the *Sidereus Nuncius*, and more than 50 to Father Zupi of our Society with a

Fontana telescope, and about as many appeared to us too. Pétau gives more, and learnedly, on the rising and setting of the Vergiliae in the *Uranologion*, Book 2 of the dissertations, ch. 9 and 10. For although these stars are small and occupy a small space, namely a single degree in longitude and 37½′ in latitude, they nevertheless have been and are indicators of great things. And rightly Aratus sings, in Cicero's translation:

[Margin: Notable points on the rising and setting of the Vergiliae.]

> *…placed in a small part, you will see the Vergiliae with a faint light. These faint ones shine, gliding with a small light; yet the name of the sign is great and is called famous, because they make bright the beginnings of summer, and afterwards, spreading their rising in the winter season, they warn mortals to entrust seeds to the lands.*

But Festus Avienus distinguishes the morning rising and setting more expressly, thus:

> *For if the Vergiliae lift themselves from the deep, it is the days for plying curved sickles on the yellow fallows; if they hide their flames in the sea, it is the time to break the earth with the pressed plough.*

Understand here, with Pétau, the morning rising, though the heliacal one, and the cosmical setting, because of those verses of Hesiod, Book 2 of the *Works* [Greek: ἔργων (ergōn, "of the Works")]:

> *When the Pleiades, born of Atlas, are rising, begin the harvest; but the ploughing when they are setting. These indeed for forty nights and days are hidden; but again, as the year turns round, they appear, as soon as the iron is sharpened.*

For in Hesiod's age, between the heliacal setting of the first or westernmost of the Pleiades and the heliacal rising of the same star or of the bright star of the Pleiades, that is, from about the beginning of Aries to 11° of Taurus, 40 days intervened; and at that same time the bright star of the Pleiades rose heliacally on May 11, but the last of the Pleiades on May 16, which was harvest time in Greece. Virgil's passage in *Georgics* 1, however,

> *Let the Eoan Atlantides hide from you, and the Cnossian star of the burning Crown depart, before you entrust the due seeds to the furrows, etc.,*

must be understood of the morning cosmical setting, not the heliacal, as Servius ignorantly thought, as if the Vergiliae were in Scorpio, where the Sun is in November, when it is time for sowing. Virgil, *Georgics* 4, on bees, also marks two times for gathering honey, one after the morning rising of the Pleiades, at the end of spring, the other after the morning setting, at the start of autumn:

> *Twice they gather the heavy produce: two times of harvest: as soon as Taygete the Pleiad shows the earth her comely face and with her foot has spurned the scorned streams of Ocean, or when the same, fleeing the constellation, etc.*

But Pétau, in that ch. 8 and 9, supplies far more, and corrects many writers who are false or obscure on the rising and setting of the Vergiliae. See also Stöffler on Proclus, p. 95.

[Margin: What are the Pleiades in Sacred Scripture?]

It remains to explain that saying of God to Job, ch. 38, v. 31: *Will you be able to join the sparkling stars, the Pleiades, or will you be able to scatter the circuit of Arcturus?* In Hebrew Arcturus is here called Chesil, of which we shall speak in ch. 4, no. 5. The Pleiades are here called Chimà or Kimà, which among Hebraists means Septistellium, that is, the knot and binding of seven stars; granted that Pineda here thinks that the Pleiades are stormy, and suspects that the Greek Chimà, which means winter or storm, is derived from this original name Chimà. But Pagninus [Sante Pagnini] translates: *Will you be able to bind the delicate fruits of the Vergiliae, or will you make the late fruits of Orion open?* The Tigurine version has it thus: *Will you hold back the delights of the Pleiades, or open the contractions of Orion?* In the same place, however, Vatablus [François Vatable] and the Royal Bible [Regia] translate *the delights of the Pleiades*; for in Hebrew it is Ghadanoth, from Ghadan, which means to delight. It is as if God were saying that no one can prevent, at the end of spring, about the rising of the Pleiades, the flowers and the first yield of fruit and grain from coming to birth; or can bring it about that in the autumn season, about the rising of Orion, the earth and the plants open for bearing fruit. The Septuagint, however, translates thus: *Have you understood the bond of the Pleiad?* On this passage Olympiodorus interprets most aptly as follows: *That chain, so to speak, and that fitting together of the stars which the Pleiades make up: is it the work of your wisdom and understanding? Or do you hold the reason why those same stars cohere so aptly with one another?* For, as St Gregory adds in the same place: *They were founded so near to one another and yet so divided, that they are both together and yet can in no way be joined.* That is why Seneca aptly said in the *Medea*, chorus of Act 1, *the dense flocks of the Pleiades*, and Manilius calls them *Sidus glomerabile*, a star-group massed like a ball. For this reason the Complutensians, Caietanus [Cajetan] and others, in Pineda on this passage, read: *Will you gather, or bind, the chains or the applications of the Pleiades?* as though deriving *Ghadanoth*, by transposition, from *Ghanad*, which means "I surround" or "I bind". But what would they have said if they had known that in so narrow a space, which is scarcely 85′, more than 40 stars have been divinely gathered, as the telescope has revealed?

[Margin: 3. Gemini.]

5. GEMINI, the third constellation of the Zodiac, Δίδυμοι [Greek: Didymoi, "the Twins"], or the Tyndaridae, or the Ledaean youths, or the Spartan offspring, or the Dioscuri, namely Castor and Pollux, as Hyginus has it; or Apollo in the preceding head and Hercules in the following one, as Varro thinks; or Triptolemus and Iasion, as others in Hyginus have it; or Amphion and Zethus, or the Samothracian gods, as others in Bayer have it. The preceding head of Gemini, namely Castor or Apollo, is called in

Arabic *Ras Algeuze* or rather *Ràs Elgiautze*, and, by a corrupted Greek name, *Apullùn*; in Chaldean *Anhelar*. The following head, namely Pollux or Hercules, is in Chaldean *Abrachaleus* or, by a corrupted name, *Garacles*, but better *Elhenaat* or *Ketpholtsumman*, that is, the shoulders. Both stars, of the second magnitude, are called by the Arabs by the common name *Elhakaach*. There is likewise a star of the second magnitude at the top of Pollux's left foot, which Proclus and most others call πρόπους [Greek: propous, "forefoot"], although Bassus and Hyginus call it πρόπος [Greek: propos], because it is nearest to Cancer and to the summer turning. This constellation consists of 25 stars according to Ptolemy, counting 7 unformed, but of 30 according to Kepler and 33 according to Bayer.

[Margin: The tale of the Twins.]

The little tale of the Twins, from Ovid, *Fasti* 5, and others, is this. Jupiter, taking the form of a swan, lay with Leda, the daughter of Cebalus and wife of Tyndareus. She laid an egg, from which were born Castor, Pollux and Helen. Castor and Pollux were bound to each other by such goodwill and concord that they never contended over rule and did nothing without common counsel. Indeed, as Homer, Hyginus and Lactantius say, when Castor was killed in the war of the Lacedaemonians against the Athenians, or when Lynceus and Idas were attacking Sparta, Pollux obtained from Jupiter that he might grant half of his life to his brother, so that they lived on alternate days; according to that passage of Virgil, *Aeneid* 6:

If Pollux ransomed his brother by alternate death, And goes and returns along the road so many times.

Therefore, as a monument of this devotion and brotherly friendship, Jupiter raised them into the heavens, where they shine in a mutual embrace, and indeed on alternate days, as Homer imagines and Hyginus relates. Hyginus adds that Neptune gave them horses and the power to calm storms and prevent shipwrecks.

[Margin: The flames of the Twins, salutary to sailors.]

For, on the testimony of Porphyrio, when they show themselves to sailors they offer hope of safety; for those twin flames that kindle about the yards during storms were taken to be Castor and Pollux. For as Pliny the Elder says, book 2, ch. 37, *they are grievous when they have come singly, and sink ships, and if they fall into the lowest part of the keel they burn them up; but twin ones are salutary and heralds of a prosperous voyage, at whose coming, they say, that dread and threatening one, called Helen, is put to flight; and for that reason they assign this divinity to Pollux and Castor and invoke them as gods at sea. The heads of men too shine round about in the evening hours, with great portent. All this for an uncertain reason, and hidden in the majesty of nature.* So sailors thought a single flame was Helen, the occasion of the burning of Troy, and unlucky for sailors, and a twin flame salutary. And Seneca, in the *Natural Questions*: *In a great storm they appear like stars settling on the sail; those in danger then believe that they are helped by the*

divinity of Pollux and Castor. Horace too mentions them, praying for a fortunate voyage for Virgil, book 1, Ode 3:

So may the goddess who rules Cyprus, So may the brothers of Helen, shining stars, And the Father of the winds guide you, etc.

and Propertius in that verse:

To behold and to seek the longed-for Tyndaridae.

[Margin: Fire of St Elmo.]

The Greeks call this meteor πολυδεύχεα [Greek: polydeuchea]; the Spaniards *El fuego de Sant'Elmo*. I have read in the narrative of Antonius Pigafetta [Antonio Pigafetta], in Ramusius [Giovanni Battista Ramusio], that it appeared more than once, with a fortunate light, in the storms of the Magellanic voyage. On the cause of this meteorological impression natural philosophers say much in their *Meteorics*: for Xenophanes, in Plutarch, *De placitis*, book 2, ch. 18, thought they were little clouds kindled above the ships by the motion of the storm, whereas Metrodorus thought they were flashings of the eyes of those hallucinating from fear.

[Margin: 4. Cancer.]

6. CANCER, the fourth constellation of the Zodiac, Κάρκινος [Greek: Karkinos, "crab"]; or ὀκτάπους [Greek: oktapous, "eight-footed"], that is, *Octipes*; or ὀπισθοβάμων [Greek: opisthobamōn], that is, walking backward; likewise *Nepa*, *Astacus*, *Cammarus*; in Arabic *Elsartan*, not *Asartan*. It consists of 13 stars according to Ptolemy, counting 4 unformed, of 17 according to Kepler and 35 according to Bayer. Two of these are of the third magnitude: one on the arm, or claw, or acetabulum, called in Arabic *Azubene*; the other on the last southern foot. In addition, in the quadrilateral there are two following stars of the fourth magnitude, which the Greeks call ὄνους [Greek: onous, "asses"] or ὀνίσκους [Greek: oniskous, "little asses"], *Asini* or *Aselli*, as Proclus has it; but Manilius calls them *Iugulae*; in Arabic *Elnatret*. Finally there is a famous star, likewise of the fourth magnitude, on the breast of Cancer, called *Nebulosa*, *Praesepe* and *Praesepium*; in Greek τὸ νεφελοειδὲς [Greek: to nepheloeides, "the cloudy one"], and φάτνη [Greek: phatnē, "manger"], and συστροφὴ [Greek: systrophē, "gathering"]; and in Arabic *Meeleph* or *Melleff*; in Chaldean *Pesebre*. This nebulous star, examined with Galileo's telescope, contains 36 little stars; and astrologers say that it portends ruin to the eyes if it is in the Ascendant with the Luminaries, or in any circle of position.

[Margin: The little tale of Cancer.]

Cancer is feigned to have been set in the heavens through the prayers of Juno, because it had been killed by Hercules, whose foot it had seized with its bite while he was fighting against the Hydra on the bank of the Lernaean swamp. The Aselli with their manger are said to have ascended to the heavens because, as Hyginus reports from Eratosthenes, Liber, Vulcan and the Satyrs, riding on asses, more easily defeated the Giants, who were terrified by the

uncouth braying of the asses; or because they carried Liber across a swamp in such a way that he did not touch the water.

[Margin: The gate of Cancer and of Capricorn.]

As to why the Platonists imagined two gates of the Sun in the heavens, one of Cancer, through which descending souls were poured into bodies, and the other of Capricorn, through which those souls that were passing into gods returned to those above, see Macrobius on the *Dream of Scipio*, book 1, ch. 12; Stœflerus [Johannes Stöffler] on Proclus, p. 89; and the most learned Iacobus Mazzonus [Jacopo Mazzoni] in the *Defence of Dante*, book 3, ch. 42.

[Margin: 5. Leo.]

7. LEO, the fifth constellation of the Zodiac, Λέων [Greek: Leōn, "lion"], the Herculean Lion, Cleonaean, Nemean, set in the heavens through the prayers of Juno, who was always hostile to Hercules, by whom it had been killed. In Arabic or Chaldean it is called not *Alezet* or *Alatid*, but *Ellessed*. It consists of 27 stars, besides 8 unformed, in Ptolemy; but of 40 in Kepler and 43 in Bayer. Among these, two are notable and of the first magnitude. The first is in the breast, called *Cor Leonis* or *Regulus*; in Greek βασιλίσκος [Greek: basiliskos, "little king"], or βασιλικὸς ἀστὴρ [Greek: basilikos astēr, "royal star"], *Stella Basilica*, that is, the regal star; in Arabic *Kalbellessed* or *Fetzanis*; among the Moroccan and the other Nubian astrologers *Kalbelasid*, which means Heart of the Lion to them as well; and it is the Prince of the celestials, as Theon says. The other is at the extremity of the Tail, in Arabic *Deneb-ellessed*, or among the Nubian astrologers *Nebolassid*, that is, Tail of the Lion, or in al-Ṣūfī *Alazet*. I do not discuss here the image of Leo which some people impress in gold for cordial medicines and for curing diseases of the kidneys, when the Sun in the heart of Leo culminates in the heavens, following Marsilius Ficinus [Marsilio Ficino], *De vita caelitus comparanda*, book 3, and Stöffler on Proclus, p. 90½.

[Margin: 6. Virgo.]

8. VIRGO, the sixth constellation of the Zodiac; πάρθενος [Greek: parthenos, "virgin"], Astraea, Erigone, Atargatys, but in Avienus Isis; in some writers Ceres, the Spike-bearing Virgin, Pax; in Arabic *Elaadari*, in others *Adrenedefa*. It has 26 stars in Ptolemy, with 6 unformed, 39 in Kepler, 42 in Bayer. Among these two are most famous, one of the first magnitude in the left hand of the Virgin, called *Arista* or *Spica*, in Greek στάχυς [Greek: stachys, "ear of grain"]; and in the Persian Tables of Chrisococca [Georgios Chrysokokkes] called μικρὸς κοντάρατος [Greek: mikros kontaratos]; in Arabic *Azimech*, or in Hermes *Alhaiseth*, as Bayer has it. But according to Kircher it is called among the Arabs *Sombalet Elaadari*, that is, *Spica Virginis*, or *Eltsamach*; and among the Nubian astrologers *Elaazelet*, and corruptly *Alaazel* or *Alzimon*, that is, *fusus* of the Virgin [*fusus*: spindle]. The other is the preceding star in the right wing, called *Praevindemiatrix*, *Vindemiator*, in Vitruvius,

book 9, ch. 6, *Provindemia*; in Greek προτρυγητὴρ [Greek: protrygētēr, "fore-harvester"], or in Proclus προτρυγητὴς [Greek: protrygētēs]; in Arabic *Alaraph*, *Almuredin Alcalft*.

[Margin: The tale of the Virgin.]

She is said to have been the daughter of Jupiter and Themis, if we believe Hesiod; or of Astraeus and Aurora, if Aratus; or, as others in Hyginus have it, Erigone, daughter of Icarius. As the golden age was ending in the iron, she turned away from the earth because of the injustice and avarice of men and flew off into the heavens, according to that line of Virgil's *Georgics*:

Astraea, last of the heavenly ones, left the earth.

But Albumasar [Abū Maʿshar] the Arab, in the *Introductorium in Astronomiam*, book 6, ch. 2, says that she is the Virgin Mother of God.

[Margin: 7. Libra.]

9. LIBRA, or *Iugum* in Cicero, in Greek στάθμος [Greek: stathmos, "balance"] or στατὴρ [Greek: statēr, "balance"]; in Hebrew *Mizan*; in Arabic *Azubene*. According to Ptolemy it consists of 8 stars, with the unformed ones around it 9; but according to Bayer of 15, and according to Kepler of 18. Among these are two notable stars of second rank, called *Lances*, the Pans, or the Claws of Scorpius, and by Ptolemy *Forfices*, the Pincers. The first is the southern Pan, χηλὴ νότιος [Greek: chēlē notios, "southern claw"], in Arabic *Zubeneschemalì*; the second is the northern Pan, χηλὴ βόρειος [Greek: chēlē boreios, "northern claw"], in Arabic *Zubenelgenubi*, as Bayer has it. But Kircher says that the southern is better called *Vazneganubi* and the northern *Vazneschemali*; this one, however, is in reality on the beam, though nearer the northern Pan. There is no separate tale by which Libra was carried up into the heavens apart from Scorpius, because among the Egyptians and Chaldeans Scorpius with its Claws was a single constellation, and so there were eleven Signs, speaking of visible asterisms and signs. Hence Ovid on Scorpius:

He stretches his limbs over the space of two signs.

[Margin: To whom are Libra and Scorpius one Sign?]

And Aratus calls it μέγα θήριον [Greek: mega thērion], a great animal; and Hyginus said: *This Scorpius, because of the size of its limbs, is divided into two signs, the figure of one of which our people have called Libra.* Rightly, therefore, Martianus Capella, *De Nuptiis Philologiae*, book 8, chapter on the Fixed Signs: *The Zodiac completes twelve equal portions of Signs, but it has eleven signs: for Scorpius occupies its own space with its body just as it occupies that of Libra with its Claws.* This explains that verse of Virgil, *Georgics* 1:

Where there is room between Erigone and the following Claws.

[Margin: Of Julius Caesar and Augustus [?].]

There he wishes for Augustus a place between the Virgin and the Claws of Scorpius, as one who would fill that place and would stand in the place of a twelfth asterism; and there Servius rightly notes that Virgil followed the opinion of the Chaldeans, who make up eleven starry signs. Moreover Pliny

the Elder and Suetonius, in *Caesar*, penultimate chapter, say that in that space between the Virgin and the Claws, after the murder of Gaius Julius Caesar, a hairy star appeared for 7 days while Augustus was consecrating games to him; it was believed to be the soul of Caesar received into heaven. Perhaps, then, Virgil had this in view, that Augustus should occupy a place in the heavens kindred and neighbouring to Gaius Julius. Finally, this Sign is most famous, both because it was in the horoscope of the City of Rome when it was founded, according to the calculation of Lucius Taruntius Firmanus, as Cicero testifies, *De divinatione* 2, Plutarch in *Romulus*, and Solinus, ch. 2; and because it was formerly nearest the autumnal section, in which both the Equinox occurs and a kind of equilibrium, as it were, between day and night, as Virgil sings, *Georgics* 1:

When Libra has made the hours of day and sleep equal, And now divides the globe midway between light and shadows.

[Margin: 8. Scorpius.]

10. SCORPIUS, or Scorpio, the eighth constellation; in Quintus Cicero *Nepa*; Σκόρπιος [Greek: Skorpios, "scorpion"]; in Arabic *Hacerab*, *Alatrab* or *Alacrab*, as Bayer has it, but as Kircher says it is called *Aakrab* or, with the article, *Elaakrab*. In Ptolemy's Catalogue it has 21 stars with three unformed, in Bayer 29, in Kepler 10 Tychonic and 18 semi-Tychonic. The ruddiest of these, and of first rank, on the breast of Scorpius, is called *Cor Scorpii*; in Greek ἀντάρης [Greek: antarēs], Antares; in Hesychius ἀντάρτης [Greek: antartēs], Tyrant; among the Arabs *Kalbelaakràb*, that is, Heart of Scorpius; in Sophocles *Vespertilio*, the Bat. Ptolemy wrongly places it among the stars of the second magnitude. The star on the brow of Scorpius that is the most northerly of three is very famous in the observations of the ancients. The one that precedes on the sting of the tail is called *Lesath*, or *Moshlek*, or *Alascha*; but by the Arabs, as Kircher affirms, it is called *Lessaa Elaakràb*, that is, the bite of Scorpius, or *Denneb Elaakràb*, the Tail of Scorpius.

[Margin: The little tale of Scorpius.]

Since Orion sets when Scorpius rises, the mythographers feigned that Scorpius brought death upon Orion while he was boasting that there was no animal so fierce that he could not overcome it, and that it was therefore raised into the heavens, to be a portent for restraining the display and boasting of men. So, roughly, Hyginus.

[Margin: 9. Sagittarius.]

11. SAGITTARIUS, the ninth constellation of the Zodiac, Centaur, Chiron, Philerides, Thessalian arrows, etc. In Greek τοξευτὴρ [Greek: toxeutēr, "archer"], οἱ ῥύτωρ τόξου [Greek: rhytōr toxou, "drawer of the bow"]; in Arabic *Elkusu* or *Elkausu*, or, as Kircher says, it is called *Kàus* and, with the article, *Elkaus*, that is, the bow. It has 31 stars in Ptolemy, 32 in Bayer, and in Kepler 16 Tychonic and 18 semi-Tychonic. Among these are two

of the second magnitude, one on the left hock of the right forefoot, the other on the leg of the same foot.

[Margin: The tale of Sagittarius.]

Some in Hyginus say that this is Crotus, added to the heavens by the prayers of the Muses, because he was the son of Euphene, their nurse, very swift in hunting on horseback and very keen in poetry; and that both these qualities are signified by the half-horse body and the arrows. But Macrobius, *Saturnalia*, book 1, ch. 21, says: *Sagittarius, which is the lowest and last of all the domiciles of the Zodiac, therefore degenerates from man into beast through its hinder limbs, as if thrust down by its last parts from the higher regions to the lower; yet it shoots an arrow, which indicates that even then the life of all things depends on the ray of the Sun, even as it comes from the lowest part.* He calls it lowest because it is the last of the six descending Signs, with respect to northern inhabitants.

[Margin: 10. Capricornus.]

12. CAPRICORNUS, Αἰγόκερως [Greek: Aigokerōs, "goat-horned"], *Caper*, *Aegipan*, *Hircus aequoris* (He-goat of the sea), *Pelagi procella* (Storm of the deep), *Imbrifer* (Rain-bringer); in Nonnus ἀθαλπὴς [Greek: athalpēs, "unwarmed"]; in Arabic *Elgedi* or *Alcantarus*. The star in the Tail of Capricornus is called *Denneb Elgedi*, that is, tail of Capricornus. It consists of 29 stars according to Ptolemy and Kepler; of 29 according to Bayer.

[Margin: Its tale.]

Hyginus narrates that, in fear of the giant Typhon, many of the Gods who had gathered in Egypt fled clothed in various forms, and that Pan turned himself into a Capricorn, but half fish, so as to be safe on land and sea; because of this invention he was received into heaven by Jupiter. At that time Apollo turned himself into a crane, Mercury into an ibis, Venus and Cupid into fishes, Diana into a cat. Macrobius, *Saturnalia*, book 1, ch. 21, says that this sign imitates the climbing of goats, when it brings the Sun back from the lowest parts to the upper, since from Capricorn the Sun begins to ascend towards the north.

[Margin: Capricorn, the natal Sign of Augustus.]

Capricorn is said to have been the natal star or sign of Augustus; for Suetonius says of him: *Augustus soon had such confidence in his fate that he published his horoscope and struck a silver coin with the mark of the star of Capricorn, under which he was born.* The same author, in ch. 5, testifies that he was born in the consulship of Marcus Tullius Cicero and Antonius, on September 23 [IX. Kal. Octobris]. Understand this of the old year, dislocated by the faulty intercalations of the priests; otherwise Capricorn could not have been the horoscope in the month of September of the Julian year, shortly before sunrise. On this controversy, however, see Bayer in the *Uranometria* at this constellation, and the dispute on this matter between Thesaurus [?] and Monodus [?]. Certainly Manilius, who was living then, *Astronomica*, book 2, ch. 14, sings:

He loves Erigone; Capricorn in turn turns his gaze Upon her; for what greater thing will he marvel at Than that he shone, auspicious, on the birth of Augustus?

And on this same matter there survive these verses of Germanicus:

Here, Augustus, with your natal star, your divinity, Amid astounded nations and a trembling fatherland, It bore into the heavens, and brought among your mother's stars.

[Margin: 11. Aquarius.]

13. AQUARIUS, the eleventh visible Sign, Ὑδρόχοος [Greek: Hydrochoos, "water-pourer"], Ganymede, the pourer of water; in Apian *Hydridurus*; in Horace Tyrant of the water; in others Deucalion, Aristaeus or Cecrops; in Arabic *Edeleu*. In Ptolemy it consists of 42 stars with 3 unformed, but in Bayer of 41, and in Kepler of 45. Among these one is of the first brilliance, shining in the outpouring of the water, and commonly and corruptly called *Fomahant*, although the Arabs say *Phomaut* or *Phomelhaut*, that is, mouth of the fish, as Kircher says. The star on the left shin is called *Scheat Edeleu*.

[Margin: The tale of Aquarius.]

Very well known indeed is the tale and the disgrace of Jupiter, who is feigned to have carried off to heaven, by means of the Eagle, Ganymede, son of Troilus and Callirrhoe, while he was hunting on Mount Ida, and to have made him cupbearer of the Gods. Note, from Vitruvius, book 9, ch. 6, that the outpouring of Aquarius is called ἑρμηδόνην [Greek: hermēdonēn], that is, the gift of Mercury, or his delight.

[Margin: 12. Pisces.]

14. PISCES, the twelfth constellation of the Zodiac, οἴχθυες [Greek: oichthyes, "the fishes"], one southern, the other northern; *Proles Dercia*, *Dea Syria*, Διώνη [Greek: Diōnē]; Venus and Cupid; in Arabic *Haut* or, with the article, *Elhaut*, *Sameh* or *Elhautine*; the northern, however, is called *Haut Elschemali*, and the southern *Haut Elgenubi*. In Ptolemy it has 34 stars with 4 unformed, in Bayer 37, in Kepler 59.

[Margin: The tale of Pisces.]

of which the most famous, though of third rank, is the one on the knot of the Line, which line Aratus calls σύνδεσμον [Greek: syndesmon, "bond"], or knot. Venus and Cupid turned themselves into these fishes when, fleeing Typhon, they were carried to Syria; and for that reason the Syrians once abstained from eating fish, so as not to seem to devour Gods. See the little tale in Hyginus and in Ovid, *Fasti* 2.

SCHOLIA.

Scholia.

[Margin: On the distinction of the twofold Zodiac.]

1. I have said on more than one occasion as it arose, especially in Book 1, ch. 14, and Book 3, ch. 14, in the Scholia, that there are at least two Zodiacs. One of them is Rational, that is, thinkable only by reason or imagination, but of itself invisible and ἀνάστρον [Greek: anastron, "starless"]. It is the band of the highest heaven divided into 12 equal segments, which the Ecliptic bisects, and it is wide on either side of the Ecliptic by at most 10

degrees, which is the greatest latitude of the Planets, according to what was said in Scholium 1 of Chapter 14 of Book 1. Its segments are called celestial Domiciles, and by the Greeks δωδεκατημόρια [Greek: dōdekatē-moria, "twelfth-parts"], and each occupies 30 degrees in longitude. The other is the visible and starry Zodiac, filled with the images of twelve constellations, from which the twelve segments of the rational zodiac took their name, when the beginnings of the visible Signs, or of these constellations, coincided with the beginnings of the dodecatemories, or occupied their spaces with the greatest part of themselves. But since the ancient astrologers, in order to comply with the common people and to indicate the seasons of the Year and the changes of the weather to citizens, farmers, sailors and physicians by signs visible in the heavens, used the asterisms of the Zodiac rather than the dodecatemories, great misuse and great confusion arose among writers as a result. This was because of the progress of the Fixed Stars in longitude and their recession from the segments of the rational Zodiac, which is so great that the first star of Aries was found by us in the year of Christ 1644 in rational ♈ at 28° 12′ 52″, and according to our motion of the Fixed Stars it was at the beginning of rational ♈, or on the Colure drawn through the vernal section, in the year 388 before Christ. On that confusion of the Zodiac and the Signs, review what I said in Book 3, ch. 14, in the Scholia.

It remains, then, for me to set down here, from our table of longitudes and latitudes of the Fixed Stars, how much of the heavens these twelve constellations occupy in length and in breadth, so that they may be more easily distinguished from the Segments of the rational Zodiac. I shall add the Year before the Epoch of Christ in which the beginning of these asterisms, that is, their first star, according to Tycho's opinion and mine on the motion of the Fixed Stars, was at the beginning of its own Dodecatemory, the one named after it, that is, on the great circle drawn through the poles of the Ecliptic and through the beginning of each Sign of the invisible Zodiac; and also where it was at the beginning of the World according to Kepler or ourselves. I hope that this will be very pleasant and equally very useful to my Readers. I suppose that the World was founded in the Year 4063 before CHRIST, even if Kepler favours the Year 3993, according to what was said in Book 3, ch. 25, no. 5 and 6. Note, moreover, that in the following table longitude is counted from the westernmost to the easternmost star of each constellation.

[Margin: The places here are taken from our hypotheses.]

Constellations of the Zodiac, in order	Extent of longitude	Greatest latitude, north	Greatest latitude, south	Year before (A) or after (P) Christ in which the first star of the constellation was at the beginning of its dodecatemory: Tycho	The same: ourselves	Place in the rational Zodiac of the first star of the constellation in the first Year of the World	The same in the first Year of Christ
Aries	20° 13′	12° 25′	4° 1′	349 A	388 A	♒ 8° 57′	♈ 5° 23′
Taurus	33° 38′	5° 45′	14° 30′	507 P	480 P	♒ 26° 55′	♈ 23° 21′
Gemini	26° 6′	10° 58′	10° 9′	191 A	225 A	♈ 6° 42′	♊ 3° 8′
Cancer	16° 40′	10° 23′	7° 5′	82 A	120 A	♉ 5° 14′	♋ 1° 40′
Leo	39° 9′	17° 40′	7° 39′	916 P	904 P	♉ 21° 1′	♋ 17° 20′
Virgo	46° 46′	16° 15′	9° 16′	348 P	324 P	♉ 29° 4′	♋ 25° 30′
Libra	18° 37′	8° 35′	10° 57′	986 P	976 P	♌ 20° 2′	♎ 16° 28′
Scorpius	8° 17′	1° 54′	8° 27′	325 A	344 A	♍ 8° 16′	♏ 4° 42′
Sagittarius	27° 22′	3° 6′	3° 50′	200 A	236 A	♎ 6° 50′	♐ 3° 16′
Capricornus	23° 19′	7° 16′	6° 58′	314 A	352 A	♏ 8° 28′	♑ 4° 54′
Aquarius	38° 32′	15° 23′	16° 31′	1162 P	1154 P	♏ 17° 32′	♑ 13° 58′
Pisces	41° 9′	23° 3′	9° 4′	680 P	649 P	♐ 6° 50′	♒ 20° 48′

2. From the preceding table I infer that, if the first star, that is, the westernmost of each of the twelve images of the Zodiac, is the one which now, in recent figures according to the catalogue of the Rudolphine Tables, is the first of them, the ancient astronomers who flourished before Christ did not look to that star in beginning all the Signs of the Rational Zodiac; since some of those stars were at the beginning of their dodecatemory after Christ. According to Kepler, moreover, the first star of Aries in the first Year of the World was in ♒ at 8° 23′ 25″, and in the first Year of the Christian Epoch in ♈ at 4° 57′.

[Margin: Foreign names of the Signs of the Zodiac, from Kircher.]

3. Now, Reader, receive the Arabic, Chaldean, Hebrew and Egyptian names of the Zodiac and of the Signs, which Father Athanasius Kircher, at my request, recently sent me. The Zodiac, then, was once called among the Egyptians ταμετούρο ἐντενὶφθα [Greek: tametouro enteniphtha], that is, the realm of the 12 Gods; among the Arabs *Pholck Albarugi*, that is, the Sphere of the 12 citadels; among the Chaldeans *Hadronitho Demalusche*, the Orb of the Signs, or the circuit of the 12 mansions; among the Hebrews *Galgal hammataloth*, the sphere of the Signs.

Signs of the Zodiac	Arabic	Chaldean	Hebrew	Egyptian, with its meaning
1 Aries	Hammel	*Emro*	Telch	ταμετούρο Αμνοὺ [recte: Αμοὺν, per the book's errata], of the Kings of Ammon
2 Taurus	Tur	*Tauro*	Schor	ὥριας, Station of Horus
3 Gemini	Gieuz	*Tammch*	Theomim	Κλύσος, Enclosure of Horus
4 Cancer	Sartàn	*Sartono*	Sartan	Κλαρία, beast or station of Typhon
5 Leo	Essed	*Ario*	Arieh	πιμεντεκέων, cubit of the Nile
6 Virgo	Sombalet	*Btultho*	Betulàh	ἄσφολια, station of love, which the Gods showed at the increase of the Nile
7 Libra	Mizan	*Masathe*	Miznaim	λαμβαδία, station of propitiation
8 Scorpius	Aakrab	*Acrobo*	Akrab	Ισιας, station of Isis
9 Sagittarius	Elkaus	*Kertho*	Kescheth	πιμάηρε, station of delight
10 Capricornus	Elgedi	*Gadio*	Gedi	ὀπέυτυς, arm of sacrifice
11 Aquarius	Edali	*Daulo*	Delli, that is, a bucket	ὑπευθέριαν, beneficent arm
12 Pisces	Elhaut	*Dagiotho*	Dagim	πικοτώριων, fish of Horus

For the mystical reasons, see the *Oedipus Aegyptiacus* of the same Athanasius, in the treatise on Hieroglyphic Astrology.

CAPVT IV.

Chapter 4.

De Astris Borealibus extra Zodiacum.

On the Northern Constellations outside the Zodiac.

[Margin: 1. Ursa Minor.]

1. URSA Minor, or the Lesser Wagon [Plaustrum minus], or *Cynosura* and *Phoenice*, so named because the Phoenicians and Sidonians especially used it in navigating. In Greek it is ἄρκτος μικρᾶ [Greek: arktos mikra, "little bear"], that is, the Small Bear, and ἅμαξα [Greek: hamaxa, "wagon"], namely the Wagon (understand: the Lesser), and κυνόσουρα [Greek: kynosoura, "dog's tail"]; and in Arabic *Alrukabah*, which means Wagon. It consists of 7 stars, with an eighth unformed, in Ptolemy, but of 8 in Bayer and 20 in Kepler. Among these two are of the second magnitude. One is the upper of the preceding stars in the quadrilateral, or on the right shoulder of Ursa Minor, and is called by the Arabs *Dhub Elezguar*. The other is at the extremity of the tail, called by the Ishmaelites *Ruccabah*, the name being taken from the whole asterism; commonly *la Tramontana*, the Star of the Sea, and the Pole Star, since in these centuries it is the nearest of all to the pole of the world. It was not so formerly, however, about the times of Christ, as I shall say in the appendix on the Pole Star and its declinations, to be added shortly. The two remaining little stars, which are in

the tail of Ursa Minor, are called *Vigiles* and *Circitores*, the Watchmen and the Patrollers, and by Hyginus χορέυται [Greek: choreutai, "dancers"]. Of this constellation's ἀνάστρωσει [Greek: anastrōsei; glossed by the author as "relation among the stars"], that is, its relation among the stars, and its difference from Ursa Major, I shall speak shortly, after Ursa Major.

[Margin: 2. Ursa Major.]

2. URSA Major, the Greater Wagon, Arctos Maior, *Helice*, *Callisto* or *Megisto*, and by patronymic *Parrhasis*, *Maenalis*, *Erymanthis*, *Nonacrine*. In Greek it is ἄρκτος, μεγίστη [Greek: arktos, megistē, "bear, greatest"], or ἑλίκη [Greek: helikē, "the turning one"], and ἅμαξα μείζων [Greek: hamaxa meizōn, "greater wagon"] or μέγα [Greek: mega, "great"], that is, the Greater Wagon; and in Homer ἑλικῶπις [Greek: elikōpis]. In Arabic it is *Dubbeh*, that is, Bear, and *Alrukabah*, that is, Wagon (understand: the Greater). Hence, by a corrupted word, *Dubbhelacbar*, although it ought rather to have been called *Dubbherukabah*, as though Bear-Wagon [Ursiplaustrum]. It consists of seven more notable stars, which are called *Septem Triones*, as though *teriones*, from treading [terendo] a path around the pole, going and returning around it in a restless revolution that is always visible to us. In all, however, there are 27 in Ptolemy, with 8 unformed, but 32 in Bayer and 56 in Kepler. I am surprised that these two placed the four brighter stars of the quadrilateral among the stars of the second magnitude, since it is evident that the one at the root of the tail is scarcely of third rank, as Ptolemy has it. Now, among the seven already mentioned, certain Arabs call the one on the shoulder, as the chief of them, *Dubbeh*; the first of the tail *Alhaiath*, that is, life-giving; the middle one *Mizar*; and the last *Alalicth*, that is, nocturnal, or *Benenath* or *Benenaim*.

[Margin: Arabic proverb.]

And near the middle star of the tail it has a little star barely visible, called in Arabic *Alcor*, from which comes the Arabic proverb, reported by Apian, about a man who sees tiny things and does not see, or pretends not to see, greater ones: *You saw Alcor but not the full Moon.* The Christian Arabs, however, as Father Athanasius Kircher informed me by letter, call this Greater Wagon *Naasch Laazar*, that is, the bier of Lazarus, made up of the four stars of the quadrilateral; but the three brighter stars of the tail, or wagon-pole, they call *Benath Naasch*, that is, the daughters of the Bier: the first of the tail is Mary Magdalene, the middle of the tail Martha, and the last the maidservant, for them *Ellamath*.

[Margin: Tale of both Bears.]

Further, Aratus, Avienus and Germanicus hold that Cynosura and Helice were Jupiter's nurses in Crete, when Ops was bringing him up in secret from Saturn, most voracious devourer of his sons, while the Corybantes prevented the wailing of Jupiter from reaching Saturn's ears by the clashing of bronze; and that Jupiter therefore carried each of his two nurses up into the heavens, which Agatosthenes also followed in his *Asiatic poems*. But Hesiod, the comic writer

Amphis, Ovid in the *Metamorphoses* and Hyginus in the *Poetic Astronomy* tell the tale that Ursa Major, or Callisto, was the daughter of Lycaon and a follower of Diana, who consented to debauchery with Jupiter on Mount Nonacris in Arcadia; that she was therefore turned into a Bear, either by Diana or by Juno; that when hunters pursued her she fled into a temple which it was forbidden to enter, and for that reason was to be killed, had not Jupiter, taking pity on her, carried her up into the heavens; and that she does not set because Tethys, the nurse of Juno, refuses to receive her, lest Tethys be defiled by her. Ovid referred to this in *Metamorphoses* 2, fable 6, singing:

Forbid the seven oxen the blue gulf, And drive off the stars received into heaven as the price of debauchery, Lest the concubine be dipped in the pure water.

Neat, too, is that line of Virgil, *Georgics* 1:

The Bears that fear to be dipped in the water of Ocean.

and this from Ovid, *Fasti* 3:

That there are two Bears, of which this one is called Cynosura By the Sidonians, and the Greek keel marks out Helice. Beasts great and lesser, of which the one steers Greek ships, The other Sidonian, and both themselves dry.

[Margin: Why Ursa Minor served the Sidonian and Ursa Major the Greek ship-masters.]

For to those sailing in the Mediterranean, to the Greeks in the Archipelago and in the Pontic or Euxine sea, Ursa Major did not set; but from the more southerly Sidonians, Phoenicians and Carthaginians some part of Ursa Major was hidden, either in itself and by reason of its position in the sphere, or incidentally through the vapours of the horizon; whereas Ursa Minor, turning around the pole in a tighter circuit, set for them in neither way. Therefore the Greeks directed the course of their navigation by the Greater Bear, and the Sidonians and Phoenicians by the Lesser, measuring from their revolution the time of night and the quarter of the journey completed. Germanicus, following Aratus, sang this elegantly in the following verses:

Here the nurses of Jupiter, Helice and Cynosura, shine; Helice gives the Greeks their courses by greater stars; Cynosura guides the Phoenicians. But bright all over And with liquid splendour shines Helice; no other sooner, When the Sun has hidden his gleaming face in the ocean, Sparkles as a star in the sky than the Cretan one with its seven flames. Yet Cynosura is surer for those who furrow the sea, For in a short circuit she turns her whole self about the Hinge, And, once observed, has never misled a Sidonian ship.

With this Manilius agrees, book 1 of the *Astronomica*, ch. 3:

And the greater Helice traces out the greater Bear [arcton]: Seven stars turning mark her with their light, Under whose guidance the Greek keels set sail through the waves. Small Cynosura wheels in a narrow circle, Lesser both in space and in light, but in the Tyrian's judgement she surpasses The greater: for the Phoenicians she is the surer authority As they seek on the sea a world not in view. Nor are they set with their fronts level: each turns her tail Toward the other's muzzle, and follows her follower.

Appendix De Stella Polari

Eiusq. accessu ad Polum, & erroribus Auctorum aliquorum circa illam.

Appendix on the Pole Star, its approach to the Pole, and the errors of certain authors concerning it.

[Margin: Whether the star that is now the Pole Star was also the Pole Star formerly.]

3. As we are turning this Wagon about, it seemed opportune to weigh briefly certain things said rightly or wrongly about the Pole Star: first, whether the star now called the Pole Star was formerly too, as it is now, the nearest to the North Pole among all the stars of Cynosura, and so has always deserved the name of Pole Star; next, whether it moves perceptibly, approaching the pole of the world. There are, as I said, two notable stars in the constellation of Cynosura, of the second magnitude. One is on the shoulder, the upper of the preceding stars of the quadrilateral, which the ancients used as the head of the Bear; and in the age of Eudoxus and of Hipparchus too it was the most northerly of all this asterism and the nearest to the north pole, and so it was this star that then deserved the name and office of Pole Star. The other, which sparkles at the end of the tail, was then the most southerly of all this constellation. Now, however, conversely, owing to the motion in longitude and the variation of declination, the bright star of the shoulder has become the most southerly, and the last of the tail the most northerly. That this is so will become evident first by authority, then by astronomical calculation.

Hipparchus, then, called the last star in the tail of Ursa Minor the most southerly in his time, as Marinus of Tyre attests in Ptolemy, *Geography* book 1, ch. 7. Marinus's words are: *It is related by Hipparchus that the* νοτιώτατος [Greek: notiōtatos, "most southerly"]*, that is, the most southerly star of Ursa Minor and the last of its tail, is 12° and two-fifths distant from the Pole.* So all the Greek codices have it, manuscript and printed alike, says Pétau, *Uranologion* book 3, ch. 3: for they have νοτιώτατος. So it is a wonder that so many of the Latin translators did not distinguish the periods, and, because this star is now the most northerly, corrupted this text and put "most northerly" in place of "most southerly", as Mercator, Montanus [?] and Magini did;

[Margin: Lapse of Mercator, Magini and others in the text of Ptolemy.]

because of whose authority our own Guldin [Guldinus] also hesitates here, book 2 of the Refutation of the *Elenchus* of Calvisius, ch. 10. But these men are to be corrected from the genuine Greek edition, and from a similar passage of Strabo, book 2, who says: *For the bright star at the end of the tail, which is the most southerly, is placed on the Arctic circle itself, so that it touches the horizon.* Marinus of Tyre in Ptolemy also alludes to this, saying above that the whole of Ursa Minor is enclosed by the visible circle of that region which lies 500 stadia, or one degree, north of Ocele; that the parallel of Ocele is raised 11° 24′; and that

the last star of the tail of Ursa Minor is 12° 24′ from the Pole. Magini, however, errs less here, because, although he retains the corrupt Latin edition of Ptolemy, in which it reads "most northerly", he nevertheless admits that in Hipparchus's age it was the most southerly.

Hyginus, on the contrary, writes in book 3 about Cynosura: *But among the first stars of the tail there is one from which the pole is named, as Eratosthenes says, through which place the World itself is thought to turn; and the two others are called* χορευταὶ [Greek: choreutai, "dancers"]*, because they turn around the pole.* The old scholiast on Aratus also affirms this, but confuses the seven bright stars of Ursa Major with the stars of Ursa Minor. This error, however, flowed from Aratus, or rather perhaps from Eudoxus, about whom Hipparchus of Bithynia writes thus, on Aratus's *Phenomena*, book 1, no. 5 in Pétau: *On the north pole, however, Eudoxus is mistaken, writing thus: (There is a certain star, standing always in the same place, which indeed is the pole of the World:) for no star is situated at the pole itself. But there is an empty place, beside which lie three stars, with which the point of the pole makes a four-sided figure, as Pytheas of Massalia also says.* And he goes on to refute various errors of Aratus and Eudoxus about the positions of the Fixed Stars, even though Pétau, book 3, ch. 3, tries to vindicate Eudoxus too against Hipparchus, on the ground that he spoke about another star. The author of the fragment falsely inscribed with Eratosthenes's name also says: *They name the ends of the axis of the world poles, which some have said are tiny stars, others points existing in thought alone, as does Aratus too.* And Martianus Capella, book 8, the chapter on the northern circle: *that in the very hinge of the world there is a brighter star.* Yet none of these three said which star this was. Perhaps it was either the next-to-last star of Draco or the bright shoulder of Ursa Minor, which were then about 7° from the North Pole; for no other nearer one presents itself for that time, unless you gratuitously imagine that a new star appeared in that age at the very pole and afterwards vanished. Pétau, however, in book 3 of the *Uranologion*, ch. 5, thinks that Eudoxus spoke in a popular sense rather than because he really judged it so.

4. It is established, then, from Pytheas of Massalia, from both Hipparchuses, from Strabo, Marinus of Tyre and Ptolemy, that the last star of the tail of Ursa Minor was also the most southerly of the stars of Cynosura, and was not at the pole of the world. These authors are surely to be believed rather than Eudoxus, an astronomer of a cruder age and perhaps speaking in popular terms, and Aratus, who drew from him, or the fictitious Eratosthenes, and Hyginus the Grammarian, or Capella; especially since astronomical calculation proves it. For today the last star of the tail of Ursa Minor is 2° and nearly 36′ from the Pole, as I shall show below; and there is no hypothesis of any distinguished astronomer on the motion of the Fixed Stars that does not show this star, 1,700 years ago and more, when Hipparchus of Rhodes flourished, as then the farthest from the pole of all the stars in Cynosura. And if we abide by the report of Ptolemy mentioned above, it was then 12° 24′ distant. Assuming a motion of the

Fixed Stars of one degree in 72 years, and an obliquity reckoned for that time at nearly 24°, Pétau, book 3 of the *Uranologion*, ch. 3, gathers the distance from the Pole of the following stars of Cynosura, as in this little table.

Stars of Cynosura	Distance from the Pole in the time of Eudoxus	Distance from the Pole in the time of Hipparchus
Last of the tail	14° 4′	12° 24′
Middle of the tail	12° 37′	11° 10′
Antepenultimate of the tail	11° 24′	10° 22′
Shoulder of Ursa Minor	7° 26′	8° 7′
Upper following star	9° 16′	8° 54′

[Margin: Scaliger's fabrication.]

5. Despite all this, Joseph Justus Scaliger, in the *Diatriba* on the Equinoxes and in book 4 of *De emendatione temporum*, p. 284, relying on Hyginus and on Aratus or his scholiast rather than on Pytheas, Hipparchus, Strabo, Marinus and Ptolemy, asserts that the last star of the tail of Ursa Minor both is now and formerly always was at the pole of the world. He asserts that the stars do not change their declinations with respect to this pole, but that the poles of the Equator, which he imagines as different from the poles of the world, and movable, approach the stars; that the position of the meridian line changes; and that the Fixed Stars stand still, and it is rather the equinoctial points of the Equator that move against the order of the signs [in prȩcedentia]. In a word, he overturns the entire doctrine of the motion of the eighth sphere on the emptiest and slightest of grounds, which it is not worth the trouble to report and refute here. Clavius did that, in few words, in the *Sphaera*, p. 62, but at greater length Denis Pétau in the Dissertations of the *Uranologion*, the whole of book 3, and Paul Guldin in the Refutation of the *Elenchus* of Calvisius written against the Gregorian Calendar, book 2, chs. 9 and 10. It is enough for us, as more proper to this place, to show from the distances of the last star in the tail of Ursa Minor from the Pole, observed by the highest and lowest meridian altitudes, the change of declination and the approach to the pole of the world, perceptible in this and the preceding century, let alone over several. Whether this star or another has ever been or ever will be at the very pole of the world, see below, ch. 19, Problem 5.

Obseruationes Decem Stellæ Polaris

Eiusque accessus his sæculis ad Mundi polum Borealem.

Ten observations of the Pole Star, and of its approach in these centuries to the north pole of the world.

[Margin: 1. Observation.]

6. Gemma Frisius, as he relates in the *Catholic Astrolabe*, ch. 9, observed at Leuven, in the year of Christ 1547 December 13, the greatest meridian altitude of the Pole Star as 53° 58′ and the least as 47° 43′, from which he deduced its distance from the Pole as 3° 7′ 30″. But [sed sed] according to the Tychonic approach it should have been 3° 8′ 50″.

[Margin: 2. Observation.]

Pedro Nunes [Petrus Nonius], book 2 of *De observatione*, ch. 7, after he too has censured the error of those who contend that this star was the most northerly in Hipparchus's age, since according to Ptolemy's *Geography* and the accurate version of it made by Vuernerus [Johannes Werner] and Bilibaldus [Willibald Pirckheimer] it was the most southerly, adds that in the year 1500 sailors observed it to be distant from the Pole not 4° 9′, as the Alphonsine calculation required, but 3° 30′. By Tycho's reckoning, however, it should have been 3° 24′ 40″; but those sailors may have observed with an instrument not so finely divided.

[Margin: 3. Observation.]

Paul Hainzel, as Tycho relates in *Progymnasmata*, vol. 1, p. 347, observed at Göggingen near Augsburg, in the year 1572, with a huge Quadrant, the greatest meridian altitude of the Pole Star as 51° 22′ 15″ and the least as 45° 21′ 45″, and therefore its distance from the Pole as 3° 0′ 30″.

[Margin: 4. Observation.]

Tycho himself, in the same place, p. 362, reports from both altitudes these observed distances of the Pole Star from the Pole; from which, and from Paul's observation, he concluded that it approaches the pole of the world by 20″ each year.

Year	Distance from the Pole
1577	2° 58′ 51″
1581	2° 57′ 30″
1586	2° 56′ 10″
1589	2° 54′ 50″

[Margin: 5. Observation.]

Nor does Rothmann's observation, made at Kassel in the year 1586, stand against these, in which he had taken its greatest altitude as 54° 16′ and its least as 48° 24′. For, as is evident from Tycho's letters, pp. 11, 30 and 82, and from Willebrord Snel in the *Hessian Observations*, Rothmann, having corrected his plumb line and set the instrument more exactly, found the altitude of the pole at Kassel to be 51° 19′ 20″, and hence the distance of this star from the pole 2° 55′ 40″.

[Margin: 6. Observation.]

Jacob Christmann, in his commentaries on al-Farghānī, confirms that the Pole Star approaches the pole of the world by 20″ each year, and that in the year 1618 it was 2° 44′ 30″ from it; even though he had earlier said, in book 1 of

the *Solar Observations*, ch. 4, that in the year 1596 it was 2° 51′ 50″ away, when by Tycho's reckoning it should then have been 2° 52′ 30″ away. For by a less accurate observation he had taken its greatest meridian altitude at Heidelberg as 52° 13′ 50″ and its least as 46° 30′ 10″.

[Margin: 7. Observation.]

Pierre Gassendi, in the *Judgement on the nine stars seen around Jupiter and on the reciprocation of the pendulum*, asserts that in the year

1633, toward the end of the year, at Digne, at 6^h in the morning, he observed the lowest altitude of the Pole Star as 41° 25′ 30″, and toward 6^h in the evening the highest altitude as 46° 46′, and therefore the distance as 2° 40′ 15″ and the altitude of the pole at Digne as 44° 5′ 45″. I afterwards found this observation in our Father Fournier, *Hydrographie*, book 10, ch. 27.

[Margin: 8. Observation.]

Andreas Garcia Cespedius [Andrés García de Céspedes], in the book entitled *Regimento della Navigation*, printed at Madrid in the year 1606, relates on p. 87 that he observed at Madrid in the year 1598 the greatest meridian altitude of the Pole Star as 43° 23′ and the least as 37° 24′, and therefore the difference as 5° 59′, and the half-difference, that is, the distance of the star from the Pole, as 2° 59′ 30″. But according to the Tychonic approach it should have been only 2° 51′ 50″. So Céspedes's instrument or observation was not so accurate, and neither was the observation of Rodrigo Zamorano, which the same Céspedes reports on p. 87 from Zamorano's *Breviary of Navigation*, and, from both of them, Father Fournier, *Hydrographie*, book 10, ch. 27. For Rodrigo says that he observed in the year 1584 the distance of the Pole Star from the Pole as 3° 8′, whereas from Tycho's observations it should have been 2° 56′ 30″. For ship-masters usually employ Astrolabes or hanging Rings of no great size, and a meridian line found by the magnetic needle or otherwise, but not so corrected as it should be.

[Margin: 9. Observation.]

Father Stephanus Ostoul [?], a Minim, prompted by Father Théophile Raynaud at my request, observed at Avignon in the year 1643, on January 6, 7, 8 and 9, the greatest meridian altitude of the Pole Star, in the evening, as 46° 58′, and the least, in the morning, as 40° 45′ 20″, and half the difference, that is, the distance from the Pole, as 3° 6′ 20″, from which he deduced the altitude of the pole as 43° 51′ 40″. But from Tycho's observations and ours that distance should have been 2° 36′ 30″. So there was an error of 30′ in his observation, owing to some pyramid or other that he devised as an instrument for observing.

[Margin: 10. Observation.]

[Margin: Altitude of the pole of Bologna, but at our observatory; at the middle of Bologna it would be 44° 30′.]

We, finally, at the end of the year 1641 and the beginning of 1642, with observations repeated more than forty times, in order to establish most evidently for myself and my companions the altitude of the pole of Bologna in Italy, observed with large Quadrants the least meridian altitude of the Pole Star, in

the morning, as 41° 52′, and the greatest, in the evening, as exactly 47° 7′; from which the altitude of the pole is 44° 29′ 30″ and the distance of the Star from the Pole 2° 37′ 30″, exactly as the Tychonic approach required, although while observing I was giving no thought to this. Moreover, in the year 1643 December 26, with a very large bronze Quadrant set up in the new observatory, I took this star's greatest meridian altitude as 47° 6′ 40″, and in the morning its least as 41° 52′ 20″, from which a distance from the Pole of 2° 36′ 50″ is gathered, exactly by Tycho's reckoning; I rejoiced greatly at this marvellous agreement.

[Margin: Magini's error.]

I found, moreover, in the *Ephemerides* of Nicolò Simi, mathematician of Bologna, and of Pietro Pitati, and in Biancani, the altitude of the pole of Bologna as 44° 30′. The error of Magini, therefore, who puts it at 44° 12′, came from the faulty observation of Egnazio Danti through the shadow of a Gnomon at the Equinox, in which neither the visible semidiameter of the Sun was subtracted from the Sun's altitude, nor the northern declination of about 2′, since the Equinox took place two hours before noon, as I shall teach elsewhere.

[Margin: 3. Draco.]

7. DRACO, guardian of the Hesperides, killed by Hercules and raised into heaven by Juno. In Arabic it is *Eltanin*; and in al-Ṣūfī, Mathematician of the Persians, in place of a Dragon there are five Dromedaries and two Wolves, *Abeen* or *Taeben*. It has 31 stars in Ptolemy, 33 in Bayer, 32 in Kepler. Of these the antepenultimate of the tail is of the second magnitude; and the one called the bright star of the head, which is of third rank, is called *Ras Eltanin* or *Rasaben*. There are some, as Stöffler reports, who imagine that Draco was thrown at Minerva by the Giants, but was seized by her, twisted, and flung to the heavens, and has stuck there, always entwined.

[Margin: 4. Cepheus.]

8. CEPHEUS, Κήφευς [Greek: Kēpheus], *Ceginus*, *Iasides*, Lord of the Sun, πυκαρεὺς [Greek: pykareus] *Cancaus*, and in Nonnus *Vir regius*, the Kingly Man; in Arabic, from the corrupted Greek, *Keiphus*. It has 11 stars in Ptolemy, with 2 unformed, 17 in Bayer, 11 in Kepler. Of these three are brighter, one on the left shoulder, called by the Arabs *Aderaimin*. Kircher says that three stars near the left shoulder are called in Arabic *Raar*, *Kelds* and *San*, that is, the Shepherd, the Dog and the Sheep. On the tale of Cepheus, see below, where Cassiopeia is treated.

[Margin: 5. Bootes.]

9. BOOTES, Βοωτὴς [Greek: Boōtēs, "ox-driver"], or Αρκτοφύλαξ [Greek: Arktophylax, "bear-guard"], Arctophylax, that is, guardian of the Bears; *Bubulus*, *Bubulcus*, the Oxherd; in Arabic *Theguius*, as though weeping or crying out, or *Ariamech* or *Alramech*; on the Turkish map Οἰστοφόρος [Greek: Oistophoros, "arrow-bearer"], that is, *Lanceator* or *Sagittifer*, Lancer or Arrow-bearer; in others *Lycaon*, father of the Bears, although

he is rather the son of Ursa Major; on the Persian map *Kontaratus*, and in Avenezra [Avraham ibn Ezra] *Canis latrans*, the Barking Dog. Isidore calls him the lesser Arcturus, but Hesychius calls him Orion. He carries, as Virgil sings, a crook *handsome with even knots*, that is, with stars; or a spear, as Ptolemy has it, κολόρροβον [Greek: kolorrobon]; Hesychius and the Greeks have καλαύροπον [Greek: kalauropon, "shepherd's staff"]; Hyginus ῥόπαλον [Greek: rhopalon, "club"], that is, a club; the Arabs *Incalurum* or *Alkalurops*, by a corrupted Greek word. His belt, moreover, the Arabs call *Mezer* or *Mirach*. In all, it consists of 23 stars in Ptolemy, 34 in Bayer, 28 in Kepler. Among these is one of the first magnitude, called *Arcturus*, of which I shall speak shortly, and which Ptolemy places among the unformed stars; it hangs from the belt between the knees, as Ptolemy and Proclus have it. And so Vitruvius says in book 9, ch. 6, *the middle of the knees of the guardian of the Bear* [custodis arct.], although others fasten it to the belt, with Rufus Festus, in this verse:

Where the golden belts bind the upper garments

and with Germanicus:

They call the star Arcturus where the bands make their knots.

The first star of the sickle, which he carries in his left hand, is called *Marrha*, and in Pliny the Elder *Falx Italica*, the Italian Sickle.

[Margin: Tale.]

He was not Lycaon, but the son of Jupiter and Callisto, cut into pieces by Lycaon and set before Jupiter, his guest, to eat; but he was restored by Jupiter and taken up among the stars, out of compassion, because, having pursued his mother, unrecognised under the form of a Bear, right into the temple of Lycaean Jupiter, which was a capital crime, he was to be put to death.

[Margin: Arcturus.]

ARCTURUS, Αρκτοῦρος [Greek: Arktouros], as though the tail of the Bear, since in Greek a tail is ὄυρα [Greek: oura, "tail"]; and in Homer Οψεδύων [Greek: Opsedyōn, "late-setting"]; in Isidore *Arctuzona*; in Arabic *Alkameluz* or *Kolanza*, *Alramech*; the sword or dagger of Bootes; in Pliny the Elder the *sidus horridum*, the bristling star. It is a star of the first magnitude, hanging, as I said, from the belt between the knees of Bootes.

[Margin: What is Arcturus in Amos and Job, and in the sacred interpreters?]

Now I must explain those words of the prophet Amos, ch. 5, verse 8, who praises God's power in stirring up storms: *Who makes Arcturus and Orion, and turns darkness into morning, and changes day into night, who calls the waters of the sea and pours them out upon the face of the Earth: the Lord is his name.* Our own Cornelius à Lapide expounds this passage learnedly. In Hebrew it reads: *who makes Kima and Kesil*, or *Chimà and Chesil*, or *Cecil*. Likewise those words of Job 9, verses 8 and 9: *Who commands the Sun and it does not rise, who shuts up the stars as under a seal; who makes Arcturus and Orion and the Hyades and the inner parts of the South.* In Hebrew: *who makes Nghais, Chesil and Chimà*. And those words of God himself to Job, 38, verse 31: *Will you be able to join the glittering*

stars, the Pleiades, or will you be able to break up the circuit of Arcturus? Here the name of Arcturus in Hebrew is *Chesil*, which in the same place Pagninus [Sante Pagnini] and the Zurich Bible [Tigurina] translate as Orion.

The translator of the Vulgate, therefore, renders as Arcturus now the name *Nghas* or *Nghais*, as in Job 9, now *Chesil*, as in Job 38, now the name *Chimà*, as in Amos 5. Nor is this surprising, since the Hebrew names of Animals, Plants and Stars take their origin from some property common to several species. The name *Nghas*, or *Nghais*, or *Ghas*, from the verb *Nghus*, which means "I gather", is rightly suited to any constellation aggregated from several stars, but especially to Arcturus, which not only consists of its own stars but besides seems to gather the Wagons and Bears around the Arctic pole; just as the name *Cocaû* generically signifies a star and *Cocacuim* stars. But the name *Chesil*, from the verb *Chacal*, signifies being inconstant and variable, which the Vulgate translator rightly applied to Arcturus in Job 38 because of the variability of the weather, but more rightly to Orion in Job 9 and Amos 5, because Orion is not only variable and inconstant on account of the storms it stirs up by its rising in the month of November, which from this inconstancy is called *Kasleu*, but it also sparkles with an inconstant gleam. Finally, those versed in Hebrew say that the name *Kimà* is a constellation or knot consisting of seven stars, as Pineda affirms on Job 9. Hence, therefore, the Vulgate translator not inappropriately renders this name as Hyades in Job 9 and as Pleiades in Job 38, since each of these constellations consists of seven stars; and as Arcturus in Amos 5, since Arcturus, taken figuratively for Bootes, presides over the governing of the seven stars of Ursa Minor and Major, each of which consists of seven principal and better-known stars. Although other translators have varied on these three names, they in no way conflict, however, with the generic or specific meaning. For any constellation can be called *Nghais* in Hebrew, insofar as it is gathered from several stars. If, however, it consists of seven more notable stars, as do both Bears and the Pleiades and the Hyades, it can be called *Chimà*; or if at least it comes closer to these. Finally, every star that either twinkles inconstantly or causes inconstancy and variability in the weather can be called *Chesil*. Of this kind are Orion, as we shall say in its place, and the Hyades, named from rains, as we explained in the preceding Chapter, number 4, and Arcturus, of which Pliny the Elder says in book 2, ch. 39: *The star of Arcturus hardly ever rises without stormy hail*; and in book 18, ch. 28, speaking of storms and hail, which he calls the greater force, he says: *These will come from the bristling stars, as we have said more than once, such as Arcturus, Orion and the Kids.* And before him Plautus in the *Rudens*:

Arcturus am I, of all signs by far the fiercest, Violent when I rise, more violent still when I set.

And Horace, *Odes* 2, ode 1:

The man who desires what is enough is troubled Neither by the tumultuous sea Nor by the savage onset of setting Arcturus Or of the rising Kid.

And Virgil, *Aeneid* 1:

From where come rain and fires, Arcturus and the rainy Hyades and the twin Triones.

Rightly, therefore, both Amos above and Job ch. 9 praise God's power by his ability to stir up storms at will and to vary the weather at his nod; for this is what it is to make Arcturus and Orion and the Hyades, as Pineda and Cornelius à Lapide, with most others, expound those passages. So too Job ch. 37, verse 9 says: *Out of the inner parts a storm will come forth, and cold from Arcturus*; meaning there, and in ch. 9, the inner parts of the South, from which rain-clouds and rains come, and, by the name of Arcturus, the north wind that scatters the clouds. For the Hebrew has *Mazar* or *Zarah*, which means "I scatter and winnow"; and the north wind blows from the Septentriones, which are figuratively designated by the name of Arcturus; see Pineda there.

[Margin: The opinions of the holy Fathers on these names.]

I do not know, however, from where St Thomas Aquinas on Job ch. 38, and St Gregory in the same place, book 29, ch. 29, took what they understood by the name of Arcturus: the former Ursa Major, the latter the Bears without distinction, which no one can prevent from revolving around the pole in their gyre, just as there is no one who could have composed the Pleiades, so tightly, coherently and constantly joined, as GOD did. St Basil too, on Isaiah ch. 13, verse 10, explaining the words *For the stars of heaven and their splendour will not spread forth their light*, where the Septuagint translators put Orion in place of splendour, affirms that on certain globes Orion is the same as Bootes and consists of 22 stars, or that Arcturus is placed in the middle of Orion. And St Cyril in the same place says that Isaiah *mentioned Orion or Arcturus, so as to signify by this some one of the most notable of the stars*. Less in accord, however, with the more recent nomenclatures and positions of the stars is what Pineda says on Job ch. 38, namely that the Pleiades are northern and Arcturus southern. See St Gregory, *Moralia* book 9, ch. 9, where by Arcturus he understands the Church, which never sets or sinks, though it may suffer persecutions; by Orion the holy Martyrs; and by the Hyades the holy Doctors, etc. Here now is a synopsis of the already mentioned names as used in the sacred writings, with the interpretations according to the versions noted beside them.

Passage	Name	Rendering	Authority
Job 9, verse 9	*Ng-has*	Arcturus	Vulgate
		Pleiades, or Tail of the Ram	Rabbi Mardochaeus
	Chesil	Orion	Vulgate
	Chimà	Hyades	Vulgate
		Arcturus	Septuagint
Job 37, verse 9	*Mazar*	Arcturus	Vulgate
		North wind [Aquilo]	Rabbi David and Levi, Pagninus, Vatablus [François Vatable], Zurich Bible
Job 38, verse 31	*Chimà*	Pleiades	Vulgate

Passage	Name	Rendering	Authority
	Chesil	Arcturus	Vulgate and Septuagint
		Orion	Zurich Bible, Pagninus
Isaiah 13, verse 10	*Chesil*	Splendour	Vulgate
		Orion	Septuagint
Amos 5, verse 5	*Chimà*	Arcturus	Vulgate, Vatablus, Pagninus, Arias [Benito Arias Montano]
		Pleiades	Symmachus and Theodotion
		Hyades	Chaldean
		Cynosura	Zurich Bible
	Chesil	Orion	Vulgate and Syriac
		Gemini	Arabic

[Margin: 6. Corona.]

10. CORONA Borea, the Northern Crown, or the Gnossian Crown of Theseus or of Ariadne; πρώτος στέπανος [recte: στέφανος] [Greek: prōtos stephanos, "first crown"]; in al-Ṣūfī *Parma*, the Shield; in Arabic *Elkleilchemali*, namely the Northern Crown; in Chaldean *Malphelcarte*, which means *sertum pupillae*, the garland of the *pupilla*; in the Persian table πινάκις κεκλασμένος [Greek: pinakis keklasmenos, "broken dish"], *Discus contractus*, the Contracted Disc. It has 8 stars by Ptolemy's reckoning, 20 in Bayer, and in Kepler 8 observed by Tycho. Among these one of the second magnitude is notable, called *Lucida Coronae*, the Bright Star of the Crown; in Ptolemy λαμβρὸς ἀστὴρ [Greek: lambros astēr, "bright star"]; *Alphetal* or *Alphecca*, that is, "opening", in Arabic; and *Mumir*, that is, *pupilla*. Some depict this crown as of laurel, others as of myrtle, and Bayer as of ivy with clusters of berries, from the coins of Magna Graecia and from Manilius, book 5, and Capella. For Bacchus presented his wife Ariadne with such a garland, and it is called *Alphecca* because it breaks through walls; although the tales say that a crown made in its likeness by Vulcan out of gold and gems was given to her at the wedding and afterwards set in the sky.

[Margin: 7. Hercules.]

11. HERCULES, Ενγόνασι [Greek: Engonasi, "on the knees"], that is, one who is ἐν γόνασι [Greek: en gonasi, "on the knees"], namely *in genibus*; and therefore *Ingeniculus* in Vitruvius, book 9, ch. 6, and *Nixus genu* or *genibus*, kneeling on the knee or knees, in Manilius, book 5, and in Ovid, *Metamorphoses* 8. In Arabic it is *Algethi*, or *Elzegeziale*, or *Elhathi*, that is, *geniculator*, the Kneeler; among the Persians, however, *Ternuelles*. It has 28 stars in Ptolemy, with one unformed; 48 in Bayer; in Kepler 28 observed by Tycho. Of these 9 are of the third magnitude, among which the one on the head is called *Ras-Algethi* or *Ras-elbathi*; the middle one on the left hand is called in Arabic *Marfic*, and the one on the right elbow *Maasym*. There are some who would have this be the image of Theseus,

or of Ixion, or of Thamyris, blinded by the Muses, and for that reason on his knees like those who supplicate and beg to be spared punishment; or of Orpheus, or of Prometheus.

[Margin: 8. Lyra.]

12. LYRA, or *Vultur cadens*, the Falling Vulture; in Greek χέλυς [Greek: chelys, "tortoise"] or γύψ κατειμένος [Greek: gyps kateimenos, "vulture let down", i.e. descending]; or *Deferens Psalterium*, the Bearer of the Psaltery; in Chaldean *Asanges*, or rather *Mesanguo*; in Arabic *Albegalà* or *Alahore*: whether this lyre be Apollo's or Orpheus's, according to Ovid, *Metamorphoses*, books 10 and 11. It contains 10 stars in Ptolemy, as many, that is, as there are strings on the ten-stringed psaltery; but in Bayer there are 13 and in Kepler 11. Among them the most brilliant, and of the first magnitude, is the one called *Fidicula* or *Lucida Lyrae*, the Bright Star of Lyra, and in Arabic *Brinek* or *Vuega* or *Vagieh*, that is, face.

[Margin: 9. Cygnus.]

13. CYGNUS, κύκνος [Greek: kyknos, "swan"] or ὄρνις [Greek: ornis, "bird"]; in Ptolemy *Gallina*, the Hen, but in Ovid *Miluius*, the Kite; in Arabic *Eldegiagich*, that is, *Gallina*, or corruptly *Adigege*, in others *Degige* or *Hierezim*, as if fragrant of a rose or lily. It has 17 stars in Ptolemy, with 2 unformed; 36 in Bayer; in Kepler 27 Tychonic stars. Among these one of the second rank is notable, on the Tail, called *Deneb Adigege* or *Arided*; the one on the beak is called *Albireo*, and the one at the tip of the tail *Azelfage*.

[Margin: New star in Cygnus.]

On the Breast of this Swan, in the year 1600, a new star of the 3rd magnitude appeared, having a northern latitude of 55° 37′ and a longitude of 16° 18′ of Aquarius; see Biancani, part 3 of the *Sphaera*, treatise 13. It lasted until the year 1621.

[Margin: Little tale.]

Further, Leda, by Jupiter turned into a Swan, bore an egg, from which came Helen, Castor and Pollux; for that reason the Swan was placed in the heavens, in the Milky Way.

[Margin: 10. Cassiopeia.]

14. CASSIOPEIA, among the Greeks *Cathedra* or *Thronus*, the Chair or Throne, or *Sedes regia*, the Royal Seat, or *Mulier sedis cum Siliquastro*, the Woman of the Seat with the Chair; the seat itself is called in Arabic *Dath Elkarsi*. It has 13 stars in Ptolemy, 25 in Bayer, but Tycho observed 45, besides the new one, of which shortly. Among these 5 of the third magnitude are more notable, of which the one on the breast is called in Arabic *Schedar* or *Seder*, that is, breast, and by the Persians the dyed hand, χεὶρ βεβαμμένη [Greek: cheir bebammenē, "dyed hand"].

[Margin: New star in Cassiopeia.]

On 1572 November 9 [ad 5. Idus Nouemb.] a new star appeared in Cassiopeia, equal to Venus in visible size. Through December it rivalled the star

of Jupiter. In 1573 January it was a little greater than stars of the first magnitude, but equal to them in February and March; equal to stars of the second magnitude in April and May; in June and July it was equal to stars of the third rank; and at the end of the year it scarcely exceeded the fifth magnitude. At last, in 1574 March, it vanished. At its beginning it had a northern latitude of 55° 45′ and a longitude of 6° 54′ of Taurus. Now if it had no sensible parallax, at its beginning it was at least 361 times greater than the Earth and 2 2/[?] times greater than the Sun, according to Tycho, who has more on it in volume 1 of the *Progymnasmata*, from p. 301; on it see also Biancani in the *Sphaera*, part 3, treatise 13, and Claramontius [Scipione Chiaramonti], *De Novis Stellis*.

[Margin: Little tale.]

Cassiopeia is said to have been the mother of Andromeda and the wife of Cepheus, King of the Ethiopians. When she boasted that the Nereids were surpassed by her in beauty, she was carried away from here and placed among the stars as a warning, sitting shamefully with her head thrown back, in her chair. Cepheus too was placed there, by the favour of Minerva, with Andromeda and Perseus, so that their whole family might be immortal.

[Margin: 11. Perseus.]

15. PERSEUS, *Inachides*, *Cyllenius*; in Arabic *Chelub* or *Cheleub*. It has 26 stars in Ptolemy, with 3 unformed; in Bayer there are 38, in Kepler 33. Among these only one is of the second magnitude, shining on the flank, called *ElGenab*, that is, flank, or corruptly *Genib* or *Chenib*. For the one that shines on the shield, called *Raas Algol*, that is, head of the Demon, or *Aloue*, or Head of Medusa, and set at the second magnitude by Ptolemy and Bayer, is in reality now of the third magnitude to Tycho and to us when we inspect it. The Nubian astrologers call it head of the Demon from its malign effects, for they say it threatens hanging or the cutting off of the head or hand, as Ptolemy has it in the *Centiloquium*, aphorism 73.

[Margin: Tale.]

Perseus was the grandson of Acrisius, King of the Argives, and the son of Danaë and Jupiter. He was carried up to heaven with the help of Minerva because he had conquered the Gorgon and also, the marriage having been agreed, had freed Andromeda, who because of her mother Cassiopeia's boasting had been exposed to the sea-monster Cetus and bound near a rock, by killing Cetus. All of these were added to the heavens, but with Cetus banished to the South.

[Margin: 12. Auriga.]

16. AURIGA, or *Erichthonius*, *Myrtilus*; among the Arabs *Mulus clitellatus*, the Saddled Mule; Driver of chariots; ἡνίοχος [Greek: hēniochos, "rein-holder"], *Heniochus*; in Isidore *Mavors*, in others *Maforte*. It consists of 14 stars in Ptolemy, 32 in Bayer, 27 in Kepler. Among them the best known and of the first rank is *Capella Olenia*, the Olenian She-goat, or *Capra Amalthea*, the Goat Amalthea, nurse of Jupiter, called in Arabic *Albaiot* or

Albatod; with the two Kids it imitates the figure of an acute-angled and oblong isosceles triangle. After it the famous one is the star of the second magnitude on the following shoulder; for the one under the foot, likewise of the second rank, belongs rather to the tip of the northern Horn of Taurus. Jupiter took this man into the heavens for his invention of the four-horse chariot: read Ovid, *Metamorphoses* book 2, and St Augustine, *City of God* book 18, ch. 12. Pliny the Elder has more on Capella in book 18, chs 26, 27, 28, 29; for Capella is a stormy star to Rufus Avienus, or, as Germanicus sings here, is hostile to sailors.

[Margin: 13. Serpentarius.]

17. SERPENTARIUS, or Ophiuchus, Ὀφιοῦχος [Greek: Ophiouchos, "serpent-holder"]; Aesculapius; in Aratus Μογερὸς [Greek: Mogeros, "toiling one"]; *Alhague* or *Alangue*; among the Moors the Crane, or the Stork standing on a serpent. It has 24 stars in Ptolemy, with 5 unformed; but in Bayer 30, in Kepler 37. Of these only one is notable, of the second magnitude, the more northerly one on its Head, called in Arabic *Raas alangue* or *Ras alhauge*. But for how wrongly several stars of this constellation are drawn on globes, with their positions interchanged, see Kepler, the Rudolphine Tables, p. 68.

[Margin: New star in Serpentarius.]

In this constellation, on 1604 October 9, a new star appeared, wonderfully ruddy, of almost 2° northern latitude and at 18° of Sagittarius, with no parallax, equal to Venus in apparent size; and it vanished from sight in 1606, at the beginning of February. Kepler has much on this star in his book *De Nova Stella in Serpentario*: see also [?] Biancani, part three of the *Sphaera*, treatise 13, and Chiaramonti, *De Novis Stellis*. It is a remarkable thing that the new star of Cassiopeia, of Cygnus and of Serpentarius appeared in the Milky Way or at its border.

[Margin: Tale.]

Aesculapius, moreover, was the son of Apollo and Coronis, called up to heaven to immortality because by the art of medicine, especially by means of a herb shown him by a snake, he restored people to life, whether dead, like Glaucus, or half-dead.

[Margin: 14. Serpens.]

18. SERPENS of Ophiuchus, Ὄφις [Greek: Ophis, "serpent"], *Alangue*. It has 18 stars in Ptolemy, with 5 unformed; but 37 in Bayer, and 13 Tychonic stars in Kepler. Of these the bright one of the Neck is of the second magnitude, but in Ptolemy of the third.

[Margin: 15. Sagitta.]

19. SAGITTA, Οἰστὸς [Greek: Oistos, "arrow"]; in Arabic *Schaham* or *Alhance*; among the Turks *Orfercalem*. It has 5 stars in Ptolemy, 8 in Bayer, 8 in Kepler.

[Margin: Tale.]

This was the arrow with which Hercules killed the vulture that was eating Prometheus's liver; for when Prometheus had been received back into favour by Jupiter, Jupiter willed that the arrow be set in the sky. In Cicero it is called *Musator*, in others *temo meridianus*, the southern Wagon-pole.

[Margin: 16. Aquila.]

20. AQUILA, or *Vultur volans*, the Flying Vulture, abductress of Ganymede, Αετὸς [Greek: Aetos, "eagle"]; in Arabic *Altair* or *Atair*, that is, bird. It contains 9 stars in Ptolemy, 32 in Bayer, because he adds Ganymede; but Kepler gives 12. Of these the bright one on the shoulders, or at the projection of the neck, is of the second magnitude. In the Persian Tables it is γὺψ πετόμενος [Greek: gyps petomenos, "flying vulture"], *vultur volans*.

[Margin: 17. Antinous.]

21. ANTINOUS, in place of Ganymede, has 7 stars in Kepler, 11 in Bayer. He was the darling of the Emperor Hadrian for his beauty, was born at Claudiopolis in Bithynia, and was given celestial honours.

[Margin: 18. Delphinus.]

22. DELPHIN, Delphinus, carrier of Arion; in Pliny the Elder *Hermippus*; Δελφὶν [Greek: Delphin, "dolphin"] or δελφὶς [Greek: delphis, "dolphin"]; *Rhomboides*. It consists of 10 stars in Ptolemy, and as many in Bayer and Kepler. Cicero calls it *Currus*, the Chariot; the Arabs, borrowing the name from the Greeks, say *Dalphin*.

[Margin: 19. Equiculus.]

23. EQUULEUS, the Lesser Horse; in Arabic *Alpharas Elmac*. It has only 4 stars, of the fourth magnitude, and those faint. In Proclus it is προτοπὴ [recte: προτομὴ] τοῦ ἵππου [Greek: protomē tou hippou, "forepart of the horse"], *sectio equi*, the section of the horse, or *Equiculus*.

[Margin: 20. Pegasus.]

24. PEGASUS, the Greater Horse, *Medusaeus*, *Gorgoneus*, *Bellerophonteus*, *Alpheratz*. It has 20 stars in Ptolemy, 23 in Bayer, likewise 23 in Kepler, of which 4 are of the second magnitude. Of these the one on the shoulder of the preceding wing is called *Markab*, that is, chariot; the one at the tip of the wing *Algenib*; the one at the projection of the leg *Scheat Alpheraz*; and the one on the navel, which is shared, is the Head of Andromeda: Ομφαλος ἵππου [Greek: Omphalos hippou, "navel of the horse"], navel of Pegasus.

[Margin: 21. Andromeda.]

25. ANDROMEDA, in Arabic *Marat musalseleth*, that is, the Chained Woman; on her, see under Cassiopeia. In all it has 23 stars in Ptolemy and as many in Kepler, but Bayer has 27. Among these three are noteworthy and of the second magnitude, though in Ptolemy they are of the third: the first on the Head, which is also called the navel of Pegasus; the second on the belt or navel, called *Mizar* or *Mirach*; the third on the southern foot, called *Allamak* by the Arabs.

[Margin: 22. Triangulum.]

26. TRIANGULUM Boreale, the Northern Triangle, Δελτῶτον [Greek: Deltōton, "delta-shaped"], or τρίγωνος [Greek: trigōnos, "three-cornered"]; Gift of the Nile; Δέλτα [Greek: Delta]; in Arabic *Mutlathum* or *Mutlat*. It has 4 stars by Ptolemy's reckoning, 5 in Bayer, 4 in Kepler.

[Margin: 23. Coma Berenices.]

27. COMA BERENICES, in Bayer *Spicarum manipulus*, the Handful of Ears of Grain, and τρίχες [Greek: triches, "hairs"], or *crines*, hair. It has 7 stars in Proclus and Stöffler, 15 in Kepler.

[Margin: Little tale.]

This Berenice was the wife of Ptolemy Euergetes. She vowed that if her husband returned victorious from Asia she would cut off her hair and hang it in the temple of Venus. When she had done so, and the next day it had not appeared, Conon the mathematician, to console the king, showed the hair set among the stars; and so Callimachus the poet, in Olympiad 136, and afterwards Catullus, wrote a poem on the hair of Berenice. Pliny the Elder, however, seems to place the Berenicean hair among the southern constellations, book 2, ch. 72. Further, in Ptolemy the three stars he calls τρίχες, that is, *crines*, or *Plocamos*, that is, *cincinnus*, a lock, outside the constellation of Leo, at its tail toward the north, together with four others observed by Conon, are the hair of Berenice, to which more recent authors have added another 8.

CAPVT V.

Chapter 5.

De Astris extra Zodiacum Australibus, præcipuè de Canicula, & de nominato in sacris literis Orione.

On the Southern Constellations outside the Zodiac, especially on the Dog Star, and on the Orion named in the sacred writings.

[Margin: 1. Cetus.]

1. CETE, or Cetus, Κῆτος [Greek: Kētos, "sea-monster"], Whale, *Pistrix*, πρῆστις [Greek: prēstis, a sea-monster], Sea Beast, Sea Lion or Sea Bear, *Orphas*, Ὀρφὼς [Greek: Orphōs]; in Arabic *Elkaitos* or *Alkeitos*. It has 22 stars in Ptolemy, 27 in Bayer, and in Kepler 21 Tychonic and 4 semi-Tychonic. Of these the bright one on the jaw or nostril, called in Arabic *Menchar* or *Menkar Elkaitos*, that is, beak of Cetus, and the brighter and more southerly one on the tail, called in Arabic *Demeb Kaitos*, are in reality now of the second magnitude, although in Ptolemy's table they are placed at the third magnitude. The middle, more northerly one of the belly is called *Baten Elkeitus*, belly of Cetus.

This Beast was sent by Neptune to devour Andromeda, but it was killed by Perseus: see what we said under Andromeda, and Manilius, who in book 5, ch. 40, describes the fight of Perseus with Cetus.

[Margin: 2. Orion.]

2. ORION, Ωριων [Greek: Ōriōn], or Ωάριων [Greek: Ōariōn], *Hyriades* or *Hyrides*, from his parent Hyreus; the Hunter; *Iugula* in Plautus and Varro, but in Manilius, book 5, *Iugulae*; in Hebrew *Chesil* or *Kesil*; in Arabic *Elgeuze* or *Almahbarrab*, that is, mighty warrior; and, from hunting or searching out, *Algebra*; or *Elgebar*, that is, Giant. He is called Orion as if Urion, from urine, which in Greek is οὖρορ [recte: οὖρον] [Greek: ouron, "urine"], because of the little tale that follows shortly; or from ὀρίνειν [Greek: orinein], which means to disturb. It has 38 stars in Ptolemy, 49 in Bayer, and in Kepler 62 Tychonic stars, which I too once counted. Among them two are of the first magnitude: one on the following shoulder, called in Arabic *Bectelgeuze*, that is, house of the Giant, or *Bedalgeuze*, which Aratus calls γλήνεαν [Greek: glēnean]; the other at the tip of the right foot, called *Rigel* or *Regel Elgebar*, that is, foot of the Giant. The one on the preceding shoulder, of the second magnitude, is called *Bellatrix*, to which Kepler makes the bright one of the following shoulder equal in size. In the Persian Tables translated by Chrysococca the foot of Orion is called ποῦς Διδύμων [Greek: pous Didymōn, "foot of the Twins"], and the bright one of the following shoulder ὦμος Διδύμων [Greek: ōmos Didymōn, "shoulder of the Twins"]. The three of the belt, or, as they are commonly called, the staff of St James, are of the second magnitude.

[Margin: A huge multitude of stars detected by the telescope.]

Further, Galileo, as he relates in the *Sidereus Nuncius* from p. 16, having undertaken to draw the whole constellation of Orion with the spyglass, was deterred by the huge abundance of stars and, for lack of time, put off the work; and he adds: *For more than five hundred stand beside and around the old ones, scattered within the limits of one or two degrees: therefore to the three in the belt and the six in the sword, noted long since, we have added eighty others adjacent to them, recently seen; and we have preserved their intervals as exactly as we could.*

[Margin: Little tale.]

Now from Ovid, *Fasti* book 5, Hyginus and others, the tale is told that Orion, otherwise Urion, at the prayers of the childless Hyreus, was born from the urine of three Gods whom Hyreus had received as guests, namely Jupiter, Neptune and Mercury, collected in a leather vessel; that he was an outstanding hunter and companion of Diana, but suspect to Apollo for his excessive familiarity with Diana; and that Apollo therefore challenged Diana to aim at a certain black target emerging from the sea, which was in fact the head of Orion, swimming. And so Orion was pierced with arrows, but was received into heaven to console Diana, and between the Dog and the celestial Hare he seems even now to be hunting. It is, moreover, a stormy constellation, according to that line of Virgil, *Aeneid* 1:

When suddenly, rising on the swell, stormy Orion

and of Horace, *Odes* book 1, where he calls the rainy south wind the companion of setting Orion:

The south wind, swift companion of declining Orion.

[Margin: What is Orion in Hebrew and in the sacred writings?]

In Hebrew it is called *Kesil* or *Chesil*, from the verb *Cacal*, which means to be varied and inconstant. And Orion either twinkles inconstantly because of its very ruddy sparkling, or rather causes varied and inconstant weather at its rising or setting, especially in the month of November, a month which for that reason is called *Kasleu*, from the same root. Hence in the sacred writings the power of GOD is praised in stirring up storms at his pleasure and varying the weather of the year, as when it is said in Job 9, verse 9: *Who makes Arcturus and Orion and the Hyades*, etc., and in the Prophet Amos, ch. 5, verse 5, where he praises God as *Making Arcturus and Orion*. In both places, however, the Hebrew has *Kesil*, although, as we said in ch. 4, no. 9, they are confused with Arcturus by some sacred writers. Yet, as I said, *Kesil* can be drawn to the variability of the sparkling and splendour with which Orion twinkles, since in Isaiah 13, verse 10, where the Vulgate has *For the stars of heaven and their splendour will not give their light*, the Septuagint translators render the name *Kesil*, in place of splendour, as Orion, according to that line of Virgil, *Aeneid* 3:

And he looks around at Orion, armed with gold

and of Lucan, book 1:

The flank of sword-bearing Orion shines too brightly.

Finally, one should see St Gregory, *Moralia* book 9, ch. 9, where by the name of Orion he understands the holy Martyrs; Pineda on Job ch. 9, and Cornelius à Lapide on Amos ch. 5; and, on the power of Orion, Pliny the Elder, book 18 throughout, especially ch. 28, and Manilius, book 5, from the verse *Maximus Orion*, Greatest Orion, etc.

[Margin: 3. Eridanus.]

3. ERIDANUS, the river Po, ποταμὸς [Greek: potamos, "river"]; in others the Nile, Gyon; among the Fetzani *Vardi*, that is, River; among the Mauritanians *Guad*; in Arabic *Nahar*. It has 34 stars in Ptolemy, 43 in Bayer, and in Kepler 19 Tychonic and 20 semi-Tychonic. Among these one is notable and of the first magnitude, at the end of the river, called *Acharnahar*, that is, the end of the river, or *Acarnaharim*. Its southern latitude, for navigators, is 59° 30′, and its longitude for the year 1600 is 9° 45′ of Pisces; and so it cannot be observed on our horizon. The constellation has no stars of the second magnitude. The last star of Eridanus is called in the Persian Tables of Chrysococca αὖλαξ [Greek: aulax, "furrow"], that is, furrow, as Boulliau shows at the end of his *Astronomia*. But the one at the bend of Eridanus is called in Arabic *Argentenar*.

Those who recognise this constellation as Eridanus, with Ovid, *Metamorphoses* book 2, place it in the heavens because, when Phaethon was plunged into it, the burning of the world was extinguished; those who think it the Nile

judged it worthy of an image in the heavens for the outstanding benefits it brings to Egypt.

[Margin: 4. Lepus.]

4. LEPUS, λαγωὸς [Greek: lagōos, "hare"], *Levipes*, Light-foot; among the Sicilians λέπορος [Greek: leporos]; in Nicander δερκευνὴς [Greek: derkeunēs]; in Arabic *Harneb* and *Elharneb*. It consists of 12 stars in Ptolemy, 13 in Bayer, 13 in Kepler, and has none of the first or second magnitude. It was put in the sky either for its speed, or because Orion, to please Diana, who delighted in hare's blood, took the greatest delight in hunting the hare.

[Margin: 5. Canis Maior.]

5. CANIS Maior, *Laelaps*; in Arabic *Scheer Eliemini*, from which in the Persian Tables σιάηρ ἰαμανῆ [Greek: siaēr iamanē]; and in Homer, *Iliad* book 10, ἀστὴρ ὀπώρινος [Greek: astēr opōrinos, "autumn star"], the star near autumn, or the one ending the summer. It has 18 stars in Ptolemy, with 11 unformed; 19 in Bayer; in Kepler 13 Tychonic and 16 others semi-Tychonic. Among them there is one which, as far as appearance goes, is the greatest of the fixed stars and equal to Jupiter at perigee, namely the one that shines on the cheek or tongue of the Dog, called *Sirius*, in Arabic *Alhabor* or *Elscheer* or *Elkabar*; but the one that glows red at its left ear is called *Isis*.

[Margin: Little tale.]

Some say that both dogs belonged to Orion the hunter and were carried up to Heaven with him by the prayers of Diana; others that this one was given to Cepheus by Aurora and endowed with such speed that it outran all wild beasts, but that, when it was about to encounter at Thebes a fox endowed by Jupiter with the same speed, it was raised to heaven so that the fates should not be made void.

[Margin: Sirius: what it is and where the name comes from.]

That splendid star, further, was called Σείριος [Greek: Seirios], either from σείρειν [Greek: seirein], which means to gape, because under this star the ground is made to gape open from dryness; or from σειριάω [Greek: seiriaō], which means I dry up. For this reason also the Sun itself is called Sirius by Hesiod in book 2 of the *Works and Days*, according to the common opinion of the interpreters; and Hesychius says expressly Σείριος, ὁ ἥλιος, καὶ ὁ τοῦ κυνὸς ἀστὴρ [Greek: Seirios, ho hēlios, kai ho tou kynos astēr], that is, *Sirius is the Sun, and the Dog Star*; and he adds that the word is used by Archilochus for the Sun, and by Ibycus for any star whatever.

[Margin: The Sothic period and the Canicular Year.]

This Dog the Egyptians called Sothis, and they gave it first place among the stars, and from its rising they drew conclusions about the rise of the Nile and the weather of the whole year, as Horus, Apollo and Hephaestio report. Indeed, from it they named that great Sothic period, or great Cynic or Canicular year; and, beginning it from its rising on the first day of the first month, Thoth, they

defined it as 1461 Egyptian years, at the end of which the first day of the month Thoth returned to the same degree of the Zodiac, and so, as they themselves thought, to the same rising of the Dog; for they had not yet learned that the Fixed stars are moved forward in the order of the signs. See what we said on the Canicular year and its Epoch in book 3, ch. 13, no. 9. Whether Sirius is the Canicula for everyone, and whether the effects of the dog days are to be attributed to it, will be argued shortly, after the explanation of Canis Minor. On what days it used to rise and set in former times, and rises and sets now, we shall discuss at length in ch. 22, under the word *Canicula*.

[Margin: 6. Canis Minor.]

6. CANIS Minor, the Little Dog, *Procyon*, or *Procynis*, or *Praecanis*; in Cicero *Anticanis*, in that verse:

Anticanis, who is called by the Greek name Procyon;

in Arabic *Kebelazguar*. It has 2 stars in Ptolemy, 8 in Bayer, 5 in Kepler. Of these the bright one of the breast or thigh is called, from the name of the whole constellation, *Procyon*; in Arabic *Algomeiza* or *Algomisa*. To Ptolemy and Bayer it is of the first magnitude, but to Tycho of the second; certainly if it is compared with Sirius, Lyra and Arcturus, not, however, if with the tail of Leo. On its rising and setting too there is more below, in ch. 22.

[Margin: Little tale of the Dog.]

Some, moreover, have told the tale that this dog was carried up into heaven with Erigone, that is, the Virgin, because it had wasted away with grief for its master Icarius, killed by drunken peasants, and for Erigone, who had hanged herself because of the murder of her father Icarius,

Vter Canum sit Canicula, & quinam & vnde sint effectus dierum Canicularium.

Which of the Dogs is the Canicula, and what the effects of the Dog Days are and where they come from.

[Margin: Authors who recognize Canis Minor as the Canicula.]

The name of the Canicula is as famous as it is ambiguous. At first sound it seems to suit the smaller dog, which is taken for the Canicula by the Alphonsine Tables; by Bede in his little work on the sphere; by Adrianus Turnebus [Adrien Turnèbe] in his *Adversaria*; by Raderus [Matthäus Rader] on Martial, book 11, epigram 49; by Carolus Stephanus [Charles Estienne] in his *Poetic Dictionary*; and by Kepler in the *Epitome of Copernican Astronomy*, p. 391. Hyginus also favours this when he attributes to Procyon what Germanicus attributes to Sirius, namely conjunction with the Sun in the centre of the sky. For whether you take the centre of the sky to mean the solstitial point, or the culmination on the meridian, or the zenith in a barbarian horizon, Procyon was then both nearer to the beginning of Cancer and culminated earlier with the Sun and as the nearer star in its passage across the meridian; and because of its northerly declination it can appear, and could appear, closer to the zenith than Sirius for

more people on this side of the Equator. Horace chimes in with these, book 3, ode 29:

> … Now Procyon rages, And the star of frenzied Leo, As the Sun brings back the dry days.

[Margin: Pliny the Elder's opinion.]

But Pliny the Elder says so much more clearly in book 18, ch. 28, in these words: "In Egypt Procyon is hot in the morning; this star has no name among the Romans, unless we wish this one to be understood as the *canicula*, that is, the lesser Dog, as indeed it is depicted among the stars." He confirms this from its relevance to, and nearness with, the rising of the greater Dog, for he adds at once: "But it is very relevant, as we shall show a little later: on the 3rd day before the Nones [III. Nonas] the Crown sets in the morning for the Chaldeans; for Attica the whole of Orion rises on that day. On July 14 [Pridie Idus Iulij] Orion finishes rising for the Egyptians. On July 17 [XVI. Kalend. Augusti] Procyon rises for Assyria; then on the following day comes the star agreed on among all peoples almost everywhere, which gives the signal we call the rising of the Dog, when the Sun has entered the first part of Leo." And in ch. 29 he distinguishes the Canicula from Canis Major more plainly, saying: "By the varying observation of the nations the Dog sets on April 28 [IV. Calendas Maij], a star both violent in itself and one before which the Canicula must set." Granted that our Pétau, in book 7 of the *Dissertations* of the *Uranologion*, ch. 2, because Procyon does not set before the Dog in any setting but after it, may take this passage to concern not the heavenly *canicula* but an earthly red bitch, which the Romans used to kill at the Catularia gate, as Festus attests, to appease the star of the Canicula. The Rubigalia, moreover, were held on April 25 [VII. Cal. Maij] to turn rust away from the crops, as Pliny the Elder relates in the same place.

[Margin: For whom both Dogs are the Canicula?]

The old interpreter of Aratus, however, and Bayer in his *Uranometria* call both dogs the Canicula.

[Margin: Authors who recognize Canis Major as the Canicula.]

Yet more others understand by the name Canicula Sirius, the most brilliant part of Canis Major: first of all Germanicus and his commentator Bassus; Iulius Firmicus [Firmicus Maternus], book 8; Apian, Magini and Argolus [Andrea Argoli] in their catalogues of stars; Pierius Valerianus [Pierio Valeriano], *Hieroglyphica*, book 49; Stöffler on Proclus's *Sphere*, p. 125; Henricus Stephanus [Henri Estienne] in the *Thesaurus linguae Graecae*; the author of the *Thesaurus linguae Latinae*; Pétau in the *Uranologion*, book 2, *Dissertations*, ch. 10, book 5, chs 5 and 6, and book 7, ch. 2; and finally those who attribute the canicular changes to the rising of the Dog or of Sirius. For although both Dogs rise, both cosmically and heliacally, in the summer season and with the summer already advanced, and although greater heat arises from the Sun's approach to both, or even to Regulus, still this assessment appears to come more manifestly from

the heliacal rising of Sirius; and it was certainly then that the Nile used to swell more abundantly.

[Margin: Effects of the Canicula [?].]

Further, on the effect of the Dog Days, and on their observance, which Hippocrates and Aëtius urge, there are discussions, besides the physicians, by Pierius, *Hieroglyphica*, book 43; by Simon Maiolus [Simone Maioli] in the preface to his work on the Dog Days; by Delrius [Martin Del Rio] on the first act of the *Oedipus*; by Laelius Bisciolius [Lelio Bisciola] in his *Horae subsecivae*, volume 2, book 12, ch. 2; and by the commentators, both on Persius, satire 3, at this verse:

> See, what are you doing? The frenzied Canicula, the dry harvests, etc.

and at this verse of Horace, book 3, ode 13:

> You the fierce hour of the blazing Canicula

and on this from Ovid, *Fasti* 5:

> There is a Dog, they call it Icarian, at whose star's rising The scorched earth thirsts and the crop is snatched away. In place of the starry Dog, this dog is laid on the altars; And why it is done has nothing but the name.

Let Pliny the Elder stand for all, book 18, ch. 28: "Then on the following day comes the star agreed on among all peoples almost everywhere, which gives the signal we call the rising of the Dog, when the Sun has entered the first part of Leo. This happens on the 23rd day after the solstice. Seas and lands feel it, and many wild beasts too, as we have said in their places; nor is the veneration of it less than that of the stars assigned to the gods; and it kindles the Sun and is a great cause of the heat." For in book 2, ch. 40 he had said: "For who does not know that the Sun's vapours are kindled at the rising of the Canicula? The effects of this star are felt most widely on earth: seas seethe as it rises, wines surge in the cellars, pools are stirred. Egypt gives the name oryx to a wild beast which, it reports, stands facing the star at its rising and gazes at it, and as it were worships it when it has sneezed. As for dogs, there is no doubt that throughout that whole period they are driven most of all to madness." And in book 9, ch. 16 he says of the silurus, a fish: "Among river fish the silurus is stricken at the rising of the Canicula, and at other times is stunned by lightning. They think this also happens at sea to the cyprinus; and in any case the whole sea feels the rising of that star, which is most apparent in the Bosphorus. For seaweed and fish are carried to the surface, and everything is turned over from the bottom." For this reason Hippocrates forbade purging medicines during these days, saying: "Under the Dog and before the Dog treatments are difficult."

[Margin: Abundance and excellence of honey at the rising of Sirius.]

Yet the Canicula by its rising brings this benefit, that a greater quantity of honey is then produced, as Mercurialis [Girolamo Mercuriale], *Variae lectiones*, book 2, ch. 24, gathers from Aristotle, *History of Animals*, book 5, ch. 22. He brings forward the text from an ancient codex of Scipio Garteromachus

[Scipione Forteguerri], in which it is read that honey falls from the air μαλίστα ἐν ταῖς τῶν ἄστρων ἐπιτολαῖς, καὶ ὅταν κατασκήψη ὅ ξείριος [recte: ὁ Σείριος] [Greek: malista en tais tōn astrōn epitolais, kai hotan kataskēpsē ho Seirios], that is, especially at the risings of the stars, and when Sirius is seen; for in great dryness and a clear sky this aerial nectar exudes onto the flowers. Pliny the Elder states this expressly in book 11, ch. 14: "A second kind is summer honey, which is called *horaeum* from its especially good season, when Sirius itself blazes out, about 30 days after the solstice"; and a little later: "But if this is kept when Sirius is rising, and if the rising of Venus, or of Jupiter or Mercury, happens to fall on the same day (as often), its sweetness and its power for calling mortal ills away from death prove to be nothing other than those of divine nectar." So the saliva of the heavenly dog is then sweet and life-giving, while the saliva of earthly dogs turned to madness is poison.

[Margin: Singular opinion of Geminus and Petavius.]

Notwithstanding all this, Geminus in the *Elements of Astronomy*, ch. 14, and with him, more or less, Pétau in the *Uranologion*, book 2, ch. 10, contend that the effects already mentioned are indeed signified, but not produced, by these or other rising stars, being produced rather by the Sun, which is stronger at that time and falls in the days of such risings. But this is to struggle against the stream, or rather the torrent, of authors, and to make far too idle the creation and arrangement, divinely accomplished, of so many stars.

[Margin: 7. Argonavis.]

7. ARGONAVIS, the Ship of Jason; in Catullus *Currus volitans*, the Flying Chariot; Noah's Ark; *Markeb*. It comprises 45 stars in Ptolemy, 63 in Bayer, and in Kepler 11 Tychonic and another 42 semi-Tychonic. Of these only one, at the end of the rudder, is of the first magnitude, called Κάνωβος [Greek: Kanōbos], *Canobus* in Proclus, or *Canopus*, and in the Persian Tables *Soail Iamani*, σοαὶλ ιαμανῆ [Greek: soail iamanē]; in Arabic *Suhel*. At Rhodes it seems to graze the horizon, perhaps because hills stand in the way, as Snel conjectures in *Eratosthenes Batavus*, book 1, ch. 16. Kepler, on the contrary, in the *Astronomia Optica*, thinks that it could not appear even at Rhodes except by the benefit of refractions; on which I shall speak in volume 2 of the *Almagest*, book 4. This constellation has besides 6 stars of the second magnitude in Ptolemy, 7 in Bayer and Kepler.

[Margin: 8. Centaurus.]

8. CENTAURUS, Chiron, the son of Philyra, who died from an arrow of Hercules falling by accident; Minotaur; *Albeze* or *Asmeat*. It consists of 37 stars in Ptolemy, 40 in Bayer, and in Kepler 4 observed by Tycho and the remaining 52. Of these 5 are of the second magnitude; and for Kepler and Ptolemy one is of the first magnitude, namely the one at the top of the forward foot, while the one on the hip of the hind foot is likewise of the first magnitude for Bayer. The Cross, moreover, commonly *el Crosiero*, has its top and bottom stars of the second magnitude and its side

stars of the third magnitude, and it is set on the hind feet of the Centaur. The Centaur's spear was called *Thyrsus* or *Thyrsolochus*, according to Proclus and Bassus.

[Margin: 9. Lupus.]

9. LUPUS, the Beast of the Centaur; in Capella the Panther; among the Persians *Bridemif*; in Arabic *Asida*. It has 19 stars in Ptolemy, 20 in Bayer, 19 in Kepler, none of them of the second rank.

This wolf the Centaur is said to have sacrificed to the gods as a victim, but above all to Bacchus.

[Margin: 10. Hydra.]

10. HYDRA, ἀσβία [Greek: asbia], *Asbia*; in Arabic *Elhauich*. It has 25 stars in Ptolemy, with 7 unformed; 29 in Bayer; in Kepler 24 Tychonic and the remaining 9. Of these the one that shines on the Heart, called *Alphard*, is of the first magnitude to Tycho and most others, but of the second in Ptolemy's table. This Hydra is not a trophy of Hercules, but was added by Apollo to frighten the raven, suffering from thirst, away from the water.

[Margin: 11. Crater.]

11. CRATER, the Bowl, κρήτηρ [Greek: krētēr, "mixing bowl"]; in Arabic *Elkis* or *Eluarad* or *Pharmaz*. It has 7 stars in Ptolemy, 11 in Bayer, 8 in Kepler, none of the third magnitude, let alone of the second or first. Is this the cup of Bacchus? Or of Apollo, as Hyginus prefers, which Apollo gave to the Raven to fetch water?

[Margin: 12. Corvus.]

12. CORVUS, Κόραξ [Greek: Korax, "raven"]; in Arabic *Algorab*. It has 7 stars in Ptolemy, and likewise in Bayer and in Kepler. This raven was carried up into heaven by Apollo. See Hyginus.

[Margin: 13. Thuribulum.]

13. THURIBULUM, the Censer; *Lar*; *Acerra*; in Arabic *Almegrameth*; in others *Ara*, or altar, made by the Cyclopes, on which the gods are said first to have made sacrifice and their conspiracy against the Titans. It has 7 stars in Ptolemy and Kepler, but 8 in Bayer.

[Margin: 14. Corona Australis.]

14. CORONA Australis, οὐρανίσκος [Greek: ouraniskos, "little heaven"], *parvum caelum*, the Little Sky; in Arabic *Ekleil Elgenubi*; *Rota Ixionis*, the Wheel of Ixion; in Hipparchus the Caduceus; in Proclus κηρυκεῖον [Greek: kērykeion, "herald's staff"]. It has 13 stars in Ptolemy, Bayer and Kepler. But if the star of the second magnitude in front of the knee of the left foot of Sagittarius is assigned to it, as it conveniently can be, this Crown too will have its bright gem, as Corona Borealis has. This Crown was carried down to the underworld by Bacchus.

[Margin: 15. Piscis Notius.]

15. PISCIS Notius, the Fish of Capricorn. It has 12 stars in Ptolemy and Bayer, who count among these *Fomahant*, a star of the first rank; but we

have counted that one in the outflow of Aquarius. Ptolemy adds another 6 unformed; in Kepler there are 17. This fish is said to swim in the sky because it saved Phacetis, daughter of Venus, who had fallen into the pool of Boëth. [?]

Sequuntur Duodecim noua Astra Australia, partim ab Americo Vespucio, partim ab Andrea Corsalio, & Petro Medinensi obseruata & à Petro Theodori Nauclero peritissimo, Friderico Houtmanno, ac Iacobo Barthschio ad normam Astronomicam redacta.

The twelve new southern constellations follow, observed partly by Amerigo Vespucci, partly by Andrea Corsali and Pedro de Medina, and brought to an astronomical standard by that most expert navigator Petrus Theodori [Pieter Dirkszoon Keyser], by Frederik de Houtman, and by Jakob Bartsch.

[Margin: 16. Grus.]

16. GRUS consists of 13 stars in Kepler and Bayer, of which three are of the second magnitude: one on the head, another at the start of the tail, the third on the more southerly wing.

[Margin: 17. Phoenix.]

17. PHOENIX, of 15 stars, with only one of the second rank, on the neck, and two nebulous ones.

[Margin: 18. Indus.]

18. INDUS, of 12 stars, in human form.

[Margin: 19. Pavo.]

19. PAVO has 16 stars in Bayer, 23 in Kepler with the unformed ones; among these is one on the head of the second magnitude, and two nebulous ones.

[Margin: 20. Apus.]

20. APUS, or *Avis Indica*, the Indian Bird, of 12 stars in Bayer, 11 in Kepler.

[Margin: 21. Apis.]

21. APIS, or *Musca*, of only 4 stars.

[Margin: 22. Triangulum Australe.]

22. TRIANGULUM Australe, with five stars, of which the three at the vertices or angles are put at the second magnitude by Bayer, at the third by Kepler.

[Margin: 23. Chamaeleon.]

23. CHAMAELEON, of 10 stars in Kepler, 9 in Bayer; all are of the fifth rank.

[Margin: 24. Piscis volator.]

24. PISCIS volans, the Flying Fish, or *Passer*, of 7 stars.

[Margin: 25. Dorado.]

25. DORADO, the Swordfish [Piscis Xiphias], with 6 stars, but of these two belong to the nearby little cloud. The star of the belly was distant from

the pole of the ecliptic in the year 1600 by 1° 48′, and it is of the fifth magnitude.

[Margin: 26. Anser Americanus.]

26. ANSER Americanus, the American Goose, called *Toucan*, of 8 stars, of which four are notable and of the second magnitude in Bayer, of the third in Kepler: one on the head, another on the beak, the third on the rump, the fourth on the wing or breast.

[Margin: 27. Hydrus.]

27. HYDRUS, of 15 stars in Bayer, 20 in Kepler besides the little cloud. Of these one, on the head, is of the second magnitude in Bayer, of the third in Kepler. The one almost at the tip of the tail, closest to the Antarctic pole of the world, is of the fifth magnitude; it was distant from that pole in the year 1600 by about 2° 30′, and now it is less distant.

CAPVT VI.

Chapter 6.

De Numero Astrorum & Stellarum in illis aut extra illa, tum relatiuè ad sex magnitudinis apparentis classes, aut ad Mundi Plagas, tum absolutè: & de Figuris ac Nominibus situue eorum mutandis.

On the number of the constellations and of the stars in them or outside them, both relative to the six classes of apparent magnitude or to the quarters of the world, and absolutely; and on changing their figures and names, or their position.

[Margin: Number of the constellations according to the moderns.]

1. The first question is how many constellations [Astra seu Constellationes] there once were and how many there are now. I reply briefly that the ancient ones are 50 and the new ones 12, and so 62 in all. Among them are the 12 Signs or constellations of the zodiac, reviewed in ch. 3; the 23 northern constellations named in ch. 4; and the 27 southern ones related in ch. 5. Among the older writers, however, the number is smaller or larger, whether from the omission of some whole constellation, or from the reduction of two to one, or from the division of one into two or more; and all the ancients left out the 12 new ones.

[Margin: 43 constellations.]

Ausonius and Bede, in the little work on the sphere and in the *Eclogarium* on the heavenly signs, list 43, with Equuleus, Lupus, Crater, Corvus, Serpens, Coma Berenices and Antinous omitted or merged with others.

[Margin: 44 constellations.]

Germanicus Caesar, even though he is the interpreter of Aratus, sets down 44, with Sagitta, Corona Australis, Procyon, Serpens, Coma Berenices and Antinous omitted; perhaps some fragments of his have been lost.

[Margin: 45 constellations.]

Vitruvius, book 9, chs 6 and 7, lists 45, with Equuleus, the Vulture, Corona Australis, Coma Berenices and Antinous omitted; but the man who added pictures to Vitruvius set down 48.

[Margin: 46 constellations.]

Manilius, *Astronomica* book 1, Avienus, Hyginus and Apian count 46, with Equuleus, Coma Berenices, Antinous and Corona Australis omitted. Tycho, volume 1 of the *Progymnasmata*, p. 258, and Lansbergen in his tables, p. 161, agree on this number, having deliberately passed over the southern Lupus, Ara, Corona and Piscis, because these cannot be observed in their climate.

[Margin: 47 constellations.]

Martianus Capella, *De Nuptiis Mercurii et Philologiae*, book 8, says that there are 35, and with the zodiacal ones 47; but he gives the names of only 46, leaving out Serpens, Equuleus, Coma Berenices and Antinous.

[Margin: 48 constellations.]

Ptolemy, book 7, ch. 5, with the Platonists, counts 48, with Equuleus omitted, which he reduces to Pegasus, and Antinous, which he puts among the unformed stars of Aquila, and Coma Berenices, three of whose stars, however, he puts among the unformed ones around the tail of Leo, under the name τρίχες [Greek: triches, "hairs"] or πλόκαμους [Greek: plokamous, "locks"], namely of hair or of a curl. Ptolemy was followed by Alfonso, Copernicus, Piccolomineus [Alessandro Piccolomini], Iunctinus [Francesco Giuntini] and Clavius in their catalogues.

[Margin: 50 constellations.]

Griembergerus [Christoph Grienberger], after his catalogue of the fixed stars, sets down 50 figures, adding Antinous and Coma Berenices to the Ptolemaic ones; but in the catalogue he reviews only 48.

[Margin: 52 constellations.]

Proclus in the *Sphere* counts 52, adding to the Ptolemaic ones Coma Berenices, the Water of Aquarius, Thyrsolochus, namely the spear of the Centaur, and the Caduceus, a constellation he set down following Hipparchus of Nicaea, and which is thought to consist either of 6 stars of Corona Australis set in a straight line, or of five of Eridanus likewise set in one straight line, as Stöffler explains on Proclus, p. 133.

[Margin: 60 constellations.]

Johann Bayer in his *Uranometria* sets down 48 Ptolemaic ones and 12 of the recent navigators.

[Margin: 62 constellations.]

Johannes Kepler in the Rudolphine table of stars, p. 105, sets down 50 of the older constellations, adding Coma Berenices and Antinous separately to the Ptolemaic ones. He then adds 12 new constellations observed or set in order by navigators who crossed beyond the Equator, namely Amerigo Vespucci, Andrea Corsali and Pedro de Medina, but above all Frederik de Houtman, who observed them on the island of Sumatra; all of them, however, were put in order

by Petrus Theodori and Jakob Bartsch, who were followed by the makers of star globes, Iansonius [?], Hondius and Jacob Floris van Langren. So for these men the constellations are 62, a number accepted by Biancani in the *Sphaera*, book 17, and by Boulliau in his tables, p. 180.

[Margin: 72 constellations.]

Pliny the Elder, however, in book 2, ch. 41, counts 72. But he passed over their names with too dry a foot, says Stöffler on Proclus, p. 82; and he seems to have counted parts of constellations as whole constellations: for instance the Kids as distinct from Auriga, the Head of the Gorgon or Medusa as distinct from Perseus, Arcturus from Boötes, the Pleiades and Hyades as distinct from Taurus, the Aselli from Cancer, Canopus from Argonavis, and the Urn from Aquarius. But perhaps Pliny the Elder had something else in mind.

[Margin: Should the names and figures of the ancients be changed in the constellations?]

2. From this arises another question: whether it is permissible, or even expedient, to change over to other figures and forms of the stars, which sailors, farmers and beginners in astronomy could more easily observe and commit to memory, or at least to change their profane names into sacred ones, the sky being purged, so to speak, and the minds of Christians drawn away from the turpitude or vanity of the ancient tales.

[Margin: Ptolemy's example in this.]

For indeed Ptolemy was allowed to alter the Hipparchean figures, as he himself testifies in the *Almagest*, book 7, ch. 5, saying: "In many places, therefore, assigning names better suited to the figures themselves, we have changed the usage of the ancients, as for example the figures which Hipparchus places on the shoulders of Virgo etc." Tycho was allowed to bring the stars which in Ptolemy are σποράδες [Greek: sporades], that is, Scattered, or ἀμόρφωτοι [Greek: amorphōtoi], that is, Unformed, within the forms of a nearer constellation.

[Margin: Bede's attempt.]

Bede too began from the constellations of the zodiac, and gave a sample of the way in which one ought to substitute sacred names and holier figures for profane ones, in that trochaic poem which Stöffler also reports on Proclus, p. 86, and whose opening is:

> Truly astonishing and savage is the madness of the pagans, Who
> transferred such diverse beasts into the sky, When it is agreed that
> on Olympus are the ranks of the Angels etc.

[Margin: Schiller's invention.]

Kepler too, in the Rudolphine Tables, p. 118, indicates that Jacob Bartsch of Lusatia took his globe from the *Uranographia* of Christian figures by some Schiller or other [Julius Schiller], who had ordered its publication in his will.

[Margin: Copernicus's opinion.]

Nevertheless Copernicus, book 2, ch. 14, indicates that the old forms and ancient names are to be retained, in these words: "For the stars were formed into images, according to the opinion of Theon the Younger in his exposition

of Aratus, for no other reason than that so great a multitude of them should be distinguished part by part and could be designated part by part with certain names; an institution ancient enough, since it is agreed that even in Job some were already named, and we also read the Pleiades, the Hyades, Arcturus and Orion expressly named in Hesiod and Homer. Therefore in their etc."

[Margin: Tycho's view.]

But Tycho says it more clearly, in volume 1 of the *Progymnasmata*, p. 256, affirming that these ancient names and figures are to be retained, even though they smell of heathenism and of the tales of the poets, for three reasons: first, because under their wrappings they are not without a certain abstruse knowledge; second, because some of them are mentioned not only in the most ancient poets but also in the holy Job and Amos; third, because "It is not fitting lightly to abolish and cancel what has been confirmed by long approval, unless something better is ready at hand to be put back in their place. And it is to be feared that, if the old names of the stars are set aside because they rest on fictions and tales, it may not be possible to introduce others anew, unless here too one is allowed to invent and shape whatever one pleases. For the true names of the stars, by which the supreme Maker calls each of them, as David testifies in Psalm 146, are entirely unknown to us; perhaps something about these was known to Adam and the Fathers, or at least they used more solid and truer appellations for them."

Nor is there any doubt that, if we knew the properties of the constellations, it would be best to seek a nomenclature from them.

[Margin: Our view.]

It is false, however, what Copernicus and Tycho suppose, that the names of the Pleiades, the Hyades, Arcturus and Orion, as they sound among us, are in the Hebrew text, since the names there are different, namely *Nghais*, *Kimà*, *Kesil*; in their place the Vulgate and other versions have put those names which had already gained currency among the Greeks and Latins, as is clear from what was said in chs 3, 4 and 5 where those constellations were treated. And so, if the necessity did not bind us to retain the ancient names of the stars after all, so that ancient observations can be compared with more recent ones and the motion of the fixed stars established, I would come close to abolishing most of them and to coining new names and new forms from a holier store. Perhaps, however, I shall indicate names, in case anyone outside the business of astronomy should wish to use them at some time, where I set out the catalogue in the tables.

[Margin: Position.]

3. But you must necessarily be warned, Reader, that the parts of the celestial figures which hold the right-hand position in some authors are on the left in others, and that those which are upper for some are lower for others, according to how the viewer is imagined to contemplate them, turned towards this quarter or that, either from beneath the concavity of the sky or from above the convex surface of their globe. The illustrations of Bayer, moreover, the painter or engraver represented with their position

interchanged and inverted, against the will and to the grief of the author, as Kepler says in the book *De Stella Nova*, p. 68. Our Grienberger, however, in his catalogue of the fixed stars, added a double perspective of the celestial figures, one from the poles of the world and the other from the centre of the world, and represented them in both ways.

[Margin: The number of stars seen with the naked eye?]

4. The number of stars seen without the telescope differs among authors. First, because of the different climates in which they were observed: and so Tycho and Lansbergen, who noted only those observable in Denmark and Zeeland, have fewer than others, and all the older authors present fewer than the more recent, since observations made beyond the Equator by the explorers of the New World were lacking. Second, because of the different season of the year, since on clear winter nights more seem to appear; or because of greater or lesser diligence and circumspection in counting the stars and in not confusing them with stars already counted in another, nearer asterism. Third, because of the greater or lesser sharpness of the eye: for how many there are who cannot make out Alcor in the tail of Ursa Major, or the seventh of the Pleiades, or other little stars of this kind! Fourth and last, because of errors of calculation, or printers' mistakes.

Now, to bring the variety of opinions into a brief synopsis, I have chosen four catalogues out of very many. The first is Ptolemy's, followed by al-Battānī, Alfraganus [al-Farghānī], Alfonso, Copernicus, Apian, Piccolomini, Giuntini and Clavius, who corrects the text of Ptolemy from a manuscript codex, as well as the errors that crept into the Arabic versions, which Alfonso followed, or into the translation of Georgios Trapezountios, even though it was revised by Luca Gaurico. The second is that of our Christoph Grienberger, which he put together both from the catalogue of Tycho, in which there are 767 stars most accurately observed and arranged; and from that of Francesco Pifferi, who from a double manuscript of Tycho has another 218, not so accurate, nor polished by Tycho's final file; and from Clavius, supplying the remaining 240 omitted by Tycho. The third is that of Johann Bayer, who compiled his *Uranometria* from Ptolemaic, Tychonic, Arabic and Turkish catalogues or maps, and from the more recent navigators. The fourth is that of Kepler, and after him of Boulliau, which is divided into three parts. The first part has a thousand more accurate Tychonic stars from Tycho's papers, whereas in the published catalogues of Longomontanus and Tycho no more than 858 are recorded. The second contains both a few other, less accurate Tychonic stars, and the Ptolemaic stars omitted by Tycho and corrected from the Greek codex rather than from the Arabic version, those which, namely, Ptolemy put in order and emended from the old catalogue of Hipparchus. The third part presents the stars that cannot be observed in our temperate zone but were observed by the recent navigators, with the addition of those Bayer did not know.

[Margin: Opinion of Pliny the Elder and others on the number of the stars.]

In the summaries that follow, moreover, you will see four opinions on the number of stars seen with the naked eye, to which add Rabbi Kimhi in Bayer, who counted 1,089. But it seems certain that there are many more, since Pliny the Elder says in book 2, ch. 41: "The immense vastness of the sky pleads for it, its extent being divided into 72 signs [duo atq. LXX.]. These are the images of things or of living creatures into which the experts have arranged the sky. In these they have recorded 1,600 stars, those namely that are notable for their effect or their appearance." So if the rest are added, which are not so notable to sight, and those which were always hidden from those early observers who lived on this side of the Torrid Zone, they will become far more than 1,600. What if those very constellations we have reviewed should be observed by very sharp eyes? For, as Kepler relates in the *Dissertatio cum Nuncio Sidereo*, p. 9, Mästlin counted in the Pleiades 14 stars without the tube, which others scarcely count as 6 or 7; and he tells of a man in religious orders known to him who on a moonless night counted more than 40 in the Shield of Orion, in which others note scarcely 11 or 12; and the Chinese have far more on their count than we do. But as for the Hebrews counting, as Kepler says on the same p. 9, more than twelve thousand, or, as Bayer reports at the end of the *Uranometria*, as the *Cabela in Drach* [?], cited on Isaiah ch. 49, nu- mber 29,000 myriads of stars and 140 besides, that has been derived from cabalistic fabrications rather than from observation by the eye. Conversely, when Clavius, on ch. 1 of the *Sphere*, p. 140, contends that the stars contained within the six common magnitudes are no more than 1,022, because of the diligence he thinks astronomers have applied in counting them, and that sight is deceived when in winter it counts more around the North Pole than at other times, owing to the greater scintillation by which many seem to arise from one, he must be understood as speaking of stars observed in the northern zones. Otherwise he does not deny that all the stars, counting those smaller than the sixth magnitude, are innumerable.

STELLARVM NVMERVS Sine Teleſcopio obſeruatus.

The number of the stars, observed without the telescope.

Group	Order of the constellations [ASTRORVM ORDO]	Ptolemy, Alfonso, Copernicus, Clavius [Ptolemæus Alfōs⁹ Copernicus Claui⁹.]	Grienberger from Clavius, Tycho and Pifferi [Griēbergerus ex Clauio Tych. & Piffero.]	Johann Bayer from many [Ioānes Bayerus ex multis.]	Kepler and Boulliau from many [Keplerus & Bullialdus ex multis.]
Northern zodiac [Zodiaci Boreal.]	Aries	18	22	29	23
	Taurus	44	52	48	52
	Gemini	25	30	31	30
	Cancer	13	16	35	17
	Leo	35	40	43	40
	Virgo	32	41	42	41

Group	Order of the constellations [ASTRORVM ORDO]	Ptolemy, Alfonso, Copernicus, Clavius [Ptolemæus Alfōs⁹ Copernicus Claui⁹.]	Grienberger from Clavius, Tycho and Pifferi [Griēbergerus ex Clauio Tych. & Piffero.]	Johann Bayer from many [Ioānes Bayerus ex multis.]	Kepler and Boulliau from many [Keplerus & Bullialdus ex multis.]
Southern zodiac [Zodiaci Auſtr.]	Libra	17	20	15	20
	Scorpius	24	24	29	27
	Sagittarius	31	31	31	31
	Capricornus	28	28	29	28
	Aquarius	45	45	41	45
	Pisces	34	40	37	42
Northern, outside the zodiac [Borealia extra Zodiacum]	Ursa minor	7	21	8	20
	Ursa maior	35	57	32	56
	Draco	31	30	33	32
	Cepheus	13	13	17	12
	Bootes	23	29	34	29
	Corona	8	8	20	8
	Hercules	28	31	48	31
	Lyra	10	11	13	11
	Cygnus	19	20	35	28
	Cassiopea	13	26	25	45
	Perseus	29	35	38	34
	Auriga	14	27	32	27
	Ophiuchus	29	24	30	56
	Serpens	18	13	37	26
	Sagitta	5	8	8	8
	Aquila	15	12	32	12
	Antinous	0	7	0	7
	Delphin	10	10	10	10
	Equiculus	4	4	4	4
	Pegasus	20	23	20	24
	Andromeda	23	26	26	26
	Triangulum	4	4	5	4
	Coma Berenices	0	14	0	15
Southern, outside the zodiac [Auſtr. extra Zod.]	Cetus	22	25	27	25
	Orion	38	62	49	62

Group	Order of the constellations [ASTRORVM ORDO]	Ptolemy, Alfonso, Copernicus, Clavius [Ptolemæus Alfōs⁹ Copernicus Claui⁹.]	Grienberger from Clavius, Tycho and Pifferi [Griēbergerus ex Clauio Tych. & Piffero.]	Johann Bayer from many [Ioānes Bayerus ex multis.]	Kepler and Boulliau from many [Keplerus & Bullialdus ex multis.]
	Eridanus	34	39	42	39
	Lepus	12	13	13	13
	Canis maior	29	29	19	29
	Procyon	2	5	8	5
	Argonavis	45	50	63	53
Southern, outside the zodiac [Auſtralia extra Zod.]	Centaurus	37	37	40	37
	Lupus	19	19	20	19
	Hydra	27	34	29	33
	Crater	7	8	11	8
	Corvus	7	7	7	7
	Ara, Thus	7	7	8	7
	Corona Australis	13	13	13	13
	Piscis Australis	18	17	12	17
New southern, outside the zodiac [Auſtralia noua extra Zod.]	Grus			13	13
	Phoenix			14	15
	Indus			12	12
	Pavo			16	23
	Apus			12	11
	Apis			4	4
	Chamaeleon			8	10
	Triangulum Australe			5	5
	Piscis volans			7	7
	Dorado, Xiphias			7	7
	Toucan, Anser Americae			8	8
	Hydrus			15	21

SVMMÆ STELLARVM
Totals of the stars.

Of magnitude [Magnitudinis]	Ptolemy [Ptole.]	Grienberger [Griēb.]	Bayer [Bayer.]	Kepler [Kepl.]
1	15	15	17	15
2	45	55	63	58
3	208	201	196	218
4	474	456	415	494
5	217	304	348	354
6	49	186	341	240
Obscure and nebulous [Obſc. & Nebul.]	14	8	3	13
Scattered stars [Sparſiles]: zodiacal [Zodiaci]			45	
Scattered stars: northern [Boreales]			200	
Scattered stars: southern [Auſtrales]			81	
Total [Summa]	1022	1225	1709	1392

SVMMÆ ALIÆ.
Other totals.

	Ptolemy	Grienberger	Bayer	Kepler
Northern zodiac [Zodiac. Bor.]	167	199	239	203
Southern zodiac [Zodiaci Auſt.]	179	190	208	193
Whole zodiac [Zodiaci totius]	346	389	447	396
Northern, outside the zodiac [Borealium extra Zod.]	358	471	701	495
Southern, outside the zodiac [Auſtralium extra Zod.]	317	365	561	501
Northern [Borealium]	525	670	940	698
Southern [Auſtralium]	497	555	769	694
All [Omnium]	1022	1225	1709	1392
Pliny the Elder [Plinio]	1600			
Rabbi Kimhi	1089			
The Hebrews [Hebræi]	12000	And more [Et amplius]		

[Margin: Total of all.]

V. To anyone who asks about the absolute number of all the stars, I reply that they are indeed not infinite, as Iordanus Brunus [Giordano Bruno] contends in the book *De infinito et innumerabilibus*, saying that there are as many worlds as stars, and that these are infinite in number; against him, Kepler in the book *De Stella Nova*, ch. 21. But I reply that they are nevertheless beyond counting by human beings. First, because even if they are viewed with the naked eye, yet, given the differing keenness of eyes, and endurance and diligence, there will always be someone who notes a different number from the others; and so they are not numerable in such a way that there is a settled number of them on which all finally agree. Second, because it is now clear from the telescope that there are far more than those observed with the naked eye. For Galileo, as we have already reported from the *Sidereus Nuncius*, detected with the glass 21 stars in the nebula of Orion, and 36 in the nebula of Praesepe, and more than

40 in the asterism of the Pleiades, and at least 80 within the space of the belt and sword of Orion; and within the space of one degree or another he counted more than 500 in Orion, so much so that he was deterred from observing the whole constellation of Orion, which he had decided to draw. And of the Galaxy, or milky circle, he says: "For the Galaxy is nothing other than a mass of innumerable stars planted together in clusters: to whatever region of it you direct the spyglass, at once a vast crowd of stars presents itself to view, many of which appear quite large and very conspicuous, but the multitude of the small ones is wholly beyond exploring." From this he concludes that the Milky Way, and other small areas like it, and the nebulous stars are nothing other than a mass of stars. Kepler endorses this in the *Dissertatio*, p. 9½, as do Blancanus [Giuseppe Biancani] in book 17 of the *Sphere*, ch. 2, and Mastrius [Bartolomeo Mastri], disputation 2 *De caelo*, no. 87. Nor does Galileo doubt that the stars go beyond ten thousand. Add Antonín Maria Šírek, who on p. 197 of his *Radius Sidereomysticus* affirms that in Orion alone about two thousand stars were counted by him with a Belgian telescope, and Father Giovanni Battista Zupi, who with Fontana's telescope counted more than 50 in the Pleiades and about as many in the Sword of Orion and in the Hyades.

[Margin: Whether the stars are infinite?]

[Margin: The innumerability of the stars. First argument.]

[Margin: Second argument, from the telescope.]

[Margin: What the Galaxy is.]

Now if Orion occupies very nearly 500 square degrees of the sky, as will be evident to anyone measuring its area on an Aratean globe, and if within a square space whose side is 2 degrees there are at least 500 stars, as Galileo noted, there will be 62,500 stars in the whole of Orion; but let us set 60,000. Again, suppose that as the 63 stars of Orion seen with the naked eye stand to the roughly 60,000 that would be revealed by the telescope, if one had the leisure and did not mind counting them, so the number of any constellation counted with the naked eye, and set down in the last columns of the preceding table, stands to the number of stars observable by telescope: then more than a million stars will be gathered, not counting those in the Milky Way. And so if anyone should think that there are more than two million stars, he will, to my mind at least, have imagined nothing unbelievable.

[Margin: Third argument, from sacred authority.]

Finally, this seems to agree very well with the assertion of God himself, in Genesis 12, saying to Abraham: "Look up at the sky and count the stars if you can." And in ch. 22: "I will bless you and multiply your seed like the stars of the sky and like the sand that is on the seashore." Mindful of this promise, the holy Patriarch said in Genesis 32: "You said that you would do me good and spread my seed like the sand, which for its multitude cannot be counted." And lastly it is said in Jeremiah 33: "As the stars of the sky cannot be counted and the sand of the sea cannot be measured, so I will multiply the seed of David my servant." For although some think that these things were said to suit the understanding

of the common people, and of a number of stars that human beings cannot easily take in one or a few looks, yet since no necessity compels us to depart from the sense the letter conveys, and the majesty of the Creator requires of us that we think magnificently of his works, we shall do much better to interpret these passages of some huge number of stars, and so great that it is not smaller than either the number of the elect or that of the whole stock of Abraham and David signified in those promises; and thus we shall proclaim with the Psalmist, in Psalm 146, that God alone is he "who counts the multitude of the stars, and calls them all by name."

[Margin: Whether there are more stars toward the north or toward the south?]

VI. Therefore, if you ask further whether there are more stars in the northern or in the southern hemisphere: if you are asking about all of them together, including those invisible to us, you ask in vain, and it would be no less rash for any of us to determine this than whether their number is odd or even. But if you are asking about those counted by astronomers, I reply that according to all authors there are more in the northern than in the southern region; according to Kepler and Boulliau, however, there are about as many in each, and a little more in the northern, although Amerigo Vespucci affirms that more are noted beyond the equator than on this side of it.

[Margin: Stars of first magnitude.]

VII. Moreover, the stars of the first rank or magnitude (for their number is usually sought with special care) are at least 15 in all authors, and 18 in Bayer, but they are not the same in all. Such without dispute are:

[Margin: Certain.]

1. Acarnar, which lies at the end of Eridanus. 2. Aldebaran, or the southern eye of Taurus. 3. Arcturus, or the boss of the fringe of Bootes. 4. Canopus, on the rudder of Argonavis. 5. Capella, on the shoulder of Auriga. 6. The topmost of the Centaur, on the right forefoot. 7. Fidicula of Lyra. 8. Fomahant, or the last in the outpouring of Aquarius, which Piscis Notius appears to swallow. 9. The tail of Leo. 10. The heart of Leo, or Regulus. 11. Rigel, on the foot of Orion. 12. Sirius, or Alhabor, in the mouth of Canis maior. 13. Spica.

[Margin: Disputed.]

The disputed ones, however, are: 1. Antares, or the Heart of Scorpius, for it is of first magnitude to Ptolemy and his followers, of second to others. 2. The bright one of the Centaur on the hind hip, which to Bayer is of first rank, to others of second. 3. The heart of Hydra, which to Ptolemy is of second brilliance, to others of first. 4. The following shoulder of Orion, which to Ptolemy and Bayer is of first class, to others of second. 5. Procyon, which for Ptolemy and Bayer stands among the first, among others in the second, and to me seems no larger than the bright star of Aquila. On this, moreover, depends the variation in the arc of the Sun's depth that Ptolemy assigned for the heliacal

rising and setting of the fixed stars, which varies among the stars with their varying apparent magnitude and brilliance.

[Margin: How many nebulous stars?]

VIII. Finally, one could ask about the number of the nebulous stars by themselves. On this I reply that there are at most 14, and they are: 1. The nebulous star on the breast of Cancer, called Praesepe. 2. The nebulous star on the sting or behind the sting of Scorpius. 3. The double nebulous star in the eye of Sagittarius; and on these all agree. To them Ptolemy and his followers add 4. a nebulous star on the head of Perseus, or at the tip of the right hand, as Grienberger has it, and 5. one on the head of Orion, not placed there by others. Grienberger and Kepler, moreover, add to these 6. one at the tip of the foot or on the calf of Hercules, 7. one on the forehead of Capricorn, 8. one on the eastern horn, and 9. one on the western horn of Capricorn. Finally Kepler adds to these 10. one in Capricorn, 11. and 12. in Phoenix, and 13. and 14. in Pavo, to say nothing of the two small clouds near the Antarctic Pole. So there are about as many nebulous stars as stars of first magnitude; but in Kepler 13, in Grienberger 8, in Ptolemy and Clavius 5, though 14 with the obscure ones. Bayer, however, being less careful in distinguishing these, lists only the nebulous stars of Hercules, Cancer and Sagittarius.

SCHOLIA.

Scholia.

I. M. Manilius describes all the constellations in order, but in many verses, in book 1 of the *Astronomica*, ch. 4. In elegance in describing the signs of the zodiac, as I reported in ch. 3, he surpasses Ausonius; but Ausonius surpasses Manilius in the brevity with which he reviews all the signs of his age, which he does in the *Eclogarium* in the following verses:

> *Ad Boreæ partes Arcti iunguntur & Anguis; Poſt has Arctophylax, pariterq. Corona, Genuq. Prolapſus, Lyra, Auis, Cepheus, ac Caſſiopea: Auriga, & Perſeus, Deltoton & Andromedæ aſtrum, Pegaſus, & Delphin, Telūq. Aquila, Anguitenenſq. Signifer inde ſubeſt, bis ſex quem ſidera complent. Hic Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libraque, Scorpius, Arcitenēs, Capricornus, & Vrnam Qui tenet, & Piſces. Hi ſunt in partibus Auſtri: Orion, Procyon, Lepus, ardens Sirius, Argo, Hydrus, Chiron, Thuribulum quoq. Piſcis, & ingens Hunc ſequitur Piſtrix, ſimul Eridaniq. fluenta.*

To the regions of Boreas the Bears are joined, and the Serpent; after these Arctophylax, and likewise the Crown, and the one fallen on his knee, Lyra, the Bird, Cepheus and Cassiopea; Auriga, and Perseus, Deltoton and the constellation of Andromeda, Pegasus, and the Dolphin, the Arrow, the Eagle, and the Serpent-holder. Then below lies the zodiac, which twice six constellations complete. Here are Aries, Taurus, Gemini, Cancer, Leo, Virgo, and Libra, Scorpius, the Archer, Capricorn, and he who holds the Urn, and Pisces. These

are in the regions of the south: Orion, Procyon, Lepus, blazing Sirius, Argo, Hydrus, Chiron, the Censer too, the Fish, and huge Pistrix follows this, together with the streams of Eridanus.

II. Since there was an old opinion, passed down from Ptolemy onward, that the stars in all are 1,022, Erycius Puteanus [Erik van der Putte] left to posterity, as a monument of his piety and talent, this most ingenious line:

Tot tibi ſunt dotes Virgo, quot ſidera cælo.

You have as many gifts, O Virgin, as there are stars in the sky.

This verse, with the quantity of its syllables preserved and the meaning of the same thought kept, is converted into 1,022 verses by transposing the words, to each of which he added its own particular gift befitting the Mother of God. Nobody will find this difficult, if he knows how many combinations or different arrangements can be made even of a few terms, since the combination of the immediately preceding number, multiplied by the next number of terms, yields the combination of the following number. For example, because two terms have a twofold arrangement, so that if A and B are given the orders AB and BA can be made: if three terms are given, these multiplied by 2 make 6, that is, the number of ways in which six [recte: tres] terms can be arranged; and so on with the rest, as Clavius teaches in the *Sphere*, p. 34. For these, here for you in a small table are the numbers of combinations corresponding to the number of terms, up to 12.

[Margin: Rule of combinations.]

Terms [Termini]	Possible combinations [Combinationes Poſſibiles]
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800
12	479001600

[Margin: Protean verses.]

So if verses are made of the kind that from their capacity for turning are called Protean, which have the first or last syllable common and consist of as many monosyllables as possible, the same verse could be permuted, with the laws of grammar intact, into so many others of the same metre and meaning that they exceed the number of all ver- ses that Greek and Latin poets alike once composed. Let a specimen of such verses be, especially in the second line, the following distich, which we put together on another occasion:

Vi metri tibi me en nunc hîc Tethy Prothea ſacro; Sum Strix, Glis, Grus, Sphinx, Mus, Lynx, Sus, Bos Capra & Hydrus.

By the force of metre, behold, here and now, Tethys, I consecrate myself to you, a Proteus; I am screech-owl, dormouse, crane, sphinx, mouse, lynx, sow, ox, goat and water-snake.

Here you see that before those names Capra and Hydrus there are 9 monosyllables that can be freely permuted, so that 362,880 verses can be made. Again, they can be ended in such a way that a dactyl and a spondee stand in last place, *Bos Hydrus & Mus*, beginning the verse *Sum Capra*, etc.; and so with innumerable others, which it may suffice to have indicated.

CAPVT VII.

De Loco & Diſtantia Fixarum à Terra, adeòq. de illarum Parallaxi, & de Infinitate huius diſtantiæ, ac velocitate Fixarum.

Chapter 7. On the place and distance of the fixed stars from the Earth, and hence on their parallax, and on the infinity of this distance, and on the speed of the fixed stars.

I. Just as it is God who counts the multitude of the stars, so he is the same who has measured their distance from the Earth and knows it with certainty. For so it is asserted in Ecclesiasticus, ch. 1: "Who has measured the height of the sky, and the breadth of the Earth, and the depth of the abyss?" And again in ch. 17: "He himself sees the power of the height of the sky." For this reason the warning of the same book in ch. 3 seems able to apply here: "Do not seek things higher than yourself." And so, in order to separate here what is certain from what is uncertain, we must first set out the methods by which parallax, and from it, by Problem 1 of ch. 7 of Book 3, the altitude of the fixed stars, seems able to be investigated by observations.

[Margin: First method of observing the distance of the fixed stars from the Earth.]

The first method is the one I spoke of in Book 3, ch. 7, no. 5: namely, if someone placed under the equator, with a very exact pendulum whose oscillations correspond exactly to single seconds of time of the primum mobile, were to count the time from one altitude of a star placed on the equator to another altitude, taken with a huge quadrant showing single seconds, provided that the altitudes lie beyond the hazard of sensible refractions. For if the seconds of the counted time are equal to the intercepted arc, that is, to the difference of the altitudes converted into seconds of time, there will be no sensible parallax; but if that difference is greater than the time counted, the excess will give the parallax, or at least the difference of parallax. But when will all those requirements ever come together: that someone be certain both that he is precisely under the equator, and that the fixed star is on the equator itself, and that it lies beyond the hazard of refractions, independently of parallax, and that he have a quadrant showing single seconds, and a pendulum that does not stray by even one second of time after a count of several hours? Certainly never.

[Margin: Second method.]

The second is by means of two meridian altitudes of a star circling the Pole of the World, if the altitude of the pole were known from elsewhere, independently of the fixed stars, without any sensible error, and the star at both places were beyond all sensible refraction, and the altitude were taken with a quadrant showing single seconds, and the meridian line were very exact. For with these things coming together, if, for example, the distance of the same star from the Pole of the World were found to differ in one of the meridian altitudes, it would furnish a difference of parallax and some evidence of parallaxes. But neither will all these things ever come together; and let the same judgement be made of the other methods, which depend on an altitude taken of a star, or on a motion of the fixed stars known beforehand independently of what we are seeking.

[Margin: Third method.]

The third method is from the difference of refractions, which have been found to be smaller in the fixed stars than in the planets. But since refraction is not known precisely, except either with the parallax computed, which it cancels, or with no parallax assumed, parallax itself cannot be known from refraction alone; although from this, as we shall say, I may have drawn a sufficient argument against the infinity of the distance, and indeed against the huge distance asserted by some Copernicans. Let the following theorem therefore stand as certain, namely:

The parallax and distance of the fixed stars cannot be grasped by human means through certain and evident observation.

This will be further confirmed from what was said of the uncertain parallax and altitude of the Sun itself, namely in Book 3, ch. 7, in the scholium.

[Margin: The fixed stars are above Saturn.]

II. Yet it seems certain that those stars that are occulted from us by some planets, as are almost all those within the zodiac, are higher than the planets, at least at the time when they are occulted by them. Hence, from the constancy of the distance that the fixed stars perpetually keep among themselves, and from the same measure of motion, it becomes very probable that all the fixed stars are higher than Saturn; and this indeed is the common view of all, both astronomers and natural philosophers, in their books on the heavens. But it is uncertain whether some or all of the fixed stars lie immediately above the heaven of Saturn, or whether there is a notable interval between Saturn and the fixed stars and how great; and whether all lie on the same spherical surface described from the centre of the world, or whether some are higher than others. All those who determine altitudes of the fixed stars rely on mere conjectures and on the choice of hypotheses, within the limits of probability.

[Margin: The opinion of the ancients on the equal distance of the fixed stars.]

Now, as regards the equality of the distance of the fixed stars from the Earth: Xenocrates held that the stars move along one surface, while the other Stoics

held that some are lower and higher than others, as Plutarch relates in *De placitis philosophorum*, book 2, ch. 15.

[Margin: Tycho's opinion on the unequal distance of the stars.]

The opinion of the Stoics has been followed above all by Tycho in vol. 1 of the *Progymnasmata*, p. 470, where he says: "It can therefore happen that some that appear smaller to us are equal to some that in truth appear larger, even though they appear smaller to us because of their higher position." And more clearly on p. 481: "Since it seems more likely that some are placed higher and others lower."

[Margin: And Galileo's.]

Galileo too agrees, in Dialogue 3 on the world system, saying: "For I do not believe that the stars are scattered on a spherical surface equally distant from the centre; rather, I think that their distances from us are so varied that some can be two or three times more remote than others. Therefore if at some time a very small star were found with the telescope, very close to some larger one, and yet that one were very high, it could happen that some sensible change would occur between them, corresponding to the change of the superior planets."

[Margin: And Kepler's.]

Kepler agrees with both, in the book *De Stella Nova*, ch. 21, and in the *Epitome of Copernican Astronomy*, book 1, part 2, from p. 34 on, where he thinks it probable that some fixed stars look smaller than others because they are farther from us. To this applies the line of Manilius in book 1 of the *Astronomica*, ch. 4, who says that the nebulous or obscure stars on the forehead of Orion look that way

Non quòd Clara minùs, ſed quòd magis alta recedant.

not because they are less bright, but because they recede higher up.

Yet Kepler judges it more probable that all the stars that appear to us of equal magnitude are equally distant from us, and form a concave surface around us. And in that ch. 21 he does not indeed affirm, but concedes as not absurd, that all the stars are of equal magnitude, yet that some look smaller because of their lesser distance from the Earth. At greater length, however, he contends in both treatises that the sphere of the fixed stars is so bounded toward us that the space between us and the nearest fixed stars is nevertheless very ample, and that we are, as far as sense can tell, in the middle of it, and see equally the hemispheres of stars divided by the Milky Way. Al-Farghānī too puts as much from the concave of the firmament and the least distance of the fixed stars to the convex as there is from the Earth to the concave of the firmament, as Clavius has it in the *Sphere*, p. 21[?], although others, of whom see the end of no. 6, put a smaller thickness.

[Margin: Whether the distance of the fixed stars is infinite.]

III. But is the distance from us of all, or of some, of the fixed stars visible to the naked eye or the eye equipped with telescopes infinite? This is much the same as if you asked whether this visible world is of infinite bulk. For, as Plutarch relates in *De placitis philosophorum*, book 2, ch. 1, Seleucus thought

the world infinite; Diogenes thought the universe infinite but the world finite; just as, in the same place, the Stoics held that the universe is infinite, counting in it the void or empty, but that the world, subsisting apart from the empty, is finite. Pliny the Elder, however, speaks very ambiguously in book 2, ch. 1, when he says: "The world, and this which it has pleased us to call by another name the sky, by whose encircling all things are covered, etc., is sacred, eternal, immense, whole in the whole, nay rather itself the whole: finite and like the infinite." Here Milich explains that it is like the infinite because of its immense bulk and the distance of the highest heaven from human beings, which is incomprehensible. But Giordano Bruno, in the book *De Maximo & Immenso*, and in *De Infinito & innumerabilibus*, argues for this infinity, which Aristotle in Book 1 of *De caelo* reports was the view of some of the ancients. Aristotle refutes them from text 33 to 75 with the well-worn argument from motion. For if stars, say A and B, were an infinite interval from the Earth's centre C, then the straight lines drawn to them from the centre of the world C, namely AC and CB, would be of infinite length; and so the chords subtending the arc AB would be infinite, and the arc AB itself infinite, let alone the whole circumference. Therefore its infinite space could not be traversed in a finite time. But we see, says Aristotle, that the whole sky with its stars moves in the finite time of 24 hours. Therefore neither the sky nor the distance of the stars is infinite.

Diagram. Set into the left column beside the argument from motion: two concentric circles about a centre, with two straight lines drawn from the centre out through the inner circle to the outer, forming a V.

- A (upper left, on the outer circle, at the end of the left-hand line)
- B (upper right, on the outer circle, at the end of the right-hand line)
- M (on the left-hand line, at the inner circle)
- N (on the right-hand line, at the inner circle)
- C (the centre, at the vertex of the two lines)

Someone might perhaps evade this argument by saying that an infinite space can be traversed in a finite time, but by the force of an infinite power and with infinite velocity in the mover; or, with the Copernicans, by denying that the sky or the fixed stars move, and assigning the daily motion to the Earth rather than to the sky and the fixed stars. Others therefore reason better as follows. Since arcs of circles described from the same centre and enclosed within the same straight lines drawn from the centre are similar or proportional, there will then be some infinite arc AB similar to a finite arc MN; and this seems absurd, unless one says that proportions of this kind demonstrated by geometers hold so long as one stays within the limits of finite quantity, but do not hold in the infinite. Others will say that it is impossible either for images and lights to reach our eye from an infinite interval, decreasing in a uniformly difform manner; or for the infinite distance between two stars to be seen under a finite angle; or, with Kepler in ch. 21 of *De Stella Nova*, that what is seen as having a shape, and so is enclosed within certain boundaries, as the stars are, cannot

be infinite. These and similar arguments we once discussed in physics, and we now leave them to the natural philosophers to treat at greater length. The argument that seems to me more astronomical, though with the help of optics, is the one supplied by refractions, of which I shall speak in Scholium 3.

[Margin: 1. The opinion of Pliny the Elder.]

IV. The first and truest opinion is that of Pliny the Elder, who holds that the distance of the fixed stars is uncertain, as we too have taught in number 1. Indeed he adds that it is rash to want to define it. For in Book 2, ch. 1, after saying of the world: "It is finite and like the infinite," he goes on: "It is madness that certain people have turned its measure over in their minds and have dared to publish it." And soon after: "It is madness, truly madness, to go out of it and, as if its interior were by now fully known, to search in this way what lies outside: as if someone could take the measure of anything who does not know himself, or the mind of man could see what the world itself does not contain." And below, in ch. 23, when he was offering a conjecture on the distance of the Sun, saying: "Not as a measure, for to want that is the mark of an almost demented leisure, but only so that an estimate for conjecture may stand in the mind," he added: "It is astonishing how far the shamelessness of the human heart goes, lured on by some tiny success, as in the matters above reason supplies an occasion for impudence: and having dared to divine the distances of the Sun to the Earth, they do the same for the sky, since the Sun is in the middle, so that straightway the measure of the world itself too comes to their fingers, etc., as if the measure of the sky were established quite by the plumb line."

[Margin: 2. The opinion of the Pythagoreans, and their method from harmonic intervals.]

V. The second opinion and method is that of the Pythagoreans, who determine the distance of the stars from harmonic intervals. For Pliny the Elder reports it thus in Book 2, ch. 21, saying: "But Pythagoras, a man of shrewd mind, inferred that from the Earth to the Moon is one hundred and twenty-six thousand stadia; from her to the Sun double that; from there to the twelve Signs three times that: and Gallus Sulpicius too was of this opinion." Then in ch. 22 he adds: "But Pythagoras sometimes, on a musical principle, calls the distance of the Moon from the Earth a tone; from her to Mercury half of that space; and from him to Venus about the same; from her to the Sun one and a half times. From the Sun to Mars a tone, that is, as much as from the Earth to the Moon; from him to Jupiter a half; and from there one and a half times to the zodiac [Signiferum]: thus seven tones are produced, the harmony they call diapason, that is, the totality of concord." Censorinus, *De Die Natali*, ch. 11, is in accord with Pliny the Elder's account. I shall quote that passage whole, since it also serves for the intervals of the planets. He says, then:

"So Pythagoras held that from the Earth to the ☾ is about 126 thousand stadia (understand: Italian ones), and that this is the interval of a tone; and from the Moon to the star of Mercury, which is called [Greek: στίλβων (stilbōn, "the

gleaming one")], half of that, like a [Greek: ἡμιτόνιον (hēmitonion, "semitone")]; from there to [Greek: φόσφορον (phosphoron, "the light-bearer")], which is the star of Venus, about the same, that is, another semitone [ἡμιτόνιον]; then further on to the Sun three times as much, as it were a tone and a half: so that the star of the Sun is three and a half tones from the Earth, which is called Diapente; and two and a half from the Moon, which is Diatessaron; and from the Sun to the star of Mars, whose name is [Greek: πυρόεις (pyroeis, "the fiery one")], there is as much interval as from the Earth to the Moon, and this makes a tone [τόνον]; from there to the star of Jupiter, which is called [Greek: φαέθων (phaethōn, "the blazing one")], half of that, which makes a semitone [ἡμιτόντον] [recte: ἡμιτόνιον]; the same from Jupiter to the star of Saturn, whose name is [Greek: φαίνων (phainōn, "the shining one")], that is, another semitone [ἡμιτόνιον]; from there to the highest sky, where the Signs are, likewise a semitone [ἡμιτόνιον]; and so from the highest sky to the Sun the interval [diastema] is a Diatessaron, that is, two and a half tones, and to the summit of the Earth from the same sky there are six tones; in which there is the concord [Symphonia] [Greek: διὰ-πασῶν (dia pasōn, "through all")]. Besides this he related many things that musicians discuss: he showed that the stars and this whole world are [Greek: ἐναρμόνιον (enarmonion, "harmonious")]. For this reason Dorylaus wrote that the world is the instrument of God."

[Margin: The world, the instrument of God.]

So dividing the 126000 stadia by 8 gives 15750 Italian miles. On this basis the distances of the stars from the Earth and from one another, according to Pythagoras's opinion, are those we have gathered into the following table.

[Margin: The Pythagorean intervals of the stars.]

INTERVALLA SIDERVM PYTHAGORICA

The Pythagorean intervals of the stars

Order of the stars	Distance from the Earth, in Italian miles	Distance between them, in Italian miles
☾	15750	15750 Tone
☿	23625	7875 Semitone
♀	31500	7875 Semitone
☉	55125	23625 Tone and a half [Sesquitonus]
♂	70875	15750 Tone
♃	78750	7875 Semitone
♄	86625	7875 Semitone
✶	94500	7875 Semitone

But since an excessive parallax follows from this, especially in the fixed stars, which would be no further from the Earth than about 24 semidiameters of the Earth, one of which contains 4139 modern Italian miles [Milliaria Italica recentia], from what was said in Book 2, ch. 7, their parallax would be 2° 23′; and it is certain that the lunar parallax itself is not so great. For that reason both this and the Archimedean measurement of the intervals were rejected by the Platonists, as Macrobius asserts in Book 2 on the *Dream of Scipio*, ch. 3. If

only he had set out the Archimedean measures for us! For he says only: "This measurement of Archimedes, however, was rejected by the Platonists, as not keeping the double and triple intervals: and they laid down that this is to be believed: that as far as it is from the Earth to the Moon, it is twice that from the Earth to the Sun; and as far as it is from the Earth to the Sun, it is three times that from the Earth to Venus; and as far as it is from the Earth to Venus, it is four times as much from the Earth to the star of Mercury; and as far as it is to Mercury from the Earth, it is nine times as much from the Earth to Mars; and as far as it is from the Earth to Mars, it is eight times as much from the Earth to Jupiter; and as far as it is from the Earth to Jupiter, it is twenty-seven times as much to the orb of Saturn." Therefore if we take the Pythagorean distance of the Moon, which is about 4 semidiameters of the Earth, or the Ptolemaic, which runs at about 60 (for this roundness of numbers is enough for what we intend), the intervals of the stars would have to be arranged according to the Platonists as you see in the following table, in which, however, Macrobius puts ☿ higher than Venus at apogee.

[Margin: The Platonic intervals of the stars.]

INTERVALLA SIDERVM PLATONICA

The Platonic intervals of the stars

Order of the stars	If the distance of the ☾ from the Earth is 4: distance from the Earth, in semidiameters of the Earth	But if it is 60: distance from the Earth, in semidiameters of the Earth
☾	4	60
☉	8	120
♀	24	360
☿	96	1440
♂	864	12960
♃	69125	103680
♄	1866375	27993654

Finally our Bettini, drawing the harmonic intervals from another source, from the divisions of the octochord, in *Apiarium* 10, Progymnasma 1, propositions 1 and 3, gives the distances that you have here, copied from him, in the following table.

INTERVALLA SIDERVM AC CÆLORVM IVXTA BETTINVM

The intervals of the stars and heavens according to Bettini

Distances from the Earth, in semidiameters of the Earth	Semidiameters of the Earth	Names of the musical intervals, and proportions:			Of the whole interval
The Earth					
☾ Apogee	381 6/9	Fà fà	Nete	Major tone	9th part
♀ Perigee	687	Mì mì	Paranete	Major ditone	5th part

Distances from the Earth, in semidiameters of the Earth	Semidiameters of the Earth	Names of the musical intervals, and proportions:			Of the whole interval
☿ Perigee	858 3/4	Rè rè	Trite	Diatessaron	4th part
☉ Mean	1145	Sol	Paramese	Diapente	3rd part
♂ Mean	1374	Fà	Mese	Major hexachord	Two-fifths
♃ Perigee	1603	Mì	Lycanos	✝	Seven-fifteenths
♄ Perigee	1717 1/2	Rè	Parhypate	Diapason	A half
✶ Fixed stars	2290	Ut	Hypate	Diapason-diapente	Two-thirds
Crystalline heaven	2574 3/4			Disdiapason	Three-quarters
Empyrean	3435				The whole interval

But just as the distance of the Moon is absurd and too great, against the evident observations of the lunar parallaxes, and destroys all lunar eclipses, as we said in Book 4, ch. 14, Scholium 2, so the distance of the fixed stars is too small, and it subjects the fixed stars to a parallax of 1′ 30″ [sesquiminuti], which not even the Sun has in the truer opinion, although no instruments have yet been able to detect this parallax of the fixed stars. A truer astronomy also requires the distance of ♃ to be about five times, and that of ♄ about ten times, the Sun's distance from the Earth, etc. So you see how uncertain, and in some places how evidently deceptive, is a distance of the stars devised from harmonic intervals alone. I pass over for now the harmonic intervals of the planets established by Kepler, since they are more fittingly dealt with in the book of the Systems, where the Harmonic System is treated.

[Margin: Bettini's error in the intervals of the stars.]

[Margin: 3. Opinion and method.]

VI. The third opinion and method rests on the distance of Saturn at apogee from the Earth, than which the distance of the fixed stars is agreed to be greater. How the distance of Saturn is investigated from the hypothesis and theory of its motions will have to be told in the next book, although all astronomers concur in this, that Saturn's distance from the Earth is about ten times the distance of the Sun from the Earth. So here we shall only add the authors who have determined the distances of the fixed stars by this method. They are:

[Margin: Authors who have used this third method.]

al-Battānī, ch. 50; al-Farghānī, chapter [differentia] 21; Jean Fernel, *Cosmotheoria*, Book 1, ch. 11; Erasmus Reinhold on Peuerbach's *Theoricae*; Francesco Maurolico at the end of the *Cosmographia*; Giovanni Antonio Magini in the *Primum Mobile*, Book 11, Problem 29; Christoph Clavius on the *Sphere*, p. 215;

Tycho, *Progymnasmata*, vol. 1, p. 481, and vol. 2, p. 465, and in the *Epistolae Astronomicae*, p. 167; Longomontanus, *Astronomia Danica*, Book 1, *Theorica*, ch. 1; Francesco Giuntini on the *Sphere*, and from him Baranzanus [Redento Baranzano], *Uranoscopia*, p. 120; Barocius [Francesco Barozzi], *Cosmographia*, Book 1, p. 49; and Giuseppe Biancani, *Sphaera*, Book 17, ch. 14.

Among these, al-Battānī and al-Farghānī declare that they deduced the distances from the Ptolemaic hypothesis. But al-Farghānī doubles the least distance to obtain the thickness of the eighth sphere, or the greatest distance of the fixed stars; and he sets the former at 20220 semidiameters of the Earth and the latter at 40440. Fernel, Barozzi, Magini and Clavius follow these numbers fairly closely, while Giuntini follows al-Battānī. Tycho, however, and following him Longomontanus, Biancani and Baranzano, do not double the distance; yet they assign a greater distance to some stars and a lesser to others, and they define the least as 13000 and the greatest as 14000 semidiameters of the Earth. But we shall set out their opinions at the end of number 14, in a table together with others.

[Margin: 4. Opinion of Copernicus.]

VII. The fourth opinion is that of Copernicus. In Book 1, ch. 10, he places the Sun at the centre of the world and the Earth as movable around the Sun, with an annual motion through the same orbit through which the Ptolemaics carry the Sun round. So that no sensible parallax should arise from such a hypothesis in observations of the fixed stars, through the Earth's approach to the fixed stars or its recession from them, he holds the distance of the fixed stars from the centre of the world to be so great that the diameter of the Earth's annual orb has no apparent or sensible difference and proportion to the diameter of the sphere of the fixed stars. And thus he acknowledges a certain vast space empty of stars between Saturn and the fixed stars, which he confirms in the same ch. 10 in these words: "For that there is still a very great space from Saturn, the highest of the wanderers, to the sphere of the fixed stars, is shown by their twinkling lights. By this sign above all they are distinguished from the planets; and between things moved and things not moved there had to be the greatest difference. So great indeed is this work of the Best and Greatest." For he makes the fixed stars immobile and the daily motion the Earth's; and the apparent motion of the fixed stars in longitude towards the east he assigns to a motion of the equinoctial points towards the west, by which the intersections of the terrestrial equator with the celestial ecliptic are advanced towards the setting, as we have already set out in Book 3, ch. 28. But how great that distance is, Copernicus nowhere determined.

To show the reasoning for so vast a distance in the Copernican hypothesis, let the vertical line AB in the following figure be drawn through the centre of the world C, where Copernicus places the Sun immobile, and around this centre let the Earth move with the annual motion in the great orbit, whose semicircle is DEF and its diameter DF. And about the same centre C let the

semicircle of the sphere of the fixed stars AIB be described, in which let there be a star G or H, on the meridian or on some other vertical AIB.

Diagram. Set into the right column at the left of the text: a vertical diameter AB with a large semicircle running from A round to the right down to B, and a small semicircle about the same centre on the diameter's middle part, bulging right from D to F. Straight lines run from D to G, to H and to I; from F to G, to H, to I and to K; and from C to I. G and H are drawn as stars.

- A (top of the vertical line)
- G (on the large semicircle, upper right, star)
- H (on the large semicircle, right of G and below it, star)
- D (on the vertical line, upper end of the small semicircle)
- E (on the small semicircle, at its right, among the lines)
- C (on the vertical line, the centre)
- I (on the large semicircle, at the right, level with C)
- F (on the vertical line, lower end of the small semicircle)
- K (on the large semicircle, below I, level with F)
- B (foot of the vertical line)

Now let the Earth be at F, and its sensible horizon FK; and let the distance of the star G from the zenith A, which is the arc AG, be taken with some instrument, the angle AFG on the instrument measuring it. Then six months later let the Earth be at D, and from there again, if not on the same meridian or vertical, at least on a vertical standing off from the meridian line by an equal azimuth, let the distance of the same fixed star G from the zenith be taken through the angle ADG. For if the diameter of the annual orb DF has a sensible proportion to FG, the distance of the fixed stars, some sensible parallax will certainly appear in the distance of the fixed star G, and it will seem greater when viewed from D than from F, because the angle ADG, being exterior, is greater than the interior AFG, by proposition 32 of Euclid's first book; and that parallax will be the angle G, by which, by the same proposition 32, the angle ADG differs from the angle AFG, and this is called the parallax of the annual orb. Similarly, the star H, which seen from the Earth placed at D would appear on the physical horizon DH, would appear, seen from the Earth placed at F, raised above the physical horizon FK by the whole arc HK. Moreover, if the distance GH of two stars G and H were taken with some large sextant from the Earth at D, through the angle GDH, and then the distance of the same stars were taken from the Earth placed at F, through the angle GFH, then, if their distance from the Earth were of sensible proportion to the diameter DF, their distance would appear greater to the eye nearer at D than to the eye more remote at F, according to the laws of optics, because the angle GDH would easily be shown to be greater than the angle GFH. But since it is commonly assumed by astronomers that no parallax in the fixed stars has been observed or is observable with any instruments, whether their declination through meridian altitudes, or through armillary spheres, or their mutual distance, or the altitude of the pole is taken with them, the Copernicans, so that no observable parallax

from the Earth's annual motion can perhaps be demonstrated, lay it down that the diameter DF has no apparent ratio to the distance of the fixed stars FA or FB, and that the annual orb itself is like a point with respect to the sphere of the fixed stars.

VIII. So, supposing now the hypothesis of the Earth carried round through the annual orb, let us see how great the distance of the fixed stars from the Earth, say FH, must be, for the parallactic angle H to vanish to the senses and to be observable by no human instruments. For some put it greater, others smaller. [Margin: The Copernican distance of the fixed stars, from Magini.] First, Magini says in the *Primum Mobile*, Book 11, Problem 29: "To leave aside the opinion of Copernicus, who maintains that between the sphere of Saturn and the eighth there is so immense a vastness that the eighth sphere is removed from the Earth by at least 7,850,000 semidiameters." I do not recall having read this anywhere in Copernicus; but I think it was deduced from his hypothesis by Tycho, from whom Magini may have drawn it. [Margin: And from Tycho.] For Tycho says in the letter to Christoph Rothmann of 1589 November 24: "As for the Earth's other motion, the annual one, which would remove the eighth sphere so far that the orbit this motion traces vanishes with respect to it, explain whether it seems likely to you here that the space from the Sun, then the centre of the universe, to Saturn is contained more than 700 times between Saturn and the sphere of the fixed stars [Affixarum]; and all of that filled with no stars. Yet this must happen, if the Earth's annual orb is to appear even the size of 1′. Indeed, even then the fixed stars of the third magnitude, which have 1′ in diameter, will necessarily be equal to this whole annual orb: that is, they will take in 2284 semidiameters of the Earth in their diameter; for they will be very nearly 7,850,000 of the same semidiameters distant. What shall we say of the stars of the first magnitude, some of which occupy 2′ and some nearly 3′ in visible diameter? And what if the eighth sphere were removed still higher, so that the Earth's annual motion vanished there altogether?"

[Margin: From Longomontanus.] Nor does Longomontanus depart much from this measure, in the *Theorica*, Book 1, ch. 1, where he draws a second argument against the annual motion of the Earth from the incredible gap between the sphere of the fixed stars and the Earth, which the Copernican hypothesis brings in. For in the preceding figure, if the angle DIF, the parallax of the annual orb, were 1′, and the angle CID ½′ [semiminuti], then with CD, the semidiameter of the annual orb, that is, the Tychonic mean distance of the Sun, put at 1150 semidiameters of the Earth, he deduces by the doctrine of triangles the distance CI as 7,906,818 semidiameters of the Earth. If you subtract from this the distance of Saturn, which is about 12000 semidiameters of the Earth, there remains, he says, between Saturn and the fixed stars a gap of 7,894,818 semidiameters of the Earth, utterly empty. Moreover, from almost the same figure, if a star is taken to be FED, and its apparent semidiameter under the angle CID is 1′, as in stars of the first magnitude, and the distance CI 7,906,818, he infers DC, the true semidiameter of the star, as 2300 semidiameters

of the Earth, and the diameter DF as 4600. But of these things below, where the true size of the fixed stars is treated.

[Margin: Two distances of the fixed stars in the hypothesis of Copernicus, from Scheiner.]

IX. But our Scheinerus [Christoph Scheiner], in the *Disquisitiones Mathematicae*, number 13, taking from Michael Mästlin's appendix to Rheticus the semidiameter of the annual orb as 1208 semidiameters of the Earth, and the distance of Saturn as 10872, makes: as the Earth's semidiameter 1 is to Saturn's distance 10872, so the semidiameter of the annual orb 1208 is to the distance of the fixed stars, 13,133,376 semidiameters of the Earth, so that in this way the parallax of the fixed stars at the extremes D and F of the annual orb vanishes. And in the triangle CID, from CD 1208 and CI 13133376, there follows the angle CID 20″, and the whole parallax of the annual orb DIF 40″. [Margin: The parallax of the annual orbit from this distance.] But if, according to Tycho, *Progymnasmata*, vol. 1, p. 480, the distance of Saturn is 12900, and this has to be multiplied about eight hundred times, so that in the hypothesis of Copernicus the semidiameter of the annual orb vanishes with respect to the semidiameter of the sphere of the fixed stars, then, as Scheiner infers, the distance of the fixed stars will be 10,320,000, multiplying 12900 by 800; from which there results CID 24″, and the whole parallax DIF or DFH 48″.

[Margin: The Copernican distance of the fixed stars, from Galileo.]

X. But Galileo, in Dialogue 3 of the *De Systemate Cosmico*, infers the distance of the fixed stars from the proportion of the periodic motions, thus. If Saturn's motion of 30 years requires a distance of Saturn from the Earth nine times greater than the greatest distance of the Sun from the Earth, whose motion is of one year, then the motion of the fixed stars, which in Ptolemaic terms is of 36000 years, requires a distance of the fixed stars 10800 times greater than the distance of the Earth from the Sun. For as 30 is to 9, so 36000 is to 10800. Multiplying by these the Earth's distance from the Sun, 1208, as Galileo uses it here, the distance of the fixed stars from the Earth becomes 13,046,400 semidiameters of the Earth, and hence the angle DIC 20″, but the whole parallax of the annual orb DIF or DHF 40″.

[Margin: The distance of the fixed stars, from Kepler.]

XI. With similar ingenuity Kepler, in the book *De Stella Nova*, ch. 16, and in the *Epitome of Copernican Astronomy*, p. 490, investigates the distance of the fixed stars. Having placed the Sun immobile at the centre of the world and as the source of planetary motion, and the sphere of the fixed stars as immobile, as the place containing all things, he says that the Movable, that is, the interval of the planets, or the distance of highest Saturn, is the mean proportional between the mover and the immobile place. He gives the Sun's semidiameter 6 semidiameters of the Earth, and the distance of the Earth from the Sun 1432 semidiameters of the Earth, and to the distance of Saturn ten times this, namely 14320. On this basis he makes: as 6 is to 14320, so 14320 is to 34,077,066 2/3, the distance of the fixed stars; from which he denies that a

parallax of the annual orb greater than 9″ follows, and the perception of that he grants to none of the astronomers. [Margin: Kepler's slip on the parallax of the annual orbit.] But he is mistaken. For although the parallax CID, which the Earth's motion makes when it has completed a quadrant of the zodiac, is 8″ 27‴, yet after the semicircle FED is completed the whole DIF or DHF is 16″ 54‴.

But in the *Epitome*, p. 479, he gives the Sun's semidiameter 15 semidiameters of the Earth, and the distance of the Earth from the Sun 3469, so as to make the Sun's parallax almost insensible. Saturn's distance, however, he makes a little less than ten times the Earth's distance from the Sun, that is, in round figures 30000 semidiameters of the Earth; and he makes this same distance the mean proportional between the Sun's semidiameter and the distance of the fixed stars. Accordingly he makes: as 15 is to 30000, so 30,000 is to 60,000,000 semidiameters of the Earth, namely to the distance of the fixed stars; from which he denies that a parallax of the annual orb greater than 12″ follows, and on this he says that no practitioner who asserts that 12″ can be perceived with instruments is to be believed. [Margin: Kepler's second slip on the same parallax.] But here too he is mistaken, or perhaps conceals the error. For granted that the parallax CID comes to 11″ 56‴ from these measures, yet the whole parallax DIF or DHF is 23″ 52‴, and greater than the preceding, because of the far greater distance of the Earth from the Sun assumed here than there.

Further, to show that the distances he assumes do not conflict with Tycho's observations, he recounts that in 1586, at midnight after the autumn equinox, Tycho found the greatest altitude of the pole star to be 58° 51′, and that he found the same on December 26, in the evening at about the sixth hour, near the solstice, although the Earth had then completed the quadrant ED, and the horizon DH in the later observation was higher than the horizon CI in the earlier by the whole semidiameter CD of the annual orb. Therefore, if Tycho did not perceive a parallax of even 1′, much less can a parallax of 12″ be perceived. [Margin: Kepler disparages Tycho's instruments.] Yet Tycho in his *Mechanica Astronomica* asserted that with his largest quadrant a sixth part of 1′, that is, 10″, can be perceived in the altitudes of the stars; so much the more, then, 24″. And in *Progymnasmata*, vol. 1, pp. 347 and 362, he recounts the altitude of the pole star observed down to these seconds.

[Margin: Two distances of the fixed stars, from Lansbergen.]

XII. Let us now hear Philips van Lansbergen, and with him Hortensius [Maarten van den Hove], who is itching to be a Copernican [Copernicoturientem]. Formerly Lansbergen, in his commentaries on the motion of the Earth, supposing a parallax of the annual orb of 30″, had inferred the distance of the fixed stars as 68,754,937 parts of which the radius of the annual orb is 10000, that is, 10,312,227 semidiameters of the Earth; since the semidiameter of the annual orb, or the mean distance of the Earth from the Sun, is according to Lansbergen 1498 1/2 semidiameters of the Earth, from what was said in Book 3, ch. 7. Later, in the *Uranometria*, he chooses a distance of the fixed stars of 280,000,000 parts of which the radius of the annual orb is 10000, [Margin:

Book 3, Element 7.] that is, 41,958,000 semidiameters of the Earth; from which he deduces the parallax of the annual orb as 7″ 22‴. This is true indeed, if one is speaking of the parallax CID made by the semidiameter of the annual orb; [Margin: Lansbergen's slip on the parallax of the annual orbit.] but if of the whole DIF or DHF, made by the diameter, it is 14″ 44‴. Lansbergen's basis is, first, that if a smaller distance is assumed, the parallax of the fixed stars caused by the Earth's annual motion will turn out sensible, "contrary to the experience of every age"; and second, that he puts the revolution of the fixed stars at 28000 years, and reasons thus: "As the periodic time of the Earth, of one year, is to the radius of its orbit, 10000, so the periodic time of the fixed stars, of 28000 years, is to the radius of their sphere, 280,000,000." He adds that between Saturn and the fixed stars there is a space of 279,900,696 parts of which the radius of the an- nual orbit is 10000, empty of stars. From this it comes about that, just as the Sun, the centre of the world, is a point with respect to the sphere of the highest planet, so the sphere of the planets is a point with respect to the sphere of the fixed stars, and, viewed from the fixed stars, does not appear greater than 1′ 49″; and that this befits the immensity and infinity of God the Architect. Nor does he blush to say *that the ancient and more recent astronomers have erred by the whole heaven*, who made out of the sphere of the planets and that of the fixed stars one heaven, or a contiguous one, although these are separated from each other by a vast interval. But this reasoning comes down to this: the distance of the fixed stars is 28000 semidiameters of the annual orb because the period of the fixed stars is 28000 years. To be consistent, then, the distance of Saturn ought to have been 30 semidiameters of the annual orb, because the period of ♄ is 30 years. And yet in Book 3 of the *Uranometria*, Element 6, he makes it 11½ parts of which the radius of the annual orb is 1. But if the parallax of ♄ stands in the way of the proportion between the distance and the period of ♄, what if a parallax of the fixed stars, known to God, stands in the way of the proportion between their distance and period?

[Margin: Hortensius's opinion on the distance of the fixed stars, and his lapse on the parallax of the annual orbit.]

XIII. I come to *Hortensius*, who, in his dissertation with Gassendi, chooses Lansbergen's earlier opinion on the distance of the fixed stars: 68,754,937 parts of which the radius of the annual orb is 10000, that is, 10,312,227 terrestrial semidiameters. This is because from it, he says, there follows a parallax of the annual orb of only 30″, which he thinks can be admitted without any evident conflict with the observations. Yet the whole parallax DHF, or DIF, is 1′, a full minute, even granted that its half, CID, is 30″.

[Margin: Hérigone's opinion on the distance of the fixed stars.]

XIV. *Pierre Hérigone* has a stronger stomach. In volume 5 of the *Cursus Mathematicus*, p. 615, he assumes that the semidiameter of the annual orb, or the distance of the Earth from the Sun, which he makes 1200 semidiameters of the Earth, is the mean proportional between the semidiameter of the Earth and the semidiameter of the sphere, or the distance of the fixed stars, which

he accordingly makes 144000 [recte: 1440000, per the book's errata; on this copy a hand has also added the final 0]. From this he infers that the parallax of the fixed stars produced by the annual orb is as great as the greatest parallax of the Sun, namely 3′. He denies, however, that it can be perceived, except in stars near the pole of the zodiac, and only after an interval of six months. But because, he says, after this interval, if it was night at the first observation it will be day at the second, it therefore cannot be perceived absolutely, since the fixed stars are then hidden.

[Margin: Hérigone's twofold error.]

But he errs twice over. *First*, because the parallax CID, in the figure of no. 7, is indeed 3′, given CD 1200 and CI 144000 [recte: 1440000, per the book's errata]; but the whole parallax DIF, or DHF, produced by the Earth's annual motion of six months through the semicircle DEF, is 6′. *Second*, because the altitude of the Pole Star can be observed both in the evening near the summer solstice and in the morning near the winter solstice, with an interval of six months between them; and in this way that great parallax, not only of 3′ but even of 6′, would certainly be perceived. I do not know with what stomach Hérigone would swallow that.

[Margin: Corollary against the asserters of the Copernican hypothesis.]

XV. All these asserters or supporters of the Copernican hypothesis, then, have concealed from us half of the parallax of the annual orb, whether by deceit or rather through inadvertence, so that they would neither lose the beauty of their analogies nor, in keeping those analogies, end up with a parallax of the fixed stars too great against the observations. And what deceived them, I believe, was the way of deriving the Sun's horizontal parallax, for which we usually employ only the Earth's semidiameter, since of course we are seeking that variation of view which arises with the eye supposed now at the centre, now on the surface of the Earth. But if we were seeking the variation of view that the whole diameter of the Earth produces for us and our Antipodes, or would produce for us if we walked around a semicircle of the Earth, it would certainly be twice as great as that from the centre to the surface. For my part, in the hypothesis of Copernicus, I would not admit a parallax of the annual orb in the fixed stars greater than 10″, since this difference, and indeed even a smaller one, can be clearly perceived with exactly divided instruments, using the very fine threads of the dioptras, set far from the eye. Below, therefore, in the last table, I shall give the distance of the fixed stars required so that the parallax DHF does not come out greater than 10″, according to the doubled distance of the Earth from the Sun, or DF, the diameter of the annual orb asserted by the supporters of the Copernican hypothesis.

[Margin: 5. Rheita's method: his opinion and fallacy.]

XVI. The fifth opinion and method is that of *Antonín Maria Šírek de Rheita*, who on p. 193 of his *Radius Sidereomysticus* derives the distance of the Sun and of the fixed stars. He says that he observed the Sun's diameter to be 34′ 41″, whose sine is 100 of the parts of which the radius in the tables of sines is

10000. Therefore, he says, the Sun's diameter is to the distance of its orbit as a square root is to its square. Then he adds that just as the Earth's diameter is contained a thousand times in the radius of the Sun's orbit, so the diameter of the Sun's body is contained a thousand times in the radius of Saturn's orbit, and the diameter of Saturn's sphere a thousand times in the radius of the sphere of the fixed stars; so that the radius of the fixed stars contains 1,000,000 diameters of the Sun and 10,000,000 diameters of the Earth, and the distance of the fixed stars is therefore 20,000,000 semidiameters of the Earth. But earlier, on the same p. 193, he had gratuitously assumed that the Earth's diameter is to the true diameter of the Sun as 10 to 100. No less gratuitously does he assume the other analogies already mentioned, since other analogies no less probable can be invented, as is now clear from the preceding opinions.

[Margin: 5 principal methods are weighed.]

XVII. Of all the ways brought forward so far, five might seem the most probable of all, setting aside those that rest on the mobility of the Earth.

[Margin: 1. Method from the smallest of the perceptible parallaxes.]

The *first* is to posit a distance of the fixed stars at least as great as a parallax of 2″ or 3″ requires, within which it is certain that all the refinement of observations lies. For although instruments for taking the altitudes of stars can be built so large that, by transversal lines, they admit a subdivision of each minute into 2″ or 3″ each, yet the cylinders or strings of the dioptras cannot be so fine as to make us certain of any greater refinement in sighting at the centre of the stars. And the instrument and the observer must be very exquisite indeed if one is to be certain of not having committed an error of 5″, let alone 3″ or 2″. Now if in a right-angled rectilinear triangle the Earth's semidiameter is assumed as 1, and the parallactic angle opposite it as 2″, there will come out, for the base, a distance of the fixed stars from the Earth of at least 100000 semidiameters; which seems very probable to me, supposing the distance of Saturn, of which more below.

[Margin: 2. Method from the proportion of the motions.]

The *second* method rests on the proportion of the more certain periodic motions and distances. Now no star's distance is more certain than the Moon's, because of the evidence of its parallaxes. For us that distance is 59 semidiameters of the Earth, while its revolutions in one Julian year are 12⅓. One revolution of the fixed stars, on the other hand, requires 26000 years in our opinion, in which there are 321360 lunar months. So if we form the proportion, as 1 month to the distance of 59 semidiameters of the Earth, so 321360 months to another, there will be 18,960,240 semidiameters of the Earth. But if we used the distance of the Sun, not now our own but the smallest among those that can be entertained, which is at least 1000 semidiameters of the Earth, and formed the proportion, as 1 year to 1000, so 26000 years to another, the distance of the fixed stars would be 26,000,000 semidiameters of the Earth. That is very enormous and absurd, as is the former, because of the excessive gap between the fixed stars and Saturn;

for between them and the system of the planets there would be an immense space, empty of stars and idle.

[Margin: 3. Method from the difference of refractions.]

A *third* method could rest on the difference of refractions, of which we shall speak again in Scholium 8. For the Moon's horizontal refraction is greater than the Sun's refraction by very nearly 1′, and the difference of the intervals of the Moon and the Sun from us is 7268 terrestrial semidiameters; while the refraction of the fixed stars is less than the solar by 4½′ at the horizon. If we form the proportion, as the difference of refraction, 1, to the difference of distance, 7268, so 4½ to another, the difference of distance between the Sun and the fixed stars will be 32706; and if you add to it the Sun's mean distance, 7327, the distance of the fixed stars will be 40033 semidiameters of the Earth. But although a probable argument can be taken from this for their finite distance, it is hardly very probable for the determinate measure of that distance, both because of the headlong increase of refractions near the horizon, which does not follow the decrease of the inclination, and because our distance of the Sun from the Earth rests on a more solid foundation, and from it is derived a distance of Saturn, and hence of the fixed stars, far greater than 40033 semidiameters of the Earth.

[Margin: 4. Method from the distance of Saturn.]

The *fourth* method, then, the most likely of all those reported so far, uses the distance of Saturn alone, giving no thought meanwhile to its shadow. Now it is established from the parallaxes of the annual orb, and from the equations necessary to reconcile Saturn's mean motion with its true motion and with the places observed by instruments, that the distance of Saturn from the Earth is ten times the distance of the Sun from the Earth, or less than that by an imperceptible small difference, so that for this purpose we may safely use the tenfold ratio; *Kepler* too noted this in the *Epitome of Copernican Astronomy*, Book 4, p. 455. Now the Sun's mean distance, for us, is in round figures 7300 semi- diameters of the Earth; tenfold that, then, which is the distance of Saturn, will be 73000. This should not seem too much to anyone who examines the foundations prudently, especially since for Posidonius and Wendelen, for whom the Sun is distant from the Earth by about 15000 terrestrial semidiameters, Saturn must be raised to nearly 150000 semidiameters. For this reason we cannot assign the fixed stars a distance less than 73000 semidiameters of the Earth, even if we give no thought to the shadow of Saturn. But the greatest distance of ♄ is, for us, 90000 and more; therefore the least distance of the fixed stars is about 100000.

[Margin: 5. Method from the distance and shadow of Saturn.]

The *fifth* method, a very probable one, joins the distance of Saturn with the length of Saturn's shadow. Though that shadow is one length for Biancani and another for others, as we shall say in Book 7, section 6, ch. 14, for me it is 118680 semidiameters of the Earth, from what will be said there. So when my greatest distance of Saturn, 90155, and the axis of Saturn's shadow are joined, the distance of the fixed stars will be not less than 200000 semidiameters of the Earth, and at least 208835; that is, so that it lies beyond all hazard of Saturn's shadow.

For although there can be no fear for the fixed stars from that shadow, since they shine of themselves and not from the Sun, whatever *Biancani* may have suspected about their eclipse in Book 15 of the *Sphaera*, ch. 2, it is nevertheless probable that God raised them beyond this limit; especially since the ascents of comets, as *René Descartes* notes in part 3 of his new *Philosophy*, or new stars, seem to require a great space between the planets and the fixed stars. For us, then, let the minimum be 210000 semidiameters of the Earth, a little above the shadow of Saturn with its companions, while the maximum is uncertain; for the thickness of the heaven of the fixed stars is uncertain, in which they are unequally removed from us, in their lowest part as well as in their centre.

FIXARVM DISTANTIA A TERRA

The distance of the fixed stars from the Earth

Authors	Semidiameters of the Earth: minimum	Semidiameters of the Earth: maximum	Horizontal parallax of the fixed stars
al-Battānī, Giuntini	19000	✢	10″ 58‴
al-Farghānī, Barozzi	20220	40440	10″ 14‴
Maurolico	20077	20086	10″ 16‴
Fernel, Clavius	22612	45225	9″ 8‴
Magini	20110	40220	10″ 15‴
Tycho	13000	14000	15″ 6‴
Mario Bettini	2290	✢	90″ 0‴
Antonín Maria Šírek de Rheita	20000000	✢	0″ 1‴
We, from method 4	100000	✢	2″ 0‴
from method 5	210000	✢	1″ nearly

The parallaxes placed in the third column correspond to the minimum distances; otherwise the maximum distance of al-Farghānī, Magini, Fernel and Clavius agrees with a parallax of only 5″ or 6″.

DISTANTIA FIXARVM A TERRA A Copernici Sectatoribus asserta.

The distance of the fixed stars from the Earth, as asserted by the followers of Copernicus

Followers or supporters of the Copernican hypothesis	Distance in semidiameters of the Earth	Greatest parallax of the fixed stars produced by the Earth's annual motion: half, from the semidiameter	Greatest parallax of the fixed stars produced by the Earth's annual motion: whole, from the diameter of the annual orb
Copernicus	indefinite	✢ ✢	✢ ✢
Galileo	13,046,400	0′ 20″	0′ 40″
Kepler, formerly	34,077,067	0′ 9″	0′ 17″
Kepler, afterwards	60,000,000	0′ 12″	0′ 24″
Lansbergen, formerly	10,312,227	0′ 30″	1′ 0″
Hortensius	10,312,227	0′ 30″	1′ 0″

Followers or supporters of the Copernican hypothesis	Distance in semidiameters of the Earth	Greatest parallax of the fixed stars produced by the Earth's annual motion: half, from the semidiameter	Greatest parallax of the fixed stars produced by the Earth's annual motion: whole, from the diameter of the annual orb
Lansbergen, afterwards	41,958,000	0′ 7″	0′ 15″
Hérigone	144,000 [recte: 1440000, per the book's errata]	0′ [?]″ [recte: 3′ 0″, per the book's errata; corrected by hand to 3′ 0″ in this copy]	6′ 0″

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DISTANTIA FIXARVM A TERRA Posito Telluris motu annuo circa Solem & Distantia Terræ à Sole Copernicæa.

The distance of the fixed stars from the Earth, assuming the Earth's annual motion around the Sun and the Copernican distance of the Earth from the Sun

From the calculation of	Semidiameters of the Earth	Total parallax of the fixed stars, etc.
Tycho and Magini	7850,000	1′ 0″
Longomontanus	7906,818	1′ 0″
Scheiner	13,133,376	0′ 40″
Or	10,320,000	0′ 48″

DISTANTIA FIXARVM A TERRA Ex noſtro calculo, poſito motu Terræ & Parallaxi Fixarum maxima 10″. & diſtantijs infraſcriptis.

The distance of the fixed stars from the Earth, from our calculation, assuming the motion of the Earth, a greatest parallax of the fixed stars of 10″, and the distances written below

Authors of the distance of ⊙ and the Earth	Distance of ⊙ and the Earth in semidiameters of the Earth	Distance of the fixed stars in semidiameters of the Earth
Copernicus	1150	47,439,800
Mästlin, Galileo	1208	49,832,416
Kepler	3469	142,746,428
Lansbergen	1498½	61,616,122
Boulliau	1460	60,227,920
Hérigone	1200	49,502.400
Wendelen	14656	604,589,312

[52] In Hérigone's row, the final 0 of the distance and the figures 3 and 0 of the half parallax are written by hand on this copy, over the print. Under the pen 3 the type is a printed 0, to which the hand has added a hook and a tail; the type under the pen 0 of the seconds was not made out. The figure 1440000, given twice in the text on p. 418, ends in a similar dark, slanted 0 run up against the next word, and has received the same hand's added 0: the type prints 144000 at both places.

Velocitas Fixarum.

The speed of the fixed stars.

[Margin: Method of deriving the speed of the fixed stars.]

XVIII. Double the greatest distance of the fixed stars from the Earth, and you will have the diameter of the sphere of the fixed stars. Multiply it by 314 and divide the sum by 100, and you will have the circumference, or perimeter, in semidiameters of the Earth, which the fixed stars traverse in one day by the diurnal motion. Divide these by 24, and you will have the number of terrestrial semidiameters that the fixed stars traverse in one hour. Divide these again by 60, and you will have the number of them that they complete in one minute. Finally divide these by 60, and you will have the number of terrestrial semidiameters that the fixed stars complete in one horary second. If you want to express this speed in miles, consult Book 2, ch. 7, and multiply these semidiameters by the miles contained in a semidiameter of the Earth.

Fixarum Velocitas in motu Primi Mobilis, expreſſa in Semidiametris terræ, quas Fixa quælibet in Æquatore poſita, & in maxima ſui à terra diſtantia percurrit.

The speed of the fixed stars in the motion of the primum mobile, expressed in semidiameters of the Earth, which any fixed star placed on the equator, and at its greatest distance from the Earth, traverses

Authors	1 hour: semidiameters of the Earth	1 minute: semidiameters of the Earth	1 second: semidiameters of the Earth
Ptolemy according to Fernel	11834	197 1/4	3 17/60
al-Battānī according to Giuntini	4972	82 5/6	1 25/60
al-Farghānī according to Maurolico	5240	87 1/3	1 27/60
al-Farghānī according to Clavius	11834	197 1/4	3 17/60
al-Farghānī according to Barozzi	10582	176 1/3	2 56/68 [recte: 2 56/60]
Tycho	3663	61	1 0
We	549500	9158 1/3	152 2/3

If you want these measures in miles or stadia, consult Book 2, ch. 7. The remaining authors either did not determine the distance of the fixed stars, or denied this motion to them.

SCHOLIA.

Scholia.

[Margin: Whether a parallax of the pole and of the fixed stars is to be feared in the hypothesis of Copernicus; and in which stars it is greater.]

I. Since the hypothesis of Copernicus admits an infinite distance of the fixed stars, there is no danger in it that any parallax be detected in the altitude of the pole, observed by means of the altitudes of stars in the same place on the Earth but at different times of the year, because in that hypothesis the meridian altitude of the fixed stars does not vary perceptibly. It should be noted, however, from Galileo, Dialogue 3 on the System of the World, that in this hypothesis the equator and its axis and poles are in the Earth itself, and that only in imagination are the poles of the equator feigned in the heaven at those points where the terrestrial axis of the equator would fall, if it were produced as far as the highest heaven. Now since in this hypothesis this axis always remains parallel to itself, and describes a cylindrical surface with an inclination of 23½° to the plane of the ecliptic, it follows: first, that wherever the Earth is in the annual orb, the elevation of the feigned pole in the heaven is always the same; second, that the parallax in the altitude of the fixed stars is indeed imperceptible, given their Copernican distance, but is nevertheless greater in stars nearer the pole of the ecliptic, smaller in those more remote, and smallest in stars of the ecliptic; third, that the parallax of the apparent size of the fixed stars is indeed imperceptible, but is nevertheless greater in stars of the ecliptic than in stars nearer the pole of the ecliptic, as Galileo notes in the same place. He adds, however, that the parallax of the fixed stars is imperceptible with customary instruments; but if instruments were used whose side contained several miles, say 50 or 60, so that each degree would be 1 mile, each minute about 50 cubits, and each second nearly 1 cubit, perhaps that parallax would become perceptible. See what we said in the preceding chapter, nos. 12 and 13.

II. Now we must show what I promised in no. 3: that from the refractions of the fixed stars neither can their distance be determined demonstratively, nor can the infinity of their distance be solidly undermined. For this purpose, let AB be a quadrant of the terrestrial meridian described from the Earth's centre C; and from the same centre, in the same plane, let there be described a quadrant DE of the atmosphere suited to refracting the rays of the stars perceptibly, and a quadrant FH of the celestial meridian in the sphere of the fixed stars, with a small appendage HV. Let the observer's eye be at A, and his sensible horizon AG, but the rational horizon CH, each of which makes right angles with the perpendicular CF, produced as far as the zenith F. This being laid down, I shall show that, even if the refraction of the fixed stars is said to be known, and the altitude of the refractive air, and the Earth's semidiameter, and the true altitude of the star, then, as long as the parallax is unknown separately from the refraction, or is supposed to be nil, the distance of the fixed stars cannot be known, nor can the infinity of their distance be convicted of falsehood by this means. I demonstrate this first *a posteriori*, from three cases that seem best suited to it, and then *a priori*.

Diagram. Set into the left column, with the text of para II running down its left side. A vertical line rises from the centre C at lower left to F at the top; a large arc, the celestial quadrant, runs from F down the right side through N,

P, K and G to H, and continues below H to V. From C a horizontal line runs to H, and another line slants down to V. A horizontal line runs from A to G. Two smaller concentric arcs stand about C, one from the region of T to B and one from the region of I and O to E. Straight lines run out from C through I towards L and through O towards M, and lines run from A towards K and towards G, from I to H, and from O to V. Two dotted lines run up and to the left from the region of I and O to R and S on the vertical line.

- F (top of the vertical line)
- N (a star on the arc, just right of F)
- P (on the arc, upper right)
- K (a star on the arc, right)
- G (a star on the arc, at the end of the horizontal from A)
- H (a star on the arc, at the end of the horizontal from C)
- V (a star at the lower end of the arc, below H)
- S, R, D (on the vertical line, in that order downward, above A)
- A (on the vertical line, the observer's eye)
- C (the centre, at the foot of the vertical line)
- T (on the small arc, just right of A)
- B, E (below the horizontal CH, on the two smaller arcs)
- I (intersection point, below L)
- L, M (above I and O, on the lines from C)
- O (intersection point, right of I and lower)

[Margin: First case.]

III. *First*, let the altitude of the pole P, the arc PH, be exactly 45°, and let a fixed star also have a declination of 45°, so that in culminating it passes through the zenith itself, F. For 12^h before or after the culmination you will be certain that it is at H, a point of the rational horizon. About that time, with the instrument now placed in the meridian line, let its apparent altitude be observed; for unless more opaque vapours obstruct, it will appear at K, by the force of a refraction of about 30′, which may become known from this or a similar observation. So the ray HI, drawn into the denser air, does not go straight to R but, refracted at the point I, is carried to the eye A, so that the refracted ray is IA, along which, produced to K, the star is seen at K; and let the angle of refraction of 30′ be RIA, to which, by proposition 15 of Book 1 of Euclid, the angle KIH is equal. For the star's apparent distance from the zenith, to the eye A, by the instrument, will be the angle FAK, 89° 30′, and therefore the angle IAC, by proposition 13 of Book 1 of Euclid, will be 90° 30′. Now through the point of incidence I, from the Earth's centre C, let the straight line CIL be drawn, so that the triangle ACI arises, in which the angle CAI, 90° 30′, is now given. Also given is the Earth's semidiameter AC, to which CT is equal, of 4139 Bolognese miles, from what was said in Book 2, ch. 7. And let TI, the altitude of the refractive air, be assumed to be at most 100 miles (for it is much less, as is clear from what was said in Book 1, ch. 31), so that the whole of CI is 4239 miles. For from these three givens you will derive, by the doctrine of

triangles, the angle ACI, 11° 58′ 48″, and AIC, that is, its vertical angle LIK, 77° 31′ 12″. This, joined to the angle KIH, 30′, makes the angle LIH, namely the angle of inclination, 78° 1′ 12″. But ICH will be found equal to this, as is clear to anyone subtracting the angle ACI, 11° 58′ 48″, from the right angle ACH. So IH and CH will be parallel, by proposition 28 of Book 1, and will not meet at H, as otherwise they ought to meet; nor will the triangle CIH be formed, in which, from the givens CI and the angles at C and I, the distance CH could be derived or demonstrated to be finite. The reason for this parallelism is that the parallactic angle H is supposed nil, and therefore is not reckoned in the angle FAK, and the apparent distance from the zenith is taken by the force of the refraction raising the star, just as if no parallax at all depressed it below H. And so that you do not think this happens from things wrongly assumed, vary what is assumed, for in any case you will find the angle LIH equal to the angle ICH. For instance, if with Kepler, p. 287 of the *Optics*, you assume the altitude TI to be only 2 Italian miles, so that the whole of CI becomes 4141 miles of which CA is 4139: then, keeping the refraction at 30′, you will find the angles ACI, 1° 20′ 58″ 30‴, and AIC, that is LIK, 88° 9′ 1″ 30‴, and hence LIH, 88° 39′ 1″ 30‴. But ICH will be equal to this, namely when ACI is taken from the right angle ACH; and so for the rest.

[Margin: 2nd case.]

IV. *Second*, let there be a star N which, as its declination has it, when it has returned to the meridian below the rational horizon and is at V, nevertheless appears, by the force of refraction, at G, a point of the physical horizon, so that its true ray VO does not go straight to S, but is refracted at O and carried to A, the refracted ray being AO, along which the star is seen at G. And through O let the straight line COM be drawn, and let there be supposed known both the straight lines AC and CO, as above, and the angle of refraction SOA, that is VOG, known from elsewhere, 30′. For in the triangle CAO, right-angled at A, one can find both the angle ACO, and when this is taken from the right angle ACH, OCH will be known; and also the angle AOC, that is MOG, which, joined with the angle of refraction GOV, will give the angle MOV; and this, subtracted from two right angles, will give the angle COV, in the triangle OCV, in which OC is supposed known. From the star's declination, moreover, its distance from the pole of the world will be known; and from the altitude of the pole, here too presupposed, the arc HV becomes known, namely by taking the altitude of the pole HP from the star's distance VP from the pole. Hence the angle HCV becomes known, which, added to OCH already found, will give the angle OCV. In the triangle OCV, then, nothing seems to be lacking for finding the distance CV of the fixed star V, since CO and the angles OCV and COV are given. Let us come to an example, and take Capella, whose northern declination in the year 1644 was 45° 34′, and hence its distance from the pole 44° 26′. Let its horizontal refraction according to Tycho be 30′, represented by the angle VOG, and at an altitude of the pole of 43° 56′ 10″ let it appear on the physical horizon, or at the point G, when its true place is at V. For when

the altitude of the pole HP is taken from the star's distance from the pole VP, there will remain the arc HV, and the angle HCV, 29′ 50″, which is to be set aside. Next, in the triangle ACO, let the side AC be 4139 miles and the base CO 4239. The angle ACO will be 12° 28′ 41″, and AOC, that is MOG, 77° 31′ 19″. So when the refraction of 30′, that is, the angle VOG, is added, the angle MOV becomes 78° 1′ 19″, and COV 101° 58′ 41″. And when ACO is taken from the right angle ACH, there remains OCH, 77° 31′ 19″; if you add to this HCV, 29′ 50″, the angle OCV becomes 78° 1′ 9″. The sum of the angles OCV and COV will then be 179° 59′ 50″, and accordingly the angle V is 10″. With this laid down, the distance CV will be derived as 87,323,400 miles of which CO is 4239, or 21097 terrestrial semidiameters. But if you had assumed with Kepler an altitude of the refractive air of only 2 miles, CV would finally come to about 20630 miles. We are certain, however, that the altitude of that air neither exceeds 100 miles nor falls short of 2 miles.

and so that the distance of the fixed stars deduced from this hypothesis does not exceed 22000 of the Earth's semidiameters, and indeed does not even reach them. But what uncertainty lies hidden in the hypothesis I shall soon reveal.

[Margin: The uncertainty of the second case.]

V. In the hypothesis of the second case proposed, it is supposed without warrant that the star's declination and the altitude of the pole were observed elsewhere independently of the parallax of the fixed stars; that the star can appear on the physical horizon itself by force of refraction, so that it can be seen in spite of the thickness of the vapours, and does not need, as usually happens, a greater altitude in order to be seen; and that its refraction, or that of another fixed star, was observed from some other source. Indeed, refraction is not known beforehand separately from parallax; otherwise we could obtain the distance at once from the parallax alone. Therefore the refraction is not exactly known either. From the star's declination, the altitude of the pole and the star's appearance on the physical horizon one can conclude that the refraction is greater than the arc HV of true depression below the rational horizon, but one cannot know how much greater it is, that is, how great is the arc HG, which is the difference of the physical horizon from the rational, and is always as great as the star's horizontal parallax. Tycho, moreover, and anyone else to whom the parallax or distance of the fixed stars was not known beforehand, could not, and will not be able to, determine their refraction from observation except by neglecting the parallax and supposing it to be nil; and on that supposition there is likewise no certain limit of a finite distance. And so in our case nothing else could be concluded from those three givens [Greek: διδόμενοις (didomenois, "things given")] than that the refraction is greater than 29′ 50″, even granting that by hypothesis it is assumed to be 30′.

[Margin: An argument from refraction against the annual motion of the Earth, but an invalid one.]

VI. Still, someone could argue *ad hominem*, as the dialecticians say, against Kepler, Lansbergen, Boulliau and others who accept the annual motion of the

Earth and yet receive the declinations and refractions of the fixed stars as exactly known, in this way. The declination of Capella in the year 1560 was 45° 26′. Therefore, at a pole altitude of 44° 4′ 10″, it had to be, at the lowest point of the meridian, below the rational horizon by the arc HV, 29′ 50″. Now if its horizontal refraction, by force of which it appears at G unless the opacity of the vapours obstructs sight, is 30′, as Kepler and Boulliau lay down with Tycho, then the arc HG, that is, the horizontal parallax, is 10″. Given this, it follows that the distance of the fixed stars is less than 22000 terrestrial semidiameters; and from this and Kepler's diameter of the annual orb there would follow a parallax of the annual orb of several degrees, whereas none is observed. But the Copernicans will deny that the arc HG is 10″, not only because the angle GOV does not measure the arc VG, but because it is feigned without warrant that a star of so great a declination appears on the horizon and on the meridian at the same time. This is also to be noted in the example set out above.

[Margin: 3rd case.]

VII. There remains the third case, in which the star, as to its true place at its lowest altitude on the meridian, is neither on the rational horizon, as in case 1, nor below it, as in case 2, but above it, say at G or some other nearer point, and by force of a known refraction appears higher, as at K; and from its declination and the altitude of the pole its true altitude is known. But in this case too, so long as the parallax of the fixed stars, or the angle K, is supposed nil, the angle LIK will be found equal to the angle ICK, and the triangle ICK cannot be formed so that the distance CK may be derived from it. Instead, IK and CK will be parallel, and an infinite distance cannot be overthrown.

[Margin: An argument for the finite distance of the fixed stars from the difference of the refractions.]

VIII. But surely a great, if not an evident, argument for the finite distance of the fixed stars, and for its proportion with the finite distance of the Moon and the Sun, seems to be the proportion of the refractions. For the horizontal refraction of the Moon is greater than the refraction of the Sun by one or two minutes, granted that Tycho (*Progymnasmata*, vol. 1, pp. 79 and 124) makes the Moon's refraction at one time greater, at another smaller, than the Sun's, as Kepler also does in the Rudolphine Tables, p. 115, while Lansbergen, p. 46, makes them equal. But the refractions of the fixed stars are, for Tycho (p. 280) and for Kepler (p. 124), smaller than the solar ones by 4½′. Now it is certain that where the inclination of a ray from the perpendicular is smaller, the refraction is smaller; and it is likewise certain that, other things being equal, the inclination of a ray from a star more remote from the Earth is smaller, and hence its refraction smaller, than that of a nearer star, as the more expert writers on optics concede, and as we shall show in the Optical Book, which is the third of volume 2 of the *New Almagest*. Therefore, just as the difference of one or two minutes in the refractions of ☾ and ☉ arises from the difference of distance, a difference that is great but nevertheless finite and known within definite limits, so the difference of 4½′ between the refractions of the Sun and

of the fixed stars seems to arise from a difference of distance that is, however great, in the end finite. But see above, no. 17, why I rejected the distance of the fixed stars badly deduced from this proportion.

CAPVT VIII.

Chapter 8.

De Refractionibus Siderum Inuestigandis earumque vſu.

On investigating the refractions of the stars, and on their use.

I. By all those methods by which we taught the refractions of the Sun to be observed, in Book 3, chapter 9, from number 2 to number 5, the refractions of the fixed stars can be observed more easily, since there is no need to wait, as there, for the solstitial days, and it suffices that the altitude of the pole and the star's declination be known beforehand. But for stars to appear on the physical horizon itself, the horizon must be clear of the thicker and more opaque vapours; otherwise the stars will not appear unless they have risen by several degrees. Beyond the methods just mentioned there are now three others, indicated in the Scholia of the preceding chapter, and the figure set out there is to be taken up again from them.

[Margin: First method of observing the refractions of the stars.]

II. *First*, then, where HP, the altitude of the pole P, is exactly 45°, and the declination of some fixed star, such as F, is likewise 45°, so that its distance from the pole, namely the arc FP, or HP, is 45°, you will be certain that when it has reached the bottom of the meridian FPH it will be at H, a point of the rational or astronomical horizon. So observe from A, with an instrument, its apparent distance from the zenith, namely the arc FK, by means of the angle FAK; for it will most certainly appear above the physical horizon AG, unless the thickness or opacity of the vapours hinders the view of it. For the difference of the apparent distance from a whole quadrant, that is, the amount by which it falls short of 90°, will be the refraction sought, and it will be as great as the arc HK, on the supposition, as is supposed for the fixed stars, that the arc GH and their parallax are nil or insensible. But it will rarely happen that these three things coincide, namely the altitude of the pole and the declination of the star exactly equal, and a clear horizon.

[Margin: 2nd method.]

III. *Second*, if the distance of some fixed star from the pole is greater than the altitude of the pole, but does not exceed it by more than 28′ or 30′, it will be able to appear above the physical horizon at its lowest depression, as at K. Let there be, then, a star N of this kind, whose distance from the pole, namely the arc NP, is known from its declination and the altitude of the pole; and let the altitude of the pole HP be, for example, as at Bologna, 44° 29′ 30″, and the distance of the star from the pole 44° 59′; and on a night longer than 12 hours let the star culminate in the evening at N, for in the morning, 12 hours

later, it will be at the lowest point of the meridian, V. Or if it culminates in the morning, it will be at the lowest point V after sunset, and it will not be difficult to know the time of the lowest depression. At that time, then, observe its apparent distance from the zenith, namely the angle FAK; subtract this from 90°, and to the remainder add the arc HV, that is, the star's true depression below the rational horizon, which becomes known by subtracting the altitude of the pole HP from the star's true distance from the pole, which is the arc VP. For the sum of the arc HV, or the angle HCV, and the remainder of the apparent distance from the zenith will be the star's refraction, its parallax being neglected here too as insensible.

[Margin: 3rd method.]

IV. *Third*, choose a star whose distance from the pole is less than the altitude of the pole, but by as few minutes as possible, so that at its lowest meridian depression it is above both horizons, and then observe its apparent altitude; for its excess over the true altitude will be the refraction sought. Its true altitude becomes known by subtracting its distance from the pole, or the complement of the declination, from the altitude of the pole. Such a star was Capella in the year 1644 at Bologna: its declination was 45° 34′ and the complement of the declination 44° 26′. When this was taken from the pole altitude of 44° 29′ 30″, there remained its true altitude above the horizon, 3′ 30″. With its apparent altitude set at 32′ 30″, the refraction was 29′, which belongs to fixed stars at a true altitude of 3′ 30″.

[Margin: The use of refractions.]

V. The use of the refraction of the fixed stars is frequent and necessary for correcting their altitudes, by subtracting, that is, the refraction from the apparent altitude so that the true altitude remains, but chiefly in observing the declinations of those fixed stars which at their highest meridian altitude are subject to a sensible refraction. Of this kind are undoubtedly Sirius and Antares on the horizon of Denmark; and then in correcting the apparent distances of the minor planets from fixed stars nearer the horizon, when other stars cannot be used, so that the places of the planets may be observed by means of the fixed stars. How this is to be done we shall say in Book 7. Meanwhile, from many observations of ours, no necessity is established for departing sensibly from the refractions which Tycho observed in Denmark, although some of them have indicated some difference. So, before we weigh many other observations, which are to be brought forward in volume 3 of the *Almagest*, we shall use the little table of refractions that Tycho provided in *Progymnasmata*, vol. 1, p. 280.

CAPVT IX.

Chapter 9.

De Apparenti & Vera Magnitudine Fixarum.

On the apparent and true magnitude of the fixed stars.

I. We are not inquiring in this place into how many classes the apparent magnitudes of the fixed stars are divided, and which stars are of the first magnitude, which of the second, and so on, for enough has been said of this in chapter 6. We ask rather how many minutes and seconds in the heavens their apparent diameter contains, whose measurement and observation are very difficult, because of the stars' apparent smallness and the scintillation of their rays. Yet both the attempts of others and our own must be set out. But first, Reader, you must be warned that stars appear larger than usual, or than others, for six reasons.

[Margin: The causes for which stars appear larger or smaller than they should.]

First, on the horizon, because of a double refraction or because of a deception of the imagination. *Second*, when the sky has cleared after rains, or when east winds are blowing, as Aristotle noted in the *Meteorologica*, Book 3, t. 18, and in the *Problems*, section 26, q. 54, and from him Kepler in the *Optics*, from p. 133. *Third*, because of a white colour: thus Sirius, because it is whiter than Arcturus, appears far larger than Arcturus to the naked eye, through the predominance of the image of whiteness in the eye, than when both are observed with a telescope, as Scheiner too noted in the *Disquisitiones Mathematicae*. *Fourth*, because of weakness of sight: thus the same Sirius seemed to Kepler equal to the Moon, when the rays glittering around it were counted in, as he himself admits in the *Dissertatio cum Nuncio Sidereo*, p. 9. *Fifth*, they vary because of the curls and hair of the rays, by which the disc of the stars seems to be enlarged, whether from innate scintillation, or from the vaporous sphere, or from some other source. *Sixth*, in the case of the planets, because of their approach to and withdrawal from the Earth. For the contrary reasons, or because of the nearness of a greater brightness, they appear smaller, as do stars viewed at dawn or near the full Moon. With this laid down, we must give the methods of observing the diameters of the fixed stars and of the minor planets, next of deriving from them the true magnitude, and finally the opinions on both.

[Margin: 1st method of observing the diameters of stars.]

II. The *first* method of observing the diameters of stars rests on estimation by eye and on comparison either with the Moon's diameter or, lest its brightness interfere, with the distance from one another of the Pleiades, the Hyades or the Kids, otherwise known: one considers, that is, how many such stars would be required to fill that interval. This conjecture is undoubtedly exposed to error, especially because of the extraneous rays that enlarge the stars and diminish the intervals, so much so that the star in the middle of the tail of Ursa Major seems a single star, though there are in fact two, as the telescope has disclosed. For that reason Galileo, in Dialogue 3 of the *De Mundi systemate*, judges it better that diameters be observed in luminous surroundings, near the Sun or the Moon, because they are then stripped of those spurious rays, as appears in Venus seen by day and in the little flames of lamps seen from afar by day; and I have quite often found this by experience at full Moons.

[Margin: 2nd method.]

The *second* method, which al-Battānī indicates in chapter 50, is by two apertures in dioptras, of which the one to which the eye is applied is very narrow, like a point, while the other, remote from the eye, is somewhat wider, but exactly as wide as the apparent diameter requires. For from the distance of the dioptras, and from the semidiameter of the wider aperture, known in the same parts, the angle subtending the semidiameter of the star is derived: namely, if one makes, as the distance of the dioptras is to the radius, so the semidiameter of the wider aperture to the tangent of the angle already mentioned; or if the logarithm of the semidiameter of the aperture and the residual of the logarithm of the distance are added together, so that the sum is the mesologarithm of the angle already mentioned. Besides the fact, however, that the visual angle is formed not at the smaller aperture but in the middle or at the back of the eye, the distance of the dioptras can be varied by many spans and yet the other aperture be seen to match exactly the diameter of the star, as I said in a similar case about Hipparchus's dioptra in Book 3, chapter 10, number 4.

[Margin: 3rd method.]

III. A *third* method is given by Galileo in Dialogue 3 of the *De systemate Cosmico*, though he admits that it is not wholly exact. Hang up, he says, a thread opposite some star (he himself used the bright star of Lyra), and by moving towards and away from it seek the position in which the thickness of the cord exactly covers the star; and measure the distance of that position from the cord in very small parts, say in hundredths of the inches of a known foot. Next wind the cord around a plate an inch wide, so that you may know how many cords exactly cover one inch: for from this you will derive what fraction of an inch matches the thickness of the thread, and half of it will be the semidiameter of the thread. With this and with the distance just mentioned, as the sides about a right angle, you will find the angle subtending the star's apparent semidiameter. But Galileo adds that this distance must be somewhat increased, because the meeting of the rays that forms the angle occurs, on account of refractions, not in the eye but beyond the eye; and he gives a method of finding this meeting point, of which we shall speak in the Optical Book, comparing it with the method of Archimedes and of Kepler. For I do not wish to be needlessly longer here, since this method is subject to the same uncertainty and fallacy as the second, and you can move several feet towards and away from the thread with no sensible difference as regards covering the star exactly. What is the point, then, of that scruple over a little addition to the distance on account of the meeting of the rays beyond the eye, when we are uncertain of the proper distance between the thread and the eye, and the suspicion of an error of anything short of whole feet makes this practice uncertain? I have found, however, that it is carried out somewhat more perfectly if the thread is not fibrous and is lit by a rather near small lamp, whose glare must nevertheless not strike your eyes. Now Galileo asserts there that the apparent diameter of the bright star of Lyra, which Tycho estimated at no less than 2′, was found by him in this way to be not greater

than 5″, and from this he accuses Tycho and others of a notable error, arising, as he thinks, from the rays spread about the discs of the stars in the manner of a curl, which, he says, the thread in covering the star takes away not from the star but from the eye, to which they are proper. But in truth Galileo's observation is suspect to me, both because of the fallacy already explained and because he is wholly bent on building up the probability of the Copernican hypothesis; and lest an incredible bulk of the fixed stars should follow from it on account of their insane distance, he studiously diminishes their diameters, or more willingly embraces observations that favour that diminution.

[Margin: 4th method.]

IV. A *fourth* method is estimation by eye, but with the stars viewed through a fine telescope, for this shows the discs of the stars shorn of the hair of adventitious rays. Galileo tells us that he used this method, in Dialogue 3 of the *De systemate Cosmico* and in the *Sidereus Nuncius*, where, on p. 16, he rightly advises that stars viewed through the spyglass [Perspicillo] are by no means enlarged in the same proportion as other objects; for if other objects are enlarged a hundredfold, the stars are enlarged barely fourfold. The cause, he says, is that before the use of the spyglass we were accustomed to view the stars radiant and hairy with certain brightnesses, especially as night deepens, and the visual angle is then bounded not by the star's little body alone but also by the splendour poured around it. Since, then, in darkness and to the naked eye the stars, unshorn, are seen so large, it is no wonder if through the telescope they are not seen so large, since it enlarges only the disc of the stars and not those extraneous rays, which are in reality not in the object but in our eye. He admits, however, that the planets do appear exactly round by means of the telescope, except when Venus appears horned; but that the fixed stars are by no means shown bounded by a circular periphery, but as flashes, darting rays all around, though now their own, yet so that their disc appears much larger than to the naked eye, so much so that a little star of the fifth or sixth magnitude seems to equal Sirius (if, however, you place over the lens farther from the eye a thin plate perforated in the middle with a small hole, you will see them round). Kepler subscribes to this in the *Dissertatio cum Nuncio Sidereo*, p. 9, as do Scheiner and Hortensius, as regards this effect of the telescope in shearing off the hair of adventitious rays. And Kepler adds, in the *Epitome of Copernican Astronomy*, p. 498, that fixed stars viewed through more perfect telescopes look like radiant points.

[Margin: The diligence of Martinus Hortensius in exploring the capacity of the tube.]

Martinus Hortensius, as he himself reports in his dissertation with Gassendi, explored the capacity of his telescope in order to estimate the diameters of the stars better, by the method treated in Book 7, section 6, chapter 10, scholium 4. Comparing other distances with this capacity, he found two contiguous stars in Capricorn to be distant from each other by one eighth part of the tube, that is, 5′ or 5½′; the contiguous stars of the Hyades about 5′ or 4½′; and two of the Pleiades 31′. Comparing the discs of the planets and fixed stars, viewed through

the tube, with these distances, he estimated their diameters. We, for our part, found the capacity of our tube to be 30′, because it took in the Moon from its eastern limb as far as the middle of the Caspian spot when the Moon's diameter was 33′.

[Margin: 5th method.]

V. The *fifth* method, used by the same Hortensius on Jupiter, by Scheiner on Venus and by Gassendi on Mercury, is by receiving the bright image of these planets, passed through the lenses of the telescope, onto very clean paper; by measuring the diameter of that image, or, if Mercury is under the Sun, of its little shadow; and by comparing it with the Sun's diameter, taken in the same way. But Hortensius admits that the edges of the image of Jupiter appear somewhat fainter. Further, with Jupiter's diameter now obtained, Hortensius compared the diameters of the fixed stars viewed directly through the tube, and found Sirius six times smaller than Jupiter.

[Margin: The danger of deception in this method.]

But this estimate, like the previous one, is exposed to the danger of error unless you very carefully distinguish the magnitude of the circle viewed from the magnitude of the diameter alone. For if Sirius, for example, appears eight times smaller than Jupiter as to its whole disc, it is easy for us to estimate its diameter also as eight times smaller than Jupiter's, when in such a case it would be only twice smaller; for circles are to one another as the squares of their diameters, by proposition 2 of the twelfth book of the *Elements*. So if the diameter of Sirius is 2 and that of Jupiter 5⅘, Sirius would appear eight times smaller than Jupiter, that is, as many times as 4, the square of 2, goes into the number 32, which is approximately the square of the number 5⅘.

[Margin: 6th method.]

VI. The *sixth* method, our own, is by radii made of black plates, these being fixed at right angles to a small beam of 15 Roman feet, which turns up and down and crosswise, as is done in the supports of telescopes. The first radius, which I call triangular, is a black plate ABC, formed into an isosceles triangle, but much narrower than in this diagram. Its base AC was of 11 particles, of which an inch of the Roman foot contains 200, and so it subtended an angle of 2′ when viewed from a distance of 15 feet, as is established from trigonometry. The little white line DE, marked on the black of the plate, was of 9 particles, so as to subtend an angle of 1′ 40″; the little line FG, of 7 particles, subtending 1′ 20″; the little line HK, of 6 parts, subtending 1′ 0″; and the little line LM, of 4 particles, subtending 40″. In the same way the plate NPO of the other, stepped radius [scalaris] was cut so that QR contained 11 parts, ST 9, and so on. Now, with the plates already mentioned turned towards the stars, and by raising or lowering the small beam, you will observe by which of their breadths they exactly cover the stars: whether by the first, AC or QR, or by the second, DE or ST, and so on for the others, or by a breadth intermediate between these. In this way you will observe that the apparent diameters of all the fixed stars, and also of the superior planets, are covered by a portion of the plate DE, and hence

that the apparent diameter of none of these exceeds 1′ 40″; and that by the breadth HK no planet of the three superior ones, placed at perigee, is wholly covered. But a lamp is needed at the edges of the plates, whose splendour, however, must not strike the eyes of the observer.

Diagram. Two solid black woodcut figures side by side, set into the left column with the text wrapped at their left. On the left, a tall narrow isosceles triangle, the triangular radius, crossed by four thin white horizontal lines. On the right, a stepped pyramid of black, the stepped radius, narrowing upward through a series of steps to a narrow column at the top.

- Triangle, apex: B
- Triangle, foot of base, left and right: A, C
- Triangle, white lines from the base upward, left and right ends: D, E; F, G; H, K; L, M
- Stepped figure, top: P
- Stepped figure, foot, left and right: N, O
- Stepped figure, steps from the foot upward, left and right ends: Q, R; S, T; V, X; Y, Z

[Margin: 7th method, the best of all.]

VII. The *seventh* method, which first came to the mind of Father Francesco Maria Grimaldi S.J., on a truly happy occasion, and was used by Don Vicenç Mut, who also devised it by his own ingenuity at quite the same time, is by the apparent diurnal motion of Jupiter and Saturn, and by the distance of its limb from a fixed star, estimated with the telescope, some hours, or even a day, immediately before and after its conjunction with that star, such that both the planet and the fixed star fall within the capacity of the tube. But since this method is proper to these two planets only, and must be explained by use itself, we shall set it out more fully in Book 7, section 6, chapter 10, and illustrate it with examples in the same place, nos. 2 and 9.

[Margin: 8th method, the best after the 7th.]

VIII. The *eighth* method, the best after the seventh, and presupposing either it or the fourth, is carried out by drawing on paper several little circles of different sizes, viewing the stars through the telescope, and at once looking back repeatedly at those circles, until it is evidently established which of them most nearly matches the disc of the star viewed. When this has been done, first with Jupiter or Saturn, whose diametral size will already have been established by method 7 or 4, and then with other stars, the diameter of Jupiter or Saturn is divided into very tiny equal particles, a division that comes out clearer if it is made by transversal lines. Finally, with a very sharp pair of compasses taking in the diameter of the circle that matches the star's disc, one investigates what proportion it has to the diameter of Jupiter or Saturn.

[Margin: Why the observations of the diameter of the fixed stars are deferred to Book 7.]

We, therefore, have established the apparent diameters of the planets, and next of the fixed stars, chiefly by methods 4, 7 and 8. Their observations,

because of the dependence which the fixed stars have in this on the planets, we shall present in Book 7, section 6, chapter 11; and it will be enough here, taking those as supposed, to give in this place the tables of the apparent and true magnitude of the fixed stars.

Problema. Data diſtantia & ſemidiametro apparenti Stellæ, Veram eius Magnitudinem inquirere.

Problem. Given the distance and apparent semidiameter of a star, to find its true magnitude.

IX. With the apparent diameter known, and the distance of the fixed stars assumed from what was said in chapter 7, the true magnitude is derived by Problem 1 of chapter 11 of Book 3. For let the star be HDFE, the eye on the Earth G, the straight line passing through the centre of the eye and of the star HBG, and let the portion of the heavens in which the star's centre lies be ABC; and let two rays, GD and GE, be drawn touching the globe of the star at D and E. For by proposition 18 of the third book of Euclid they will make a right angle at the contact with the semidiameters BE or BD. Therefore, in the triangle GBD, right-angled at D, given the angle DGB, subtending the star's apparent semidiameter, and the distance BG as base, the true semidiameter BD will become known in parts of which GB was given. The semidiameter doubled will give the diameter; and when this is multiplied by 314 and the product divided by 100, you will obtain the circumference, according to what was said in Book 1, chapter 4. Again, if the distance BG, and hence the star's diameter, is known in diameters of the Earth, you will easily find the volume of the star, since spheres are in the triplicate ratio of their diameters, from what was said in Book 1, chapter 6.

Diagram. A woodcut set at the right of the right column, with the text wrapped at its left. A circle, the star's globe, stands above a point on the Earth. A vertical straight line runs from the top of the circle through its centre and down to the point below; two straight lines rise from that point to touch the circle on either side, and a curved line crosses the circle through its centre and runs out on both sides.

- Top of the circle: H
- Centre of the circle: B
- Left and right on the circle, where the tangents touch: D, E
- Bottom of the circle: F
- Curved line, far left and far right ends: A, C
- Point below, where the tangents meet: G

X. Before I cast into tables the opinions on the apparent and true magnitude of the fixed stars, the authors from whom we have taken them must be indicated. Among them there is a remarkable diversity, arising partly from the varying distance of the fixed stars, on which see the table in chapter 7, and partly from differing estimates of the diameter of the stars, made by Tycho and the older

authors with the naked eye, and by other, more recent ones with the telescope; nor all of them the apparent, but only the true magnitude do they report.

[Margin: Authors on the true or the apparent diameter of the fixed stars.]

These authors are as follows. Al-Battānī, chapter 50. Al-Farghānī, difference 22. Maurolico, at the end of the dialogues of his *Cosmographia*, from al-Farghānī. Fernel, *Cosmotheoria*, Book 1, chapter 11. Clavius, in his *Sphere*, p. 187, from Maurolico. Barozzi, *Cosmographia*, Book 1, at the end, from Clavius and Maurolico, though content with round numbers. Magini, *Theoricae Planetarum*, Book 1, chapter 19, and in the *Isagoge* to his *Ephemerides*, p. 19. Tycho, *Progymnasmata*, vol. 1, p. 481, and from him Biancani, at the end of his *Sphere*, and Bayer, *Uranometria*, table 31, and from the same source Hérigone, *Cursus Mathematicus*, vol. 5, Book 2 of the *Theorica*, p. 619, although in vol. 4, in the *Sphere*, chapter 4, he reports the opinion of Hortensius. Galileo, *Sidereus Nuncius*, p. 16, and dialogue 3 on the Cosmic System. Kepler, in the book *De Stella Nova*, chapters 16 and 21, before the use of the telescope, and in the *Epitome of Copernican Astronomy*, p. 484. Longomontanus, *Theorica*, Book 1, chapter 1. Lansbergen, *Uranometria*, Book 3, from element 20 to 31. Martinus Hortensius, in his dissertation with Gassendi on Mercury seen under the Sun; with him Gassendi, *Institutio Astronomica*, Book 3, chapter 11. Baranzano, *Uranoscopia*, part 2, from p. 114 to 119, reporting the tenets of the ancients and of Tycho. For our own opinion, see the end of the tables that follow.

[Margin: Hortensius, however, assigns 10″ to Sirius. Galileo gives 5″ to the bright star of Lyra.]

I. TABVLA. DIAMETER APPARENS FIXARVM.

Table I. Apparent diameter of the fixed stars.

Magnitude classes / Authors	First	Second	Third	Fourth	Fifth	Sixth
Magini	10′ 0″	5′ 30″	4′ 0″	3′ 0″	2′ 0″	1′ 0″
Tycho, Longomontanus, Biancani	2′ 0″	1′ 30″	1′ 5″	0′ 45″	0′ 30″	0′ 20″
Lansbergen	1′ 0″	0′ 40″	0′ 30″	0′ 20″	0′ 10″	0′ 5″
Hortensius	0′ 8″	0′ 6″	0′ 5″	0′ 4″	0′ 3″	0′ 2″
Kepler, formerly	3′ 0″	2′ 0″				

[Margin: The first three from al-Farghānī.]

II. TABVLA. VERA DIAMETER FIXARVM Terræ diametrum continet vicibus

Table II. True diameter of the fixed stars: how many times it contains the Earth's diameter.

Magnitude classes	First	Second	Third	Fourth	Fifth	Sixth
Maurolico & Clavius	4 3/4	4 20/90	4 1/6	3 4/5	3 11/38	2 5/8
Fernel	4 3/4	4 1/2	4 7/60	3 3/4	3 1/4	2 7/12
Tycho	4 1/13	3 1/18	2 1/5	1 19/36	1 1/49	0 2/5
Lansbergen	40712	27132	20356	13580	6776	3388

III. TABVLA. SOLIDITAS FIXARVM AD TERRAM.

Table III. Volume of the fixed stars in relation to the Earth.

Magnitude classes	First: As	To	Second: As	To	Third: As	To	Fourth: As	To	Fifth: As	To	Sixth: As	To
al-Farghānī	100	1	90	1	72	1	54	1	36	1	18	1
Fernel, Maurolico, Clavius	107	1	90	1	72	1	55	1	36	1	18	1
Tycho, Bayer, Biancani	68	1	28 1/2	1	11	1	4 1/2	1	1 1/18	1	0 1/3	1

IV. TABVLA. MAGNITVDO APPARENS FIXARVM EX NOSTRA OBSERVATIONE Cum Vera Magnitudine deducta ex infrascriptis distantijs Fixarum à Terra.

Table IV. Apparent magnitude of the fixed stars from our own observation, with the true magnitude derived from the distances of the fixed stars from the Earth written below.

Order of magnitude	Names of the fixed stars	Apparent diameter	Our greatest distance of the smallest stars, 210000 semidiameters of the Earth, from chapter 7, method 5: true diameter has diameters of the Earth	Same distance: body contains the Earth, times	Our least distance of the smallest stars, 100000 semidiameters of the Earth, from chapter 7, method 4: true diameter has diameters of the Earth	Same distance: body contains the Earth, times	Greatest distance of the Ptolemaics, 40000 semidiameters of the Earth: true diameter has diameters of the Earth	Same distance: body contains the Earth, times	Tycho's greatest distance, 14000 semidiameters of the Earth: true diameter has diameters of the Earth	Same distance: body contains the Earth, times
1	Sirius	18″ 0‴	17 45/100	5355	8 7/10	815	3 1/2	42	0 61/100	0 1/5
1	Arcturus	16″ 42‴	16 2/5	3932	8 0	512	3 1/5	32	0 56/100	0 1/7
1	Aldebaram	15″ 24‴	14 1/5	2810	7 3/5	402	3 0	27	0 52/100	0 1/8
1	Spica	15″ 5‴	13 9/10	2660	7 1/3	374	2 7/10	18	0 47/100	0 1/10
1	Regulus	14″ 5‴	13 3/5	2202	6 4/5	249	2 6/1[?]	16	0 43/100	0 1/13
1	Regel [Rigel]	13″ 40‴	13 0	2197	6 2/5	220	2 1/2	15 1/2	0 41/100	0 1/16
2	Procyon	12″ 20‴	12 0	1728	6 0	216	2 3/10	12	0 39/100	0 1/17
2	Aquila	11″ 0‴	10 3/5	1120	5 7/25	137	2 1/10	8 2/5	0 37/100	0 1/20
2	Polaris	7″ 53‴	7 3/5	402	3 9/10	62	1 55/100	3 1/4	0 27/100	0 1/50

Order of magnitude	Names of the fixed stars	Apparent diameter	Our greatest distance of the smallest stars, 210000 semidiameters of the Earth, from chapter 7, method 5: true diameter has diameters of the Earth	Same distance: body contains the Earth, times	Our least distance of the smallest stars, 100000 semidiameters of the Earth, from chapter 7, method 4: true diameter has diameters of the Earth	Same distance: body contains the Earth, times	Greatest distance of the Ptolemaics, 40000 semidiameters of the Earth: true diameter has diameters of the Earth	Same distance: body contains the Earth, times	Tycho's greatest distance, 14000 semidiameters of the Earth: true diameter has diameters of the Earth	Same distance: body contains the Earth, times
3	Algol	7″ 3‴	6 3/5	260	3 2/5	34	1 35/100	2 1/5	0 24/100	0 7/500
4	Propus	6″ 10‴	6 0	216	2 9/10	26	1 15/100	1 1/2	0 20/100	0 4/500
5	Pleias	5″ 16‴	4 4/5	92	2 7/10	18	1 8/100	1 1/4	0 18/100	0 3/500
6	Alcor	4″ 24‴	4 0	64	2 15/100	9	0 86/100	0 7/10	0 15/100	0 3/1000

If you want the magnitude of the fixed stars for a distance of the fixed stars asserted, or to be asserted, by the Copernicans, consult the tables below, Book 7, Section 6, chapter 11, at the end.

SCHOLIA.

Scholia.

I. Galileo, in dialogue 3 on the Cosmic System, says more than once that the fixed stars are so many Suns, of the same kind as our Sun. Giordano Bruno also held this, in the book *De maximo & immenso*, and so did Kepler, in the *Dissertatio cum Nuncio Sidereo*, p. 9. How true this is becomes clear from their bulk, treated in the preceding small tables, when it is compared with the bulk of the Sun, treated in Book 3, chapter 11; for the Sun, at least in our opinion, is much larger than any fixed star.

II. Now if you should ask how many stars of the size of Arcturus or Aldebaram, for example, could fill the hollow surface of the firmament while touching one another, you will obtain it as follows. Make, as the diameter of a fixed star is to 1, so the diameter of the firmament, that is, the doubled distance of the fixed stars, to another number; and you will have the number of stars contained in the diameter of the firmament. Next make, as 100 is to 314, so the number of stars of the diameter of the firmament to another number, and you will have the number of stars contained in the circumference of the firmament. Multiply this by the stars of the diameter of the firmament, and you will have the number of stars filling the spherical surface of the firmament, according to the rules given in Book 1, chapters 4 and 6. So, if on our opinion you assume stars such as Arcturus, and a distance of 100000, the diameter of the firmament will contain

2273 Arcturuses, the circumference 7137, and the whole concavity 16,222,401. But Clavius, in his *Sphere*, p. 189, using a diameter of the firmament of 22612 1/2 diameters of the Earth, and assigning with Maurolico 4 3/4 diameters of the Earth to stars of the first magnitude, gathers 4760 stars in the diameter of the firmament, 14960 in the circumference, and 71,209,600 on the concave surface.

[Margin: To whom the infinite bulk of the stars?]

III. Giordano Bruno, in *De Infinito & innumerabilibus*, asserts that the fixed stars are infinite in number. Kepler rises against him in the book *De Stella Nova*, chapter 21, inferring from this that any one of them is of infinite magnitude, and yet, because they are shaped and bounded on all sides, that they are finite, and so their infinite distance cannot stand. But it is not our task at this point to settle this controversy about the infinite, and we have indicated our opinion sufficiently in physics.

[Margin: The magnitude of the fixed stars on the Copernican hypothesis.]

IV. Among the arguments with which the Copernican hypothesis, requiring an enormous distance of the fixed stars, is usually attacked is the insane bulk of the fixed stars that follows from it. For thus Longomontanus, *Theorica*, Book 1, chapter 1, assigning to the Copernican hypothesis a distance of the fixed stars of 7,906,818 semidiameters of the Earth, and to fixed stars of the first magnitude 2′ of apparent diameter, teaches that one star of the first magnitude would be so large as to contain the Earth 12,167,000,000 times, and the Sun at least 86,907,143 times. But Scheiner, in the *Disquisitiones Mathematicae*, p. 26, from the distance of the fixed stars on the Copernican hypothesis, which he thinks is 13,133,376, and from an apparent diameter of fixed stars of the first magnitude of 2′, has the diameter of any such fixed star containing 7972 semidiameters of the Earth, and the diameter of the great orb more than 3 times, and the volume of the great orb more than 33 times. But more on this argument in Book 9. Meanwhile I say that such a bulk does not follow from our distance of the fixed stars, but with our diameters of the stars posited; indeed, with those very diameters diminished, as Galileo and Hortensius do, it does not follow. Now if anyone wishes to know the ratio of the great orb to any fixed star, let him consult Hortensius in the dissertation with Gassendi, and Gassendi himself in the *Institutio Astronomica*, Book 3, chapter 11. From these, take the small table set below.

STAR of magnitude	Smaller than the great orb according to Lansbergen, in Hortensius	Smaller than the great orb according to Hortensius	But for Lansbergen himself it was once larger
Sirius	☩	217	
1	6 1/3	422	20053
2	15	1012	5935
3	25	1725	2506
4	50	3385	144
5	122	8245	92
6	412	27826	11 1/2

STAR of magnitude	Smaller than the great orb according to Lansbergen, in Hortensius	Smaller than the great orb according to Hortensius	But for Lansbergen himself it was once larger
	times	times	times

CAPVT X.

Chapter 10.

De Diſtantijs Fixarum Stellarum inter ſeſe, quarum multæ à Tychone, plures à nobis, paucæ à Metio, aut Naucleris recentioribus obſeruatæ, & nonnullæ calculo inuentæ ab Herigonio vel alijs.

On the distances of the fixed stars from one another, of which many were observed by Tycho, more by us, a few by Metius or by recent navigators, and some found by calculation by Hérigone or others.

[Margin: Why they are called fixed stars.]

I. It is now admitted among astronomers that all the non-wandering stars keep the same distance from one another perpetually, and that saying of Ptolemy's in *Almagest*, Book 7, chapter 1, is well known: *Before all else, then, it must be asserted that this name rightly suits these stars, so that they are called non-wandering, because they are always seen to preserve both similar alignments and figures and equal distances from one another.*

[Margin: The constancy of the fixed stars in the immutability of their distance.]

Ptolemy shows this in the same place, from the fact that in his own time they stood in the same position relative to one another in which they had been seen by Hipparchus, and indeed by Aristyllus and Timocharis, that is, over more than 400 years. Nor did Hipparchus hold the opposite. For although, on account of the few observations before him, which Ptolemy above calls *insufficiently explained and ambiguous*, he suspected that the stars of the zodiac alone among themselves, and the rest among themselves, but not with the stars of the zodiac, keep the same distance, he afterwards followed a different hypothesis, as is gathered from the same chapter 1 of Ptolemy. But this truth became evident much more clearly when, after so many centuries, they were found in the same position, figure and distance that is deduced from the tables and delineations not only of Aratus and Hyginus but also of Hipparchus and Ptolemy, by Tycho, as he himself attests on p. 43 of the *Epistolae Astronomicae* and in *Progymnasmata*, vol. 1, p. 233, and by us and others who have been seized by some care and ardour for observing the sky. But so that we do not leave posterity without any specimen of this matter, we will list some stars that are now in the same line, or very nearly so, as they once were in the age of

Hipparchus and Ptolemy, so that a similar judgement may be made about the others. Then we will set out a Table of Distances for the more notable stars.

[Margin: Examples of the constancy already mentioned.]

In the same line, then, or very nearly the same, are the 2 more northerly stars of the head of Aries, the bright star on the knee of Perseus, and Capella. Likewise the leading shoulder of Aquarius, the brighter of the 2 on the neck of the Horse, and the Head of Andromeda. Likewise the bright star on the breast of the Horse, the one at the gape of the Horse, the one at the gape of the Southern Fish, and the bright star on the shoulders of the Horse. Likewise the Goat, the one on the northern horn of Taurus, and the leading shoulder of Orion. Likewise the 3 in the belt of Orion. Likewise the bright star on the neck of Hydra and the 2 bright stars on the heads of Gemini. Likewise Spica and the 2 on the wings of Corvus. Likewise the middle star of the tail of Ursa Major, Arcturus, and the bright southern star on the forehead of Scorpius; and so for many others that Ptolemy lists.

[Margin: On the distances of the fixed stars observed by us.]

II. I come now to the distances observed by us. When I considered, then, that these were to be preferred to all observations of the fixed stars, inasmuch as they are perpetual and, once accurately determined, could always serve for deducing more exactly the positions of the planets through the Problems to be given in Books 7 and 10, I selected 70 more notable stars, that is, all those of the first and second magnitude and some others more worthy of note that can be viewed on our horizon, especially beyond the hazard of sensible refractions (for the distance of very few, seen within the refractions, had to be converted into the true one by Problem 8, to be given in chapter 11). These I observed not merely once or twice but repeatedly, with 2 different large sextants, and sometimes with a quadrant or a hemicycle, together with my most diligent colleague, highly practised in this, Father Francesco Maria Grimaldi, over 4 years, with incredible patience, so that the distances of stars of the first and second magnitude were repeated 10 times, 12 times, and some more often, in a very convenient position. Any that we observed only once or twice, or in a less favourable position, I have distinguished from the others by adding an asterisk, and I do not consider them so certain that an error of about 2′ or 3′ could not underlie them. I used not these but others in deducing, along with the declination, the equatorial distance; just as I also did not use distances taken in the past under a sky that was not quite clear. For example, Sirius and Procyon, amid reddish half-vapours, seemed to be 25° 39′ or 40′ apart, but 25° 42′ when the sky was thoroughly clear. Likewise Spica, when lower, appeared to be 34° 58′ from the Tail of Leo, whereas when higher it was 35° 2′ distant. This difference arose from the variation of refraction.

[Margin: On the Tychonic distances.]

III. As for the Tychonic distances collected from Tycho's *Epistolae Astronomicae*, p. 50, and *Progymnasmata*, vol. 1, chapter 2, no one should be surprised if they sometimes disagree with ours, especially in stars lower on the Danish

horizon, since Tycho himself was sometimes inconsistent on this. For the heel of Gemini is distant from Regulus, for Tycho, now 54° 35′, in the *Epistolae*, p. 50, now 54° 33′ 45″, in the *Progymnasmata*, p. 198. Moreover, as Willebrord Snel relates in the observations, Tycho saw Spica distant from Regulus by 54° 2′ at Uraniborg, but near Prague by 54° 1′ 30″. And the bright star of Aries is placed in the *Epistolae* at 41° 19′ 30″ from the bright star of the Chair, but in the first volume of the *Progymnasmata* at 41° 20′ 40″. Next, the Northern Scale and Antares are, for Tycho, 24° 10′ apart, and the Scales from one another 9° 8′ 20″, although from their Tychonic longitude and latitude Gassendi derives the former as 24° 9′ 12″ and the latter as 9° 12′. Spica also and Antares are, for Tycho, 45° 51′ 30″ apart, but Magini, in the *Primum Mobile*, gathers from the Tychonic longitude and latitude a distance of 45° 52′, whether refraction produced this or something else in observing. Finally, I advise the reader that, after deriving from declination and distance the differences of right ascension by the Problems of the primum mobile, I then made the return, and from the difference of right ascension and the declination confirmed the distance by analysis of triangles, and I have found that these differ from one another by no seconds or by very few. So the following table, unless love of my own work deceives me, is to be held in great value. When I say *for Tycho* [Tychoni], know that the distance was observed with instruments by Tycho; when I say *Tychonically* [Tychonicè], that it was derived from data given by Tycho.

TABVLA DISTANTIÆ FIXARVM Inter se.

Table of the distances of the fixed stars from one another.

Stars in alphabetical order	Distance
ANDROMEDÆ — *of Andromeda*	
Head from the bright star of Aries, *for me*	27° 9′ 0″
for Tycho	27° 7′ 30″
Head from Markab of Pegasus, *for me*	20° 11′ 35″
Head from the southern foot of Andromeda, *for me*	26° 40′ 0″
Bright star of the belt from the bright star of Aries, *for me*	17° 27′ 40″
for Tycho	17° 27′ 15″
Bright star of the belt from the Head of Andromeda, *for me*	14° 29′ 30″
Bright star of the belt from the southern foot, *for me*	14° 26′ 0″
Southern foot from Aldebaran, *for me*	41° 24′ 0″
Southern foot from the bright star of Aries, *for me*	18° 48′ 30″
for Tycho	18° 48′ 0″
ANTINOI — *of Antinous*	
Leading hand from the bright star of Aquila, *for me*	10° 57′ 0″
for Tycho	10° 56′ 0″
AQVARII — *of Aquarius*	
Fomahant [Fomalhaut] from the bright star of Aries, *for me*	70° 0′ 0″
Fomahant from the bright star of Aquila, *for me*	59° 5′ 20″
Leading shoulder from the following one, *for me*	10° 0′ 0″
for Tycho	10° 1′ 0″
Leading shoulder from the bright star of Aquila, *for me*	29° 1′ 0″

Stars in alphabetical order	Distance
Leading shoulder from the northern horn of Capricorn, *for Tycho*	19° 25′ 0″
Leading shoulder from the southern horn of Capricorn, *for Tycho*	19° 37′ 0″
Right shoulder from the bright star of Aquila, *for me*	34° 54′ 0″
for Tycho	34° 53′ 30″
Right shoulder from Markab of Pegasus, *for me*	21° 1′ 0″
Leading hand, from the right shoulder, *for Tycho*	21° 27′ 30″
AQVILÆ — *of Aquila*	
Bright star from Spica, *for Tycho*	97° 50′ 0″
Bright star, from the bright star of Lyra, *for me*	34° 9′ 0″
and for Tycho	34° 9′ 0″
Bright star from the tail of Cygnus, *for me*	38° 4′ 0″
for Tycho	38° 4′ 30″
Bright star from Antares, *for me*	60° 14′ 0″
from the bright star of Aries, *for Tycho*	90° 15′ 0″
✶ Bright star from the northern one of the wing, *for me*	2° 7′ 0″
✶ Bright star of the throat from the southern bright one, *for me*	2° 33′ 0″
ARIETIS — *of Aries*	
First in the leading horn from Aldebaran, *for me and for Tycho*	38° 36′ 0″
Following one in the horn from the belt of Andromeda, *for me*	17° 44′ 0″
Following one in the horn from Aldebaran, *for Tycho*	38° 21′ 40″
The one above the tail from Aldebaram, *for Tycho*	26° 45′ 0″
Bright star of ♈ from the bright star of the Pleiades: see Taurus	
Bright star of Aries from Aldebaram, *for me*	35° 32′ 0″
for Tycho in the *Epistolae*	35° 32′ 0″
but in the *Progymnasmata*, page 200	35° 32′ 10″
Bright star of Aries from Capella, *for me*	44° 5′ 0″
for Tycho	44° 6′ 36″
Bright star of Aries from the foot of Andromeda, *for me*	18° 48′ 30″
for Tycho	18° 48′ 0″
Bright star of Aries from the Head of Andromeda, *for me*	27° 9′ 0″
for Tycho	27° 7′ 30″
Bright star of Aries from Fomahant, *for me*	70° 0′ 0″
AVRIGÆ — *of Auriga*	
Capella from the bright star of Aries, *for me*	44° 5′ 0″
for Tycho	44° 6′ 30″
Capella from Aldebaran, *for me*	30° 42′ 30″
Capella from Procyon, *for me*	51° 7′ 0″
Capella from the bright shoulder of Orion, *for me*	39° 29′ 0″
Capella from the head of Pollux: see Pollux	
Right shoulder from Procyon, *for me*	45° 10′ 40″
Right shoulder from the bright star of the Pleiades, *for me*	33° 57′ 30″
Right shoulder from Capella, *for me*	7° 42′ 30″
✶ Leading foot from Procyon, *for me*	46° 48′ 0″
✶ The same foot from the bright shoulder of Orion, *for me*	29° 6′ 0″
The same foot from the head of Pollux, *for Tycho*	36° 23′ 30″
✶ Foot or tip of the Kids from the Goat, *for me*	3° 6′ 0″
✶ The same from the bright foot of ♊, *for me*	34° 9′ 0″
BOOTIS — *of Boötes*	
Arcturus from Regulus, *for me*	59° 49′ 0″

Stars in alphabetical order	Distance
Arcturus from the Tail of Leo, *for me*	35° 25′ 30″
Arcturus from Spica Virginis, *for me*	33° 2′ 0″
Arcturus from the bright star of Lyra, *for me*	58° 52′ 30″
✶ Bright star on the hip from Arcturus, *for me*	10° 13′ 0″
✶ Unformed star between Arcturus and Ursa Major, of the 2nd magnitude, from Arcturus	9° 57′ 0″
✶ The same from the bright star of the Crown, *for me*	11° 0′ 0″
CANCRI — *of Cancer*	
Northern Ass from Regulus, *for Tycho*	22° 29′ 30″
Northern Ass from the bright foot of ♊, *for Tycho*	30° 4′ 39″
✶ Southern arm from the Tail of ♌, *for me*	19° 0′ 0″
✶ The same from the bright star of Hydra, *for me*	16° 41′ 0″
CANIS MAIORIS — *of Canis Major*	
Sirius from Procyon, *for me*	25° 42′ 0″
for Tycho	25° 41′ 30″
Sirius from Regel of Orion, *for me*	23° 42′ 0″
for Tycho	23° 41′ 40″
Sirius from Aldebara, *for Tycho*	45° 57′ 30″
CANIS MINORIS — *of Canis Minor*	
Procyon, from Regel of Orion, *for me*	38° 38′ 0″
for Tycho	38° 37′ 30″
Procyon from Sirius, *for me*	25° 42′ 0″
for Tycho	25° 41′ 30″
Book 8 of the *Primum Mobile*, from Tycho's data, *for Magini*	25° 40′ 0″
Procyon from Regulus of Leo, *for me*	37° 20′ 0″
for Tycho	37° 19′ 30″
Procyon from Aldebaram, *for me*	46° 22′ 0″
and Tychonically	46° 22′ 0″
Procyon from the Heart of Hydra, *for me and for Tycho*	30° 20′ 0″
Procyon from Capella, *for me*	51° 7′ 0″
✶ Procyon from the bright shoulder of Orion, *for me*	26° 2′ 0″
✶ Bright star in the Neck of Canis Minor from the bright foot of Gemini, *for me*	14° 30′ 0″
CAPRICORNI — *of Capricorn*	
Leading horn from Fomahant, *for me*	40° 37′ 0″
Leading horn from the left shoulder of Aquarius, *for Tycho*	19° 25′ 0″
Leading horn from the bright star of Aquila, *for me*	22° 24′ 30″
for Tycho	22° 25′ 0″
Southern horn from Fomahant, *for me*	39° 5′ 0″
The same from the left shoulder of ♒, *for Tycho*	19° 37′ 0″

TABVLA DISTANTIÆ FIXARVM Inter se.

Table of the distances of the fixed stars from one another.

CAPRICORNI — *of Capricorn*	Distance
The same horn from Scheat of Pegasus, *for me*	50° 29′ 0″
The following star of the tail, from the bright star of Aquila, *for me and for Tycho*	38° 5′ 0″
CASSIOPEÆ — *of Cassiopeia*	
Head from Capella, *for me*	43° 49′ 0″

CAPRICORNI — *of Capricorn*	Distance
from the belt of Cassiopeia, *for Tycho*	4° 17′ 0″
from Schedar, *for Tycho*	2° 41′ 0″
from the hollow of the knee of Cassiopeia, *for Tycho*	9° 6′ 0″
from the haunch of Cassiopeia, *for Tycho*	13° 53′ 0″
from the tip of the foot, *for Tycho*	18° 54′ 0″
Schedar in the breast of Cassiopeia, from the bright star of Aries, *for me*	36° 39′ 0″
for Tycho	36° 38′ 45″
The same from the tail of Cygnus, *for me*	37° 59′ 50″
from the Pole Star, *for Tycho*	32° 47′ 45″
from the head of Pollux, *for Tycho*	75° 11′ 15″
from the belt of Cassiopeia, *for Tycho*	1° 42′ 0″
and in the *Epistolae*	1° 40′ 0″
from the bend, *for Tycho*	4° 40′ 0″
from the hollow of the knee of Cassiopeia, *for Tycho*	6° 56′ 0″
from the leg of Cassiopeia, *for Tycho*	11° 33′
Belt from the hollow of the knee of Cassiopeia, *for Tycho*	5° 19′ 30″
from the leg of Cassiopeia, *for Tycho*	9° 53′ 0″
from the tip of the foot, *for Tycho*	14° 45′ 0″
from Schedar, *for Tycho*	1° 42′ 0″
Flank of Cassiopeia from the head of Cassiopeia, *for Tycho*	7° 20′ 0″
from the belt, *for Tycho*	3° 3′ 0″
from the hollow of the knee, *for Tycho*	3° 35′ 0″
from the leg, *for Tycho*	7° 19′ 0″
from the Pole Star, *for Tycho*	28° 36′ 0″
Upright of the seat from the head, *for Tycho*	9° 3′ 0″
from the belt, *for Tycho*	5° 23′ 0″
from the hollow of the knee, *for Tycho*	6° 48′ 38″
from the leg, *for Tycho*	9° 5′ 30″
from the Pole Star, *for Tycho*	26° 26′ 0″
Leg of Cassiopeia from Schedar, *for Tycho*	11° 33′ 0″
from the tip of the foot, *for Tycho*	5° 9′ 0″
from the hollow of the knee, *for Tycho*	4° 50′ 0″
from Aldebaran, *for Tycho*	55° 23′ 30″
Bright star of the Chair from the bright star of Aries, *for Tycho*	41° 20′ 40″
but in the *Epistolae*	41° 19′ 30″
from the Pole Star, *for Tycho*	30° 13′ 30″
from the head of Cassiopeia, *for Tycho*	6° 33′ 30″
from the foot of Cassiopeia, *for Tycho*	13° 18′ 0″
from Schedar, *for Tycho*	4° 59′ 0″
but in the *Epistolae*	4° 57′ 30″
from the belt, *for Tycho*	5° 19′ 0″
but in the *Epistolae*	5° 17′ 30″
from the hollow of the knee, *for Tycho*	9° 42′ 0″
but in the *Epistolae*	9° 41′ 0″
Bright star of the Chair from the tail of Cygnus, *for me*	33° 32′ 0″
Bright star of the Chair from the Head of Andromeda, *for me*	27° 57′ 0″
Left knee of Cassiopeia from the bright star of Aries, *for Tycho*	37° 27′ 0″
from Aldebaran, *for Tycho*	55° 23′ 30″
from the Head of Andromeda, *for Tycho*	33° 40′ 0″

CAPRICORNI — *of Capricorn*	Distance
from the bend, *for Tycho*	3° 35′ 30″
CENTAVRI — *of Centaurus*	
✶ Leading shoulder from Antares, *for me*	41° 0′ 0″
✶ Following shoulder from Antares, *for me*	31° 41′ 0″
CEPHEI — *of Cepheus*	
Belt from the tail of Cygnus, *for me*	25° 56′ 30″
from the bright star of Lyra, *for me*	38° 42′ 50″
Right shoulder from the bright star of Lyra, *for me*	33° 59′ 0″
from the tail of Cygnus, *for me*	18° 4′ 0″
CETI — *of Cetus*	
Jaw from the bright star of Aries, *for me*	23° 33′ 0″
from Aldebaram, *for me*	26° 17′ 30″
from Regel of Orion, *for me*	35° 17′ 40″
from the bright shoulder of Orion, *for me*	43° 18′ 0″
and for Tycho	43° 18′ 0″
CETI — *of Cetus*	
Northern star in the tail, from the bright star of ♈, *for me*	41° 37′ 40″
Southern star in the tail, from Fomahant, *for me*	26° 45′ 0″
from the bright star of Aries, *for me*	46° 14′ 30″
from the jaw, *for me*	40° 38′ 0″
CORONÆ Borealis — *of Corona Borealis*	
Bright star from Arcturus, *for me*	19° 26′ 0″
from the bright star of Lyra, *for me*	39° 40′ 30″
from Spica Virginis, *for me*	49° 5′ 0″
CORVI — *of Corvus*	
✶ Beak from Spica, *for me*	21° 20′ 0″
✶ Leading wing from Spica, *for me*	17° 58′ 0″
✶ Following wing from Spica, *for me*	14° 27′ 0″
✶ Breast from Spica, *for me*	17° 15′ 0″
CYGNI — *of Cygnus*	
Beak from the tail of Cygnus, *for me*	22° 16′ 0″
from Scheat of Pegasus, *for Tycho*	46° 36′ 40″
from the bright star of Aquila, *for me*	19° 39′ 30″
for Tycho	19° 40′ 0″
Breast of Cygnus from the bright star of Aquila, *for me*	32° 10′ 30″
from Scheat, *for Tycho*	35° 12′ 30″
Tail of Cygnus from the Pole Star, *for Tycho*	44° 39′ 30″
from the bright star of Aquila, *for me*	38° 4′ 0″
for Tycho	38° 4′ 30″
from the bright star of Lyra, *for me*	23° 51′ 40″
for Tycho	23° 53′ 0″
Middle star in the left wing, from Scheat, *for Tycho*	46° 1′ 30″
✶ Elbow [Ancon] of the northern wing, from Aquila, *for me*	28° 8′ 0″
✶ Elbow of the southern wing, from Aquila, *for me*	36° 23′ 0″
DRACONIS — *of Draco*	
Leading bright star of the head, from Lyra, *for me*	17° 44′ 0″
ERIDANI — *of Eridanus*	
✶ The one at the feet of Orion, from Sirius, *for me*	26° 24′ 0″
✶ from Aldebara, *for me*	23° 2′ 0″

CAPRICORNI — *of Capricorn*	Distance
✶ Lowest of the brighter ones, from Regel, *for me*	19° 36′ 0″
from the bright shoulder of Orion, *for me*	35° 50′ 0″
GEMINORVM — *of Gemini*	
Head of Castor from Aldebaram, *for me*	43° 12′ 0″
for Tycho	43° 12′ 30″
from Regulus, *for Tycho*	40° 32′ 0″
from Procyon, *for me*	26° 37′ 0″
from Capella, *for me*	29° 58′ 0″
from Rigel of Orion, *for me*	52° 9′ 15″
from the bright shoulder of Orion, *for me*	33° 38′ 10″
from the head of Pollux, *for me*	4° 32′ 0″
Head of Pollux from Procyon, *for me*	22° 44′ 0″
from Regel of Orion, *for me*	51° 26′ 50″
from Aldebaran, *for Tycho*	45° 5′ 0″
from Regulus, *for me and for Tycho*	36° 59′ 40″
but in the *Progymnasmata*, page 200	36° 59′ 30″
from the heel of the foot, *for Tycho*	19° 28′ 0″
from the tip of the foot of ♊, *for Tycho*	21° 14′ 0″
from Saica [?], *for Tycho*	90° 46′ 20″
from Capella, *for me*	34° 19′ 30″
for Tycho	34° 18′ 30″
from the bright shoulder of Orion, *for me*	33° 15′ 0″
for Tycho	33° 16′ 0″
from the bright star of Aries, *for Tycho*	74° 45′ 30″
Bright foot of Gemini from the Goat, *for me*	34° 6′ 0″
from Procyon, *for me*	18° 49′ 0″
from Regulus, *for Tycho*	51° 11′ 0″
from Aldebaran, *for me and for Tycho*	29° 9′ 0″
from the northern horn of Taurus, *for me*	20° 27′ 20″
from the bright star of Aries, *for Tycho*	58° 22′ 0″
Left heel of Castor, from Aldebara, *for me and for Tycho*	25° 53′ 0″
from Pollux, *for me and for Tycho*	19° 29′ 0″
from Regulus, *for Tycho*	54° 33′ 45″
but in the *Epistolae*, page 50	54° 35′ 0″

TABVLA DISTANTIÆ FIXARVM Inter se,

Table of the distances of the fixed stars from one another.

GEMINORVM — *of Gemini*	Distance
✶ The same heel from the Dog Star [Canicula], *for me*	25° 15′ 0″
Propus on the foot of Castor, from Aldebaran, *for Tycho*	24° 3′ 30″
Tip of the foot of Pollux from the head of Pollux, *for me*	21° 14′ 0″
from Regulus, *for Tycho*	56° 26′ 30″
HERCVLIS — *of Hercules*	
Head from the bright star of Lyra, *for me*	30° 23′ 0″
from the bright star of Aquila, *for Tycho*	38° 32′ 30″
HYDRÆ — *of Hydra*	

GEMINORVM — *of Gemini*	Distance
Heart from Procyon, *for me*	30° 20′ 0″
and for Tycho	30° 20′ 0″
from Spica, *for me*	58° 27′ 45″
from Regulus of Leo, *for me*	23° 0′ 20″
LEONIS — *of Leo*	
Heart, Regulus, from Spica, *for me*	54° 1′ 20″
and for Tycho	54° 2′ 0″
from Procyon, *for me*	37° 20′ 0″
and for Tycho	37° 19′ 30″
from the Tail of Leo, *for me*	24° 39′ 0″
for Tycho	24° 39′ 45″
from Arcturus, *for me*	59° 49′ 0″
from the Heart of Hydra, *for me*	23° 0′ 20″
Tail of Leo, from Arcturus, *for me*	35° 23′ 40″
for Tycho	35° 24′ 40″
from Spica Virginis, *for me*	35° 0′ 30″
for Tycho	35° 1′ 45″
from Regulus: see above	
Bright star of the mane from Spica, *for me*	55° 10′ 0″
for Tycho	55° 12′ 0″
from the Tail of Leo, *for me*	22° 0′ 10″
from Pollux, *for Tycho*	35° 59′ 0″
from Vindemiatrix, *for Tycho*	40° 3′ 10″
Bright star of the loins from Spica, *for me*	45° 11′ 20″
from Regulus, *for me*	17° 54′ 45″
from the Tail of Leo, *for me*	10° 17′ 40″
from the bright star of the mane, *for me*	12° 43′ 0″
for Tycho	12° 44′ 0″
Hip and Heart of Leo, *for Tycho*	17° 51′ 0″
Hip and neck of Leo, *for Tycho*	8° 31′ 0″
Hip and thigh of Leo, *for Tycho*	12° 44′ 0″
LEPORIS — *of Lepus*	
✶ Brighter star of the back from Sirius, *for me*	17° 27′ 0″
✶ from Rigel of Orion, *for me*	10° 0′ 0″
LIBRÆ — *of Libra*	
Northern Scale from Spica, *for me*	27° 33′ 0″
for Tycho	27° 34′ 0″
from Antares, *for me*	24° 10′ 0″
and for Tycho	24° 10′ 0″
from the bright star of Aquila, *for Tycho*	70° 27′ 0″
from the Head of Ophiuchus, *for Tycho*	40° 41′ 30″
from the Southern Scale, *for Tycho*	9° 8′ 20″
but for Gassendi, from the Tychonic longitude and latitude	9° 12′ 0″
Southern Scale from Spica, *for me*	21° 22′ 0″
and for Tycho	21° 22′ 0″
from Antares, *for me*	25° 7′ 40″
LYRÆ — *of Lyra*	
Bright star from Arcturus, *for me*	58° 52′ 30″
from the bright star of Aquila, *for me*	34° 9′ 0″

GEMINORVM — *of Gemini*	Distance
and for Tycho	34° 9′ 0″
from data given by Tycho	34° 10′ 20″
from the tail of Cygnus, *for me*	23° 51′ 40″
for Tycho	23° 53′ 0″
from Scheat of Pegasus, *for Tycho*	55° 30′ 40″
for Longomontanus	55° 31′ 0″
OPHIVCHI — *of Ophiuchus*	
Head from the bright star of Lyra, *for me*	29° 33′ 34″
from the bright star of Aquila, *for me*	33° 33′ 0″
OPHIVCHI — *of Ophiuchus*	
Head from Arcturus, *for me*	48° 2′ 20″
from the bright star of the Crown, *for me*	31° 30′ 10″
from the Northern Scale, *for Tycho*	40° 41′ 30″
from the northern hand of Ophiuchus, *for Tycho*	25° 46′ 0″
More southerly star of the left hand of Ophiuchus, from Aquila, *for Tycho*	55° 17′ 20″
from Spica, *for Tycho*	42° 33′ 0″
More northerly star of the left hand, from Spica, *for Tycho*	42° 33′ 30″
from Aquila, *for Tycho*	55° 17′ 0″
ORIONIS — *of Orion*	
Rigel, or the left foot, from Procyon, *for me*	38° 38′ 0″
for Tycho	38° 37′ 30″
from Sirius, *for me*	23° 42′ 0″
for Tycho	23° 41′ 40″
from Aldebaram, *for me*	26° 36′ 0″
for Tycho	26° 32′ 0″
from the head of Castor, *for me*	52° 9′ 15″
from the head of Pollux, *for me*	51° 26′ 50″
Leading shoulder from Procyon, *for Tycho*	33° 13′ 0″
from the bright star of Aries, *for Tycho*	50° 31′ 20″
✶ from Aldebaran, *for me*	15° 45′ 0″
Following and brighter shoulder of Orion from Regel, *for me*	18° 45′ 0″
from the head of Castor, *for me*	33° 38′ 10″
from Procyon, *for me*	26° 2′ 0″
and for Tycho	26° 2′ 0″
from the head of Pollux, *for me*	33° 15′ 0″
for Tycho	33° 16′ 0″
from Aldebaran, *for me*	21° 24′ 50″
for Tycho	21° 23′ 0″
First star of the belt of Orion, from Aldebaran, *for me*	21° 54′ 0″
but note, *for Tycho*	21° 49′ 30″
from the bright star of Aries, *for Tycho*	55° 7′ 40″
✶ from the bright shoulder of Orion, *for me*	9° 43′ 0″
✶ from Sirius, *for me*	22° 57′ 0″
Middle star of the belt from Procyon, *for me*	31° 25′ 0″
✶ from Aldebara, *for me*	23° 6′ 0″
✶ from Sirius, *for me*	22° 56′ 0″
✶ from the bright shoulder, *for me*	9° 48′ 0″
Last star of the belt, from Aldebaran, *for me*	24° 24′ 0″
✶ from Sirius, *for me*	21° 38′ 30″

GEMINORVM — *of Gemini*	Distance
✶ from the bright shoulder, *for me*	10° 50′ 0″
Brighter star of the sword ✶ from Aldebaran, *for me*	26° 46′ 40″
✶ from Sirius, *for me*	20° 10′ 0″
Right knee ✶ from Aldebaran, *for me*	31° 36′ 0″
✶ from Sirius, *for me*	15° 42′ 0″
PEGASI — *of Pegasus*	
Markab, from the bright star of Aries, *for Tycho*	43° 37′ 15″
from the Head of Andromeda, *for me*	20° 11′ 36″
from the bright star of Aquila, *for me*	47° 48′ 40″
for Tycho	47° 49′ 40″
from Algenib, *for me*	16° 29′ 0″
from the right shoulder of Aquarius, *for me*	21° 1′ 0″
Algenib from the bright star of Aries, *for me*	28° 2′ 0″
✶ from the Head of Andromeda, *for me*	13° 57′ 0″
✶ from Scheat, *for me*	20° 34′ 0″
Scheat of Pegasus, from the bright star of Aquila, *for me*	49° 14′ 45″
from the bright star of Aries, *for Tycho*	41° 16′ 30″
from the bright star of Lyra, *for Tycho*	55° 30′ 40″
from the beak of Cygnus, *for Tycho*	46° 36′ 30″
from the breast of Cygnus, *for Tycho*	35° 12′ 30″
Mouth of Pegasus from the bright star of Aquila, *for me*	27° 57′ 0″
Knee of Pegasus, from the Head of Andromeda, *for Tycho*	18° 28′ 0″
PERSEI — *of Perseus*	
Bright side, from Aldebaran, *for me*	36° 14′ 40″
from Capella, *for me*	18° 56′ 0″
Head of Medusa from Aldebara, *for me*	30° 54′ 0″
and for Tycho	30° 54′ 0″
from Capella, *for Tycho*	23° 37′ 0″
✶ from the foot of Andromeda, *for me*	12° 0′ 0″
✶ Leading right foot from Aldebaran, *for me*	30° 53′ 0″

TABVLA DISTANTIÆ FIXARVM Inter se.

Table of the distances of the fixed stars from one another.

PISCIVM — *of Pisces*	Distance
The one of the third magnitude at the knot of the line, from Aldebaran, *for me*	40° 13′ 30″
for Tycho	40° 13′ 45″
The leading one on the back, from Aquila, *for Tycho*	53° 55′ 0″
Head of the southern Fish, from Aquila, *for Tycho*	51° 32′ 0″
Mouth of the southern Fish, from Aquila, *for Tycho*	48° 16′ 30″
The leading one in the jaw, from the mouth of Pegasus, *for me*	20° 41′ 0″
SAGITTARII — *of Sagittarius*	
Middle star of the head, from Antares, *for me*	35° 37′ 30″
✶ Hand from the Head of Ophiuchus, *for me*	43° 22′ 0″
✶ from Aquila, *for me*	37° 22′
✶ from Antares, *for me*	32° 34′
SCORPII — *of Scorpius*	

PISCIVM — *of Pisces*	Distance
Heart, Antares, from Spica, *for me*	45° 54′ 0″
for Tycho	45° 51′ 30″
from Arcturus, *for me*	56° 4′ 10″
from the Southern Scale, *for me*	25° 7′ 40″
from the Northern Scale, *for me*	24° 10′ 0″
and for Tycho	24° 10′ 0″
from Aquila, *Tychonically*	60° 10′ 0″
for me	60° 14′ 0″
Northern star in the forehead of Scorpius, from Spica, *for me*	39° 28′ 40″
for Tycho	39° 26′ 30″
✶ from the Southern Scale, *for me*	18° 2′ 0″
Southern star in the forehead of Scorpius, from Spica, *for me*	39° 8′ 40″
for Tycho	39° 6′ 30″
✶ from Antares, *for me*	6° 52′ 0″
Middle star in the forehead of Scorpius, from Spica, *for me*	38° 39′ 30″
for Tycho	38° 40′ 0″
✶ from the Southern Scale, *for me*	17° 34′ 0″
✶ Region of the heart [Præcordium] from the Southern Scale, *for me*	23° 12′ 0″
✶ In the bend of the tail, from the point of ♐, *for me*	16° 18′ 0″
✶ The same from the lowest star of Sagittarius, *for me*	19° 17′ 0″
SERPENTIS — *of the Serpent*	
Bright star of the neck from Arcturus, *for me*	24° 50′ 0″
from the bright star of Lyra, *for me*	50° 29′ 40″
✶ from Spica, *for me*	38° 46′ 0″
✶ from the bright star of the Crown, *for me*	20° 23′ 0″
✶ The one in the temples of the Serpent, from Aquila, *for me*	55° 10′ 0″
TAVRI — *of Taurus*	
Aldebaran, or the southern eye	
from the bright star of Aries, *for me and for Tycho*	35° 32′ 0″
from Procyon, *for me and for Tycho*	46° 22′ 0″
from the Head of Medusa, *for me and for Tycho*	30° 54′ 0″
from the bright foot of ♊, *for me and for Tycho*	29° 29′ 0″
from the left heel of ♊, *for me and for Tycho*	25° 53′ 0″
from Sirius, *for me*	45° 59′ 0″
for Tycho	45° 57′ 30″
from the first star of Aries, *for me*	38° 37′ 0″
for Tycho	38° 36′ 0″
from the head of Castor, *for me*	43° 12′ 0″
for Tycho	43° 12′ 30″
from the knot of the line of Pisces, *for me*	40° 13′ 30″
for Tycho	40° 13′ 45″
from the bright shoulder of Orion, *for me*	21° 24′ 50″
for Tycho	21° 23′ 0″
from the first star of the belt of Orion, *for me*	21° 54′ 0″
for Tycho	21° 49′ 0″
from Capella, *for me*	30° 42′ 30″
from the jaw of Cetus, *for me*	26° 17′ 30″
from the bright side of Perseus, *for me*	36° 14′ 40″
from the southern foot of Andromeda, *for me*	41° 24′ 0″

PISCIVM — *of Pisces*	Distance
from Propus of Gemini, *for Tycho*	24° 3′ 30″
from the head of Pollux, *for Tycho*	45° 5′ 0″
from the northern foot of Auriga, *for Tycho*	33° 3′ 30″
from Regel of Orion, *for me*	26° 36′ 0″
for Tycho	26° 32′ 0″
Northern horn from the bright foot of Gemini, *for me*	20° 27′ 20″
TAVRI — *of Taurus*	
The same horn from the bright star of Aries, *for Tycho*	33° 8′ 30″
from the head of Pollux, *for Tycho*	30° 48′ 0″
✶ from Aldebaran, *for me*	16° 45′ 0″
✶ from Capella, *for me*	17° 30′ 0″
✶ from the Dog Star, *for me*	39° 14′ 0″
✶ from Regel of Orion, *for me*	36° 58′ 0″
✶ Southern horn of Taurus from Aldebaran, *for me*	17° 16′ 0″
✶ from Capella, *for me*	13° 24′ 0″
✶ from the Dog Star, *for me*	46° 53′ 0″
✶ Northern eye from Aldebara, *for me*	3° 9′ 0″
for Tycho	3° 10′ 0″
✶ from the bright star of Aries, *for me*	33° 8′ 0″
✶ Twin star of the Hyades from Sirius, *for me*	46° 56′ 0″
✶ Middle star of the Hyades from Sirius, *for me*	49° 2′ 0″
✶ Last star of the Hyades, in the nostrils, from Sirius, *for me*	48° 14′ 0″
Bright star of the Pleiades from Aldebara, *for me*	13° 43′ 0″
from the bright star of Aries, *for me*	22° 56′ 0″
for Tycho	22° 56′ 30″
from the Head of Medusa, *for Tycho*	18° 44′ 0″
✶ from Capella, *for me*	28° 24′ 0″
✶ from the bright shoulder of Orion, *for me*	34° 50′ 0″
VIRGINIS — *of Virgo*	
Spica, from Arcturus, *for me*	33° 2′ 0″
from Antares, *for me*	45° 54′ 0″
for Tycho	45° 51′ 30″
from Regulus, *for me*	54° 1′ 20″
for Tycho at Uraniborg	54° 2′ 0″
for Tycho at Prague	54° 1′ 30″
for Magini, Tychonically	54° 2′ 0″
Spica from the tail of Leo, *for me*	35° 0′ 30″
for Tycho	35° 1′ 45″
from the mane of Leo, *for me*	55° 10′ 0″
for Tycho	55° 12′ 0″
from the bright star of the loins of Leo, *for me*	45° 11′ 20″
from the Heart of Hydra, *for me*	58° 27′ 45″
from the Southern Scale, *for me*	21° 22′ 0″
from the Northern Scale, *for me*	27° 33′ 0″
for Tycho	27° 34′ 0″
Spica from the northern star in the forehead of Scorpius, *for me*	39° 26′ 0″
from the southern star in the forehead of Scorpius, *for me*	36° 6′ 0″
for Tycho	36° 6′ 30″
Vindemiatrix from Arcturus, *for me*	20° 24′ 0″

PISCIVM — *of Pisces*	Distance
from the Tail of Leo, *for me*	17° 57′ 0″
Tip of the left wing from Spica, *for me*	14° 29′ 0″
and for Tycho	14° 29′ 0″
Third star of the southern wing, from Spica, *for Tycho*	14° 39′ 15″
from Regulus, *for Tycho*	40° 22′ 0″
VRSÆ MAIORIS — *of Ursa Major*	
Of the leading stars, the one in the quadrilateral nearer the pole of the world, from Regulus, *for me*	50° 47′ 0″
from Arcturus, *for me*	53° 46′ 0″
Of the leading stars in the quadrilateral, the one that is nearer the equator, from Regulus, *for me*	45° 35′ 0″
from Arcturus, *for me*	51° 34′ 30″
Of the following stars in the quadrilateral, the one that is nearer the pole of the world, from Regulus, *for me*	51° 58′ 0″
from Arcturus, *for me*	42° 18′ 0″
Of the following stars in the quadrilateral, the one nearer the equator, from Regulus, *for me*	46° 37′ 36″
from Arcturus, *for me*	43° 49′ 0″
First star of the tail from Regulus, *for me*	54° 22′ 0″
from Arcturus, *for me*	39° 42′ 30″
✶ from the Pole Star, *for me*	34° 45′ 0″
Middle star of the tail, from Regulus, *for me*	57° 23′ 10″
from Arcturus, *for me*	36° 24′ 20″
✶ from the bright star of the Crown, *for me*	36° 52′ 0″
Last star of the tail, from Regulus, *for me*	58° 20′ 30″
from Arcturus, *for me*	30° 29′ 0″
✶ from the bright star of the Crown, *for me*	30° 35′ 0″

TABVLA DISTANTIÆ FIXARVM Inter ſe.

Table of the distances of the fixed stars from one another.

VRSÆ MINORIS. — *of Ursa Minor*	Distance
Polaris, now the last star of the Tail, from Capella, *for me*	43° 23′ 20″
from Arcturus, *for me*	71° 19′ 20″
from the bright star of Lyra, *for me*	51° 36′ 40″
for Tycho	51° 34′ 40″
from the last star of the Tail of Ursa Major, *for me*	41° 22′ 40″
from the Tail of Cygnus, *for Tycho*	44° 39′ 30″
from Schedar of Cassiopeia, *for Tycho*	32° 47′ 45″
or	32° 48′ 30″
from the bright star of the Chair, *for Tycho*	30° 13′ 30″
Bright star of the shoulder, from Arcturus, *for me*	55° 0′ 10″
Tychonically	55° 0′ 46″
from the bright star of the Crown, *for me*	47° 51′ 30″

APPENDIX DISTANTIARVM Quas ex Longitudine & Latitudine *Tychonica*, deduxit *Henrico Gallus*

Appendix of distances, which Henrico Gallus [?] derived from the Tychonic longitude and latitude.

Stars	Distance
Aldebaran, from the first of Aries	38° 39′ 0″
from the bright star of Aries	35° 32′ 10″
from the head of Pollux	45° 5′ 0″
from the head of Castor	45° 12′ 30″
from the heel of the foot of Gemini	25° 53′ 0″
from the bright foot of Gemini	29° 9′ 0″
from the Propus of Gemini	24° 3′ 30″
from the bright shoulder of Orion	21° 23′ 0″
from Rigel of Orion	26° 32′ 0″
from the bright star of the Pleiades	13° 29′ 0″
from Canicula, that is, Procyon	46° 22′ 0″
from the head of Medusa	30° 54′ 0″
from the bright star of the knot of Pisces	40° 13′ 45″
from Sirius	45° 57′ 30″
Bright star of Aquila, from the bright star of Lyra	34° 9′ 0″
from the tail of Cygnus	38° 4′ 30″
from the bright star of the tail of Capricorn	22° 25′ 0″
from the upper horn of Capricorn	20° 25′ 0″
from the right shoulder of Aquarius	34° 55′ 30″
from the left shoulder of Aquarius	29° 0′ 30″
from the hand of Antinous	10° 56′ 0″
from the head of Hercules	38° 32′ 30″
from the hand of Ophiuchus	55° 17′ 20″
from the Northern Scale	70° 27′ 0″
from the bright star of Aries	90° 15′ 0″
Arcturus, from the bright star of the Crown	19° 26′ 0″
from the tail of Leo	35° 24′ 40″
from Spica Virginis	33° 2′ 0″
Capella, from the bright star of Aries	44° 6′ 36″
from the head of Cassiopeia	43° 44′ 30″
from the head of Medusa	23° 37′ 0″
from the head of Pollux	34° 18′ 30″
The Scales of Libra, from each other	9° 8′ 20″
Northern Scale, from Spica Virginis	27° 34′ 0″
Southern Scale, from Spica	21° 22′ 0″
Northern Scale, from the head of Ophiuchus	40° 41′ 30″
Northern Scale, from Antares	24° 10′ 30″
Bright star of Aries, from Schedar	36° 38′ 45″
from the head of Pollux	74° 45′ 30″
from the heel of the foot of Gemini	58° 22′ 0″
from Scheat of Pegasus	41° 16′ 30″
from the head of Andromeda	27° 7′ 30″
from the left shoulder of Orion	50° 31′ 20″
from the bright star of the Pleiades	22° 56′ 30″
Bright star of Lyra, from the bright star of Aquila	34° 9′ 0″

Stars	Distance
from the Pole Star	51° 34′ 40″
from Scheat of Pegasus	55° 50′ 40″
from the tail of Cygnus	25° 53′ 0″
The shoulders of Orion, from each other	7° 30′ 40″
Right shoulder of Orion, from Pollux	33° 16′ 0″
the same from Procyon	26° 2′ 0″
the same from the jaw of Cetus	43° 8′ 0″
Procyon, from Sirius	25° 41′ 30″
from Regel of Orion	28° 37′ 30″
from Aldebaran	46° 22′ 0″
Procyon, from the left shoulder of Orion	33° 13′ 0″
from the bright star of Hydra	30° 20′ 0″
Regulus, from the tail of Leo	24° 39′ 45″
from Procyon	37° 19′ 30″
from the head of Castor	40° 32′ 0″
from the head of Pollux	36° 59′ 40″
from the bright star of the foot of Gemini	51° 11′ 0″
from the Northern Ass	22° 29′ 30″
Polaris, from the tail of Cygnus	44° 39′ 30″
from Schedar of Cassiopeia	32° 47′ 45″
Sirius, from Procyon	25° 41′ 30″
from Regel of Orion	23° 41′ 40″
from Aldebaran	45° 57′ 30″
Spica Virginis, from Regulus	54° 2′ 0″
from the bright star in the mane of Leo	55° 12′ 0″
from the northern star in the forehead of Scorpius	39° 26′ 30″
from the southern star in the forehead of Scorpius	39° 6′ 30″
from the middle star of the forehead of Scorpius	38° 40′ 0″
from the bright star of Aquila	97° 50′ 0″
from Pollux	90° 46′ 20″
from the northern star in the left hand of Ophiuchus	42° 33′ 30″
from Arcturus	33° 2′ 0″
from Antares	45° 51′ 30″

DISTANTIÆ per Radium captæ ab *Hadriano Metio*, ex libro eius 3. de vſu globi.

Distances taken with the radius by Adriaan Metius, from his third book on the use of the globe.

Stars	Distance
Antares, from the Northern Scale	24° 10′ 0″
Capella, from the head of Cassiopeia	43° 49′ 0″
Capella, from the heel of the foot of Gemini	27° 1′ 0″
Capella, from the bright star of Aries	44° 6′ 0″
The mane of Leo, from Pollux	36° 0′ 0″
Bright star of Aquila, from the left shoulder of Aquarius	20° 0′ 0″
Bright star of Aries, from the northern eye of Taurus	33° 8′ 0″

Stars	Distance
Bright star of Aries, from Aldebaran	35° 32′ 0″
Markab of Pegasus, from the left shoulder of Aquarius	31° 3′ 0″
Regulus, from the bright foot of Gemini	51° 11′ 0″
Spica, from the Northern Scale	27° 34′ 0″

DISTANTIÆ aliquot Stellarum inſignium à Naucleris obſeruatæ in noſtro Horizonte latentium.

Distances of some notable stars hidden below our horizon, observed by navigators.

Stars	Distance
Canobus, from Sirius, by the radius	36° 50′ 0″
Canobus, from the bright star of Hydra	57° 52′ 0″
Right forefoot of the Centaur, of the first magnitude, from Antares	38° 20′ 0″
The same foot of the Centaur, from Spica	52° 10′ 0″
Haunch of the Centaur, of the 2nd magnitude, from Spica	44° 40′ 0″
the same from the bright star of Hydra	54° 26′ 0″
Top of the Cross, of the 2nd magnitude, from Spica	48° 20′ 0″
the same from Antares	51° 25′ 0″
Foot of the Cross, from Spica, of the 2nd magnitude	52° 50′ 0″
the same from Antares	53° 15′ 0″
Leading arm of the Cross, from the following arm	3° 55′ 0″
the same from the foot of the Cross	4° 25′ 0″
Following arm of the Cross, from the foot of the Cross	4° 54′ 0″
Left knee of the Centaur, of the 2nd magnitude, from Antares	44° 0′ 0″
the same star from Spica	49° 30′ 0″
Eye of the Peacock, of the 2nd magnitude, from Antares	51° 45′ 0″
the same from the right forefoot of the Centaur	40° 25′ 0″
Bright star of the Southern Crown, from Antares	36° 10′ 0″
the same, of the 2nd magnitude, from Aquila	50° 45′ 0″
Foot of Sagittarius, of the 2nd magnitude, from Antares	38° 40′ 0″
the same from the bright star of Aquila	55° 10′ 0″
Northern wing of the Crane, of the 2nd magnitude, from the eye of the Peacock	18° 15′ 0″
the same from the bright foot of Sagittarius	29° 5′ 0″
Breast of the Crane, of the 2nd magnitude, from the eye of the Peacock	23° 18′ 0″
the same from the bright foot of Sagittarius	34° 50′ 0″
Head of the Crane, from the eye of the Peacock	24° 20′ 0″
the same, of the 2nd magnitude, from the foot of Sagittarius	29° 55′ 0″
First of Eridanus, of the first magnitude, from the eye of the Peacock	39° 25′ 0″
Alcarnar, the same star, from Fomahant	39° 45′ 0″

Vltimæ quæ per Radium captæ ſunt à Naucleris, vellem iterum per Sextantes caperentur.

I should like these last distances, which the navigators took with the radius, to be taken again with sextants.

CAPVT XI.

Chapter 11.

De Fixarum Declinatione & Aſcenſione Recta obſeruandis, aut ex obſeruatione colligendis.

On observing the declination and right ascension of the fixed stars, or deriving them from observation.

[Margin: What is declination?]

I. The declination of a star is its distance from the equator along an arc of a great circle drawn through the poles of the equator and the centre of the star; in other words, it is the arc of a great circle drawn through the poles of the equator, intercepted between the centre of the star and the equator. It can be observed with equatorial and declination armillaries [Armillas Æquatorias ac Declinationum], as we shall teach in the *Liber Organicus*. Yet the simplest and most certain way of all to observe it is by altitudes, especially true meridian altitudes, taken with large quadrants that show at least single minutes and tens of minutes.

[Margin: Method of observing the declinations of the fixed stars.]

And at the centre of the instrument it is better to use either a very fine needle or threads of cithara strings, set up perpendicular to the plane of the quadrant and lit by a lamp placed close by, in such a way that a lantern keeps back the glare that would otherwise hinder the observer's eyes, than to use Tycho's cylinders, as I shall teach at greater length in the *Liber Organicus*. I said *true meridian altitudes* because if the altitude of the observed star is less than 20°, it is subject to sensible refraction, which must be subtracted from its apparent altitude so that the true altitude remains. With these things laid down, let there be:

Problema 1. Data Vera Altitudine Meridiana Sideris, & Altitudine Poli, Declinationem eiuſdem Sideris Inquirere.

Problem 1. Given the true meridian altitude of a star and the altitude of the pole, to find the declination of the same star.

II. In the present figure let the horizon be HR and the meridian HZRN, on which let the zenith be Z and the nadir N, and let there be the pole of the equator AQ, the northern being B and the southern P. Let the altitude of the north pole be the arc BR, to which the arc AZ, namely the distance of the equator from the vertex, is always equal; and let the complement of the altitude of the pole to a quadrant be the arc BZ, to which the altitude of the equator HA is always equal.

Diagram. A small circle standing as the meridian, set into the left column at the right of the text of paragraph II. Straight lines cross at its centre: the

horizon from H to R, the equator from A to Q, and the axis from B to P. Small stars are drawn at K, G, C and M on the circumference.

- K (top, left of Z): star position, with a small star
- Z (top): zenith
- G (top, right of Z): star position, with a small star
- A (upper left): end of the equator
- C (left, below A): star position, with a small star
- H (left): end of the horizon
- B (upper right): north pole
- M (right, below B): star position, with a small star
- R (right): end of the horizon
- Q (lower right): end of the equator
- N (bottom): nadir
- P (lower left): south pole

Labels read: K, Z, G, A, C, H, B, M, R, Q, N, P. Labels not read: none.

[Margin: Seven cases of the meridian altitude.]

There are seven cases of the true meridian altitude of a star. First, if it equals the altitude of the equator HA, as when the star is at A, the star has no declination. Second, if the star is at the zenith Z itself, that is, if its altitude HZ is 90°, the declination AZ equals the altitude of the pole. Third, if it were at the pole B or P itself, its declination would be 90°, northern in the one case and southern in the other. Fourth, if the star's true altitude is less than the altitude of the equator, that is, less than the complement of the altitude of the pole, as when it is at C, subtract the star's altitude HC from the altitude of the equator HA, and the star's declination AC will remain, southern. Fifth, if the star's altitude is greater than the altitude of the equator but towards the south, as when the star is at K, subtract the altitude of the equator HA from the star's altitude HK, and the star's declination AK will remain, but northern. Sixth, if the star is beyond the zenith towards the pole B, and its true meridian altitude is less than the altitude of the pole, as when the star is at M, its altitude will be not the greatest but the least. So by doubling its distance BM from the pole you will have the arc MG, which added to the least altitude RM will give the greatest, RG; work with this as in case 7. Seventh, then, if the altitude towards the pole B is greater than the altitude of the pole, as when the star is at G, subtract from it the altitude of the pole, and its distance from the pole, namely the arc GB, will remain; subtract this from the 90° of the arc AB, and the star's northern declination AG will remain. Work similarly for stars around the southern pole P.

EXEMPLVM.

Example.

[Margin: Example in the southern eye of Taurus.]

On 1642 February 12, 13, 14 and 16, the meridian and true altitude HK of Aldebaran at K was observed by us, with the instruments in agreement, as 61° 14′ 40″, at Bologna, where the altitude of the equator HA, observed by us more than 40 times, is 45° 30′ 30″. When this was subtracted from the altitude of Aldebaran, there remained its northern declination AK, 15° 44′ 10″.

Problema 2. Declinationem Sideris Inueſtigare, Data Poli Altitudine, & obſeruatis Sideris Altitudine & Azimutho vero.

Problem 2. To investigate the declination of a star, given the altitude of the pole, with the star's altitude and true azimuth observed.

III. Let the horizon be HR and the meridian HZR, on which let there be the zenith Z and the pole of the equator P, and let the given altitude of the pole be PR, whose complement to the quadrant RZ is the distance of the pole from the zenith, that is, the arc PZ. Let the equator be EQ, and the vertical circle, or circle of altitude, AZ. Let AB be the star's altitude, observed free of sensible refraction, for its complement to a quadrant will be the arc BZ. And finally let the azimuth of the star B be the arc HA of the horizon, for this measures the angle HZA, since it is described from the pole of the horizon. When this angle is subtracted from two right angles, that is, from 180°, the angle BZP will be known. So in the spherical triangle BZP we are given the vertical angle at Z and the sides containing it, namely ZP, the distance of the pole from the zenith, and BZ, the true distance of the star from the vertex. Therefore, by the analysis of triangles the base BP will not stay hidden; in this example it is the complement of the northern declination (if P is the north pole). So when BP has been subtracted from 90°, the star's declination BN will be known.

Diagram. A small circle standing as the meridian, set into the right column at the right of the text of paragraph III. The horizon runs from H to R, the equator from E to Q, a line runs from Z down to A through B and the crossing near G and C, and arcs run from P to N, G and I.

- Z (top): zenith
- E (upper left): end of the equator
- B (top, inside the circle): the star
- P (upper right): the pole
- N (inside, just below E): foot of the star's declination
- G (inside, centre left): point of the equator
- C (inside, right of G): foot of the declination of the star at I
- I (inside, below G): second position of the star
- H (left): end of the horizon
- A (inside, just below the horizon near H): foot of the vertical circle
- R (right): end of the horizon
- Q (lower right): end of the equator

Labels read: Z, E, B, P, N, G, C, I, H, A, R, Q. Labels not read: none.

But if that base came out as PG, exactly 90°, the star would be at G, a point of the equator, and would have no declination. If, however, the star were at I and its true altitude were AI, the base would be PI, and would be found greater than a quadrant, that is, than 90°. So with those 90° subtracted from the base PI, the star's declination CI would remain, but towards the other pole, that is, southern, if P is the north pole.

[Margin: Warning about refraction.]

If the star's observed altitude is less than 20°, and so subject to sensible refraction, while the refraction of the fixed stars is known from elsewhere, as when the star's apparent place would be at B but its true place at C, subtract the refraction, that is, the arc CB, from the apparent altitude, and the true altitude AC will remain; then do the rest as above.

[Margin: Method of observing azimuths.]

You will observe the star's azimuth most exactly if you set the plane of our thread triangle (on which see Book 5, chapter 15, no. 16) at a determined degree of distance from a most exact meridian line, and notice that the star's centre is going to reach it shortly. As soon as the star's centre is in that plane, let the star's altitude be taken by yourself or by another most skilled observer, with the instrument already directed there and set up by the plumb line on a separate observing table, but in the same azimuth. Otherwise, an observation of azimuths made by other methods is unreliable.

Problema 3. Declinationem Sideris Inquirere, Data Poli Altitudine, & obſeruatis aut Altitudine vera & momento Temporis; aut Azimutho & momento Temporis.

Problem 3. To find the declination of a star, given the altitude of the pole, with either the true altitude and the moment of time observed, or the azimuth and the moment of time.

IV. In the figure set out above, let the star be at B. For in the triangle BZP, ZP is given, the complement of the altitude of the pole. So if BZ, the complement of the true altitude AB, is given, and the angle BPZ, from the time before or after noon converted into degrees of the equator, three sufficient quantities will be given, so that the arc BP can be derived by the rules of spherical triangles. But if in the same triangle, besides ZP and the angle BPZ known from the time, the angle BZP is given, which is the complement to two right angles of the angle HZA, that is, of the observed azimuth, then by the rules of the same triangles the arc BP will not stay hidden, and this is the complement of the declination. So when it has been subtracted from 90°, the star's declination NB will become known; it will be northern if the pole P is northern, and southern if P is southern. But if the arc found were GP, of 90°, the star would be at the point G of the equator, lacking all declination; and finally, if the arc found were greater than 90°, as the arc PI is, the star would be at I, beyond the equator, and so when 90° are subtracted from the arc PI, the star's declination IC would

remain, southern if the pole P is northern, and so on. How the moment of time is observed, whether by the altitude of another known fixed star, or by the count of time with the help of our pendulum (on which see Book 2, chapter 20), counted from the transit, or before the transit, of a known fixed star through the midheaven, or from an altitude of the Sun taken before its setting, or before an altitude of the Sun to be taken after its rising, we shall say in the Problems of the primum mobile, Book 10.

[Margin: Problem 1 is preferred to the others.]

But I advise you to take the declinations of the fixed stars by Problem 1, as we have done, rather than by 2; or, if at any time you cannot use that one, to use the second rather than the third. The remaining problems that belong here do not depend on direct observation but on other data that presuppose observations, or else they are more deceptive. Here, however, we set out only those that are resolved by the first and simpler principles of observations.

Problema 4. Differentiam Aſcenſionis Rectæ inter duo Sidera, Data illorum Declinatione & Diſtantia, Inueſtigare.

Problem 4. To investigate the difference of right ascension between two stars, given their declination and distance.

[Margin: Four cases of the Problem.]

V. In the following figure let the south pole of the equator be A and the north pole B, from which let the arc EQ of the equator be described, which the angle B or A therefore measures. First, let one star be at C and the other at D, through which let the declination circles BCA and BDA be drawn; and let their distance CD be given, either from observation or from our table in the preceding chapter. For in the spherical triangle CBD three sides will be given, namely the distance CD and the arcs CB and BD, which are the complements of each star's own declination to a quadrant. So by the rules of triangles the angle B will become known, which measures the equatorial distance EQ, that is, the difference of right ascension between the stars C and D.

Diagram. A pointed lune between two curved sides meeting at B above and A below, set into the left column at the right of the text of paragraph V. Across it the straight line EQ joins the two sides at the middle. Further lines run between the sides: from C to D, from C down to F, from E down to F, and from G to F.

- B (top vertex): north pole
- D (right side, upper): star
- C (left side, upper): star
- E (left side, middle): end of the equator
- Q (right side, middle): end of the equator
- F (right side, lower): star
- G (left side, lower): star
- A (bottom vertex): south pole

Labels read: B, D, C, E, Q, F, G, A. Labels not read: none.

But if the stars were G and F, and the declination of both southern, the triangle GAF would have to be solved, from the given distance GF and the complements of the declinations AG and AF; and the angle A, once found, would be the measure of the difference of right ascension. If, however, one star is at E, lacking declination, and the other at F, solve the triangle EAF, seeking the angle A from the given distance EF, the arc FA, which is the complement of the declination of star F, and the quadrantal arc AE, that is, of 90°. Finally, if both stars lacked declination, and one were at E and the other at Q, their distance itself would be the arc EQ of the equator and the difference of right ascension.

Problema 5. Aſcenſionem Rectam Stellæ alicuius Fixæ more Antiquiorum Aſtronomorum Obſeruare.

Problem 5. To observe the right ascension of some fixed star in the manner of the older astronomers.

[Margin: First method, most deceptive.]

VI. The first method, an ancient one but now antiquated, was by a count of time made with the help of clepsydras, from the transit of the Sun or the Moon through the middle of the sky up to the transit, or the rising or setting, of the star. They converted that time into an arc of the equator, and added that arc of the equator to the right ascension of the Sun or the Moon if the star was further east than they were, and subtracted it if further west, and the sum or remainder was the right ascension of the star. But enough has been said about the deceptiveness of clepsydras in Book 2, chapter 20, proposition 15, and Book 3, chapter 10, no. 6.

[Margin: Second method, but deceptive.]

The second method, used by Hipparchus and Ptolemy (*Almagest* Book 7, chapters 2 and 4), by al-Battānī (chapter 51), by Regiomontanus and Bernhard Walther (as found in their observations), and by Copernicus (Book 2, chapter 14), was by the equatorial distance of the Moon from the Sun, taken with the Sun near the horizon at a definite moment of time known from the Sun's altitude or from a sundial; and then by the equatorial distance of the Moon from a fixed star, taken at night with the armillary spheres of the astrolabe. For from the first observation they deduced the right ascension of the Moon, and from the second the right ascension of the fixed star, through the difference of right ascension, in the first case between the luminaries, in the second between the Moon and the fixed star; and this difference was known from the arc of the equator taken between the stars. But Tycho rightly rejects this method (*Progymnasmata* volume 1, chapter 2, from p. 140): both because of the deceptiveness of the armillary spheres and their division carried no further than tens of minutes, on which we say more in our *Liber Organicus*; and because of the refractions of

the Sun when closer to the horizon, unknown to the ancients or neglected by them; and because of the complications of lunar parallaxes.

[Margin: Third method, likewise deceptive.]

The third method, used by Walther and Cardanus [Girolamo Cardano], was by the distance of a fixed star from Venus, after first verifying the place of Venus and the right ascension of Venus relative to the Sun, since Venus can be seen within the same natural day both by day and by night. But because they obtained that distance with the astrolabe and armillary spheres in the old manner, the observation was not so exact. And so for this reason, or perhaps from inexperience in observing the places of the stars exactly, or from neglected refractions and parallaxes, Cardano erred by 1° 40′ in fixing the place of the Southern Scale by means of Venus, more than the Alphonsine calculation required, which he was nevertheless striving to correct, as Tycho reports (*Progymnasmata* volume 1, chapter 2).

Problema 6. Aſcenſionem Rectam Stellæ Fixæ Tychonis more obſeruare, cum correctione Obſeruationis Tychonicæ.

Problem 6. To observe the right ascension of a fixed star in Tycho's manner, with a correction of the Tychonic observation.

VII. So Tycho, rejecting the preceding methods, used the distance of the Sun from Venus, sometimes morning and sometimes evening, taken with his sextants, and then the difference of right ascension taken with the equatorial armillaries, or the distance of Venus from a fixed star taken at night. The moments of both times were observed, with the altitudes and declinations of the Sun and Venus; the declination of that fixed star was known, or observed shortly afterwards; and both places of the planet were corrected, with parallaxes and refractions taken into account. It is better, however, to teach this practice by an example than by a longer precept. So let the example be taken from Tycho's *Progymnasmata*, volume 1, chapter 2, on the bright star of Aries, observed in 1586 on December 27 of the Julian calendar, but on January 6 of 1587 of the Gregorian. On that day the earlier observation, from the true altitude of the Sun, was made at Uraniborg at $2^h\ 48^m$ after noon, and the later at $4^h\ 58^m$, by Tycho's clock.

EXEMPLVM.

Example.

Ex Tychone tomo 1. Progymn. pag. 176. — *From Tycho, Progymnasmata volume 1, p. 176*	Value
At $2^h\ 48^m$, altitude of the Sun	3° 30′ 0″
Altitude of ♀	23° 0′ 0″
Distance of ♀ from the centre of the Sun, taken with the sextant	46° 22′ 0″
Southern declination of ♀, taken with the armillary spheres	10° 49′ 30″
Southern declination of ☉, from Tycho's tables	22° 34′ 37″

Ex Tychone tomo 1. Progymn. pag. 176. — *From Tycho, Progymnasmata volume 1, p. 176*	Value
Place of ☉ from the same tables, in Capricorn	15° 52′ 34″
Right ascension of the Sun, from the same tables	287° 14′ 1″
So the difference of right ascension between the Sun and Venus, by Problem 4	47° 3′ 6″
So the right ascension of ♀, which was further east than the Sun, was	334° 17′ 7″
Then, at 4^h 48^m after noon, the equatorial distance between ♀ and the bright star of Aries, taken with the armillary spheres, was	51° 32′ 0″
Altitude of ♀	20° 0′ 0″
Time elapsed between the observations, of hours	2^h 0^m 0^s
Motion of ♀ corresponding to the preceding time	0° 4′ 5″
So the right ascension of ♀, increased by 4′ 5″, was	334° 21′ 12″
Add the equatorial difference of ♀ and the bright star of Aries, the further east, and the result is	385° 53′ 12″

	Value
Subtract the whole circle, and there remains the right ascension of the bright star of Aries	25° 53′ 12″

With 50″ subtracted for the almost entire year, however, the right ascension of the bright star of Aries at the end of 1585 remains 25° 52′ 22″.

[Margin: Tycho's caution.]

But in observations of this kind both the Sun and, not rarely, Venus undergo a sensible refraction because they are close to the horizon, the Sun more so, and the Sun's greater refraction reduces the true distance of the Sun from Venus. As a result the right ascension of ♀ in the morning, being then further west, is apparently increased, and that of ♀ in the evening, being then further east than the Sun, is decreased. For this reason Tycho, in his *Epistolae Astronomicae*, p. 96, prudently advises that several observations be made by way of Venus, both morning and evening, with ☉ and ♀ at the same altitude as nearly as possible, so that the defect of right ascension arising from the evening refractions may be offset by the excess arising from the morning ones and, once the arithmetic mean has been chosen, a more correct right ascension of the star may be obtained. So the same Tycho, as stands in *Progymnasmata* volume 1, chapter 2, p. 193, from fifteen observations of the bright star of Aries by way of morning Venus and as many by way of evening Venus, made from 1582 to 1588, derived for the end of 1585 a right ascension of the bright star of Aries of 26° 0′, with the seconds at the least 4″ and at the most 44″. He therefore chose 26° 0′ 30″, which differs insensibly from the 26° 0′ 24″ derived from the arithmetic mean.

[Margin: Tycho's method suspected by Kepler.]

However, the morning and evening refractions, and the altitudes of ☉ and ♀, are rarely equal, and only very rarely is Venus, at the same distance from the horizon, at the same distance from the centre of the Earth. For that reason Kepler, in the prologue to his *Ephemerides*, p. 15, held this method suspect, and it is certainly a too scattered labour, because of the stretch of time needed to get two observations suitable for offsetting the excess of one with the defect of the other. Kepler's words are these: *"There remains the matter of the six minutes*

of the Sun's motion from the fixed stars, a dispute between the Landgrave and Brahe left undecided, and the testimonies waver this way and that. For the parallax of Venus cannot be as great as Brahe assumed in the Progymnasmata*, adapted to the parallax of the Sun handed down by the ancients, which is without doubt smaller; and even if we suppose the parallax of Venus to be that great, its two parallaxes at the same altitude from the horizon are still not exactly offset, unless the interval of Venus and of the Earth is also the same."*

Problema 7. Aſcenſionem Rectam Stellæ Fixæ per diſtantiam eius ab vno trium Superiorum Planetarum, ſed præcipuè Saturno aut Ioue Stationarijs, Quorum tranſitus per M. C. paulò ante vel poſt obſeruatus ſit, Inuenire.

Problem 7. To find the right ascension of a fixed star by its distance from one of the three superior planets, but especially from Saturn or Jupiter when stationary, whose transit through the M. C. has been observed a little before or after.

VIII. Saturn or Jupiter, nearly stationary, should be chosen: not indeed of necessity, but so that a smaller and insensible error may enter into its right ascension because of the different moment at which the distance of the fixed star was observed, since when they are stationary their proper motion varies insensibly over several hours. But it is necessary that the planet's meridian altitude can be observed that same night, that from this, with the altitude of the equator, its declination be made known by Problem 1, and that from the declination and its latitude, on which the astronomers' tables usually agree very closely, the right ascension of the same planet be derived by the problems of the primum mobile. For by its distance, taken on the same night, from whatever fixed star you prefer, and by the declination of that fixed star, already observed or to be observed by Problem 1, there is easily found by Problem 4 the difference of right ascension, to be added to the planet's right ascension if the planet is further west than the fixed star and subtracted if it is further east, so that the right ascension of the fixed star results or remains. We carried this out, on both the bright star of Aries and Aldebaran, by way of Jupiter in 1643, on the very clear night preceding August 15, with Jupiter close to its station, since it had begun to be retrograde on the 12th. The altitude of Jupiter was taken with three different quadrants, while the declination of the bright star of Aries was taken at the end of August, and that of Aldebaran at the beginning of October of that year.

EXEMPLVM DVPLEX.

Double Example.

Anno 1643. Auguſti 15. illuceſcente — *1643, as August 15 was dawning*	Value
True meridian altitude of Jupiter	53° 50′ 30″

Anno 1643. Auguſti 15. illuceſcente — *1643, as August 15 was dawning*	Value
Altitude of the equator at Bologna	45° 30′ 30″
Therefore the true northern declination of ☊	8° 20′ 0″
True latitude of ☊ at that time, southern	1° 28′ 0″
And the true place of ☊	♈ 24° 34′ 0″
Therefore the right ascension of ☊	23° 43′ 28″
Distance of the bright star of Aries from ☊, as taken	13° 43′ 0″
Distance of Aldebaran from ☊, as taken	39° 53′ 0″
Northern declination of the bright star of Aries	21° 45′ 36″
Northern declination of Aldebaran	15° 44′ 25″
Therefore, by Problem 4, the difference of right ascension between ☊, the further west, and the bright star of Aries	3° 5′ 32″
And between ☊, the further west, and Aldebaran	40° 10′ 10″
Therefore the right ascension of the bright star of ♈	26° 49′ 0″
And the right ascension of Aldebaran	63° 53′ 38″
But at the end of 1644, the right ascension of the bright star	26° 50′ 8″
And the right ascension of Aldebaran	63° 54′ 47″

Assuming the advance of these fixed stars, on which Tycho, *Progymnasmata* volume 1, p. 276, and we ourselves below.

[Margin: Right ascension of the bright star of Aries and of Aldebaran at the end of 1644.]

Problema 8. Aſcenſionem Rectam Sirij per diſtantiam eius à Sole acquirere.

Problem 8. To obtain the right ascension of Sirius by its distance from the Sun.

[Margin: Sirius seen by day.]

IX. Sirius is so bright that in this century, around the equinoxes, it can be seen eight minutes and more before sunset in March and after sunrise in September, and its distance from the centre of the Sun can be taken with a quadrant or a hemicycle, since sextants are not sufficient. From the altitude of the Sun then observed, and corrected by adding parallax and subtracting refraction, the true altitude is derived, and from that the moment of time. From the Sun's meridian altitude immediately preceding or following are derived both the declination and the right ascension of the Sun, the altitude of the pole and the obliquity of the ecliptic being now given; these are adjusted to the time of the observed distance between the Sun and Sirius, according to the difference of time before or after noon converted into degrees of the equator. Next, from the altitudes of the Sun and Sirius converted into true ones, the true distance is derived by the method to be given shortly, by correcting the apparent or observed distance. Finally, from the true distance and the declination of the Sun and of Sirius, taken or to be taken by Problem 1 near the time of this observation, the difference of right ascension between the Sun and Sirius is derived by Problem 4. This, added to the Sun's right ascension if Sirius is

further east, or subtracted if further west, gives the right ascension of Sirius for that time.

[Margin: Method of converting an apparent distance into a true one.]

Now let Sirius appear at C but truly be at F, and let the Sun appear at D but truly be at G. Through their centres let the vertical circles VCA and VDB be drawn from the zenith V down to the horizon. Let the observed altitude of Sirius be AC, whose complement will be the arc CV, and the apparent altitude of the Sun BD, whose complement is the arc DV, and let the apparent distance between Sirius and the Sun be CD. For from the three sides of the spherical triangle CVD you will easily derive, by the analysis of triangles, the angle CVD, which is the same as the angle FVG. Therefore in the triangle FVG, with the already mentioned angle known and the complements of the true altitudes, FV for Sirius and GV for the Sun, you will derive by the rules of the same triangles the base FG, that is, the true distance between the Sun and Sirius.

Diagram. A woodcut set into the right column against the words of this paragraph: a tall, narrow spherical figure with its point at the top right. One curved side runs from V down and to the left through C and F to A and on to H; the other runs from V almost straight down through D, G and B to R. Two arcs cross the foot: AB, and below it HR. Fine lines cross the interior from C and F towards D and G, and a long line runs from F down to R. The course of the fine lines is described only as far as seen on the scan.

- V (top right): point of the figure, the zenith in the text
- C (left side, upper): Sirius apparent
- F (left side, just below C): Sirius true
- D (right side, upper): the Sun apparent
- G (right side, below D): the Sun true
- A (left side, lower): foot of the vertical circle VCA, on the arc AB
- B (right side, lower): foot of the vertical circle VDB, on the arc AB
- H (bottom left): end of the lowest arc
- R (bottom right): end of the lowest arc, the Sun's place when below the horizon

But if Sirius suffers no sensible refraction, being higher than 20°, and is at C, its true place, then to derive the angle at V use the complement of the true altitude, CV, and the remaining items as above. Then in the triangle CVG, with the angle at V known, again use the arc CV and, as above, the arc VG, and derive the true distance CG. It can happen, however, that the Sun's true place is below the horizon AB, as when the Sun is at R but appears at G, and then the arc VR, greater than a quadrant, must be used. Finally, the apparent place of one may be at F and, because of parallax, the true place at C, without any refraction, and yet the apparent place of the other may be at D, higher than the true place G, because of refraction. In this case the apparent distance would be FD, and the true distance CG.

[Margin: Method of making up for the observation of the distance.]

But if you cannot observe the distance for lack of instruments, at least take the altitudes and the azimuths of both, since the difference of the azimuths will be the angle V. With this, and with the complements of the altitudes, work as above to find the true distance.

Now let the example be from our own observation of 1644 September 13, in the morning, with a very clear horizon. The declination of Sirius was observed repeatedly on the days following the autumnal equinox.

EXEMPLVM.

Example.

Anno 1644. Septembris 13. manè — *1644 September 13, morning*	Value
Apparent altitude of the Sun	0° 16′ 0″
⊙, our parallax	0° 0′ 28″
⊙, refraction	0° 31′ 0″
True depth of the Sun below the horizon	0° 14′ 32″
Apparent distance of Sirius and ⊙	75° 35′ 0″
Time before noon, in hours	7^{h} 1^{m} 0^{s}
True northern declination of the Sun at that time	3° 41′ 0″
True right ascension of the Sun at that time	171° 29′ 40″
True altitude of Sirius	25° 0′ 0″
True distance of Sirius from ⊙	75° 47′ 30″
Southern declination of Sirius	16° 17′ 20″
Difference of right ascension	74° 2′ 6″
Therefore the right ascension of Sirius	97° 27′ 30″
But at the end of 1644	97° 27′ 40″
For converting the apparent distance into the true one:	
FV, complement of the altitude of Sirius	65° 0′ 0″
GV, complement of the apparent altitude of ⊙	89° 44′ 0″
FG, apparent distance of Sirius from ⊙	75° 35′ 0″
Therefore the angle FVG	74° 11′ 0″
Again FV	65° 0′ 0″
And the arc VR	90° 14′ 32″
And the angle FVR	74° 11′ 0″
Therefore the true distance FR	75° 47′ 30″

Problema 9. Aſcenſionem Rectam Fixarum adipiſci per tempus numeratum inter tranſitum earum per Meridianum, & Momentum obſeruatæ Aſcenſionis Rectæ Solis.

Problem 9. To obtain the right ascension of the fixed stars by the time counted between their transit through the meridian and the moment of the observed right ascension of the Sun.

X. The counting of the time is to be carried out with our pendulum, on which see Book 2, chapter 20, Propositions 11 and 12, and the transits of the fixed stars through the meridian are to be observed with the thread triangle

already described by us in Book 5, chapter 15, number 16, since otherwise the observation can hardly be exact. Furthermore, a time should be chosen at which the fixed star passes through the meridian near sunrise or sunset, not a whole hour away from it, so that the count of the time between the star's culmination and the Sun's altitude, taken or to be taken, may be as short as possible and exposed to as little tedium and risk of error as possible. For an error of 4^s in the time produces an error of a whole 1′ in the right ascension of the stars. In addition, at the nearest noon the Sun's meridian altitude is to be taken with large quadrants and corrected by adding parallax, so that from the true altitude and the altitude of the equator the declination of ☉ may become known by Problem 1. From the declination of the Sun and the obliquity of the ecliptic the right ascension of the Sun is to be derived by the solution of a right-angled spherical triangle.

So if the time is counted between the star's transit through the M. C. before morning and the following noon, or between the star's transit towards evening and the preceding noon, that time, converted into degrees of the equator, will give the difference of right ascension, to be subtracted in the first case and added in the second to the Sun's right ascension, so that the right ascension of the star is obtained. But it is advantageous to count the time between the fixed star's transit through the M. C. and the altitude of the Sun taken a little before sunset or after sunrise, and, with the Sun's altitude corrected, to derive from it the moment of time, and for that moment the Sun's right ascension, from the difference of time between the nearest noon and the moment at which the solar altitude was taken near the horizon.

In this way, with incredible endurance on our part and that of our colleagues, whom we found most practised and most trustworthy, we observed at Bologna the right ascension of the following stars, together with their declination by Problem 1. These we reduced to the end of 1644 by Tycho's table in *Progymnasmata* volume 1, p. 276, on which see below, that is, by the proportional part owed to the difference of right ascension and declination arising in this century over an interval of 100 years. To give at least one example, let it be on the bright star of Aries, which Tycho takes as the foundation for the others.

EXEMPLVM.

Example.

In 1643, on August 24, in the morning before sunrise, the meridian altitude of the bright star of Aries was taken with three quadrants as 67° 16′ 10″ at Bologna, where the altitude of the equator is most certainly 45° 30′ 30″. Therefore the star's northern declination at that time was 21° 45′ 40″. From the transit of the bright star of Aries through the meridian up to the apparent altitude of the risen Sun, 1° 25½′, the most thoroughly corrected times of $1^h\ 5^m\ 55^s$ were counted with the pendulum (on which see Book 2, chapter 20, Proposition 11), to which 16° 28′ 50″ of the equator correspond. But when the count of

oscillations was continued up to noon, they came to $8^h\ 25^m\ 17^s$, to which 126° 19′ of the equator correspond. At noon the apparent altitude of the Sun was 56° 38′, but the true altitude by our parallax was 56° 38′ 10″. Therefore the Sun's northern declination was 11° 7′ 40″, and so, from our obliquity of the ecliptic, the Sun's right ascension at noon was 153° 8′. If you take from this those 126° 19′, there remains 26° 49′ as the right ascension of the bright star of Aries.

Again, because the apparent altitude was 1° 25′ 30″, but with nearly 30″ of parallax added it was 1° 26′, and with 26′ of refraction subtracted the true altitude was exactly 1°, it was therefore then exactly $7^h\ 19^m\ 20^s$ before noon, to which 109° 50′ of the equator correspond. With these taken from the right ascension of the following noon, there remains 43° 18′ as the right ascension of the Sun observed a little after sunrise. Subtracting from this the arc of the time elapsed, which was 16° 28′ 50″, there remains 26° 49′ 10″ as the right ascension of the bright star of Aries. This value we chose, because in the oscillations of the pendulum over 1 hour a far more insensible error could creep in than in the oscillations over 8 hours and more. From that time to the end of 1644 there are 16 months, for which 68″ must be added, from Tycho's small table in *Progymnasmata* volume 1, p. 276. So the right ascension of the bright star of Aries at the end of 1644 was 26° 50′ 18″. But from the same small table, at the end of 1600, according to Tycho's observations, it was 26° 13′, and at the end of 1700 it will be 27° 38′. Therefore at the end of 1644, according to Tycho, it was 26° 50′ 24″, greater than ours by 6″.

Synopsis Ascensionum Rectarum & Declinationum obseruatarum & ad finem Anni 1644. Reductarum.

Synopsis of the right ascensions and declinations observed and reduced to the end of 1644.

Order of the preceding problems	Stars observed	Years of Christ: year, month, day	At the time of observation: right ascension	At the time of observation: declination	At the end of 1644: right ascension	At the end of 1644: declination
VII.	Bright star of ♈	1643 August 15	26° 49′ 0″	21° 45′ 36″	26° 50′ 8″	21° 46′ 0″ B
IX.	The same	1643 August 24	26° 49′ 10″	21° 45′ 36″	26° 50′ 18″	21° 46′ 0″ B
IX.	Arcturus	1642 May 12	209° 55′ 4″	21° 6′ 18″	209° 57′ 0″	21° 5′ 31″ B
VII.	Aldebaran	1643 August 15	63° 53′ 38″	15° 44′ 25″	63° 54′ 48″	15° 44′ 38″ B
IX.	Aquila	1643 August 31	293° 22′ 41″	8° 0′ 38″	293° 22′ 53″	8° 0′ 48″ B
VIII.	Sirius	1643 September 13	97° 27′ 30″	16° 17′ 20″	97° 27′ 40″	16° 17′ 30″ A
IX.	Regulus	1644 April 18	147° 21′ 30″	13° 41′ 23″	147° 21′ 50″	13° 41′ 12″ B

Order of the preceding problems	Stars observed	Years of Christ: year, month, day	At the time of observation: right ascension	At the time of observation: declination	At the end of 1644: right ascension	At the end of 1644: declination
IX.	Spica ♍	1644 June 3	196° 38′ 33″	9° 15′ 30″	196° 39′ 3″	9° 15′ 20″ A

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Problema 10. Aſcenſionem Rectam Stellæ Fixæ per Aſcenſionem Rectam alterius ſtellæ vicinæ cognitam, Obſeruare.

Problem 10. To observe the right ascension of a fixed star by the known right ascension of another star nearby.

XI. The principal method is by the distance of the stars and their declination, on which see Problem 4. The secondary one is as follows. With the oscillations of a pendulum, but one exhibiting exactly the times of the primum mobile, of the kind we treat in Book 2, chapter 20, Proposition 12, count the time between the transit through the meridian of the star whose right ascension is known and the transit of another nearby star through the same meridian. Let this time be converted into an arc of the equator, and let that arc be added to the right ascension of the known star if this star is further west, and subtracted if it is further east, and the desired right ascension of the star will become known. But the transit through the M. C. is to be observed by our thread triangle, on which see Book 5, chapter 15. A nearby star should be used, so that the intervening time does not reach an hour, or exceeds it only slightly, lest a small error in the pendulum's times become more sensible over a longer stretch of time; and stars closer to the equator should be used, since they cross more quickly and do not seem to stick on the meridian, as those nearer the poles do. See, moreover, what we said in Book 2, chapter 20, in the examples to Proposition 11.

By this technique, with the colleagues indicated in the same place, I observed the difference in right ascension of all the fixed stars of the first and second magnitude visible in our climate, and of many also of the third and fourth rank, using in succession stars closer to one another. We bore this labour over many nights, and in three different years we summoned the whole starry circuit of the sky to this count, so that in this way at least their right ascension should be as nearly certain as possible, until it could be brought to a more certain polish by distances and declinations, according to the method given in Problem 4. I have thought it superfluous, however, to fill pages with those observations or with the meridian altitudes of the stars, since all of them are in the end contained in effect in the table to be given in Chapter 14.

[53] B and A in the last column stand as printed: B northern (*Borea*), A southern (*Australis*). The leaf sets no key to the two letters.

Problema 11. Data Longitudine & Latitudine ſideris Quærere illius Aſcenſionem Rectam & Declinationem.

Problem 11. Given the longitude and latitude of a star, to seek its right ascension and declination.

XII. On this, see Chapter 12, Problem 3.

SCHOLIA.

Scholia.

[Margin: Tycho's diligence.]

I. Tycho, in *Progymnasmata* volume 1, chapter 2, from p. 195, examines the right ascensions of selected fixed stars from the sum of the differences of right ascension, which he derives, by our Problem 4, from their mutual distances and from their declinations as observed by him in 1585. He asks, namely, whether their sum exactly equals the circuit of the whole equator; and he does this in a threefold combination, which it pleases me to show in the following small table, in which the declinations are northern except for Spica and the hand of Ophiuchus.

Ordo Stellarum — *Order of the stars*	Distance from the preceding star	Declination of 1585	Difference from the preceding right ascension
Bright star of ♈	☩	21° 28′ 30″	☩
Southern head of ♊	74° 45′ 30″	28° 57′ 0″	83° 57′ 33″
Spica ♍	90° 46′ 20″	8° 56′ 20″	85° 54′ 18″
Bright star of the Vulture	97° 50′ 0″	7° 51′ 20″	96° 45′ 9″
Bright star of ♈	90° 15′ 0″	21° 28′ 30″	93° 22′ 58″
1. Sum of the differences of right ascension			359° 59′ 58″
Bright star of ♈	☩	21° 28′ 30″	☩
First of the wing of Pegasus	43° 37′ 15″	13° 0′ 40″	44° 58′ 0″
Bright star of the Vulture	47° 49′ 40″	7° 51′ 20″	48° 25′ 0″
Spica ♍	97° 50′ 0″	8° 56′ 45″	96° 45′ 9″
Heart of Leo	54° 2′ 0″	13° 58′ 0″	49° 19′ 20″
Heel of the foot of ♊	54° 33′ 45″	22° 38′ 30″	57° 4′ 10″
Bright star of ♈	58° 22′ 0″	21° 28′ 30″	63° 28′ 30″
2. Sum from the differences of right ascension			360° 0′ 9″

Ordo Stellarum — *Order of the stars*	Distance from the preceding star	Declination of 1585	Difference from the preceding right ascension
Bright star of ♈	☩	21° 28′ 30″	☩
Eye of ♉	35° 32′ 10″	15° 36′ 15″	37° 3′ 15″
Southern head of ♊	45° 5′ 0″	28° 57′ 0″	46° 53′ 53″
Heart of Leo	36° 59′ 30″	13° 58′ 0″	36° 34′ 47″
Spica ♍	54° 2′ 0″	8° 56′ 20″	49° 19′ 20″

Ordo Stellarum — *Order of the stars*	Distance from the preceding star	Declination of 1585	Difference from the preceding right ascension
Northern left hand of Ophiuchus	42° 33′ 30″	2° 33′ 15″	42° 19′ 20″
Bright star of the Vulture	55° 17′ 20″	7° 51′ 20″	54° 26′ 21″
First of the wing of Pegasus	47° 49′ 40″	13° 0′ 40″	48° 25′ 0″
Bright star of ♈	43° 37′ 15″	21° 28′ 30″	44° 58′ 0″
3. Sum from the differences of right ascension			359° 59′ 56″

[Margin: Tycho's semi-paralogism.]

II. Since, then, the first sum falls short by only 2″ and the third by 4″, while the second exceeds the equator by 9″, an imperceptible difference, Tycho asserts that the places of the stars are clear evidence both of rightly established right ascensions and of his instruments' being free of every fault in observing distances and declinations. I, however, say that this is indeed very probable, but by no means evident. For the defect of one star, however large and sensible, could have been offset by an equal excess of another, or of other stars, with the integrity of the equator, as represented by the sum, still preserved.

CAPVT XII.

De Longitudine ac Latitudine Fixarum Obſeruanda aut ex Obſeruatione deriuanda, & inde Aſcenſione Recta ac Declinatione ad alios multos annos extendenda. Cum 8. Problematibus ad id ſpectantibus.

Chapter 12. On observing the longitude and latitude of the fixed stars, or deriving them from observation, and then extending the right ascension and declination to many other years. With 8 problems relating to this.

[Margin: Three methods of observing the longitude and latitude of the fixed stars.]

I. There are chiefly three methods of obtaining by observation the longitude and latitude of the fixed stars and of other stars. The first is by the zodiacal armillaries enclosed in the astrolabe, on which see Ptolemy, Book 7, chapter 2; the older astronomers used it from Ptolemy up to Tycho. But on the insufficiency of those armillaries for showing the exact place of the stars, enough has been said by Tycho, in *Progymnasmata* volume 1, chapter 2, pp. 140 and 204, and by us in the *Liber Organicus*. The second is by the applications of the Moon's horns or edges to the stars, of which we have not a few examples, partly in Ptolemy and Copernicus, partly in Lansbergen, in Class 7 of the lunar observations in the *Treasury of Observations*. But this method is both entangled in the intricacies of the parallaxes, which have to be resolved into longitude and latitude, and serves only for stars placed near the zodiac; and it serves rather for establishing the Moon's place by the already known places of

the fixed stars than for discerning the places of the fixed stars themselves by the Moon, unless perhaps the Moon's appulse to some fixed star occurs in a total eclipse of the Moon, during which even the smallest stars appear. This method, approved by Cardanus [Girolamo Cardano], Tycho all but rejected in *Progymnasmata* volume 1, p. 146. The third is by the star's declination and right ascension, given or already found, or by the star's distance from two stars of already known longitude and latitude; this is the most universal and most exact of all. The others will be indicated in the problems of the primum mobile. But here, where the most certain foundations of the fixed stars are to be laid, only the third is to be chosen and set out by us, somewhat more accurately than the other astronomers before us have done. For Copernicus, in Book 2, chapter 4, and our Petavius [Denis Pétau], in Book 1, chapter 6, of the dissertations of the *Uranologion*, rather from a given longitude and la- titude teaches how to find the right ascension and declination, as if the former were established by observation more certainly and earlier than the latter. Tycho, however, who handed down this method most diligently of all in vol. 1 of the *Progymnasmata*, chapter 2, from p. 203 to 205, Kepler in the *Epitome of Copernican Astronomy*, p. 357, Longomontanus in Book 2 of the *Sphaerica*, chapter 2, Magini in Book 8 of the *Primum Mobile*, Problems 13 and 14, Osualdus [Erasmus Oswald Schreckenfuchs], and the others who have touched on this method have not covered all the cases. For there are 22 of them, of which 10 do not need trigonometry. The rest do need it, unless someone were to try to solve them by mechanical use of the compasses or of armillary spheres, with large planispheres or globes showing minutes and some subdivisions of minutes, by an insane mass of instruments, and were to spare no expense in order to escape the labour of trigonometry. For these, then, let the following diagram serve.

Problema 1. Longitudinem ac Latitudinem Stellæ Inquirere, datis Declinatione Aſcenſione Recta, & Obliquitate Eclipticæ.

Problem 1. To find the longitude and latitude of a star, given the declination, the right ascension and the obliquity of the ecliptic.

II. Let ABCD be the solstitial colure, in which B is the north and D the south pole of the equator, from which let the equator A♈C be described, and through which let the circle of declination BSDZB be drawn. In the same colure let G be the north and H the south pole of the ecliptic, from which let the ecliptic E♈F be described, and let arcs of a circle of latitude be drawn through the centres of the stars, as will be said below. And let the distance of the poles, BG or HD, be given, which is as great as the angle of the ecliptic and the equator, A♈E or F♈C; and equal to this is the greatest declination of the ecliptic, namely the arcs AE and CF. In our opinion this is always 23° 30′.

Diagram. A circle (the solstitial colure ABCD) set in the left column, crossed by the equator from A to C and by the ecliptic from E to F, which meet at the

centre at ♈. Curved arcs run from the poles at the top (B, G) to the poles at the foot (D, H), two of those from G in dotted line. Stars are marked by small star signs at several lettered points.

- B (top of the circle): north pole of the equator
- G (top, right of B): north pole of the ecliptic
- K (upper left, with star sign)
- E (left, on the circle), with the sign ♋ beside it
- P (upper interior, with star sign)
- M (right, with star sign)
- I (left, with star sign)
- R, X, S, T (interior, left of centre, close together)
- A (left, on the equator)
- λ, V (interior, below the equator, left of centre)
- ♈ (centre), with a sign like ♎ directly below it
- C (right, on the equator)
- N (right, with star sign)
- L (lower left, with star sign)
- Y (lower interior, with star sign)
- ♑ and F (right, at the end of the ecliptic)
- O (lower right, with star sign)
- H (foot, left of D): south pole of the ecliptic
- D (foot): south pole of the equator
- Z (upper right interior, with star sign)
- Q (on the ecliptic between E and R, italic)
- δ, θ (on the equator, right of centre)
- ε (interior, right of centre, below the equator, with star sign)
- β, μ (interior, right of centre, just below the ecliptic)

[Margin: 5 cases, when the right ascension of a star is 90°.]

III. With this established, if the given right ascension of the star is exactly 90°, its longitude is the very beginning of ♋, and the colure is at once a circle of declination and of latitude. So either the star is at A, lacking all declination, and then its south latitude is AE, and as great as the obliquity of the ecliptic; or it is at E, with declination AE equal to the obliquity of the ecliptic itself, and then it has no latitude; or it is at I, having a north declination AI, less than the obliquity of the ecliptic, and then, with the declination subtracted from the obliquity of the ecliptic, its south latitude EI will be known; or it is at K, having a north declination AK, greater than the obliquity of the ecliptic, and then, with the obliquity of the ecliptic subtracted from the declination, the north latitude EK will be known; or finally it is at L, having a south declination AL, and this, added to the obliquity of the ecliptic AE, will make up the south latitude EL.

[Margin: 5 other cases, when the right ascension is 270°.]

But if the star's right ascension is exactly 270°, it will be at the beginning of ♑ as regards longitude, and the solstitial colure will likewise be a circle of

declination and of latitude. So either the star is at C, free of all declination, and then its latitude is north, FC, and as great as the obliquity of the ecliptic; or it is at F, having a south declination equal to the obliquity of the ecliptic, and then it has no latitude; or it is at N, having a south declination less than the obliquity of the ecliptic, and the former subtracted from the latter will give the latitude NF, north; or it is at M, with a north declination, and the obliquity of the ecliptic, or the arc CF, added to this makes up the north latitude FM; or finally it is at O, with a south declination greater than the obliquity of the ecliptic, and the latter subtracted from the declination leaves the south latitude FO. And these are the ten cases in which there is no need for the analysis of triangles: rare indeed, but such as ought not to have been passed over altogether.

[Margin: On the 12 cases to be solved by trigonometry.]

IV. The remaining 12 cases arise from the difference of the 4 quadrants of the equator in which the right ascension is reckoned. In any one of these the star can be either with no declination, or with a north one, or with a south one; and so, the three cases of declination being multiplied by the 4 quadrants, 12 result. For we do not enumerate here the cases that concern the conclusions rather than the data, though we shall nevertheless point them out. A star belongs to the first quadrant of the circle when it is within 90°, to the second when it is beyond 90° but short of 180°, to the third when it is beyond 180° but short of 270°, to the fourth when it is beyond 270°. With this known beforehand:

[Margin: Case 1, in quadrant 1, without declination.]

First, let the star be at V, a point of the first quadrant of the equator, without any declination, and through it, from either pole of the ecliptic, say from H, let the arc of latitude HX be drawn as far as the ecliptic. There will then be a triangle VX♈, right-angled at X, in which the angle at ♈ is given, which is the obliquity of the ecliptic, and the base V♈, which is the right ascension itself. Therefore by the rules of right-angled spherical triangles both the longitude ♈X and the south latitude VX will become known.

[Margin: Case 2, of quadrant 1, with north declination.]

Second, let the star be at P, whose right ascension is λ♈, marked at λ by the arc of declination BPλ drawn through the star, and let it belong to the first quadrant of the equator. For the complement of the right ascension to the quadrant, that is the arc λA, measures the angle ABλ; and this subtracted from 180° gives the angle PBG. Therefore, with the arc of latitude GPQ drawn, in the triangle PBG the angle at B will be known, and the side BG, the distance of the poles, and BP, the complement of the north declination Pλ. So the base PG will not remain hidden, which is the complement of the north latitude PQ; nor will the angle BGP, which is the measure of the arc of the ecliptic EQ, whose complement to the quadrant will be the longitude sought. But if the base opposite angle B were of 90°, as is the arc GR, the star would have no latitude, and the longitude would be the arc R♈, that is the complement of the arc RE, known from the angle RGB. Finally, if it were greater than 90°, as is the arc GS, the star would be at S, and the excess above 90°, namely the arc ST,

would be the south latitude, and the longitude would be the arc T♈, that is the complement of the arc TE, known from the angle SGB. Understand these cases to be possible also in the other three quadrants.

[Margin: Case 3, of quadrant 1, with south declination.]

Third, let the star be at Y, with south declination Yλ and right ascension λ♈. For when this is subtracted from 90° the arc Aλ, or the angle ADλ, will become known; and when the arc of latitude HYX has been drawn through the star, the triangle HYD will arise, in which, as I said, the angle HDY will be known, and the side HD, the distance of the poles, and DY, the complement of the declination. And therefore by means of trigonometry the base HY will become known, which is the complement of the south latitude to the quadrant, and the angle YHD, that is XHF. So when 90°, that is the arc F♈, is removed, the arc ♈X will remain, which will be the star's longitude.

[Margin: Case 4, in quadrant 2, with no declination.]

Fourth, let the star be at V, but as a point of the second quadrant, without any declination. For in the triangle VX♎, right-angled at X, the angle X♎V will be given, which is the obliquity of the ecliptic, and the base V♎, which is the complement of the right ascension to the semicircle; and therefore the arc VX, of south latitude, will become known, and X♎, which, subtracted from the semicircle or from 180°, will leave the star's longitude.

[Margin: Case 5, in quadrant 2, with north declination.]

Fifth, let the star be at P, with north declination Pλ, but let the right ascension belong to the second quadrant of the equator. For this, subtracted from 270°, will give the angle PBG of the triangle PGB, in which BG, the distance of the poles, is given, and BP, the complement of the declination. So the base GP will become known, which subtracted from 90° will give the north latitude PQ; and the angle PGB will become known, measuring the arc EQ, which is to be added to 90° to obtain the star's longitude from ♈.

[Margin: Case 6, in quadrant 2, with De]

Sixth, let the star be at Y, but with the right ascension belonging to the 2nd quadrant, with south declination Yλ. For the excess of the right ascension above 90°, that is the arc

[Margin: clination, south.]

Aλ, will give the angle HDY in the triangle HYD, in which HD, the distance of the poles, is given, and DY, the complement of the declination. So the base HY will become known, the complement of the south latitude, and the angle YHD, which, subtracted from 270°, will leave the star's longitude.

[Margin: Case 7, in quadrant 3, with no declination.]

Seventh, let the star be in quadrant 3 with no declination, as at δ. For when an arc of latitude, say Gδβ, has been drawn through it from either of the poles of the ecliptic, there will be a triangle ♎δβ, right-angled at β, in which the angle at ♎ is the obliquity of the ecliptic, and ♎δ, the base, is the excess of the right ascension above the semicircle. So by analysis of this triangle the north

latitude βδ will easily become known, and the arc ♎β, which, added to 180°, will make up the star's longitude.

[Margin: Case 8, in quadrant 3, with north declination.]

Eighth, let the star be at Z, with north declination Zθ, and let its right ascension belong to the 3rd quadrant. For this, subtracted from 270°, will make known the angle ZBG. So when the arc of latitude GZβ has been drawn, there will be a triangle BGZ, in which the already mentioned angle at B will be known, with the sides enclosing it, namely BG, the distance of the poles, and BZ, the complement of the declination. So the base GZ will become known, which, subtracted from 90°, will give the north latitude Zβ, and the angle BGZ, measuring the arc of the ecliptic Eβ; if you add 90° to this, you will have the star's longitude.

[Margin: Case 9, in quadrant 3, with south declination.]

Ninth, let the star be at ε, with south declination εθ, and through the centre of the star let the arc of latitude Hμε be drawn. For the triangle HεD will then arise, in which the side HD, the distance of the poles, is given, and Dε, the complement of the declination; and the angle HDε becomes known by subtracting 90° from the right ascension. So the base Hε will not remain hidden, which in our case is greater than a quadrant, and therefore the excess με above the quadrant will be the north latitude. In other cases, however, it can be either a quadrant, as if the star were at μ, without latitude, or less than a quadrant, with south latitude. The angle εHD will also become known, which measures the arc μF, and which, subtracted from 270°, leaves the star's longitude.

[Margin: Case 10, in quadrant 4, without declination.]

Tenth, let the star be at δ, but as a point of the fourth quadrant, without any declination. For in the triangle ♈βδ, right-angled at β, the angle at ♈, the obliquity of the ecliptic, is given, and the base ♈δ, which becomes known when the right ascension is taken from 360°. So the north latitude βδ will not remain hidden, nor the arc ♈β; when this is subtracted from 360° the longitude will be known.

[Margin: Case 11, in quadrant 4, with north declination.]

Eleventh, let the star be at Z, with north declination Zθ and with right ascension belonging to the 4th quadrant. For by subtracting 270° from it the arc θC will be known, that is the angle ZBG of the triangle BGZ. Since in this triangle BG, the distance of the poles, is given, and BZ, the complement of the declination, the base GZ will also become known, which, subtracted from 90°, will give the north latitude Zβ, and the angle BGZ, which measures the arc Eβ. So its complement to 180°, added to 270°, will make up the star's longitude.

[Margin: Case 12, in quadrant 4, with south declination.]

Twelfth, let the star be at ε, and the rest as above in case 9, but with right ascension belonging to the 4th quadrant. For if 270° is taken from the right ascension, the angle εDC will be known, whose complement to 180° will give the angle HDε in the triangle HεD. In this triangle, with HD, the distance of the poles, and Dε, the complement of the declination, now given, the base Hε will

become known. Since in this figure it is greater than a quadrant, its excess με above the quadrant will be the star's north latitude. That base could, however, be a quadrant, without latitude, or less than a quadrant, and the complement of a south latitude. Finally the angle εHD will become known, which measures the arc μF, and which, added to 270°, will make up the star's longitude that was sought.

Problema 2. Longitudinem ac Latitudinem Stellæ Inquirere, Data eius diſtantia à duabus ſtellis notæ Longitudinis, Latitudinis, & Diſtantiæ mutuæ.

Problem 2. To find the longitude and latitude of a star, given its distance from two stars of known longitude, latitude and mutual distance.

V. We anticipate this problem here from our Book 10, and from Regiomontanus's *Epitome of the Almagest*, from prop. 7, because it serves marvellously not only for the fixed stars but also for planets whose distance has been observed from two stars of known latitude, longitude and mutual distance, either from observation or from the table of chapter 10.

So in the figure below let A be the south pole of the ecliptic and B the north; let EC be a portion of the ecliptic, and let the three semicircles of latitude be BEA, BHA and BCA.

Diagram. A lens-shaped figure set at the right of the right column, formed by two curved arcs that meet at the top and the foot, with a straight vertical line between them from top to foot and a horizontal line across the middle. Further arcs join the lettered points in the upper half and run down to the lower right. A small dot stands in the lower interior. No star signs.

- B (top)
- G (upper interior, on the vertical line)
- F (upper right, on the arc)
- D (left, on the arc, above E)
- E (left end of the horizontal line)
- H (centre)
- C (right end of the horizontal line)
- K (lower right, on the arc)
- A (foot)

[Margin: Case 1, a star at D, and stars G and F.]

First, let the star be at D, whose observed distance is DG from the star G and DF from the star F, both of north latitude. Let the latitude of these be known, HG, and hence its complement GB; and the latitude FC, and therefore its complement FB; and the distance GF. From the difference of longitude, moreover, let the arc HC be known, or the angle HBC. So in the triangle DGF, from the three given sides, which are the three distances, find the angle DGF. Then in the triangle BGF, from the given distance FG, the complement of latitude BF, and the angle GBF, which is the same as the angle HBC, find the

angle BGF. Join this to the angle DGF that was found, and subtract the sum from four right angles, or from 360°. For there will remain the angle DGB, with which, in the triangle DGB, and with the distance DG and the complement of latitude GB, find both the base BD, which will be the complement of the latitude of the star D, and the angle GBD, which will be the difference of longitude, to be added to the longitude of the star G if that star were more westerly, or subtracted if more easterly, so that the longitude of the star D may be known.

[Margin: Case 2, a star at F, and stars D and G.]

Second, let the star for which the work is being done be at F, and the two other stars D and G. For in the triangle DGF, from the three distances, gather the angle DGF as above; and in the triangle DBG, from the distance DG, the complement of latitude BD, and the angle DBG, which is the difference of longitude of the stars D and G, find the angle DGB. Subtract the sum of this and the angle DGF from 360°, and the angle BGF will be known. With this, and with the complement of latitude BG and the distance GF, find the base BF, for it is the complement of the latitude of the star F, and the angle GBF, which measures the arc HC, to be added, etc., to the longitude of the star G, so that the longitude of the star F may become known.

[Margin: Case 3, a star at G, and stars D and F.]

Third, let the star for which the work is being done be at G, and the other stars at D and F. First, then, in the triangle DGF, from the three distances given or observed, find the angle DFG. Then in the triangle DBF, from the three given sides, DF, the distance of the stars, and DB and FB, the complements of the latitudes, or from the given angle DBF, which is their difference of longitude, find the angle DFB. From this subtract the DFG that was found, and the remaining GFB will become known. With this, in the triangle GFB, and with the distance GF and the complement of latitude BF, find both the side BG, which is the complement of the latitude of the star G, and the angle GBF, which measures the arc HC, that is the difference of longitude to be subtracted from the longitude of the star F, so that the longitude of the star G may be known.

[Margin: Case 4, a star at D, and stars G and K.]

Fourth, let the star you are concerned about be at D, and one of the known stars at G, to the north, the other at K, to the south. First, then, in the triangle DGK, from the three known distances, find the angle DGK. Then in the triangle GKB, from the known distance GK and the side BK, which is the aggregate of the latitude CK and the quadrant BC, and with the angle GBK, that is HBC, which is the difference of longitude of the stars G and K, investigate the angle BGK. Join this to the angle DGK, take the sum of these from 360°, and the angle DGB will become known. With this, finally, in the triangle DGB, with the distance DG and the complement of latitude BG, find both the base BD, which is the complement of the latitude of the star D, and the angle

DBG, which measures the difference of longitude, to be subtracted from the longitude of the star G, so that the longitude of the star D may become known.

[Margin: Case 5, a star at G, and stars D and K.]

Fifth, let the star you are after be at G, and one of the known stars at D, to the north, the other at K, to the south. First, in the triangle DAK, from the angle DAK, which is the difference of the known longitudes, the distance DK and the complement of latitude AK, find the angle KDA. Afterwards in the triangle DGK, from the three known distances, find the angle GDK. Join this to the angle ADK, subtract the sum from two right angles, or from 180°, and the angle BDG will remain. With this, finally, in the triangle DGB, with the distance DG and the complement of latitude DB, find both the base BG, which is the complement of the latitude of the star at G, and the angle DBG, which is the measure of the difference of longitude, to be added to the longitude of the star D, so that the longitude of the star G may become known.

[Margin: Case 6, a star at D, and stars at F and K.]

Sixth, let the star be at D, and the other two of the same longitude, but one of north latitude at F, the other of south latitude at K. So in the triangle DFK, in which the distances DF and DK are given, and FK, either as a distance or as the aggregate of the latitudes of the stars F and K, find the angle DFK, which, subtracted from 180°, will give the angle DFB in the triangle DFB, in which the distance DF and the complement of latitude BF are given. So in that triangle investigate both the base DB, namely the complement of the latitude of the star D, and the angle DBF, namely the difference of longitude to be added to the longitude of the stars F and K if the star is, as in our figure, more easterly, so that the longitude of the star D may be known.

[Margin: Use of the table of Chapter 10.]

From these six principal cases, moreover, it will not be difficult to shape figures for the other cases, and in them to find the longitude and latitude of the star wanted. But if the distance of the two stars adopted here were neither observed nor given, one would have to hunt it out by solving one or another further triangle, and carry out the rest as above. So our table in chapter 10, showing so many distances of the fixed stars, relieves us of this labour.

Problema 3. Data Longitudine ac Latitudine Stellæ Fixæ, Inquirere Aſcenſionem Rectam & Declinationem eiuſdem.

Problem 3. Given the longitude and latitude of a fixed star, to find its right ascension and declination.

VI. It will be useful to read here, in advance, the rules for the angles at the poles, which we shall set down in chapter 13, no. 4. Then let the figure set out and explained in Problem 1 be repeated here, and let everything be done that we said was to be done in that problem, but use latitude in place of declination, or the complement of latitude in place of the complement of declination, and the arc or angle of longitude in place of the arc or angle of right ascension, in

adapting the data, and in this way you will dispatch all 22 cases. For example, if the star is at P, and its latitude PQ is given, and the star is in the first quadrant of the ecliptic, the angle PGB will be known from the complement of the longitude to the quadrant. So with it, and with the complement of latitude PG and the distance of the poles BG, find both the base BP, which will be the complement of the north declination Pλ, and the angle PBG, which measures the arc λC. So when 90°, or the arc ♈C, is subtracted, λ♈ will remain, namely the right ascension of the star P; and so for the others. For to someone already practised in that figure and in the theory of triangles the remaining cases will not remain hidden. For these, remember that a star is in the 1st quadrant of the ecliptic if its longitude is less than 90°, but in the 2nd if greater than 90° and less than 180°, in the 3rd if greater than 180° but less than 270°, and in the 4th if greater than 270°. But if it is exactly 90° or 270°, that is at the beginning of Cancer or Capricorn, the solstitial colure will be at once a circle of latitude and of declination, and from the latitude alone and the declination of the tropics the star's declination can be gathered, without trigonometry.

Diagram. The figure of Problem 1 (p. 514) printed again in the left column, as the text calls for: the solstitial colure as a circle, crossed by the equator from A to C and by the ecliptic from E to F, which meet at ♈, with arcs from the poles at the top (B, G) to those at the foot (D, H), two of those from G in dotted line, and star signs at several lettered points.

- B, G (top)
- K (upper left, with star sign)
- E (left, on the circle), with the sign ♋ beside it
- P (upper interior, with star sign)
- M (right, with star sign)
- I (left, with star sign)
- R, X, S, T (interior, left of centre, close together)
- A (left, on the equator)
- λ, V (interior, below the equator, left of centre)
- ♈ (centre), with a sign like ♎ directly below it
- C (right, on the equator)
- N (right, with star sign)
- L (lower left, with star sign)
- Y (lower interior, with star sign)
- ♑ and F (right, at the end of the ecliptic)
- O (lower right, with star sign)
- H, D (foot)
- θ (interior, right of centre, above the equator)
- ε (interior, right of centre, below the equator, with star sign)
- β (interior, right of centre, below the ecliptic)
- Z (upper right interior, with star sign)
- Q (on the ecliptic between E and R, italic)
- δ (on the equator, left of θ)

• μ (right of β, just below the ecliptic, below ε)

Problema 4. Data Longitudine, ac Declinatione Stellæ, cum Polorum Diſtantia, ſeu Obliquitate Eclipticæ; Latitudinem & Aſcenſionem Rectam Inquirere.

Problem 4. Given the longitude and declination of a star, with the distance of the poles or the obliquity of the ecliptic, to find the latitude and right ascension.

VII. In the same figure of Problem 1 let the star be at P, and let the complement of the declination BP and the distance of the poles BG be known, and, from the star's longitude, not greater than 180°, its distance from the beginning of Cancer, namely the arc EQ, or the angle PGB. For from these three data in the triangle PBG trigonometry will make known to you the base PG, which is the complement of the latitude, and the angle PBG, which is the argument of the right ascension: for it measures the arc of the equator λC. When the arc ♈C, of 90°, is subtracted from this, λ♈ will remain, the right ascension, if the star is in the first quadrant. But if it is in the 2nd, subtract λ♈ from 180°, and the right ascension will remain. But if the star were at Z, the triangle suited to the purpose would be BGZ, and the complement of the declination BZ, and from the longitude the angle BGZ would be known, as in case 8 of Problem 1, and hence GZ, the complement of the latitude, and ZBG, the argument of the right ascension, etc.

Problema 5. Data Aſcenſione Recta & Longitudine cum Diſtantia Polorum, Inuenire Declinationem & Latitudinem ſideris.

Problem 5. Given the right ascension and longitude, with the distance of the poles, to find the declination and latitude of the star.

VIII. In the same figure of Problem 1 let the star be at P. For in the triangle BPG, given the distance of the poles BG, the angle PBG from the right ascension, and the angle BGP from the longitude, by the distance from the equinoctial or solstitial points, both BP, the complement of the declination, and GP, the complement of the latitude wanted, will easily be found. When these are subtracted from 90°, the declination and latitude will be known: and so in the other 12 cases, on which see Problem 1.

Problema 6. Data Aſcenſione Recta & Latitudine Stellæ, cum Diſtantia Polorum, Longitudinem ac Declinationem eius Indagare.

Problem 6. Given the right ascension and latitude of a star, with the distance of the poles, to search out its longitude and declination.

IX. In the same figure of Problem 1 let the star be at P. For in the triangle PBG, given BG, the distance of the poles, GP, the complement of the latitude, and the angle PBG from the right ascension, by the equatorial distance from the solstitial or equinoctial points, BP, the complement of the declination, and the angle PGB, the argument of the longitude, cannot remain hidden from the trigonometer: and so in the other cases, on which see Problem 1.

Problema 7. Data Latitudine ac Declinatione cum Diſtantia Polorum, & Quadrante ad quem Stella pertinet, Longitudinem atque Aſcenſionem Rectam eius deducere.

Problem 7. Given the latitude and declination, with the distance of the poles and the quadrant to which the star belongs, to deduce its longitude and right ascension.

X. In the same figure of Problem 1 let the star be at P. For in the triangle PGB, given the distance of the poles BG, the complement of the latitude GP and the complement of the declination BP, the analysis of triangles will yield the angle at the pole of the world B, which is the argument of the right ascension, and the angle at the pole of the ecliptic G, which is the argument of the longitude: and similarly in the other cases of Problem 1.

Problema 8. Conſtituta Aſcenſione Recta, & Declinatione Fixarum pro aliquo tempore, ad alia tempora intra vnum circiter ante vel poſt ſæculum eam Extendere.

Problem 8. Given the right ascension and declination of the fixed stars as established for some time, to extend it to other times within about one century before or after.

XI. For example, let the right ascension and declination of Spica be established for the end of the year 1600 as a root, together with its longitude and latitude, by Problem 1 or 2, and suppose you wish to extend it to any time within this century, up to the end of the year 1700. First, then, in accordance with the opinion you have chosen on the motion of longitude, add to the longitude of Spica for the completed year 1600 the motion appropriate to 100 years, and you will have the longitude of Spica at the end of the year 1700. The latitude, however, will of itself be the same, unless perhaps it varies because of a variation in the obliquity of the ecliptic. Therefore by Problem 3, from the longitude and latitude of the year 1700, find the right ascension and declination of Spica, and the difference of each from the right ascension and declination of the year 1600, subtracting the smaller from the greater, etc. For that difference, distributed by proportional part over the intervening time, added to the root if the right ascension or declination increases and subtracted if it decreases, will give the right ascension and declination wanted, although the result is had more exactly by trigonometry and Problem 3. By this device Tycho, in vol. 1 of the *Progymnasmata*, p. 276, constructed a table of the right ascension

and declination of selected fixed stars for the end of the years 1600 and 1700, inserting the secular difference appropriate to them.

CAPVT XIII.

Chapter 13.

In quo Regulæ Incrementi & Decrementi in Aſcenſione Recta, & Declinatione Fixarum, cum aliquot Proprietatibus earumdem.

In which the rules for the increase and decrease in the right ascension and declination of the fixed stars, with some of their properties.

I. We suppose here that the fixed stars move not only by the diurnal motion, that of the primum mobile, on the poles of the equator, describing diurnal arcs parallel to the equator, but also by a proper motion in longitude towards the east, on the poles of the ecliptic, as will be said in chapter 16; and that meanwhile they perpetually keep among themselves the same distance that they once obtained from the beginning by divine act, as was said in chapter 10. So it comes about that by this motion they describe arcs parallel to the ecliptic, always keeping the same latitude, either with respect to the mean ecliptic or with respect to the temporary ecliptic, if that changes its obliquity; on this subject, chapter 15. With these things laid down, we must give the rules for telling whether the right ascension and declination increase or diminish as the longitude increases.

II. As regards declinations, from the suppositions made above and from inspection of the globe itself there arise the following propositions, which can serve as very certain rules for practice.

Proposition 1. *For fixed stars moving by their motion in longitude from Capricorn through ♈ to Cancer, north declination increases and south declination decreases.* Ptolemy, Book 7, chapter 3. *Reinhold*, in the *Theorica* of the eighth sphere, p. 230.

Proposition 2. *For fixed stars moving by their motion in longitude from Cancer through ♎ to Capricorn, north declination decreases, but south declination increases.*

[Margin: Error of certain astronomers.]

III. As regards right ascension, it has seemed to some that it increases continually as the longitude of the fixed stars increases. But this is false, since there are some stars whose right ascension decreases for many years as their longitude increases, although at other times it increases. Such in this century is the bright star in the Wain, or shoulder, of Ursa Minor. For its right ascension in the year 1600 was 222° 59′ 12″, and yet it was found by us in the completed year 1644 to be 222° 54′, and for the year 1700, 222° 47′ 7″, as will be clear to anyone deriving it trigonometrically by Problem 3 of the preceding chapter from its longitude and latitude, even though those are Tycho's. For its longitude in the

year 1600 was ♌ 7° 16′ 30″, and in the year 1700 it will be ♌ 8° 41′ 30″, while its north latitude at both times is 72° 51′ 30″.

[Margin: A suspicion about the hesitation of Tycho or of the Tychonians.]

From this I suspect it came about that Tycho or his calculators, when they had come upon this star, found its right ascension smaller in the year 1700 than in the year 1600, and yet believed that it too must increase as the longitude increases, chose to cut this knot, by cutting the shoulder of Ursa Minor out of the catalogue of right ascensions, rather than set about untying it. For no more probable cause can be offered for their having passed over this star, which is of the second magnitude and very conspicuous.

The rules of this increase and decrease, together with other propositions most worth knowing, will be given more conveniently and demonstrated in chapter 19, in the Corollaries of the last Problem.

[Margin: Rule on the angles at the poles.]

IV. Since, however, the angles at the pole of the ecliptic made by the solstitial colure and by a circle of latitude, or the angles at the pole of the equator made by the same colure and by a circle of declination, often come into use in the Problems of the preceding chapter and elsewhere, take the rules for these angles, in which the given right ascension is taken for the angle at the pole of the equator, and the longitude for the angle at the pole of the ecliptic. When the longitude or the right ascension is exactly 0°, 180° or 360°, the angles, each at its own pole, are right on both sides; otherwise one is acute and the other obtuse; but at 90° and 270° there is no angle.

De Angulis ad Polum Ecliptic æ factis à Coluro Solſtitiorum & à Circulo Latitudinis.

On the angles at the pole of the ecliptic made by the solstitial colure and by a circle of latitude.

Rule 1. In the first quadrant of the ecliptic, namely from the beginning of ♈ to the beginning of ♋, subtract the longitude from 90°, and the acute angle at the pole of the ecliptic will be left. Add 90° to the longitude, however, and the obtuse angle will result.

Rule 2. In the second quadrant, subtract 90° from the longitude, and the acute angle will be left; subtract the longitude from 270°, however, and the obtuse angle will remain.

Rule 3. In the third quadrant, subtract the longitude from 270°, and the acute angle will be left; subtract 90° from the longitude, however, and the obtuse angle will result.

Rule 4. In the fourth quadrant, subtract 270° from the longitude, and you will have the acute angle; subtract the longitude from 450°, however, and you will have the obtuse angle.

De Angulis ad Polum Æquatoris.

On the angles at the pole of the equator.

Do everything as in the preceding rules, but in place of the longitude use the right ascension of the star.

CAPVT XIV.

Chapter 14.

De Tabula Aſcenſionis Rectæ, Declinationis, Longitudinis, ac Latitudinis ſelectarum Fixarum à nobis conſtituta, eiuſque diſcrimine à Tychonica & ab alijs.

On the table of right ascension, declination, longitude and latitude of selected fixed stars established by us, and its difference from Tycho's and from others.

I. Since from Chapter 11 up to this point we have set out the method of constructing a table of this kind, and its most solid foundations were laid by us in chapters 11 and 12, it remains for us to present the table of selected fixed stars, which indeed differs only very slightly from Tycho's. It is no wonder, moreover, that so small a divergence has arisen between our observations and those of that great astronomer, or the conclusions drawn from the observations, since we did not use his dioptras but our own, which we shall describe in the *Liber Organicus*, and which, in the judgement not only of ourselves but of those who were present when we were observing and to whom we showed both the Tychonic method and our own, give a somewhat clearer and finer observation.

[Margin: The cause of the disagreement indicated.]

Indeed, a greater difference is found between Tycho and the Landgrave of Hesse. In addition, Tycho uses a greater parallax of the Sun and a greater obliquity of the ecliptic than we do; and so a slight difference has arisen in the right ascension of the Sun, on which the ascension of the fixed stars depends. Let us indeed hear Tycho himself, on p. 93 of the *Epistolae Astronomicae*, writing thus to the Landgrave of Hesse: *"That we nevertheless differ in the longitudes of the stars by about five minutes, by which mine are found to stand further forward, deserves its excuse among those who have sufficiently surveyed the perplexing labyrinths of this subtle inquiry, etc.*

since those ancient leaders of astronomy hardly attained the truth in these matters to within a sixth part of a degree." These, then, are those five or six minutes about which Kepler said, in the groundwork of his *Ephemerides*, p. 15: *There remains undecided, concerning six minutes of the Sun's motion from the fixed stars, a dispute between the Landgrave and Brahe, and the testimonies waver this way and that. For the parallax of Venus cannot be as great as Brahe assumed in the* Progymnasmata*, fitted to the parallax of the Sun handed down by the ancients.* For we have already seen in chapter 11, Problem 6, that Tycho observed the right ascension of the bright star of ♈, with which he began the others, by its distance from

Venus. Accordingly Longomontanus, who uses a smaller parallax of the Sun than Tycho, always falls short of Tycho's catalogue in his catalogue of fixed stars by a single minute in their longitude, as he himself also attests in Book 1 of the *Theorica*, chapter 4, saying: *In restoring the fixed stars, observations repeated where needed, until they settled at as nearly as possible the same minutes; and because we had diminished the mean longitude of the Sun by more than a minute, we subtracted as much, star by star, from the longitude of the stars in Tycho's canon.* Granted, Kepler sets down Tycho's numbers in his catalogue, notwithstanding the reduction of the parallax.

[Margin: Examples of the disagreement among the older astronomers on the places of the fixed stars.]

But far greater is the disagreement among the older astronomers, both with one another and with the more recent ones. Noticing this, that shrewd man Petrus Nonius [Pedro Nunes] says in Book 2, *De Observatione*, p. 49: *But either through the deceptiveness of their instruments, or because the latitudes of the places where they made their observations were not sufficiently ascertained, they disagree among themselves. For Vernerus [Johannes Werner] found Spica at 16° 54′ of Libra, but Copernicus, using the same method, at 17° 14′ of the same sign; and Girolamo Cardano says that in Italy, twenty-two years later, he found the same star again, by eleven observations he himself made, at 16° 18′ of Libra. We meanwhile, although we make constant observations of the stars, since we do not yet have such instruments as we could confidently use, affirm nothing for certain and hold with al-Battānī.* But as a specimen of this disagreement, let there stand the longitude and latitude of ten of the more notable stars observed in the years 1585 and 1586 by the Landgrave of Hesse and by Tycho, with the Alphonsine and Copernican calculation, as found in Tycho, in the *Epistolae Astronomicae*, pp. 39, 52, 55. To these I have added the place owing to the same stars by the calculation of Ptolemy, who fixes his roots in the year of Christ 132, and of al-Battānī, in the year 880; of these the former requires 100 years for one degree, the latter only 66.

[Margin: A specimen of the same disagreement in 10 stars.]

Longitude of the fixed stars from the beginning of Aries (the first six value columns); latitude (the last three).

In the year of Christ 1586. 10 stars	Long.: Ptolemy	Long.: al-Battānī	Long.: Al-fonso	Long.: Coper-nicus	Long.: Land-grave	Long.: Ty-cho	Lat.: Ptolemy, al-Battānī, Alfonso, Copernicus	Lat.: Land-grave	Lat.: Ty-cho
Aldebaran	57° 12′	64° 40′	63° 5′	63° 57′	64° 6′	64° 1′	5° 10′ A	5° 28′	5° 30½′
Orion, following shoulder	76° 32′	84° 0′	82° 25′	83° 17′	83° 5′	83° 1′	17° 0′ A	16° 3½′	16° 6½′
Rigel of Orion	64° 22′	71° 50′	69° 35′	70° 27′	71° 12′	71° 5′	31° 30′ A	31° 9′	31° 11′
Sirius	92° 12′	99° 40′	98° 5′	98° 57′	98° 28′	98° 22⅓′	39° 10′ A	39° 28′	39° 31′
Regulus	147° 12′	144° 30′	142° 55′	143° 47′	194° 11′	194° 6′	0° 10′ B	0° 32′	0° 27′
Heart of Hydra	134° 32′	142° 0′	140° 25′	141° 17′	141° 38′	141° 34′	20° 30′ A	22° 20′	22° 24′
Tail of Leo	149° 2′	156° 30′	164° 55′	165° 47′	166° 0′	165° 51⅓′	1° 50′ B	12° 24½′	12° 19′
Spica	191° 14′	198° 40′	197° 5′	197° 57′	198° 9′	198° 4′	2° 0′ A	1° 56′	1° 59′

In the year of Christ 1586. 10 stars	Long.: Ptolemy	Long.: al-Battānī	Long.: Al-fonso	Long.: Coper-nicus	Long.: Land-grave	Long.: Ty-cho	Lat.: Ptolemy, al-Battānī, Alfonso, Copernicus	Lat.: Land-grave	Lat.: Ty-cho
Arcturus	191° 32′	199° 0′	197° 25′	198° 17′	198° 35½′	198° 27′	31° 30′ B	31° 7½′	31° 2′
Heart of Scorpius	237° 12′	244° 40′	243° 5′	243° 57′	244° 5′	244° 1′	4° 0′ A	4° 25′	4° 27′

There also exists, at the end of the works of Cardinal Cusanus, a table of 53 fixed stars verified to the year 1440 from his own observation and that of the Cardinals of Nicaea and of Ailly, which departs not a little from the Alphonsine and from the truth.

[Margin: Authors of the catalogue of right ascensions and declinations. ✳]

II. The principal authors who set out catalogues of the fixed stars with right ascensions and declinations are: Io. Stadius [Johannes Stadius], p. 215 of the *Canones Bergenses*; Magini, in the *Introduction to the Ephemerides*, part 4, chapter 5, from the Prutenic Tables for the year 1581, together with the mediations of the heaven [Mediationibus cæli]; Origanus, vol. 1 of the *Ephemerides*, p. 204, from Copernicus and Tycho, with the mediations of the heaven, for the year 1620; Grienberger, in his catalogue from p. 57, from Tycho, Piffero [?] and Clavius, for the year 1600; Argoli, in the *Canones* of his *Ephemerides*, after chapter 29, from Tycho, for the year 1640; Francesco Montebruni, at the end of his *Ephemerides*, partly from Tycho and partly from Lansbergen, for the year 1650; Tycho himself, vol. 1 of the *Progymnasmata*, p. 276. On the observations of declinations made by Aristillus, Timocharis, Hipparchus and himself: Ptolemy, Book 7, chapter 3; Tycho, vol. 1 of the *Progymnasmata*, especially pp. 155, 276, 361, chapter 2 passim, and in the *Epistolae Astronomicae*, p. 52; and Lansbergen in the *Treasury of Observations*, Class 7.

Our table of 70 stars, verified to the completed year 1644, with right ascension, declination, longitude and latitude, and with an equation to be added or subtracted so that it may serve back to the year 1500 and forward to 1800, is at the end of this Sixth Book, so that it may be found more readily.

CAPVT XV.

Chapter 15.

An Stellæ Fixæ Latitudinem mutent per ſe, aut per accidens ad mutationem Obliquitatis Eclipticæ.

Whether the fixed stars change their latitude of themselves, or incidentally, with a change in the obliquity of the ecliptic.

I. It is an old opinion, held ever since Hipparchus, that the non-wandering stars move by a proper motion in circles parallel not to the equator but to the ecliptic, and described from the poles of the ecliptic. Hipparchus and Ptolemy thought it a sufficient argument for this that they had moved in longitude,

according to the succession of the signs, in such a way as always to keep the same distance from the ecliptic in the circles of latitude, that is, the same latitude.

[Margin: Hipparchus's opinion on the immutable latitude of the fixed stars.]

And of Hipparchus, indeed, Ptolemy says as follows in Book 7 of the *Magna Syntaxis*, chapter 3: *Hipparchus too appears to have agreed that this motion takes place about the poles of the zodiac. For in his treatise on the displacement of the solstitial and equinoctial points he gathered, from the observations of Timocharis and his own, that Spica has kept the size of its distance in latitude with respect not to the equinoctial circle but to the ecliptic, and that both formerly and afterwards it was 2° farther south than the circle through the middle of the signs. For this reason, in his treatise on the length of the year he assumes that this motion takes place on the poles of the zodiac. Yet he was in doubt (as he himself admits), since he neither regarded the observations made in the time of Timocharis as certain, because they had been taken too crudely, nor was the time that had elapsed in the meantime sufficient for a perfect understanding of this matter.* Adding his own opinion straight after these words, he goes on to say: *But we, finding this through observations over a longer time, and in nearly all the non-wandering stars, affirm that their motion takes place on the poles of the zodiac:*

for observing their distances from the zodiac in latitude, in a great circle described through the poles of the zodiac, we find them nearly the same as and equal to those gathered and written down by Hipparchus, or with the smallest difference, such indeed as can creep in through error on the part of any observer. And a little later: *But from all of these the same latitude with respect to the circle through the middle of the signs is found.* He confirms this, first, from the third, or middle, following star of the Pleiades, whose north latitude was observed both by Timocharis at Alexandria in the year of Nabonassar 465 and by Agrippa in Bithynia in the year of Nabonassar 840, and was found by both to be 3° 40′. Second, from Spica, whose south latitude, very nearly 2°, was observed both by Timocharis in the year of Nabonassar 466 and by Menelaus at Rome in the first year of Trajan, that is, of Nabonassar 845. Third, from the northern star in the brow of Scorpius, whose north latitude of 1° 12′ he says was observed both by Timocharis in the year 454 of Nabonassar and by Menelaus in the year 845 of Nabonassar; although he also uses these three pairs of observations to confirm the motion of 1° in 100 years.

[Margin: Al-Battānī's opinion in favour of the immutability of latitude.]

II. This is the opinion of Ptolemy, to which al-Battānī subscribed, who, though nearly a thousand years after Hipparchus, nevertheless found no variation in the latitude of the fixed stars. For he says in chapter 51: *The motion of the fixed stars has been found to be on the two poles of the circle of the signs; and since the time their motion was detected it has in no way departed from this, nor have their latitudes been altered.* [Margin: And Alfonso's.] So too Alfonso in his Tables assigned to the fixed stars the same latitude as Ptolemy. [Margin: Regiomontanus's.] Regiomontanus holds the same in the *Epitome of the Almagest*, Book 7, Proposition 3, and [Margin: Peuerbach's.] Peuerbach with all

his commentators, in the *Theorica* on the motion of the eighth sphere, among whom [Margin: Reinhold's.] Reinhold says, p. 230: *These stars are gradually carried to other places on the poles of the ecliptic, not of the equator. A clear argument of this is that in that progress they change their declination, not their latitude.* [Margin: Copernicus's.] Nor did Copernicus himself think otherwise, since he took all motion away from them, and in Book 2, chapter 14, attributed to the fixed stars the same latitudes as Ptolemy, and in Book 3, chapter 2, asserted this immutability of latitude as beyond doubt. Finally, Augustinus Riccius [Agostino Ricci] defends the same immutability in his treatise on the motion of the eighth sphere, p. 31, and Kepler in the Rudolphine Tables, chapters 12 and 34, precept 195, whose words we shall give below. But Boulliau, in Book 5 of the *Astronomia Philolaica*, chapter 4, whose words and reasons we shall likewise adduce shortly, pronounces it uncertain.

[Margin: By whom a change in the latitude of the fixed stars has been asserted.]

III. The contrary opinion, however, was first discovered, so far as I know, by Pomponius Gauricus [Pomponio Gaurico], or rather by Thebit [Thābit ibn Qurra] and Regiomontanus in the treatise on the motion of the eighth sphere. [Margin: The opinion of Pomponio Gaurico.] Praising them, Gaurico affirms that the north latitudes of the fixed stars have decreased since the time of Ptolemy by two-fifths of a degree, and that the south latitudes have increased by the same amount. [Margin: The doubt of Stadius.] So Io. Stadius reports in his Commentary on the fixed stars, chapter 1, adding his own opinion in these words: *But this will have to be ascribed either to the obliquation of the ecliptic, narrower by two-fifths since the time of Ptolemy up to now, or the matter itself, otherwise of doubtful credit, was to be propped up by observations, and hence by geometrical demonstrations.*

[Margin: Christoph Rothmann's.]

These were succeeded by Christoph Rothmann, who, as Tycho reports in a letter to the Landgrave of Hesse dated 1587 January 18 and in another to Rothmann dated January 20 of the same year, affirmed that the latitudes of the fixed stars had been detected by him to have varied from the amount assigned to them in antiquity.

[Margin: The opinion of Tycho, Longomontanus and Lansbergen.]

Tycho himself, however, in the same place and in vol. 1 of the *Progymnasmata*, chapter 2, pp. 233 and 247, as also Longomontanus at the end of his commentary on the fixed stars, and Lansbergen in his tables, precept 12 and Canon 3, which is found on p. 103, and in the *Theorica motuum*, chapter 8, consider that the fixed stars have undergone no change of latitude other than what comes to them from an external cause and incidentally, through the approach or withdrawal of the ecliptic as it changes its obliquity. If any greater change is found in their latitude than the change of the ecliptic allows, Tycho thinks this is to be ascribed partly to less exact observation, partly to calculation and a slip in solving the triangles, partly to the negligence of copyists, or to inexperience in

a matter of such subtlety. And he attests that Rothmann himself was of the same opinion, in the letter already mentioned, p. 65, in these words: *That you have noticed that the fixed stars change their latitudes, and this proportionally, according to the variation of the obliquity of the ecliptic, pleases me greatly. For I too had long been gathering to myself that something of the kind had happened:* etc. And a little later: *And since this matter is of great moment, and has been noticed by no one else, so far as I know, except you alone,* etc. But Tycho is mistaken, if what Stadius reports from Thābit about Gaurico is true, that he long ago caught the scent of this variation.

[Margin: Tycho's foundations and method.]

IV. Tycho's foundation, in that second letter and in the *Progymnasmata*, and his method of establishing this change, are as follows. First, because he held suspect the numbers noted in the table and in the common version of Ptolemy, on the ground that they do not agree with the unvarying distance of the fixed stars, he chose Spica, as more certainly observed by Timocharis, Hipparchus and Ptolemy, both because of its nearness to the equinoctial point and because Ptolemy often mentions it. And from the declinations of Spica observed by them, which Ptolemy gives in Book 7, chapter 3, assuming the obliquity of the ecliptic in that age as 23° 51′ 20″ and the south latitude of Spica as 2°, he deduced the longitude of Spica by triangles, as in the following small table [laterculo].

With the obliquity of the ecliptic set at 23° 51′ 20″ and the south latitude of Spica at 2°.

And with the observed declination of Spica, for	Declination	Longitude of Spica
Timocharis	1° 24′ Bor.	♍ 21° 53′
Hipparchus	0° 36′ Bor.	♍ 23° 53′
Ptolemy	0° 30′ Austr.	♍ 26° 38′

Second, from the difference in longitude of several selected stars from the longitude of Spica, observed by himself, he gathers their longitude appropriate to the time of Timocharis, Hipparchus and Ptolemy. Third, from that longitude, together with their declination observed by the same men and reported by Ptolemy in Book 7, chapter 3, and with the distance of the poles, or the obliquity of the ecliptic, appropriate to their age, he computes their latitudes by Problem 4 of our Chapter 12. Comparing these with one another and with the latitude appropriate to them in the year of Christ 1600, he gathers that as great a difference has arisen in it as the change in the obliquity of the ecliptic requires, apart from Aldebaran, Sirius and the bright star of the Vulture. Thus, for example, the Head of Castor is distant in longitude from Spica 93° 35′ by Tycho's observation; therefore in the time of Timocharis it was in ♊ 18° 18′, since Spica was then in ♍ 21° 53′, etc. But so that Tycho's principles and conclusions can be surveyed at one glance, here is a table put together from p. 69 of his *Epistolae Astronomicae* and from vol. 1 of the *Progymnasmata*, from

p. 236, in which, however, he sets the ancient latitude of the bright star of the Pleiades and of the more northerly star in the brow of Scorpius from the very observation of the ancients and the calculation of Ptolemy, Book 7, chapter 3, not from his own.

Synopsis of Tycho's calculation for the change in the latitude of the fixed stars.

Names of the stars	Observing astronomers	Tychonic longitude	Observed declination	Deduced latitude	Difference of latitude from the Tychonic
Head of Castor	Timocharis	♊ 18° 18′	33° 0′ B	9° 43′ B	19′
	Hipparchus	♊ 20° 18′	33° 10′ B	9° 42′ B	20′
	Ptolemy	♊ 23° 3′	33° 24′ B	9° 45′ B	17′
	Tycho	♋ 14° 41′	32° 41′ B	10° 2′ B	0′

Remainder of the synopsis of Tycho's calculation for the change in the latitude of the fixed stars.

Names of the stars	Observing astronomers	Tychonic longitude	Observed declination	Deduced latitude	Difference of latitude from the Tychonic
Head of Pollux	Timocharis	♊ 21° 20′	30° 0′ B	6° 26′ B	12′
	Hipparchus	♊ 23° 20′	30° 0′ B	6° 19′ B	19′
	Ptolemy	♊ 26° 5′	30° 10′ B	6° 22′ B	16′
	Tycho	♋ 17° 43′	28° 55′ B	6° 38′ B	0′
Bright star of the Vulture	Timocharis	♐ 29° 50′	5° 48′ B	29° 39′ B	19′
	Hipparchus	♑ 1° 50′	5° 48′ B	29° 39′ B	19′
	Ptolemy	♑ 4° 35′	5° 50′ B	29° 41′ B	17′
	Tycho	♑ 26° 9′	7° 54′ B	29° 21½′ B	0′
Left shoulder of Orion	Timocharis	♉ 19° 0′	1° 12′ B	17° 12′ A	19′
	Hipparchus	♉ 21° 0′	1° 48′ B	17° 6′ A	13′
	Ptolemy	♉ 23° 45′	2° 30′ B	17° 3′ A	10′
	Tycho	♊ 15° 23′	5° 55′ B	16° 53′ A	0′
Right shoulder of Orion	Timocharis	♉ 26° 49′	3° 50′ B	16° 23′ A	17′
	Hipparchus	♉ 28° 49′	4° 20′ B	16° 17′ A	11′
	Ptolemy	♊ 1° 34′	5° 15′ B	15° 54′ A	12′
	Tycho	♊ 23° 12′	7° 16′ B	16° 6′ A	0′
Heart of Leo	Timocharis	♋ 27° 54′	21° 20′ B	0° 23′ B	3′
	Hipparchus	♋ 29° 54′	20° 40′ B	0° 9′ B	17′
	Ptolemy	♌ 2° 39′	19° 50′ B	0° 4′ B	22′
	Tycho	♌ 24° 17′	13° 53′ B	0° 26½′ B	0′
Heart of Scorpius	Timocharis	♏ 7° 0′	18° 20′ A	4° 14′ A	13′
	Hipparchus	♏ 9° 45′	19° 0′ A	4° 14′ A	13′
	Ptolemy	♏ 12° 25′	20° 15′ A	4° 35′ A	8′
	Tycho	♐ 4° 13′	25° 26′ A	4° 27′ A	0′
Aldebaran	Timocharis	♉ 7° 49′	8° 45′ B	5° 56′ A	25′
	Hipparchus	♉ 9° 49′	9° 45′ B	5° 33′ A	2′
	Ptolemy	♉ 12° 34′	11° 0′ B	5° 8′ A	23′

Names of the stars	Observing astronomers	Tychonic longitude	Observed declination	Deduced latitude	Difference of latitude from the Tychonic
	Tycho	♊ 4° 12′	15° 38′ B	5° 31′ A	0′
Bright star of the Pleiades	Timocharis	♈ 29° 0′		3° 40′ B	15′
	Agrippa	♉ 3° 0′		3° 40′ B	15′
	Tycho	♉ 24° 45′		3° 55′ B	0′
More northerly star in the brow of Scorpius	Timocharis	♏ 2° 0′		1° 20′ B	15′
	Menelaus	♏ 6° 0′		1° 20′ B	15′
	Tycho	♏ 23° 20′		1° 5′ B	0′
Sirius	Timocharis	♊ 12° 12′	16° 20′ A	39° 18′ A	13′
	Hipparchus	♊ 14° 12′	16° 0′ A	39° 10′ A	21′
	Ptolemy	♊ 16° 57′	15° 45′ A	39° 7′ A	24′
	Tycho	♋ 8° 35′	16° 11′ A	39° 31′ A	0′

[Margin: Tycho's foundation is shaken.]

V. Yet this whole great apparatus of 11 stars has only four that favour Tycho's opinion: namely the Head of Castor, the Bright star of the Pleiades, the northern star of Scorpius and, if you stand by the Tychonic reduction, the Bright star of the Vulture. For the ancient latitude of these stars differs from the Tychonic, and very nearly from ours, by as much as the obliquity of the ecliptic is said to have been greater at those times, namely 19′ or 20′, or, if we are speaking of the times of Agrippa and Menelaus, 15′. It has two stars, though, that protest strongly against Tycho, namely the Heart of Leo and Aldebaran, whose ancient latitude differs from that of our own time by no more than 2′ or 3′, if for the former you follow Timocharis and for the latter Hipparchus. The remaining five, since they do not keep the proportion owed to the variation of the obliquity, but have a greater difference of latitude when the difference of obliquity was less, and a lesser one when it was greater, certainly do not vote for Tycho, but rather against him.

Further, to draw the 7 stars that resist him round to what he wishes, Tycho tries to cast suspicion on the declinations observed by the ancients, or on the figures for them reported by Ptolemy; and in some places he does this with a probable conjecture. But why does he not call into suspicion the declinations of the four former stars? Surely because, taken as they stand, they support his opinion. I ask likewise why he rejects those noted by Ptolemy and substitutes others, sought out by his own calculation, in their place, and yet accepts the latitudes of the Bright star of the Pleiades and of the northern star in the brow of Scorpius as Ptolemy reports them, just as he accepts the latitude of Procyon on p. 71 of the *Epistles*? Certainly for no other reason than that the former harm an opinion he has already formed, while the latter favour it. So Tycho concludes with too much exaggeration, on p. 246 of the *Progymnasmata*, that *through the stars listed it has been proved by sufficiently evident testimony that the latitude of the fixed stars changes according to what the change in the obliquity of the ecliptic requires.*

VI. To give my own view on this matter, then, I shall set it out distinctly in three propositions.

[Margin: 1st conclusion.]

Propoſitio 1. Si Obliquitas Eclipticæ mutatur, Latitudines tamen Fixarum non mutantur neceſſariò.

Proposition 1. If the obliquity of the ecliptic changes, the latitudes of the fixed stars nevertheless do not necessarily change.

This is proved. For if the fixed stars by their proper motion describe circles parallel to the ecliptic, they can certainly direct their course by the moving ecliptic and its poles, that is, by the ecliptic that the centre of the Sun describes at the time by its annual motion. Indeed, it is necessary to use this moving ecliptic whenever the latitude and longitude of the fixed stars are deduced from their right ascension, as observed through the distance from the Sun: for then the right ascension of the Sun, which is sought as the foundation for those quantities, depends on the obli- quity of the current ecliptic, and is deduced trigonometrically from its angle and the declination of the Sun.

[Margin: 2nd conclusion.]

Propoſitio 2. Si Obliquitas Eclipticæ Solaris mutatur, & pro ratione huius mutationis mutatur Latitudo Fixarum, ob Eclipticæ acceſſum ac receſſum ad ſtellas Fixas; Ecliptica cui Fixæ parallelῶs incedunt, diuerſa eſt ab Ecliptica ſolari.

Proposition 2. If the obliquity of the Sun's ecliptic changes, and the latitude of the fixed stars changes in proportion to this change, because the ecliptic approaches and recedes from the fixed stars, the ecliptic to which the fixed stars run parallel is different from the solar ecliptic.

This is proved, because if the fixed stars by their motion in longitude described a path parallel to the Sun's moving ecliptic and followed it, they would retain the same latitude: as it receded from the equator they would recede, and as it approached they would likewise approach. Since, then, the opposite is posited, it follows that the ecliptic the fixed stars have regard to of themselves is not the Sun's moving ecliptic.

[Margin: Kepler's corollaries from Tycho's hypothesis on the extrinsic change in the latitude of the fixed stars.]

Kepler, moreover, seems to have indicated these two distinct ecliptics in chapter 12 of the Rudolphine Tables, where he says: *Although Tycho did not entirely deny that the obliquity was greater under Ptolemy and Hipparchus, he nevertheless detected this: that the fixed stars do not undergo this variation together with it. For the ecliptic is not fixed beneath the fixed stars, so as to depart from and approach the equinoctial together with them; rather, the ecliptic alone nods away from or towards both the fixed stars and the equinoctial. If this is so, then that Prutenic variation of*

declinations and ascensions becomes altogether useless and superfluous, at least for the practical treatment of the fixed stars. For however great the declination and ascension of a fixed star comes out through the ancient obliquity of the ecliptic and the star's ancient latitude from it, exactly so great does each come out through both modern values as well. From this Tychonic hypothesis the same Kepler adds, in the last chapter of the Rudolphine Tables, precept 195: *For although the latitudes of the fixed stars perhaps change from the ecliptic, this nevertheless happens not by a real motion of the fixed stars, but by the departure of the ecliptic itself from the fixed stars and the equator: where the fixed stars and the equator, with respect to this change at least, do not wink at one another.*

In this hypothesis, however, it would be better to note down the latitudes that belong to the fixed stars in themselves and immutably with respect to their own proper ecliptic, whether that is the ecliptic midway between the least and the greatest solar ecliptic, which Kepler calls the Royal Road, or not, than to put only the current latitudes in the catalogue, provided there were a way of distinguishing it from the Sun's ecliptic.

[Margin: 3rd conclusion.]

Propoſitio 3. Probabilius eſt Fixarum ſtellarum Latitudinem non mutari: Poſita verò Eclipticæ obliquitatis mutatione, aut probabilior eſt Latitudinis Fixarum Immutabilitas aut incerta Mutabilitas.

Proposition 3. It is more probable that the latitude of the fixed stars does not change; but, given a change in the obliquity of the ecliptic, either the immutability of the latitude of the fixed stars is the more probable, or its mutability is uncertain.

First, because if any foundation exists, above all the Tychonic one discussed above, it rests on the change in the obliquity of the ecliptic; but it is more probable that the obliquity of the ecliptic is immutable, as we have sufficiently established in Book 3, chapter 27.

Second, because there are some fixed stars whose ancient latitude has been found the same as the one recently observed, however much our opponents assume the ancient obliquity in the time of Hipparchus and Ptolemy to have been greater by about 22′. Of this kind are not only the latitude of Capella, which is now 22° 50½′ and in Hipparchus's time was 22° 50′, and that of Aldebaran, which under Hipparchus was 5° 33′ and is now 5° 31′, and Regulus, whose latitude under Timocharis was 23′ and is now 26′, as Tycho himself deduces and admits, in the places cited above, from the ancient obliquity, from the declinations they observed and from the corrected longitude; but also far more other stars, as anyone running through Ptolemy's catalogue and comparing it with Tycho's will find on every side. From this a probable argument is also taken against the change in the obliquity of the ecliptic. Tycho, on p. 246 of the *Progymnasmata*, may think that, other things being equal, it is more incredible that the ancients erred in observing the obliquity of the ecliptic

than in the declinations of the fixed stars, since they could easily obtain the obliquity by the parallactic rulers. But Tycho does not notice that the obliquity of the ecliptic was observed by Ptolemy with a quadrant of no great size, as is clear from Book 1 of the *Almagest*, chapter 11, whereas the rules were devised by him later, when he began to observe the parallaxes of ☾, as is evident from Book 5 of the *Almagest*, chapter 12.

Third, there are many stars whose latitude at the time of the ancient observations is found to be greater than the recent latitude, when it should have been less, and some whose latitude is found to be less, when it should have been greater, if it had to change according to the variation in the obliquity of the ecliptic. Of this kind are not only Aldebaran, Sirius and the Northern Scale, as Tycho himself admits above, but far more others passed over in silence by Tycho, which meet anyone comparing Ptolemy's catalogue with Tycho's on every side. Such are the Bright star of the Vulture, Scheat of Pegasus, the Head of Andromeda and the Heart of Hydra.

Fourth, there are many stars whose latitude should have increased and did increase, or should have decreased and did indeed decrease, but neither exactly nor nearly by as much as the change in the obliquity admitted by our opponents required. Of this kind are not only some enumerated in the preceding catalogue, but very many others in Ptolemy's catalogue compared with Tycho's: such as the three in the tail of the Great Bear, the Bright star of Lyra, of Corona and of Aries, the Tail of Cygnus, the Side of Perseus, the Shoulder of Auriga, the Loins of Leo[54], the Shoulders of Orion, Fomahant [Fomalhaut], and very many others of lesser note.

Fifth, since in the places adduced Tycho sometimes accepts latitudes or declinations as they stand in Ptolemy, namely when they support a latitude of the fixed stars that changes in step with the change of the obliquity, and sometimes does not accept them, and calls them into suspicion of error in the ancient observations or in the transcription of the numbers, one may reason as follows. Either it is more probable that most of those noted by Ptolemy are false or erroneous, or that fewer are. If most, then from the ancient observations compared with more recent ones it either remains doubtful whether the latitude of the fixed stars changes or not, or else, in such a doubt, the pronouncement should rather be for immutability, because that agrees better with the immutable distance in longitude and with a motion parallel to the ecliptic itself known to us; nor should a new motion be charged to the heavens without sufficient foundation. But if it is more probable, as it certainly seems to me, that not most but fewer of those noted by Ptolemy suffer from faults or errors, then, since most of Ptolemy's do not support the mutability of latitude, it is more probable that latitude does not change.

[Margin: Boulliau's prudent judgement.]

[54]the page prints *Luni bi Leonis*, for *Lumbi Leonis*

This controversy can certainly never be settled evidently unless the numbers of the latitudes that Tycho, we and other more recent astronomers hold as ratified and evidently observed reach posterity uncorrupted, and posterity after many centuries observes the motion of the fixed stars with equal evidence. This is very nearly the opinion of Boulliau, Book 5, chapter 4, who, having acknowledged that the latitudes of few stars stand for Tycho and of many against him, said prudently: *From the contrariety of these observations we can certainly draw nothing. Nor are the earlier examples cited sufficient to establish that change of latitude. When, therefore, after some centuries there have arisen astronomers who will treat the heavenly matters with the same diligence and expense with which Tycho Brahe, a man never sufficiently praised, took them up in the past age, they will be able to pass a certain judgement on this variation of latitude.* In the same place he affirms that in a very old codex of Ptolemy, kept in the library of the Most Christian King, some numbers are noted in capital Greek letters otherwise than they commonly circulate, and he adduces four examples of this ambiguity. Yet expense on the Tychonic scale is not necessary for this or other astronomical work, as I shall show in the *Liber Organicus*.

SCHOLION.

Scholium.

[Margin: Two cautions for comparing the latitudes of Ptolemy and of Tycho.]

I. *If anyone wishes to weigh Ptolemy's latitudes and compare them with Tycho's, to see which they are that favour Tycho's opinion and which oppose it, he must note two things. First, that those near the equinoctial points must have undergone a small variation from the time of Ptolemy to Tycho, even if the obliquity of the ecliptic is taken to have varied by almost 20′. So if a latitude has varied markedly, as with Arcturus, whose north latitude is 31° 30′ for Ptolemy and 31° 3′ for Tycho, it opposes Tycho; for it ought to be a little greater or almost equal, as Boulliau too points out, Book 5, chapter 4. Second, a distinction must be made between stars whose longitude belongs to the northern semicircle and those whose longitude belongs to the southern. For among the northern stars, a north latitude should have increased and a south latitude decreased, if Tycho's opinion is true; among the southern stars, a south latitude should have increased and a north latitude decreased; and indeed, near the solstitial points, by about 20′. But when the difference should not exceed 5′, nothing can be made out of Ptolemy's table, because in it no minutes are set below multiples of five, and indeed in most entries only tens of minutes are noted, as the imperfection of the astrolabe dictated, which he admits he used, Book 7, chapter 2.*

CAPVT XVI.

Chapter 16.

Quantus ſit Motus Fixarum in Longitudinem, & Quanta illarum Periodus.

How great the motion of the fixed stars in longitude is, and how great their period.

[Margin: 1st opinion, of Hipparchus and Ptolemy.]

I. The first opinion was that of Hipparchus, in Ptolemy, Book 7 of the *Magna Constructio*, chapters 2 and 3, which Ptolemy confirms there from the places of three stars. For the Heart of Leo was observed at ♌ 29° 50′ by Hipparchus, in year 50 of the third Callippic period, that is, in the year 128 before Christ, but at ♌ 2° 30′ by Ptolemy, in year 2 of Antoninus, or the year 138 of Christ. So if it advanced 2° 40′ in 265 years, it seems to have completed one degree in 100 years: thus chapter 2. But from chapter 3 of Ptolemy: Timocharis, in year 36 of the first Callippic period, namely 294 before Christ, observed Spica at ♍ 22° 20′. Menelaus, in year 1 of Trajan, which Ptolemy says is year 845 of Nabonassar, that is, 97 of Christ, observed the same star, by its conjunction with the Moon, at ♍ 25° 45′, according to Ptolemy's reckonings. Therefore it advanced 3° 25′ in 391 years, and therefore 1° in 114 years, granted that Ptolemy says it advanced 3° 55′ in 391 years, so as to infer 100 years for one degree. Again, Timocharis, in year 36 of the first Callippic period, by contact with the Moon and Ptolemaic calculation, observed the northern star in the brow of Scorpius at ♏ 2°. Menelaus, likewise from the Moon's application, in year 1 of Trajan, observed the same star at ♏ 5° 55′, if you accept Ptolemy's calculations: so in 391 years it progressed 3° 55′, and thus in 100 years very nearly 1°.

[Margin: The annual motion and period of the fixed stars according to Hipparchus and Ptolemy.]

From this it follows further that, if the motion of the fixed stars is uniform, in the opinion of Hipparchus and Ptolemy they will complete the period of the zodiac only in 36000 years, and move 36″ in one year. Afterwards al-Farghānī subscribed to Ptolemy, *Differentia* 13, as did Proclus in the *Hypotyposes*.

[Margin: Al-Battānī's opinion.]

II. The second opinion was that of al-Battānī, chapter 51, where he asserts that the fixed stars advance in longitude one degree in 66 years. For he says that the northern star of the brow of Scorpius, found by Menelaus at 5° 55′ of Scorpius in year 845 of Nabonassar, was found by himself in year 1627 of Nabonassar at 17° 50′ of Scorpius, and so in 783 years it advanced 11° 50′ in round figures, or 11° 55′. On this basis he rightly derives that it advanced one degree in 66 years. If, however, you compare al-Battānī's observations with Ptolemy's, the years will be 64 1/6. For Ptolemy, in year 886 of Nabonassar, records that he observed Regulus at ♌ 2° 30′ and the northern star of the brow of Scorpius at ♏ 6° 20′. But al-Battānī, chapter 51, in year 1627 of Nabonassar, observed Regulus at ♌ 14°, as the common codices have it, granted that Copernicus, Book 3, chapter 2, reads ♌ 14° 5′, and the northern star of the brow of Scorpius at ♏ 17° 50′. Now the interval of years is 741, and the advance of each star 11° 30′, or 690′. So as 690′ is to 741 years, so 1°, or 60′, is to 64 12/69 years.

[Margin: The period and annual motion of the fixed stars.]

Further, if 66 years are supposed to be owed to one degree, the motion of the fixed stars comes out at 23760 years and the annual motion at $54'' 32''' 43''''$ 38^{v} 20^{vi}.

[Margin: Alfonso's earlier opinion.]

III. Most, however, subscribed to al-Battānī up to Copernicus, and above all King Alfonso, although he had earlier followed a different opinion, as Agostino Ricci testifies in his treatise on the motion of the eighth sphere, chapters 5, 38, 42 and 43, and, from him, Erasmus Reinhold in the Peuerbachian *Theorica* of the eighth sphere, pp. 235 and 244.

[Margin: Who is the author of the Alphonsine Tables?]

For when Moors and Jews skilled in astronomy had been summoned to Toledo by Alfonso, the chief among them was Isahac Hazan [Yitzchak ibn Sid], that is, Cantor: for he was Cantor of the synagogue of Toledo, and the Hebrews of Seville call him the author of the Alphonsine Tables.

[Margin: A fiction of the Cabalists on the motion of the fixed stars.]

Accustomed to cabalistic mysteries, then, he said that by the sabbatical year God signified the motion of the fixed stars that is called access and recess, and that its revolution is therefore completed in seven thousand years, because God, in Exodus 21, Deuteronomy 15 and Jeremiah 34, commands the Jews to rest every seventh year from agriculture and from exacting loans, and to release their slaves, etc.; and so this year was called *Scemita*, that is, the year of remission, and sabbatical, from the rest of the land. And since the same Lord commanded there that, when 7 weeks of sabbatical years had passed, that is, at the completion of every 49 years, a Jubilee year should be celebrated, so called from *Iobel*, that is, from the blast of the trumpet, a year in which everything sold returned to its former owners, Isahac therefore invented that these 49 years signify the restitution of the absolute motion of the fixed stars and of the auges of all the planets (since Ptolemy too, Book 9, chapter 5, thought that the fixed stars and the apogees of the five lesser planets move with equal motion), and that this revolution is therefore completed in forty-nine thousand years, and that all things then return to their former state, as in a universal Jubilee, as though a Great Year had run its course. Accordingly, on the Alphonsine view, the periodic time of the eighth sphere as it trepidates towards rising and setting is 7000 years; but the motion of the ninth sphere, which carries the fixed stars and the auges forward in the order of the signs, is of 49000 years, and so in one year by such a motion they complete about $26'' 26'''$[55] $54''''$: as much, that is, as the Sun advances in the time by which the solar year falls short of 365 days and six whole hours, which time for the Alphonsines is about 10^{m} 44^{s}, as Reinhold above, p. 235, Clavius in the *Sphere*, p. 56, and Maurolico, p. 88 of the *Cosmographia*. And this was the earlier opinion of the Alphonsines and of Alfonso himself.

[55] the page marks this figure with two primes

[Margin: Alfonso's later opinion, and his agreement with al-Battānī.]

IV. But Alfonso, 4 years after the Alphonsine planetary tables had been composed, namely in the year 1256 from the Incarnation, saw a book composed by Albuaſſim [?], translated from Arabic into the Sevillian tongue by Rabbi Iuda and presented to him by that same man, in which book the opinion of al-Battānī on the places and motion of the fixed stars was approved. So, *having recalled his former opinion, he embraced that of al-Battānī; and he placed the fixed stars, which are seen today set down in their places in Alfonso's tables, according to this same opinion, and wrote that the stars are carried in a continued course always towards the rising. The time of the roots of these same stars, moreover, is the year 1256, not 1252, as those note who have mixed the earlier Canons of Alfonso with the new roots of the stars. All this Abraham Zaguth [Avraham Zacuto] reports in his great composition.* So Ricci has it, chapter 46 on the motion of the eighth sphere. He himself there, and Reinhold, p. 244, both confirm that Alfonso adhered to al-Battānī, because Menelaus put the northern star of the brow of Scorpius at a distance of 35° 55′ from the autumnal equinox, while Alfonso has it at 53° 28′ from the same point. So if in the 1160 years that elapsed between Menelaus and Alfonso the star advanced 17° 33′, it certainly completed one degree in 66 years, granted that the earlier Alphonsine Canons put its advance at 15° 59′. Furthermore, al-Battānī observed and recorded the same star at ♏, 17° 50′, and Alfonso at ♏, 23° 28′. So according to Alfonso it advanced 5° 38′ in 381 years, and 1° in 66 years, granted that the same earlier Alphonsine Canons show it as having advanced only 4° 32′ in this time.

[Margin: Agreement of Pedro Nunes with al-Battānī.]

[Margin: And of Agostino Ricci.]

[Margin: And of Rabbi Levi.]

V. This same opinion of al-Battānī and the later one of Alfonso were followed by Pedro Nunes, Book 2 of *De observatione*, whose words I reported in chapter 14, near the end of number 1, and by Agostino Ricci, on the motion of the eighth sphere, chapters 5, 43, 46, 47, who affirms that both his own master Avraham Zacuto and that most diligent contemplator of the stars, Rabbi Leui [Gersonides], were of the same opinion. Rabbi Levi flourished 84 years after Alfonso, or in the year of the Incarnation 1336; for after many considerations on the stars, in the fifth treatise of the book *Milhamot heſcem* [*Milchamot HaShem*], that is, *of the Defences of God*, chapter 62, he affirms that the star Assimec, that is, Spica, as found by him, and the Heart of Leo, as observed, had advanced 6° 58′ since the time of al-Battānī, that is, in 460 years, and so had completed each degree in 66 years.

[Margin: And of Abraham Zaguth.]

Abraham, however, Zacuto, who taught astronomy at Salamanca, observed Spica, in the year of the Incarnation 1474, or about 223 years after Alfonso, by the application of the Moon to it, at ♎ 17° 10′. Alfonso in his own time put it at ♎ 13° 48′. So in 223 years it advanced 3° 22′, and therefore one degree in 66 years, although the earlier Alphonsine Canons assign only 2° 41′ to this interval.

Admittedly, for that figure of 66 to come out exactly, one must assume 219 years, which are those between Alfonso's later construction and the observation of Avraham Zacuto. Having reported these things, *Ricci* concludes that the fixed stars traverse one degree in any 66 solar years, and that their annual motion is 54″ 32‴ 43⁗ 38⁗⁗ 20⁗⁗.

[Margin: Annual motion of the fixed stars according to al-Battānī.]

VI. Before and after al-Battānī, however, there were not a few who, asserting a trepidation of the fixed stars, that is, an access and recess, now towards the east and now towards the west, brought in another, different opinion on the quantity of this motion, though on no sufficient foundations, as I have already shown in Book 3, chapter 28, where I reported those authors and their doctrines from number 6 and refuted them from number 11. So I will not repeat that opinion, just as I will not repeat that of *Fracastoro*, *Turrianus* [Giovanni Battista della Torre] and *Copernicus*, which *Reinhold*, *Magini*, *Longomontanus* and *Lansbergen* support, wholly or in part.

[Margin: The opinion of della Torre and Fracastoro.]

For *della Torre*, *Fracastoro* and *Copernicus*, in order to reconcile all the observations and opinions of the ancients, said that the fixed stars at times move most slowly, and then complete one degree in 100 years; at times most swiftly, and then one degree in 66 years; at times moderately, and then finish one degree in 72 years, which is very nearly the rate at which *Copernicus*, Book 3, chapter 2, affirms he found the fixed stars to have moved in his own century.

[Margin: The opinion of Copernicus.]

There, after surveying the observations of Ptolemy and al-Battānī, which we treated above, he adds that in the year 1525 he observed the meridian altitude of Spica at Frombork as very nearly 27°, under a pole altitude of 54° 19½′, and that Spica's south declination was therefore then 8° 40′. From this, by a most laborious calculation through rectilinear triangles, he derives Spica's place at ♎ 17° 21′. And from the declination of Spica of 30′ observed by Ptolemy, he derives that in Ptolemy's time Spica was at ♍ 26° 40′. So since it has advanced 20° 41′ in about 1390 years, he distributes those years so that from Ptolemy to al-Battānī, in 741 years, he gives 65 years to the completing of one degree, and from al-Battānī to his own time, or in 645 years, he assigns 71 years to one degree. But from the ancient observations of Spica, and also of Regulus and of the northern star of the brow of Scorpius, from Timocharis to Ptolemy, in 432 years, he attributes 100 years to one degree.

However, since the Tychonians found the pole altitude at Frombork to be 54° 22′ from the circumpolar stars, as we related in Book 3, chapter 27, number 2, the declination of Spica was 8° 38′. From this, with Spica's south latitude of 2° and the truer obliquity of the ecliptic, 23° 30′, there follows a place for Spica at that time of ♎ 17° 13′ by our Problem 7 of chapter 12, or ♎ 17° 14′ as *Tycho* has it, Volume 1 of the *Progymnasmata*, chapter 2. Granted that Lansbergen, in the *Treasury of Observations*, p. 156, using the uncorrected pole altitude of Frombork, 54° 19′, and diminishing Spica's altitude by 2′ 30″, brings

the Copernican declination to 8° 44′, and from it, with a latitude of Spica of 1° 58′ and an obliquity of the ecliptic of 23° 30′, derives a place for Spica of ♎ 17° 35′; but there is no need of a subtler correction, since Copernicus says he took the altitude of Spica approximately, not precisely.

[Margin: Copernicus's erroneous observation of Spica.]

VII. *Tycho Brahe*, Volume 1 of the *Progymnasmata*, p. 253, affirms that the fixed stars complete by their motion exactly 51″ every year. Therefore the tropical year, which is of $365^{d}\ 5^{h}\ 48^{m}\ 45^{s}$, is exceeded by the sidereal, which is of $365^{d}\ 6^{h}\ 9^{m}\ 26^{s}\ 43‴$, by a time of $20^{m}\ 41^{s}\ 43‴$, because in that much time the Sun completes 51″ in the zodiac and catches up with the fixed star with which it was in conjunction at the beginning of the year.

[Margin: Tycho's opinion.]

To establish this, *first*, he takes from Copernicus, Book 3, chapter 2, Spica's declination of 8° 36′, observed at Frombork in the year 1515, which, however, he corrects according to the truer pole altitude; and from it, with the obliquity of the ecliptic and the latitude of the star, he derives that Spica was at ♎ 17° 3′ 30″. So since he himself found the same star in the year 1585 at ♎ 18° 3′, and it advanced 1° 59′ 30″ [recte: 59′ 30″] in 70 years, it follows that it moved forward precisely 51″ each year.

[Margin: 1. Tycho's foundation.]

Second, Hipparchus, in *Ptolemy*, Book 7, chapter 3, in year 50 of the third Callippic period, which was the 196th from the death of Alexander and 128 before Christ, found Regulus at ♌ 29° 50′. *Tycho*, in the year of Christ 1585, that is, in the year 1713 after Hipparchus, found it at ♌ 24° 5′. So if it advanced 24° 15′ in 1713 years, the ratio of proportion requires that it advanced 50″ 9‴ 47⁗ in each year. And lest anyone suspect a large inequality in so great an interval of time, he takes from al-Battānī the place of Regulus at ♌ 14° 5′, observed about the year of the Lord 880. For if from Hipparchus to al-Battānī, namely in 1006 years, it advanced 14° 15′, and from al-Battānī to Tycho, in 705 years, it moved forward 10°, then certainly in one year it advanced 51″ or 50″ 56‴.

Nor does it stand against Tycho that Timocharis, in the year 454 of Nabonassar, found Spica at ♍ 22° 20′, Ptolemy, in the year of Christ 139, found it at 26° 40′ of ♍, and Tycho, in the year 1585, namely the year 2333 of Nabonassar, found it at ♎ 18° 3′, so that in the earlier interval, between Timocharis and Tycho, of 1879 years, it advanced 25° 43′, and 49″ 15‴ in each year, and in the later interval, of 1446 years, 21° 23′, and 53″ 15‴ in each year. For from this, rather, by bounding both and taking the mean, the annual motion of 51″ is confirmed.

[Margin: Tycho disposes of the objections.]

Tycho does not, however, claim that this answers to all ages, but at least to the nearest three or four centuries, without sensible difference. In these centuries, therefore, the fixed stars complete one degree in 70 years and seven months; and if they always moved so, their period would be of 25806 years.

Kepler carried this opinion into the Rudolphine Tables, p. 42, granted that he himself thinks the fixed stars do not move.

[Margin: The annual motion and period of the fixed stars.]

VIII. I come to *Longomontanus*, who in Book 1 of the *Theorica*, in the Commentary on the fixed stars, corrects, from his own hypothesis of the Moon's motion, the ancient observations of four stars taken from Ptolemy, Book 7, chapters 3 and 4, compares them with the Tychonic ones, and concludes that the mean annual motion of the fixed stars is 49″ 45‴, so that they complete their period in 26040 years and one degree in 72⅓ years. The stars, with their places and the times of the observations, are as set out below.

[Margin: The opinion and foundation of Longomontanus.]

[Margin: The mean annual motion and period of the fixed stars.]

Observers	Years of Nabonassar	Place in the zodiac
Timocharis	466	Spica ♍ 22° 30′
Hipparchus	630	Spica ♍ 24° 25′
Tycho	2350	Spica ♎ 18° 16′
Ptolemy	886	Regulus ♌ 3° 53′
al-Battānī	1626	Regulus ♌ 14° 27′
Tycho	2350	Regulus ♌ 24° 17′
Timocharis	454	Northern brow ♏ 1° 58′
Tycho	2350	of Scorpius ♏ 27° 36′
Timocharis	465	Of the Pleiades ♈ 29° 32′
Tycho	2350	the bright star ♉ 24° 24′

From these intervals of time and advances of the stars he gathers the annual motion of the fixed stars as below.

Annual motion of the fixed stars	″	‴
From Ptolemy to *al-Battānī*	51	0
From Hipparchus to *Tycho*	49	46
From Ptolemy to *Tycho*	49	37
From Timocharis to *Tycho*	49	10
From al-Battānī to *Tycho*	48	14
From Timocharis to *Ptolemy*	46	0
From Timocharis to *Hipparchus*	44	0

In this treatise, however, Longomontanus admits that the places of the fixed stars have been emended by means of the prosthaphaeresis of the equinoxes, as if this were better known and more certain than the unequal motion of the fixed stars which he is trying to establish, whereas the inequality of this motion ought rather to be set down first, as one of the chief foundations of that prosthaphaeresis. Moreover, this author could have gathered from the reckonings already mentioned a mean motion of a round 50″, with no less probability, as I shall show at number 11, argument 2.

IX. *Lansbergen* agrees with Longomontanus as regards the inequality of the motion of the fixed stars, which needs a prosthaphaeresis, but differs greatly in the measure of the mean motion. For in the

[Margin: The opinion of Lansbergen.]

Treasury of Observations, p. 160, he affirms that the fixed stars move 8‴ 25⁗ 12⁗′ 32⁗′′ every day and complete their period in 28000 years, and so 1° in 77 7/9 years. He does not, however, determine from which observations chiefly he derives this. But from some observations he himself set out beforehand a different motion of the fixed stars is not rarely gathered. For he puts Spica, observed by Ptolemy in the year of Christ 139, at ♍ 26° 28′ 4″, and by himself in the year 1599 at ♎ 18° 19′ 28″; the northern star in the brow of Scorpius, observed by Ptolemy in the same year 139, at ♏ 5° 51′ 4″, and by himself in the year 1601 at ♏ 27° 44′; and the Heart of Scorpius, by Ptolemy likewise in the year 139, at ♏ 12° 38′, and by himself in the year 1600 at ♐ 4° 30′. So from these intervals of years and advances of the stars it follows that they completed 53″ 52‴ or 54″ each year. But for Lansbergen it follows from the period of 28000 years that the fixed stars complete 46″ 2/7 each year, while from the daily motion he himself assigned they complete 51″ 15‴, as a careful calculator will find.

[Margin: Lansbergen's disagreement with himself.]

X. *Ismaël Boulliau*, Book 5 of the *Astronomia Philolaica*, chapter 3, rejects the observations of Timocharis, since Hipparchus too held them suspect as rather crude, on the testimony of Ptolemy, Book 7 of the *Almagest*, chapter 3, and selects the observations of Regulus and Spica made by Hipparchus, Ptolemy, al-Battānī and Tycho. The Ptolemaic one he corrects, because Ptolemy, Book 7, chapter 3, sets out the manner and circumstances of his observation, and so the star's place can be emended from a truer motion of the Moon. But he does not correct the observations of Hipparchus and al-Battānī, because it is not known by what methods and in what circumstances they observed the places of the stars. So the selected observations of Regulus are those set out below.

[Margin: Boulliau's opinion and foundation.]

Observers	Years of Nabonassar	Place of Regulus
Hipparchus	621	♊ [recte: ♋] 29° 50′
Ptolemy	886	♌ 3° 42′
al-Battānī	1627	♌ 14° 5′
Tycho	2350	♌ 24° 17′

Hence the annual motion of the stars:

From / to	″	‴	⁗
From *Hipparchus* to *Ptolemy*	52	31	42
From *Hipparchus* to *al-Battānī*	50	59	38
From *Hipparchus* to *Tycho*	50	54	29

From / to	″	‴	⁗
From *Ptolemy* to *al-Battānī*	50	26	27
From *Ptolemy* to *Tycho*	50	36	53
From *al-Battānī* to *Tycho*	50	47	18

Boulliau adds that from the given latitude of Spica of 2° south and the north declination of 36′, found by Hipparchus in the year of Nabonassar 621, its place follows at ♍ 23° 59′. From this, and from the place of Spica at ♎ 18° 16′ found by Tycho in the year of Christ 1601, which was of Nabonassar 2350, it comes about that in 1729 years Spica advanced 24° 17′, and that the annual motion was 50″ 33‴ 39⁗ 39⁗′ 52⁗′′ 22⁗′′′. But he concludes: *Nevertheless we shall retain the annual motion of the fixed stars from the motion of the Heart of Leo from Hipparchus to Tycho, in which we are little away from the limit of the motion of the fixed stars fixed by Tycho.* So in the *Philolaic Tables*, p. 10, he puts the annual motion of the fixed stars at 51″, following Tycho.

[Margin: Annual motion.]

In Book 5, chapter 3, however, for the sake of learning, he gives an observation of Regulus made by *Azophi*, or *Ebennen-Sophim*, in the year of the Arabs 325, or of Christ 936 November 18, the seventh day of the week, and reported by him in the book on the places of the stars, where Regulus is then put at ♌ 15° 12′. And he says that in the same volume is contained a catalogue of 47 fixed stars verified in the year of Christ 1364, in which Regulus is put at ♌ 20° 40′. Next he says that in the Persian Tables, which Georgios Chrysokokkes translated into Greek and which are kept in the library of the Most Christian King, there is found a catalogue of some fixed stars with this title: *This table [canonium] is adapted to the beginning of the Arab year 509, through 68 of whose years they advance one degree.* And there Regulus is put at ♌ 17° 30′. Now the Arab year 509, as computed by Hyginus, began in the year of Christ 1115 June 6, the fifth day of the week. So by comparing these years, and the place of Regulus observed by al-Battānī at ♌ 14° 5′ in the year of Christ 879, and at ♌ 24° 17′ by Tycho in the year 1601, Boulliau gathers the following motions of Regulus.

[Margin: Motion of the fixed stars from the Persian and Arabic tables.]

Annual motion of the fixed stars	″	‴	⁗
From *al-Battānī* to *Azophi*	69	18	37
From *al-Battānī* to the Persian Tables	52	7	7
From *al-Battānī* to the year 1364	48	51	57
From *Azophi* to the Persian Tables	46	31	0
From *Azophi* to the year 1364	45	5	20
From the Persian Tables to *Tycho*	50	14	49
From the Persian Tables to the year 1364	45	46	59

Noſtra Opinio de Motu Fixarum.

Our opinion on the motion of the fixed stars.

XI. Having reckoned up all that has been said so far, and other observations, of which shortly, I have chosen an annual motion of the fixed stars of precisely 50″, and 72 years for completing one degree, and therefore a period of 25920 Julian years. Yet I do not deny that the Tychonic motion is almost equally as probable as ours; that you may use either of them; and that no certain and evident observation exists which could convict them of falsity. Indeed, since each motion has been gathered partly from evident and partly from far more probable observations, the places of the stars so far observed by others can rather be emended from it, especially with the addition of the difference of longitude, which is established with certainty for Tycho from his observations, as for us from ours. Further, I have been led to choose the motion already mentioned by the arguments set out below; before them, however, I set down the following observations.

[Margin: Our annual motion of the fixed stars, and period.]

First, Timocharis, as *Ptolemy* relates, Book 7 of the *Almagest*, chapter 3, in the year of Nabonassar 454, that is, 296 before Christ, observed Spica at ♍ 22° 20′, as Ptolemy gathers from the application to the Moon reported by him; and in the year of Nabonassar 466, or 284 before Christ, he observed it at ♍ 22° 30′. So if in 12 years it completed 10′, then certainly in 72 years it made 1°, and it moved forward 50″ each year. But since this interval is too short, let us compare his observation with ours. For at the end of the year of Christ 1644 we found Spica at ♎ 18° 54′ 20″. So the advance from the first observation of Timocharis was 26° 34′ 20″, and the interval is of 1940 years; from the second it is 26° 24′ 20″, and the interval of 1928 years. From the first there follows an annual motion of 49″ 26‴; from the second, 49″ 18‴. But since Ptolemy deduced the place of Spica from the place of the ☾ with no great certainty, and calls the observations of *Timocharis*, Book 7, chapter 1, *little explained and ambiguous*, and, together with Hipparchus, Book 7, chapter 3, *uncertain and too simple*, let us move on to others.

[Margin: 1. Observation.]

Second, Hipparchus, on the testimony of *Ptolemy*, Book 7, chapter 2, observed the Heart of Leo at ♌ 29° 50′ in year 50 of the third Callippic period, which was 128 before Christ. But since Ptolemy there relates that his own observations of the fixed stars were made with the astrolabe, by which he took the distance of the Moon from the Sun at sunset, and the distance of the fixed star from the Moon once the Sun had set, it is probable that he learned this from Hipparchus and that Hipparchus observed in a similar way. So since the refraction of the setting Sun, being nearly horizontal, was about 30′, and diminished the true longitude of the Moon by about as many minutes, it is probable also that the true longitude of Regulus was greater by about 30′, and thus that it was rather at ♌ 0° 20′. We, however, observed Regulus in the completed year 1644 at ♌

24° 55′ 20″. So in 1772 years it advanced 24° 35′ 20″, or 88520″. Dividing these by 1772 gives an annual advance of 49″ 1692/1772, or very nearly 50″.

[Margin: 2. Observation.]

Third, al-Battānī relates, chapter 51, that in the year of Nabonassar 1627, which was of Christ 879, Regulus was observed by him at ♌ 14°, as the common Latin codices have it. Add to these 30′ for refraction, on the reasoning by which we corrected observation 2 (since indeed, down to Tycho exclusively, the more common method of observing the places of the stars was by the distance of the ☾ from the setting Sun, and of the ☾ from the fixed stars on the following night), and the corrected place becomes ♌ 14° 30′, granted that *Longomontanus*, as I said at number 8, corrects it and brings it to 14° 27′. We, however, found Regulus in the completed year of Christ 1644 at ♌ 24° 55′ 20″. So in an interval of 766 years it advanced 10° 25′ 20″, that is, 37520″; dividing these by 766 gives an annual motion of very nearly 49″. But if we use al-Battānī's place, which Copernicus gives as 14° 5′ of ♌, the advance would be 10° 50′ 20″

[Margin: 3. Observation.]

and the annual motion 51″. So, holding to the mean, it will be 50″.

Fourth, comparing the observation of Hipparchus, of which we spoke in the second observation, and that of al-Battānī, of which we spoke in the third, whether you add 30′ to each for refraction or not, the advance of Regulus was 14° 10′ in 1006 completed years. So in one year it advanced 50″ 42‴.

[Margin: 4. Observation.]

Fifth, Hipparchus observed the north declination of Aldebaran, or the Eye of Taurus and brightest of the Hyades [succularum], at 9° 45′, as *Ptolemy* has it, Book 7 of the *Almagest*, chapter 3. But in chapter 2 he had said: *From the times of Hipparchus's observation to the reign of Antoninus, in which we chiefly observed very many advances of the non-wandering stars, about 265 years are gathered.* Now the first year of Antoninus was 137 of Christ, so the observation of Hipparchus was, not indeed certainly, but not improbably, in the year 128 before Christ. The south latitude of this star was found by us to be 5° 30′. From this, with the more probable obliquity of the ecliptic, 23° 30′, and the declination of 9° 45′, there follows by Problem 7, chapter 12, a place for Aldebaran at that time of ♉ 10° 27′. But we, in the completed year of Christ 1644, observed Aldebaran at ♊ 4° 48′. So in 1771 years this star advanced 24° 21′, and therefore in one year it advanced 49″ 30‴.

[Margin: 5. Observation.]

Sixth, in the first year of Antoninus, that is, 137 of Christ, *Ptolemy*, from Book 7 of the *Almagest*, chapter 3, observed the north declination of the same Aldebaran at 11°, and its south latitude is known to us as 5° 30′. From this and the constant obliquity of the ecliptic, 23° 30′, by Problem 7, chapter 12, the place of Aldebaran at that time comes to ♉ 14° 2′. So from Hipparchus to Ptolemy, from what is said here and in the preceding observation, it advanced 3° 35′ in 265 years, and very nearly 49″ in one year. But this interval is too small, and the observations are not so certainly known to us. Assuming, however, the

greater interval of 1506 years, which run to our observation of the year 1644, and the advance of Aldebaran, which was 20° 46′, the annual motion comes to 49″ 40‴.

[Margin: 6. Observation.]

Seventh, the north declination of the same Aldebaran was observed at 15° 37′ in the year of the Lord 1586, both by *Tycho* at Uraniborg, through a meridian altitude of 49° 42′, as found in his *Epistolae Astronomicae*, p. 52, since the altitude of the equator there is 34° 5′, and by *Lansbergen* in the *Treasury of Observations*, p. 151; and its south latitude is 5° 30′. From these and the obliquity of the ecliptic, 23° 30′, by Problem 7, chapter 10, the place of Aldebaran at that time is gathered as ♊ 4° 2′. But the Ptolemaic observation of the year of the Lord 137 gives its place at ♉ 14° 2′. So in an interval of 1449 years it crept forward 20°. Therefore, keeping the proportion, the annual motion is 49″ 40‴.

[Margin: 7. Observation.]

Eighth, the Persian mathematicians observed, in the year of the Lord 1115 June 6, Regulus at ♌ 17° 30′, and *Boulliau* holds that they are to be trusted on account of their diligence, as was related at number 10. We, however, in the completed year 1644 had Regulus at 24° 55′ 20″, and *Tycho* in the completed year 1600 at ♌ 24° 17′. So in the first interval, of 529½ years, it advanced 7° 25′ 20″, and in the second, of 485 years, from the Persians to Tycho, 6° 47′. Hence from the first there follows an annual motion of 50″ 28‴, and from the second 50″ 15‴. So, gathering the annual motion from the eight comparisons already mentioned, the result is as in the following table.

[Margin: 8. Observation.]

Annual motion of the fixed stars

Observations	″	‴
1	49	26
2	50	0
3	50	0
4	50	42
5	49	30
6	49	40
7	49	40
8	50	28
or	50	15

First, then, the argument for choosing a motion of 50″ rather than the Tychonic 51‴ [recte: 51″], which is otherwise very probable, was on account of the observations already mentioned, since it either answers to them or is a mean between the extreme ones, namely between 1 and 5 on one side and 8 on the other, and nearly between 4 and 6 or 7.

[Margin: 1. Argument.]

The *second* argument was that the round motion of 50″ is very nearly the mean between the extremes gathered from the reckonings of *Longomontanus* at

number 8 and the motion of 51″ of *Tycho* above at number 7 and of *Boulliau* at number 10. For, setting aside the uncertain observations of Timocharis, the extreme figures in *Longomontanus* are 51″ and 48″ 14‴. Their difference is 2″ 46‴ and its half 1″ 23‴; taking this from 51″ leaves 49″ 37‴. But by offsetting the uncertainty of Timocharis against the length of the interval from Timocharis to Tycho, he gathered 49″ 10‴, and from Ptolemy to al-Battānī 51″; the mean of these is 50″ 5‴. And the mean between 49″ 37‴ and 51″ is 50″ 18‴. So he could have chosen a round motion of 50″ no less probably than the 49″ 45‴ he did choose, since 50″ is the mean between 50″ 18‴ and 49″ 45‴, or between 49″ 10‴ and 51″, without sensible difference.

[Margin: 2. Argument.]

The *third* argument I drew from the data of *Lansbergen*, of which at number 9. For from his daily motion an annual motion of 51″ 15‴ is gathered; from the period of the fixed stars of 28000, an annual motion of 46″ 2/7; but from the places of the stars observed by him, and those of the ancients approved by him, 54″ is gathered; and the mean of these is 50″. So although this disagreement in him may displease, it is nevertheless permitted to show that he cannot by any right recoil from the motion of 50″.

[Margin: 3. Argument.]

The *fourth*, finally, was that an annual motion of 50″, or one degree in 72 years, is owed both to the times of Callippus, as *Reinhold* affirms in the *Theorica* of the eighth sphere, p. 241½, and before him *Regiomontanus*, Book 7 of the *Epitome*, proposition 5, and very nearly to the time from al-Battānī to us, as Copernicus admits, who gathers about 71 years to a degree in that interval, as I said at number 6. These two extreme intervals enclose the interval from Ptolemy to al-Battānī.

[Margin: 4. Argument.]

Not far from this are *Rabbi Iosue* [Rabbi Yehoshua] in the Talmud, book Horaioth, *Rabbi Moyses* [Maimonides] in the book Misnetora [Mishneh Torah], or of the repeated law, and *Rabbi Auenezra* [Avraham ibn Ezra] in the book Taamin, or of reasons, who, as *Agostino Ricci* reports in his treatise on the eighth sphere, chapter 4, said that the fixed stars complete one degree in 70 years; and Haly [ʿAlī ibn Riḍwān] too held this, Book 2 of the *Quadripartitum*. Granted that for other Moors and a few Jews, in Regiomontanus, the mean annual motion was 45″, and 1° of 80 years, and that for al-Zarqālī [Arzaëli] and Thābit they complete 1° in 75 years by mean motion, as the same *Ricci* has it.

[Margin: The opinion of the ancients and the rabbis.]

Finally, I see that this motion pleased our *Pétau*, Book 1 of the *Uranologion*, in the dissertations, chapter 3. Nor is there any danger that, if you use our motion rather than the Tychonic, you will be refuted by the risings and settings of the stars reported in Hesiod, who flourished 2400 years ago, as I shall say in the Scholia. But the fact that neither the Tychonic motion nor ours satisfies to the letter, whether with the less certain observations of the ancients, or with the more certain Tychonic ones and our own compared with each other, but

over too narrow an interval, ought to be no obstacle at all for those wise in astronomical matters. See the table with a synopsis of the opinions at the end of the Scholia.

SCHOLIA.

Scholia.

I. Johannes Kepler's *opinion I did not set down in the preceding chapter, both because he himself holds absolutely, with Copernicus, that the fixed stars do not move, but that their apparent motion is to be attributed to the equinoctial points creeping against the order of the signs, on account of the motion of the Earth; and because this same motion, which for Tycho and for us, with very many others, belongs to the fixed stars, he derives from no observation but follows Tycho, as is clear* first *from the Rudolphine table, p. 43, in which he puts the annual motion of the fixed stars at 51″.* Second *from the* Rudolphine Tables, *p. 115, where, speaking of the restitution of tropical and sidereal years, which he calls the great celestial year, he says that 25411 sidereal years equal 25412 tropical years of the Sun, and that the restitution of Julian and tropical years is nearly twice as slow. Granted that* Wendelen *in the* Idea of the Atlantic Tables, *p. 28, says that 25519 sidereal years equal 25520 tropical. For from this it follows that in one sidereal year the Sun completes the zodiac and 51″ besides, or 50″ 47‴, which is as much as the fixed stars are moved forward on the Tychonic reckoning.* Third *from the* Epitome of Copernican Astronomy, *p. 396, where he says that in the 2400 years from Hesiod to the year 1618 the fixed stars advanced about 34°, that is, 122400″, which, divided by 2400 years, give an annual motion of 51″.* Gassendi *too, in the* Institutio Astronomica, *by putting the fixed stars' completion of their period in 25000 years, gives them an annual motion of 51″ 19‴ 24⁗ and 70 1/3 years to one degree.*

[Margin: Kepler's opinion.]

[Margin: Wendelen's opinion.]

[Margin: Gassendi's opinion.]

II. *Let no one fear for our motion or the Tychonic from what the most ancient poet* Hesiod *sings of Arcturus in the books* Works and Days: *for he says that Arcturus in his age used to rise in the evening, 60 days after the winter solstice. Hesiod, moreover, flourished on Mount Helicon near Athens about the beginning of the Olympiads, that is, about the year 776 before Christ. So to the year of the Lord 1610 there are 2386 years, and to the year 1624, 2400 years. First, therefore, on this basis,* Longomontanus, *in the* Astronomia Danica, *Book 2 of the* Sphaerics, *chapter 4, Problem 2, gathers that the place of Arcturus at that time was at ♍ 12° 16′, taking the altitude of the pole on Mount Helicon as 37° 45′, according to Ptolemy, the apogee of the Sun at ♉ 20° 10′, and the place of the Sun, sixty days after the solstice of Capricorn, at ♓ 1° 10′. He then adds that in the year 1610, by a recent observation, Arcturus was at 18° 47′ ♎. So Arcturus would have advanced 36° 31′, or 131460″; dividing these by 2386 years, the annual motion would be 55″. But to draw this passage over to his own opinion on the precession of the equinoxes and the motion of the fixed stars, he says*

that about 2658 years are required, so that Hesiod is reporting the rising of Arcturus not of his own time, but of a time earlier by 272 years. He nevertheless has recourse to the refraction of Arcturus at the horizon, 30′, which cuts this interval by nearly half. But how easy it would be for us, with so much freedom to limit the small details of what is assumed, to defend Hesiod in such a way that the annual motion came out as 50″ or 51″! It is not worthwhile, however, to play at such labour on these matters, and the diversity among the authors who have examined this passage is quite enough to show this freedom, as will soon be clear.

III. *Now* Kepler, *in the* Epitome Astronomiæ, *at the end of Book 3, that is, p. 396, also quoting the verses of Hesiod, supposes 2400 years from Hesiod to the year of the Lord 1618, and an advance of the stars of 34°, from which, as I said in Scholion 1, an annual motion of 51″ follows. From this and from various assumptions he gathers that in Hesiod's time Arcturus used to rise in the evening on March 3 Julian, with the Sun placed at ♓ 5° 11′. But if anyone wants opinions on the rising of Canicula and of Arcturus in the times of Eudoxus or Pliny the Elder or others, let him consult* Kepler *in the already mentioned* Epitome, *from p. 378 to p. 398, and* Pétau *in the Dissertations on the* Uranologion, *Book 1, chapter 8, and Book 2, chapter 8.*

Tabula MOTVS ANNVI ac PERIODI Fixarum.

Table of the annual motion and period of the fixed stars.

Of the fixed stars, according to the authors		Annual motion ✶ ″	‴	⁗	V.	One degree, in Julian years	Whole period, of Julian years
Hipparchus, Ptolemy, Proclus, al-Farghānī	True	36	0	0	0	100	36000
della Torre and Fracastoro	Mean	36	0	0	0	100	36000
Alfonso earlier, and the Alphonsines, Isaach Hazan, etc.	Mean	26	26	54	0	136 1/9	49000
al-Battānī, Albuhassim, Alfonso later	True	54	32	43	38 1/5[?]	66	23760
Rabbi Levi, Avraham Zacuto, Ricci, Nunes	True	54	32	43	38 1/3	66	23760
al-Zarqālī and Thābit	Mean	48	0	0	0	75	27000
The Moors, a few Jews, and the Indians	Mean	45	0	0	0	80	28800
Rabbi Yehoshua, Rabbi Moshe, ibn Ezra, and Haly	True	51	25	42	51	70 nearly	25200
The Persian mathematicians, in the Persian Tables	True	52	28	14	7	68	24480
Copernicus, Reinhold, Mästlin, and formerly Magini	Mean	50	12	5	0	71 nearly	25798
Tycho, Kepler, Boulliau	True	51	0	0	0	70 3/5	25806
Christianus Longomontanus	Mean	49	45	0	0	72 1/3	26040
Philips van Lansbergen	Mean	46	17	0	0	77 7/9	28000
Gottefridus Vendelinus [Govaert Wendelen]	M. V.	50	47	0	0	70 8/9	25520
Petrus Gassendus [Pierre Gassendi]	True	51	19	24	0	70	25000

Of the fixed stars, according to the authors		Annual motion ✶″	‴	⁗	V.	One degree, in Julian years	Whole period, of Julian years
Ourselves and Denis Pétau	True	50	0	0	0	72	25920

CAPVT XVII.

Chapter 17.

Qualis ſit Motus Fixarum & Quotuplex, Directuſne ſemper an Retrogradus cum Trepidatione, & Aequaliſne an Inæqualis.

What the motion of the fixed stars is like and of how many kinds; whether it is always direct, or retrograde with a trepidation; and whether it is equal or unequal.

I. We have already discussed the questions named in the title of this chapter, on the occasion of the precession of the equinoxes, in Book 3, chapters 28 and 30, setting out various opinions together with their authors and foundations. We showed that the fixed stars, besides the daily motion of the primum mobile, have a slow proper motion eastward on the poles of the ecliptic, as *Ptolemy* shows from the changed declinations of the fixed stars, Book 7 of the *Almagest*, chapter 3. This is true not only of the stars within the zodiac, as Hipparchus thought in his earlier hypothesis, but of all of them, because the unchangeableness of their distances has been found in them all; and in the end Hipparchus conceded it of all of them, on the testimony of Ptolemy, Book 7, chapter 1. Granted, *Joseph Justus Scaliger* invented the view that it takes place on the movable poles of the equator, and that the equinoctial points are moved westward on the equator with the motion of the equator; but we have reported and rejected his empty foundation in Book 3, chapter 28, number 10, and in this Book 6, chapter 4, at the passage on Ursa Minor and the changed declination of the Pole Star, a change that *Scaliger* quite ignorantly denied. We showed besides, in Book 3, chapters 28 and 30, that the motion of the fixed stars in longitude is always direct, without any trepidation or libration toward the east and west, and that it is uniform and equal. All of this should be reread at this

[Margin: 1. Argument for the equal [?] motion of the fixed stars.]

point, so that you may have the full doctrine of the motion of the fixed stars. Since, however, we adduced there chiefly two foundations for an inequality in the motion of the fixed stars, one from the inequality of the year, which we have now overthrown, and the other from an apparent variation in the motion of the fixed stars gathered from observations — observations by which, if you choose to abide by them as they stand in the ancient astronomers, the motion of the fixed stars appears to have been very slow from Timocharis and Hipparchus down to Menelaus and Ptolemy, so much so that they barely completed one degree in 100 years; to have begun to quicken from Ptolemy, and to have become so swift by the time of al-Battānī that they ran through one degree in *66* years; and from then on to have been slowed again, so that

in the century of Copernicus they completed one degree in nearly 71 years. We promised the overthrow of this foundation in those places, and deferred it to this one. Our pledge must therefore be redeemed, and it must be proved from the observations themselves that this foundation is not solid, and that the equality of the motion of the fixed stars is far more probable than its inequality, as will be made plain by the following arguments.

II. *First*, it is clear from what was said in chapter *16*, from number 7 to 11, that the annual motion of the fixed stars, gathered from numerous and more select observations, from Timocharis or Hipparchus to Ptolemy, from there to al-Battānī, and from al-Battānī to us, is always very nearly 50″ or 51″. These observations certainly outweigh those few on which *Copernicus* relied in establishing the inequality, both in number and in certainty, since they are either those of more recent astronomers, or those of the ancients, it is true, but corrected in places.

[Margin: 2. Argument from the false longitude of the ancients.]

III. *Second*, the motion of one degree in *66* years was gathered by al-Battānī from only one or two pairs of observations, and the motion in 100 years was deduced by Ptolemy from only three. But the ancient longitude of the fixed stars that was used is most certainly fallacious, since it does not agree with the difference of longitude which, by clear observations, we in this century and Tycho in the pre- ceding have found. Nor indeed do Ptolemy, al-Battānī or Copernicus deny that the fixed stars perpetually keep the same difference of longitude among themselves. The fallacy already mentioned cannot be shown more clearly than by setting out, in the small table below, the difference of longitude found by Tycho and by us between the stars used to establish the inequality of the motion, together with the difference between them when the longitude is placed as reported by *Ptolemy*, Book 7 of the *Almagest*, chapters 2 and 3, and by *al-Battānī*, chapter 51, from whom *Copernicus* copied them, Book 3, chapter 2.

[Margin: You have the Tychonic longitude and ours in the tables of the longitude of the fixed stars.]

Observers, with the years of Nabonassar	Pairs of stars observed	Longitude ✶ from the ancients	Difference of longitude: of the ancients	Ty- chonic	Ours
Timochares Year 465	Spica	♍ 22° 30′			
	Pleiades	♈ 29° 30′	143° 0′	143° 52′	143° 52′ 30″
Timochares Year 454	Spica	♍ 22° 20′			
	Northern brow of Scorpius	♏ 2° 0′	39° 40′	39° 20′	39° 20′ 50″
Hipparchus Year 620	Spica	♍ 24° 0′			
	Regulus	♌ 29° 50′	54° 10′	53° 59′	53° 59′ 0″
Menelaus Year 845	Spica	♍ 25° 45′			
	Northern brow of Scorpius	♏ 5° 55′	40° 10′	39° 20′	39° 20′ 50″

Observers, with the years of Nabonassar	Pairs of stars observed	Longitude ✶ from the ancients	Difference of longitude: of the ancients	Tychonic	Ours
Ptolemy Year 886	Spica	♍ 26° 30′			
	Regulus	♌ 2° 30′	54° 0′	53° 59′	53° 59′ 0″
al-Battānī Year 1627	Regulus	♌ 14° 0′			
	Northern brow of Scorpius	♏ 17° 50′	93° 50′	93° 19′	93° 19′ 50″

So whether the errors already mentioned in the difference of longitude flowed from the instruments, or from refraction being left out of account, or from an erroneous obliquity of the ecliptic, or from the motion of the luminaries, or from all of these together, it is certain that a motion of the fixed stars deduced from them cannot stand. The Ptolemaic observation does seem to bear the mark of truth in Spica and Regulus, because its difference of longitude departs by a single minute from Tycho's and ours; but the others taken with it for comparison exceed or fall short by a full 50′, 30′ or 20′. *Tycho* pointed out this disagreement, Volume 1 of the *Progymnasmata*, p. 255.

[Margin: 3. Argument from the same fallacy.]

IV. *Third*, neither the longitude given by Ptolemy for Spica, Regulus, the Pleiades and the northern star of the brow of Scorpius, which the authors already mentioned used in order to introduce an unequal motion of the fixed stars, nor the longitude of these or other stars deduced geometrically from their declination and latitude as observed by the same men and reported by Ptolemy, Book 7 of the *Almagest*, chapters 2 and 3, or in the table of chapter 5, supposing for the obliquity of the ecliptic in those times 23° 52′, which our opponents suppose along with Ptolemy — that longitude, I say, agrees neither with the true difference of longitude, which is certain to Tycho and to us, nor with the difference of longitude deduced from other observations of the ancients in the preceding argument. These longitudes are therefore fallacious, and Copernicus had no solid foundation on which to build upon them the anomaly of the precession of the equinoxes, or of the motion of the fixed stars. To make this clear at once, here in the following table [canone] is the longitude both of the four stars already mentioned and of the remaining stars of the first magnitude whose declination, observed by himself or by the ancients, Ptolemy gives with the latitude in the place already mentioned. It is deduced by the solution of triangles according to our problem 7 of chapter 12, assuming the obliquity of the ecliptic at 23° 52′, with the difference of longitude arising from this between the stars added, and compared with the Tychonic difference, which nearly equals ours. I shall compare stars nearer one another, and stars observed by Hipparchus and Ptolemy.

Observers	Stars	Observed declination	Observed latitude	Deduced longitude	Difference of longitude: ancient	Tychonic
Hipparchus	Bright star of the Pleiades	15° 10′ N	3° 40′ N	♉ 0° 12′		
	Aldebaran	9° 45′ N	5° 10′ S	♉ 8° 42′	8° 30′	9° 48 1/2′
Ptolemy	Bright star of the Pleiades	16° 15′ N	3° 40′ N	♉ 3° 11′		
	Aldebaran	11° 0′ N	5° 10′ S	♉ 12° 40′	9° 29′	9° 48 1/2′
Hipparchus	Aldebaran	9° 45′ N	5° 10′ S	♉ 8° 42′		
	Sirius	16° 0′ S	39° 10′ S	♊ 14° 16′	35° 34′	34° 23′
Ptolemy	Aldebaran	11° 0′ N	5° 10′ S	♉ 12° 40′		
	Sirius	15° 45′ S	39° 10′ S	♊ 17° 38′	34° 58′	34° 23′
Hipparchus	Arcturus	31° 0′ N	31° 30′ N	♍ 23° 48′		
	Capella	40° 24′ N	22° 30′ N	♉ 22° 54′	120° 54′	122° 23′
Ptolemy	Arcturus	29° 50′ N	31° 30′ N	♍ 25° 50′		
	Capella	41° 10′ N	22° 30′ N	♉ 25° 34′	120° 16′	122° 23′
Hipparchus	Regulus	20° 40′ N	0° 10′ N	♌ 0° 2′		
	Spica	0° 36′ N	2° 0′ S	♍ 23° 59′	53° 57′	53° 59′
Ptolemy	Regulus	19° 50′ N	0° 10′ N	♌ 3° 42′		
	Spica	0° 30′ S	2° 0′ S	♍ 26° 50′	53° 8′	53° 59′
Hipparchus	Antares	19° 0′ S	4° 0′ S	♏ 10° 26′		
	Spica	0° 36′ N	2° 0′ S	♍ 23° 59′	46° 27′	45° 57′
Ptolemy	Antares	20° 15′ S	4° 0′ S	♏ 14° 18′		
	Spica	0° 30′ S	2° 0′ S	♍ 26° 50′	47° 28′	45° 57′

Nor can one take refuge in uncertainty about the time of the observed declination, on the grounds that Ptolemy, Book 7, chapter 3, does not record the time but gives the bare declinations. For in chapter 2 he had said that his very many observations of the fixed stars were made in the reign of Antoninus, and in the same place he indicates about 265 years between his own and those of Hipparchus. Nor could there have been so much time between the observations of the same astronomer as to excuse so enormous a difference of longitude.

V. *Fourth*, if *Copernicus* and his followers want to accept the observations of longitudes exactly as they stand in Ptolemy and al-Battānī, even though *Ptolemy* deduced most of them from observed applications of the Moon to Spica, the bright star of the Pleiades and the northern brow of Scorpius, assuming his own hypotheses on the motion of the Sun and Moon, then with far greater right they ought to accept the longitudes deduced by trigonometry from the declinations and latitudes of the fixed stars given by the same astronomers, together with the obliquity of the ecliptic that the same men supposed. For this method is far more certain and simpler, and it rests on a spherical triangle in which all the sides are given, as is clear from our Problem 7 of chapter 12.

Now from the longitude of the ancients deduced in this way there does not follow, from Hipparchus to Ptolemy, a motion of the fixed stars of one degree in 100 years, but either one far slower, or a very swift one of 66 years, or a moderate one differing greatly from 100 years, provided you grant that very nearly 265 years passed between the declination observations of Hipparchus and those of Ptolemy, as *Ptolemy* indicates, Book 7, chapter 2. Or if you contend that as many years passed as are required to establish 100 years for one degree, you must necessarily interpose an improbable interval. So that this may appear at once, I shall repeat in the following table the longitudes of the fixed stars deduced from the declination, latitude and obliquity of the ecliptic that the preceding table treats, adding the years thereby required for 1°, and the years required between the observations for the advance of the stars to correspond in proportion to an advance of one degree made in 100 years.

Stars observed	Observers	Deduced longitude ✶	Advance of the stars	If there are 265 years between the observations: years to 1°	If 1° is made in 100 years: years between the observations
Bright star of the Pleiades	*Hipparchus*	♉ 0° 12′			
	Ptolemy	♉ 3° 11′	2° 59′	88 7/9	298
Aldebaran	*Hipparchus*	♉ 8° 42′			
	Ptolemy	♉ 12° 40′	3° 58′	66 19/24	397
Arcturus	*Hipparchus*	♍ 23° 48′			
	Ptolemy	♍ 25° 50′	2° 2′	130 1/3	204
Capella	*Hipparchus*	♉ 22° 54′			
	Ptolemy	♉ 25° 34′	2° 40′	99 3/8	266
Regulus	*Hipparchus*	♌ 0° 2′			
	Ptolemy	♌ 3° 42′	3° 40′	72 3/11	367
Spica	*Hipparchus*	♍ 23° 59′			
	Ptolemy	♍ 26° 50′	2° 51′	93	285
Sirius	*Hipparchus*	♊ 14° 16′			
	Ptolemy	♊ 17° 38′	3° 22′	78 7/10	337
Antares	*Hipparchus*	♏ 10° 26′			
	Ptolemy	♏ 14° 18′	3° 52′	69	387

So if the advance of Aldebaran is accepted, the motion from Hipparchus to Ptolemy would be very swift, one degree in *66* years, while over the same

interval it is elsewhere put at 100 years, and very slow. Nor, to excuse this, can one admit between the observations of Hipparchus and Ptolemy an interval of 397 years, as would be necessary, since *Ptolemy*, Book 7, chapter 2, indicates 265, and that his own were made in nearly the same year. In the others too there is a notable disagreement with 100 years, except for Capella and, nearly, Spica. Finally, a sufficient indication of rather crude observations is that in Ptolemy's catalogue the longitudes are noted only by degrees and decades of minutes, and the latitudes by pentads, that is, by fives; yet a neglected error of 10′ or 5′ can seriously corrupt the annual motion of the fixed stars.

[Margin: Authors asserting the equality of the motion of the fixed stars.]

VI. Let it stand, then, that from the observations of the ancients an inequality in the motion of the fixed stars cannot be gathered without contradiction between those observations themselves and the hypotheses of the same men. Therefore, so long as the inequality is not proved on other grounds, after observations over a very long time to be undertaken by future posterity, if the world lasts so long, or if there are people whom a passion for watching the stars has seized, it is rather the equality of this sidereal motion that is to be retained. So it was wisely held by *Hipparchus* and *Ptolemy*, Book 7 of the *Almagest*; *al-Battānī*, chapter 51; *al-Farghānī*, *differentia* 13; *Albuhassim* the Moor, in the book on the fixed stars; King *Alfonso*, in his later opinion, of the year 1256; *Haly* [ʿAlī ibn Riḍwān], on Book 2 of the *Quadripartitum*; *Rabbi Levi*, 84 years later than Alfonso, in his Milhamot hescem, that is, in the book of the defences of God, chapter *62*; *Rabbi Yehoshua* in the Talmud, book Horaioth; *Rabbi Moses* in the Mishneh Torah [Miſnetorà], or book of the repeated law; *Rabbi Auenezra* in the book Taàmin, or of reasons; *Abraham Zaguth* in his great composition, and his disciple *Agostino Ricci* in the treatise on the eighth sphere, chapters 5, 43, 46, 47; *Kepler* in the Rudolphine Tables, chapter 34; and *Boulliau*, Book 5 of the *Astronomia Philolaica*, chapter 2, along with the ancient Persian mathematicians.

[Margin: Tycho's opinion.]

Tycho, for his part, in Volume 1 of the *Progymnasmata*, chapter 2, p. 255, although he speaks timidly, favours us rather than the contrary opinion, since he says that the inequality of the motion of the fixed stars asserted by Copernicus is refuted by more recent observations, which show a much quicker motion of the fixed stars in this century than his hypothesis allows. And he adds:

[Margin: If you assert it, why do you say "hardly"?]

This, however, I do not hesitate to assert here for just reasons: that so enormous an anomaly as is detected from the observations of Timocharis and Ptolemy compared with Hipparchus and al-Battānī hardly underlies the motion of the fixed stars. For it is not likely that they sometimes traverse one degree in 100 years, as Ptolemy gathered, and at other times in 66, as al-Battānī wished. Rather, in the observations of the practitioners themselves some fault no doubt lay hidden. He confirms this from the errors indicated in the difference of longitudes, though not as we have demonstrated them above. Finally, on p. *256*, he says: *Nor yet would I think*

that the considerations of the ancients regarding the fixed stars were so far astray that it could not in truth be gathered from them that some inequality of motion underlies them; although I judge that this occurs from some external cause and by accident, and is not rightly to be imputed to them. On this matter, however, I am not yet pleased to pronounce an absolute judgement, thinking it more prudent to reserve it for a comprehensive treatment of astronomy and to hold it back for several more years. Nothing could have been said either more timidly or more cautiously; and I seem to myself to see in Tycho the cast of mind of Ptolemy, from Copernicus, as from his master Hipparchus, hanging again and again so closely that he does not dare abandon him entirely, and is fearful from time to time.

[Margin: Tycho's timidity.]

Now, I think that this external cause is the unequal motion of the equinoctial points, from which comes some inequality of the year, though a small one, and one that Tycho admitted.

[Margin: Authors in doubt about the inequality of this motion.]

But Simon Stevin too, in Hérigone, volume 5 of the *Cursus Mathematicus*, p. 579, and Pedro Nunes, Book 2 *De Observatione*, ch. 4, and Erasmus Reinhold at the opening of the *Theorica* on the eighth sphere, and Gassendi, *Institutio Astronomica*, Book 2, ch. 8, call into question the inequality of this motion and of the precession of the equinoctial points, as something that arose rather from error in the observations, or that remained undiscovered because of the slowness of the motion.

[Margin: Regiomontanus's suspension of judgement, and his advice.]

And before them that great man Johannes Regiomontanus had said, in the *Epitome of the Almagest*, Book 7, proposition 6: *Whether, then, the multitude of instruments has introduced this variation, or nature has implanted in the fixed stars some motion still hidden from us, it is and will be very difficult to work out the character of this motion, because of its slowness. For if our forebears were deceived by their instruments, we too shall necessarily be deceived, since our considerations, unless they are compared with the considerations of the ancients, can never teach anything. But if we judge that this hidden motion is present in the stars, it will be expedient to keep an eye constantly on the fixed stars, and for posterity likewise to weigh the considerations written down.* These are almost the very words of al-Battānī, ch. 52.

SCHOLION.

Scholion.

From what was said in the preceding chapter, that distribution and anomaly of the motion of the fixed stars which Fracastoro sets out in the *Homocentrica*, section 1, ch. 16, certainly collapses. *There he says that the whole period of the fixed stars is completed in 36000 years, but its anomaly in 3600 years; that the motion of the Aplanes, taken by itself, completes 1° in 100 years, and was of that size at the time of Christ; that it becomes faster and faster over 900 years, up to the fastest, as*

from Christ to al-Battānī; that from then on it began to diminish, and will return to the mean in the year of Christ 1800; that afterwards, over another 900 years, it will be slower and slower, until in the year 2700 it is at its slowest; and that from then, receding from its extreme slowness over another 900 years, it reaches the mean in the year of Christ 3600. So 900 years before Christ it was slow, but with its slowness diminishing. Those numbers of the anomalies treated in Book 3, chs 28 and 29, collapse as well; I did not set down this Fracastorian anomaly there, because it is not common to the precession of the equinoxes, but proper to the fixed stars. Look, then, at its measures in the following small table, that is, at how much of the four quadrants of this anomaly the fixed stars are said to complete in each century. To it I shall append the times before and after Christ to which such quadrants correspond.

Quadrants of the Fracastorian anomaly, with the motion above or below the mean, which is 1° in 100 years.

Order of the centuries	Quadrant 1	Quadrant 2	Quadrant 3	Quadrant 4
1	1° 5′ 20″	1° 48′ 0″	0° 54′ 40″	0° 17′ 20″
2	1° 10′ 40″	1° 42′ 40″	0° 49′ 20″	0° 22′ 40″
3	1° 16′ 0″	1° 32′ 0″	0° 44′ 0″	0° 28′ 0″
4	1° 21′ 20″	1° 26′ 40″	0° 38′ 40″	0° 33′ 0″
5	1° 26′ 40″	1° 21′ 20″	0° 33′ 20″	0° 38′ 20″
6	1° 32′ 0″	1° 16′ 0″	0° 28′ 0″	0° 44′ 0″
7	1° 42′ 40″	1° 10′ 40″	0° 22′ 40″	0° 49′ 20″
8	1° 48′ 0″	1° 5′ 20″	0° 17′ 20″	0° 54′ 40″
9	2° 0′ 0″	1° 0′ 0″	0° 12′ 0″	1° 0′ 0″
Years	**from year – to year**	**from year – to year**	**from year – to year**	**from year – to year**
Before Christ	3600 – 2700	2700 – 1800	1800 – 900	900 – 0
Before Christ	7200 – 6300	6300 – 5400	5400 – 4500	4500 – 3600
Of Christ	0 – 900	900 – 1800	1800 – 2700	2700 – 3600
Of Christ	3600 – 4500	4500 – 5400	5400 – 6300	6300 – 7200

CAPVT XVIII.

Chapter 18.

De Theorica motuum Octauæ Sphæræ, & Conciliandis eius apparentibus motibus in Oppoſitas Mundi plagas.

On the theory of the motions of the eighth sphere, and on reconciling its apparent motions in opposite quarters of the world.

I. We have dealt with this matter in Book 3, chs 28, 29 and 30, because of the connection that the motion of the fixed stars has seemed to many to have with the variation of the obliquity of the ecliptic, with the precession of the equinoxes and of the equinoctial points, and with the motion of the auges of the

five or six planets above the Moon. To explain these changes some introduced 10 spheres and others 11, the highest of which was to be the primum mobile, imparting the diurnal motion of 24 hours to all the lower spheres. We have already settled the controversy over the change in the obliquity of the ecliptic in Book 3, chs 27 and 29, and have likewise dispatched the question of the anomaly of the year and of the precession of the equinoxes in Book 3, chs 28, 29 and 30; and what is usually sought from the unequal motion of the fixed stars in order to establish that inequality we have sufficiently weakened in Book 3, ch. 28, no. 11, and in this Book, in the preceding chapter. It now remains for us to bring together and briefly set out the opinions on the number and kind of the motions that appear in the fixed stars, and to indicate by what hypothesis, and not a fictitious one, they can not only be drawn on paper but actually be performed in the heavens.

[Margin: 1. Opinion on the single motion of the fixed stars.]

II. The first opinion, then, was that of the Babylonians, the Egyptians and the Greeks, above all Eudoxus and Callippus, as is clear from Aristotle in the books *De caelo* and in Book 12 of the *Metaphysics*: that the fixed stars are carried to the west by a single diurnal motion. For that reason the Babylonians defined the year as *the revolution of the Sun to the same fixed star*, as to a fixed point of the sky, that is, one movable by no other motion than the diurnal and common motion of 24 hours. Yet some think that in those words of Cicero in the *Dream of Scipio*, taken from Plato's *Timaeus*, *when the eight heavens have completed their circuits*, the particle *their* indicates a proper motion of the eighth sphere in longitude, distinct from the common one. Granted, he recognised no more than eight orbs, as will be said below at no. 18. Scaliger, *De emendatione temporum*, Book 4, p. 283, judges that Thales of Miletus, in the two books that on the testimony of Diogenes Laertius he wrote on the solstices and equinoxes, and then Meton and Euctemon, and afterwards Aristarchus and Timocharis, suspected a difference between the tropical and the sidereal year, and so some proper motion, either of the fixed stars in the order of the signs, or of the equinoctial and solstitial points against the order of the signs. Ptolemy, however, attributes this suspicion rather to Hipparchus, Book 7, ch. 1.

[Margin: 2. Opinion of Hipparchus and Ptolemy.]

III. The second opinion, then, was that of Hipparchus and Ptolemy. For as Ptolemy relates in Book 7 of the *Almagest*, ch. 1, Hipparchus came to suspect a twofold motion of the fixed stars. These are Ptolemy's words: *Hipparchus too, from what he had at that time, had a suspicion of both of them, so that he conjectured about the longer time rather than affirmed; for he found very few observations of the fixed stars before his day, and virtually only those that Aristyllus and Timocharis had written down, which are little explained and ambiguous as well.* And in ch. 2 he adds about the same man: *For Hipparchus too, in the treatise on the displacement of the solstitial and equinoctial points, having set out lunar eclipses, partly observed in his own age and partly long before by Timocharis, gathers that the star Spica stood,*

in his own time, 6° from the autumnal section against the order of the signs, but in Timocharis's time very nearly 8°. For in concluding thus he says:

[Margin: Hipparchus's words.]

So Spica formerly preceded the autumnal point by 8° in longitude, but now it precedes it by only 6°. And he shows that the other fixed stars have moved by the same interval in the order of the signs. Ptolemy adds in the same place that Hipparchus at first recognised this motion only in those fixed stars that are in the zodiac, but that from the distances of all the other fixed stars from one another and from the zodiacal stars, the same motion is to be ascribed to all. For he says: *The same figures and lines of the fixed stars are seen quite unchanged even now as were observed at the time of Hipparchus, and not only of those in the zodiac among themselves, or of those outside the zodiac with respect to neighbouring stars (which would certainly happen if, according to Hipparchus's first hypothesis, only those in the zodiac advanced in the order of the signs), but of all without qualification, even of those in the zodiac when they are compared with those that lie far from it.* So what Hipparchus suspected because of the short interval and the fewness and uncertainty of the observations, Ptolemy affirmed outright, having obtained a longer interval and more observations, especially those of Hipparchus, which in ch. 1 he calls *most clearly set out*. Moreover Ptolemy, in ch. 3, discovered, not merely ambiguously as Hipparchus did, but with certainty, that the motion of the fixed stars in the order of the signs takes place on circles parallel to the ecliptic, and on the poles of the ecliptic, not of the equator. He shows this in Book 7, ch. 3, from the fact that the fixed stars, always keeping the same distance from one another, never changed their former latitude, or distance from the ecliptic, but did change their declination, or distance from the equator, so much that many that were in the southern hemisphere later crossed the equator and became northern, and conversely those in the northern hemisphere turned out southern; and that those near the equinoctial points underwent a greater and sudden change in declination, while those near the solstitial points underwent a smaller and slower one. He demonstrates this by adducing many observations of Hipparchus and of his own. This motion in longitude he thought to be always equal and direct.

Ptolemy was followed by al-Battānī, ch. 51; Haly [ʿAlī ibn Riḍwān], Book 2 of the *Quadripartitum*; al-Farghānī, *differentia* 13; Albuhassim; King Alfonso in his later hypothesis; and various rabbis, above all Moses, Levi, Yehoshua, ibn Ezra and Avraham Zacuto; likewise Agostino Ricci, Kepler and Boulliau, with the Persian mathematicians, in the places cited in the preceding chapter, no. 6.

[Margin: 3. Opinion on both the inequality and the trepidation of the motion of the fixed stars.]

IV. The third opinion was that of those who thought that the proper motion of the fixed stars in longitude is both unequal and not always direct, but for some years retrograde toward the west, and for some others direct again toward the east, with a slow trepidation or libration in longitude. Of this kind were, first, certain people before al-Battānī, as is clear from his ch. 52, above all

Hermes, as Isaach Israelita [Yitzchak Yisraeli] testifies in the book *Ieſſod holam*, or the foundation of the world, and Alpetragius [al-Biṭrūjī]; then Theon, on the testimony of al-Biṭrūjī in the *Theorica Physica*; then al-Zarqālī and Thābit, and al-Biṭrūjī himself, though he explains this access and recess differently, as I shall say at no. 9. For the rest see Book 3, ch. 28, no. 6, where we rejected this fiction, because no fixed star from Hipparchus down to us has so far been seen to go back and have a smaller longitude from the beginning of Aries than it had before.

[Margin: 4. Opinion on the inequality of the motion and on a certain other trepidation.]

V. The fourth opinion was that of Alfonso in his earlier hypothesis, and of the Alphonsines, who thought that the mean motion of the fixed stars is always direct, but the true motion unequal, now slow, now fast, so that because of its speed they appeared direct, and because of its slowness stationary or retrograde; but not to such a degree, nor in the way that the third opinion holds. They therefore admitted a motion of trepidation or libration in another sense. Very many followed the Alphonsines, especially Gherardus Cremonensis [Gherardo da Cremona] and Georg von Peuerbach with his commentators; Maurolico in dialogue 3 of the *Cosmographia*; and Clavius on ch. 1 of the *Sphere*, from pp. 56 and 63. Magini seems to have agreed with them in Book 1 of the *Theoricae Planetarum*, but in reality he belongs rather to the following opinion.

[Margin: 5. Opinion on the inequality of a motion always direct, without trepidation.]

VI. The fifth opinion was that the motion of the fixed stars in longitude is always direct, with no trepidation that would make it retrograde, but that it is nevertheless unequal. This at length was the view, above all, of Giovanni Battista della Torre, and with him of Fracastoro in the *Homocentrica*, section 2, from ch. 9 to ch. 17; of Copernicus, Book 3 of the *De revolutionibus*, and with him of Reinhold in the Prutenic Tables, precepts 10 and 11; and of Rheticus in his *Narratio*, of Mästlin in his notes on that *Narratio*, of Magini, who in Book 1 of the *Theoricae Planetarum* sets out a libration of the spheres without retrogradation, of Longomontanus, Book 1 of the *Theorica*, chs 2 and 4, and of Lansbergen in his *Theoricae*, chs 1, 7 and 8, with precept 4 joined to 11. For although not all of these attribute this motion to the fixed stars in reality, as Fracastoro, Magini and Lansbergen do, but some grant it to them only as apparent and in reality attribute it to the Earth, as Copernicus, Mästlin, Rheticus and in part Longomontanus do, nevertheless all posit this apparent motion as always direct but unequal. For the remaining matters bearing on the measure and periods of this anomaly, see Book 3, ch. 28, no. 8, and ch. 29.

Since, however, the inequality of this motion, let alone its retrogradation, has been rejected by us in Book 3, chs 28 and 30, and in this Book, in the preceding chapter, there remain only three motions of the fixed stars that are true, either in reality or as to true appearance: namely the diurnal motion to the west, the proper motion to the east, and the variation of declination. The

various hypotheses that have been devised, or could be devised, to reconcile these without contradiction or violence I shall now review.

Hypotheſes Motus Fixarum.

Hypotheses of the motion of the fixed stars.

[Margin: 1. Hypothesis.]

VII. The first hypothesis was that of very many who admit solid orbs. It places above the eighth sphere another, distinct sphere, called the primum mobile, which mediately or immediately carries the eighth along with it from east to west, by a motion described around the poles of the equator, while meanwhile the eighth sphere of the fixed stars, or Aplane, moves eastward on the poles of the ecliptic, gradually striving against the motion of rapture [raptus] and of the primum mobile. But some have said that the primum mobile is the ninth sphere, others the tenth, others the eleventh.

[Margin: Sphere 9, the primum mobile for some.]

For some of the older astronomers thought that the ninth sphere performs the office of the primum mobile, especially those who thought that the motion of the fixed stars is uniform and perpetually direct and that the obliquity of the ecliptic does not vary. So Johannes de Sacrobosco, ch. 1 of the *Sphere*, and many Jews and Arabs, chiefly Rabbi Moshe, Rabbi Yehoshua, al-Farghānī, ibn Ezra and Haly, in Agostino Ricci. Yet Ptolemy, whatever Scheiner says in the *Disquisitiones Mathematicae*, p. 50, never mentions a ninth sphere, as I shall show below. But later more people assigned the tenth sphere to the primum mobile; and the ninth to the motion toward the east imparted by it to all the lower spheres: to the eighth with respect to the motion of the fixed stars, and to the rest, the Moon excepted, with respect to the motion of the auges, each to be completed in 49000 years. The eighth, or firmament, they assigned to the motion of access and recess, or trepidation, to the east and west, to be completed in 7000 years. So, indeed, the Alphonsines, Georg von Peuerbach and Johannes Regiomontanus, to whom Maurolico subscribed in the *Cosmographia*, dialogue 1, p. 24, and dialogue 3, p. 89, and Clavius on ch. 1 of the *Sphere*, pp. 44 and 72, in the first edition.

[Margin: Sphere 11, but motionless.]

To these must be added those who posited an eleventh sphere, but one wholly motionless, in order to save the differences of the influences that dominate in particular regions and produce different plants and animals in different places. In this way Pierre d'Ailly, questions 2 and 9 on the *Sphere*, posits an eleventh, resting sphere, and a tenth for the motion of the primum mobile, and a ninth for the motion to the east, but one to be completed in 36000 years, and an eighth for the motion of access and recess.

[Margin: Sphere 11, but movable.]

Indeed Clavius himself, with Bede, Strabo and other theologians, admits an eleventh heaven, motionless, whether watery or empyrean. Others, however,

added an eleventh sphere that is movable and destined for the office of the primum mobile; the tenth they assigned to the motion of the first libration and trepidation, in latitude, by force of which they said the obliquity of the ecliptic is changed; the ninth they assigned to the motion of the second libration or trepidation, to the east and west, to preserve the anomaly both of the motion of the fixed stars and of the year and of the precession of the equinoxes; and the eighth to the proper motion of the fixed stars in longitude toward the east. So Johannes Werner, in Osualdus [?] on the theory of the eighth sphere, and Magini, Book 1 of the *Theoricae Planetarum* and in the *Secunda Mobilia*, canon 18, though using the measures of the Copernican motion. Leupoldus Austriacus [Leopold of Austria] adhered to this view in his compilation published in the year 1489, and Clavius in the second edition of the *Sphere*, p. 36, as Scheiner has it in the *Disquisitiones Mathematicae*, p. 49. In line with this opinion, the theory of the eighth sphere is set out by Deusingius [Anton Deusing] and by Gassendi in the *Institutio Astronomica*. But we have said enough on the tenets of these authors concerning the quantity and period of the motion of trepidation in Book 3, the whole of ch. 29, above all at nos 12 and 13, where we also treat the opinion of Thābit and al-Zarqālī and of others who have written on the trepidations of the eighth sphere. All the authors already mentioned therefore require at least two distinct total spheres, one of which sweeps the fixed stars and the other lower orbs from east to west, by a motion performed around the axis of the equator, while the other pushes the fixed stars eastward on the axis and poles of the ecliptic.

[Margin: 2. Hypothesis.]

VIII. The second hypothesis is that of Giovanni Battista della Torre and of Girolamo Fracastoro in the *Homocentrica*, section 1, chs 15, 16 and 17. There, having said that it is more probable that the Aplanes is always carried direct beneath the ecliptic of the highest heaven, though with a motion now slow, now fast, now moderate, he places 6 orbs above the Aplanes, namely the primum mobile, then the *Circumducens*, which carries all the lower orbs around in breadth from north to south and conversely; beneath this the *Circitor*, by force of which the Aplanes is moved eastward, but unequally; and the *Controvectus* and the *Anticircitor*, so that the latitude of the fixed stars should not change, or should change only insensibly. He says that the anomaly of the motion, from fastest back to fastest, is completed in 3600 years, but the whole period of the motion of the fixed stars in 36000 years. You have a specimen of this anomaly in the scholion of the preceding chapter.

[Margin: 3. Hypothesis of Alpetragius and Achillini.]

IX. The third hypothesis was that of al-Biṭrūjī in the *Theorica Physica* and of Alessandro Achillini in the book *De orbibus*: that there are not two motions in any stars, one eastward and the other to the west, but a single motion, westward and diurnal. It is fastest in the primum mobile, as in the idea of motions, which the other spheres are to emulate, but in the rest it is slower and slower the further they are from the primum mobile. Because of this slowness they do not

match the speed of the primum mobile, nor do they reach the same meridian at the moment the primum mobile does, and they therefore seem to recede toward the east, because they do not reach the meridian so quickly in moving west. From this, they say, it has come about that as the shortfalls multiply, the distance from the meridian becomes perceptible and grows greater and greater from day to day, and gives occasion for feigning a second motion toward the east; but that motions are not to be multiplied without necessity, especially ones that conflict with one another or are violent. This hypothesis is refuted by Agostino Ricci in the treatise on the eighth sphere, ch. 10, and by Clavius on the *Sphere*, p. 48, because they think that this retardation takes place on the same poles and along the same line, and so the declinations of the stars would not vary, though it is agreed that they do vary.

[Margin: Alpetragius defended against the objections of Clavius and Ricci.]

But it appears that neither examined the text of al-Biṭrūjī. For al-Biṭrūjī was not so unskilled as not to know of the change of the declinations; indeed, to explain them, he said that this motion takes place on different poles surrounding the pole of the world. So al-Biṭrūjī says, on p. 9 of the *Theorica Physica*, that the primum mobile moves the lower orbs, not by sweeping them along through identity, continuity or the immediate conjunction of contact, but by imprinting on them a power and impetus that is separated from it, just as one who throws a stone or arrow is separated from the stone or arrow; and this power in the lower orbs is more and more relaxed and fails, so that they cannot keep up with the speed of the primum mobile. He then says that the poles of the eighth sphere are different from the poles of the primum mobile, that they move on their own small circles around the poles of the equator toward the east, and that they are as far from the poles of the primum mobile as the greatest declination of the Sun. So from the motion of the poles of the eighth sphere arises the variety of the declinations, and a motion mixed of a motion in longitude, but a slower one to the west, and one in latitude toward the poles of the equator. Since the motion of the poles of the eighth sphere is not uniform, it follows that the fixed stars always move to the west, but sometimes faster, a motion that Hermes, Theon and al-Zarqālī call recess against the order of the signs; and sometimes more slowly, a motion they call access in the order of the signs; whereas in this hypothesis the former should rather be called access and the latter recess. So al-Biṭrūjī has it, though in a longer discussion, from p. 10 to p. 15. This hypothesis retains the motion toward the east only in the poles, for this does not conflict so much with the motion of the primum mobile, since it is the motion of only two points, and the whole eighth sphere revolves with a single motion, following and imitating the motion of the primum mobile.

[Margin: Hypothesis of Alpetragius rejected in part.]

In the planets, to be sure, the apparent motion to the east does seem explicable by the retardation already mentioned, because we can compare their diurnal motion with the faster motion of a higher planet, and the diurnal motion of Saturn with the diurnal motion of the fixed stars, which return to the same

meridian sooner than Saturn and the other planets. But we cannot perceive the slowness of the fixed stars themselves, because we have no visible sign in the heavens above the fixed stars that moves faster than they do toward the west and with which we might compare their motion; and below them all the planets are posited as moving more slowly than the fixed stars. Moreover, if the motion of the primum mobile is continuous, as is fitting, and if it imprints the diurnal impetus on the eighth sphere contiguous to it continually and not once only, it does not seem that the impetus can ever be relaxed, and so this is not the physical cause of the slower motion of the eighth sphere. Finally, both the sphere of the primum mobile and its motion are multiplied in vain, since the eighth sphere itself can be the primum mobile and the first measure of all times, at least of those perceptible to us, as we shall say below, given, however, the other things that al-Biṭrūjī posited, or their equivalents.

[Margin: 4. Hypothesis of Agostino Ricci.]

X. The fourth hypothesis is that of Agostino Ricci in the treatise on the motion of the eighth sphere, chs 1, 13 and several others following. In these he teaches that there are only eight movable spheres, and that each is moved by a twofold motion, one to the west and the other to the east; and that primum mobile can be taken in two senses. In one it means some celestial body that is moved per se and first by the first mover, and in this sense there is no other primum mobile than the aggregate of all eight spheres, in which the parts are not moved per se, while the whole is moved per se, according to the doctrine of *Physics* 6, text 86. The other is that in which primum mobile is said of what is first in nobility and order in such a kind of motion, and in this sense the eighth sphere is the primum mobile; whereas the ninth, tenth or eleventh, feigned by others for the common diurnal motion, is a being of reason [ens rationis]. He goes on to confirm this opinion of his with great erudition and subtlety, and answers the arguments brought against it. Orontius [Oronce Finé] subscribes to Ricci, in Clavius on the *Sphere*, ch. 1, p. 50. Clavius, however, says that Ricci's hypothesis cannot be grasped even in thought, since it is impossible for a whole, an animal for example, and each and all of its parts to be moved by two contrary motions, if there is no other body by which it is moved. Unless, then, it is explained how the same sphere can be moved by two contrary motions without physical conflict, this hypothesis cannot be sustained.

[Margin: 5. Hypothesis of Joseph Scaliger.]

XI. The fifth hypothesis is that of Joseph Justus Scaliger in the *Diatriba de Aequinoctiis*, and *De emendatione temporum*, Book 4, p. 284. He relates that the Basilas at Olympia and the hierophants at Athens defined the year as 365 1/4 days; but that Hipparchus recognised that the tropical year is shorter than the hierophantic or Julian, and the astric or sidereal year longer than either, and that the solstitial and equinoctial points had receded from the fixed stars to the west. He asserts that this motion must be attributed to those points, and to the whole equator with its parallels, creeping toward the west, on movable and different poles from those of the world.

He confirms this from the immobility of the Pole Star, for he says: *That star which is in the tail of Cynosura was believed by the ancients to be the arctic pole of the world, and not without reason: for it never seems to us to move, not even in the least.* And he contends, on the testimony of Hyginus and Germanicus, that this star was once, otherwise than now, the most northerly among the stars of Cynosura. This star once stood 12° 24′ from the pole of the equinoctial, as *Ptolemy* reports in Book 1 of the *Geography*; it was not believed to be the pole of the world by Pytheas of Massalia and by Hipparchus, as it was believed by Eudoxus; and it now stands 3° 24′ from the pole of the equinoctial circle. All this, he says, came about not through a motion of the star itself approaching the pole of the world, but through a motion of the poles of the equator approaching the star itself; and because these poles have come nearer to that star through the ecliptic's [Greek: λόξωσιν (loxōsin, "obliquity")], which is now small, in former times, when the obliquity of the ecliptic was greater, the pole of the equator stood farther from that star; yet that distance will go on decreasing until the pole of the equator meets the pole of the world. He confirms the same from the star Canopus, which even now grazes the horizon of Rhodes, just as it once skimmed it on the testimony of Pliny the Elder, Book 2, chapter 70; and this is a sign that the declination has not changed and that the motion of the stars does not take place on the poles of the ecliptic. Finally, Scaliger holds that the fixed stars are not carried round and do not move in the order of the signs. He calls the motion of the eighth sphere a monstrosity, and the opinion about the motion of the fixed stars an old wives' tale and a trifle; the fixed stars, he insists more than once, retain the same distance from the poles of the world. Instead, he holds, the equinoctial and solstitial points are carried round by a proper and inexplicable motion, and it is they that bring about the [Greek: μετάπτωσιν (metaptōsin, "shift")].

But Scaliger's fabrication has already been refuted, besides Boulliau in Book 5, chapter 4, by four most learned men of our Society: *Clavius* on the *Sphere*, p. 62; *Paulus Guldinus* [Paul Guldin] in his refutation of the *Elenchus* written by Seth Calvisius against the Gregorian Calendar, Book 2, chapters 9 and 10; *Georgius Furnerius* [Georges Fournier], *Hydrography*, Book 10, chapter 26; and, most eruditely, *Pétau* in his dissertations on the *Uranologion*, the whole of Book 3. For the present, what we said in Book 3, chapter 28, no. 10, and in this Book, chapter 4 — where we treated Ursa Minor and the approach of the Pole Star to the pole of the world, observed through its greatest and least meridian altitudes — is quite enough to confute Scaliger. For it is all too evident from observations that the fixed stars describe now larger, now smaller circles about the poles of the world and of the equator. Whether a certain third motion, distinct from the daily motion and from the motion of declination, follows from this can be disputed, even if Pétau in chapter 4 asserts that it is demonstrated from this with the greatest certainty *that the eighth heaven moves in the order of the signs, whereas the equinoctial points are in no way carried against the order of the signs.* On this, see below.

[Margin: 6. Hypothesis of Copernicus.]

XII. The *sixth* hypothesis is that of Copernicus, Book 1, chapters 10 and 11, and Book 3, chapter 1, where he says that the sphere of the fixed stars, as the place containing all things, is motionless, although it seems to us to move because of the motion of the Earth. For he attributes to the Earth the threefold motion that appears in the fixed stars. The daily motion he attributes to the daily revolution of the Earth about its own centre toward the east. The motion of declination he attributes to the deflection of the axis of the terrestrial equator, which is oblique to the celestial ecliptic; through the varying inclination of this axis, he says, the obliquity of the ecliptic varies, and so do the declinations of the fixed stars from the equator, or rather of the terrestrial equator from the fixed stars. Finally, he ascribes the apparent motion of the fixed stars in the order of the signs to a motion of the equinoctial points against the order of the signs. So the fixed stars seem to us to advance toward the east, receding from the equinoctial points, when in fact and physically the fixed stars stand still; it is rather the equinoctial points themselves, which lie in the terrestrial equator, that little by little recede from the fixed stars against the order of the signs, toward the west, because of the libration of the terrestrial axis. This third motion, he says, is the Precession of the Equinoxes, and it is anomalous; about its anomaly we said much in Book 3, chapter 28, no. 9, and chapter 29. This hypothesis was followed by *Reinhold*, *Rheticus*, *Mästlin*, and many others of the Copernican sect, against whom we must argue at greater length in Book 9.

[Margin: 7. Hypothesis of Longomontanus.]

XIII. The *seventh* hypothesis, akin to the Copernican, is that of *Longomontanus* in the *Astronomia Danica*, Book 1 of the *Theorica*, chapters 2 and 4. He fears that, if any motion of conveyance is granted to the fixed stars, they may be torn apart and may alter the distance between them, which is agreed to be unchanging, or at least that they may be forced to suffer, if not a conflict of motions, at any rate some violence. He therefore attributes the annual motion to the Sun, not to the Earth, and in this he departs from Copernicus. But with Copernicus he nevertheless attributes the daily motion not to the stars but to the Earth, turning about its own axis toward the east in the space of 24 hours; and he likewise attributes to the Earth their apparent motion in the order of the signs, which he ascribes to a precession of the equinoxes that takes place in the motion of the Earth, though otherwise than Copernicus did. For Copernicus places on the surface of the Earth an equator oblique to the celestial ecliptic, and holds that the axis of this equator librates in such a way that the sections of the equinoxes shift little by little against the order of the signs. Longomontanus, however, places on the terrestrial surface not only an equator but also an ecliptic, lying beneath the celestial ecliptic; and he represents the precession of the equinoxes and the apparent motion of the fixed stars in the order of the signs by means of the poles of the Earth going back gradually, with a very slow motion, about the axis of the terrestrial ecliptic.

[Margin: 8. Hypothesis of Lansbergen and Boulliau.]

XIV. The *eighth* hypothesis is that of *Lansbergen* in the Tables, precept 11, and of *Boulliau* in the *Astronomia Philolaica*, Book 5, chapter 2. They attribute the annual motion of the Sun to an annual motion of the Earth about the Sun through the great orb, and the daily motion of all the stars to a daily rotation of the Earth toward the east about its own centre, and in this they agree with Copernicus. But they attribute the motion of the fixed stars in the order of the signs to the fixed stars themselves, and to their sphere, carried round gradually toward the east on the poles of the ecliptic. For *Lansbergen* thinks it fitting that, just as in the microcosm the brain, which is in the highest part, moves with a slow and barely perceptible motion, so the eighth sphere, the highest part of the macrocosm, should not be motionless but should move with a slow motion, which it communicates to the lower spheres. Nor does he think it likely that the sections of the ecliptic and the terrestrial equator gradually move against the order of the signs; otherwise the quarters of the world at each place on Earth would by now be other than they were at the beginning of the world, which he considers absurd. In the *Theoricae* of the celestial motions, chapter 7, however, he refers the inequality of the motion of the fixed stars to an unequal motion of the vernal equinox, from which the longitude of the fixed stars is counted; and so he counts the mean motion of the fixed stars from the mean equinox, in which he agrees with Longomontanus. It is in this sense that I said in Book 3, chapter 28, no. 9, that both attribute to the fixed stars a motion reckoned from the mean equinox. Otherwise they differ from each other, because Lansbergen attributes a real motion to the fixed stars, and attributes the inequality of the motion of the fixed stars to the inequality of a point that is not terrestrial but celestial.

Boulliau, on the other hand, grants the fixed stars a real but uniform motion, so that that body, which is farthest from the centre of the world, that is, from the place of rest, should not be without motion. He judges that every body set about the centre of the world is owed some motion about that centre by nature, which preserves all things in motion and never lets them grow torpid. He also gives the reason why the motion of the fixed stars does not in fact come about through a slow motion of the terrestrial poles, carried round like a spinning top about the poles of the zodiac against the order of the signs. Granted, he says, this could be admitted if no planet existed other than the Sun; but once the other planets are posited, as they exist in nature, it cannot. Otherwise, through that spinning of the Earth, the remaining planets would appear to approach the zenith and to recede from it by some peculiar motion, which nevertheless does not appear in them. In the fixed stars, however, because of their slowness, he grants that in that fictitious hypothesis this approach and recession would go unperceived, especially since there is no other body above them moved in a different manner with which the motions of the fixed stars could be compared. These, then, are the many and so diverse hypotheses we have so far had devised for reconciling the motions of the fixed stars with one another.

Noſtra Opinio & Hypotheſis de Motu Fixarum.

Our Opinion and Hypothesis on the Motion of the Fixed Stars.

[Margin: 1. Assertion.]

XV. *First* I assert that if the solid sphere of the fixed stars is posited as moving toward the east on the poles of its ecliptic, and meanwhile, turned round by another, higher sphere about the poles of the world and of the equator, as moving toward the west — whether by mere carrying-round and the sweep of a contiguous orb, or by an impressed impetus — no conflict, or violence, or other absurdity follows. For there is no danger that those two bodies will be worn away by long friction, or catch fire and melt, or give out a sound perceptible to us at so great a distance, or a sound perceptible indeed, but to which our dulled ears went deaf immediately after our first cries, as is imagined in the *Dream of Scipio*.

Nor can it be violent for a part to be moved in one direction for the good of the universe while, tending meanwhile in the other direction because of a particular function assigned to it by nature and by God, it experiences a certain mixed motion.

[Margin: Various comparisons for motions in contrary directions.]

The conflict, however, that some perceive in these two motions, or in this one motion mixed from two, is commonly removed by various comparisons: that of an ant crawling in the contrary direction on a wheel turned round the other way; or of fish making their way up a river against the current, which nevertheless carries them down, though not in so headlong a rush as if they strove with no effort in the contrary direction; or of a sailor walking back from prow to stern while the prow of the ship is being borne toward the opposite quarter; or of the ship itself, which, carried to the west by the course of the ocean or a river, yet driven back to the east by the force of oars or winds, either overcomes or retards its being swept to the west; or of a glass sphere full of water, which, turned round in one direction, communicates the same impetus to the water enclosed in it, while the sphere itself, after that impression, is turned back by a reciprocal rotation in the opposite direction, the former motion of the water still persisting toward the earlier quarter; or finally by the comparison of a basin, likewise full of water, which you turn to the right with one hand, gripping its handle, about a pole fixed beneath it, while the water, driven round meanwhile by a finger of your other hand thrust into it, is carried with its eddies in the contrary direction, to the left, and a cork or something else floating on top rolls the same way.

[Margin: Correction of the imagining of two motions in contrary directions.]

In these and similar cases, however, we must not, from the false imagination of the vulgar, conceive of two real continuous motions of the same movable point, really or even formally distinct, one of which is an approach toward a given fixed quarter of the world and the other a recession from that same quarter

of the world. That manifestly conflicts with the essence and definition of local motion, and *Clavius*, on the *Sphere*, p. 52, rightly said that there is no instant in which the fixed stars do not move toward the west, or in which they approach farther toward the east. We must therefore conceive of one motion made from two that are distinct only equivalently, or extrinsically, by reference to the different quarters of the world and by dependence on two efficient principles. One of these prevails absolutely, moves toward one side, and really brings into act the motion it intends. The other, striving against it in the opposite direction, does not actually move the movable body toward the side it strives for, but resists, so that the body is not swept along by the first principle with as great an impetus as it otherwise would be; and, if that resistance is oblique, so that the body is not swept along exactly the same line along which it would otherwise be swept. Therefore, if the impetus of the primum mobile to the west on the one hand were equal to that of the eighth sphere resisting to the east on the other, the eighth sphere would move in neither direction but would remain in the same state. But if the impetus of the eighth sphere, striving obliquely toward the east on the poles of the ecliptic, were greater than the impetus of the primum mobile, the fixed stars would move continually toward the east, but obliquely. So a star placed at the beginning of Capricorn and on the ecliptic itself — such as Præregulus nearly is, that is, the little star preceding the Heart of Leo — seen at the setting-point of the winter solstice, would gradually rise from the horizon, and in 6480 years, according to my motion, would arrive through the ecliptic at the meridian of the horizon already mentioned and at the equator itself. Then, descending step by step, after another 6480 years, having completed a semicircle of the ecliptic, it would set for those people at the point of the summer rising. Yet because of the resistance of the primum mobile, striving in the opposite direction on the poles of the equator, it would not be carried through the ecliptic at the same speed, nor along exactly the same path, as it would go without such resistance.

Now, however, the impetus of the primum mobile (in this hypothesis, at any rate, which we are setting out) is far greater than that of the eighth sphere, and the latter's effort aims at a motion otherwise feasible on the poles of the ecliptic. It does not actually obtain this motion, but it does bring about that no fixed star is swept along by the primum mobile with so great an impetus, and that none describes the same and perfect daily circle parallel to the equator, but that each is twisted rather into a spiral or helicoidal line, ever looser and looser over some centuries and tighter and tighter over others, according as the declination from the equator turns out at one time smaller, at another larger. But *Clavius*, on chapter 1 of the *Sphere*, p. 53, accustomed to the ten spheres of the Alphonsines, tries to explain the very thing we have said in another way, but by a motion physically unintelligible: by a motion in a qualified sense [secundum quid] and an approach to the eastern signs of the zodiac of the ninth sphere.

[Margin: What if there are two motions, divided and interrupted.]

XVI. But if anyone were to say that the two motions already mentioned are really distinct, yet not continuous but discrete — so that through imperceptible brief pauses the primum mobile rests, and meanwhile allows the eighth sphere to be advanced ever so little on the poles of the ecliptic in the order of the signs, whether that pause were single in one natural day or multiple, so that between any two perceptible parts of the day there came an imperceptible pause — he, in my judgement at least, would be saying nothing absurd, nor could he be refuted by any observations, since he would be positing the motion of the primum mobile as physically continuous to the senses, or at any rate interrupted by those intermissions, to some of which he might perhaps ascribe the twinkling of the fixed stars. Yet he too would have no argument from observations by which he could prove this motion to be interrupted and composed of two really distinct motions, rather than mathematically continuous and only virtually double. He would, however, have to concede that those brief pauses, taken together, produce a perceptible change over the course of time.

On my reckoning, moreover, the fixed stars advance daily by 8‴ 13⁗ 9⁗′ 3/73. But let us suppose, for ease of calculation, that it is only 8‴, so that to each hour of one day there fall 20⁗, to each minute 20⁗′, and to each second 20⁗′, that is, sixths. Therefore, if anyone were to say that these spheres complete their motion in alternate seconds, and that the natural day is so distributed that of the 86400^{s} in it half falls to the primum mobile and half to the eighth sphere, so that in one second of time the eighth sphere would accomplish 40⁗′ toward the east, and in the next second the primum mobile would accomplish 30″ of the equator — no one could discern, from observed altitudes or courses of the stars, the 40⁗′ made toward the east, nor whether an interruption of the motion of the primum mobile occurred through alternate seconds. How much less, if it were through alternate thirds or fourths of time, which are already not perceptible to us?

[Margin: 2. Assertion.]

XVII. *Secondly*, however, I say that a primum mobile distinct from the whole sphere of the fixed stars is not necessary, but that their apparent motions can really be carried out through that very sphere of the fixed stars or beneath it, and indeed in three ways above all, which I have recently devised and now propose.

[Margin: 1. Way of the motion of the fixed stars within one heaven.]

The *first way* is if the heaven of the fixed stars is either penetrable by the fixed stars, or rather fluid, and this heaven itself moves toward the west on the poles of the equator, carrying round with it the fixed stars as if swimming in that huge ocean, while the stars themselves, meanwhile, by their own effort on the poles of the ecliptic, resisting against that aetherial torrent, retard their being swept along. They do not allow themselves to be carried off through circles strictly parallel to the equator, but let themselves be carried round in daily gyres through spiral semi-helices. Each star may undertake this resistance by its own internal force; or the resistance may proceed from God, or from

individual Intelligences, or from a single Intelligence that governs all the stars, bound together by a bond hidden from us, like lamps or carbuncles fastened on a spheroidal lattice. For make or imagine for yourself a little net of iron wires, shaped into a spherical surface, with little balls turned from cork inserted at its knots or at certain parts of the wires, by whose lightness the net floats within water enclosed in a spherical glass vessel. Let the water, turned round to the right, carry the net round, yet in such a way that, with someone resisting toward the left and obliquely, those balls are carried to the right indeed, but more slowly and through one whorl after another. What you would see in that motion can be transferred exactly to the appearance of the motion of the fixed stars, if it takes place in a similar way.

[Margin: Whether the fluidity of the heavens is probable.]

For the perpetual constancy of the fixed stars in the same latitude and difference of longitude, and in their distance from one another, is indeed a probable argument that they are fixed like golden nails or fire-spewing gems in solid adamant; and the solidity of the heavens in general is favoured by that saying — not indeed of God, or of the canonical writer himself, but of Elihu in Job, chapter 37: *Perhaps you have fashioned the heavens, which are most solid, as if cast in bronze.* Even so, fluidity is not improbable; indeed in the planetary heaven, as will be said in Book 9, it is more probable; and that saying could be understood metaphorically of the constancy in distances already mentioned, or of incorruptibility, or of the perenniality of the motion.

[Margin: 2. Way.]

The *second way* is if, admitting likewise the fluidity of the Aplanes rather than mutual penetrability, the heaven itself of the fixed stars is not primarily moved of itself so as to sweep the fixed stars to the west, but, so far as lies in it, is motionless, like water in a pool, its parts being moved only accidentally, upon the motion of the fixed stars by which they are pushed forward; while the fixed stars themselves, each turned round either by an intrinsic force or by an Intelligence, undergo a single daily motion through spiral helices about the equator, as is required by the variety of declinations that appears in them and by the immutability of their latitude with respect to the ecliptic alone. But the Sun itself too, of itself or by an Intelligence assisting it, traces with its centre its helicoidal spirals in its own heaven, intersecting the equator twice a year at points ever different and ever farther west, and returning each day more slowly to the same meridian in which it was on the previous day with or beneath one of the fixed stars. With these things posited together, through one daily motion, single in kind and quasi-spiral, in each of the stars and in the Sun, the fixed stars will appear ever farther to the east of the equinoctial points, and the variety of declinations and the other phenomena we observe in them will be saved. Nor will a motion be posited from two principles, one moving and prevailing, the other resisting and retarding, but from a single principle without any resistance, and therefore worthier of the heaven.

[Margin: The objection of Clavius is dissolved.]

As for *Clavius*, on the *Sphere*, p. 47, who denies that the stars can move of themselves like birds in the air or fish in the sea, so far as to pronounce: *It is impossible that the stars should move of themselves, if the things that appear in their motions are true; rather, they must be carried round only at the motion of the orb in which they are* — this ought to have been demonstrated, if it could be demonstrated; and it must be forgiven in a man steeped in the ancient opinion of the Alphonsines and the followers of Peuerbach, and long accustomed to picturing solid orbs. The examples, however, that he himself supplies on p. 54 — of the motion of a ship's master on a ship, or an ant on a wheel, or water enclosed in a glass sphere or a basin, toward the east, while the ship, wheel, sphere or basin is carried toward the west — presuppose the fluidity of air and water, which do not resist.

[Margin: 3. Way.]

The *third way* is if, in keeping with the commoner understanding of astronomers and physicists, we wish to retain the solidity of the eighth sphere, with the fixed stars set in it like gems in crystal, and we conceive of some Mover moving any one of the fixed stars on the spherical surface through helices and, as it were, daily spirals toward the west, not on the poles either of the equator or of the ecliptic, but about the centre of the world; yet in such a way that it has regard to the equator in varying the declinations of that star, to the ecliptic in preserving its latitude, and to the equinoctial points in increasing the star's distance from them. For it is certain that if it moves even one star so as to reproduce its apparent motions, all the others fixed in the same sphere will equally be moved, with the appearances to be preserved being preserved, because of the perpetual distance they keep among themselves. What is difficult for our apprehension, however, we must not judge [Greek: ἀδύνατον (adynaton, "impossible")], or difficult for the moving Intelligences; especially since we can move a globe half-enclosed in a bowl with such ease that, by carrying one point of its convex surface round the same centre, we can trace lines so varied and admirable and exhibit the motions of the fixed stars already mentioned by a single motion. We could do the same with a free sphere, enclosed in no orb or bowl, if its centre, taken mathematically, were supposed motionless and fixed. The poles of the world, however, and the equator and the ecliptic will have to be conceived as outside the eighth sphere.

[Margin: A notable corollary on the primacy of the sphere of the fixed stars.]

XVIII. It is evident, therefore, that the motions of the fixed stars can be explained without a motion of the Earth, and indeed not by multiplying spheres, but in and through the single sphere of the fixed stars; and so there is no necessity to posit another moving heaven beyond the sphere that they call the Eighth, from the excess of its number over the path and place of the seven planets. Hence it comes about that the eighth sphere itself can be called the primum mobile, by priority of speed and by eminence over all the movable spheres; the first mover of the other celestial bodies, not efficiently but exemplarily, in

that the other planets or their movers strive to imitate it in their daily spirals or helices; and, finally, the first measure of all times.

[Margin: Eight heavens in Plato, Nembroth and Ptolemy.]

Finally, it is not necessary to multiply motions in this sphere. Plato said this, though in another sense, in Book 10 of the *Republic*, where he introduces the Pamphylian [Pamphilium], killed in war and returning from the underworld, and asserting *that he saw eight tracks, of which the eighth was driven in the swiftest course.* And in the *Epinomis* he says: *The philosopher must not be ignorant of how the seven circuits revolve beneath the first.* Nembroth the Chaldaean too, in his astronomical work, ascribes only a single motion to the signs of the heaven. Nor does Ptolemy describe the equator and the zodiac in different spheres, in Book 1 of the *Almagest* or elsewhere, nor does he anywhere mention a ninth or tenth sphere, even if *Averroes* on *De caelo* 2, commentary 67, imposes on Ptolemy a number of nine heavens, and *Albertus Magnus* in the same place, treatise 3, chapter 11, a number of ten. *Abraham Abenezra* [Avraham ibn Ezra], moreover, at the opening of the little book entitled *Miſpate hamazaloth*, that is, *On the Judgements of the Signs*, says: *The great and glorious orb, where the glorious hosts of God are, is called the heaven of heavens, because beneath it are the orbs of the seven planets, etc.* But a number of theologians hold that the heaven of heaven is another, motionless heaven.

SCHÓLIA.

Scholia.

[Margin: A new question about the fixed stars, if the planets did not exist.]

I. It could be asked what would happen if there were no planets, and the fixed stars alone moved with the motion with which they now move: that is, whether a double motion in longitude could still appear in them and be established for us. For it is certain that it would then be possible, from two greatest digressions, or from two equal altitudes of the same star on either side toward east and west, to obtain an arc of the horizon, by bisecting which a meridian line could be drawn; that on this line, from two meridian altitudes of the same never-setting star, the altitude of the pole, and so of the equator, could be obtained; and lastly that from meridian altitudes of the same fixed star, observed in different centuries and preserved in writing or by tradition, a change of declination in the fixed stars could be recognised, with the approach of some to the visible pole and the recession of others. On these premises I shall answer the question briefly in the following scholia.

[Margin: 1. Response. On the ecliptic of the fixed stars.]

II. *First* I say that in such a case there would be neither the ecliptic that now exists, nor equinoctial points, nor right ascensions or longitudes that could be counted from them. If, however, some one or more of the fixed stars lacks all latitude — as perhaps does the little star immediately preceding Regulus, although in the Rudolphine table it has a southern latitude of ½′ —

that star would mark out for us the circle along which the centre of the Sun now goes, a circle that in such a case we may call the ecliptic of the fixed stars. Its greatest declination would be as great as the obliquity of the ecliptic, which for me is always 23° 30′. And such a star would arrive from its greatest southern declination at no declination in 6480 years, in my opinion; from its greatest southern at its greatest northern in 12960 years; and finally from its greatest southern back to its greatest southern in 25920 years. From that point of the equator which such a star would reach, then, just as from the equinoctial intersections, the right ascensions and longitudes of the other fixed stars could not unfittingly be counted, and the latitudes could be counted from the same.

[Margin: 2. Response. On the oblique circles of the others.]

III. *Secondly* I say that all stars whose latitude, reckoned now from the ecliptic of the Sun and then from the ecliptic of the fixed stars, is 23° 30′ will, in changing declination, come to touch the equator, but will not cross it; indeed they will return from there again to their greatest declination toward the same pole. Such is a certain little star following in the hand of Ophiuchus. Those whose latitude is less than 23° 30′ will cut the equator; and all will describe circles parallel to the ecliptic of the fixed stars, that is, to the circle described by any star without latitude, smaller and smaller for greater and greater latitude. The remaining stars, whose latitude is greater than 23° 30′, will reach neither contact with the equator nor its section; and any that have a latitude of 66° 30′, as nearly does the little star following in the bend of Draco, will reach the pole of the world at their greatest declination.

CAPVT XIX.

Chapter 19.

In quo Selecta & identidem noua Problemata ſpectantia potiſſimùm ad Declinationes & Aſcenſiones Fixarum tum Rectas tum Obliquas.

In which are selected and, now and again, new problems, concerning chiefly the declinations and ascensions of the fixed stars, both right and oblique.

In chapters 11 and 12 we set out in advance certain problems, and in chapter 13 certain rules, which concern the declinations and ascensions of the fixed stars, and indeed their longitudes and latitudes, because the order of the teaching so required. The other matters of longitude or latitude present scarcely any difficulty, since the latitude of the fixed stars, at least of itself, does not change, any more than does the difference of longitude; and it is very easy, by adding to or subtracting from the root according to their annual progress, to find the longitude of the fixed stars for any future or past time. Certain other matters that remain, however, seem to me to belong partly to this place, and partly, as being more general, to Book 10, of Problems of the primum mobile; although in this sorting it is very hard to satisfy everyone.

Problema 1. An & quando ſtella datæ Longitudinis & Latitudinis peruentura ſit ad Aequatorem, aut Tropicum ſuum, vel etiam illum tranſitura.

Problem 1. Whether and when a star of given longitude and latitude will reach the equator or its tropic, or will even cross it.

I. *First*, a star without latitude reaches the equator when it reaches the beginning of Aries or Libra. So seek the arc of longitude remaining to the equinoctial points, and convert it into the time that corresponds to it. For example, the star immediately preceding Regulus either lacks all latitude or has a latitude of scarcely half a minute, and in the year 1644 it stood at ♌ 22° 21′ 50″. To the beginning of Libra these fall short by 37° 38′ 10″; and if one degree requires 72 years, these in every case require 2710 years.

[Margin: Example of case 1.]

Second, if the latitude of a star is greater than the obliquity of the ecliptic, it will never reach the equator; and if it is greater than double the obliquity, it will not reach the tropic.

Third, if the latitude of a star is equal to the obliquity of the ecliptic, it will reach the equator when it reaches the beginning of Cancer, if the latitude is south, or when it reaches the beginning of Capricorn, if the latitude is north; and from there it will go on receding from the equator. So from the distance of such a star from the beginning of ♋ or ♑, seek the time corresponding to that distance.

Fourth, if a star's latitude is less than the obliquity of the ecliptic, it will at some time cross the equator, and will change south declination into north, or north into south, etc.

Problema 2. An & quæ Stella capax ſit tranſitus per Verticem habitantis extra Zonam Torridam.

Problem 2. Whether a star, and which, is capable of passing through the zenith of someone living outside the Torrid Zone.

[Margin: 1. Rule.]

II. *First*, a star without latitude never passes through the zenith of the inhabitant already mentioned; just as a star having south latitude does not pass through the zenith of northerners, nor a star having north latitude through the zenith of southerners.

[Margin: 2. Rule.]

Second, a star whose north latitude is less than the distance of the zenith from the tropic of ♋ does not pass through the zenith of northerners, and always remains south. Likewise, a star whose south latitude is less than the distance of the zenith from the tropic of ♑ does not pass through the zenith of southerners, and always remains north.

[Margin: 3. Rule.]

Third, a star whose north latitude is equal to the distance of the tropic of ♋ from the zenith passes through the zenith of northerners when it is at the beginning of ♋; otherwise it is always south. Similarly, a star whose south latitude equals the distance of the tropic of ♑ from the zenith passes through the zenith of southerners when it is at the beginning of ♑; otherwise it is always north.

[Margin: 4. Rule.]

Fourth, a star whose north latitude is greater than the sum of the distance of the equator from the zenith and the declination of the tropic does not come to the zenith of northerners, but is always north for them. Similarly, a star whose south latitude is greater than the sum already mentioned does not come to the zenith of southerners, but is always south for them.

[Margin: 5. Rule.]

Fifth, a star whose north latitude is equal to the sum of the distance of the equator from the zenith and the declination of the tropic passes through the northern zenith when it is at the beginning of ♑. Similarly, a star whose south latitude equals the sum already mentioned passes through the southern zenith when it is at the beginning of ♋.

[Margin: 6. Rule.]

Sixth, therefore, every star whose latitude is neither less than the distance of the zenith from the nearer tropic nor greater than the distance of the farther tropic from the zenith is capable of passing through the zenith of northerners, if the latitude is north, and of southerners, if south, who live outside the Torrid Zone.

Problema 3. An & quæ Stella capax ſit tranſitus per verticem habitantis in Zona Torrida.

Problem 3. Whether a star, and which, is capable of passing through the zenith of someone living in the Torrid Zone.

[Margin: 1. Rule.]

III. *First*, a star without latitude passes at some time through the zenith: through that of those living under the equator when it is at the beginning of ♈ and ♎, and through that of those living under the tropics when it comes to their tropic.

[Margin: 2. Rule.]

Second, a star whose latitude does not exceed the distance of the zenith from the farther tropic is capable of passing through the zenith, that of northerners if the latitude is north, that of southerners if south. But if it exceeds the distance already mentioned, it is not capable of such a passage, but is always north for northerners, if the latitude is north, and south for southerners, if the latitude is south.

[Margin: 3. Rule.]

Third, a star whose north latitude is equal to the distance of the zenith from the farther tropic comes to the zenith of a northern inhabitant when it is at the beginning of ♎. Similarly, a star whose south latitude equals the distance of the zenith from the farther tropic comes to the zenith of a southern inhabitant when it is at the beginning of ♋.

[Margin: 4. Rule.]

Fourth, a star whose latitude toward the opposite quarter does not exceed the distance of the zenith from the nearer tropic is capable of passing through the zenith; but if it exceeds it, it is not capable.

[Margin: 5. Rule.]

Fifth, a star whose south latitude equals the distance of the zenith from the nearer tropic passes through the zenith of northerners when it is at the beginning of ♋; just as a star whose north latitude equals the distance of the zenith from the nearer tropic passes through the zenith of southerners when it is at the beginning of ♎.

Problema 4. Inuenire Tempus, quo ſtella Fixa capax tranſitus per verticem loci dati, tranſijt, aut tranſitura eſt per illum; ſi detur ſtellæ Latitudo & Longitudo.

Problem 4. To find the time at which a fixed star capable of passing through the zenith of a given place has passed, or will pass, through it, if the star's latitude and longitude are given.

IV. Let a star capable of passing through the given zenith be found by Probl. 2 or 3. We must then seek the longitude, or place on the ecliptic, that the star must reach in order to pass through that zenith; for from the given present longitude the time will easily be gathered. So let ABCD be the meridian and solstitial colure, and AC the equator, whose pole is B; let EL be the ecliptic, whose pole is K; and let the zenith be V, through which let VG be drawn parallel to the equator. For it is certain that whenever the star reaches that parallel, it will pass through the zenith V at its culmination, as if it is at F or at H. So we must seek the longitude of the star corresponding to such a position. Now suppose that the star must be at F, and through its centre let there be drawn both the circle of declination BFM and the circle of latitude KFI. Then in

Diagram. A circle set into the right column at the foot of the text, the meridian and solstitial colure of Problem 4, crossed by straight and curved lines representing the equator, the ecliptic, the parallel through the zenith, and circles of declination and latitude through the star's place.

- A (top left, on the circle): point of the meridian
- E, V (top, side by side, on the circle): the end of the ecliptic and the zenith
- B (right, on the circle): pole of the equator
- I (inside, upper left): on the circle of latitude

- F (inside, upper centre): the star's position
- M (left, inside the circle): on the circle of declination
- D (left, on the circle): point of the meridian
- ♎ and ♈ (centre, side by side): the equinoctial points
- K (right, on the circle): pole of the ecliptic
- H (inside, lower right): a second position of the star
- G (lower right, on the circle): end of the parallel through V
- P (inside, lower left): not named in the text on this page
- N (lower centre, inside the circle): not named in the text on this page
- C (lower right of centre, on the circle): point of the meridian
- L (bottom, on the circle): end of the ecliptic triangle FBK find the angle at the pole of the ecliptic, namely FKB; for it will be the argument of the longitude sought. All the triangle's sides are given: BK, the distance of the poles; KF, the complement of the star's latitude; and FB, the complement of the distance of the parallel from the equator, which is always equal to the complement of the altitude of the pole. So the angle FKB will not remain hidden, which measures the ecliptic arc EI, namely the star's distance in longitude from the beginning of ♋, E, both on this side in the first quadrant and beyond it in the second. From this the desired longitude will at once be known. When it is compared with the present longitude, the difference of longitudes will be established, and from that, according to the motion of the fixed stars, the time required for the star to have come or to come to the parallel VG, and so to have passed or to be about to pass through the zenith V. But if the star is supposed at H, draw the arcs of declination BHN and of latitude KHP; and in triangle BHK, in which are given the distance of the poles BK, the complement of the star's latitude KH, and the complement of the altitude of the pole, which is BH, find the angle BKH, which measures the arc EP of the ecliptic. By adding 90° to it you will have the desired longitude of a star of the third quadrant; by subtracting it from 450° the desired longitude of the star will be known, but in the fourth quadrant of the ecliptic. Do likewise in the other cases.

[Margin: Corollary.]

From what has been said you will gather that it is possible for a fixed star, in the whole period of its longitude, if the world lasts so long, to pass twice through a given zenith, according to two of the four quadrants of the ecliptic in which it can come to the given parallel.

EXEMPLVM.

Example.

V. *At Bologna the arc AV, the distance of the equator from the zenith, is 44° 29′ 30″, and its complement VB, that is FB, 45° 30′ 30″; BK, the distance of the poles, is 23° 30′; and the north latitude of the bright star of Lyra, namely the arc FI, 61° 46′*

30″; therefore the arc FK is 28° 13′ 30″. So by the analysis of triangles the angle FKB, and hence the arc EI, is 55° 22′ 45″. Therefore when Fidicula [the Lyre] has been, or will be, at ♉ 4° 37′ 15″ or at ♌ 25° 22′ 45″, it has passed or will pass through the zenith of Bologna. But since in the year 1644 it stood at ♒ 10° 25′ 40″, the difference of longitude to ♉ 4° 37′ 15″ is 114° 11′ 35″, for which there are required, in my opinion, 8222 years; going backward, however, to ♌ 25° 22′ 45″, it is 135° 2′ 55″, for which there are required about 9721 years. So on the true chronology Fidicula has never yet passed through the zenith of Bologna, nor will it pass except 6062 years from now.

Problema 5. An ſtella Polaris, aut alia fuerit aut futura ſit vnquam in ipſo Mundi Polo.

Problem 5. Whether the Pole Star, or another, has been or will ever be at the very pole of the world.

VI. Two conditions are required for a star to be at the very pole of the world. The *first* is that its latitude be precisely as great as the complement of the distance of the poles; the *second*, that if the latitude is north, the star be at the beginning of Cancer, and if south, at the beginning of Capricorn. In our view, then, the star's latitude ought to be 66° 30′, and I find none such in the corrected catalogues. According to Copernicus and his followers, however, the greatest distance of the poles is 23° 52′ and the least 23° 28′; so the star would have to have a latitude not greater than 66° 32′ nor less than 66° 8′. Such is the star of the 4th magnitude in Cygnus, the leading one of the two toward Lyra, whose latitude is 66° 15′. But when will these conditions coincide, so that this star is at the beginning of Cancer at the very time when, as the variation of the obliquity of the ecliptic brings it about, the star's latitude is precisely equal to the complement of the obliquity, or of the distance of the poles? The latitude of the star now the Pole Star, that is, of the last one in the tail of Ursa Minor, is 66° 2′. So it has never been, nor will be, at the very north pole of the world, even though it is going to approach it as closely as it ever will, and to reach the beginning of Cancer in the year of Our Lord 2092 according to Tycho, 2103 according to us; and it will then be about 7′ from the pole of the world, after which it will begin to recede from it. We, however, once misled by the authority of Regiomontanus, used to think that this star would reach the very pole of the world.

[Margin: The Pole Star never at the pole of the world.]

[Margin: Regiomontanus's deception.]

Problema 6. Data ſtellæ Declinatione, & Altitudine Poli, ſcire vtrum ſtella Oriatur & Occidat, necne.

Problem 6. Given a star's declination and the altitude of the pole, to know whether the star rises and sets or not.

VII. *First*, if the complement of a south declination is less than the altitude of the north pole, or the complement of a north declination is less than the altitude of the south pole, the star will always lie hidden below the horizon.

Second, if the complement of a south declination is equal to the altitude of the north pole, or the complement of a north declination is equal to the altitude of the south pole, the star will graze the horizon, and will never appear of itself, but only by the aid of refraction.

Third, if the complement of a south declination is less than the altitude of the south pole, or that of a north declination less than the altitude of the north pole, or equal to it, the star will always appear above the horizon; for although, when they are equal, it should graze the horizon, refraction will nevertheless lift it up.

Fourth, if the complement of a north declination is greater than the altitude of the north pole, or the complement of a south declination is greater than the altitude of the south pole, and likewise if a south declination is less than the complement of the altitude of the north pole, or a north declination less than the complement of the altitude of the south pole, the star will rise and set.

Problema 7. Data Poli altitudine, & ſtellæ latitudine versùs illum Polum, ſcire an & quando ſtella quæ Oritur & Occidit, euaſura ſit Semper Apparens, & è conuersò.

Problem 7. Given the altitude of the pole and a star's latitude toward that pole, to know whether and when a star that rises and sets will become always visible, and conversely.

VIII. *First*, we must find the declination of the always-visible circle that grazes the horizon, which is done by subtracting the altitude of the pole from 90°. So for a star that rises and sets to become always visible, its latitude toward your pole must be so great that the sum of the latitude and of the distance of your tropic from the equator equals or exceeds the declination of the always-visible circle. For if it equals it, the star will reach contact with the circle of always-visible stars; if it exceeds it, the star will pass inside that circle.

Second, if a star is to turn from always visible to rising and setting, its latitude must be less than the sum of the declination of the circle of always-visible stars and the distance of the opposite tropic from the equator.

[Margin: Finding the desired time.]

Third, from what has just been said, ask whether the star is capable of such a change. Then use the figure of Problem 4, but take the parallel VG for the circle of always-visible stars; and in triangle BFK, with BK, the distance of the poles, with KF, the complement of the star's latitude, and with BF, an arc that always equals the altitude of the pole, or the complement of the declination of the circle of always-visible stars, seek the angle at the pole of the ecliptic, namely BKF. For it will be the argument of the longitude, as we said there. When the star reaches that longitude (which will become known from the

present longitude and the difference from the longitude found, and from the motion in longitude, or the time owed to that difference), it will also reach the circle of always-visible stars, etc.

Problema 8. Mediationem Cæli ſeu Punctum Eclipticæ Culminans cum Stella Fixa Inuenire.

Problem 8. To find the mediation of the heaven, or the point of the ecliptic culminating with a fixed star.

IX. Seek the star's right ascension in the usual tables of right ascensions corresponding to the points of the ecliptic, but in the body of the tables. For opposite it, in the left margin, you will find the degrees, and under the head of the table the sign of the ecliptic with which the star mediates heaven, or crosses the meridian. Or, from the given right ascension and the obliquity of the ecliptic, seek that point by the analysis of a right-angled spherical triangle. For let AIL be the equator and A♋LV♑ the ecliptic, and let the right ascension given in the first quadrant of the equator be AD. When the arc of declination GD is drawn, the triangle ADG is formed, right-angled at D, in which, with the side AD given, and the angle GAD of the obliquity of the ecliptic, find the base AG, and you will have at G the point of the ecliptic culminating with the star, reckoned from the beginning of Aries. *Second*, let the right ascension be AH, in the second quadrant; for its complement to 180° will be the side HL, in triangle HLV, right-angled at H. So with the angle at L seek the base VL, and subtract it from 180°; for the arc of the ecliptic AV will remain, and the point of the ecliptic, reckoned from the beginning of Aries, with which the star mediates heaven will be known. *Third*, let the right ascension be in the third quadrant; for when 180° is taken from it, HL will remain. With that side and the angle HLS, namely with the obliquity of the ecliptic, find the base SL; for when it is added to 180° it will give the longitude from Aries of the point of the ecliptic culminating with the star. *Fourth* and last, let the right ascension be in the fourth quadrant of the equator; for its complement to 360° will be the arc AD. With this, in triangle ADP, right-angled at D, and with the angle of obliquity DAP, seek the base AP; subtract it from 360°, and there will remain the longitude of the point of the ecliptic crossing the meridian with the star.

Diagram. A lens-shaped figure set into the head of the left column, text wrapped on its left: two arcs of the ecliptic meeting at the ends of a straight line, the equator, with three perpendiculars dropped to that line from the upper and lower arcs.

- G (upper arc, left): the culminating point of the first quadrant
- ♋ (upper arc, centre): beginning of Cancer
- V (upper arc, right): the culminating point of the second quadrant
- A with ♈ below it (left end): beginning of Aries
- D (on the line, left): foot of the perpendicular from G and to P
- I (on the line, centre): midpoint of AIL

- H (on the line, right): foot of the perpendicular from V and to S
- L with ♎ beside it (right end): beginning of Libra
- P (lower arc, left): the culminating point of the fourth quadrant
- ♑ (lower arc, centre): beginning of Capricorn
- S (lower arc, right): the culminating point of the third quadrant

Problema 9. Diem inquirere quo Stella cum Sole tranſibit meridianum.

Problem 9. To find the day on which a star will cross the meridian with the Sun.

X. Seek by Probl. 8 the point of the ecliptic culminating with the star, and, from ephemerides or other astronomical tables, the day on which the Sun will reach that point of the ecliptic; and you will have the desired day.

Problema 10. An & cuius ſtellæ Fixæ Aſcenſio Recta poſſit decreſcere etiam ſi creſcat Longitudo eius.

Problem 10. Whether a fixed star's right ascension, and which star's, can decrease even if its longitude increases.

XI. In most stars the right ascension also increases as the longitude increases; in a few, however, it decreases although the longitude increases. Capable of this retrogradation is every star, and that star only, whose complement of latitude is less than the distance of the poles of the ecliptic and the equator. Now the rules for the increase and decrease of the right ascension of the fixed stars, which we of necessity indicated in advance in chap. 13, no. 3, so that no one should err in using the table of right ascensions assigned to the fixed stars by thinking that they always increase: those rules, gathered from here and from the following problem, you have more conveniently at the end of the chapter, in the Corollaries.

Problema 11. Tempus Inuenire Quo ſtellæ Fixæ Aſcenſio recta incipiet decreſcere, durabitq. in tali decremento donec iterum creſcere incipiat; ſi ſit Capax talis decrementi: & Quanta ſit Maxima Aſcentio eius recta. Datis Longitudine ac Latitudine Stellæ vnà cum Diſtantia Polorum.

Problem 11. To find the time at which the right ascension of a fixed star will begin to decrease, and will continue in that decrease until it begins to increase again, if it is capable of such a decrease; and how great its maximum right ascension is. Given the longitude and latitude of the star together with the distance of the poles.

XII. If the star's longitude is in the 1st or 4th quadrant of the ecliptic, use the former of the diagrams that follow; but if in the second and third quadrant, use the latter. In both, let AT be the solstitial colure, and on it let A be the pole of the equator and B that of the ecliptic; let the equator CE and the ecliptic DF be

described from them; and from the pole of the ecliptic B, as centre, with an interval smaller than the distance of the poles BA, describe a small semicircle parallel to the ecliptic, HMOP, which we shall call the semicircle of division [ſemicirculum diſcriminis].

[Margin: 1. Case.]

First, let the star's longitude be in the first quadrant of the ecliptic, ♈F, or in the fourth, D♈; and from the pole of the equator A draw a quadrant of a circle of declination to the equator so that it touches the semicircle of division, namely AMI, touching it at M. For as long as the star proceeds along the arc PM of the semicircle, that is, from the beginning of Capricorn as far as Aries and beyond, short of the point of contact M, its right ascension will increase as the longitude increases. This will be clear to anyone who describes through the star the quadrants of the circles of latitude and declination of the star, and who gathers its two right ascensions from two given longitudes by Probl. 3 of Chapter 12. Thus if the star is at O, draw the quadrant of latitude BS and of declination AL, so that the triangle BOA results. In it, from the given longitude, the angle OBA will be known, and BA, the distance of the poles, and BO, the complement of the star's latitude. So the angle BAO will not remain hidden, which measures the arc CL, the star's equatorial distance reckoned from the beginning of ♎. Next let the star be at M, or at a point between O and M; and when the quadrants of latitude BR and of declination AI are drawn, in triangle BMA you will gather from the given longitude the angle MBA, with which, and with BA, the distance of the poles, and BM, the complement of latitude, you will gather the angle MAB, which measures the arc of the equator CI, to be reckoned from the beginning of ♎. For the arc CI will be greater than CL, and so on for the others, as long as the star is within POM.

Diagram. A large circle occupying the upper part of the right column, the solstitial colure AT of Problem ii, with the equator CE and the ecliptic DF as chords-like curves, the small semicircle of division about B, and quadrants of latitude and declination, some dotted, drawn from B and from A.

- P (top left, outside the small semicircle's end): end of the semicircle of division
- B (top, on the circle): pole of the ecliptic
- H (top right, on the circle's rim): end of the semicircle of division
- O (upper left inside): a position of the star on the semicircle
- N (upper right inside): a further position of the star on the semicircle
- A (upper right, on the circle): pole of the equator
- M (centre, upper): point of contact on the semicircle
- C (left, on the circle): end of the equator
- L (left, on the equator): foot of the quadrant of declination from A through O and N
- I (left of centre, on the equator): foot of the quadrant of declination AMI
- D with ♎ below it (far left, on the circle): end of the ecliptic, beginning of Capricorn

- S (on the ecliptic, left of centre): foot of the quadrant of latitude BS
- ♈ (on the ecliptic, centre): beginning of Aries
- R (on the ecliptic, right of centre): foot of the quadrant of latitude BR
- ♋ with F beside it (far right, on the circle): end of the ecliptic, beginning of Cancer
- a letter-like sign at the foot of the dotted quadrant through N, on the ecliptic just inside F: probably the Q of the quadrant BNQ cited in the prose [?]
- E (lower right, on the circle): end of the equator
- T (bottom, on the circle): end of the colure AT

But on the contrary, when the star has passed the point of contact M, along the whole arc MH, and so in the whole remainder of the first quadrant of the ecliptic, the right ascension will decrease as the longitude increases. This will be clear to anyone who gathers from two given longitudes the right ascensions corresponding to them. Thus suppose in triangle BMA you gather, as above, the angle MAB, measuring the arc of the equator CI, and next, with the star advancing to N, the quadrant of latitude BNQ and that of declination ANL are drawn through it: you will always find the angle at the pole of the equator less than the preceding one, and so the arc CL less than the arc CI; and therefore the right ascension will decrease although the longitude increases.

We must now find the point R of the ecliptic, in the first quadrant, on reaching which the star already mentioned will be at the contact M. So in the spherical triangle MAB, right-angled at the contact M, by 5 of the second and 15 of the first of the *Sphaerics* of Theodosius, since AB, the distance of the poles, and BM, the complement of the star's latitude, are given, you will find the angle MBA, which measures the arc RF of the ecliptic. When it is subtracted from 90° there will remain the arc ♈R, that is, the longitude on reaching which the star already mentioned will be at the contact M; and thereafter, in the whole remainder of the first quadrant, its right ascensions will decrease.

[Margin: Maximum right ascension of a star.]

Again, you will find the angle MAB, which measures the arc of the equator CI; this, added to 270°, will give the maximum right ascension that the star already mentioned can have.

[Margin: Time at which it will happen.]

Moreover, since its present longitude is given, you will easily gather, from the difference of longitudes and the time corresponding to it, when the star came or will come to the contact M, and whether it is on the arc PM of the semicircle, which is the arc of increase, or on the arc MH, which is the arc of decrease of the right ascension. I will give an example when I have explained the use of the second diagram.

[Margin: 2. Case.]

XIII. *Second*, let the star be in the second or third of the ecliptic's

quadrants, as in the semicircle FD of the diagram that follows. For when a quadrant AMI of a circle of declination is drawn from A, the pole of the

equator, touching the semicircle of division [ſemicirculum diſcriminis] at M, then so long as the star already mentioned is on the arc HM, its right ascension will decrease while its longitude increases. After that contact, however, it will begin to increase, as will become clear to anyone who works out the right ascensions as above. For if the star is at N, and the quadrants BNQ of latitude and ANL of declination are drawn, then in the triangle ANB you will find the angle NAB, that is, the arc LC. But when the star has advanced to M, and BMR and AMI are drawn, you will find the angle MAB, that is, the arc IC, a larger one to be subtracted from 270°; and so the right ascension of the star placed at M will be smaller than of the star placed at N. Once the contact M has been passed, however, as when the star is at O, the arc LC to be subtracted will be smaller than the arc IC; and so the right ascension will now increase.

Diagram. A large circle set into the left column, breaking the sentence at `arcum | LC`: the second diagram of Problem ii, with the equator EC and the ecliptic FD crossing inside it, the small semicircle of division through N, M and O, and straight and curved quadrants of latitude and declination, some dotted, drawn from B and from A.

- A (top, on the rim, left of centre): the pole of the equator
- H (top, on the rim): end of the semicircle of division
- N (just inside H): a position of the star
- B (top right, on the rim): a pole, the other end of the distance of the poles AB
- P (upper right, on the rim): end of the semicircle of division
- M (upper centre, inside): the contact of the quadrant AMI with the semicircle
- O (upper right, inside): a further position of the star on the semicircle
- F (left, outside the rim), with the sign ♋ beside it: end of the ecliptic
- Q (left, on the ecliptic just inside F, set turned): foot of the quadrant of latitude BNQ
- E (left, outside the rim): end of the equator
- R (left of centre, inside, on the ecliptic): the point sought in the second quadrant
- ♎ (centre, at the crossing of equator and ecliptic, set slanted): the beginning of Libra
- I (right of centre, on the equator): foot of the quadrant AMI
- L (right, on the equator): foot of the quadrant ANL
- C (right, outside the rim): end of the equator
- S (lower right, on the ecliptic): foot of the dotted line through O
- a turned sign on the ecliptic just inside D, resembling ♑: [?]
- D (lower right, on the rim): end of the ecliptic
- T (foot of the circle, outside the rim)

[Margin: Beginning of the increase of right ascension.]

Here too we must find the point R of the ecliptic, belonging to its second quadrant, on reaching which the star will see the decreases cease, and from

which the increases of the right ascensions will begin again. So in the triangle AMB, right-angled at the contact M, given the distance of the poles AB and the complement of the star's latitude BM, seek the angle MBA, which measures the arc FR. Add it to 90°, and you will have the longitude sought, and from it the time, as above.

[Margin: Duration of the already mentioned increase or decrease.]

Finally, by joining the arcs RF of the earlier diagram and FR of the later one, or by doubling either of them (for they are equal), you will have the arc of the ecliptic, and from it the time, through which the decrease of the right ascension will persist, even though the longitude is increasing. The remainder to the whole circle will be the arc in which it increases.

EXEMPLVM.

Example.

[Margin: The hesitation or omission of the Tychonians.]

XIV. While examining the bright star in the shoulder of Ursa Minor, we came upon this very beautiful and new problem, and from it we conjectured why either Tycho or the Tychonians left this star out of their catalogue of right ascensions, although it is of the second rank [fecundi honoris]: because, namely, its right ascension, appearing to them to decrease from the year 1600 to 1700, raised a scruple in them, since they did not know the cause. I should like to give an example in this star, and indeed from the data supplied by Tycho. Now the northern latitude of the bright star in the shoulder of Ursa Minor is 72° 51′ 30″. So in the diagrams above, the arc BM is 17° 8′ 30″, and AB, according to Tycho, is 23° 31′ 30″. So in the spherical triangle AMB, right-angled at M, the angle MAB, that is, the arc CI reckoned from the beginning of ♎, is 47° 25′ 49″. Adding 270°, then, gives the greatest of the right ascensions this star can have, 317° 25′ 49″. Next, in the same triangle the angle MBA, that is, the arc of the ecliptic RF, is found to be 44° 53′ 14″, and accordingly ♈R is 25° 6′ 46″. On reaching this point of Aries the star was at the contact M, and from there its right ascension began to go back and diminish. Now in the year 1600 this star was at ♌ 7° 16′ 30″. So the difference of longitude between this place and the ♈ 25° 6′ 46″ just found is 102° 9′ 44″, for which more than 7150 years are required, and it is certain that the world has not existed for that many years. So this star was never on the arc of increase of the ascensions; rather, its right ascension has always been retrograde and has continually decreased while its longitude increased. For the arc RF, doubled, makes 89° 47′ 28″, which is the arc of decrease. And when 44° 53′ 14″ are added to the beginning of ♋ in the second diagram, one arrives at ♌ 14° 53′ 14″. When this star comes to that point it will be at the second contact M, and its right ascensions will then begin to increase. Now from 7° 16′ 30″ of ♌, where it stood in the year 1600, to 14° 53′ 14″ there are 7° 36′ 44″, for the completion of which Tycho requires 537⅓ years. Therefore the right ascension of this star has always been

decreasing since the beginning of the world, while its longitude increases, and it will go on decreasing until the year of the Lord 2137⅓. This has until now been unknown to astronomers, or, so far as I know, published by no one. The arc of decrease, moreover, is of nearly 6465 years.

COROLLARIA & PROPOSITIONES insignes ex Problemate 11.

Notable corollaries and propositions from Problem 11.

1. Prop. *Those stars whose complement of latitude is less than the distance of the poles of the ecliptic and the equator do not always increase in right ascension, even though their longitude increases.*
2. Prop. *The right ascension of those of the stars already mentioned that are northern increases continually throughout the whole southern semicircle of the ecliptic and in some part of the northern semicircle, while in some other part they decrease.*
3. Prop. *The right ascension of the stars already mentioned, but the southern ones, increases continually in the northern semicircle of the ecliptic; in a part of the southern semicircle they increase and in a part they decrease.* This is clear to anyone who draws, from the southern poles of the ecliptic and the equator, the arcs of latitudes and declinations and the small semicircle touching the circle of declination, on the model of the two diagrams given above in Problem 11.
4. Prop. *The right ascension of the northern stars already mentioned begins to decrease in the first quadrant of the ecliptic and stops decreasing in the second; that of the southern ones begins to decrease in the 3rd quadrant and stops in the 4th, when they come to the mutual contact of their circle of declination and the parallel of the ecliptic along which they travel.*
5. Prop. *The right ascension of the bright star in the shoulder of Ursa Minor has continually decreased since the beginning of the world, and will go on decreasing until about the year of the Lord 2137.*
6. Prop. *The stars whose right ascension decreases, or can decrease, while the longitude increases, number in the Rudolphine catalogue 59 northern and 35 southern*, because their latitude is greater than the complement of the distance of the poles. They lie in the asterisms of the table below.

Northern asterisms [Aſtra Boreal.]	Number of stars [Numerus Stellarum]	Southern asterisms [Aſtra Auſtral.]	Number of stars [Numerus Stellarum]
Ursa Minor	14	Argo Navis	4
Draco	29	Chamaeleon	8
Cepheus	5	Piscis Volans	7
Hercules	4	Doradus	7
Cygnus	7	Hydrus	9
Total [Summa]	59	Total [Summa]	35

CAPVT XX.

Chapter 20.

In Quo Problemata ſpectantia ad Fixarum Ortus Occaſuſq. Aſcenſiones ac Deſcenſ. Obliq. Amplitud. Ortiuas, Puncta Eclipticæ coorientia aut cooccidentia &c.

In which are problems concerning the risings and settings of the fixed stars, their oblique ascensions and descensions, their ortive amplitudes, the points of the ecliptic rising or setting with them, etc.

Problema 1. Differentiam Aſcenſionalem aut Deſcenſionalem ſtellæ aut Puncti cuiuſuis reperire, Data eius declinatione, & altitudine Poli.

Problem 1. To find the ascensional or descensional difference of any star or point, given its declination and the altitude of the pole.

[Margin: What the ascensional and descensional difference is.]

I. The ascensional and descensional difference [DIFFERENTIA Aſcenſionalis & Deſcẽſionalis] is the arc of the equator intercepted between the horizon and the circle of declination of a star standing on an oblique horizon. So in the figure that follows, let APQ be the meridian, on which P is the north pole of the equator, HMR the horizon, and AMQ the equator. Now let the star be at B, the rising point on the horizon, with BI its southern declination. The ascensional difference will then be the arc MI, which becomes known in the spherical triangle BMI, right-angled at I, since the declination BI is given, and the angle BMI, which measures the altitude HA of the equator, or the complement of the altitude of the pole. But if the star is at D, with the northern declination DL, the ascensional difference will be the arc LM, which becomes known by analysis of the triangle MLD, right-angled at L, since in it are given the declination DL and the angle LMD, which equals the angle HMA, the measure of the complement of the altitude of the pole. If, however, the points D or B were on the western semicircle of the horizon, the differences would be descensional. By the same device the ascensional differences are sought for points of the ecliptic whose declination is given. In brief, add the 2nd mesologarithm of the declination to the mesologarithm of the complement of the altitude of the pole, and the logarithm of the ascensional difference will result. Or say: as the radius is to the 2nd tangent of the complement of the altitude of the pole, so is the tangent of the declination to the sine of the ascensional difference.

Diagram. A circle set into the right side of the left column beside the opening of paragraph I, the figure of Problem 1 and of Problems 3 and 4, which refer back to it: the meridian as the rim, the horizon HMR across it, the equator AMQ, and straight and curved arcs through B and D.

- C (top, on the rim)
- A (upper left, on the rim): end of the equator
- K (left, on the rim): end of the parallel KN
- P (upper right, on the rim): the north pole of the equator
- H (left, on the rim): end of the horizon
- I (inside, above the horizon, on AMQ): foot of the declination BI
- M (centre, on the horizon): intersection of horizon and equator
- B (left of centre, just below the horizon line): the star on the rising side, with southern declination
- D (right of centre, on the horizon): the star with northern declination
- R (right, on the rim): end of the horizon
- L (below D, on AMQ): foot of the declination DL
- T (lower right, on the rim): end of the parallel CT
- Q (lower right, at the foot of the rim): end of the equator
- N (foot of the rim, left of Q): end of the parallel KN

Aliter per Tabulas.

Another way, by tables.

II. In the *Tables of Directions* [Tabulis Directionum] of Regiomontanus and of Reinhold there are tables headed "Of Ascensional Differences", and in Magini's *Directions of the Primum Mobile* there is such a table, that of the Fourth Analogy, from page 30. In these, entering with the altitude of the pole at the head or the foot and with the declination at the sides of the table, you will find the ascensional difference in the common cell. But the table of Regiomontanus, which Clavius copied out in his *Sphere*, p. 340, does not go beyond 32° of declination, because he constructed it for the planets, which never have a greater declination; nor does it extend beyond 60° of altitude of the pole.

[Margin: Example.]

I will give an example at number VIII.

Problema 2. Aſcenſionem ac Deſcenſionem Obliquam ſtellæ aut puncti Eclipticæ, cuius data ſit Declinatio, & Aſcenſio Recta, data Poli Altitudine reperire.

Problem 2. To find the oblique ascension and descension of a star or of a point of the ecliptic whose declination and right ascension are given, the altitude of the pole being given.

III. Find the ascensional difference by Problem 1, and subtract it from the right ascension if the declination is northern, or add it if southern, and the oblique ascension will result. Conversely, if the declination is northern, add the ascensional difference to the right ascension; if southern, subtract it, and the oblique descension will result. The table of oblique ascensions was constructed by this technique, but for points of the ecliptic, and it also serves stars with no greater declination than the ecliptic has. There is one such table in Clavius's

Sphere, from p. 349, which begins at 36° of altitude of the pole and goes up to 60°, assuming an obliquity of the ecliptic of 23° 30′. Another is in Magini, in the tables of the *Primum Mobile* or of directions, from 1° to 66° of altitude of the pole, for a least obliquity of 23° 28′ and a greatest of 23° 52′. It is followed by a table from 1° to 79° of altitude of the pole, but for the Tychonic obliquity of 23° 31′ 30″, which Renieri cut down to an altitude of 62° in his most recent *Tabulae Mediceae*, p. 316.

[Margin: Example.]

In these tables you will find the oblique descension if you look up the oblique ascension of the opposite point and take 180° away from it, borrowing a whole circle if need be. An example will come below at number VIII.

Problema 3. Amplitudinem Ortiuam Stellæ aut Puncti inquirere Datis eius Declinatione & Altitudine Poli.

Problem 3. To find the ortive amplitude of a star or point, given its declination and the altitude of the pole.

IV. The ortive amplitude [Amplitudo Ortiua] is the arc of the horizon intercepted between the rising star or point and the equator: in the diagram of Problem 1 it is the arc BM, and MD. It is found by analysis of the spherical triangle right-angled at I or at L, since in these are given the angle at M, which is the measure of the complement of the altitude of the pole, and the declination, BI if southern or DL if northern.

[Margin: A shortened form of the rule.]

In brief, join the logarithm of the declination to the residual of the logarithm of the complement of the polar altitude, and the logarithm of the ortive amplitude will result. Or say: as the radius is to the second secant of the complement of the altitude of the pole, so is the sine of the declination to the sine of the ortive amplitude. I will give an example at number VIII.

Problema 4. Arcum Semidiurnum ac Seminocturnum ſtellæ aut puncti Eclipticæ reperire; & inde Tempus, quod ſtella aut punctum conſumit ab ortu ad occaſum ſui, aut quo latet ſub Horizonte, Data Aſcenſionali Differentia.

Problem 4. To find the semidiurnal and seminocturnal arc of a star or of a point of the ecliptic, and from it the time the star or point takes from its rising to its setting, or during which it lies hidden below the horizon, given the ascensional difference.

V. Here the star is supposed to be one of those that neither always appear nor always lie hidden, which is known by Problem 6 of Chapter 19. With this laid down, recall the diagram of the first problem of this chapter, and let the star or point be at B, with southern declination. Its semidiurnal arc [arcus ſemidiurnus] on the parallel of the equator KN will be the arc BK, which is similar to the arc of the equator AI. So when the ascensional difference IM is

taken from the 90° that the arc AM encloses, the semidiurnal arc will remain. But if the star is at D, with northern declination, add the ascensional difference ML to 90°, and the arc AL will result, to which the semidiurnal arc CD on the parallel CT is similar. The complement of these to a semicircle will be the seminocturnal arc, and by doubling the arcs already mentioned the diurnal or nocturnal arc will result. By converting them into times of the equator you will have the time the star takes from its true morning rising to the meridian, and from there to its setting. When, however, a star rises in the evening with a true rising, the arc is called diurnal improperly, since it is the arc the star runs through by night above the horizon. I will give an example at number VIII. For points of the ecliptic, and for the Sun standing at them, tables of semidiurnal arcs and the like are to be had everywhere.

Problema 5. Punctum Eclipticæ, cum quo ſtella Oritur aut Occidit, ſeu cum quo eleuatur ſupra aut deprimitur infra Horizontem, Data eius Aſcenſione aut Deſcenſione obliqua, & Obliquitate Eclipticæ, cum Altitudine Aequatoris, Inueſtigare.

Problem 5. To find the point of the ecliptic with which a star rises or sets, that is, with which it is raised above or depressed below the horizon, given its oblique ascension or descension and the obliquity of the ecliptic, together with the altitude of the equator.

VI. Let HVR be the meridian, HGR the horizon, AGQ the equator, and ELCPE the ecliptic, whose intersection with the equator is K or M. So in the oblique-angled triangle KGL, given the star's oblique ascension KG, the angle of the obliquity at K, and the angle KGL, which is the complement to 180° of the angle AGH that measures the altitude of the equator, you will find by the analysis of triangles the arc of the ecliptic KL, counted from the vernal section

Diagram. A circle set into the right side of the right column beside the opening of paragraph VI, with the meridian as the rim, the horizon HGR, the equator through G, and the ecliptic through L and M; a dotted line runs from G to D.

- V (top, on the rim)
- A (upper left, on the rim): end of the equator
- E (left, on the rim, below A): where the ecliptic meets the rim
- K (inside, just below E): an intersection named in the prose
- D (inside, right of centre): end of the dotted line from G
- L (right of D, on the horizon's right part): end of the arc KL
- R (right, on the rim): end of the horizon
- H (left, on the rim): end of the horizon
- P (left of centre, below the horizon)
- G (centre, on the horizon)
- M (below G, inside): the other intersection of ecliptic and equator
- C (lower right, on the rim): a point of the ecliptic ELCPE

• Q (lower right, at the foot of the rim): end of the equator

K; and so the point of the ecliptic with which the star rises will be known. But if KG were the oblique descension, the point L would be the one with which the star sets. Similarly in the triangle PMG, when the oblique ascension is greater than 270°, if it is subtracted from 360°, the arc GM will remain. With this, the angle of the obliquity M, and the angle PGM, which is the complement to 180° of the angle HGA subtending the altitude of the equator, you will find the arc of the ecliptic PM. Subtracted from 360°, it will give the arc of the ecliptic counted from the beginning of Aries, with whose terminal point the star rises, etc. You have an example at number 8.

Aliter per Tabulas.

Another way, by tables.

VII. Look up the oblique ascension of the star in the tables named in Problem 2, but in the body of the table [arealiter]; for at the head of the tables you will find the sign of the ecliptic, and in the left margin the degree of the sign, that rises with the star. But if you look up in them, in the body of the table, the oblique descension first increased by 180°, you will find the point of the ecliptic diametrically opposite the point with which the star sets.

EXEMPLVM Problematum Præcedentium in Sirio & in Procyone ad Annum Domini 1644. in Bononienſi Horizonte.

Example of the preceding problems in Sirius and in Procyon, for the year of the Lord 1644, on the horizon of Bologna.

[Margin: Example of Problem 1.]

VIII. In the year already mentioned the southern declination of Sirius is 16° 17′ 30″, and its right ascension 97° 27′ 40″. This, looked up in Magini's table (on which see number 2) under an altitude of the pole of 44° 29′ 30″, gives an ascensional difference of 16° 41′.

[Margin: Example of Problem 2.]

Since the declination is southern, this difference added to the star's right ascension makes the oblique ascension 114° 8′ 40″; subtracted from it, the oblique descension 80° 46′ 40″.

[Margin: Example of Problem 5.]

The oblique ascension already mentioned, looked up in the body of Clavius's table under the altitude of the pole already mentioned, shows the point of the ecliptic with which Sirius rises, ♌ 9° 37′. The oblique descension, increased by 180°, gives ♐ 1° 24′ 30″ of the ecliptic; opposite it is ♊ 1° 24′ 30″, with which Sirius sets.

[Margin: Example of Problem 4.]

Again, since the star's declination is southern, when the ascensional difference of 16° 41′ is taken from 90°, there remains a semidiurnal arc of 73° 19′. To this

correspond 4^h 53^m 16^s, namely the amount of time the star takes from its rising to its culmination.

[Margin: Example of Problem 3.]

Finally, for the southern ortive amplitude, from the said complement of the altitude of the pole, 45° 30′ 30″, and the declination of Sirius, 16° 17′ 30″, the arc of the horizon, or ortive amplitude, is found to be 23° 9′ 27″.

Keeping to the same method: since the northern declination of Procyon in that year is 6° 6′ 40″ and its right ascension 110° 11′ 38″, its oblique ascension is found to be 104° 10′, and the point of the ecliptic rising with it ♌ 2° 27′. Its oblique descension is 116° 14′, and the point of the ecliptic setting with it ♋ 0° 52′. And so for the rest.

Problema 6. Ortum & Occaſum verum Stellæ tam matutinum quàm veſpertinum, ſeu Coſmicum & Acronyctum reperire, ſi Stella ſit ex orientibus & occidentibus.

Problem 6. To find the true rising and setting of a star, morning as well as evening, that is, cosmic and acronychal, if the star is one of those that rise and set.

[Margin: Cosmic rising.]

IX. You will find the true morning rising, or cosmical rising [Ortum Coſmicum], of a star if by Problem 5 you find the point of the ecliptic with which the star rises, and from an ephemeris the day on which the Sun reaches that point.

[Margin: Cosmic setting.]

You will find the true morning setting, or cosmical setting, if by Problem 5 you find the point of the ecliptic with which the star goes down, and from an ephemeris the day on which the Sun reaches the point of the ecliptic diametrically opposite it.

[Margin: Acronychal rising.]

You will find the true evening rising, or acronychal rising [Ortum Acronyctum], if by Problem 5 you find the point of the ecliptic with which the star rises, and from an ephemeris the day on which the Sun reaches the point diametrically opposite that point.

[Margin: Acronychal setting.]

You will find the true evening setting, or acronychal setting, if by Problem 5 you find the point of the ecliptic with which the star goes down, and from an ephemeris the day on which the Sun reaches that point.

I am assuming, moreover, the definitions of the risings and settings already mentioned, given in Book 1, Chapter 32, with their diagrams.

EXEMPLVM IN SIRIO. Ad Annum 1644. Bononiæ.

Example in Sirius, for the year 1644, at Bologna.

X. Since at number eight the points of the ecliptic were found with which Sirius and Procyon ascend above the horizon of Bologna in the year 1644, and

with which they descend, and since from Montebruni's ephemeris, constructed for the meridian of Bologna, the day is established on which the Sun reaches these points and the points diametrically opposite them, I have easily gathered the points of the ecliptic and the days of the risings and settings already mentioned into this little table.

Year 1644, Bologna [Anno 1644. / Bononiæ]	Sirius: place of ☉	Sirius: month	Sirius: day	Procyon: place of ☉	Procyon: month	Procyon: day
Cosmic rising	♌ 9° 37′	Aug.	2	♌ 2° 27′	July	25
Cosmic setting	♐ 1° 25′	Nov.	23	♍ 0° 52′	Dec.	22
Acronychal rising	♒ 9° 37′	Jan.	29	♒ 2° 27′	Jan.	23
Acronychal setting	♊ 1° 25′	May	22	♋ 0° 52′	June	22

Problema 7. Horam diei vel noctis inuenire, qua Stella qualibet earum quæ Oriuntur & Occidunt, emergat ab Horizonte, vel infra illum deſcendat.

Problem 7. To find the hour of the day or night at which any star among those that rise and set emerges from the horizon or goes down below it.

XI. First weigh whether on the given day the star precedes the Sun, that is, reaches the horizon first, or follows it, which will be established from an ephemeris and by inspecting a globe. Next, by Problem 2, seek the oblique ascension and descension of both the Sun and the star, and convert the difference of both into time of the equator. Finally, by Problem 4, seek the diurnal and nocturnal arc of the Sun. With these prepared, keep to the following rules.

[Margin: Rising of a star preceding the Sun.]

First. If the star precedes the Sun, subtract the time of the difference of the oblique ascensions from the nocturnal arc (taking in, if need be, the diurnal arc reckoned from the rising of the preceding day). For the remainder will be the hour from the beginning of the night at which the star emerges, if the subtraction was made from the nocturnal arc alone; but it will be the hour from the rising of the preceding day, or Babylonian hour, if the diurnal arc was taken in.

[Margin: Rising of a star following the Sun.]

Second. If the Sun precedes the star, add the time of the difference of the ascensions to the time of the Sun's rising, and the result will be the time at which the star will emerge from the horizon on that day, or on the following night.

[Margin: Setting of a star preceding the Sun.]

Third, for the setting. If the star precedes the Sun, subtract the time of the difference of the oblique descensions from the time of the Sun's setting reckoned from noon, if this is possible, or from the same time counted from the preceding midnight, and you will have the time at which the star goes

down below the horizon, reckoned either from noon or from the preceding midnight.

[Margin: Setting of a star following the Sun.]

Fourth. If the Sun precedes the star, add the time of the difference of the oblique descensions to the time of the Sun's setting, and the result will be the time at which the star sets on the following night, or after the rising of the day.

When, however, the star's rising or setting precedes or follows the Sun's rising or setting by several hours, gather the oblique ascension of the Sun corresponding to one degree, multiply it by those several hours, divide the sum by 24, and add the quotient to the difference of the oblique ascensions if the star rises or sets later than the Sun, but subtract it if earlier.

Another way, from Magini in the *Directions*, Canon 17. Subtract the oblique ascension of the Sun from the oblique ascension of the star; for if the remainder is less than the Sun's diurnal arc, the star will rise by day; if greater, by night. When this has been done, sub- tract the Sun's semidiurnal arc (for so it should have been, although there it is wrongly *diurnum*, "diurnal") from the difference of the oblique ascensions of the Sun and the star. For if nothing remains, the star will rise at noon; but if 180° remain, it will rise at midnight. But if the remainder is greater or less than a semicircle, convert it into hours, and you will have the time of the star's rising from noon; and if you take away the diurnal arc, it will be the time from the Sun's setting.

For the star's setting, subtract the oblique descension of the Sun from the oblique descension of the star, taking in a whole circle if the subtraction cannot be made; and the remainder, converted into hours, will be the time from the Sun's setting at which the star sets. If it exceeds the Sun's nocturnal arc, the star will set during the following day; but if it is less, it will set by night.

EXEMPLVM IN SIRIO. Anno 1644. Septembris 22. Bononiæ.

Example in Sirius, 1644 September 22, at Bologna.

XII. The longitude of Sirius in the year 1644 is ♋ 9° 19′. The place of the ☉ on September 22, at Bologna, at noon, I have as ♍ 29° 45′; the star therefore precedes the Sun. The oblique ascension of Sirius, from what was said at number 8, is 114° 9′, and its descension 80° 47′. The Sun's oblique ascension at that time is 179° 42′, and its oblique descension 179° 53′. The difference of the ascensions is therefore 65° 33′, to which correspond $4^h\ 22^m\ 12^s$; and the difference of the descensions is 99° 6′, whose time is $6^h\ 36^m\ 24^s$. The Sun's diurnal arc, like its nocturnal arc, is 12 hours. From these I subtract $4^h\ 22^m\ 12^s$, and there remains the time from the Sun's setting at which Sirius emerges from the horizon, namely $7^h\ 37^m\ 48^s$ Italian.

Again, since the star's setting precedes the Sun's setting, and I cannot subtract the time of the difference of the oblique descensions from the time of the Sun's setting reckoned from noon, which is 6^h, I take the time of the Sun's setting

from midnight, and this makes 18 hours. From these I subtract $6^h\ 36^m\ 24^s$, and there remain $11^h\ 23^m\ 36^s$ from midnight, at which Sirius goes down below the horizon.

If one wishes to work more precisely: since the Sun's oblique ascension corresponding to one degree is 78′ on that day, this multiplied by 7⅔ hours gives about 598, and when this is divided by 24 it makes 25′, whose time is $1^m\ 40^s$. I subtract this from the time of the difference of the oblique ascensions, and there remain $4^h\ 20^m\ 32^s$. When these are subtracted from the Sun's nocturnal arc, or from 12^h, there remains the time of the emergence of Sirius, $7^h\ 39^m\ 20^s$ from the Sun's setting.

Problema 8. Dato Puncto Eclipticæ, cum quo Stella Fixa oritur, aut occidit Coſmicè, vel Acronycè, & Latitudine Stellæ Poliq. altitudine; Inquirere locum ſtellæ, & inde Sæculum quo ſubijt ortum illum occaſumq.

Problem 8. Given the point of the ecliptic with which a fixed star rises or sets cosmically or acronychally, and the latitude of the star and the altitude of the pole, to find the star's place, and from it the century in which it underwent that rising and setting.

XIII. This problem, certainly an ingenious one and not useless for examining ancient observations of the fixed stars, is given by Longomontanus, Book 2 of the *Sphaerics*, Chapter 4, Problem 2, and by our own Pétau in the dissertations of the *Uranologion*, Book 1, Canon 19.

Let the evening rising of Arcturus be examined. For this, in the following diagram, let ABZ be the meridian, B the pole of the world, C that of the zodiac, and GHL the horizon, on which let Arcturus, rising for the sake of example, be D. Let FHI be the equator and EKO, etc., the ecliptic, with its equinoctial section at P. When an arc of latitude CDM has been drawn through the star D, the star's latitude will be the arc MD, and the star's place with respect to the ecliptic will be at M; and the point of the ecliptic with which the star rises will be K, whose declination from the equator is KS. First

Diagram. A large circle (the meridian ABZ of the text), set into the lower part of the left column with the text running in a narrow measure to its left. A small circle is centred at B on the upper right of the great circle and another at the Z-shaped label on its lower left; these are joined by straight and curved arcs crossing the interior, which meet at D, K, H, the point labelled Q and the point labelled R.

- A (top of the circle, outside): point on the meridian
- B (upper right, inside the small circle): pole of the world, per the text
- C (right of the upper small circle, on its rim): pole of the zodiac, per the text
- E (upper left on the rim): end of the ecliptic EKO, per the text
- F (left on the rim, above G): end of the equator FHI, per the text
- G (left on the rim): end of the horizon GHL, per the text

- T (upper interior): not cited on this page
- R (left interior, below T): not cited on this page
- Q (left interior, on the line from G): not cited on this page
- H (centre left, on the horizon line): intersection of horizon and equator, per the text
- S (below H): foot of the declination KS, per the text
- K (centre, above S): co-rising point of the ecliptic, per the text
- D (right of centre, on the horizon line): Arcturus, per the text
- L (right on the rim): end of the horizon, per the text
- P (lower right interior): equinoctial section, per the text
- N (lower right interior, right of P): not cited on this page
- M (lower right interior, below P): the star's place on the ecliptic, per the text
- I (right, outside the rim, below L): end of the equator, per the text
- O (lower right, outside the rim): end of the ecliptic, per the text
- V (lower left on the rim): not cited on this page
- Z (lower left, inside the small circle; a looped, tailed sort): end of the meridian ABZ, per the text

it is necessary to seek the arc KM, that is, the difference between the star's place on the ecliptic and the co-rising point of the ecliptic, which will be done as follows. In triangle KPH are given KP, the distance of the co-rising point from the nearer equinox; the angle KHP, equal to the altitude of the equator; and HPK, the obliquity of the ecliptic. The angle HKP will therefore not be hidden, and its complement to 180° will be the angle MKD, in triangle MKD, right-angled at M, in which the star's latitude MD is given. Hence by the analysis of this triangle the arc of the ecliptic KM will not be hidden, namely the difference between the known co-rising point of the ecliptic, K, and the point of the ecliptic M, which is the star's required place. And from the place of the same star taken from a star catalogue for the time of the epoch, you will gather the difference of the two longitudes, and from it the interval of time.

EXEMPLVM IN ARCTVRI. Ortu veſpertino, ex Heſiodo.

Example in the evening rising of Arcturus, from Hesiod.

Hesiod, who flourished on Mount Helicon near Athens at about the beginning of the Olympiads, or about the year 775 before Christ, sings thus of Arcturus in Book 2 of the *Works and Days*:

> When sixty days after the turnings of the Sun, Days of winter,
> Jupiter has completed, then indeed the star Arcturus, leaving the immense flood of Ocean, First appearing whole, rises in the evening.

Here he clearly describes the true evening rising of Arcturus 60 days after the winter solstice, as Longomontanus rightly has it above, and Kepler in the *Epitome of Copernican Astronomy*, p. 398. At that time (for one needs to know that century roughly for a more precise working) the Sun's apogee was at

about ♉ 20°, and therefore the Sun's corrected motion in 60 days is 61⅙°, as Longomontanus says. When this is added to the place of the winter solstice, that is, the beginning of Capricorn, the Sun's place comes to ♓ 1⅙°, which is the place opposite the place with which Arcturus rose in its true evening rising. Therefore point K, co-rising with Arcturus, was ♍ 1⅙°. Hence its distance from the nearer equinox, or arc KP, is 28° 50′; the altitude of the equator at Athens, that is, angle PHK, is 52° 15′, from Ptolemy's *Geography*; and the obliquity of the ecliptic, HPK, which Longomontanus uses, is 23° 32′. Therefore angle PKH is 107° 43′ and MKD 72° 17′.

The latitude of Arcturus, or side MD, is 31° 3′ north; therefore arc KM is 11° 5′, which, added to point K, that is, to ♍ 1° 10′, gives the place M of Arcturus, ♍ 12° 15′. But at the completed year 1644 the place of Arcturus has been found by us to be ♎ 18° 19′. Arcturus has therefore advanced from Hesiod's time to ours by 36° 4′, and to traverse these, according to our motion of the fixed stars, 2597 years are required. From these I subtract the 1644 years of Christ, and there remain 953 years before Christ. Hesiod therefore seems to speak of the rising of Arcturus not of his own time but of the second century preceding him. This working, however, is rendered uncertain both by the refraction of Arcturus, which hastens its rising, and by that of the Sun, which delays the Sun's setting, and by the day of the winter solstice having been computed in a civil and rather coarse way. So this practice is not to be trusted much, unless you have more certain indications from ancient observations.

EXEMPLA ALIA INDICATA.

Other examples pointed out.

XIV. In Chapter 16 of the *Elements of Astronomy*, Geminus says that, according to Euctemon, Arcturus used to rise on days 10 and 19 of Parthenon; hence the first rising was the cosmic, the second the heliacal. And Pliny the Elder, Book 18, Chapter 25, relates that Thales set the morning setting of the Pleiades on the 25th day after the autumnal equinox, which Pétau says is to be understood of the cosmical setting, Book 1 of the *Dissertations* of the *Uranologion*, Canon 19. There, by the preceding problem, he gathers that in the century of Euctemon Arcturus was in ♍ 19° 7′, and the bright star of the Pleiades in the time of Thales in ♈ 23° 53′, and thus that Euctemon flourished about the year 4188 of the Julian Period, and Thales in 4116; that is, Euctemon in the year 526 before Christ, and Thales in 598.

Problema 9. Ortum & Occaſum Heliacum Stellæ inueſtigare In Data Altitudine Poli.

Problem 9. To investigate the heliacal rising and setting of a star at a given altitude of the pole.

XV. On this problem, carefully, Regiomontanus, Book 8 of the *Epitome of the Almagest*, Proposition 14; Magini

[Margin: Authors of this problem.]

in the *Tables of the Primum Mobile*, or of Directions, Canons 20, 21 and 22, and in Book 8 of the *Primum Mobile*, Problem 22; Longomontanus, Book 2 of the *Sphaerics*, Chapter 4, Problem 3; Kepler in the *Epitome of Copernican Astronomy*, from p. 371; and Pétau, Book 1 of the *Dissertations* of the *Uranologion*, Canon 14; and before these, Ptolemy, Book 8, Chapter 6; Regiomontanus in the *Epitome*, Book 8, Proposition 11. From these, keep to the following method.

First, find the arc of vision, that is, of a vertical circle, by which the Sun must be depressed below the horizon so that the Sun's nearness and the twilight do not hinder the star's appearance. This arc varies with the varying apparent magnitude and brilliance of the stars, and with the varying depth of the Sun required for the first twilight, which for some is 18° and for others 19°, according to what was said in Book 1, Chapter 31; we shall establish this depth in the Book on Optics, once the refractions have been settled. For now let it suffice to present this arc from what is commonly observed, as the authors already mentioned use it, in the following little table, in line with the view of Ptolemy, Book 8, Chapter 6, and Book 13, Chapter 7.

[Margin: Arc of vision.]

Arc of vision for the stars [ARCVS VISIONIS PRO STELLIS]

Planets	Arc	Fixed stars, magnitude	Arc
♄	11° 0′	First	12°
♃	10° 0′	Second	13°
♂	11° 30′	Third	14°
♀	5° 0′	Fourth	15°
☿	10° 0′	Fifth	16°
		Sixth	17°
		Least	18°

Second, find the point of the ecliptic with which the star truly rises or sets, by Problem 5, and by the common problems of the primum mobile seek the angle that the ecliptic makes with the horizon at that point co-rising or co-setting with the star; or else by tables made for that purpose, which Magini gives in the Tables of Directions from p. 200, from 30° to 51° of altitude of the pole, but for all altitudes of the pole Kepler in the Rudolphine Tables, p. 26, and from him Boulliau in the tables, p. 158; or else hunt it down by the analysis of triangles in the following scheme.

Diagram. A large circle (the meridian ABC of the text), set into the lower part of the left column with the text running in a narrow measure to its left. A small circle centred at B stands on its upper right. The horizon HFR crosses the great circle from left to right, and the ecliptic EFC crosses it from upper left to lower right, the two meeting at F; further arcs descend from A and from the small circle to the region of F, G, I, L, M, S and N.

- A (top, outside the rim): vertex, per the text
- P (top, above the small circle): a pole of the ecliptic, per the text

- B (upper right, inside the small circle): a point of the meridian ABC, per the text
- Q (right of the small circle): a pole of the ecliptic, per the text
- E (upper left, on the rim): end of the ecliptic EFC, per the text
- H (left, on the rim): end of the horizon HFR, per the text
- R (right, on the rim): end of the horizon, per the text
- G (centre, on the horizon line): foot of the vertical arc on the horizon, per the text
- I (centre, immediately right of G): star on the horizon, per the text
- L (right of I): star on the horizon, per the text
- F (centre, below G): intersection of horizon and ecliptic, per the text
- M (below F): foot of the latitude MI on the ecliptic, per the text
- S (below M): the Sun, per the text
- N (below and right of S): foot of the latitude LN on the ecliptic, per the text
- C (lower right, on the rim): end of the ecliptic and of the meridian ABC, per the text

In it let ABC be the meridian, HFR the horizon, and EFC the ecliptic, whose poles are either at P or at Q, as the star's position requires. Let the star be at point I or L of the horizon, rising or setting, and through it let a circle of latitude QIM or PLN be drawn to the ecliptic; for the latitude of the star I will be MI, and of the star L, LN. Since, therefore, in triangle FMI, right-angled at M, if the star is at I, the latitude MI is given, and point F of the ecliptic, co-rising or co-setting with the star, when compared with the point of the star's longitude, which is M, shows the arc of the ecliptic FM, the angle MFI, which the ecliptic makes with the horizon at the point co-rising or co-setting with the star, will not be hidden from a trigonometer. Likewise in triangle FNL, right-angled at N, from the star's latitude NL and from the difference of point F, co-rising or co-setting, from point N, the star's place, that is, from arc FN, you will gather the angle NFL, which the ecliptic makes with the horizon, etc.

Third, from the vertex A draw the arc of a vertical circle AS down to the Sun S, placed below the horizon, so that the arc of vision given above is GS. For in triangle FGS, right-angled at G, with the angle at F now found and the arc of vision GS, you will find by the solution of triangles the arc of the ecliptic FS. This is, and is called, the interval of the ecliptic for the star's appearance or occultation [Interuallum Eclipticæ Apparitionis vel Occultationis], which serves to determine the star's heliacal rising and setting; for from an ephemeris one can learn the day on which the Sun reaches the degree of the ecliptic distant from the point F of the ecliptic, co-rising or co-setting with the star, by just as much as the interval FS.

[Margin: Shortcut of the operation.]

But if the altitude of the pole is 43°, 44° or 45°, use Magini's table in the *Directions of the Primum Mobile*, p. 195. In it, under the arc of vision noted at the head of the table, and with the sign and point of the ecliptic with which

the star rises or sets looked up in the left or right margin of the table, you will find in the common area the interval of appearance suited both to the planets and to fixed stars of any magnitude.

Fourth, since the star rises with the Sun in its true morning rising, or cosmical rising, before it rises out of the Sun in its heliacal morning rising; and conversely sets heliacally in the evening before it sets truly, or acronychally, it will be easy, from the interval of appearance and the days in which the Sun traverses it on the ecliptic, to know by how much time the heliacal morning rising follows the cosmical rising, or the heliacal setting precedes the true evening setting.

EXEMPLVM IN SIRIO ET PROCYONE Per Tabulam Maginii, pro Anno 1644. Bononiæ.

Example in Sirius and Procyon, by Magini's table, for the year 1644, at Bologna.

XVI. Sirius, from what was said at number VIII, in the year 1644 at Bologna, where the altitude of the pole is 44° 29′ 30″, rises cosmically with ♌ 9° 37′ and sets acronychally with ♊ 1° 24′ 30″. To the former point, under the altitude of the pole already mentioned and under a star of the first magnitude, such as Sirius is, or under an arc of vision of 12°, corresponds an arc of appearance of 13° 55′; when these are joined with ♌ 9° 37′, the result is ♌ 23° 32′. In that year the Sun reached this degree on August 16. To the latter point, of the setting, or 1° 25′ of ♊, corresponds an arc of occultation of 23° 27′; when these are subtracted from ♊ 1° 25′, there remains ♉ 7° 58′, which the Sun reached in that year on April 27, on which day Sirius sets heliacally in the evening, whereas before it was appearing on the horizon.

Next, the point co-rising with Procyon is ♌ 2° 27′, and the arc of appearance 15° 56′; therefore the heliacal rising occurs with the Sun placed in ♌ 18° 23′. The co-setting point, however, is ♋ 0° 52′, and the arc of occultation 20° 16′; therefore the heliacal setting will be with the Sun in ♊ 10° 36′. For Procyon is of the second magnitude. You will find nothing different if you seek these risings or settings by the analysis of triangles. So, combining all the risings and settings of Sirius and Procyon from what has been said here and at number X, let there be a second little table, fuller than the one placed there.

Year 1644, Bologna [ANNO 1644. / Bononiæ]	Sirius: ☉ in	Sirius: month	Sirius: day	Procyon: ☉ in	Procyon: month	Procyon: day
Cosmic rising	♌ 9° 37′	Aug.	2	♌ 2° 27′	July	25
Heliacal rising	♌ 23° 32′	Aug.	16	♌ 18° 23′	Aug.	11
Cosmic setting	♐ 1° 25′	Nov.	23	♑ 0° 52′	Dec.	22
Acronychal rising	♒ 9° 37′	Jan.	29	♒ 2° 27′	Jan.	23
Heliacal setting	♉ 7° 58′	Apr.	27	♊ 10° 36′	June	1
Acronychal setting	♊ 1° 25′	May	22	♋ 0° 52′	June	22

CAPVT XXI. In Quo Definitiones ac Regulæ Ortuum & Occaſuum Explicantur ampliori Notione quàm anteà.

Chapter 21. In which the definitions and rules of risings and settings are explained in a fuller sense than before.

I. In Book 1, Chapter 32, indeed, when instructing the beginner, I preferred to explain only four species of the rising of stars, and as many of setting, along with most of the recent writers of little books on the sphere. Now, however, it has seemed right to set them out a little more fully for a better-instructed reader, whom I nonetheless refer to that chapter for the diagrams representing rising and setting. Not all the ancients, moreover, use either the same names or the same distribution and number.

[Margin: Opinion of Theophrastus.]

For Theophrastus, in the book *On the Signs of Rain*, mentions only two settings and two risings. The former setting he calls [Greek: ἀφανισμὸς δύσις (aphanismos dysis)], namely a setting that does not appear, which, he says, occurs when [Greek: σονδύνῃ τῷ ἡλίῳ τὸ ἄστρον (sondynēi tōi hēliōi to astron)], that is, when the star sets together with the Sun.

[Margin: Pétau corrected.]

Pétau, Book 1 of the *Dissertations* of the *Uranologion*, Chapter 1, interprets this of a heliacal setting occurring a little after the Sun's setting, but not correctly, since at that time the star and its setting appear, and the setting does not occur with the Sun; whereas Theophrastus says that it occurs with the Sun and does not appear. The latter setting is [Greek: ὅταν ἀνατέλλοντι τῷ ἡλίῳ δύνῃ (hotan anatellonti tōi hēliōi dynēi)], *when it sets as the Sun rises*, which the more recent writers call the cosmical setting. The former rising is [Greek: ἀνατολὴ ἑῴα (anatolē heōia)], *morning rising*; but he shows that he is speaking of the heliacal rising when he adds [Greek: ὅταν πρωανατέλλῃ τοῦ ἡλίου τὸ ἄστρον (hotan prōanatellēi tou hēliou to astron)], *when the star rises before the Sun*. The latter rising he calls [Greek: ἀνατολὴ ἀκρόνυχος (anatolē akronychos)], *evening rising*, which he teaches occurs [Greek: ὅταν ἀμαδυομένῳ τῷ ἡλίῳ ἀνατέλλῃ (hotan amadyomenōi tōi hēliōi anatellēi)], when the star rises together with the setting Sun; and this is the acronychal evening rising.

[Margin: Opinion of Achilles Tatius.]

Achilles Tatius, in the *Isagoge* to the *Phaenomena* of Aratus, touches on just as many risings and settings, but only true ones: namely the morning rising with the Sun and the evening rising at the Sun's setting, and the evening setting with the Sun and the morning setting at the Sun's rising. But this rising, which has reference to the Sun, he calls [Greek: ἐπιτολὴ (epitolē, "rising with reference to the Sun")], while the absolute one, which occurs without reference to the Sun, he calls [Greek: ἀνατολὴ (anatolē, "absolute rising")], a distinction that Geminus also uses, as below.

[Margin: Opinion of Ioannes Tzetzes.]

Ioannes Tzetzes, on Book 2 of Hesiod, agrees on nearly the same number, but not on the species. For he says that a star preceding the Sun by 15° makes [Greek: ἑῴαν ἀνατολὴν (heōian anatolēn)], that is, a morning rising; but when it is less than 15° from it, [Greek: ἑῴαν δύσιν (heōian dysin)], a morning setting. When, however, it follows the preceding Sun by fifteen degrees, it makes [Greek: ἀνατολὴν ἑσπερίαν (anatolēn hesperian)], namely an evening rising; and when it is less than 15° distant, [Greek: ἑσπερίαν δύσιν (hesperian dysin)], or an evening setting. He adds, however, that when a star goes down below the earth diametrically opposite the Sun, it is called [Greek: ἀκρονυχίαν (akronychian, "acronychal")]. So the first four species are heliacal emergences from the Sun's rays, or immersions, while the fifth is the true morning setting [recte: *verus*; the page prints *verſus*]. On this Pétau notes that the heliacal evening rising suits only Mercury and Venus, but the Moon should have been added.

Julius Firmicus, Book 2, Chapter 8, also offers us a fourfold division, saying: *The five stars, namely* ♄, ♃, ♂, ♀ *and* ☿, *become morning or evening, or western, or hidden, which by the Greeks are called* [Greek: ἀφανῆται καὶ συνοδικαὶ (aphanētai kai synodikai)]*, or* [Greek: ἀκρονύκται (akronyktai)]. He then explains: *Morning stars in rising are those that rise preceding the orb of the* ☉*; evening stars in rising are those that follow the rising Sun; hidden stars are those that the orb of the* ☉ *covers;* [Greek: ἀκρονύκται] *are those that rise when the Sun sets.* Thus the first is the heliacal morning rising and the second the heliacal evening rising; in the others he indicates risings and settings with the Sun that do not appear, and in the last place the true evening rising.

[Margin: Distribution of Geminus and, nearly, of Proclus.]

II. Geminus, however, has distinguished more species for us, in the *Elements of Astronomy*, Chapter 11, and Proclus, expounding Book 2 of Hesiod, nearly follows him. Geminus says that [Greek: ἀνατολὴν (anatolēn)] is the daily rising of a star without reference to the Sun, but [Greek: ἐπιτολὴν (epitolēn)] with reference to the Sun. Next, according to him there are four species of rising and as many of setting, namely:

1. [Greek: ἐπιτολὴ ἑῴα ἀληθινὴ (epitolē heōia alēthinē)], *true morning rising*.
2. [Greek: ἐπιτολὴ ἑῴα φαινομένη (epitolē heōia phainomenē)], *apparent morning rising*.
3. [Greek: ἐπιτολὴ ἑσπερία ἀληθινὴ (epitolē hesperia alēthinē)], *true evening rising*.
4. [Greek: ἐπιτολὴ ἑσπερία φαινομένη (epitolē hesperia phainomenē)], *apparent evening rising*.
5. [Greek: δύσις ἑῴα ἀληθινὴ (dysis heōia alēthinē)], *true morning setting*.
6. [Greek: δύσις ἑῴα φαινομένη (dysis heōia phainomenē)], *apparent morning setting*.
7. [Greek: δύσις ἑσπερία ἀληθινὴ (dysis hesperia alēthinē)], *true evening setting*.

8. [Greek: δύσις ἑσπερία φαινομένη (dysis hesperia phainomenē)], *apparent evening setting*.

Here the epithet *apparent* is not taken to mean feigned or false, nor is it thus distinguished from *true*; rather, it is so called because the star then appears in its rising or setting, and the nearby rays of the Sun do not obstruct the sight of it.

[Margin: Pliny the Elder's view on this.]

Pliny the Elder, Book 18, Chapter 25, does not depart far from this when he says: *Risings and settings are understood in two ways. For either the stars are hidden by the Sun's approach and cease to be seen, or they show themselves at its departure. Custom would better have called this an emergence than a rising, and that an occultation rather than a setting. In another way, the day on which they begin to appear, or cease to, at the rising or setting of the Sun, named morning and evening according as either of these happens to them in the morning or at twilight.* He restricts himself, however, to the fixed stars, saying: *And the whole discussion concerns those stars which we have said adhere to the heaven.*

Kepler subscribes to Pliny the Elder in the *Epitome of Copernican Astronomy*, p. 366, saying that the heliacal rising is more correctly called an appearance or emergence than a rising, and the heliacal setting more correctly an occultation than a setting. He confirms this from the Greeks, who call the former [Greek: φάσιν (phasin)], that is, appearance, and the latter [Greek: κρύφιν (kryphin)] or [Greek: κρίψιν (kripsin)], that is, occultation or hiding.

But if, at the same moment at which they emerge from the Sun's rays, stars ascend above the horizon, and at the moment they begin to be immersed within its brilliance they descend, it seems to me that for a double reason the former deserves the name of rising, and the latter that of setting; so far is it from being the case that those names do not fit. So in this I subscribe neither to Pliny the Elder nor to Kepler.

[Margin: Pliny the Elder and Kepler corrected.]

When, however, a star begins to emerge from the rays of the Sun under which it previously lay hidden, yet at that time does not ascend from the horizon but rather descends, as happens in the first phase of the Moon and sometimes in Mercury and Venus as evening stars; or when it begins to be submerged under them and yet ascends from the horizon, then it seems doubtful whether that appearance should be called a rising and this occultation a setting. Geminus certainly calls the former an apparent evening setting, while the latter he passes over; more on this below, when we have heard Ptolemy.

[Margin: Ptolemy's teaching on these.]

III. For Ptolemy seems to have been more generous and more careful than the others, since in Book 8 of the *Magna Constructio*, Chapter 4, after listing the aspects of the stars (on these, our Book 1, Chapter 33), he considers not only more risings and settings than the others but also mediations of the heaven. I believe this is because he was contemplating the astrological powers of the stars at the four cardines of the day and of the heaven, according as on these they

are opposed to or joined with the Sun, exactly or [Greek: πλατικῶς (platikōs, "broadly")].

[Margin: 9 Ptolemaic schematisms.]

He therefore brings these comparisons back to nine schematisms [Schematifmos], or configurations with the Sun, from whose subdivision 24 species spring, and they are those written below. Regiomontanus also points them out in the *Epitome of the Almagest*, Book 8, Proposition 5; Reinhold on Peuerbach's *Theoricae*, p. 149; and Pétau, Book 1 of the *Dissertations* of the *Uranologion*, Chapter 2.

[Margin: 1. Rising of a star with the Sun, before the Sun and after the Sun.]

The first [Greek: σχηματισμὸς (schēmatismos, "configuration")] is called [Greek: πρωινὸς ἀπηλιώτης (prōinos apēliōtēs)], that is, the morning east wind [Subfolanus], and it is threefold. Either the star climbs the horizon at the same moment as the Sun, and this is called [Greek: ἑῴα συνανατολὴ ἀληθὴ (heōia synanatolē alēthē)], *true morning co-rising*; or it rises at dawn before the Sun and begins to appear in the east, when previously it did not appear in the morning because of the Sun's nearness, and this is called [Greek: ἑῴα προανατολὴ φαινομένη (heōia proanatolē phainomenē)], *apparent morning pre-rising*; or, the Sun having risen a little before, it ascends from the horizon but is not seen, and Ptolemy calls this [Greek: ἑῴα ἐπανατολὴ μὴ φαινομένη (heōia epanatolē mē phainomenē)], namely *non-apparent morning sub-rising*. For a rising of this kind to have a definite distance from the Sun, however, the star seems to have to be at so great an interval that it begins to be immersed in its rays, or as great as it stands from the Sun at its first morning appearance.

[Margin: [?] Mediation of the heaven by a star, with the Sun rising, risen, or about to rise soon.]

The second schematism is [Greek: πρωινὸν μεσουράνημα (prōinon mesouranēma)], *morning mediation of the heaven*, or midheaven [medicælium], or occupation of the meridian, both above and below the earth. This is threefold, according as the star holds the meridian either at the Sun's rising, and this is called [Greek: ἑῷον συμμεσουράνημα ἀληθινὸν (heōion symmesouranēma alēthinon)], *true morning mediation of the heaven*; or with the Sun having risen a little before, and therefore it ceases to appear, and it is called [Greek: ἑῶον μεσουράνημα μὴ φαινόμενον (heōon mesouranēma mē phainomenon)], *non-apparent morning mediation of the heaven*; or with the Sun not yet risen but about to rise shortly, and it is called [Greek: ἑῶον προμεσουράνημα (heōon promesouranēma)], *morning pre-mediation of the heaven*.

[Margin: Setting of a star with the Sun rising, or risen, or about to rise shortly.]

The third schematism is called [Greek: πρωινὸς λὶψ (prōinos lips)], *morning setting*, and it is threefold. Either the star descends below the horizon as the Sun rises, and this is called [Greek: ἑῴα συγκατάδυσις ἀληθὴ (heōia synkatadysis alēthē)], *true morning setting*; or it plunges down with the Sun now risen, and

this is called [Greek: ἑῴα ἐπικατάδυσις μή φαινομένη (heōia epikatadysis mē phainomenē)], *non-apparent morning sub-setting*; or with the Sun about to rise a little later, and this is called [Greek: ἑῴα πρόδυσις φαινομένη (heōia prodysis phainomenē)], *apparent morning pre-setting*.

[Margin: Rising of a star at noon or at midnight.]

The fourth schematism is called [Greek: μεσημβρινὸς ἀπηλιώτης (mesēmbrinos apēliōtēs)], *noon eastern rising*, and it is twofold. Either the star climbs the horizon at noon and therefore does not appear, and this rising is called [Greek: ἡμερινὸς μὴ φαινόμενος (hēmerinos mē phainomenos)], *non-apparent diurnal*; or it does so at midnight and appears, and is called [Greek: νυκτερινὸς φαινόμενος (nykterinos phainomenos)], *apparent nocturnal*.

[Margin: Culmination or lowest transit [imatio] of a star, at noon or at midnight.]

The fifth schematism is called [Greek: μεσηβρινὸν μεσουράνημα (mesēbrinon mesouranēma)], *noon mediation of the heaven*, and it is twofold: either diurnal, when the star culminates at noon with the Sun or is opposed to it at the bottom of the heaven; or nocturnal, when at midnight it either holds the bottom of the heaven or culminates. This configuration could have been divided in four ways.

[Margin: Setting of a star at noon or at midnight.]

The sixth schematism is called [Greek: μεσημβρινὸς λὶψ (mesēmbrinos lips)], *noon setting*, and it is twofold, according as the star slips below the horizon either at noon or in the middle of the night.

[Margin: Rising of a star with the Sun setting, or set, or about to set shortly.]

The seventh schematism is called [Greek: ὀψινὸς ἀπηλιώτης (opsinos apēliōtēs)], *evening rising*, and it is threefold. Either the star climbs the horizon as the Sun sets, and this is called [Greek: ἑσπερία συνανατολὴ ἀληθινὴ (hesperia synanatolē alēthinē)], *true evening rising*; or it climbs the horizon with the Sun not yet set but about to set shortly, and therefore does not appear, and this is called [Greek: ἑσπερὶα προανατολὴ μὴ φαινομένη (hesperia proanatolē mē phainomenē)], *non-apparent evening pre-rising*; or it climbs the horizon with the Sun having slipped below the horizon a little before, and this is called [Greek: ἑσπερία ἐπανατολὴ φαινομένη (hesperia epanatolē phainomenē)], *apparent evening sub-rising*.

[Margin: Mediation of the heaven by a star, with the Sun setting or set, or about to set shortly.]

The eighth schematism is called [Greek: ὀψινὸν μεσουράνημα (opsinon mesouranēma)], *evening mediation of the heaven*, and it is threefold, according as the star holds the meridian as the Sun sets, and this is called [Greek: ἑσπερινὸν συμμεσουράνημα ἀληθινὸν (hesperinon symmesouranēma alēthinon)], *true evening mediation of the heaven*; or with the Sun now having set a little before, and this is called [Greek: ἑσπερινὸν ἐπιμεσουράνημα φαινόμενον (hesperinon epimesouranēma phainomenon)], *apparent evening post-mediation of the heaven*; or with the Sun about to set shortly, and this is called [Greek:

ἑσπέριον προμεσουράνημα μὴ φαινόμενον (hesperion promesouranēma mē phainomenon)], *non-apparent evening pre-mediation of the heaven*.

[Margin: Setting of a star with the Sun setting, or set, or about to set shortly.]

The ninth schematism is called [Greek: ὀψινὸς λὶψ (opsinos lips)], *evening setting*, and it is threefold, inasmuch as the star descends below the horizon either together with the Sun, and this is called [Greek: ἑσπερία συγκατάδυσις ἀληθινὴ (hesperia synkatadysis alēthinē)], *true evening co-setting*; or with the Sun having slipped below the horizon a little before, and this is called [Greek: ἑσπερία ἐπικατάδυσις φαινομένη (hesperia epikatadysis phainomenē)], *apparent evening sub-setting*; or finally it occurs with the Sun not yet setting but about to set a little later, and this is called [Greek: ἑσπερὶα πρόδυσις μὴ φαινομένη (hesperia prodysis mē phainomenē)], *non-apparent evening pre-setting*. So far Ptolemy. Let us come to the authors of the middle period.

[Margin: Opinion of Alkindus, Campanus, Sacrobosco, Stöffler, Clavius and Biancani.]

IV. Alkindus [al-Kindī] in the book *me motu diurno* [recte: *de motu diurno*, *On Daily Motion*]; Campanus in Maurolico, Dialogue 3; Johannes de Sacrobosco in Chapter 3 of the *Sphere*, and there his commentators, especially Clavius, p. 314; on Proclus's *Sphere*, Johannes Stöffler, pp. 33 and 36; Biancani on the *Sphere*, Book 17, Chapter 10. These writers divide the rising and setting, both of signs and of stars, into astronomical and poetic. They further subdivide the astronomical into apotelesmatic [Apotelesmaticum], that is, judicial, and inapotelesmatic [Inapotelesmaticum]; and this last they again split in two, into the horizontal, or absolute from the Sun, and the solar, or that which has reference to the Sun. The poetic, so called because poets almost always use it, is in fact the same as the astronomical solar. They subdivide it into the cosmic or mundane [Mundanum], that is, the morning one, because in the morning the world and its adornment seem to be renewed; 2. into the chronic or temporal, that is, the evening one (for so Martianus Capella, at the end of Book 8, and others called chronic what they should have called [Greek: ἀκρονύκτον (akronykton)] or [Greek: ἀκρόνυχον (akronychon)], that is, at the very first edge of the night, as is plain from what was said in number 2); 3. into the heliacal, which they finally subdivide into morning and evening. From the poetic, therefore, there spring eight species, just as in Geminus too. These are also used by Fernel, p. 40 of the *Cosmotheoria*; Magini in the *Theoricae*, Book 2, Chapter 14; Origanus in Part 2 of the *Ephemerides*, Chapter 8; Reinhold on Peuerbach's *Theoricae*, p. 149; and Magini in the *Secundum Mobile*, Canon 98.

The eight are: 1. Cosmic rising [Ortus Coſmicus], that is, the true morning rising. 2. Cosmic setting [Occaſus Coſmicus], that is, the true morning setting. 3. Chronic rising [Ortus Chronicus], that is, the true evening rising. 4. Chronic setting [Occaſus Chronicus], that is, the true evening setting. 5. Morning heliacal rising [Ortus Heliacus matutinus], that is, the morning emergence from the Sun's rays. 6. Evening heliacal rising [Ortus Heliacus veſpertinus],

that is, the evening emergence from the Sun's rays. 7. Morning heliacal setting [Occaſus Heliacus matutinus], namely the morning immersion in the rays of the Sun. 8. Evening heliacal setting [Occaſus Heliacus veſpertinus], namely the evening immersion beneath the rays of the Sun.

V. To us, however, it seems that the mediations of the heaven should be omitted, since they do not bear on the rising of stars, but that all the other species should be retained, so that nothing may be left out in this matter that could serve agriculture, medicine, navigation, and the illustrating or understanding of writers, whether they were poets or not. I shall therefore not be reluctant to go over again, from the authors named above, the arrangement of risings and settings with its Greek nomenclature, adding rules that establish to which wandering or fixed stars each applies. The risings and settings of the stars (though they can be extended to the signs as well) are thus divided into absolute, or purely horizontal, and those having reference to the rising or setting of the Sun. For although in the absolute ones one considers the ascension and descension, right if they occur in a right sphere and oblique if in an oblique sphere, and there is reference to the parts of the equator with which they co-ascend or co-descend, so that the time of these risings and settings can be known, or even reference to the point of noon or of midnight, no account is taken of the Sun rising or setting at that time or very near it. This is the sense in which we take the name "having reference to the Sun" [Reſpectiui ad Solem]. First, then, the 8 species of the absolute, and next the 12 of those having reference.

[Margin: Of the absolute.]

Ortus & Occaſus Abſolutus, Quid & Quotuplex, Quibuſq. conueniat.

Absolute rising and setting: what it is, of how many kinds, and to which stars it applies.

[Margin: 1. Species.]

The absolute noon rising [Ortus abſolutus Meridianus] is the ascension of a star above the horizon at noon, which therefore does not appear; Ptolemy calls it [Greek: ἀνατολὴ μεσημβρινὸς ἡμερινὸς μὴ φαινομένη (anatolē mesēmbrinos hēmerinos mē phainomenē)]. It applies to all stars except Mercury and Venus on those horizons where, with the point of the winter solstice rising, the arc of the ecliptic between the east and the meridian is greater than their greatest digression from the Sun, unless a latitude toward the hidden pole helps a little.

[Margin: 2. Species.]

The absolute midnight rising [Ortus abſolutus Medinocturnus] is the ascension of a star above the horizon at midnight, and hence apparent; Ptolemy calls it [Greek: ἀνατολὴ μεσημβρινὸς νυκτερινὸς φαινομένη (anatolē mesēmbrinos nykterinos phainomenē)]. It applies to all stars except ♀ and ☿, on the horizons mentioned above.

[Margin: 3. Species.]

The absolute noon setting [Occaſus abſolutus Meridianus] is the descension of a star from the horizon at noon; Ptolemy calls it [Greek: μεσημβρινὸς λὶψ ἡμερινὸς μὴ φαινομένη (mesēmbrinos lips hēmerinos mē phainomenē)]. It applies to all stars except ♀ and ☿ on the horizons already mentioned.

[Margin: 4. Species.]

The absolute midnight setting [Occaſus abſolutus Medinocturnus] is the descension of a star from the horizon at midnight; in Ptolemy's terms, [Greek: μεσημβρινὸς λὶψ νυκτερινὸς φαινομένη (mesēmbrinos lips nykterinos phainomenē)]. It applies to all stars except ♀ and ☿ on the horizons already mentioned.

[Margin: 5. Species.]

The absolute diurnal rising [Ortus abſolutus Diurnus] is the ascension of a star from the horizon made in the daytime, without reference to the four cardines of the day; in Greek, [Greek: ἀνατολὴ ἡμερινὸς μὴ φαινομένη (anatolē hēmerinos mē phainomenē)]. It applies to all stars.

[Margin: 6. Species.]

The absolute nocturnal rising [Ortus abſolutus Nocturnus] is the ascension of a star from the horizon made at night, and hence apparent, without reference to the four cardines of the day; in Greek, [Greek: ἀνατολὴ νυκτερινὸς φαινομένη (anatolē nykterinos phainomenē)]. It applies to all stars.

[Margin: 7. Species.]

The absolute diurnal setting [Occaſus abſolutus Diurnus] is the descension of a star from the horizon made in the daytime, without reference to the four cardines of the day; in Greek, [Greek: δύσις ἡμερινὸς μὴ φαινομένη (dysis hēmerinos mē phainomenē)]. It applies to all stars.

[Margin: 8. Species.]

The absolute nocturnal setting [Occaſus abſolutus Nocturnus] is the descension of a star from the horizon made at night, without reference to the four cardinal points of the day; in Greek, [Greek: δύσις νυκτερινὸς φαινομένη (dysis nykterinos phainomenē)]. It applies to all stars.

So much for the first 8 species of risings and settings, namely the absolute. Now to be explained are those risings and settings of stars that concern the rising or setting of the Sun.

Quotuplex ſit Ortus & Occaſus Reſpectiuus ad Solis ortum occaſumue, & Quibus conueniat.

Of how many kinds is the rising and setting having reference to the Sun's rising or setting, and to which stars each applies.

[Margin: 1. Species.]

The cosmical rising [Ortus Coſmicus] is the true morning rising, but not apparent because of the Sun's presence; that is, it is the ascension of a star from the horizon as the Sun rises. For Geminus and Tatius it is [Greek: ἐπιτολὴ ἑῴα

ἀληθινὴ (epitolē heōia alēthinē)], but for Ptolemy [Greek: ἑῴα συνανατολὴ ἀληθὴ (heōia synanatolē alēthē)]. It applies to all stars.

[Margin: 2. Species.]

The morning heliacal rising [Ortus Matutinus Heliacus], or apparent morning rising, is the ascension of a star from the horizon a little before the Sun's rising, with its first appearance, that is, its emersion from the rays of the Sun, under which it previously lay hidden. For Geminus and Achilles Tatius it is [Greek: ἐπιτολὴ ἑῴα φαινομένη (epitolē heōia phainomenē)], but for Ptolemy [Greek: ἑῴα προανατολὴ φαινομένη (heōia proanatolē phainomenē)]. It applies to all stars except the Moon, because the Moon is faster than the Sun and cannot be left behind by the Sun, as the Sun advances eastward, in such a way that it begins to appear in the morning on the eastern horizon when previously it lay hidden under the Sun's rays. Yet before the Sun's rising the Moon can be in its last phase and at its first immersion, and thus it is rather setting heliacally. In the others, however, it earns the name of rising on two counts.

[Margin: 3. Species.]

The subsolar morning rising [Ortus Matutinus ſubſolaris], or the morning rising that is neither true nor apparent, occurs a little after the Sun's rising, at about as great an interval of the ecliptic as there is from the cosmical rising to the heliacal morning rising. It applies to all stars faster than the Sun. For if they are slower than the Sun, as are the fixed stars and the three superior planets, it is the first morning immersion beneath its rays; but if they are faster than the Sun, as are the Moon and sometimes Mercury and Venus, it comes with the first morning emersion from the rays of the Sun, and so it does not forfeit the name of rising. Ptolemy calls it [Greek: ἑῴα ἐπανατολὴ μὴ φαινομένη (heōia epanatolē mē phainomenē)].

[Margin: 4. Species.]

The acronychal rising [Ortus Acronychus], called by others, half-barbarously, the chronic rising [Ortus Chronicus], is the true evening rising, but not apparent because of the presence of the Sun standing opposite; that is, it is the ascension of a star from the horizon as the Sun sets. For Geminus it is [Greek: ἐπιτολὴ ἑσπερία ἀληθινὴ (epitolē hesperia alēthinē)]; for Ptolemy [Greek: ἑσπερία συνανατολὴ ἀληθὴ (hesperia synanatolē alēthē)]; for Theophrastus [Greek: ἀνατολὴ ἀκρόνυχος (anatolē akronychos)]; for Firmicus the [Greek: ἀκρόνυκτος (akronyktos)] rising. It applies to all stars, usually except ♀ and ☿; I shall give the reason in the Scholium.

[Margin: 5. Species.]

The evening rising that is apparent but not true [Ortus Veſpertinus apparens ſed non verus] is the ascension of a star from the horizon a little after the Sun's setting, at as great an interval as is required for it, in spite of the twilight, to be able to appear for the first time on the eastern horizon, when on the preceding days it lay hidden; and for Geminus it is [Greek: ἐπιτολὴ ἑσπερία φαινομένη (epitolē hesperia phainomenē)], but for Ptolemy [Greek: ἑ[?]περία ἐπανατολὴ φαινομένη (he[?]peria epanatolē phainomenē)]. This, moreover, is one of the

more notable and commonly observable risings. I know, however, that most people understand in its place that heliacal evening rising in which the star descends from the horizon yet emerges for the first time from the rays of the Sun, as sometimes happens to the Moon, Venus and Mercury alone. But this we now relegate to the evening setting, on which see species 11. The evening rising in the first sense, moreover, applies to all stars, usually except ♀ and ☿, according to what will be said in the Scholium.

[Margin: 6. Species.]

The evening rising that is neither true nor apparent [Ortus Veſpertinus nec verus nec apparens] is the ascension of a star from the horizon a little before the Sun's setting, at as great an interval as the acronychal evening rising stands from the apparent evening rising. For Ptolemy it is [Greek: ἑσπερία προανατολὴ μὴ φαινομένη (hesperia proanatolē mē phainomenē)]. It applies to all stars, usually except ♀ and ☿, etc.

[Margin: 7. Species.]

The cosmical setting [Occaſus Coſmicus], or true morning setting, is the descension of a star from the horizon as the Sun rises, and therefore it does not appear. Geminus and Tatius call it [Greek: δύσις ἑῴα ἀληθινή (dysis heōia alēthinē)], but Ptolemy [Greek: ἑῴα συγκατάδυσις ἀληθὴ (heōia synkatadysis alēthē)]. It applies to all stars, usually except ♀ and ☿, etc.

[Margin: 8. Species.]

The morning setting that is apparent but not true [Occaſus Matutinus apparens ſed non verus] is the descension of a star from the horizon a little before the Sun's rising, with its first appearance in this descent, when on the preceding days it did not appear because of the Sun's presence or the excessive light of the morning twilight. For Geminus it is [Greek: δύσις ἑῴα φαινομένη (dysis heōia phainomenē)], but for Ptolemy [Greek: ἑῴα πρόδυσις φαινομένη (heōia prodysis phainomenē)]. This species applies to stars slower than the Sun, that is, the fixed stars and ♄, ♃ and ♂, but to ☾ never, and to ♀ and ☿ almost never. For although Mercury and Venus are sometimes slower than the Sun, they nevertheless do not digress from the Sun in such a way that, with the Sun rising or a little later about to rise, they set, except on very few horizons, as below in the Scholium. For this species, however, not a few substitute the heliacal morning setting, which is nevertheless rather a rising, according to what was said in species 3.

[Margin: 9. Species.]

The morning setting that is neither true nor apparent [Occaſus Matutinus nec verus nec apparens] is the descension of a star from the horizon a little after the Sun's rising, at as great an interval as there is from the acronychal setting to the apparent morning setting. For Ptolemy it is [Greek: ἑῴα ἐπικατάδυσις μὴ φαινομένη (heōia epikatadysis mē phainomenē)]. It applies to all, usually except ♀ and ☿, etc.

[Margin: 10. Species.]

The acronychal setting [Occaſus Acronychus], half-barbarously the chronic setting [Occaſus Chronychus], is the true evening setting, that is, the descension of a star from the horizon with the Sun likewise setting. For Geminus and Tatius it is [Greek: δύσις ἑσπερία ἀληθινὴ (dysis hesperia alēthinē)], but for Ptolemy [Greek: ἑσπερία συγκατάδυσις ἀληθὴ (hesperia synkatadysis alēthē)], and for Theophrastus [Greek: ἀφανισμὸς δύσις (aphanismos dysis)].

[Margin: 11. Species.]

The apparent evening setting [Occaſus veſpertinus apparens] is the descension of a star from the horizon after the Sun's setting, with its first appearance if the star is faster than the Sun, as ☾ is and sometimes ♀ and ☿; or with its last evening appearance, and the first evening occultation following soon after, if it is a star slower than the Sun, as are the fixed stars and the three superior planets. In the former case, therefore, not a few more recent writers call it the heliacal evening rising because of the first emersion from the rays of the Sun, but it is simply a setting. For that reason Geminus and Tatius call it [Greek: δύσις ἑσπερία φαινομένη (dysis hesperia phainomenē)], and Ptolemy [Greek: ἐπικατάδυσις φαινομένη (epikatadysis phainomenē)]. In the second case, however, it earns the name of setting on two counts.

[Margin: 12. Species.]

The evening setting that is neither true nor apparent [Occaſus veſpertinus nec verus nec apparens] is the descension of a star from the horizon a little before the Sun's setting, at as great an interval as there is between the acronychal setting and the apparent evening setting. For Ptolemy it is [Greek: ἑσπερία πρόδυσις μὴ φαινομένη (hesperia prodysis mē phainomenē)]. It applies to all stars.

[Margin: Corollary.]

So we have 10 species of rising and as many of setting, 20 in all, of which 8 are absolute and 12 have reference to the Sun's exact or nearest rising or setting.

Regulæ aliæ de Ordine ac Interuallis Ortuum & Occaſuum Reſpectiuorum.

Further rules on the order and intervals of risings and settings having reference to the Sun.

Rule 1. Among morning risings, the one that is neither true nor apparent comes first, the cosmic follows, and the apparent or heliacal comes after that, if the stars are slower than the Sun.

Rule 2. Among evening risings, the apparent comes first, the true or acronychal follows, and then the one that is neither true nor apparent, if the stars are slower than the Sun.

Rule 3. Among morning settings, the one that is neither true nor apparent comes first, the true or cosmic follows, and then the one that is apparent but not true, if the stars are slower than the Sun.

Rule 4. Among evening settings, the apparent comes first, the true or acronychal setting follows, and then the one that is neither true nor apparent, if the stars are slower than the Sun.

In the Moon, however, the order of the phases is already known; in ♀ and ☿ it is too varied, so much so that general rules can hardly be committed to memory.

On the order among the six principal risings and settings, see a specimen in the example of Sirius and Procyon below in Chapter 22, on the rising and setting of Canicula. But beware of that looser rule of Pliny the Elder, who says in Book 18, Chapter 25: *Those intervals require at the least three-quarters of an hour before the Sun's rising or after its setting, so that the stars can be seen.*

Rule 5. In fixed stars having no latitude or a very small one, the true risings and settings stand about six months apart in time.

Rule 6. In all fixed stars, the acronychal rising stands from the morning heliacal rising that follows it by an interval greater than 6 months, whatever Geminus may say to the contrary in the *Elements of Astronomy*, Chapter 11, and however Pétau may hesitate in the *Uranologion*, Book 1 of the *Dissertations*, Chapter 2.

Rule 7. In fixed stars having a great northern latitude, the cosmical rising stands from the following cosmical setting, and the cosmical setting from the acronychal rising, by a greater span than six months; but if the latitude is southern, by a smaller span than six months. In these, however, the span from the cosmical setting to the cosmical rising is greater than six months, in those smaller.

Rule 8. Stars endowed with latitude, in an oblique sphere, do not rise or set with that point of the ecliptic in which they lie according to longitude. If the latitude is northern, they rise with an antecedent degree; with a following degree they set. They do so in roughly this order: first they rise cosmically, then heliacally; next they set heliacally, afterwards acronychally; then they rise acronychally, and finally they set cosmically, as can be seen in Arcturus, Capella and the other northern stars. But if their latitude is southern, they rise with a following degree of the ecliptic and set with an antecedent one. First they set heliacally, then acronychally; next they rise cosmically, afterwards heliacally; after that they set cosmically; afterwards they rise acronychally. These things are to be understood [Greek: πλατικῶς (platikōs)].

Rule 9. Not all stars that rise together also set together, nor do all that rise first also set first. For the rest, see Autolycus, Book 2.

SCHOLION.

Scholium.

[Margin: ♀ and ☿: where the acronychal rising and the cosmic setting can apply.]

I said in species 4 that on some horizon Venus and Mercury can at times rise acronychally as the Sun sets: namely on that horizon where the north pole of the ecliptic coincides with the zenith. For the Sun in the southern signs can set so close to noon that, in spite of the small digression of these planets (which can nevertheless reach about 50° in ♀ and 30° in ☿), either of these planets, placed on the ecliptic, rises. This can also occur on a horizon whose zenith declines little from the pole of the ecliptic on one side or the other, provided these planets have a sufficient latitude to one side or the other. Say the same when the south pole of the ecliptic is at the zenith or near it, provided Venus and Mercury are either on the ecliptic or have some sufficient latitude, as will be plain to anyone who turns a sphere. From this you will understand that these planets too can at times set cosmically, or rise a little after the Sun's setting with their first appearance, the species of rising I spoke of in species 5. Since, however, these things do not happen in the whole of the Torrid Zone or in the greatest part of either temperate zone, and since within these zones the smallness of the digressions does not allow them to set cosmically or rise acronychally, astronomers therefore commonly deny this rising and setting to Venus and Mercury, and we too have followed them in the instruction of beginners, Book 1, Chapter 32.

CAPVT XXII. De Ortu & Occaſu Siderum apud Poetas & Hiſtoricos inſigni tum ad Annum primum Iulianum, tum ad alios expenſis, cum Auctoribus qui Fixarum Ortus & Occaſus in Tabulas digeſſere aut in Faſtos.

Chapter 22. On the risings and settings of stars notable in the poets and historians, weighed both for the first Julian year and for others, with the authors who have set out the risings and settings of the fixed stars in tables or in calendars.

I. We have chosen first, with Pétau, this Julian year because Ovid and Virgil flourished around it, and because from that time a judgement can easily be passed on that whole century and indeed on the nearest ones as well. For within one century the fixed stars do not move from their place a full degree and a half, and it is enough for us to know, more or less, the day on which they rose or set. But if the poets or historians seem to depart markedly from that year, it will have to be said either that they copied the risings and settings from a century or an altitude of the pole very different from the century in which Christ was born, and [?] from the Roman altitude of the pole; or that they had regard to the common appearance, which, because of the clouds and vapours of the horizon, differs greatly from the rising and setting of stars considered in itself; or that, because of the dislocation of the calendar and the inconsistency of intercalations, the days of the old year do not agree with the days of the Julian year; or that for some other reason an error of one or more days crept in from calendars [Faſtis] composed for civil use only; or that they confused the rising of the signs with the rising of the stars.

For ease of finding, we shall proceed in alphabetical order in reviewing the stars rather than in the order of days and months, so that whenever a

controversy arises about the ancient rising or setting of some star it can be discussed at once from what is to be said, although a calendar is to be drawn up for these afterwards. We assume that the motion of the fixed stars is 50″ annually, and one degree in 72 years. On this assumption, our own Pétau computed the following risings and settings, for an obliquity of the ecliptic of 23° 32′ and the Roman altitude of the pole, for the first Julian year, that is, the year 45 before the first year of Christ. He prudently set first the true ingress of the Sun into the twelve signs for the same year, computing the hours from midnight, which was as follows.

The true ingress of the Sun into the signs at Rome in Julian year 1, that is, 45 before the birth of Christ.

Signs	Months	Days	Hours and minutes
♒ Aquarius	January	22	7^h 52^m
♓ Pisces	February	21	9^h 13^m
♈ Aries	March	22	23^h 47^m
♉ Taurus	April	23	12^h 10^m
♊ Gemini	May	24	12^h 22^m
♋ Cancer	June	24	23^h 3^m
♌ Leo	July	26	5^h 39^m
♍ Virgo	August	26	2^h 37^m
♎ Libra	September	25	11^h 11^m
♏ Scorpius	October	25	5^h 54^m
♐ Sagittarius	November	23	15^h 54^m
♑ Capricornus	December	22	23^h 54^m

From the day of the Sun's ingress into any sign you will easily infer the day on which the Sun in that year was at the other degrees of the same sign, since the Sun completes very nearly one degree in the zodiac each day. For example, the bright star of Aquila set heliacally in that year with the Sun placed at 10° 29′ ♑, while the Sun's ingress into ♑ occurred on December 22 at 23^h 54^m. If you add ten and a half days to this, you will arrive at January 2 and a half. Therefore Aquila sets heliacally there between January 2 and 3; and so with the others.

Here now is Pétau's table, to which we have only added the day and month. As for the synonyms of the stars: in place of Canis Major look for Sirius; in place of Canis Minor, Procyon; in place of Antares [Antare], the heart of Scorpius; in place of Aldebaran [Aldebara] or Palilicium [Palilicio], see the bright star of the Hyades or Suculae, or the southern eye of Taurus; in place of the bright star of the Pleiades, the bright star of the Vergiliae.

Table of the risings and settings of the stars at Rome in the year 45 before the first of Christ.

Names of the stars	Place	Rising: cosmic	Rising: heliacal	Rising: acronychal	Setting: cosmic	Setting: heliacal	Setting: acronychal
Bright star of Aquila [Aquilæ lucida]	♒ 3° 39′	♐ 8° 45′; Dec. 2	♐ 27° 8′; Dec. 20	♊ 8° 45′; June 2	♋ 26° 20′; July 21	♒ 10° 29′; Jan. 2	♒ 26° 20′; Jan. 18, o
Arcturus	♍ 25° 48′	♍ 13° 32′; Sept. 9	♍ 26° 16′; Sept. 21, o	♓ 13° 32′; Mar. 6	♊ 3° 16′; June 28	♏ 10° 48′; Nov. 5	♐ 3° 16′; Nov. 27
Bright star of Aries above the head [Arietis lucida ſupra Caput]	♈ 9° 15′	♓ 17° 0′; Mar. 9	♈ 22° 8′; Apr. 18	♍ 17° 0′; Sept. 12, o	♎ 12° 45′; Oct. 8	♓ 27° 45′; Mar. 20	♈ 12° 45′; Apr. 3

Names of the stars	Place	Rising: cosmic	Rising: heliacal	Rising: acronychal	Setting: cosmic	Setting: heliacal	Setting: acronychal
Right shoulder of Auriga [Aurigæ humerus dexter]	♊ 3° 2′	♈ 2° 0′; Mar. 25	♉ 5° 18′; Apr. 29	♎ 2° 0′; Sept. 27	♐ 26° 55′; Dec. 21	♊ 12° 13′; June 4	♊ 26° 55′; June 20
Capella	♉ 23° 25′	♓ 16° 50′; Mar. 9	♈ 16° 43′; Apr. 9	♍ 16° 50′; Sept. 12	♐ 13° 52′; Dec. 7	♉ 28° 12′; May 21	♊ 13° 52′; June 7
Right forefoot of Centaurus [Centauri pes dexter anter.]	♎ 6° 36′	♎ 26° 32′; Oct. 22	♏ 9° 26′; Nov. 3	♈ 26° 32′; Apr. 19	♒ 4° 15′; Dec. 27	♊ 15° 54′; June 9	♋ 4° 15′; June 29
Bright star of Corona Borealis [Coronæ Boreæ lucida]	♎ 14° 47′	♍ 25° 55′; Sept. 21	♎ 9° 37′; Oct. 5	♓ 25° 55′; Mar. 18	♋ 3° 51′; June 29	♏ 15° 16′; Nov. 9	♒ 3° 51′; Dec. 27
Right wing of Corvus [Corui ala dextra]	♍ 10° 57′	♍ 16° 10′; Sept. 11	♎ 0° 59′; Sept. 26	♓ 16° 10′; Mar. 8	♒ 27° 40′; Feb. 19	♌ 9° 8′; Aug. 4	♌ 27° 40′; Aug. 23
Tail of Cygnus [Cygni cauda]	♒ 7° 2′	♏ 21° 22′; Nov. 15	♐ 6° 19′; Nov. 30	♉ 21° 22′; May 15	♍ 14° 6′; Sept. 9	♓ 0° 19′; Feb. 21	♓ 14° 6′; Mar. 6
Preceding head of Gemini [Geminor. caput præced.]	♊ 21° 50′	♊ 13° 33′; June 7	♋ 0° 25′; June 25	♐ 13° 33′; Dec. 7	♐ 27° 32′; Dec. 21	♊ 12° 56′; June 6	♊ 27° 32′; May 21
Following head of Gemini [Geminor. caput ſequens]	♊ 24° 52′	♊ 15° 39′; June 9	♋ 8° 31′; July 4	♐ 15° 39′; Dec. 9	♒ 2° 38′; Dec. 25	♊ 12° 27′; June 5	♋ 2° 38′; June 27
Bright star of the Hyades, Palilicium [Hyadum lucida, Palilicium]	♉ 11° 21′	♉ 20° 39′; May 14	♊ 11° 1′; June 5	♏ 20° 39′; Nov. 15	♏ 8° 53′; Nov. 3	♈ 25° 43′; Apr. 18	♉ 8° 53′; May 2
Heart of Hydra [Hydræ Cor]	♋ 29° 16′	♌ 11° 20′; Aug. 6	♌ 24° 58′; Aug. 20	♒ 11° 20′; Feb. 2	♒ 3° 35′; Dec. 27	♊ 15° 31′; June 9	♋ 3° 35′; June 29
Northern Scale [Lanx Borealis]	♎ 20° 57′	♎ 17° 38′; Oct. 13	♏ 1° 35′; Oct. 27	♈ 17° 38′; Apr. 10	♉ 9° 7′; May 3	♎ 8° 2′; Oct. 3	♏ 9° 7′; Nov. 3
Southern Scale [Lanx Auſtralis]	♎ 16° 40′	♎ 16° 34′; Oct. 12	♏ 11° 14′; Nov. 5	♈ 16° 34′; Apr. 9	♈ 17° 19′; Apr. 9	♍ 23° 39′; Sept. 19	♎ 17° 19′; Oct. 13

Names of the stars	Place	Rising: cosmic	Rising: heliacal	Rising: acronychal	Setting: cosmic	Setting: heliacal	Setting: acronychal
Tail of Leo [Leonis Cauda]	♌ 23° 12′	♌ 16° 55′; Aug. 12	♍ 0° 21′; Aug. 26	♒ 16° 55′; Feb. 8	♓ 22° 18′; Mar. 14	♌ 21° 36′; Aug. 17	♍ 22° 18′; Sept. 17
Heart of Leo, Baſiliſcus [Leonis Cor Baſiliſcus]	♌ 1° 25′	♌ 0° 58′; July 27	♌ 17° 44′; Aug. 13	♒ 0° 58′; Jan. 23	♒ 2° 33′; Jan. 25	♊ 26° 58′; June 20	♌ 2° 33′; July 29
Middle ſtar of Lepus [Leporis media]	♉ 22° 58′	♋ 12° 24′; July 7	♌ 0° 6′; July 26	♍ 12° 24′; Jan. 4	♏ 1° 32′; Oct. 27	♈ 16° 19′; Apr. 8	♉ 1° 32′; Apr. 25
Northern Aſs of Cancer [Cancri Aſellus Boreus]	♋ 9° 6′	♋ 13° 10′; July 8	♌ 8° 10′; Aug. 3	♍ 13° 10′; Jan. 5	♍ 4° 55′; Dec. 28	♊ 9° 22′; June 2	♋ 4° 55′; June 30
Bright ſtar in the tail of Delphinus [Delphini lucida in cauda]	♍ 15° 41′	♐ 17° 0′; Dec. 11	♍ 5° 37′; Dec. 29	♊ 17° 0′; June 11	♌ 5° 0′; July 31	♍ 14° 15′; Jan. 6	♒ 5° 0′; Jan. 27
Bright ſtar of Lyra, Fidicula [Lyræ lucida Fidicula]	♐ 16° 52′	♎ 28° 32′; Oct. 24	♏ 11° 28′; Nov. 5	♈ 28° 32′; Apr. 21	♌ 19° 9′; Aug. 14	♒ 5° 49′; Jan. 28	♒ 19° 9′; Feb. 10
Preceding ſhoulder of Orion [Orionis humerus prçcedēs]	♉ 22° 33′	♊ 15° 39′; June 10	♋ 7° 13′; July 2	♐ 15° 39′; Dec. 9	♏ 13° 57′; Nov. 8	♈ 29° 25′; Apr. 21	♉ 13° 57′; May 7
Following ſhoulder of Orion [Orionis humerus ſequens]	♊ 0° 21′	♊ 20° 42′; June 12	♋ 11° 20′; July 6	♐ 20° 42′; Dec. 12	♏ 21° 9′; Nov. 15	♉ 6° 16′; Apr. 30	♉ 21° 9′; May 14
Middle ſtar of Orion's belt [Orionis cinguli media]	♉ 25° 3′	♊ 26° 24′; June 18	♋ 16° 29′; July 11	♐ 26° 24′; Dec. 18	♏ 11° 55′; Nov. 6	♈ 27° 22′; Apr. 19	♉ 11° 55′; May 4
Navel of Pegaſus, head of Andromeda [Pegaſi Vmbil. cap. Androm.]	♓ 15° 56′	♍ 27° 48′; Jan. 20	♒ 22° 10′; Feb. 13	♋ 27° 48′; July 23	♍ 25° 7′; Sept. 20	♓ 11° 24′; Mar. 3	♓ 25° 7′; Mar. 17
Left foot of Orion, Rigel [Orionis pes ſiniſter Rigel]	♉ 18° 26′	♊ 29° 37′; June 22	♋ 17° 31′; July 13	♐ 29° 37′; Dec. 22	♏ 4° 21′; Oct. 29	♈ 21° 24′; Apr. 13	♉ 4° 21′; Apr. 28
Procyon, Canis Minor [Procyon Canis minor]	♊ 27° 27′	♋ 11° 0′; July 6	♋ 27° 54′; July 23	♍ 11° 0′; Jan. 3	♐ 13° 24′; Dec. 6	♉ 29° 2′; May 22	♊ 13° 24′; June 6
Left knee of Sagittarius [Sagittarij genu ſiniſtrum]	♐ 14° 29′	♍ 7° 35′; Dec. 30	♍ 28° 14′; Jan. 20	♋ 7° 35′; July 3	♉ 3° 46′; Apr. 27	♎ 5° 53′; Oct. 2	♏ 3° 46′; Oct. 29
Heart of Scorpius, Antares [Scorpij Cor Antares]	♏ 11° 22′	♏ 13° 42′; Nov. 8	♏ 27° 17′; Nov. 21	♉ 13° 42′; May 7	♉ 2° 0′; Apr. 26	♎ 3° 11′; Sept. 29	♏ 2° 0′; Oct. 27, o
Northern Claw of Scorpius [Scorpij Chela Borealis]	♎ 19° 56′	♎ 16° 49′; Oct. 11	♏ 0° 23′; Oct. 25	♈ 16° 49′; Apr. 9	♉ 7° 17′; Apr. 30	♎ 8° 10′; Oct. 2	♏ 7° 17′; Nov. 1

Names of the stars	Place	Rising: cosmic	Rising: heliacal	Rising: acrony-chal	Setting: cosmic	Setting: heliacal	Setting: acronychal
Southern Claw of Scorpius [Scorpij Chela Auſtralis]	♎ 15° 29′	♎ 15° 21′; Oct. 10	♎ 29° 32′; Oct. 24	♈ 15° 21′; Apr. 7	♈ 16° 24′; Apr. 8	♍ 5° 14′; Aug. 31	♎ 16° 24′; Oct. 10
Bright side of Perseus [Perſei lucidū latus]	♉ 3° 27′	♒ 7° 44′; Jan. 30	♓ 4° 20′; Feb. 25	♌ 7° 44′; Aug. 3	♏ 23° 5′; Nov. 17	♉ 8° 0′; Apr. 30	♉ 23° 5′; May 15
Sirius, Canis Major [Sirius Canis maior]	♊ 15° 44′	♋ 22° 48′; July 18	♌ 7° 43′; Aug. 3	♑ 22° 48′; Jan. 15	♏ 19° 14′; Nov. 13	♉ 5° 31′; Apr. 29	♉ 19° 14′; May 11
Bright star of the Vergiliae or Pleiades [Virgiliarū ſeu Pleiadū luc.]	♉ 1° 24′	♈ 22° 51′; Apr. 15	♉ 28° 3′; May 20	♎ 22° 51′; Oct. 17	♏ 3° 12′; Oct. 28	♈ 18° 3′; Apr. 10	♉ 3° 12′; Apr. 27
Vindemiator	♍ 11° 32′	♍ 5° 3′; Aug. 31	♍ 20° 5′; Sept. 15	♓ 5° 3′; Feb. 26	♈ 20° 0′; Apr. 12	♍ 25° 47′; Sept. 21	♎ 20° 0′; Oct. 14
Spica [Virginis Spica]	♍ 25° 25′	♍ 26° 0′; Sept. 21	♎ 8° 31′; Oct. 3	♓ 26° 0′; Mar. 18	♓ 21° 5′; Mar. 13	♌ 20° 38′; Aug. 16	♏ [recte: ♍] 21° 5′; Sept. 16

Hiſtoria Selecta Ortus & Occaſus Siderum ex Plinio lib. 18. à cap. 26. ad 30. Varrone, Columella lib. 11. cap. 2. Ouidij Faſtis, Virgilij Georgicis, & Alijs multis Scriptoribus.

A selected history of the rising and setting of the stars, from Pliny the Elder, Book 18, Chapters 26 to 30; Varro; Columella, Book 11, Chapter 2; Ovid's Fasti*; Virgil's* Georgics*; and many other writers.*

II. Some things about these are reported by Stöffler on the *Sphere* of Proclus, from p. 35; Clavius on the *Sphere* of Sacrobosco, p. 314; Kepler in the *Epitome of Copernican Astronomy*, from p. 378; and Pétau in the books of dissertations of the *Uranologion*.

[Margin: Aquila.]

AQUILA. Varro, *De re rustica* Book 2, Chapter 2, and Pliny the Elder, Book 8, Chapter 47, say that the time for admitting the male to breed young runs from the setting of Arcturus to the setting of Aquila, that is, as the same authors say, from May 13 [tertio Idus Maias] to July 20 [13. Kal. Auguſti]. They are speaking of the cosmical setting, or true morning setting. Ovid, however, in *Fasti* 5, sings of the acronychal or evening rising at the end of May, saying:

You will see the beak of the tawny bird, welcome to Jove.

[Margin: Arcturus.]

ARCTURUS. Theophrastus, *Historia plantarum* Book 4, Chapter 12, indicates the heliacal rising of Arcturus in the month of Boedromion, that is, in September. Ptolemy, however, reports its rising on September 26 and October 3. Pliny the Elder assigns its evening rising to February 23 [7. Kal. Martij], Columella to February 21 [9. Kal. Mart.]. Firmicus, on the other hand, is far astray when, in Book 8, Chapter 14, he puts its rising in the 5th degree of ♐:

see Pétau in the *Uranologion*, Book 2 of the dissertations, Chapter 8. Ovid, for his part, in *Fasti* 3, sang thus of the apparent evening setting on March 5:

When the wife of Tithonus begins to shed dew from her saffron cheeks and brings on the time of the fifth day, whether it is Arctophylax, or whether that sluggish Bootes, it will sink and escape your sight.

But in *Fasti* 5 he speaks of the cosmical setting about May 26, though from faulty calendars, or very ancient ones:

Aurora, as she comes, will take Bootes from your eyes.

On the evening rising, see what was said from Hesiod in Chapter 20, Problem 8. Kepler, in the *Epitome of Copernican Astronomy*, p. 178, from data given by Tycho about the beginning of this century and from the calculation of Mästlin, sets out the risings and settings of Arcturus as you see in the following small table, in which we give the days in the Gregorian style.

Arcturus		Sun in
Rises in the evening [Oritur veſpere]	March 23	♈ 2° 24′
Sets in the morning [Occidit manè]	June 16	♊ 25° 18′
Rises with the Sun [Oritur cum Sole]	Sept. 25	♎ 2° 24′
Emerges from the rays of ☉ [Emergit è radijs ☉]	Oct. 9	♎ 15° 40′
Culminates with ☉ [Culminat cum ☉]	Oct. 25	♏ 1° 34′
Is hidden in the evening [Occultatur veſperi]	Nov. 25	♐ 3° 2′
Sets with the Sun [Occidit cum Sole]	Dec. 17	♐ 25° 18′

[Margin: Aquarius.]

AQUARIUS. That verse of Ovid, *Fasti* 2,

Now light Aquarius sinks with his tilted urn; you who are next, Fish, receive the horses of the ether,

is interpreted by Stöffler, Clavius and Biancani as referring to the heliacal rising, on the ground that the Sun, entering Pisces, leaves behind Aquarius, which it was overwhelming with its brightness, so that Aquarius can now appear in the morning; and for that reason Pisces begins to set heliacally. But the poet is to be understood of the sign rather than of the stars, although at that time the visible stars [Aſtra viſibilia] were not very far from the invisible signs of the zodiac of the same name. On the heliacal setting about January 16, however, this verse of the same Ovid in *Fasti* 1 holds good:

When these have passed, Phoebus, with Capricorn left behind, you will run through the signs of the youth who rules the waters.

ALDEBARAN. See the Hyades.

[Margin: Delphin.]

DELPHIN. Ovid, in *Fasti* 1, sings of the heliacal rising of the whole Dolphin about January 13 [circa Idus Ianuarij]:

Meanwhile the Dolphin, a bright star, rises above the ether and thrusts out its face from its native shallows.

But of the acronychal rising on June 10 or 11 he sings thus in *Fasti* 6:

The sailor, sitting on the stern, says: we shall see the Dolphin when damp night has risen and the day has been driven off.

And a little later, on the rising of the whole constellation:

The Dolphin will be visible through the whole night.

[Margin: Cancer.]

CANCER. On this, Ovid, *Fasti* 1:

So when the third night is at hand before the coming Nones, and the ground is wet, sprinkled with heavenly dew, the arms of eight-footed Cancer will be sought in vain; headlong he will go down under the western waters.

He is speaking of the cosmical setting on January 2, although for Pétau, strictly, it fell on December 28. Of the heliacal setting you will form a conjecture from *Fasti* 6, where the Sun's ingress into ♋ is placed on June 17, for he says:

Now six and as many days of the month remain; yet add one day to this number. The Sun departs from Gemini, and the signs of Cancer redden.

Here you will note an error of six days, since in fact the Sun entered ♋ about June 24: unless you refer that *Now six* to months, as if he had meant to say that six months remain, and of the month of June six days, for in that way he would rightly have said that the Sun enters ♋ on June 24 or 25. But in fact the former interpretation must be retained, because the ancients thought that the ingresses of the signs take place in the eighth degree before the visible signs.

[Margin: Canicula.]

III. CANICULA. Whether this is Sirius or Canis Major, or rather Procyon or Canis Minor, is disputed, both because of the nearness of their risings and settings and because of the similarity of their effects. And what is to be said, if you look to authority, is no less doubtful, since almost as many authors in number, and of equal authority, stand for Procyon as for Sirius, as I showed in Chapter 5, number 6, where I also touched on the effects of the dog days. Now we must deal with the rising and setting of each. And since the ancient writers speak of this confusedly and variously, we shall set the certain before the uncertain, and we shall choose three centuries above all: namely the century of Hippocrates the physician, who presses the observation of the dog days, and who flourished not long before Meton and Eudoxus, whom Aratus followed in his *Phaenomena*, especially since the Eudoxan calendars are repeatedly drawn on by Varro, Columella and Pliny the Elder; the second century will be that of Caesar and Augustus, and in particular the first

Julian year; the third century will be our own, or the one immediately preceding.

[Margin: Risings etc. of Canicula in the time of Hippocrates and Eudoxus.]

In the time of Hippocrates, therefore, and almost of Eudoxus, that is, about 400 years before Christ, the fixed stars stood about 28 degrees further back [anteriores] than now. On this basis Kepler, in the *Epitome of Copernican Astronomy*, Book 3, p. 388, taking Sirius at that time in ♊ 10° 36′ with southern latitude 39° 30′, and Procyon in ♊ 12° 19′ with southern latitude 15° 57′, for climate 4, whose parallel runs through Rhodes, that is, at an altitude of the pole

of 36°, derives for each star the risings etc. written below, by the Problems on which we treated in Chapter 20, with the months and days of the Julian year projected back, but with the addition of the days on which the Sun is now at those points of the zodiac.

About 400 years before Christ: order	Sirius, old style: months of the Julian year	Sirius: ☉ in	Sirius, old style: where the Sun is today, on the day	Procyon, old style: months of the Julian year	Procyon: ☉ in	Procyon, old style: where the Sun is today, on the day
Heliacal setting [Occaſus Heliacus]	May 2	♉ 7° 24′	April 17	May 22	♉ 25° 40′	May 6
Acronychal setting [Occaſus Acronychus]	May 16	♉ 20° 25′	May 1	June 7	♊ 11° 39′	May 23
Culmination with ☉ [Culminatio cum ☉]	June 11	♊ 15° 38′	May 27	June 19	♊ 22° 41′	June 4
Cosmical rising [Ortus Coſmicus]	July 11	♋ 13° 49′	June 26	July 1	♋ 4° 37′	June 16
Heliacal rising [Ortus Heliacus]	July 26	♋ 28° 17′	July 11	July 18	♋ 21° 49′	July 4
Cosmical setting [Occaſus Coſmicus]	Nov. 18	♏ 20° 25′	Nov. 3	Dec. 9	♐ 11° 39′	Nov. 24
Acronychal rising [Ortus Acronychus]	Jan. 9	♑ 13° 49′	Dec. 25	Dec. 31	♑ 4° 37′	Dec. 16

[Margin: Risings etc. of Canicula in the time of Julius Caesar.]

In Julian year 1, however, that is, 45 before Christ, at Rome, from the calculation of Denis Pétau, as is plain from his table [Canone] already set out at the end of number 1, the risings and settings of each Dog were as follows, in the old style.

Year 45 before Christ: order	Sirius: months, days	Sirius: ☉ in	Procyon: months, days	Procyon: ☉ in
Heliacal setting [Occaſus Heliacus]	April 29	♉ 5° 31′	May 22	♉ 29° 2′
Acronychal setting [Occaſus Acronychus]	May 11	♉ 19° 14′	June 6	♊ 13° 24′
Cosmical rising [Ortus Coſmicus]	July 18	♋ 22° 48′	July 6	♋ 11° 0′
Heliacal rising [Ortus Heliacus]	Aug. 3	♌ 7° 43′	July 23	♋ 27° 54′
Cosmical setting [Occaſus Coſmicus]	Nov. 13	♏ 19° 14′	Dec. 6	♐ 13° 24′

Year 45 before Christ: order	Sirius: months, days	Sirius: ☉ in	Procyon: months, days	Procyon: ☉ in
Acronychal rising [Ortus Acronychus]	Jan. 15	♒ 22° 48′	Jan. 3	♒ 11° 0′

[Margin: Risings etc. of Canicula at the beginning of this century.]

For the year of Christ 1600, further, Kepler in the *Epitome of Copernican Astronomy*, p. 387, from data given by Tycho, for an altitude of the pole of 48° 30′, that is, for climate 7, derives the following risings of each Dog; here the days are given in the old style, so that the difference between the ancient and the recent risings may be more readily apparent.

Year of Christ 1600: order	Sirius: month, days	Sirius: ☉ in	Procyon: month, days	Procyon: ☉ in
Heliacal setting [Occaſus Heliacus]	April 21	♉ 11° 7′	May 16	♊ 4° 16′
Acronychal setting [Occaſus Acronychus]	May 7	♉ 26° 40′	June 9	♊ 17° 36′
Cosmical rising [Ortus Coſmicus]	July 27	♌ 13° 32′	July 17	♌ 3° 40′
Heliacal rising [Ortus Heliacus]	Aug. 11	♌ 28° 12′	Aug. 4	♌ 21° 7′
Cosmical setting [Occaſus Coſmicus]	Nov. 9	♏ 26° 40′	Nov. 29	♐ 17° 36′
Acronychal rising [Ortus Acronychus]	Jan. 23	♒ 13° 32′	Jan. 13	♒ 3° 40′

[Margin: Risings of Canicula for the year 1644.]

Lastly, for the year 1644, we have already in Chapter 20, number 16, derived the risings and settings of each Dog from our own observations at Bologna, at an altitude of the pole of 44° 29′ 30″. It is good to repeat them here, but with the days given in the old style, so that a comparison can be made with the ancients, even though there, at that number 16, we followed the days of the Gregorian calendar: for they are easily reduced to Gregorian days by adding 10 days.

Year of Christ 1644: order	Sirius: months, days	Sirius: ☉ in	Procyon: month, days	Procyon: ☉ in
Heliacal setting [Occaſus Heliacus]	April 17	♉ 7° 58′	May 22	♊ 10° 36′
Acronychal setting [Occaſus Acronychus]	May 12	♊ 1° 25′	June 12	♋ 0° 52′
Cosmical rising [Ortus Coſmicus]	July 23	♌ 9° 37′	July 15	♌ 2° 27′
Heliacal rising [Ortus Heliacus]	Aug. 6	♌ 23° 32′	Aug. 1	♌ 18° 23′
Cosmical setting [Occaſus Coſmicus]	Nov. 13	♐ 1° 25′	Dec. 12	♒ 0° 52′
Acronychal rising [Ortus Acronychus]	Jan. 19	♒ 9° 37′	Jan. 13	♒ 2° 27′

From what has been said it is plain that the places and positions of the risings etc. have changed; and from the days given above it will be possible to form

a judgement on the statements of the poets or of other writers. For if they disagree by several days, the blame will have to be laid either on the uncertainty of the days of the ancient calendar, which was dislocated; or on a different altitude of the pole; or on corrupted calendars, expressed in too popular a fashion, or confusing the old risings with more recent ones; or on ignorance of the motion of the fixed stars; or finally on the carelessness of the writers.

First, then, Virgil presents himself, singing in *Georgics* Book 1:

When the white Bull opens the year with his gilded horns [carnibus; recte: cornibus], and the Dog, yielding to the star opposite, sets.

And indeed aptly, for about that time, around the end of April, the Sun, entering Taurus, opened the year, and

with its warmth it roused the earth to gladder sprouting. Then, however, Sirius was setting heliacally at the coming of the Sun opposed to it: not hostile on the ground that the Sun drives dogs to madness, as Kepler interprets it, but because the Dog is turned, in its celestial figure, towards the west, and so appears opposed to the Sun coming to meet it, as if barking at it; and yet, overwhelmed by the Sun's rays as it set, it seemed to give way. Or else the reading should be *auerſo* [turned away], which Servius allows, so that the Dog gives way to the Argo following at its back. The inexperience of Salmasius [Claude Saumaise], who takes Virgil to mean the cosmical setting, Pétau has amply refuted in the *Uranologion*, Book 7, Chapter 3. But what are we to do with Ovid? In *Fasti* Book 4 he says plainly, of April 24 or 25:

When you have six days of April remaining, the season of spring will be at the middle of its course. And you will look in vain for the beast of Athamantian Helle; the signs give rains, and the Dog rises.

And in *Fasti* Book 5, of May 21, where he sings:

On the night following the day, the Erigonian Dog comes out.

For neither Dog, on any of those days, rose even approximately by a rising reckoned relative to the Sun; rather, it was setting around the middle of spring. Homer, in Book 10 of the *Iliad*, places the rising of the Dog at the close of summer, meaning the heliacal rising, which most of the ancients kept to, foretelling the changes of the weather from the first morning sighting of the Dog. Granted that Theon on Aratus seems to start the dog days from the cosmical rising of Sirius, this rather suits later centuries, in which, the fixed stars having advanced [Greek: ἐις τὰ ἑπόμενα (eis ta hepomena, "towards the following signs")], the effects and indications of the dog days drifted away from the heliacal rising of the Dog to the cosmical, as Pétau observes in the *Uranologion*, Book 2, Chapter 10. Manilius, whom some contend flourished under Augustus, others under Constantine or a little after, says in Book 5 of the *Astronomica* that Sirius rises in a part of Leo. Scaliger, correcting him in his notes, says: *He would have done better to say that the Dog rises at the beginning of Cancer on the barbarian horizon. In Manilius's age it was rising in the 7th degree of Cancer,* meaning the age of Augustus, as Scaliger supposes. But Pétau, cited above, rebukes Scaliger, because the true rising of Canicula at Alexandria then

took place at ♋ 12° 27′, and the heliacal at 26° 9′. Many of the ancients, however, placed the heliacal rising of Canicula at the beginning of ♌, which agrees with the age of Hipparchus and Geminus, as they themselves testify; but later it moved away from that first degree. And yet, since the Sun's entries into the cardinal points were commonly thought to occur in the eighth degrees of the signs, they also put its entries into ♋ and ♌ forward by the same number of days. The solstice had been fixed by Julius Caesar at June 24; so they reckoned that the Sun enters ♋ 8 days before, namely on June 17, and ♌ on July 18. This way of counting carried over into the start of the dog days, as Pétau notes in the place cited, and Kepler in the *Epitome of Copernican Astronomy*, p. 392. For Varro indeed, Book 1, Chapter 28, and Columella, Book 9, Chapter 14, count 29 or 30 days from the solstice to the rising of Canicula; reckoned from the Julian solstice, these would fall on July 23 or 24. Palladius, however, in Book 7, Title 9, records that Canicula rises on July 19, and Pliny the Elder, Book 2, Chapter 47, says: *In the hottest season of summer the star of Canicula rises, as the Sun enters the first degree of Leo, which is July 18 [XV. ante Auguſti Kalendas].* And in Book 18, Chapter 28, he records that Canicula rises on the 23rd day after the solstice, that is, July 18 or 19, the day to which Hephaestion of Thebes also assigns its rising. Censorinus, in Chapter 21, or as I have it from the Manutian emendation, Chapter 8, says: *The 13th day before the Kalends of August [Ante XIII. Kal. Auguſti], at which time Canicula usually makes its rising in Egypt*, that is, July 20. So too Firmicus, Book 8, says it rises in the first degree of ♌ (at any rate not in his own age), and Manilius, Book 5, at ♋ 27°. So great is the variety among those writers about this rising, whether for the reason just given, or because of the difference of the climates from which they transferred their calendars to others; or because they confused the cosmical rising with the heliacal; or followed the Eudoxan or other older calendars.

[Margin: On the setting of the Dog according to the ancients.]

IV. Nor is there less variety about the setting of Canicula, which Columella (Book 11, Chapter 2), Manilius (Book 1) and Pliny the Elder (Book 18, Chapter 29) believed stirs up cold and frosts. Columella indeed says: *On April 30 [Pridie Kalendas Maias] the Dog hides itself in the evening; this signals a storm.* Yet on April 29 it was setting heliacally in the first Julian year, that is, on the third day before the Kalends of May [3. Kal. Maij]. Pliny the Elder, in the place just cited, reports as follows: *Numa instituted the Robigalia in the 11th year of his reign, which are now held on April 25 [ad VII. Kal. Maij], since that is about when rust takes hold of the crops. Varro fixes this time as when the Sun holds the tenth degree of Taurus, as the reckoning then required. But the true reason is that nineteen days after the spring equinox, across that four-day period, by the varying observation of the peoples, on April 28 [in IV. Kal. Maij], the Dog sets, a star both vehement in itself and one before which Canicula must set.* He seems to be speaking of the heliacal setting, yet with a looser use of a day or two: for a century before Pliny the Elder it fell on April 29. He errs, however, in thinking that Canicula must set before the Dog: for by the very fact that Procyon precedes the Dog in heliacal rising and has a

northern declination, it must follow Canis Major, which declines to the south, in heliacal setting. So Kepler rightly rebukes him in the *Epitome of Copernican Astronomy*, p. 392. Pétau, in the *Uranologion*, Book 7, Chapter 2, does indeed rebuke Salmasius for building Canicula's earlier setting on Pliny the Elder's testimony, but he interprets Pliny the Elder as speaking of the canine sacrifice to be offered beforehand to appease the star Canicula, red dogs being killed at the Catularian Gate. Further, Geminus states that in Euctemon's calendar the Dog sets in the 2nd degree of ♉, but in Eudoxus's table in the 4th degree.

[Margin: Philander's error.]

Philotheus [?], on Section 4 of the *Aphorisms* of Hippocrates, says that the Dog rises on July 19. But Philander [?], on Book 9, Chapter 7, says: *It is established that for them the Dog rose on July 27 [ſexto Kal. Auguſti]; for us it rises on July 1 [calendis Iulijs]*; which is utterly false. So much for the more ancient writers.

[Margin: The opinion of more recent writers on the rising of the Dog.]

After them, Stöffler, on the *Sphere* of Proclus, says that the rising of Canicula in the time of Hippocrates, that is, about 400 years before CHRIST, in climate 4, was around July 14; but in the time of Avicenna of Seville, that is, in the year of CHRIST 1100, in the same climate, around July 17; and that around the year of Our Lord 1500, in climate 7, Canicula rises around St James's day. Stadius, in the *Canones*, p. 199, says that at Bergis in Flanders Sirius rises with the Sun at 16° ♌, but heliacally on August 25 [8. Kal. Septembris]. Magini, in his *Directions of the primum mobile*, Canon 22, for a polar elevation of 45° in the year 1600, gathers from data given by Tycho the evening occultation of Canis Major on May 6; the morning one on June 6; the cosmical rising on August 2; the heliacal rising on August 17; the evening emersion on January 16.

[Margin: Length of the dog days.]

Argoli, in the *Pandosion Sphaericum*, Chapter 23, states that at Rome the dog days now begin on July 23 and end on August 18. Kepler, however, in the *Epitome of Copernican Astronomy*, p. 395, notes that the dog days were formerly held to be at the least 30 but usually 45, because the Etesian winds blew for the same number of days after the rising of the Dog; but he adds, as his own opinion, that they last until the rising of Arcturus, which brings on contrary and evident weather. What if they begin from the morning pre-rising with the first immersion into the rays of the Sun, with which the Dog begins to be conjoined platically [platicè], and last only until the morning heliacal rising; or rather from the cosmical rising of Canis Minor until the heliacal rising of Canis Major, that is, in the new style, from July 25 to August 16?

[Margin: Capella.]

V. CAPELLA rose on May 1 [Kalendis Maij], according to Columella, and to Ovid, singing in *Fasti* 5:

Let the work rise from Jove: on the first night I am to see the star that served in the cradle of Jove. The rainy constellation of the Olenian She-goat rises.

But Pliny the Elder says it rises on the eighth day of May, which was true of the heliacal rising in Egypt, not at Rome.

[Margin: Centaurus.]

CENTAURUS rises on the third day of May, according to Ovid, saying in *Fasti* 5:

On the night short of the fourth, Chiron will bring out his stars.

[Margin: Corona.]

The heliacal rising of CORONA Borealis is indicated by Virgil, *Georgics* 1, in that verse:

And let the Gnosian star of the blazing Crown withdraw.

It took place on October 5. But its acronychal rising, which then occurred around March 18, is set by Ovid, *Fasti* 3, on the eighth day of March, in this line:

At once, on the night that comes, you will see the Gnosian Crown.

[Margin: Corvus.]

CORVUS, Crater and Anguis rise on February 14 [16. calendas Martij], according to Ovid, *Fasti* 2:

Three stars continuous in place, the Raven and the Snake, and the Bowl lies hidden in the middle between the two: on the Ides they lie hidden; they rise on the following night.

This was taken from excessively old calendars, for in Ovid's age they rose acronychally around the 8th day of March, and cosmically and heliacally in the month of November.

[Margin: Hyades.]

HYADES, or Suculae, of which the brightest is the southern Eye of Taurus, called Palilicium and in Arabic Aldebaran: it set cosmically on November 2 according to Columella and Geminus. Pliny the Elder, Book 18, Chapter 31, said that the Suculae rise with the Sun on October 31 [pridie Kal. Nouembris], but he should have added *with the setting Sun.* Ovid places the heliacal setting of the Hyades around April 18, in *Fasti* 4, and rightly, but describes it in a phrase too ambiguous, saying:

But when the fourth Morning Star looks back on the Ides now past, on this night the Hyades hold Doris.

But in Book 6 he records the heliacal rising on June 2, thus:

The next day calls forth the Hyades, the horns of the Bull's brow, and the earth grows wet with much water.

[Margin: Lyra.]

LYRA. What Ovid sings for January 5 [Nonis Ianuarij], in *Fasti* 1, about its rising is most uncertain:

The Nones will give signs as Lyra rises.

And in the same place, around January 23 [10. Kal. Febr.], he places the setting of Lyra, namely the heliacal:

When the seventh rising from now has let itself down into the waves, no Lyre will any longer shine in all the sky.

Columella puts the same setting on January 22. Ovid thought it rises on May 12, saying in *Fasti* 5:

The ninth day was at hand, when you, most just Chiron, had your body girt with twice seven stars. The curved Lyre would wish to follow him, but the way is not yet fit; the third night will be the proper time.

[Margin: Orion.]

ORION. Polybius, Book 1, writes that the Romans crossed from Sicily into Africa between the rising of Sirius and that of Orion, in unfavourable weather. He seems to be meant of the heliacal rising of each, for Orion rose a little after the solstice, and Sirius at the start of August, and so the voyage was made in the month of July: thus Pétau, above, Book 2, Chapter 8. But what unfavourable weather was there then? Is he then rather to be understood of the acronychal rising of each, since the middle star of Orion's belt rose by such a rising around the middle of December, and Sirius around the middle of January, and the time between was truly unfavourable? Ovid places the setting of Orion around the middle of April, saying in *Fasti* 4:

Sword-bearing Orion will be sunk in the sea.

So this was the heliacal setting, as is clear from Pétau's table. But the acronychal setting is meant when, around May 10, he says in *Fasti* 5:

If in the midst of these you look for Boeotian Orion, you will be deceived.

Pliny the Elder and Ovid rightly said that the Belt of Orion rises heliacally on June 28 [4. Kal. Iulij], if we take the older times, and the first star in the belt; for the middle one rose heliacally on July 11 at Rome in the age of Ovid, whose words in *Fasti* 6 are:

Your Belt is hidden now, and perhaps will be hidden tomorrow; after that, Orion, it is to be seen by me.

[Margin: Pegasus.]

The acronychal rising of PEGASUS Ovid, *Fasti* 3, places a little after the start of March, saying:

And when the stars dapple the dark-blue sky, look up: you will see the neck of the Gorgonian Horse.

[Margin: Pisces.]

The acronychal setting of PISCES the same Ovid assigns to March 3, in these words, *Fasti* 3:

When the third night, sent down, has set its risings moving, one of the twin Fishes will be hidden.

[Margin: Pleiades.]

The PLEIADES, or Vergiliae, daughters of Atlas, set cosmically about the start of the sowing; and to this belongs that passage of Virgil, *Georgics* 1, where he calls them *Eoas* [of the dawn], because the setting was in the morning:

Let the Eoan daughters of Atlas be hidden from you before you commit the owed seed to the furrows.

Pliny the Elder also, Book 18, Chapter 25, says that Thales fixed the morning setting of the Pleiades on the 25th day from the autumnal equinox. Virgil,

moreover, took the precept from Hesiod, *Works and Days*, Book 2, where it says:

When the Pleiades, born of Atlas, are rising, begin the harvest, and the ploughing when they are setting; they indeed are hidden for forty nights and days, etc.

But the bright star of the Pleiades in Hesiod's time rose heliacally at 11° ♉ on May 11, and set heliacally in the 4th degree of ♈ on April 4: for in that age the Sun entered Aries on March 31. It is astonishing, however, how they place the rising of the Vergiliae either 44 days after the spring equinox (Varro, Book 1) or 48 (Columella, Book 9, Chapter 14, and Pliny the Elder, Book 18, Chapter 25), which at Rome and in the ages of these writers was utterly false, but true in the age of Meton and at Athens: for the bright star of the Pleiades then rose heliacally at ♉ 18° 46′, or about May 15, and the equinox was on March 27, so that 48 days intervened. You see how ill-advisedly that Metonic reckoning, brought into Latium after so many years, has upset farming, and has deceived the common people, indeed the most eminent of the poets. Ovid, *Fasti* 5, puts the rising of the Pleiades on May 13 [3. Idus Maij]:

You will see all the Pleiades, and the whole troop of sisters, when one night remains before the Ides.

That is, on the 57th day from the spring equinox, although it occurred there around May 20. Manilius, Book 5, Firmicus, Book 8, Chapter 7, and Pliny the Elder, Book 2, Chapter 47, put their rising in the 6th degree of Taurus.

Varro, Book 1, Chapter 28, puts the setting of the Pleiades at a distance of 32 days from the autumnal equinox; but Columella, Book 9, Chapter 14, at 40 days; and Pliny the Elder, Book 2, Chapter 47, at 44 days. And in Book 18, Chapter 25, he puts their setting in Hesiod's time at the equinox, but in Thales's time on the 25th day, in Anaximander's the 29th, in Euctemon's the 48th. But Anaximander is to be taken of the heliacal setting and Thales of the true setting, as Pétau learnedly shows, Book 2, Chapter 9. In Hesiod's case no reckoning stands firm, and perhaps Pliny the Elder took the morning setting for the heliacal. Geminus, from Callippus, says that the Pleiades set at ♏ 16°; for Eudoxus at 19°; for Euctemon at 15°; he is speaking of the apparent setting. Their evening setting, according to Geminus, was for Euctemon at 10° ♈, for Eudoxus at 13°. Ovid put it on April 2, saying in *Fasti* 4:

The Pleiades will begin to lighten their father's shoulders, who are called seven, yet are usually six.

The same poet is to be understood of the evening rising in *De Ponto*, Book 1, Elegy 9:

The risen Pleiad makes four autumns.

Finally, Virgil is to be understood of the morning rising and the evening setting, when, in accordance with Aristotle, *History* Book 9, Chapter 40, he prescribes two times for gathering honey, *Georgics* 4:

Twice they gather the heavy yield; there are two seasons of honey: as soon as Taygete the Pleiad shows the earth her comely face, and with her foot has spurned the scorned streams of Ocean.

[Margin: Regulus. Heart of the Lion.]

REGULUS, or Basiliscus, the Heart of the Lion, set cosmically on January 24 in the first Julian year. Ovid has the same thing in *Fasti* 1, saying:

From this star, on the coming night, the fire that flashes in the middle of the Lion's breast will be sunk.

Columella, however, puts it on the 27th; Geminus, at ♒ 2°, that is, January 23; but Ptolemy on January 27. Its rising Pliny the Elder, Book 18, Chapter 28, assigns to July 30 [tertio Kal. Auguſti], if *emergit* [emerges] is read there for *immergitur* [is immersed], as Pétau wishes, above, Chapter 8.

[Margin: Scorpius.]

SCORPIUS began to set on March 16 [17. Kal. Aprilis], and set whole on April 1 [Kalendis Aprilis], cosmically; and so Ovid, marking also the acronychal rising, says in *Fasti* 3:

When the next Dawn has refreshed the tender grasses, Scorpius will be visible from its first part. The third day after the Ides is, etc.

And in *Fasti* 4:

While I speak, Scorpius, to be feared for the sting of its raised tail, plunges into the green waters.

In *Fasti* 5 the same poet rightly sings, in May, of the Heart of Scorpius rising acronychally around May 7 [circa Nonas Maij]:

Scorpius in the sky, when we say that tomorrow the Nones dawn, will be visible from its middle part.

[Margin: Taurus.]

TAURUS, according to Ovid, rises cosmically on May 15 [Idibus Maij], for so in *Fasti* 5:

On the Ides the earlier one indicates that Taurus lifts his starry face.

[Margin: Vindemitor.]

VINDEMITOR rises acronychally on March 6, according to Ovid, *Fasti* 3, where after March 5 he has this:

When Tithonus's wife begins to wet her saffron cheeks with dew, and brings on the hours of the fifth day, whether it is Arctophylax, or that sluggish Bootes, it will sink, and will escape your sight. But Vindemitor will not escape, etc.

Auctores qui Catalogos Ortuum & Occaſuum Siderum condiderunt.

Authors who have compiled catalogues of the risings and settings of the stars.

VI. One may consult: Geminus, in the *Elements of Astronomy*, Chapter 16, where he gathers a patchwork from the calendars of Callippus, Eudoxus, Meton, Euctemon, Dositheus and Democritus; Ptolemy, *On the Appearances of the Fixed Stars and Their Indications*, edited from a Greek codex by Pétau, far more accurately than the version that circulates under Ptolemy's name in the translation of Niccolò Leonico Tomeo; the old calendar of Julius Caesar, which survives in Johannes Rosinus, in the book of *Roman Antiquities*, and in

Gyraldus [Lilio Gregorio Giraldi], at the end of the short work *On Years and Months*; the old Roman calendar of Denis Pétau, collected from Ovid's *Fasti*, Columella, Book 11, Chapter 11, and Pliny the Elder, Book 18, from Chapter 18 to 30, which he has in the *Uranologion*, p. 102; the Roman calendar drawn up in the time of Constantine the Great, which the same man publishes from the collection of Johann Georg Herwart von Hohenburg; Aratus's *Phaenomena*, with its interpreters; Stadius, in the *Canones* of motions, in the commentary on the fixed stars, Chapter 10, where he has a catalogue of the risings and settings of 27 fixed stars for the year of Our Lord 1558, from 16° to 63° of polar altitude, with a table of the rising and setting of the fixed stars at Alexandria and Rome for the ancient age; Origanus, *Ephemerides*, Volume 1, Part 2, Chapter 8, where he gives the rising and setting, both morning and evening, of very many fixed stars for the year 1620, from both Tycho and Copernicus, but at a polar altitude of 52° 20′; Magini, in the *Introduction to the Ephemerides*, Part 4, where from Copernicus's data he gives two tables, one of 170 fixed stars, with rising and setting, from 6° to 56° of polar altitude, the other of 34 fixed stars from 1° to 56° of polar altitude; Longomontanus, *Sphaerics*, Book 2, Chapter 4, where he has a table of the rising and setting of 100 fixed stars for the year 1630 and a polar altitude of 56°; and finally Francesco Montebruni, at the end of the *Ephemerides*, where he gives the rising and setting of 245 stars for the year 1650 and the polar altitude of Bologna, or 44° etc., and indeed from data given by Tycho and sometimes by Lansbergen.

CAPVT XXIII.

Chapter 23.

De Galaxia feu Lacteo Circulo.

On the Galaxy, or Milky Circle.

[Margin: Authors on the Milky Circle.]

I. So far we have dealt with the fixed stars or asterisms, for the most part separately, or as belonging to distinct figures. It remains, in imitation of Ptolemy, to treat of the mass of innumerable stars that make up the Milky Circle. Many have written on it, but above all: Aristotle, *Meteorologica* Book 1, text 43, and his interpreters there; Plutarch, *De placitis philosophorum* Book 3, Chapter 1; Ptolemy, *Almagest* Book 8, Chapter 2; Ruffus Festus; Ovid, *Metamorphoses* 1, fable 6; Hyginus, *Poetica Astronomica* Books 2 and 4; Marcus Manilius, *Astronomica* Book 1, Chapter 9; Macrobius, on the *Dream of Scipio*, Book 1, Chapters 4, 12 and 15; Proclus, at the end of the *Sphere*, and Stöffler there, from p. 69; Albertus Magnus, *Meteorologica* Book 2, Tractate 2, from Chapter 3 to 6; Giorgio Valla, *Astronomy* Book 16, Chapter 1; Clavius, on the *Sphere*, p. 306; the Coimbra commentators on the *Meteorologica*, Tractate 4, Chapter 2; Io. Cottunius [Ioannis Kottounios] on *Meteorologica* Book 1, Lecture 34; Libertus Fromondus [Libert Froidmont], *Meteorologica* Book 2, Chapter 5,

Article 2; Bartholomæus Mastrius [Bartolomeo Mastri], on the *Meteorologica*, Disputation 2, Question 2, Article 3, from number 79; Aversa, Question 32, Section 3; and Nicolaus Cabæus [Niccolò Cabeo], on *Meteorologica* Book 1, text 43, sole question; Antonio Rocco, Antonius Polus [?], Cremoninus [Cesare Cremonini], Licetus [Fortunio Liceti] and Amicus, in the places to be cited at number 5.

De Nomine ac Notione Viæ Lacteæ.

On the name and notion of the Milky Way.

[Margin: Names of the Milky Way.]

II. By the Greeks it is called [Greek: Γαλαξίας (galaxias, "the milky one")], or [Greek: κύκλος γαλακτίος (kyklos galaktios, "milky circle")], or [Greek: γαλακτικὸς (galaktikos, "milky")], that is, the Milky Circle, or, as Cicero puts it in the *Dream of Scipio*, the Milky Orb, from the whiteness of milk, which it imitates; for in Greek milk is called [Greek: γάλα (gala, "milk")], and in the oblique case [Greek: γάλακτος (galaktos, "of milk")], which is also why the gem galactites, of this colour, has its name. In Arabic, if we believe Albertus Magnus and Stöffler, and indeed the old interpreter of Ptolemy, it is called *Maiarati*, that is, a movable area. But Father Athanasius Kircher, in his letter to me, states that among the Arabs and almost all the Orientals it is called *Almegiret*, which means a track of scattered chaff, or a course in the sky, called the way of straw; and he confirms this by the authority of Aben-Ragel [Ibn Abī al-Rijāl] in his *Astronomy*. He adds that the Hebrews call it *Nedibath theben*, the Chaldeans *Schabil tabno*, the Ethiopians *Chasara tsamangadu*, and finally the Egyptians [Greek letters: πινωιτεν τέπιτοχ], or the way of straw; for they feign that Typhon scattered in the sky the bundle of ears of grain thrown in his way by Isis as she fled. Others call it the Way of Rome or the Way of St James. This is a great circle, as Proclus notes, since it is described from the centre of the world; or rather a Zone conspicuous in the sky, of varying breadth and density, running obliquely across the equator and the tropics. For it is narrower around Cassiopeia and Centaurus; broader at Heniochus, Gemini and Sagittarius; but nowhere is it broader than 10°. It is usually held within about 7° or 6°, in some places within 8° or 9°, where, however, it does not divide into a fork.

Situs Viæ Lacteæ in Cælo.

The position of the Milky Way in the sky.

III. No one has been more careful in this than Ptolemy, Book 8, Chapter 2, in describing it word by word; but it is better to contemplate it on more recent globes or maps or hemispheres, and better still to note its position carefully with the eyes, as Ptolemy did, and as we once did at Parma.

[Margin: The asterisms through which the Milky Way passes.]

Beginning, then, from the segment of the colure that passes through the vernal intersection, the course of the Milky Way runs through Cassiopeia,

Perseus, Erichthon, the feet of Gemini, the Club of Orion, the Tail of Canis Major, the Argo, and the feet of Centaurus. Beyond these, over against Ara, like a river embracing an island, it divides into two branches, or a fork. Its more easterly part passes through Ara, the end of the Tail of Scorpius, the more easterly foot of Ophiuchus, and the bow of Sagittarius; its more westerly part through the fore part of the tail of Scorpius and the knee of Ophiuchus, beyond which it is joined to the eastern branch by a crosswise path. But the division continues by a straight path towards the Poles, and its more easterly part runs through the feet of Antinous, through Aquila, and the following wing of Cygnus; its more westerly part through the bend of the Serpent, the right hand of Ophiuchus, and the Neck of Cygnus, at whose shoulders and breast it is joined to the eastern branch. Then in one single track it passes through the tail of Cygnus and the Chain and right hand of Andromeda, and completes its circuit in Cassiopeia, where we began it; although Ptolemy starts it from the feet of Centaurus. The more notable stars that shine in the Milky Way are the Breast of Cassiopeia; the bright side of Perseus; Capella, on the eastern border; the Northern Horn of Taurus, on the western border; Propus of Gemini; the bright stars on the thwarts and at the foot of the mast of the Argo; the Southern Cross, or the four bright stars at the feet of Centaurus; the sting of Scorpius; the knee and right hand of Ophiuchus; the Bright Star of Aquila; the Breast and Tail of Cygnus. So it runs from Graecus towards Notolybicus.

[Margin: Manilius's description of the Milky Way.]

Not inelegantly, Manilius, Book 1, Chapter 9, after the zodiacal circle, describes this other, the Milky one, in these verses:

The other moves up towards the Bears set opposite, and with its circuit draws its threads back a little from the north, and passes through the stars of inverted Cassiopeia. From there, descending obliquely, it touches the Swan, and cuts the summer bounds, and supine Aquila, and the circle that equals the seasons, and the Zone that bears the Sun's horses, within the tail where Scorpius burns and the furthest left hand and arrow of Sagittarius. From there it winds its bends through the legs and feet of the other Centaur, and again begins to climb the sky, and cuts the Argive ship through the top of its stern-ornaments, and the middle circle of the world, and the signs at lowest Gemini; it enters under Heniochus, and, making for Cassiopeia, from where it set out, passes over Perseus himself, and closes in her the orb begun from her.

Fabellę de Via Lactea.

Little fables about the Milky Way.

IV. The little fable of the straw and the bundle of ears of grain I have already indicated at number 2. Another fiction concerns the milk from the breasts of Juno, which Manilius, Book 1, Chapter 9, hints at, saying:

[Margin: Fable of Juno's milk.]

nor must I conceal the gentler tale of ancient rumour, spread abroad, that from the snowy breast of the queen of the gods a liquid of milk flowed and stained the sky with

its liquid; for which reason it is called the Milky Circle, and the name descends from that very cause.

And Rufius Festus Avienus:

Look up at the stars: the appearance of gleaming milk is stretched out across the sky; the colour established its name for it, from its first origin, etc.

[Margin: Fable of Phaëthon.]

The third fable concerns Phaëthon and the horses of the Sun, which strayed from the zodiac and, driven accordingly crosswise, burned up the whole of that part of the sky in which the Milky Circle is seen. So Ovid sings, *Metamorphoses* Book 1, Fable 6, from those verses:

There is a high road, manifest in a clear sky (it bears the name Milky), notable for its very whiteness. By this way [inter] [recte: iter] the gods go to the roof of the great Thunderer and his royal house, etc.

And Manilius, above:

Or does that belief better endure through the ancient ages, that the horses of the Sun went that way on a different course and wore another road, and that through long ages the scorched seats, and the stars baked by the flames, changed their blue appearance, the colour turned, and that ash was poured over the place, and the world buried?

[Margin: Fable of the road to the gods above.]

The fourth fable comes from the springs of Plato. It tells of a footpath worn by the gods and by heroes and brave and just men going back and forth that way; or of souls that have slipped down along it into bodies and return again along the same path to their former seats, placed in the Milky Circle. Hence that passage in the Dream of Scipio in Cicero: *That life is the way to heaven, and to this assembly of those who have already lived and, released from the body, inhabit that place which you see. It was a circle shining out among the flames with the most splendid whiteness, which you, as you have received it from the Greeks, call the Milky Circle.* On these words Macrobius, Book 1, Chapter 4, says: *For it must be understood that the place in which Scipio seemed to himself to be during his sleep is the Milky Circle, which is called* γαλαξίας [Greek: galaxias, "the milky one"]. This he confirms from those words of Cicero: *Moreover he was pointing out Carthage from a certain high place, full of stars, bright and clear.* On the gate of Cancer, through which the Platonists imagined that souls go out, and that of Capricorn, through which they return to the gods above, see what we said in Chapter 3, number 6. Following these, Manilius, Book 1, Chapter 9, sings:

Or do brave souls, and names deemed worthy of heaven, released from their bodies and given back by the earth, migrate here from the world, and dwelling in a heaven of their own live ethereal years and enjoy the Universe?

Then, having listed many Greek and Latin men distinguished for wisdom and uprightness, he adds:

Higher, where the circle of the ethereal orb gleams white, is that seat of the gods; this one, nearest to them, is of the divine ones who by their virtue touch on footsteps like theirs.

[Margin: Stöffler's pious allegory.]

Stöffler indeed, piously to be sure, turned this fiction into an allegory worthy of a Christian man, teaching that this is that way to Heaven of which Psalm 14 speaks: *Lord, who shall dwell, etc. He who walks without stain and works justice*; and again Psalm 23: *Who shall ascend the mountain of the Lord, or who shall stand in his holy place? The innocent in hands and clean of heart, etc.* This way is indeed steep and narrow, but, as Ovid sings, *manifest in a clear sky*; that is, to an intellect directed by prudence and, I add, divinely illuminated and free from disturbances.

[Margin: Fable of the scar and of the joint of two hemispheres.]

The last fable (for such it seems, rather than a philosophical opinion) is that of Theophrastus, of which Macrobius, Book 1 on the *Dream of Scipio*, Chapter 15, says: *Theophrastus said that the Milky Circle is the joint by which the sphere of the heaven is made solid out of two hemispheres; and that therefore, where the edges meet from either side, a notable brightness is seen.* Manilius too, above, did not neglect this, saying:

Or does the Universe come together, and do the extremities of its double hollow meet, and touch the edges of the sky and the stars; and does a manifest scar, through those very bonds, join them, making the cleft of the world, while the crowded orb, turned by its bright seam into airy mist, forces into wedges the foundations of the lofty sky?

As if God, in the way our craftsmen do, had glued two hemispheres together with gypsum or some other white cement!

Opiniones Phyſicorum aut Aſtronomorum de Lacteo circulo.

Opinions of the natural philosophers or astronomers on the Milky Circle.

V. Plutarch, *De Placitis Philosophorum* Book 3, Chapter 1, and Macrobius, Book 1 on the *Dream of Scipio*, Chapter 15, report these opinions of the ancients. Some of the Pythagoreans, and Metrodorus, said that this was once the path of the Sun, retaining traces of its former burning; as if the zodiac now glowed like that from the Sun! Others of the Pythagoreans said it is the light of the Sun reflected from the sky as from a mirror, like the light in the rainbow; if that were so, it would change position with the motion of the Sun. Anaxagoras said it is the shadow of the Earth when the Sun is below the Earth; as if either a shadow were white, or the Earth were not smaller than the Sun and its shadow rose beyond 300 semidiameters of the Earth. Parmenides said it is a mixture of dense and rare, from which a whitish colour arises; Posidonius, a fabric of Fire brighter than a star, denser than splendour; Aristotle, a continuous mass of exhalations kindled in the highest air, from the matter out of which he thinks comets are made.

[Margin: Aristotle's opinion is rejected.]

But first, it seems incredible that so great a mass of exhalations should exist only under that tract of the sky, should last perpetually with so constant a figure, and should move with the motion of the primum mobile. Next, since

on Aristotle's own view exhalations do not pass beyond the elementary sphere, the Milky Way, seen in different regions, would undergo a most manifest variation of view [aſpectûs diuerſitatem], or parallax, and would be seen under widely different fixed stars; whereas in fact all people everywhere on earth see it under the same ones. For that reason Averroes here deserted his Aristotle, because both at Córdoba in Spain and at Marrakesh [Marochi] in Africa he saw the bright star of Aquila in precisely the same position in the Milky Way. Moreover, if it were a sublunary meteor, it would darken the planets moving above it, which it does not do. Notwithstanding all this, Aristotle is defended by Cremoninus [Cesare Cremonini] in his *Apology on the Milky Way*; Antonio Rocco on *Meteorologica* 1, question 4; Antonius Polus [?] in his *Digression on the Milky Circle*; Amicus [Giovanni Battista Amico], Treatise 4, question 7, doubt 4; Cottunius [Ioannis Kottounios] on *Meteorologica* 1, lecture 34; and Fortunius Licetus [Fortunio Liceti], *De novis astris* Book 2, Chapter 56. And to escape the argument from parallax, and that from the telescope, which shows a mass of stars in the Milky Way, Kottounios and Licetus in particular say that in Aristotle there are two Galaxies: one celestial, set in the firmament, spread with very many little stars and shining, which Aristotle called *Sporades*, or scattered stars [ſparſiles]; the other elementary, made of vapours and breaths perpetually drawn upwards by the celestial Galaxy and set on fire, and therefore keeping the same position and figure beneath it. But how can stars so far removed have so great a power of preserving beneath them, in such a figure, a fluid matter, when the nearer Sun has no such power of preserving the clouds and mists raised by its own force? Besides, if there are two Galaxies wholly alike in colour and figure, we would certainly see one of them without parallax and the other with a manifest and large parallax. For although it is true that halos, rainbows and the like, which appear by refraction, are not seen with parallax, because each eye sees its own rainbow (a comparison by which Cremonini thinks he triumphs over parallax), yet, as Father Cabæus [Niccolò Cabeo] rightly presses him, at the time when we see a rainbow, say at Bologna, those who are in Africa or Germany do not see a rainbow. Nor is that vapour, which hangs under certain fixed stars with respect to one city or region, also hanging under them with respect to all other places, in which the Milky Circle is nevertheless seen perpetually under the same fixed stars. Finally, the two Galaxies are multiplied to no purpose, and for no other reason than that Aristotle may be granted never to have erred. On that reasoning one would have to invent two Torrid Zones: one that Aristotle denied to be habitable, the other that evident experience has shown to be habitable. Many other things I leave to the natural philosophers; for us the ocular and astronomical foundation from the Dutch tube, or telescope, is enough, of which shortly.

VI. The true view, then, is the one that Democritus reached, before the use of the telescope, by conjectures and merely probable reasons, when he asserted, in Plutarch, *De Placitis* Book 3, Chapter 1, *that the Milky Circle is a collection and*

condensation of splendour proceeding from many stars, small and contiguous, shining together; and Manilius, Book 1, Chapter 9, indicated it in those verses:

Or does a greater crowd of stars, in a dense crown, weave together its flames, and glow white with thick light, and does the circle shine out brighter with their gathered brilliance?

So too Ptolemy, Book 8, Chapter 2, placing this circle in the sphere of the non-wandering stars and saying that it varies in the density of its stars; and Albertus Magnus, Chapter 5, and Stöffler, above. After the use of the telescope, moreover: Galileo in the *Sidereus Nuncius*, p. 16; Kepler in the dissertation with that *Nuncius*; Fromondus [Libert Froidmont], Cabeo, Aversa and Mastrius [Bartolomeo Mastri], in the places cited at the beginning. Nor would Clavius have denied it, if he had come by the telescope before the earlier editions of his *Sphere*, for he would not then have said that these are denser parts of the sky, apt to receive the light of the Sun. The true view, I say, is that the Milky Way is a mass of very many small stars, which on account of their enormous distance cannot be seen individually, distinct from the others, but only together with others; just as white grains of sand viewed from afar cannot be seen separately, although their heap may be seen. For the telescope shows us the Galaxy as just such a thing, and likewise the nebulous stars, which the naked eye judges to be whitish clouds, but which the spyglass reveals to be a mass of stars.

[Margin: The Milky Way near the Antarctic Pole.]

That part of the Milky Circle around the southern pole which, as the eye-witness Josephus Acosta [José de Acosta] relates in his *History of the New World*, Book 1, Chapter 2, shines wonderfully, and in places glows red with rust-coloured patches like the redness of the eclipsed Moon, consists of very many little stars of a somewhat reddish colour, imitating Mars in their light.

[Margin: A new star in the Milky Way.]

Finally, it is worth noting that all the new stars that appeared in this and the preceding century appeared in the Milky Way. For in the year 1572, about the beginning of November, a new star appeared on the northern border of the Milky Way, among the stars in the chair of Cassiopeia; and it lasted there until March of the year 1574. Then in the year 1600 there came forth that star which still shines today at the boundary of the neck and breast of Cygnus. Lastly, on 1604 October 9 there first shone out in Serpentarius that flame-spewing star which imitated the colours of the rainbow, namely on the left knee of Ophiuchus and at the edge of the Galaxy; and it persisted there until the year 1606 and the beginning of February of that year, when it was extinguished. It is just as if the Galaxy were a storehouse from which God, when he wishes, sends these bright portents to the lower parts of this world.

CAPVT XXIV.

Chapter 24.

Tabula Fixarum 70. à nobis verificatarum ad Annum CHRISTI 1644. completum, cum eius vſu & extenſione ad alia tempora.

Table of 70 fixed stars verified by us, for the completed year of Christ 1644, with its use and its extension to other times.

I. There follows at last the table promised at the end of Chapter 14. So that it can be extended to all ages as regards longitude, it has attached a small table of the motion of the fixed stars in longitude. We made this motion most probable in Chapter 16, from number 11; and its perpetual uniformity we showed to be much more probable in Chapter 17. We also taught in Chapter 15 that the latitude is the same and unchangeable. Further, so that the right ascension and declination of these fixed stars can be extended to other times, we give two methods. The first and principal, serving all ages and stars, is by Problem 3 of Chapter 12, given of course the longitude, the latitude and the obliquity of the ecliptic. The other is by the annual equation that we have added to the fixed stars listed below. This equation serves for only one and a half centuries, or at most for two; that is, from the year of Christ 1644 up to the year 1800 going forward, and back to 1500 going backward. The equation of right ascension is always to be added, going forward, to the right ascension of the year 1644, and subtracted going backward, except for the bright star in the shoulder of Ursa Minor [humero lucido Vrſę minoris], according to what was said in Chapters 13 and 19. But the equation of declination is to be applied, according to the letters A and S, which indicate addition or subtraction, in the forward progress of the years; in going backward the contrary is to be done, subtracting where A is set and adding where S is set.

II. The longitude and latitude of the remaining stars, however, will be set out in the catalogue at the end of the second part, preserving nevertheless the difference of longitude from the stars we observed, and in other respects imitating Kepler and Boulliau in their catalogue of fixed stars.

TABVLA LXX. FIXARVM, A NOBIS OBSERVATARVM Ad Annum CHRISTI MDC XLIV. Completum.

Table of 70 fixed stars observed by us, for the completed year of Christ 1644.

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Northern star of the tail of Cetus [Ceti caudæ Borealior]	*Denneb Elkaitus Schemah*, that is, northern tail of Cetus	10° 47′ ♒	23′ 19″ ♓ A	26° 0′ 10″ ♓	10° 1′ 40″
Bright star of the breast of Cassiopeia [Caſſiopeæ lucida pectoris]	*Scheder*, that is, breast	54° 36′ ♑ B	12′ 0″ ♉	2° 54′ 0″ ♉	46° 36′ 0″

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Southern star of the tail of Cetus [Ceti caudæ Auſtralior]	*Denneb Elkaitus genubi*, that is, southern tail of Cetus	19° 57′ 0″ A	25° 0′ 0″	27° 34′ 0″	20° 46′ 50″
Polaris, last star of the tail of Ursa Minor [Polaris, vltima caudæ Vrſę min.]	*Ruccabah*	87° 24′ 30″ B	32° 10′ 0″	23° 38′ 0″	66° 2′ 0″
Bright star of the belt of Andromeda [Andromedæ cinguli lucida]	*Mizar* or *Mirach*	33° 47′ 0″ B	26° 26′ 30″	25° 26′ 0″	26° 0′ 37″
Leading star on the horn of Aries, Head of Aries [Arietis in cornu præcedens, Caput Arietis]	*Ras Hammel*	17° 32′ 0″ B	33° 32′ 30″	28° 12′ 5″	7° 8′ 13″
Southern foot of Andromeda [Andromedæ pes Auſtralis]	*Allamak*	40° 35′ 30″ B	34° 10″	9° 16′ 0″	27° 47′ 0″
Knot of the Line of Pisces [Nodus Lini Piſcium]		1° 3′ 0″ B	25° 56′ 0″	24° 27′ 0″	9° 4′ 30″
Bright star above the Head of Aries [Lucida ſupra Caput Arietis]		21° 46′ 0″ B	26° 50′ 10″	2° 43′ 20″	9° 58′ 0″
Brighter star of the jaw of Cetus [Ceti mandibulæ lucidior]	*Mēkar Elkeitus*, that is, beak of Cetus	2° 40′ 0″ B	40° 58′ 0″	9° 21′ 16″	12° 36′ 42″
Head of Medusa [Meduſæ Caput]	*Ras Algol*, that is, head of the demon	39° 33′ 0″ B	41° 11′ 54″	21° 12′ 0″	22° 25′ 15″
Bright star of the side of Perseus [Perſei lucidum latus]	*Elgenab*, that is, side	48° 31′ 10″ B	45° 0′ 43″	26° 51′ 0″	30° 2′ 0″
Middle and brighter star of the Pleiades [Pleiadum media & lucidior]	*Altorieh*, not *Athoraie*	22° 59′ 30″ B	36′ 18″	25° 2′ 0″	4° 2′ 35″
Southern eye of Taurus, Palilicium [Tauri oculus Auſtr. Palilicium]	*Aldebaran*, that is, the ruling star	15° 44′ 30″ B	73° 54′ 47″	4° 48′ 0″	5° 29′ 45″
Capella, or Hircus, on the left shoulder of Auriga [Capella, ſeu Hircus, in humero ſiniſtro Aurigæ]	*Alhaioth*, that is, Beast of Auriga	45° 34′ 0″ B	72° 36′ 38″	16° 54′ 0″	22° 51′ 18″

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Leading foot of Orion [Orionis pes præcedens]	*Regel Elgebar*, that is, foot of the giant	8° 39′ 40″ A	74° 23′ 31″	11° 55′ 0″ B[56]	31° 11′ 0″
Northern horn of Taurus [Tauri cornu Boreale]		28° 15′ 30″ B	75° 58′ 41″	17° 38′ 0″	5° 21′ 30″
Leading shoulder of Orion [Orionis humerus præcedens]		5° 58′ 30″ B	76° 32′ 46″	15° 59′ 2″	16° 52′ 16″
First star of the belt of Orion [Orionis balthei prima]		0° 36′ 0″ A	78° 31′ 54″	17° 26′ 5″	23° 36′ 0″

[56] on this copy the printed B is struck through in pen, with A written beside it by hand

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Second of the belt [Balthei ſecunda]		1° 27′ 24″ A	79° 34′ 30″	18° 30′ 54″	24° 32′ 0″
Third star of the belt of Orion [Orionis balthei tertia]		2° 9′ 50″ A	80° 43′ 16″	19° 42′ 56″	25° 20′ 0″
Right shoulder of Auriga [Aurigæ humerus dexter]		44° 51′ 48″ B	83° 30′ 30″	26° 30′ 0″	21° 28′ 30″
Following shoulder of Orion [Orionis humerus ſequens]	*Bectelgeuze*, that is, house of the giant	7° 17′ 54″ B	84° 2′ 7″	23° 52′ 0″	16° 4′ 30″
Bright foot of Gemini, Propus [Geminorum lucidus pes, Propus]		16° 39′ 30″ B	91° 16′ 45″	4° 10′ 0″	6° 46′ 40″
Sirius, splendid star of the mouth of Canis Major [Sirius, Splendida oris Canis maioris]	*Alhabor*, or *Scheer Eliemini*	16° 17′ 39″ A	97° 27′ 20″	9° 19′ 0″	39° 32′ 58″
Castor, northern or leading Head of Gemini, Apollo [Caſtor, Geminor. Caput Bor. ſeu præcedens, Apollo]	*Ras Elgiautze*, that is, Head of Gemini	32° 37′ α″ B	107° 53′ 25″	15° 15′ 0″	10° 3′ 40″
Procyon, bright star of Canis Minor [Procyon, Canis minoris lucida,]	*Algomeiza*, or *Kelbelazguar*	6° 6′ 40″ B	110° 11′ 38″	20° 57′ 20″	15° 55′ 51″
Pollux, southern or following Head of Gemini, Hercules [Pollux, Geminorum Caput Auſtr. ſeu ſequens, Hercules]	*Ketpholtsuman*, that is, shoulder of Gemini	28° 51′ α″ B	110° 54′ 22″	18° 22′ 0″	6° 40′ 36″
Heart of Hydra [Hydræ cor]	*Kalb Elhauich*, heart of Hydrus	7° 8′ 0″ A	137° 33′ 40″	22° 26′ 18″	22° 22′ 0″
Heart of Leo, or Regulus, Basiliscus, etc. [Leonis Cor, ſeu Regulus, Baſiliſcus &c.]	*Kalb Ellessed*, that is, Heart of Leo	13° 41′ 12″ B	147° 21′ 30″	24° 55′ 20″	0° 27′ 0″
Bright star of the mane of Leo [Lucida iubæ Leonis]		21° 38′ α″ B	150° 3′ 7″	24° 34′ 0″	8° 47′ 37″
Leading lower star of Ursa Major in the □ [Vrſæ maioris in □ præcedens inferior,]	*Dubhe*	58° 17′ α″ B	159° 54′ 41″	14° 15′ 0″	45° 6′ 30″
Leading upper star of Ursa Major in the □ [Vrſæ maioris in □ præced. ſuperior]		63° 40′ α″ B	160° 21′ 6″	10° 15′ 0″	49° 40′ 32″
Bright star in the loins of Leo [Lucida in lumbis Leonis]		22° 28′ α″ B	163° 48′ 11″	6° 22′ 0″	14° 21′ 9″
Tail of Leo [Leonis Cauda]	*Denneb Ellessed*	16° 32′ α″ B	172° 43′ 13″	16° 44′ 0″	12° 17′ 10″
Following lower star of Ursa Major in the □ [Vrſę maioris in □ Sequēs inferior]		55° 38′ α″ B	173° 39′ 40″	25° 27′ 0″	47° 4′ 20″
Following upper star of the same [Sequēs ſuperior]		59° 0′ 0″ B	179° 25′ 42″	26° 4′ 0″	51° 37′ 48″
First star of the tail of Ursa Major [Vrſæ maioris prima caudæ]	*Alhaiath*, life-giving	57° 56′ α″ B	189° 31′ 11″	3° 49′ 30″	54° 18′ 30″
Vindemiatrix, Protrigetis	*Alaraph*	12° 55′ α″ B	191° 9′ 54″	5° 6′ 0″ B	16° 16′ 30″

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Spica in the hand of Virgo [Spica in manu Virginis]	*Sombalet Elaadri*, or *Eltsamech*, Spica of ♍	9° 15′ 18″ A	196° 39′ 9″	♎ 18° 54′ 20″	1° 59′ 40″ A
Middle star of the tail of Ursa Major [Vrſæ maioris media caudæ]		56° 49′ 10″ B	197° 21′ 43″	♍ 10° 36′ 0″	56° 22′ 24″ B
Last star of the tail of Ursa Major [Vrſæ maioris vltima caudæ]	*Alalieth*, that is, nocturnal, or *Ellamath*	51° 8′ 35″ B	203° 21′ 0″	♍ 21° 52′ 30″	54° 25′ 20″ B
Arcturus in the fringe of Bootes [Arcturus in fimbria Bootis]	*Alramech*, Bootes himself	21° 5′ 31″ B	209° 57′ 0″	♎ 19° 18′ 0″	31° 2′ 30″ B
Southern Pan of Libra [Libræ Lanx Auſtralis]	*Vazneganubi*, that is, pan of the south	14° 29′ 40″ A	217° 51′ 0″	♏ 10° 8′ 24″	0° 25′ 21″ B
Bright star of the shoulder of Ursa Minor [Vrſæ minoris lucida humeri]		75° 41′ 22″ B	222° 54′ 0″	♎ 7° 58′ 0″	72° 52′ 40″ B
Northern Pan of Libra [Libræ Lanx Borealis]	*Vazne Schemali*, that is, northern pan	8° 0′ 40″ A	224° 30′ 22″	♎ 14° 25′ 18″	8° 34′ 0″ B
Bright star of Corona Borealis [Coronæ Boreæ lucida]	*Alphetah*, that is, opening	27° 56′ 40″ B	229° 54′ 36″	♎ 7° 16′ 24″	44° 22′ 0″ B
Bright star of the neck of Serpens [Serpentis lucida colli]		7° 36′ 40″ B	231° 45′ 0″	♏ 17° 10′ 48″	25° 36′ 0″ B
Southern star in the forehead of Scorpius [Scorpij in fronte Auſtralis]	*Aakràb genubi*	20° 13′ 0″ A	235° 34′ 0″	♏ 28° 4′ 10″	5° 21′ 30″ A
Northern star in the forehead of Scorpius [Scorpij in fronte Borealis]	*Aakràb schemali*	18° 46′ 22″ A	236° 12′ 46″	♏ 28° 15′ 18″	1° 4′ 30″ B
Heart of Scorpius, Antares [Scorpij Cor, Antares]	*Kalb & Aakràb*, that is, heart of Scorpius	25° 34′ 40″ A	241° 55′ 43″	♐ 4° 48′ 40″	4° 30′ 30″ A
Head of Hercules [Herculis caput]	*Ras Elhhathi*, that is, head of the Kneeler	14° 52′ 0″ B	254° 35′ 55″	♐ 11° 9′ 0″	37° 21′ 40″ B
Head of Ophiuchus or Serpentarius [Ophiuchi, ſeu Serpētarij Caput]	*Ras Alhague*, head of Serpentarius	12° 53′ 0″ B	259° 36′ 14″	♐ 17° 29′ 0″	35° 56′ 0″ B
Bright star of the head of Draco [Draconis Lucida Capitis]	*Ras Eltanin*, head of Draco	51° 36′ 72″ B	267° 7′ 12″	♑ 4° 31′ 20″	75° 1′ 0″ B
Bright star of Lyra, Fidicula, Chelys [Lyræ lucida, Fidicula, Chelys]	*Vagieh*, *Alahore*, that is, face of Lyra	38° 30′ 27″ B	276° 14′ 18″	♑ 10° 25′ 48″	61° 46′ 30″ B

For the remainder of this table, see the following page. [Huius Tabulæ Reſiduum vide in ſequenti pagina]

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Beak of Cygnus [Cygni roſtrum]	*Mēkar Eldigiagich*	27° 14′ 52″ B	289° 6′ 38″	♑ 26° 25′ 18″	49° 0′ 10″ B

Stars observed: Latin names [STELLÆ Obſeruatæ / Nomina Latina]	Arabic names corrected, from Athanasius Kircher or Bayer [Nomina Arabica Correcta Ex Athanaſio Kircher, vel Bayero]	Declination	Right ascension	Longitude	Latitude
Bright star of Aquila or the Vulture [Aquilæ vel Vulturis lucida]	*Altair*, that is, bird	8° 0′ 10″ B	293° 22′ 8″	♑ 29° 22′ 0″ 26° 43′ 30″ B	
Leading star of the horn of Capricorn [Capricorni Cornu præcedens]		13° 33′ 0″ A	299° 35′ 8″	♑ 7° 1′ 15″ 28° 55′ 28″ B	
Breast of Cygnus, new [Cygni pectus *noua*]		39° 9′ 0″ B	302° 25′ 8″	♒ 57° 8′ 30″ 20° 6′ 28″ B	
Tail of Cygnus [Cygni cauda]	*Denneb Eldigiagich*, that is, tail of the hen	44° 2′ 0″ B	307° 19′ 10″	♒ 59° 56′ 0″ 0° 29′ 0″ B	
Leading shoulder of Aquarius [Aquarij humerus præcedens]		7° 5′ 0″ A	318° 12′ 38″	♒ 8° 39′ 20″ 18° 29′ 28″ B	
Leading star of the tail of Capricorn [Capricorni caudæ præcedens]	*Dēneb Elgedi*, that is, tail of Capricorn	18° 9′ 50″ A	320° 6′ 0″	♒ 2° 27′ 0″ 16° 52′ 0″ A	
Girdle of Cepheus [Cephei cingulum]		69° 2′ 0″ B	320° 55′ 41″	♒ 71° 8′ 0″ 0° 46′ 52″ B	
Following shoulder of Aquarius [Aquarii humerus ſequens]		2° 4′ 0″ A	326° 54′ 42″	♒ 10° 38′ 47″ 28° 22′ 22″ B	
Mouth of the Fish, outflow of Aquarius [Os piſcis, effuſio Aquarii]	*Phombaut* or *Phomelhaut*, that is, mouth of the fish	31° 30′ 0″ A	339° 24′ 20″	♒ 21° 4′ 25″ 28° 48′ 0″ A	
Leg of Pegasus [Pegaſi crus]		26° 10′ 0″ B	341° 40′ 41″	♓ 31° 7′ 30″ 24° 29′ 20″ B	
First star of the wing of Pegasus [Pegaſi alæ prima]	*Markeb*, that is, chariot	13° 19′ 0″ B	341° 48′ 0″	♓ 19° 26′ 0″ 18° 36′ 0″ B	
Head of Andromeda, navel of Pegasus [Andromedæ Caput, vmbilicus Pegaſi]		27° 9′ 5″ B	357° 32′ 33″	♓ 25° 42′ 0″ 9° 22′ 32″ B	
Bright star of the Chair of Cassiopeia [Caſſiopeæ lucida cathedræ]	*Dath Elkarte*, that is, seat	57° 13′ 0″ B	357° 38′ 0″	♓ 51° 15′ 36″ 0° 10′ 0″ B	
Outermost star of the wing of Pegasus [Pegaſi alæ extrema]	*Algenib*	13° 12′ 59″ B	358° 47′ 20″	♓ 12° 35′ 0″ 4° 14′ 40″ B	

TABELLA ÆQVATIONIS

Small table of the equation.

Annual equation of right ascension, to be added to the right ascensions of the fixed stars of the year 1644 down to 1800, unless it is marked as to be subtracted; to the declinations, however, it is either to be added or to be subtracted, as will be indicated by the letters A or S. [Æquatio Aſcenſionis Rectæ Annua, addenda Aſcenſionibus Rectis Fixarum Anni 1644. vſq. ad 1800. niſi indicetur Subtrahenda; Declinationibus autem vel addenda, vel ſubtrahenda, vt indicabitur litteris A vel S.]

Names of the stars [Nomina Stellarum]	Right ascension [Aſcenſ. Rectæ]	Declination [Declinatio]
Northern star of the tail of Cetus [Ceti cauda Boreal.]	46″ 48‴	20″ 24‴ S
Breast of Cassiopeia [Caſſiopeæ pectus]	49″ 12‴	20″ 24‴ A
Southern star of the tail of Cetus [Ceti cauda Auſtr.]	46″ 12‴	20″ 24‴ S
Polaris in the tail of Ursa Minor [Polaris in cauda Vrſę min.]	142″ 12‴	20″ 24‴ A
Bright star of the belt of Andromeda [Andromedæ cinguli lucida]	49″ 48‴	19″ 48‴ A
Leading star on the horn of Aries [Præcedens in cornu Ariet.]	49″ 48‴	19″ 12‴ A
Southern foot of Andromeda [Andromedę pes Auſtr.]	53″ 24‴	18″ 0‴ A
Knot of the Line of Pisces [Nodus lini Piſcium]	46″ 48‴	18″ 0‴ A
Bright star above the Head of Aries [Lucida ſupra Cap. Ariet.]	51″ 0‴	18″ 0‴ A
Brighter star of the jaw of Cetus [Ceti mandibulæ lucidior]	45″ 0‴	15″ 0‴ A
Head of Medusa [Meduſæ Caput]	46″ 12‴	15″ 0‴ A
Bright star of the side of Perseus [Perſei lucidum Latus]	52″ 48‴	12″ 36‴ A
Bright star of the Pleiades [Pleiadum lucida]	53″ 24‴	12″ 36‴ A
Aldebaran, Palilicium [Aldebaram, Palilitium]	51″ 54‴	9″ 0‴ A
Capella	65″ 24‴	6″ 0‴ A
Leading foot of Orion, Rigel [Orionis pes præced. Rigel]	45″ 18‴	5″ 42‴ S
Northern horn of Taurus [Tauri cornu Boreum]	58″ 12‴	4″ 48‴ A
Leading shoulder of Orion [Orionis humerus præced.]	47″ 24‴	4″ 48‴ A
First star of the Belt of Orion [Orionis Balthei Prima]	46″ 12‴	4″ 12‴ S
Second [Secunda]	46″ 12‴	3″ 36‴ S
Third star of the Belt of Orion [Orionis Balthei Tertia]	45″ 36‴	3″ 0‴ S
Right shoulder of Auriga [Aurigæ humerus dexter]	69″ 0‴	2″ 24‴ A
Following shoulder of Orion [Orionis humerus ſequens]	49″ 12‴	2″ 24‴ A
Bright star of the foot of Gemini [Geminorum pes lucidus]	52″ 48‴	1″ 12‴ S
Sirius, Canis Major [Sirius, Canis maior]	40″ 12‴	2″ 24‴ A
Castor, leading head of Gemini [Caſtor, Geminor. cap. præc.]	62″ 24‴	6″ 36‴ S
Procyon, Canis Minor [Procyon Canis minor]	48″ 0‴	7″ 12‴ S
Pollux, following head of Gemini [Pollux, Caput Gem. ſequēs]	56″ 24‴	7″ 12‴ S
Heart of Hydra [Hydræ Cor]	45″ 0‴	15″ 0‴ A
Heart of Leo, Regulus [Leonis Cor, Regulus]	49″ 30‴	17″ 6‴ S
Bright star in the mane of Leo [Lucida in iuba Leonis]	51″ 18‴	17″ 24‴ S
Lower leading star of Ursa Major [Vrſę maioris inferior præc.]	58″ 12‴	19″ 12‴ S
Upper leading star [Superior præced.]	60″ 36‴	19″ 12‴ S
Bright star in the loins of Leo [Lucida in lumbis Leonis]	52″ 12‴	20″ 24‴ S
Tail of Leo [Leonis Cauda]	47″ 24‴	20″ 24‴ S

Names of the stars [Nomina Stellarum]	Right ascension [Aſcenſ. Rectæ]	Declination [Declinatio]
Lower following star of Ursa Major [Vrſę maioris inferior ſequēs]	49″ 48‴	20″ 24‴ S
Upper following star [Superior ſequens]	48″ 0‴	20″ 24‴ S
First star of the tail [Prima caudæ]	41″ 24‴	19″ 48‴ S
Vindemiatrix	46″ 12‴	19″ 48‴ S
Spica [Spica Virginis]	47″ 42‴	19″ 30‴ S
Middle star of the tail of Ursa Major [Vrſæ maioris media caudæ]	37″ 48‴	19″ 30‴ S
Last star of the tail [Vltima caudæ]	37″ 12‴	19″ 30‴ S
Arcturus	42″ 36‴	17″ 42‴ S
Southern Pan [Lanx Auſtralis]	49″ 48‴	16″ 12‴ A
Bright star of the shoulder of Ursa Minor [Vrſę minoris humerus luc.]	8″ 24‴, to be subtracted [8 Subtr. 24]	14″ 12‴ S
Northern Pan [Lanx Borealis]	48″ 54‴	14″ 24‴ A
Bright star of Corona Borealis [Coronæ Boreæ lucida]	39″ 0‴	12″ 36‴ S
Bright star of the neck of Serpens [Serpentis colli lucida]	45″ 0‴	12″ 36‴ S
Southern star in the forehead of Scorpius [Auſtralis in fronte Scorpij]	52″ 48‴	11″ 24‴ A
Northern star in the forehead of Scorpius [Borealis in fronte Scorpij]	52″ 48‴	11″ 24‴ A
Heart of Scorpius, Antares [Scorpij Cor, Antares]	55″ 12‴	9″ 36‴ A
Head of Hercules [Herculis Caput]	40″ 48‴	4″ 48‴ S
Head of Ophiuchus [Ophiuchi caput]	42″ 36‴	4″ 12‴ S
Bright star of the head of Draco [Lucida capitis Draconis]	21″ 0‴	1″ 12‴ S
Bright star of Lyra [Lyræ lucida]	30″ 0‴	2″ 24‴ A
Beak of Cygnus [Cygni roſtrum]	36″ 36‴	6″ 36‴ A
Bright star of Aquila [Aquilæ lucida]	46″ 12‴	7″ 48‴ A
Leading star of the horn of Capricorn [Capricorni cornu præced.]	51″ 0‴	9″ 36‴ S
Breast of Cygnus [Cygni pectus]	32″ 6‴	10″ 48‴ A
Tail of Cygnus [Cygni cauda]	30″ 54‴	12″ 18‴ A

Names of the stars [Nomina Stellarum]	Right ascension [Aſcenſ. Rectæ]	Declination [Declinatio]
Leading shoulder of Aquarius [Aquarij humerus præced.]	48″ 36‴	15″ 36‴ S
Leading star of the tail of Capricorn [Capricorni caudæ præced.]	51″ 36‴	15″ 36‴ S
Girdle of Cepheus [Cephei cingulum]	13″ 12‴	15″ 36‴ A
Following shoulder of Aquarius [Aquarij humerus ſequens]	48″ 0‴	17″ 24‴ S
Outflow of Aquarius, Fomalhaut [Aquarij effuſio, Fomahant]	51″ 0‴	18″ 36‴ S
Leg of Pegasus, Scheat [Pegaſi crus, Scheat]	43″ 12‴	20″ 24‴ A
First star of the wing of Pegasus, Markab [Pegaſi alæ prima, Marchab]	45″ 0‴	20″ 24‴ A

Names of the stars [Nomina Stellarum]	Right ascension [Aſcenſ. Rectæ]	Declination [Declinatio]
Head of Andromeda [Andromedæ caput]	46″ 12‴	20″ 24‴ A
Bright star of the Chair of Cassiopeia [Caſſiopeæ lucida Cathedr.]	45″ 0‴	20″ 24‴ A
Outermost star of the wing of Pegasus, Algenib [Pegaſi alę extrema, Algenib]	45″ 36‴	20″ 24‴ A

MOTVS FIXARVM In Longitudinem, ex opinione noſtra.

Motion of the fixed stars in longitude, according to our opinion.

Years [Anni]	Motion [Gr. /. //.]
1	0° 0′ 50″
2	0° 1′ 40″
3	0° 2′ 30″
4	0° 3′ 20″
5	0° 4′ 10″
6	0° 5′ 0″
7	0° 5′ 50″
8	0° 6′ 40″
9	0° 7′ 30″
10	0° 8′ 20″
20	0° 16′ 40″
30	0° 25′ 0″
40	0° 33′ 20″
50	0° 41′ 40″
60	0° 50′ 0″
70	0° 58′ 20″
80	1° 6′ 40″
90	1° 15′ 0″
100	1° 23′ 20″
200	2° 46′ 40″
300	4° 10′ 0″
400	5° 33′ 20″
500	6° 56′ 40″
600	8° 20′ 0″
700	9° 43′ 20″
800	11° 6′ 40″
900	12° 30′ 0″
1000	13° 53′ 20″
2000	27° 46′ 40″
3000	41° 40′ 0″
4000	55° 33′ 20″
5000	69° 26′ 40″
6000	83° 20′ 0″
7000	97° 13′ 20″
8000	111° 6′ 40″
9000	125° 0′ 0″

Years [Anni]	Motion [Gr. /. //.]
10000	138° 59′ 20″
20000	277° 58′ 40″
25920	360° 0′ 0″

End of the Sixth Book. [LIBRI SEXTI FINIS.]

LIBER SEPTIMVS DE QVINQVE PLANETIS MINORIBVS.

Book Seven: On the five lesser planets.

SECTIO PRIMA DE PLANETIS MINORIBVS IN COMMVNI.

Section One: On the lesser planets in general.

CAPVT I.

Chapter 1.

De Nominibus, Characteribus, Attributionibus, & Analogijs Planetarum cum alijs rebus Septenario præſertim inſignibus.

On the names, signs, attributions and analogies of the planets with other things especially marked by the number seven.

I. In this Book 7 we are going to treat only of the five lesser wandering stars, which the Greeks call [Greek: πλανήτας (planētas, "wanderers")] or [Greek: πλανόντας (planontas, "wandering")], or, with Ptolemy, [Greek: πλανώμενα ἄστρα (planōmena astra, "wandering stars")], that is, wanderers or roamers. Even so, in this chapter it has pleased us to take in the names and characteristic signs of all seven planets, together with the properties they have or that are attributed to them, and their manifold analogies.

[Margin: Names of Saturn.]

Saturn, then, is called in Hebrew *Schabtai*, that is, satiety or rest; in Greek [Greek: κρόνος (kronos, Saturn's Greek name)], from the slowness of time with which it completes its period beneath the zodiac, a period that requires nearly 30 years. But by Plato in the *Timaeus* and by Martianus Capella, Book 8, it is called [Greek: φαίνων (phainōn, "appearing")], that is, appearing; for of all the planets it lies hidden least beneath the rays of the Sun, and because of the slowness of its motion it is left behind by the Sun and emerges sooner. By the Egyptians it is called *Rephan*, that is, god of time; by the Chaldeans *Caun*; by the Arabs *Tzogel*; by astrologers the Greater Infortune [Infortuna maior]. Its astronomical sign is ♄, namely a sickle, with which time mows down all things; and in Plutarch, in the opuscule *On the Face in the Orb of the Moon* [de facie

Lunæ], it is called [Greek: Νυκτιούρος (Nyktiouros, "guardian of the night")], or guardian of the night.

[Margin: Names of Jupiter.]

Jupiter is called in Hebrew *Tzedek*, that is, justice or equity, by which it is believed to govern lower things; in Greek [Greek: Ζεύσ (Zeus)]; but by Plato in the *Timaeus* and by Capella, Book 8, [Greek: φαέθων (phaethōn, "shining")], from its splendour; by the Egyptians *Picheus*, that is, god of life; by the Chaldeans *Taus*; by the Arabs *Elmesteri*; by astrologers the Greater Fortune [Fortuna maior]. Its sign is the first letter of the Greek name, intersected thus: ♃.

[Margin: Names of Mars.]

Mars, or Gradivus, is called in Hebrew *Maadìm*, that is, red like blood; in Greek [Greek: Ἄρης (Arēs)]; by Plato and Capella [Greek: Πυρόεις (Pyroeis, "fiery")], from fire; by the Egyptians *Meloch*, that is, god of destruction; in Chaldean *Aris*; in Arabic *Elmarigh*; by astrologers the Lesser Infortune [Infortuna minor]. Its sign is ♂, that is, an arrow with a shield. Pliny the Elder, Book 2, Chapter 8, writes that some call it the star of Hercules.

[Margin: Names of the Sun.]

The Sun is called in Hebrew *Chammah* or *Schemasch*, from its heat; in Greek [Greek: Ηλιος (Hēlios, "Sun")], or [Greek: φοίβος (phoibos, "bright")], as if [Greek: φῶς τοῦ βίου (phōs tou biou, "light of life")], that is, light of life; by the Egyptians *Potiris*, that is, holy god; in Chaldean *Schemso*; in Arabic *Elschems*; and its mark is ☉.

[Margin: Names of Venus.]

Venus is called in Hebrew *Nogah*, that is, light; in Greek [Greek: Αφροδίτη (Aphroditē)], from the foam of the sea. But by Plato in the *Timaeus* and by Capella, when in the morning it goes before the Sun, it is called [Greek: φώσφορος (phōsphoros, "light-bearer")], that is, Lucifer, and [Greek: ἑώσφορος (heōsphoros, "dawn-bearer")], as if east-bearing [orientifer]; when, however, in the evening it shines after the Sun, it is [Greek: ἕσπερος (hesperos, "of the evening")], that is, Hesperus or Vesper or Vesperugo, as Pliny the Elder has it, Book 2, Chapter 8, and Vitruvius, Book 9, Chapter 4. By the Egyptians it is called *Suroth*, that is, mistress of love; by the Chaldeans *Astaroth*; by the Arabs *Elzahareh*; by astrologers the Lesser Fortune [Fortuna minor]. Its sign is ♀, namely a round mirror with a handle.

[Margin: Names of Mercury.]

Mercury, or Hermes, is called in Hebrew *Cochau* or *Cochab*, that is, shining star, or *Choteb*, that is, scribe and man of letters; in Greek [Greek: Ερμῆς (Hermēs)], as if interpreter; but by Plato in the *Timaeus* and by Capella [Greek: Στίλβων (Stilbōn, "glittering")], from the flashing, quivering light with which it gleams red; by the Egyptians *Hermanibis*, that is, god of speech; by the Chaldeans *Nobu*; by the Arabs *Adared*; by astrologers Fortune by aspect [Fortuna per aſpectum], because on account of its versatile nature it is reckoned malefic with the malefics and benefic with the benefics. Its sign is the caduceus, or ☿,

namely a rod entwined by two serpents gazing at each other. For, as Hyginus says in Book 2, Mercury was presented with this rod by Apollo in return for the seven-stringed lyre, tuned to the number of the Pleiades or Atlantides; and coming upon two dragons fighting in Arcadia, he restrained them and at once bound them to each other by a mutual bond of alliance. Pliny the Elder, Book 2, Chapter 8, says that some call it the star of Apollo.

[Margin: Names of the Moon.]

The Moon, finally, is called in Hebrew *Lebanah*, from its whiteness, or *Ianeah*; in Greek [Greek: Σελήνη (Selēnē)], or [Greek: Σελαναία (Selanaia)], from the new splendour with which it is renewed after the interlunium; by the Egyptians *Isis Pcochos Actephcom*, that is, mistress of the sea and of moist things; by the Chaldeans *Schaoro*; in Arabic *Elkamer*. Its sign is ☾. But we have said more about the other names of the Moon and the Sun in Book 3, Chapter 1, and Book 4, Chapter 1. The more elegant astronomers, however, following Plato, have not infrequently named Saturn *Phænona*, Jupiter *Phætonta*, Mars *Pyroënta*, Venus *Phosphoron* and *Hesperon*, and Mercury *Stilbonta*, with a half-Greek inflection of the word.

[Margin: Seven angels presiding over the 7 planets.]

II. Further, it is the view of many Rabbis and Christian philosophers and theologians that seven Intelligences or angels were divinely set over the seven planets from the beginning of the world, to move and direct them, as the Scholastics teach on Book 2, Distinction 2 [in 2. diſt. 2.]. St Thomas Aquinas, *De potentia*, Question 6, Article 3, in the body of the article, says: *But it is the teaching of the faith that the angels by their command move locally not only the heavenly bodies, but also other bodies, as God ordains, etc.* And in Opuscule 10, Article 3: *But I remember reading that none of the Saints or philosophers denied that the heavenly bodies are moved by a spiritual creature.* And in Opuscule 11, Article 2, he affirms that this has been demonstrated by physical reasons. Trithemius concurs. Near the beginning of his book on the 7 Intelligences that move the orb, addressed to the Emperor Maximilian I, he says: *It is the view of the ancients that this lower world is governed, by the ordinance of the First Intellect, who is GOD, through secondary Intelligences. Assenting to their opinion, the Conciliator of physicians says that seven Spirits have been set over the seven planets from the beginning of heaven and earth.* Some refer to this the words of Apocalypse 1, *And from the seven Spirits who are in the sight of his throne*, as can be seen in Cornelius à Lapide on that passage. St Jerome notes that the seven-lamped candlestick [cādelabrum illud heptalychnon] of Exodus 25 was a figure of these angels, with their planets as its lamps.

[Margin: Names of the 7 angels.]

The names of these 7 Spirits, as our own Serarius notes on Chapter 12 of Tobit, are gathered by some: Michael, Gabriel, Raphael and Uriel from Books 3 and 4 of Esdras, Schaltiel from Genesis 16, Jehudiel from

Exodus 23, and Barachiel from Genesis 18. These names, divinely revealed in the year of the Lord 1460 to Blessed Amedæus, Cornelius relates in the place

cited above, adding their interpretations: Michael, that is, *Who is like God?*; then Gabriel, that is, *Strength of God*; next Raphael, that is, *Medicine of God*; there follows Uriel, namely *Light* or *Fire of God*; after him Scaltiel, or *Prayer of God*; then Jehudiel, namely *Praise* or *Confession of God*; and finally Barachiel, that is, *Blessing of God*.

[Margin: The epithets and emblems of the 7 Angels.]

Lastly, these names were found in their church at Palermo, with their images, epithets and emblems, in the year 1516, as Father Cornelius à Lapide reports, in the passage cited, from the public records of Palermo. They were as follows. 1. *Michael the Victorious*, treading Lucifer underfoot, carrying a green palm in his left hand and in his right a lance, at whose tip a white banner, with a red cross woven into it, wound itself around the shaft. 2. *Gabriel the Messenger*, carrying in his right hand a small torch enclosed in a lantern, and in his left a mirror of green jasper dotted with red spots. 3. *Raphael the Physician*, leading by the right hand the boy Tobias, who carries a fish seized by the mouth, while in his raised left hand he bears a box. 4. *Uriel, Companion of the Strong*, who in his right hand gripped a drawn sword held crosswise before his breast, while on his left, near his feet, a flame shone upon him. 5. *Jehudiel the Rewarder*, holding out a crown of gold in his right hand and in his left a scourge set off with three black cords. 6. *Barachiel the Helper*, carrying white roses in the fold of his cloak. 7. *Scaltiel the Intercessor*, who, with face and eyes lowered and palms joined at the breast, presented the appearance of one praying. On the church erected at Palermo to the seven princes of the Angels, and on the other church at Rome, at the Baths of Diocletian, dedicated by Pius IV in honour of St Mary of the Angels, see the same Cornelius.

[Margin: 7 dukes of the Persians.]

[Margin: 7 Deacons of the Church.]

[Margin: 7 Cardinal Deacons.]

Therefore, just as Ahasuerus, following the custom of the Kings of the Persians, had *Seven dukes of the Persians and Medes, who saw the King's face and were accustomed to sit first after him*, as is told in Esther 1; and just as it is established from the book *On the Roman Pontiffs* that, ever since Evaristus, as many Cardinal Deacons were substituted for the Seven Deacons of the primitive Church, and used to attend the supreme Pontiff; so the Seven Angelic Spirits, whether they are those who preside over the 7 Planets or are distinct from them, appear to hold the chief place before GOD in administering the affairs of this World. And as Clement of Alexandria says in Book 6 of the *Stromata*: *There are seven, whose power is greatest, the first-born princes of the Angels.* This has been said on the occasion of the name of the Planets, and of their first and noblest analogy with the seven Spirits of God.

III. [Margin: Praises of the number seven.] Among the many things that Macrobius brings forward, in Chapter 6 of Book 1 on the *Dream of Scipio*, to show that the number seven, composed of even and odd, is full, and to illuminate that saying of Cicero's about the number seven in the same Dream,

a number which is virtually the knot of all things, he first adduces the judgement of Hippocrates, who says that what has been conceived in the womb moves from the seventieth or the ninetieth day, and that this day, multiplied by three, points to a birth in the seventh or in the ninth month. He then adds that the Ocean wanes for 7 days from the interlunium and waxes again on the eighth day. Next, from the same Hippocrates, *who knows neither how to deceive nor how to be deceived*, he marks out the seven weeks by which the human foetus is formed and the newborn is strengthened, explaining what is brought to completion in each of them, as I shall say at no. 6. But in Chapter 12, closer to our subject, he teaches from the Platonic fables that the soul descends from the upper part of the World and from the Milky Circle through the gate of Cancer, and that in passing through the seven spheres of the Planets it is imbued by them with a particular faculty and acquires:

[Margin: Platonic fictions about the descent of souls through the 7 spheres.]

in the sphere of Saturn, reasoning and understanding, which they call λογιστικὸν θεωρητικὸν [Greek: logistikon theōrētikon, "the reasoning, contemplative [faculty]"]; *in Jupiter's, the power of acting, which is called* πρακτικὸν [Greek: praktikon, "the active [faculty]"]; *in Mars's, the ardour of spirit, which is named* θυμιτικὸν [Greek: as printed, "the spirited [faculty]"]; *in the Sun's, the nature of sensing and of forming opinion, which they call* ἀισθητικὸν [Greek: aisthētikon, "the sensitive [faculty]"] *and* φανταστικὸν [Greek: phantastikon, "the imaginative [faculty]"]; *and the motion of desire, which is called* ἐπιθυμητικὸν [Greek: epithymētikon, "the desiring [faculty]"], *in Venus's; that of pronouncing and interpreting what it senses, which is called* ἑρμηνευτικὸν [Greek: hermēneutikon, "the interpretive [faculty]"], *in the orb of Mercury;* φυτικὸν [Greek: phytikon, "the vegetative [faculty]"], *that is, the nature of planting and increasing bodies, on entering the globe of the Moon, etc.*

IV. [Margin: The Harmony of the World in the spheres of the 7 Planets.] But nothing in praise of the seven Planets sounds more pleasant to the ears of not a few than that celebrated harmony of divine Providence through the heptachord of the seven spheres. For Dorylaus, in Censorinus, Chapter 11, shrewdly declared that *the World is the Organ of God*. With this Plutarch agrees, near the end of his little work on Music, saying: *Pythagoras, Archytas, Plato and the other ancient Philosophers proclaimed that the motions of all things and the revolutions of the stars can neither come about nor persist without Music: for they maintain that the Demiurge and maker, God, fashioned all things with harmony.*

[Margin: 8 Muses or Sirens.]

Indeed the ancient theologians of the Gentiles, as is found in Macrobius on the *Dream*, Book 2, Chapter 3, assigned to eight or 9 spheres as many Muses, and Plato as many Sirens, or Songstresses of God. They also attributed to them musical intervals, and to each its own tones, which Cicero carried over from Platonic sources into what is his own dream more than Scipio's, and which Macrobius expounded in Book 2, Chapter 4. And before them Pythagoras did so, as is clear from Pliny the Elder, Book 2, Chapter 22, from Censorinus,

Chapter 11, and from Macrobius on the *Dream*, Book 2, Chapter 3. Granted that Pliny the Elder says this was done *with a charm more agreeable than necessary* [iucunda magis quàm necessaria vtilitate]; that Macrobius says that to treat harmonics at greater length on this occasion *is the mark of one showing off, not teaching*; and that Glareanus [Heinrich Glarean], in Book 1 of the *Dodecachordon*, p. 11, says that these things have seemed to many *frivolous dreams*. Even so, I ought not to pass over in this place that distribution, both of those who assigned the Muses to the seven Planets and of those who assigned the strings or tones. Of these, some, holding that the highest sphere moves most swiftly and the lowest most slowly, considering, that is, the daily motion to the west, gave to the fixed stars Nete, the sharpest of the strings, and to the Moon Hypate, the deepest. Conversely, others, who considered the proper motion to the east, and recognised that the slowest is in the highest sphere and the swiftest in the lowest, gave Hypate to the highest and Nete to the lowest.

[Margin: Authors from whom the following little tables are drawn.]

Besides the authors already reviewed, those who write on these matters are, in the first place, Nicomachus in the *Enchiridion of Harmonics*; Boethius, *On Music*, Book 1, Chapter 20, and Book 4, Chapter 10; Servius and the other commentators on that line of Virgil, *Eclogue* 2:

I have a pipe made of seven unequal hemlock stalks [Est mihi disparibus septem compacta cicutis Fistula]

and on that line of *Aeneid* 6:

He accompanies in measure the seven distinctions of notes [Obloquitur numeris septem discrimina vocum].

For the seven-stringed lyre, invented by Apollo in imitation of the seven Planets, or devised by Mercury in imitation of the seven-star group of the Pleiades, is recorded by Hyginus, Book 2 [recte: Book 1, per the book's errata]; by Vitruvius, Book 5, Chapter 4, and there by Daniel Barbarus [Daniele Barbaro] and Gulielmus Philander [Guillaume Philandrier]; also by Ficinus [Marsilio Ficino] on Plato, on poetic frenzy; by Glareanus in Book 2 of the *Dodecachordon*, Chapters 8 and 13; by Guido Aretinus [Guido of Arezzo] in his *Scala Musica*; by Petrus Valla [?] on Pliny the Elder, Book 2, Chapter 22; by Zarlinus [Gioseffo Zarlino], Part 2 of the *Institutiones Harmonicae*, Chapters 6 and 29; and by Bettini in the *Apiarium* 10, Progymnasma 1, Proposition 3. From these, let the distribution of the Muses, Strings, Tones and Notes that are attributed to the Planets according to the differing opinions be set out in the following tables.

MVSÆ (Muses): Ficinus & Zarlinus, after the mind of Plato	MVSÆ (Muses): Glareanus, after the mind of Hesiod	Strings, Notes and Tones, or Phthongi: Cicero & Macrobius in Valla	Strings, Notes and Tones, or Phthongi: Cicero in Glareanus & Zarlinus	[Margin: The ancient order of the spheres]
Calliope	Calliope			Ninth
Vranie	Vranie		Μέση, sharpest [acutissima]	✶

MVSÆ (Muses): Ficinus & Zarlinus, after the mind of Plato	MVSÆ (Muses): Glareanus, after the mind of Hesiod	Strings, Notes and Tones, or Phthongi: Cicero & Macrobius in Valla	Strings, Notes and Tones, or Phthongi: Cicero in Glareanus & Zarlinus	[Margin: The ancient order of the spheres]
Polhymnia	Polhymnia	Nete	λιχανὸς ὑπατῶν	♄
Terpsichore	Erato	Paranete	παρυπάτη μεσῶν	♃
Clio	Terpsichore	Paramese	ὑπάτη μεσῶν	♂
Melpomene	Melpomene	Mese	λιχανὸς ὑπατῶν	☉
Erato	Thalia	Hyphypate	παρυπάτη ὑπατῶν	♀
Euterpe	Euterpe	Parhypate	ὑπάτη ὑπατῶν	☿
Thalia	Clio	Hypate	προσλαμβανόμενος	☾

CHORDÆ VOCES TONI &c.
Strings, Notes, Tones, etc.

Nicomachus and, for the most part, Boethius	Boethius in Glareanus & Zarlinus	[Margin: The ancient order of the spheres]
	λιχανὸς ὑπατῶν — A, rè	✶
Hypate	ὑπάτη μεσῶν — B, mi	♄
Parhypate	παρυπάτη μεσῶν — C, fà, vt	♃
Hyperhypate	λιχανὸς μεσῶν — D, sol, rè	♂
Mese	μέση — E, là, mì	☉
Paramese	τρίτη συνεμμένων — E, fà, vt	♀
Paranete	παρανέτη συνεμμένων — G, sol, rè, vt	☿
Nete	νέτη συνεμμένων — E, là, mì, rè	☾

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[Margin: The ancient order of the spheres]	VOCES &c. (Notes, etc.): Father Mario Bettini	INTERVALLA (Intervals): Pythagoras in Pliny the Elder, Censorinus, Macrobius, Zarlinus
✶	Hypate — Vt	Lesser semitone [Hemitonium minus]
♄	Parhypate — Rè	Greater semitone [Hemitonium maius]
♃	Lichanos — Mì	Lesser semitone [Hemitonium minus]
♂	Mese — Fà	Tone [Tonus]
☉	Paramese — Sol	Tone with a lesser semitone [Tonus cū hemitonio minori]
♀	Paranete — Mi mì	Lesser semitone [Hemitonium minus]
☿	Trite — Rè rè	Greater semitone [Hemitonium maius]

[57]The Greek string names are given as printed. They mean: ὑπάτη, the "highest" string, which is the deepest in pitch; παρυπάτη, the string next to it; λιχανὸς, the "forefinger" string; μέση, "middle"; τρίτη, "third"; νέτη with παρανέτη, the "last" string and the one next to it; προσλαμβανόμενος, the "added" note; ὑπατῶν, μεσῶν, συνεμμένων, "of the highest", "of the middle", "of the conjunct" tetrachord. In the first table the third column has no entry in its first two rows, and the fourth none in its first; in the second table the first column has no entry in its first row. The planetary signs stand in the margin outside the ruled frame, level with the rows. ✶ is a printed star sign for the fixed stars.

[Margin: The ancient order of the spheres]	VOCES &c. (Notes, etc.): Father Mario Bettini	INTERVALLA (Intervals): Pythagoras in Pliny the Elder, Censorinus, Macrobius, Zarlinus
☾	Nete — Fà fà	Tone to the Earth [Tonus ad Terram]

But note that Bettini uses the Moon at apogee, the Sun and Mars at a mean distance, and the rest at perigee.

V. [Margin: Planetary hours.] As one passes from the heptachord to the hebdomad, one meets a varied economy of times, arranging hours, days, months and years by the number seven. As regards the Planetary hours, from which the lordship of the days of the week is then believed to arise, see what we said in Book 1, Chapter 30.

[Margin: The days of the week distributed to the Planets.]

Cassius Dio, in Book 37, says: *That the days were arranged according to the seven stars which the Greeks call Planets and the Latins Wanderers was first established by the Egyptians, and then spread to the other nations. There is a twofold reason for this, and it is not difficult. For if someone refers that harmony called* διατέσσαρον [Greek: diatessaron, "the fourth"]*, in which the ancients believed the foundation of all music to be contained, to those stars by which the whole ornament of the heaven is varied and divided, and to the order in which each of them moves;*

[Margin: 1. Reason for this distribution.]

and then, taking his beginning from the outermost orb, that of Saturn, and leaving out the next two, counts the ruler of the fourth orb; then, passing over two others, reaches the seventh; and, running through the same stars and their ruling Gods in the same manner, applies them to the days: he will find that they all agree in a kind of musical way with the ornament of the heaven. This, then, is how this reason stands. Now the ancient order of the Planets is the one I reported above, and it is shown by that well-worn line:

Cynthia, Mercury, Venus, and Sun, Mars, Jove, Saturn [Cynthia, Mercurius, Venus, & Sol, Mars, Ioue, Saturn.]

with these figures printed beneath the names in the same order: 3, 5, 7, 2, 4, 6, 1.

So, entering on the week from the Sabbath and setting Saturn over it, go down, always leaving out the two in between, and you will find that the Sun presides over the following day, that is, the second after the Sabbath, and the Moon over the third. Then go back to Saturn and, leaving him out along with Jupiter, you will find that Mars presides over the fourth day and, by the same jump, Mercury over the fifth. Finally, going down through the Moon and going up again through Saturn, you will find Jupiter set over the sixth day after the Sabbath and Venus over the seventh.

[Margin: 2. Reason for this distribution.]

Cassius Dio goes on to the second reason, saying: *The other reason is of this kind. Review all the hours of the day and night, in such a way, however, that Saturn holds the first place, Jupiter the second, Mars the third, the Sun the fourth, Venus the fifth, Mercury the sixth, the Moon the seventh. For the Egyptians judge this to be the*

order of the orbs. If you do this with the twenty-four hours reckoned up, you will find the first hour of the following day assigned to the Sun; again, if you go on through the same 24 hours in the manner stated above, you will assign to the Moon the first hour of the third day; lastly, if you pursue the rest in this way, each day will have its own Gods, suited to it; namely, that Planet which has obtained the first hour of the day. We followed this order, represented in a little table, in Book 1, Chapter 30.

[Margin: 3. Reason from the aspects.]

In the same place we set out the third reason for this distribution, drawn from the aspects. You come back to the same thing if, beginning from the Sun, as Bettini does, with Scheiner, in the *Apiarium* 10, Progymnasma 1, Proposition 4, you assign to it the first hour of the first day, going down through ♀ ☿ ☾, and then going up from there to ♄, etc.

[Margin: The emptiness of this distribution.]

But these reasons assume that that old system stands, which Io. Picus [Giovanni Pico della Mirandola] rejects in Book 10 *Against the Astrologers*, Chapter 4. Certainly Mercury is in reality nearer the Sun than Venus is, and both, for a great part of the time in which they run through their epicycle, are above the Sun. That foundation is therefore empty; and much emptier is the superstition of those who hold that those days are ruled by the Planets as by divinities. St Isidore rightly detests them in Book 5 of the *Origines*, Chapter 30, mocking the folly of the Gentiles, *who from these seven stars gave names to the days, saying that the spirit is had from the Sun, the body from the Moon, wit and speech from Mercury, pleasure from Venus, blood from Mars, temperance from Jupiter, and humour from Saturn.* And he adds that the days of the week are better begun from the Lord's Day, and the rest called, according to the order of the feriae, the second, third, fourth feria, etc., up to the Sabbath. Philastrius too held this, in his book *On Heresies*. The Emperor Constantine introduced this custom into the Church at the urging of Sylvester, the Supreme Pontiff, as Bede reports in his book *On the Nature of Things*, Chapter 8.

VI. [Margin: The order of the Planets in forming the foetus.] Nor was it believed that the Planets preside only over the hours and the days of the weeks, but also over the months in which the infant is formed in the womb and, once born, is strengthened and made firm, and finally over the gradatory or climacteric years, which the common people call scalar. So indeed Macrobius, partly from the sources of Hippocrates, in Book 1 on the *Dream of Scipio*, Chapter 6, and partly from those of Plato Marsilio Ficino, in the book *De vita longa*, Chapter 19. Ficino says that Saturn presides over the first month of the forming foetus, Jupiter over the second, Mars over the third, the Sun over the fourth, Venus over the fifth, Mercury over the sixth, and the Moon over the last.

[Margin: The order of the Planets in the years of our life, and on the climacterics.]

In the contrary order, the Moon presides over the first year after the boy is born, Mercury over the 2nd, Venus over the 3rd, the Sun over the 4th, Mars over the 5th, Jupiter over the 6th, and Saturn over the 7th; so that every seventh year, which is climacteric and decisive and in which the greatest change of the body occurs, namely the year 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, etc., is under Saturn.

But Macrobius, speaking of the number seven, says: *This, finally, is the number that causes a human being to be conceived, formed, brought forth, to live, to be nourished, to be handed on through all the stages of the ages to old age, and altogether to subsist.* Then, from Hippocrates, Strato and Diocles, he teaches that in the first week a little sac covered with a thin membrane is formed; in the second, blood begins on the outside; in the third, it is filled with blood within; in the fourth, it curdles, and the distinctions of flesh and blood appear; in the fifth, if the birth is to be in the seventh month, the limbs are shaped, and the outlines of all the limbs are moulded, though within the size of a bee. Otherwise, if the birth is to be in the ninth month, and female, this happens in the sixth week; if male, in the seventh. The born child, if it endures the breath of the air for 7 hours, is viable; at the end of the first week it casts off the remains of the navel; at the end of the second it moves to see the light; at the end of the seventh it freely turns its whole face and its pupils to look at each thing. Again, after 7 months the teeth come through; after 14 the child sits without fear of falling; after 21 its sound breaks out into words; after 28 it stands firmly and walks; after 35 it turns from the nurse's milk. As for years, in year 7 it puts in more solid teeth and speaks readily; at 14 it reaches puberty; at 21 it becomes a young man and stops growing in length; at 28 it stops growing in breadth; at 35 it gains strength for the last time, without hope of acquiring greater strength any further; in year 42 a decrease of strength begins, though an imperceptible one, while judgement is brought to completion, up to year 49, in which age is perfected in every way, and a man is fit for counsel and no stranger to the exercise of his strength. But in year 63 or 70 one comes to the goal of living, or the perfected span of human life: *Whoever has passed it, free of every duty, gives himself to the exercise of wisdom alone, and has his whole use in giving counsel, venerable for his release from other offices*, says Macrobius, who judges that all these things flow from the seven spheres of the heaven.

VII. There are many other things that the astrologers, and in part the Spagyrists and Chemists, attribute to the Planets, collected from the sewage of Arabic books, or from the masters of the Pyronomic sect, namely Hermes, Geber, Ruperissa [?], Arnoldus de Villanoua [Arnau de Vilanova], Raymundus Lullus [Ramon Llull], Rogerius Bacho [Roger Bacon], Albertus Magnus, Isaacius Hollandus [Isaac Hollandus] and Theophrastus Paracelsus; they are to be read with a grain of salt and with antidotes against superstition. Besides these, what concerns metals and gems is touched on by Ficino in the book *De vita coelitus comparanda*; by Cardano in Book 5 of *De subtilitate*; by Tycho in the letter to Rothmann of 1588 August 17, where he calls spagyric art terrestrial

Astronomy on account of its analogy with the Planets; and by Kepler in the *Epitome of Copernican Astronomy*, p. 488. These things, and others besides, are covered by Redemptus Baranzanus [Redento Baranzano] in Part 2 of the *Uranoscopia*. The rest, which concerns the Houses, Exaltations, Detriments and Falls of the Planets, is found throughout the Introductions to the Ephemerides. From these I have selected, for the sake of learning rather than from my own judgement and opinion, the little tables set out below.

[Margin: Table 1.]

True order [Ordo verus]	Tycho and most others	Baran-zano and others	Ficino, for the most part Baranzano, and others	Colours	Humours and Spirits
♄	Lead [Plumbum] — Spleen [Splen]	Lead [Plumbum]	Onyx, jasper, topaz [Onyx, Iaspis, Topacius]	Black [Niger]	Black bile [Atra bilis]
♃	Tin [Stannum] — Liver [Iecur]	Bronze [Æs]	Sapphire, emerald, Pracius [?] [Sapphirus, Smaragdus, Pracius]	Yellow [Flauus]	Radical moisture [Humidū radicale]
♂	Iron [Ferrum] — Gall [Fel]	Steel [Chalybs]	Amethyst, jacinth, garnet [Amethystus, Iacinthus, Granatus]	Red [Rubeus]	Yellow bile [Flaua bilis]
☉	Gold [Aurum] — Heart [Cor]	Gold [Aurum]	Diamond, carbuncle, balas [Adamas, Carbūculus, Ballacius]	Saffron [Cro-ceus]	Vital spirit [Spiritus vital.]
☿	Quicksilver [Argentū viuum] — Lung [Pulmo]	Copper [Cuprum]	Agate, chrysolite, beryl [Acathes, Chrysolithus, Berillus]	Blue [Cæruleus]	Animal spirit [Spiritus animal.]
♀	Copper, bronze [Cuprum Æs] — Kidneys [Renes]	Tin [Stan-num]	Pearl, turquoise, chrysoprase [Margarita, Turquesia, Chrysoprasus]	Green [Viridis]	Blood [Sanguis]
☾	Silver [Argentum] — Brain [Cerebrum]	Silver [Argen-tum]	Coral, chalcedony, selenite [Corallus, Chalcedonius, Selenites]	White [Albus]	Phlegm [Pituita]

But Kepler differently, as follows: ♄ *a very hard, dense gem, as 324;* ♃ *a magnet, as 438;* ♂ *iron, as 810; the Earth silver, as 1000;* ♀ *lead, 1175;* ☿ *quicksilver, 1605; the Sun gold, 1800.* And in the *Astronomia Optica*, p. 261, he says that *if a black surface is strongly lit, it gives the colour of Mars; if a bright red, of* ♃; *if leaden or white, of* ♄; *if blue, of* ☿; *if yellow, of* ♀. Then he compares the light of Mars, Jupiter, Venus, Mercury and Saturn to the light of a burning coal, of a very transparent ruby, of a yellow but very bright body, of sapphire, and of coarser crystal: for if the crystal is fine, the light is Mercury's.

[Margin: Table 2.]

Order [Ordo]	Houses of the Planets: Diurnal [Domus Planetarum: Diurna]	Houses of the Planets: Nocturnal [Nocturna]	Detriment of the Planets [Detrimentum Planetarum]	Exaltation of the Planets [Exaltatio Planetarum]	Depression or Fall [Depressio seu Casus]	Joy or Throne [Gaudium seu Thronus]	Sex of the Planets [Sexus Planetarum]
♄	♒	♑	♌ ♋	♎ 19	♈ 19	♒	Male [Masculus]
♃	♐	♓	♊ ♍	♋ 15	♑ 15	♐	Male [Masculus]
♂	♈	♏	♎ ♉	♑ 28	♋ 28	♏	Male [Masculus]
☉	♌		♒ —	♈ 19	♎ 19	♌	Male [Masculus]
☿	♊	♍	♐ ♓	♍ 15	♓ 15	♍	Hermaphrodite [Hermaphroditus]
♀	♎	♉	♈ ♏	♓ 27	♍ 27	♉	Female [Fœmina]
☾		♋	— ♑	♉ 3	♏ 3	♋	Female [Fœmina]

CAPVT II.

Chapter 2.

De Figura Quinque Planetarum, Maculisq. ac Zonis aliquorum Telescopio detectis.

On the Figure of the Five Planets, and the Spots and Belts of Some, Detected with the Telescope.

[Margin: New phenomena indicated.]

I. By a plainly secret design, that supreme maker of the stars, *who shuts up the stars as under a seal*, at the end of the preceding century and the start of the present one, gave so many and such new phenomena in the heaven to be viewed, partly by naked eyes, partly by eyes armed with the telescope. Redoubling in the heavens the spectacles of a New World, as he had most recently done on earth, he admitted into the vast theatre of wonder and contemplation the latest Argonauts there, and Prometheuses here. So at about the same time, and at no very long interval of years, there came forward on the stage of heaven: besides the hosts of almost infinite small stars, ancient but once undistinguished, in the nebulous stars and along the whole tract of the Galaxy, the outstanding splendours of three new stars, in Cassiopeia, in Serpentarius and in Cygnus; then the face of the Moon, and the Sun himself, gleaming all round with innumerable shadows of spots and lights of faculae; and at once Venus, with

her horned brow, and perhaps unlovely with caverns or spots; and half-flashing Hermes, with his sickle-shaped radiance; then the fire of Mars, with a humped and blunted boss, not quite round, yet grim and dreadful still with its rust-coloured spot; then the tetrastric star of Jupiter, surrounded by the splendours of four petty kings rather than satellites, and girt crosswise with a belt that is seldom single, more often double, and more rarely triple; and then, finally, Saturn, the Geryon of the Planets, marvellous for the portent and astonishment of a three-bodied star. So true is it that neither the choir of the unwandering stars nor any of the Planets has appeared without new array and new insignia. Since these matters belong above all to this place, they must be set out from what we have partly observed ourselves with our own eyes and spyglasses, and have partly received from the books of others or from letters written to us.

Mercurij ac Veneris Figura.

The Figure of Mercury and Venus.

II. All stars, whatever their figure may be, appear round when viewed from afar, just as a flame does, which is established to be pyramidal: for their angles are optically compressed into the narrowness of the visual angle, so that they do not appear. But viewed with the spyglass, once Belgian and later, by the craftsmanship of Galileo, of Fontana [Francesco Fontana], of Torricelli [Evangelista Torricelli] and of Eustachio [Eustachio Divini], now Italian, they appear otherwise. Thus Mercury and Venus, after their morning emersion from the rays of the Sun, appear sickle-shaped a little before the second station, as at position I of the following figure; then, around their greatest digression from the Sun, split in two, imitating the look of the Moon at dichotomy, as at G; afterwards, approaching with direct motion their heliacal morning immersion, they appear gibbous and gradually brought round to the roundness of a disc, as at E. From this an argument is drawn that, if the brightness of the Sun S did not stand in the way, they would be seen by us from the Earth T as perfectly round, when viewed at the conjunction in apogee C.

Diagram. Set into the right column, with the text running down its left side. A large circle with the Sun S at its centre carries eight small planet-discs along its circumference, each showing a phase by a dark part and a light part. From S a vertical line runs down through K to T, the Earth, at the foot of the figure; a curved line joins F and G through the region of S; two further straight lines run up from T to the dark discs at H and I.

- C (top of the circle): open disc, wholly light; the conjunction in apogee.
- D (upper left): disc largely light, dark on its outer side; nearly round.
- E (upper right): disc largely light, dark on its outer side; gibbous.
- S (centre): the Sun.
- F (left): disc half dark; bisected.
- G (right): disc half dark; bisected.
- H (lower left): disc mostly dark, the light part sickle-shaped; horned.

- I (lower right): disc mostly dark, the light part sickle-shaped; sickle-shaped.
- K (bottom of the circle): wholly dark disc; the conjunction at perigee.
- T (foot of the figure): the Earth.

When they have come down from the summit of their Pericycles and, having become evening stars, have emerged from the rays of the Sun, they again appear nearly round, as at D; and around the greatest evening digression bisected, as at F; and near the next station horned again, as at H.

In conjunction with the Sun occurring at perigee K, and on the days nearest on either side, they lie hidden under the splendours of the Sun and are not seen except very rarely: either covering, because of their small latitude, a little part of the Sun like a very black spot,

[Margin: Mercury seen under the Sun.]

in which way we have already told of Mercury seen under the Sun in Book 3, Chapter 3, Scholium 3, and Scaliger testifies the same of himself in Exercitation 72, and after him the Coimbra College, on Book 2 of *De caelo*, Chapter 2, Question 4, Article 2; or else, because of its enormous latitude, Venus, which is visible in some measure.

[Margin: Venus seen under the Sun but outside the Sun.]

For, by the testimony of Longomontanus in Book 2 of the *Theorica*, Chapter 18, and of Kepler in the *Optics*, p. 261, Tycho saw Venus near the perigee conjunction on the Danish horizon, when she had a northern latitude of almost 9°. These things being so, a very strong argument is made that these Planets circle the Sun S, and that by it they are illu- minated. For if they always trailed along below the Sun, never departing from it beyond 29 or 30 degrees in the case of ☿ and beyond 50 in the case of ♀, they would always appear in the telescope sickle-shaped, or at most dichotomous; but if they always moved above the Sun, they would always appear round, or as good as round, as will be plain from inspecting the figure itself and from drawing the lines of illumination and of reflection from there to our eyes according to the rules of optics. And if they shone of themselves on all sides, they would be seen as always round. But now each of them must be treated individually.

De Mercurij Figura seorsim.

On the Figure of Mercury Individually.

[Margin: Scheiner's opinion.]

III. Our colleague Scheiner, in the *Disquisitiones Mathematicae*, no. 38, has this: *Of Mercury it is established that it often shines out spherical, so that it is as good as certain that it moves above the Sun*; and shortly after: *But some also assert that Mercury becomes crescent; since we have not yet experienced this, we leave the matter undecided.* And in fact, because of its small digression from the Sun and the vapours of the horizon, the opportunity of seeing it sickle-shaped rarely arises, especially since the quivering of its intensely twinkling light, on account of which the Greeks called it [Greek: στίλβων (stilbōn, "the glitterer")], hinders

that phase unless the eyes are strong and the glass [specillum] outstanding. Yet from that liveliness of brightness, communicated from nearer the Sun, by which it surpasses Venus herself and Jupiter,

[Margin: And Galileo's.]

Galileo, in Dialogue 3 of *De Mundi systemate*, thinks a very firm argument is drawn that Mercury is closer to the Sun than Venus and runs around it in a tighter circuit; whereas in the *Sidereus Nuncius*, p. 16, he had said that the Planets appear exactly round, without scintillation.

[Margin: The testimony of Hortensius.]

Martinus Hortensius, in the Dissertation with Gassendi on Mercury seen under the Sun, where he dealt with certain matters concerning the telescope, affirms that Mercury, no differently from Venus, when it lowers itself beneath the Sun, was observed by him to pass into horns, but when ascending towards the apogee to become gibbous, and finally round.

Diagram. Three discs of Mercury stacked one above another, inset in the left column, the text narrowed beside them and a printed rule dividing the first two. Each disc is a circle with its dark part solidly stippled and its lit part left white.

- Upper disc, sign at upper left: ☿. Label at upper right: `Vespr- / tinus` (evening star). The dark part fills the upper left and centre; the lit part is at the lower right. Dates beneath it: 1639 May 23; 1646 January 26 (`1646. Ian26.`, the `I` not separated from an `L`); 1647 April 30 (printed `Apru`).
- Middle disc, sign at upper left: ☿. Label at upper right: `Mattuti / nus` (morning star). The dark part fills the upper right and centre; the lit part is at the lower left. Dates beneath it: 1643 August 25; 1644 August 6.
- Lower disc, sign at upper left: ☿. Label at upper right: `Vesper / tinus` (evening star). The dark part is a cap across the top; the lit part fills the lower portion (gibbous). Date beneath it: 1643 March 5.

[Margin: Kepler's and Simon Marius's.]

Kepler testifies to the very same in the *Epitome of Copernican Astronomy*, p. 762, also adducing the testimony of Simon Marius; and so does Biancani in Book II of the *Sphere*, Chapters 3 and 4, who however holds Mercury, like all the planets alike, to be rough, lest, if it were turned to a polished roundness, it should reflect to us a single ray of the Sun or very few.

[Margin: Father Giovanni Battista Zupi's observations.]

From the letters, too, that Father Niccolò Zucchi of the Society of Jesus wrote to me, and from the observations of Francesco Fontana, Tract 5, Mercury was observed by Father Giovanni Battista Zupi, mathematician of our Society, at Naples in the year 1639 May 23, three quarters of an hour after sunset, and indeed with a tube of Fontana's, which inverts objects by means of a small glass globe near the eye; but I show it to you upright in the figure above, as the same man later saw it in the year 1646 January 26, one quarter of an hour after sunset. In the earlier observation it stood about 22° from the Sun, and in the later 18°.

[Margin: And ours.]

We too saw it like this, only very slightly smaller, as an evening star in the year 1647 April 30, when it was 20½° east of the Sun. And we saw the same planet as a morning star in the year 1643 August 25, when it was 19° from the Sun, and in the year 1644 August 6, when it was 20° from it, in the shape and position that you see in the second of the preceding figures; and in the last of them as an evening star and gibbous, in the year 1643 March 5, since it was no more than 15° from the Sun and still in the upper part of the Epicycle. Nor is there any need to add other observations; for among recent writers skilled in the telescope and in observing it is now beyond dispute that Mercury is illuminated by the Sun in the same way as the Moon would be, if the Moon went around the Sun rather than the Earth in its circuit.

De Veneris Figura seorsim.

On the Figure of Venus Individually.

IV. Venus, however, appears now round or gibbous, now bisected, now sickle-shaped, so frequently and so plainly that it would be pointless to pile up witnesses here. The same ones adduced for Mercury in the preceding number are as good as enough: among them Galileo in Dialogue 3 and in *De maculis Solis*, p. 15; Biancani in Book 12 of the *Sphere*, Chapter 2; and Scheiner in the *Disquisitiones*, no. 38, and in the second letter to Markus Welser. They testify that in various cities of Italy she was seen gibbous at one and the same time, bisected at another such time, and horned at another. And they add that she is seen smallest but rounder when, moving direct, she is heading for her morning conjunction with the Sun, and largest in length but narrower in breadth, departing most from the roundness of the disc, when, retrograde, she is heading for her evening conjunction; and that she then appears forty times larger (understand Galileo as speaking of the area).

[Margin: The varying size of Venus.]

Scheiner adds, and from him Hortensius in the Dissertation with Gassendi, that after her second quadrature (understand: when she appears bisected and imitates the Moon in □; otherwise she is never 90° from the Sun) her horns appear very blunt, but then gradually taper to points until they become very sharp; yet that when she is two-horned [bifida] she lights up the lands with a very strong radiance, so that she then serves very well for reading letters and casts a more evident shadow, because she has grown in apparent bulk and in light. For although her luminous sickle is a portion of a large circle, it is nevertheless slenderer, and falls far short in vigour of light of the light with which she shines when dichotomous. And indeed we too have very often observed Venus sickle-shaped, and more than once two-horned or gibbous, but always flame-spewing: especially in the year 1643 January 9, with Fontana's tube lent to me, in the position and shape that can be seen in the present diagram.

Diagram. One large disc of Venus set in the right column between the lines of text. The circle has its dark part solidly stippled, occupying the upper

left and centre, and its lit part left white as a sickle along the right and lower right.

- Upper left: ♀ (sign).
- Upper right: `Vesper.` (evening star).
- Left of the circle, set vertically: `Oriens.` (east).
- Right of the circle, set vertically: `Occid.` (west).
- Beneath: `1643 Ianuarij 9.` (1643 January 9).

Toward the Sun she was ruddy, in the middle yellowing, and in the part turned away from the Sun blue-green; but that variety probably came from the glass of the tube. The bright half-ring by which she was crowned from behind perhaps came from Jupiter and Saturn illuminating her, since they were more easterly. She was observed both at other times and most frequently in the year 1644, when, having already slipped down in the evening from the Apogee of her Epicycle, she had appeared after her evening heliacal rising. From May 7 she was clearly gibbous, as seen through larger tubes, until June 20; around the beginning of July she was [Greek: διχότομος (dichotomos, "cut in two")], or very like the Moon at quadrature, but strongly flame-spewing towards the Sun and the lands; then, until August 4, when we stopped observing her, she grew from day to day more sickle-shaped and narrower in breadth, but more drawn out in length,

Diagram. A very large and very thin sickle of Venus filling the width of the left column, its convex edge to the right and its two needle-sharp horns reaching back to the left at the top and foot. The inscription stands inside the concavity.

- At the upper horn: `Occidens` (west).
- At the lower horn: `Occidens.` (west).
- Inside the concavity, above the inscription: ♀ (sign).
- Inside the concavity, below the inscription: ♀ (sign).
- Inscription: *VENERIS Vltima Phasis vespertina, cùm esset proxima Occasui Heliaco, & in Apogeo Eccentrici. Anno CHRISTI 1649. Iunij die 29. Post quem Soli coniuncta fuit Die 7. Iulij.* (The last evening phase of Venus, when she was very near her Heliacal Setting and in the Apogee of the Eccentric. In the year of Christ 1649, June 29. After which she was conjoined with the Sun on July 7.)

so that she rivalled the Moon in its first phase, just as she rivals the last phase of that same Moon when she rises heliacally in the morning. This was above all most splendidly evident in the year of the Lord 1649 June 29, on the evening of which Vesperugo was last seen through the tube, very close to the evening heliacal setting that soon followed; seen, I say, exactly as you see in the figure above, with very sharp horns and both edges very clean, without any filaments of rays, let alone that head of hair mentioned above. The telescope, moreover, was one familiar to us, though not ours but belonging to the Most Illustrious Lord Giuseppe Maria Grimaldi, of whom more elsewhere. Venus was very near the Apogee of the Eccentric and was rushing headlong to the Perigee of

the Epicycle, in which she was due to be conjoined with the Sun on July 7. In exactly the same phase, though in the opposite position, ♀ was observed by us after her ♂ with ☉: not immediately, because the clouds did not allow it, but on July 18 of the same year 1649, when she was already appearing in the morning in the East, sickle-shaped, her very cleanly sharpened horns turned to the West, with a weak and diminished light, although already Lucifer or Phosphoros. So much from our own observations.

But in the observations of Francesco Fontana, Tract 5, Chapter 2, I read that ♀ was seen through his tube as an evening star, oblong and nearly half a foot long, and as large as the Moon is seen without the telescope, half-rough on its concave side and darting rays, especially when she imitates a parabolic figure; and (something unheard of until now) with one or another blackish little globe, now outside, now inside the body of Venus, who is certainly unlovely on that account, as can be seen in the following figures, though on a reduced scale. If these things are true (far be it from us to call into suspicion the good faith of those who so affirm), it seems we must say either that it was some meteor, for example a wisp or a small cloud lying between or set around Hesperus; or that they are spots like the Solar ones, blown out from the body of Venus and as it were bubbling up; or spots like the Lunar ones, of caverns and mountains, more or less illuminated according to Venus's varying position with respect to the Sun, or perhaps her rotation or libration. For I would not dare to say that they are planets accompanying Venus, until time teaches something more certain on this matter. Certainly I have never been able, nor has Father Grimaldi ever been able, nor Gassendi, as is plain from Book 3 of the *Institutio Astronomica*, to view those little globes on Venus or near Venus with any telescope whatever.

Diagram. A ruled box in the right column holding four sickles of Venus in two rows of two, each with its convex edge to the right drawn with a fringe of short rays and its concave edge jagged and irregular. Dark oval globes stand beside or within them.

- Head of the box: ♀ Vesprtine 4. Phases (the four evening phases of Venus).
- Upper left sickle, `1645 Nou. 11.` (1645 November 11): one dark oval within the concavity.
- Upper right sickle, `1645.Nou / 15.` (1645 November 15): one dark oval above the upper horn, and one dark oval beside the lower horn, standing next to the figure `25.` of the next label.
- Lower left sickle, `1645. Dec. 25.` (1645 December 25): one dark oval beyond the lower horn.
- Lower right sickle, `1646. Ian 22.` (1646 January 22): one dark oval within the concavity.

The last thing is to set forth that most ancient portent which St Augustine reports from Varro; for in Book 21 of *De Civitate Dei*, Chapter 8, he has this: *It is in the books of Marcus Varro entitled* De Gente populi Romani*, and I shall set it down here in the very words in which it is read there: In the heaven, he says,

there appeared a marvellous portent; for in the most noble star of Venus, which Plautus calls Vesperugo and Homer Hesperos, calling it most beautiful, Castor writes that so great a portent appeared that it changed its colour, size, shape and course; which neither before nor after has happened in this way. Adrastus of Cyzicus and Dion of Naples, noble mathematicians, said that this happened in the reign of Ogyges. This* *certainly Varro, so great an authority, would not call it a portent unless he saw that it was against nature.* What sort of portent this was cannot be judged from the position of the Stars, since neither the year nor the season of the year has been noted. What if there was nothing extraordinary in Venus herself? What if it was rather a Comet or a New Star near a place which at that time was thought least to suit Venus, yet within the limits of the digression of ♀ from the Sun? Or surely some chasm, or a Meteorological apparition?

De Martis Figura.

On the Figure of Mars.

V. Kepler, in Book 6 of the *Epitome of Copernican Astronomy*, affirms that Mars, on approaching its quadrature aspect with the Sun and having come nearer the Earth, appears almost [Greek: διχότομον (dichotomon, "cut in two")] or gibbous; and he thinks this also happens with Jupiter and Saturn, inasmuch as they are illuminated by the Sun. But whether he draws this about Mars from his own experience or from that of others is uncertain. Gassendi, in Book 3 of the *Institutio Astronomica*, denies that he saw Mars with a red spot, although he used Galileo's glass. Francesco Fontana, on the other hand, in his observations, Tract 6, Chapter 1, says that Mars appeared to him by means of the telescope as large as the full Moon appears to the naked eyes, but with a black spot in the middle, which others think a boss and he himself a cavity; that is, in the year 1636; and shining more dimly around the limb,

[Margin: Mars gibbous and perhaps spotted.]

as can be seen in figure A, which we have reduced from three inches to one.

Diagram. A circular disc of Mars inset in the left column. A broad ring of short radial strokes runs inside the outer circle, around a clear field holding a small round hatched spot at the centre.

- A (above the central spot): the figure's letter.
- Beneath: `Anno 1636.` (in the year 1636).

And in the year 1638 August 24 he saw it gibbous with a spot, which appeared narrower from day to day in step with the decrease of the Martian disc, from which he and Father Zupi infer the gyration of Mars around its own centre. You have a specimen of it in figure B, but reduced, with the portion CEDF of the disc illuminated by the Sun.

Diagram. A gibbous disc of Mars inset in the left column, beneath the previous figure. The circle is cut off at the top by a shallower arc running from C through E to D. A band of radial strokes follows the curved edge from C

down round to D, inside the lower circle, and an almond-shaped hatched spot lies in the middle.

- C (upper left, at the end of the top arc).
- E (above the top arc).
- D (right, at the other end of the top arc).
- B (below the almond-shaped spot).
- F (beneath the circle).
- Beneath: `Anno 1638. Aug. 24.` (in the year 1638, August 24).

At that time Mars was at about 19° ♐ and the Sun at the beginning of ♍, and so, having just passed the first □, it was heading for △. Father Niccolò Zucchi of our Society testifies to me in his letters that it was observed by him at Rome in exactly the same figure in the year 1640 May 23, but without a spot of any kind, either black or red. The Sun was then at 2° ♊, and ♂ at 14° ♒; and so, the trine aspect being passed, it was heading for the second quadrature with the Sun.

[Margin: The retinue of Mars doubtful.]

Finally, Antonín Maria Šírek, in the preface to Book 4 of the *Radius Sidereomysticus*, has these words: *Mars perhaps, as we have often observed, at a certain time stealthily attended as it were by its companions.* He said "perhaps" because he was perhaps doubtful whether they were Fixed stars nearer to Mars; and in Chapter 4 he repeats that it is uncertain whether it is attended by satellites and whether it shines with its own light. Yet it often appeared to him maned, no differently from the Greater Dog; but he depicts this mane as though really distinct from its edge. Therefore, of everything reported so far, the more certain things are the gibbous figure between the quadrature and trine aspects, and, from that, the light communicated by the Sun; the rest is less certain, especially the satellites.

[Margin: Mars seen with a double spot.]

Father Daniello Bartoli of our Society, now most celebrated throughout all Italy for his talent, his eloquence, and his manifold and uncommon erudition, wrote to me from Naples in the year 1644 December 24 that he had seen Mars, with the outstanding telescope of Father Sersale, not with a spot in the middle but with two spots in the lower part of the disc. Many things therefore remain to be observed, which God, their Storekeeper or rather their Founder, will one day, as their Dispenser, grant to us, or to you, O posterity.

De Iouis Figura.

On the Figure of Jupiter.

[Margin: The atmosphere of Jupiter doubtful.]

VI. Let us leave aside the vaporous sphere [vapidâ sphærâ] which Šírek, on p. 177 of his *Radius Sidereomysticus*, asserts he saw around Jupiter with a large spyglass. Pierre Gassendi, in Book 3 of the *Institutio Astronomica*, denies that he could see the bands of Jupiter, although he was using Galileo's tube. But they were seen first, as far as I know, by Father Giovanni Battista Zupi of our Society, and by Francesco Fontana and Father Bartoli, all at Naples. I have

seen them very often, and more often still has Father Francesco Maria Grimaldi, who with his usual sharpness of sight and wonderful circumspection observed not only their number but also their situation with respect to the Ecliptic and the Equator. We have here many full sheets of figures, but lest we burden the book and the reader with superfluities, we display in the attached small table six figures containing nearly all the variety of this phenomenon. In this table, however, the size of Jupiter's disc is not that which our larger tubes represent.

Diagram. A heavy ruled box across the full width of the page holding six circular discs of Jupiter in two rows of three, each numbered above in Roman, crossed by hatched bands.

- I (upper left): three hatched bands, the topmost narrower than the two below.
- II (upper centre): two hatched bands, the upper narrower than the lower.
- III (upper right): one hatched band near the top, with a thin line under it.
- IV (lower left): one hatched band, with a thin line above and a thin line below.
- V (lower centre): one broad hatched band, with a thin line above and a thin line below.
- VI (lower right): two parallel hatched bands running obliquely from lower left to upper right.

[Margin: Explanation of Figure 1.]

The first figure shows Jupiter with three Zones, the topmost however narrower. It was seen so in the year 1639 May 22, one day before its Opposition with the Sun. Three bands, too, seen at Naples, are reported in Fontana's observations, Tract 6, in the year 1643 and in 1646 January 28, of which the third was on the eastern edge; and there had been an Opposition of ♃ with ☉ in the year 1645 December 21; but at that time it was seen with only two zones, as I shall say shortly.

[Margin: Figure 2.] The second figure has Jupiter with two bands set very far apart from each other, their convexity turned downward and their concavity upward. Jupiter was seen in this form at Naples by Father Zupi in the year 1644 January 20, 21, 22, when it was more easterly than the Sun by nearly 80°. But the figure of Jupiter that he sent me at once, with great generosity, is far larger, and equals five inches of the Roman foot given in Book 2, Chapter 7. From a drawing by Father Niccolò Zucchi communicated to me, I gather that it was seen in the same form at Rome in the year 1630 May 17, around its □ aspect with the Sun. It was seen so at Bologna in the year 1647 January 23, on which night it was opposite the Sun. At Naples too it is reported as seen in the same form, in Fontana's observations, in the year 1645 January 21, at $2\frac{1}{2}^{h}$ of the night, on which night its opposition with the Sun took place; and again in that form at the same place it was observed on December 31, and once more in the year 1646 on January 1 and 22. On January 28, however, a third zone emerged on the eastern edge of Jupiter, the one I spoke of under the first figure.

[Margin: Figure 3.] The third figure of Jupiter is with two bands, but closer to each other, and the lower one very slender, seen by us at Bologna in the year 1643 from the beginning of October, over several nights, up to the opposition of ♃ with ☉, which fell on the 13th.

[Margin: Figure 4.] The fourth figure offers Jupiter with a broader zone, lying a little lower than before, enclosed on either side by two very slender, parallel zonelets. It was seen so at Bologna a little after the opposition with the Sun, namely in the year 1643, from the 15th to the 20th of October, always near the Meridian. So much for the curved zones.

[Margin: Figure 5.] The fifth figure shows us a more level Jupiter, now girt with straight belts, these lying nearer the middle, and the middle one of the zones wider, with two thinner, parallel ones on either side. In this configuration it appeared to Father Grimaldi at Bologna in the year 1643 from the beginning of November to December 22, but especially on November 10 at 4^h of the night, and on the 12th at 4^h, and on the 13th at 5^h, and on the 14th, 15th, 23rd and 30th, not far from the Meridian.

[Margin: Figure 6.] The sixth figure gives Jupiter as observed by Father Grimaldi in the year 1648 from February 24 to March 16, but most evidently on the 11th at $2½^h$ after sunset, when Jupiter, in the eastern quadrant and moderately distant from the Meridian, appeared with two entirely straight bands. The upper of these, turned toward Arctapeliotes [the north-east wind], that is, the more easterly, was narrower and darker; the more westerly, turned toward Libycus [the south-west wind], was seen as more spread out and less dark, while the part of Jupiter's disc enclosed between them shone far more brightly than all the rest of the disc. The opposition of ♃ with ☉, moreover, was on February 22.

[Margin: Parallelism of Jupiter's zones with the ecliptic.] As for the parallelism of the bands, and very nearly of the satellites, with the ecliptic, I read these very words in the notes and drawings of the same Father Francesco Maria Grimaldi: *"At Bologna we have often observed the satellites, very nearly, and the bands of Jupiter keeping along the length of the ecliptic; but especially on February 24 of the year 1648, at 3^h after the setting of ☉. For at that time, if a line parallel to the bands and to the path of the satellites had been drawn from the bright star of the Lion's tail, it would have had to be drawn a little above the bright star of the neck of the same Lion. This observation was repeated on the 26th of the same month and on March 11."* And on the next page: *"The length of the bands was parallel to the ecliptic, as was gathered from an imaginary line drawn from the tail to the mane of the Lion, which was nearly parallel to the bands themselves."*

[Margin: Rotation or libration of Jupiter.] Now from this phenomenon it is gathered plainly enough that Jupiter moves, by either rotation or libration about its own centre, nodding roughly toward the north and south; and that for this reason its zones appear sometimes as three, sometimes as only two, now close together and slight, now farther apart and wider, and finally now curved,

now straight — whether they are cavities or valleys shadowed by higher parts and not enjoying the light of the Sun, or rows of spots, or something else.

[Margin: Roughness of Jupiter.] That Jupiter is rough, however, and indeed even around its edges, with mountains and swellings standing out most evidently, appears from a drawing sent to Italy from Flanders, where it is stated to have been seen in that form through the excellent telescope of Dom Leander Bandtius [?], Abbot *Disbergensis* [?], in the year 1643 November 2 at $8\frac{1}{2}^{h}$ after noon, the opposition with the Sun having come before it, on October 12. In this drawing, too, two satellites were seen below Jupiter, darkening it like spots, of which the more northerly occupied a sixteenth part of Jupiter's diameter; and in the same drawing two great spots or cavities, one round, the other oval, equalling in length a seventh part of the same diameter.

De Saturni Figura.

On the Figure of Saturn.

VII. At last the keen sight of the optical tube stretched as far as Saturn. With Galileo the first to observe it, as one finds in his first letter to M. Velser [Markus Welser], pp. 25 and 148, then Scheiner (*Disquisitiones Mathematicae*, no. 44), and afterwards a great many others at Rome, Venice and Naples, and we ourselves at Parma and Bologna, that sight revealed Saturn as either three-bodied, or enormously oblong, or very closely propped on either side by two companions or attendants [lateronibus]. These at times seem to form one and the same body with it, but an oblong one, the intervals on each side having contracted toward the extreme tips; at other times, with arms stretched out and an interval left in the middle, they cling to Saturn like little handles; at other times they are entirely separate from it, either like round and smaller planets, as in the first of the following figures, or like pears, as in the second, or like Moons, gripping it with their horns as if with pincers, as in figures 3 and 4. For although Galileo formerly ascribed changes and alternations of this kind to an imperfection of the eye or of the telescope, which could not distinguish the companions and represented them as if they were a single thing, the case is otherwise. For Scheiner at Ingolstadt in 1614 February 12, 13 and 14 and often at other times, and our colleagues at Rome and Parma, at Ingolstadt, and at Naples with Francesco Fontana, as is clear from his observations, Tract 7, and we at Bologna with excellent spyglasses, have very often observed it as the five figures below display it, and that with the glass lenses exchanged, so that no one should dare to ascribe it to bubbles in the glass.

Diagram. A heavy rectangular frame across both columns at the foot of the page, holding four numbered figures of Saturn in two rows of two.

- I (top left): a large plain disc with a smaller round disc standing clear of it on each side.
- II (top right): a large plain disc with an oval body touching or nearly touching it on each side.

- III (bottom left): a large disc with a plain outlined circle overlapping it on each side; within each side circle, against the disc, a hatched crescent whose horns run toward the disc's top and bottom.
- IV (bottom right): the same arrangement, with the crescents narrower.
- Or (between III and IV): the East.
- Occ. (right of IV, inside the frame): the West.

[Margin: Figure 1.] The first figure is of Saturn separated from entirely round companions, as we saw him at Bologna in the year 1643, three days after his opposition with the Sun, that is, from the beginning of October for some days, with the tube of Galileo and also with that of Fontana. But earlier Fontana had seen him so in the year 1630 June 20.

[Margin: Figure 2.] The second figure gives Saturn separated indeed from his companions, but with these now having taken on the shape of a gourd or of a rather large pear. Father Niccolò Zucchi wrote to me from Rome that Saturn was seen in this form in the year 1640 May 23, near his □ aspect with ☉; and we ourselves saw it in the year 1643 from October 29.

[Margin: Figures 3 and 4.] The third and fourth figures embrace Saturn closely joined with companions that are now crescent-shaped and, because of their nearness, shadowed by him and half-hidden behind his back. Fontana observed him in this way in the years 1633, 1634 and 1636, and nearly so did Father Bartoli at Naples in the year 1644, in December, and we in the year 1646 October 10, when he was five signs, or 150°, from the Sun and near the Meridian; but at that time the more easterly companion was smaller than the other and nearer the body of Saturn. [Margin: Parallelism of Saturn's companions with the equator.] Both lay on a line parallel to the equator, which was evident because a line drawn through their centres was parallel to a line drawn from the bright star of Aries to the southern eye of Taurus. *"This too was observed rather often by us,"* adds Father Francesco Maria Grimaldi in his autograph. And in the year 1647 on February 3 and 4, at which time, with the same man observing, the line of the companions appeared parallel to a line drawn from the bright star of Aries to the Pleiades, and thus parallel to the equator rather than to the ecliptic. So too Galileo had once observed. Father Bartoli adds that they appeared curved into a rather parabolic figure, and the upper one smaller.

Diagram. Figure V of Saturn, set in the left column: an oval outline, longer from side to side, within which stand two large solid black crescents, horns inward toward top and bottom, one on each side of a plain centre.

- V (above the oval, at centre): the figure's number.

[Margin: Figure 5.] The fifth figure shows the companions now united behind Saturn and touching each other, or as it were intersecting, so that, with him nearly round in the middle, they seem shaped into a kind of ellipse. For we saw him so in the year 1648 October 15, around the trine aspect with ☉, and in the year 1649 toward the end of March, and then very often, both as western up to his heliacal setting and as eastern after his heliacal rising, namely on July 20.

A line drawn through the greater diameter of this ellipse was parallel to a line drawn through the Pleiades and through the centre of Saturn himself, placed at ♊ 32[58], with south latitude 1° 32′, as the same Father Grimaldi noted. [Margin: Parallelism to the equator confirmed.] Thus in the year 1647 October 15 a line extended through the extreme points of the companions passed through the southern horn of Taurus and through the most southerly of the Pleiades; and from these observations the parallelism of these companions to the equator was confirmed. Giuseppe Biancani, in the *Sphaera*, Book 15, Chapter 7, testifies that he saw him in this form in the year 1616, in October, and in 1619, in November.

From this some gather that these planets do not move around Saturn. For then at some time they would, by their size, cast their shadow onto his middle, since from Saturn's shadow cast upon them when they are nearer it is very nearly established both that they lack light of their own as much as Saturn does, and that they shine only from the Sun. Rather, they hold, behind and at the back of Saturn, by alternate approaches to him and recessions from him, they present such phases to us, especially if they are egg-shaped; and this too seemed more probable to Scheiner than that they are generated anew or perish. [Margin: Saturn's solitude without companions.] For at times they are hidden, and then the attendants truly lie wholly concealed behind Saturn's back, and then he himself appears round and solitary (the fables would repeat "devourer of his children"). Galileo first observed this, not without astonishment, in the year 1612 toward the end of autumn. From this, because the winter solstice was near, he timidly ventured to conjecture and predict that Saturn's companions would reappear around the summer solstice of the year 1613, and afterwards be hidden until the winter solstice of the year 1614; that once this had passed they would show themselves again for some months, and hide once more around the following winter solstice; that afterwards, on emerging, they would not withdraw in part before the solstice, and that the summer solstice, of the year 1615; and that when these turns were completed they would grow larger and appear continually for many years. But by no observation, that I know of, was that prediction confirmed.

Pierre Gassendi, however, in his *Iudicium* on the 9 stars seen around Jupiter, no. 6, affirms that he always saw Saturn with handles in the year 1633 in the month of June, and in 1634 in April, and in 1636 in November, and in 1637 in February, July and November, and in 1638 in December; but that in the year 1642, around the beginning of August, Saturn, soon to be acronychal, was found with Galileo's telescope to be without handles, and that from then on he began, on as many nights as he could, to watch whether and when they would be restored; and that this had still not happened in the whole of the time that elapsed up to Saturn's heliacal setting, which took place in the year 1643, in February, near the conjunction of ♄ with ☌. What then became of the handles, and of their observation, he does not record. He only advises shrewdly, in

[58] so printed; a degree above 30 within one sign cannot stand, and the intended figure is not recoverable from the page

Book 3 of the *Institutio Astronomica*, that we must wait until the whole diversity of appearances and periods has been observed, which is to be hoped for from the Providence of the Divine Power gradually instructing mortals.

For although Seneca, when speaking of comets, said in his *Natural Questions*: *"How many things besides these go their way in secret, never rising for human eyes! For neither has God made all things known to human eyes. How small a part of so great a work is entrusted to our eyes?"*, [Margin: Seneca's twofold prophecy, as it were.] the same author nevertheless declares as follows: *"How many things have we come to know for the first time in this age? And indeed the people of the coming era will know many things unknown to us. Many things are reserved for ages yet to come, when the memory of us will have faded. The universe is a paltry thing, unless the whole universe has in it something to seek."* And in Book 7 of the same *Questions*, speaking of the eclipse of the Moon: *"THE TIME WILL COME when the day, and the diligence of a longer age, will draw into the light those things that now lie hidden; THE TIME WILL COME when our descendants will wonder that we did not know things so plain."* To this oracle of his, if it is permitted to speak so, what we have said in this whole chapter lends marvellous support; just as the discovery of the New World confirmed another prophecy of the same Seneca, *Medea*, Act 2, Chorus:

> *... In late years there will come an age when Ocean will loosen the bonds of things, and the vast earth will lie open, and Tiphys will uncover new worlds, and Thule will not be the last of lands.*

Meanwhile two things are very hard to reconcile. For the shadow of Saturn to produce the phase we see, it seems that it must be calathoid [calathoides] and that the companions ought to lie very far from ♄, if indeed those black Moons, or crescent-shaped shadows, are segments of Saturn's shadow. But for that shadow to be calathoid, and its segments perceptible, Saturn must be larger than the Sun; and this is not allowed by its distance, ten times the Sun's distance from the Earth, unless you increase its apparent diameter beyond 3′. It is in fact smaller, as will be shown below in the last section, even granting that on a hypothesis and a certain probable observation I conceded it 3′ in Book 6, Chapter 7, No. 14.

CAPVT III.

Chapter 3.

De Satellitibus Iouis, seu Stellis Mediceis, eaq. occasione de Numero Planetarum.

On the Satellites of Jupiter, or the Medicean Stars, and on that occasion on the Number of the Planets.

I. Since the very treatment of the figure of the planets has led us to Saturn's companions, and we have explained them sufficiently at the end of the preceding chapter, it remains for us to deal separately with the satellites of Jupiter. Galileo

of Florence, their first discoverer, named them the MEDICEAN STARS, so as to consecrate in a family heaven the surname and house arms of Cosimo, truly Grand Duke of Tuscany. [Margin: Authors who treat of Jupiter's companions.] He discusses them both in the *Sidereus Nuncius*, from p. 17 to the end, reporting 65 observations, and in the letters to Markus Welser on the Sun's spots, at the end of which he sets an ephemeris of their motions; and in dialogue

3 *De systemate Mundi*; Simon Marius in his *Mundus Iovialis*; Christophorus Scheinerus [Christoph Scheiner] in the *Disquisitiones Mathematicae*, no. 39; Biancani in the *Sphaera*, Book 14, Chapters 1 and 7; Kepler in the *Dissertatio cum Nuncio Sidereo* from p. 10, in the notes to Chapter 16 of his *Mysterium Cosmographicum*, and in the *Epitome of Copernican Astronomy*, pp. 538 and 553; Pierre Hérigone in volume 5 of the *Cursus Mathematicus*, pp. 580, 663 and 857; Pierre Gassendi in his *Iudicium* on the nine stars seen around Jupiter; Antonín Maria Šírek of Rheita, Capuchin, in the *Oculus Enoch et Eliae*, or *Radius Sidereomysticus*, Book 4; Francesco Fontana in his observations, Tract 6, Chapter 2; and finally Govaert Wendelen and Father Giovanni Battista Zupi in their very recent letters to me; and in the same letters Dom Vincenzo Renieri, an Olivetan, Galileo's pupil and chief mathematician of the University of Pisa. Renieri, diligently devoting himself to observing these for ten whole years, had composed most complete ephemerides and tables and had sent me a specimen of them. [Margin: Vincenzo Renieri's work on the motion of Jupiter and of its companions, and its loss.] But alas, while he was preparing the publication of the book *De motu stellae Iovis et quatuor Comitum ac recentioribus caeli phaenomenis* [On the motion of the star of Jupiter and of its four companions, and on the more recent phenomena of the sky], which he had promised me, in his letter of the year 1647 September 11, would be begun after one month, I learned in the month of November that he had died. And because he was then living outside the monastery on account of his affairs, those who seized his effects either lost his writings or did not find them, lost or stolen as they were; so that they could not be recovered by any search of ours, nor indeed even of the Grand Duke of Tuscany. A truly lamentable loss, both of these writings and of three most distinguished men of Galileo's school, who departed from the living within two months in the same year: namely Fathers Vincenzo Renieri and Bonaventura Cavalieri, whose astronomical friendship I enjoyed and profited from, and Evangelista Torricelli. But let us return to our own subject and gather in a few words what we know has so far been published on the number, position, motions, periods, eclipses, etc., of these stars. And let us not trouble ourselves at all about Martinus Horckius [Martin Horký], barking with futile effort at Galileo's discovery and at the whole *Sidereus Nuncius*, as at a fiction or a trick of the eyes.

De Numero Iouialium Comitum.

On the Number of Jupiter's Companions.

II. Galileo, the first to do so, discovered near Jupiter first three stars, in the year 1610 January 7, at the first hour of the night, and thereafter never more than four, on the same line parallel to the ecliptic, now closer to each other and to Jupiter, now farther off, yet never departing from it beyond 14′ or 17′, and at times nowhere to be seen. From this he found that they were not fixed stars but certain little planets, companions of Jupiter. With him Kepler, Biancani, Hérigone, Gassendi, Renieri, Scheiner and Wendelen affirm this set of four. For although Scheiner, in his first letter to Welser, suspects that there are more than four, yet in the *Disquisitiones* he acknowledges four.

[Margin: Antonín Maria Šírek, asserter of five other companions of Jupiter.] But Šírek asserts that in the year 1642 December 29, and afterwards on other occasions, besides the four Galilean and inner attendants of Jupiter, another five, outer or more remote and much larger, were observed with a new telescope that takes in a larger space of the sky at a single look; for with other, narrower tubes, he says, this phenomenon cannot be seen. These five he named the Urbanoctavian stars, after Urban VIII, the Supreme Pontiff then reigning, and he wrote of this novelty to Erik van der Putte. A copy of that letter, sent from Belgium to Gabrielem Naudæum [Gabriel Naudé], roused Gassendi to call into dispute this addition of five circum-Jovial planets.

[Margin: Gassendi's judgement on the 5 new companions of Jupiter, and his first reason for doubting.] First, because on the same night of December 29 of that same year 1642 he observed Jupiter's four companions all more easterly than Jupiter, which Šírek's drawing nevertheless represented as western; either because Šírek used a glass that inverts objects, or because, as he was using both eyes at once, the sections of the glasses produced an exchange of vision similar to that by which, when we look with each eye alternately at our nose or at a finger held close to the eye, we see that it covers the left part of a more distant object for the right eye, and the right part for the left. And the drawing represented them not only as western, but also the outermost of the Medicean Stars as more southerly than it had appeared to him.

[Margin: Gassendi's second reason.] Second, because he noticed that those five Urbanoctavian stars are non-wandering, namely of those that in Tycho are in the Effusion of Aquarius, nos. 24, 25, 26, 27, 28 in the catalogue, or others nearer Jupiter from the constellation of Aquarius.

[Margin: Gassendi's third reason.] Third, because it seems improbable, and against the analogy of motions, that those five stars should move around Jupiter in an entirely contrary direction, as Šírek says: namely, above, from east to west, whereas the four Medicean planets, Saturn, Jupiter, Mars, Venus and Mercury around the Sun, and the solar spots themselves, by what is now common opinion, are carried in the upper part of their pericycle from west to east.

[Margin: Gassendi's fourth reason.] Fourth, because it is easy, unless you are very circumspect, to take fixed stars near Jupiter, whether visible to the naked

eye or only with the telescope, for new satellites on account of their nearness. He confirms this by the example of Propus in Gemini and of certain small stars on the brow of Scorpius, which he saw occulted by Jupiter. Yet at the end of this *Iudicium* he withholds assent, until Šírek either seeks fuller credit for his observations, or, once the error about these new wanderers has been exposed, sings a palinode.

III. But indeed Šírek, in Book 4 of the *Radius Sidereomysticus*, Chapter 1, Member 2, replies to Gassendi, as before him Io. Caramuel [Juan Caramuel], Suffragan of the see of Mainz, had replied: that those Urbanoctavian stars changed their distance from one another, and therefore could not have been fixed stars; and that it is no wonder they later vanished, since just as they had been newly generated, so they could perish. For he says he observed around Jupiter a vaporous atmosphere, an indication of changes such as occur in our air when meteoric chasms are kindled and afterwards extinguished. He also cites Io. Stampion [Johan Stampioen], mathematician to the Prince of Orange, who with Prince Willem of Nassau observed around Jupiter for two months two new planets, which he thought were breathed out and generated by Jupiter from itself, just as spots are from the Sun; since Jupiter, viewed through larger tubes, appears like the Sun.

[Margin: Francesco Fontana's opinion on this number, and his observations.] I find, moreover, in Francesco Fontana's observations, Tract 6, Chapter 2, that he saw Jupiter in the year 1630 with 9 stars never distant from Jupiter more than 10 Jovian diameters; and in the years 1636 and 1643 with 8; and in the year 1645, December 21 and 31, at $2\frac{1}{2}^{h}$, with 7; and in the year 1646, January 1, at 2^{h} of the night, with 7; and in the year 1646, January 22, at 1^{h}, and January 28, with 6 satellites, which did not keep the same distance from one another and are not seen around Mars or Saturn; and that to see these stars it was necessary to extend the tube, whereas to see the fixed stars perfectly one must contract the tube.

[Margin: Father Giovanni Battista Zupi's observation on these.] But it is worth reporting what Father Giovanni Battista Zupi, S.J., in two letters to me dated at Naples in the year 1644 January 23 and February 4, wrote that he had observed with an excellent tube made by Fontana, adding a drawing of the stars observed around Jupiter. His words, put into Latin, are these: *"As regards the five satellites that, as I learned a few months ago, are written from France to have been seen around Jupiter besides the usual four, we saw one or another besides the 4; and because there was as yet no suspicion of novelty, we thought them to be fixed stars, either known ones or of those scattered over the whole sky and invisible without the telescope, such as those in the Milky Way; especially since from the telescope they acquired neither enlargement nor steadiness, which is a sign that they were very high."* And a little later: *"On January 22, returning to the observation of Jupiter, I saw six stars near Jupiter, of which the two new ones, or fifth and sixth, had not changed position relative to each other but had, however, approached Jupiter; I think they are in the heaven of Jupiter."* This is in the letter written the following day, that is,

January 23. But in the letter of February 4 he writes as follows: *"I owe Your Reverence much for the occasion first given to me of observing, in these regions, many more than 4 satellites around Jupiter, just as I was the first to note bands on it. I say, therefore, to Your Reverence that it is established for me that not only 6 but many more satellites wander around Jupiter, since I have counted not only 9 but 12, whose figure I send to Your Reverence, though drawn with a rather coarse hand, because, since they could not be seen by me at a single look, it was necessary to beg their intervals and proportion from estimation itself. It is true indeed that among the latest observed I do not dare to list all among the wanderers; yet four of those new ones, because they did not twinkle and acquired some change from the telescope, seem to me to be wandering."* When I had told Dom Vincenzo Renieri about these, he flatly denied to me that in the whole ten years of his most frequent observations he had ever seen more Jovian little stars than 4.

Therefore, since the truth, which should outweigh friendship, is not yet clear to me, I do not wish to settle a dispute between friends or to pronounce any judgement on this matter. Rather, with the Roman praetors, I would bring myself back to that ancient formula: *Amplius* ("further hearing"), because *Non liquet* ("it is not clear").

De Situ Satellitum Iouis.

On the Position of the Satellites of Jupiter.

[Margin: Whether their path is parallel to the ecliptic?]

IV. First, the path of these satellites, by which they move from west to east, does seem to be parallel to the ecliptic itself, as it seemed earlier to Galileo and to Biancani, and to others, who drew a line through them and through Jupiter and two, or at least one, of the fixed stars. Yet at times, as the latitude of Jupiter or of the satellites themselves requires, they depart somewhat from it on an oblique track. Scheiner admits this slight inclination to the ecliptic in the *Disquisitiones Mathematicae*, p. 87, and so does Gassendi, above, p. 15; and the drawings of the observations themselves prove it.

[Margin: Whether in different circles?]

Second, it is certain among all the authors already mentioned that they do not move in one circle, or on the circumference of one circle, but in different ones, which this diagram of Scheiner and Biancani represents.

Diagram. Set in the left column, with the text running in a narrow measure down its left side. Four concentric circles stand around a small central disc, Jupiter, with a horizontal diameter through the centre and a vertical line from the top of the outer circle down to the Earth's centre below; small six-pointed stars mark the satellite positions where lines cross the circles. A hatched band, the shadow of Jupiter, runs obliquely from the central disc up and to the left, beyond the outer circle, to its cut-off end at R. Below, an arc passes through M to a small circle, the Sun, at lower right; lines run from B to the top of the figure and from the Sun's circle up to the Jupiter system.

- Top of the shadow band: R
- Along the left edge of the shadow band, top to bottom: a, b, c, d
- Above the outer circle, left to right: C, D, E
- Upper interior, right of the band: e; f below it
- Upper right, on the three inner circles' upper parts: g, h, m
- Horizontal diameter, left to right: F, H, P, Q, A (at the central disc), L, K, I, G
- Lower left interior, top to bottom: n, o, p
- Lower right interior, top to bottom: b, z, q
- Foot of the outer circle: r at left; u at right
- On the arc below the circles: M
- Foot of the figure: B; N; S (in the small circle); T (right of the small circle)

In it A is Jupiter and B as the centre of the Earth, around which lies the heaven of the Sun, MS, with the Sun at S, illuminating Jupiter and touching it with the rays NR and TR. AR is the shadow of Jupiter, which, though cut short here, in fact ends in a conical apex. Around Jupiter's centre A four circles are described. The largest is DFG, in which the outermost of the satellites, G, revolves in the upper part of its pericycle in the order of the signs, namely from G to D. Next is the circle gHI, in which the next-to-outermost satellite, I, turns from I to g; then the circle hPK, in which the next-to-innermost satellite, K, turns from K to h; and lastly the circle LmQ, in which the satellite innermost to Jupiter, or nearest it, L, turns from L to m. This difference is inferred both from their different periods, since the innermost completes its period far faster than the outermost, and because at their greater digressions from Jupiter they are never seen joined but well separated, as at F, H, P, Q, or at G, I, K, L; whereas near Jupiter they appear crowded together, as at E, g, h, m.

[Margin: Whether in the same plane?]

Third, they are indeed thought all to lie in one plane, because when Jupiter has no latitude, or an imperceptible one, they appear without any declination from the ecliptic. Otherwise, however, the outermost undergoes a greater deviation from the plane of the ecliptic than the next-to-outermost, this one a greater than the next-to-innermost, and this one a greater than the innermost. It is probable that they have this deviation from Jupiter's latitude: "for just as they accompany it in its motion in longitude, so it is likely that they accompany it in latitude," says Scheiner. But because we are not in the plane of these small circles, the digressions do not occur in their lower part but on the diameter itself, FG or HI, etc.

[Margin: In which position they decline to the south and in which to the north.]

Fourth, Galileo and Scheiner agree that when they are above, as at E, g, h, m, they appear to us southerly with respect to Jupiter and in the antarctic quarter, because there they approach or tend towards their own eclipse; but when they are below, as at b, z, q, u, they decline to the north and, at an equal

digression from Jupiter, appear larger. Here, however, one must be very much on guard against a delusion of the imagination. For when they are really in the upper part of the pericycle (which is inferred from their motion towards the east), they then appear depressed below Jupiter towards the south, and are so represented in the drawings of the observations, as being below the line FG. Conversely, when they are really below Jupiter and nearer the Earth than it is, they nevertheless appear to us higher and carried up to the north, as the rules of optics require. We, however, in the preceding figure represent them according to the position they actually have, so that the outermost satellite at C is above Jupiter, and at r below it. Hence it comes about that when several or all of the Jovian stars converge with Jupiter on one straight line in longitude, and towards one quarter of longitude, as when they are on the line AG, to the west, or on AF, to the east, then, with Jupiter placed on the meridian or in the nonagesimal, they appear to us hardly perceptibly above it or below it; and at that time they are close to the moments of their stations. But when they appear on one straight line in latitude, which cuts the line of longitude at right angles, as on the line BD, or nearly BE, then if all appear in the northern quarter, the outermost appears most northerly and the innermost northerly by the least possible amount; if they are seen depressed to the south, the outermost appears most southerly and the innermost southerly by the least possible amount. And in either case, that is, when they are on one line in latitude, they are seen to be less distant from one another and from Jupiter, and can more easily be taken in together with Jupiter at a single glance of the astronomical eye, that is, the telescope, and their apparent diameters compared with Jupiter's diameter.

De Iouialium Satellitum Motibus, Digressionibus à Ioue, Coniunctionibus, & Periodis.

On the Motions of the Jovian Satellites, Their Digressions from Jupiter, Conjunctions and Periods.

[Margin: Motion in circles or near-circles; where it takes place in the order of the signs.]

V. First, that they move around Jupiter describing small circles or ellipses, in whose upper part they are carried from west to east and in the lower part from east towards west, is inferred from this: from their greatest western digression from Jupiter, such as, for example, that of the outermost at G, at which they seem stationary, they next approach Jupiter towards the east and then appear to us lower towards the south, until they join it and are hidden. From there, emerging again towards the eastern quarter and heading for their greatest eastern digression, say at F, they appear less and less depressed towards the south; and having completed there a stay, or second station, they go back towards Jupiter to the west, and then, as far as appearance goes, they seem to us more and more carried up to the north, and they join Jupiter again. Hence, once the rules of optics and astronomy are considered, it follows that they

move in the way already described. For if they only approached and receded from Jupiter along one line of longitude, that motion could be accounted for by a mere libration on the diameter FG. But since in approach and recession they apparently change position towards the arctic and antarctic quarters, they must describe either circles or ellipses or figures very like these. And since, when at an equal digression from Jupiter they appear larger than themselves and carried up to the north, they then approach Jupiter towards the west, and when smaller and depressed to the south they approach it heading east, they must be carried in the upper part of their pericycle in the order of the signs and in the lower part against the order of the signs. We are speaking, however, of the four ordinary and certain companions of Jupiter; for the five remaining Urbanoctavian ones, if we believe Šírek, appear to move with the contrary motion, namely in the upper part against the order of the signs and in the lower part in the order of the signs, in the way the Moon is imagined to move on its epicycle. And when they appear carried further up to the north, or, as far as appearance goes, higher than Jupiter, they appear smaller, but when depressed to the south, larger: *"which would indeed be astonishing,"* says Šírek in his letter to Erik van der Putte; by these words he indicates that he doubts this alternation. On account of their anomalous motion, the same author holds (Book 4, ch. 4) that all the satellites move in circles eccentric to Jupiter.

Second, the greatest digressions of the satellites, in various ways, [Margin: The greatest digression from Jupiter.]

are determined. For most authors use the apparent diameters of Jupiter as the measure here. But Šírek (Book 4, ch. 4), after first giving the distance of Jupiter from the Earth as, on the Copernican reckoning, 10798 semidiameters of the Earth and, on the Tychonic, 12798, also measures the greatest digressions in semidiameters of the Earth, and adds that the apparent diameter of Jupiter, from his own observation, is 3′. So look at the table set out below, in which Kepler and Hérigone follow Simon Marius, and Wendelen follows Galileo. If you want these intervals in minutes and seconds, work them out from what is to be said below in Section 6 on the apparent diameter of Jupiter, which we shall show there does not reach 2′.

Greatest Digression of the Satellites of Jupiter from Jupiter [Digressio Maxima Satellitum ♃ à Ioue.]

Satellites	Diameters of Jupiter: Galileo	Diameters of Jupiter: Marius	Diameters of Jupiter: Šírek	Semidiameters of the Earth
1	3	3	3	21
2	5	5	4	27
3	8	8	6	41
4	14	13	10	69

[Margin: The period of the satellites of Jupiter.]

Third, the period of the motions is not so certainly obtained from the stations, or greatest digressions, and the satellite's return to them, as Simon Marius did it, because the satellites linger in these for several hours. It is better, therefore, as Scheiner advises in the *Disquisitiones Mathematicae*, p. 82, to observe the time at which the satellite is joined to Jupiter, or begins to be hidden, and again the time at which it emerges, and to take half as the time of central conjunction; and so to observe the times that elapse between one conjunction and another of the same kind, that is, both upper or both lower. For these, let there be a second table, in which Biancani, Scheiner, Kepler, Hérigone and Šírek either follow or report the opinion of Simon Marius, although some, such as Kepler and Šírek, use rounder numbers. In these tables we use "first" for the innermost, the one nearest to Jupiter.

Periods of Revolution of the Satellites of Jupiter around Jupiter [Periodi Reuolutionum Satellitum ♃ circa Iouem.]

Satellites	Period	Or
1	$1^d\ 18^h\ 28^m\ 30^s$	$1^d\ 18^h\ 30^m$
2	$3^d\ 13^h\ 18^m\ 0^s$	$3^d\ 13^h\ 20^m$
3	$7^d\ 3^h\ 56^m\ 34^s$	$7^d\ 4^h\ 0^m$
4	$16^d\ 19^h\ 9^m\ 15^s$	$16^d\ 19^h\ 9^m$
Next-to-outermost of the new ones, according to Šírek		$67^d\ 0^h\ 0^m$
Third-from-outermost, according to the same		$33^d\ 12^h\ 0^m$

[Margin: The diurnal and hourly motion of the satellites.]

On these suppositions Hérigone, dividing 360° by the periodic motions, obtains their diurnal motion in their own circles or pericycles; and, dividing the diurnal motion by 24, he obtains the hourly motion, as in the following table.

Motion of the Satellites of Jupiter in Their Own Pericycles [Motus Satellitum ♃ in proprijs Pericyclis.]

Satellites	Diurnal	Hourly
1	203° 25′ 0″	8° 28′ 0″
2	101° 17′ 22″	4° 11′ 0″
3	50° 14′ 57″	2° 5′ 37″
4	21° 29′ 4″	0° 53′ 43″

[Margin: The irregularity of this motion.]

Fourth, the motion of these satellites is judged irregular, whether because it takes place on a circle eccentric to Jupiter, as Šírek thinks; or because they are ordered not primarily to Jupiter but to the Sun, by which they are illuminated, and the motion is not equal with respect to lines drawn from the centre of the Earth through Jupiter, but through the Sun, as Kepler thinks with Simon Marius; or because, as Hérigone holds, this apparent inequality arises from the

parallax, or angle at Jupiter, which changes somewhat each day, owing to the excess by which the Sun's motion exceeds Jupiter's. From this it would follow that to know the moment of conjunction of the Jovian stars with Jupiter, one must first know the mean motion of the Sun and of Jupiter.

De Illuminatione, Diminutione luminis, & Eclipſibus Satellitum Iouis

On the Illumination, Diminution of Light, and Eclipses of the Satellites of Jupiter.

[Margin: Occultations and eclipses of the satellites.]

First, it is now the common opinion, introduced by Galileo in his last letter to M. Velser [Markus Welser] on the sunspots, p. 161, and confirmed by Scheiner in the *Disquisitiones Mathematicae*, p. 83, and by others and by frequent experiments, that the satellites of Jupiter, when they come very close to conjunction with it, whether lower, which occurs when they are carried westward in the lower part of their pericycle, or upper, when they approach it towards the east, are not seen because of Jupiter's brilliance, and set, so to speak, *Zeusiacally*. Indeed, outside conjunctions they sometimes suffer eclipse and lie hidden in it for a longer or shorter stay, according to the different inclinations of the cone of Jupiter's shadow and its different latitude, and the different nearness of the satellites to Jupiter: for those nearer fall into a denser shadow, as does satellite m at d, but with its faster motion it gets clear of the shadow.

[Margin: The evidence for these eclipses.]

The evidence for these eclipses is that, for example, after the opposition of Jupiter with the Sun, at an equal distance from Jupiter along the line of longitude, when they are carried towards Jupiter lying further to the west they appear, whereas when receding from it towards the east they do not appear until they have gained a greater distance, evidently because they were lying hidden in Jupiter's shadow. For example, satellite a does not emerge or first shine out except at a great distance from Jupiter, whereas on its way back to the west it usually appears at a much smaller distance from Jupiter; and likewise with the others. Moreover, those further from Jupiter are hidden when, depressed to the south, they approach Jupiter, yet they appear when they approach it carried up to the north; and this has been observed only before the opposition of Jupiter with the Sun. Finally, when the hours and moments of the occultations that occur in these satellites heading east, and the position of the Sun relative to Jupiter, are considered, it is established that these occultations occur when the satellites are in that part of the sky towards which the shadow of Jupiter, illuminated by the Sun, is directed.

[Margin: Corollaries on the light of Jupiter and of the satellites.]

From this it is rightly inferred that Jupiter lacks light of its own, and therefore cannot impart light to its satellites hidden behind its back; and that they likewise shine from no other source than the Sun. Granted that, against the common

opinion, Šírek says that the first and third scintillate so much that this is sufficient proof that they shine with their own light: but if that were so, they would shine even in Jupiter's shadow, at least the third and more remote one, as at e and b, where nevertheless it is hidden.

[Margin: Why they are larger than themselves at an equal distance from Jupiter.]

Second, since the Medicean Stars, although they complete very narrow rotations around Jupiter, nevertheless sometimes appear twice as large as themselves at nearly equal distance from Jupiter, Galileo, at the end of the *Sidereus Nuncius*, holds that there is a vaporous atmosphere and a denser aether, because of which the satellites appear smaller near apogee, owing to the greater thickness of that denser body interposed between them and our eyes, but larger towards perigee, owing to its lesser thickness. Simon Marius, however, holds that this comes from the light of Jupiter, which is increased in them when they are near perigee. Scheiner, however, both regards that vaporous sphere as an arbitrary fiction and holds that the increase of the light communicated from Jupiter is imperceptible to us across so great an interval. He adds, however, that the light of twilight helps the pupil, and that therefore both the satellites and Venus are then better seen; just as they are better discerned when they are indeed far from Jupiter but are nevertheless seen together with Jupiter at one glance, because Jupiter illuminates the eye and the air; for a moderately illuminated medium assists vision.

Problema. Iouis Diſtantiam à Terra ope Satellitum obſeruare.

Problem. To observe Jupiter's distance from the Earth by means of the satellites.

[Margin: First method of the problem.]

VII. Let the figure of no. 4 be repeated here. In it Biancani (Book 14, ch. 1) wants observed the time between the first occultation under Jupiter's rays above (which he thinks occurs on the line BD, as if the outermost of the Jovian stars were at D) and the first emersion from Jupiter's shadow, say at a, so that the arc Da may be known. For if we make: as the time of the whole period, which we suppose known, to the 360° of the whole circle DFuG, so the time already mentioned to another quantity, the arc Da will be known, which subtends at Jupiter's centre A an angle similar to itself. When this angle is known, together with the angle ABS, known from the distance of the Sun and Jupiter in the zodiac, and with the distance SB of the Sun S from the Earth B also known, you will have the triangle ABS, with three data sufficient for finding the side AB, that is, the distance of Jupiter from the Earth.

Diagram. The figure of p. 490 is printed again here, set in the left column with the text running in a narrow measure down its left side, as the text says it is to be repeated. Four concentric circles stand around a small central disc, Jupiter, with a horizontal diameter through the centre; small six-pointed stars mark the satellite positions. A hatched band, Jupiter's shadow, runs obliquely

up and to the left beyond the outer circle to its cut-off end at R. Below, an arc passes through M to a small circle, the Sun, at lower right; lines run from B up to the circles and from the Sun's circle up to the Jupiter system.

- Top of the shadow band: R
- Along the left edge of the shadow band, top to bottom: a, b, c, d
- Above the outer circle, left to right: C, D, E
- Upper interior, right of the band: e; f below it
- Upper right, on the upper parts of the circles: g, h, m
- Horizontal diameter, left to right: F, H, P, Q, A (at the central disc), L, K, I, G
- Lower left interior, top to bottom: n, o, p
- Lower right interior, top to bottom: b, z, q
- Foot of the outer circle: r at left; u at right
- On the arc below the circles: M
- Foot of the figure: B; N; S (in the small circle); T (right of the small circle)

Yet the first occultation does not occur on the line BD but on the line BE, tangent to Jupiter, or even a little short of it; nor does the first emersion occur on the axis of the shadow (which my engraver omitted here) but on the Sun's ray Na, likewise tangent to Jupiter; nor does the arc Ea exactly measure that angle made at Jupiter's centre.

[Margin: Second method of the problem.]

Second, Scheiner, in the *Disquisitiones Mathematicae*, p. 82, directs that one observe the first lower occultation of the satellite, say at p, and the first emersion from Jupiter's rays, say at q, so that from the proportion of the whole periodic time to the whole circle HpI, the arc pq may be known. For since the rays Be and Bg, touching Jupiter, are sensibly parallel because of the enormous distance of the eye at B, just as the Sun's rays TR and NR are too, or else have sensibly equal convergences, it follows that, once the arc pq is known, the arc eg is sensibly equal to it, as is also the shadowed arc eb. On these assumptions, from the time between the first occultation at p, or emersion at q, and the first emersion from the shadow at b, you will be able to find the arc pqb, or qeb, and from this be helped towards knowing the angle at Jupiter's centre made by the axis of the shadow and the straight line BD. For if from the whole known pqb you subtract pg, which is supposed here to be of 180°, and half of both the arc ge and the shadowed arc eb, the remainder will be the arc intercepted between the axis of the shadow AR and the line AD, which measures the angle made by the lines already mentioned at the centre A. Equal to this will be the angle vertical to it, made at the centre A by the axis SA (to be supplied here in imagination) and the straight line AB. With this angle, therefore, and with the distance of the Sun BS, and with the angle ABS, the distance of Jupiter from the Sun in the zodiac, AB, the distance of Jupiter from the Earth, will become known in the rectilinear triangle ABS. In this truly excellent and ingenious

inquiry you see that geometrical rigour is not kept throughout, as being by no means necessary for what is intended.

SCHOLIA.

Scholia.

[Margin: The number of the planets according to Kepler.]

I. On one side the telescope, by detecting new wanderers around Saturn and Jupiter, has called the number of the planets into doubt; on the other, Kepler has done so in the *Mysterium Cosmographicum*, contending that there are six primary planets and seeking the cause of this number from the five regular, or Platonic, solids. He assumes, however, that the Sun is not a planet but is immobile at the centre of the universe as regards motion of translation, and that the Earth is a planet, carried along the annual orb with the Moon, its secondary planet. On these suppositions he says in the preface to the reader: "The Earth is the circle that measures all the others. Around it circumscribe a dodecahedron; the circle enclosing this will be Mars" (that is, the orb of Mars). "Around Mars circumscribe a tetrahedron; the circle enclosing this will be Jupiter. Around Jupiter circumscribe a cube; the circle enclosing this will be Saturn. Now inscribe an icosahedron in the Earth; the circle inscribed in it will be Venus. Inscribe an octahedron in Venus; the circle inscribed in it will be Mercury." But because the intervals of some planets favour this opinion more closely and others do not, he tries to correct the latter so that they may be drawn to the desired measure, one fitting this mystery. And when he had heard of the four satellites of Jupiter detected by Galileo, he makes use of these very satellites both in the *Dissertatio cum Nuncio Sidereo*, on the last page, and in the notes to ch. 16 of the *Mysterium Cosmographicum*; and he increases the semidiameter of the Jovian orb by as much as the greatest digression of the outermost satellite from Jupiter comes to, which is very nearly 14′. In the same way he includes the Moon within the Earth's annual orb, and adds to the semidiameter of the great orb the distance of the Moon at apogee. If satellites should be discovered around Mars, he says, a more just interval arises from this for the commensuration of the planetary system, fitted to the number of the five regular solids. But more on these in Book 9, where the system of the world is treated. Meanwhile you see, reader, that Kepler has struck from the roll of the primary planets the two great luminaries: the Sun, which he will not have to be a planet, and the Moon, which he will have to be a secondary planet and the Earth's companion, or circumterrestrial planet.

[Margin: A skirmish against Kepler.]

II. But from this there at once arises the question of the notion or class of planets: which are to be counted primary, and which not. Kepler, however, will answer immediately that only those are primary that are not companions of another. If you say that Venus and Mercury are companions of the Sun and yet primary, he will deny that they are companions of the Sun as of a planet,

because he will not have the Sun to be a planet, but substitutes the Earth in its place, so as to make the Moon its companion. And that is why he embraced the hypothesis of Copernicus, because otherwise, with this removed, the whole foundation of the *Mysterium Cosmographicum* is undermined. For if the Earth stands still, the Moon turns out to be a primary planet, but its interval from the Earth fits none of the regular solids inscribed in or circumscribed about the lunar orb. Yet in this way he increases the improbability of the Copernican system. For Copernicus does not make the Sun a planet, but this man makes neither the Sun nor the Moon a planet, and makes the Moon only a periplanet, or secondary. Then, what has the number of the regular solids to do with the number of the planets, when the determination of that number arises not from the diameter of an inscribed or circumscribed circle, but from the solid bodies themselves, in so far as they are such bodies, and from the equality of the surfaces bounding them? Certainly these things have nothing to do with the bare intervals of orbs, or rather of circles, or, as Kepler prefers, of ellipses, in which they move irregularly. But of this elsewhere. Meanwhile, however, it is established that on the hypothesis of a stationary Earth there are seven primary planets, namely those which are not companions of another; or, if they are, as Venus and Mercury are of the Sun, they are nevertheless visible to the naked eye without the aid of the telescope, they move away from it by a notable interval, and they have very distinct effects on sublunary things.

[Margin: Whether the satellites of Jupiter are a hindrance to the ancient astrology.]

The companions of Saturn and Jupiter do not have such effects, because of their excessive nearness to Saturn or Jupiter: they do not stand more than 14′ from the latter, and so the diameter of the circle they complete does not equal in the sky so great an apparent space as the Sun does, which subtends more than 28′. Therefore, as Kepler rightly thought on this point in the *Dissertatio cum Nuncio Sidereo*, p. 11, although they concur with Jupiter by aspect in stirring the sublunary faculties, the difference of their effect from the effects of Jupiter is imperceptible, nor does any danger not to be despised threaten the ancient astrology from that quarter.

[Margin: Wendelen's ingenious speculation on the motions and intervals of the satellites of Jupiter.]

III. No less ingenious than Kepler's is the observation of that most sagacious astronomer Wendelen on the proportion of the periods and intervals of the Jovian satellites, which he communicated to me with the greatest generosity in a very long and very learned letter. For just as in the major planets the mean distances of the planets from the Sun are to one another in the sesquialteral ratio of their periodic times, so the distances of these little Jovian planets from Jupiter, which are 3′, 5′, 8′, 14′, are to one another in the sesquialteral ratio of the periods, which are 1 769/1000 days for the innermost, 3 554/1000 days for the next-to-innermost, 7 164/1000 days for the next-to-outermost, and 16 756/1000 days for the outermost. But of these in the harmonic system. Meanwhile he

adds cleverly that, given this proportion, if there were a fifth little Jovian planet whose period was 29 531/1000 days, or $12^{h}\ 44^{m}\ 3^{s}\ 12'''$, its interval from Jupiter at acronychal position would be 20′ for us who live on the Earth. He further adds that if the Sun were to look upon the Earth in central conjunction with Jupiter, it would come about that the Sun looks upon the four or five orbs of Jupiter's satellites under angles proportional to the periodic times: that is, the orb of the innermost under an angle of 2′ 20″, of the next-to-innermost under 3′ 40″, of the next-to-outermost under 5′ 50″, of the outermost under 10′ 10″, of the fifth under 15′ 0″; and that it impresses on them a motion and time proportioned to those angles.

[Margin: The rotation of Jupiter about the centre of its own body.]

IV. Vuackerius [Johann Matthäus Wacker von Wackenfels] maintained, according to Kepler in the *Dissertatio cum Nuncio Sidereo*, p. 12, that Jupiter has a rotation about the centre of its own body, by which it carries its four Companions around eastward. Kepler himself holds the same, both there and in the *Epitome of Copernican Astronomy*, p. 549, where he assigns this motion to all the planets: to the Earth, as it carries the Moon around; to Venus, which seems to unfold different spots one after another in its daily scintillation; and even to the Sun, which by a rotation of about 27 days leads all the planets around. But Jupiter's eastward rotation seems to conflict with its rotation toward the poles, by which it changes the appearance of its bands, according to what was said in chapter 2, near the end of no. 6, unless you would rather call that a libration than a rotation.

[Margin: The distance between meridians can be corrected by means of the eclipses of the Jovian satellites.]

V. Not content with discovering the four satellites around Jupiter, Galileo's sagacity scented out from this, above all, the following: if the moment of one and the same eclipse of one of the Jovian satellites (for instance, the moment at which the outermost begins to emerge from Jupiter's shadow) is observed with an excellent telescope by two observers in different places, the difference of longitude between those two places on the Earth can be derived from it. When this reached our ears at Parma, Biancani noted it in the margin of his manuscript *Sphere*. Galileo himself proposed it to the Catholic King in the year 1631, as Michael Florent van Langren testifies in his *Selenographia*, and also to the States of Holland through D. Deodatus of Lucca [Élie Diodati], as Father Bonaventura Cavalieri, Galileo's pupil, assured me. Yet Pierre Hérigone, in volume 5 of the *Cursus Mathematicus*, p. 872, affirms that it had occurred to him two years before Galileo made this discovery public. That is neither incredible nor surprising, for it has not rarely happened to me too that I thought out certain things which I later learned had been thought of by others at almost the same time. He adds that nothing in Morinus's [Jean-Baptiste Morin's] objection stands in the way of this discovery, namely that Jupiter is absent for almost three months on account of the Sun's rays: for if all the intervals in each month during which the Moon does not appear on account of the

Sun's presence are added together, they will amount in a whole year to a little less than three months. But he would have answered better by saying that lunar eclipses, which cosmographers have so far used to measure the distances between meridians, occur far more rarely than eclipses of Jupiter's satellites. For an eclipse of the Moon happens once or twice, and very rarely three times, in the same year; whereas the eclipses of Jupiter's four satellites that can happen in one year, taken together, are almost as many as the sums of their periods, namely about 480: for the innermost has 180, the next-to-innermost 130, the next-to-outermost 50, the outermost 20.

[Margin: The number of eclipses of the Jovian stars.]

CAPVT IV.

Chapter 4.

De Ordine ac Syſtemate Planetarum.

On the Order and System of the Planets.

I. What we said in Book 3, chapter 6, and Book 4, chapter 10, together with the diagrams, should be reread here. The rest, which concerns the universal constitution of the World, we reserve for Book 9. As for the ancient arguments by which the order of the planets is usually confirmed (namely the length of the shadow, parallax, the speed of the periods, and the eclipse of one planet hidden by a lower planet), we have set them out sufficiently in Book 4, chapter 10, which you should likewise go over again. Now in Book 3, chapter 6, we said that in the more recent system (which nevertheless stems as far back as Plato), both according to Copernicus and Tycho and according to all recent astronomers, the Sun is encircled by Venus and Mercury, so that they are now above it, now below; and we pointed out the ancient authors on this matter. We now wish to report their words, and those of others.

[Margin: Plato's opinion on the place of the planets.]

Plutarch, then, in *De placitis philosophorum*, Book 2, chapter 15, which is entitled "On the order of the stars", has this: "Xenocrates holds that the stars move on one surface. The rest of the Stoics hold that some are lower and higher than others. Democritus arranges them in this order: first the fixed stars, then the wanderers, among which the Sun, Lucifer and the Moon. Plato, after leaving the non-wandering stars, made the nearest the star of Saturn, which he calls Phaenon, and which we could, I think, call 'the brilliant one'; then Phaëton, unless you prefer 'the flashing one', which is Jupiter; next to this Pyroeis, that is, 'the fiery one', the star of Mars; then Lucifer, which is assigned to Venus; fifth the star of Mercury, which he calls Stilbon, as if to say 'the gleaming one'; and certain of the mathematicians subscribe to Plato. Others put the Sun back in the middle. Anaximander, Metrodorus of Chios and Crates assign the highest place to the Sun and set the Moon next below it, and below these they place the rest of the stars, fixed and wandering." So crude

and stammering, or so hot-headed and lawless, was the philosophy of certain of the ancients in astronomical matters. Yet when what Plato teaches in the *Timaeus*, in the *Epinomis* and in Book 10 of the *Republic* is compared, it appears that for him Venus and Mercury move now above, now below the Sun.

[Margin: Aristotle's opinion.]

Some have been in doubt about Aristotle's opinion on this. For in *De caelo*, Book 2, chapter 10, or text 60, after saying it is reasonable that the bodies nearer to the primum mobile, which is carried by a single motion, should be moved by fewer motions, he adds: "But as it is, the opposite occurs: for the Sun and the Moon are moved by fewer motions than some of the wandering stars, although these are farther from the centre and nearer to the first body than the Sun and the Moon." And to confirm that the Moon is lower than those stars he says: "For we have seen the Moon, at dichotomy, pass under Mars, which was hidden by its dark part and emerged from its bright part. And in like manner, concerning the other stars too, the Egyptians and Babylonians tell, who long ago observed them over very many years, and in whom we place much trust concerning each of the stars."

[Margin: Mars occulted by the Moon, with Aristotle observing; Kepler says, in his *Commentaries on Mars*, last chapter, that this happened on the evening of 357 BC April 4.]

So although he does not expressly say that Venus and Mercury are farther from the centre of the Universe than the Sun, he nevertheless says it implicitly, since he subscribes to the Egyptians, who, as is agreed, placed the Sun below these planets. This is confirmed from *Meteorologica* 1, chapter 4, where he says: "Moreover, if, as is shown in the astrological theorems, the size of the Sun is greater than that of the Earth, and the distance of the stars from the Earth is much greater than that of the Sun, just as that of the Sun from the Earth is greater than that of the Moon, then the cone which the Earth casts from the Sun's rays does not rise very far from the Earth, nor will the shadow of the Earth, which is called night, reach the stars; rather, the Sun must look around all the stars, and the Earth must obstruct none of them." Having considered these things, Clavius nevertheless says in the last edition of the *Sphere*, p. 42: "Aristotle alone, in the little book *De Mundo* to Alexander (if indeed it is his), places Venus immediately above the Sun and below Mercury."

[Margin: Apuleius's opinion.]

Scheiner, in the *Disquisitiones Mathematicae*, no. 38, asserts that Apuleius in his book *De Mundo* is in accord with this. Yet it seems to not a few interpreters that Aristotle, in *Metaphysics* 12, chapter 7, followed the system of Eudoxus and Callippus and with them placed Venus and Mercury below the Sun. For that reason we cited him for the Ptolemaic system in Book 3, chapter 6, no. 4, especially since he holds that these planets move not around the Sun but around the Earth; granted that he can be cited for the Copernican and Tychonic system, in that he seems elsewhere to have placed Mercury and Venus above the Sun.

[Margin: Vitruvius's opinion on the place of ♀ and ☿.]

Most explicitly, however, Vitruvius says in Book 9, chapter 4: "The stars of Mercury and Venus, about the Sun's rays, crowning the Sun itself with their paths as with a centre, make their regressions backward and their retardations; and through that circling they also linger at stations in the spaces of the signs. That this is so is known above all from the star of Venus: when it follows the Sun, appearing in the sky after the Sun has set and shining very brightly, it is called Vesperugo; at other times, running ahead and rising before daylight, it is called Lucifer." Here Daniel Barbarus [Daniele Barbaro], with little expertise and long accustomed to the Ptolemaic or the Plinian system, suspects an error in the manuscript of Vitruvius, because in his view it does not agree with the others. If only he had read Plato and Plutarch on this order, all that suspicion would have vanished.

[Margin: Martianus Capella's opinion on the place of ☿ and ♀.]

Next comes Martianus Capella, who in Book 8 of *De Nuptiis Philologiae et Mercurii*, that is, the book on astronomy, in the chapter on the orbs of the planets, says: "But Venus and Mercury do not go around the Earth." And in the next chapter, whose title is "That the Earth is not the centre for all the planets", he goes on: "For Venus and Mercury, although they show daily risings and settings, nevertheless their circles do not go around the Earth at all, but they circle around the Sun in a looser compass. In short, they set the centre of their circles in the Sun, so that they are carried sometimes above it and, more often, inside it, nearer to the Earth; and from it Venus stands apart by one sign and a half (that is, by 45°). But when they are above the Sun, Mercury is nearer to the Earth; when inside the Sun, Venus, since it curves in a purer and more extended orbit."

II. But we must consult Macrobius, on the *Dream of Scipio*, Book 1, chapter 19, where he gives the reason why

[Margin: Macrobius's view on the place of ♀ and ☿.]

Cicero, with the Chaldeans and with Archimedes, gave the Sun a different place among the wandering stars, namely the fourth and middle one, whereas Plato gave it the sixth place and the Moon the seventh; and he reconciles the two opinions. For he says: "Plato followed the Egyptians, the parents of all the branches of philosophy, who hold that the Sun is placed between the Moon and Mercury, yet in such a way that they discovered by reason, and declared, why the Sun is believed by some to be above Mercury and above Venus." And shortly after: "So there was no dissension among the ancients either about the order of the three upper planets, which their immense distance marks off plainly and clearly, or about the region of the Moon, which has withdrawn far from them all. But the order of these three that are close to one another, Venus, Mercury and the Sun, their proximity has confused, though among others. For the reason did not escape the ingenuity of the Egyptians, and it is this. The circle through which the Sun runs is surrounded by the circle of Mercury, as the lower one; and the higher circle of Venus encloses that one too. So it comes about that when these two stars run through the upper summits of their circles,

they are understood to be placed above the Sun; but when they travel through the lower part of the circle, the Sun is thought to be above them. So those who said that their spheres are below the Sun took this view from that course of the stars which sometimes, as we have said, appears lower, and which is also truly the more noticeable, because they then appear more freely: for when they hold the upper part, they are more hidden by the rays. Hence that conviction grew strong, and this order was accepted into use by almost everyone. A more penetrating observation, however, detected a better order, which, besides the investigation of sight, this reasoning also recommends, and so on."

So keen, without a spyglass, was the eye of some that from the faint light of Venus and Mercury they inferred that the planets' distance from the Earth is greater than the Sun's, while from their abundant light and the more frequent phases of both they inferred that it is less. What if they had seen with the telescope what has been so clearly revealed to the eyes of our age: namely, that Venus and Mercury are illuminated by the Sun in such a way that they look like other Moons, and moreover with the alternation that the circuit of both around the Sun toward the East requires, as we said in chapter 2, nos. 2, 3 and 4? Thus no one may any longer doubt the order of these planets, provided the place in the universe of the Sun, which is the centre of the circles of Mercury and Venus, is established.

[Margin: This order is confirmed by the illumination of ♀ and ☿.]

Granted, Macrobius thinks that the five minor planets shine with their own light, and from this confirms that they are above the Sun, and the Moon below it because it shines by the Sun's light. Yet there is no manifest disagreement between Cicero and Plato, for Cicero's words in that *Dream* are: "Of these, one globe is held by that star which on Earth they call Saturn's. Next comes that radiance, propitious and healthful to the human race, which is called Jupiter's; then the red one, dreadful to the lands, which you call Mars; then, below, almost the middle region is held by the Sun, leader, prince and ruler of the other lights, mind and regulator of the world, of such size that it illuminates and fills all things with its light. Him, like companions, the courses of Venus on the one hand and of Mercury on the other follow; and in the lowest orb the Moon turns, kindled by the rays of the Sun." He said "companions", not "lower ones"; and when he said that the Sun holds "almost the middle region", he perhaps added that particle to indicate that it usually is or appears to be in the middle, but sometimes is not. So in the same way the Greeks call them [Greek: σύνδρομοι (syndromoi, "running together"), ἰσόδρομοι (isodromoi, "running at an equal pace"), καὶ ὁμόδρομοι (kai homodromoi, "and running on the same course") τῷ ἡλίῳ (tōi hēliōi, "with the Sun")], that is, the Sun's fellow runners, or equal runners.

[Margin: Bede's view on the place of ♀ and ☿.]

III. There is also, in the first volume of the works of Bede, a book *De Mundi caelestis terrestrisque constitutione*, and in it a chapter on epicycles and intersecting circles, p. 383 in my copy, in which it is said: "Mercury, when it has come to

the 12th degree, is seen, as is the Moon too. Venus, when it has withdrawn from the Sun to the 7th degree of a sign, is seen as if already larger. The Moon too would be seen there, if it had its own light. That they are carried above and below the Sun is shown by conjecture in three ways: either through the intersection of the circles; or because they are epicycles, that is, supercircular, and, not having the Earth as centre, make the Sun as it were the centre of their course; or they measure out the height of the Sun with obtuse circles, or with acute windings"; that is, with either circles or ellipses, unless I am mistaken.

[Margin: Ptolemy's opinion on the position of ♀ and ☿.]

Before we hear al-Biṭrūjī, however, we must first give Ptolemy's opinion, which al-Biṭrūjī refuted. For Ptolemy says in the *Magna Syntaxis*, Book 9, chapter 1: "The spheres of Venus and Mercury were placed below the solar sphere by the ancients; but by some of the more recent they are also placed above it, because the Sun has never been seen to suffer an eclipse from these planets. But this reasoning seems weak to us: for some planets can be below the Sun and yet not be on the line that runs through them, the Sun and our sight, but on another, and for that reason not be interposed between us and the Sun, as happens in most new Moons. However, since knowledge of this matter cannot be had from any other source, because none of these stars produces a sensible parallax, the one phenomenon by which distances are taken, the system of the ancients seems to me more probable and more natural, in which the Sun, in the middle, separates the planets that can recede from it by any distance whatever from those that do not behave so, but are always led around it."

[Margin: Alpetragius's opinion.]

With this set out first, al-Biṭrūjī, in his *Physical Theory of the Planets*, chapter 9, first rebuts Ptolemy's answer, saying: "Both these planets, Venus and Mercury, necessarily run along lines that sometimes pass through our sight and the Sun, as the roots and motions he himself has laid down concede; and Mahumedes Geber son of Afflah [Jābir ibn Aflaḥ] has already brought forward a demonstration on this matter, criticising Ptolemy's order." That Mercury has certainly been seen below the Sun we have already shown in Book 3, chapter 3, Scholium 3. al-Biṭrūjī then adds his own view, which is that Venus is above the Sun and Mercury below it, because in the daily motion toward the West Venus is faster, reaches the meridian sooner and falls short of the first motion of the highest orb less than the Sun does; whereas Mercury, on the contrary, is slower in this than the Sun and falls short more. To those who put Mercury too above the Sun, because they have never seen or read that it has brought an eclipse on the Sun, he replies that this has happened because Mercury shines of itself on all sides, and so, although interposed between the Sun and us, it does not bring on an eclipse.

[Margin: Geber's opinion on the place of ♀ and ☿, and his refutation of Ptolemy.]

But I ought to have reported first the view of Geber son of Afflah, of Seville. In Book 7 of his *Astronomy*, chapter 1, he convicts Ptolemy of a threefold

contradiction. First, that he said Mercury and Venus have no sensible parallax and yet placed them below the Sun, to which he gave a parallax of 2′ 51″, and relied on it in constructing his tables and in solar eclipses. Second, from the commensurations of the orbs given by Ptolemy, he shows that a parallax of about 3′ belongs to Venus at perigee and of 7′ to Mercury. Third, from what Ptolemy gives in Book 13 on the latitude and motion of Venus and Mercury, he demonstrates that each can be interposed between the Sun and our sight if they are lower than the Sun, because they are sometimes found on the same line that passes from our eyes through the centres of the Sun and of the planets themselves. Finally, he concludes that it is more probable that Venus and Mercury are above the Sun and closer to the three upper planets, because of the likeness in the composition of their orbs and of their motions of double anomaly, and in retrogradation and station. For if Ptolemy's reasoning held, which places above the Sun the planets that digress from it by a whole semicircle, the Moon would have to be placed above the Sun. In Book 3, chapter 6, no. 1, however, I cited Jābir ibn Aflaḥ for the Ptolemaic system, with regard to the centre of the planetary system: for although Jābir ibn Aflaḥ places Venus and Mercury above the Sun, he thinks they move not around the Sun but around the Earth.

[Margin: The opinion of Theon, and of Summarius and Aristotle, on the same planets.]

Of the same opinion as Jābir ibn Aflaḥ was Ptolemy's interpreter Theon, on Book 9 of the *Almagest*, chapter 1, who teaches that the Sun is to be placed immediately above the Moon, and Mercury and Venus above the Sun. For this opinion stands also Summarius [?], and even Aristotle, according to Io. Picus Mirandulanus [Giovanni Pico della Mirandola], Book 10 *Against the Astrologers*, chapter 4, where he contends that the order of the planets is uncertain.

[Margin: Pico's opinion.]

He adds, however, that Venus and Mercury, interposed between the Sun and us, can fail to bring on an eclipse because of the smallness of their bodies, or their nearness to the Sun, by whose rays they are overwhelmed, or because they are endowed with their own light and with it make up for what they intercept of the Sun's, or because they are diaphanous. At last he concludes: "Averroes, in his paraphrase of Ptolemy's *Great Composition*, says that he once noted in the Sun two blackish spots, as it were, and that when he had worked through the numbers, Mercury was found at that time to be opposite the Sun's rays. Rightly, then, did Moses the Egyptian [Maimonides], also on the testimony of Abubachesis [?], pronounce the position and order of the planets uncertain."

[Margin: Averroes sees Mercury below the Sun.]

[Margin: Regiomontanus's opinion on the place of Venus and Mercury.]

IV. Regiomontanus saw and reported the view of al-Biṭrūjī in the *Epitome of the Almagest*, Book 9, proposition 1. But he rejected it, and retained with Ptolemy Venus and Mercury below the Sun, lest the space between the Sun at

perigee and the Moon at apogee, namely, as he himself sets it, 1006 semidiameters of the Earth, should be empty of every star. He denies, however, that what Venus can withdraw from the surface of the Sun from our sight is sensible, because if its visual diameter is one-tenth of the Sun's diameter, as the ancients supposed, it follows that its apparent disc is one-hundredth of the Sun's disc in surface. For let the Sun's diameter be 30′. Then, by what was said in Book 1, chapter 4, the circumference will be 94′, and a fourth part of these 23½′, which, multiplied by 30′, gives the area of the solar disc, 705. Likewise, if the diameter of Venus is 3, the circumference will be 9⅖, and a fourth part of this 2⅖, which, multiplied by 3, makes the area of the disc of Venus 7⅕, which is to the area of the solar disc, 705, very nearly as 1 to 100. He says it is uncertain which of these planets is the lower, because the greatest parallax of Mercury, when it is at the perigee of its epicycle, cannot be observed on account of the Sun's glare, and because we do not have full precision in reckoning its motions, any more than those of Venus.

[Margin: Refutation of Regiomontanus's arguments.]

But if these two planets are supposed to go around the Sun, they will fill now the space between the Sun and the Moon, now the space between the Sun and Mars at apogee; and from the digressions of each from the Sun, together with the Sun's distance, the distance of each from the Sun and from the Earth can be derived, from which, finally, the parallax is obtained, as we shall show below. Nor would the portion of the Sun covered by Venus be wholly insensible, if its diameter were 3′ and the Sun's 30′: for there would be an eclipse of 1 digit 12′, which is sensible; especially since Averroes saw Mercury below the Sun, as did others, of whom in Book 3, chapter 3, in the Scholia.

[Margin: The opinion of Clavius and the Coimbra commentators.]

So I cannot assent to Clavius in the *Sphere*, and to the Coimbra commentators on *De caelo*, Book 2, chapter 7, question 4, article 2, who say that this eclipse cannot be perceived by the natural sharpness of the eye, if they exclude the telescope. It must be judged, however, that they were speaking of vision effected with the naked eye alone.

[Margin: The opinion of Copernicus, Tycho and the more recent astronomers.]

V. At last Copernicus, in Book 1, chapter 10, having brought forward the opinions of Plato, Ptolemy, al-Biṭrūjī and, though he withholds the name, Regiomontanus, and having rejected the reasons of those who place Venus and Mercury always below the Sun, concludes with Martianus Capella and certain unnamed Latin writers that they go around the Sun as the centre of their motion. This has since been endorsed by Tycho, *Progymnasmata* volume 1, p. 477, and volume 2, chapter 8, p. 189; Longomontanus, *Theorica*, Book 1, chapter 1; Kepler in the *Epitome of Copernican Astronomy*, p. 535, who cites Campanus for this view and confirms it from the use of the telescope, with which Galileo observed Venus, and Simon Marius Mercury, to be sickle-shaped and illuminated by the Sun in the manner that a circuit around the Sun requires; Galileo, in Dialogue

3 of *De Mundi systemate*; Biancani in the *Sphere*, Book 2, chapter 1, Book 11, chapter 1, and Book 12, chapter 1; Argoli in the *Pandosion*, chapter 3; Gassendi in the *Institutio Astronomica*; Scheiner in the *Disquisitiones Mathematicae*, nos. 22, 23 and 38; Hérigone in volume 5 of the *Cursus Mathematicus*, p. 546; and the rest of the more recent astronomers.

[Margin: Four classes of opinion on the place of ♀ and ☿.]

It is clear, then, from what has been said that there have been principally four opinions on the place of Mercury and Venus. The first was that of the Chaldeans, Archimedes, Ptolemy, Regiomontanus and probably Cicero: that Mercury and Venus are always below the Sun. The second, that of Jābir ibn Aflaḥ, Theon, Summarius, Apuleius and perhaps Aristotle, who place them above the Sun. The third, that of al-Biṭrūjī, who placed Venus above the Sun and Mercury below it. The fourth, that of the Egyptians, Plato, Vitruvius, Martianus Capella, Macrobius, Bede, Copernicus, Tycho and all the more recent astronomers, who affirm that they move around the Sun and are carried now above it, now below it; and rightly so. For that reason we were right to call a system of this kind Platonic in Book 3, chapter 6. For if Venus and Mercury were always above the Sun, they would always appear round in the telescope; and if always below, they would be seen sickle-shaped around both conjunctions with the Sun. But around the upper or apogee conjunction they are seen round and small, and around the perigee conjunction sickle-shaped, from what was said in chapter 2, nos. 2 and 3. The reason why Venus is said never to have been seen below the Sun, however, is the great latitude that Venus mostly has at its perigee conjunction with the Sun, or the forgetfulness or carelessness of observers at the time. So Hortensius, in the *Dissertatio cum Gassendo de Mercurio sub Sole viso et Venere invisa*, said "unseen", not "invisible". Yet Scheiner, in *Apelles post tabulam*, Letter 1, relates that ♀ was seen below the Sun by someone, in the form of a certain spot; and in Letter 2 he reports that he himself lay in wait in vain for such a phenomenon, which Magini's calculation promised.

CAPVT V.

Chapter 5.

De Lumine Planetarum Minorum.

On the Light of the Minor Planets.

I. Above, in Book 6, chapter 2, no. 4, when I was asking whether the fixed stars shine of themselves or from the Sun, the need for comparison forced me to touch on the planets as well. Now that question must be taken up separately for the planets. On it there are principally four views.

[Margin: First view: that all the stars shine of themselves. Macrobius.]

The first is that of those who think that all the stars shine of themselves, though most of them except the Moon. So Macrobius, Book 1 on the *Dream*

of Scipio, chapter 19, confirms that Mercury and Venus are carried not only below but also above the Sun, because they do not shine from the Sun as the Moon does, saying: "A more penetrating observation, however, detected a better order, which, besides the investigation of sight, this reasoning also recommends: that the Moon, which lacks light of its own and borrows it from the Sun, must be below the source of its light. For this reasoning makes the Moon have no light of its own and all the other stars shine with theirs: because those, being placed above the Sun, are in the purest aether itself, in which everything, whatever it is, is natural light and its own."

[Margin: Aguilonius.]

And Aguilonius [François d'Aguilon], *Optics*, Book 5, proposition 81: "Picture in your mind the globes of the stars, which I, for my part, judge to shine of their own."

[Margin: Cardano.]

And before him Cardano, *De subtilitate*, Book 3, p. 138, after saying of the Moon: "Whose light does not, as some think, depend wholly on the Sun, and much less does that of the other stars", adds: "Therefore above the aether all things shine and flash to such a degree, etc."

[Margin: Kepler's earlier view.]

Before the use of the telescope, Kepler defended this view in his theses published in the year 1602, and in the *Astronomia Optica*, p. 261. There, having supposed from the view of Copernicus and Tycho the circuit of Venus around the Sun, carried now above and now below it, he says: "It would therefore be fitting that the light of Venus, like that of the Moon, should be extinguished when she comes between the Sun and us, and appear fullest when Venus flees to the upper conjunction", if, that is, she shone from the Sun, as al-Battānī and Vitellio [Witelo] would have it, whom Kepler attacks there. And he adds: "But this is not so: for she lies hidden for a long time at the upper conjunction, and when she is very high she spreads a feeble light and casts a scant shadow from bodies. Then, the lower she descends, the brighter she becomes, and she darts a light that rivals the Moon's; and finally she is of such brightness that at times, at the very moment of conjunction, she can be seen like a great star, as I find noted in the Brahean observations, and she is not obliterated by the almost immense brightness of the air diffused all around, she herself occupying a single point. By that solitude she moves the sight and turns it toward herself, and so she certainly has a very strong light. She must therefore have this exceptional light as her own." He confirms this thesis both from twinkling, which indicates light of its own, and from those sublunary things in which there is some little light of their own; how much more in superlunary things. He changed his view, however, as I said in Book 6, chapter 2, and shall shortly report in no. 3.

[Margin: Alpetragius and Longomontanus.]

Finally, al-Biṭrūjī, in his *Physical Theory*, chapter 9, denies that Mercury and Venus receive light from the Sun, and so does Longomontanus, *Theorica*, Book 2, chapter 18, on account of Venus seen by Tycho in conjunction with the Sun.

[Margin: Second view: that all the stars shine from the Sun. Metrodorus, Plato.]

II. The second view declares that all the stars, wandering and non-wandering, shine wholly or primarily from the Sun. So Metrodorus of Chios seems to have held, according to Plutarch, *De placitis*, Book 2, chapter 19, and Plato in the *Timaeus*, on whom Copernicus, Book 1, chapter 10, reports as follows: "So those who follow Plato, since they think that all the stars, otherwise dark bodies, shine with light received from the Sun, etc."

[Margin: Al-Battānī, Vitellio.]

Of the same view were al-Battānī, chapter 30, and Vitellio, Book 4, proposition 77, whose words I reported in Book 6, chapter 2, no. 4, where he attributes the reason why the planets do not sensibly change shape and phase, even though they are illuminated by the Sun, to their immense distance, on account of which any flame, which is truly pyramidal, nevertheless appears round.

[Margin: Reinhold, the Coimbra commentators, Biancani, Scheiner.]

Finally, the same is held by Reinhold on Peuerbach's *Theoricae*, p. 163; the Coimbra commentators on *De caelo* 2, chapter 7, question 4; Biancani, *Sphere*, Book 17, chapter 12; and Scheiner, *Disquisitiones Mathematicae*, no. 35, who says: "If we admit that light flows to the wanderers from the Sun, why we should deny the same to the fixed stars, there is no cause." And in no. 38, where he proves from this that Venus and Mercury sometimes move above the Sun, because they sometimes appear spherically illuminated over the whole roundness of their disc, and small. This would not happen, he says, if they moved below it, for they would always appear halved, because they are opaque. When they are below the Sun, however, he says they shine sickle-shaped or crescent, and he testifies that he himself observed this, not indeed in Mercury as others have, but in Venus. And in no. 41 he drew this thesis from the eclipses of the Jovian satellites: "From these we have, first, that Jupiter lacks light of itself; next, that it is a body opaque like the Moon. Moreover, that all its light redounds upon it from the Sun. Further, that the condition of the Jovian satellites is the same, for otherwise they would shine in Jupiter's shadow."

[Margin: Third opinion: that all the stars shine partly of themselves, partly from the Sun. Boulliau's opinion.]

III. The third opinion is that of Ismaël Boulliau, *Astronomia Philolaica*, Book 1, chapter 11. He asserts there that all the stars, and especially the planets, are indeed lit by the Sun as regards that more intense light which strikes our eyes, and more so those that are nearer to it, of which kind are Mercury, Venus and the Earth in his hypothesis; but that they nevertheless have something of their own native light, which is greater in the more remote planets, namely Saturn, Jupiter and Mars, because on account of their greater distance they would have more need of a light of their own.

Boulliau's ground is twofold.

[Margin: Boulliau's first ground.]

The first is the differing colour of the planets. "For Saturn looks somewhat pale, Jupiter most splendid, Mars suffused with red, the Moon silvery, Venus yellowish, Mercury somewhat reddish." He denies that these colours can come from the Sun's rays; and since colour can nowhere exist without light, if they have a colour of their own rooted in their bodies, they also have some light. But the answer to this argument is that it is enough for them to have a differing opacity, and differing colours in first act [in actu primo] as regards visibility, which become visible only when they are illuminated, as in gems, plants and so on. We do not see these on a cloudy night, and yet we do not judge that they lose their colour. But we have argued these matters more fully in the *Physics*, in the treatise *On the Soul*, where the object of the eye is dealt with.

[Margin: Boulliau's second ground.]

The second ground is the huge distance, especially of Jupiter and Saturn, from the Sun. He says it is so great that the Sun, seen from Saturn, appears no greater than 3′ 24″, and from Jupiter no greater than 6′ 10″. "It cannot be, therefore," he says, "that so tiny a light, appearing to them like a shining point, should communicate to them as much light as they transmit to us, and much less so if it is called not direct but reflected." Granted that Venus is very strongly illuminated by the Sun, because it sees the Sun under a diameter of 42′ 28″. Since, therefore, the Sun appears to Jupiter seven times smaller than to Venus in diameter, and forty-eight times smaller in the area of its disc, Jupiter, he says, ought to be lit forty-eight times less than Venus, whereas they are illuminated almost equally; it is therefore a sign that there is native light in Jupiter, Saturn and the rest. But because Venus appears through the tube like the Moon, he admits that it, and Mercury too, do not have so much native light that they could strike the pupil effectively by themselves; the three superior planets, however, he contends are endowed with a native and remarkable light.

But the conclusion first laid down does not seem to cohere with this second ground. For if that primary light which strikes our eyes comes from the Sun to them, and is reflected from them to us, then the Sun's distance, or the smallness of the diameter it presents to them, does not stand in the way of that intensity and of so strong an illumination. But if the Sun's excessive distance stands in the way of sufficient illumination, it will also stand in the way of sufficient intensity (sufficient, I mean, to move our sense), and they will have from themselves that primary light which strikes our eyes. But if this is so, how then do Jupiter and Saturn overshadow their satellites with the shadows of their bodies? Does Boulliau doubt this? Or will he say that they lack their own light in the part of the globe turned away from us? Moreover, by what reasoning does Mars, near quadrature (☐) with the Sun (☉), appear gibbous like the Moon?

[Margin: Fourth opinion: that the fixed stars shine of themselves, the planets from the Sun. Of Giordano Bruno. Of Kepler.]

IV. The fourth opinion is that the fixed stars indeed shine of themselves, as we said in Book 6, chapter 2, no. 4, whereas the planets are opaque of themselves but are illuminated by the Sun. So held Giordano Bruno in the book *De Infinito*

& innumerabilibus, saying that the fixed stars are Suns and the planets Moons illuminated by the Sun; and Kepler in the *Dissertatio cum Nuncio Sidereo*, p. 15. There, having considered the observations Galileo made with the spyglass, and his words in the *Nuncius*, p. 16, which are: "The difference between the appearances of the planets and of the fixed stars also seems worth noting. For the planets present their globes exactly round and as if drawn with compasses, and appear orbicular, like little Moons flooded with light on every side; whereas the fixed stars are by no means seen bounded by a circular periphery, but as flashes darting rays all around and twinkling strongly." Having considered these, I say, he says: "Another most delightful observation of yours is the rayed shape of the fixed stars, differing from the circular shapes of the planets. What else shall we gather from this, Galileo, than that the fixed stars emit their lights from within, while the planets, being opaque, are painted from without; that is, to use Bruno's words, that the former are Suns, the latter Moons or Earths?"

Yet in the *Epitome of Copernican Astronomy* he suspends judgement as regards the fixed stars, and confirms it for the planets in these words: "The distance of the fixed stars is so great that the Sun appears to them 18000 times smaller than to us, or appears to them under the six-hundredth part of one minute, which is 6‴. It is doubtful, therefore, to Galileo and to me, whether from so great a distance they can receive so much light from the Sun as to reflect it to us. Nor do I affirm anything; but they can have it, just as certain animals, and gems by a chemical force, receive a constant light. Of ☾ and ♀ it is certain that they are illuminated by the Sun. Jupiter casts shadows, with which it darkens the satellites when it is interposed between them and the Sun. Mars, Jupiter and Saturn always appear brighter on the side from which they look more nearly at the Sun, and darker and coloured on the opposite side. Mercury appears brighter when it is higher than the Sun, and shows that same part of its body which is also bounded by the Sun's light; and Simon Marius says he saw it sickle-shaped."

[Margin: Galileo's opinion.]

Galileo stands for this opinion in Dialogue 3 of *De Mundi systemate*, where, however, he says that Mercury is indeed seen sickle-shaped, but not so easily as Venus, namely because of the smallness of its disc, its nearness to the Sun, and the liveliness of its darting light; the result is that the telescope is not enough to strip off its curls so that it appears shorn all round.

[Margin: And of René Descartes.]

Renatus Cartes [René Descartes] subscribes to it in the principles of the new philosophy, part 2, nos. 9, 20 and 23. For the planets, because they are near the Sun and shine with a duller light, are illuminated by the Sun; but the fixed stars shine of themselves, because they flash with a very vivid light and, set at a vast interval from the Sun, cannot be illuminated by it.

[Margin: And of Šírek.]

The same in Antonín Maria Šírek in his *Radius Sidereomysticus*, p. 177: namely, that with respect to the fixed stars the Sun is like a point, so that it cannot impart light to them as it does to the planets.

[Margin: Our opinion.]

V. Our opinion, therefore, is that all the planets receive light from the Sun. For it stand all the authors of the second and third opinions, and the very many reasons they have pointed out: namely, that they shine with a duller light than the fixed stars, although they are nearer to us, and are bounded all round; and that not only do we see Venus and Mercury sickle-shaped and diminished in light, as their illumination by the Sun requires, but with larger and clearer telescopes Mars too, around the quadrate aspect with the Sun, has appeared like the Moon, ἀμφίκυρτος [Greek: amphikyrtos, "gibbous", curved on both sides]; and Jupiter eclipsing its satellites with the shadow of its body; and between Saturn and its companions, blackish shadows of differing shape according to its differing situation relative to the Sun, as we showed above, with figures set before the eye, in the individual expositions of each planet, chapter 2.

I judge, however, that the planets are not opaque as the Earth is, which is what Kepler thought, but that in their opacity they resemble metals, or rather gems steeped in some colour and very dense: Saturn, say, onyx; Jupiter, sapphire; Mars, pyrope or amethyst; Venus, chrysoprase; Mercury, beryl or agate; the Moon, the larger pearl-shells or large pearls themselves; and that they are something intermediate between mirrors and the merely opaque. For tracts of land illuminated by the Sun, although far nearer to our eyes, by no means shine so as the planets do. The planets, therefore, are not like earths, nor overlaid with so foul an opacity, but are better suited to drink in light and to send it back more vividly and more ruddily.

[Margin: A threefold objection is put forward to be answered.]

VI. We must, however, answer a threefold objection, partly against the second opinion, partly against the fourth and our own. For if Mercury and Venus do not shine of themselves, and turn now below, now above the Sun, three things follow, which experience nevertheless seems to oppose. First, as they approach the Sun in the upper part of the epicycle, they must appear with round and whole light but a smaller disc, while at their greatest digressions they must shine with only half of themselves, namely on the part that inclines toward the Sun itself; further down, however, they must appear sickle-shaped or curved into horns, like the Moon at its third or fourth day, but greater in the length of the visual diameter, so much so that Venus would appear nearly forty times greater in the lower than in the upper part of its epicycle; and in the lowest part of its cycle they must shine for us either not at all or with a very thin light. Second, in their lower conjunction with the Sun, when latitude does not prevent it, some part of the Sun would be covered as the opaque body of Mercury or Venus ran beneath it. Third, Tycho would not have seen Venus joined to the Sun below it on 1587 February 24, as Longomontanus testifies in Book 2 of the *Theorica*, chapter 18.

Yet the contrary of these things occurs, since they seem neither to change their shape or apparent magnitude, nor to hide any particle of the Sun. For this reason al-Biṭrūjī, who puts Mercury below the Sun, admits that if it did not shine with its own light it would sometimes be seen by us as oblong and arched and falling short of roundness, and would take up a particle of the Sun; but that because it shines with its own light, it shines in the same way near to and far from the Sun, and with its own light makes good as much of the Sun as it covers from us; or else, certainly, that it is diaphanous and transmits the Sun's rays. Copernicus, moreover, in Book 1, chapter 10, although he seems to lean toward al-Biṭrūjī's answer, in fact shows himself undecided, when he says that these planets "shine either with their own light or with the Sun's, with which their whole bodies are steeped, and that for this reason the Sun is not obstructed; or that, because of their smallness, so tiny a spot is not easily seen under a most excellent light. Although Averroes too, in his paraphrase of Ptolemy, recalls that he saw something blackish when he found by the numbers that a conjunction of the Sun and Mercury had been set out."

The answer, briefly, is that all those things do occur and appear, but to one who views these planets with the telescope at suitable times. Venus, however, because of its latitude, has not yet been seen in this or the previous century to run beneath between us and the Sun and darken it; but it has been seen sickle-shaped, though very large in length; without the telescope, however, it appears round, just as flames viewed by us from afar appear.

[Margin: Longomontanus's objection answered.]

That Tycho saw Venus in perigee almost joined to the Sun was the work of the obliquity of the Danish horizon and of ♀'s northern latitude of nearly 9°, as Longomontanus himself states in the passage cited; so he tries in vain on this ground to claim a light of their own for Venus and the other planets, though condensed from the Sun's light. The Moon, on the other hand, at 9° from the Sun can seldom be seen, because it is not as near to the Sun as Venus, nor so apt to drink in and reflect light. But it is clear from what was said in Book 4, chapter 3, no. 5 that the Moon, although near the Sun, has sometimes been seen; and so Longomontanus's objection, taken from the Moon's being invisible near the Sun, is answered.

[Margin: Whether the planets twinkle, and from what source.]

The remaining points on the twinkling of the planets were dealt with sufficiently in Book 6, chapter 2, no. 6.

[Margin: What the argument from the distance of the Sun is worth.]

VII. There remains the argument by which, as we said here in no. 5 and in Book 6, chapter 2, no. 4, Galileo, Kepler, Renatus [Descartes] and Antonín Maria Šírek deny that the fixed stars are illuminated by the Sun, and grant it of the planets: namely, their great distance from the Sun. But for this distinction to hold, the difference between the distance of the fixed stars from the Sun and that of the planets must be as remarkable as the followers of the Copernican hypothesis make it. But because that empty interval between Saturn and the

fixed stars does not seem so vast to us, that argument does not seem so effective. It must therefore be helped in another way, from truer distances, if it is to be worth anything, taking into account, however, also the quantity, and indeed the quality, of the light drawn from the Sun into another star. If, therefore, as we said in Book 6, chapter 7, no. 14, Saturn very probably stands at 73000 semidiameters of the Earth at its mean remoteness from the Earth, and at about 80000 at its greatest, and the fixed stars stand at 100000, it follows that the Sun, viewed according to its visible diameter, appears to Saturn as nearly 3′, and to the fixed stars as 2′ 20″. So if it communicates so leaden and languid a light to Saturn, it will not be able to communicate to the fixed stars a light as vivid and intense as that which comes from them to us; and so the argument from distance holds not of light absolutely, but of light of such a kind and such a quantity. Unless perhaps someone should try to attribute this distinction to a difference of substance or of accidents, by which the fixed stars, although more remote, are nevertheless better suited to drink in the Sun's light and spew it back upon us.

CAPVT VI.

Chapter 6.

De Modo & Difficultate obſeruandi exactè loca quinque Minorum Planetarum, Vnde in ipſorum calculo enormes antiquiorum Tabularum errores.

On the method and difficulty of observing exactly the places of the five minor planets, the source of the enormous errors of the older tables in calculating them.

I. Ptolemy was unwilling to enter the labyrinth of these planets without a preface and a protestation of the difficulty, which he sets out in Book 9, throughout the second chapter. Tycho professes the same difficulty in the preface to the *Epistolae*; Kepler in the *Mysterium Cosmographicum*, chapters 18 and 19, and in the foundation of the *Ephemerides*, p. 15. That difficulty arises from many causes, which pertain either to the objects to be observed themselves, or to the observers themselves and their instruments, places or times; among them Ptolemy expressly names "the error of instruments, the variation of the air, and the difference of the observers' eyes."

The first difficulty, then, lies in the motions of these planets themselves, which are entangled not in one but at least in two inequalities, and in a varied and manifold maze of stations and retrogradations, latitudes, inclinations and deviations, through unimaginable windings and bends. Moreover, the three superior planets can indeed be observed conveniently at their acronychal shinings, when, namely, being opposite the Sun, they rise in the evening, because then they both shine all night and wander with only the single, first inequality of motion; and so through three such observations their motions can be investigated after a fashion. But because they are slow-paced, especially

Saturn, which needs nearly 30 years, and Jupiter, which needs nearly 12, to run through the zodiac once, a small error in observation brings about a large error in their place; and a shortfall or excess of a few minutes in longitude delays or hastens their conjunctions and other aspects by a difference of several days, let alone hours. A much greater occasion of going astray lies in observing the moments of the stations, both because of the double anomaly and because of the longer stay during which they seem to stick in the same places.

[Margin: Mars, an unobservable star.]

Mars, moreover, seems so lawless and exorbitant that with coils scarcely to be untangled it has wearied the wits of all astronomers; so much so that Pliny the Elder, Book 2, chapter 17, said: "In the star of Mars, whose course is most unobservable."

[Margin: Rheticus's despair over the motion of Mars.]

And there is a report, related by Kepler in the dedicatory letter of the *Commentaries on Mars*, that Georg Joachim Rheticus, the pupil of Copernicus, while he was stuck in wonder at the motion of Mars, importunately questioned his familiar spirit about these matters; but that, seized by the hair by the spirit and dashed head first by turns against the overhanging ceiling, he was at once let go and hurled to the floor, with this answer added: "This is the motion of Mars." Unless perhaps, says Kepler, he himself, frustrated of success in his speculations, in a raging and disordered state of spirit struck his head against the wall, as Augustus once did when five legions were lost through Quintilius Varus, and the report then spread among the people grew as it went, all the way to that fiction. Kepler himself, for his part, although like another Vulcan he bound this Gradivus in chains and did not let him roam as freely as before, nevertheless says in the Introduction to the already mentioned commentaries: "I myself, who am called a mathematician, on rereading this work of mine grow faint in the powers of my brain, while I recall from the figures to my mind the meanings of the demonstrations, which I had first brought from my mind into the figures and the text."

[Margin: The difficulty of observing Venus and Mercury.]

II. As for Venus and Mercury: although some, with Copernicus, Book 5, chapter 20, think that they can measure their orbs more easily from their greatest digressions from the Sun than those of the superior planets from observation at ἀκρονυχία [Greek: akronychia, the acronychal rising, at nightfall], a more precise observation of them is hindered both by a parallax greater than in the three superior planets, and especially by the lack of suitable fixed stars from which their distance could be taken, and by the vapours of the horizon and the refractions arising from them, on account of which

Bernhard Walther in his observations, and Michael Mästlin, thesis 58, were surprised that Venus appeared to them much higher than it should.

[Margin: The peculiar difficulty of Mercury.]

Mercury, moreover, mostly lies hidden under the rays of the Sun, because of its small digressions from the Sun, and is almost always exposed to great

refractions of the horizon; and, as Regiomontanus notes in Book 9 of the *Epitome of the Almagest*, proposition 1, it cannot be observed in most climates, because even at its greatest elongations from the Sun it is overwhelmed by the light of morning or evening twilight, as the obliquity of the horizon demands. Copernicus, in Book 5, chapter 30, laments and admits that this happened to him, because of the vapours of the Vistula and the slope of the horizon at Frombork, so much so that he could never observe it there and was forced to take refuge in the observations of Bernhard Walther. And he adds: "For this reason this star has tormented us with many windings and much labour, as we searched out its errors." For with no lesser wrappings of coils than his caduceus has in the twisting bends of its snakes, this Hermes, the craftiest of the planets, steals himself away from the efforts of astronomers. As for Tycho's observations of this planet, hear what Kepler testifies before the *Ephemerides*, p. 15: "In Mercury the difficulty is greater, although the occasions for detecting an error are very rare. All its observations recorded by Brahe over 20 years (those, that is, which are reliable and undoubted) fall within a single third of Mercury's eccentric; the other two thirds remain unobserved, in this century at any rate, and in our hemisphere."

III. Now if a small number of observations is exposed to these difficulties one by one, how much more so is the great number needed to finish the business of these five wanderers, when the opportunities of observing are so rare, and either life is so short, or one's manner and way of life does not allow as many years as would be needed for observations of this kind.

[Margin: The time required for these observations.]

Tycho, in the preface to his *Epistolae Astronomicae*, lays it down thus: "At least 4 or 5 lustra are therefore required before the appearances of all the planets can be sufficiently observed, in so many and such different places as may be needed, with suitable instruments." And a little later: "Therefore, for the places and motions both of the wandering and of the unwandering stars to be conveniently brought from celestial observations into the knowledge of numbers, and fitted to common use, and finished so far as one man can accomplish at his age, about three decades of years, if not more, seem to be required." For, as is said in the same place, for the motion of the three superior planets to be corrected from acronychal observations in each sign of the zodiac, 30 years are required for Saturn, 12 for Jupiter, and for other reasons 24 for Mars. We can, however, by trusting and using the observations of others carefully made, and by comparing them with our own, make up for the small number of ours.

IV. No one, then, should be surprised if, on the testimony of Ptolemy, Book 9, chapter 2, Hipparchus, lacking suitable observations from his predecessors, did not attempt to treat these five planets. For hear Ptolemy:

[Margin: Hipparchus φιλαληθέστατος [Greek: philalēthestatos, "most truth-loving"].]

"On account of these difficulties I judge that Hipparchus proved a very great friend of truth. Because of them, and because he did not have from the ancients

as many exact observations as he himself has furnished to us, he did indeed investigate the business of the Sun and the Moon, and demonstrated, so far as could be done, that it takes place through equal and circular motions; but the business of the five planets, so far as is shown from his commentaries, he did not even take up, but only collected their observations into a better order and showed by means of them that the phenomena do not agree with the hypotheses of the mathematicians of that time." And after a few intervening words he continues: "But some demonstrated nothing at all, others did not reach the end. He therefore judged that it did not befit him, who had advanced all the mathematical disciplines exactly and honestly, to begin, as the rest had done, things that seemed by no means able to be brought to a conclusion." And finally he adds: "I think, therefore, that this seemed most difficult to him also; which we have said not for the sake of display."

[Margin: Ptolemy's excuse.]

So it comes about that Ptolemy himself too, in the same place, more than once fortifies himself with an excuse before posterity, when he says: "We should make much of whatever we attain in this matter. This business is most difficult for many reasons: first, because it has not yet been rightly considered by earlier writers; next, because in considering the periods of the individual motions, a minute error can arise in observations made by instruments, which produces a sensible difference afterward all the sooner when the later observation has been made at a shorter interval after the earlier. And now the time from which we have recorded observations of the planets is so short, compared with the magnitude of the things, that it makes a prediction for a long time weak." Lastly, he admits that, under the compulsion of the things themselves, he has used some devices contrary to reason, and fictitious circles in the plane of the ecliptic, as if they did not decline from it at all, for convenience; and certain hypotheses as first principles, not plainly evident but supported by a certain congruence; and that he has sometimes used right-angled triangles for oblique-angled ones, and did not think that an error needing attention could arise from this; and finally that he used that astrolabe of his in taking the distances of the planets from the Moon or from the fixed stars. Kepler will give us a similar excuse for Copernicus below.

[Margin: On the errors of the tables in the 5 planets.]

V. Nor is it to be wondered at if, down to Tycho, the calculation of any astronomers whatever, especially in these planets, has gone so enormously astray from the heaven and the manifest phenomena. So that no one may think we say this from a lust to accuse, let us call together some witnesses to be questioned.

[Margin: Errors of the Alphonsine Tables in these planets.]

Johannes Stöffler had predicted from the Alphonsine Tables that the great conjunction of Saturn and Mars would take place in the year of the Lord 1536, on May 25; but Pietro Pitati observed that it took place on the 20th. Gemma Frisius the younger, in chapters 19 and 20 of the *Radius Astronomicus*, affirms that he observed Mars on 1544 October 31 at 7^h at $16^\circ\ 22' \approx$, with a southern

latitude of 1° 39′, and that the Alphonsine Tables nevertheless showed it then at 18° 38′, and at other times went astray from the truth by three degrees and more. Franciscus Iunctinus [Francesco Giuntini], in the preface to his astronomical tables, giving the reason why he preferred the Prutenic Canons to the Alphonsine Tables, says that the former differ from the latter at times by 11° in Mercury indeed, and by nearly 4° in Mars; and that Giuliano Ristori of Prato, a Carmelite, doctor of sacred theology, a most excellent mathematician and his teacher, observed in 1536, on May 19, about the second hour of the night, the meeting of Saturn and Mars, with Saturn almost covered by Mars, whereas according to Stöffler and the Alphonsine Tables it should have occurred on May 25; and that in 1542, on the night following January 29, at 9^{h}, Saturn and Mars were in conjunction; and that on 1563 August 24 at Orange [Aurangæ] he himself observed the conjunction of Saturn and Jupiter at 14^{h} 30^{m} after noon, the more northerly Jupiter covering almost the whole of the more southerly Saturn, at the end of degree 28 of ♋; whereas the Alphonsine table put that conjunction on September 25, at 4° 12′ of Leo. And he adds: "There are many others, indeed innumerable and intolerable, errors in the motion of Mercury."

Tycho, moreover, in the *Mechanica Astronomica*, p. 39 in my copy, speaking of the Alphonsine and Prutenic tables, says: "I was led to the point of detecting most manifestly the intolerable errors of both calculations, which that great conjunction of the year 1563, of which I spoke at the beginning, also betrayed more than enough in the motion of Saturn and Jupiter: since it deceived the Alphonsine numbers by a whole month; those of Copernicus also by some days, though only a few." He had said that he observed it in the 16th year of his age, at Leipzig, at the end of Cancer.

[Margin: Alfonso's error of a whole month in ♄.]

With Giuntini and Tycho joined as witnesses, then, an error of a whole month, owing to the slowness of the motion of ♄ and ♃, was detected in the numbers of Alfonso. The same Tycho, in the same work, p. 43, says of the Alphonsine and Prutenic tables: "The result is that the planets are sometimes southern in the heavens when their numbers show them northern, and the reverse." In the *Epistolae Astronomicae*, p. 56, comparing his observations made in January of the year 1587 with the Alphonsine and Copernican calculation, he gathers an error at times of 2° in ♄, 2° ½ in ♂, and nearly 3° in ☿. And in Volume 1 of the *Progymnasmata*, p. 648, against Apian, who trusted the Alphonsine calculation too much, he says: "Since the motion of ♂ especially, to say nothing now of ♄, has not so far been so well explored that their conjunction could safely be made known to within three or four days from the calculation, especially the Alphonsine, which he used, no, not even the Copernican, which is a little more exact."

[Margin: And of 7° in ☿.]

His follower and assistant Longomontanus, in the *Astronomia Danica*, Book 2 of the *Theorica*, chapter 20, says that he has detected between the Alphonsine and Copernican calculations an immense discrepancy in ☿, so great that it sometimes

rises beyond 7°; and in chapter 13 he adduces the following disagreement of the Alphonsine and Prutenic tables from his observations made at Uraniborg.

[Margin: Disagreement of the Alphonsine and Prutenic tables from the observed places in ♄ ♃ ♂.]

Planets observed	Year, month, day (old style)	After noon at Uraniborg	Longitude: observed	Longitude: Alphonsine	Longitude: Copernican	Latitude: observed	Latitude: Alphonsine	Latitude: Copernican
♂	1587 January 18	$15^h\ 56^m$	♎ 4° 1′	1° 32′	5° 5′	North 3° 13′	2° 58′	3° 1′
♄	1591 March 17	$7^h\ 30^m$	♊ 22° 43′	25° 7′	22° 23′	South 0° 56½′	0° 56′	1° 35′
♃	1591 April 14	$11^h\ 0^m$	♏ 14° 16′	12° 32′	14° 40′	North 1° 23′	1° 51′	1° 58′

Giovanni Antonio Magini too, in a letter to Richard Fasterus of London [?] [Rchardum Fasterum] [recte: Richardum] of 1601 December 15, which is found in Magini's *Directions*, p. 95, prefers Tycho's observations to the Prutenic calculation and to the Alphonsine, which, except for the Sun, he affirms errs more than the Copernican.

[Margin: Copernicus accused and excused.]

And Kepler, in the *Mysterium Cosmographicum*, chapter 18, accuses Copernicus in such a way as to excuse him for neglecting hours and quarters of a degree in computing the places of Mars, Venus and the others of the 5 planets. For he says: "And so he would seem justly to incur censure, had he not done it deliberately, because it was better to have an astronomy that was in some way imperfect than none at all. For difficulties of this kind will indeed arise as long as the stars run their course; and to overcome them, and to aspire unhindered to the establishment of the science with the least loss, as Copernicus dared to do, is the mark of a brave man; to evade them is the mark of the idle; to despair and to cast aside all this care, that of the timid. Thus he neither hides about himself these errors [Greek: σφάλματα (sphalmata, "errors")] just listed, nor confesses them with shame. He arms himself with the example of Ptolemy and the ancients, he excuses himself by the difficulty of observing, etc."

[Margin: Error of the Prutenic tables: 12 days in ♄, 2° in ♂.]

And in chapter 19 he relates that in the year 1594 on November 12, new style, Saturn was seen by him exactly between the Neck and the Heart of the Lion, where according to the Prutenic calculation it should have been on October 31, the difference in longitude being about 37′. Mästlin, however, in the preface to the Ephemeris for the year 1577, testifies that "the errors of Mars from the calculation cannot be forced within the narrow bounds of two degrees."

[Margin: The difficulty of observing Mercury, and the errors of the Ephemerides.]

And speaking of Mercury he adds: "This one, certainly, is the planet that most brings the astrologers' reputation into disrepute and upsets all reckoning

of meteorological phenomena; and in predicting winds (which it most certainly stirs up, whenever it stands in suitable places) it is often off by so constant a number of days that I can very nearly correct its circle as then faultily given in the Ephemeris. And so, if I saw any astronomer applying himself too anxiously to searching out the errors of this planet, I would warn him to spend that time more properly." And a little later: "Meanwhile, if errors concerning the motions of the Earth and the Moon deserve indulgence, the errors in Mercury will deserve it much more, since it is both more remote from us and almost always hides under the Sun."

[Margin: Error of the Prutenic tables: 4° in ♂.]

Kepler likewise, in the introduction to the *Commentaries on Mars*, says that in the year 1608, in the month of August, the Prutenic place of Mars fell short of the true one by almost 4°; that in the year 1593, in August and September, it fell short by nearly 5°; and again that it fell short throughout the year 1625, but in October by almost 5°, owing to a wrong prosthaphaeresis.

[Margin: Errors of the various tables in ☿ ♂ with ☉.]

Moreover, in the year 1631 on November 7, Mercury was observed under the Sun and joined to the Sun at 7^{h} 58^{m} by Pierre Gassendi, as I reported in Book 3, chapter 3, scholium 3. From that moment and place the Ptolemaic tables departed by 4° 25′, the Copernican by 5°, the Danish by 7° 13′, those of Lansbergen by 1° 21′, the Rudolphine by 14′ 24″. And so Natalis Duret [Noël Duret], a follower of Lansbergen in his Ephemerides, here deserted to the Rudolphine camp.

Martinus Hortensius, for his part, in the dissertation with Gassendi on Mercury seen under the Sun, testifies that in the year 1632 on July 31 he observed the conjunction of ♀ and ☿ at about the hour 11½ of the night, whereas according to Longomontanus's canon these planets ought then to have been 2° 53′ apart from each other in longitude. Finally Antonín Maria Šírek [Anton. Maria de Rheita], in Book 2 of the *Radius Sidereomysticus*, chapter 2, notes an error in Tycho's hypotheses in ♄ ♃ ♂ and ☿ of 5, 6, 7 degrees, and in ♀ of 3, about which I shall see elsewhere. But let there now be an end of these errors and mistakes [Greek: σφαλμάτων, καὶ ἁμαρτημάτων (sphalmatōn, kai hamartēmatōn, "of errors and of mistakes")].

VI. Everything said so far tends to this: that the indulgence others ask of us, they should grant to us in the same way, if even in Tome 3, where we shall offer more observations, and ones brought back under a finer file, we shall not have attained as much as the avidity of astronomers demands; mindful of that saying of Horace in the *Ars Poetica*:

We know this, and this indulgence we both desire and grant in turn.

And:

For no one is born without faults; the best is he
who is burdened by the least.

Next, so as not to hold anyone back through despair, but rather to sharpen those eager to observe, so that they may not be content with ordinary diligence: I think we must here say farewell to astrolabes and armillary spheres and torqueta, to astronomical radii and to such instruments of the roughest observations [Greek: ὁλοσχεροτάτων τηρίσεων [recte: τηρήσεων] (holoscherotatōn tērēseōn, "of the roughest observations")], for reasons to be set out in the *Liber Organicus*.

[Margin: A method of observing the places of the 5 planets exactly.]

The places of the five planets, however, are to be investigated only with large quadrants, or geometric or astronomical squares, or parallactic rulers that do not wobble at all, or larger sextants, by taking their meridian altitude and by counting most exactly the seconds of time between their transit through midheaven and that of an accurately known fixed star, so that from the declination of the planets and the right ascension their longitude and latitude may be found trigonometrically, according to Problem 1, which we gave in Book 6, chapter 12.

But if the planet cannot be observed on the meridian, take with the instruments already mentioned their altitude, as far from the horizon as you can, and the azimuth, but wait for the star to reach a determined degree, and use the thread triangle, on which see Book 6, chapter 11, no. 3. Or else, with skilfully built sextants divided into minutes and tens of seconds, take the distances of the planets not from the Moon, so as not to run into the entanglements of parallaxes, but from two fixed stars placed far from the horizon, avoiding, however, those that form too acute-angled a triangle with the planet, and choosing a time when the planet is as high as possible.

So that the swift setting of Venus or Mercury hastening to the horizon may not slip away, the instruments must be ready beforehand, and several observers at the same moment, so that as soon as the twilight allows them to be seen, two may take the distance of the planet from one of the fixed stars, and two others the distance of the same planet from the other of those fixed stars that have already been assigned and chosen for this; and finally two others may take, one the altitude of the planet, the other the altitude of that fixed star, so that, the refraction being known from this, the apparent distance may be converted into the true one, and at the same time the moment of time may be known through the problems of the *Primum Mobile* to be given in Book 10.

If, however, the good fortune of an opportunity should occur that the planet covers some fixed star of known longitude and latitude, the determination of the planet's place will be most exact and most ready; whereas, on the contrary, it will not be so exact if the planet is covered by the horns or limbs of the Moon, on account of the parallaxes and the manifold motions of the Moon.

An observation will also be exact if you direct the thread triangle mentioned above at the planet, and, as the planet passes through the plane of the triangle, take the planet's altitude; then let someone else count the times of the pendulum until a fixed star passes through that same plane, and at that moment take the

star's altitude. For from these you will learn the planet's place, if the declination and right ascension of the fixed star are known, as I shall teach in the problems of the *Primum Mobile*.

CAPVT VII.

Chapter 7.

De Motuum ſpeciebus ac varietatibus obſeruatis in quinq. minoribus Planetis.

On the kinds and varieties of motion observed in the five minor planets.

I. Since from the beginnings of astronomy, the first observations being cruder and made almost without instruments, there was a gradual advance to observations of the motions carried out with one instrument after another, so too must the novice be led by the hand by us, in the same order, to the knowledge of these motions.

[Margin: Proper motion in longitude.]

First, then, there was observed, besides the diurnal motion from east to west that is common to all the stars, a proper motion in longitude, owing to the withdrawal of these planets from the fixed stars towards the east, greater and greater on most nights; and so a slower daily return from the east to the same meridian than in the fixed stars. In this motion, of the three superior planets, Saturn appeared slower than Jupiter, and Jupiter than Mars, because each day Saturn moves away from the same fixed star towards the east less than Jupiter, and Jupiter less than Mars.

[Margin: Order of risings and settings in ♄ ♃ ♂: cosmical rising; morning heliacal rising; cosmic setting; acronychal rising; evening heliacal setting; acronychal setting.]

II. Second, it was noted that risings and settings occur in this order. In the three superior planets, after the cosmical rising comes the morning heliacal rising; then (as the Sun withdraws from them up to a full semicircle of the zodiac) they become eastern and morning planets from ♂ with the Sun up to ♌, in which the cosmical setting occurs, and presently the acronychal rising, after which they shine all night; and they are western and evening planets from ♌ to ♂ with the Sun. Next, as the Sun approaches them, the evening heliacal setting occurs, which the acronychal setting must necessarily follow. You see this order noted in the margin.

From this and from the earlier motion it became clear that these planets do indeed move in the order of the signs, but more slowly than the Sun, since they cannot catch up with the Sun once it has been joined to them and is then withdrawing; on the contrary, the Sun catches up with them, overwhelms them with its rays, and leaves them behind overwhelmed. And from this, again, was confirmed the greater slowness of Saturn than of Jupiter, and of Jupiter than of Mars, since Saturn hides under the rays of the Sun less than Jupiter, and

Jupiter less than Mavors, the Sun withdrawing more quickly from the former than from the latter.

[Margin: Why is Mars Gradivus? Why is Saturn Phaenon?]

Mars is rightly called Gradivus, because, marching with longer strides, it follows the Sun, which is indeed swifter than it, yet more swiftly than ♄ and ♃ do. Saturn is called Phaenon, that is, "appearing", because its stay under the Sun, from the evening heliacal setting to the morning heliacal rising, is shorter.

[Margin: Order of risings and settings in ♀ in ☿: morning heliacal rising; last morning heliacal setting; cosmical rising; evening heliacal emersion; evening heliacal setting; acronychal setting.]

Venus and Mercury, moreover, have been observed never to move so far from the Sun as to set cosmically or rise acronychally, since they are never set opposite the Sun across the diameter of the world. But after they have risen heliacally in the morning, they digress further from the Sun each day, and at its rising appear higher and higher above the horizon, up to certain limits of greatest digression, which in Venus does not exceed about 50°, nor in Mercury 30°. Then, approaching the Sun again, they are hidden heliacally in the morning, and lie concealed under the Sun's brilliance, until they emerge heliacally in the evening; and on the following nights, at sunset, having become higher and higher above the horizon, they digress from it up to the limits of greatest elongation already mentioned. When this is completed, having come to meet the Sun, they set in the evening with a heliacal setting. From this is gathered both the order of these phases, which you see in the margin, and that these planets are sometimes faster than the Sun, namely when after the morning digression they approach it, or in the evening after the heliacal rising withdraw from it; and sometimes slower, namely when after the evening digression they fall back towards it, or in the morning after the heliacal rising withdraw from it.

[Margin: Motion in altitude.]

III. Third, there was observed a marked decrease of light and of apparent size when they approach superior conjunction with the Sun and the heliacal setting, evening if they are the three superior planets, or morning if the two inferior.

[Margin: Planets *Hipaugi*, diminished and increased in light.]

Hence they are then said to be *Diminished in light*, until, hidden under the Sun, they are called in Greek [Greek: ὑπάυγοι (hypaugoi, "under the rays")], in Latin *Combusti* or *subradiati*. Conversely, when either the three superior planets approach opposition with the Sun and the acronychal rising, or the two inferior planets, through greatest digression, approach evening conjunction with it, they become noticeably *Increased in light*. This, then, is judged to happen, if not necessarily, yet far more probably, from withdrawal from the Earth when they decrease and from approach when they increase.

[Margin: Planet apogee, perigee, aphelion, perihelion.]

From this a planet is called [Greek: ἀπόγειος (apogeios)], that is, standing off from the Earth, and sometimes [Greek: ἀπογειότατος (apogeiotatos)], that is, most remote from it; and [Greek: ἀφήλιος (aphēlios)] or [Greek: ἀφηλιότατος (aphēliotatos)], that is, remote or most remote from the Sun; and conversely [Greek: περίγειος (perigeios)] or [Greek: περιγειότατος (perigeiotatos)], namely around or near the Earth, or nearest to it; and [Greek: περιήλιος (periēlios)] or [Greek: περιηλιότατος (periēliotatos)], that is, near or very near the Sun.

[Margin: Revolution of ♄ ♃ ♂ to the Sun, and its time.]

IV. Fourth, by counting the intervals of time in which the Sun returns to opposition with the superior planets, from which a conjecture was drawn about the return to conjunction, it was observed that in the same planet this revolution is not equal in the number of days and hours, but is sometimes shorter, sometimes longer. Yet, taking the mean revolution of the planet to the Sun, or rather of the Sun, as being the swifter, to the planet, it is established that this is about 378 days in ♄, 399 in ♃, 780 in ♂. From this, again, it became clear that Saturn is slower than Jupiter and Jupiter slower than Mars, since the Sun catches Saturn more quickly than Jupiter, and Jupiter more quickly than Mars.

[Margin: Revolution of ♀ and ☿ to ☉, and its time.]

Venus too was not seen to return equally from one greatest digression from the Sun to another of the same kind, for instance from morning to morning, and neither was Mercury. Yet, considering the mean revolution, Venus returns in about 580 or 583 days; and Mercury in about 115 days, or returning three times within one year to the same syzygies with the Sun.

[Margin: Why Mercury is winged and the messenger of those above.]

On account of this speed the mythographers not inappropriately added wings to his shoulders and soles, and made him up, above the others, as the messenger of those above and those below, that is, the interpreter of changing weather.

[Margin: Inequality of the revolution to the Sun.]

V. Fifth, as astronomy advanced, in this very revolution of the Sun to the planet, commonly called the revolution of the planet to the Sun, besides the inequality of one whole revolution compared with another, a marked inequality was detected within the same revolution, by comparing its parts with its parts, even if the Sun is in the same sign of the zodiac.

[Margin: Direct planet.]

For sometimes they appear faster than themselves from day to day, and indeed fastest at conjunction with the Sun (understand apogee conjunction in ♀ and ☿). From this a planet was called Direct, or going forward directly, and in Greek [Greek: ὑπολειπτικὸς (hypoleiptikos)], that is, following, or proceeding in the order of the signs; and that succession [Greek: ὑπόλειψις (hypoleipsis)].

[Margin: And retrograde.]

Sometimes, however, with a crab-like regress they seem to fall back further towards the west each day, namely when they approach opposition with the Sun, or, if the planet is an inferior one, the evening perigee conjunction with it. Then, as regards the motion in the order of the signs, they are slow, and at the very moments of opposition or of evening perigee conjunction [copulæ] slowest, although fast in retrogression, yet not as much as at conjunctions, etc. For example, Mars on the days nearest conjunction on either side completes directly 47′ a day in the zodiac, but in opposition retrogresses 24′ a day. From this a planet was called in Latin *Retrogradus*; in Greek [Greek: προηγούμενος (proēgoumenos)] or [Greek: προηγητικὸς (proēgētikos)], that is, going before, or withdrawing against the order of the signs; and that motion [Greek: προηγήσις (proēgēsis)], namely precession.

[Margin: Stationary planet.]

Sometimes, namely just at the beginning of and after the end of retrogradation, they seem to stick for some days or hours in the same place of the zodiac, neither going forward nor going back. From this a planet was called Stational or Stationary, in Greek [Greek: στηρήζων [recte: στηρίζων] (stērizōn)], and that stay a Station, or, as in Cicero in the *Tusculans*, *Institio*; and the first station [Greek: πρώτος στήριγμος (prōtos stērigmos)], the second [Greek: δεύτερος στήριγμος (deuteros stērigmos)].

[Margin: The time that the station, direction and retrogradation of the 5 planets lasts.]

When the intervals of time they spend in the arc of direction, in the arc of retrogression and in the stay of the stations were observed, it became clear that these are not equal but sometimes shorter, sometimes longer, and that in them a sometimes shorter, sometimes longer arc of the zodiac is completed. Yet, taking a mean time, the station of ♄ is of nearly 8 days, of ♃ 4, of ♂ 2, of ♀ 1, of ☿ ½. Direct progress of ♄ is of about 244 days, of ♃ 284, of ♂ 705, of ♀ 542, of ☿ 93. Retrogradation, lastly, of ♄ is of almost 136 days, of ♃ 119, of ♂ 75, of ♀ 42, of ☿ 22. A far greater inequality, however, appears in Mars, then in Venus and Mercury. And ♄ becomes stationary [Sationarius] [recte: Stationarius] between the □ and △ aspects with ☉, but ♃ in △ exactly, and ♂ between eighths and two-fifths of its distance from the Sun; and the higher each planet is, the longer it remains retrograde.

[Margin: Revolution of the planets to the fixed stars and to the same point of the zodiac.]

VI. Sixth, as the patience of astronomers extended to observations over a longer time, begun by themselves or by their elders, and when the revolutions of these planets were compared both to the same fixed star with which they had once been joined, or with which they had climbed the meridian, and to the same point of the zodiac, it became clear that these are not indeed equal in the number of days and hours, even in the same planet. Yet, taking a mean restitution, Saturn passes through the zodiac in 29 simple or Egyptian years

and about 174 days, but it does not return to the same fixed star except in 29 years and nearly 184 days.

[Margin: The time of these revolutions.]

Jupiter completes the zodiac in 11 Egyptian years and almost 315 days, and reaches the same fixed star in 11 years and about 318 days. Mars, however, flies through the zodiac in one Egyptian year and almost 321 days, and reaches the same fixed star on the following day or night. This could not be so readily determined for Mercury and Venus, because they always accompany the Sun. When the daily or frequent motions and the parts of this revolution were also compared, and the longitudes of the planets were taken from time to time with instruments, it became plain that they move unequally, and in equal times complete unequal arcs of the zodiac, even if they were at the same distance from conjunction with the Sun.

[Margin: Twofold inequality.]

VII. Seventh, therefore, from the observations indicated in nos. 4, 5 and 6, there was at last recognized in these five planets, besides the common motion, a twofold motion in longitude, each irregular with its own inequality, and a twofold inequality or anomaly, which, merged into one, had formerly appeared undifferentiated. For the revolution to the Sun is shorter in those planets in which the revolution to the fixed stars and to the same point of the zodiac is longer; and, on the contrary, longer to the Sun in those in which it is shorter to the zodiac. Moreover, revolutions of the same planet to the Sun compared with each other, and its parts compared with parts, for instance retrogradations with retrogradations, appear unequal. So too revolutions to a point of the zodiac in the same planet compared with each other, and its parts with parts, for instance the time spent in completing one sign compared with the time in another sign, have shown their own separate inequality.

[Margin: First inequality.]

The first inequality, then, is free of any respect to the Sun, especially in the three superior planets, and is restored with the planet's return to the same point of the zodiac. It was called first, not indeed by origin of observation, for perhaps it came later; but because of itself it first belongs to the planet, even if there were no Sun, or if neither the Sun itself nor the Earth in its place were moving, or, speaking of the inferior planets, even if they moved only with mean motion; and because it appears plainly in conjunctions and oppositions with the Sun, whether true or mean, in which, however, the anomaly of motion relative to the Sun is extinguished, or appears barely perceptibly.

This inequality, as will be explained later, Ptolemy, either by his own invention or in imitation of the ancients, attributed to an eccentric circle enclosing the Earth: both because he saw that more time passes from the slowest motion of this anomaly to the mean than from the mean to the fastest, and because the places of this slowest and fastest motion lie diametrically opposite in the zodiac, two properties that suit an eccentric more than an epicycle. And so he laid down that the greatest slowness of the planet's motion in the zodiac occurs

at the apogee of the eccentric, and the greatest speed at the perigee. This was discussed in no. 6.

[Margin: Second inequality.]

The second inequality and anomaly is tied to the respect to the Sun, and this was discussed in nos. 4 and 5. For according to it the planet so adjusts its motion to the Sun's proximity that as the Sun approaches it becomes faster than usual, and as the Sun goes off to opposition, or, in the inferior planets, to evening conjunction, it becomes slower, and in the intermediate times stationary. This anomaly is restored with the return of the Sun to the planet. It was called second because of itself it belongs second to the planet, or even accidentally and from the extrinsic motion of the Sun or the Earth. Indeed, if one posits with the Copernicans the Earth's annual motion, which we nevertheless reject, stations and retrogradations are in reality nothing, but mere changes and accidents of our sight, as we shall set out later.

This anomaly, further, was attributed to an epicycle carried by the eccentric, because in it more time passes from the fastest motion to the mean than from the mean to the slowest, which is proper to an epicycle; and because this alone seemed suitable to represent the stations and retrogradations of the planets. So, in the contrary fashion from the Moon, the first and free anomaly could in the Ptolemaic manner be called eccentric, and the second, which is tied to the Sun, epicyclic. Thus Ptolemy decreed that the greatest speed of this anomaly occurs at the apogee of the epicycle and the greatest slowness at its perigee. But in the inferior planets the mean motion is the same as the mean motion of the Sun, and in this respect their first motion is connected with the Sun.

[Margin: The opportunity of separating the first inequality from the second is pointed out.]

From this I would now like novices to scent out why astronomers, in order to know more exactly both the mean motion of the first anomaly and its equations separately from the second, have studiously chosen in the three superior planets oppositions with the Sun, or acronychal shinings, and in the inferior planets the greatest digressions. For in conjunctions, no differently from oppositions, the second inequality ceases; but conjunctions cannot be observed, the planet being hidden at those times for some days under the rays of the Sun. Therefore oppositions of the three superior planets with the Sun had to be chosen, and in the inferior planets, from the pairs of greatest digressions from the Sun on either side, as close a conjecture as possible had to be drawn about the time of the conjunctions. But more fully on these matters below.

[Margin: Motion of the apogee, and its quantity.]

VIII. Eighth, when the study of observations was carried back further into more ancient times, it was decided to inquire into the places of the zodiac in which, formerly and later, these planets had held the apogeism [apogeismū] of their eccentric, or the greatest slowness of their first motion in longitude. And at last, after an interval of very long time, it was established that this does not occur at the same point of the zodiac, but at different points according to

the sequence of the signs, and that this apogee therefore moves in the order of the signs, but with so slight a motion that the astronomers up to Copernicus, with Ptolemy, Book 9, chapter 5, thought it equal to the motion of the fixed stars. And thus the auges of the five planets, according to the motion of the eighth sphere, complete 1° in 100 years, as the Ptolemaists hold, and complete the period in 36000 years; or, as the Alphonsines and Peuerbach's followers hold, in 49000 years. Copernicus, however, thinks that the apogee of ♄ by a peculiar motion of its own moves uniformly under the fixed stars by 36″ 40‴ 48⁗ a year, and completes its period in 35333 Egyptian years; the apogee of ♃ by 10″ 49‴ 26⁗ a year, completing its period in 119734 years; that of ♂ by 28″ 44‴ 37⁗ a year, and revolves to the fixed stars in 45088 years; the apogee of ☿ by 57″ 50‴ 38⁗ a year, but under the signifer, completing its period in 22700 years; but that the apogee of ♀ does not change under the fixed stars. Others hold otherwise, on whom see below.

[Margin: Motion in latitude?]

IX. Ninth, in the latitudes of these planets, taken from time to time with instruments, a remarkable diversity appeared. For their path was detected to be inclined to the ecliptic, with its nodes, as in the Moon, which they call in Greek [Greek: συνδέσμους (syndesmous, "bonds")], and limits, which they call [Greek: πέρατα (perata, "limits")] or umbilici or bellies. Further, as became clear when observations were compared with each other, the three superior planets, whenever they are around the apogee of the eccentric, have northern latitude, and around the perigee, southern. Mars has its greatest in the very apogee and perigee, or nearly; Saturn on this side of the apogee and Jupiter beyond the apogee have their greatest northern, just as the former on this side of the perigee and the latter beyond it have their greatest southern. The three superior planets stay longer in northern latitude than in southern; conversely ♀ and ☿ stay more in southern than in northern. And in ♀ the greatest latitudes are greater in the north, the contrary in ☿.

But in this too a twofold inequality appeared. For on comparing the greatest latitudes of several revolutions with each other, it became clear that they become greater still when the planet is placed in the perigee of the epicycle, and smaller in the apogee.

[Margin: Inclination of ♄ ♃ ♂: what it is and how great.]

Accordingly the ancient astronomers attributed the first cause of the greatest latitude to the inclination of the eccentric from the plane of the ecliptic, which was called [Greek: ἔγκλισις ἐκκέντρου (enklisis ekkentrou, "inclination of the eccentric")], that is, inclination of the eccentric, and by the Latins *Deviation*. It is measured by the angle made at the centre of the world by straight lines drawn from there, one through the plane of the ecliptic, the other through the plane of the eccentric; and its greatest, among the ancients, is in ♄ 2° 26′, in ♃ 2° 24′, in ♂ 1° 0′.

[Margin: Deviation of ♄ ♃ ♂: what it is and how great?]

The second cause of the varied latitude they attributed to the inclination of the epicycle, turning about the diameter of the mean longitudes, or about the diameter cutting the line of apsides orthogonally at the centre of the epicycle. By force of this libration its apogee is inclined towards the ecliptic when the centre of the epicycle is in the northern segment of the eccentric, and therefore it diminishes the northern latitude of a planet placed at the apogee of the epicycle, whereas, on the contrary, at the perigee of the epicycle it is increased; just as in the southern segment the southern latitude is diminished at the apogee and increased at the perigee of the epicycle. This, then, was called by the Greeks [Greek: ἔγκλισις ἐπικύκλου (enklisis epikyklou, "inclination of the epicycle")], that is, inclination of the epicycle, and by the Latins simply *Inclination*, understood of the epicycle to the plane of the eccentric. It is measured by the angle made at the centre of the epicycle by the diameter of the apsides of that epicycle and by a straight line drawn from the centre of the epicycle to the centre of the eccentric; and the greatest inclination reckoned by the ancients is in ♄ 4° 30′, in ♃ 2° 26′, in ♂ 2° and 15′.

Meanwhile, so that you may not be without any specimen of the observations of the ancients, the greatest latitude of the three superior planets is among them as you see in this little table, from which, however, recent observations differ not a little.

GREATEST LATITUDE

Planets	Northern in the epicycle: Apogee	Northern in the epicycle: Perigee	Southern in the epicycle: Apogee	Southern in the epicycle: Perigee
♄	2° 3′	3° 3′	2° 1′	3° 5′
♃	1° 6′	2° 5′	1° 4′	2° 8′
♂	0° 5′	4° 20′	0° 2′	7° 30′

[Margin: On the latitude of ♀ and ☿.]

X. Tenth, however, in Venus and Mercury, beyond the deviation of the eccentric circle from the plane of the ecliptic and the inclination of the epicycle from the plane of the eccentric towards north or south, a certain twisting [detorsio] of the plane of the epicycle from the plane of the eccentric has been observed, crosswise, to the right here and to the left there. This obliquity is called by the Greeks [Greek: λόξωσις ἐπικύκλου (loxōsis epikyklou, "obliquity of the epicycle")] and by the Latins reflexion [Reflexio] or reflection [Reflectio]. From these three causes and differences is believed to have arisen that far more remarkable variety of latitudes in these two planets which the observations have indicated. For once the places under the zodiac and the times of the latitudes, especially of the greatest, had been observed, three varieties were found.

[Margin: Deviation of ♀ and ☿.]

The greatest *deviation* of the eccentric, then, is reckoned at 10′ in ♀ and 45′ in ☿, approximately, and it occurs always when the centre of the epicycle is in the apogee of the eccentric or in the perigee. But it is not invariable, as it is in the three superior planets; on the contrary, it decreases from greatest to none

as the plane of the eccentric comes to coincide with the plane of the ecliptic, which happens when the centre of the epicycle is at the node. As the centre of the epicycle moves away towards the apogee of the eccentric's equant, the deviation grows, until at the apogee it becomes greatest, that of ♀ towards the north by about 10′, that of ☿ towards the south by about 45′. From there it decreases again until at the other node it comes to nothing; and from there it grows again, until at the perigee of the eccentric it becomes greatest, ♀ always to the north, about 10′, and ♀ [recte: ☿] always to the south, about 45′.

[Margin: Inclination of ♀ and ☿.]

The *inclination* of the epicycle to the plane of the eccentric, by which the diameter through the apogee and perigee of the epicycle is inclined to the north or to the south upon the diameter of the epicycle's mean longitudes, is at its greatest nearly 2° 30′ in ♀ and nearly 6° 15′ in ☿. This inclination is adjusted to the motion of the eccentric and of the longitude by this rule: when the centre of the epicycle occupies the apogee of the equant and the deviation becomes greatest, the inclination of the epicycle is none, and its plane lies in the plane of the eccentric. But when the centre of the epicycle recedes from the apogee of the equant, the inclination grows continuously, so that the apogee of the epicycle declines to the north in ♀ and to the south in ☿; and at the descending [deuehente] node of the eccentric with the ecliptic, where the deviation ceases, the inclination becomes greatest. As the centre of the epicycle recedes from the southern node, however, the inclination begins to diminish continuously, until, with the centre of the epicycle carried to the perigee of the equant, the inclination becomes none. As the centre recedes from there towards the northern node of the eccentric, the inclination of the epicycle's apogee is to the south in ♀ and to the north in ☿, and it grows continuously until it reaches its greatest at the northern node of the eccentric; and at last it decreases from there, until at the northern limit of the eccentric and at the apogee of the equant it is none. Therefore, when the deviation of the eccentric is none, the inclination of the epicycle is greatest, and when the deviation of the eccentric is greatest, the inclination of the epicycle is none. And although the deviation is always towards the same quarter, namely that of ♀ to the north and that of ☿ to the south, yet the inclination of the epicycle of ♀ is not to the north except in the northern semicircle of the eccentric, nor that of ☿ to the south except in the southern semicircle. The greatest latitude, which is derived trigonometrically from the greatest inclination, is in ♀ at apogee nearly 1° 3′ and at perigee nearly 7° 12′; in ☿ at apogee nearly 1° 46′ and at perigee nearly 4° 5′, according to the older authorities. But these are corrected by the reflexion, of which presently.

[Margin: Reflexion of ♀ and ☿.]

The *reflexion*, by which the diameter of the mean longitudes of the epicycle is moved about the diameter through the apogee and perigee of the same epicycle, and is thus bent crosswise so that the plane of the epicycle cuts the plane of the eccentric, is at its greatest close to 2° 30′ in ♀; in ☿ it is 2° 15′ at the apogee

of the eccentric and 2° 45′ at the perigee. The libration of this motion is so adjusted to the motion of the centre of the epicycle that it is none at the nodes of the eccentric and the ecliptic, that is, when the deviation is none. But as the centre of the epicycle ascends towards the apogee of the eccentric and the northern limit, the reflexion of the left or eastern semicircle of the epicycle is to the north in ♀ and to the south in ☿; and it grows until at the apogee of the eccentric it becomes greatest. From there it diminishes as far as the other node, where it dies out; and as the centre of the epicycle descends from there towards the perigee of the eccentric and the southern limit, the left portion of the epicycle is bent to the south in ♀ and to the north in ☿, and so on.

[Margin: Greatest latitude of ♀ and ☿.]

Once all the differences of deviation, inclination and reflexion that can concur to increase the latitude have been computed, the greatest latitude according to the older authorities is what you see in this little table, with the planets placed in the apogee or the perigee of the epicycle.

Planet	At the node of the eccentric	Or in the equated anomaly of the eccentric	Greatest latitude in the epicycle: Apogee	Greatest latitude in the epicycle: Perigee
♀	Northern	9ˢ	South 1° 3′	North 6° 22′
♀	Southern	3ˢ	North 1° 3′	South 6° 22′
☿	Northern	9ˢ	North 1° 46′	South 4° 5′
☿	Southern	3ˢ	South 1° 46′	North 4° 5′

[Margin: Motion of the nodes and limits.]

XI. Eleventh, it has been observed that the nodes and limits of greatest latitude of the five planets do not stay fixed under the same point of the zodiac, but advance with a slow motion in the sequence of the signs, if their motion is reckoned from the vernal equinox. If it is reckoned from the first star of Aries, however, they seem to move in the antecedents, because they are so slow that the apparent motion of the fixed stars in the sequence is faster than theirs; with the exception, however, of the motion of Mercury's node, which appears faster than the fixed stars, if what Ptolemy reported in his day about the place of that node is true. Copernicus, moreover, thinks the node of Venus stationary under the fixed stars. From Ptolemy down to our own time the node of ♄ has advanced 22°, of ♃ 5°, of ♂ 17°, of ♀ 18°, of ☿ 35½°. The Alphonsines, however, trusting in the slowness of this motion, thought that both the nodes and the apogees of these planets move with the motion of the eighth sphere.

[Margin: Motion of rotation.]

XII. Twelfth, a motion of rotation about the centre of its own body is attributed to all the planets by Hérigone, tome 5, p. 566; by Kepler, *Dissertatio cum Nuncio Sidereo*, p. 12, and in the *Epitome of Copernican Astronomy*, p. 549; and by Antonín Maria Šírek in his *Radius Sidereomysticus*, Book 4, chapter 2, member 3, on account of the conjecture I reported in Book 3, chapter 4, number 3. See also what we touched on here, chapter 3, Scholium 3.

[Margin: Variation of the eccentricity and the epicycle.]

XIII. Thirteenth, not only in Mercury formerly and later in Mars but, as it has seemed to us, in all these five planets, a variation of the eccentricity has been noticed, and indeed also a variation in the size of the semidiameter of the epicycle, on which more in chapter 6 of Section 2.

[Margin: Epiphonema.]

So many motions, then, and so wonderful, has the Best and Greatest implanted in these planets, and so great a knowledge of them has he granted to mortals, that by inquiring further they may advance in the worship of him and in veneration of his most certain Providence.

[Margin: Summing-up of the motions.]

Seven of them, to be exact. They are: the common motion of 24 hours; the proper motion in longitude, with a twofold anomaly and restitution, one free and referred to the zodiac, the other tied to the Sun; the motion in altitude; the motion of the apogees; the motion of the nodes; the motion of latitude, with a twofold anomaly in ♄ ♃ ♂, namely deviation and inclination, but a threefold one in the inferior planets, namely deviation, inclination and reflexion; and perhaps the motion of rotation and the change of the eccentricity and the epicycle.

SCHOLIA.

Scholia.

I. In reviewing the motions of the planets and their measure, we know that opinions vary; we shall give these below, in their proper places, in a single view. For now, let us hear what Vitruvius, Pliny the Elder and Martianus Capella, drawing from I know not what sources, have reported about them.

[Margin: Opinion of Vitruvius on the motions of the planets.]

So then Vitruvius, Book 9, chapter 4, after mentioning the circuits of Venus and Mercury around the Sun, their regressions backwards, their retardations and their stations, added that Mercury returns in 360 days to the place of the same sign from which it had begun its course, and that Venus stays about 40 days in any given sign, and after 485 days returns to the same place from which it began its journey; that Mars completes its revolution in about 683 days, and after its first station is swifter; that Jupiter, climbing with a gentler step against the daily motion, stays nearly one year in each sign, and accomplishes its period in 11 years and 363 days; further, that Saturn stays in one sign for 29 months and a few days more, and completes its restitution in 29 years and 160 days.

[Margin: Cause of station and retrogradation, from Vitruvius.]

He next adds that the three superior planets make a station when they have come to the trine aspect of the Sun with that degree of the zodiac in which they were joined to the Sun, and from there make their regressions, until the same Sun has crossed from that trine into another sign. He denies that the cause of this is that the Sun, by its distance, does not make the paths of the wandering

stars bright; for the Sun, even when the planets stand or retrogress, lights the whole world with open and evident splendour, so that it is seen by us, who are nonetheless less near to it. Instead, he brings forward another cause, saying: "Rather, therefore, this reasoning stands firm with us: just as heat calls out all things and draws them to itself (as we also see it raise fruits rising from the earth to a height through warmth, no less than the vapours of water from springs to the clouds through arcs), by the same reasoning the vehement impetus of the Sun, with its rays stretched out in the form of a trigon, draws to itself the stars that follow, and those running ahead it does not allow to advance, as if by reining them in and holding them back, but compels them to go back towards it and to be in the sign of another trigon." And he says that so great a force resides in the trigonism of the rays because of the equality of the sides, and so on. The Copernicans, however, assign the cause of stations and retrogressions to the annual motion of the Earth, through which it comes about that the planets appear to go back although in reality they are always advancing, as will be explained below.

[Margin: Opinion of Pliny the Elder on the motions of the 5 planets.]

II. Pliny the Elder, Book 2, chapter 8, first thinks that all the wandering stars are driven with a contrary and leftward motion against the headlong motion of the world to the right, "so that the air, rolled round in the same direction by the eternal rotation of the world, may not grow torpid as an inert mass, but may be split by the opposing blow of the stars, separated and spread out." He adds that Saturn is carried round in about 30 years, Jupiter in 12, Mars in 2; and that Jupiter is wholesome for this reason, that it is tempered on the one side by the rigour of Saturn and on the other by the heat of Mars, which is fiery from the neighbouring Sun and is called by some the star of Hercules. He next subjoins: "The star of Venus, wandering in alternating course and by its very surnames a rival of the Sun and the Moon: for, going before and rising ahead of the morning, it receives the name of Lucifer, as a second Sun, hastening the day; by contrast, shining out from the west, it is called Vesper, as prolonging the light and taking the Moon's turn.

[Margin: The time of Pythagoras, and V. C.]

Pythagoras of Samos first detected this nature of it, about the 42nd Olympiad, which was the 142nd year of the city of Rome." Before that, some indeed had thought Lucifer and Hesperus to be two distinct stars. He adds that its brightness is so great "that shadows are cast by the rays of this one star." Kepler, however, affirms in the *Optics*, p. 263, that distinct lights on a wall were seen by him, not only of Venus but of Jupiter and Spica, which we too have seen more than once with ♃ and ♀. Nonetheless the surrounding light of twilight, bounding the shadow, helps the distinct seeing of Venus's shadow. Pliny the Elder goes on and says that, according to a tenet of Timaeus, Venus never stands off from the Sun by more than 46°; and that the star of Mercury, called by some the star of Apollo, in a lower circle shines for 9 days on the one hand before the rising of the Sun and as many on the other after its setting, and is not distant from the

Sun beyond 23°, in the opinion of Sosigenes. This he repeats also in chapter 17, where he wraps up rather than unfolds many things about these five planets, I believe from the opinion of Sosigenes.

[Margin: Things worth noting about the planets, from Pliny the Elder.]

These are the clearer points: that Mars never makes a station with Jupiter in trine, and rarely in sextile with ♃, and does not rise except in ♋ and ♌; that Mercury has its evening risings rarely in ♓ and most frequently in ♍, its morning risings frequently in ♊ and in ♒ and most rarely in ♌; that it does not become retrograde in ♉ and ♊, nor in ♋ on this side of the 25th degree; that Saturn and Mars lie hidden, at the most, for 170 days; Jupiter 36, but at the least 10; Venus 69, or at the least 52; Mercury 13, or at the most 17. In these passages he is not always speaking of heliacal occultation, as Jacob Milich thought, who nonetheless rightly adds that Pliny the Elder here speaks in a loose manner. I think he had regard to the rather crude observations of his own or of the preceding age.

[Margin: Opinion of Martianus Capella.]

III. Finally, Martianus Capella, *De Nuptiis Philologiae et Mercurii*, Book 8, towards the end, hammers out on these matters many things that call for the file. The more notable are these: that Mercury cannot stand off from the Sun beyond 32 degrees, nor be away from the Sun's light within 20 moments [momenta], and completes its period every fourth month; that Venus has a latitude of 12°, at fifty moments from the orb of the Sun, but does not depart beyond 46°; that it goes direct for eleven months; that it alone among the five planets casts a shadow and is seen by day; that at its morning rising it generally stays for four months, at its evening rising never more than twenty days; that its revolution is renewed in 19 months. That Mars runs out to a latitude of 5°, has its apogee at 29° ♑, and makes both stations in quadrate aspect with the Sun. That Jupiter ranges to a latitude of 5° and has its apsis at ♋ 15°. Lastly, that Saturn has its apsis at the 20th degree of ♎, that it makes its morning station at a digression of 120° from the Sun, and its evening station when it is the same number of degrees from the Sun. Finally he too, with Vitruvius, refers the causes of these diversities to the Sun; for he concludes Book 8 with these words:

[Margin: Physical cause of the retrogradation, etc., of the planets, from Capella.]

"But the diversities of course and the causes of altitude, of standing, of going back and of advancing are brought to all the above by the shining ray of the Sun, which, striking them, either lifts them on high, or presses them down into the depth, or makes them decline in latitude or go retrograde."

[Margin: Opinion of Kepler, Wendelen and Šírek on the physical cause of the slow and swift motion of the planets is indicated.]

IV. These things, to be sure, partly favour the opinion of Kepler, who refers them to the magnetic tractions of the Sun, by which it seizes the planets and leads them round, in the *Commentaries on Mars*, chapter 57, and in the *Epitome of Copernican Astronomy*, from p. 571. Not to mention the opinion of Wendelen,

who, as Antonín Maria Šírek reports in Book 4 of the *Radius Sidereomysticus*, chapter 2, held that the Sun once ejaculated the remaining planets, as Vesuvius and Etna or Mount Hekla eject fiery stones. He himself, however, thinks that the planets are carried round by the Sun, and that those nearer to it are therefore carried round faster: the spots, for instance (which he calls solar stars), in 27 or 30 days, Mercury in 87, and so on. I said "partly", because on other points Kepler dissents; indeed, in the *Commentaries on Mars*, chapter 6, he convicts of falsity the opinion of the Latins, who attribute the speed and slowness of the planets to the aspects and rays of the Sun, because Saturn begins to retrocede in quadrate of the Sun or beyond, Jupiter in trine, Mars in biquintile or before the sesquiquadrate [ſeſquadrum], all at an inconstant interval.

CAPVT VIII.

Chapter 8.

Indicantur Hypotheſes quibus Planetarum quinque Minorum motus Explicari, ac de Machinis quibus repræſentari ſolent.

The hypotheses are indicated by which the motions of the five minor planets are usually explained, and the machines by which they are usually represented are discussed.

I. I find among the authors eight species of hypotheses, by which astronomers have until now been accustomed to draw and set forth, in the astronomical dust, the variety and anomaly of the motions indicated in the preceding chapter.

[Margin: 1st species of hypotheses.]

The first is by means of solid but homocentric orbs, that is, concentric with the world, accelerated and retarded by various circitors [circitoribus] and anti-circitors [anticircitoribus]. These were employed by Callippus, Eudoxus and Aristotle, *Metaphysics* 12, with very many Peripatetics; and Girolamo Fracastoro in his *Homocentrica*, Giovanni Battista Amico in his opuscule *De motibus corporum caelestium*, and a few others have tried to revive them. But they have been rejected by the better astronomers, because they do not maintain the motion up and down from apogee to perigee, which is contrary to the evident difference of the parallaxes, at least in the Moon; and because solid orbs conflict with the passages of comets through the aether.

[Margin: 2nd species.]

The second is that of Hipparchus and Ptolemy, which most of the Arabs, Greeks and Latins embraced down to Copernicus. They use a double eccentric circle, by which they set forth the first motion, free from the syzygies with the Sun and from relation to it, and its first anomaly. One eccentric indeed carries the centre of the epicycle, but it does not move uniformly in the arc of its own circumference, that is, with respect to its own centre, but with respect to the centre of the other eccentric; and this other eccentric is called the equant for this reason, that its equal arcs are the measure of the mean motion with which the former eccentric carries the centre of the epicycle. The epicycle

carries the planet, and by its own motion the second motion of the planet, and its second inequality, and the revolutions of the planet to the Sun, it brings to completion. Some Arabs, Sacrobosco, Peuerbach, and Magini in their *Theoricae* have stuffed this hypothesis with matter and solid orbs. In this hypothesis, one thing indeed is praiseworthy: that it makes the planet's motion perfectly circular over a completed period. But this has seemed unpraiseworthy: that the planet's uniformity of motion is begged from another's eccentric, in which it is not itself and by which it is in no way carried, while meanwhile it moves with a true and physical inequality in its own proper eccentric. This seemed unworthy to Copernicus and unbefitting the simplest bodies, which require uniform motion. Moreover, for this hypothesis the second inequality ceases at the mean ♂ and ♌ with the ☉, which ♂ and ♌ are something fictitious and scarcely ever really occur.

[Margin: 3rd species.]

The third species, therefore, devised by Copernicus, Book 5, chapter 4, explained by Mästlin in the appendix to Kepler's *Prodromus*, and indicated by Kepler in the *Commentaries on Mars*, chapter 4, uses a single eccentric and a single epicycle, with the orbs so proportioned and the motions so ordered that the motion of each is uniform with respect to its own centre. But although this preserves uniformity of motion, it nevertheless twists the planet's path into various bends and drives it out of a perfectly circular way. Further, it measures the 2nd inequality of the planet from the mean ♂ and ♌ with the ☉, as from termini, when the true ones should rather have been considered; and finally it supposes an annual motion of the Earth.

[Margin: 4th species.]

The fourth species, also of Copernicus, and indicated by the same authors as the third, contains only two eccentrics with one small circle varying the eccentricity. By this, likewise, the uniformity of motions in the proper circle is indeed saved, but the circular path is not saved with respect to the line which the planet describes by its motion; and here too the second inequality is ascribed to the mean rather than the true motion of the ☉, and to the annual revolution of the Earth.

[Margin: 5th species.]

The fifth species, discovered by the same Copernicus, explained more fully above by Kepler, and selected by Tycho and Longomontanus and adopted as their own, is by means of one concentric with two epicycles, with their motions and the symmetries of their semidiameters so ordered that it is as nearly as possible equivalent to the Ptolemaic, but retains uniformity of motion about its proper circle; granted that it departs from a perfectly circular path of the planet and multiplies circles excessively, and so on.

[Margin: 6th species.]

The sixth species is that of Lansbergen in the *Theoricae*, who uses a single eccentric whose centre is carried round by a small circle placed near the centre of the universe. This too keeps uniformity of motion with respect to its proper

centre safe and sound, but it draws the planet's path away from the course of a perfect circle; and it ties the second anomaly of the planets to the annual motion of the Earth rather than of the Sun, and to the mean motion rather than the true. Hérigone, for his part, transferred it to the hypothesis of a stationary Earth.

[Margin: 7th species.]

The seventh species is that of Kepler in the *Commentaries on Mars* and in the *Epitome of Copernican Astronomy*, and also of Boulliau in his *Astronomia Philolaica*, towards which Hérigone also inclines. This species both rests on the annual motion of the Earth and makes neither a physical uniformity nor a perfectly circular path of the planet. Instead, it supposes that the planet moves in an ellipse, with the Sun standing in one of the umbilici or foci; moves about it, I say, with a physical inequality, so that in equal times it completes unequal arcs of the elliptical circumference on this side of the centre of the ellipse, and is retarded or accelerated in its motions according to the ratio of greater or lesser distance. Yet because of the simplicity of the planetary path, which accomplishes everything with a single circle or a single ellipse, and because it has regard to the true motions of the Sun, this hypothesis has pleased many.

[Margin: 8th species.]

The eighth species is our own. In it the uniformity of the first motion about its proper centre is preserved, and the eccentricity is not bisected, but varied by a libration of the eccentric's centre up and down according to second sines corresponding to the mean anomaly of the eccentric. The motion of the second inequality is accomplished in an epicycle, but in one that is different from time to time, made through a spiral and varying its semidiameter, according to ratios similar to the variation of the eccentricity.

[Margin: 9th species.]

The ninth species, indicated long ago by al-Biṭrūjī in his *Theorica Physica Planetarum*, but accepted not only by Franciscus Patritius [Francesco Patrizi], by Io. Antonius Delphinus [Giovanni Antonio Delfino] in his book *De caelestibus globis et motibus*, by Biancani in the *Sphaera* throughout, and by not a few more recent authors, gathers all the motions of a planet into one bundle and reduces them to a single spiral motion made from east towards west, but slower in the inferior and faster in the superior planets, so that another motion into the east, contrary to the first, only appears to exist and does not. This spire, since it takes place up and down and to the sides of the world, nevertheless embraces in itself all the motions of the planet. This hypothesis, though perhaps it is the truer, and the one which the Intelligences actually use in moving the planets, as we shall say in Book 9, and may therefore be called Angelic, cannot, so to speak, be called Human; that is, accommodated to human capacity and to demonstrating geometrically the motions of the planets and their equations, so that from uniform motions the apparent inequality at any given time can be shown by linear descriptions and diagrams, explained by numbers and extended to future times. For it perpetually disagrees with itself, so much so that no spire

answers to another as regards the interval of time, and none passes into the neighbouring one by a bend of the same size.

[Margin: Kepler's opinion on the spires of the planets.]

On this spire Kepler, *Commentaries on Mars*, chapter 1, speaks thus: "If anyone should put all these together in one bundle, and at the same time believe that the Sun really moves through the zodiac in the space of a year, as Ptolemy and Tycho Brahe believed, then it is necessary to concede that the circuits of the three superior planets through the aethereal space, composite as they are of several motions, are really spiral; not, as before, in the manner of wound thread, with the spires ordered next to one another, but more truly in the figure of a Lenten loaf [panis quadrageſimalis], in about this manner." The same author sets out this figure in

Diagram. A woodcut filling the width of the right column, without a printed title: a single continuous spiral line winding round a small central circle, forming a dense interlaced rosette within an outer boundary formed by its own widest turns. Around the central circle the line makes small inward loops. Labels read:

- centre of the small circle: `a`, with a dot beside it
- within the small circle, left of `a`: a sort that reads as `6` on the block; the text below cites this circle as `a b`, and this sort is not otherwise labelled [?]
- inner loops, right of the small circle: `d`, `e`, `f`
- star on the line, right of the centre: `c`
- star on the line at the outer left: `h`
- lower left, cut in the block: in h est / finis 1596. (at h is the end, 1596)
- lower right, cut in the block: in c / initium 1580. (at c is the beginning, 1580)

Labels not read: none known. No label `b` is separately identifiable on the block unless it is the sort listed above.

notes in the margin, in these words: "This is an accurate delineation of the motions of the star of Mars, which it runs through the aethereal air from the year 1580 to the year 1596, if it is true that the Earth stands still, as Ptolemy and Brahe hold. To continue those motions further would have been tangled, for the connection is infinite, never returning into itself." And after saying that the small circle a b encloses around the Earth a the sphere of Venus, of Mercury and of the elements, he added: "We are forced to ascribe similar spires also to the remaining four, and to Venus indeed much more tangled ones, if the Earth stands still, etc. Copernicus, by attributing one annual motion to the Earth, altogether strips all the planets of these most tangled spires, leading all the planets into single, quite bare orbits, as nearly as possible circular. This one and the same orbit it runs through, in the time just mentioned, as many times as you see here little crowns twisted towards the centre, with one subtracted, namely nine times, while meanwhile the Earth runs through its circle sixteen times." The spire of Mars began in the year 1580 at c and ended in the year 1596

at h. It has been our pleasure to indicate these hypotheses, which we are going to set out one by one in the following Sections.

[Margin: Planetary automata.]

II. From the hypotheses already mentioned, and especially from the ancient or Ptolemaic and Alphonsine one, many have tried to construct machines and automata by which to put the motions of the planets before the eyes, through various armillaries and circles, for any moment of time, and to rival the glass sphere of Archimedes. Especially Ioannulus

[Margin: Automaton of Iannellus of Cremona.]

Torrianus, commonly known as Iannellus of Cremona [Giovanni Torriani], the most ingenious craftsman of Charles V, who built an automaton consisting of fifteen hundred copper circles, gilded all over, with such subtlety that it represented all the celestial motions exactly.

[Margin: Automaton of Christian, King of Norway.]

No less celebrated is that silver automaton [Greek: αὐτόματον (automaton, "self-moving")] of the celestial motions, containing a noble synopsis of them, which Christian, King of Norway and Denmark and founder of the Academy of Copenhagen, sent among other gifts to the Caesar, Duke of the Muscovites, to obtain peace from him. But the barbarous Muscovite, boasting that he could arm two hundred thousand horsemen, kept the other gifts and sent the automaton back to the Dane, declaring that it was useless for him to concern himself about heaven when the two of them had to fight it out with arms over the earth.

[Margin: Automaton of Maximilian and Ferdinand.]

Of a more civilized temper, certainly, Suleiman, Emperor of the Turks, most gladly accepted a very similar instrument from the Emperor Ferdinand, though one built from the design of the Emperor Maximilian.

[Margin: The *Astronomicum Caesareum* of Peter Apian.]

[Margin: The *Aequator Planetarum* of Schöner and Sarzosus.]

[Margin: The *Monalosphaerium* of Fernel.]

III. At less expense, however, though with no less ingenuity, others tried to display the same motions on planispheres by means of paper wheels, and to lessen the labour of astronomical calculation: Petrus Apianus [Peter Apian], in that most artful work which he named the *Astronomicum Caesareum* and dedicated to the Emperor Charles V and his brother Ferdinand, King of the Romans, besides the instrument of the primum mobile; Franciscus Sarzosus [Francisco Sarzosa] of Cella, an Aragonese, in his *Aequator Planetarum*, and Johannes Schöner in his *Aequatorium Astronomicum*; and Ioannes Fernelius [Jean Fernel] of Amiens in his *Monalosphaerium*. All of these, however, rest on Ptolemy's theorics and the Alphonsine Tables. Reinhold attempted something similar in his work on Peuerbach's *Theoricae*, p. 95, and, more successfully and most recently,

[Margin: The *Rota Planetaria* of Cavalieri.]

[Margin: Hypothesis of Šírek.]

Father Bonaventura Cavalieri in his *Rota Planetaria*, which rests on the tables and hypotheses of Lansbergen. Nor does the hypothesis of Širek [Schyrlæi], which uses a transferred ecliptic, serve the heavens; it serves those charts of his, and the calculation to be completed by means of them.

[Margin: Kepler's censure of Apian and the automaton-makers.]

[Margin: Eulogy of Apian.]

IV. But Kepler, having concluded in his *Commentaries on Mars* [Commentario de ſtella Martis], chapter 14, that the planes of the planetary eccentrics are *Atalanta* — that is, that they do not librate or nod this way and that toward north and south but remain in perfect equilibrium, that their inclination to the ecliptic does not vary at all, and that the plane of the Ptolemaic epicycle keeps a perpetual parallelism to the ecliptic — attacked Apian and his imitators, the instrument-makers [organopœios], with a censure perhaps too severe, in these words:

Who now will give me a fountain of tears with which to bewail, as it deserves, the pitiable industry of Apian, who in his Caesarean work, following the authority of Ptolemy, spent so many good hours and squandered so many most ingenious meditations, in order to express with spirals, little crowns, helices and volutes, and that whole labyrinth of most intricate windings, figments of men which nature plainly does not recognize as her own? But that man shows us that with the divine gifts of a most penetrating intellect he could easily have been nature's equal. For the rest, he delighted his own mind with these juggleries (in which he had challenged nature herself), most bravely overcome and cast into diagrams, and from them won the palm of lasting fame, whatever damage that fortune may have done to the works themselves. But what shall we say of the empty art [Greek: κενοτεχνίᾳ (kenotechnia, "empty artifice")] *of the automaton-makers, who fabricate six hundred, indeed twelve hundred wheels, so that they may triumph over latitudes (that is, over human figments) expressed in their works, and raise the price of them?*

So he writes.

SECTIO SECVNDA DE MOTV LONGITVDINIS SATVRNI IOVIS ET MARTIS Eiuſque Theorica, & Hypotheſeos Fundamentis.

Section Two: on the motion in longitude of Saturn, Jupiter and Mars, and its theory, and the foundations of the hypothesis.

CAPVT I. De Antiqua ſeu Ptolemaica Hypotheſi vſq. ad Copernicum vſitata.

Chapter 1: on the ancient or Ptolemaic hypothesis, in use up to Copernicus.

[Margin: Followers of the Ptolemaic hypothesis.]

I. This hypothesis of which we are speaking, begun by Hipparchus, was completed and perfected by Ptolemy, who set it out in Book 9 of [Greek:

περὶ τῆς μαθηματικῆς συντάξεως (peri tēs mathēmatikēs syntaxeōs, "On the Mathematical Composition")], from chapter 5. The Arabs received and developed it, first among them al-Battānī, chapter 45; al-Farghānī, Differentia 12; Jābir ibn Aflaḥ, Book 7 of his *Astronomy*; then, among the Greeks, Proclus, chapter 4 of the *Hypotyposes*; and very many of the Latins, foremost among them Johannes de Sacrobosco and his commentators, listed at the start of Book 1; Georg Peuerbach in his *Theoricae*, with his scholiasts Nonius, Osualdus [Erasmus Oswald Schreckenfuchs], Reinhold and Wurstisen. Likewise Regiomontanus in the *Epitome of the Almagest*, Book 9; Luca Gaurico in his notes on the *Almagest*; Fernel, *Cosmotheoria*, Book 1 from chapter 5, and Book 2, chapter 2; Francesco Maurolico, *Cosmographia*, Dialogue 3, from p. 93; Alexander Piccolomineus [Alessandro Piccolomini] in the *Theorica Planetarum*, from p. 36 to p. 45; Johannes Schöner in the *Aequatorium Astronomicum*; Egnatius Dantes [Egnazio Danti] in the *Tabulae Mathematicae*; Giovanni Antonio Magini in the *Theoricae Planetarum* and in Canon 31 of the *Secunda Mobilia*, though he transferred the numbers of the motions from Copernicus into Ptolemaic form. Besides these, some who are not committed to this hypothesis nevertheless explain it, such as Antonius Deusingus [Anton Deusing]; Kepler, *On Mars*, chapters 2 and 4; Longomontanus, *Theorica*, Book 1, chapter 2, and Book 2 throughout; Petrus Herigonus [Pierre Hérigone], *Cursus Mathematicus*, volume 5, *Theorica*, Book 1, proposition 8; and Pierre Gassendi, *Institutio Astronomica*, Book 2, from chapter 6.

Schema Ptolemaicæ Hypotheſis.

Diagram of the Ptolemaic hypothesis.

[Margin: Deferent.]

II. In the following figure let A be the centre of the world and AB the semidiameter of the ecliptic, and with that interval describe the ecliptic BHCPB; and let the whole CAB be the line of apsides. On it take a centre D, a little distant from the world's centre A, and with the eccentric's semidiameter DF describe the eccentric FKMLF, which is called the deferent [Deferens], because it carries [defert] the epicycle.

[Margin: Epicycle.]

[Margin: Equant.]

Then at the points F and M of its circumference, where it is cut by the line of apsides, make centres, and with the interval of the semidiameter FG on the one side and MV on the other describe the epicycle GQ and, equal to it, OV. After that, above D on the line of apsides, take another centre E, as far from the deferent's centre D precisely as the centre D is from the world's centre A; and with the semidiameter ER, equal to the deferent's semidiameter DF, describe another circle eccentric to the centre of the world, and let it be KOLR. This eccentric is called the equant [Æquans], because the centre F of the epicycle,

carried in the order of the signs by the deferent, moves uniformly about E, the centre of this eccentric, or traverses

Diagram. Woodcut set in the left column: the Ptolemaic hypothesis of the three superior planets. A large outer circle (the ecliptic) has its centre at A. A vertical line of apsides runs from B at the top through G, R, F, E, D, A, O, M and V to C at the foot, with a horizontal diameter through A. Inside are two overlapping eccentrics about D and E. Three epicycles are drawn: one at the top on the line of apsides, with centre F; one at the foot, with centre M; and one displaced to the upper left, with centre I. Straight lines run from A and E to the left-hand side of the outer circle, with a dashed line among them.

- B (top of the outer circle), with a zodiac sign beside it read as ♋
- G (top of the upper epicycle)
- R, F (on the vertical line, inside the upper epicycle)
- b, d (left and right of F, on the upper epicycle)
- Q (foot of the upper epicycle)
- T, u, S, X, Z (upper left, at or near the outer circle, in that order downward)
- I (centre of the displaced epicycle)
- n (lower edge of the displaced epicycle)
- K, N, a (left, one below another, at the inner circles)
- E, D (on the vertical line above A)
- L, Y, m (right, one below another, at the inner circles)
- A (centre)
- H (left end of the horizontal diameter), with a zodiac sign beside it read as ♎
- P (right end of the horizontal diameter), with a zodiac sign beside it read as ♈
- O (top of the lower epicycle)
- M (centre of the lower epicycle)
- V (foot of the lower epicycle)
- C (foot of the outer circle), with a zodiac sign beside it read as ♑

Labels cited in the text but not found on the block: none.

in equal times equal arcs, not of the deferent but of this eccentric, intercepted by lines drawn from the centre E through the centre of the epicycle to the circumference of this second eccentric; such is the arc IR.

[Margin: Why it is called the equant.]

We explained this with another diagram in Book 3, chapter 22, on the occasion of the equant introduced by some into the theory of the Sun. For this reason Ptolemy calls the centre E the point of equality, and the circle described from it [Greek: ἔκκεντρος ὁμαλῆς κινήσεως (ekkentros homalēs kinēseōs, "eccentric of even motion")], that is, the eccentric of regular motion.

De Excentricitatibus & Apogeis.

On the eccentricities and apogees.

[Margin: What the eccentricity is, and how many kinds there are.]

III. With these things laid down, the eccentricity, that is, the distance from the world's centre A, will be, for the centre of the deferent, the line AD; and for the centre of the equant, the line AE, which is always double the eccentricity AD, except in the hypothesis of Mercury. The apogee, or highest apsis, or aux, is considered threefold in this theory.

[Margin: The apogee of the eccentric and its line.]

1. The apogee of the deferent eccentric, which is the point F. Its longer longitude, or greatest distance from the world's centre, is the line AF; opposite it stand the perigee M and the nearer longitude AM.

[Margin: What the mean apogee is, and what its line is.]

2. The mean apogee of the epicycle, which is the point G. Its longer longitude is the line EG; opposite it stands the mean perigee of the epicycle, which is the point O, and its nearer longitude, the line EO. The mean apogee of the epicycle is the point on the epicycle's circumference on which there falls a straight line produced from the equant's centre through the epicycle's centre, as is not only the line EG drawn through F but also E g drawn through I; for it is considered outside the line of apsides as well, in whatever position the epicycle's centre may be, and it is not unique, as the apogee of the eccentric is, but multiple in number.

[Margin: What the true apogee of the epicycle is, and what its line is.]

3. The true apogee of the epicycle, which is the point on the epicycle's circumference on which there falls a straight line produced from the world's centre through the epicycle's centre, such as the line AFG and A I u. So in the first example the same point G is both the mean and the true apogee of the epicycle, just as the point O is both the mean and the true perigee of the epicycle; for they coincide whenever the epicycle's centre is in the apogee or the perigee of the eccentric. But in the second example the true apogee of the epicycle is the point u; hence this too is multiple, and as many in number as the mean apogee is. The point B, however, under which the apogee of the eccentric lies, is not the apogee but the place of the apogee in the ecliptic. For Mercury's eccentricity, however, a small circle must be conceived that carries the equant's centre on its circumference, and whose semidiameter is as large as the least eccentricity.

De Duodecim Lineis in hac Hypotheſi Memorabilibus.

On the twelve lines worth noting in this hypothesis.

[Margin: 1. Line of apsides.]

IV. The line of apsides is the straight line through the centres of the world and of the eccentrics, and through the apogee and perigee of the eccentrics,

produced on both sides as far as the ecliptic; such is the line BC, drawn through the centres A, D, E and the points F, M.

[Margin: Diameter of apsides of the eccentric.]

This line, restricted to the eccentrics, is called the diameter of apsides of the eccentric, as FDM is.

[Margin: 2. Line of the mean apogee.]

The line of the mean apogee is the straight line from the equant's centre, produced through the epicycle's centre as far as the epicycle's circumference; such are both EFG and E I g.

[Margin: 3. Line of the true apogee.]

The line of the true apogee is the straight line from the world's centre, produced through the epicycle's centre as far as the epicycle's circumference, as is both the line AFG and the line A I u.

[Margin: 4. Line of mean motion in the equant.]

The line of the mean motion of the epicycle's centre, considered in the equant, is the straight line from the equant's centre, produced through the epicycle's centre as far as the equant's circumference; such is the straight line EFR, and EI, if you consider the letter I as on the equant's circumference.

[Margin: 5. Line of mean motion in the zodiac.]

The line of the mean motion, or mean place, of the epicycle's centre in the zodiac is the straight line from the world's centre to the zodiac, produced so as to be parallel to the line of mean motion considered in the equant, that is, to the straight line produced from the equant's centre through the epicycle's centre, as far as the zodiac if you please; such is the straight line AZ, which is to be parallel to the line EIS. For in order to have in the zodiac an arc similar to the arc of the equant, it must be measured by an equal angle, and therefore one made at the centre of the zodiac or of the world. So if the epicycle's centre has come from F to I and has completed the equant's arc RI, measured by the angle IER, of 45° for example, then if you produce the straight line EI to S, the arc SB is not of 45°, because the angle BES does not measure it, since it is not made at the centre from which the arc BSZ etc. has been described. But if you draw the already mentioned parallel from A, the angle BAZ measures the arc BZ and is equal to the angle BES on account of the parallelism, by 29 of Book 1 of the *Elements*. The use of this arc will become clear later.

[Margin: 6. Line of the true place of the epicycle's centre.]

The line of the true place of the epicycle's centre, or of the true longitude of the epicycle's centre, is the straight line from the world's centre, produced through the epicycle's centre as far as the zodiac; such is the line AFB, if the epicycle's centre is at F, and AIT, if it is at I.

[Margin: 7. Line of the true place of the planet.]

The line of the true place of the planet is the straight line from the world's centre, produced through the centre of the planet's body as far as the zodiac; such is the line A n X, if the planet is at the point n of the epicycle's circumference.

[Margin: Corollary.]

It is clear, then, that when the epicycle's centre is in the apogee of the eccentric, all of the first six lines run together into one and the same line, though not taken at its full extent; indeed the seventh does too, if the planet is then in the apogee or perigee of the epicycle. And say the same, proportionally, if the epicycle's centre is in the perigee of the eccentric.

[Margin: 8. Line: diameter of apsides of the epicycle.]

The diameter of apsides of the epicycle is the straight line drawn from the true apogee of the epicycle to its true perigee through its centre, in the plane of the circle around whose circumference the planet moves; such is G F Q. This line does not always coincide with the diameter of apsides of the eccentric; indeed, it is not always in the plane of the eccentric that carries the epicycle, but is mostly inclined to it, as we said in Section 1, chapter 7, numbers 9 and 10.

[Margin: 9. Line: diameter of mean longitudes of the epicycle.]

The diameter of mean longitudes of the epicycle is the straight line drawn to opposite points of the circumference through the epicycle's centre, in the plane in which the epicycle's diameter of apsides lies, and at right angles to that same diameter; such is the straight line b F d. It is called "of mean longitudes" because its end-points b and d lie at the middle of the elongation from the true apogee, and divide in two the semicircle of the whole elongation, G b Q or G d Q.

[Margin: 10. Line: diameter of mean longitudes of the eccentric.]

The diameter of mean longitudes of the eccentric is the straight line drawn to opposite points of the circumference through the deferent's centre, in the plane of the deferent, at right angles to the line of apsides; such is the straight line NDY. It is so called both because at its ends the epicycle's centre stands midway between the apogee and perigee of the eccentric, and because at these points occurs the [Greek: πάροδος μέσος (parodos mesos, "mean passage")], that is, the mean transit from the greatest of the equations of the centre or toward the greatest; for Reinhold shows in his work on Peuerbach's *Theoricae*, p. 100, that this reaches its greatest at these points.

[Margin: 11. Line of mean longitude, or distance.]

The line of mean longitude, taken for the average distance of the epicycle's centre from the world's centre, is the straight line drawn from the world's centre to the epicycle's centre and equal to the eccentric's semidiameter; such is the line A a or A m, if the epicycle's centre is at a or at m. These points are determined by a straight line drawn at right angles through the line of apsides and bisecting the eccentricity of the deferent; such is the straight line a m, which some therefore take for the line of mean longitudes.

[Margin: 12. Line: the radius.]

Finally, the radius is the semidiameter of the eccentric, and it too is equal to the average distance of the epicycle's centre from the world's centre: as are DF, DN and DY.

The remaining definitions, concerning the arcs of the motions and of the equations, we shall give more suitably below.

Eadem Hypotheſis per Orbes Solidos explicata.

The same hypothesis explained through solid orbs.

V. Here we shall set out solid orbs: not those concentric with the world which Aristotle with Eudoxus assumed, or those which Fracastoro assumed in his *Homocentrica*, but those which the Arabs and their principal imitator Peuerbach introduced into Ptolemy's bare hypothesis, abstracted from this solidity. In the following diagram, then, let there be

Diagram. Woodcut set in the left column: the same hypothesis in solid orbs. A heavy black crescent lies between the outer circle and an inner eccentric circle, thin at the top and broad at the foot. Inside it is a white ring holding the two epicycles, at the top and at the foot on the vertical line. Inside that is a second black crescent, broad at the top and thin at the foot, around a white central circle. A horizontal line runs across the figure through A, a shorter one through D from N to Y, and a vertical line through the centres. Two thin circles run through the white ring.

- B (top, above the black ring)
- G (top of the upper epicycle)
- R, F (on the vertical line, inside the upper epicycle)
- Q (foot of the upper epicycle, at the edge of the inner black crescent)
- K, N (left, one below the other, in the white ring)
- Z (left, at the outer edge of the inner black crescent)
- E, D (on the vertical line, E at the edge of the central circle, D below it)
- T (right, at the outer edge of the inner black crescent)
- L, Y (right, one below the other, in the white ring)
- H (left end of the horizontal line, outside the black ring)
- A (centre)
- P (right end of the horizontal line, outside the black ring)
- X (foot of the central circle)
- O (top of the lower epicycle)
- M (centre of the lower epicycle)
- V (foot of the lower epicycle)
- C (foot, below the black ring)

Labels cited in the text but not found on the block: none.

the line of apsides BAC, on which lie the centres of the world A, of the deferent D, and of the equant E; and let the straight line HP cut it in two at right angles at the centre A. From the centre A describe the circumference BHCP, and from the centres D and E the eccentrics as above, namely the deferent FKML and the equant RKOL; then from the points F and M the equal epicycles GQ and OV. After that, from the centre D, with the interval DG, describe the circumference GV, passing through G and V, the apogees of the epicycles, and colour the space between it and the circumference BHCP black. From the same centre D describe another circumference QZOT through Q and O, the perigees of the epicycles; and from the centre A describe the circumference EX,

which encloses the three centres within its circle, in such a way that the upper part EQ is wider than the lower XO; and fill the space enclosed between it and the circumference QZOT with black. Finally, through the centre D draw NDY, the diameter of mean longitudes of the eccentric. When this has been done, 5 solid orbs are to be conceived in this plane figure, belonging to the theory of Saturn, Jupiter, Mars and Venus; for Mercury requires several more.

[Margin: 1. Orb.]

The first orb is the innermost and white, EX, which consists of the lower spheres enclosed in the sphere of these planets, namely the elementary sphere, that of the Moon, etc.

[Margin: 2. Orb carrying the apogee.]

The second orb is the outermost and black, concentric with the world in its convex surface BHCP but eccentric in its concave surface GV. This orb carries the apogee of the eccentric in the order of the signs from B to H, about the centre A of the ecliptic and of the world, and it moves in the plane of the ecliptic, granted that F, the apogee of the eccentric, is not in this plane but in the plane of the deferent, which, as I said in Section 1, chapter 7, numbers 9 and 10, is inclined to the plane of the ecliptic. And although the plane of the ecliptic is cut by the plane of the deferent at the centre of the world, a common section indicated by the line HAP, yet because the centre D of the deferent is outside the centre of the ecliptic, it follows by Euclid's definitions in Book 3 of the *Elements* that the upper segment of the deferent is the larger — the one, that is, in which the centre D lies, and whose height is AF — and the lower segment, from A to M, the smaller.

[Margin: Path of the apogee.]

And because the apogees of the three superior planets are perpetually toward the north, it comes about that the centre of the deferent is toward the north, and the greater part of the deferent's axis likewise toward the north. Finally, it follows from this that the apogee of the eccentric, under the motion of the orb BHCP, describes a circle parallel to the ecliptic, and never passes through the ecliptic.

[Margin: 3. Orb carrying the perigee.]

The third orb is the innermost of the black ones, concentric with the world in its concave surface EX but eccentric in its convex surface QZOT. It carries the perigee M of the eccentric in the order of the signs; indeed, by supporting the orb of the eccentric, it helps the second orb to move the apogee at an equal pace. Moreover, the perigee of the three superior planets is always to the south, and describes a circle parallel to the ecliptic, and never crosses the ecliptic. And the thicker part of this orb always lies beneath the narrower part of the outermost orb, and the reverse.

[Margin: 4. Eccentric orb carrying the epicycle.]

The fourth orb is white, and simply eccentric to the world, in both its convex surface GV and its concave surface QZOT. In its thickness, through the middle, is described the eccentrepicycle, or [Greek: ἐπικυκλόφορος (epikyklophoros,

"epicycle-carrier")], or deferent, carrying the epicycle in the order of the signs from F to K; and in the same thickness the equant RKOL is imagined as drawn. The equant, however, requires no really distinct orb, because no motion is performed through it; only the arcs are marked out in it which are the measure of the uniform motion made by the deferent through the force of the fourth orb.

[Margin: 5. Orb: the epicycle.]

The fifth orb is the epicycle GQ or OV, wholly solid and hollowed out by nothing except the globe of the planet, whose centre, fixed in its circumference, it carries in the order of the signs, performing its motion about the centre of the epicycle itself; and with the motion of the eccentric there moves in it also the mean and true apogee of the epicycle. And this too is eccentric to the world.

De Motu Apogeorum eorumq. Aequatione.

On the motion of the apogees and their equation.

[Margin: Motion of the apogee of the eccentric.]

VI. The motion of the apogee of the eccentric is uniform and needs no equation. It is the motion of which I spoke in Section 1, chapter 7, number 8, and of whose path and instruments, through solid orbs 2 and 3, I spoke a little earlier in number 5. This motion is counted from the beginning of Aries, P, in the order of the signs, that is, from the vernal equinox.

[Margin: Motion of the mean apogee of the epicycle.]

The motion of the mean apogee of the epicycle, by contrast, is irregular and needs an equation; and this motion is counted from the true apogee of the same epicycle in the order of the signs. Here I assume the definitions of these apogees given at the end of number II, and those of lines 2 and 3 given in number III. Now, so that this motion and its equation may be grasped, in the following figure let BAG be the line of apsides, on which lie three centres: of the world A, of the deferent D, and of the equant E. From the world's centre describe the circumference GHBK, and from the centre D the deferent FTLN. Taking as centres the four cardinal points of its circumference, that is, the apogee F, the perigee L, and the points of mean longitudes T and N, describe four equal epicycles GQ, CR, PO and CS; and, choosing four centres I around the middles of the eccentric's four quadrants, describe four other epicycles equal to the first. Then from the centre D draw two circumferences, one through the apogees or apices of the epicycles, which is to be GCPCG, the other through the perigees or lowest curvatures, which is to be QRO SQ. Next draw the diameter of mean longitudes, but as far as the circumferences of two epicycles, which is to be C TDNC; and from the centre D, through the centres I of the other epicycles, draw the straight lines D I C. Finally, through the centres of all the epicycles, as far as their circumferences, draw pairs of straight lines: one from the world's centre, namely the line of the true apogee of the epicycle, which are to be AIV,

AT V, ANV; and the other from the equant's centre, namely the line of the mean apogee of the epicycle, which are to be EIM, ETM, ENM.

[Margin: What the point of contact or of concavity is.]

When this has been done, the point on the epicycle's circumference on which there falls a straight line drawn from the deferent's centre through the epicycle's centre is called the point of contact and of concavity; such are the points G, P and the six points C. It is called the point of contact because this point alone of the epicycle's circumference always touches the concave surface of the outermost solid orb, nor is that surface touched by the mean apogee or the true apogee, or other points, except when they come together at this point, as at G and P. It is also called the point of concavity,

Diagram. Woodcut set in the left column: the eight-epicycle figure of the motion of the mean apogee. A heavy black crescent lies between an outer circle and an inner eccentric circle, thin at the top and broad at the foot. The vertical line of apsides runs from G at the top to B at the foot, and a horizontal line of mean longitudes runs through D. Inside the black crescent stands a ring of eight small circles, the epicycles: one at the top around F, one at the foot around L, one each at left and right around T and N, and four on the diagonals, each around a centre I. Each of the six epicycles other than the top and foot carries the three letters V, C, M in a group on its outer edge. Straight lines cross the central circle from the epicycles' centres to the two points E and A on the vertical line. Two thin circles pass through the epicycles.

- G (top, on the vertical line, above the black ring)
- F (on the vertical line, inside the upper epicycle)
- Q (on the vertical line, at the foot of the upper epicycle)
- E (on the vertical line, directly below Q)
- D (centre of the figure, on the vertical line, at the crossing of the horizontal line)
- A (on the vertical line, below D)
- O (on the vertical line, at the head of the lower epicycle)
- L (on the vertical line, inside the lower epicycle)
- P (on the vertical line, at the foot of the lower epicycle)
- B (foot, on the vertical line, below the black ring)
- H (far left, outside the black ring)
- K (far right, outside the black ring)
- T (left epicycle, its centre)
- N (right epicycle, its centre)
- R (inner edge of the left epicycle)
- S (inner edge of the right epicycle)
- I (four times: the centres of the diagonal epicycles, upper left, upper right, lower left, lower right)
- V, C, M (a group on the outer edge of each of the six epicycles other than the top and foot)

because with this point, as with a tip, the semidiameter of the eccentric seems to hollow out or mark out the outermost solid orb, or because it travels along that orb's concave surface GCPCG. With these things laid down, the rules for the motion of the mean apogee are to be given.

[Margin: 1. Rule on the motion of the mean apogee.]

First: when the epicycle's centre is placed at the deferent's apogee F or perigee L, the mean apogee and the true apogee and the point of contact come together in one point, G or P. No equation is needed, and they stand in the same place under the zodiac, under which the apogee or perigee of the eccentric stands.

[Margin: 2. Rule.]

Second: when the epicycle's centre is placed at the points of mean longitudes, T or N, the point of contact C stands off by an equal arc of the epicycle from the mean apogee M on one side and from the true apogee V on the other, and by the greatest arc by which it can stand off. Therefore the mean apogee is at its greatest distance from the epicycle's true apogee, and needs the greatest equation to be reduced to the true one.

[Margin: 3. Rule.]

Third: when the epicycle's centre is placed in the eccentric's first quadrant, which is FT, the mean apogee recedes further in the order of the signs from the point of contact than the point of contact does from the true apogee. In the second quadrant, TL, the mean apogee approaches the point of contact, and is carried against the order of the signs until it joins it at L. In the third quadrant, LN, the mean apogee M recedes from the point of contact C, likewise against the order of the signs. In the fourth, finally, NF, it again approaches the point of contact, going in the order of the signs, until it joins it at F.

[Margin: 4. Rule and 2 corollaries.]

Fourth, therefore: in the upper part of the eccentric, NFT, the mean apogee moves along the epicycle's circumference in the succession of the signs; in the lower part, TLN, against the series of the signs.

[Margin: Why and where the planet's motion on the epicycle is faster or slower.]

Moreover, it follows from this that the planet's motion on the epicycle, which is uniform with respect to the mean apogee, from which it recedes through equal arcs of the epicycle in equal time, nevertheless becomes irregular because of the irregular motion of the mean apogee. It becomes faster in the upper part of the eccentric and slower in the lower, because there the mean apogee goes along with the motion of the epicycle's centre, whereas here it resists that motion and is carried in the opposite direction. Wurstisen explains this with an excellent comparison: "For if we must walk," he says, "on the condition that you always keep a thousand paces ahead of me, then surely, if I follow you quickly, you must run on ahead faster, so as to keep an equal interval." The same is therefore necessary for the planet when the mean apogee, from which it recedes uniformly, pursues it faster in the upper hemisphere of the eccentric,

the motion being made in the same direction in which the epicycle's centre moves. But when the mean apogee draws back in the opposite direction, as happens in the lower hemisphere, the planet must move more slowly, so as to preserve the uniformity of the motion by which it stands off from the mean apogee. Although these points also concern the planet's own motion, of which below, they had to be touched on here beforehand because of their affinity.

[Margin: Equation of the centre on the epicycle.]

VII. The equation of the mean apogee [Aequatio Apogei medij], or the prosthaphaeresis of the motion of the mean apogee, is called the equation of the centre on the epicycle [Æquatio Centri in Epicyclo]. It is the smaller arc of the epicycle's periphery between the mean and the true apogee, or the difference of these two apogees counted in the already mentioned arc, such as the arc MCV; or it is the reduction of the mean apogee to the true one, made by adding or subtracting the already mentioned arc.

[Margin: Rule for this equation.]

When the epicycle's centre is placed at the eccentric's apogee F or at its perigee L, no equation of this kind is needed; but at the points of mean longitudes, T and N, it is at its greatest. Next, in the first semicircle of the eccentric, counted in the order of the signs, that is, in the semicircle FTL, this equation is to be subtracted from the mean apogee, so that it may be reduced to the true one; in the second, LNF, it is to be added.

[Margin: The angle that measures this equation.]

The immediate measure of this equation is the angle formed at the epicycle's centre by straight lines drawn out from it, one to the mean apogee and the other to the true apogee, namely the angle MIV, or MTV, or MNV. Its mediate measure is the angle vertically opposite and equal to that one, which is formed at the epicycle's centre by straight lines drawn from it, one to the centre of the equant and the other to the centre of the world, such as the angles AIE, ATE and ANE. But since the equation of the centre outside the epicycle also consists in these angles, and is of the same quantity as the equation of the centre on the epicycle, we shall more conveniently teach below the way of finding these angles, after the motion of the epicycle's centre has been explained.

De Motu Centri Epicycli, ſeu Anomaliæ Excentrici, ſeu Longitudinis eiuſdem, Et ipſius Aequatione.

On the motion of the epicycle's centre, or the anomaly of the eccentric, or its longitude, and on its equation.

VIII. The motion of the epicycle's centre, carried by the deferent in the order of the signs, in Greek [Greek: περιδρομὴ κατὰ μῆκος (peridromē kata mēkos, "circuit along the length")], has various names and senses, according to whether it is considered as mean and uniform or as apparent to us and non-uniform; and either in the equant or in the zodiac; and finally according to the different terms from which the start of the count is taken. So the first figure set out in

number 2 is repeated here, with most of the definitions explained in numbers 3 and 4; go on to the following definitions.

Diagram. Woodcut set in the right column: the Ptolemaic hypothesis repeated, with an outer circle bearing signs of the zodiac at its four cardinal points, an eccentric circle, an equant circle, and three epicycles: one at the top around F, one at the foot around M, and one at upper left around I. A vertical line of apsides runs from the top to the foot through E, D and A. Straight lines run from A, D and E up and to the left through or beside the epicycle at I to the outer circle, one of them dashed, and horizontal or nearly horizontal lines run from the vertical line to the left and right of the figure.

- B, with the sign ♋ (top, outside the outer circle)
- G (top of the upper epicycle)
- R, F (on the vertical line, inside the upper epicycle)
- b, d (left and right edges of the upper epicycle)
- Q (on the vertical line, below the upper epicycle)
- T, u, S, X, Z (upper left, at or outside the outer circle, going downward)
- g (at the left edge of the upper-left epicycle, beside X and S)
- n, beside a small star marking the planet (left, at the foot of the upper-left epicycle, beside Z)
- I (centre of the upper-left epicycle)
- K, N, a (left, one below the other, at the ends of the horizontal lines)
- E, D, A (on the vertical line, going downward)
- L, Y, m (right, one below the other, at the ends of the horizontal lines)
- H, with the sign ♎ (left, outside the outer circle)
- the sign ♈, with P (right, outside the outer circle)
- O, M, V (on the vertical line, in the lower epicycle, going downward)
- the sign ♑, with C (foot, outside the outer circle)

[Margin: Mean anomaly of the eccentric, or mean centre.]

The mean anomaly of the eccentric [Anomalia media Excentrici], or, as the Alphonsine astronomers say, the mean centre [Centrum Medium] (understand this of the epicycle's centre), is the mean motion of the epicycle's centre counted from the eccentric's apogee in the order of the signs. If it is viewed in the circle of the equant, it is the arc of the equant counted from the equant's apogee as far as the line of mean motion of the epicycle's centre, such as the arc RI, contained between the lines ER and EI. The same motion viewed in the zodiac is the arc of the zodiac counted from the line of apsides in the order of the signs as far as the line of mean motion in the zodiac, that is, as far as the parallel to the line drawn from the equant's centre through the epicycle's centre, this parallel being produced from the world's centre as far as the zodiac: such is the arc BZ, between the lines AB and AZ, AZ being parallel to EIS. Why this parallel is used I said in number 4, definition 5.

The true anomaly of the eccentric [Anomalia vera Excentrici], or, with the Alphonsine astronomers, the true centre [Centrum verum], or coequated centre [centrum coæquatum], is the true motion of the epicycle's centre, counted in

the order of the signs from the eccentric's apogee; or it is the arc of the zodiac counted in the order of the signs from the line of apsides as far as the line of true motion of this centre, that is, as far as the line produced from the world's centre through the epicycle's centre to the zodiac, such as the arc BT, between the line AB and AIT.

[Margin: True anomaly of the eccentric, or true centre.]

[Margin: Mean longitude of the eccentric, and its mean place.]

The mean longitude of the epicycle's centre [Longitudo Media centri Epicycli], or mean longitude of the eccentric [Longitudo media Excentrici], which is also called the planet's mean longitude, is the mean motion of the epicycle's centre counted in the order of the signs from the beginning of longitude, that is, either from the vernal equinox or, with Copernicus, from the first star of Aries; or the sum of the longitude of the eccentric's apogee and the mean anomaly of the eccentric; or it is the arc of the zodiac counted in the order of the signs from the beginning of longitude as far as the line of mean motion, such as the arc PBZ, if P is the beginning of longitude and the point Z is the mean place of the epicycle's centre.

[Margin: True longitude of the eccentric.]

The true longitude of the epicycle's centre [Longitudo vera centri Epicycli], or true centric longitude [longitudo vera centrica], or coequated longitude of the centre [longitudo centri coæquata], is the true motion of the epicycle's centre counted in the order of the signs from the beginning of Aries as far as the line of the true place of the same centre; or the sum of the true longitude of the eccentric's apogee and the true anomaly of the same eccentric; or it is the arc of the zodiac counted in the order of the signs from the beginning of Aries as far as the line of true motion of the epicycle's centre, such as the arc PBT.

[Margin: True place of the epicycle's centre.]

And the point T is the true place of the epicycle's centre under the zodiac, or the planet's true eccentric place.

[Margin: Corollary.]

Hence it happens that if the mean longitude of the eccentric is known, then when the motion of the apogee, or the arc PB, which is found in the tables, is taken away from it, the mean anomaly of the eccentric, or the arc BZ, becomes known. Or, if this is known, adding the motion of the eccentric's apogee to it composes the mean longitude of the eccentric. Likewise, if the motion of the apogee is taken away from its true longitude, its true anomaly becomes known; and conversely, if the motion of the eccentric's apogee is added to the true anomaly, the true longitude of the eccentric becomes known.

[Margin: What the equation of the centre in the zodiac is.]

The equation of the centre in the zodiac [Aequatio centri in Zodiaco], or equation of the anomaly of the eccentric [Aequatio Anomaliæ Excentrici], which when applied to the eccentric's mean longitude is called the equation of the longitude of the eccentric, in Greek [Greek: προσθαφαίρεσης μήκους (prosthaphairesis mēkous, "prosthaphaeresis of longitude")] [recte:

προσθαφαίρεσις], is the difference between the mean and the true anomaly of the epicycle's centre, or between the mean and the true centre. Referred to longitude, it is the difference between the mean and the true longitude; or it is the arc of the zodiac between the mean and the true place, or between the lines of the mean and the true place of the epicycle's centre, such as the arc TZ, between the lines AZ and AT.

[Margin: The angle that measures this equation.]

Its measure is the angle formed at the world's centre by the lines of the mean and the true place, etc., such as the angle TAZ; and to this the angle AIE is equal, because of the parallelism of the lines EIS and AZ.

[Margin: Rule for this equation.]

The equation of the centre in the zodiac is to be subtracted from the mean anomaly in the first semicircle of this anomaly, that is, in the semicircle BHC, because T, the true place of the epicycle's centre, precedes the mean place Z, or stands off from it against the order of the signs. So, when the arc ZT is subtracted from the arc ZB, the true anomaly TB remains. But in the second semicircle it is to be added, because the true place follows after the mean, or stands off from it in the order of the signs.

[Margin: 2. Rule.]

The equation of the centre in the zodiac is nil when the epicycle's centre is at F, the eccentric's apogee, or at M, the perigee. It grows from the eccentric's apogee until the epicycle's centre is at the first point of mean longitude, namely at the point N, where it becomes greatest, as Reinhold has demonstrated on Peuerbach's *Theoricae*, p. 100. From there, once it has passed that point, it decreases until at M it again becomes nil; and from there, taking increase again, it grows as far as the point Y, of the second mean longitude, where it becomes greatest, and from there it decreases continuously until it vanishes at F.

[Margin: 3. Rule.]

Finally, at points of the eccentric diametrically opposite it is equal; as also at points equally distant either on both sides from the apogee or on both sides from the perigee of the eccentric.

Yet the true motion of the epicycle's centre is slowest at the apogee and fastest at the perigee of the eccentric. This will be clear even from the following: if, having divided the equant into equal arcs (since the uniformity of this motion is viewed in it), you draw through the points of division the lines of true motion, that is, straight lines through the epicycle's centre to the zodiac, you will see that the arcs of the zodiac nearer the apogee are smaller, and larger near the perigee.

De Motu Planetæ in Epicyclo, ſeu de Anomalia Orbis, eiuſque Aequationibus.

On the planet's motion on the epicycle, or on the anomaly of the orb, and its equations.

[Margin: Mean anomaly of the orb, or mean argument.]

IX. The mean anomaly of the orb [Anomalia Orbis media], or the anomaly of commutation [Anomalia Commutationis] as Copernicus says, or, as the Alphonsine astronomers say, the mean argument [Argumentum Medium], in Greek [Greek: κίνημα τῆς ἀνωμαλίας (kinēma tēs anōmalias, "motion of the anomaly")], is the planet's mean motion on the epicycle about the epicycle's centre, counted in the order of the signs from the epicycle's mean apogee; or it is the arc on the epicycle's periphery, counted in the order of the signs from its mean apogee as far as the centre of the planet's body. In the preceding figure, such is the arc Gb, if the epicycle's centre is at F and the planet at b; or the arc gn, if the epicycle's centre is at I and the planet at n.

[Margin: What the true argument is.]

The true anomaly of the orb [Anomalia orbis vera], or true argument [Argumentum verum], or coequated argument [Argumentum coæquatum], is the planet's true motion on the epicycle, counted in the order of the signs from the true apogee; or it is the arc of the epicycle's periphery counted in the order of the signs from the epicycle's true apogee as far as the planet, such as the arc Gb, when the epicycle's centre is at F, the eccentric's apogee, for then the mean and the true apogee of the epicycle are the same, at G; or the arc ugn, if the epicycle's centre is at I and the planet at n.

[Margin: The harmony between the planet's motion on the epicycle and the motion of the Sun.]

Further, the planet's motion on the epicycle, although when counted from the true apogee it is irregular, and quicker so long as the epicycle's centre is in the upper semicircle of the eccentric, the one containing the equant's centre, and slower in the lower, for[59] the reason already set out at the end of number 6, is nevertheless uniform when counted from the epicycle's mean apogee, and is tempered in admirable harmony to the Sun's mean motion, by which the Sun is elongated from the mean conjunction with the planet. For the planet's mean motion on its epicycle is as great as the Sun's elongation, according to its mean motion, from the mean conjunction with the planet. From this there follow notable corollaries and 6 rules.

[Margin: 1. Rule.]

1. At every mean conjunction of the planet with the Sun, that is, one viewed according to the Sun's mean motions, the planet is at the true apogee of the epicycle; and at every mean opposition with the Sun, the planet is at the true perigee of the epicycle. So Ptolemy, Book 10, chapter 6, and there Regiomontanus, propositions 9 and 11; granted that Peuerbach, and following him Reinhold, Wurstisen, Magini and others, have said that this happens at the mean perigee. I shall refute them in Scholium 5.

[Margin: 2. Rule.]

[59] *ob* is printed twice here, at a line end and at the start of the next line

2. One whole revolution of the planet on the epicycle takes as much time as passes from one of its mean conjunctions to the next mean conjunction with the Sun, or from one mean opposition to the other opposition.

[Margin: 3. Rule.]

3. The planet's two mean motions, namely of longitude and of argument, or of the epicycle's centre on the eccentric and of the planet's centre on the epicycle, taken together, equal the Sun's mean motion made in the same time.

[Margin: 4. Rule.]

4. When the planet's mean motion in longitude, or the mean longitude of the epicycle's centre, is subtracted from the Sun's mean motion made in the same time, there remains and becomes known the planet's mean motion on the epicycle, or mean argument, or mean anomaly of the orb. Therefore no special tables are needed for it; the tables of the mean motions of the Sun and of the epicycle's centre in longitude suffice. Conversely (which serves as a check of the calculation), if you subtract the planet's mean argument from the Sun's mean motion, there remains the planet's mean motion in longitude, or the mean motion of the longitude of the eccentric; and if to the planet's mean motion in longitude you add the mean argument, the result is precisely the Sun's mean motion in longitude.

[Margin: 5. Rule.]

5. From these things, and from what was said at the end of number 8 about the slower motion of the epicycle's centre in the upper part of the eccentric and the faster in the lower, and from what was said about the quicker motion of the planet on the epicycle in the upper and the slower in the lower semicircle of the epicycle, it follows that when and by how much the motion of the epicycle's centre on the eccentric is slower, then and by so much the planet's motion on the epicycle becomes faster, because the Sun then joins the planet sooner. Conversely, when and by how much the motion of the epicycle's centre is faster, then and by so much the planet's motion on the epicycle becomes slower, and the Sun overtakes the planet more slowly.

[Margin: 6. Rule.]

6. The line drawn from the epicycle's centre to the planet at every mean conjunction or opposition [☌, ☍] with the Sun coincides with the line of mean motion of the Sun; but outside conjunction and opposition [☌, ☍] it is parallel to that line. So Ptolemy, Book 10, chapter 6, and Regiomontanus there, in the *Epitome*, proposition 10. An example, in the diagram to be set in Scholium 5, is the line IR, compared with the line AH of the Sun's mean motion.

[Margin: What the equation of the argument is.]

The equation of the argument [Aequatio Argumenti] is not the equation of the epicycle's centre, although it partly depends on it. It is the equation by

which the planet's mean motion in longitude is finally equated, so that its true place in the zodiac may be known. This equation is the difference between the true place of the epicycle's centre and the true place of the planet in the zodiac; or it is the arc of the zodiac between the true place, or the line of the true place, of the epicycle's centre and the true place, or the line of the true place, of the planet, such as the arc XT, between the lines AIT and AnX.

[Margin: The angle that measures this equation.]

Its measure is the angle formed at the world's centre by two straight lines produced as far as the zodiac, one through the epicycle's centre and the other through the centre of the planet's body: such an angle is TAX. The rules for this equation are written below.

[Margin: 1. Rule for the equations of the argument.]

1. When the epicycle's centre is at F, the eccentric's apogee, and the centre of the planet's body at G, the apogee, or at Q, the mean perigee of the epicycle, it is also at the true one, and no equation of the centre or of the argument is needed. The mean place of the epicycle's centre is the true place both of that centre and of the planet, and the same as the place in the zodiac under which the eccentric's apogee stands. Say the same if the epicycle's centre is at M, the eccentric's perigee, and the planet at V, the apogee, or at O, the mean and then also true perigee of the epicycle; but then the planet's true place is the one under which the eccentric's perigee stands.

[Margin: 2. Rule.]

2. When the epicycle's centre is at F, the eccentric's apogee, or at M, the perigee, but the planet is away from the epicycle's mean apogee, as if it is at b or at d, no equation of the centre is needed, only the equation of the argument. But this is then obtained more easily by solving one triangle, in which there are given: the distance of the epicycle's centre from the world's centre, namely either AF, consisting of the radius DF and the deferent's eccentricity AD, or AM, the remainder of the radius DM after the eccentricity AD has been taken from it; the epicycle's semidiameter, as the other side; and the angle contained by these, which is the complement to a semicircle of the argument, true and mean at that time, if the argument does not exceed 6^s, or its excess above a semicircle, if the argument exceeds 6^s. From these data the angle opposite the epicycle's semidiameter is gathered, and this is the equation of the argument; for it, consult rule 4, to be given in number 10.

[Margin: 3. Rule.]

3. When the epicycle's centre is away from the eccentric's apogee and perigee, and the planet away from the epicycle's true apogee or perigee, an equation is needed both of the centre and of the argument, or orb: as if the centre is at I and the planet at g or at n.

[Margin: 4. Rule.]

4. The equation of the argument is greatest when the epicycle's centre is at M, the eccentric's perigee, and the planet is at the point of the epicycle's periphery at which a line drawn from the world's centre to the centre of the planet's body touches the epicycle, as we shall say in chapter 11, where there will be a diagram for this matter.

[Margin: 5. Rule.]

5. The equation of the argument is to be added to the true place of the centre in the first semicircle of the anomaly of the orb, or of the epicycle; but in the second it is to be subtracted, so that the planet's true place under the zodiac, as seen by a terrestrial observer, may be had. For instance, if the planet is at n, add to the arc PBT the arc XT, so that the arc PBX results, which is the planet's true longitude, counted from the beginning of Aries, P. Nothing else, therefore, is required, except that we give the whole series of the geometrical calculation for finding the planet's true motion in longitude, assuming the mean motions and the commensurations of the orbs, which are to be set out at the end of the chapter.

Methodus Calculi Geometrici Pro Vero Motu Planetæ in Longitudinem.

Method of the geometrical calculation for the planet's true motion in longitude.

X. Let the explanation of the diagram given in number 2 of this chapter be repeated here; and with this assumed, let the planet be placed at n and the epicycle's centre at I. Then follow this method.

[Margin: 1. Step of the calculation.]

First, for the given time, find from the Ptolemaic or Alphonsine roots, and from the tables of mean motions, the mean or uniform motion both of the eccentric's apogee, namely the arc BP, and of the longitude or centre, namely the arc PBZ. Subtract the apogee's motion from the centre's mean motion, and there will remain the mean anomaly of the centre, that is, the arc BZ, and so also the angle BAZ, to which the angle REI is equal; and the complement of this to two right angles, or to 180°, is the angle IED.

[Margin: 2. Step.]

Second, in the triangle IED there are given: the angle IED, from the first step; and, from the measures of the hypothesis to be given at the end of the chapter, both the eccentricity DE, which is as great as AD, and the radius DI, in the same kind of parts. So by the rules of rectilinear triangles find the angle DIE, which is the physical part of the equation of the centre; keep it in reserve.

[Margin: Physical equation.]

The sum of this and the angle IED is equal to the external angle ADI, by proposition 32 of Euclid's first book.

[Margin: 3. Step.]

Third, in the triangle ADI there are given the angle ADI, the eccentricity AD and the radius DI. So by the rules of triangles find both the base AI, that

is, the distance of the epicycle's centre from the centre of the universe, which keep in reserve, and the angle AID, which is the optical part of the equation.

[Margin: Optical equation.]

Join this to the physical equation, and there will result the whole equation of the centre in the zodiac, that is, the angle AIE, to which, because of the parallelism, the angle TAZ is equal by proposition 29 of Euclid's first book, measuring the arc TZ of the zodiac. Since, then, the mean anomaly was in the first semicircle BHC, or less than 6^s, subtract the equation of the centre, that is, the arc TZ, from the mean anomaly, and you will have the true or equated anomaly of the centre, namely the arc BT. Again subtract the same equation from the mean longitude of the centre, and you will have the true longitude of the centre, or the true centre, that is, the arc PBT. But if the anomaly had been in the second arc CPB, the equation would have had to be added.

[Margin: 4. Step.]

Fourth, having drawn from the world's centre A the straight line AnX through the centre of the planet's body, and to that centre the epicycle's semidiameter In, find in the proper tables the mean motion of the anomaly of the orb, which Ptolemy calls the motion of the inequality; or, if the tables lack this motion, as the Alphonsine do, find for the given time the Sun's mean motion, and from it take away the true motion of the centre, and there will remain the true argument, that is, the arc ugn of the epicycle, counted from the true apogee u to the planet. Or (which comes to the same thing, but by a longer way) take away from the Sun's mean motion the mean motion of the centre, and there will remain the mean argument, that is, the arc gn, counted from the mean apogee g. To this add the equation of the centre, that is, the arc gu, which the angle gIu measures, as great as the angle AIE vertically opposite to it, which we said is the equation of the centre in the zodiac; and there will result the true or coequated argument, namely the arc ugn, as above. But if the epicycle's centre were in the second arc CPB, then the equation of the centre on the epicycle would have to be subtracted from the mean argument, so as to have the true argument.

[Margin: 5. Step.]

Fifth, subtract the true argument from 180°, and you will have the angle AIn in the triangle AIn, by proposition 13 of Euclid's first book. With this, and with the epicycle's semidiameter In, which is given from what was said in number 9, and with the distance AI found above in step 3, find by the analysis of triangles both the planet's distance from the Earth, that is, the side An, which serves for finding parallaxes, and the angle IAn, that is, TAX, measuring the zodiac's arc TX, that is, the equation of the argument. Since we are dealing with the first semicircle of the anomaly, add this to the true longitude of the centre, or the true centre PBT, and there will result the planet's true longitude in the zodiac, that is, the arc PBX. But if the epicycle's centre were in the second arc of the anomaly, the equation of the centre in the zodiac would have to be added,

while the equation of the centre on the epicycle, just like the equation of the argument, would have to be subtracted.

The reduction of the planet's place in the zodiac from its orbit to the ecliptic depends on the planet's latitude, of which below.

Menſuræ & Numeri ad Ptolemaicam & Alphonſinam Hypotheſim ſpectantes.

Measures and numbers pertaining to the Ptolemaic and Alphonsine hypothesis.

XI. In reporting these measures from the tables of Ptolemy and Alfonso, the variety among writers is incredible as regards the semidiameters of the Earth, since different writers pursue different distances of the Sun from the Earth, which become mixed up in this business:

for in parts of which the radius of the planet's orb is 60, nearly all agree with Ptolemy. Ptolemy gives the numbers of the motions in Book 9, chapters 3 and 5, and those of the equations in Book 11, chapter 10, as the Alphonsines do in their tables. He gives the measures of the epicycles and eccentricities in Book 10, chapters 7 and 8, for ♂; in Book 11, chapters 1 and 2, for ♃; and in chapters 5 and 6 for ♄. This is so although he sets the eccentricity of ♂ now at 12°, now at 13° 7′, and although the former number is kept by al-Battānī, chapter 50; al-Farghānī, differentiae 16 and 21; Regiomontanus in the *Epitome of the Almagest*, Book 10, proposition 18, and Book 11, propositions 7 and 16; Reinhold on Peuerbach, p. 139, and in the same place Wurstisen, pp. 132, 145, 149; Maurolico at the end of the *Cosmographia*; Clavius at the end of the *Sphere*, who at times carries the planet's distances from the Earth over from Maurolico to the semidiameters of the heavens, neglecting the addition of the semidiameter of the planet's body; and Barozzi, *Cosmographia*, Book 1, p. 48. These authors report these and other measures. But the latter opinion, of 13° 7′, is held by Thābit and Campanus in their *Theoricae*, by Fernel in Book 1 of the *Cosmotheoria*, and by Danti in part 2 of the *Astrolabium* and in his tables. Besides these, the present measures and distances are adduced by Mauro Fiorentino, by Giuntini in his own *Sphere* and from him Baranzano in part 1 of the *Uranoscopia*, by Schöner in the *Aequatorium Astronomicum* from p. 10, by Osualdus on Peuerbach, p. 425, by Tycho in volume 1 of the *Progymnasmata*, p. 468, following al-Battānī, and by Kepler in the *Epitome of Copernican Astronomy*, p. 494, who however follows the Copernican correction of the Ptolemaic intervals. And Mästlin, in Kepler's *Prodromus*, p. 92, gathers from Reinhold's data that Saturn's greatest distance is 24718 semidiameters of the Earth. So take first the measures in which all agree with Ptolemy, then the distances from the Earth in which they do not agree. On the distances from the Sun, which are to be determined one way in Ptolemy's time and another way at other times, consult Scholia 7 and 8 below; and to picture the lines, look at the figure set out at nos. 2 and 8.

[Margin: Authors who deal with these numbers and measures.]

Measures pertaining to the Ptolemaic and Alphonsine hypothesis

[Margin: See the figure set out at no. 2 and 8.]

In parts of which the radius DF is	Planet	60° 0′	or 100000
Eccentricity of the deferent AD	♄	3° 25′	5694
	♃	2° 45′	4583 ½
	♂	6° 0′	10000
	or	6° 33′	10850
Eccentricity of the equant AE	♄	6° 50′	11388
	♃	5° 30′	9167
	♂	12° 0′	20000
	or	13° 7′	21700
Semidiameter of the epicycle FG	♄	6° 30′	10833
	♃	11° 30′	19166
	♂	39° 30′	65833
Greatest distance of the planet from the centre of the Earth: AG	♄	69° 55′	116527
	♃	74° 15′	123749
	♂	105° 30′	175833
	or	106° 3′	176683
Least distance of the planet from the centre of the Earth: AO	♄	50° 5′	83473
	♃	45° 45′	76251
	♂	14° 30′	24167
	or	13° 57′	23317
Mean distance	of all	60° 0′	100000

Eccentricity of the deferent, derived from Ptolemy by the authors, in semidiameters of the Earth

[Margin: Double this, and it will give the eccentricity of the equant AE.]

	Planet	Author	Value
Eccentricity of the deferent AD	♄	Fernel	1284
		Maurolico and Clavius	981
	♃	Fernel	599
		Maurolico and Clavius	532
	♂	Fernel	611
		Maurolico and Clavius	503

Semidiameter of the epicycle, derived from Ptolemy by the authors, in semidiameters of the Earth

	Planet	Author	Value
Semidiameter of the epicycle FG	♄	Erasmus Reinhold	2298
		Fernel	1922
		Maurolico and Clavius	1866
	♃	Erasmus Reinhold	2743
		Fernel	2507
		Maurolico and Clavius	2226
	♂	Erasmus Reinhold	4085

	Planet	Author	Value
		Fernel	3714
		Maurolico and Clavius	3313

Distances of the planets and heavens from the Earth in the Ptolemaic manner, reported by the authors in semidiameters of the Earth and their minutes

[Margin: See the figure set out at no. 2 and 8.]

[Margin: All these distances are taken from the centre of the Earth.]

[Margin: The mean distance from the centre of the Earth is the same as the radius, or semidiameter, of the eccentric.]

[Margin: The last column holds the semidiameters of the convex and the concave of the heaven, with the planet's body counted in.]

Planet	Author	Distance	Semidiameter
	Mean distance DF		
♄	al-Battānī, Tycho	15800	
	al-Farghānī, Barozzi	17312	
	Jean Fernel	19403	
	Giuntini, Mauro Fiorentino	15373	
	Maurolico	17225 16′	
	Clavius	17225 16′	
	Copernicus, Kepler	22000	
♃	al-Battānī, Tycho	10423	
	al-Farghānī, Barozzi	11638	
	Jean Fernel	13079 20′	
	Giuntini, Mauro Fiorentino	10704	
	Maurolico	11611 31′	
	Clavius	11611 31′	
	Copernicus, Kepler	14750	
♂	al-Battānī, Tycho	4584	
	al-Farghānī, Barozzi	5045	
	Jean Fernel	5641 51′	
	Giuntini, Mauro Fiorentino	4704	
	Maurolico	5032 4′	
	Clavius	5032 4′	
	Copernicus, Kepler	6393	
	Greatest distance AG		**Semidiameter of the convex**
♄	al-Battānī	18094	
	al-Farghānī, Barozzi		20220
	Jean Fernel		22612
	Giuntini, Mauro Fiorentino	17571	
	Maurolico	20072 13′	20076 43′
	Clavius		22612 30′
	Copernicus, Kepler	25737	25742 0′
♃	al-Battānī	12420	
	al-Farghānī, Barozzi		14405

Planet	Author	Distance	Semidiameter
	Jean Fernel	16185 41′	16190 15′
	Giuntini, Mauro Fiorentino	17571	
	Maurolico	14369 15′	14373 49′
	Clavius		14378 19′
	Copernicus, Kepler	18253	18258 0′
♂	al-Battānī	8018	
	al-Farghānī, Barozzi		8876
	Jean Fernel	9967 16′	9968 26′
	Giuntini, Mauro Fiorentino	8232	
	Maurolico	8848 3′	8849 13′
	Clavius		8853 47′
	Copernicus, Kepler	11248	11243 0′
	Least distance AO		**Semidiameter of the concave**
♄	al-Battānī	12420	
	al-Farghānī, Barozzi		14405
	Jean Fernel	16091	16090 15′
	Giuntini, Mauro Fiorentino	13071	
	Maurolico	14378 19′	14373 49′
	Clavius		14378 19′
	Copernicus, Kepler	18263 0′	18259

Remainder of the preceding table

Planet	Author	Distance	Semidiameter
	Least distance AO		**Semidiameter of the concave**
♃	al-Battānī	8024	
	al-Farghānī, Barozzi		8876
	Jean Fernel	9972 59 3/5′	9968 25 3/5′
	Giuntini, Mauro Fiorentino	8237	
	Maurolico	8853 47′	8849 13′
	Clavius		8853 47′
	Copernicus, Kepler	11248	12243 0′
♂	al-Battānī	1176	
	al-Farghānī, Barozzi		1220
	Jean Fernel	1316 25 5/6′	1315 15 5/6′
	Giuntini, Mauro Fiorentino	1176	
	Maurolico	1216 5′	1214 55′
	Clavius		1216 5′
	Copernicus, Kepler	1543	1545

According to Mästlin's reasoning, Saturn's greatest distance, from Reinhold's epicyclic semidiameter, is 24718 semidiameters of the Earth.

Mean or uniform motion

Daily	Planet	Authors	Deg.	′	″	‴	⁗	V	VI
Of the centre of the epicycle, or of the longitude from ♈	♄	Ptolemy	0	2	0	33	31	28	51
		Alfonso	0	2	0	35	17	40	21
	♃	Ptolemy	0	4	59	14	26	46	31
		Alfonso	0	4	59	15	27	7	24
	♂	Ptolemy	0	31	26	36	53	51	33
		Alfonso	0	31	26	38	40	50	0
Of the planet in the epicycle	♄	Ptolemy	0	57	7	43	41	43	40
		Alfonso	0	57	7	44	19	38	53
	♃	Ptolemy	0	54	9	2	46	26	0
		Alfonso	0	54	9	4	10	11	50
	♂	Ptolemy	0	27	41	40	19	20	58
		Alfonso	0	27	41	40	56	29	14
Annual									
Of the centre of the epicycle	♄	Ptolemy	12	13	23	56	30	30	15
		Alfonso	12	13	34	40	50	27	45
	♃	Ptolemy	30	20	22	52	52	58	35
		Alfonso	30	20	28	39	59	59	59
	♂	Ptolemy	191	16	54	27	38	35	45
		Alfonso	191	15	25	13	50	25	0
Of the planet in the epicycle	♄	Ptolemy	347	32	0	48	50	38	20
		Alfonso	347	32	4	44	26	32	25
	♃	Ptolemy	329	15	1	52	28	10	0
		Alfonso	329	15	10	16	1	59	10
	♂	Ptolemy	168	28	30	17	42	32	50
		Alfonso	168	28	34	3	37	49	10
Of the apogee of the eccentric, of all		Ptolemy	0	0	36	0	0	0	0
		al-Battānī	0	0	59	4	0	0	0
		Alfonso	0	0	26	26	54	0	0

[Margin: How great, too, is the motion of the fixed stars.]

Revolutions, or periods of the motions

[Margin: See also below, chapter 8.]

	Planet	Authors	Egyptian years	Days	Hours	′	″
Of the centre of the epicycle in the zodiac	♄	Ptolemy	29	162	7	36	0
		Alfonso	29	155	8	0	0
	♃	Ptolemy	11	315	17	14	0
		Alfonso	11	313	19	0	0
	♂	Ptolemy	1	321	22	24	0
		Alfonso	1	321	23	0	0

Remainder of the preceding table

	Planet	Authors	Egyptian years	Days	Hours	′	″
Of the planet in the epicycle	♄	Ptolemy	1	13	2	30	0
		Alfonso	1	13	2	23	13
	♃	Ptolemy	1	33	21	20	0
		Alfonso	1	33	21	12	9
	♂	Ptolemy	2	49	23	0	0
		Alfonso	2	49	22	22	40
Of the apogee of the eccentric		Ptolemy	36000	0	0	0	0
		al-Battānī	21966	0	0	0	0
		Alfonso	49000	0	0	0	0

[Margin: How great, and that of the fixed stars.]
Roots of the uniform motions

	Planet	Authors	Signs	Deg.	′	″	‴
Of the apogee of the eccentric	♄	Ptolemy	7	14	10	0	0
		Alfonso	4	2	35	20	41
	♃	Ptolemy	5	2	9	0	0
		Alfonso	2	42	48	38	41
	♂	Ptolemy	3	16	40	0	0
		Alfonso	2	4	23	51	41
Of the centre, or of the longitude	♄	Ptolemy	9	26	43	0	0
		Alfonso	4	24	44	33	26
	♃	Ptolemy	6	4	41	0	0
		Alfonso	0	16	16	15	14
	♂	Ptolemy	0	3	32	0	0
		Alfonso	3	1	41	36	19
Of the planet in the epicycle	♄	Ptolemy	10	27	13	0	0
		Alfonso	8	21	52	39	14
	♃	Ptolemy	4	26	4	0	0
		Alfonso	1	0	20	57	25
	♂	Ptolemy	1	4	2	0	0
		Alfonso	10	14	55	36	20

The Ptolemaic roots are fixed to Alexandria in Egypt, at noon of the first day of Thoth in the first year of Nabonassar; the Alphonsine roots to Toledo, at noon of the last day of December in the year 1252. Just as the mean motion of the anomaly of the orb, or of the planet in the epicycle, is obtained by subtracting the mean motion of the centre from the mean motion of the Sun, so too its root is obtained by subtracting the root of the centre from the root of the Sun, that is, from the mean motion of the Sun placed at the root.

[Margin: For the places of the apogees in Ptolemy's time, see below, chapter 9.]

SCHOLIA.

Scholia.

I. In the figure of the preceding chapter, set at nos. 2 and 8, the centre of the epicycle revolves on the circumference of the deferent from F to I, but it does not move uniformly on it, for the measure of the uniform motion of the eccentric is taken from the arcs of the equant, counted from R towards I. So if we imagine, for example, that the centre I of Mars's epicycle has in 86 days completed the arc RI on the equant (for in this diagram, because of their closeness, the letter I serves both eccentrics) of 45°, and in another 86 days another 45°, and thus a quadrant of the equant in 172 days, it will nevertheless not have completed equal arcs on the circumference of the deferent. For the angle IER, which measures the arc IR of the equant, is not equal to the arc IDF, which measures the arc IF of the deferent, since it is greater than this by propositions 16 and 32 of Euclid's Book 1. The Ptolemaic hypothesis therefore seems to have two imperfections. The first is that the uniformity of the motion is begged from a foreign and external centre, namely the centre E of the equant, which has nothing to do with the subject in which it is, nor with the instrument by which the centre of the epicycle is carried, but is assumed by a fiction, so that equal arcs are held to be traversed in equal times with respect to it. The second is a physical and real irregularity or inequality in the motion of the celestial body, viewed not indeed as a whole, but in part and as half of itself. For even if no eye is placed at A, or anywhere else, still

[Margin: Objections to Ptolemy's hypothesis.]

[Margin: Physical inequality.]

the centre of the epicycle in this hypothesis really does move non-uniformly with respect to the centre D, and on the circumference of the deferent it traverses unequal arcs in equal times. Accordingly the equation of this anomaly, that is the angle DIE, is called the physical equation, as was also said in Book 3, chapter 22. Let the other part of the inequality, with respect to the centre of the world A, where the eye is, be called the optical inequality because of its appearance, and its equation, that is the angle AID, the optical equation. So if the eye were at the centre D of the deferent, there would be one and the same inequality, at once physical and optical, and an eye placed at the centre of the deferent would see the planet move uniformly, since the motion of the planet in the epicycle is computed from the true apogee of the epicycle, which is irregular because of the irregularity of the motion of the centre of the epicycle, according to what was said at no. 5.

[Margin: Optical inequality.]

II. What then was the reason why Ptolemy, knowing the imperfections already mentioned, admitted them? For it was not, as Kepler had said in the *Mysterium Cosmographicum*, chapter 22, and Mästlin in the Appendix in the same place, that he introduced the equant led by a blind conjecture; he did it deliberately, as Kepler later admits in the *Commentaries on Mars*, chapter 16. First, so that the planet's motion in longitude would be perfectly circular with respect to some centre. This perfection in the hypothesis is acknowledged by Mästlin in the Appendix to Kepler's *Prodromus*, by Kepler in the *Commentaries*

on Mars, chapter 4, and by Hérigone in volume 5 of the *Cursus Mathematicus*, p. 507. Second, because Ptolemy saw that the equations of the eccentric's motion require an eccentricity as large as, or not much smaller than, the one the equant has, that is, the circle that is the measure of the uniform motions; but that if all of it were used for the difference between the greatest and least longitude, it would be too much, and, when applied to the centre of the epicycle, would yield incorrect equations of the argument. He therefore bisected the eccentricity, so that the greatest longitude would not turn out too great or the least longitude too short, and so that the centre of the epicycle would come nearer to the Earth at apogee and recede at perigee than a simple eccentric would otherwise require, and so that the centre of the deferent would stand midway between the centre of uniform motion and the centre of sight. Kepler touches on this reason, in other words and fitted to his own mysteries, in the *Mysterium Cosmographicum*, chapter 22, and in the *Commentaries on Mars*, chapters 16 and 32. But Regiomontanus, in the *Epitome*, Book 10, proposition 8, speaking of the bisected eccentricity in ♄ ♃ ♂ ♀, says: "But what reason drove him to this is not sufficiently clear, unless that experience agrees with the position." In the perigee of Mars, however, the Ptolemaic hypothesis is more deceptive, as we shall say in chapter 3, scholium 4.

[Margin: The reason for the equant introduced by Ptolemy.]

[Margin: 1. Reason for the equant.]

[Margin: 2. Reason for the equant.]

III. Giovanni Antonio Magini in his *Theoricae Planetarum* presents the Ptolemaic form to us, but by means of solid orbs, and he borrowed the measures of the epicycles and eccentricities from Ptolemy, although he also indicated the Copernican eccentricity of Mars and followed Copernicus for the numbers of the motions. So there is no need for us to set out Magini's hypothesis separately. In the *Supplement to the Ephemerides*, however, he follows Tycho in the motion of the Sun, which is connected with the planets' second inequality, Kepler in the motions of Mars, and in the rest the numbers of Copernicus; and he bisects the eccentricity with Kepler.

[Margin: Magini's hypothesis.]

IV. Ptolemy and all who followed him, even Copernicus and Lansbergen, take the measure of the second inequality from the mean conjunction of the planet with the Sun, or its mean opposition, and require the mean motion of the Sun to be used in finding it. This does serve ease of calculation, but it is something fictitious, since the mean and true motions of the Sun very rarely coincide; nor is it consonant with nature that the movers of the planets should in their revolutions look to a position and motion of the Sun that is for the most part fictitious, rather than the true one. Moreover, it seems improbable that these same movers should look to the centre of the orbs, which is an empty point in the world, lacking any prerogative; for it seems that what is set as the centre of the planetary orb ought rather to be either the Earth itself, or the Sun itself, or something else of the kind. But of these things below, chapter

5, nos. 5 and 6, where we shall teach that the celestial motions are not exactly represented unless we use the true place of the Sun.

[Margin: Other objections to Ptolemy's hypothesis.]

V. In the preceding chapter, no. 8, in Rule 1, I said that many, with Peuerbach, erred in assigning to the mean apogee of the epicycle what should be assigned in the Ptolemaic hypothesis to the true one. For Peuerbach, in the *Theorica* of the superior planets, part 2, p. 49, says this: "So that at every mean conjunction such a centre of the planet's body is in the mean aux of the epicycle; hence also at every such mean opposition it will be in the opposite of the aux of the epicycle." Reinhold teaches the same with him, in that place, p. 91,

[Margin: Slip of Peuerbach, Reinhold, Wurstisen, Magini and Langius.]

and Wurstisen, p. 142, and Magini in his *Theoricae*, p. 20, and our Langius, p. 375. But Ptolemy, *Almagest* Book 10, chapter 6, and Regiomontanus in the *Epitome of the Almagest*, Book 10, propositions 9 and 11, expressly teach that the superior planets, when they are in mean conjunction with the Sun, are in the true apogee of the epicycle; when they are in mean opposition with the Sun, in the true perigee of the epicycle; and conversely, when they are in the true apogee or perigee of the epicycle, they are on the line of the Sun's mean motion. And rightly so, because according to the opinion of the whole of old astronomy down to the last days of Tycho, or down to Kepler's speculations, whenever these planets fall on the line of the Sun's mean motion, that is in mean ☌ or ☍ with the Sun, they are stripped of the second inequality, and the planet's place under the zodiac is the same as that of the centre of the epicycle, also under the zodiac. So the centre of the planet's body and that of the epicycle are on the same line drawn from the centre of the world to the zodiac, by which line the true apogee and perigee of the epicycle are determined, as Peuerbach's own definitions teach. Therefore, when the centre of the epicycle is placed away from the apogee and perigee of the eccentric, the planet never occupies the line of the Sun's mean motion while set in the mean apogee or perigee of the epicycle, but only in the true one. That this is so the very diagram of Ptolemy and Regiomontanus will teach, which is as follows.

Diagram. The Ptolemaic scheme of the deferent and epicycle, set into the right column with the text running down its left side. A large circle (the deferent) carries a vertical diameter through the points B, F, E, D and A down to C. A small circle centred at F stands at the top, reaching up to B. A second small circle, the epicycle, is drawn on the deferent's left side with its centre at I. Solid lines run from E through I to M, from A through I to V, and from A down to H and to G on the lower arc of the large circle. Dotted lines run from A towards M and towards K. A further solid line runs from I down to R, and the circumference of the large circle passes through I, the centre of the epicycle.

- B (top of the vertical diameter, on the small circle): point labelled B
- F (centre of the small circle at the top of the deferent): point labelled F
- E (on the vertical diameter): point labelled E
- D (on the vertical diameter, below E): point labelled D

- A (on the vertical diameter, below D, where the lines converge): point labelled A
- C (foot of the vertical diameter, on the large circle): point labelled C
- G (lower right on the large circle): point labelled G
- H (lower left on the large circle): point labelled H
- I (centre of the epicycle): point labelled I
- V (upper left on the epicycle): point labelled V
- M (far left on the epicycle): point labelled M
- R (lower left on the epicycle): point labelled R
- L (bottom of the epicycle): point labelled L
- P (lower right on the epicycle): point labelled P
- K (right on the epicycle, on the line from E): point labelled K

Let FC be the diameter of the deferent and the line of apsides, in which the centre of the deferent is D, and A the centre of the world, and E the centre of the equant. If the centre of the epicycle is at F, the apogee of the eccentric, and the planet at B, the apogee of the epicycle, and a mean conjunction with the Sun then occurs, the same line BC will be the line of the Sun's mean motion, and B will be the true and at the same time the mean apogee. Now let the centre of the planet be at I, through which let the line EIM be drawn from the centre of the equant; then the mean apogee of the epicycle will be M, and the mean perigee K. But when the straight line AIV is drawn from the centre of the world, the true apogee of the epicycle will be V, and the true perigee P, from the definitions of Peuerbach and our own, indeed those of all astronomers. Then if a mean conjunction of the planet with the Sun occurs, the planet is at V; but if a mean opposition, it is at P; and the line of the Sun's mean motion is VAG, which at the same place of the zodiac indicates the true place of the centre of the epicycle and at the same time that of the planet. So there is then no need of any other equation than the equation of the centre, that is the angle AIE, which is equal to the angle MIV, by which the true apogee is distant from the mean. Hence Regiomontanus aptly teaches, Book 10, proposition 12, that if at the extreme shinings the planet's place observed by instruments is opposite the place of the zodiac where the mean place of the Sun is then found, the place of the epicycle, or of its centre, has been found. But if, as Peuerbach's followers teach, the planet in mean ♂ with the ☉ were at M, the mean apogee, the line AM would show a different place from the place of the centre I and would be different from the line AI, nor would the planet be free of the second inequality, but the equation of the angle MAI would have to be added to the place of the centre; and likewise, if in mean ♌ it were at K, the mean perigee, the angle IAK would have to be subtracted. The occasion for the error, moreover, was that they thought that the conjunction and opposition with the Sun had to be regarded as mean according to the mean motions of both the planet and the Sun, whereas the mean motion of the planet in the epicycle is counted from the mean apogee.

VI. The greatest and least distances of ♄♃♂ from the centre of the Earth do not vary in Ptolemy, but the distances from the Sun do vary, both because of the Sun's eccentricity, if its variation is admitted, and because of the Sun's unequal annual distance from the Earth, and the motion of the apogee of the Sun and of the planets. The planet's greatest distance from the Sun occurs when it is in conjunction with the Sun and the centre of the epicycle is at the apogee of the eccentric, for then the planet is at the apogee of the epicycle, since it is then most apogean, in Greek [Greek: ἀπογειότατος (apogeiotatos, "farthest from the Earth")]. But the least distance occurs when the planet is opposed to the Sun and the centre of the epicycle is at the perigee of the eccentric, for then the planet is at the perigee of the epicycle and is most perigean, in Greek [Greek: περιγειότατος (perigeiotatos, "nearest the Earth")]. Therefore, when the distance of the Sun from the Earth that belongs to that time of conjunction is taken from the planet's greatest distance from the Earth, what remains is the planet's greatest distance from the Sun. Conversely, the distance of the Sun from the Earth at the time of the already mentioned opposition, added to the planet's least distance from the Earth, makes up the planet's least distance from the Sun. Yet at least for the distances of ♄ and ♃, one could without perceptible inconvenience always use the mean distance of the Sun from the Earth, which in Ptolemy is 1168 semidiameters of the Earth, subtracting it at the already mentioned ☌ and adding it at the already mentioned ☍.

[Margin: On the greatest and least distances of the planets from the Sun.]

If anyone should wish to represent these distances for the time at which the line of apsides of the Sun coincided or will coincide with the line of apsides of the superior planets, a hypothetical time would have to be sought, long before the creation of the world or after it. For since the world was made those two lines have not yet coincided, nor is it probable that it will last so long that they should coincide. For only in the year of Christ 4000 will the apogee of the Sun be joined to the perigee of Saturn, at about 17° ♌, and the perigee of ☉ to the apogee of ♄, at 17° ♒. And not until about the year of Christ 76300 will the perigee of ♂ reach the apogee of ☉, at 14° ♒, and the apogee of the former the perigee of the latter, at 14° ♌. Finally, the apogees of ☉ and ♃ will not be joined until the year of Christ 24000, and that at the beginning of ♌. But for the apogee of ☉ to coincide with the apogee of ♄ or ♂, many more myriads of years are required; and more still, if the apogees of ☉ and of these three planets were all to run together at the same point of the zodiac.

VII. From what has been said, a twofold way can be gathered of setting out the greatest and least distances of the planets from the Sun. One is for a hypothetical time, in which, to obtain the greatest possible distance of the planet from the Sun under this hypothesis, the Sun at the time of its conjunction with the planet would be at the perigee of its eccentric, and the planet at the apogee of its eccentric and epicycle; and to obtain the least, at the time of opposition ☉ would likewise be at the perigee of its eccentric, but the planet at the perigee of its eccentric and epicycle. The other way is for the time of Ptolemy, when

the apogee of ☉ was at 5° ♊, that of ♄ at 23° ♏, that of ♃ at 11° ♍, and that of ♂ at 25½° ♌. Accordingly, with the Sun's anomaly taken into account, the distance of ☉ from the Earth to be used for ♄ was 1147, for ♃ 1166, and for ♂ 1181. Therefore, if we choose the planetary distances of Maurolico and Clavius, the greatest and least distances at that time were as you see in the following small table.

Distance of ♄ ♃ ♂ in semidiameters of the Earth, from Ptolemy, in Ptolemy's time

Planet	Greatest, from the Earth	Greatest, from ☉	Least, from the Earth	Least, from ☉
♄	20072	18925	14378	15525
♃	14370	13204	8854	10020
♂	8848	7667	1216	2397

VIII. To put the size of the epicycle of ♂ before your eyes in the old hypothesis, here is a diagram from Fernel, Book 1 of the *Cosmotheoria*, chapter 8. In it C is the centre of the deferent AB; E the centre of the equant OP; and D the centre of the world. The centre of the Martian epicycle is sometimes at A, the apogee of the eccentric, sometimes at B, its perigee, and the epicycle's semidiameter is BG or AQ. Therefore the outermost orb enclosing the heaven of Mars is FH, KM, and so on, the whole thickness of the globe of Mars itself being counted in, even though it is not shown in the diagram.

Diagram. Fernel's scheme of the Martian orb, set into the lower right of the left column with the text of paragraph VIII running beside it. A single vertical line runs down the middle of the figure. Two large circles overlap on it, and two equal smaller circles sit at its upper and lower ends, the upper around A and the lower around B. An arc from F to H passes over the top of the upper small circle, and an arc from K to M passes under the bottom of the lower small circle.

- Q (top of the vertical line, just inside the upper small circle): epicycle's far point, as cited in the text (AQ)
- F (upper left, at the end of the upper arc): end of the arc FH
- H (upper right, at the end of the upper arc): end of the arc FH
- O (on the vertical line, at the top of one large circle): cited in the text with P as the equant OP
- A (on the vertical line, at the top of the other large circle): apogee of the eccentric; one position of the epicycle's centre
- E (on the vertical line): centre of the equant
- C (on the vertical line, below E): centre of the deferent
- D (on the vertical line, below C): centre of the world
- L (on the vertical line, below D, near the top of the lower small circle): not cited on this page
- P (on the vertical line, below L): cited in the text with O as the equant OP

- B (on the vertical line, below P): perigee of the eccentric; the other position of the epicycle's centre
- G (foot of the vertical line): end of the semidiameter BG
- K (lower left, at the end of the lower arc): end of the arc KM
- M (lower right, at the end of the lower arc): end of the arc KM

CAPVT II.

Chapter 2.

De Triplici Hypothesi Copernici ac Sectatorum eius.

On the threefold hypothesis of Copernicus and his followers.

I. Up to this point all astronomers, along with Ptolemy, except the very few who have dug up the ancient homocentrics (who, not even admitting motion in height, are refuted by the parallaxes of the Moon, which regularly vary most plainly, and are forced into silence), all of them, I say, by admitting the equant had admitted a physical inequality into the motion of the planets, something that ears accustomed to Peripatetic axioms judged especially discordant. For this reason Copernicus, in Book 4, chapter 2, and Book 5, chapter 2, thought it unworthy to ascribe to a heavenly body any real inequality in its own motion, and unbecoming for the mover of the planet itself, whoever that might be, to beg uniformity from another's circle and centre; that is, to borrow the rule of uniformity of motion not in the circle in which the planet moves, nor from the centre of the circle that carries it, but from another, extraneous circle and centre. And he burst out, whether in wonder or in indignation, into this question: "What kind of uniformity, then, will that be, in an alien circle in which its motion does not exist, but in its own eccentric? Thus indeed we marvel," etc. And after a few intervening words: "If now we should wish to establish the inequality of the appearance from unequal motions, it is permissible to observe what kind of argument it will be. For what else shall we do but give a handle to those who disparage this art?"

[Margin: From Copernicus, Book 4, chapter 2.]

"For someone could rise up against the astronomers in this way: Either your observations, by which the motion of the planets appears unequal to you, are fallacious; or, if they are not fallacious, either that inequality is not only apparent to you but real in the world, and this is absurd, since it is proclaimed in the schools and accepted by the wise that a simple and uniform motion befits a heavenly and simple body; or else it appears to you, viewing it from an alien centre, but is not real, and to what end, then, do you pour part of this inequality back into the heavens? It would surely have been better to attach to a heavenly body one single and simple inequality than two, and those mixed with uniformity."

[Margin: Argument against the inequality of the Ptolemaic deferent.]

Fearing these or similar weapons, Copernicus began to think how, from two or more uniform motions in the heavens, and with respect to the circles and centres of such motions, he might demonstrate the inequality that appears to us, or even refer it to a mere accident of sight. He indicated to us three hypotheses for the five minor planets in Book 5, chapter 4. These are set out by Rheticus in his *Narratio*; by Mästlin in the appendix to Kepler's *Prodromus*, that is, to the *Mysterium Cosmographicum*; by Kepler himself, in the same place, chapters 1, 14 and 15, and in the *Commentaries on Mars*, chapters 4, 5 and 6; and by Longomontanus in the *Astronomia Danica*, Book 1 of the *Theorica*, chapter 1, and throughout Book 2. But none of them did so fully or in order, with everything brought together that is necessary for equivalence with the Ptolemaic hypothesis, or for avoiding fallacies or ambiguities in understanding these hypotheses.

[Margin: Authors explaining these hypotheses of Copernicus.]

The first is by a single eccentric with an epicycle, that is, by an eccentrepicycle. The second is by two eccentrics with no epicycle at all. The third is by two epicycles carried on a concentric, that is, by a homocentrepicepicycle [Homocentrepicepicyclum]. Before we set them out, four foundations, as it were, must be laid first, so that you do not keep stumbling on account of less perfect diagrams.

II. The first is that for Copernicus the Sun, at the centre of the world and exempt from all translatory motion, is the centre of the orbs both of the five minor planets and also of the great orb of the Earth, which circles the Sun with an annual motion, as was indicated in Book 3, chapter 6, number 3. It is, I say, a centre not from which the orbs themselves are to be described, but from which the eccentricities, in the hypothesis of eccentrics, are to be measured, as from a starting point, if you examine Copernicus's first idea and intention; whereas Ptolemy reckoned them from the centre of the Earth.

[Margin: Four foundations laid down first for understanding the Copernican hypothesis.]

The second is that Copernicus nonetheless, so as not to seem too fastidious to those accustomed to Ptolemy and to the eccentricity of the Sun, eccentricities of the planets he reckoned not from the very centre of the Sun's body but from that point of the universe around which he posited that the Earth moves uniformly. This point is as far from the Sun as, in the hypothesis of a resting Earth, the centre of the Sun's orb is distant from the centre of the Earth, that is, as great as the eccentricity of the Sun's orb. This distance is indeed so small in respect of the distance of the Sun from the Earth that for this reason some say everywhere that for Copernicus the Sun is the centre of the orbs, and so on. This point, then, around which the Earth is imagined to be turned uniformly each year, is what Copernicus, in reforming the Ptolemaic hypothesis, uses as the centre of the universe. He takes it as the centre of the concentrepicepicycle [Concentrepicepicycli], or as the starting point from which he measures

the eccentricities of the eccentrepicycle, or of the eccentrhypocycle [Eccentrhypocycli], that is, of the biseccentric [Bifeccentrici], as Kepler notes in the *Epitome of Copernican Astronomy*, page 449.

[Margin: Second foundation.]

[Margin: Some call this centre the centre of vision.]

The third is that Copernicus first tried to represent the Ptolemaic hypothesis as to its measures while avoiding a physical inequality of motions, and therefore divided the whole of Ptolemy's eccentricity into 8 parts. He assigned 6 of these to the eccentricity of the eccentrepicycle and the remaining 2 to the semidiameter of the epicycle. So too in the form of the biseccentric he assigned 6 of them to the mean eccentricity of the true eccentric, and the remaining 2 to the semidiameter of the small circle that varies the eccentricity from greatest to least. And finally, in the form of the homocentrepicepicycle, he assigned those 6 to the semidiameter of the greater epicycle and the remaining 2 to the semidiameter of the lesser epicycle. In this way he represented for us the equivalence of his hypothesis with the Ptolemaic. Later, however, weighing the observations of his own time, he took for himself some liberty of correcting the Ptolemaic eccentricity somewhat.

[Margin: Third foundation.]

The fourth is that the annual, or great, orb of the Earth does for Copernicus whatever the epicycles do for Ptolemy. So in this respect Copernicus has no need of those epicycles by which Ptolemy displays the stations and retrogradations, granted that he has converted part of the Ptolemaic eccentricity into other epicycles.

[Margin: Fourth foundation.]

Prima Forma Copernicanæ Hypotheſeos.

The first form of the Copernican hypothesis.

III. In the following figure let E be the Sun, or rather that centre of the universe around which the Earth is to move through the great orb OPQR toward the east. (For since the apogees of the superior planets always tend toward the north, Mästlin not inappropriately turns the eye toward the north in this diagram, and has the east on the right.) Let the eccentricity EA be 6 parts, of which the whole eccentricity of the Ptolemaic equant is 8. From A, at the interval AB, as great as the semidiameter of the Ptolemaic eccentric, describe the eccentric BCD. From the points B, C, D on its circumference as centres, with the semidiameter BG, of 2 parts of which the eccentricity AE was 6, describe three epicycles, one at each of these three places;

[Margin: First form of the Copernican hypothesis.]

Diagram. Mästlin's scheme, set across the left column between two lines of paragraph III. A vertical line runs from F at the top to H at the bottom. Small circles (epicycles) stand at B on top, at C on the right and at D at the bottom. Several large overlapping circles and arcs surround the figure. Within them is

a middle-sized circle around E, and a small circle around A. Horizontal lines run from N and from A to the right, reaching L and C. Straight lines radiate from E to the left and right, and straight lines join G and H to points on the left and right of the middle circle.

- F (top, above the upper epicycle)
- B (centre of the upper epicycle)
- G (below B, at the foot of the upper epicycle)
- O (on the vertical line, above N)
- N (on the vertical line, at the top of the small circle)
- A (left of the vertical line, inside the small circle)
- M (on the vertical line, below A)
- E (on the vertical line, below M; lines radiate from this point)
- V (left, on the middle circle, above R)
- R (left, on the middle circle, level with E)
- T (left, below R)
- X [?] (lower left, below T; the sort is not a clear X: a single diagonal with a foot serif and a short heavy stroke at its base, nearer an italic A in form; the text on the next page cites a point X here)
- S (right, above P)
- P (right, on the middle circle, level with E)
- L (right, above the right epicycle)
- C (centre of the right epicycle)
- K (right, below the right epicycle)
- Q (on the vertical line, at the foot of the middle circle)
- I (on the vertical line, below Q)
- D (centre of the lower epicycle)
- H (bottom, below the lower epicycle)

[Margin: Scheme of Mästlin and in part of Copernicus.]

and let the line of apsides be BAD. Let the eccentric move in the order of the signs toward the east from B to C, and meanwhile let the epicycle, carrying the planet on its circumference, move around its own centre with equal speed and in an equal revolution, in the order of the signs as to its upper part, but against the order of the signs as to its lower part. And let this be on the condition that when the centre of the epicycle is at the apogee B, the planet is at the perigee of the epicycle, G, and when the centre is at D, the planet is at the apogee of the epicycle, H; but there, with the Earth placed at O, B will be the perigee of the eccentric and D the apogee.

With this motion maintained, when the eccentric, having completed the quadrant BC, which the right angle BAC measures, has carried the centre of the epicycle to C, the planet, having completed a quadrant of the epicycle, will be at L. So when NL is drawn parallel to AC, AN will be equal to the epicycle's semidiameter CL. Moreover, the path that the planet describes by its motion is GLH, which is not a perfect circle but a quasi-circle. For if a perfect circle were to be had, the circle GCH would have to be described from the point

M, as far from A as N is from A, that is, a third of the whole eccentricity AE; but the planet does not proceed along this circle. And Copernicus himself, who proposes this form in Book 5, chapter 4, with a very similar and single diagram, demonstrates that the planet goes not along a perfect circle but along a quasi-circle.

The same form is set out by Longomontanus, in Book 1 of the *Theorica*, chapter 1, with this other diagram, in which A is the centre of the eccentric, B the centre of the universe, the eccentricity AB is 6 parts of the 8 that the eccentricity of the Ptolemaic equant has, and CG, a third part of the eccentricity AB, serves as the semidiameter of the epicycle. But the left-hand side is the east, and while the eccentric moves from C to D, the epicycle with equal motion puts the planet from G at H; when the centre of the epicycle is at E, the planet is at I; and then at K, and finally at G. Therefore the path of the planet is GHIKG, departing somewhat from a perfect circle. Yet he adds that in ♂ and ☿ GC is not a third part of the eccentricity.

[Margin: Longomontanus's scheme for Copernicus.]

Diagram. Longomontanus's scheme, set into the right column with the text of the same paragraph running down its left side. A solid circle around A carries four small epicycles at its top, left, bottom and right, with C, D, E and F inside them. A dotted circle stands just inside or crossing the epicycles, through G, H, I and K. A vertical line runs from C through G, A, L and B into the lower epicycle; a horizontal line joins D, A and F; straight lines run from B up to the left and right epicycles.

- C (top, in the upper epicycle)
- G (on the vertical line, below C)
- H (upper left, at the left epicycle)
- D (left, in the left epicycle)
- K (upper right, at the right epicycle)
- F (right, in the right epicycle)
- A (centre, right of the vertical line, above the horizontal line)
- L (on the vertical line, below A; not cited on this page)
- B (on the vertical line, below L)
- E (bottom, in the lower epicycle)
- I (bottom, below the lower epicycle, on the dotted circle)

Secunda Forma Copernicææ Hypotheſis.

The second form of the Copernican hypothesis.

IV. Now, in Mästlin's earlier diagram, for the second form, which Copernicus indicated but did not illustrate with a diagram, let there be the point A, distant from the centre of the universe E by 6 parts, of which the eccentricity of the Ptolemaic equant is 8. From it describe the small circle MN, whose semidiameter AM is to be 2 parts. Let there be a single eccentric BCD, described from A, which is to move the true eccentric that carries the planet, along the small

circle of the eccentricity, in the order of the signs; so that this small circle is understood to move in the order of the signs at N, but against the order of the signs at M. Whenever the planet is at the apogee or perigee of its eccentric, the centre of the true eccentric is to be at M, and the position of the true eccentric GCH; but when the planet is a quadrant from the apogee or perigee, as at L, the centre of the true eccentric is to be at N, and the position of the eccentric along the path FCI.

[Margin: Second form of the Copernican hypothesis, and its first part, for the first inequality.]

For from this too it follows that the planet's path is GLH, very like a circle but not entirely so; and that N is a point which is indeed movable, but a point of uniformity, around which the planet moves uniformly. For with the planet placed at H or G, a straight line is drawn to the planet from M, the centre of the true eccentric, through N, because it lies on the same straight line; outside G and H, if it is at L, then the centre of the true eccentric is united with N; and with the planet and the centre of the true eccentric placed elsewhere, a straight line is nevertheless always drawn from the planet through N. It is necessary, then, that as the planet's motion around the centre of the true eccentric is uniform, so it should be around N; and that this point should be equivalent to the centre of the Ptolemaic equant, as regards accounting for the first inequality, the eccentric one, that is, the one not tied to the Sun.

As for the second inequality and anomaly, which to the motion of the Sun, or in Copernicus's opinion to the motion of the Earth, is attuned, it is the great orb OPQR that exhibits it, because the Earth, viewing the planet from different sides by its annual motion, produces various angles of apparent motion. This variation Copernicus calls the commutation [commutationem] and parallax of the great orb toward the planet. For if the planet is at G or H, and the Earth at O or Q, the same straight line GEH is the line of the true and of the apparent place: of the true place with respect to the centre of the world E, and of the apparent place with respect to the Earth at O or Q. But if the Earth is in the semicircle QRO, say at R, and the planet at G or F, then, when the straight line RG is drawn, the straight line EG will indicate the true place and the straight line RG the apparent place, and the planet will be seen farther on, in the order of the signs. And with the Earth in the semicircle OPQ, it will be seen farther ahead, against the order of the signs, than the line EG requires.

[Margin: Second part, for the second inequality.]

Moreover, the motion of the Earth from the point P, or a little before it, through Q as far as R or a little beyond, makes the planet appear to move faster in the order of the signs than the line EG demands. But when the Earth has revolved from R, approximately through O, to P, the planet appears slower than the line EG requires, because the Earth, being swifter than the planet, comes to meet it when moving through ROP, and draws away from it through PQR. Hence the planet appears to us not only slow but even retrograde, although in reality it always proceeds directly. And so this one motion of the Earth does

what the Ptolemaic epicycles do, and frees the heavenly body from that second inequality, transferring it to mere appearance.

The greatest equation of the centre, or of the eccentric, is the angle ELN, based on the greatest eccentricity, from which, together with the radius NL, it is gathered. The greatest equation of the parallax of the annual orb, if the planet is at apogee, is the angle EGV, which is obtained from the radius of the annual orb EV and the planet's greatest distance EG; but if it is at perigee, it is EHX, which the radius EX and the least distance EH give. But GV and HX ought to have been tangents to the annual orb at the points V and X.

[Margin: Note on the Copernican hypothesis.]

Tertia Forma Copernicææ Hypotheſeos.

The third form of the Copernican hypothesis.

V. The third form, which Copernicus indicates without a diagram in Book 5, chapter 4, and which alone among the Copernican forms Kepler sets out with a diagram, in the *Commentaries on Mars*, chapter 4, works by a circle concentric with the centre of the universe, though eccentric to the Earth, with two epicycles. For this form let A be the Sun, or near the Sun, that is, the centre of the universe. From it describe the concentric BCDI, and let the line of apsides be DF, on which, from the centres B and M, describe one greater epicycle each, whose semidiameter BF is to be 6 parts, of which the whole eccentricity of the Ptolemaic equant has 8. With these parts draw from C and E the two other equal epicycles, that is, positions of the greater epicycle.

[Margin: Third form of the Copernican hypothesis.]

Diagram. The third form, set into the lower part of the left column with the text of paragraph V running down its left side. A circle around A carries four larger circles, on its top, left, bottom and right, with B, C, D and E in them. A smaller circle stands on each of these: above the top one around F, on the upper left around G, inside the bottom one around H, and on the upper right around I. A vertical line runs from the top circle through A to the foot of the bottom circle; a horizontal line runs through C, A and E; straight lines run up from A toward L on the left and N on the right.

- F (top, in the uppermost small circle)
- K (on the vertical line, below F)
- B (on the vertical line, below K, in the upper larger circle)
- L (upper left, outside the small circle around G)
- G (upper left, in a small circle)
- C (left, on the horizontal line, in the left larger circle)
- N (upper right, by the small circle around I)
- I (upper right, in a small circle)
- E (right, on the horizontal line, in the right larger circle)
- A (centre, just below the horizontal line)
- T (below and to the right of A; not cited on this page)

- H (on the vertical line, in the lower small circle)
- M (on the vertical line, below H)
- D (on the vertical line, in the lower larger circle)

Then, with the centre placed at F, a point on the circumference of the greater epicycle, describe the lesser epicycle, whose semidiameter FK is to be 2 parts, of which BF is 6, that is, about a third of it. Let the centre of the greater epicycle B move around the centre A, with the motion of the concentric, in the order of the signs from B to C; and meanwhile let F, the centre of the lesser epicycle, move with an equal revolution against the order of the signs above and in the order of the signs below, striving against the motion of the concentric. But let the planet, beginning its motion from K, move with a doubled revolution against the order of the signs below and in the order of the signs above, resisting the motion of the centre F but following the motion of the eccentric.

For so it will come about that, with the centre F placed at the apogee, the planet holds K, the perigee of its epicycle; and at D, the perigee of the orb, it holds M, the apogee of the epicycle. And when B has completed a quadrant and has been carried to C, the planet, having completed a semicircle of its epicycle, is at L, as far as possible from the centre C. Therefore the arc CL, and its measure the angle CAL, will be the greatest equation to be subtracted from the mean place indicated by the line AC, so that the true place, indicated by the line AL, may be had. But when B, having completed the semicircle BCD, is at D, then the planet, having completed a semicircle of its epicycle, will be at M; and when B has come through CD to E, three quadrants now being finished, the planet, having completed a semicircle and a half, will be at N, and the greatest additive equation will be the arc EN, or the angle EAN.

On these assumptions, as Kepler demonstrates and Copernicus does not deny, and as will be evident to anyone tracing the planet's path mechanically, the planet does not describe a perfect circle in the heavens, but one departing from a perfect circle outward from K to L, and inward from L to M. For another diagram see below, chapter 3, number 4.

Indicatur Methodus Calculi Geometrici.

The method of geometrical calculation is indicated.

VI. The method of the first form is similar to the Ptolemaic method as regards representing the place with respect to the centre of the universe, provided that Copernican measures are used and that the motions are numbered from the first star of Aries, to which the precession of the equinoxes, when added, produces the motion from the equinox. To obtain the place seen from the Earth, imagine the Earth at P, as in the figure of number 3, and the planet at G. For when the centric place of the planet is subtracted from the mean motion of the Sun, or the latter from the former, you will have the anomaly of the orb, or the angle at the Sun, which is GEP. With this, and with the radius of the annual orb EP, or rather with the true distance of the Sun from the Earth, and with EG, the

distance of the planet from the Sun found by the earlier procedure, you will have the equation of the orb EGP, and its elongation from the Sun, or the angle EPG. But in the hypothesis of the moved Earth this method will occur more often; see, therefore, what is to be said in chapter 3, number 5, in the second procedure. The method of the second form, because it is Lansbergen's, is given in chapter 4. The procedure for the third form, but transferred to a standing Earth, will be indicated in chapter 3, numbers 4 and 5.

De Numeris ac Menſuris ad Copernici Hypotheſim ſpectantibus.

On the numbers and measures pertaining to the hypothesis of Copernicus.

VII. The numbers of the motions are found in Copernicus, Book 5, and in the Prutenic Tables, from which Giuntini, Stadius and Magini took their canons. Copernicus indeed gives only the motion of the commutation, or anomaly of the orb, and wants the mean motion of longitude to be gathered by subtracting the motion of commutation from the mean motion of the Sun, and to be numbered from the first star of Aries. But the Alphonsines, omitting the motion of anomaly, give the mean motion of the planet in longitude, numbered from the equinox, and want the motion of the anomaly of the orb, or, as they themselves call it, the motion of the argument, to be gathered by subtracting the planet's mean motion in the zodiac from the mean motion of the Sun. But Reinhold, imitating Ptolemy, arranged three motions in the Prutenic Tables: those of longitude, of the apogee, and of the anomaly or commutation.

[Margin: Difference between the Alphonsine and Prutenic Tables.]

As for the measures of the orbs, Copernicus exhibits or indicates them in one way, in Book 5, chapters 5, 6 and 9 for ♄, chapters 9, 10 and 11 for ♃, and chapters 14, 15 and 16 for ♂. Reinhold gives them in another, since, as is clear from the preface to the Prutenic Tables, he considered that they had to be corrected before he built the edifice of his tables on them. Therefore Mästlin tracked those measures down from the Prutenic prosthaphaereses, and reported them partly in the notes to Kepler's *Prodromus*, or *Mysterium Cosmographicum*, after chapter 15, and partly in the appendix to the same *Prodromus*.

[Margin: Diversity of the measures in Copernicus, Reinhold and Mästlin.]

For in the figure of number 3 consider the triangle ELN, right-angled at N, and look up in the Prutenic Tables the angle L of the equation of the centre, at an anomaly of 90°. For the tangent of that angle will be the quantity of the whole eccentricity EN, in parts of which the radius NL of the planet's orb has been taken, whether as 10000 with Copernicus or as 100000 with Mästlin. The half, EM, will be the least eccentricity, and a fourth part of the whole EN will be AM or GB. Therefore by adding the line EM to the radius one gets EG, the planet's greatest distance from E, the centre of the universe; and by subtracting, EH, the least. Then in the triangle EGP, right-angled at E, there is given from the Prutenic Tables, at an anomaly of the orb of 90°, the angle EGP of the

annual parallax, or commutation, or prosthaphaeresis of the orb, and from it GPE, which is the complement of the other to a right angle, whose tangent EG is the apogee distance of the planet from the centre of the world, but in parts

[Margin: Method of fishing out these measures from the tables.]

of which the radius EP of the Earth's annual orb shall have been 10000 or 100000. But by adding to the angle EGP the whole and greatest excess set down in the tables of the prosthaphaereses of the orb, one gets the angle EHP, whose complement to a right angle is the angle EPH. Its tangent, or what is equal to the tangent, is the line EH, that is, the perigee distance, in parts of which the radius EP of the annual orb has been taken. Half of the whole GH will be the radius AC of the planet's orb in the same parts. The difference between EG and EH will be the whole eccentricity EN; a fourth part of it will be AM or BG; and ME will be half of the whole eccentricity. Mars, however, is an exception: in it Copernicus in his own time found EM smaller than MN, and the whole EN smaller than Ptolemy did, so that as the Sun's eccentricity decreased Mars seemed to Copernicus to have diminished its own eccentricity as well. He thought the same about the eccentricity of Venus.

[Margin: Exception for the eccentricity of ♂.]

In the first place we shall give the measures derived by Mästlin from the Prutenic Tables, and from them we shall append the others he omitted, adding the same measures in semidiameters of the Earth, which he passed over, setting only the mean distance of the Sun from the Earth at 1160 semidiameters of the Earth; by this we shall determine the others. Then we shall set out the measures deduced from Copernicus himself, and we shall use a distance of the Sun from the Earth of 1142 at the mean, but of 1190 at its greatest, at the greatest eccentricity, according to what was said in Book 3, chapters 7 and 24. By these we shall increase or diminish the distances of the planets from the Earth, supposing that the line of apsides of the Sun or Earth and of each individual planet is the same, so that it may appear how far these measures can possibly be extended; granted that such a meeting will never occur, as I said in chapter 1, scholium 6. And because Mars has varied its eccentricity, we shall distinguish its eccentricities and distances at the greatest eccentricity by the letter M, and at the least by the letter m. How the greatest distances from the Earth are to be established for another time will be clear from what is to be said in chapter 5, number 20, § *Tertiò* [Thirdly].

Measures from Mästlin and the Prutenic Tables for the time of Copernicus, the same mean distance of the Sun from the Earth being assumed and applied. Then from Copernicus himself, but for the time of the meeting of the line of apsides with the Sun's line of apsides, and with the greatest of its possible distances applied.

See the figure of numbers 3 and 5 for these intervals	Planets	Parts of which the radius of the planet's orb is 100000: Mästlin	Parts of which the radius of the planet's orb is 100000: Copernicus	Parts of which the radius of the annual orb is 100000: Mästlin	Parts of which the radius of the annual orb is 100000: Copernicus	Semidiameters of the Earth: Mästlin	Semidiameters of the Earth: Copernicus
Eccentricity EN, greatest or whole, in form 2	♄	11390	11390	104300	104520	1210	1394
	♃	9150	9170	48000	47420	556	542
	♂	19530	M. 20000	30410	M. 29210	352	M. 334
			m. 19530		m. 29820		m. 318
Eccentricity EM, least, in form 2	♄	5695	5690	52150	52260	605	667
	♃	4575	4590	24000	23710	278	271
	♂	9530	M. 10000	15205	M. 15180	176	M. 173
			m. 9670		m. 14680		m. 168
Mean eccentricity AE in form 2; or whole in form 1; or semidiameter of the larger epicycle BF in form 3	♄	8543	8543	78225	78390	907	1045
	♃	6863	6870	36000	35560	417	406
	♂	14530	M. 15000	22807	M. 22780	264	M. 260
			m. 14600		m. 22160		m. 243
Semidiameter of the epicycle BG in form 1, or of the small circle AM in form 2, or of the smaller epicycle KF in form 3	♄	2848	2850	26075	26130	302	348
	♃	2288	2290	12000	11850	139	135
	♂	5000	M. 5000	7602	M. 7590	88	M. 87
			m. 4930		m. 7480		m. 85
Radius or semidiameter of the planet's orb AC, or mean distance from ☉ or from the Earth	♄	100000	100000	916450	920960	10933	10517
	♃	100000	100000	524600	521150	6085	5951
	♂	100000	100000	152040	151820	1764	1733
Radius EP of the Annual or Great Orb of the Earth	♄	10912	10860	100000	100000	1160	1142
	♃	19062	19190	100000	100000	1160	1142
	♂	65772	65870	100000	100000	1160	1142
Greatest distance EG from the Sun, the centre of the world [Margin: Aphelion distance.]	♄	105695	105690	968600	973220	11538	11214

See the figure of numbers 3 and 5 for these intervals	Planets	Parts of which the radius of the planet's orb is 100000: Mästlin	Parts of which the radius of the planet's orb is 100000: Copernicus	Parts of which the radius of the annual orb is 100000: Mästlin	Parts of which the radius of the annual orb is 100000: Copernicus	Semidiameters of the Earth: Mästlin	Semidiameters of the Earth: Copernicus
	♃	104575	104590	548600	544860	6363	6222
	♂	109530	M. 110000	166530	M. 166430	1940	M. 1906
			m. 109670		m. 165720		m. 1901
Least distance EH from the Sun, the centre of the world [Margin: Perihelion distance.]	♄	94305	94310	864300	868700	10328	9820
	♃	95425	95410	500600	497440	5807	5681
	♂	90470	M. 90000	137550	M. 137220	1588	M. 1560
			m. 90330		m. 137900		m. 1565
Greatest distance GQ from the centre of the Earth [Margin: Apogee distance, with the planet in ♂ with ☉.]	♄	116607	116550	1068600	1073220	12698	12356
	♃	123637	123780	648600	644860	7523	7364
	♂	175302	M. 175870	266530	M. 267000	3100	M. 3048
			m. 175540		m. 266500		m. 3043
Least distance HQ from the centre of the Earth [Margin: Perigee distance, with the planet in ☍ with ☉.]	♄	83393	83450	764300	768700	9168	8678
	♃	76363	86220 [recte: 76220]	400600	397440	4767	4538
	♂	24698	M. 24130	37550	M. 36640	428	M. 418
			m. 24460		m. 37140		m. 423

Because Mästlin takes the radius of the annual orb as 1160 semidiameters of the Earth, larger than Copernicus, who takes it as 1142, the measures of the distances in semidiameters of the Earth therefore come out larger from the former than from the latter. Copernicus, Book 5, chapters 5, 6, 10, 11, 15 and 16, sets EN at 1139 parts in Saturn, 917 in Jupiter and 2000 in Mars, of which AC is 10000. But EA he sets at 854 in Saturn, 687 in Jupiter and 1460 in Mars, and

BG at 285 in Saturn and 229 in Jupiter, always in parts of which AC is 10000. We, however, have reduced them to a radius of 100000.

[Margin: Copernicus's own measure.]

Mean or equal motion.

Daily	Planet	′	″	‴	⁗	V	VI
Of the apogee, from the first star of ♈	♄	0	0	6	1	46	27
	♃	0	0	1	46	45	27
	♂	0	0	4	43	30	0
Of longitude, from the first star of ♈	♄	2	0	27	17	53	49
	♃	4	59	7	34	45	14
	♂	31	26	30	58	57	38
Of longitude, from the apogee [Margin: Anomaly of the eccentric.]	♄	2	0	21	16	7	22
	♃	4	59	5	47	59	47
	♂	31	26	26	15	27	38
Of commutation, from the apogee [Margin: Anomaly of the orb.]	♄	57	7	44	4	22	22
	♃	54	9	3	47	30	57
	♂	27	41	40	23	18	34

60

Annual	Planet						
Of the apogee, from the first star of ♈	♄	0	0	36	40	48	0
	♃	0	0	10	49	26	0
	♂	0	0	28	44	37	0
Of longitude, from the first star of ♈	♄	12	12	46	3	52	23
	♃	30	19	41	6	25	10
	♂	191	16	18	28	40	34
Of longitude, from the apogee	♄	12	12	9	23	4	23
	♃	30	19	30	16	59	10
	♂	191	15	49	44	3	34
Of commutation, from the apogee	♄	347	32	3	6	16	6
	♃	329	25	8	4	3	18
	♂	188	28	30	31	47	54

Revolution, or period of the motions, in Egyptian years.

	Planet	Years	D.	H.	I.	II.	III.
Of the apogee, under the fixed stars	♄	35333	0	0	0	0	0
	♃	119734	0	0	0	0	0
	♂	45088	0	0	0	0	0
Of the apogee, under the zodiac	♄	14917	0	0	0	0	0

[60] the annual rows that follow in the same frame carry no printed order labels; the column heads are left blank here accordingly.

	Planet	Years	D.	H.	I.	II.	III.
	☿	21237	0	0	0	0	0
	☌	16416	0	0	0	0	0
Of longitude, under the fixed stars	♄	19 [recte: 29]	174	4	58	25	30
	☿	11	317	14	49	31	56
	☌	1	321	23	31	56	49
Of longitude, under the zodiac	♄	29	161	22	28	3	22
	☿	11	315	15	2	10	30
	☌	1	321	22	19	49	48
Of longitude, from apogee to apogee	♄	29	183	4	55	0	0
	☿	11	318	0	58	0	0
	☌	1	321	23	56	0	0
Of commutation, from the apogee	♄	1	13	6	24	57	26
	☿	1	33	21	13	15	33
	☌	2	49	22	28	57	49

Copernicus reckons his motions from the first star of Aries. Therefore, when the motion of the fixed stars is added to the motion of the apogee or of longitude, one gets the motion of the same, but computed from the mean equinox. The motion of longitude reckoned from the apogee of the planet is called the anomaly of the eccentric, and is had by subtracting the motion of the apogee from the motion of longitude. The motion of the planet on the Ptolemaic epicycle, or of the anomaly of the orb, is transferred by Copernicus to the annual motion of the Earth, which produces this parallax, and it is therefore called the motion of commutation, or of the annual parallax. This motion is had by subtracting the motion of longitude under the fixed stars from the mean motion of the Sun, likewise under the fixed stars.

Roots for the first year of Christ, at the midnight preceding 1 January, at Königsberg in Prussia.

	Planet	Signs	Deg.	′	″	‴	⁗
Of the apogee, from the first star of ♈	♄	7	15	0	5	6	0
	☿	5	3	55	54	31	0
	☌	3	17	45	32	0	0
Of longitude, from the 1st star of ♈	♄	2	6	41	51	25	54
	☿	5	24	14	3	24	41
	☌	1	4	7	46	34	59
Of commutation, from the apogee	♄	6	25	48	0	7	53
	☿	3	8	15	48	10	7
	☌	7	28	22	4	57	49

SCHOLIA.

Scholia.

I. I see that six objections especially can be raised against the hypotheses of Copernicus. The first is that in none of his forms does a planet trace a perfect

circle, and that in this he is outdone by Ptolemy. For it seems as absurd to attribute to a heavenly body a path of irregular figure, as Copernicus does, as to attribute to it an irregular motion, which Copernicus detests in Ptolemy. Further, as Boulliau notes in Book 1, chapter 14, the planet does not move in the very circle that is the index of the mean motion but on epicycles, as far as forms 1 and 3 are concerned.

[Margin: Objection 1 against Copernicus.]

II. The second is the disagreement with Ptolemy in the calculation, especially of Mars, by almost one degree, as Kepler shows by examples in his *Mars*, chapter 4; and that the calculation of the first inequality is easier in the Ptolemaic form than in the Copernican, as Kepler shows in the same place, chapter 5. But that disagreement is perhaps no fault, if it agrees with the heavens rather than with Ptolemy.

[Margin: Objection 2.]

III. The third is that if Copernicus admits solid orbs, he too multiplies the movers, as many as the circles he employs. But if, as Tycho establishes from the trajections of comets and from other arguments, there are no solid orbs, something almost impossible is imposed on either one mover or several: namely, that they should look to centres that have no stable or known seat in any body. And in forms 2 and 3 there is a conflict of motions, since while one small circle advances another goes back. Kepler noted this in Copernicus in part above, and Boulliau in part in Book 1, chapter 14.

[Margin: Objection 3.]

IV. The fourth is that he uses the mean motion of the Sun rather than the true in separating the second inequality from the first and in investigating the anomaly of commutation. We shall show in chapter 5, numbers 5 and 6, that this is neither probable a priori nor in agreement with observations.

[Margin: Objection 4.]

V. The fifth is that the variety of effects in sublunary things, evident from the stations and retrogradations of the planets, is attributed to the Earth itself, which receives their rays in one position after another, going to meet them or receding from them. Yet it seems more consonant with reason that this cause should lie in the heaven itself and in the motion of the planets; although the Copernicans will easily make light of this objection.

[Margin: Objection 5.]

VI. The sixth is the absurdity of the Earth's motion and the rest of the Sun, against the senses and against the authority of Holy Scripture, without any evident reason to the contrary. For his chief reason for devising new hypotheses was so as not to admit a motion of a heavenly body that is unequal about its own centre and equal about another's, as is clear from his own words in Book 5, chapter 2, where he says of the ancients: “They grant, therefore, here too that the equality of circular motion can occur about a centre that is another's and not its own, and likewise that this happens all the more in Mercury. But this has already been sufficiently refuted in connection with the Moon” (namely

in Book 4, chapter 2). "These and similar things gave us the occasion for thinking about the mobility of the Earth and about other ways by which the equality and the principles of the art might remain, and the reason of the apparent inequality might be rendered more constant." And so immediately in chapter 3 he explains away, by the annual motion of the Earth, the stations and retrogradations apparent in the planets. But that very equality of motion about its own centre

[Margin: Objection 6.]

[Margin: The occasion for introducing the motion of the Earth.]

is upheld by Tycho, Longomontanus and ourselves, below, without the motion of the Earth; and, on the contrary, Kepler and Boulliau admit the motion of the Earth, but in such a way that they nonetheless admit a physical inequality of motion, and for this reason they bisect the eccentricity, as will be clear below. The motion of the Earth was therefore not necessary, nor, once it is posited, is an inequality of motion about its own centre necessarily excluded. But more against Copernicus in Book 9.

VII. Although Copernicus, out of all three forms, finally chose the eccentrepicycle for the three superior planets, as is clear from his statement in Book 5, chapter 4, where he says: "But we have here chosen the eccentrepicycle", nonetheless for the two inferior planets, Mercury and Venus, he chose the eccentric of the eccentric [Eccentri Eccentrum]; for he says of them in Book 5, chapter 22: "and this will be demonstrated more easily and conveniently by means of the eccentric of the eccentric." I wonder, therefore, why for the three superior planets Mästlin proposes the eccentric of the eccentric [Eccentri Eccentricum], and Kepler epicepicycles, as Copernicus's view, with the other forms set aside, as if that were the chief one in Copernicus's judgement.

[Margin: Which of the three forms is to be chosen according to Copernicus's mind.]

CAPVT III.

Chapter 3.

De Tychonis & Longomontani Hypotheſibus.

On the hypotheses of Tycho and Longomontanus.

I. Tycho himself too, in volume 1 of the *Progymnasmata*, p. 479, and in volume 2, p. 188, detesting "an incongruous coherence with mathematical regularity", to use his words, that is, the physical inequality of motion admitted by Ptolemy in introducing the circle of the equant, treated in the scholia of chapter 1, chose the third form of Copernicus, sufficiently explained in chapter 2, number 5, and to be repeated below in number 6, which has a concentric with two epicycles. He kept their measures and the kind of their motions, and indeed their quantity too, until he might correct them, should he survive. Since God did not grant him this, on his deathbed he asked Kepler to demonstrate

everything in this form of his as well, as Kepler himself attests in his *Mars*, chapter 6. For although it is equivalent to the Copernican and the Ptolemaic, it nonetheless differs from them in certain respects and has its own characteristic features.

It agrees with the Ptolemaic hypothesis both because it keeps the same measures, though transferred to the semidiameters of two epicycles, and because it lays down that not the Earth but the Sun moves about the centre of the universe in an annual revolution. Again, it agrees with the Copernican both because, having rejected real inequality, it recognises in all the circles it employs an equal motion with respect to their own centres, and transfers the whole inequality to our sight, which is outside those centres, so that it is all apparent or optical, not physical; and because it would have the planets move about the Sun as about the centre of their orbs. In these two conditions it differs from Ptolemy, who both attributes half of the inequality to a physical irregularity and measures the eccentricity of the planetary orbs from the centre of the Earth, not from the Sun or from a point near the Sun.

[Margin: Agreement of the Tychonic hypothesis with the Ptolemaic and the Copernican.]

It differs from Copernicus both as regards the annual motion of the Earth, which it denies, as I have already said in Book 3, chapter 6, number 4, where I indicated the Tychonic system; and as regards the cause of the second inequality, which it attributes to the motion of the planet about the Sun and of the Sun about the Earth, so that the orb of the Sun serves Tycho as a great epicycle, just as the annual orb of the Earth serves Copernicus in the manner of the Ptolemaic epicycle. And so in volume 2, p. 188, he says: "From this an evident cause is given why the simple motion of the Sun is necessarily mixed with the motions of all five planets in a particular and fixed tenor, so that all the celestial appearances direct themselves by the norm of the Sun, and it governs the whole harmony of the choir of the planets like Apollo in the midst of the Muses." In volume 1, p. 478, he had said from his own observations that the Copernican symmetries of the orbs are somewhat different, but with a very small discrepancy, which he could not explain to us. We shall therefore turn to Longomontanus.

[Margin: Difference of the Tychonic hypothesis from the Ptolemaic and the Copernican.]

[Margin: Why the Sun is Apollo.]

II. Now then, Longomontanus, in the *Astronomia Danica*, Book 1 of the *Theorica*, chapter 2, towards the end, and throughout Book 2, prefers the Tychonic form to the others, although he also presents the Copernican of the third form, and the Ptolemaic, but reformed. We shall briefly indicate these here from him.

Ptolemaica Hypotheſis à Longomontano Reformata.

The Ptolemaic hypothesis as reformed by Longomontanus.

This form, applied to Saturn, Longomontanus proposes in Book 2 of the *Theorica*, chapters 2, 3 and 4, for the three superior planets, and indeed for the other two planets in chapters 17 and 20. On that occasion he also indicates a method of geometrical calculation.

[Margin: First form of the Longomontanic hypothesis.]

Diagram. A figure set into the right column, with the text running in a narrow measure to its left. A straight vertical line runs from B at the top through A to P at the bottom. A large arc runs from B up and to the left and down around to P. On the upper left of this arc stand three circles of different sizes, overlapping. The largest passes near I and K at the upper left and near L at the right. A middle-sized circle lies below it, with E on its left. A small circle stands between them, with D inside it, G above it and H to its right. F and C stand side by side below the small circle, with M to the right of them. Straight lines run from A up and to the left into the group of circles, and a dotted line runs from A towards L. Within the arc, below the circles, stand the planetary signs ♄ ☉ ♂, and below them the word *Ptolem*.

- I (upper left of the largest circle)
- K (just below I)
- G (top of the small circle)
- H (right of G)
- D (inside the small circle)
- E (left of the middle-sized circle)
- F (below the small circle)
- C (right of F)
- L (right of the largest circle)
- M (below L)
- A (on the vertical line, right of the circles)
- B (top of the vertical line)
- P (foot of the vertical line)
- ♄ ☉ ♂ (inside the arc)
- *Ptolem* (inside the arc, below the signs)

In the accompanying figure, then, let A be the centre of the Earth, about which is the concentric BCP etc., on which the simple revolution of the planet through the whole zodiac is completed by a motion in the order of the signs from B to C. At some point on its circumference, say C, let there be a centre about which you are to describe the larger epicycle DE etc., whose semidiameter in Saturn is to be 8721 parts, of which the radius AB of the orb is 100000. Let this epicycle move against the order of the signs in its upper part and in the order of the signs in its lower part, with a motion commensurable with the motion of the concentric itself, so that if the centre of this epicycle has been carried by the concentric from B to C through an arc of 60°, the point E, that

is, the centre of the smaller epicycle, of which more shortly, has come to D, an arc ED of 60° having been completed. Therefore from the point D, on the circumference of the epicycle, describe the smaller epicycle GHF, whose semidiameter DG is to be a third part of the semidiameter CD, that is, 2907 parts (this epicycle Longomontanus calls the equant). Let this move with a double revolution, or twice as fast as the larger epicycle, but at the top in the order of the signs from F, its perigee, through H to G; and let the point that was at F when the centre C was at B be, in the case of the present figure, at H, an arc FH of 120° having been completed.

Although by these epicycles alone the anomaly of the three superior planets can be saved in most respects, yet so that their acronychal position may be better saved, because they then come closer to the Earth, from the point H on the circumference of the small epicycle describe the largest epicycle IEL, which Longomontanus calls the annual. Its semidiameter HI Longomontanus, Book 2, chapters 3 and 4, finds from acronychal observations of Saturn to be now 10398½ parts, now 10453½ parts; and he therefore chooses the mean, of 10426 or 10425 parts, of which the radius AB is 100000. For he says that this satisfies the observations, namely those Tycho has in the *Epistolae Astronomicae*, p. 56, in such a way that no sensible difference, that is, one reaching a whole minute, is detected; and that on account of the varied eccentricity of the Sun, with whose increase the semidiameter HI ought to decrease and with whose decrease it ought to increase, no sensible difference is however found in the place of Saturn and Jupiter, which nevertheless, if the proportion of increase and decrease were kept, should have reached a fifth part of a degree.

[Margin: The variation of the Sun's eccentricity does not sensibly vary the place of ♄ and ♃.]

Accordingly in the acronychal position, when the planet is opposed to the Sun in true ☍, if the centre of this largest epicycle is at H, the planet will be at M; but outside it, elsewhere, say at L or K; and in true conjunctions with the Sun it will be at the apogee I. The motion of the planet on this annual epicycle is to be in the order of the signs from I to K, and as great as the elongation of the Sun from the planet, or as great as the motion of the anomaly of the orb in the tables.

Methodus calculi Geometrici in hac Forma.

The method of geometrical calculation in this form.

III. First, let us suppose, as above, that the centre of the first epicycle has been carried from B to C by the planet's mean motion in longitude, and that the centre of the smaller or second epicycle is at H by a double and contrary motion; and let the straight line CH be drawn. For in triangle CDH the sides CD and DH are given from the semidiameters of the epicycles, and so is the angle CDH, which measures the arc FH, double the arc ED or CB, and in our case of 120°. Therefore by trigonometry the side CH will be given, and the

angle DCH, which in this figure and case is to be added to the angle ECD, or the arc ED, to make the angle ECH. With the straight line AHI then drawn through H, the triangle ACH arises, in which the angle ACH is given, since it is the complement to two right angles of the angle ECH just found; the side CH is given too, in parts of which the given or assumed side AC, or radius, is 100000. Therefore, by a further solution of this triangle, the angle CAH will be given, which is the equation of the centre [Æquatio Centrica], or of the first inequality. In the first semicircle BCP, as you see, it is to be subtracted from the mean motion in longitude, and the true motion results at H, as shown by the line AHI, which is the line of the true place of the centre. Indeed, if an acronychal opposition occurs, it will be the line of the true place of the planet, and the planet will be at M. Outside the acronychal position, however, the prosthaphaeresis of the second inequality has to be sought.

[Margin: 1. Step.]

Second, therefore, in the triangle ACH, from the data above, find the side AH, which is the distance of the centre H from the centre of the Earth. And let the planet be at L (to which draw the straight lines AL and HL), where it has completed the arc of anomaly IKEL from the apogee I, which in this figure exceeds the semicircle IEM by the arc ML. The angle MHL, that is, the angle AHL, will therefore be known. But the side HL is given, which is the semidiameter of the annual epicycle; so the angle HAL, that is, the equation of the second inequality, will not remain unknown. Subtracted here from the true place of the centre, it will show the true place of the planet in the zodiac. But if the planet were at K, one would have to draw to it the straight line HK, and from A the straight line AK (which is missing from the figure); and the arc of anomaly IK would give the angle IHK, whose complement to a semicircle, measuring the arc KEM, would have to be used as the angle contained by the sides HK and HA. From these the equation KAH would be gathered, to be added to the true place of the centre so as to have the true place of the planet in the zodiac. Longomontanus says that this form also suits Jupiter and Mars, if the semidiameters of the epicycles and the radius of the annual orb suited to each are applied, according to the measures to be given below after number 8.

[Margin: 2. Step.]

Copernicæa Hypotheſis à Longomontano Propoſita.

The Copernican hypothesis as proposed by Longomontanus.

IV. This is the third form of Copernicus, already set out in chapter 2, number 5, where we indicated the motions and measures of the epicycles. As regards the first two epicycles, these agree with the motions and measures of the preceding form set out in number 2. We shall therefore be briefer here, as Longomontanus too is in Book 2 of the *Theorica*, chapters 2, 5, 6 and 7, where he proposes it. Let A, then, be the Sun, for Copernicus the centre of the universe, about which is the orb of the planet BCP, in which the mean motion under the zodiac

is performed from B to C. And from C describe the larger epicycle (for we suppose that centre to have come from B to C by the arc BC, which let be, for example, 60°). Meanwhile, on the circumference of that epicycle, let the centre of the smaller epicycle have come from E to D, by a motion equally swift but contrary, the arc ED having been completed; and let the planet, on the circumference of the smaller epicycle, by a doubled motion but in the order of the signs, have come from its perigee F to H, an arc FH of 120° in our figure having been completed.

Diagram. A figure set into the foot of the left column, with the text running in a narrow measure to its left. A straight vertical line runs from B at the top, past A, to P at the bottom. A large arc runs from B up and to the left and down around to P. On the upper left of this arc stand two overlapping circles: a smaller one with G above it, D inside it, F inside it below D and H on its right, and a larger one with E on its left and C at its lower part. Short straight lines join the points inside these circles. From the group of circles a solid line and a dotted line run down and to the right to a small circle about A, whose circumference carries M at the upper left, L at the left and K and I at the lower right; a line runs on through A to the lower right. Within the arc, below the circles, stand the planetary signs ♄ ♃ ♂, and below them the word *Coper*.

- G (above the smallest circle)
- D (inside the smallest circle)
- H (right of the smallest circle)
- E (left, where the two circles overlap)
- F (below D)
- C (lower part of the larger circle)
- B (top of the vertical line)
- M (upper left of the circle about A)
- L (left of the circle about A)
- A (centre of the small circle, beside the vertical line)
- K (lower right of the circle about A)
- I (below K)
- P (foot of the vertical line)
- ♄ ♃ ♂ (inside the arc)
- *Coper* (inside the arc, below the signs)

And let two straight lines be drawn from A, one to the centre of the larger epicycle, which is to be AC, the other to the planet, which is to be AH. For AC will be the line of the true place of the centre as seen from the Sun, and AH the line of the true place of the planet, likewise as seen from the Sun. Then, about A, or about a point distant from the Sun by just as much as the Sun's eccentricity at the given time, describe the Earth's Annual or great orb MLIK. For if a true opposition of the planet H with the Sun occurs, the Earth will be at M; but if a true conjunction, the Earth will be at I; at other times it will be in other places, say at K or at L. The semidiameter AM of the annual orb, however, must be equal to the semidiameter of the annual epicycle discussed

in the preceding form, namely, in Saturn, 10425 parts of which AB is 100000, so that this hypothesis may be equivalent to that reformed Ptolemaic one. From this you see that, given the mean distance of the Sun from the Earth in semidiameters of the Earth, AB is given too, and also the semidiameters of the epicycles, CE and FD, in semidiameters of the Earth, together with the distances of the planet from the Earth and from the Sun, if they are given in parts of which AB is 100000, as they will be given below in a table.

[Margin: Foundation of the measures by semidiameters of the Earth.]

Indicatur Calculus Geometricus in hac Forma.

The geometrical calculation in this form is indicated.

V. First, let there be a true opposition or conjunction of the planet H with the Sun A, so that the whole second inequality ceases and only the equation of the centre, or of the first inequality, is needed; and let the line IAMH be the line of the true place, and of the place seen both from the Sun A and from the Earth at M, in ☍, or at I, in ☌ of the planet with the Sun. For then, with the straight line AH drawn, and from C to the planet H the straight line CH, and from D to the same planet the straight line DH, one must first solve the triangle CDH. In it are given the semidiameters of the epicycles, CD and DH, and the angle they contain, CDH, as great as the arc FH, which is known from the planet's mean motion in longitude, but doubled: thus, if BC was 60°, let FH, that is, the angle CDH, be 120°. So, with trigonometry as your guide, find the side CH and the angle DCH; add this to the angle ECD, that is, to the planet's mean motion, and the angle ECH results, whose complement to two right angles is the angle ACH. With this angle, in the triangle HCA, and with the side CH just found, and the radius AC, seek both the angle of the equation CAH, which here is to be subtracted from the mean motion to make the true place both of the centre and, in this first case, of the planet; and the side AH, which is the distance of the planet from the Sun. If, in acronychal ☍, you subtract from this AM, the distance of the Sun from the Earth, MH will be known, the distance of the planet from the Earth at M; and in ☌, when the distance IA of the Earth at I from the Sun is added, HI will become known, the distance of the planet from the Earth.

[Margin: 1. Step in true ☍ and ☌ of a planet with the ☉.]

Second: if there is not a true opposition or conjunction of the planet with the Sun, but the Earth is, for example, at L, count the anomaly of the orb from the apogee of the annual orb, which is I, in the order of the signs through K and M, so as to have the arc IKML. And if, as in this figure, it exceeds 180°, when these have been cast off there will remain the arc ML, which measures the angle HAL. Therefore, when LH is drawn, the triangle LAH will arise, in which are given the angle at A, as I said, and the side AH, found by the previous step, and AL, the semidiameter of the annual orb. So the angle LHA will not remain unknown, which is the prosthaphaeresis of the parallax, or of

the commutation, or of the second inequality. In this case it would have to be subtracted from the true place of the centre H found above, so that the true place of the planet H in the zodiac, but as seen from the Earth, might become known. But if the Earth were at K, the arc of anomaly IK would give the angle IAK, whose complement to a semicircle would have to be used as the angle contained by the semidiameter AK of the annual orb and the distance HA of the planet from the Sun, so that from these data the angle at H might be gathered, which would have to be added to the true place of the centre so as to have the true place of the planet in the zodiac.

[Margin: 2. Step outside ☍ and ☌.]

Tychonica Hypotheſis à Longomontano Propoſita.

The Tychonic hypothesis as proposed by Longomontanus.

VI. This third form, finally, which is plainly Tychonic (for the first is Tycho-Ptolemaic, the second Tycho-Copernican, or altogether Copernican), Longomontanus proposes in Book 1 of the *Theorica*, towards the end of chapter 1, and in Book 2, chapter 2, at the end, and chapter 9. In it let A be the centre of the body of the Sun, M the centre of the Earth, and ARL the annual orb of the Sun; and about A describe the orb of the planet BCP etc., concentric with the Sun, in which its mean motion in longitude is performed under the zodiac; and from C describe the larger epicycle, and from D the smaller, whose revolutions and measures are to be altogether of the same quantity and kind as in the two preceding forms. For when the planet H is truly opposed to the Sun at A, or truly conjoined with the Sun at L, the second inequality will cease, and nothing will be needed but the equation of the centre.

Diagram. A figure set into the head of the left column, with the text running in a narrow measure to its left. A straight vertical line runs from B at the top, past R and A, to P at the bottom. A large arc runs from B up and to the left and down around to P. On the upper left of this arc stand two overlapping circles: a smaller one with G above it, D and F inside it and H on its right, and a larger one with E on its left and C at its centre. Straight lines run from C towards E and towards H, and from the region of the circles down and to the right to a circle about M, which passes through A at its foot and has L at its upper left and R at its upper right. A dotted line runs from H to R, and a short dotted line from R towards M. Within the arc, below the circles, stand the planetary signs ♄ ♃ ♂, and below them the word *Tycho*.

- G (above the smaller circle)
- D (inside the smaller circle)
- H (right of the smaller circle)
- E (left, at the larger circle)
- F (inside the smaller circle, below D)
- C (centre of the larger circle)
- B (top of the vertical line)

- L (upper left of the circle about M)
- R (on the vertical line, upper right of the circle about M)
- M (inside the circle)
- A (foot of the circle, on the vertical line)
- P (foot of the vertical line)
- ♄ ♃ ♂ (inside the arc)
- *Tycho* (inside the arc, below the signs)

Indicatur Methodus Calculi Geometrici in hac Forma.

The method of geometrical calculation in this form is indicated.

VII. When the planet H is in true opposition with the Sun at A, or in conjunction with the Sun at L, the line AMLH of the place seen from the Sun and from the Earth is the same; and then, with CH drawn, the side CH is sought in the triangle CDH. For in it are given the sides CD and DH, which are the semidiameters of the epicycles, and the angle CDH, which is as great as the arc FH, that is, double the planet's mean motion. Further, the angle DCH is sought, which, added here to the angle ECD, that is, to the planet's mean motion, makes up the angle ECH, whose complement to two right angles is the angle HCA. Therefore, in the triangle CAH, with the angle at C, and the side CH found, and the radius of the orb AC, the angle CAH is found, that is, the equation of the centre, or of the first inequality, to be subtracted, in this semicircle BCP, from the mean motion of the centre C; and the true place both of the centre and of the planet in the zodiac results. Further, the side AH is to be sought, so as to have the distance from the Sun at A, in opposition; or, with the diameter of the Sun's orb AL subtracted, the distance from the Sun at L, in conjunction. From these, by subtracting the distance of the Sun from the Earth in the first case and adding it in the second, the distance of the planet from the Earth will be known.

[Margin: 1. Step, for a planet in ☌ or ☍ with the ☉.]

But if a true ☌ or ☍ does not take place, and the Sun is, for example, at R, and the motion of the anomaly counted from the apogee L in the order of the signs is the arc LAR, its complement to a circle will be the arc RL, measuring the angle LMR. So, with RH and MR drawn, the triangle HRM will be formed, in which the angle at M is known from the anomaly or its complement; the side MH was already found in the first step; and MR is the semidiameter of the Sun's orb, or rather the distance of the Sun from the Earth. So the angle at H will not remain unknown, to be added in this case to the true place of the centre, so as to make the true place of the planet in the zodiac.

[Margin: 2. Step outside ☌ and ☍.]

De Menſuris & Numeris ad Tychonis aut Longomontani Hypotheſim ſpectantibus.

On the measures and numbers pertaining to the hypothesis of Tycho or of Longomontanus.

VIII. We have derived the Tychonic measures, as many as we could, from the data given by Tycho in volume 1 of the *Progymnasmata*, pp. 476 and 480. The Longomontanic measures, on the other hand, we have derived from the data given by Longomontanus in Book 2 of the *Theorica*, chapters 2 and 3, where he assesses the semidiameters of the epicycles of Saturn; chapters 5 and 6, those of Jupiter; and chapters 8 and 9, those of Mars, together with the semidiameters of the annual orb, in parts of which the radius of the orb of each planet is 100000. Since, moreover, in Book 1 of the *Theorica*, chapters 6 and 8, he has defined the semidiameter of the annual orb, that is, the Sun's mean distance from the Earth, as 1288 semidiameters of the Earth, the greatest as 1334 and the least as 1242, the semidiameters of the Earth corresponding to the semidiameters of the epicycles and to the radius of the planetary orb will easily be gathered by the golden rule of proportions, and from these the distances from the Earth and from the Sun.

[Margin: Method of gathering the distances of the planets from the Sun and from the Earth.]

In Mars, however, two things are to be noted specially from Longomontanus, Book 2 of the *Theorica*. The first is from chapter 8, where he says that the semidiameter of the equating small circle [circelli æquatorei], or smaller epicycle, is not to be given a third part of the semidiameter of the larger epicycle (as was done in Saturn and Jupiter), but only a fourth, as is gathered from the prosthaphaereses of Mars that agree with Tycho's observations. The second is from chapters 9 and 12, where he affirms, from observations treated geometrically, that the semidiameter of the annual orb, in proportion to the radius of the orb of Mars, comes out much greater when the Sun and Mars are at the apogee of their eccentric than when both are at perigee, so much so that in the former case it is 67962½, in the latter 63028½, of which the radius of the orb of Mars is 100000; the mean, therefore, is 65497. But if we always used this mean, an error of nearly two degrees could be committed at the apogees and perigees of both bodies; and so it is not permissible to use it in the prosthaphaereses of Mars, but a more exact value, suited to each place, must be used. This variation, moreover, he attributes not to the orb of Mars itself but to a certain libration, either of the eccentric or of the annual orb, by which it would imitate the distances of the Sun from the Earth.

[Margin: The smaller epicycle of Mars is unlike the others.]

[Margin: The Annual Orb of Mars is one at apogee, another at perigee.]

IX. For an easier grasp of the measures written below, from the hypothesis of Longomontanus according to Tycho's mind, we shall add three diagrams, one for Saturn and Jupiter, the second and third for Mars.

Diagram. A tall, narrow figure set into the right column, with the text running in a narrow measure to its left. A single straight vertical line runs through all its parts. At the top a small circle with F overlaps a larger circle carrying K and B; an arc of the planet's orb crosses at B, ending in G at left and G at right. Below on the line stand the planetary signs ♄ ♃, then A at the top of a circle with T inside it and C at its foot. At the bottom a small circle with H overlaps a larger circle carrying M and D; a second arc crosses here, ending in G at left and G at right.

- F (inside the top small circle)
- K (below F)
- B (on the upper arc)
- G, G (ends of the upper arc)
- ♄ ♃ (on the vertical line)
- A (top of the middle circle)
- T (inside the middle circle)
- C (foot of the middle circle)
- H (inside the lower small circle)
- M (below H)
- G, G (ends of the lower arc)
- D (lower part of the bottom circle)

In these diagrams A is the centre of the body of the Sun and of the planets' orbs, and T the centre of the Earth, or at least of the solar or annual orb, namely the orb AC. Let the orb of Saturn and Jupiter be understood as described from A through the arcs BGGD, even though here they are not continued; let the orb of Mars BGDG, which cuts the annual orb at the points R and S, be continued. So when the centre of the smaller epicycle is at F, the apogee of the larger, the planet is at K, the perigee of the smaller, and then the planet's distance from the Sun is greatest, namely AK; and if a conjunction with the Sun is supposed then to occur, the distance KT from the Earth will be greatest. Conversely, when the centre of the smaller epicycle is at H, the perigee of the larger, the planet is at M, the apogee of the smaller, and the distance from the Sun, AM, is least; indeed, in acronychal oppositions the least distance from the Earth is TM. Therefore Mars then approaches the Earth most of all, and descends to the heaven of the Sun, below the Sun, as the parallax of Mars at perigee requires, which has been found greater than the parallax of the Sun, as will be said in Scholium 4.

[Margin: Parallax of Mars at perigee.]

Diagram. A figure filling the lower part of the right column. A straight vertical line runs down the whole figure. At the top a small circle with F overlaps a larger circle carrying K and B. A large circle, the orb of Mars, runs from B down both sides, with G at left and G at right, and ♂ on the vertical line within it, above A. From A a second large circle, the annual orb, runs down around T, with C at its foot. The two large circles cross at R on the left

and S on the right. Near the foot a small circle with H overlaps a larger circle carrying M and D.

- F (inside the top small circle)
- K (below F)
- B (at the top of the large circle)
- ♂ (on the vertical line)
- G (left of the large circle)
- G (right of the large circle)
- A (top of the second large circle)
- T (on the vertical line, inside the second large circle)
- R (left crossing of the two large circles)
- S (right crossing of the two large circles)
- H (inside the lower small circle)
- M (below H)
- D (below M)
- C (foot of the second large circle)

Finally, so that another position of Mars in the acronychal place, according to the

Tychonic hypothesis may be drawn, from Longomontanus, Book 2 of the *Theorica*, chapter 8: let A be the Sun, the centre of the orb of Mars BCP, which carries the larger epicycle, or its centre C; and let this carry the centre D of the smaller; and let M be the Earth, about which describe the annual orb of the Sun passing through A. And let Mars be at H, close to the opposition with the Sun that occurred on the line AH when that line passed through M; and let the equation of the centre be the angle CAH, and the equation of the orb MHA. The rest is clear from what was said above.

Diagram. A figure set into the head of the left column, with the text running in a narrow measure to its left. A straight vertical line runs from B at the top, through A, to P at the bottom. A large circle about M runs from the upper left past B and down to A, and round below M. A large arc runs from B to the left and down to P, crossing that circle. In the upper left stand a small circle about D, with G above it and H on its right, and a larger circle with F in its upper part and C at its centre. Straight lines run from C to the points on these circles and down to the right towards M and A, and lines run from H towards M and A. Within the arc, near its foot, stand the sign ♂ and the word *Tycho*.

- G (above the small circle)
- D (inside the small circle)
- H (right of the small circle)
- B (top of the vertical line)
- F (inside the larger circle, upper part)
- C (centre of the larger circle)
- M (below C, right of the larger circle)
- A (on the vertical line, at the large circle)

- P (foot of the vertical line)
- ♂ (inside the arc)
- *Tycho* (inside the arc, right of the sign)

Measures from Tycho, volume 1 of the Progymnasmata*, pp. 476 and 480.*

See number 9, figure 1 or 2		Parts of which the radius of the planet's orb is 100000	Semidiameters of the Earth
Semidiameters KF	♄	2849	300
Semidiameters BF	♄	8537	900
Mean distance, or radius AB	♄	100000	10550
	♃	100000	3990
	♂	100000	1745
Radius AT of the annual orb	♄	10900	1150
	♃	28140	1150
	♂	65903	1150
Distance from ☉, AK [Margin: Greatest.]	♄	105688	11150
Distance from ☉, AM [Margin: Least.]	♄	94312	9950
Distance from the Earth, TK [Margin: Greatest.]	♄	116588	12300
Distance from the Earth, TM [Margin: Least.]	♄	83412	8800
Distance of the centre B from the Earth, BT [Margin: Greatest.]	♄	110900	11700
	♃	128140	5140
	♂	165903	2895
Distance of the centre D from the Earth, DT [Margin: Least.]	♄	89100	9400
	♃	61860	2840
	♂	34097	595

From this it is clear that the least distance of Jupiter and of Mars from the Earth is less than the distance of the centre D. But Tycho determined for us neither these nor the epicycles.

Measures pertaining to the hypothesis of Longomontanus.

See number 9, figure 1 or 2, for these lines	Planets	Parts of which the radius of the planet's orb is 100000	Semidiameters of the Earth
Semidiameter KF of the smaller epicycle	♄	2907	358
	♃	2385	158
	♂	3710	74
Semidiameter BF of the larger epicycle	♄	8721	1075
	♃	7155	473
	♂	14840	297
Radius AB of the planet's orb, or mean distance	♄	100000	12354
	♃	100000	6605

See number 9, figure 1 or 2, for these lines	Plan-ets	Parts of which the radius of the planet's orb is 100000	Semidiameters of the Earth
	♂	100000	1967

The remainder of the preceding table.

See number 9, figure 1 or 2, for these lines	Plan-ets	Parts of which the radius of the planet's orb is 100000	Semidiameters of the Earth
Mean radius AT of the annual orb [Margin: Mean distance.]	♄	10426	1288
	♃	19349	1288
	♂	65497	1288
☉ and ♂ at their apogees	♂	67962	1288
☉ and ♂ at their perigees	♂	63028	1288
Greatest distance AK from the Sun [Margin: Aphelion distance.]	♄	105814	13070
	♃	104770	6920
	♂	111130	2190
Least distance AM from the Sun [Margin: Perihelion.]	♄	94186	11637
	♃	95223	6290
	♂	88870	1744
Distances from the centre of the Earth, at mean distances of the Sun from the Earth: greatest, TK [Margin: Apogee.]	♄	116240	14358
	♃	124119	8208
	♂	176627	3478
At mean distances: least, TM [Margin: Perigee.]	♄	83760	10349
	♃	75474	5002
	♂	23373	456
At least distances of the Sun from the Earth: greatest, TK	♄	112670	14312 [recte: 14313, per the book's errata]
	♃	120549	8162 [recte: 8163, per the book's errata]
	♂	173057	3432
At least distances: least, TM	♄	87330	10395
	♃	79044	5048
	♂	26943	502
At greatest distances of the Sun from the Earth: greatest, TK	♄	119810	14404 [recte: 14405, per the book's errata]
	♃	127689	8254 [recte: 8255, per the book's errata]
	♂	180197	3524
At greatest distances: least, TM	♄	80190	10303
	♃	71904	4956
	♂	18803	410

See number 9, figure 1 or 2, for these lines	Plan-ets	Parts of which the radius of the planet's orb is 100000	Semidiameters of the Earth

Mean or equal motion.

Daily	Planet	Deg.	′	″	‴	⁗	V	VI
Of the apogee	♄	0	0	0	13	10	31	17
	♃	0	0	0	9	30	22	10
	♂	0	0	0	12	18	0	7
Of longitude	♄	0	2	0	35	24	43	46
	♃	0	4	59	15	54	13	8
	♂	0	31	26	39	28	14	20
Annual								
Of the apogee	♄	0	1	20	12	0	0	0
	♃	0	0	57	52	0	0	0
	♂	0	1	14	52	0	0	0
Of longitude	♄	12	13	35	23	33	16	50
	♃	30	20	31	44	0	0	0
	♂	191	17	10	6	0	0	0

Root of the motions for the year 1 of Christ, at Copenhagen, at noon on 1 January.

	Planet	Root
Of the apogee	♄	$7^{s}\ 20^{\circ}\ 48'\ 12''$
	♃	$5^{s}\ 11^{\circ}\ 48'\ 50''$
	♂	$3^{s}\ 25^{\circ}\ 25'\ 20''$

Remainder of the preceding table.

	Planet	Root
Of longitude	♄	$2^{s}\ 12^{\circ}\ 32'\ 51''$
	♃	$5^{s}\ 29^{\circ}\ 56'\ 46''$
	♂	$1^{s}\ 9^{\circ}\ 39'\ 14''$

These motions, like the places of the apogees, are reckoned from the mean equinox. The motion of the anomaly of the eccentric, which Longomontanus sets down in his tables contrary to the custom of others, is obtained by subtracting the motion of the apogee from the mean motion of longitude. The anomaly of the orb, however, is not set down, because in this hypothesis it is obtained by subtracting the planet's coequated eccentric place from the true motion of the Sun; both of these are irregular, and it is therefore excluded from tables of mean and regular motions.

SCHOLIA.

Scholia.

[Margin: Censure of the Danish hypothesis.]

I. This Danish hypothesis has the following advantage: it preserves the uniformity of motion in each of the circles about their own proper centres; it does not fall back on an annual motion of the Earth; it uses the true motion of the Sun; and for the centre of the planets it has a definite and conspicuous point in the world, namely the centre of the Sun. But it does not carry the planet round along a perfect circle, and, by in fact employing an orb eccentric to the centre of the world, though concentric with the Sun, it seems to multiply circles, namely an eccentric with two epicycles, and these indeed having no definite centre in the world but one perpetually uncertain and unstable, with motions that conflict with one another.

II. It is one thing to determine the highest altitude of a planet, and another to determine the highest elevation of the heaven assigned to it. In the first way Tycho, tome 1, p. 480, sets Saturn's greatest distance from the Earth at 12300 terrestrial semidiameters; in the second way at 12900 semidiameters, that is, with the diameter of the lesser epicycle added, granted that in his scheme the planet is never at the apogee of the lesser epicycle when the centre of that epicycle is at the apogee of the greater.

[Margin: Baranzanus's error in the Tychonic distances.]

But Baranzanus, part 1 of the *Uranoscopia*, p. 121, ascribes to the planet itself the Tychonic greatest distance of 12900. He errs far more intolerably when he attributes to Tycho a least distance of Mars from the Earth of 1176 semidiameters, for it is much smaller. Since the mean distance of Mars, as he himself agrees, is 1745, and this same figure is the semidiameter of Mars's orb, then when Mars is opposite the Sun one must subtract the Sun's distance from the Earth, which at its mean is 1150. And so the centre D of Mars's greater epicycle, as in the figure placed above at no. 9, is then distant from the Earth by the quantity of the line TD, that is, by only 595 terrestrial semidiameters. From this distance two-thirds of the semidiameter of the greater epicycle have still to be subtracted, so that the least distance of Mars itself remains, which is certainly less than 595. We must therefore beware of mere collectors of this sort, who copy out figures without examining the hypotheses.

[Margin: Athanasius Kircher's Tychonic measure.]

Nor do I find correct the Tychonic measures of Father Athanasius Kircher, which he sets out at the end of the *Ars Magna Lucis et Umbrae*, from p. 762, where he gives the mean distance from the Earth, in semidiameters of the Earth, as ♄ 10550, ♃ 3990, ♂ 1745. But for their distances from the Sun, the Sun however at apogee, and in oppositions, in order to calculate their shadows, he gives ♄ 10588, ♃ 3652, ♂ 1628, and the Sun's apogeal distance from the Earth as 1181. These figures cannot hang together. For in oppositions the planet is at M, and the distance TM of ♄ from the Earth is then, in Tycho's terms, 8800;

if you add AT, 1181, it will make 9981, not 10588. And so with the rest, which need correcting.

III. Although Longomontanus, with the Sun and Mars at perigee, had set the semidiameter of the annual orb at 63028 parts, of which the radius of Mars's orb is 100000, nevertheless in Book 2 of the *Theorica*, at the end of chapter 12, he concludes that it is more correct and better suited to the phenomena of Mars if a radius of 63100 parts is always used for the least semidiameter of the annual orb. Students of this hypothesis should bear this in mind.

[Margin: The parallaxes of Mars.]

IV. Kepler reports in his work on Mars, chapter 11, that in 1582, when Mars was opposite the Sun in ♋, Tycho, with incredible diligence and by observation over several nights, morning and evening, hunted for the parallax of Mars through its distance from stars near the ecliptic but remote from Mars, so that their refraction should not interfere, and that he ordered his assistants to seek out the parallax by comparing the morning and evening observations. But they, Kepler says, fitted the observed places to the Copernican scheme, using a concentric and a double epicycle, and by an immense labour of triangles worked out how much parallax the Copernican scheme would produce; and perhaps it came about from this that Tycho stated that the parallax of Mars when acronychal is notably greater than the Sun's.

[Margin: Kepler's opinion on the parallax of Mars.]

But Kepler, examining these passages most diligently and reading through the volume of Tycho's observations, affirms that the parallax of Mars, even at perigee, is very small, yet somewhat greater than the Sun's, and that both are scarcely perceptible; and that solar eclipses require the parallax of the Moon from the Sun to be almost nil, because the Sun's is almost nil. Kepler teaches this in the *Ephemerides*, p. 2, in the *Epitome of Copernican Astronomy*, p. 479, and in the already mentioned work on Mars, chapters 11 and 64. There he proves it, first, because in that volume of Tycho's it is noted in Tycho's own hand that the matter sometimes turned out the opposite way, that is, that the effect found from the observations was at times the contrary of what a parallax of Mars greater than the Sun's would require.

[Margin: The method of observing the parallaxes of Mars, etc.]

Next, because parallax must throw retrograde Mars toward the east in the evening, and in the morning, when it is setting, throw it toward the west, and so accelerate its daily retrograde motion. Therefore, if its parallax is perceived near the acronychal position, that is, when it is retrograde, the retrograde motion must be perceived as excessively increased. But when the daily motions after the first station and during the time of retrogradation were examined from Brahe's observations of 1582, from 1582 November 23 and 24, on which days the distances from the fixed stars, found the same at different hours, showed the station, up to 1583 January 17, the retrograde motion was found either not increased at all, or scarcely perceptibly, or even at times diminished. Therefore the parallax of Mars, even at perigee, is either nil or scarcely perceptible. Lastly,

from the observation of the 17th he demonstrates that it is much less than 4′, and scarcely 2′, and therefore that the Sun's parallax is about 1′.

[Margin: Tycho's opinion on the parallaxes of Mars.]

Granted, Tycho, in a letter to Rothmann [Christoph Rothmann] of 1587 January 20, affirms from observations of 1582 that the parallax of acronychal Mars, and indeed of Jupiter too, is perceptible. And in a letter of 1588 to Peucer [Caspar Peucer], which Mästlin reports in his notes to the *Narratio* of Rheticus, p. 135, he says, again from observations of 1582, when Mars, up all night, was opposite the Sun in Cancer and had north latitude, that he found the daily motion of retrograde Mars against the order of the signs to be faster than the Alphonsine and Ptolemaic calculation allows, and almost as the Copernican calculation requires; that this acceleration arose from no other cause than a distance from the Earth smaller than the Sun's distance; that therefore in that position the parallax of Mars is greater than the Sun's; and that in this the Ptolemaic hypothesis gives way to the Copernican.

[Margin: Mästlin's opinion on the parallax of Mars.]

In that place, however, Mästlin, moved by Kepler's authority, does concede to Copernicus, Rheticus and Tycho that the parallax of acronychal Mars is greater than the Sun's, yet he affirms that it is very small and scarcely observable.

[Margin: Other authors for the nearness of acronychal Mars.]

After these men, the doctrine that Mars at perigee is nearer the Earth than the Sun is was taught by Longomontanus, Book 2 of the *Theorica*, chapter 9; Biancani in his *Sphaera*, book 13, chapter 1; Argoli in the *Pandosion Sphaericum*, chapter 3; Lansbergen, precept 14; Boulliau in his hypotheses; Galileo in Dialogue 3 *On the Cosmic System*, where he adds that Mars at perigee comes eight times nearer to the Earth, and appears six times larger, than when it is at apogee; and Antonín Maria Šírek, book 4, chapter 4, who adds that in ♌, when the Sun is in ♒ [recte: in ♒, when the Sun is in ♌, per the book's errata, which exchange the two signs; the corrected order is also written by hand in the margin of this copy], it sinks below the Sun. But it does this also in ♓ and in the opposite signs, as Lansbergen notes; and then, because of the nearness, the Sun's eccentricity bears a perceptible ratio to the radius of Mars's orb and cannot be neglected without error in the places of Mars.

[Margin: ♒ ♌, handwritten]

CAPVT IV.

Chapter 4.

De Hypotheſi Philippi Lansbergij.

On the hypothesis of Philips van Lansbergen.

[Margin: The source from which Lansbergen drew his hypothesis.]

I. This hypothesis, which by means of an eccentric with a small circle varying the eccentricity, together with the annual orb of the Earth, Copernicus

indicated and Mästlin expounded, as I reported in chapter 2, no. 4, Lansbergen offers us as if it were new, and expounds all too briefly in his

Theoricae of the heavenly motions, chapter 9, where he calls it "new and true and in agreement with all ages". But let the same judgement hold for it as for the second Copernican form, according to what was said in the scholia of chapter 2.

Let A, then, be the centre of the universe, about which the Earth revolves through the annual or great orb TRN, in the order of the signs, from R to N; and let the greatest eccentricity of the planet's orb be AD, the least AB, and the mean AC, so that AB is as large as BD, although the engraver has made BD somewhat larger.

Diagram. Lansbergen's scheme, set in the left column. A large circle, the eccentric orb, carries O at the top, K at the upper right, P at the foot, and Q at the left. Inside it are the signs ♄ ♃ ♂ (upper left). A vertical diameter runs from O down through G and E to P. A horizontal line runs from G through H to K. A second vertical runs from H down through the small circle's letters to A and on to I. A smaller circle about A, the annual orb, carries T at the upper right, R with a rayed disc at the lower left, N at the lower right, and I near its foot. A small circle stands above A, passing through a small letter at its top, through E with a rayed disc at its left, and through B at its foot, with F and C on its vertical diameter. Straight lines join E to K, A to K, E to F and E to C, and straight lines pass through A from R to T and from N to E. A line joins N to K, and a dashed line runs from T to K.

- O (top of the large circle): O
- G (on the vertical diameter, upper): G
- H (on the horizontal line): H
- K (right end of the horizontal line): K
- T (upper right of the annual orb): T
- D (top of the small circle): D
- E (left of the small circle, with a rayed disc): E
- F (on the small circle's vertical): F
- C (below F): C
- B (foot of the small circle): B
- A (centre of the annual orb): A
- N (lower right of the annual orb): N
- R (lower left of the annual orb, with a rayed disc): R
- I (foot of the second vertical): I
- P (foot of the large circle): P
- Q (left edge of the large circle): Q, drawn with a long tail turning right
- Inside the large circle, upper left: ♄ ♃ ♂

Then, having bisected BD at C, with semidiameter BC describe the small circle of the eccentricity DEB, on whose circumference let the centre of the planet's orb lie. This centre is to revolve on it in the order of the signs from B, through D, toward E, beginning from B, but with a double motion, that is,

twice as fast as the motion by which the planet, fixed on the circumference of the eccentric OQPK, moves with the motion of longitude in the order of the signs from the apogee O to Q. For in this way, when the planet is at apogee and at perigee, the centre of the orb will be at B.

Further, let the diameter of the great orb AD be the line of apsides, or of the mean apogee, to which a straight line drawn through the centre of the planet's orb is always parallel. Thus in our scheme, because the centre of the orb is at E, the straight line PEO is parallel to AD. Let this parallel be called the line of the true apogee, and let the planet's apogee be understood to move with the very slow motion of the line AD in the order of the signs. For so it will come about that when the centre of the planet's orb is at D the eccentricity is then greatest, and when at B least; and that if the planet is on the line of apsides it is at apogee, or below at perigee.

In the scheme, with the planet supposed at K, its apogee is at O, and the semidiameter of the planet's orb is EO. The line drawn from the centre of the orb through the planet, as here the line EK, is called the line of the mean place of the centre; but the line drawn from the centre of the universe through the planet is the line of the true place of the centre, as the line AK. Finally, the line drawn from the Earth to the planet is the line of the true place of the planet in the zodiac as seen from the Earth: if the Earth is at T, the line TK will be the line of the true or apparent place of the planet.

Therefore the equation of the centre is the angle AKE, to be subtracted from the mean motion of longitude in the first semicircle of the anomaly, OQP, and added in the second, PKO, so as to obtain the true centre. But the equation of the planet's orb, or of the second inequality, which arises from the annual motion of the Earth, is the angle AKT. In the first semicircle of the anomaly it is added to the true place of the centre, and in the second subtracted from it, so as to obtain the planet's true place in the zodiac with respect to the Earth, but reckoned from the mean equinox. If the true precession of the equinoxes is added to this, the result is the planet's true place reckoned from the true equinox.

[Margin: Why Mars needs a special correction.]

II. Since Mars, in travelling through the signs of Aquarius, Pisces, Aries and Taurus, comes very close to the Earth and at the same time to its acronychal opposition with the Sun, and in that position the Sun's eccentricity has a very perceptible proportion to the radius of Mars's orb, it follows that Mars then needs a special equation. Lansbergen has not yet set out the theory of this equation, but only a practical precept, which is number 14 among the precepts of his tables; we deal with it in the precepts of the tables. He supposes that the centric longitude of Mars must be corrected by a second equation: but what if this variation arises from the equations of the orb?

Methodus Calculi Geometrici in Hypotheſi Lansbergij.

Method of geometrical calculation in Lansbergen's hypothesis.

[Margin: 1. Step of the calculation.]

III. First, from Lansbergen's tables gather the planet's mean motion of longitude and the motion of its apogee, and subtract the latter from the mean motion of longitude; the result is the mean anomaly of the centre. For example, if Saturn's mean motion of longitude is 153° and that of its apogee 226°, then take 360° and join them to the 153°, which makes 513°, so that the subtraction can be made; and when 226° are taken away, there remains a mean anomaly of the centre of 287°. Double this anomaly and count it from the perigee of the small circle of the eccentricity in the order of the signs, and you will obtain the point on its circumference at which the centre of the eccentric lies. So in our case, when 287 has been doubled and made 574, a whole circle of 360 must be subtracted, and there remain 214°. Counted from B through D, these show the place of the eccentric at E, from which describe the eccentric orb OQPK. Through E draw a straight line parallel to the diameter AD of the great orb, so that the planet's apogee is at O. Counting from O in the order of the signs toward Q the anomaly of the centre, that is, in our case 287°, you will have the planet's place on the eccentric, which in our scheme is K. To it draw the straight line EK from the centre of the orb, and from the planet to OEP, the diameter of the orb, draw the perpendicular GK, and extend to it the diameter AD of the annual orb as far as H. Since, then, the arc KO is given, which is the complement of OQPK, that is, of the anomaly of the centre, to the full circle, and which in our case is 73°, its right sine GK is also given, 9563 in parts of which the semidiameter or radius of the orb EO is 10000; and the second sine GE will be 2924. Keep this in reserve.

[Margin: 2. Step.]

Second, in the small circle of the eccentricity the arc DE is given, which in our case is the excess over a semicircle, of 34°, and its sine EF, 5592, and its second sine CF, 8290, in parts of which the radius CE is 10000. CE is likewise given, because it is a third part of the supposed mean eccentricity, as in Saturn BC or CE is 285 of the parts of which EO is 10000, and AC is 855. Let it be done, then, as CE, 10000, to 285, so the sine FC, 8290, to another, and FC will come out at 236 of the parts of which EO is 10000. Therefore the whole AF will be 1091; and when FH, equal to the sine GE, 2924, is added, the whole AH becomes 4015 parts. Let it be done again as CE, 10000, to 285, so the sine EF, 5592, to another, and EF, that is GH, will come out at 159 of the parts of which EO is 10000. When this is taken from GK, 9563, there remains KH, 9404. And so in the triangle AKH, right-angled at H, the sides about the right angle are given, namely HK, 9404, and AH, 4015, from which by the analysis of triangles you will gather the angle HAK, 66° 58′, and the base AK, 9448. But the angle OEK was 73°. To it, because of the parallelism of the lines OE and HA, the angle HDK is equal, by 29 of the First Book; and this angle, by

32, is equal to the two interior and opposite angles DAK and AKD. Therefore, when DAK, 66° 58′, is taken from HDK, 73°, there remains AKD, 6° 2′, which is the equation of the eccentric. Here it is to be added to the planet's mean longitude, which was 153°, and so its centric longitude will be 159° 2′.

[Margin: 3. Step.]

Third, count from D, the planet's apogee, toward R, the distance of the planet's mean apogee from the mean motion of the Sun, and suppose it to be 117° 15′, that is, if the mean motion of the Sun is 117° 15′. For the remaining arc to the semicircle, RI, will be 62° 45′, equal to the arc DT, that is, to the distance of the Earth T from the planet's mean apogee D, and so the angle DAT will be known. Subtracted from the angle HAK found above, 66° 58′, it leaves the angle TAK, 4° 13′, in the triangle TAK, in which the side AK was found to be 9448, and the semidiameter of the annual orb AT is supposed known, in parts of which EO is 10000; in Saturn it is 1007. Therefore the angle AKT will not be hidden. This is the equation of the orb, to be taken away here from the longitude of the centre, so that the planet's true longitude from the mean equinox results; and when the prosthaphaeresis of the equinoxes is applied, the planet's true longitude from the true equinox will be obtained.

[Margin: 4. Step, for ♂.]

Fourth, if the place of Mars is to be computed, Lansbergen's precept 14 must be consulted, which we hold over for the precepts of the tables, because it concerns a merely logistic calculation and does not teach a way of hunting out this equation geometrically for the place of Mars.

Menſuræ ac Numeri ad Hypotheſim Lansbergij ſpectantes.

Measures and numbers concerning Lansbergen's hypothesis.

IV. Lansbergen has given the measures of the eccentricities and of the radius of the planets' orb, but only in parts of which the radius of their orb is 10000, in the *Theoricae* of the heavenly motions, chapter 9, and at the end of the *Treasury of Observations*. From these, and from the Sun's distance from the Earth, which is the same as the semidiameter of the annual orb, and which he gives in the *Uranometria*, book 2, Element 9, as 1498 35/60 terrestrial semidiameters, we have gathered the distances of the three planets from the Sun and from the Earth, the greatest, the mean and the least, which, however, he himself indicates in book 3 of the *Uranometria*, Elements 4, 5 and 6.

Recall that the centre of the planet's orb is at B, with the planet both at apogee and at perigee; and therefore that when the least eccentricity AB and the radius of the orb EO are joined, the planet's greatest distance from the Sun A results, and when AB is subtracted from the same radius, the least distance from the Sun is left. Therefore, if to the former you add, and from the latter subtract, the radius of the great or annual orb, the result will be, in the former case, the greatest, in the latter, the least distance of the planet from the Earth. For Lansbergen uses the Sun's mean distance from the Earth, because the radius

of the great orb is equal to it. These distances are, moreover, those that are possible in this hypothesis, if the lines of apsides of any planet and of the Sun were to coincide, and the planet at apogee were to be joined with the Sun at apogee, or the planet at perigee opposed to the Sun at apogee: for in the former case the distance of the planet from the Earth would be the greatest among those possible in this hypothesis, and in the latter the least; though [?] we rightly said in chapter 1, scholium 6, that these have never coincided and never will.

[Margin: Phocylides's opinion on the eccentricity of Saturn.]

Furthermore, Johannes Phocylides Holwarda, in his *Examination of Lansbergen's Astronomy*, contends from acronychal observations of Saturn that the greatest eccentricity of ♄ is 1140 and the mean 860. Everyone sees that the other objections that can be brought against the second form of Copernicus can also be brought against this one.

Measures concerning Lansbergen's hypothesis.

Measures of the circles from Lansbergen	Planet	In parts of which the radius of the planet's orb is 10000	Semidiameters of the Earth
Semidiameter of the small circle BC	♄	285	424½
	♃	229	185
	♂	485	111
Mean eccentricity AC	♄	855	1273
	♃	687	555
	♂	1455	332
Greatest eccentricity AD	♄	1140	1698
	♃	916	741
	♂	1940	442
Least eccentricity AB	♄	570	849
	♃	458	370½
	♂	970	221
Radius EO of the planet's orb	♄	10000	14880
	♃	10000	8091
	♂	10000	2275
Radius AI of the great orb	♄	1007	1498½
	♃	1852	1498½
	♂	6586	1498½
Greatest distance of the planet from the ☉	♄	10970	15729
	♃	10458	8461½
	♂	10970	2496
Least distance of the planet from the ☉	♄	9430	14031
	♃	9542	7720
	♂	9030	2054
Greatest distance of the planet from the Earth	♄	12121	17227½
	♃	12768	9960

Measures of the circles from Lansbergen	Planet	In parts of which the radius of the planet's orb is 10000	Semidiameters of the Earth
	♂	18526	3994

[Margin: beside the greatest distance from the ☉: With AB and EO joined.]
[Margin: beside the least distance from the ☉: With AB taken from EO.]
[Margin: beside the greatest distance from the Earth: With IA, AB and EO joined.]
Remainder of the preceding table.

Measures of the circles from Lansbergen	Planet	In parts of which the radius of the planet's orb is 10000	Semidiameters of the Earth
Least distance of the planet from the Earth	♄	9563	12534
	♃	8606	6222
	♂	2444	556

[Margin: beside the least distance from the Earth: With IA and AB taken from EO.]
Mean or equal motion.

Daily	Planet	Deg.	′	″	‴	⁗	V	VI
Of the apogee	♄	0	0	0	12	53	18	50
	♃	0	0	0	9	53	41	4
	♂	0	0	0	13	9	51	4
Of longitude	♄	0	2	0	35	22	46	34
	♃	0	4	59	15	54	46	23
	♂	0	31	26	39	28	13	20
Of the anomaly of the orb	♄	0	57	7	44	22	12	41
	♃	0	54	11	3	50	12	52
	♂	0	27	41	40	16	45	55
Annual								
Of the apogee	♄	0	1	18	24	19	34	10
	♃	0	1	0	11	34	43	15
	♂	0	1	20	4	55	39	20
Of longitude	♄	12	13	35	13	33	16	50
	♃	30	20	31	48	12	9	55
	♂	191	17	10	6	41	6	40
Of the anomaly of the orb	♄	347	32	5	26	7	1	10
	♃	329	25	8	52	18	8	5
	♂	168	28	30	33	49	11	20

Roots for the year 1 of Christ, at Goes, at noon on the first day of January.

	Planet	Root
Of the apogee	♄	$7^{s}\ 21^{\circ}\ 3'\ 0''$
	♃	$5^{s}\ 6^{\circ}\ 22'\ 42''$
	♂	$3^{s}\ 19^{\circ}\ 55'\ 9''$
Of longitude	♄	$2^{s}\ 12^{\circ}\ 15'\ 0''$
	♃	$5^{s}\ 29^{\circ}\ 48'\ 2''$
	♂	$1^{s}\ 9^{\circ}\ 16'\ 27''$
Of the anomaly of the orb	♄	$6^{s}\ 26^{\circ}\ 21'\ 34''$
	♃	$3^{s}\ 8^{\circ}\ 48'\ 32''$
	♂	$7^{s}\ 29^{\circ}\ 20'\ 7''$

SCHOLIA.

Scholia.

[Margin: Objections against this hypothesis.]

I. Lansbergen's hypothesis, which is in fact that of Copernicus, does keep the motions equal in the circles about their own proper centres. But the centres of the eccentric or of the small circle of the eccentricity are not a definite point in the world, or one worthy of the planet's mover having regard to it; the planet's path in the world is not perfectly circular; and it is bound to the annual motion of the Earth. These are the things I see can be objected to it. It does, however, contain the equivalent of the Ptolemaic equant, since it has bisected the whole eccentricity into two equal parts.

[Margin: Hérigone's semi-Lansbergian hypothesis.]

II. Further, Pierre Hérigone, in tome 5 of the *Cursus Mathematicus*, in the *Theoricae*, p. 536, transferred this hypothesis to the hypothesis of a stationary Earth, which I shall not be reluctant to set out here. Let A be the centre of the motionless Earth, about which let there be the circle CT, carrying the apogee in the order of the signs from C toward T; and let the line of apsides be FAY; and from the point C, where that line cuts the circumference of the circle CT, describe the small circle

BDE, which is to be called that of the anomaly of the eccentric; and on its own circumference let it carry the centre E of the eccentric PGYH, with a double motion, that is, one twice as fast as the motion of the centre G from the mean apogee I, the motion beginning at D toward E, and from there to B, etc. Next, from G describe the circle of the anomaly of the orb, or epicycle, NML, on whose circumference let the planet be, as at L, and let it move in the order of the signs from the true apogee of the epicycle, N, by as much as the mean motion of the Sun exceeds the mean motion of the centre G. Let the line FY, drawn through the centres of the Earth and of the small circle of the anomaly of the centre, be the line of the true apogee; and let IE, parallel to it and drawn through the centre E of the eccentric, be the line of the mean apogee I, from which the anomaly of the centre is counted. So when the centre of the epicycle is at the apogee F or the perigee Y, the centre of the eccentric will be at D; the lines of the mean and the true apogee will coincide, there will be no need

of any equation of the centre, and its true place in the zodiac will be the same as that of the apogee or the perigee. Now, however, since the centre of the eccentric is supposed to have completed the arc DE at E, say of 100°, while the centre G, departing from the apogee I, has completed the arc IG toward G, of 50°, let the straight line EM be drawn from the centre E through the centre G of the epicycle; for it will be the line of the mean motion of the centre. Parallel to it let AK be drawn from the centre of the Earth, and through the centres of the Earth and of the eccentric the straight line VAEP, so that the angle IEC [recte: IEG] is equal to the angle FAK. And from A let the straight line AGN be drawn through the centre of the epicycle, to be the line of the true place of the centre G; for the angle AGE will be the equation of the centre, to be subtracted, in the first semicircle of the anomaly, from the mean motion of the centre G, so that the true motion may be had. Finally, since we have supposed the planet at L, let the straight line AL be drawn to it, and the semidiameter of the epicycle GL; for the angle GAL will be the equation of the anomaly of the orb, to be added, in the first semicircle of the anomaly, to the true place of the centre G, so that the true place of the planet may result, counted from H, which is the point of the mean equinox, or from R, that of the true equinox.

Diagram. A figure set into the left column, for the construction just described. A large circle (the eccentric) has a vertical line running from F at its top down through the small circle and A to Y at its foot. A smaller circle stands inside it about A, and a small circle bearing B, C, D and E lies just above A. An epicycle centred on G sits on the upper left of the large circle, with N, M and L on its circumference. Straight lines run from the region of A and E to the points on the circumference.

- N, M (upper left, on the epicycle): epicycle points
- G (centre of the epicycle)
- L (left side of the epicycle)
- F (top of the vertical line)
- I (immediately right of F): mean apogee
- P (upper right of the large circle)
- K (left, outside the large circle)
- B, C, D, E (small circle above A)
- H (right, on the large circle)
- R (right, below H)
- T (left of A, by the inner circle)
- A (centre of the Earth)
- V (lower left)
- Y (foot of the vertical line)

Labels read: N, M, G, L, F, I, P, K, B, C, D, E, H, R, T, A, V, Y. Labels not read: none known. T is on the figure and is not cited in the text on this page.

[Margin: Method of geometrical calculation for the equation of the centre.]

III. With these things laid down, if the mean motions are given from the tables, and the measures of the semidiameter of each of the circles, the

prosthaphaereses will be gathered by this method. Subtracting the motion of the apogee from the mean motion of longitude gives the anomaly of the eccentric, or of the centre, that is, the angle IEG, to which FAK is equal. Twice this is the arc DE, that is, the angle ACE in the triangle ACE, in which the semidiameters AC and CE are given (CE is missing in the figure). So the eccentricity AE will not be hidden, nor the angle CAE, to which the angle IEP is equal; added to the angle IEG, it makes up the angle PEG, whose complement to two right angles is the angle AEG. Therefore, in the triangle AEG, with the angle just mentioned, the eccentricity AE and the semidiameter of the eccentric EG, you will find the angle AGE, to which, because of the parallelism, the angle GAK is equal, that is, the equation of the centre. Subtract this from the mean motion of longitude in the first semicircle of the anomaly and add it in the second, and the true place of the centre G in the zodiac will result, counted from the mean equinox; and, with the prosthaphaeresis of the equinoxes, from the true equinox.

[Margin: Method of calculation for the equation of the orb and the distance of the planet from the Earth.]

IV. Then, in the triangle AGE, from the data already mentioned, seek the side AG, that is, the distance of the centre of the epicycle from the Earth; and by subtracting the mean motion of the centre G, or of longitude, from the mean motion of the Sun, obtain the anomaly of the orb, or the motion of the planet L on the epicycle from the aux M. For when MGN is added, the arc NML will be known, whose complement to a semicircle in this case will make known the angle AGL. So, this being given, with the side AG and the semidiameter of the epicycle GL, you will hunt out in the triangle AGL the angle GAL, which is the equation of the anomaly of the orb, to be added to the true place of the centre G in the first semicircle of this anomaly and subtracted in the second, so that the true place of the planet may be established. And, if you wish, you can find in the same triangle the planet's distance from the Earth, AL.

[Margin: The measures pertaining to this hypothesis.]

V. As for the measures of the circles or semidiameters, consult the first table of the preceding page, placed after number 4. For, as Hérigone would have it, the semidiameter of the small circle CD is as great as the semidiameter of the small circle of the eccentricity in Lansbergen, namely in ♄ 285, of which the radius of the eccentric EG is 10000, etc.; and the semidiameter of the deferent of the apogee, or AC, is as great as the eccentricity midway between the greatest and the least, for example in ♄ 855 parts. Finally, the semidiameter of the epicycle GM is as great as the radius of the great orb in Lansbergen's hypothesis, that is, in ♄ 1007.

[Margin: A mistake in Hérigone.]

But Hérigone's numbers in ♂, as regards the semidiameter of the small circle CD, are corrupt: he has 247 ½, when 485 ought to be put in its place, for CD in Lansbergen is a quarter of the greatest eccentricity, which is 1940. Unless perhaps Hérigone, led by some reason of his own, thought Lansbergen needed

correcting on this point; but in that case he ought at least to have said so. The numbers of the motions and the greatest equations, however, are exactly as in Lansbergen.

[Margin: The same hypothesis fitted to ♀.]

Finally, he adds that this same theory suits Venus, assigning for ♀ 102 to CD itself, likewise 102 to AC, and 7193 to GM, of which EG is 10000; the greatest equation of the centre 2° 32′; and the orb 45° 10′ at the apogee and 46° 51′ at the perigee.

CAPVT V.

Chapter 5.

De Hypotheſi Kepleri, & Bullialdi.

On the hypothesis of Kepler and Boulliau.

[Margin: Summing-up of the preceding hypotheses.]

I. Until now astronomers had remained divided, as it were, into two classes. The older of these, from Ptolemy down to Peuerbach and beyond, affirmed that the inequality which appears in the motion of the planets is partly physical and real, partly apparent; and it held that half of it, namely the apparent part, rests on half the eccentricity of the circle that carries the planet or its epicycle. But it used nothing but circles; indeed, it led the planet around a perfect circle. Finally, it measured the second inequality from the mean conjunctions of the planet with the Sun and by the mean motion of the Sun. The other class, from Copernicus onward, together with the earlier opinion of Tycho and Longomontanus and together with Lansbergen, rejected the physical inequality and maintained that the motions of the planets about the centres of their orbs are equal. It said that the whole inequality appearing in their motion is optical, or appears such to us, because we view these motions from a position outside the centre of the celestial orbs; and so it kept the entire eccentricity as the foundation of this irregularity. For that reason, although it used perfect circles, it nevertheless carried the planet around along the path of an imperfect circle. Of these, some used the mean motions of the Sun and others the true motions in measuring the second anomaly.

[Margin: The new hypothesis of Kepler, and his followers.]

II. Both were succeeded by Johannes Kepler, who broke away from both, agreed with both on certain points, and introduced an entirely new hypothesis. He pursues it at length in the *Commentaries on the Motion of Mars* and in the *Epitome of Copernican Astronomy*, Books 5 and 6. Hérigone leans toward it in tome 5 of the *Cursus Mathematicus*, from p. 550 to p. 588; Ismaël Boulliau embraced the same hypothesis wholeheartedly (if you except the planet's motive force) in his *Astronomia Philolaica*; and so did Magini, as regards the motions of Mars, in the *Supplement to the Ephemerides*. Kepler, then, emboldened by the authority of the former class, granted that the second inequality is optical

and arises from the motion of the Earth, but he asserted that the first is wholly physical and real, even though, if the eye is transferred to the Sun, he may have called part of it optical; so that the planet, about the centre of its orbit and indeed about the Sun as well, completes unequal arcs in equal times. And yet he taught that the eccentricity must be bisected. From the second class he drew the imperfection of the planetary path, or its deviation from a perfect circle; and, daring to break up perfect circles and the whole structure of eccentrics and epicycles, he carried each planet around along the simple circumference of a single ellipse, in one of whose foci, or umbilici, is the Sun. About the Sun, likewise motionless as regards motion of translation, he revolved the Earth in an annual motion as one of the planets. The second inequality, however, from the true or apparent conjunction and motion of the Sun, he measured. Although we have already given a foretaste of this hypothesis in connection with the Sun and the Moon in Book 3, chapter 23, no. 3, and more fully in Book 4, chapter 27, up to no. 17, and in the scholia of the same chapter from 1 to 10, and I have touched on many things that belong here and are to be reread here, it must nevertheless be set out in this place, with its properties and, as it were, its characteristic marks distinctly added, by which it is distinguished from the other hypotheses. I shall do this more clearly if I first provide a diagram suited to representing it.

III. In the following figure let the Sun be A, at the centre of the universe, from which let the ecliptic BDCE be described, whose diameter BC is to be the line of apsides of the Sun and the Earth; and on it the centre G, from which describe the Earth's ellipse PNQT. Along it let the Earth move toward the east, from the aphelion P to N, with an annual motion; let the Earth's perihelion be Q, and let the eccentricity of this ellipse from the Sun be AG. Now let us imagine the aphelion or apogee of one of the three superior planets to be under the same point of the zodiac B as the aphelion of the Earth, say under the beginning of ♈, and therefore the line of apsides of the planet and of the Earth to be the same, with the centre on it at F. From F describe the planet's ellipse XHZY, on whose circumference let the planet move in the order of the signs from the aphelion X to H. Let the eccentricity of this ellipse from the Sun be AF, and the greater semidiameter, or semiaxis, FX; the lesser, or conjugate, semidiameter FH, which is to be perpendicular to the axis XZ; and let the Sun A be at one of the foci, or umbilici, of the ellipse. Let the centre of the Sun be joined to the end H of the lesser semidiameter by the straight line AH. For by the properties of the ellipse demonstrated by Apollonius, AH will be equal to the greater semidiameter FX, and therefore the planet at H, as also at the opposite point, will be at its middle distance from the Sun, while its greatest distance from the Sun will be AX and its least AZ. The planet's greatest distance from the Earth will be QX, if, with the planet placed at X, the Earth is at Q; yet it would come out greater if, with the planet at the apogee X, the Earth, at Q, were at aphelion, and AQ were as great as AP, that is, as great as the greatest distance of the Earth from the Sun. The planet's least distance from

the Earth will be ZQ, which would come out smaller still if, with the planet placed at perihelion Z, the Earth at Q were at aphelion and, receding from the Sun, were approaching the planet. But on this meeting see what was said in chapter 1, Scholium 6.

[Margin: Four situations of the Earth, the planet and the Sun.]

With these things laid down, four possible situations of the Sun, the Earth and the planet taken together must be considered. The first is if, at the same moment at which the planet appears to the Earth conjoined with or opposite the Sun, it is at aphelion or at perihelion. Then both inequalities cease and no equation is needed; the place in the zodiac of the centre F and of the planet is the same, and the mean and the true motion coincide: as when, with the Earth placed at P or at Q, the planet is at X or at Z.

[Margin: 2. The situation in which the first inequality ceases.]

The second is if the planet is at its aphelion X or at its perihelion Z, but the Earth is outside the planet's line of apsides, which is always passed through the centre of the Sun. Then the planet's first inequality ceases and there is no need of the equation of the centre, but the second inequality comes in, as when the Earth is at T; and the equation of the second anomaly, or prosthaphaeresis of the orb, is needed. For the true place of the centre F will be under B or C, but the apparent place of the planet X will be the point V in the zodiac, which is marked by the line of true motion TXV, drawn, that is, from the Earth through the planet.

[Margin: 3. The situation in which the second inequality ceases.]

The third situation is if the planet, away from its aphelion and perihelion, is conjoined with or opposed to the Sun in true ☌ or ☍. Then the first inequality will come in and the equation of the centre will be needed, but the second anomaly will cease: as when, to the Earth at T, the planet appears either conjoined with the Sun at S or opposite it at Y.

[Margin: 4. The situation in which both inequalities show themselves.]

The fourth situation is if the planet is away from its aphelion and perihelion and neither joined to nor opposite the Sun. In this case a twofold irregularity comes in, and an equation is needed both of the centre, or of the eccentric place, and of the anomaly of the orb, or of commutation: as when it is at H and the Earth at T.

[Margin: Correction of an imagining, and a sketch of a truer diagram.]

IV. But so that no one, misusing the present diagram, may carry over the most perfect idea of this hypothesis from mere possibility to actual use, it must be understood that it has never happened, nor, granted the end of the world (which by probable conjecture is expected after a few centuries), will it ever happen, that the aphelion of the Earth and that of any of the three superior planets are under the same point of the zodiac; and we judge it impossible for the aphelia of all of them together to meet at the same point, because of the incommensurability in their motion. For the apogee of the Sun at the beginning of the world was under the beginning of ♈ and is now a little after

the beginning of ♋; the Earth's aphelion was then in ♎ and is now in ♍; Saturn's aphelion was then about the end of ♏, and is now about the end of ♐.

Diagram. A figure set into the right column, for the hypothesis of the planetary ellipse. A large outer circle, the ecliptic, is ringed outside by the twelve zodiac signs. At the centre stands a rayed Sun at A. Inside the outer circle are a smaller and a larger closed curve (the Earth's and the planet's ellipses) and a vertical diameter from B at the top to C at the foot. Straight and dotted lines run through A across the figure to the signs and the planet symbols.

- ♍ with V and K (top, outside the circle)
- ♒ (upper left), ♓ (left), ♈ (left), ♉ (lower left), ♊ with D (lower left), ♋ with R and C (foot), ♌ (lower right), ♍ (right, lower), ♎ (far right), ♏ (upper right), ♐ with E (upper right)
- B (top of the vertical diameter)
- X with ♄ (below B)
- P, F, G (descending on the vertical above A)
- A (centre, in the rayed Sun)
- Y (upper right, inside the outer circle)
- T (right of the vertical)
- H (left, end of a horizontal line from F)
- M (left, below H)
- N (left of A)
- L (right of A)
- ♃ with γ (right, on a dotted line through A)
- δ (left, at the outer end of the same dotted line, below ♈; cut with a descending tail)
- I with ♂ (lower right of A)
- Q, Z (below A on the vertical)
- S (lower left of A)

Labels read: all those listed. Labels not read: none known.

Jupiter's aphelion was then near the end of ♋, and is now a little after the beginning of ♎. Finally, Mars's aphelion was then at the middle degrees of ♉, and is now at the end of ♌ or the beginning of ♍. As for a future meeting, if the world were to last, I have already spoken of it in chapter 1, Scholium 6. So, that you may be able to draw a diagram fitting any time, once the planet's aphelion is given, a straight line must be passed through the centre of the Sun A to the places of the zodiac in which the aphelion lies on the one side and the perihelion on the other, and the eccentricity from the Sun must be taken on it, so that you obtain the centre of the ellipse to be drawn. Accordingly, to give an example fitted to this time of ours, the line of apsides of Saturn will be KAR, its aphelion under K and its perihelion under R, and its eccentricity approximately AF. So from the centre F Saturn's elliptical orbit must be described, so that Saturn at aphelion is at ♄ and at perihelion almost at Z. The line of apsides of Jupiter, on the other hand, will be γ A δ, its aphelion under γ, its perihelion under δ, and its eccentricity from the Sun AL. Therefore from the centre L you

will describe Jupiter's ellipse, so that Jupiter at aphelion is at ♃ and at perihelion at N. Finally, the line of apsides of Mars will be ♍ A ♓, its aphelion nearly under ♍ and its perihelion nearly under ♓, and its eccentricity from the Sun AI. So from the centre I you will draw the ellipse of Mars, in such a way that Mars at aphelion is at ♂ and at perihelion approximately at M. But such will be the proportion between the Earth's annual orb and the ellipse of Mars that Mars in opposition to the Sun, especially at perihelion, is much nearer to the Earth than the Earth is to the Sun. Now we must come down to the more notable properties of this new theory.

De Proprietatibus & Characteribus huius Hypotheſeos.

On the properties and characteristics of this hypothesis.

Propoſitio 1. Apſidum Linea Quinque minorum Planetarum tranſit per Centrum Corporis Planetæ, & Solis.

Proposition 1. The line of apsides of the five minor planets passes through the centre of the body of the planet and of the Sun.

[Margin: 1st property.]

V. So Kepler teaches in the *Mars*, chapters 5, 6, 33 and 52, and in the *Epitome of Copernican Astronomy*, pp. 539 and 763; Boulliau, in Book II, theorems 2, 6, 11, 12. For in the ancient hypothesis the line of apsides of these planets passed either through the centre of the Earth or through the centre of the annual orb of the Sun, and the centre of the planetary system was the point about which the Sun was supposed to move uniformly. But Copernicus put that centre at the point about which he thought the Earth moves uniformly, and therefore near the Sun, distant from the Sun only by the amount of the eccentricity attributed to the Sun; and from that limit he computed the eccentricities of the planets. Ty- cho, however, placed the same centre near the Sun, because they wished to represent in their hypotheses the equivalence of the Ptolemaic hypothesis. But Kepler and Boulliau placed the centre of the planetary system in the very centre of the body of the Sun, on a twofold argument.

[Margin: 1st argument, a posteriori.]

The first is a posteriori, from observations, especially of Mars in its acronychal shinings. These observations, properly handled, testify that the line of apsides, that is, the line that cuts the planet's orbit into two semicircles equal in the quantity and speed of motion, or that passes through the two points at which the slowness on one side and the speed on the other are greatest, passes through the body of the Sun. The observations by which this is urged are presented and weighed, for Mars, by Kepler, chapters 51 and 52 of the *Mars*, and by Boulliau, Book II, theorem 2; for Saturn, theorem 6; and for Mercury, theorem 12. For when two observations are chosen at which the planet is equally distant from the aphelion on either side, that is, before and after, or from the point of greatest slowness, the lines drawn from them to the centre of the Sun are found to be

equal, as they must be, while those drawn to any other point are unequal. And the angles of the equations of both inequalities are never correctly found, that is, they never agree exactly with the observations, unless the line of apsides is drawn through the centre of the Sun itself.

[Margin: 2nd argument, a priori.]

The second argument is a priori: it is not likely that the planet's moving power, in dispensing its motions, looks to a point in the world that is empty, of no account and unstable, especially for Copernicus, who set the Earth's eccentricity as different in different centuries, and finally a point inhering in no body of the world and resting on imagination alone (for Tycho destroyed solid orbs from the trajectories of comets). Indeed, it is more probable that, if the centre is near the Sun, it is rather the Sun itself, whether the Sun is the centre of the universe, as in Copernicus, or as it were the centre of the planetary system, as in Tycho; for it is the worthiest centre and the fountain of light. And so these planets move around it in order to be lit by it; for we have already all but proved, in Section 1, chapter 5, that they are lit by the Sun. So if you remove the body of the Sun from the centre of the planetary system, you do just as if you took the heart out of its body. Let it stand, then, that the line of apsides, that is, the line that bisects the planet's course into two semicircles equal in the quantity and speed of motion, is passed through the centre of the Sun's body, and that the planet's orbit is cut by the ecliptic at places that appear opposite from the centre of the Sun and not from any other point. Granted that Longomontanus, Book 2, chapter 9, Mars excepted, doubts about the others and seems to persist in the opposite opinion: let it stand, I say, in the hypothesis of a motionless Sun; for in Ptolemy's hypothesis that line must be passed through the centre of the Earth.

Propoſitio 2. In ſecunda Inæqualitate, ſeu in Anomaliæ Commutationis menſura, non Media Planetarum cum Sole oppoſitio vel coniunctio, nec Medius motus, ſed Vera Veruſque ſeu apparens Solis motus adhibendus eſt.

Proposition 2. In the second inequality, that is, in measuring the anomaly of commutation, it is not the mean opposition or conjunction of the planets with the Sun, nor the Sun's mean motion, but the true opposition or conjunction and the true or apparent motion of the Sun that must be used.

VI. This proposition is akin to the first and connected with it, yet different from it, because that one concerns the first inequality and this one the second. This proposition is shown, both a posteriori and a priori, by Kepler in the *Mars*, chapters 6, 7, 9, 18, 52, in the letter to Magini that Magini reports in the *Supplement to the Ephemerides*, from p. 242, and in the *Epitome of Copernican Astronomy*, p. 535; and by Boulliau, Book 11, theorems 3, 7, 8 and 11. For their arguments to be understood, the procedure of the ancients must be gone over again here.

[Margin: Two reasons for which the ancients chose the mean motions of the Sun here.]

So Ptolemy, from Book 9 of the *Almagest*, and with him Regiomontanus in the *Epitome of the Almagest*, Book 10, propositions 9 and 10, and Copernicus, Book 5, chapters 5, 6, 10, 11, 15 and 16, and Lansbergen in his *Theoricae*, thought that these planets are stripped of the second inequality, which is tied to the motion of the Sun, whenever they fall at the points of mean conjunction or opposition with the Sun, because the difference of the planet's place from its centric place then appears either nil or barely perceptible; and it was customary for Ptolemy to disregard slight differences of this kind, or to refer them to the fault of the instruments and observations. On the other hand, the calculation came out far easier if the mean motion of the Sun was used.

[Margin: The procedure of the ancients in separating and establishing the first inequality.]

Confident in this persuasion, then, they observed the superior planets in their acronychal shinings, when it is established that they are close to opposition to the Sun. If, at the moment of the observation, the planet's place had been opposite the place that the tables of mean motions then assigned to the Sun, they concluded that at that very point the mean opposition with the Sun took place, and that at the same point the planet was under the same place as the centre of the epicycle, at the very middle of its retrogradation, that is, of that greatest slowness which appears in the motion of the second inequality. But if the planet was not then at the point of mean opposition with the Sun, they compared other observations made on other nights, and the daily motions of the planet and the Sun, and hunted out the point of mean opposition and the moment at which it had taken place, or was to take place, a little before or after. When they had done this in various places of the zodiac, and had compared the acronychal observations and the equations needed to make the planet's place derived from the tables of mean motions agree, at the points of mean opposition, with the place observed by instruments, they began to investigate, on the one hand, in which places of the zodiac it needed no equation, and at these they said the apogee and perigee of the eccentric lay; and, on the other, in which places equations had to be subtracted or added. Throughout the time in which they had to be subtracted, they said the planet was receding from the apogee of the eccentric toward the perigee; and throughout the time in which they had to be added, that it was receding from the perigee and climbing toward the apogee. Finally, from the angles of the equation and of the mean motion, and from the radius of the eccentric, they worked out the eccentricity, and thus established the hypothesis of the first inequality. Passing from here to the second inequality, they took its measure from the planet's elongation from the mean place of the Sun. For when they saw that, outside the conjunctions and oppositions with the Sun, the usual equations of the eccentric were not enough to make the planet's place drawn from the tables agree with the place observed in the sky, they distributed the increase and decrease of those equations into

tables, according to the greater or lesser elongation of the planet from the mean place of the Sun. And so, to find the anomaly of the orb, or of the epicycle, some ordered the planet's mean motion in longitude to be subtracted from the mean motion of the Sun, and others the true motion of the centre, that is, the motion of longitude coequated by the first equation. Finally, they affirmed that the line drawn from the centre of the epicycle to the centre of the planet's body, when the planet is at the apogee or perigee of the epicycle, coincides with the line of the Sun's mean motion; but that outside those two points it is a different line, though always parallel to the line of the Sun's mean motion.

VII. But Kepler and Boulliau, in the places indicated above, show that the second inequality ceases exactly when the planet is at the points of true, or apparent, conjunction or opposition with the Sun; and that therefore, to find the anomaly of the orb, or of commutation, that is, of the second inequality, the coequated centre of the planet, or the planet's eccentric place, must be subtracted from the motion of the Sun, not the mean but the true, as Tycho did in the case of the Moon. After reading the work on Mars, Longomontanus subscribed to Kepler's view, Book 2 of the *Theorica*, chapter 9.

[Margin: 1st argument, a posteriori, for the true place of the Sun.]

This, I say, they show first a posteriori, from observations. For although near the conjunctions or oppositions with the Sun the difference between the planet's observed place and the place derived from the tables is very slight, for those who use the Sun's mean motion in finding the anomaly of the orb or of commutation, yet outside these points a quite perceptible difference is found, especially with the Sun at perigee, as it now is at about 6° ♑, and greater still if the line of the mean longitudes of the annual orb coincides with the line of apsides of Saturn, Jupiter and Mars.

[Margin: The difference between those who use the true and those who use the mean place of the Sun.]

So much so that the difference between those who refer the places of the planets to the true place of the Sun and those who refer them to the mean can then run up to about 5° 19′ in Mars, to 15′ in Saturn and to 30′ in Jupiter, that is, to an error of several days, because of the slowness of the motion. Boulliau shows this difference in Book 11, theorem 8, with a diagram of this kind, in which the Sun is at P, the eccentricity of the annual orb is PR, the annual orb MKI is described from the centre R, the Earth's line of apsides is MI, with its aphelion in ♑ when the Sun appears to the Earth in ♋, and the line of the mean longitudes of the annual orb is LPN, because it cuts the line of apsides perpendicularly at the centre of the Sun. The orb

of ♄, ♃ or ♂, now, is to be ACFH, and let their line of apsides be BPE, drawn through the Sun. Parallel to it let ARF be drawn through the centre of the Earth's mean motion, representing the apsides of the planets as they pass through the mean motion of the Earth, or rather of the Sun. Let the planet move from A to B, that is, from ♌ to ♋, and the Earth from L to M (for the viewer of this diagram is supposed to be facing north, toward which the apogees

of the three superior planets tend), and let the planet be supposed at D, and let the line PD mark its eccentric place. On this line the Earth will be at L if the planet is truly opposite the Sun, or at N if it is in conjunction with it; just as, if the planet is at G, it will appear truly opposite the Sun P to the Earth at N, but in conjunction with the Sun P to the Earth at L.

Diagram. A figure set into the left column, for the comparison of true and mean opposition. A large outer circle (the planet's orb) carries zodiac signs at its rim. A smaller circle (the annual orb) stands inside it. A vertical line runs from the top of the large circle down to its foot, and a second, nearly parallel vertical line beside it. Oblique lines cross the small circle through its centre region and run out to the large circle, from D and C at the upper left to G and H at the lower right, and from M at the upper right to I at the lower left.

- Top of the large circle: sign ♌, with A and B just below it.
- Upper right, outside the circle: sign ♋.
- Upper left, on the large circle: C and D, with a zodiac sign beside D [?].
- On or near the small circle: L, K and T at the upper left, S and M at the upper right, I at the lower left with a zodiac sign below it [?], N and O at the lower right.
- Centre: R and P.
- Lower right: G, with the sign ♈ beside it.
- Foot of the large circle: F, E and H.

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, R, S, T, ♌, ♋. Labels not read: the three signs marked above. T stands on the figure but is not cited in the text on this page.

Next draw CRH parallel to the line DPG, passing through R, the centre of the Earth's mean motion; for according to the old astronomy the acronychal point [Acronychium], that is, the mean opposition of the Sun and planet, will occur when the planet is at C and the Earth at K, or when the planet is at H and the Earth at O. The question now is how far the places of the planet among the fixed stars, as seen from the Earth at L and at K, or from the Earth at N and at O, differ: that is, the angle KDL if the planet is at D, or OGN if it is at G. This angle is found in the triangle KLD, by the following method. Given the planet's place at D, its distance from the aphelion B is given, that is, the angle BPD, to which the angle ARC is equal because of the parallelism. The planet's distance from the Sun, PD, is then derived, in parts of the radius of the annual orb, from the given eccentricity of the planet, PS, and the radius SD, which is here to be supplied in the figure in imagination. Next the distance of the Earth at L from the Sun, that is, PL, is assumed as given in the tables or as found; and when it is subtracted from DP, the side DL remains known. Moreover, the difference between the true and the mean place of the Sun, or rather of the Earth, is given, that is, the arc KL, and its sine, to be understood here in parts of which the radius of the annual orb, RK, is known; for the arc KL is taken here for a straight line making a right angle at L, since it differs imperceptibly from one. Therefore in the triangle KLD, right-angled at L, the

angle KDL will become known, which was to be done. Since, therefore, the celestial motions cannot be represented with rigour, nor observations exactly reproduced by calculation, unless they are referred to the true place of the Sun, it is clear that the ancient astronomers, as many as used the mean, strayed from the heaven and from the truth. Remember, however, that at true ☌ and ☍ the planet's place must be reduced from the ecliptic to the orbit, as Kepler teaches and does in the *Mars*, chapter 9; otherwise an error of 9′ can be committed.

[Margin: 2. Reason a priori.]

VIII. The second reason is a priori, from the causes of the planet's motive force. Whether this is the Sun, as Kepler holds, or an intrinsic force and the planet's own form, as Boulliau holds, no probable reason persuades us that the planets, which circle the Sun so as to be illuminated by it, and which with wonderful harmony quicken or slacken their motion about it, so much so that in ☌ they are direct and swiftest and in ☍ retrograde and slowest, look to a point near the Sun, empty and unstable, rather than to the very centre of the Sun's body, which is the heart and source of the planetary light; and that they regulate their courses by a fictitious line and by the Sun's mean motion, which is true in only two cases and in all the others is false and fictitious, not really existing in the heaven but only drawn, from our imagination, in our tablets and papers. Kepler adds, in the *Mars*, p. 53, that this hypothesis fits better with the intervals he had determined in his *Mysterium*.

[Margin: Tycho's last opinion, and Longomontanus's.]

Finally, just as Copernicus and Tycho placed the centre of the planetary system not at the centre of the Earth or at a point near the Earth, but at a point near the Sun, so as to represent the motions of the planets more exactly and more elegantly, so it was more in keeping with their view that they should set it in the very Sun itself. Tycho finally acknowledged this in his last days, as Kepler testifies in the *Mars*, chapter 6, p. 47, and Longomontanus in Book 2 of the *Theorica*, chapter 3, at the end of chapter 11, and chapter 13; granted that Boulliau, in Book 11, theorem 11, lumps him in with the ancient astronomers who use the Sun's mean motion.

[Margin: To whom Boulliau attributes the opposite.]

Propoſitio 3. Planetæ iter eſt per lineam Ooidem ideſt Ouiformem, ſeu Ellipticam; ſeu Orbis illum deferens non eſt perfectus circulus, ſed Ellipſis.

Proposition 3. The planet's path runs along an ooid line, that is, an egg-shaped or elliptical line; that is, the orb that carries it is not a perfect circle but an ellipse.

IX. Kepler urges this proposition in the *Mars*, chapters 22, 23, 39 and from chapter 40 to 50, in the *Epitome of Copernican Astronomy*, Book 5, from p. 658, and in the Rudolphine Tables, chapter 20. So does Boulliau, in Book 11 of his *Astronomia*, Book 10, chapter 1, where he says: *I held it a matter of religious scruple to depart from the circular hypothesis, and it had not occurred to me that the*

path of the planets could be elliptical; apart from which, if anyone tries to twist the planets' progress into other orbits, he will waste his oil and his labour by doing violence to them, and will roll the stone of Sisyphus. And in Book II, theorems 7 and 13, he admits that, because the difference from a circle of the ellipse attributed to them is scarcely perceptible, it cannot readily be shown from particular observations of Saturn, Jupiter and Venus that their path is elliptical, but that no one will be left in doubt about this if he reads his arguments set out in Book I, chapter 15; and in Book II, theorem 13, he tries to show it in the case of Mercury.

[Margin: 1. Reason a posteriori.]

The sum of the a posteriori arguments is this. If, by geometrical calculation from the given eccentric anomaly, eccentricity and radius of the eccentric, the eccentric place of the planet and its distance from the Sun are found, and from these, with the anomaly of the orb or of commutation, the planet's place, using a perfect circle and lines drawn from the centre of the circle to the circumference; and if this is compared with the place given by the more exact observations of more recent astronomers, for instance Tycho, Gassendi, etc., a perceptible difference is found, which almost wholly vanishes if we use an ellipse, in such a way that the planet at aphelion or perihelion is at the ends of the axis of the ellipse, while around the mean longitudes it is at the ends of the conjugate diameter.

Thus Boulliau, in that theorem 13, reports that he observed Mercury at Loudun [Iulioduni] in 1630 December 12, in the morning, at $6^h\ 50^m$ after midnight, by means of its distance from Spica, and with the place corrected for refraction, at 29° 13′ of Scorpio; whereas its place, sought geometrically on a perfect eccentric circle, came out at 28° 41′ 39″, but on the ellipse at 29° 8′ ♏, and, applying the curtation on account of latitude, at 29° 12′ 59″ ♏. Again, in 1635 December 1, at $5^h\ 30^m$ after noon, at Digne, Mercury by distances from the stars of Ophiuchus stood at 0° 26′ ♐, with the refraction computed, for it was 1⅔° above the horizon. The geometrical calculation resting on the circle gives the place of ☿ as 0° 47′ 35″ ♐, but resting on the ellipse it comes very close to the observed place, that is, it puts ☿ at 0° 23′ ♐. Since, then, Mercury was observed around its morning and evening digressions, in nearly opposite places, and since, with the radius of the eccentric, the eccentricity and the anomaly otherwise equal, the planet's place comes out far more exactly from the ellipse than from the circle, Boulliau concludes that Mercury is carried round not along the circumference of a circle but along that of an ellipse. By the same argument, Kepler, in the *Mars*, from chapter 40, tries to show from the observed places of the Sun and Mars that the planet's path, as it approaches the mean longitudes, does not stray outward from the circle, as it does in the epicycles of Copernicus and Tycho, but inward, approaching the centre of the eccentric; and that it is therefore ooid or oval, since otherwise the equations do not answer to the observed places.

[Margin: This a posteriori reason is not evident.]

X. Notwithstanding all this, Boulliau himself, in Book 1, chapter 13, and in Book 11, at the end of theorems 7 and 13, frankly admits what Kepler had confessed about the Sun's orbit in the *Mars*, chapter 40: namely, that it cannot be discerned from calculation alone and the comparison of observations whether these planets move in an ellipse rather than in a circle: the Sun and Venus because of their slight eccentricity, which makes the minor diameter of the ellipse to be attributed to them by hypothesis scarcely perceptibly smaller than the axis, or major diameter, so that their ellipse departs imperceptibly from the circle; Saturn and Jupiter because of the slowness of their motion, which makes a very slight error in observation give rise to a larger error than in the others,

and therefore it cannot be evidently established whether the discrepancy between the geometrical calculation and observation arises from the figure used in the calculation being circular rather than elliptical, or else from the fallibility of observations, which easily creeps up on any observer, however careful. For Mars it is indeed inferred with greater conjecture, and in a way demonstrated; but not absolutely, nor in such a way that the disagreement of the observations with a calculation assuming a circle could not be referred partly to the Sun's eccentricity, partly to other causes, rather than to the neglect of the ellipse. *In Mercury the ellipse is most perceptible, but it is not seen, in our parts, at those places from which we could infer with full certainty that the matter is so.* These are Boulliau's words, alluding to the nearness of the Sun and of the horizon, close to which Mercury is always observed, with the suspicion of a refraction whose quantity in its case has not yet been reliably determined.

[Margin: 2. Reason a priori for the ellipse.]

XI. But as for the second argument, a priori, we so far have none that is evident. Kepler himself, in the *Mars*, admits that he sang an empty triumph, and says at the end of chapter 55: *And so the physical causes of Chapter 45 go up in smoke.* Neither there nor in the *Epitome* does he adduce any that are apodictically derived from what all grant or must grant; so that Boulliau, Book 1, chapter 13, rightly said: *But the physical reasons that Kepler adduces display ingenuity of mind; they do not reveal the truth.*

[Margin: Boulliau's censure of Kepler.]

Boulliau, in the same place, tries to establish the elliptical path of the planets by other reasons, from prior and general conditions of the celestial motions.

[Margin: Boulliau's foundations for the ellipse.]

For he assumes that the planets move with a single regular motion, compounded of regular motions, perpetual and returning into itself, and therefore along a single line returning into itself, but one arising from circular motions, because it is accomplished by the laws of circles; and that their revolutions have a principle of equality, which they regard of themselves; but that the apparent inequality must be referred to the centre of the zodiac, that is, to the Sun as the point of inequality. Half of this inequality is to be attributed to the eccentricity of the orbit, or to the distance between the points of apparent and mean motion, the other half being thrown on another cause, which makes the planet slower

at aphelion and swifter at perihelion, with the equality of motion about its own centre preserved. And therefore the equal motion answers to several parallel circles, growing from aphelion to perihelion, all of which the apparent motion embraces in one surface, but eccentric to all of them and inclined to all, so that the individual parts of the apparent motion answer to all the parts of the equal motion. These conditions, he assumes, cannot stand together unless the planet describes an ellipse about a scalene cone. He proves this by excluding the sphere, the elliptical spheroid, the parabolic and hyperbolic solids, and the cylinder: although an ellipse can be cut in a cylinder, yet all the circles described in it are equal, and some of them concentric with the ellipse, whereas they must be unequal. These circles Boulliau calls equant circles, which we have already indicated in Book 3, chapter 23, no. 6; read that passage again. On the resolution of the ellipse into two circles we have spoken in the same place, no. 10, and will speak below, when we treat of the equations of the first inequality.

[Margin: Another reason for the ellipse.]

To others this elliptical path might, in brief, seem probable for the following reason. Supposing the necessity, or the very great probability, of an orbit eccentric to the world, so that the apparent inequality of a motion equal in itself may be referred to the distance of the centre about which the equal motion takes place from the centre from which the motion is viewed, and supposing the bisection of the eccentricity, of which in the following proposition: neither in a circle nor in any other figure bounded by a single line returning into itself is there found any point outside the centre worthy of being regarded by the planet or by the planet's mover, and from which a reason could be given why the motion is also ordered about that point and not only about the centre of the figure. But in the ellipse alone, besides its centre, there are two notable points, which Apollonius calls the points from the comparison, Kepler the foci and Boulliau the umbilici, endowed with notable properties. Therefore this figure seems of itself most suited to the motions of the planets, provided that between the centres of apparent and of mean or equal motion stands the very centre of the ellipse, and one umbilicus is the centre of mean motion, and the other the centre of apparent motion, at the centre of the world. But we ought not to be so devoted to this figure as not to think that the motions of the planets can be carried out and explained by a perfect circle; especially since Copernicus in Book 1, chapter 4, Tycho in tome 1, p. 11, and Longomontanus in Book 2, chapter 9, adopt as an astronomical axiom that the celestial motions are either circular or compounded of circular motions, and do not subscribe to Kepler on this.

[Margin: Axiom on the circular figure of the celestial motions.]

Propoſitio 4. Eccentricitas, in qua fundatur Prima Inæqualitas motus Planetarij biſecanda eſt bifariàm; ſeu dimidium illius inæqualitatis Eccentricitati Deferentis tribuendum eſt, alterum verò dimidium cauſæ alteri phyſicè ac realiter retardanti vel acceleranti motum Planetæ; ac proinde opus eſt Aequante.

Proposition 4. The eccentricity on which the first inequality of planetary motion is founded must be bisected in two; that is, half of that inequality must be attributed to the eccentricity of the deferent, and the other half to another cause physically and really retarding or accelerating the planet's motion; and therefore there is need of an equant.

XII. How Ptolemy bisected the eccentricity and introduced the equant, whose centre, or point of equal motion, is as far from the centre of the planet's deferent as the centre of the deferent is from the centre of the world, we have already explained in chapter 1, scholia 1 and 2. That the same must be done in the orbit of every planet is taught by Kepler in the *Mars*, chapters 19, 24, 32, and in the *Epitome of Copernican Astronomy*, p. 712, and by Boulliau in Book 1, chapters 13 and 22, and Book 11, theorems 1, 4 and 5. For you to understand their foundations, it must be explained again more clearly what it is to bisect the eccentricity and to use an equant.

[Margin: What the bisection of the eccentricity is.]

This is nothing other than to place the centre of the orbit that carries the planet, either by itself or by means of some other device such as an epicycle, exactly midway between the point of apparent motion, or the centre of the universe (which for Ptolemy and Tycho is the centre of the Earth, and for Copernicus, Kepler and Boulliau the centre of the Sun's body), and the point of mean or equal motion. Therefore if, by means of the mean motion compared with the apparent, the eccentricity of the point of equality (which is therefore called the centre of the equant) from the centre of vision, or of the world, has been found to be, for example, 3600 parts, of which the radius of the planet's orb is 100000, the eccentricity of the planet's deferent (whether it is a circle or an ellipse), or the distance of the centre of the deferent from the centre of the world, must have only half of that, namely 1800 parts. And so the difference between the planet's greatest distance from the centre of the world, which is made up of the radius of the eccentric orb, or mean distance, and the eccentricity of that orb, and its least distance, which remains on subtracting the eccentricity from the radius, or mean distance, is only half of that eccentricity which is deduced from the mean motion compared with the true or apparent. Conversely, if the eccentricity of the planet's deferent is gathered from the planet's distance and the equations needed to reconcile the mean motion with the apparent, it must be doubled to obtain the eccentricity of the point about which the mean motion takes place. From this it follows that the foundation of the whole inequality, which is contained in the planet's motion in longitude independently of the

syzygies of the Sun, is the whole eccentricity, and the angle opposite it is the difference between the mean and the apparent motion; but the foundation of the merely apparent, or optical, inequality is half the eccentricity, and the other half is the foundation of the physical and real inequality, arising from some cause that now speeds the planet up and now holds it back in the heaven, independently of the observer's eye or of the centre of vision, whatever that cause may be.

[Margin: Arguments for the bisection of the eccentricity.]

XIII. With these things laid down: of the arguments by which the proposition stated above is built up, some are proper to individual planets, some common to all or to several. In the case of the Moon, Tycho, from distances observed through parallaxes, finding their difference smaller by almost half and the intervals contracted, as we said in Book 4, chapter 26, from no. 7, imitated the bisection of the eccentricity. In the case of the Sun, what moved Tycho to suspect, and Kepler and his followers to assert, that its eccentricity must be bisected I touched on in Book 3, chapter 22, no. 5, but did not approve; for the difference of only 1′ in the Sun's apparent diameter at apogee and perigee, which is one of Kepler's chief foundations, conflicts with observations evident to me and to very many others.

[Margin: Arguments against the bisection.]

Nor, in eclipses of the Moon, which, as Kepler says in the *Epitome*, p. 712, seem to demand a smaller variation of the shadow from the Sun at apogee to the Sun at perigee, is the bisection of the Sun's eccentricity and of the dif-

ference of the distances founded securely enough, as is clear from what was said in Book 3, chapter 7, no. 8.

[Margin: ☿ and ♀: bisection of the eccentricity.]

As for Venus and Mercury, no one has so far demonstrated it; and Kepler asserted that their eccentricity must be bisected only topically and through general considerations, in the *Mars*, chapter 32, saying: *Nothing prevents us from believing the same of Venus and Mercury.* In Mars, however, Kepler in the place cited above and Boulliau in theorem 1, and in Saturn in theorem 5, seem to have shown it, chiefly from observations of Mars in □ aspect with the Sun, which cannot be reconciled except with a geometrical calculation using half the eccentricity; otherwise the parallaxes of the annual orb come out very different from those observed.

[Margin: ♂: by whom its eccentricity is not bisected.]

Yet Copernicus, Book 5, chapter 16, did not exactly bisect the eccentricity of Mars, and he attributed the reason for not bisecting it not, indeed, to Ptolemy's errors, for he thought that in Ptolemy's time it should be bisected, but to the variation and diminution of the Earth's eccentricity; and he held that Mars in its eccentricity imitates the Earth's eccentricity. Longomontanus also thought this of ♂, Book 2 of the *Theorica*, chapter 8; and therefore, in his own hypothesis and the Tychonic one, he assigned to the second small circle not a third part of the whole eccentricity, as in ♄ and ♃, but only a fourth. And before him

Tycho, as Kepler relates in the *Mars*, chapter 16, did not assume the eccentricity of Mars but investigated it from his own observations, and made the centre of the deferent eccentric 13680 parts distant from the centre of vision, or of the universe, and the centre of the equant 3780 from the centre of the deferent; which, in the Ptolemaic radius of the orb, would be to make the former to the latter as 9900 to 7560, and so the whole eccentricity 17460, but the eccentricity of the equant alone 7560.

[Margin: ♄: whether its eccentricity is to be bisected.]

Finally, Johannes Phocylides Holwarda, in his examination of Lansbergen's astronomy, p. 143, declares that he has demonstrated from acronychal observations that the eccentricity of ♄ should not be bisected, and that in Lansbergen's form the least eccentricity of Saturn is 8600, but the greatest not double this, but 11400. So to this day this bisection seems to be disputed.

[Margin: The physical cause moving the planets, according to Boulliau.]

The remaining points that seem proper to this hypothesis have partly been discussed in Book 3, chapter 23, and Book 4, chapter 27, where we treated of the Sun's magnetic force carrying the planets round, according to the arrangement of the magnetic fibres that are in the planets and according to the principles of the balance and the lever, causes which Boulliau nevertheless refutes, Book 1, chapter 12; they will partly be discussed shortly, when we treat of the equations; and they partly concern the harmonic proportions of the intervals, of which we will speak either in the last section of this book or in the harmonic system, Book 9.

De Prima Inæqualitate eiuſque Aequationibus.

On the first inequality and its equations.

XIV. Let the figure of Book 4, chapter 27, no. 3 be repeated here. In it the centre of the Sun and of the universe, and one umbilicus of the ellipse, is A; the line of apsides and axis of the ellipse is PR, in which B is the centre of the circle PDRC, in which is inscribed the ellipse PERV, on whose circumference the planet moves from the aphelion P, through HEO, toward the perihelion R.

Diagram. A figure set into the left column, interrupting the sentence between P and R. A large circle with a vertical diameter from P at the top to R at the foot encloses an ellipse inscribed along the same axis. A small circle is drawn about the point A on the axis. Horizontal lines cross the figure through B and through A, and further oblique straight lines run from A, and a dotted line runs down to the lower right.

- Top: P, with F and S below it on the axis.
- Upper left: G and H.
- Left: D and E, level with B.
- Right, level with B: V and C.
- Right, below that: K, L and T.
- Left, below D: M, then Z, then N and O at the lower left.

- Centre: B, with I below it and A below that.
- Lower axis: Q.
- Lower right: Y.
- Foot: R.

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q, R, S, T, V, Y, Z. Labels not read: none.

When perpendiculars are drawn through the centres A and B to the circumferences, EV is called the Diacentros and the minor or conjugate diameter of the ellipse, and MT the Dihelios. The other umbilicus, or focus, is S, as far from B as is the eccentricity of the centre B from the Sun A. This umbilicus is the centre of mean motion and stands in place of the centre of the equant.

[Margin: The physical cause moving the planets, according to Kepler.]

The Sun, in Kepler's opinion, by a rotation of about 27 days about the centre of its body toward the east, sends out from itself a certain image spread with its light in every direction, which resembles magnetic effluvia; and this, seizing the planets, carries them round eastward, faster or slower according to their lesser or greater distance from the Sun, and their lesser or greater passive resistance, or inclination or inertia toward such a motion, and according to the arrangement and position of the magnetic fibres, whose diagrams we have already set out when treating of the Moon, Book 4, chapter 27, and in its scholia. For Kepler imagines in the bodies of the planets two poles, about which the planets rotate, one of them in the Sun-seeking part, or the part friendly to the Sun, which at the beginning of the world was turned toward the Sun; the other in the Sun-fleeing, or hostile, part, then turned away from the Sun toward the fixed stars. So when, in the periodic motion in longitude, the part friendly to the Sun is turned toward the Sun, the planet is attracted by the Sun and descends from the aphelion, and becomes nearer and nearer to the Sun, and nearest at the perihelion R; once this is passed, the hostile part begins to turn toward the Sun, and then the planet is repelled by the Sun and moves away toward the aphelion. But Boulliau thinks these are empty fictions, Book 1, chapter 12, and holds that the planets move on the ellipse by their own intrinsic form; we will say more of this in Book 9. Meanwhile they agree that the planet moves unequally on the circumference of the ellipse, and is librated in proportion to its distances from the Sun, so that the ratio of the greater distance to the lesser is the same as that of the planet's slower daily motion to its faster daily motion; and that the daily motions of the same planet about the Sun, with the eye placed in the Sun, appear in the duplicate ratio of their distances from the Sun.

[Margin: The measure of the planetary stay, from the area of the ellipse.]

But the measure of the stay and time on any arc of the ellipse is taken not from the arc itself but from the area and areal segment of the ellipse. For if the circumference of the circle circumscribed about the ellipse is divided into equal parts by lines applied in order to the axis, or perpendicular to the axis, so that if the planet is at H the division is made by GHF, the arcs of the ellipse are found to be unequal among themselves, smaller around the apsides P and R,

largest around the mean longitudes, that is, around the points E and V. At these points, a quadrant having been completed, the nearly greatest equations occur, and AE, the planet's distance from the focus or Sun A, is the mean, or equal to the major semidiameter of the ellipse, BP; as is clear from the properties of the ellipse demonstrated by Apollonius. Therefore the time and stay of the planet at H, on the arc PH, are measured by the area enclosed by the straight lines AP and AH and the curve HP. For Kepler shows in the *Mars*, chapter 59, and in the *Epitome of Copernican Astronomy*, from p. 665, that as the planet's whole periodic time is to the whole area of the ellipse, divided into equal parts, say 360, so is the part of the time and stay on an arc of the ellipse to the part of the area enclosed by that arc and the straight lines drawn from the Sun to the ends of the arc.

XV. For ease of calculation, however, Kepler assumes a circle circumscribed about the ellipse, and tries to demonstrate the equipollence of the circular plane with the plane of the ellipse, because the elliptical circumference cannot be divided geometrically into equal parts. Therefore in the *Mars*, chapter 59, and in the *Epitome of Copernican Astronomy*, p. 671, he teaches that as the area of the sector PGA is to the area of the whole circle, so is the arc of the elliptical segment PHA to the area of the whole ellipse, and likewise for the rest. So he would have it that, if the circular arc PG is of 30°, the arc PH too is denominated 30° and is in power as great as the circular arc PG, because by means of the area of the ellipse enclosed by it we arrive at knowledge of the angle HAP, which measures the planet's distance from the aphelion. With these things supposed, the remaining terms and definitions used by Kepler for the equation of the first inequality are these.

[Margin: What is the libration of a planet?]

Libration of the planet is the difference of distance between the mean longitude, where the motion is swiftest, and the other longitudes, in which the motion is slower; such is the difference between AE, the mean longitude of the planet placed at E, and AH, the longitude of the planet situated at H.

[Margin: Mean anomaly.]

Mean anomaly is the span of time that the planet consumes in completing an arc of its orbit, reckoned from the aphelion, reduced to parts of which the whole anomaly is worth 360. Its measure is the area of the sector of the circle that denominates and determines the arc of the elliptical orbit: an area enclosed by the arc of the circle and the straight lines connecting the ends of that arc with the Sun, and reduced to parts of which the planet's whole periodic time is worth 360. So if the planet is at H and the arc PG denominates and determines the arc PH, the area of the sector enclosed by the arc PG and the straight lines PA and GA is the measure of the mean anomaly, because of its equivalence with the area HAP.

[Margin: Eccentric anomaly.]

Eccentric anomaly is the arc of the eccentric circle counted from the aphelion in the order of the signs as far as the perpendicular drawn through the body

of the planet to the line of apsides. So if the planet is at H and you draw GHF through it perpendicular to PR, the arc PG is the eccentric anomaly, because it is equivalent to the arc PH.

[Margin: Coequated anomaly.]

Coequated anomaly is the angle made at the centre of the Sun by the line of apsides and another straight line, drawn through the body of the planet, if the anomaly does not exceed two right angles; but if it does exceed them, it is the complement of that angle to four right angles. Thus if the planet is at H, the coequated anomaly is the angle HAP; but if it were at Y, it would not be the angle YAP but that angle's complement to 360°.

[Margin: Eccentric place.]

The *eccentric place of the planet* is the point of the zodiac on which falls the straight line drawn from the Sun through the planet: such is the straight line AH, if it is produced all the way into the zodiac.

[Margin: Equation of the eccentric place.]

The *equation of the eccentric place* is the difference between the mean anomaly and the coequated anomaly. Thus if the mean anomaly, or the value of the area PAG, is 45°, and the angle PAH, of the equated anomaly, is 40°, the equation will be 5°. This equation is served by the equating triangle [triangulum Æquatorium] GAB, because of its equivalence with the triangle HAB; and the area of this triangle is called by Kepler the *physical equation*, while its angle at or near the planet is called the *optical equation*: namely the angle AGB.

[Margin: Physical and optical equation.]

Kepler teaches in the *Epitome of Copernican Astronomy*, from p. 694, that in the upper semicircle CPD, above the Diacentros, the physical equation is a little greater than the optical, but in the lower, DRC, a little smaller; that the physical equation, taken by itself, is greatest at the points D and C; but that the optical, taken by itself, would be greatest at the points of the Dihelios line, that is, at O and at Y, if the orbit were a circle, whereas because it is an ellipse it is greatest between D and O and between Y and C. The one compounded of both, however, is greatest between D and E and between V and C.

Methodus Aequationis Primæ Inæqualitatis ex Keplero.

Method of the Equation of the First Inequality, from Kepler.

XVI. Kepler gives this method in the *Mars*, chapter 60, in the *Epitome of Copernican Astronomy*, Book 5, part 2, and in the Rudolphine Tables, chapter 20; it runs as follows.

[Margin: 1. Step, for the distance of the planet from the Sun.]

First, find from the tables the mean motion in longitude of the planet and that of the aphelion or apogee, and subtract the latter from the motion in longitude, for what remains will be the eccentric anomaly, that is, the arc PG (if the planet is at H). Find its second sine, which is BF, in the tables of sines; multiply this by the given eccentricity and divide the total by the radius BP,

for the quotient will be the libration. Add this to the radius BP as long as the planet is in the upper semicircle, or above the Diacentros DBC, but subtract it if the planet is below, and you will have the distance of the planet from the Sun.

[Margin: 2. Step, for the coequated anomaly.]

Second, add the second sine already mentioned, that is, BF, to the eccentricity AB, and the result is AF, which is a side of the triangle AFH, right-angled at F, in which the base AH, that is, the distance of the planet from the Sun, has already been found. So the angle HAF will not remain unknown; this is the coequated anomaly, and the place of the apogee added to it will give the planet's eccentric place.

[Margin: 3. Step, for the greatest physical equation.]

Third, multiply half of the eccentricity AB by the radius BD, and you will have the area of the greatest equating triangle ABD, in parts of which the whole area of the circle is, for Kepler, 31,415,926,536, or of which the radius BP is 100000. Then let it be: as the area of the whole circle is to the time, or span, of 360° resolved into seconds, that is, to 1296000″, so is the area of the triangle ABD to another quantity; and you will find the value of the greatest triangle, or of the greatest physical equation, which is equipollent to the value of the triangle AEB.

[Margin: 4. Step, for the other physical equations.]

Fourth, since the arc PG is given, as above, the value of the area of the sector GBP is also given; and it is of as many degrees, of those of which the whole area of the circle is 360, as there are degrees in the arc PG. In addition, the sine of the arc PG is given, which is GF. Now as DB, the whole sine, is to the sine GF, so the area of the greatest triangle ABD is to the area of the equating triangle AGB. Therefore multiply the value of the greatest triangle, found above in the third step, by the sine GF, and divide it by the radius BD, 100000, or strike five figures from the end of the total, and there will remain the degrees and minutes of the value of the triangle AGB, or the physical equation, which is equivalent to the triangle AHB.

[Margin: 5. Step, for the mean anomaly.]

Fifth, add the physical equation to the sector GBP, that is, to the degrees and minutes of the arc PG, if the arc PG does not exceed a semicircle; but if it does exceed it, subtract the equation from the sector. This will make or leave the mean anomaly, that is, the whole area GAP, in parts of which the whole circle is 360°.

[Margin: 6. Step, for the optical equation.]

Sixth, in the triangle AGB the eccentricity AB is given, and the radius BG, and the angle ABG, because it is the complement to two right angles of the angle PBG, which measures the given arc PG. So the angle AGB will not remain unknown; it is the measure of the optical equation, and is always somewhat smaller than the angle AHB.

So much for this method, if you wish to construct tables geometrically or quasi-geometrically and to distribute in them the anomalies and equations already mentioned. Otherwise, to obtain the planet's eccentric place, the first and second steps of the calculation suffice, since if you add the place of the apogee to the coequated anomaly you will have the planet's eccentric place.

[Margin: Imperfection of Kepler's method.]

I said "quasi-geometrically" because geometrical rigour is not preserved in these, since the elliptical circumference cannot of itself be cut geometrically into equal parts, as Kepler admits in the *Epitome*, p. 780; just as, given the mean anomaly, there is no geometrical method of finding the coequated anomaly, and from there the eccentric anomaly. The same author admits this in the *Mars*, chapter 60, concluding in these words: "This is my view; and the less geometrical beauty it will seem to have, the more I urge geometers to solve for me this problem: TO CUT THE AREA OF A SEMICIRCLE from any point of the diameter in a given ratio. It is enough for me to believe that it cannot be solved a priori, because of the ἑτερογένειαν [Greek: heterogeneia, "difference in kind"] of the arc and the sine. Whoever shows me the way, wandering as I am, will be a great Apollonius to me." In the same way, in the letter to Magini that survives in Magini's *Supplement to the Ephemerides*, p. 244, he had admitted the difficulty in these words: "But the calculation will not be made easier for that. On the contrary, the plainer this form of astronomy is to grasp, the more laborious it is to compute, and the more intricate to discover." After this he implores Magini's help. So it is not undeservedly that Boulliau says of Kepler, Book II, theorem 14: "It is true that that man was most ingenious and an astronomer; but I pity him in this place and grieve at the ἀγεωμετρίαν [Greek: ageōmetria, "lack of geometry"] with which he often struggles."

Methodus Eiuſdem Aequationis ex Bullialdo.

Method of the Same Equation, from Boulliau.

XVII. Boulliau proposes two methods of equating the first inequality in the planets, in Book I of the *Astronomia Philolaica*. One is in chapters 17, 18 and 19; we have already set it out in treating the Sun, Book 3, chapter 23, number 8, and he himself admits at the end of chapter 19 that it is rather long and too troublesome. He therefore substitutes the other in chapter 20, resolving the ellipse into two circles; I indicated this too briefly in Book 3, chapter 23, number 10, and now wish to set it out more exactly. Let A be the Sun and umbilicus of the ellipse to be described or already described, B its centre, C the other umbilicus, and DF the axis or major diameter; and let MN be the conjugate diameter, or minor diameter. Let both of these be given, and so let their difference be DG, or KM, and half of it EG, or KH. Use this half as the semidiameter of the small circle or epicycle DG, OQ, KM, and of similar and equal ones to be described from their centres, taken on the periphery of the circle EHSI, whose semidiameter BH exceeds the conjugate semidiameter BM

by the quantity HM, that is, by half of the difference between the semidiameters of the ellipse. Now let the planet be at D, at the aphelion and at the apogee of the epicycle DG, and let it move at double the motion of the centre E, but in the opposite direction, so that while the centre E travels to T, in the order of the signs, the planet moves from D to V. For in this way it will come about that, when the centre of the epicycle has completed the semiquadrant ET, the planet will have completed a quadrant OP of the epicycle and be at P. When the quadrant ETH has been completed, let the planet be at M, the perigee of the epicycle; when the semicircle ETS has been completed, let the planet be again at F, the apogee of the epicycle, where however it will be perihelial; and when the arc EHSI has been completed, the planet will again be at N, the perigee of the epicycle. With these things laid down, Boulliau demonstrated in that place that the planet describes an ellipse by its motion, or that if a curved line is drawn through the planet, set in any number of positions, for example through the points D, P, M, F, N, D, this line is the elliptical periphery; and consequently that the ellipse has been resolved into two circles, namely EHSI and the epicycle already mentioned, and that the method of calculation has been reduced to the customary solution of triangles.

Diagram. Boulliau's resolution of the ellipse into two circles, set into the left column with the opening lines of text running down its left side. A vertical diameter runs from D at the top to F at the bottom, and a horizontal line crosses it at B from K to L. A large circle is drawn about B, and small circles (epicycles) stand on it at the top, left, bottom and right and at a position in the upper left. Straight lines run from O through T to B, from T to P, and to P from B and from A, and a dotted line runs from P to C.

- D (top of the vertical line, on the rim of the upper epicycle)
- E (centre of the upper epicycle)
- V (right side of the upper epicycle)
- G (on the vertical line, just below the upper epicycle)
- C (on the vertical line, end of the dotted line from P)
- B (intersection of the vertical and the horizontal)
- A (on the vertical line, below B)
- R (on the vertical line, just above the lower epicycle)
- S (centre of the lower epicycle)
- F (bottom of the vertical line, on the rim of the lower epicycle)
- O (rim of the upper-left epicycle)
- P (rim of the upper-left epicycle)
- T (centre of the upper-left epicycle)
- Q (lower part of the upper-left epicycle)
- K (left end of the horizontal, on the rim of the left epicycle)
- H (centre of the left epicycle)
- M (right rim of the left epicycle)
- N (left rim of the right epicycle)
- I (centre of the right epicycle)

• L (right end of the horizontal, on the rim of the right epicycle)

[Margin: 1. Step of the calculation.]

First, then, let the planet be given at P, and let there be from the tables the eccentric anomaly ET, or the angle EBT; for double this will be the angle OTP, which measures the arc OP and whose complement to two right angles will be the angle BTP, in the triangle BTP, in which TP, the semidiameter of the epicycle, is given, and the radius BT. So neither the side BP nor the angle TBP will remain unknown; in this place the angle TBP, subtracted from the angle of anomaly EBT, will leave the angle EBP. But if the centre of the epicycle were below the Diacentros HI, it would have to be added.

[Margin: 2. Step of the calculation.]

Second, in the triangle APB the eccentricity AB is given, as I suppose, and the side BP, found above, and the angle ABP, which is the complement to two right angles of the angle EBP just found. So neither the distance of the planet from the Sun, AP, will remain unknown, nor the angle APB, which is the optical part of the equation. Subtracted here from the angle EBP, this will give the angle EAP, that is, the coequated anomaly; and this, added to the place of the aphelion D, will give the planet's eccentric place, seen from the Sun A. But if you were to draw the line CP from the other umbilicus, the angle BPC would be the physical equation, which can be known because in the triangle BPC the eccentricity BC, equal to AB itself, is given, and the side BP, found above, and the angle CBP, that is, EBP, likewise found above. Note, however, that the compound equation is not correct if you double the optical, as Kepler notes in the *Mars*, chapter 40, pp. 194 and 195. Further, Boulliau regularly calls the equations arising from the greater and lesser circles already mentioned the equations of the equant circles.

De Calculo Geometrico pro Aequatione Orbis, ſeu ſecundæ Inæqualitatis.

On the Geometrical Calculation for the Equation of the Orb, or of the Second Inequality.

XVIII. The annual motion of the Earth produces the second inequality. For this, let the figure be repeated from number 4, and let Saturn be at X and its centric place B, namely on the line drawn from the Sun A through the planet X; but let the Earth be at T, and let the planet's place as it appears to the Earth be V, on the line TXV drawn through the planet from the Earth. Then, according to Kepler in the *Epitome of Copernican Astronomy*, Book 6, part 2, from p. 750, three angles are to be considered.

[Margin: Angle at the Sun, or anomaly of commutation.]

The *first* is the angle at the Sun, which is called the *anomaly of commutation.* It is the arc of the ecliptic counted from the true place of the Sun to the eccentric place of the planet in the ecliptic, following the order of the signs, if it does not exceed a semicircle; otherwise its complement to the circle is taken in its

place. Thus, because in our case the eccentric place of Saturn is B, and the place of the Sun A, seen from the Earth T, is D, the angle at the Sun will be BAD, which becomes known by subtracting from the true place of the Sun the planet's eccentric place reduced to the ecliptic, or the place of the Sun from the planet's eccentric place.

[Margin: Angle at the Earth, or elongation from the Sun.]

The *second* angle is at the Earth and is called the *elongation of the planet from the Sun*. It is the arc of the ecliptic counted from the true place of the Sun to the apparent place of the planet, following the order of the signs, unless it exceeds 180°; otherwise it is its complement to the circle. Thus, because in our case the true place of the planet, as apparent to the Earth T, is V, and that of the Sun D, the elongation from the Sun is the angle DTV.

[Margin: Angle at the planet, or parallax of the orb.]

The *third* angle is the angle at the planet, such as AXT, which is called the *parallax of the great or annual orb*, and it is the difference of the angles of commutation and elongation. This parallax is least in Saturn, moderate in Jupiter, greatest in Mars; and in each of them it is smaller at aphelion, greater at perihelion. And if the motion of the apsides were unequal, the parallax would not be the same in all ages. This parallax, or annual commutation, is the equation of the orb, which Ptolemy calls the equation of the epicycle, the Alphonsines and Magini the equation of the argument, and others simply the equation of the orb.

XIX. In the triangle ATX, then, the angle at the Sun or its complement is given, for when the planet's eccentric place is subtracted from the true place of the Sun, or the latter from the former, the angle TAX becomes known. The distance AT between the Sun and the Earth is supposed given, and the side AX, the distance of the planet from the Sun, has been found, or in Boulliau's figure the side AP. So with the aid of trigonometry you will find the parallactic angle AXT. Add this to the angle TAX and subtract the sum from 180°, for the angle ATX, of the true elongation of the planet from the Sun, will become known. Add this to the true place of the Sun if the planet's eccentric place is greater than the true place of the Sun; subtract it if smaller; and the true place of the planet as seen from the Earth will result.

How the elongation already mentioned is found in the Rudolphine Tables by means of the index, or mystical, number, and how tables of the equations of the orb are constructed, I shall say in chapter 12, number 15.

De Menſuris ad hanc Hypotheſim ſpectantibus.

On the Measures Pertaining to This Hypothesis.

XX. The periods of the motions are given in Kepler's *Epitome of Copernican Astronomy*, Book 6, p. 731, and the other numbers of the motions in the Rudolphine or Philolaic Tables. The measures of the radii, eccentricities and so on are given in part by Kepler in the *Epitome of Copernican Astronomy*, p.

733, and in the tables of equations of the Rudolphine Tables; by Boulliau in the *Astronomia Philolaica*, Book 1, chapter 9, Book 6, chapter 1, Book 7, chapter 2, Book 8, chapter 4; and by Hérigone in volume 5 of the *Cursus Mathematicus*, p. 580. In part we have computed them from what they give, and we have added the semidiameters of the Earth. But you must be warned of several things.

First, Hérigone, taking the intervals and eccentricities from Kepler's *Epitome of Copernican Astronomy*, reports them as though they were referred to the same radius, although Kepler expressed the intervals in parts of which the radius of the great or annual orb is 100000, but the eccentricities in parts of which the radius of each planet's orb is 100000. So Boulliau, Book 1, chapter 9, more prudently derives other eccentricities from Kepler in parts of the radius of the annual orb.

[Margin: Kepler's disagreement with himself.]

Second, Kepler himself in the Rudolphine Tables gives the aphelial distance from the Sun as ♄ 1005147 and ♃ 545074, the perihelial as ♄ 896860, ♃ 494926, and the mean as ♃ 520000, in parts of which the radius of the annual orb is 100000. This does not agree with the measures given in the *Epitome*, nor does the mean distance hold the arithmetical mean. But I have decided that the figures of the *Epitome* should be followed, since they are derived from the parallaxes of the annual orb and agree better with the intervals established by him in the *Mysterium Cosmographicum* and in Book 5 of the *Harmonica*. But for the distances that depend on the intervals of the Sun from the Earth, we have built the last two columns: one that supposes, from the *Ephemerides* and the *Commentaries on Mars*, a greatest distance of the Sun from the Earth of 1800 and a mean distance of 1768 semidiameters of the Earth; the other that supposes, from the *Epitome* and the Rudolphine parallaxes, a greatest distance of the Sun from the Earth of 3469 and a mean of 3381 semidiameters of the Earth, following what was said in Book 3, chapter 7, number 15. Boulliau, however, sets the mean at 1460 and the greatest at 1486.

[Margin: Threefold case of distances from the Earth, as one may choose.]

Third, the greatest and least distances from the Sun are constant in this hypothesis, but the distances from the Earth vary. So if anyone requires the greatest and least distances of the planets from the Earth, with the mean distance of the Sun from the Earth supposed everywhere, let him seek them from the antepenultimate and pre-antepenultimate column of the following table, where we use the radius GQ. But if he wants them with the greatest distance of the Sun from the Earth supposed, that indeed will never happen, because the lines of apsides of the Sun and of these planets, and the places of the apogees, have never concurred since the foundation of the world and never will, according to what was said in chapter 1, scholium 6. Since such concurrences are nevertheless possible in this hypothesis, we have set them in the penultimate and last triads of the table. Finally, if anyone wants the greatest and least distances of the planets from the Earth that belong to any given time, then for the greatest he must look to the planets in conjunction with the Sun, occurring when the planet

is at the apogee of its eccentric, and for the least to their opposition, when the planet is at the perigee of its eccentric. Then, from the place of the Sun standing under the planet's aphelion, he must derive the elongation of the Sun from its own apogee, that is, its anomaly, so as to learn from this the distance of the Sun from the Earth. For if this is added to the planet's aphelial distance, or subtracted from its perihelial, the result will be, in the one case, the apogeal, in the other, the perigeal distance of the planet from the Earth.

[Margin: Rule for the minor semidiameter of the ellipse.]

Fourth, the minor semidiameter of the ellipse, FH, is derived from the major, FX, in the figure set out at numbers 3 and 4, if the eccentricity from one of the foci, that is, AF, is given. For by proposition 49 of Book 1 of Euclid, if you subtract the square of the line AF from the square of the line AH, the square of the line FH remains; and from Apollonius in the *Conics*, AH is equal to FX, that is, to the major semidiameter, or semiaxis, of the ellipse.

MENSVRÆ AD KEPLERI ET BVLLIALDI Hypotheſim ſpectantes.

Measures Pertaining to the Hypothesis of Kepler and Boulliau.

For these lines see the figure set out at numbers 3 and 4	Planets	In parts of which the radius of the planet's orb is 100000: Kepler	In parts of which the radius of the planet's orb is 100000: Boulliau	In parts of which the radius of the annual orb is 100000: Kepler	In parts of which the radius of the annual orb is 100000: Boulliau	Semidiameters of the Earth: Boulliau	Semidiameters of the Earth: Kepler, in the *Ephemerides* and the *Commentaries on* ♂	Semidiameters of the Earth: in the *Epitome of Copernican Astronomy* and the Rudolphine Tables
Semidiameter of the ellipse FX, or mean distance AH, both from the ☉ and from the Earth	♄	100000	100000	9510000	9541980	13931	16795	32156
	♃	100000	100000	519650	522520	7628	9188	17569
	♂	100000	100000	152350	152350	2224	2694	5150
Minor semidiameter FH, or conjugate of the ellipse	♄	99832	99832	9508403	9540377	13907	16767	32102
	♃	99910	99882	519183	521903	7619	9180	17554
	♂	99570	99589	151695	151724	2215	2693	5125
Eccentricity AF of the centre of the ellipse from the Sun	♄	5700	5774	54207	54911	804	938	1805
	♃	4822	4856	25058	25234	370	443	847
	♂	9265	9239	14112	14075	206	250	327

For these lines see the figure set out at numbers 3 and 4	Planets	In parts of which the radius of the planet's orb is 100000: Kepler	In parts of which the radius of the planet's orb is 100000: Boulliau	In parts of which the radius of the annual orb is 100000: Kepler	In parts of which the radius of the annual orb is 100000: Boulliau	Semidiameters of the Earth: Boulliau	Semidiameters of the Earth: Kepler, in the *Ephemerides* and the *Commentaries on* ♂	Semidiameters of the Earth: in the *Epitome of Copernican Astronomy* and the Rudolphine Tables
Radius GP, or GQ, of the annual orb, or the great orb of the Earth	♄	10515	10480	100000	100000	1460	1768	3381
	♃	19244	19138	100000	100000	1460	1768	3381
	♂	65638	65638	100000	100000	1460	1768	3381
Aphelial distance AX, or greatest from the Sun	♄	105700	105774	1005207	1009109	14735	17734	33961
	♃	104822	104856	544708	547754	7998	9631	18416
	♂	109263	109239	166465	166425	2430	2943	5477
Perihelial distance AZ, or least from the Sun	♄	94300	94226	896793	899387	13127	15857	30351
	♃	95178	95144	494592	497286	7258	8745	16722
	♂	90737	90761	138235	138275	2018	2444	4823
Apogeal distance, consisting of AX and GQ, or greatest from the Earth [Margin: At the mean distance of the Earth from the Sun.]	♄	116215	125018	1105207	1109109	16195	19502	37342
	♃	115337	124100	644708	647754	9458	11399	21797
	♂	119778	128507	266465	266425	3890	4711	8858
Perigeal distance, GQ taken from AZ, or least from the Earth [Margin: At the mean distance of the Earth from the Sun.]	♄	83785	83746	796793	799387	11667	14089	26970
	♃	75934	76006	394592	397286	5798	6977	13341
	♂	2599	25123	38235	38275	558	676	1442
Apogeal distance, from AX and AP, if AQ were as great as AP [Margin: At the greatest.]	♄	116414	116441	1107007	1110595	16221	19534	37430

For these lines see the figure set out at numbers 3 and 4	Planets	In parts of which the radius of the planet's orb is 100000: Kepler	In parts of which the radius of the planet's orb is 100000: Boulliau	In parts of which the radius of the annual orb is 100000: Kepler	In parts of which the radius of the annual orb is 100000: Boulliau	Semidiameters of the Earth: Boulliau	Semidiameters of the Earth: Kepler, in the *Ephemerides* and the *Commentaries on* ♂	Semidiameters of the Earth: in the *Epitome of Copernican Astronomy* and the Rudolphine Tables
	♃	124412	124340	646508	649240	9484	11431	21885
	♂	176082	175009	268265	267911	3916	4743	8947
Perigeal distance, AP taken from AZ [Margin: At the greatest.]	♄	83586	83558	794993	797901	11641	14057	26882
	♃	75588	76660	392792	395800	5772	6945	13253
	♂	23918	24091	36435	36789	532	644	1354

If with Boulliau the ellipse is resolved into two circles, the semidiameter of the orb comes out for Boulliau as Saturn 99916, Jupiter 99941, Mars 99795, and the semidiameter of the epicycle, in such parts, as Saturn 84, Jupiter 59, Mars 205: parts, that is, of which the semiaxis of the ellipse is 100000.

[Margin: In these tables K stands for Kepler and B for Boulliau.]

MOTVS Medius ab Æquinoctio ex Kepl.& Bullialdo

Mean Motion from the Equinox, after Kepler and Boulliau: Daily.

DIVRNVS (daily)	Planets	Deg.	′	″	‴	⁗	V	VI
Of the apogee or aphelion	♄ K	0	0	0	12	25	53	1
	♄ B	0	0	0	18	45	14	20
	♃ K	0	0	0	7	45	2	17
	♃ B	0	0	0	14	38	10	44
	♂ K	0	0	0	10	59	50	18
	♂ B	0	0	0	12	56	47	24
Of longitude	♄ K	0	2	0	35	25	21	19
	♄ B	0	2	0	35	13	37	44
	♃ K	0	4	59	15	59	8	42
	♃ B	0	4	59	15	58	32	44
	♂ K	0	31	26	39	11	41	11
	♂ B	0	31	26	39	8	21	5

MOTVS Medius ab Æquinoctio ex Kepl.& Bullialdo

Mean Motion from the Equinox, after Kepler and Boulliau: Annual.

ANNVVS (annual)	Planets	Deg.	′	″	‴	⁗	V	VI
Of the apogee or aphelion	♄ K	0	1	15	36	27	29	4
	♄ B	0	1	54	5	12	10	4
	♃ K	0	0	47	8	58	56	27

ANNVVS (annual)	Planets	Deg.	′	″	‴	⁗	V	VI
	♃ B	0	1	29	2	15	17	59
	♂ K	0	1	6	54	0	59	30
	♂ B	0	1	18	45	28	23	13
Of longitude	♄ K	12	13	35	29	14	39	34
	♄ B	12	13	34	17	54	31	49
	♃ K	30	20	32	14	47	54	29
	♃ B	30	20	32	11	9	7	17
	♂ K	191	17	8	26	5	31	55
	♂ B	191	17	8	5	50	18	35

RADIX Motuum in Meridie Diei 1. Ianuarij Anni 1. CHRISTI Vraniburgi.

Root of the Motions at Noon on 1 January of the Year 1 of Christ, at Uraniborg.

	Planets	Sig.	Deg.	′	″	‴	⁗	V
Of the aphelion	♄ K	7	22	19	36	0	0	0
	♄ B	7	5	15	26	0	0	0
	♃ K	5	15	53	50	0	0	0
	♃ B	4	28	25	26	0	0	0
	♂ K	3	29	14	40	0	0	0
	♂ B	3	23	58	15	0	0	0
Of longitude	♄ K	2	12	36	23	0	0	0
	♄ B	2	13	7	46	0	0	0
	♃ K	5	29	49	33	0	0	0
	♃ B	5	29	54	47	0	0	0
	♂ K	1	10	32	10	0	0	0
	♂ B	1	10	41	52	0	0	0

Ad initium verò Mundi vt cenſet Keplerus, Anno ante CHRISTVM 3993. in meridie diei 24. Iulij Vraniburgi Radix motuum fuit.

At the beginning of the world, however, as Kepler holds, in the year 3993 before Christ, at noon on 24 July, the root of the motions at Uraniborg was:

	Planet	Sign	Deg.	′	″	‴	⁗
Of the aphelion	♄	♑	28	24	6	0	0
	♃	♋	23	34	18	0	0
	♂	♉	15	0	0	0	0
Of longitude	♄	♎	5	29	57	0	0
	♃	♍	7	3	21	0	0
	♂	♋	10	43	52	0	0

PERIODVM ſub Fixis ſtellis, ex Kepleri Epitomes lib.6. part.2. Abſoluunt

They complete the period under the fixed stars, from Kepler's Epitome*, Book 6, part 2:*

	Planet	Egyptian years	Days	Hours	′	″	‴
Period of longitude under the fixed stars	♄	29	174	4	58	25	30
	♃	11	317	14	49	31	56
	♂	1	321	23	31	56	49

Subtract the motion of the apogee from the motion of longitude, and the eccentric anomaly will remain. The motion of the anomaly of commutation is not given in the tables, because it is obtained by subtracting the true motion of the centre from the true motion of the Sun, and each of these is irregular, or unequal.

SCHOLIA.

Scholia.

[Margin: The hypothesis of Kepler and Boulliau: its merits and defects.]

I. The hypothesis of Kepler and Boulliau has many perfections. These are the simplicity of the planetary path, along one single elliptical line; physical causes of the first inequality, from or around points of the orbit that are not idle but notable, of which kind are the centre of an ellipse and its two foci or umbilici; and the relation of the second inequality to the true place of the Sun, and true syzygies with it. And those physical causes were certainly devised with the highest ingenuity: the Sun, namely, swinging the planet round in a circuit by the laws of the lever and the steelyard, in proportion to the distance and to a magnetic disposition in the planet's body, and doing this through a magnetic power diffused in all directions together with light. But against it Ptolemy would object its contempt for the perfect circular path without evident necessity, as is clear from what was said in ch. 5, nos. 10 and 11. Copernicus and Tycho would object the admission of a physical inequality in celestial motion; Tycho, in addition, the motion of the Earth, on which this theory rests; and all of them the difficulty and ageometry in the calculation, and the vain attempt to demonstrate by a geometrical method the equations and places of the planets, as was said in nos. 15 and 16, at the end, even granting that Boulliau brought some remedy to this difficulty by resolving the ellipse into two circles, as we said in no. 17. Finally, they measure the mean motion not from the other focus of the ellipse but from the centre of the ellipse, although they maintain that that umbilicus is equivalent to the point of equality.

[Margin: The hypothesis of David Fabricius.]

II. Nor did the hypothesis of David Fabricius please Kepler, in the *Epitome of Copernican Astronomy*, p. 674, or in the Rudolphine Tables, ch. 20. For Fabricius librated the centre of the eccentric ellipse by means of a small circle along the transverse diameter of the ellipse, or along a line parallel to it, so as to represent the planet's ingress at the sides toward the centre of the orbit; but at the same time he librated the apsides up and down, and dislocated the eccentric. Moreover, he needed a motion compounded of contraries, and he attained

neither pure uniformity of motion, nor precision or ease of calculation, nor, finally, the force of the physical causes moving the planet. He therefore fixed a cross upon ingenuity, and imposed blindness on the eye of reason as it asks for physical causes.

[Margin: Cavalieri's attempt at the equations in elliptical form.]

III. Nor again did Father Bonaventura Cavalieri, in his *Directorium*, ch. 4, p. 149, apply a sufficient remedy to the ageometric equations of this elliptical hypothesis, although he brought some help and came closer to geometrical truth. In this figure, he says, let the line of apsides be AP, and on it let the centre of the semicircle ANP be C, whose eccentricity CS from the Sun S is given, in parts of which the radius CM is 100000. And let the mean anomaly be given, for which take the arc AN, counted from the aphelion A, or the angle ACN. For when NS is drawn, in triangle CNS the angle NCS will be given, being the complement of angle ACN to two right angles, and also the sides containing it, CN and CS. Therefore, by the laws of triangles, both the planet's distance SN from the Sun and the angle CNS will become known, and this angle he affirms to be very nearly the physical equation. To bring it almost to a geometrical polish, he draws CM parallel to SN, joins the point M to S by the straight line MS, and draws the chord MN, which he bisects at V; and he connects the point V with C and S. He says, then, that the space or area MSA is very nearly equal to the sector NCA, and that if the little space SVM is added to it, the space ASV is most nearly equal to the sector NCA, which he demonstrates in the same place. To obtain its quantity he squares the little area NVM, multiplying the second sine of the arc NM that has been found into the first sine, and subtracting the product from what results from multiplying the radius into half the arc MN, reduced to parts of the radius; for in this way the little area NVM remains. Next, as the area of the circle, which in Kepler is 31415926536 in parts of which the radius is 100000, is to the little area found, so he makes 360 to a fourth term, that is, to the arc MV; and thus, he says, is the whole circle to the sector VCM. Then, as the radius is to the difference of the sines of the arcs AM and AV, so he makes the greatest equating triangle, of which we spoke in ch. 5, no. 16, step 3, to the small difference to be subtracted from the arc MV; so that there remains a most thoroughly corrected small equation to be taken from the arc MN, so that the arc NV remains, measuring the physical equation, that is, the triangle CVS; and let AV be the eccentric anomaly, as great as the sector ACV is. He confirms this by an example in Mars.

Diagram. A semicircle set into the right column beside the text of paragraph III, its straight diameter upright at the right with A at the top and P at the bottom, and its arc bulging to the left. C and S stand on the diameter, C near the middle and S below it. On the arc, reading down from A, stand M, V, N and O. Solid lines run from M to C and to S, from N to C and to S, from O to C (level) and from O towards S; the chord joins M and N, and two dotted lines run from the chord near V to C and to S.

- A (top of the diameter): aphelion
- M (upper arc)
- V (on the arc side of the chord MN)
- N (arc, below V)
- O (arc, level with C)
- C (diameter): centre
- S (diameter, below C): Sun
- P (foot of the diameter)

[Margin: The hypothesis of Antonín Maria Šírek.]

IV. Of the hypothesis of Antonín Maria Šírek, devised not for the real cause of the motions but for calculation, by means of a transferred ecliptic, you have a specimen under the Moon, ch. 27, no. 20.

CAPVT VI.

Chapter 6.

De Noſtra Trium Superiorum Hypotheſi Duplici.

On our double hypothesis of the three superior planets.

I. I have already indicated, in Book 4, ch. 27, scholium 9, and ch. 28, no. 3, and in this Book 7, sect. 1, last ch., that the physical cause of the inequality and apparent variation in the motion of the planets is not to be sought from the material necessity of describing a geometrical figure, or of purely mathematical proportions, or of mutual magnetic attraction, but above all from the end and good that GOD destined for sublunary things from the beginning. And just as cithara-singers, singers and dancers keeping time, although by their motions they do eventually trace certain figures in the air, nevertheless do not essentially and primarily geometrize, or have some point fixed beforehand in their minds around which they move tongue, hands or feet regularly or in a uniformly difform manner, but rather follow rhythmic laws and harmonic proportions, so the Intelligences that move the planets act in accordance with a harmony, not of the ears but of the intellect and of Divine Providence, foreknown through images divinely infused. They raise or lower the planets, urge them on or hold them back, advance or draw them back, according to the diversity of those effects that GOD has willed to come about at fixed times in this nature of things; and they do not essentially and primarily strive for the elegance of some figure to be described in the aether. Yet since GOD has willed, for his own glory, to impart to men some knowledge of these motions, in which they should advance more and more as the centuries pass, and has so ordered the planetary motions that men could devise various hypotheses by which the motions could be brought back to fixed laws and then carried over into tables for the common use of calendars and ephemerides, it has therefore seemed good to divide these hypotheses into two kinds.

[Margin: The difference between Angelic and human hypotheses.]

The first kind comprises the real and true hypotheses, which we call Angelic because the Intelligences alone know them perfectly and carry them out; they are entangled in an incredible winding of spirals and in labyrinths of twists almost inextricable to us. The second contains the human hypotheses, which are mere fictions sprung from the fertility of our ingenuity, but which imitate, or at any rate represent without great errors, the revolutions of the celestial motions. So that our own effort should not be missing in this place, although we have found almost all the forms already taken by astronomers, we have nevertheless devised a certain new hypothesis, and a twin one at that: one for the hypothesis of the Earth moved around the Sun, of which we shall speak in the Scholia; the other for the truer hypothesis of the immobile Earth, whose properties I wish to set out in distinct propositions.

Propoſitio 1. Motus Planetarum præcipui per circulos potiùs quàm per alias figuras regulares explicandi ſunt.

Proposition 1. The principal motions of the planets are to be explained by circles rather than by other regular figures.

II. This is urged not only by the authority of Aristotle and of all ancient astronomy, which laid down that this figure suits the heaven and the celestial motions, as being the most apt for the revolutions of motions returning into themselves and for framing their laws and rules, but also by the very appearance of the observations, which have found that these motions take place through circles, or just as if they took place through circles.

[Margin: Rejection of rectilinear figures in the motions of the planets.]

Therefore, if anyone were to try to reduce these motions to rectilinear figures, such as hexagonal, pentagonal and so on, then either he would say that their angles are imperceptible to us because of their excessive distance, and so would fall back on a figure circular as far as the senses go; or, if he made those angles perceptible, he would introduce sudden changes of motion that conflict with the observations. I afterwards found this noted in Kepler, in ch. 16 of the *Commentaries on Mars*, where he says: *For if a planet were to proceed along straight lines forming angles (as if it walked the sides of a five-angled figure, thoughts I once entertained), then, by reason of the lines, there would sometimes be a sudden change of faster motion into slower with an evident difference, and this would happen not at one but at several places of the zodiac, according to the number of the sides.* By this, however, I wish to be understood here either perfect circles, or spirals composed of one circle after another, rather than the ellipses of Kepler or Boulliau. For just as they rightly say that the point which is the centre of the planetary system must, in the hypothesis of the moving Earth, be set in the Sun itself rather than in another point outside the Sun but very close to it, so we, having noticed that the ellipses they posit depart almost imperceptibly from a circle, have considered that there is no reason why we should depart from perfect circles; especially since the eccentricity is not to be bisected, and

since they devised the ellipse to represent this bisection and its causes, or tried to show the bisection of the eccentricity in order to establish the ellipse in these motions, as we shall presently say.

Propoſitio 2. In Hypotheſi Humana motus Regulares Planetarum potiùs quàm Irregulares circa propria centra ponendi videntur.

Proposition 2. In the human hypothesis, regular rather than irregular motions of the planets seem to be what should be posited around their own centres.

[Margin: The choice of regular motion.]

III. For, other things being equal, it is easier to grasp, more amenable to laws, less absurd to the ears of ordinary astronomers, and less exposed to those complaints with which Copernicus and Tycho press the hypothesis of Ptolemy, who admits a physical inequality. Granted that the true and Angelic hypothesis, all things reckoned, may in fact consist of unequally equal motions, and that this inequality is not merely not unfitting but highly congruent with the end of Divine Providence, as I shall show clearly in Book 9. But let this be said chiefly of the motion of the centre of the epicycle, for if the motion of the planet in the epicycle follows the true motion of the Sun, it is certainly unequal.

Propoſitio 3. Abſidum linea in hypotheſi Terræ ſtantis per centrum Terræ ducenda eſt.

Proposition 3. In the hypothesis of a stationary Earth, the line of apsides is to be drawn through the centre of the Earth.

[Margin: Through which point of the world the line of apsides is to be drawn.]

IV. For by the same arguments, or similar ones, by which Kepler and Boulliau taught that in the hypothesis of the Earth going round the Sun the line of apsides is to be drawn through the very centre of the Sun, as was said in ch. 5, no. 5, it can be taught that in the hypothesis of the motionless Earth it is to be drawn through the centre of the Earth, which is the centre of the universe. In addition, the minor planets in their first inequality are free from the Sun's relations and aspects, and they move under the zodiac and the fixed stars, and complete their periods, just as if there were no Sun. Hence the eccentricity of the eccentric orbs, if it is used to set out the anomaly of this motion, must be sought from the centre of the Earth itself rather than from the Sun itself, or from a point around which the equal motion either of the Sun or of the Earth is supposed to take place. See Kepler on Mars, ch. 16, p. 105, where he says he has shown that the planet moves slowest at the apogee of the eccentric if we follow the Ptolemaic hypothesis, but at the aphelion if the Copernican.

Propoſitio 4. Secundæ Inæqualitatis motus ex Vero Solis motu potiùs quàm ex Medio metiendus eſt.

Proposition 4. The motion of the second inequality is to be measured from the true motion of the Sun rather than from the mean.

[Margin: The true motion of the Sun is to be preferred to the mean in estimating the second motion of the planets.]

V. This has been sufficiently shown, both by Kepler and by Boulliau, ch. 5, no. 6, and their arguments hold good in the hypothesis of a stationary Earth as much as of one going round. For that mean motion, and the mean conjunction or opposition with the Sun, are fictions, which neither in fact agree with the true motion except very rarely, nor, when assumed by hypothesis, satisfy the observations. If, on the other hand, the motion of the second inequality and the anomaly of the epicycle, commonly called the anomaly of the orb, is taken by subtracting the coequated first motion from the true motion of the Sun, the equations of the orb are found to be much more exact and closer to the observations. Therefore, just as in the first and slower motion they look to and imitate a stationary Earth, so in the second and swifter motion they look to and imitate a moving Sun.

Propoſitio 5. Eccentricitas horum Planetarum Non eſt neceſſariò Biſecanda, ſed potiùs varianda, ſi non varietur Epicycli ſemidiameter.

Proposition 5. The eccentricity of these planets need not necessarily be bisected, but should rather be varied, if the semidiameter of the epicycle is not varied.

VI. Suppose that the first inequality of the motion of these planets is to be explained by an eccentric. If the whole and always the same eccentricity of the deferent carrying the planet or the epicycle is used to investigate the equations of the centre, these often come out wrong and do not agree with the observations, but around the perigee of the eccentric are greater than is right. Again, if the whole eccentricity is employed in determining the greatest and least distance of the centre of the epicycle from the centre of the universe, and from these distances of the orb or epicycle or second motion the equations are constructed, always using a semidiameter of the epicycle of one and the same perpetual quantity, they are found to disagree with the observations, and, much more evidently, to be greater around the perigee of the eccentric than the observed places of the planets allow.

[Margin: The origin of the bisected eccentricity.]

Hence it came about that Ptolemy, to moderate their excess, did indeed use the whole eccentricity in regulating the first motion, with respect to the centre of the motion made or feigned equal, which is called the centre of the equant; but with respect to the centre of the deferent of the epicycle, the vehicle of the second inequality, he cut the eccentricity in two and allowed only half of

it to the deferent, offering no demonstration, for the three superior planets, of why he divided it in two rather than in three or in some other proportion. Copernicus and Tycho, however, rejecting the equant and the bisection of the eccentricity, used another division of it, yet in such a way as to represent the equivalence of that quantity. Since, moreover, an ellipse has two foci or umbilici equally distant from its centre, and the bisection of the eccentricity greatly favours that equidistance, Kepler and Boulliau, moving the planets on an ellipse, strove with all the zeal and effort we have seen, ch. 4, to urge the bisection of the eccentricity, as may be seen even from ch. 42 alone of the *Commentaries on Mars*. But if we set aside the partiality that comes from love of the ellipse, the bisection of the eccentricity is by no means necessary, although the same eccentricity ought not always to be used, but a lesser one for the equations of the centre around the perigee, and a greater around the apogee of the eccentric.

[Margin: The eccentricity is not to be bisected but varied in another way.]

Indeed Kepler himself, on Mars, ch. 16, from 4 acronychal observations and using a perfect circle, had concluded thus: *Let the whole eccentricity, then, remain of 18564 parts, that of the eccentric 11332, and that of the equant 7232.* Therefore the distance of the centre of the equant from the centre of the eccentric deferent is much less than the distance of the centre of the deferent from the centre of the world. And in ch. 20 he shows that one eccentricity results at the apogee, another at mean longitude, and another at the perigee, especially when acronychal observations are considered, even if he uses this to pave his way to the bisection. Let these words of his be noted: *Hence it is certainly concluded that there is no certain and fixed point in the planet's eccentric around which the planet perpetually completes equal angles in equal times.* And earlier too Copernicus had detected the inequality of this distance in Mars, Book 5, ch. 16, even if he attributed the cause to a variation of the solar eccentricity brought on by Mars. And Johannes Phocylides Holwarda, in his examination of Lansbergen's astronomy, shows the same thing from the acronychal observations of Saturn: namely, that the distance of the centre of the deferent from the centre of the world is less around the perigee of the eccentric than around the apogee.

We, however, after long and much weighing of the equations of the centre needed for the place of these planets, observed near the true conjunctions with the Sun and at the true oppositions themselves (in which they are stripped of the second inequality), to agree with the calculation of the tables, have not only recognized this variation of the eccentricity, though it is less than a bisection, but also that the anomaly of the eccentricity follows the anomaly of the second sines corresponding to the mean anomaly of the eccentric, as we shall explain below, through a libration up and down. After I had devised the hypothesis, I found this libration indicated but rejected by Kepler, ch. 20 of *Mars*, in these words about the centre of the eccentric: *For it would altogether (if indeed we were to keep the other of the assumptions about the circular orbit of the star) have to be librated by us on the line of apsides up and down: and how this can be reconciled with*

natural reasons I do not see. It appears, therefore, that if a circular figure is chosen, a variation of the eccentricity and a libration of the centre of the eccentric up and down often satisfy the heavens; and they would have satisfied Kepler himself, if he had judged that the physical cause of unequal motion was to be sought from the end of these motions rather than from a magnetic attraction of the planets by the Sun and from the necessity of a geometrical figure. But if the semidiameter of the epicycle is varied, the necessity of varying the eccentricity is not yet evident.

Propoſitio 6. Epicycli Semidiameter in Hypotheſi Ptolemæi reformata, Varianda eſt, præſertim ſi Eccentricitas non varietur.

Proposition 6. In Ptolemy's reformed hypothesis the semidiameter of the epicycle is to be varied, especially if the eccentricity is not varied.

VII. I have quite often wondered why Ptolemy, in using the whole of one and the same eccentricity, on finding a conflict between the equations of the eccentric on the one hand and of the epicycle on the other, took nothing from the epicycle and laid the whole cause on the eccentric and its eccentricity. Yet astronomical equity seemed to demand that this contest and conflict be settled more or less evenly, and that one portion of that variation be assigned to the eccentric, the instrument of the first inequality, and the other to the epicycle, the instrument of the second: that is, that the semidiameter of the epicycle too be varied.

[Margin: The occasion on which the anomaly in the semidiameter of the epicycle was detected.]

Indeed, even in my very apprenticeship in astronomy, when I heard and repeatedly read that the greatest digressions of Venus and Mercury from the Sun are not perpetually of the same quantity, I used to picture this variation to myself in diagrams by a line like a spiral, along which the planet would describe around the centre of the epicycle now a looser circle, now a tighter one. Later, having examined the greatest equations of the orb or epicycle, at the apogee of the eccentric on the one side and at its perigee on the other, which astronomers lay down, and having used there the greatest, here the least distance of the centre of the epicycle from the centre of the world, by the analysis of a right triangle, of which I shall speak in the penultimate chapter of this section, I recognized that a different semidiameter of the epicycle results in the Ptolemaic form: evidently greater at the apogee of the eccentric, and less at the perigee. Before I had read of Kepler's suspecting this, therefore, I had resolved within myself to introduce an anomaly of this kind into the theory of the five minor planets.

[Margin: Kepler's suspicion of this anomaly.]

[Margin: And Tycho's opinion.]

I was afterwards confirmed by Kepler in the *Commentaries on Mars*, ch. 22, which is entitled: *The epicycle or annual orb does not move equally around the point of equality*; and by the words of Tycho that he reports in the same place from Tycho's letter sent in the year 1598 to Kepler in Styria, where he wrote thus: *The annual orb according to Copernicus, or the epicycle according to Ptolemy, does not seem to be always of the same magnitude with respect to the eccentric itself, when the comparison is made; but it brings in an alteration, perceptible in all three superior planets, so much so that the angle of difference in Mars grows to 1° 45′.* Tycho himself indicates this in the appendix to the *Mechanica*, and Kepler concludes it from some observations: namely, that the Ptolemaic epicycle, whose role in the Copernican hypothesis is performed by the single annual orb of the Earth, is not of the same measure in the same planet, but wider around the apogee and narrower around the perigee. Yet he attributes this to other causes. Nor did I grieve that what I had thought was first devised by me had occurred to the minds of others; rather, glad of the agreement out of love of the truth, I began more keenly, from observations especially of Mars, of which there are a great many in Kepler, to investigate the measure of this anomaly, until it appeared to satisfy most observations and the more select ones. Accept this hypothesis, then, and use it securely for as long as the evidence of other most certain and corrected observations has not proved that its measures must be emended, though with the substance of the figure retained. But consult what I shall say in Scholium 5.

Proponitur Nouæ Hypotheſis Trium Superiorum Prima forma.

The first form of the new hypothesis of the three superior planets is set out.

[Margin: The theory of the first inequality.]

VIII. In the attached figure let the line of apsides be DAC, on which A is the centre of the Earth and of the world; and let I be the centre of the small circle BZ, which contains the laws of the anomaly of the eccentricity. On its diameter BZ let the centre of the eccentric be librated up and down, according to the increase and decrease of the second sines corresponding to the degrees of the mean anomaly of the eccentric; on this condition, that when the centre of the epicycle is at C, the apogee of the eccentric, the centre of the eccentric is then at the apogee of the small circle already mentioned, that is, at B, and the eccentricity is then greatest, AB. When the centre of the epicycle is a quadrant distant from the apogee, the centre of the eccentric is at I, the centre of the small circle, and the eccentricity is then mean, namely AI. Finally, when the centre of the epicycle is at D, the perigee of the eccentric, the centre of the eccentric is at Z, the perigee of the small circle, and it then has the least eccentricity, AZ. Therefore, for the prosthaphaereses of the first inequality, or for the equations of the centre, one eccentricity after another will be needed.

IX. For the other inequality, however, let there be described on the periphery of the eccentric (which here, as a single one, or in a single position

[Margin: Theory of the second inequality.]

to avoid confusion we have described from centre I) other epicycles, larger and smaller. At the apogee of the eccentric the largest is GE, whose semidiameter is CG or CE, and at the perigee the smallest is KL. To conceive the whole variation of the epicycles, however, describe both at the apogee the smallest epicycle HN and at the perigee the largest PF, so that the whole difference of the epicyclic semidiameter is GH or PL, and the variation of the spiral keeps within these limits. It is carried out on this law: when the centre of the epicycle is at C, the apogee of the eccentric, the planet describes the largest epicycle, whether the planet is then at G, the apogee of the epicycle, or elsewhere, as at E; but when the centre of the epicycle has been carried through O to the perigee D, in the order of the signs, it describes the smallest. And in describing it, it is likewise carried in the order of the signs on the upper part, that is, from G to E, by a motion as great as the elongation of the epicycle's centre from the true conjunction with the Sun. Further, the decrease of the epicycle's semidiameter is to imitate the decrease of the eccentricity, and the increase of the former the increase of the latter, to this extent: that it grows or shrinks according to the growth or diminution of the second sines corresponding to the mean anomaly of the eccentric, although elsewhere I may have judged that the coequated anomaly should rather be used. Lastly, at every true conjunction with the Sun, let the planet be at the true apogee of the epicycle then described, or temporary epicycle, and at every true opposition at its perigee. For example, if the centre of the epicycle is at O and the Sun's annual orb is VS, and the apparent conjunction of the planet with the Sun takes place with the Sun placed at V, the planet is to be at R; but if the opposition takes place with the Sun placed at S, the planet is to be at T. The line drawn from the centre of the Earth through the centre of the epicycle marks the epicycle's true perigee T and its apogee R, and it is from this apogee that the anomaly of the orb is to be counted.

Diagram. Standing in the left column, interrupting the sentence at "describe". A large eccentric, on which stand four small epicycles and, at its top and foot, a pair of concentric circles each. A vertical line runs from the top of the upper pair down through the small circle at the middle and through the world's centre to the foot of the lower pair. Around the point A is drawn a broken (dotted) circle, the Sun's annual orb. A small circle with centre I stands above A on the vertical line. Straight lines join the points on the small circle and A to the epicycle centres, and a dotted line descends from E towards the small circle.

- Top pair, on the vertical line: G (top of the outer circle), H (top of the inner circle), C (centre, the eccentric's apogee), N (foot of the inner circle), with E on the outer circle at its left
- Upper left epicycle: R (top), O (centre), Q (left), T (lower right)

- Upper right epicycle: δ (upper right), Y (centre), M (lower left)
- Small circle on the vertical line: B (top), κ (below B), I (centre), ω (below I), Z (foot)
- Broken circle: V (upper left), S (lower right); A at its centre
- Lower left epicycle: X (upper left), θ (centre), λ (lower left)
- Lower right epicycle: γ (centre), ε (right, printed like a reversed 3), β (below)
- Bottom pair, on the vertical line: P (top of the outer circle), L (top of the inner circle), D (centre, the eccentric's perigee), F (foot of the outer circle), with K on the inner circle at its left

To determine the greatest prosthaphaereses of the orb, lines tangent to the epicycle are to be drawn from A, the centre of the world; for when the planet is placed at the points of contact already mentioned, the epicycle's greatest equation for that position on the eccentric occurs. Hence the greatest at the apogee of the eccentric is the angle CAE, and at the perigee DAK.

It is no wonder, then, if Mars, which quickly completes the variation of its epicycles, describes few and rather loose spirals, and undergoes that evident difference which many astronomers have until now begged from an uncertain variation and distance of the Sun's eccentricity; whereas Saturn describes very many epicycles, most closely akin to one another, which produce a sensible difference only with time.

Methodus Calculi Geometrici in hac forma.

The method of geometrical calculation in this form.

[Margin: ist step, for the true place of the epicycle's centre.]

X. Let the centre of the eccentric be at κ, the centre of the epicycle at O, and the planet at Q. *First*, then, find the eccentricity Aκ, as follows. When the motion of the eccentric's apogee has been subtracted from the planet's mean motion, which is given in the tables, there remains the anomaly of the eccentric, or the arc CO, which the angle CκO measures. For this anomaly, then, look up the second sine in the tables of sines, multiply it by BI, the semidiameter of the small circle, which we shall shortly give in tables, and divide the total by the whole sine. Add the quotient to the mean eccentricity AI, likewise to be set out shortly in tables, and the result will be the eccentricity Aκ. But if the anomaly of the eccentric were in the fourth quadrant, as if the centre of the epicycle were at Y, the second sine of the arc CY, that is, of the complement of the anomaly to the circle, would have to be used, and the rest carried out as above. If, however, the anomaly were in the second or third quadrant of the eccentric, as if the centre of the epicycle were at θ or at γ, then in the former case the complement of the anomaly to the semicircle is to be used, in the latter its excess over the semicircle; the second sine of these is to be multiplied by the semidiameter IZ, the total divided by the whole sine, and the quotient

subtracted from the mean eccentricity AI, so that the eccentricity Aω becomes known.

[Margin: 2nd step.]

Second, when the eccentric's radius κO has been drawn, and from the world's centre the line AOR, of the true apogee, through the epicycle's centre, the triangle AκO arises. In it the radius κO is given, in the parts we have been accustomed to use, 100000, and in the same parts the eccentricity Aκ has been found; and the angle contained by these, AκO, is known, for it is the complement of the mean anomaly to the semicircle if that anomaly does not exceed 180°, or to the circle if it exceeds 270°, or it is the excess over the semicircle if the anomaly exceeds 180° but is less than 270°. These three being given, find by the doctrine of triangles the angle AOκ, which measures the equation of the centre, to be taken away from the planet's mean motion if the centre of the epicycle is in the prior semicircle of anomaly COD, so that the true place of the epicycle's centre, seen from the Earth along the zodiac by way of the line AO, is obtained. Thus, if the centre were at Y, the equation would be AYκ; if the centre were at θ, the equation would be Aθω; if the centre were at γ, the equation would be the angle Aγω; and similarly for the rest. Using the occasion of these same data, look for the distance AO of the epicycle's centre from the centre of the world, and keep it.

[Margin: 3rd step, for the equation of the orb.]

Third, when the epicycle RQT has been described from O, its semidiameter OQ is to be found. So take the second sine, mentioned above, of the mean anomaly of the eccentric when it does not exceed a quadrant, multiply it by the difference, set out by us in the tables, between the mean and greatest epicyclic semidiameters, and divide the total by the whole sine. Add the quotient to the mean semidiameter of the epicycle given in the tables, and the semidiameter of the epicycle suited to that position of its centre on the eccentric will be known. The same is to be done if the anomaly of the eccentric is greater than 270°, but the second sine of the complement of the anomaly to the circle must then be used. But if the anomaly of the eccentric is greater than a quadrant, use the second sine either of the complement to the semicircle or of the excess over the semicircle, multiplying a sine of this kind by the difference of the semidiameters and dividing the total by the whole sine; and in both these cases subtract the quotient from the mean semidiameter, so as to obtain the epicycle's semidiameter sought.

[Margin: 4th step.]

Fourth, compile from the corrected tables of the Sun the Sun's true place, and from it (adding a whole circle if that is needed for the subtraction) subtract the true place of the planet's centre, already known by the 2nd step; for the remainder will be the anomaly of the epicycle, which astronomers now commonly call the anomaly of the orb, the Alphonsines the true argument, and the Copernicans the anomaly of commutation. You will count this anomaly on the epicycle, in the order of the signs, from the true apogee of the epicycle, and

in this way the point of the epicycle at which the planet stands will become known. For example, if this anomaly is 50°, it will be the arc RQ, counted from the apogee R, and the planet will be at Q; and to it from A, the centre of the Earth, draw the line of the planet's true place, in our case the line AQ.

Fifth and last, in the triangle AOQ you have now found, by the

[Margin: 5th step.]

second step, the side AO, and by the third the side OQ; and by the fourth the angle AOQ is known, as being the complement of the anomaly of the orb, which the angle QOR measures. So with the help of trigonometry find the angle OAQ, namely the equation of the orb, to be added to the planet's eccentric place, or true place of the centre, if the anomaly of the orb is in the first semicircle of the epicycle, as if the planet is at Q or at M; but to be taken away if it is in the other, as if it is at X or at β. And thus the planet's true place along the zodiac, in its own orbit, seen from the Earth, will be known. It is afterwards to be reduced to the ecliptic by means of the latitude, as will be said in Section 4.

EXEMPLVM in ♂.

Example, in Mars.

[Margin: Example from Tycho's observation of Mars.]

XI. Kepler relates, in his *Mars*, ch. 53, that Tycho observed Mars at Uraniborg at ♋ 17° 40′ 30″, with north latitude 4° 7′, in the year of the Lord 1582 December 26, in the style of the old calendar, at 8^{h} 30^{m} after noon. For that moment the motions of the Sun and of Mars, from our tables, stood as follows.

	Value
True place of ☉	9^{s} 14° 59′ 57″
Mean motion of ♂	3^{s} 8° 31′ 30″
Motion of the apogee of ♂	4^{s} 28° 39′ 43″
Therefore, anomaly of the eccentric	10^{s} 9° 51′ 47″

[Margin: 1st step, example.]

First, then, in the preceding figure the anomaly was the arc CODY, of 10^{s} 9° 51′ 47″, and so the arc CY, as also the angle CκY, was 50° 8′ 13″; the centre of the epicycle was at Y and that of the eccentric at κ. Further, the second sine of 50° 8′ 13″ is 64095. Multiplied by the semidiameter BI, which for me is 1365 of the parts of which the radius κY is 100000, it gives the total 87,489,675. When this is divided by the whole sine 100000, there remains the difference κI, of 875 parts, to be added to the mean eccentricity of Mars, which for me is 18635; and so the eccentricity Aκ at this position comes out at 19510 parts.

[Margin: 2nd step, example.]

Second, when AYδ and κY have been drawn, there are given in the triangle AκY the eccentricity Aκ, the radius κY, and the angle AκY of 129° 51′ 47″, as being the complement of the angle CκY. Therefore, as the geometry of

triangles establishes, the equation of the centre comes out, that is, the angle AYκ, of 7° 34′ 55″. Added at this position to the mean motion of Mars, it makes the true eccentric place, or the place of the centre, 3ˢ 16° 6′ 25″. And in the same triangle the distance AY is found, of 115720 parts, which is to be kept aside for the time being.

[Margin: 3rd step, example.]

Third, the mean semidiameter of the epicycle of Mars is for me 66057 parts, and the greatest 70957, so that their difference is 4900. By this, then, the second sine of the anomaly of the eccentric mentioned above, that is 64095, is multiplied, giving the total 314,065,500. When this is divided by the whole sine there remains the difference 3141, to be added at this position to the mean semidiameter; and so the epicycle's semidiameter MY becomes 69198 parts, of which the eccentric's radius is 100000.

[Margin: 4th step, example.]

Fourth, since the true place of the Sun was 9ˢ 14° 59′ 57″, and the true place of the centre of Mars was found to be 3ˢ 16° 6′ 25″, when the latter is subtracted from the former there remains the anomaly of the orb, 5ˢ 28° 53′ 32″, namely the arc δM, in the prior semicircle of the epicycle. So the angle MYδ is of that size, and its complement to two right angles, by proposition 32 of the first book of Euclid, is the angle AYM, 1° 6′ 28″; and Mars was at the point M.

Fifth, when the line AM has been drawn the triangle AYM arises, in which are now given the distance AY, by the second step, of 115720 parts, MY of 69198 parts, and the angle they contain, AYM, of 1° 6′ 28″. Trigonometry therefore gives the angle MAY as 1° 38′ 52″, which is of course the equation of the orb, to be added at this position to the eccentric place of Mars, namely 3ˢ 16° 6′ 25″.

[Margin: True place of Mars.]

And so at length the true place of Mars becomes 3ˢ 17° 45′ 17″, or ♋ 17° 45′ 17″. But Tycho found it at 3ˢ 17° 40′ 30″. The calculation therefore exceeds the observation by 4′ 47″, an excess that can be referred no more to our hypothesis than to the observation.

[Margin: The difference between us, Kepler and Magini.]

Kepler, however, in that chapter 53, computed from his hypotheses the place of Mars at ♋ 17° 44′ 19″. But Magini, examining his equations of the orb in the *Supplement to the Ephemerides*, p. 17, gathered from this observation ♋ 17° 40′ 48″. This place is, moreover, a slippery one because of the smallness of the angles MAY and MYA; so if there is an error of some minutes in the Sun's place, and hence in the anomaly of the orb, a notable error will also result in the place of Mars. By a similar method we examined other places of Mars observed by Tycho and related by Kepler in chs. 42 and 53, at various positions on the eccentric, so as to test the reliability of our hypothesis; and in some places, I admit, it does not satisfy them exactly. But it must be remembered that Tycho, in determining the places of the planets from distances of the fixed stars, used a theory of the Sun that assumes excessive solar parallaxes; and that

in the distances of fixed stars taken by him and by us a discrepancy of one or more minutes is sometimes found, which can alter the places of the planets. We therefore await with great longing the edition of Tycho's observations, so that we may examine them more deeply, and bring the measure of our hypotheses, from those observations and from our own, which we are still pursuing, to the final polish. Meanwhile, take the measures we have obtained.

[Margin: Second form of our hypothesis.]

XII. The other hypothesis is that the same eccentricity is always retained, namely the one we have called mean, so that the hypothesis is simpler; nor, without evident necessity, does it seem that this labour should be imposed on astronomers, that to work out the equations of the centre they must first seek one eccentricity after another. And the observations that favour this supposition are not few. In that case, however, the anomaly of the epicycle's semidiameter will perhaps have to be set up differently, so as to satisfy all or nearly all observations. On this matter we cannot pass a definite judgement until we have obtained more trustworthy observations at more positions on the eccentric.

XIII. There now follows the table of our measures and numbers. As to the numbers, we have found the Danish tables accurate enough for the mean motion of Saturn and Jupiter, and the Rudolphine for the mean motion of Mars; the rest we have built up from our own investigations, by a method partly to be indicated and partly to be set out more fully hereafter. We also lay down, roughly with Copernicus and with Kepler in the *Epitome of Copernican Astronomy*, that the mean distance of Saturn from the Earth is tenfold the Sun's mean distance from the Earth, that of Jupiter fivefold, and that of Mars about one and a half times that same distance of the Sun. What the Sun's mean distance is for us appears from Book 3, ch. 7.

MENSVRÆ NOSTRÆ

Our measures.

In ſemidiametris Terræ & Partibus Radij.

In semidiameters of the Earth and parts of the radius.

For these letters see the figure set out at no. 9	Plan-ets	Of which the radius of the planet's orb is 100000	Of which the Earth's semidiameter is one
Semidiameter of the small circle of the anomaly of the eccentricity, or BI	♄	700	511
	♃	430	157
	♂	1365	150
Diameter of the small circle already mentioned, or BZ	♄	1400	1022
	♃	860	314
	♂	2730	300
Least eccentricity from the centre of the world, or AZ	♄	10800	7884
	♃	9100	3321
	♂	17270	1900

For these letters see the figure set out at no. 9	Planets	Of which the radius of the planet's orb is 100000	Of which the Earth's semidiameter is one
Mean eccentricity, or AI	♄	11500	8395
	♃	9530	3478
	♂	18635	2050
Greatest eccentricity, or AB	♄	12200	8906
	♃	9960	3635
	♂	20000	2200
Least semidiameter of the epicycle, or DL	♄	10100	7373
	♃	18460	6738
	♂	61157	6727
Mean semidiameter of the epicycle	♄	10700	7811
	♃	19390	7077
	♂	66057	7266
Greatest semidiameter of the epicycle, or CG	♄	11300	8249
	♃	20320	7417
	♂	70957	7805
Difference between the mean and greatest semidiameters of the epicycle	♄	600	438
	♃	930	339
	♂	4900	539

Reſiduum Tabulæ præcedentis.

The remainder of the preceding table.

For these lines see the figure set out at no. 9	Planets	Of which the radius of the planet's orb is 100000	Of which the Earth's semidiameter is one
Difference between the greatest and least semidiameters of the epicycle	♄	1200	876
	♃	1860	679
	♂	9800	1078
Mean distance, of both the centre and the planet, from the centre of the Earth	♄	100000	73000
	♃	100000	36500
	♂	100000	11000
Longer longitude of the epicycle's centre, AC	♄	112200	81906
	♃	109960	40135
	♂	120000	13200
Nearer longitude of the epicycle's centre, AD	♄	89200	65116
	♃	90900	33179
	♂	82730	9100
Greatest distance of the planet from the centre of the Earth and of the world, AG	♄	123500	90155
	♃	130280	47552
	♂	190957	21005
Least distance of the planet from the centre of the Earth, AL	♄	79100	57743

For these lines see the figure set out at no. 9	Plan-ets	Of which the radius of the planet's orb is 100000	Of which the Earth's semidiameter is one
	♃	72440	26441
	♂	21573	2373

MOTVS MEDIVS ſeu ÆQVALIS.
Mean or equal motion.

		Deg.	′	″	‴	⁗	V	VI
Of longitude								
Daily	♄	0	2	0	35	25	21	19
	♃	0	4	59	15	59	8	42
	♂	0	31	26	39	11	41	11
Annual	♄	12	13	35	29	14	39	34
	♃	30	20	32	14	47	54	30
	♂	191	17	8	26	5	31	55
Of the eccentric's apogee								
Daily	♄	0	0	0	16	26	23	0
	♃	0	0	0	14	50	0	0
	♂	0	0	0	11	30	0	0
Annual	♄	0	1	40	0	0	0	0
	♃	0	1	30	0	0	0	0
	♂	0	1	10	0	0	0	0

RADIX Ad Annum 1. CHRISTI ineuntem in vltimo meridie Decembris Præcedentis, ſub Meridiano Bononienſi.
Root for the beginning of the year 1 of Christ, at the last noon of the preceding December, on the meridian of Bologna.

Motion	Planet	Value
Of longitude	♄	$2^s\ 12°\ 33'\ 50''$
	♃	$5^s\ 29°\ 45'\ 0''$
	♂	$1^s\ 10°\ 0'\ 50''$
Of the apogee	♄	$7^s\ 11°\ 33'\ 23''$
	♃	$4^s\ 27°\ 30'\ 2''$
	♂	$3^s\ 27°\ 43'\ 22''$

Of the greatest equations I shall speak more suitably in ch. 12; of the motion of the north node, in Section 3.

SCHOLIA.

Scholia.

[Margin: On the geometry of our epicycle.]

I. I know, of course, that our hypothesis does not have geometrical exactness [acribiam] in the measurement of the epicycle's semidiameter, since it cuts that

continuous spiral into several circles as if they were distinct. But I preferred to serve the ease of a calculation that comes as close as possible to the truth rather than to multiply small circles or lines by some other, more intricate scheme. However, that difference among the epicycles could be represented by a small circle which would either roll the epicycle round on its circumference or librate it up and down on its diameter, according to the increases of the second sines; and it would then have to be weighed whether the sines of the coequated anomaly of the eccentric would be more useful than those of the mean anomaly.

[Margin: Another form of our hypothesis.]

One could also imitate the webs of spiders, which, by a kind of leaping passage from a smaller to a larger circle, sometimes weave their webs from several circles, or from chords of several circles. But I have judged that we should abstain from these and similar windings. What should rather be considered is whether some other anomaly of the semidiameter can be found which alone, without the anomaly of the eccentricity, would satisfy very many observations, all of them. For that, however, we need more observations than we have, and I have therefore judged that this business should be deferred to volume 3.

[Margin: The hypothesis of two concentrics, without any epicycle.]

II. Among the hypotheses that can be devised, one seems to have been omitted which consists of two concentrics without any epicycles. But this one cannot represent the apparent stations and retrogressions of the planets unless, in place of the epicycles, it substitutes the annual orb of an Earth going round the Sun; and for this reason I have passed it over. Still, I should like to set out what occurred to me in this hypothesis of a moved Earth. In the following figure let the line of apsides be BC, on which let S be the Sun at the centre of the world, and let the mean eccentricity of the planet's orb be AS, and half of it AO. From A let two circles concentric with each other be described: one deferent of the planet's body, DLEM, the other drawn through the outermost surface of the planet's body, BHCN, standing so far from the deferent that BD equals AO, half the eccentricity.

[Margin: Note on the size of the planet's body.]

With centre D let the small circle BG be described, and with centre E the small circle CF, containing the whole substance of the planet, which consists of the nucleus, that denser and more opaque part which we see, and of the remaining thinner part, which we perhaps experience in its effects but do not perceive with the eyes. For just as flame and the whole air have a twofold portion, one denser, fatter and visible, the other rarer and more transparent, which we do not see with the eyes, so it does not seem incongruous with the vastness of the heavens and the power of the planets that they should occupy with the substance of their body a far larger sphere than the one that appears to our eyes. On these assumptions, the physical cause of the whole eccentricity from the Sun would be the diameter of the planet's body BG, as large as AS. Unless someone should likewise prefer to imagine, in the substance of the solar

globe, which is much greater than what appears to us as a nucleus, three regions or markedly differing portions: one farthest from the centre, on whose surface would be the centre of Saturn's eccentric; another nearest the centre of the Sun, which would turn the centre of Mars's eccentric; and one midway between these, which would turn Jupiter's. And so the Sun's power, communicated to these planets along straight lines, would be weaker in Saturn, more vigorous in Mars and more temperate in Jupiter, and this would be the cause of the eccentricity's being greater in Saturn than in Jupiter, and in Jupiter than in Mars. But whatever may be the case with these fictions, let us proceed to the measures and the form of the calculation.

Diagram. Standing in the right column, interrupting the word *Eccentricitas* at the line end. Two large concentric circles. On the vertical diameter stand two small circles, each hatched with rays round a dark central nucleus, one at the top between the two large circles and running inwards, one at the foot likewise. Straight lines cross the figure among the labelled points on the circles and meet about the centre and below it; a horizontal line runs across the inner circle through O.

- Vertical line, top to foot: B (top of the outer circle and of the upper small circle), D (centre of the upper small circle), G (foot of the upper small circle), A, O, S, F (top of the lower small circle), E (centre of the lower small circle), C (foot of the lower small circle and of the outer circle)
- Upper left: H (on the outer circle), I (on the inner circle)
- Upper right: N (on the outer circle), K (on the inner circle)
- Interior: R (left), T (right)
- Horizontal line: L (left, on the inner circle), O, M (right, on the inner circle)
- Lower left: P
- Lower right: Q
- Lower right of S: V, at the end of a short line from S

[Margin: The method of calculation in]

III. Let AB be, then, the radius of 100000 parts, by which the remaining lines are to be measured; but the distance of the centre of the pla- [Margin: this form, for the equations of the centre.]

net AD is to be the remainder of the radius AB once BD, that is, half the eccentricity, has been taken away. The given anomaly of the eccentric, then, is to be counted in the inner orb from the apogee D, in the order of the signs, towards I. Thus, if the centre of the planet has reached I, the arc will be DI, measuring the angle DAI, whose complement to two right angles is the angle SAI. With this angle, the eccentricity AS and the line AI, which is the remainder of the radius shortened by half the eccentricity, the angle AIS is to be found. This is the equation of the centre, to be subtracted in the earlier semicircle of the anomaly and added in the later, so as to give the planet's centric place as seen from the Sun. Moreover, the whole eccentricity that is to be used and bisected here is not the one that other hypotheses call the whole, but the one we call

the mean: for instance, in Mars, 18635 of the parts of which AB is 100000. And so the line AI, like AP, AQ and AK, will always be 9317 parts shorter than the radius, that is, of 90683 parts, so that the equations of the centre may come out more correct; though would that they answered the observations everywhere in this form!

[Margin: Method for the equations of the orb.]

IV. For the equations of the second inequality, let us place the planet at I, and the Earth at V at the planet's conjunction with the Sun, and at R at its opposition. For then the same line IRSV will be the line of the centric place and of the planet's place, and indeed of the place of the Sun or its opposite, as seen from the Earth, and no equation of the orb will be needed. Now let the planet be outside the true ☌ and ☍ with the Sun, say at K, and the Earth at T, which is to be joined to the Sun and the planet by the straight lines ST and TK. For in the triangle KST, the planet's centric place subtracted from the Sun's true place will give the anomaly of the orb, from which you will learn the angle at the Sun, S, and at the Earth, T, exactly as we showed in the hypothesis of Copernicus and Kepler. But through the triangle ASK, by which you found the equation of the centre, or the angle AKS, you must work out from the same data the planet's distance from the Sun, that is, the side KS, and take from the solar tables the Earth's distance from the Sun, that is, the side ST. For from these two sides and the angle S or T you will be able to find the angle TKS, namely the equation of the orb, to be added to the planet's eccentric place in the earlier semicircle of the anomaly of the orb and subtracted in the later, so as to give the planet's apparent place as seen from the Earth; here, as seen along the line TK.

The whole difficulty of this hypothesis lies in the measure of the eccentricity AS, which must satisfy all the equations of the centre and, by its half, establish the correct semidiameter of the inner circle.

CAPVT VII.

Chapter 7.

De Mutua Transformatione Hypotheſium Præcedentium, traductis vniuſcuiuſq. menſuris ad aliam quamuis formam Hypotheſeos. Vbi Epilogus Hypotheſium, & Hypotheſis Electiua mediocritatis inſinuatur.

On the Mutual Transformation of the Preceding Hypotheses, the measures of each being carried over to any other form of hypothesis. In which a recapitulation of the hypotheses and an Elective Hypothesis of the mean are introduced.

I. Two things above all are considered in a hypothesis of the planets. The first is the form itself, or figure, by which their motions are represented; this is, as it were, the substance and essential part of the hypothesis. The other is

the size of the orbs, the eccentricities and so on, which is an accident, but a principal one. Now it may be that one hypothesis is better, or better suited to someone's cast of mind, as regards form, but is at fault in its measures, and that a student of astronomy wishes to investigate whether the measures of another hypothesis, when used in that form, satisfy the celestial phenomena. To satisfy him, therefore, we present in the following table the measures of each of the rather more celebrated astronomers, to be carried over to the form of each. For this purpose it must be recalled that

[Margin: The Ptolemaic hypothesis.]

Ptolemy gave the whole eccentricity to the circle around whose centre the planet's equal motion is imagined, the circle called the equant, but gave half of it to the deferent that carries the centre of the epicycle bearing the planet's body.

[Margin: The hypothesis of Copernicus.]

Copernicus, however, correcting Ptolemy's whole eccentricity somewhat from his own observations and dividing it into four parts, gave three of them, or three-quarters [dodrantem], to the eccentricity of the deferent of the epicycle, and the remaining part to the semidiameter of the epicycle, in his first form.

[Margin: And that of Lansbergen.]

In the second form, which is also Lansbergen's, he gave three-quarters of the whole eccentricity to the mean eccentricity, and the remaining quarter to the semidiameter of the small circle that carries the centre of the eccentric and produces the anomaly of the eccentricity, so that the minimum eccentricity was half the maximum, except in Mars.

[Margin: The hypothesis of Tycho and Longomontanus.]

Lastly, in the third form, which Tycho and Longomontanus followed, though with corrected measures, he assigned three-quarters of the whole eccentricity to the semidiameter of the larger epicycle, and the remaining quarter to the semidiameter of the smaller epicycle.

[Margin: The hypotheses of Kepler and Boulliau.]

Kepler and Boulliau, however, assign half of the whole corrected eccentricity to the centre of the ellipse that carries the planet on its periphery, and they place the centre of the world in the motionless Sun, which they locate at one of the foci, or umbilici, of the ellipse, and from which they measure half the eccentricity; the other focus they imagine to be the point about which the planet moves equally. Hence, from the properties of the ellipse demonstrated by Apollonius, the square of the distance of the foci from the centre, or of the semi-eccentricity, taken from the square of the radius, or major semidiameter, leaves a square whose root is the minor semidiameter of the ellipse. Furthermore Boulliau, for ease of calculation, resolves the ellipse into two circles and, bisecting the difference of the major and minor semidiameters of the ellipse, on the one hand gives half of it to the semidiameter of the epicycle carrying the planet, and on the other adds it to the minor semidiameter of the ellipse so as to make the semidiameter of the larger circle. The centre of this circle is the

same as the centre of the ellipse, and stands off by half the eccentricity on one side from the Sun, the centre of the world, and on the other from the point of equal motion.

[Margin: Which is the centre of the world and of the planetary system?]

II. It must further be recalled that the centre of the universe and at the same time of the planetary system, from which, that is, as from a limit, the eccentricities of the orbs are taken or measured, is for Ptolemy and for us the centre of the Earth. For Copernicus and Lansbergen, however, it is that point which stands as far from the centre of the solar body as the eccentricity of the Ptolemaic solar eccentric stands from the Earth, although they do not recoil from the Sun itself; indeed this agrees better with their hypothesis. Therefore for Kepler and Boulliau that centre is the centre of the very body of the Sun. But with Tycho and Longomontanus the centre of the world is indeed the centre of the Earth, but the Sun, or a point near the Sun, is the centre of the system of the five minor planets. In addition, all the old astronomy, as well as Lansbergen, and Tycho and Longomontanus in their first exercises, thought that the planet is stripped of its second inequality at mean conjunctions and oppositions with the Sun: that is, when the planet's true place was under that place of the zodiac, or under its opposite, under which, or under whose opposite, lay the place of the Sun, not the true place but the one marked by the Sun's mean motion. But the more recent astronomers, namely Tycho and Longomontanus in their later studies, Kepler and Boulliau, and we along with them, have judged that the planet is free of the second inequality at the moments of true ☌ or ☍ with the Sun. And so, to obtain the anomaly of the orb, the planet's eccentric or first-coequated place must be subtracted from the true motion of the Sun, and the planet's second motion must be reckoned not from the line of the mean but from that of the true solar motion, or from the true line of the syzygies. Finally, in the hypothesis of a moved Earth the annual orb takes the place of the Ptolemaic epicycles.

III. But since someone might wish to choose out of all the measures the one that is the mean between the extremes, and to use it in any form of the hypotheses, as the safer course, with the lesser error, we have added the last two columns: in the second-to-last are the measures absolutely mean between the old and the recent, and in the last the means among the most recent, from Tycho inclusive. This hypothesis may indeed be called the Elective Hypothesis of the mean [Electiua mediocritatis].

IV. Further, so that the measures of my newest hypothesis may be transferred to the forms of others, not our whole but our mean eccentricity, which comes closer to the whole eccentricity of the others, is to be bisected or cut into four, so that three-quarters of it and a quarter may be given to whatever each form requires. Likewise the mean semidiameter of the Ptolemaic epicycle between our extremes is to be used. The conversion of other measures to our form, however, is somewhat more laborious. For to obtain the difference of the epicycles, one must, from the maximum equation of the orb set down in each

author's tables, both at the apogee and at the perigee of the eccentric, find by analysis of triangles the maximum and minimum semidiameter of the epicycle. The anomaly of the eccentricity, for its part, is deduced only with difficulty from the eccentricities of others. Yet if anyone wants it close to the truth, let him take of the whole eccentricity found among the others the 17th part for ♄, the 24th for ♃ and the 14th for ♂, and assign it to the semidiameter of the small circle of the anomaly of the eccentricity.

V. However, so that the measures set out below may be applied more readily to each form, it seemed best to use capital letters, as characteristic marks of the hypotheses; the rules for them will be given after the table, together with their use.

SYNOPSIS MENSVRARVM & ÆQVIPOLLENTIÆ HYPOTHESIVM

Synopsis of the Measures and Equivalence of the Hypotheses.

In partibus, qualium Radius Orbis Planetæ eſt 100000.

In parts, of which the radius of the planet's orb is 100000.

Letters of the forms	Planet	Ptolemy	Copernicus	Lansbergen	Longomontanus	Kepler	Bouliau	Ourselves	Elective, absolute	Elective, of the more recent
A	♄	11388	11390	11400	11628	11400	11548	11500	11508	11514
A	♃	9167	9150	9160	9540	9644	9712	9530	9431	9621
A	♂	20000	19530	19400	18550	18530	18478	18635	19239	18556
B	♄	5694	5695	5700	5814	5700	5774	5750	5754	5757
B	♃	4584	4574	4580	4770	4822	4856	4765	4715	4810
B	♂	10000	9665	9700	9775	9265	9239	9318	9619	9278
C	♄	10000	10912	10070	10426	10515	10480	10700	10456	10563
C	♃	19167	19068	18520	19349	19244	19138	19390	18955	19264
C	♂	65833	65772	65860	65497	65638	65638	66057	65777	65777
D	♄	8541	8543	8550	8721	8550	8661	8625	8631	8635
D	♃	6874	6862	6870	7155	7233	7284	7147	7072	7215
D	♂	15000	14530	14550	13912	3897	13858	13977	14428	13917
E	♄	2847	2848	2850	2907	2850	2887	2875	2877	2878
E	♃	2292	2288	2290	2385	2411	2428	2382	2357	2405
E	♂	5000	5000	4850	4637	4632	4619	4659	4809	4639
F	♄	99838	99837	99836	99830	99836	99832	99835	99835	99834
F	♃	99896	99898	99897	99884	99910	99882	99898	99888	99909
F	♂	99499	99523	99528	99522	99570	99589	99500	99545	99522
G	♄	81	81½	82	85	82	84	82½	82½	83
G	♃	52	51	51½	58	45	59	50½	56	46
G	♂	250½	238½	236	239	215	205	250	227	239
K	♄	99919	99918	99918	99915	99918	99916	99917	99917	99917
K	♃	99948	99949	99948	99942	99955	99941	99938	99944	99955
K	♂	99750	99862	99764	99761	99785	99794	99750	99772	99761
L	♄	669	669	671	684	671	679	700	701	701
L	♃	382	382	382	397	402	405	430	393	401

Letters of the forms	Planet	Ptolemy	Coper-nicus	Lans-ber-gen	Lon-gomon-tanus	Ke-pler	Boul-liau	Our-selves	Elective, absolute	Elective, of the more recent
L	♂	1429	1395	1386	1324	1321	1323	1365	1374	1335

The mean semidiameter of Mars's epicycle for Longomontanus is 65497, but with the Sun and Mars at apogee 67962, and at the perigee of both 63028. For us, however, the mean semidiameter of Mars's epicycle is 66057, the maximum 70957 and the minimum 61157.

Regulæ Pro Vſu præcedentis Tabulæ.

Rules for the Use of the Preceding Table.

[Margin: Rule 1, for the hypothesis of Ptolemy.]

VI. The hypothesis of PTOLEMY. For the eccentricity of the equant, consult letter A; for the eccentricity of the deferent, letter B; and for the semidiameter of the epicycle, letter C. But measure the eccentricities from the centre of the Earth.

[Margin: Rule 2, for forms 1, 2 and 3 of Copernicus.]

The hypothesis of COPERNICUS, in the first form. For the eccentricity of the deferent see letter D, and for the semidiameter of the epicycle letter E. In the second form, for the maximum eccentricity letter A; for the minimum eccentricity letter B; for the mean eccentricity letter D; and for the semidiameter of the small circle that varies the eccentricity letter E. Lastly, in the third form, for the semidiameter of the larger epicycle see letter D, and for the semidiameter of the smaller epicycle letter E. But in the first two forms measure the eccentricities from the Sun, or from the point about which the Earth is imagined to move with its annual motion around the motionless Sun; in the third, let the centre of the concentric be the Sun or the point already mentioned.

[Margin: Rule 3, for the hypothesis of Lansbergen.]

The hypothesis of LANSBERGEN. See the second form of the Copernican hypothesis, and measure the eccentricities from the Sun, etc.

[Margin: Rule 4, for the hypothesis of Tycho and Longomontanus.]

The hypothesis of TYCHO and LONGOMONTANUS. See the third form of the Copernican hypothesis, and let the centre of the concentric be the Sun, but let the centre of the universe be the centre of the Earth.

[Margin: Rule 5, for the hypothesis of Kepler and Boulliau.]

The hypothesis of KEPLER and BOULLIAU. For the whole eccentricity, or the distance of the foci, or umbilici, of the ellipse from each other, see letter A; for the distance of the centre of the ellipse from either focus, letter B; for the major semidiameter of the ellipse, the radius 100000; and for the minor semidiameter of the ellipse, letter F. But at one of the foci let the Sun stand, as the motionless centre of the universe, and measure the eccentricities from it.

[Margin: Rule 6, for the second hypothesis of Boulliau.]

The later hypothesis of BOULLIAU. For the whole eccentricity, or the distance of the point of equality from the centre of the universe, that is, from the Sun, see letter A; for the eccentricity of the centre of the ellipse from either umbilicus, letter B; for the minor semidiameter of the ellipse, letter F; for the semidiameter of the epicycle (if you resolve the ellipse into two circles), letter G; and for the semidiameter of the larger circle, letter K.

OUR hypothesis. For the mean eccentricity consult letter A; for the semidiameter of the small circle of the anomaly of the eccentricity, letter L; for the mean semidiameter of the epicycle, letter C. For the difference of the semidiameter of the epicycle, consult the foot of the table or the advice given at number 4, taking from the table of chapter 12 the maximum equations of the orb at the apogee of the eccentric, and you will have the angle opposite the semidiameter sought. To the radius add half the eccentricity found among the others, in the row of letter B, and you will have the base. Then add together the logarithm of the angle already mentioned and the logarithm of the base, and the sum will be the logarithm of the semidiameter of the epicycle placed at the apogee of the eccentric. Again, from the last table of chapter 12 take the maximum equation of the orb at the perigee, and you will have the opposite angle; from the radius, however, subtract half of the eccentricity, and you will have the base. Therefore add the logarithm of the angle to the logarithm of the base, and the result will be the logarithm of the semidiameter of the epicycle at the perigee of the eccentric. In this way the difference of the semidiameters will be known, half of which comes into use in our hypothesis.

Vſus Præcedentis Tabulæ Exemplis declaratus.

The Use of the Preceding Table Explained by Examples.

[Margin: Example 1, of the Ptolemaic scheme in the measures of Kepler.]

VII. Suppose someone wishes to derive the true motions of Saturn by geometrical calculation from the Ptolemaic scheme, according to the method given in chapter 1, number 10, yet does not wish to use Ptolemaic measures, but Keplerian ones, that is, such as Kepler, from his own observations or from considerations of Tycho's observations, would supply to a Ptolemy wishing to represent the motions of the planets by two eccentrics with an epicycle. Then from Rule 1, at number 6, he needs the measures indicated by the letters A, B and C. So, proceeding rightward from letter A as far as the column headed with the name of Kepler, he will find the whole eccentricity of the equant answering to ♄ at 11400 parts, of which the radius or semidiameter of Saturn's orb is 100000. From letter B, likewise under Kepler, he will find the eccentricity of the deferent of ♄, 5700 parts. And finally from letter C he will find under Kepler the semidiameter of the epicycle of ♄, 10515 parts, of which the radius of Saturn's orb is 100000. With these measures, then, according to Kepler's thinking but in the Ptolemaic scheme, the calculation for ♄ is to be completed,

but taking the mean motion of longitude and of the apogee, the root of both, and the Sun's distances from Kepler or from the Rudolphine Tables.

[Margin: Example 2, of the Keplerian scheme in the measures of Ptolemy.]

VIII. Conversely, let there be someone who would like to undertake the geometrical calculation of the motion of ♃ in the scheme of Kepler or Boulliau, that is, by the ellipse, but prefers, for the sake of experiment, to use Ptolemaic measures. Now, from what was said in chapter 5, from number 17 on, where this calculation is treated, and from Rule 5 of this chapter, he needs the measures indicated by the letters A, B and F. So, from A, under Ptolemy's column, he will find the whole eccentricity, or the distance of the foci in the ellipse of ♃, 9167 parts; from B, likewise under Ptolemy, the bisected eccentricity, or the distance of the centre of the ellipse of ♃ from one of the foci, 4584 parts; and from letter F, under Ptolemy, the minor semidiameter of the ellipse of ♃, 99896 parts, of which the major semidiameter, or radius of the elliptical orbit, is 100000. Let him therefore use these measures, with the mean motions, roots and solar distances taken from Ptolemy, and carry out the calculation in the form of the Keplerian hypothesis.

[Margin: Example 3, of the Lansbergian scheme in the measures of Longomontanus.]

IX. Now suppose someone likes the second form of Copernicus, or the scheme of the Lansbergian hypothesis, but wants, in tracking down the place of ♂, to use the numbers and measures of Longomontanus. Now, from what was said in chapter 4, number 3, and from Rule 3 of this chapter, he needs the letters A, B, D, E: that is, the maximum eccentricity of ♂, which from letter A he will find under Longomontanus as 18550; the minimum eccentricity, which from letter B he will find under Longomontanus as 9775 parts; the mean eccentricity, which from letter D he will see to be 8721 under Longomontanus; and the semidiameter of the small circle, which from letter E he will get under Longomontanus as 2907 parts, of which the radius of the orb of ♂ is 100000. Therefore, with these measures and the other numbers taken from the *Astronomia Danica* and transferred to Lansbergen's scheme, let him seek the place of ♂.

[Margin: Example 4, of the Tychonic scheme in the measures of Lansbergen.]

X. Conversely, suppose someone wishes, in order to investigate the place of ♄, to use form 3 of Copernicus, which is also that of Tycho and Longomontanus, but prefers to use the measures that Lansbergen would prescribe for this scheme from his own store. Since, from what was said in chapter 3, number 7, and from Rule 4 of this chapter, he needs the measures indicated by the letters D and E, that is, the semidiameters of the two epicycles, he will therefore at once see, from letter D, under the name of Lansbergen, the semidiameter of the larger epicycle of ♄, 8550 parts, and from letter E, likewise under Lansbergen, the semidiameter of the smaller epicycle of ♄, 2850 parts, of which the radius of

Saturn's orb is 100000. So, taking the remaining numbers from Lansbergen's tables, let him complete the calculation of the motion of ♄.

[Margin: Example 5, of Boulliau's scheme in the measures of Copernicus.]

Lastly, there will be no lack of someone who is wonderfully pleased with that form of the ellipse resolved into two circles, which we gave from Boulliau in chapter 5, number 17, but who is pleased to use Copernicus's measures and to seek in them the place of ♃. Now, from what was said there and from Rule 6 of this chapter, he needs the whole eccentricity, or the distance of the umbilici, which from letter A, under the name of Copernicus, he will see to be 9150 parts for ♃; next the eccentricity of the centre of the resolved ellipse, which from letter B under Copernicus he will have for ♃ as 4574 parts; third, he needs the semidiameter of the epicycle, which from letter G he will find under Copernicus as 51 small parts [particularum]; and finally the semidiameter of the larger circle, which from letter K he will find answering to ♃ as 99949 parts, of which the major semidiameter of the ellipse is always to be 100000. So, taking the remaining numbers from Copernicus or from the Prutenic Tables, let him carry out the calculation in Boulliau's scheme. And so for the rest.

CAPVT VIII.

Chapter 8.

De Methodo inueſtigandi Primam Inæqualitatem trium Superiorum Planetarum; videlicet Motum Medium, Eccentricitatem, & Locum Apogei vel Aphelij.

On the Method of Investigating the First Inequality of the Three Superior Planets; namely the Mean Motion, the Eccentricity, and the Place of the Apogee or Aphelion.

[Margin: The difficulty of this method, and its cause.]

I. This method is much more laborious in these three planets than in the others. In the Sun, which undergoes a single inequality, the inquiry is made by a simple eccentric or epicycle. In the Moon, the manifest variation of the parallaxes, and hence of the distances, reveals the eccentricity to us at once; its mean motion is accurately deduced from eclipses, as that of the Sun is from the comparison of equinoxes. Lastly, in Mercury and Venus, the fixed digression from the Sun and the concomitance by which they move with the Sun offer an easier way of reaching the eccentricity and the other elements of the first inequality. But these three superior planets are so entangled in a double inequality of motion that they are not released from the second, which is tied to the syzygies of the Sun, except at conjunctions and oppositions with the Sun, whether mean according to the old astronomy or true according to the recent. The moments of the conjunctions, however, cannot be observed because of the nearness of the Sun, which with its brightness hides the planets from their heliacal setting to their heliacal rising. And the moments of the oppositions themselves have not yet, so far as I know, been learned by immediate

observation, or with the certainty with which full Moons are learned from the beginning and end of eclipses of the ☾. For it would be necessary that, at the very moment at which the apparent place of these planets is observed with instruments in their acronychal shinings, the opposition with the Sun should occur, that is, that the Sun should then be at the opposite place of the ecliptic. But all the observations that we have so far obtained among others, or have made ourselves, have fallen outside the moments of opposition, and were taken some hours or minutes before or after the opposition with the Sun. Consequently, in order to reduce them to the moments of the observations, it was necessary to know beforehand, in some fashion, the mean motion and the anomaly, or distance of the planet from the apogee, and so the place of the apogee, and from these and the eccentricity to gather the prosthaphaereses of the eccentric place, or the equations of the first inequality; yet these are the very things we are seeking. So it seems that one falls into a vicious circle and supposes as known what is being investigated as unknown; or at least that these things cannot be known except from the drawbacks that would follow if what is so supposed stood very differently; or that, by trying which quantity of mean motion and of eccentricity, and which place of the apogee supposed in the calculation, better satisfies the observed places of the planets, one comes nearer to the truth through smaller and smaller differences of the equations, but, as in the thirst of Tantalus, never grasps it.

[Margin: Johannes Regiomontanus's confession on the difficulty of this method.]

II. Hence that old complaint, or candid confession, of astronomers, especially of Regiomontanus in Book 10 of the *Epitome of the Almagest*, proposition 13, where he says: *But there is no way of knowing these small arcs unless the place of the aux of the eccentric is had: the one indeed depends on the other. It will, however, be easier and more certain, since there is no power of proceeding by a straight and exact path, to find these small arcs from the place of the aux known by estimate, than with these small arcs taken by estimate, to seek the place of the aux, and to test whether the rest agree with experiments.*

[Margin: And of Copernicus.]

Copernicus himself felt nothing different about the ancients, and experienced nothing different. For in Book 5, chapter 5, he said: *Those men, then, deprived of the means of demonstration, strove a posteriori and by roundabout ways towards those things to which no access lay open directly and a priori.* And in chapter 6: *Although this does not suffice, yet, brought to close quarters, we reach the truth more easily.*

[Margin: Kepler's confession on the same matter.]

Kepler stuck on the same sandbanks in the *Commentaries on Mars*, chapter 16, when, anxious about this method, he says: *The solution is not geometrical.* And soon after: *There will arise keen geometers like Vieta* [François Viète]*, who will think it a great thing to demonstrate this method's* [Greek: ἀτεχνίαν (atechnian, "artlessness, lack of art")]*; for that is what was charged against Ptolemy, Copernicus and Regiomontanus in this business by Vieta. Let them go, then, and solve the figure*

geometrically themselves, and they will be great Apollos to me. For my part it is enough, for building up four or five conclusions from one argument, in which four observations and two hypotheses are contained, that is, for retracing the way out of the labyrinth, to have shown, in place of geometrical light, an [Greek: ἄτεχνον (atechnon, "artless")] *thread (by which you may nevertheless be guided to the exit). If the method is difficult to grasp, the investigation without a method is much more difficult.*

[Margin: Boulliau's censure of Kepler.]

And these, surely, are the words Boulliau alluded to when, in Book 1 of the *Astronomia Philolaica*, chapter 22, having rejected the method of Ptolemy and Copernicus, he added: *Kepler, in his Commentaries on the star of Mars, what great things he labours at? What divinity does he not invoke? He awaits some god* [Greek: ἐκ μηχανῆς (ek mēchanēs, "from the machine")] *to determine an indeterminable problem; and at last, after long windings of calculation, and with every by-way surveyed, he fixed the places of the aphelia and perihelia.* As if Boulliau himself supposed nothing in this treatise, and did not try, by a long and often repeated calculation, using the arithmetic mean between various excesses and defects, to come as near as possible to a truer place of the aphelion and a more correct eccentricity.

Astronomers must therefore be forgiven, and the circle is not to be reckoned useless, if you can first know beforehand in a coarser fashion, on the one hand the difference of motion, from the smallness of the equations of the second inequality required in order that the places of the planet observed near oppositions with the Sun may answer to the calculation; and on the other the mean motion, from some ancient opposition together with a recent one, separated from each other by a long interval, in much the same way as we sought the Sun's mean motion from two equinoxes in Book 3, chapter 17; and then to inquire into these same things more exactly, with a greater approach to the truth day by day.

So it was not unskilfully that Ptolemy, in Book 9, chapter 3, and Copernicus, in Book 5, chapter 1, before the more subtle examination of the apogee, the eccentricity and the mean motion, set out first the tables of the mean motion of the five minor planets; and then tried to polish those three things: Ptolemy for ♂ in Book 10, chapters 8, 9 and 10, for ♃ in Book 11, chapters 1, 2, 3, and for ♄ in Book 11, chapters 5, 6 and 7; Regiomontanus in the same places in the *Epitome*; Copernicus in Book 5, chapters 6, 7, 11, 12, 16 and 17; and the others soon to be adduced.

Primus Modus inueſtigandi Apogeum trium Planetarum Superiorum, & Eccentricitatem inquirendi aut examinandi ex Copernico.

First Way of investigating the apogee of the three superior planets, and of inquiring into or examining the eccentricity, from Copernicus.

III. Ptolemy, in Books 9, 10 and 11 of the *Almagest*, and Regiomontanus, in the *Epitome* of those books, lacked a means of demonstrating the eccentricity and the apogee a priori, both for the causes indicated in numbers 1 and 2, and because, in using mean oppositions with the Sun, they were using unknown angles as though they were known. Therefore, as Copernicus observes in Book 5, chapter 5: *They strove a posteriori and by roundabout ways towards those things to which no access lay open directly and a priori. Thus Ptolemy, in carrying these things out in lengthy discourse, spread himself over a huge multitude of numbers, which I think it tiresome and superfluous to review, especially since in what follows in our own work too we shall imitate almost the same method.* So, passing over that farrago of Ptolemaic figures, which attain the goal no more happily, we shall begin from the Copernican method, from which one will nevertheless be able to judge sufficiently of the Ptolemaic.

Copernicus, then, in Book 5, gave the warning in chapter 1 that the motions of the planets are to be numbered not from the equinoctial points but from the first star of Aries, and that in measuring the revolutions of the planets one should use not equinoctial or tropical solar years but sidereal ones, which he thought more uniform and constant than those; he set out first the mean motions of the 5 minor planets; in chapter 2 he condemned the Ptolemaic equant and the physical inequality Ptolemy admitted in these planets; and in chapter 4 he substituted three hypotheses that preserve equality of motion, but that, as to the measure of the orbs and the difference of the distances, are equivalent to the bisection of the eccentricity, because he had made the semidiameter of his epicycle a third part of the remaining eccentricity of the eccentric. These things, I say, being supposed, in chapters 5, 6, 10, 11, 15 and 16 he examines three acronychal oppositions of each of the three superior planets with the Sun, once observed by Ptolemy, and another three observed by himself. He assumes the whole eccentricity of these planets as found by Ptolemy and, dividing it into four equal parts, assigns three of them to the eccentricity of his eccentric and the fourth to the semidiameter of the epicycle. He then investigates, from the difference of the mean motions from the true, both the place of the apogee and whether the presupposed eccentricity agrees with the equations, that is, yields the equations necessary for the mean motions, when reduced to the true, to agree with those observed. And so he professes that he is examining and testing a posteriori an eccentricity already known, not hunting down an unknown one a priori. In Mars, however, he found that this measure must be varied to some extent, as we have already said in chapter 5, number 13.

Diagram. Woodcut set in the right column, with the text running down its right side. A large eccentric circle centred at D, with the line of apsides drawn vertically from F at the top through D and E to G at the bottom. Three small epicycle circles stand on the circumference, centred at A (upper right), B (upper left) and C (lower left). Straight lines join A, B and C to the centres D and E; further lines run from E out to the points N, O and P on the epicycles, and short radii join A to N, B to O and C to P.

- F (top of the vertical line): apogee
- G (foot of the vertical line): opposite end of the line of apsides
- D (on the vertical line, above E): centre of the eccentric
- E (on the vertical line, below D): centre of the great orb
- A (upper right): centre of the epicycle; N and K on or beside that epicycle
- B (upper left): centre of the epicycle; O and L on or beside that epicycle
- C (lower left): centre of the epicycle; P and M on or beside that epicycle

Let there be, then, three centres of the epicycle, called the centric places of the planet, observed at the three mean oppositions with the Sun, A, B, C, on the circumference of the eccentric described from centre D; and let the line of apsides be FDG, in which the centre of the great orb of the Earth is E; and let those points A, B, C be connected by straight lines with the centres D and E, so that the lines of mean motion are AD, BD and CD, and the lines of the true centric place are AE, BE and CE. Then let the true place of the planet on the epicycle be, in the first observation the point N, in the second the point O, in the third the point P, which are connected with the centre E by the straight lines NE, OE, PE. And from the centre of the epicycle to the planet let the semidiameters AN, BO and CP be drawn, which is a third part of the eccentricity DE. With these things laid down, you will carry out the business through the following steps.

[Margin: 1st step.]

First, in triangle ADE, the eccentricity DE is given from Ptolemy, in parts of which AD is 100000, as we set out in chapter 1, number 11, in the table; and also the angle ADE, as being the complement to two right angles of the angle FDA, which is known from the mean anomaly of the eccentric. So by the aid of trigonometry you will find both the side AE and the angles DEA and DAE.

[Margin: 2nd step.]

Second, in triangle AEN: since the arc NK, that is, the motion of the planet on the epicycle, is as great as the arc AF, the motion of the centre of the epicycle from the apogee of the eccentric F, and so the angle NAK is known from the mean anomaly of the eccentric, if you add to it the angle found, DAE, the whole angle NAE will be known in that same triangle, in which the side AE has been found and AN is given, a third part of DE. So the angle AEN will not lie hidden. When it is subtracted from the angle AED, found earlier, the angle NED, or NEF, will become clear, by which the apogee F, but under the fixed stars, is distant from the true place of the planet observed at N. Therefore, when the true place of the planet is known and the angle FEN is added to it, it will be known whether the place of the apogee observed under the fixed stars answers to the place of the apogee assumed.

[Margin: 3rd step.]

Third, in the second acronychal observation and the triangle BDE, the sides ED and DB are given, as above, and the angle BDE, the complement of the anomaly of the eccentric, or of the angle FDB. Seek, then, the side BE and the angles DBE and BED.

[Margin: 4th step.]

Fourth, in triangle BEO, since the arc LO, that is, the angle

LBO, is as great as the arc FB of the mean anomaly, if you add to it the angle DBE, already found, the whole angle OBE will be known in triangle OBE, in which you have already found the side BE, and BO is given, a third part of the eccentricity DE. So the angle BEO will not lie hidden; when it is taken from the angle BED, the angle OEF will be manifest, by which the apogee F is distant, under the fixed stars, from the place of the planet observed at O.

[Margin: 5th step.]

Fifth, in the third acronychal observation and in triangle CDE, DC and DE are given as above; and the angle CDE is known, because it is the complement to a semicircle of the angle FDC, known from the mean anomaly of the eccentric. So by the solution of triangles you will easily find the side CE and the angles DCE and CED.

[Margin: 6th step.]

Sixth, in triangle PCE, the arc MP, that is, the angle PCM, is as great as the arc FBC of the mean anomaly of the eccentric. So when the angle found, DCE, is added, the angle PCE will become known in triangle PCE, in which the side CE was found, and PC is given, a third part of DE. Accordingly the angle CEP will become manifest; when it is subtracted from the angle CED, what remains will be the angle PED, by which the apogee F is distant from the true place of the planet P.

[Margin: Conclusion of the practice.]

When this is done, combine the angles FEN and FEO into one angle NEO. For if it is equal to the difference between the true places of the planet observed in the first and second acronychal observations, the measure of the eccentricity and the semidiameter of the epicycle assumed are correct. This will be confirmed if the angle OEP too is equal to the difference between the true places of the planet observed in the second and third acronychal observations. Copernicus, in Book 5, chapters 5, 10 and 15, affirms, indeed teaches by geometrical calculation, that in the three acronychal observations of Saturn, Jupiter and Mars made by Ptolemy, from the hypothesis already mentioned, as assumed, the already mentioned angles come out equal to the difference of the true places observed by Ptolemy. So if those observations are to be stood by, the measure of the Ptolemaic eccentricity and the place of the apogee he found are correct. Yet you may see in Copernicus the numbers for Saturn very corrupt, whether by a fault of the autograph or by a fault of the printer. For he puts the whole Ptolemaic eccentricity of Saturn at 1016 parts, of which the radius of the orb is 10000, and says that a quarter of it is 285, and three quarters now 854, now 864, which are false. In Mars, however, he thinks the cause of the discrepancy arose from a variation in the Sun's eccentricity.

But the same Copernicus, in Book 5, chapters 6, 11 and 16, having found that the numbers and measures of the Ptolemaic hypothesis did not answer to the places of these three planets observed by himself from the year of Christ 1513 to

1539, was forced to correct somewhat the Ptolemaic eccentricities and the places of the apogees. And so that no one should attribute this discrepancy to the form of the Copernican hypothesis, in which the motion of the centre of the epicycle is equal around the centre of the deferent, he first examined his three observations of each planet in the form of the Ptolemaic equant. Noticing that not even in that form did the measures and apogees answer to the observations, he corrected both as far as the agreement of the observations required. Then, distributing the whole eccentricity of the Ptolemaic equant into four parts, he gave three of them to the eccentricity of his deferent and the fourth to the semidiameter of the epicycle, so as to transform it into his own hypothesis. But the way in which he hunted down the eccentricity and the apogee in the Ptolemaic form is longer and more obscure, though similar to the second way, to be given below from Hérigone; therefore we have passed it over and will substitute Hérigone's way for it.

[Margin: Imperfection of this method, from Copernicus's own confession.]

Copernicus also confesses, in Book 5, chapter 5, that in this method the angle NEO, of the true places of the planet, is not measured by the arc of the mean motions BA, known from the time between the two observations A and B; just as the measure of the angle PEO is not the arc of mean motion BC, because they are not described from the same centre. He confesses too that the arc AB subtends an unknown angle AEB, just as BC subtends an unknown BEC; and that the differences of the angles AEN, BEO and CEP cannot be known unless the arcs AF, FB and FAC are established through the mean anomaly, which supposes a known mean motion from the mean apogee of the eccentric, and hence a place of the apogee known approximately; and that these things thus contain a mutual dependence. Therefore one must take refuge in demonstrations a posteriori, and explore by feigned suppositions whether through them the true places observed in the planets can be represented, as we said above in number 1. Yet a great part of this imperfection arose from these astronomers' having chosen the mean oppositions of the planets; for if they had chosen the true ones, the planet would have been for them at the points K, L and M, and they would have come nearer to the truth.

Secundus Modus Eccentricitatis & Apogei inueſtigandi ex Herigonio.

Second Way of investigating the eccentricity and the apogee, from Hérigone.

[Margin: Hérigone, volume 5 of the *Cursus Mathematicus*, in the *Theorics*, Book 1, chapter 3, proposition 12, and Book 3, proposition 27.]

IV. In the following diagram let there be three places of the same planet, observed at the moment of true opposition with the Sun: let them be, I say, A, B and C, on a circle whose centre of the equal motion made from A to B is E. And from the conjecture I spoke of in number 3, let the place of the apogee H be approximately known, from which through E let the line of apsides HEF be

drawn, in which let the eccentricity being sought be ED; and let the point, or centre of the world, from which the centric place of the planet, or in Ptolemaic terms the centre of the epicycle, is viewed, be D. And let the three observed places be connected with each centre, E and D, by the straight lines AE and AD, then BE and BD, then CE and CD; and with AD produced as far as G, let the point G be connected with the points B and C by the straight lines GB and GC; and finally let the three chords AB, AC and BC be drawn. Next, from the interval of time between the observations, let the mean motions be known, and hence their differences, namely the arc AB, which measures the angle AEB, and the arc BC, which measures the angle BEC. And from the true places of the planet under the zodiac (understand it here as described from D), let the difference of the arcs of the zodiac between those three places be known, and hence the angle ADB, known from the difference of the true places under which the planet was when at A and when at B; and the angle BDC, known from the difference of the true places under which the planet was when it stood at B and at C. From these givens [Greek: δεδόμενοις (dedomenois, "things given")], then, by the solution of five triangles, you will obtain a more exact place of the apogee and the eccentricity desired.

Diagram. Woodcut set in the right column, with the text running down its right side. A circle with A at the upper left, H at the top, B at the upper right, C at the right, G at the foot, F at the foot just left of G, and M at the lower left, all on the circumference. A vertical line runs from H through E and D to F. E stands on it above D, with K just to the right of D. Solid lines join A, B and C with E and with D, and the chords AB, AC and BC are drawn; a line runs from C through D to M. Dashed lines run down to G from E, from B and from C.

- H (top): place of the apogee
- A, B, C (circumference): the three observed places
- E (on the vertical line, upper): centre of equal motion
- D (on the vertical line, lower): centre of the world
- F (foot of the vertical line): other end of the line of apsides
- G (foot of the circle): end of AD produced
- K (beside D), M (lower left): named on the block; not named in the text on this page

[Margin: 1st step.]

First, then: since the angle ADB, the difference of the true place A and B, is given, its complement to two right angles BDG is also given [Margin: by 13 of the first of Euclid]; and since the angle AEB, the difference of the mean motions of the planet placed at A and B, is given, the angle at the circumference AGB is also given, because it is half of that one [Margin: by 20 of the third of Euclid]. So in triangle BDG the angles BDG and BGD are known. Assume, then, the side BD as 200000 parts (for you will reduce these afterwards to the radius of the circle), and from these three givens seek the side DG.

[Margin: 2nd step.]

Second, in triangle DCG, the side DG has been found, and the angle DGC at the circumference is known, because it is half of the angle AEC at the centre, known from the difference of mean motion between A and C. The angle GDC is also known [Margin: by 20 of the third of Euclid], because it is the complement to two right angles of the angle ADC, known through the difference of the true places of the planet observed at A and C. So from this seek the side DC.

[Margin: 3rd step.]

Third, in triangle BDC, the side DC has been found, and BD has already been assumed as 200000 of the same parts, and the angle BDC is known from the difference of the true places B and C. Seek, then, both the side BC and the angle DBC.

[Margin: 4th step.]

Fourth, in the isosceles triangle BEC, the base BC has been found, and the angle BEC is known from the difference of mean motion between B and C. So the sum of the angles at the base is known, as being the complement to two right angles, and hence each of the angles at the base, as being equal to each other [Margin: by 32 of the first of Euclid and by 5]. Seek, then, the side BE, and from the angle EBC subtract the angle DBC found in the 3rd step: for the angle DBE will be left.

[Margin: 5th step.]

Fifth, in triangle EDB, there are known both the side DB, assumed as 200000, and the side EB, and the angle contained by them, through the 4th step. So seek DE and the angle EDB, which measures in the zodiac the distance of the true place of the planet observed at B from the true place of the apogee H. Therefore, since the point of the zodiac under which the planet observed at B stood is known, the point of the zodiac under which the planet's apogee stood at that time will also become known; it will be called the aphelion if the Sun is supposed at the centre D. Further, if it is made: as EB, in the parts found, to DE, so the radius EB, of 100000 parts, to another, the eccentricity DE will be known, in parts of which the radius of the eccentric circle is 100000. These were the two things to be investigated.

[Margin: Confirmation of the conclusion found.]

It would also be possible, for confirmation, with the side EC found (which is equal to the side EB and AE) in the isosceles triangle AEC, and with the angles at the base known from the angle AEC, that is, from the difference of the mean motions of the planet at A and C, and with the base AC found, for the angle DAE to be known, if the angle DAC had been found; for EAC subtracted from this would give DAE. Now the angle DAC can be found with the base AC found, as above, and with the angle ADC known, which is the difference of the true places A and C, and with the side DC found, as in the 2nd step; and in the same triangle the side AD can be found. So with the sides AD and AE known, together with the contained angle DAE, the eccentricity ED would not lie hidden, nor the angle ADH, measuring the distance of the true place

of the planet A from the true place of the apogee, but in the zodiac. You see now that in this method two things are supposed as known beforehand, about which, however, the question stands, namely the mean motion of longitude and the place of the apogee; and that it is only a posteriori that one explores whether they agree in every respect with the observed places.

EXEMPLVM.

Example.

[Margin: Example in ♂ from the observations of Tycho.]

V. *As Longomontanus reports in Book 2 of the* Theorica*, chapter 8, Tycho observed Mars at Uraniborg in three acronychal shinings and true ☍ with the ☉: in the year 1585 January 30, at $19^h\ 15^m$ p.m., in ♌ 21° 35′ 10″; in the year 1587 March 6, at $7^h\ 20^m$, in ♍ 25° 42′; and lastly in the year 1589 April 14, at $6^h\ 20^m$, in ♎ 4° 23′. Let these three places, in order, be A, B, C in the preceding figure. So the difference of the first and second true places, that is, the angle ADB, was 34° 6′ 50″, which is the amount from the place in ♌ to the already mentioned place in ♍. And the difference of the second true place from the third, that is, the angle BDC, was 38° 41′. The time from the first to the second observation was 2 common years, 34 days, $12^h\ 5^m$, to which there corresponds from the Rudolphine Tables a mean motion of Mars, that is, the angle AEB, after the casting out of the whole circle, of 40° 39′ 13″. From the second opposition to the third there were 2 years, $39^d\ 23^h$, counting the leap day of the year 1588, to which there corresponds a mean motion of 43° 30′ 45″, which was the size of the arc CB, or the angle BEC. Let them therefore be set in the following synopsis.*

Diagram. Woodcut set in the left column, with the italic text of the example running down its left side. A circle with A at the upper left, H at the top, B at the upper right, C at the right, G at the foot, F at the foot just left of G, and M at the lower left, all on the circumference. A vertical line runs from H through E and D to F, with E above D and K just to the right of D. Solid lines join A, B and C with E and with D, and the chords AB, AC and BC are drawn; a line runs from C through D to M. Dashed lines run down to G from E, from B and from C. It has the same labels and arrangement as the figure of the second way.

- H (top), F (foot of the vertical line), G (foot of the circle), M (lower left)
- A, B, C (circumference): the three places of ♂ in the example
- E (on the vertical line, upper), D (on the vertical line, lower), K (beside D)

Angles and sides (ANGVLI, LATERA).

[Margin: 1. For triangle DBG.]

Angles	° ′ ″	Sides	Parts
ADB	34° 6′ 50″	BD	20000
BDG	145° 53′ 10″		

Angles	° ′ ″	Sides	Parts
AEB	40° 39′ 13″		
AGB	20° 19′ 36″		
DBG	13° 47′ 14″	Therefore DG	13721

[Margin: 2. For triangle DCG.]

Angles	° ′ ″	Sides	Parts
AEC	84° 9′ 59″	DG	13721
DGC	42° 4′ 59 1/[?]″		
ADC	72° 47′ 50″		
DCG	30° 6′ 51″	Therefore DC	18329

[Margin: 3. For triangle BDC.]

Angles	° ′ ″	Sides	Parts
BDC	38° 41′ 0″	BD	20000
		DC	18329
Therefore DBC	63° 34′ 42″	And BC	12792

[Margin: 4. For triangle BEC.]

Angles	° ′ ″	Sides	Parts
BEC	43° 30′ 45″	BC	12792
EBC	68° 14′ 38″		
DBC	63° 34′ 42″		
Therefore DBE	4° 39′ 56″	And BE	17256

[Margin: 5. For triangle EBD.]

Angles	° ′ ″	Sides	Parts
DBE	4° 39′ 56″	BE	17256
		DB	20000
Therefore EDB	25° 36′ 52″	And ED	3247

[Margin: Apogee of ♂ in the year 1587.]

♂ was, moreover, in the second opposition B, in ♍ 25° 42′. So with the angle HDB, 25° 36′ 52″, taken away, there remains ♍ 0° 5′ 8″, in which stood the apogee H of ♂ in the year 1587 March 6, old style, at 7^h 20^m after noon at Uraniborg.

[Margin: Eccentricity of ♂.]

Next, if it is made: as EB 17256 to the same radius EB 100000, so ED 3247 to another, the eccentricity ED will be 18817 parts, of which the radius, or semidiameter of the planet's orb, is 100000; and its half 9408. But this way is slippery, as I shall soon say.

Tertius Modus Eccentricitatis & Apogei inquirendi ex Keplero.

Third Way of inquiring into the eccentricity and the apogee, from Kepler.

[Margin: Imperfection of the first and second ways.]

VI. The first and second ways set out above, relying on only three acronychal observations, are judged slippery by Longomontanus, in Book 2 of the *Theorica*, chapter 2, by Kepler, in the *Commentaries on Mars*, chapter 16, and by Boulliau, in Book 8 of the *Astronomia Philolaica*, chapter 3. For if in the already mentioned figure you solve the triangles CDE and ADE, you will find neither the same eccentricity DE nor the same place of the apogee.

[Margin: Boulliau's opinion of those ways.]

So Boulliau there, from three acronychal and true oppositions of ♂ with the ☉ observed and reported by Longomontanus in Book 2 of the *Theorica*, chapter 8, page 342, gathers, but by his own method, a whole eccentricity of ♂ of 19402 parts, and a half of 9702, of which the radius of the orb of ♂ is 100000. But the place of the apogee from the first opposition, which was in the year 1604 March 28, at 16^h 20^m p.m., with ♂ placed in ♎ 18° 36′, he gathers to be in ♍ 1° 28′ 28″; but from the second, of the year 1608 July 24, at 2^h 0^m, with ♂ in ♒ 11° 10′, he gathers it in ♍ 1° 25′ 59″; and lastly from the third, of the year 1610 October 8, at 16^h 50^m, with ♂ in ♈ 25° 10′, he gathers it in ♍ 1° 9′ 4″. He thinks this place of the apogee too far advanced, because, to be sure, in chapter 1, from four other acronychal shinings observed by Tycho, he had obtained the aphelion of ♂, from three of them in ♌ 29° 0′, and from the fourth in ♌ 29° 4′. But he confesses that this disagreement found in the three Longomontanian observations arose from the admission of a small error of about 3′ in the observation; and nevertheless he condemns this method, *Because it is completed with only three points on the circle*; and he concludes: *Hence it is manifest that three places assumed are by no means sufficient for investigating the first inequality*. This he had charged against Ptolemy and Copernicus for using three points and observations, in Book 1, chapter 22, especially because Ptolemy had used three points in such a way that he had first decided that the eccentricity was to be bisected.

[Margin: Kepler's opinion of the same.]

Kepler too, in the passage cited, says: *Since Ptolemy, with three* [Greek: ἀκρονυχίοις (akronychiois, "acronychal")] *observations, and with this preconceived opinion about the proportion of the eccentricities, established both the place of the apogee and the correction of the mean longitude, and finally also the quantity of the eccentricities, I saw that if this problem were weakened, with the axiom about the proportion of the eccentricities removed* (that is, about the doubleness of the eccentricity of the equant to the eccentricity of the deferent) *it would be uncertain, and not of a single case, and must accordingly be strengthened in turn by a fourth* [Greek: ἀκρονυχίῳ (akronychiō, "acronychal")] *observation besides.*

[Margin: Regiomontanus's view of this method, and another way.]

But the most sharp-sighted Johannes Regiomontanus had seen this earlier, in Book 10 of the *Epitome of the Almagest*, proposition 21, where he says: *All the things mentioned above suppose this one thing, never demonstrated: that the centre of the eccentric deferent is equidistant from the two centres, namely of the world and of the equant, and lies on one straight line with them. But if something uncertain is mixed into a fine demonstration, the intellect will be sickened by it.* He adds, however, a remedy, but an arduous one, saying: *But if you wish to escape this, embrace this way; yet here the difficulty will perhaps produce no less trouble than uncertainty does elsewhere. Four configurations, their extremes known, we shall observe such that the intervals of time between each pair are equal; for this condition will show the aux to be midway between the paired configurations.* So if it were given to observe four acronychal positions whose paired intervals of time were equal, the point of the ecliptic at which the arc enclosed between the extreme places was bisected would be the place of the apogee. But since hardly any astronomer will meet with such good fortune, let us consult Kepler's earlier method.

[Margin: Kepler's earlier method.]

VII. With centre B, describe the eccentric HEDG, in which the diameter of the apsides is HI; suppose it to be as if unchanged for some years, so that the place of the apogee stays sensibly at the same point of the zodiac. On that line let A be the centre of vision, and C the centre of the equant, that is, the point about which the planet is taken to move uniformly.

Diagram. A circle, the eccentric, set into the left column with the text running down its left side. Point labels stand around the circumference and on the interior vertical line. Straight lines run from the circumference points to the interior points, and chords join the circumference points.

- H (top of the circle): apogee
- F (upper left, on the circle): place of the planet
- E (left, on the circle): place of the planet
- G (right, on the circle): place of the planet
- I (bottom of the circle, left of D): perigee
- D (bottom of the circle, right of I): place of the planet
- C (interior, uppermost of the three on the vertical line): centre of the equant
- B (interior, below C): centre of the eccentric
- A (interior, below B): centre of vision

Let the 4 places of the planet in oppositions with the Sun be F, G, D, E, so that the planet, stripped of the second inequality, appears to the eye at A along the lines FA, GA, DA, EA. Let these places be joined to the two other centres, B and C, by straight lines, and indeed to one another by the chords FG, GD, DE, EF. For although only in the opinion of the ancient and Ptolemaic astronomy is our sight at A, the centre of the Earth, while in the opinion of Copernicus, Tycho and the moderns A is the Sun or a point near the Sun, yet according to these too the sight, or the centre of the Earth, is in true oppositions on the lines AF, AG, AD, AE.

With this laid down, we hold two things as certain: namely the angles at A, which are FAG, GAD, DAE and EAF, known from the difference of the true places of the planet observed in the zodiac; and likewise the angles at C, namely FCG, GCD, DCE and ECF, known from the difference of the mean motion that the interval of time elapsed between the observations yields. But the place of the apogee H, or the position of the line HI, is uncertain, or known only roughly; uncertain likewise are the mean motion and, from it, the anomaly; uncertain also the whole eccentricity of the equant, namely AC; and lastly it is uncertain whether B, the centre of the planet's deferent, is on the same line with the centres A and C, and whether at the exact midpoint, or rather, as Copernicus detected in Mars, nearer the centre of the equant than the centre of vision.

Since, however, from the ancient method, resting on three acronychal positions, both the nearly true mean motion and the place of the perigee I and of the apogee H can be known beforehand, for example for ♂ at the end of ♌ towards the end of the preceding century, or about the year 1600, let such a place be assumed that from its distance from the observed places of the planet the anomalistic angles FCH, GCH, or GCI, DCI and ECI are known; and finally let AC be assumed, for ease, as 10000 parts. With this laid down, what must be made clear is whether the assumed place of the apogee, and the angles depending on it, allow the four points F, G, D, E to be on the same circle, and B, the centre of that circle, to be on the line CA; and how great AC must be to obtain this; indeed how great AB, if B must be exactly at the midpoint of AC, as Ptolemy supposes. AC could also be assumed as a determinate quantity, in parts of which BH is 100000, known in some fashion from the earlier methods; but for the sake of easier calculation Kepler assumes it as 10000, and afterwards reduces it to parts of the radius AH. With these things laid down, then, use the following steps.

[Margin: Step 1.]

First, in the triangles CFA, CGA, CDA and CEA, seek the sides AF, AG, AD, AE, in parts of which their common side AC was assumed; for in them the angles at A and C are given. For example, in triangle CFA, AC is given as 10000, and the angle FAC from the distance of the planet observed at F from the assumed place H of the apogee, and FCH from the anomaly of the eccentric, that is, from subtracting the place of the apogee from the planet's mean longitude belonging to it by the tables at the time when it was observed at F; and likewise for the others.

[Margin: Step 2.]

Second, in the triangles FAG, GAD, DAE and EAF, in which the sides have now been found by the preceding step and the angles they enclose are given, being known from the difference of the true observed places of the planet, seek therefore each angle at the base of each triangle, and join them to the neighbouring angles so as to have four angles, namely EFG, FGD, GDE and DEF. For if all together equal four right angles, and any two opposite ones

between them equal two right angles, the four points F, G, D, E, bounding the quadrilateral FGDE, will be on the circumference of the same circle, and that quadrilateral will be inscribed in that same circle, by proposition 22 of Book 3 of *Euclid* and its scholia; and the things assumed above, up to this point, will not be false. But if those four angles do not equal four right angles, and those pairs do not equal two right angles, the assumption is false. Yet it will be uncertain which of those assumptions is false, namely the mean motion or the place of the apogee.

[Margin: Step 3.]

Third, therefore, keeping the mean motion used above, move the place of the apogee forward or back a little, and so increase or decrease the angles at A made by the line of apsides and by the lines along which the planet is seen from A; and repeat the first and second steps, until, with no sensible difference, the four angles of the quadrilateral FGDE equal four right angles, and each two opposite ones equal two right angles. And so you will have the four places of the planet on the same circumference of a circle, which was the first thing to be explored.

[Margin: Step 4.]

Fourth, to find out whether B is at the midpoint of the line AC, join into one the angles GAD and DAE, which are known from the difference of the true places of the planet observed at the three places G, D, E, so that the angle EAG becomes known; and in triangle EAG, in which the sides are now known by the first step, alone or repeated, seek the base GE.

[Margin: By 20 of Book 3 of Euclid.]

Next, since in triangle EFG the angle at F is known from the second or third step, and this must be half the angle at the centre, namely the angle EBG, therefore in the isosceles triangle EBG the base EG and the angle opposite it are given;

[Margin: By 5 of Book 1 of Euclid.]

therefore the angles at the base, BEG and BGE, are given.

[Margin: Step 5.]

Fifth, then, in triangle EBG, from the given or found EG and the angles at the base, seek the radius of the circle, that is, the side EB, in parts of which AB was assumed. And since the angles BGE and AGE have now been found, and the sides BG and AG, if you take AGE from BGE itself (or the latter from the former, if the latter should come out smaller than the former), AGB will remain. Therefore in triangle AGB, in which the sides AG and BG and the enclosed angle AGB are given, the angle BAG is sought. If it disagrees with CAG, assumed above, that is a sign that B does not fall, as it should, within the line AC.

[Margin: Step 6.]

Sixth, therefore, the mean motion, which you had not changed above, must be changed, as must the place of the apogee, and the preceding steps must be repeated for as long as it takes until the four points of the observed places are

on the circumference of the same circle, that is, until the opposite angles of the quadrilateral already mentioned together equal two right angles. Then you will work by other steps until you find BAD as great as you had assumed CAD, or HAD, in that operation. When you have achieved this, you will then take the radius BD as 100000 parts, and in triangle BAD you will seek, by means of the angles now found, the eccentricity of the deferent, AB, and the eccentricity of the equant, AC; and at last you will declare that the place of the apogee and the mean motion assumed are right.

[Margin: Imperfection of this method.]

Kepler, however, admits that this is not a geometrical solution, and that there will be keen geometers like Viète who will charge Ptolemy, Regiomontanus and Copernicus with this method's lack of art [Greek: ἀτεχνίαν], as Viète did; and that an artless [Greek: ἄτεχνον] thread, by which you may retrace your way out of this labyrinth, is better than none; and that it is nothing unusual for geometers to hunt the true from the false and, by arguments to the impossible, the truth that stands in the middle between defect and excess; and that, out of weariness with this method, the *Commentaries on Mars* were drawn out into a fifth year. Finally, having brought forward an example of this practice, from four acronychal positions of Mars he concludes the whole eccentricity of ♂, AC, to be 18564 parts, but that of the deferent alone, or AB, 11332, and BC 7232. In the Copernican and Tychonic form this would amount to giving the semidiameter of the epicycle a value, for the greater one, of 14948 parts, or, on account of the maximum equations, a more correct 14988; and for the lesser, 3616 or 3628. But he reserves the examination of these quantities for another place, namely not on a perfect circle but on an ellipse; which he does in the *Commentaries on Mars*, chapter 42, by a method soon to be set out.

Quartus Modus inueſtigandi Eccentricitatem, Aphelium, & correctionem Motus Medij ex Keplero.

Fourth way of investigating the eccentricity, the aphelion and the correction of the mean motion, from Kepler.

VIII. When Kepler, reducing the eccentric places of Mars to the law of a circle, saw that they did not yield the same aphelion, and from this came to suspect that the planet's path is not a circle, he resolved in chapter 42 to inquire, from his own observations, yet outside acronychal position, into the distances of the planet from the Sun at any place whatever, especially when it is near the aphelion and perihelion; and from them to inquire into the genuine eccentricity and proportion of the orbs, the most certain place of the aphelion, and the correction of the mean motion. This he does by the following method. In the following figure let A be the centre of the world, that is, the Sun, and IT the line of apsides; and with centre B describe the eccentric ICTF, in which I is the aphelion and T the perihelion. With centre A, moreover, describe the

annual orb of the Earth DKH; and let AK be the line of the equinoxes, along which the Earth at K sees the Sun A under the beginning of Aries.

[Margin: Kepler's later method.]

Now let the Earth, according to the various times at which the planet was observed, be at G, D, E, K, Z, M, H; of these let D, G, H be on the left-hand parts of the eccentric, and the remaining places on the right. For those times let the true place of the Sun be given, and its anomaly, with which gather from the tables of the Sun the distances of the Earth from the Sun, which are to be the lines AG, AD, AE, AK, AZ, AM, AH. And since the line IT is the line of apsides, on which now lie the planet as seen from the Sun and the planet's eccentric place, if the true place of the planet, seen along the lines GI, DI, EI, etc., is established from observation, and the true place of the Sun or Earth from calculation, then the angles at the Earth will also be established, namely the angles AGI, ADI, etc. These become known by subtracting the eccentric place of the planet from the true place of the Sun, or the latter from the former, according as the Earth is heading towards the opposition or towards the conjunction of the planet and the Sun, that is, is on the left or on the right side of the annual orb. For example, if the Earth is at M and the planet at T, the angle AMT will be known by subtracting the true place of the Sun A, seen from M, from the place of the planet T, seen from M; and you will have the angle AMT. But if the Earth is at H, subtract the planet's eccentric place from the true place of the Sun, and you will have the angle AHT. With these things supposed:

Diagram. Two circles set in the left column. The larger is the eccentric, with I at its top and T at its bottom. The smaller, inside it, is the annual orb about A, its circumference carrying the Earth's stations. Straight lines radiate from A to the stations; lines run from I to stations on the upper arc and from T to stations on the lower arc. Three planetary signs stand in the space between the two circles.

- I (top of the larger circle): aphelion
- T (bottom of the larger circle): perihelion
- C (left, on the larger circle)
- F (right, by the larger circle)
- ♂ (upper left, between the circles): planetary sign
- ♃ (upper right, between the circles): planetary sign
- ♄ (lower left, between the circles): planetary sign
- B (interior, above A): centre of the eccentric
- A (interior, on the vertical line): the Sun, centre of the annual orb
- D (upper arc of the smaller circle)
- G (upper left of the smaller circle)
- E (upper right of the smaller circle)
- K (right of the smaller circle)
- Z (right of the smaller circle, below K)
- L (lower right of the smaller circle)

- M (lower right of the smaller circle, below L)
- P (bottom of the smaller circle)
- H (lower left of the smaller circle, beside P)

[Margin: Step 1, for the true place of the aphelion.]

First, assume the distance of the planet, whether of the aphelion, AI, or of the perihelion, AT, already known beforehand in some fashion; for example for Mars AI as 166700 and AT as 138400, in parts of which the radius of the annual orb is 100000; and by means of trigonometry seek the angles at I and at T, for example the angles AIG, AID, AIE, etc. Finally, add these angles to the observed place of the planet, if the Earth is in quadrant GD and the planet at I; subtract if in quadrant EK; and you will have the place of the aphelion. But if the planet is at T and the Earth at H, subtract the angle ATH from the observed place of the planet; if, however, the Earth is at Z, or L, or M, add; and you will have the place of the perihelion. Thus either from all these operations the line TI is restored to the same place of the zodiac, that is, the same place of the aphelion or perihelion always results, and that is a sign that the assumed distance is correct; or the same does not result, but the difference is greater than the difference of the precession of the equinoxes allows, on account of the different times at which the planet was observed against the fixed stars, and another distance a little greater or smaller must be assumed, until that difference of the places of the aphelion and perihelion gathered from several observations is found to be very small. Then that little error must be distributed among the individual operations, and so the true place of the aphelion must be fixed, which for ♂ Kepler finds, in 1588 November 22, at ♌ 28° 50′ 44″.

[Margin: Step 2, for the correction of the mean motion.]

Second, since the mean motion changes when the place of the aphelion is changed, from the change of the former you will gather the change of the latter. For if, at the time when Mars, for example, was thought to fall on the aphelion and to lack all equation of the eccentric place, it had at that same time passed the aphelion by 11′, then to this anomaly there corresponds an equation of 4′ to be subtracted; and so by its mean motion it had passed that earlier mean place by 4′; and these must be added to the mean motion usually set in the tables. But if it fell short of the aphelion, and the equation was additive, they will have to be subtracted.

[Margin: Step 3, for the eccentricity.]

Third, combine the distances corrected as above, the aphelial AI and the perihelial AT, into one line IT, and its half, BI or BT, will be the radius or semidiameter of the planet's orb, in parts of which the radius of the annual orb is 100000. Then subtract the perihelial distance AT from the radius BT, and there will remain the eccentricity AB, that is, the distance of B, the centre of the circle or ellipse carrying the planet, from A, the centre of the world. Doubled, this will give the whole eccentricity, or the distance from the centre of the world of the point about which the planet is imagined to move uniformly. Finally, if you want these measures in parts of which the planet's orb BI is

100000, multiply them by 100000 and divide the sum by BI, known in the earlier parts. Accordingly, *Kepler*, in that chapter 42, concludes thus about ♂.

In parts of which the radius of the annual orb is	100000
Aphelial AI	166780
Perihelial AT	138500
Sum IT	305280
Half BI	152640
Eccentricity AB	14140

Now as 152640 is to 100000, so the eccentricity 14140 is to the eccentricity 9264, but in parts of which the radius of the orb of Mars is 100000. Its double is the whole eccentricity, 18528, which is certainly more correct than the one we gathered by Hérigone's method, that is, by the second way, where we also overshot in the place of the aphelion. Kepler, however, corrected the places of Mars observed by Tycho, and used the daily motion, taken at the time from Magini's *Ephemeris*, and the distances of the Sun from the Earth, which he had computed in chapter 30 from the bisected eccentricity of the Sun. The same eccentricity he confirmed in chapter 54 from the aphelial and perihelial distances of Mars from the Sun.

Quintus Modus Eccentricitatis & Aphelij inquirendi ex Bullialdo.

Fifth way of inquiring into the eccentricity and the aphelion, from Boulliau.

IX. Most recently *Boulliau*, in Book 1 of the *Astronomia Philolaica*, chapter 21, having shown in his hypothesis of the ellipse, which the planet describes about a scalene cone, that the first inequality is distributed through circles in the ratio of the right sines counted from the aphelion or perihelion, in chapter 22 rejects the way of Ptolemy and Copernicus, and of Kepler. He adds, however, that by another, new path, from Viète's problems, he will derive the measure of the first inequality from three or four points. Yet afterwards, in Books 6, 7 and 8, he uses four acronychal positions, and in Book 8, chapter 3, teaches that it does not suffice to employ three points of the circle, that is, three observed oppositions of the planet with the Sun, as I have already said in number 6. Moreover, Boulliau's method is the same one we set out in Book 3, chapter 24, in the 4th method of the solar eccentricity, after number 4; and so we refer the reader there, for examining three acronychal positions; but for four, let the following scheme stand, for the true oppositions of Mars with the Sun observed by Tycho and reported by Kepler in the *Commentaries on Mars* and by Longomontanus in Book 2 of the *Theorica*, chapter 8. They were as follows, the days being counted in the old style.

Phaſes Martis Acronychiæ, ſeu Oppoſitiones cum vero Solis loco, obſeruatæ Vraniburgi.

Acronychal phases of Mars, or oppositions with the true place of the Sun, observed at Uraniborg.

[Margin: Order of the observations.]

	Year	Month	Day	Time	True place of ♂
1	1582	Dec.	28	$4^h\ 0^m$	♋ 16° 54′
2	1587	March	6	$7^h\ 20^m$	♍ 25° 42′
3	1591	June	8	$7^h\ 45^m$	♐ 26° 42′
4	1593	Aug.	25	$17^h\ 30^m$	♓ 12° 15′

So let these four places be on a circle in the same order, B, C, D, E; from them, to the diameter FN, cut at right angles by the diameter AL, let the perpendiculars BG, CIS, DK and EHQ be dropped, and let the four places be joined to one another both by the chords BC, CD, DE, EB and by the diagonals BD and CE.

Diagram. A circle set into the left column, the text running down its left side. A vertical diameter runs from A at the top to L at the bottom, and a horizontal diameter from F at the left to N at the right, crossing at O. The four places B, C, D, E stand on the circumference, joined by chords and diagonals, with perpendiculars dropped to the horizontal diameter.

- Q (top, on the circle, left of A)
- A (top, on the circle): aphelion
- B (upper left, on the circle): first place
- C (upper right, on the circle): second place
- D (right, on the circle, below N): third place
- E (bottom, on the circle, left of L): fourth place
- L (bottom, on the circle): perihelion
- S (lower right, on the circle)
- F (left end of the horizontal diameter)
- N (right end of the horizontal diameter)
- O (interior, at the crossing of the diameters)
- G (on the horizontal diameter, between F and H)
- H (on the horizontal diameter, between G and O)
- I (on the horizontal diameter, right of O)
- K (on the horizontal diameter, between I and N)
- M (interior, below I)
- P (interior, upper left of O, below X)
- X (interior, near the top, between B and A)
- Z (interior, lower right, above S and left of D)

And since the place B of the first observation was in the middle of ♋, and that of the second about 26° ♍, and we know from tables and observations,

however coarsely weighed, that the aphelion of ♂ at that time was about the end of ♌, the line of the aphelion, AOL, therefore falls between B and C. The aim is to investigate the true place of the aphelion A and of the perihelion L, that is, the distance of the points B, C, D, E from the points A and L; and the quantity of the line FO, which represents the eccentricity of the centre O from F, the centre of vision, or from one of the umbilici. For this we have these Data.

[Margin: Data for the solution of the problem.]

From the first opposition to the 2nd are 4 common years, $69^{d}\ 3^{h}\ 20^{m}$; and, the circles being cast off, the apparent motion is 68° 48′, the mean motion 81° 22′ 41″, and the difference of the mean motion from the true 12° 34′ 41″.

From the 2nd opposition to the 3rd, 4 years, $95^{d}\ 0^{h}\ 25^{m}$; true motion 91°, mean 94° 56′ 25″, difference 3° 56′ 25″.

From the 3rd opposition to the 4th, 2 years, $79^{d}\ 9^{h}\ 45^{m}$; true motion 75° 33′, mean 64° 11′ 10″, difference 11° 21′ 50″.

Since, however, as *Boulliau* teaches, the whole first inequality of the planet is founded on the distance FN of the elliptical umbilici, but half of it is to be taken, the planet must therefore be considered as seen not from the umbilici but from the centre O, which is exactly midway between the umbilici. The arithmetic mean between the mean and apparent motions must therefore be taken. For example, in the first interval the half of the difference between the true, or apparent, and the mean motion was 6° 17′ 20″; these, then, added to the true motion, because it was less than the mean motion, that is, to 68° 48′, make for the interval BC 75° 5′ 20″, whose arithmetic mean likewise is 37° 32′ 40″; and so for the other intervals. Once these are corrected we shall have the angles standing on them and set at the circumference, since by 20 of Book 3 of *Euclid* they are half of such arcs. Finally we shall seek from the tables of sines both the sines of the half-arcs already mentioned and the sines corresponding to the half-differences of the apparent and mean motions. Thus, then, we shall set out these Data, with the half-differences considered on the diameter FN.

Arcs	Measure	Angles	Measure
BC	75° 5′ 20″	BEC, EDC	37° 32′ 40″
CD	92° 58′ 13″	CBD, CED	46° 29′ 6″
DE	69° 52′ 5″	DCE, DBE	34° 56′ 3″
EB	122° 4′ 22″	EDB, ECB	61° 2′ 11″
BCD	168° 3′ 33″	BCD	95° 58′ 13″
CDE	162° 50′ 18″	CDE	98° 34′ 51″
DEB	191° 56′ 27″	DEB	84° 1′ 46″
EBC	197° 9′ 42″	EBC	81° 25′ 9″

But in parts of which the radius is 100000.

Half-arcs	Sines		Half-differences	Measure	Sines
BC	60937	1	GI	6° 17′ 20″	10945
CD	72519	2	IK	1° 58′ 13″	3488
DE	57263	3	KH	5° 40′ 55″	9901
EB	87493		GK	8° 15′ 33″	14365
DB	99457		GH	2° 34′ 38″	4497
EC	98880				

[Margin: Step 1.]

First, then, with the given quantities [Greek: διδομένοις (didomenois, "the given quantities")] arranged as above, in triangle BCD let it be as GK is to BD, so IK to MD, of 23803 parts.

[Margin: By 20 of Book 3 of Euclid.]

Therefore, given in triangle MCD the sides MD and CD with the enclosed angle, which is 37° 32′ 40″, the angle MCD will be 15° 7′ 38″, and the arc SD, its double, 30° 15′ 16″. From this the nearly equated anomalies at the four oppositions of ♂ will be given, by a way that *Boulliau* does not even deign to indicate, but it is as follows. Since the arc CD is given as 92° 58′ 13″, and SD, just found, is 30° 15′ 16″, therefore the whole CDS is 123° 13′ 29″. Its complement to the semicircle is 56° 46′ 31″; this is the sum of the arcs AC and SL. But since CS is parallel to AL by construction, the arc AC is as great as LS, and so half of the sum just mentioned, that is, 28° 23′ 15″ (not 33′, as Boulliau's text has on p. 289), which is the anomaly of the place C, counted from the aphelion A. With the arcs AC and CD joined, there results the anomaly of the place D, that is, the arc ACD, 121° 21′ 28″, whose complement to the semicircle, namely the arc DL, is 58° 38′ 32″. Subtracted from the arc DE, given above as 69° 52′ 5″, this leaves the arc EL, 11° 13′ 33″. Therefore, adding the semicircle ANL, there results the anomaly of the place E, ANE, 191° 13′ 33″. Finally the arc AC, 28° 23′ 15″, subtracted from the arc BC, given as 75° 5′ 20″, leaves the arc AB, 46° 42′ 5″, whose complement to the circle is 313° 17′ 55″; this is the arc ANLB, which is the anomaly of the place B.

[Margin: Step 2.]

Second, since the arc AB has been found as 46° 42′ 5″, GO, its sine, is given as 72778; and since the arc DL was found as 58° 38′ 32″, its sine KO is therefore 85393, and the whole GK 158171. So let it be as GK is to the radius FO, 100000, so GK, estimated above at 14365 parts, is to FO, the eccentricity of the umbilicus F from the centre of the ellipse O, of 9082 parts. And since the arc AB was found as 46° 42′ 5″, and Mars at the first opposition B, observed by Tycho in 1582 December 28 at 4^{h}, was at ♋ 16° 54′, if you add the arc BA to this place the aphelion will fall at ♍ 3° 36′ 5″.

[Margin: Step 3.]

Third, in triangle BED he directs that BP be found by the same proportion, as 31135 parts, and from it the angle BEQ, 16° 3′ 5″, whose double is the arc BQ, 32° 6′ 10″. Joined to the arc EB this gives the arc QFE; and half its complement

to the semicircle gives the arc EL, which with the semicircle ANL makes up the anomaly of the place E, 192° 54′ 44″. Taking the arc ED from this, the arc AND remains, 123° 2′ 39″, for the anomaly of the place D; and taking the arc CD from this again, the arc AC remains, 30° 4′ 26″, for the anomaly of the place C. And adding the arc EL, or AQ, to BQ itself, there results AB, which is the complement to the circle of the anomaly of the place B, which will be 314° 59′ 6″; and the arc AB will be 45° 0′ 54″, whose sine GO is 70728; and the arc DL will be 56° 57′ 21″, whose sine OK is 83825. Therefore the whole GK is 154553. So let it be as GK, 154553, is to the radius FO, of 100000 parts, so GK, set earlier at 14365, is to another, and the eccentricity FO will come out as 9294 parts. And adding the arc AB, 45° 0′ 6″, to the place of ♂ at B, which was ♋ 16° 54′, the aphelion will come out at ♍ 1° 54′ 6″.

[Margin: Step 4.]

Fourth, using the same proportion for triangle CBE, you will find BX, of 25037 parts, and the angle BEQ, 16° 28′ 0″, and its double arc, 32° 56′ 0″, and QA, 12° 29′ 49″; and the anomalies: for the place B, 314° 34′ 11″; for C, 29° 39′ 21″; for D, 122° 37′ 44″; and for E, 192° 29′ 49″; and the arc AB, 45° 25′ 49″, whose sine OG is 71240; and the arc AC, 29° 39′ 21″, whose sine OI is 49483. And so the whole GI is 120723. Let it be, then, as GI, 120723, is to the radius FO, 100000, so GI, set above among the half-differences at 10945, is to another; and FO will come out at 9074 parts. And if the arc AB, 45° 25′ 49″, is added to the place of ♂ at B, as above, the aphelion will come out in ♍ 2° 19′ 49″.

[Margin: Step 5.]

Fifth, by a similar proportion for triangle CDE, you will find ZD, of 19884 parts, and the angle ZCD, 14° 35′ 57″, and its double, the arc SD, 29° 11′ 54″; and from this the anomalies: for B, 313° 49′ 37″; for C, 28° 54′ 57″; for D, 121° 53′ 10″; and for E, 191° 45′ 15″; and the sine OH, 20371, and the sine OK, 84911, and so the whole HK, 105282. If this is made to 100000 as HK, set above at 9901, is to another, FO will arise, the eccentricity, of 9404 parts. And the arc AB will be 46° 10′ 23″, which, added to the place of Mars at B, will give the aphelion in ♍ 3° 4′ 23″.

[Margin: Step 6.]

Sixth, since in the steps already mentioned different anomalies, and from them different eccentricities, have been found (which *Boulliau* says arises from the form of the ellipse sought in the circle, and from the optical inequality, which is greater in the lower part of the orbit and smaller in the upper than the part of the inequality arising from the equant circles), he therefore directs that the arithmetic mean be taken both of the anomalies found and of the eccentricity FO. And so, using the half-differences, after some limitations of the excesses and defects and a repeated calculation, he finally arrives at FO, an eccentricity of 9239 or 9242 parts, and the aphelion of ♂ at ♌ 29° 3′ 54″. Later, in chapters 4 and 5, he teaches how to seek the same things in the case where all four places of opposition given lie in the upper part of the orbit, or all in the lower; and from this he concludes the aphelion of ♂ for January of the year

1586 to be at ♌ 28° 53′ 16″, and the eccentricity to be 8988 and 9129. So, with the arithmetic mean chosen, he appears to use an eccentricity of 9056 parts. But in the table of equations for ♂ he uses an eccentricity of 9239, whose double gives the distance of the umbilici, 18478, or the difference between the greatest distance of ♂ from the Sun, which he sets at 109239, and the least, 90765.

So you see that not even this method is wholly geometrical a priori, and that, with the mean motion assumed, it strives toward the truth more and more through various excesses and defects and arithmetic means; and with how great an effort astronomy still labours to exhaust this depth.

Sextus Modus Noſter Inueſtigandi primam Inæqualitatem trium ſuperiorum Planetarum.

Our sixth way of investigating the first inequality of the three superior planets.

X. Just as in chapter 6 we proposed our hypothesis not as something new but as one that accommodates its measures to all, so now we offer no new diagram, but use indifferently the schemes of any astronomer, set out in chapters 1, 2, 3 and 4. For in Ptolemy's figure one can investigate the whole eccentricity of the equant; in Copernicus's first form, the eccentricity and the semidiameter of the epicycle; in the second form, which is Lansbergen's, the difference between the greatest and the least eccentricity; in the third, which *Tycho* and *Longomontanus* chose, the semidiameter of the greater and of the lesser epicycle; and in the form of the ellipse, the whole eccentricity of the foci, or the eccentricity of the centre of the ellipse from one of the foci. But to give an example in at least one of them, the one that seems to us simpler and easier, namely Copernicus's second form, let observations be chosen of true oppositions of the planet with the Sun, such as those which *Kepler* above all has on Mars, chapter 15; *Longomontanus* in Book 2 of the *Theorica*, chapters 2, 5 and 8; and after them *Boulliau* in Books 6, 7 and 8; or others that are Tychonic, or ours, or those of any more exact observer, when they come out. And if two can be had, with the planet placed once in the aphelion and again in the perihelion, the way will be far better and quicker. We assume, moreover, that astronomy has by this time advanced so far that no sensible error can lie hidden in the place of the aphelion, whose motion is very slow, and in the mean motion.

These things set down, in the present figure let the line of apsides be AB, in which the aphelion is A, the perihelion B, and C the centre of the eccentric ADBE; and let the centre of the world be M. And let at least four acronychal places of the same planet be observed, F and G in the upper part of the eccentric and H, K in the lower; let these be connected by straight lines with the centres C and M, and by the chords FLG and HPK.

Diagram. At the head of the right column, inset to the right of the narrowed text: a large circle, the eccentric, with the vertical diameter AB. A horizontal chord joins F and G at the upper part, crossing the diameter at L, and a horizontal chord joins H and K at the lower part, crossing it at P. On the diameter below

L stands the point C, then O, the centre of a small circle drawn about it, then I at the foot of that small circle, then M. Straight lines run from F and from G to C and to M, and from H and from K to C and to M. From D, on the left of the circumference, two dotted lines run inward, one to the small circle about O and one to M. E stands on the right of the circumference, with no line drawn to it.

- A (top of the circle, on the diameter)
- L (on the diameter, at the upper chord)
- F (upper left of the circumference)
- G (upper right of the circumference)
- C (on the diameter, top of the small circle)
- D (left of the circumference)
- E (right of the circumference)
- O (centre of the small circle)
- I (on the diameter, foot of the small circle)
- M (on the diameter, below the small circle)
- H (lower left of the circumference)
- K (lower right of the circumference)
- P (on the diameter, at the lower chord)
- B (foot of the circle, on the diameter)

First, then, let the planet be at F. For from recent tables there will be given both the mean motion in longitude and the planet's distance from the apogee, or anomaly of the eccentric, which is the arc AF, measuring the angle ACF; and therefore its sine FL and its second sine CL will be given. And since, from the apparent motion, or the true place known by observation and compared with the mean motion, the angle MFC is known, that is, the equation of the centre, or of the eccentric place (for in true oppositions no other equation comes in than that of the first inequality), the angle FMC will also be known, by 32 of Book 1 of *Euclid*; for it is the difference by which the angle MFC differs from the angle of anomaly FCL. In triangle FML, right-angled at L, from the given angle FML and the sine FL, seek the side ML, and subtract from it the second sine CL. For if MC, the eccentricity, is equal to the eccentricity which the tables suppose in this hypothesis for that time, that eccentricity is correct as regards that position. We assume in this diagram, moreover, that the eccentricity varies and librates up and down on the diameter CI of the small circle, since it is not necessary in this procedure that the centre of the eccentric always be at the apogee C or the perigee I of this small circle.

But if you do not wish to test the assumed eccentricity but to seek it, which is certainly better, use the triangle MFC, in which let there be given the radius CF, of 100000 parts, and the angle FCM, the complement of the angle FCA, known from the anomaly of the eccentric, and MFC, known from the difference of the true motion from the mean; and seek the side MC, namely the eccentricity. And do likewise with the places H, K, G, or others of the planet known in the eccentric by observation of a true opposition with the Sun. But if the planet is

at H, the angle HCM will become known from the complement of the anomaly to the semicircle; if at K, the angle KCM will be the excess of the anomaly above the semicircle; if at G, the angle GCM will become known by subtracting the complement of the anomaly ACG, to the circle, from 180°. Further, if in the triangles HMP or KMP you do not seek but test the eccentricity, you must subtract from the second sine PC the side found, MP, and from this discern whether the remainder CM is equal, as it should be, to the assumed eccentricity CM.

When this is done you will find the eccentricity greater near the apogee of the eccentric and smaller near the perigee. And if you choose a pair of acronychal positions, in one of which the planet was at the apogee of the eccentric and in the other at the perigee, or as nearly as possible, such as *Kepler* supplies for Mars in chapter 42 in the *Commentaries on Mars*, you will have the greatest eccentricity at the apogee and the least at the perigee; and their half-difference will be the semidiameter OC, that is, of the small circle, which by its libration varies the eccentricity, according to our hypothesis set out in chapter 6.

XI. You could, however, without labour follow an elective hypothesis, whose measures we have set in the last columns of the table of chapter 7, but especially the one that has been derived from the most recent observations of Tycho and of others after him by Longomontanus, Kepler and Boulliau; and choose a mean eccentricity between their extremes. In this matter very little industry indeed lies hidden, but not very little security.

EXEMPLVM.

Example.

[Margin: Example in ♂.]

XII. *In the year 1589, as Longomontanus reports in Book 2 of the Theorica, chapter 8, Tycho observed the true ☍ of ♂ with ☉ on April 14 at 6^h 20^m after noon at Uraniborg, when ♂ was in ♏ 4° 23′ and ☉ in ♉ 4° 23′. But for me, who recognize a far smaller parallax in the Sun, the Sun was then at Uraniborg in ♉ 4° 28′. So, with the daily motion of Mars, retrograde, which was about 24′, and the daily motion of the Sun computed, the truer opposition was there at the hour 4^h 35^m, in ♏ 4° 24′. For that moment the mean longitude of ♂ from the Rudolphine Tables was 7^s 14° 22′, and the aphelion 4^s 28° 47′; when this is subtracted from the mean longitude, the anomaly of the eccentric remains, 75° 35′, that is, the arc AF, or the angle ACF, with Mars at F. But the difference of the mean motion, that is, of 7^s 14° 22′, from the true motion, that is, from the 7^s 4° 24′ in which ♂ stood, is the equation of the centre to be subtracted from the mean motion, that is, the angle CFM, which, subtracted from the angle ACF, 75° 35′, leaves the angle CMF, that is, FML, in triangle FLM, right-angled at L. Now the sine of the angle FCL, or the side FL, is 96851, of parts of which CA is 100000, and the second sine CL is 24897. And by analysis of the triangle FLM the side LM is found, 43447; when the sine LC, 24897, is subtracted from it, there remains the*

whole eccentricity MC, 18550, of parts of which the radius AC is 100000. And so for the others.

In this way, from the Rudolphine Tables or from Magini's Keplerian tables for ♂, you will find at an anomaly of the eccentric of ♂ of 1° an eccentricity of 20000 parts, at an anomaly of 90° one of 18600 parts, and at an anomaly of 179° one of 17060 parts. The mean of these is 18530, which Kepler chose, and gathered by another method with the greatest industry and a most troublesome search.

XIII. Now, so that you can use observed acronychal positions, consult the observations set out below. It must be admitted, however, that in those observations, made at another time and reduced to the moments of true opposition with the Sun, some minute lies hidden, both because of the Tychonic motion of the Sun, which, owing to the excessive parallax of the Sun assumed by Tycho and not demonstrated, errs in the ecliptic by some minutes, and because of the distances of the planets from the fixed stars, whose right ascension and declination here and there need a slight correction, as can be established from the distances of the fixed stars from one another, diligently observed by us, from Book 6, chapter 10. So if Tycho's observations were extant, we could more easily apply some correction to the following acronychal positions. Meanwhile, however, we shall enjoy them, until astronomy undergoes the final file, as observations of others and our own are added; we have not a few of these, but we await more, at more places of the eccentric. Those listed below up to the year 1600 were observed by Tycho; the other acronychal positions partly by Kepler and Longomontanus, partly by David Fabricius. *Longomontanus*, moreover, adds the computation from his own tables, using the true precession of the equinoxes.

[Margin: ♂. Acronychal positions from Kepler, ch. 15.]

♂ Veræ Oppoſitiones cum Vero Loco Solis ex Kepleri Comm. de Marte; & Obſeruationibus Tychonis, Kepleri, ac Fabricij. Ad Vraniburgum habitis vel reductis.

True oppositions of ♂ with the true place of the Sun, from Kepler's Commentaries on Mars, and from the observations of Tycho, Kepler and Fabricius, made at Uraniborg or reduced to it.

[Margin, beside rows 1–12: Tycho (rows 1–9); Kepler (row 10); David Fabricius (row 11); Fabricius (row 12).]

	Old style: Year	Month	Day	Time at Uraniborg	Longitude observed	Latitude observed
1	1580	Nov.	18	$1^h\ 31^m$	♊ 6° 28′ 35″	1° 40′ B
2	1582	Dec.	28	$3^h\ 58^m$	♋ 16° 55′ 30″	4° 6′ B
3	1585	Jan.	30	$19^h\ 14^m$	♌ 21° 36′ 10″	4° 32⅙′ B
4	1587	March	6	$7^h\ 23^m$	♍ 25° 43′ 0″	3° 41′ B
5	1589	Apr.	14	$6^h\ 23^m$	♏ 4° 23′ 0″	1° 12¾′ B

	Old style: Year	Month	Day	Time at Uraniborg	Longitude observed	Latitude observed
6	1591	June	8	$7^h\ 43^m$	♐ 26° 43′ 0″	4° 0′ A
7	1593	Aug.	25	$17^h\ 27^m$	♓ 12° 16′ 0″	6° 2′ A
8	1595	Oct.	31	$0^h\ 39^m$	♉ 17° 31′ 40″	0° 8′ B
9	1597	Dec.	13	$15^h\ 54^m$	♋ 2° 28′ 0″	3° 33′ B
10	1600	Jan.	18	$14^h\ 2^m$	♌ 8° 38′ 0″	4° 30⅚′ B
11	1602	Feb.	20	$14^h\ 13^m$	♍ 12° 27′ 0″	4° 10′ B
12	1604	March	28	$16^h\ 23^m$	♎ 18° 37′ 10″	2° 26′ B

[Margin: ♄. Fifteen acronychal positions, in hours after noon, from Longomontanus, Book 2 of the *Theorica*, chapter 2.]

[Margin: Copenhagen lies under the meridian of Uraniborg, with an imperceptible difference.]

Planetarum ♄, ♃, ♂ veræ Oppoſitiones, cum Vero Solis loco, Obſeruatæ & ſupputatæ ex Tabulis Danicis.

True oppositions of the planets ♄, ♃, ♂ with the true place of the Sun, observed, and computed from the Danish Tables.

Order	Old style: Year	Month	Day	Time at Copenhagen	Place observed	Place computed
1	1582	Aug.	21	$2^h\ 0^m$	♓ 7° 26′	7° 22⅓′
2	1583	Sept.	3	$1^h\ 0^m$	♓ 19° 50′	19° 48⅓′
3	1584	Sept.	15	$6^h\ 30^m$	♈ 2° 34′	2° 35¾′
4	1585	Sept.	28	$19^h\ 30^m$	♈ 15° 39½′	15° 39¼′
5	1586	Oct.	12	$11^h\ 0^m$	♈ 29° 2′	29° 4′
6	1587	Oct.	26	$9^h\ 0^m$	♉ 12° 46′	12° 46′
7	1588	Nov.	8	$10^h\ 10^m$	♉ 26° 44′	26° 44⅔′
8	1589	Nov.	22	$14^h\ 30^m$	♊ 10° 53′	10° 53⅗′
9	1590	Dec.	6	$20^h\ 30^m$	♊ 25° 10′	25° 8⅓′
10	1591	Dec.	21	$1^h\ 0^m$	♋ 9° 24½′	9° 23′
11	1595	Jan.	30	$21^h\ 0^m$	♌ 21° 15½′	21° 18⅓′
12	1608	July	9	$3^h\ 0^m$	♌ 26° 53′	26° 57½′
13	1609	July	21	$13^h\ 0^m$	♒ 8° 31′	8° 31⅕′
14	1610	Aug.	2	$22^h\ 30^m$	♒ 20° 10′	20° 14½′
15	1611	Aug.	15	$16^h\ 0^m$	♓ 2° 12′	2° 13½′
♃						
1	1583	Sept.	6	$20^h\ 10^m$	♓ 23° 33′	23° 38⅖′
2	1584	Oct.	13	$7^h\ 20^m$	♉ 22° 0′	22° 0¼′
3	1591	Apr.	23	$19^h\ 0^m$	♏ 13° 10′	13° 9′
4	1595	Sept.	12	$6^h\ 10^m$	♓ 28° 56′	28° 54¾′
5	1596	Oct.	18	$12^h\ 0^m$	♉ 5° 40′	5° 35′
6	1607	Sept.	17	$11^h\ 10^m$	♈ 4° 10′	4° 8⅙′
7	1610	Dec.	30	$14^h\ 40^m$	♋ 19° 36′	19° 38½′
8	1613	March	1	$22^h\ 0^m$	♍ 21° 45′	21° 42¼′
♂						
1	1580	Nov.	28	$1^h\ 30^m$	♊ 6° 27½′	6° 27½′

Order	Old style: Year	Month	Day	Time at Copenhagen	Place observed	Place computed
2	1582	Dec.	28	$4^h\ 0^m$	♋ 16° 54½′	16° 55½′
3	1585	Jan.	30	$19^h\ 15^m$	♌ 21° 35⅙′	21° 37′
4	1587	March	6	$7^h\ 20^m$	♍ 25° 42′	25° 42⅙′
5	1589	Apr.	14	$6^h\ 20^m$	♏ 4° 23′	4° 26½′
6	1591	June	8	$7^h\ 45^m$	♐ 26° 42′	26° 45⅓′
7	1593	Aug.	25	$17^h\ 30^m$	♓ 12° 15′	12° 14¾′
8	1595	Oct.	31	$0^h\ 40^m$	♉ 17° 30⅔′	17° 29⅙′
9	1597	Dec.	13	$16^h\ 0^m$	♋ 2° 27′	2° 26′
10	1600	Jan.	18	$14^h\ 0^m$	♌ 8° 37′	8° 37⅙′
11	1602	Feb.	20	$14^h\ 15^m$	♍ 12° 26′	12° 24′
12	1604	March	28	$16^h\ 20^m$	♎ 18° 36⅙′	18° 36½′
13	1608	July	24	$2^h\ 0^m$	♒ 11° 10′	11° 13⅓′
14	1610	Oct.	8	$16^h\ 50^m$	♈ 25° 30′	25° 26⅔′

[Margin: ♃. Eight acronychal positions from Longomontanus, Book 2 of the *Theorica*, chapter 5.]

[Margin: ♂. Fourteen acronychal positions from Longomontanus, Book 2 of the *Theorica*, chapter 8.]

[Margin: Longomontanus's text has June for January.]

CAPVT IX.

Chapter 9.

De Motu Medio trium Superiorum Planetarum.

On the mean motion of the three superior planets.

I. When they had examined the mean oppositions of the three superior planets with the Sun, that is, with the mean place of the Sun, observed by the most ancient astronomers, and compared them with those which Hipparchus, Ptolemy and Copernicus observed in their own time, and when they had computed the intervals of years and the whole periods of the planet's revolutions both in the zodiac and with respect to the Sun, they found that as much time is required to complete the revolutions listed below (whole with respect to the Sun, but with an appendix after the whole revolutions completed under the zodiac or under the fixed stars) as you see in the following small tables, from *Ptolemy*, Book 9, chapter 3, and *Copernicus*, Book 5, chapter 1. He calls motion of commutation the motion by which the equal motion of the Earth either exceeds the mean motion of ♄ ♃ ♂ or is exceeded by the motion of ☿ and ♀, known through the greatest digressions and the revolutions with respect to them, as I shall say in Section 3, chapter 10, although it pleases me now to embrace all the planets here in one synopsis.

[Margin: Foundations of the mean motion of the planets.]

I. TABELLA EX HIPPARCHO, ET PTOLEMÆO

Table 1, from Hipparchus and Ptolemy.

Planets	In solar years: Years	Days, hours	Or: days, hours, minutes	Revolutions under the zodiac	Plus	Revolutions with respect to the Sun	Revolutions with respect to ☉, resolved into degrees of the epicycle
♄	59	1d 18h	21551d 7h 12m	2	1° 45′	57	20520
♃	70	360d 4h	25927d 14h 48m		355° 10′	65	23400
♂	79	3d 4h	28857d 21h 12m	42	3° 10′	37	13320
♀	7	362d 18h	2919d 16h 0m	7	357° 45′	5	1800
☿	46	1d 1h	16802d 9h 36m	46	1° 0′	145	52200

II. TABELLA EX COPERNICO

Table 2, from Copernicus.

Planets	In solar but sidereal years: Years	Days, hours, minutes, seconds	Revolutions in the zodiac under the fixed stars	Plus	Periods of the commutations, or revolutions
♄	69 [recte: 59, per the book's errata; corrected by hand to 59 in this copy]	1d 2h 55m 12s	2	1° 5′ 50″	57, Earth to planet
♃	70	364d 7h 32m 28s	5	blank	65, Earth to planet
♂	79	2d 9h 54m 24s	42	blank	37, Earth to planet
♀	7	363d 0h 16m 4s	12, around the Sun	357° 36′ 31″	planet to Earth
☿	46	0d 10h 0m 0s	191, around the Sun	0° 21′ 53″	45, planet to Earth

From these they gathered those mean motions of which we have shown a specimen in the tables at the ends of the chapters. Comparing these with the periods and intervals between observed oppositions of the same planet, they confirmed or emended them: *Ptolemy* for ☿ in Book 9, chapter 10; for ♀ in Book 10, chapter 4; for ♂ in Book 10, chapter 9; for ♃ in Book 11, chapter 3; and for ♄ in Book 11, chapter 7; and *Copernicus* in Book 5: for ♄ in chapter 7, for ♃ in chapter 12, for ♂ in chapter 17, for ♀ in chapter 23, and for ☿ in chapter 29. Another small table for ♀ and ☿ I shall give at the end of Section 3.

II. But since the practitioners already mentioned used oppositions with the mean place of the Sun, when the true ones should rather have been used; and since more recent observations are rightly judged more accurate than the ancient ones, and stand at a longer interval from the ancient ones, it is no wonder if the mean motions among the more recent authors have come out somewhat different from the ancient ones and more accurate. Now I said that true oppositions should be used because in them the motion of the second inequality ceases and only the first, which we are investigating, has force. Granted that, because we are investigating mean motions, a mean opposition might seem the one to choose; but it is one thing to employ an opposition with

the mean place of the Sun, which we deny should be done, and another to investigate the mean motion of the planet for the time of the true opposition, which we do not deny. And so *Longomontanus*, in Book 2 of the *Theorica*, from acronychal positions observed by Ptolemy and compared with Tychonic ones, gathers the mean motion from the interval of the mean places of the planet at the true oppositions with ☉: for ♄ in chapters 4 and 5, for ♃ in chapter 7, and for ♂ in chapter 10. We indicated these in chapter 3, in the table at the end of the chapter. In the same way *Kepler*, likewise from Tychonic acronychal positions, emended the mean motions: for ♂ in the *Commentaries on Mars*, chapters 42 and 69, and for it and the rest in the Rudolphine Tables, of which you have a specimen in chapter 5, number 20, in the table, as also of those of *Boulliau*, who indeed corrected the mean motions of ♄ in Book 6, chapter 5, of ♃ in Book 7, chapter 7, and of ♂ in Book 8, chapter 13. From which observations *Lansbergen* gathered the mean motions of these planets indicated in the table in chapter 4, he himself teaches at the end of the *Treasury of Observations*. We meanwhile, until other observations of Tycho come out and ours are more numerous than those we have, shall use mean motions derived from ancient and recent observations by a device similar to the one we used for the mean motions of the Sun in Book 3, chapter 17, with the true motions of the planets reduced to the mean ones for the moment of the true oppositions with the Sun.

CAPVT X.

Chapter 10.

De Loco & Motu Apogei & Aphelij trium ſuperiorum Planetarum, eorumq. diſcrimine.

On the place and motion of the apogee and aphelion of the three superior planets, and on the distinction between them.

I. The way of investigating the place of the apogee of the eccentric, or of the aphelion, of these planets has already been indicated in chapter 8, on the occasion of the eccentricity. Different astronomers in their own times found this place differently; from which it became plain that the motion of the auges is indeed very slow, but not as great as the motion of the fixed stars, as, with *Ptolemy* in Book 9, chapter 5, *al-Farghānī* in *differentia* 17, *al-Battānī* in chapter 47, and the whole school of the Arabs and Alphonsines, down to *Copernicus*, thought. In Book 5, chapters 7, 12 and 16, Copernicus was the first to find the motion of the apogee of Saturn and of Mars faster than the ordinary motion of the fixed stars, but that of Jupiter slower. After him this was also found by *Longomontanus*, in Book 2 of the *Theorica*, chapters 4, 7 and 10; by *Kepler* in the *Commentaries on Mars*, chapters 17, 42, 59 and 69, in which he corrects what he had said in chapter 17, and in the *Epitome of Copernican Astronomy*, p. 733; and by *Boulliau*, Book 6, chapter 2, Book 7, chapter 7, and Book 8, chapter 13. Besides these one can see *Jean Fernel*, Book 1 of the *Cosmotheoria*, p.

32; *Wurstisen* on Peuerbach's *Theoricae*, p. 255; *Magini* in the *Secunda Mobilia*, canon 35; *Hérigone*, volume 5 of the *Cursus Mathematicus*, pp. 529 and 584; and *Lansbergen* in the *Theoricae*, chapter 10. From these we have gathered the places of the apogees and aphelia for the times listed below into the following table. As for the annual motion of the apogees, we have already set it in the tables at the end of each hypothesis, from chapter 1 to chapter 5.

APOGEI, vel APHELII LOCVS In Annis CHRISTI infrà notatis & Completis.

Place of the apogee, or aphelion, in the years of Christ noted below, and completed.

Authors	Planets	Place
Ptolemy, year 136	♄ Apog.	♏ 23° 0′ 0″
	♄ Aphel.	♏ 23° 40′ 0″
	♃ Apog.	♍ 11° 0′ 0″
	♃ Aphel.	♍ 17° 31′ 0″
	♂ Apog.	♋ 25° 30′ 0″
	♂ Aphel.	♌ 4° 27′ 0″
al-Battānī, year 880	♄ Apog.	♐ 14° 58′ 0″
	♃ Apog.	♍ 24° 58′ 0″
	♂ Apog.	♌ 6° 18′ 0″

Reſiduum Tabulæ præcedentis.

The remainder of the preceding table.

Authors	Planets	Place
Copernicus, year 1525	♄ Apog.	♐ 27° 42′ 0″
	♄ Aphel.	♐ 28° 3′ 0″
	♃ Apog.	♎ 6° 21′ 0″
	♃ Aphel.	♎ 11° 30′ 0″
	♂ Apog.	♌ 27° 0′ 0″
	♂ Aphel.	♍ 4° 21′ 0″
Longomontanus, year 1600	♄ Apog.	♐ 26° 26′ 57″
	♃ Apog.	♎ 7° 31′ 58″
	♂ Apog.	♌ 28° 41′ 56″
Kepler, year 1600	♄ Aphel.	♐ 24° 57′ 36″
	♃ Aphel.	♎ 6° 52′ 1″
	♂ Aphel.	♌ 28° 59′ 54″
Lansbergen, year 1600	♄ Apog.	♐ 25° 55′ 12″
	♃ Apog.	♎ 3° 8′ 55″
	♂ Apog.	♌ 25° 32′ 6″
Boulliau, year 1600	♄ Aphel.	♐ 25° 59′ 46″
	♃ Aphel.	♎ 8° 1′ 22″
	♂ Aphel.	♌ 28° 59′ 52″
Hérigone, from Copernicus, year 1636	♄ Apog.	♐ 29° 10′ 0″

Authors	Planets	Place
	♄ Aphel.	♐ 25° 48′ 0″
	♃ Apog.	♋ 6° 40′ 0″
	♃ Aphel.	♋ 7° 0′ 0″
	♂ Apog.	♌ 28° 24′ 0″
	♂ Aphel.	♌ 29° 20′ 0″
Ourselves, year 1644	♄ Apog.	♐ 27° 13′ 20″
	♃ Apog.	♋ 8° 36′ 0″
	♂ Apog.	♌ 29° 50′ 2″

[Margin: The distinction of the apogee from the aphelion.]

II. *Kepler*, however, in the *Epitome*, p. 733, says that in Ptolemy's time, with the eccentricity of the annual orb set at 1800 of the parts of which its radius is 100000, and with a line drawn from the centre of the annual orb through the centre of the eccentric of the planets, parallel to Ptolemy's lines of apsides, the aphelion of ♄ was in ♏ 23° 34′, that of ♃ in ♍ 15° 7′, and that of ♂ in ♌ 0° 42′. But today, he says, the aphelion of Saturn is in 25° 48′ ♐, that of Jupiter in ♋ 7°, and that of Mars in ♌ 29° 20′; though he regards Ptolemy's observations as suspect, especially concerning Jupiter. But, you will say, how is the place of the aphelion to be distinguished from the place of the apogee? Let there be, then:

Problema 1. Datis Apogei loco, Eccentricitate à Terra, ac Solis à Terra Diſtantia; Inueſtigare locum Aphelij & Eccentricitatem à Sole.

Problem 1. Given the place of the apogee, the eccentricity from the Earth, and the distance of the Sun from the Earth: to investigate the place of the aphelion and the eccentricity from the Sun.

III. We shall employ Copernicus's second form, as the one best suited to this business, and, with Mästlin on chapter 15 of Kepler's *Prodromus*, we shall represent the situation of the centres of the eccentrics with respect to the Sun and the Earth, as it suits the planets at this time. So let AB be the line of apsides of the Sun, S, and of the Earth, T; and TA the line of the Sun's apogee, such that A marks about the 6th degree of Cancer; and TB the line of the Earth's aphelion; and ST the greatest distance of the Sun and the Earth. Next let the centres of the small circles that vary the eccentricity be: for ♂, C; for ♃, F; for ♄, I; for ☿, M; and for ♀, P. Through these let a pair of lines be drawn from the Sun and from the Earth as far as the ecliptic, although here, for want of room on the page, they are not extended that far. And let the lines of the aphelia drawn from the Sun be SD, SH, SO, SL, SR; and let the lines of the apogees drawn from the Earth be TE, TG, TN, TK, TQ. With this laid down, to give an example in Mars, consider the triangle CTS, in which let there be given ST, the greatest distance between the Sun and the Earth; and CT, the mean eccentricity of Mars from the Earth; and the place of the apogee of Mars, marked by the

line TCE. For if you subtract that place from the place of the Sun's apogee, the angle ETA, that is, CTS, will be known. Therefore, by solving the triangle already mentioned, one can find both CS, the mean eccentricity of the planet from the Sun, and the angle TCS, by which the place of the apogee differs from the place of the aphelion. That place will be known if the quantity of the angle at C is added to the place of the apogee already found, when the place of the planet's apogee is in the first semicircle, counted from the Sun's apogee in the order of the signs; but in the second semicircle let it be subtracted.

Diagram. In the right column, interrupting the word *lo-cus* near the head of the column. A vertical line AB, with A at the top and B at the foot. On it stand S and, below it, T. About the line are five small circles, each with its planet's sign: at upper left a circle with the sign of ♂ and the centre C, with lines running out from it to D and E; at left a large circle with the sign of ♃ and the centre F, with lines running out to G and H; below T and to the left a circle with the sign of ☿ and the centre M, with lines running out to N and O; at right of S a small circle with the centre P, with the sign of ♀ above it and Q and R beside it; and below, a large circle with the sign of ♄ and the centre I, with lines from S and T running through it to K and L, which stand side by side at the foot, left of B. Straight lines join S and T to these circles.

- A (top of the vertical line); B (foot of the vertical line)
- S (on the vertical line, upper); T (on the vertical line, lower)
- C, with ♂ (upper-left circle); D, E (ends of the lines beyond it)
- F, with ♃ (large circle at left); G, H (ends of the lines beyond it)
- M, with ☿ (circle below T); N, O (ends of the lines beyond it)
- P, with ♀ (small circle to the right of S and T); Q, R (ends of the lines beyond it)
- I, with ♄ (large circle at the foot); K, L (ends of the lines beyond it)

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N, O, P, Q, R, S, T, and the signs ♂ ♃ ☿ ♀ ♄. Labels not read: none.

Problema 2. Datis Aphelij loco, Eccentricitate media à Sole, maximaq. diſtantia Telluris à Sole, eiuſdemq. Aphelio; Inuenire Apogei locum, & Eccentricitatem à Terra.

Problem 2. Given the place of the aphelion, the mean eccentricity from the Sun, the greatest distance of the Earth from the Sun, and the Earth's aphelion: to find the place of the apogee and the eccentricity from the Earth.

IV. In the same triangle CTS, let there be given ST, the greatest distance of the ☉ and the Earth; and SC, the mean eccentricity of Mars from the Sun; and the Earth's aphelion, marked by the line TB, with its opposite, the perihelion, toward A. Subtract its place from the aphelion of Mars, marked by the line SD; for the angle ASC will be left, whose complement to two right angles will be the angle CST. From these three givens, then, seek CT, the eccentricity from the Earth, and the angle TCS, which is to be subtracted from the given place

of the aphelion when the planet's aphelion is in the first semicircle, counted from the Earth's perihelion in the order of the signs, but added when it is in the second, so that you may have the place of the apogee of the eccentric. And do not let the difference of the points T and S, from which the ecliptic is described, disturb you; for both because of the parallelism, on which see Kepler above, and because in the second Problem the centre of the world is transferred from T to S, no error is committed. From what has been said it is clear that those who have put the apogee for the aphelion in the more recent tables have not acted accurately.

[Margin: A corollary against some of the moderns.]

CAPVT XI.

Chapter 11.

De Secũda ſeu Annua inæqualitate Trium Superiorum Planetarum.

On the second, or annual, inequality of the three superior planets.

I. The second inequality of these planets depends on the places of the planet observed outside the acronychal position, and on the Sun's approach to the planet and withdrawal from it, not the mean approach, as with the ancient astronomers, but the true; enough was said about this motion in Book 3. For when the true place on the eccentric of the planet is taken from the true place of the Sun, there remains the anomaly of the orb, or of the commutation, which concerns the second inequality. It depends besides on the distance of the planet from the Sun, which we have taught how to investigate in the form of a geometric calculation, according to any one of the celebrated hypotheses; and lastly it depends on the distance of the Sun from the Earth. And since all these measures are to be reduced to the same kind, namely to parts of which the radius of the planet's orb is assumed, for example, as 100000, or of which the mean distance of the Sun and of the planet from the Earth is known in terrestrial semidiameters, the investigation of the second inequality therefore consists chiefly in seeking the proportion of the semidiameter of the annual orb of the Sun or of the Earth to the semidiameter of the planet's orb. But in the Ptolemaic hypothesis it consists in seeking the semidiameter of the epicycle, which supplies the commutations of the annual orb. For this, observations outside the acronychal position are needed, since at acronychal positions the second inequality ceases.

[Margin: The second inequality depends on the proportion between the semidiameters of the annual orb and of the planet's orb.]

[Margin: Authors of the method for finding this proportion.]

The method of seeking the proportion of the orbs from those observations is taught by Ptolemy, Book 10, chapter 8, and Book 11, chapters 2 and 6, where he demonstrates the sizes of the epicycles of ♂ ♃ ♄; by Regiomontanus, Book 10

of the *Epitome of the Almagest*, proposition 23, and Book 11, propositions 7 and 16; by Copernicus, Book 5, chapters 9, 14 and 19; by Longomontanus, Book 2 of the *Theorica*, chapters 3, 6, 9; and by Boulliau, Book 6, chapter 2, Book 7, chapter 3, and Book 8, chapter 6. Nor is this procedure difficult, if you have already learned the way of finding the equation of the second inequality; for the same triangle is to be used here, and the same figures set out earlier for the geometric calculation. I shall not repeat those figures in this place, but I shall still indicate them, together with the way of solving this Problem by means of them.

[Margin: The way by Ptolemy's figure.]

II. *First*, in Ptolemy's figure set out in chapter 1, no. 2, with the planet placed at *n*, consider triangle AI*n*. In it, from the place observed outside the acronychal position, and from the place of the centre I, gathered by tables or by the geometric calculation given in the same place, there is given the difference of the two, that is, the equation of the epicycle, or the angle IA*n*; and the side IA was found in the same place, and the angle *n*IA, which is the complement of the angle *n*IT, known from the distance of the planet from the true apogee of the epicycle, as we taught in the same place. Therefore I*n* will not stay hidden, that is, the semidiameter of the epicycle, which is the foundation of the second inequality, in parts of which DI, the radius of the eccentric, is 100000.

[Margin: The way by Copernicus's figure.]

Second, in Copernicus's second form, treated in chapter 2, no. 4, imagine the planet at G and the Earth at P; and suppose the distance GE of the planet from the Sun, given or found, and the angle EGP, which is the difference between the place observed outside the acronychal position and the eccentric place of the planet derived from the tables, and finally the angle GEP, at the Sun, or the anomaly of commutation, known by subtracting the eccentric place from the true place of the Sun. From these the radius EP of the annual orb will appear, which is the mean distance of the Earth from the Sun.

[Margin: The way by the other figure of Copernicus.]

Third, in Copernicus's form 3, which *Tycho* and *Longomontanus* chose, and which we treat in chapter 3, no. 4, let the planet be at H and the Earth at L, a point of the annual orb MI; and in triangle HLA, as we taught there in steps 1 and 2, let AH be found, which is the distance of the planet from the Sun; and through the anomaly of the orb let the angle HAL be found. Given, then, the angle already mentioned, and the distance AH, and the angle HLA, which is the complement to two right angles of the angle CLH, known from the difference between the observed place of the planet and its eccentric place, or the place of the centre C, the side AL will not stay hidden, that is, the semidiameter of the annual orb that was sought. *Tycho* and *Longomontanus*, however, adapted this form to a motionless Earth, as we said there in no. 6.

[Margin: The way by the figures of Kepler and Boulliau.]

Fourth, to set before your eyes here at least one figure ready to hand, suited to the form of Kepler and Boulliau: let A be the Sun, AC its eccentricity, and

PR the line of apsides; and from C let the annual orb RHPF be described. Let the centre of the planet's orbit be B, from which the orbit EINK is described, and let its line of apsides be DE. Further, let the place of the planet observed outside the acronychal position be at N, for a certain time, and let the Earth be at G, and the distance of the planet from the Sun be AN, along which line the eccentric place of the planet is seen from the Sun; but let the true place of the planet observed from the Earth be along the line GN; and so the angle ANG is the angle of commutation, or of annual parallax. With this laid down, for that time find, as we have set out in the form of Kepler or of Boulliau, besides the true place of the Sun and its equated anomaly, and from that the distance AG of the Sun from the Earth; find, I say, both the eccentric place of the planet, so that you may have the angle GNA by comparing that place with the observed place, and also the distance NG of the planet from the Earth, and also the angle GAN, that is, at the Sun, which becomes known from the anomaly of the orb by subtracting the true eccentric place of the planet from the true place of the Sun. For in the triangle AGN, from those three givens you will find the side AG, in parts of which BE, the radius of the planet's orb, is 100000.

Diagram. In the right column, inset to the right of the narrowed text. A large outer circle, inscribed along its arc with the signs ♄ ♃ ♂, and a smaller inner circle lying within it, its top near the top of the outer circle. A vertical line runs through the inner circle from R at its top to P at its foot, passing C and A. B stands to the right of C, on a line drawn from D at the left of the outer circle to E at its right. From A, dotted lines run in opposite directions, one up to L and down to G, the other up to F and down to H. Solid lines run from A to M, to I, to E, to N and to K; a line runs from L to M, from D to H, from F to E, from N to P and from N to G, and from G to K.

- M (upper left of the outer circle); R (top of the inner circle); L (upper left of the inner circle)
- F (upper right of the inner circle); C (above A, on the vertical line); B (right of C)
- D, I (left of the outer circle); E (right of the outer circle)
- A (centre of the figure); H (lower left of the inner circle); P (foot of the inner circle); G (lower right of the inner circle)
- N (lower left of the outer circle); K (lower right of the outer circle)

Labels read: A, B, C, D, E, F, G, H, I, K, L, M, N, P, R, and the signs ♄ ♃ ♂.
Labels not read: none.

Finally, set up the following analogy, saying: if AG, the distance of the Sun from the Earth, of so many parts of which the radius CP of the annual orb is 10000 [recte: 100000], contains so many parts of the kind of which the radius BE of the planet's orb is 100000, then the radius CP itself, of 100000 parts, will contain parts of the radius BE, etc. The same can be done in semidiameters of the Earth, if AG and the radius CP are known in semidiameters of the Earth. But the rule must be illustrated by an example.

EXEMPLVM.

Example.

[Margin: Example in ♄.]

III. Longomontanus, as he himself relates in Book 2 of the *Theorica*, chapter 3, reports that from the Earth at G the place of Saturn at N was observed in ♊ 7° 32′, in the year 1590 February 8, old style, at the 8th hour of the evening at Uraniborg. For that time the true place of the Sun on the line AL, according to Boulliau, was ♓ 0° 1′ 41″; and so the place of the Earth, G, seen from the Sun along the line AG, was ♍ 0° 1′ 41″; and the equated anomaly of the ☉ was 7^s 23° 49′; and the distance of the Sun from the Earth 98953, of the parts of which the radius CP of the annual orb is 100000; and the eccentric place of ♄ seen from the Sun along the line AN was ♊ 13° 49′ 18″. Therefore, when the place of Saturn is subtracted from the eccentric place, the angle of commutation AHG [recte: ANG] remains, 6° 17′ 18″. And the distance of ♄ from the Sun, that is AN, was 94338, of the parts of which the radius BE is 100000. And when the true eccentric place of ♄ is subtracted from the true place of the Earth, the angle NAG is found, 76° 12′ 23″. Therefore in triangle ANG the side AG comes out at 10423 parts, of which the radius BE is 100000. But AG was of 98953 parts, of which the radius CP is 100000. So it is made: as 98953 to 10423, so 100000 to 10533. Therefore the radius of the annual orb is 10533 parts, of which the radius, or semidiameter, of Saturn's orb is 100000.

IV. It will be useful, however, following the example of Longomontanus and Boulliau, to examine several other places observed in different places of the eccentric, and above all in opposite ones, and to choose the mean of the extreme measures. So Boulliau chose 10480 parts for ♄, 19138 for ♃, and, with Kepler, 65638 for ♂, as we said in chapter 5, no. 20, in the table, if anyone holds the heresy [hæresi] of thinking that the semidiameter of the annual orb in the planetary system, or of the Ptolemaic epicycles, must always be of the same measure. But I have already shown that this is false, in chapter 6, proposition 6. You will also learn this a posteriori, if from the last table of the following chapter you gather the greatest equations of the orb, and through them find the semidiameter of the epicycle according to the rule given in chapter 7, at the end of no. 6, and to be repeated in chapter 12, no. 19.

CAPVT XII.

Chapter 12.

De Maximis Aequationibus, earumq. locis, & de Tabulis Aequationum ritè conſtruendis. Vbi de imperfectione veteris Aſtronomiæ in hoc negotio.

On the greatest equations and their places, and on the proper construction of tables of equations; where the imperfection of the old astronomy in this business is treated.

[Margin: The greatest equation of the centre.]

I. First, the equation of the centre in the zodiac, or of the eccentric place, in the Ptolemaic hypothesis, is greatest at the points of mean longitude; that is, when the centre of the epicycle is at those points of the deferent from which a perpendicular can be drawn to the line of apsides, bisecting the eccentricity of the deferent; that is, in the following figure, at the points ε and Y, from which εB is drawn, cutting DA at B. For it is there that the greatest prosthaphaereses occur, as *Reinhold* demonstrated on Peuerbach, p. 100. These points are found by means of the rectilinear triangle BεD, whose side Dε equals the radius of the eccentric, and whose base BD equals BD, the half of the eccentricity AD, or the fourth part of the whole eccentricity AE; for the angle BεD is equal to the alternate angle HDε, which measures the arc εH, and by this arc the point ε, where the greatest equation of the centre occurs, exceeds the quadrant HF of the anomaly.

Diagram. In the left column, interrupting the sentence between *angulus* and BεD. A large outer circle, and within it a second circle drawn on a centre above the centre of the outer circle, its top touching the top of the outer circle. Small circles stand on the inner circle: at the top about F; at upper left about I; at left two overlapping about H and ε; at lower left about K; and at the foot about M. A vertical line runs from the top through F, a, E, D, B and A down through M and b to the foot of the outer circle. Lines radiate from A, past the small circles, to the rim of the outer circle. A dotted line runs from H to D.

- L, T, G (together at the top, outside the outer circle)
- M, L (upper left, at the rim of the outer circle)
- I (centre of the small circle at upper left); c (beside that circle on the inner circle's arc, towards F); F (centre of the small circle at the top); a (below F, on the vertical line)
- c (below the small circle about I)
- E, D, B, A (down the vertical line, A at the point from which the lines radiate)
- X (right, on the inner circle); Y (right, outside the inner circle, at the outer circle)
- N, V, L (left, at the ends of the radiating lines); Z, H, ε (in and beside the small circle at left); c (below ε)
- K (centre of the small circle at lower left); c, c (between K and M); M (centre of the small circle at the foot); c (right of M); S (below M to the right); b (below M, on the vertical line)
- O, L, P, R, Q, L (around the foot of the outer circle, left to right)

Labels read: A, B, D, E, F, G, H, I, K, M (twice), N, O, P, Q, R, T, V, X, Y, Z, L (five times), c (six times), a, b, S, and the Greek letter ε. Labels not read: none.

Since, moreover, we taught in chapter 6, no. 13, how to transform the quantities of any hypothesis into the Ptolemaic form, the place of the greatest equation of the eccentric can easily be found in each hypothesis by this one

figure, if the fourth part of the whole eccentricity is taken for the base BD. Granted, we may point out this place from time to time in explaining the hypotheses of others; and it may be more prudent to use each author's own figure.

Once the place of the greatest equation of the centre has been found, and so the arc of the anomaly of the eccentric at whose end it occurs, the greatest equation is to be investigated by exactly the same method by which we taught how to investigate the other equations of the centre, according to each one's hypothesis and the geometric calculation that rests on it.

[Margin: The greatest equation of the orb.]

II. The greatest equation of the argument, or of the orb, or of the annual or second inequality, in the Ptolemaic hypothesis, occurs at the points of contact of the epicycle; that is, with the planet placed at those points on the circumference of the epicycle through which straight lines drawn from the centre of the world are tangent to the circumference of the epicycle, as in the preceding figure if the planet is at the points C, wherever at length the centre of the epicycle may be. But when the variety in the situation of the epicycle's centre is considered, the greatest equations of the orb themselves vary, and the least of the greatest occur when the centre of the epicycle is at F, the apogee of the eccentric, and the planet at the point of contact C. The measure of this equation is the angle FAC, which is known in the triangle FAC, right-angled at C, in which are given the semidiameter of the epicycle FC and the base AF, made up of the radius DF and the eccentricity of the deferent AD. But the greatest of the greatest occur with the centre of the epicycle placed at M, the perigee of the eccentric, and the planet at the contacts C; and the quantity of this greatest equation is investigated through the triangle AMC, right-angled at C, in which are given MC, the semidiameter of the epicycle, and AM, that is, the remainder of the radius DM when the eccentricity AD has been taken away. For the angle MAC, when found, is the measure of the greatest equation of the orb. When the angle FAC has been found, its complement to a right angle is the angle AFC, measuring the arc C*a*; and this, subtracted from the semicircle GC*a*, leaves the arc GC of the anomaly of the orb or of the epicycle, at whose end C the greatest equation of the orb or of the argument occurs. So the greatest equation of the orb itself, added to a quadrant, will at once give the point of the anomaly of the epicycle, or the distance from the apogee of the epicycle, where the least of the greatest equations of the orb occurs. Similarly, with the centre placed at M, the angle MAC added to a quadrant will give the anomaly of the orb at which the greatest of the greatest equations of the orb occurs. So when the measures of any hypotheses whatever have been reduced to the Ptolemaic form, according to what was said in chapter 6, no. 13, it will not be difficult to draw the greatest prosthaphaereses from them.

III. In Copernicus's form 2 and his figure, explained in chapter 2, no. 4, which *Lansbergen* claimed for himself, and in others that use the annual orb, the greatest equations of the orb occur at the points of contact of the annual

orb; that is, when the anomaly of the orb is so great that the Earth is at those points on the circumference of the annual orb through which a straight line can be drawn from the planet tangent to the annual orb. So in the adjoining figure, in which A is the centre of the world, that is, the Sun; C the centre of the annual orb; and MGO the annual orb; let B be the centre of the planet's eccentric orb KPL, and KBL the line of apsides, in which L is the apogee of the eccentric and K the perigee. The straight lines LO, LG, KM, KI, drawn from these on each side, tangent to the annual orb at the points O, G, M and I, mark the places where the greatest equations of the orb occur, there for the apogee and here for the perigee of the eccentric. These consist in the angles at the planet, for instance in the angle OLA,

Diagram. In the right column, interrupting the sentence between OLA and *vel GLA*. An outer circle KPL, carrying the signs ♄ ♃ ♂ inverted along its lower inside arc; within it two further circles and, inside those, a small circle about C. K stands at left on the outer circle and L at right; the line KBL runs between them through A. From K a line runs to M and a line to I, and from L a line to O and a line to G, each tangent to an inner circle. Lines from the point at A run to M, N, O, S, I, H, G.

- K (left, on the outer circle); L (right, on the outer circle); P (foot of the outer circle)
- M, N (together at the top of the inner circles); S (upper right, on the inner arc); O (upper right, at the rim of the inner circle)
- C (above A); A (at the point from which the lines radiate); B (right of A, on the line KL)
- I (lower left, on the small circle); H, G (together at the foot of the inner circles); Q (below G, on the circle next inside the outer circle)

Labels read: A, B, C, G, H, I, K, L, M, N, O, P, Q, S, and the signs ♄ ♃ ♂. Labels not read: none.

or GLA, as in MKA and IKA. They become known in the triangles right-angled at the points of contact, as in the triangle LOA, right-angled at O, in which are given the base AL, that is, the greatest distance of the planet from the Sun, and AO, the radius or semidiameter of the annual orb, in parts of the same kind.

But because, especially in Mars, the proportion of the annual orb to the radius of Mars varies sensibly, just as in the Ptolemaic hypothesis reformed according to what was said in chapter 6, proposition 6, the semidiameter of the epicycle is not the same, three annual orbs are therefore marked in this figure: the mean MGO, the least IG, and the greatest MQO. For the rest, the greatest equation of the orb at the apogee and perigee will be set out below, at the end of this section, gathered from the tables of the astronomers.

[Margin: Authors who treat the construction of tables of equations.]

IV. Further, the way of constructing and applying tables of the equations of the centre, or of the eccentric place, and of the orb, is handed down by *Ptolemy*, Book 11, chapter 10; by *Copernicus*, Book 5, chapters 33 and 34; by *Fernel*, Book

2 of the *Cosmotheoria*, chapter 3; by *Reinhold* on Peuerbach, from p. 99 to 106; by *Wurstisen* in the same place, from p. 158 to 171;

by *Magini* in the *Theoricae*, chapters 6, 11 and 12, and in the *Supplement to the Ephemerides*, p. 21; by *Longomontanus* in Book 2 of the *Theorica*, chapter 12; by *Kepler* on Mars, chapter 60, and in the *Epitome of Copernican Astronomy*, Book 5, part 2, and Book 6, p. 749, and in the Rudolphine Tables, precepts 85 and 96; and by *Cavalieri* in the *Directorium Uranometricum*, from p. 144 on. And the equations of the eccentric are collected through all the degrees of the semicircle of the anomaly of the eccentric, in the way I taught in the geometrical calculation, according to each hypothesis: granted that Ptolemy collected them by sixes or by threes, and Copernicus by threes of degrees of this anomaly. In the other semicircle of the anomaly, moreover, the equations are equal at an equal distance from the apogee or from the perigee. But it is not enough to construct the equations of the orb, or of the argument, etc., for the single degrees of the anomaly of the orb, because each of them varies with the varying position of the centre of the epicycle, that is, with the varying anomaly of the eccentric. Thus in the preceding figure, no. 1, with the centre placed at F, one table would have to be constructed for the equations of the epicycle for the single degrees of the epicycle; with the centre placed at I, another table; and so for the other positions. Consequently 180 tables would have to be constructed, for the single degrees of the semicircle FHM, which

Diagram. In the left column, interrupting the sentence between *quod* and *esset*. A large outer circle, and within it a second circle drawn on a centre above the centre of the outer circle, its top meeting the top of the outer circle. Small circles stand on the inner circle: at the top about F; at upper left about I; at left two overlapping about H and ε; at lower left about K; and at the foot about M. A vertical line runs from the top through F, a, E, D, B and A, on through M and b to the foot of the outer circle. Lines radiate from A, past the small circles, to the rim of the outer circle. A dotted line runs from H towards D.

- Labels read, top of the figure: L, T, G.
- Upper left: M and L outside the rim; I in its small circle; c beside that circle and c below it.
- On the vertical line: F, a (italic), E, D, B, A, M, b (italic).
- Left: N, V, L outside the rim; H; ε; Z; c.
- Lower left and foot: K; c, c; O; L; c and S beside the lower small circle; P, R, Q, L along the foot of the rim.
- Right: X within the rim; Y outside it.
- Labels not read: none noticed.

would be far too laborious and of monstrous bulk. To avoid this, the old practitioners, from Ptolemy on, used proportional minutes [Scrupulis proportionalibus] and the excess, as is plain in the tables of *Ptolemy*, *Alfonso* and *Copernicus*, in the Prutenic Tables, and in those of *Longomontanus* and *Lansbergen*. The device is of this kind.

Problema 1. Scrupula Proportionalia in Tabula Aequationum Centri diſtribuere.

Problem 1. To distribute the proportional minutes in the table of the equations of the centre.

[Margin: On proportional minutes and the excess.]

V. *First*, with the centre of the epicycle placed at the single degrees of the semicircle FHM, or of the mean anomaly of the eccentric, say at F, I, ε, K, M, etc., collect the greatest equations of the epicycle in the triangles right-angled at C, in the way the remaining equations of the epicycle are usually collected, namely by means of the distance of the centre of the epicycle from the centre of the world, A. *Second*, value as 60 the excess of the greatest of the greatest equations, that is PR, which the angle PAR measures, over the least of the greatest, namely the arc GL, measured by the angle GAL. To that proportion reduce the excesses of the greatest equations over the least of the greatest: that is, the excess of the arc LM, or of the angle LAM, the greatest epicyclic equation when the centre is at I, over the arc GL; and when the centre of the epicycle is at ε, compare the excess of the arc NL, again over the arc GL; and when the centre of the epicycle is at K, compare the excess of the arc OL over the arc GL, reducing these excesses to sixtieth parts, or to parts such that the excess of the arc PR over the arc GL is 60 of them. These proportional minutes, or sixtieths of the excess, write in the third column of the equations of the centre, opposite the degrees of the mean anomaly of the eccentric; and the 60 minutes themselves write opposite 180°, or 6^{s}, of the anomaly.

EXEMPLVM.

Example.

[Margin: Example of proportional minutes in ♄.]

VI. Let the centre of Saturn's epicycle be at the apogee of the eccentric, F, and Saturn at the point C of the epicycle, where the greatest epicyclic equation for that position occurs; and let that equation, that is, the arc GL, be 5° 28′. Next let the centre of the epicycle be at the perigee, M, and Saturn at the contact C, where the greatest equation of the epicycle comes about; and let this, that is, the arc PR, be 6° 8′. The greatest excess between the equations at the contacts, or the greatest equations, will then be 40′. Keep this, then, and value it as 60 minutes. Now let the centre of the epicycle be at ε, and let the arc Fε, of the mean anomaly of the eccentric, be 94°; and let Saturn be at the point of contact C, where the greatest equation of the epicycle placed at this position on the eccentric occurs; and let that greatest equation be 5° 46′, so that the arc NL exceeds the arc GL by 18′. Therefore make as 40 is to 60, so 18 to another number, namely to 27: the proportional minutes to be written in the table of the equations of the centre, opposite 94°, or 3^{s} 4°, of the mean anomaly of the eccentric. And so for the rest.

Problema 2. Aequationes Orbis ſeu Epicycli in Apogeo Eccentrici poſiti, ad eiuſdem Aequationes in quocumq. ſitu Eccentrici redigere. Seu Tabulam vnicam Aequationum Orbis, cum Exceſſu earum congruenti conſtruere.

Problem 2. To reduce the equations of the orb, or of the epicycle placed at the apogee of the eccentric, to its equations at any position on the eccentric; or, to construct a single table of the equations of the orb, with the excess that agrees with them.

[Margin: Step 1.]

VII. *First*, with the centre of the epicycle placed at F, collect the equations of the orb for the single degrees of the semicircle GCa of the epicycle, counted from the apogee of the epicycle, G, in the order of the signs towards C. Write them in the second table, which is headed *Prosthaphaereses of the Orb*, opposite the degrees of the true anomaly of the orb. For example, if the planet is imagined at T, and the arc TG, that is, the anomaly of the orb, or the angle GFT, is 30°, the angle AFT, the complement to two right angles, will be 150°, in the triangle AFT. In this triangle FT is given, that is, the semidiameter of the epicycle, according to the measures of each hypothesis reduced to the Ptolemaic form by chapter 6, no. 13. Also given is the greatest longitude of the centre of the eccentric, which is the side AF, since it consists of the radius DF and the eccentricity of the deferent, AD. Therefore from these sides comprehending the known angle AFT you will collect trigonometrically the angle TAF, that is, the arc of the ecliptic GT, which is the equation of the orb, to be written opposite 30° of the anomaly of the orb in the second column. It also serves 330°, because at an equal distance of the planet from the apogee the prosthaphaereses of the orb are equal. In the first column are placed the degrees of the anomaly of the orb itself.

[Margin: Step 2.]

Second, with the centre of the epicycle placed at the perigee, M, for the single degrees of the former semicircle of the epicycle, counted from the apogee of the epicycle, b, towards C, you will collect the equations of the orb that agree with such a degree of the anomaly of the orb. Write these separately in another table, distinct from the table of the prosthaphaereses of the orb. For example, if the planet is at S, and the arc of anomaly bS is given, that is, the angle bMS, the angle AMS will be known, as the complement to two right angles. Therefore in the triangle AMS, if the semidiameter of the epicycle MS is given, and the nearer longitude of the centre, AM, that is, the remainder of the radius DM after the eccentricity AD has been taken away, the angle ASM will not stay hidden, that is, the arc RQ, which is the prosthaphaeresis of the orb that suits that degree of the anomaly of the orb.

[Margin: Step 3.]

Third, subtract the equations of the orb collected at the apogee from the equations of the orb collected at the perigee, comparing them at the same degree of anomaly; and write the excess of the perigean equations in the third column

of the prosthaphaereses of the orb, opposite the degrees of the anomaly of the orb against which you have already written the apogean, or least, equation.

EXEMPLVM.

Example.

[Margin: Example in ♄. See the table of chapter 6, no. 12.]

VIII. Let the eccentricity of the deferent of ♄, AD, from the measures of *Lansbergen*, but in Ptolemaic form, be 5700 of the parts of which the radius DF is 100000, and let the semidiameter FT of the epicycle of ♄ be 10070; and so the line AF will be 105700 parts, and AM 94300. And let Saturn be now at T, at a distance of 30° from the apogee G, and now at S, likewise at a distance from the a- pogee b, of 30°. For in the triangles AFT and AMS the angle AFT will be 150°, no differently from the angle AMS. Therefore, given FT and MS, each of 10070 parts, and AF of 105700, but AM of 94300, the angle FAT, or the arc GT, will come out as 2° 31′, but the angle MAS, that is, the arc RQ, as 2° 48′, which exceeds the arc GT by 17′. So in the table of the prosthaphaereses of the orb, in the first column you will set the degree 30 of the anomaly of the orb; opposite it, in the second column, which is titled *Prosthaph. of the orb*, 2° 31′; and in the third column, which is titled *Excess*, you will write 17′. And so for the rest. Remember, moreover, that the prosthaphaereses of the orb are to be added to the eccentric place of the planet, or the place of the centre, in the first semicircle of the anomaly of the orb, and subtracted in the second, so as to have the true place of the planet, the one apparent to those observing it from the Earth. The equation of the eccentric place, by contrast, is to be subtracted in the first semicircle of the anomaly of the eccentric and added in the second.

Problema 3. Aequationes Maximas Centri, earumq. Locum ſeu Gradum Anomaliæ ex Tabulis Aequationum colligere.

Problem 3. To collect from the tables of equations the greatest equations of the centre, and their place, or degree of anomaly.

IX. In the preceding figure, no. 1, let the centre of the epicycle be at ε, where the greatest equation of the centre occurs. Consult, then, the table of the prosthaphaereses of the centre made for each planet, running through the second column until you find in it the greatest number of degrees and minutes. That will be the greatest equation of the centre, and in the first column you will have the degree of anomaly at which that greatest equation comes about. Thus in *Lansbergen's* table of the prosthaphaereses of the centre of Saturn, which he sets out on p. 108, you will see that the greatest equation is 6° 31′, opposite 94°, or 1 sexagene 34° [Sexag.1. Gr.34.].

Problema 4. Aequationes Maximas Orbis, in quocumq. loco Eccentrici Epicyclus ſit, ex Tabulis Aequationum colligere, nec non Locum illarum in Epicyclo.

Problem 4. To collect from the tables of equations the greatest equations of the orb, at whatever place on the eccentric the epicycle may be, and also their place on the epicycle.

[Margin: Case 1. For the apogee of the eccentric.]

X. *First*, if the centre of the epicycle is at the apogee of the eccentric, the second table itself, that of the prosthaphaereses of the orb, will show the greatest equation of the orb, if, running through its second column, you observe what the greatest number of degrees and minutes is; and opposite it in the first column you will see the degree of the anomaly of the orb to which that greatest equation belongs. That place, however, will be had more exactly if the greatest equation is added to 90°. Thus in *Lansbergen's* table of the prosthaphaereses of the orb of Saturn, which he has on p. 110, the greatest equation of the orb at the apogee, that is, the arc GL, is 5° 28′, which corresponds to several degrees of the anomaly because the seconds are omitted. Therefore, by adding 5° 28′ to 90°, the truer place on the epicycle at which that greatest equation occurs is 95° 28′ of the anomaly. In the tables of *Ptolemy* and *Alfonso*, however, from the greatest equation at the mean longitude must be taken the greatest difference set in the column of the longer longitude.

[Margin: Case 2. For the perigee of the eccentric.]

Second, if the centre of the epicycle is at the perigee of the eccentric, collect as above, from the table of the prosthaphaereses of the orb, the greatest equation in the second column and the excess in the third. Add the excess to the greatest apogistic equation, and the greatest perigistic, or absolute, equation will result. Then add this to 90°, and you will have the degree of anomaly to which this greatest equation belongs and at which it comes about. Thus in the already mentioned table of *Lansbergen*, beside the greatest equation of the orb of Saturn, 5° 28′, an excess of 40′ is set in the third column. Therefore the greatest equation at the perigee of the eccentric, that is, the arc PR, is 6° 8′, and the place on the circumference of the epicycle that suits it is 96° 8′, reckoned from the apogee of the epicycle. In the tables of *Ptolemy* and *Alfonso*, however, to the greatest equation sought in the column of the mean longitude must be added the greatest difference found in the column of the nearer longitude.

[Margin: Case 3. Away from the apogee and perigee.]

Third, if the centre of the epicycle is elsewhere, say at ε, with the mean anomaly of the eccentric seek in the first table of the equations of the centre, but in its third column, the proportional minutes; and in the second table, which is that of the equations of the orb, seek as in case 2 both the greatest equation of the orb (which I call apogistic) and the greatest excess, which is to be found in the third column. Then multiply this greatest excess by the proportional minutes, and divide the resulting sum by 60. For the quotient, added to the greatest apogistic equation, will make up the greatest equation of

the orb that suits the planet for that place of the centre of the epicycle. Added to 90°, this will give the place on the epicycle to which that equation belongs. One must proceed a little differently in the tables of *Ptolemy* and *Alfonso*.

EXEMPLVM.

Example.

XI. Let the centre of the epicycle of ♄ be at ε, and Saturn at C, and let the mean anomaly of the eccentric, or the arc Fε, be 94°, to which in *Lansbergen's* table there correspond 27′ of proportional minutes. Since, from the second table of the same author, of the prosthaphaereses of the orb of ♄, the greatest equation is 5° 28′ and the greatest excess 40′, 27 multiplied by 40 makes 1080′; these divided by 60′ leave 18′. Add, then, 18′ to 5° 28′, and the greatest equation, that is, the arc NL, is 5° 46′; and the degree of the epicycle at C, counted from the apogee of the epicycle, to which this equation belongs, is 95° 46′.

Problema 5. Proſthaphæreſes tum Centri tum Orbis ex Tabulis continentibus Scrupula proportionalia, & Exceſſum colligere.

Problem 5. To collect the prosthaphaereses both of the centre and of the orb from tables containing the proportional minutes and the excess.

[Margin: Step 1.]

XII. *First*, with the mean anomaly of the eccentric seek the equation of the centre and also the proportional minutes, and keep them. Subtract the equation of the centre from the planet's mean motion in longitude in the first semicircle of the anomaly, and add it in the second. For the result will be the place of the centre of the epicycle, or the true centre, or the eccentric place of the planet.

[Margin: Step 2.]

Second, subtract the eccentric place of the planet from the mean motion of the Sun according to the old astronomy, or from the true motion according to recent astronomy, and there will remain the true anomaly of the orb, otherwise called the annual argument. With this, in the table of the prosthaphaereses of the orb, seek the equation of the orb in the second column, which is the one that would occur if the centre of the epicycle were at the apogee of the eccentric; and in the third column the excess of the equation of the orb, namely that by which the equation of the orb, with the planet placed at the same degree of anomaly but the centre of the epicycle placed at the perigee of the eccentric, exceeds the equation of the orb with the centre placed at the apogee. Multiply this excess by the proportional minutes found in step 1, and divide the resulting sum by 60. For there will result the excess, always to be added to the equation of the orb that was found, so as to have the corrected equation. This, added to the eccentric place of the planet in the first semicircle of the anomaly of the orb and subtracted in the second, will give the true place of the planet as seen from the Earth.

De Imperfectione Aequationum Orbis præcedenti methodo inueſtigatarum.

On the imperfection of the equations of the orb investigated by the preceding method.

XIII. The device already mentioned rests on a certain proportion [Analogia], close to the truth indeed, but not exactly true. For in the preceding figure let the centre of the epicycle be now at F, now at ε, now at M; and let the planet at these three places be once at the contacts C, where the greatest equation of the epicycle comes about, and at another time be at T, at Z and at S, yet in such a way that the arcs of anomaly TG, ZH and bS are equal, say of 30° each. With this laid down, the old astronomy supposes that as the greatest equation, or the arc RL, is to the greatest GL, so the equation or arc RQ is to the equation or arc GT; or that as the excess of the arc RL is to the arc GL, so too is the excess of the arc RQ to the arc GT. Similarly it holds that as the arc NL is to the arc GL, so is the arc NV to the arc GT. But this is not so, because the arcs of the epicycle that correspond to the greatest equations are not (as is here supposed) equal, wherever on the eccentric the centre of the epicycle is set. And already in the examples brought from *Lansbergen* for ♄ at no. 10 we saw that the arc GC is 95° 28′, the arc HZC 95° 46′, and the arc bSC 96° 8′.

[Margin: Cause 1 of the imperfection already mentioned.]

Although, therefore, in the method already mentioned the excesses of the other equations of the orb are compared with one another with respect to the same anomaly of the orb, the excesses of the greatest equations, and the proportional minutes derived from them, are nevertheless not with respect to the same anomaly of the orb. And this indeed is the first cause why equations of the orb collected by such a method differ by some minutes from the exact prosthaphaeresis: granted that *Ptolemy*, *Alfonso*, *Copernicus*, *Longomontanus*, *Lansbergen*, *Giuntini*, *Stadius*, *Magini* in the canons of the *Secunda Mobilia*, and almost all the older astronomers have used it.

[Margin: Authors who recognized this imperfection.]

But its imperfection, though they did not disclose the cause, was at length recognized by *Magini* in the *Supplement to the Ephemerides*, canon 12, and, in his pages, by *Ricardus Forsterus* [Richard Forster], the most learned physician and mathematician of London; and by *Cavalieri* in the *Directorium*, p. 145. *Magini's* words are: *For I am not alone in having often found by experience that the form of calculation handed down by our predecessors for computing the equations of the orb by proportional minutes and the excess is deceptive and erroneous; others have too.* He then adds words from a letter of Forster, writing to Magini himself: *I have examined the calculation of the most learned Erasmus Reinhold in ♂, and it strays by 16′ 20″ from the true value according to his own hypothesis, by the doctrine of triangles, in the parallaxes of the orb. I attribute this difference to the operation by proportional minutes and the excess, which is almost never done exactly.*

[Margin: Cause 2 of the imperfection.]

[Margin: The opinion of Boulliau and Kepler.]

The second cause is that the ancients referred the apparent motion of the planet on the epicycle to the centre of the mean motion of the Sun; and therefore, to investigate the anomaly of the orb, they subtracted the central place of the planet from the mean motion of the Sun. Hence for them the method of calculation was constant. But since their motion ought to be referred to the Sun, while the true place of the Sun varies perpetually, as does its true distance from the Earth, it comes about that the proportion of the annual orb in the planets, that is, the semidiameter of the Ptolemaic epicycle, varies perpetually. Therefore a single table cannot be constructed that would show, by proportional minutes and the excess, correct equations of the orb, especially in ♂, as *Boulliau* learnedly points out in Book 8 of the *Astronomia Philolaica*, chapter 9. And this is what *Kepler* obscurely indicated in the *Epitome of Copernican Astronomy*, p. 751, in these words: *Because it cannot be that we refer the parallaxes of the orb to the semidiameter of the orb, as is done in the old astronomy, therefore in place of the double proportional minutes and the excess of the parallax of the orb, or of the variation of the diameter, a number has been introduced indicating what the proportion is of the sum of the intervals of the Sun and the planet, and of the Earth, to the difference.* But *Longomontanus* and *Lansbergen* too tried to correct the equations of the orb of Mars, as I shall indicate in Scholium 1.

Primus Modus condendi exactiùs Tabulas Aequationum Orbis, eaſq. adhibendi ex Magino.

First way of composing the tables of the equations of the orb more exactly, and of applying them, from Magini.

[Margin: Step 1.]

XIV. *First*, from Magini in the *Supplement to the Ephemerides*, canons 12 and 13, collect for the single degrees of the mean anomaly of the eccentric the greatest equation of the orb by means of a right-angled triangle, as I indicated above in no. 5. Add it to 90°, and you will have the point of contact, which divides the half-arc of direction from the half-arc of retrogradation, that is, the half-arc of the epicycle on which the planet is direct from the half-arc on which it is retrograde.

[Margin: Step 2.]

Second, with the eccentric place of the planet subtracted from the place of the Sun, either mean, according to the old astronomers, or true, according to the more recent, obtain the true anomaly of the orb. If this is less than the half-arc of direction, or greater than the sum of the half-arc of direction and the whole arc of retrogradation, it is a sign that the planet is in the upper part of the epicycle, or direct, and that the half-arc of direction is to be used. But if the anomaly of the orb is greater than the half-arc of direction, but not greater than the sum of the half-arc of direction and the whole arc of retrogradation, the planet will be retrograde, and the half-arc of retrogradation is to be used.

[Margin: Step 3.]

Third, consult the General Table of Positions which *Magini* sets out in the Tables of Directions of the primum mobile, from p. 137 on, and in it seek in the left margin the half-arc of direction or the half-arc of retrogradation, just as if it were a semidiurnal arc; and in the body of the table [area] opposite it look for the equated anomaly of the orb. For above that, at the head of the table, you will have the head number [numerum frontalem], whose difference from the true anomaly of the orb will be the equation of the orb sought. Magini, moreover, constructed and set out in the *Supplement to the Ephemerides*, from p. 202 to 216, tables in whose third column you have the greatest equations of the five minor planets, derived from the hypothesis of Copernicus, for the single degrees of the anomaly of the eccentric. Therefore from these and from the General Table of Positions the whole business of the equations of the orb can be completed.

Secundus Modus condendi Tabulas Aequationum Orbis exactiùs, eaſq. adhibendi ex Magino & Keplero.

Second way of composing the tables of the equations of the orb more exactly, and of applying them, from Magini and Kepler.

[Margin: Magini's way 2.]

[Margin: Kepler's way.]

XV. *Magini* barely and thinly indicates this way in the *Supplement to the Ephemerides*, canons 12 and 14. It consists both in the search for the mystic number [numeri Myſtici] and in the construction of a general table of the equations of the orb. The mystic number arises from the commensuration of the orbs of the planet and of the Sun (or, for Copernicus, of the Earth), and under it, with the true anomaly of the orb, the desired equation of the orb is collected in the general table of equations. So Magini, in the same *Supplement*, from p. 202 to 216, puts the mystic number in the last column, and from p. 218 sets out the General Table of the Equations of the Orb. He calls it by that name because, given the mystic number arising from any hypothesis and measures of the orbs, the same table serves all hypotheses, and indeed for any planet. That mystic number is the proportion of the difference of the intervals or distances of the planet from the Earth and of the Sun from the Earth to the sum made up of the same intervals, but in parts of which the sum of the distances is 60, as *Magini* employs it, or in parts of which the sum of the distances is 100000, as *Kepler* employs it in the *Epitome of Copernican Astronomy*, p. 751, where he simply calls this number the *Index*.

[Margin: Step 1.]

First, therefore, collect for the single degrees of the semicircle of the anomaly of the eccentric the distance of the planet from the Sun, as we taught in the calculation by geometrical forms, according to each hypothesis. To it add the mean distance of the Sun from the Earth, or at least, for Mars, the true one, and you will have the sum of the intervals. But if you subtract that distance

of the Sun from the Earth from the distance of the planet from the Sun, you will have the difference. Multiply this difference either by 60 or by 100000, and divide the resulting sum by the sum of the intervals. For the quotient will be the index of the proportion already mentioned, in parts of which the sum of the intervals has been taken as 60 or as 100000. *Kepler* admits, however, in the same place that in the Rudolphine Tables he set, in place of the quotient already mentioned, its logarithm, under the interval of the planet from the Sun.

[Margin: Step 2.]

Second, as *Kepler* teaches there, multiply the tangent of half the true anomaly of the orb by that Index, but found in parts of which the sum of the intervals is 100000; and you will have the tangent of the arc to be added to that half in the superior planets and taken away in the inferior, so as to have the angle of elongation of the planet from the Sun. But if, as in the Rudolphine Tables, a logarithm stands in place of the Index, add to this logarithm the mesologarithm of half the true anomaly of the orb, and there will result the mesologarithm of the arc to be added to the half-anomaly itself in the superior planets, and taken from it in the inferior, so that the elongation of the planet from the Sun visible under the ecliptic results, or the angle at the Earth.

[Margin: Step 3.]

Third, if the eccentric place of the planet is less than the true place of the Sun, subtract the elongation already mentioned from the true place of the Sun; but if greater, add it; and the visible place of the planet in the ecliptic results, or the place apparent to those observing the planet from the Earth. In *Magini's* general table of the equations of the orb, however, under the mystic number you have the equations of the orb to be added to or taken from the eccentric place of the planet, as the titles at the head of the table indicate. For other, more troublesome precepts of the Rudolphine Tables, see the Rudolphine Tables, chapter 22.

Tertius Modus Condendi & Adhibendi Tabulas Aequationum Orbis ex D. Vincentio Reinero, & D. Petro de S. Legier, & Iſmaele Bullialdo.

The third way of constructing and using tables of the equations of the orb, from D. Vincenzo Renieri, D. Petrus de S. Legier [?] and Ismaël Boulliau.

XVI. *D. Vincenzo Renieri*, in the newly reissued *Tabulae Mediceae*, p. 243; *D. Petrus de S. Legier* [?] of Avignon, in a double table of the equations of the orb published at Avignon in 1640 and sent on to me from there by Father Théophile Raynaud; and lastly *Ismaël Boulliau*, in the *Philolaic Tables* from p. 98 on, have published a general table of the equations of the orb, suited to all planets affected by the second inequality and to all hypotheses, provided that the true anomaly of the orb is given, and the greatest equation of the orb corresponding to the anomaly of the eccentric. For the anomaly of the orb is looked up at the sides, the greatest equation of the orb at the head, and the equation of the orb in the

common area. As for the greatest prosthaphaeresis, *Renieri* and *Boulliau* give it in special tables of the prosthaphaereses of each planet, because it depends on the particular commensuration of the orbs themselves. That general table of the equations of the orb, moreover, is constructed by solving an oblique-angled rectilinear triangle in which there are given the proportion of two sides, namely the distance of the Sun from the Earth and of the planet from the Sun, and the angle they contain, which is called the angle of the anomaly of the orb, or of commutation. It becomes known by subtracting the true eccentric place of the planet from the true place of the Sun, or the latter from the former, etc. For one of the angles at the base found in that triangle, namely the angle at the planet, is the equation of the orb. The more exact table, however, is D. de Legier's second, because it contains degrees, minutes and seconds.

Quartus Modus Conſtruendi Tabulas Aequationum Orbis ex P. Bonauentura Caualerio.

The fourth way of constructing tables of the equations of the orb, from Father Bonaventura Cavalieri.

XVII. From Father Bonaventura Cavalieri's *Directorium Uranometricum*, part 2, chapter 4, p. 144, collect the distance of the planet from the Sun for the single degrees of the semicircle of the anomaly of the eccentric. Beneath those distances, or opposite them, write the arithmetical complement, or residual of the logarithm; and beside the distances of the Sun from the Earth write the logarithm. For the sum of the residual already mentioned and the logarithm is the logarithm of the greatest equation of the orb; and that same sum, sought as a mesologarithm, will give an angle whose residue to 45° is the mystic angle [angulus Myſticus] (for so he calls it, imitating Magini), an angle common to all the equations of the orb to be found for a given interval. Indeed, the mesologarithm of the mystic angle, joined to the mesologarithm of the half-anomaly of the orb, makes up the mesologarithm of the angle to be taken from the half-anomaly, so as to have the lesser angle, or to be added, so as to have the greater angle. In these angles the equation of the orb consists: in the lesser, when the distance of the Sun from the Earth is less than the interval of the Sun and the planet; in the greater, when it is greater.

EXEMPLVM.

Example.

[Margin: Example in ♄.]

XVIII. Let there be, from the Rudolphine Tables, as Cavalieri sets it, a distance of Saturn from the Sun of 971197, of parts of which the radius of the annual orb is 100000; a distance of the Earth from the Sun of 101546 of those parts; and an anomaly of commutation, or of the orb, of 155° 49′ 13″, with its half 77° 54′ 36″. You will therefore set out the first operation thus.

Dist. ♄ from ☉ 971197.	Res. Logar.	401273
Dist. ☉ from Earth 101546.	Logarith.	500680
Angle 5° 58′ 17″.	Mesol.	901953

Next, with the angle 5° 58′ 17″ subtracted from 45°, there remains the mystic angle 39° 1′ 43″. The second operation is therefore to be set up thus.

Mystic ang. 39° 1′ 43″.	Mesol.	990881
Half-anomaly 77° 54′ 36″.	Mesol.	1066918
Angle 75° 11′ 54″.	Mesol.	1057799

Therefore, with the angle 75° 11′ 54″ taken from the angle of the half-anomaly, 77° 54′ 36″, because Saturn's distance from the ☉ is greater than that of the Sun from the Earth, there remains the most exact equation of the orb, 2° 42′ 42″.

Quintus Modus Noſter Conſtruendi Tabulas Aequationum Orbis Maximarum.

The fifth way, our own, of constructing tables of the greatest equations of the orb.

XIX. *First*, a place of the planet observed both at the apogee and at the perigee of the eccentric, or as nearly as possible, is to be chosen, and the eccentric place of the planet and the true place of the Sun are to be sought from the tables. For the former subtracted from the latter will give the anomaly of the orb, while the difference of the eccentric place of the planet from the observed place will give the equation of the orb required. Go therefore to the general table of the equations of the orb, of which I spoke at no. 16, and in its area [arealiter] seek the equation of the orb already mentioned, opposite the anomaly of the orb: for at the head of the table you will have the greatest equation of the orb, but the proportional part must be used with exactness. Having thus obtained the greatest equation already mentioned, both apogeal and perigeal, look up the logarithm of each and reserve it.

Second, from our table of measures placed at the end of chapter 6, collect the distance of the centre of the epicycle from the centre of the Earth, both the apogeal or greatest, which is called the longer longitude, and the perigeal, which is called the nearer longitude; and look up the logarithm of each. Then add together the logarithm of the greatest equation of the orb and the logarithm of the distance of the centre of the epicycle, first at the apogee, then at the perigee, and you will have the logarithm of the semidiameter of the epicycle, greatest at the apogee and least at the perigee. The latter subtracted from the former will give the difference of the semidiameters of the epicycle; reserve half of this.

Third, with the mean anomaly of the eccentric given or assumed, look up both the distance of the centre of the epicycle from the centre of the Earth, according to the method given in chapter 6, no. 10, at the end of step 2, and the temporary semidiameter of the epicycle, or that suited to the anomaly already mentioned, according to the precept given in the same place in step 3, using half the difference of the semidiameters. For if you join into one sum the logarithm of the semidiameter of the epicycle and the residual of the logarithm of the distance of the centre of the epicycle, the result will be the logarithm of the greatest equation of the orb to be applied in that position of the eccentric, so as to obtain, by means of the general table of the equations of the orb, the equation of the second inequality.

EXEMPLVM in ♂.

Example in ♂.

[Margin: Step 1, for the greatest equation of the orb at the apogee of the eccentric.]

XX. On 1587 January 5, Old Style, as *Kepler* relates in the *Commentaries on Mars*, chapter 42, Tycho observed Mars at ♎ 2° 8′ 30″, at 9^{h} 31^{m} p.m. at Uraniborg. At that time, for me, the Sun was at ♑ 25° 24′, and the mean motion of ♂ from the Rudolphine Tables was 4^{s} 29° 27′, and the motion of the apogee of ♂ 4^{s} 28° 43′. Therefore, with the latter subtracted from the former, the anomaly of the eccentric was 0^{s} 0° 44′, so that Mars was almost at the very apogee of the eccentric. The equation of the centre from my tables was 7′ 20″. This, subtracted from the mean motion of ♂, leaves the eccentric place of ♂ as 4^{s} 29° 19′ 40″. That place, taken from the true place of the Sun, gives the anomaly of the orb, 4^{s} 26° 4′ 20″; and taken from the true place of Mars, it leaves the equation of the orb, 32° 58′ 50″. In the general table of the equations of the orb, opposite the anomaly 4^{s} 26° 4⅓′, this equation is found only under the head number 36° 17′. Therefore the greatest equation of the orb suited to Mars at a distance of 44′ from the apogee of the eccentric is 36° 17′; so at the apogee itself it is prudently set at 36° 15′.

[Margin: Step 2, for the greatest equation of the orb at the perigee of the eccentric.]

XXI. Again, on 1593 July 31, as *Kepler* reports in the same chapter 42, Tycho observed Mars at ♓ 17° 39′ 30″, in the morning, at 1^{h} 15^{m} after midnight at Uraniborg. At that time, for me, the Sun was at ♌ 17° 13′, and the mean motion of ♂ from the Rudolphine Tables, which we follow in this, was

10^{s} 26° 15′, and the motion of the apogee of ♂ 4^{s} 28° 50′. With this subtracted from the former, the anomaly of the eccentric remains 5^{s} 27° 25′, so that Mars was only 2° 35′ from the perigee of the eccentric. At that time, moreover, the equation of the centre was, for me, 32′ 20″, and therefore the eccentric place of ♂ was 10^{s} 25° 42′ 40″. This, subtracted from the true place of the Sun, leaves the true anomaly of the orb, 5^{s} 21° 30′ 20″; while the difference of the eccentric

place from the observed place, that is, the equation of the orb required, was 21° 56′ 50″. In the general table of the equations of the orb, opposite the anomaly already mentioned, this is found only under the head number 47° 36′. From this and from other similar observations I found that the greatest equation of the orb suited to Mars at the very perigee of the eccentric is 47° 40′.

[Margin: Step 3, for the greatest and least semidiameter of the epicycle.]

XXII. Further, from the table of chapter 6, the longer longitude of the epicycle of ♂ is, for me, 120000 parts, and the nearer longitude 82730. The logarithm of the number 120000, which is 507918, joined to the logarithm of the angle 36° 15′, which is 977181, makes the sum 485099 (with unity taken away on the left, as is customary). This is the logarithm to which there corresponds in the table of logarithms the apogeal semidiameter of the epicycle, 70957 parts, of which the radius of the orb of Mars is 100000. Next, the logarithm of the number 82730, which is 491766, added to the logarithm of the angle 47° 40′, which is 986879, makes the sum 478645 (with unity taken away on the left). This is the logarithm of the number 61157, which is evidently the semidiameter of the epicycle of Mars at the perigee of the eccentric. The difference of the two is 9800 parts, and half of it 4900.

[Margin: Step 4, for the distance of the centre of the epicycle and its semidiameter.]

XXIII. Now let there be given a mean anomaly of the eccentric of 100°, or 3^{s} 10°. Its complement is 80°, and their second sine is 17365; and the semidiameter of the small circle of the eccentricity of ♂, from the table of chapter 6, is 1365. Let it therefore be: as the whole sine, 100000, is to 17365, so 1365 is to 237. With these taken from the mean eccentricity of Mars, which for me is 18635, there remains the eccentricity of Mars suited to such an anomaly, 18398. With this and with the radius 100000, as sides containing the angle of 80°, there is found the equation of the centre, 10° 36′ 8″; and, with these given, the distance of the centre of the epicycle from the centre of the Earth, 98486 parts, of which the radius of the orb of Mars is 100000. Again, let it be: as the whole sine, 100000, is to the same second sine, 17365, so the half-difference of the epicyclic semidiameters, 4900, is to 851. This number, taken from the mean semidiameter of the epicycle of Mars, which for me is 66057, leaves the semidiameter of the epicycle suited to Mars in that position of the anomaly, 65206 parts, of which the radius of the orb of ♂ is 100000.

[Margin: Step 5, for the greatest equation of the orb.]

XXIV. Finally, the logarithm of the number 65206, that is, of the semidiameter of the epicycle, which is 481429, joined to the residual of the logarithm of the distance 98486, which is 500663, makes the logarithm 982092, to which in the logarithmic tables of arcs there corresponds the angle 41° 28′. So great is the greatest equation of the orb suited to Mars at an anomaly of the eccentric of 100°. Know, then, that our tables have been constructed by this device.

SCHOLIA.

Scholia.

I. Since the distance of the Sun from the Earth bears a more sensible ratio to the distance of Mars, therefore, when the Sun's distance varies, whether annually because of its recession from the apogee, or from age to age, so to speak, because of the varied eccentricity, according to many astronomers, the prosthaphaereses of the orb of Mars also vary sensibly: especially in its acronychal shinings, when it is in ♒, ♓, ♈ and ♉, in which the distance of Mars from the Earth is far less.

[Margin: Lansbergen's correction of the equations of Mars.]

For this reason *Lansbergen*, in precept 16 of his tables, advises that the equations of the centre be corrected thus: With the centric longitude [Longitudine centrica] (just as if it were the anomaly of the eccentric), look up the prosthaphaeresis of the centre, and keep it. Next, with the anomaly of the orb, just as if it were the anomaly of the eccentric, look up the proportional minutes. Multiply these by the prosthaphaeresis of the centre and divide the sum by 60: for from the quotient there results the equation to be added to or taken from the centric longitude, as the titles of the table indicate.

[Margin: And Longomontanus's.]

Longomontanus, in Book 2 of the *Theorica*, chapter 9, judged that on this account a libration should be introduced, either of the eccentric or of the annual orb of Mars, to account for this sub-anomaly of the annual anomaly; and he constructed a table in which there is the analogy of the distance of the Sun from the apogee in the annual orb of Mars. On its use consult him, chapters 9 and 12. But they needed this remedy because they had first hunted for the equations of the orb in the old way, through proportional minutes and the excess, though in vain, especially for Mars.

[Margin: Boulliau's consideration for the greatest equations of the orb.]

II. *Boulliau*, however, in Book 8, chapter 9, and in the tables, pp. 17, 33, 47, 65 and 83, considering that the least distance of the Earth from the Sun, compared with the radius of the planet's orb, appears greater at perihelion, especially of ♂, than at aphelion, called this excess the increment of the side from the Sun to the Earth [Incrementum lateris à Sole ad terram], for it grows from aphelion to perihelion; and he set it in the tables, but distinguished a first increment from a second. He calls first that by which the distance of the Sun from the Earth grows of itself from the Sun's perigee to its apogee, in parts of which the radius of the planet's orb is 100000, or in parts of which the radius is 60. He calls second that by which the ratio of the side of the distance of the Sun from the Earth grows in relation to the decrease of the distance of the planet from the Sun. With the help of the side and the increment already mentioned he hunted down the greatest parallaxes of the orb, which are necessary for the use of the general table of the prosthaphaereses of the orb. He gives the practice of using the table of equations on p. 435, and we, after him, in the practical part of this volume.

III. The longitude of the planet found through the equations of the orb must still be refined, also because of the reduction of the planet from its orbit to the ecliptic, a correction that has joined to it the curtation of the distance of the planet from the Sun. But of this, since it depends on the latitude and the motion of the nodes, we shall have to speak below, in Section 5. Meanwhile consult *Kepler* on Mars, chapter 9, and in the *Epitome*, p. 701; and *Hérigone*, volume 5 of the *Cursus Mathematicus*, p. 559.

IV. Ptolemy calls proportional minutes [Greek: ἐξήκοστα μέρη ἐπιβάλλοντα (hexēkosta merē epiballonta, "sixtieth parts falling to it")]; and the excess of the greatest equations of the orb at other places of the eccentric, over the greatest at the apogee of the eccentric, is called [Greek: ὑπεροχὴ τῶν προσθαφαιρέσεων (hyperochē tōn prosthaphaireseōn, "excess of the prosthaphaereses")]. In its place the Alphonsines and Peuerbach have the diversity of the diameter [Diuerſitas Diametri], on which see *Reinhold* on Peuerbach, pp. 105 and 132, and *Wurstisen*, p. 178. It is that by which the mean longitude, or distance of the centre of the epicycle, exceeds the least or is exceeded by the greatest. For that whole excess, converted into 60 small parts and compared with the excesses of the remaining distances, produces the proportional minutes of which I spoke at no. 5, but with this difference, that the greatest number, namely 60, is placed at the beginning of the anomaly of the centre. The Alphonsines also differ in seeking the proportional minutes by means of the coequated anomaly of the eccentric, which *Reinhold* censures there, p. 106, since *Ptolemy* uses the mean anomaly. The remaining differences in arranging the tables of proportional minutes and excesses are plain from a mere inspection of the older tables. One is notable, however: Ptolemy and Alfonso set the equations of the orb suited to the epicycle at mean longitude, with the difference on either side in two columns, one for the longer longitude, or the interval from the mean distance to the apogeal, the other for the nearer longitude, or the interval from the mean distance to the perigeal. The others, by contrast, usually set the equation of the orb fitting the apogee of the eccentric, with the excess suited to the recession from the apogee all the way to the perigee.

[Margin: On the table of the greatest equations.]

V. It is now my pleasure, at the end of this section, to append a table of the greatest equations, but with the measures brought over to the Ptolemaic form, which uses an eccentric for the first inequality and an epicycle for the second (to the former belongs the equation of the centre, to the latter the equation of the orb), so that comparison of the opinions may be easier. These equations we have derived partly from the tables of the astronomers listed below, partly from what is given by *Ptolemy*, Book 11, chapter 10; *Copernicus*, Book 5, chapter 33; *Mästlin* in the *Appendix* to Kepler's *Prodromus*; *Magini* in the *Theoricae* of these planets, chapters 6 and 11; and *Longomontanus*, Book 2 of the *Theorica*, chapter 12. Remember, moreover, that the greatest equation at the apogee of the eccentric is the least of the greatest equations of the orb. Further, for

the greatest equations of the orb on our hypothesis we have taken the greatest distance of the planet from the Sun, at aphelion, and the least, at perihelion, and the mean distance of the Sun from the Earth, 7300 semidiameters of the Earth. ♂ is an exception: for the apogeal greatest equation we took the least distance of the ☉ from the Earth, which for us is 7000, and for the perigeal the greatest, which is 7600 semidiameters of the Earth.

ÆQVATIONES MAXIMÆ Cum gradibus Anomaliæ in quibus eueniunt.

The greatest equations, with the degrees of anomaly at which they occur.

Authors	Equation of the centre: greatest equation	Equation of the centre: anomaly of the eccentric	Equation of the orb at the apogee of the eccentric: greatest equation	Equation of the orb at the apogee of the eccentric: anomaly of the orb	Equation of the orb at the perigee of the eccentric: greatest equation	Equation of the orb at the perigee of the eccentric: anomaly of the orb
♄						
Ptolemy	6° 32′ 0″	93° 16′ 0″	5° 53′ 0″	95° 53′ 0″	6° 36′ 0″	96° 36′ 0″
Alfonso	6° 31′ 0″	93° 15′ 30″	5° 52′ 0″	95° 52′ 0″	6° 38′ 0″	96° 38′ 0″
Copernicus	6° 31′ 0″	93° 15′ 30″	5° 53′ 0″	95° 53′ 0″	6° 39′ 0″	96° 39′ 0″
Prutenic Tables	6° 30′ 0″	93° 15′ 15″	5° 55′ 33″	95° 55′ 33″	6° 38′ 39″	96° 38′ 39″
Magini	6° 30′ 30″	93° 0′ 0″	5° 55′ 33″	95° 55′ 33″	6° 38′ 38″	96° 38′ 38″
Longomontanus	6° 38′ 17″	93° 19′ 8″	5° 39′ 15″	95° 39′ 15″	6° 21′ 20″	96° 21′ 20″
Kepler	6° 32′ 0″	93° 16′ 0″	5° 43′ 0″	95° 43′ 0″	6° 38′ 10″	96° 38′ 10″
Lansbergen	6° 31′ 0″	93° 15′ 30″	5° 55′ 0″	95° 55′ 0″	6° 8′ 0″	96° 8′ 0″
Boulliau	6° 37′ 10″	93° 18′ 35″	5° 47′ 0″	95° 47′ 0″	6° 30′ 0″	96° 30′ 0″
Ourselves, for the present	6° 39′ 24″	93° 19′ 12″	5° 45′ 0″	95° 45′ 0″	6° 30′ 0″	96° 30′ 0″
♃						
Ptolemy	5° 16′ 0″	92° 38′ 0″	10° 34′ 0″	100° 34′ 0″	11° 35′ 0″	101° 35′ 0″
Alfonso	5° 17′ 0″	92° 38′ 30″	10° 33′ 0″	100° 33′ 0″	11° 36′ 0″	101° 36′ 0″
Copernicus	5° 15′ 0″	92° 37′ 30″	10° 34′ 0″	100° 34′ 0″	11° 35′ 0″	101° 35′ 0″

Authors	Equation of the centre: greatest equation	Equation of the centre: anomaly of the eccentric	Equation of the orb at the apogee of the eccentric: greatest equation	Equation of the orb at the apogee of the eccentric: anomaly of the orb	Equation of the orb at the perigee of the eccentric: greatest equation	Equation of the orb at the perigee of the eccentric: anomaly of the orb
Prutenic Tables	5° 13′ 59″	92° 6′ 59″	10° 30′ 10″	100° 30′ 10″	11° 31′ 23″	101° 31′ 23″
Magini	5° 13′ 59″	93° 0′ 0″	10° 30′ 9″	100° 30′ 9″	11° 31′ 21″	101° 31′ 21″
Longomontanus	5° 26′ 59″	93° 30′ 0″	10° 38′ 33″	100° 38′ 33″	11° 43′ 2″	101° 43′ 2″
Kepler	5° 31′ 37″	93° 45′ 0″	10° 34′ 0″	100° 34′ 0″	12° 5′ 0″	102° 5′ 0″
Lansbergen	5° 15′ 0″	93° 0′ 0″	10° 12′ 0″	100° 12′ 0″	11° 11′ 0″	101° 11′ 0″
Boulliau	5° 33′ 59″	94° 0′ 0″	10° 43′ 0″	100° 43′ 0″	11° 49′ 0″	101° 49′ 0″
Ourselves, for the present ♂	5° 28′ 30″	93° 40′ 0″	10° 39′ 0″	100° 39′ 0″	11° 43′ 0″	101° 43′ 0″
Ptolemy	11° 32′ 0″	95° 46′ 0″	36° 45′ 0″	126° 45′ 0″	47° 1′ 0″	137° 1′ 0″
Alfonso	11° 24′ 0″	95° 42′ 0″	35° 32′ 0″	125° 32′ 0″	49° 13′ 0″	139° 13′ 0″
Copernicus	11° 8′ 0″	95° 34′ 0″	36° 54′ 0″	126° 54′ 0″	50° 22′ 0″	140° 22′ 0″
Prutenic Tables	11° 5′ 59″	95° 2′ 59″	36° 50′ 18″	126° 50′ 18″	46° 38′ 8″	136° 38′ 8″
Magini	11° 5′ 59″	95° 30′ 0″	36° 54′ 18″	126° 54′ 18″	46° 38′ 4″	136° 38′ 4″
Longomontanus	10° 34′ 20″	95° 17′ 10″	38° 8′ 0″	128° 0′ 0″	47° 26′ 0″	137° 26′ 0″
Kepler	10° 37′ 0″	95° 18′ 30″	36° 55′ 0″	126° 55′ 0″	48° 30′ 0″	138° 30′ 0″
Lansbergen	12° 9′ 0″	96° 0′ 0″	36° 54′ 0″	126° 54′ 0″	46° 50′ 0″	136° 50′ 0″
Boulliau	10° 35′ 46″	95° 18′ 0″	37° 42′ 0″	127° 42′ 0″	47° 24′ 0″	137° 24′ 0″
Ourselves, for the present	10° 37′ 10″	95° 30′ 0″	36° 15′ 0″	126° 15′ 0″	47° 40′ 0″	137° 40′ 0″

SECTIO TERTIA DE MOTV LONGITVDINIS VENERIS ET MERCVRII.

Section Three. On the motion in longitude of Venus and Mercury.

CAPVT I. De Hypotheſi Veneris ac Mercurij Ptolemaica, vel Antiqua vſq. ad Copernicum.

Chapter 1. On the Ptolemaic hypothesis of Venus and Mercury, or the ancient one down to Copernicus.

[Margin: Authors on the hypothesis of ♀ and ☿.]

I. The hypothesis of ☿ is explained by *Ptolemy*, Book 9, from chapter 7 to the end, and that of ♀ in Book 10 of the *Magna Constructio*, from chapter 1 to 5; by *Geber*, Book 7, chapter 2; by *al-Farghānī*, from Difference 14 to 17; by *Regiomontanus* in the *Epitome of the Almagest*, Books 9 and 10, from proposition 1; by *Jean Fernel*, Book 1 of the *Cosmotheoria*, chapters 5 and 6; by *Johannes Schöner* in the *Aequatorium Astronomicum*, pp. 14 and 17; by *Peuerbach* in the *Theorica* of ♀, and there by *Reinhold*, from p. 107 to 113; by *Wurstisen*, from p. 186 and from 207; by *Osualdus*, from p. 180; by *Alessandro Piccolomini*, Book 2 of the *Theoricae*, chapter 5; by *Langius* in the *Theoricae Planetarum*, chapter 12; by *Magini* in the *Theoricae*, from p. 41, and in the *Secundum Mobile*, canon 33; by *Christoph Clavius* at the end of the *Sphaera*; by *Hérigone*, volume 5 of the *Cursus Mathematicus*, pp. 533 and 539; and by *Gassendi*, Book 2 of the *Institutio Astronomica*, chapter 7.

This old theory, moreover, is very like the hypothesis of the three superior planets set out in chapter 1 of Section 2, to which one must go back for the figure and the definitions.

[Margin: See the figures set out in Section 2, chapter 1, nos. 2 and 5.]

The hypothesis of Venus and Mercury therefore consists of four orbs, with one circle besides: namely, two deferents carrying the apogee and perigee of the eccentric in the order of the signs [in conſequentia]; the equant eccentric, which is not an orb but a mere circle; the eccentric deferent carrying the epicycle in the order of the signs; and the epicycle carrying the planet, above in the order of the signs and below against that order [in præcedentia]. Of these, according to the opinion of the authors who use solid orbs, the outermost two are eccentric to the centre of the world in a certain respect [secundùm quid]; for the su- rfaces, the convex surface of the highest and the concave of the lowest, are concentric with the world, while the concave surface of the highest and the convex of the lowest are eccentric to the world, though concentric with each other. The deferent, within whose thickness are contained both the equant circle and the epicycle, is eccentric to the world in both its surfaces. The definition of the lines and of the apogees, and the method of the geometric calculation that calls for the equation of the centre and then that of the orb or argument, are entirely the same as in Saturn, Jupiter and Mars, provided

the eccentricity is established. There is no need to repeat them here; chapter 1 of Section 2 should be consulted. Venus also agrees with the three superior planets as regards the bisection of the eccentricity: the centre of its equant is twice as far from the centre of the Earth and the world as the centre of the deferent of the epicycle, or in other words the centre of the deferent lies precisely midway between the centres of the world and of the equant. Finally, Venus and Mercury agree with the three superior planets in being slowest at the apogee of the eccentric and swiftest at its perigee as regards true motion, just as, in the second motion, they are swift and direct towards the apogee of the epicycle, and slow and retrograde towards its perigee.

[Margin: See Section 2, chapter 1, from no. 6 for the definitions and from no. 10 for the geometric calculation.]

De Proprietatibus, quibus Venus ac Mercurius ſic Differunt à trib. Superioribus, vt inter ſeſe Conueniant.

On the properties by which Venus and Mercury so differ from the three superior planets that they agree with each other.

II. First, the apogees of the eccentrics of Saturn, Jupiter and Mars always incline to the north and their perigees always to the south, and they never intersect or cross the ecliptic. But the apogee of Venus and Mercury, and likewise the perigee of the eccentric, undergo every alternation: they take now a northern latitude, now a southern, and sometimes they coincide with the plane of the ecliptic.

[Margin: 1. Property, in the quarter of the apogee.]

Second, although *Messhala* [Māshāʾallāh], in the book *De Orbibus*, rightly said that all the planets are tied to the Sun as if by a chain or rope, the three superior planets are tied to the Sun as regards the second motion, that is, the revolution of the epicycle, and free as regards the first motion, that of the eccentric. Venus and Mercury, by contrast, are free of the Sun's motion as regards their own motion on the epicycle, and tied to the Sun as regards the motion of the eccentric. For the mean motion of the centre of the epicycle, which is carried by the eccentric deferent but moves regularly about the centre of the equant in the order of the signs, is precisely as great as the Sun's mean motion, and indeed is to be computed uniformly from the same starting point. It follows that the line of the mean motion of these planets falls on the same points of the zodiac as the line of the Sun's mean motion; that they are always in mean conjunction with the Sun; that once the Sun's mean motion has been found, the mean motion of Venus and Mercury in longitude is known at once; and that no special tables are needed for it, the canons of the Sun's mean motion being enough. *Reinhold*, moreover, in his commentary on Peuerbach, p. 117, set out the theory of the line of mean motion of the Sun, Venus and Mercury with this diagram, in which D is the centre of the world,

[Margin: 2. Property, in the motion of the eccentric.]

[Margin: Reinhold's diagram.]

Diagram. Reinhold's diagram, set in the left column between the two parts of its sentence: a large outer circle, the ecliptic, with concentric-looking and eccentric circles inside it, a vertical line from A at the top to K at the bottom, an oblique line from H at the lower left to I at the upper right, and several parallel oblique and horizontal lines joining the inner centres to points on the circles. A rayed Sun sign stands at L on the second circle, a tilted ♀ sign stands at the upper right of the Venus circle, and a tilted ☿ sign stands at the left of the small Mercury circle.

- A (top of the outer circle): start of the line of apsides ADK; beginning of Cancer in the text
- G (just below A, on the vertical line): point on the Sun's orb
- B (upper, left of the vertical line): point on the orb of Venus; centre of Venus's epicycle when the Sun is at G
- ♀ (upper right): planetary sign of Venus
- I (upper right, at the end of the oblique line): end of Mercury's line of apsides
- T (above centre, on the vertical line): centre of Mercury's epicycle when the Sun is at G
- ☿ (left of centre): planetary sign of Mercury
- E (centre, above D): centre of the Sun's eccentric
- C (small letter, directly below E): centre of Venus's eccentric
- D (centre): centre of the world
- P (just below and to the left of D, on the oblique line): centre of Mercury's eccentric
- L, with ☉ (right, on the Sun's orb): the Sun
- O (right, to the left of L): centre of Venus's epicycle when the Sun is at L
- S (right of P; the letter stands reversed): centre of Mercury's epicycle when the Sun is at L
- F (right, on the outer circle): beginning of Aries
- M (lower left, on the Sun's orb): the Sun when Mercury's epicycle stands under H
- Q (lower left, beside M): centre of Venus's epicycle in that case
- X (below centre, left of the vertical line): point on Mercury's orb
- R (bottom, on the vertical line): point on the orb of Venus
- N (below R, on the vertical line): point on the Sun's orb
- H (lower left, on the outer circle): beginning of Scorpio; Mercury's apogee
- K (bottom of the outer circle): end of the line of apsides ADK; beginning of Capricorn in the text

and from that centre the ecliptic AHKF is described, in which the beginning of Aries is F and that of Cancer is A. The centre of the Sun's eccentric is E, from which the Sun's orb GMNL is described, and the centre of the eccentric of Venus is C, from which the orb of Venus BR is described. These three

centres lie on one and the same line of apsides ADK, because Peuerbach and the Alphonsines held that the apogee A of Venus and of the Sun is the same, and the perigee K likewise, standing under the same place of the zodiac, namely the former at the beginning of Cancer, the latter at the beginning of Capricorn. But because they placed the apogee of Mercury under the beginning of Scorpio, H, the line of apsides of Mercury is HI, with the apogee on it under H, the perigee under I, and the centre of the eccentric at P, from which the orb of Mercury STX is described. So the eccentricity of the Sun is DE, that of Venus DC and that of Mercury DP. Now with the Sun at G, the centre of the epicycle of Venus is at B and that of Mercury at T. In this case the line TX is the line of the mean motion of Mercury only, while AK is at once the line of the mean and of the true motion of the Sun and of the centre of Venus's epicycle, supposing that they share the place of the apogee. Now let the Sun be at L. The line of the Sun's mean motion will be EL; DF, parallel to it from the centre of the world, will mark on the ecliptic the point F, the place of the mean motion. The line of the mean motion of Venus will be CO, with the centre of the epicycle at O, and the line of the mean motion of Mercury will be PS, with the centre of its epicycle at S. Both are parallel to the lines EL and DF, and so the mean motion of these three planets will be at the beginning of Aries, F. Now let the centre of the epicycle of Mercury be under the apogee H, that is, on the line of its apsides, and occupy the beginning of Scorpio. Then, by the same agreement, the Sun will be at M and the centre of the epicycle of Venus at Q, and the lines of the mean motions of the Sun, EM, and of Venus, CQ, will be parallel to each other and to the line DPH, which in this case is the line both of the mean and of the true motion of Mercury as regards the motion of the centre. From these you will easily learn all the other cases. In one year, therefore, each completes a single revolution on the eccentric; and just as in the three superior planets the line of the mean motion on the epicycle is parallel to the line of the Sun's mean motion, so in the inferior planets is the line of the mean motion on the eccentric.

Third, as I have already indicated, they differ from the three superior planets as regards motion on the epicycle. The three superior planets complete one revolution in one year, and once each year the Sun is joined to them in mean conjunction when they are placed at the apogee of the epicycle, and once is opposed to them in mean opposition at its perigee. But Venus and Mercury, free of this relation as regards their own motion on the epicycle, complete their synodic revolution in a different time. By its mean motion Venus completes the epicycle in about 19 months, or $583^{d}\ 22^{h}\ 15^{m}$, whereas 19 solar months contain $578^{d}\ 7^{h}\ 13^{m}$. Mercury completes its epicycle by its mean motion in about $115^{d}\ 21^{h}\ 5^{m}$, repeating the variations of the epicycle more than three times a year. So in the number of its motions Mercury is swifter than Venus, since in the time that Venus runs through its epicycle once, Mercury flies around its own about five times, even though in the distance and size of its arc it may be slower. For the arc that Venus completes in one day, taken absolutely, is greater than

the arc Mercury completes in 5 days, each on its own epicycle; and by the Ptolemaic measures one degree of Venus's epicycle almost equals ten degrees of Mercury's.

[Margin: 3. Property, in the motion of the epicycle.]

[Margin: Whether ☿ is swifter on its epicycle than ♀.]

Fourth, the three superior planets, being slower than the Sun in mean motion on the eccentric, and indeed in true motion too, are so left behind by the Sun that at times they stand a whole semicircle of the zodiac away from it and are opposite to it, and can be observed in their acronychal positions [in Acronychijs] wholly free of the second inequality. Venus and Mercury, by contrast, because with the centre of their epicycle they perpetually keep the Sun's mean motion and the mean conjunction with it, and do not revolve on an epicycle so great that it encloses the Earth within itself, can neither digress from the Sun by a whole semicircle of the zodiac nor ever be opposite to it. Indeed, Venus does not digress even 50° on either side of the Sun, nor Mercury 30°. So it comes about that they are joined with the Sun not only at the apogee of the epicycle but also at the perigee, rising there cosmically once again, whereas on the contrary the three superior planets are opposite the Sun at the perigee of the epicycle, and rise there acronychally. From this follow other properties concerning morning and evening risings and settings, but these are the chief. First, since they are freed of the second inequality only at their two conjunctions with the Sun, and at those times lie hidden under the Sun's rays, they can never be observed by us except when entangled in the second inequality. Second, they appear swiftest when in the morning they

[Margin: 4. Property, in the elongation from the Sun.]

verge to their heliacal setting, or in the evening to their heliacal rising, phases which the three superior planets lack. But when they are heading in the evening to their heliacal setting and in the morning to their heliacal rising, in which case the three superior planets are swiftest, the inferior planets on the contrary are heading for their stations, namely after the evening greatest elongations from the Sun and before the morning ones. But see Scholium 3 below.

Fifth, although according to some ancient astronomers the three superior planets are so called because they are always carried above the Sun, and Venus and Mercury are called inferior because they are carried below it, yet according to others, not only more recent but older ones as well, they revolve now above and now below the Sun, as was shown in Section 1, chapter 4.

[Margin: 5. Property, as regards order in the system.]

Sixth, if you believe Copernicus, who reformed Ptolemy and Alfonso, the eccentricity of ♀ is variable, as is that of ♂, according to the varying eccentricity of the Sun.

[Margin: 6. Property, in the eccentricity.]

Seventh, according to the same Copernicus, in the superior planets the apogees and perigees are determined by reference to the planets in the annual

orb itself, whereas conversely in these inferior ones they are determined with respect to the centre of the Earth, wherever it may be, and by the Earth's annual motion they are carried through all the places of the deferents. The epicycles of the three superior planets are commensurable with the radius of their own eccentric, whereas the epicycle of these with the radius of the great orb.

[Margin: 7. Property.]

De Proprietatibus folius Veneris.

On the properties of Venus alone.

III. Apart from the measures of the orb and the numbers of its own motions, which are treated at the end of the chapter, *Alfonso*, and with him *Peuerbach*, held that the apogee of Venus is always the same under the zodiac as that of the Sun, and shifts with an equal motion. So since the mean motion of the Sun and of Venus is the same, the mean anomaly of the centre of Venus's epicycle (which they call the mean centre) is as great as that of the Sun, that is, as great as the Sun's argument. But Ptolemy found the apogee of Venus at 25° ♉ and the apogee of the Sun at 6° ♊, and now too they are in different places, as we shall say later. Again, in Ptolemy's time, the ratio that the eccentricity of the Sun had to the radius of the solar orb was the same as the ratio that the total eccentricity, or that of the equant, had to the semidiameter of the eccentric of Venus, as Ptolemy shows. The Alphonsines too, although they changed the measure, kept this ratio, and gave the equant of Venus an eccentricity equal to the solar one, namely 2° 16′ 6″, in parts of which the radius of the eccentric is 60, and half of that to the eccentricity of the deferent, namely 1° 8′ 3″. If the centre of the equant were the carrier of the epicycle, as it is the regulator of the mean motion, this ratio would make the equations of the centre of Venus and of the Sun's argument equal; so if the place of the apogee of the Sun and of Venus were the same, the true place of the Sun and the true place of the centre of Venus's epicycle would be the same, as the Alphonsines wished. But because both of these assumptions are false, there is some difference between the equations already mentioned. Venus also has this property: what happens to Mercury when it declines to the south happens to Venus, in the same way and by the same laws, when it inclines to the north. According to Copernicus, however, it is proper to Venus that its apogee is always 48° 21′ from the first star of ♈ and does not move, unless the fixed stars are supposed to move, which he does not suppose, and that Venus at apogee is at perihelion and at perigee at aphelion. Recent astronomy requires this, as I shall say in chapter 3, no. 5, and chapter 4, no. 6.

[Margin: Whether ☉ and ♀ have the same apogee.]

[Margin: Whether ☉ and ♀ have the same eccentricity.]

[Margin: The fallacy of this analogy.]

De Proprietatibus folius Mercurij.

On the properties of Mercury alone.

IV. None of the minor planets is more entangled in its motions than Mars and Mercury, or involves greater tangles in its theory, especially as regards the shifting of the eccentricity and of the apogee, so much so that this heavenly Mercury has tormented astronomers no less than the earthly one eludes alchemists. See what Copernicus himself admits about himself in chapter 2, no. 9, below, in our text, and recall the errors of the old astronomy concerning this star, of which I spoke in Section 1, chapter 6, no. 5. To make these things understood, they must be illustrated by two or three figures. The first is the one also used by Copernicus, Book 5, chapter 25, and *Lansbergen* in the *Theoricae*, chapter 13, though these enclose it within the annual orb of the Earth. But *Peuerbach*, and *Reinhold* on him, p. 114, *Wurstisen*, p. 204, *Osualdus*, p. 205, and *Magini* in the *Theoricae*, p. 46, enclosed it within the ecliptic, stuffing the orbs of Mercury with solid matter, whereas *Ptolemy*, Book 9, chapter 7, and *Longomontanus*, Book 2 of the *Theorica*, chapter 20, represented those orbs bare and in mere circles.

Let C, then, be the centre of the Earth and the world, from which is described the convex surface ALFM of the outermost black orb and the concave surface VGX of the innermost black orb; and let D be the centre of the equant

Diagram. A figure set in the right column: a broad black ring (solid orb) with a thin white band of circles inside it, and within those a second black ring around a white central disc. A vertical line runs from A at the top to F at the bottom; two near-horizontal lines cross the whole figure, the upper from Z to P and the lower from L to M, with a third line through R and Q. A small circle stands in the white band directly under A, and a small circle (circellus) stands at the top of the central white disc on the vertical line.

- A (top, outside the black ring): upper end of the vertical line
- small circle under A (on the vertical line): O (top, left of the line), T (top, right of the line) and I (below the crossing line); a small star-shaped mark stands at its left edge
- S (upper left, in the white band): standing on the circles; not cited in the text so far on this page
- N (upper right, in the white band)
- Q (left, in the white band)
- R (right, in the white band)
- Z (far left, outside the ring): left end of the upper horizontal line
- P (far right, outside the ring): right end of the upper horizontal line
- L (far left, outside the ring, below Z): left end of the lower horizontal line
- M (far right, outside the ring, below P): right end of the lower horizontal line
- E (centre, at the top of the small circle)
- B (centre, in the small circle)

- D (centre, at the foot of the small circle)
- C (centre, below D, where the vertical and lower horizontal lines cross)
- V (white disc, left of C)
- X (white disc, right of C)
- G (white disc, on the vertical line below C)
- K (lower, on the vertical line, in the white band)
- H (lower, immediately left of K)
- F (bottom, outside the black ring): lower end of the vertical line

circle HNI, at a distance from the centre of the world of 3 of the parts of which the radius DI of the eccentric is 60. Let B, the centre of the small circle that causes the anomaly of the eccentricity, be the same number of parts from the centre of the equant, so that ED, its diameter according to Ptolemy, is 6 parts. At its apogee E let there be the centre of the deferent TQKR, 9 parts from the centre of the world. On its circumference this deferent has T, the centre of the epicycle carrying Mercury. Through O, the apogee of that epicycle, with the centre of the deferent set at the apogee E of the small circle, is described the concave surface OQHR of the outermost black orb; and through the perigee I, from the same centre at E, the convex surface IN of the innermost black orb. These black orbs are called the deferents of the apsides, or of the apogee and perigee. But each is subdivided into two portions: of both the innermost and the outermost black orb, the outer portions carry the apogee of the equant, while the inner portion of each carries the apogee of the deferent. Let LCM be the axis of the orbs carrying the apogee of the equant in the order of the signs, following the motion of the eighth sphere, or of the fixed stars; QER the axis of the deferent of the epicycle; and ZBP the axis of the orbs carrying the apogee of the eccentric. Now on the small circle of the anomaly of the eccentric let the centre of the deferent of the epicycle move above against the order of the signs and below in the order of the signs, with an annual motion equal to the Sun's mean motion, so that once a year it is united with the centre of the equant D, and once is at its greatest distance from it at E. But the line of the apogee of the equant, which is ADF, and the centres D and B on it, move very slowly, and only with the motion of the eighth sphere. The line of the apogee of the eccentric deferent of the epicycle, however, is always the one drawn from the centre of the world through the centre of the deferent of the epicycle as far as the circumference of the equant. In the part within which the centre of the equant lies, it marks the apogee of the deferent, which in this diagram is the point T, and in the other, opposite part it marks its perigee K. This line is not fixed like the line of the apogee of the equant, but is driven to and fro by a certain reciprocal motion, one that nevertheless agrees with the periodic motion of the eccentric. Moreover, the centre of the epicycle, following the motion of the deferent, moves in the order of the signs with the same mean motion as the Sun.

V. From what has been said it follows, first, that the point under which the apogee of the equant stands and that under which the apogee of the eccentric

stands are not always the same, as they are in the other planets, the point from which the anomaly of the eccentric is accordingly counted, but are usually different; and that the anomaly of the eccentric by which the equation of the centre is taken from the tables is counted from the apogee of the equant, and not, strictly speaking, from the apogee of the deferent, inasmuch as from a

[Margin: Porisms or corollaries.]

[Margin: 1. Mercury's difference from the others.]

more certain and stable point, movable only with the motion of the fixed stars. Second, it comes about that when the apogee of both is on the same line AF, the apogee of the deferent is nevertheless not farthest from the centre of the world, nor its perigee nearest. For when the centre of the deferent is united with the centre of the equant, the apogee of the deferent comes nearer the centre of the world, and the perigee farther from it, by 6 parts of the 60 of the radius, than when the centre of the deferent is at the apogee E of the small circle. Third, it comes about that the centre of the equant is not twice as far from the centre of the world as the centre of the deferent, as it is in the other planets, nor is their distance from each other always as great as the distance of one of them from the centre of the world. Rather, the centre of the deferent is usually farther from the centre of the world than the centre of the equant; once a year it is equally far, and once three times as far. But this is not yet the end of the differences and variations. There remain those that depend on the unequal and reciprocal motion of the true apogee of the eccentric deferent of the epicycle, and the resulting disagreement with the other planets in the greatest equations of the orb and in the proportional minutes. To explain these we shall use the following figure.

[Margin: 2. Difference of ☿.]

[Margin: 3. Difference of ☿.]

VI. The following figure is set out by the commentators on Peuerbach's *Theoricae*: *Reinhold*, p. 121, *Osualdus*, p. 211, *Wurstisen*, p. 227, and *Magini* in his *Theoricae*, p. 51, though not without some errors and with omission of the geometric calculation, which we shall add. In it A is

Diagram. A figure set in the left column, showing an egg-shaped closed line round the Earth's centre A, with a vertical line from the top through H, P, C, B, D and A to G. A curved crescent-shaped line runs through M, O, P, I and L beneath an arc through N, H and K. Small circles stand at H (top), F (left), E (right) and G (bottom), and a small circle round B stands above A, with straight lines radiating from A and from D to the points on the curves.

- λ (top, above the small circle at H): cited on the following page as Hλ, the semidiameter of Mercury's epicycle
- H (top, inside the small circle)
- N (upper left, on the arc)
- K (upper right, on the arc)
- P (on the vertical line, below H)
- M (left, at the end of the crescent-shaped line)

- O (left of P, on the crescent-shaped line)
- I (right of P, on the crescent-shaped line)
- L (right, at the end of the crescent-shaped line)
- a mark shaped like the numeral 2 (far left, on the outer circle of the equant) [?]
- β (far left, on the egg-shaped line, joined by a line to R through V): cited on the following page as the place of the centre of the epicycle when the centre of the deferent is at R
- V (upper left of the small central circle)
- C (top of the small central circle, on the vertical line)
- R (upper right of the small central circle)
- B (centre of the small central circle)
- S (left of the small central circle)
- D (foot of the small central circle)
- Q (right of the small central circle, in a decorated form)
- A (below D, where the radiating lines meet)
- F (lower left, in a small circle)
- a mark shaped like ε (right of F) [?]
- E (lower right, in a small circle)
- a mark shaped like a reversed ε (left of E) [?]
- Y (below A, left of the vertical line)
- Z (below A, right of the vertical line; the letter is imperfectly formed)
- X (lower left, on a small arc)
- T (lower right, on a small arc)
- G (bottom, inside the small circle)

the centre of the world, B the centre of the small circle CD of the anomaly of the eccentricity, divided into 12 signs, and D the centre of the equant, from which the equant PG is described and divided into twelve signs. The line of the apogee of the equant is GP, its apogee P and its perigee G. On this line, when the centre of the deferent is at the apogee C of the small circle, the apogee of the deferent is H and its perigee between Y and Z. But if the centre of the deferent is at V, the line of the apogee of the deferent is NZ, with the apogee at N and the perigee at Z on it. If the centre is at R, the apogee is K and the perigee Y. If it is at the point Q or S, where the line of the apogee touches the small circle, the apogee is at L in the one case and at M in the other, and the perigee at X or T. The motion of the centre of the deferent, moreover, equal to the Sun's mean motion, takes place against the order of the signs from C to R, even though the motion of the centre of the epicycle, likewise equal to the Sun's mean motion, takes place in the order of the signs. And because the semidiameter BD of the small circle is as great as AD, it comes about that the straight lines ASM and AQL, which touch the small circle, enclose the arc SDQ of 120°, or a third of the small circle, and that the upper portion SCQ of the small circle is 8^{s}, or 240°. With this set out, one must understand that the apogee and perigee of the eccentric do not make whole circuits under the zodiac, but through the

whole periodic time are contained and move back and forth within the already mentioned tangents MSAT and LQAX, that is, within the arc of the zodiac enclosed by those lines when produced, even though the centre of the eccentric runs through the whole small circle of the anomaly of the eccentricity. So when the centre of the eccentric runs through the upper part of the small circle from S, through VCR, to Q, the apogee of the eccentric deferent too is carried against the order of the signs from M, through NHK, as far as L, and the perigee likewise against the order of the signs from T, through Z, as far as X. But when the centre of the deferent is carried through the lower portion from Q, through D, to S, following the succession of the signs, the motion of the apogee takes place from L, through IPO, to M, and that of the perigee from X, through G, to T. Accordingly, when a whole period is completed, the apogee marks out a line [Greek: μηνοειδὲς (mēnoeides, "crescent-shaped")], namely MNHKLIPOM, resembling the sickle Moon, while the perigee marks out a line nearly [Greek: κογχοειδὲς (konchoeides, "shell-shaped")], that is, resembling a shell, or rather a rose petal. From this it also follows that the motion of this apogee is slowest when the centre of the eccentric is at the apogee of the small circle, but swiftest when it is at its perigee.

VII. Given, then, the double motion already mentioned, namely of the centre of the epicycle in the order of the signs and of the centre of the deferent against that order, it comes about, first, that when the centre of the deferent occupies the apogee C of the small circle, the centre of the epicycle is at H and at its farthest from the centre of the world, namely 69 parts, of 60 of the radius; and if Mercury is then at the apogee of the epicycle, it has the greatest distance from the Earth that it can have anywhere, which now happens in the month of October, about the beginning of Scorpio. But when the centre has come round to D, the apogee of the deferent is at P, the apogee of the equant. Second, when the centre of the deferent has sunk to the western point of contact Q, which now happens in February, about the beginning of ♓, after passing through the four signs of the arc CRQ, the apogee of the deferent stands as far as possible from the apogee of the equant, and the perigee from the perigee. The centre of the epicycle, having completed four signs in the order of the signs, comes to be at F, with the line FQ drawn through the centres of the equant and the deferent. And although F is at the perigee of neither eccentric (taking perigee for the point G, opposite the apogees H and P), it is nevertheless nearest the Earth, so much so that it could not be nearer, and so F is equivalently the perigee of the deferent. For the distance AF of the centre of the epicycle from the centre of the world is then 55° 33′ 30″, in parts of which DP is 60. But when the centre of the deferent is at D, the centre of the epicycle is united with the fixed perigee G of the equant, yet it stands from the centre of the world by the amount AG, that is, 57 parts, namely after subtracting from the radius DG the 3 parts of the eccentricity AD; this now happens in April, shortly after the beginning of ♉. In the same way, when the centre of the deferent has climbed to the eastern point of contact S, the point E, where the centre of the epicycle is, will be equivalent

to the perigee, and it will again come nearest to the Earth by the amount of the line AE, namely 55° 33′ 30″; this now happens in June, shortly after the beginning of ♋. Third, it follows from what has been said that in one annual revolution the centre of the epicycle is at its farthest from the centre of the Earth only once, and comes nearest to it twice, because of the double equivalent perigee, and yet at those times it is not at the fixed perigee, that is, the one belonging to the ordinary line of apsides, and indeed not at the perigees of the eccentric XT, etc. These things could seem absurd at first hearing, but they are handed down by Ptolemy, Book 9, chapter 8. Fourth, it comes about that only once in one period the centres of the equant and of the deferent meet at D, the perigees of both meeting then, once, at G; at other times those centres are always apart, and most of all at C. And so, considering the direct motions of the centre of the epicycle and the retrograde motions of the centre of the deferent, the centre of the epicycle moves more slowly towards the perigee of the equant and more swiftly towards the apogee in relation to the centre of the deferent (in which it differs from Venus and the superior planets), although with respect to the centre of the zodiac it is carried more slowly around the apogee of the equant and more rapidly around the perigee, in which it agrees with those planets. Fifth, it comes about that the movable perigee of the deferent, away from the apogee or perigee of the equant, is always in that semicircle of the zodiac in which the centre of the epicycle is, though sometimes preceding and sometimes following the centre of the epicycle. Sixth, it happens that the centre of the epicycle by its motion does not describe a circular circumference, but [Greek: ᾠοειδὲς (ōoeides, "egg-shaped")], or almost oval, wider at the apogee and narrower at the perigee, such as the line HMFGELH. Seventh, it happens that the equations of the argument or orb (which become greatest when the centre of the epicycle to the centre of the world

[Margin: 1. Corollary, and 4th difference of ☿.]
[Margin: 2. Corollary.]
[Margin: 3. Corollary, and 5th difference of ☿.]
[Margin: 4. Corollary, and 6th difference of ☿.]
[Margin: 5. Corollary.]
[Margin: 6. Corollary, and 7th difference of ☿.]
[Margin: 7. Corollary, and 8th difference of ☿.]

comes nearest) do not occur at the perigee of the eccentric deferent of Mercury's epicycle, as in the others, but at two other points, namely F and E, that is, when the centre of the deferent reaches the two points of contact of the small circle, Q and S; and the greatest digressions occur at those same places. Ptolemy, Book 9, chapter 8, finds that these occur in the morning in Aquarius and in the evening in Gemini. For the same reason it was necessary to construct tables of the equations of the orb of ☿ different from those of the other planets; and the proportional minutes of which we spoke in Section 2, chapter 12, number 5, must be arranged so that they are least at the apogee and whole, or 60, against 4 and 8 completed signs of anomaly, and decrease in

order from those places as far as the semicircle of anomaly, or the perigee, at which for Copernicus they are only 52′ 21½″. Eighth, Venus has the least and Mercury the greatest eccentricity in proportion to the radii of their orbs, so that the two eccentricities of the inferior planets equal those of the Sun and of the three superior planets. Hence the daily eccentric motion of Venus is of the least variation and that of Mercury of the greatest, so much so that Mercury's daily perihelial motion is more than double its daily aphelial motion. Therefore the old astronomy could err little in predicting the place of Venus, but a great deal in Mercury, as Kepler points out in the *Epitome of Copernican Astronomy*, page 768.

[Margin: 8. Corollary, and 9th difference of ☿.]

The remaining properties by which it differs from Venus in the Copernican hypothesis I shall indicate in chapter 2, number 7.

Methodus inueſtigandi Eccentricitatem Mercurij extra Apogeum & Perigeum.

The method of finding the eccentricity of Mercury outside the apogee and perigee.

VIII. I said above that the form of the geometric calculation by which the true place of a planet is sought is exactly the same in ♀ and ☿ as in the three superior planets, as set out in Section 2, chapter 1, from number 10, provided that the eccentricity is known. Since in Mercury it varies, it remains for us to teach the method of finding it, which the authors have omitted. So, in the figure immediately preceding, when the centre of the deferent is at C or at D, there is no need of an equation of the centre; but when it is elsewhere, an equation of the centre is needed. Let the centre of the deferent, then, be at R, with two signs of the small circle completed. For then the centre of the epicycle, by its mean motion, has completed two signs in the order of the signs, to be reckoned in the equant, that is, the arc Pγ. So, when the semidiameter of the deferent is drawn from R towards γ, the centre of the epicycle will fall at β. From β draw the straight line βA to the centre of the world, and the straight line AR, namely the eccentricity to be found; and from the centre of the small circle of the anomaly of the eccentricity draw to the centre of the deferent the straight line BR. For there will arise the triangle ABR, in which there is given AB, the eccentricity of the centre B, which is 6 parts, and the semidiameter BR of the small circle, half of it, or 3 parts, of which the radius Rβ is 60; and the angle they contain, ABR, is the complement of the angle CBR, which measures the arc CR. Therefore, with trigonometry as your guide, you will find the eccentricity AR; and with it, the radius Rβ and the angle ARβ, which is the complement of the angle IRβ, that is, of the anomaly of the eccentric counted from the apogee I, you will find in the triangle ARβ the equation of the centre, that is, the angle AβR, and Aβ, the distance of the centre of the epicycle from the centre of the world, which is needed for finding the equations of the orb. But if the centre of the deferent is at the points of contact S, Q, which are a

third of the circle apart from each other, the straight line drawn from them to the centres of the epicycle, namely SDE and QDF, passes through D, the centre of the equant. Therefore in the triangles ADQ and ADS the side AD is given, its eccentricity of 3 parts, and each of the sides DQ and DS is just as much. For since the arcs DQ and DS are 60°, the angle BDQ is 60°, and the chord DQ equals the semidiameter BQ, which likewise is 3 parts. So the eccentricity AQ, or AS, will not be hidden.

[Margin: By 15 of Book 4 of Euclid.]

IX. It remains for us, in our usual manner, to present the measures and numbers that are the foundations of the geometric and logistic calculation in this hypothesis, from Ptolemy, Books 8 and 9, from the Alphonsine Tables, and from the other authors brought forward in numbers 1 and 6. Of these, Magini takes the measures and numbers from Copernicus, although he uses the Ptolemaic or Peuerbachian hypothesis as regards the figure and the immobility of the Earth; and with Copernicus he supposes that the apogee of Venus is always 48° 21′ from the first star of ♈. When the precession of the equinoxes, or the motion of the fixed stars from the equinox, is added to these, the motion of the apogee of ♀ from the equinox becomes known. Besides these authors, the same measures are given by *al-Battānī*, chapter 50; *Egnazio Danti*, in his tables, from page 33; *Barocius*, Book 1 of the *Cosmographia*, page 48; *Maurolico*, at the end of his *Cosmographia*, and from him *Clavius*, at the end of the *Sphere*; *Baranzanus*, from Giuntini and Mauro Fiorentino, in part 1 of the *Uranoscopia*; *Tycho*, volume 1, page 468; and *Kepler*, in the *Epitome of Copernican Astronomy*, page 494. Note, however, that the measures listed below, as regards the semidiameters of the Earth, are set down according to Ptolemy by Maurolico, Clavius and Barozzi, rather than according to Alfonso, because Maurolico declares that the Ptolemaic distances of the Sun are more convenient for exhibiting the distances of the inferior planets.

[Margin: The authors from whom the following tables are taken.]

MENSVRÆ ORBIVM ♀ ac ☿ Ptolemaicæ, Alphonſinæ, aut Aliorum vetere hypotheſi vtentium, in Partibus, qualium

Measures of the orbs of ♀ and ☿, Ptolemaic, Alphonsine, or of others using the old hypothesis, in parts of which

[Margin: For ♀ see the figure of Section 2, chapter 1, numbers 2 and 10. For ☿, the figure set out in this Section, chapter 1, number 6.]

		Planet	Authors	The semidiameter of the planet's orb is 60	The radius of the planet's orb is 100000
Eccentricity	Of the equant	♀	Ptolemy	2° 30′ 0″	4166⅔
	AE	♀	Alfonso	2° 16′ 6″	3766⅔
	AD	☿	Ptol., Alph.	3° 0′ 0″	5000
	Of the deferent	♀	Ptolemy	1° 15′ 0″	2083⅓
	AD	♀	Alfonso	1° 8′ 3″	1883⅓

		Planet	Authors	The semidiameter of the planet's orb is 60	The radius of the planet's orb is 100000
	Greatest AC	☿	Ptol., Alph.	9° 0′ 0″	15000
	Least AD	☿	Ptol., Alph.	3° 0′ 0″	5000
Semidiameter	Of the epicycle FG	☿	Ptol., Alph.	43° 10′ 0″	71944⅖
	Hλ	☿	Ptol., Alph.	22° 30′ 0″	37500
	Of the small circle of the eccentric, BC	☿	Ptolemy and Alfonso	3° 0′ 0″	5000
Distance from the centre of the Earth, or of the world	AF	☿	Ptolemy	61° 15′ 0″	102083⅓
	Greatest	☿	Alfonso	61° 8′ 3″	101883⅓
	AH	☿	Ptol., Alph.	69° 0′ 0″	115000
	AM	☿	Ptolemy	58° 45′ 0″	97916⅔
	Least	☿	Alfonso	58° 51′ 57″	98116⅔
	AF	☿	Ptol., Alph.	55° 33′ 30″	97597 2/9 [recte: 92597 2/9]
	Perigeal AG	☿	Ptolemy and Alfonso	57° 0′ 0″	95000
	Of the planet	☿	Ptolemy	104° 25′ 0″	174028
	AG	☿	Alfonso	104° 18′ 3″	173828
	Greatest	☿	Ptolemy	91° 30′ 0″	152500
	Aλ	☿	Alfonso	91° 30′ 0″	152500
	Of the planet	☿	Ptolemy	15° 35′ 0″	25972
	AO	☿	Alfonso	15° 41′ 57″	26172
	Least	☿	Ptolemy	33° 3′ 30″	55097
	Aε	☿	Alfonso	33° 3′ 30″	55097
	Mean DF	☿	Ptolemy	60° 0′ 0″	100000
	Mean DP	☿	Ptolemy	60° 0′ 0″	100000

[Margin: Remember that in ☿ the perigeal distance is not the least.]

Eædem menſuræ in Semidiametris Terræ

The same measures in semidiameters of the Earth

		Planet	Authors	Measure
Eccentricity	Of the deferent	☿	Maurol. and Clavius	12 7′
		☿	Fernel	14 21
	Of the equant	☿	Maurol. and Clavius	24 14
		☿	Fernel	28 42
	Of the equant, and least of the deferent	☿	Maurolyc., Clavius	5 48
		☿	Fernel	5 51

		Planet	Authors	Mea-sure
	Greatest of the deferent	☿	Maurolyc., Clavius	17 24
		☿	Fernel	17 33
	Semidiameter of the epicycle	♀	Maurolyc., Clavius	461 41
		♀	Fernel	495 42
		☿	Maurolyc., Clavius	43 31
		☿	Fernel	43 53

DISTANTIÆ ♀ ac ☿ Eædem à Terra in Semidiam. terr. iuxta diuerſos Auctores.

The same distances of ♀ and ☿ from the Earth, in semidiameters of the Earth, according to various authors.

Planet	Authors reporting Ptolemy's measures	Semidiameters of the Earth, with minutes	
		Mean distance	
	al-Battānī and Tycho	618 0	
	al-Farghānī and Baroc.	643 30	
♀	Jean Fernel	688 59½	
	Maurolico	641 45	
DF	Giuntini and Mauro Fiorentino	622 0	
	Clavius	641 45	
	Copernic. and Kepler	808	
	al-Battānī and Tycho	115 0	
	al-Farghānī and Baroc.	115 25	
☿	Jean Fernel	117 2	
	Maurolico	116 3	
DP	Giuntini and Mauro Fiorentino	114 35	
	Clavius	116 3	
	Copernic. and Kepler	137	
		Greatest distance	*Semidiameter of the convex surface*
	al-Battānī	1070 0	
	al-Farghān. and Barozzi	1120 0	1121 0
♀	Jean Fernel	1199 2⅓	1199 29
	Maurolico	1115 33	1115 51
AG	Giuntini and Mauro Fiorentino	1078 0	1079 0
	Clavius	1115 33	1121 0
	Copernic. and Kepler	1407 0	1408 0
	al-Battānī	165 0	
	al-Farghān. and Barozzi	166 40	167 0
☿	Jean Fernel	178 28	178 30⅛
	Maurolico	167 37	167 39
Aλ	Giuntini and Mauro Fiorentino	165 0	166 0

Planet	Authors reporting Ptolemy's measures	Semidiameters of the Earth, with minutes	
	Clavius	167 37	167 57
	Copernic. and Kepler	209 0	209 0
		Least distance	*Semidiameter of the concave surface*
	al-Battānī	166 0	
	al-Farghān. and Barozzi	167 0	167 0
♀	Jean Fernel	179 2	178 30
	Maurolico	167 57	167 39
AO	Giuntini and Mauro Fiorentino	166 0	166 0
	Clavius	167 57	167 57
	Copernic. and Kepler	210	209 0
	al-Battānī	65 0	64 30
	al-Farghān. and Barozzi	64 10	64 10
☿	Jean Fernel	65 36	64 27½
	Maurolico	64 29	64 27
Aε	Giuntini and Mauro Fiorentino	64 10	64 0
	Clavius	64 29	64 0
	Copernic. and Kepler	65 0	65 0

The distances are taken from the centre of the Earth as far as the centre of the planet's body; but the semidiameters of the convexity and concavity of the heavens as far as the highest part of the planet's body in the former case, and the lowest in the latter.

[Margin: See the figures indicated above.]

MOTVS MEDIVS ſeu ÆQVALIS.

Mean or equal motion.

Motion			Deg.	′	″	‴	⁗	V	VI
			Of the planet on the epicycle, or anomaly of the orb						
Daily	♀	Ptolemy	o	36	59	25	53	11	28
	♀	Alfonso	0	36	59	27	23	59	31
	☿	Ptolemy	3	6	24	6	59	35	50
	☿	Alfonso	3	6	24	7	42	40	52
Annual	♀	Ptolemy	225	1	32	28	34	39	15
	♀	Alfonso	225	1	41	40	57	3	35
	☿	Ptolemy	53	56	42	32	32	59	10
	☿	Alfonso	53	56	46	54	38	36	20

[Margin: Ptolemy calls this motion that of the inequality; Alfonso calls it that of the mean argument.]

[Margin: After 1 period completed. After 3 periods completed.]

Radix motûs Prædicti.

Root of the motion already mentioned.

		Sign.	Deg.	′	″	‴
♀	Ptolemy	2	11	7	0	0
	Alfonso	1	15	45	55	19
☿	Ptolemy	0	21	55	0	0
	Alfonso	7	3	48	38	46

[Margin: On the place and time of the roots see what was said in Section 2, chapter 1, at the end of the last table.]

Radix Apogei

Root of the apogee

		Sign.	Deg.	′	″	‴
♀	Ptolemy	♉	16	10	0	0
	Alfonso	♉	20	37	0	0
☿	Ptolemy	♌	1	10	0	0
	Alfonso	♌	19	51	11	41

For Ptolemy the motion of the apogees of the 5 planets is of the same quantity as that of the fixed stars, namely 36″ a year, as he teaches in Book 9, chapter 7. For Alfonso, however, the motion of the apogee of Venus and of the Sun is the same, both as motion and in quantity; that of the others is the same as that of the eighth sphere, on which see Book 6, chapter 16. In his own time Ptolemy observed the apogee of Venus at 25° of Taurus and that of Mercury at 10° of Libra.

On the greatest digressions and equations it will be more fitting to speak below in chapter 7, and on the periods on the epicycle at the end of this Section.

SCHOLIA.

Scholia.

I. Among the hypotheses of the older astronomy, one is that of al-Biṭrūjī, which I touched on in the last chapter of Section 1 of this Book, and which is treated in the Book of Systems; another, commonly rejected by those more skilled in astronomy, is that of homocentric, or concentric, orbs, which Amicus, Girolamo Fracastoro and a few others tried to introduce. For even if it were equivalent to the others as regards logistic calculation, it can uphold the increases and decreases of the apparent diameter, especially in Venus and Mercury, and those phases that the telescope has revealed, only with the greatest violence,

since that hypothesis supposes that the planet's distance from the Earth is always the same. So there is no reason for us to dwell on these. On Mercury's greater nearness to the Earth at the sextants from the perigee, see what is to be said in chapter 4, Scholium 3.

[Margin: On the hypotheses of Alpetragius and Girolamo Fracastoro.]

II. That Venus and Mercury, in the Ptolemaic hypothesis, can be so near the Earth that Venus at perigee can undergo a parallax of 20′, and Mercury, at its greatest approach to the Earth, as great a parallax as the Moon at apogee, does indeed conflict with the phenomena: for it does not come nearest to the Earth at the sextile counted from the perigee, although it digresses most there. This was the occasion of Ptolemy's error, as Copernicus, Book 5, chapter 28, and Longomontanus, Book 2, chapter 20, point out. Further, the old astronomy must be pardoned, since neither through the telescope nor through other better observations did it have arguments for raising Venus and Mercury higher and arranging them around the Sun. From this, and from the difficulty of observing Mercury, and from what was said in number 7, Corollary 8, arose the error in the Alphonsine and Ptolemaic calculation of Mercury, as great as we said in Section 1, chapter 6, number 5. On the fallacy of using the Sun's mean place rather than its true place, we shall speak in the Scholia to chapter 2, since Copernicus shared it. On the inequality of motion about its own centre, which is admitted most of all in ☿, enough has been said in the Scholia to chapter 1, Section 2.

[Margin: On the excessive parallax of ♀ and ☿.]

[Margin: On the errors of this hypothesis.]

III. Among the properties by which the stars of Venus and Mercury are distinguished from the superior planets, of which we spoke in number 2, Pliny the Elder touches on the fourth, Book 2, chapter 17, in these words: "From this, too, the reversed pattern of their motions is understood. For the superior planets are carried most swiftly at their evening setting, these two most slowly; the former are at their highest from the Earth when they move most slowly, the latter when they move most swiftly, because just as in the former the nearness of the centre speeds them up, so in the latter does the extremity of the circle. The former begin to lessen their speed from their morning rising, the latter to increase it. The former run their retrograde course from the morning station to the evening one; the star of Venus from the evening one to the morning one. It begins, moreover, to climb in latitude from its morning rising, and to climb in altitude and follow the Sun from its morning station, being swiftest and highest at its morning setting. It begins to depart in latitude and to lessen its motion from its morning rising; to go backward, and at the same time to depart in altitude, from its evening station. The star of Mercury, again, begins in both respects to climb

[Margin: Pliny the Elder's view of the properties of ♀ and ☿.]

from its morning rising, but to depart in latitude from its evening one; and when the Sun has caught up with it to an interval of 15°, it stands almost motionless for four days, etc."

CAPVT II.

Chapter II.

De Hypotheſi Copernicæa Veneris ac Mercurij.

On the Copernican hypothesis of Venus and Mercury.

Theorica Veneris.

The theory of Venus.

I. If you have had a first taste of the Copernican system in Book 3, chapter 6, number 3, and have understood the theory of the three superior planets given in this Book, Section 2, chapter 2, number 4, you will more easily understand the hypothesis of Venus, which is set out by *Copernicus*, Book 5, from chapter 20 to 25; by *Rheticus*, in his *Narratio*; by *Kepler*, in the *Epitome of Copernican Astronomy*, Book 6, part 3, from page 757; by *Michael Mästlin*, in his notes on Kepler's *Prodromus*, chapter 15, and in his Appendix to the same book; and by *Longomontanus*, Book 2 of the *Theorica*, chapters 17 and 18. According to these authors, then, the Earth is moved around the Sun, which is motionless at the centre of the world, along the annual orb towards the east; and Venus, within the annual orb, along an orb eccentric to the Sun, but varying its eccentricity by means of a small circle, is likewise moved to the east, but leaping ahead of the Earth and with a swifter motion. For it completes its course in about 9 or 8½ months, but returns to the Earth in 19 months; yet in such a way that this eccentric stands in place of the Ptolemaic epicycle, while in place of the eccentric deferent, on which it is believed to be moved in its mean motion of longitude, stands the annual orb of the Earth itself. And they think this is the genuine reason why the three eccentrics of the old hypothesis, namely of the Sun, Venus and Mercury, have equal revolutions and why their mean motion is the same: namely because they themselves do not revolve, but in their place the Earth alone. So by the single annual motion of the Earth and the single motion of the epicycle, or, as Rheticus calls it, of the second eccentric, they explain all the appearances of Venus and Mercury. For if these planets stayed as fixed as the Sun, we, they say, who are carried around them by the motion of the great orb, would still think that they move like the Sun and travel through the zodiac. But because observations attest that Venus and Mercury, beyond the mean motion of the Sun, move on their orbs with their own motions, and now precede the Sun, now follow it, and are conjoined with it now above and now below, we therefore assign them epicycles, which with their centre complete the zodiac at an equal pace with the Sun. This motion, then, compounded of two equal motions but appearing unequal to us, could indeed be exhibited in the threefold

form that we set out in Section 2, chapter 2, namely by an eccentrepicycle, or, as *Tycho* and *Longomontanus* later imitated it, by two epicycles; but it is better managed by an eccentric of the eccentric [Eccentri Eccētrum], that is, by an eccentric varying its eccentricity and position. For *Copernicus* himself chose this one, Book 5, chapter 22, in these words: "This star, however (he is speaking of ♀), has something different from those (that is, from the three superior planets) in the order and commensuration of their motions; and this will be shown more easily and conveniently, in my opinion, by an eccentric of the eccentric."

[Margin: Preparation for this hypothesis.]

II. Let, then, according to the wish of Copernicus[61], in the following diagram of Mästlin, the centre of the world, that is, the Sun, be E, or the point of the Earth's mean motion, around which let the great or annual orb of the Earth be RLQ; and let the diameter RAQ be set for the line of apsides of Venus, on which let the least eccentricity of the orb of Venus from the Sun be EM and the greatest EN, double EM, and their difference MN, which, bisected at A, is to have there the centre of the small circle MN, carrying on its own circumference the centre of the eccentric in the order of the signs, on this condition: that whenever the Earth is at the apogee of Venus Q or the perigee R, or whenever it falls on the line of apsides, the centre of the eccentric of Venus is at the least eccentricity from the centre of the world, that is, at M; but when the Earth is at the mean apse, or a quadrant away from the apogee or perigee of Venus, as if it is at L, the centre of the eccentric of ♀ is at the greatest eccentricity, that is, at N. From this it necessarily follows that in the time in which the Earth once travels through its annual orb, the centre of the eccentric of ♀ revolves twice

Diagram. Mästlin's diagram for the Copernican hypothesis of Venus, set in the right column between the two parts of its sentence: a large outer circle, with a vertical diameter from R at the top to Q at the bottom; small circles on that diameter at the top (about I and D), at the centre (about A) and at the bottom (about F and B), and a fourth small circle at the right about C. Further circles, solid and dashed, lie inside the outer circle. Straight lines run from Q up to the left and to the right, from R down to the right and to Y, from L through the centre region to the upper right, from L to the diameter, and horizontally from P to the right-hand small circle.

- R (top of the outer circle): perigee of Venus in the text; the Earth at the perigee
- I (just below R, on the diameter, within the top small circle): cited in the text in IPGO
- D (below I, within the top small circle): not cited in the prose on this page
- H (on the diameter, below D): cited in the text in AH and HXG
- Y (upper left, inside the circles): foot of the tangent from R in the text

[61] the page prints *plani* here, which does not construe in this clause

- O (upper right, on the dashed circle; a small sort): cited in the text in IPGO
- T (right, above the small circle at C): not cited in the prose on this page
- N (on the diameter, above A): the greatest eccentricity EN in the text
- A (centre of the central small circle): centre of the small circle MN in the text
- C (centre of the small circle at the right): not cited in the prose on this page
- P (left, on the dashed circle): cited in the text in IPGO
- L (far left, on the outer circle): the Earth at a quadrant from the apsides in the text
- V (right, below the small circle at C): not cited in the prose on this page
- M (on the diameter, below A): the least eccentricity EM in the text
- E (on the diameter, just below M): the centre of the world, the Sun, in the text
- F (on the diameter, just above the bottom small circle): not cited in the prose on this page
- X (lower left, on an inner circle): Venus at the tangent from Q in the text
- B (on the diameter, within the bottom small circle): not cited in the prose on this page
- G (on the diameter, below B): cited in the text in IPGO and HXG
- Q (foot of the outer circle): apogee of Venus in the text

in the small circle MN. Copernicus, however, states, Book 5, chapter 22, that the greatest eccentricity EN was in Ptolemy's time 416 parts, of which the radius of the great orb is 10000, but that in his own time he found it to be 350 parts, just as, in Copernicus's opinion, the eccentricity of Mars too decreased, because of the diminished eccentricity of the Sun. And so Reinhold computed the equations of Venus on the supposition of an eccentricity of 350, as Mästlin demonstrates above from the Prutenic Tables. From what has been said it is clear that the eccentric of Venus does not describe a single path, and that there is not a single eccentric in the heaven but one after another, and that it wanders through all the places enclosed by two circles, one of which you have marked with points in the diagram, namely IPGO, while the other you are to imagine described from A at the interval AH.

III. So the greatest prosthaphaeresis of the centre occurs at 90° of the anomaly of the eccentric, computed on the annual orb from the line of apsides, as if the Earth is at L. The anomaly of the eccentric will be the arc RL, or the angle REL, and the greatest equation of the centre the angle ELN, because its foundation is the greatest eccentricity EN. The greatest prosthaphaereses of the orb, or elsewhere of the epicycle, which Copernicus calls prosthaphaereses of the parallax of the annual orb, occur when a straight line drawn from the centre of the Earth to Venus touches the orb of Venus and makes a right angle with the semidiameter of that orb. Therefore the greatest equation of the orb at

the apogee of the eccentric, Q, is, for example, the angle MQX, and it occurs with the Earth placed at Q and Venus at X (and understand the same of the other, similar place on the right), where the straight line QX touches the orb HXG and makes with the radius MX the right angle QXM. But the greatest one at the perigee, which is the greatest of all the greatest, is the angle MRY, in the triangle MYR, right-angled at Y, where RY touches the same orb of Venus as above.

[Margin: The places of the greatest equations.]

Methodus Calculi Geometrici in hac Hypotheſi Veneris.

The method of geometric calculation in this hypothesis of Venus.

IV. In the following figure let the line of apsides of Venus be ABD, on which let B, near the Sun, be the centre of the annual orb AED, and let the small circle of the eccentricity of Venus be CFL, whose centre K is distant from B by the amount BK, that is, three fourths of the whole or greatest eccentricity BL; and let the remaining fourth part be the small circle's semidiameter KF, if you follow the old hypothesis of Copernicus. For if you follow the recent one, BK will be 246 parts, which in the old was 312; but CK, in both, is 102 parts, of which the radius BE of the annual orb is 10000. Now let the Earth be at E, distant from A, the apogee of Venus according to mean motions, by the amount of the arc AE. This you will learn by subtracting the place of the apogee of ♀ from the mean place of ☉, and the mean anomaly of Venus will be known, and thus the angle ABE. Count, then, from the line AD, in the order of the signs, the arc CF of the small circle, which is to be double the arc AE; for the centre of the orb of Venus will be at F, from which you will describe the orb MING, and its eccentricity will be BF. Then from the Earth E draw three straight lines: namely EB, to the centre of the annual orb; EK, to K, the centre of the small circle; and EF, to F, the centre of the orb of ♀. Through F and through the centre B draw the straight line PFQ; and through the same centre F draw the straight line IH, parallel to the line BE of the mean motion of the Earth and of Venus (for the annual orb stands in place of the eccentric of ♀ and ☿). For H will be the mean apogee and I the mean perigee of the orb, but G the true apogee. Finally, let Venus be now at N, from which draw the straight lines NE, to the Earth, and NF, to the centre of the orb of ♀; and now at M, and draw the straight lines ME and MF. With these things laid down:

[Margin: Copernican diagram from Longomontanus, Book 2, chapters 17 and 18.]

Diagram. Set in the left column, inside the sentence naming the line EK. A large outer circle with a vertical diameter from A at the top to D at the foot. Inside it, a second circle about F, passing through N on the left, M on the right, I at the upper left, H at the lower right and G at the foot; N and M are marked by small stars. Inside that, a small circle about K, with L at its foot. From E on the outer circle at the upper left, straight lines run to N, to F, to K, to B and

to M. A dotted line runs from P on the outer circle at the upper right down through B and F to the outer circle at the lower left. Straight lines also run from N to F, from I through F to H, from F to K and from F to M.

- A (top of the outer circle)
- E (upper left of the outer circle)
- P (upper right of the outer circle)
- I (upper left of the second circle)
- B (on the vertical diameter, above the small circle)
- M (right of the second circle)
- N (left of the second circle)
- F (centre of the second circle)
- K (centre of the small circle)
- L (on the vertical diameter, foot of the small circle)
- H (lower right of the second circle)
- G (on the vertical diameter, foot of the second circle)
- Q (lower left of the outer circle, at the foot of the dotted line; a small sort with its tail crossed by the circle)
- D (foot of the outer circle)
- [?] (a very small sort just left of B, on the dotted line)

V. *First*, in the triangle BEK, BE is given, the radius of the annual orb, marking the mean place of the Earth E as seen from B; the place opposite to it in the zodiac is the mean place of the Sun as seen from the Earth. BK is also given, and the angle KBE contained by them, for it is the complement to two right angles of the angle ABE, known from the anomaly of the eccentric. From these, therefore, you will find both the angle BEK and the side KE; and the third angle, BKE, will be the complement of the other two to two right angles.

[Margin: Step 1, for the equation of the centre.]

[Margin: By Book 1, 32 of Euclid.]

Second, in the triangle KEF, the side KE has now been found, and FK is given, the semidiameter of the small circle, since it is a quarter of the whole eccentricity BL. The angle contained by them, FKE, becomes known if from the angle FKB, known from the doubled anomaly of the eccentric, you subtract the angle BKE already found. From these three givens, then, find both the side FE and the angle KEF. Add that to the angle BEK and the result will be the angle BEF: namely the whole prosthaphaeresis of the eccentricity, to be subtracted from the mean motion of the Earth, that is, from the motion opposite to the Sun's mean motion, in this first semicircle of the anomaly, AEQ, but to be added in the other, so as to give the place of the centre of ♀, coequated and seen from F, the centre of the orb of Venus; opposite to this is the place of the centre F, seen from the Earth E along the line EFG, etc.

[Margin: Step 2, for the same.]

Third, if Venus is at N, then in the triangle FEN the side FE has now been found; and FN is given from the table at the end of this chapter, otherwise the radius of the epicycle, now of the orb of Venus, in the parts in which BE,

the radius of the annual orb or the mean distance of the Earth and Sun, was known or assumed. The angle NFE is known from the anomaly of the orb, or of commutation, which here is the arc HIN, counted in the order of the signs from H, the apogee of the orb (otherwise of the epicycle). In our case, then, casting off the semicircle HMI leaves the arc NI, that is, the angle NFI, to which must be added IFE; because the lines HI and BE are parallel, IFE is equal to the angle BEF, that is, to the prosthaphaeresis of the centre already found, and so the whole NFE will be known. By means of trigonometry, therefore, both the side EN will be found, which is the distance of Venus from the Earth, and the angle at the Earth, or FEN, which is the prosthaphaeresis of the orb, or the equation of the parallax of the annual orb. In this position, namely in the later semicircle of the anomaly of the orb, it must be subtracted from the equated place of the centre, or the eccentric place of the planet, so as to give the true place of the Earth seen from Venus along the line NE, to which the true place of ♀, seen from the Earth and computed from the mean equinox, stands diametrically opposite in the zodiac. If the prosthaphaeresis of the equinoxes is applied to it, the place of the same planet from the true equinox will be obtained. Now imagine Venus at M. For the triangle MEF will have to be solved, in which the radius FM is given, and the side FE has been found, as before; and the angle MFE will become known as follows. The anomaly of the orb is the arc HM, namely the angle HFM; its complement to two right angles is the angle MFI. If from this you take away IFE, equal to the angle BEF, or the prosthaphaeresis of the centre, because the lines HI and BE are parallel, MFE will be left. You can therefore find ME, the planet's distance from the Earth, and the angle FEM; subtracting from it the angle BEF leaves BEM, the equation of the orb, to be added in the first semicircle of the anomaly of the orb to the coequated place of the centre itself, so as to give the true place of the Earth, seen from Venus along the line ME, to which is opposed in the zodiac the place of Venus seen from the Earth E, etc.

[Margin: Step 3, for the equation of the orb, or of commutation.]

[Margin: The other case, for the other position of Venus.]

VI. I pass over examples, both because I have now set out this method clearly and distinctly, so that the obscurity found until now in other authors seems to me to have been removed, and because after Section 2 has been understood I take the student of astronomy to be so trained and advanced as not always to need the support of examples. You can, however, see them sketched in *Copernicus*, Book 5, chapter 23, and in *Longomontanus*, Book 2 of the *Theorica*, chapters 17 and 18, or from me below, chapter 11, numbers 7 and 8, where this diagram is explained more clearly.

[Margin: Authors in whom examples of this method are found.]

Prior Theorica Mercurij.

The first theory of Mercury.

VII. The Copernican hypothesis of Mercury is set out by *Copernicus*, Book 5, from chapter 25 to 32; by *Rheticus* and *Mästlin*, and by *Kepler*, in the places cited for the hypothesis of ♀ in number 1; and by *Longomontanus*, Book 2 of the *Theorica*, chapters 20 and 21. Now, besides the various differences by which ☿ differs from ♀, related in chapter 1, number 7, the following must be added from the Copernican hypothesis. In it ♀, if its apogee were the same as the Sun's, would be at perihelion when at apogee and at aphelion when at perigee, contrary to the way of Mercury and the others; for when the Earth falls on the line of apsides of these planets, Venus has its least eccentricity and Mercury its greatest. The least of ♀ is half the greatest, but that of ☿ is less than half; and ♀ lacks the epicycle that ☿ has. But to grasp these and other points, in the following diagram of *Copernicus*, Book 5, chapter 25, and of *Longomontanus*, page 419, let G be the centre of the great orb AB, whose diameter and line of apsides of Mercury is AB. On it, between G and B, take GF, and divide it into four parts, or approximately so, according to what is to be said about the measures in number 11; let three of them be GI, and the fourth IF, the semidiameter of the small circle FE described from I. This is called the small circle of the anomaly of the eccentricity, and the centre of the orb of Mercury has at E its least, at I its mean, and at F its greatest eccentricity from G, the centre of the world and of the annual orb. With the centre placed at F, then, describe the eccentric of Mercury, CH (in the old theory, otherwise, the epicycle); and with the centre made at C, describe the epicycle LK, on whose diameter Mercury is to librate. Let the beginning of the revolutions be at K, the perihelion of the epicycle, when the centre of the eccentric is at F, the aphelion of the small circle, and let the planet be moved from K upward to L; but let the centre of the eccentric be moved on the small circle FE in the order of the signs, yet with a double revolution to one revolution of the Earth. For from this it will come about that whenever the Earth is at A or B, the centre of the orb of Mercury is at F, with the greatest eccentricity; but when the Earth has completed a quadrant by its mean motion, the centre of the orb of ☿ is at E, the point of least eccentricity, the semicircle FE of the small circle having been completed; and one position of the eccentric after another is marked out. To understand these, and the motions along the diameter of the epicycle, I shall set below this other figure, from Mästlin.

[Margin: Copernicus's diagram.]

Diagram. Set at the right of the right column, with the text running in a narrow measure beside it. A large outer circle with a vertical diameter from B at the top to A at the foot. Inside it, a second circle whose top is near K and C and whose foot is at H. On the diameter near the top, a small circle about C, between L above and K below. On the diameter near the centre, a very small circle, drawn clearly only on its left side, about I, between F above and E below.

- B (top of the outer circle)
- L (on the diameter, top of the small circle about C)

- C (centre of that small circle; a small sort)
- K (on the diameter, foot of that small circle)
- F (on the diameter, top of the very small circle)
- I (within the very small circle)
- E (on the diameter, foot of the very small circle)
- G (on the diameter, below E)
- H (on the diameter, foot of the second circle)
- A (foot of the outer circle)

VIII. Let the annual orb of the Earth be BCD, whose centre is A, and let its diameter, for the line of apsides, be BAD. On this let the whole eccentricity of the orb of Mercury from A be AF, and approximately three-quarters of it AE, and the quarter part EF, for the semidiameter of the small circle. When the Earth is at the apogee of Mercury, B, or at the perigee D, the centre of the orb is at F, and the orb of Mercury is HI; Mercury itself is on the epicycle DK, at the perigee K, or at the perigee L. So the path of Mercury is then KML (for in this figure the right-hand side is the east, towards which the Earth, the centre of the orb and the centre of the epicycle are all moved in the order of the signs), that is, the narrowest circuit of all in breadth.

[Margin: Mästlin's diagram.]

Diagram. Set in the left column, inside the sentence ending at K. A large outer circle with a vertical diameter from D at the top to B at the foot. Several large circles of different centres lie inside it, overlapping near the top and the foot. Small circles stand on the diameter: one near the top about H and K; one below it about P and N; one at the centre about E, with F just above it and G below it; and two near the foot, one about I and one about O, with Q below. From C on the left of the outer circle, two straight lines run to the diameter, one near E and one near A. A straight line runs through F to M at the right and on to the left. From B two straight lines rise, one to the left and one to M.

- D (top of the outer circle)
- H (on the diameter, within the topmost small circle)
- K (on the diameter, foot of the topmost small circle)
- P (on the diameter, within the second small circle)
- N (on the diameter, foot of the second small circle)
- S (upper left of the outer circle, beside a small mark)
- F (on the diameter, above the central small circle)
- M (right of the diameter, on an inner circle)
- E (within the central small circle)
- G (on the diameter, below E)
- A (on the diameter, below G)
- C (left of the outer circle)
- R (right, between the inner circles and the outer circle)
- L (left of the diameter, below A)
- I (within the upper of the two lower small circles)
- O (within the lower of the two lower small circles)

- Q (on the diameter, below O)
- B (foot of the outer circle)

But when the Earth is at mean elongation from the apsides, that is, a whole quadrant from Mercury's apogee and perigee, as at C, the centre of the orb of ☿ is at G, and the equation of the centre is the angle ACG; and its eccentric is NO. Mercury, however, holds the highest summit of its epicycle, and is either at P or at Q, and describes a path and, as it were, a very loose spire, which is PRQ. Moreover, when the Earth is at B, the apogee of Mercury, the prosthaphaereses of the orb, or of Mercury's parallax, are the least of all, because of the narrowness of Mercury's path and the very great distance from Mercury, so that the greatest of them then does not exceed the angle FBM. But with the Earth placed at the perigee D, although the orb of Mercury is at its narrowest, it nevertheless appears larger because of the nearness of the Earth. In the quadratures, as at C, that orb, although it is looser, does not appear larger, because of Mercury's great distance from the Earth. But at about a third of the circle counted from the apogee, as at S, the end of the arc BCS, or about a sixth from the perigee D, it appears greatest, in the opinion of Ptolemy and Copernicus, and requires the greatest prosthaphaereses, by which Mercury differs and digresses furthest from the Sun, that is, from its own mean place; this arises from the mutual compensation of distance and true size. Therefore, although the parallaxes of the annual orb are least at Mercury's apogee, the greatest excess is nevertheless not at the perigee but at a sixth of the circle counted from the perigee. And *Mästlin* rightly warns that proportional minutes are not to be trusted at all here, but that one must proceed by another route, in constructing the tables or in deriving the dimension of the orbs from tables already constructed. Finally, Copernicus measures the libration of Mercury on the diameter of the epicycle in the same way as the libration of the equinoctial points, according to what I said in Book 3, chapter 29, number 10. But in Book 5, chapters 28 and 30, adding a diagram, he himself weighs this libration by the rule of second sines; and he teaches that Mercury's digressions and the parallaxes of the annual orb become greater at about the side of the hexagon, that is, at a sixth of the anomaly of the eccentric counted from the perigee, than at the perigee, not because the orb of the planet comes nearer to the Earth, as Ptolemy thought, but because the planet describes a larger circle. So what gave Ptolemy occasion to err, and to place Mercury so near the Earth in that position, was the size of the equations of the orb, as *Longomontanus* also notes, Book 2 of the *Theorica*, chapter 20. You will, however, understand and admire Copernicus far better when you read, in chapter 9, his demonstrations, illustrated by me, on the commensurations of the orb of Mercury and on that hexagonal position, from problem 3 to problem 6.

[Margin: The unreliability of proportional minutes in ☿.]

[Margin: Ptolemy's error corrected.]

[Margin: Where is Copernicus illustrated below?]

Posterior Theorica Mercurij.

The later theory of Mercury.

IX. It is remarkable with what ingenuity Copernicus tried to cast Mercury, like Proteus, into the chains of astronomical geometry, though he could not catch it with his eyes and observe it. For he says in Book 3, chapter 30, speaking of Ptolemy's observations: "The ancients certainly showed us this way of examining the course of this star, but they were helped by a clearer sky, where, as they say, the Nile does not breathe such airs as the Vistula does among us. For to us who inhabit a colder region nature has denied that advantage, where calm air is rarer, and where, moreover, because of the great obliquity of the sphere, it lets us see Mercury more rarely, even at its greatest distance from the Sun. For in Aries and Pisces it does not rise into our view, nor again does it set in Virgo and Libra; nor does it show itself in any way in Cancer or Gemini, when there is only twilight or dawn at night and never true night, unless the Sun has withdrawn a good way into Leo. For this reason this star has tormented us with many windings and labours, in order that we might scrutinize its wanderings." And he adds that he was forced to take refuge in the observations of Bernhard Walther and Johannes Regiomontanus made at Nuremberg. And in chapter 32 he says: "So indeed has nature played about this star, and with so wonderful a variety, yet one she has confirmed with a perpetual, certain and immutable order." But Copernicus did not allow Mercury to remain as unconquered by him as it was unseen or unobserved, nor to slip from his hands as it did from his eyes. So in the same chapter 32 he proposes this new theory, serving chiefly the greatest digressions of ☿ from ☉, as follows.

[Margin: Mercury unseen by Copernicus, but not entirely unconquered.]

Let there be a circle GHKP, cut into four parts at the centre F by the diameters GK and HP; let the small circle ML be concentric with it. With the centre made at L, and the interval LO taken, equal to the semidiameter FG or FH, let the circle OHRP be described, and let Mercury then be at O. And let these circles be supposed to be moved about F in the order of the signs, with their intersections, that is, the lines GFR and HFP, daily by about 2° 7′, namely by as much as the motion of commutation of Mercury, or of the anomaly of the orb, exceeds the mean motion of the Earth (or, in other authors, of the Sun), counted from the apogee of Mercury's eccentric. Mercury itself, however, is to be moved from the line of apsides GK along the orb OR, and to supply the remaining motion, almost like the motion of the Earth, by an annual revolution; meanwhile, in that revolution, the centre of the orb OR, which carries the star, is to be carried up and down, by a motion of libration and reciprocation, along the diameter ML of the small circle, twice as great as the one we assumed above. With these things laid down, when the centre of Mercury's eccentric is at L and Mercury itself at O, the planet will be at its least distance from the centre F, and will describe the smallest circle, and the least semidiameter of the eccentric will be FO. But when Mercury is at about the mean apsis, it will be

at H, and at its greatest distance from F, and will describe the greatest windings and a circle whose semidiameter will be FH; for [?] then the deferent OR will coincide with the orb GH, because the centre F is the same for both. Now let the Earth proceed towards the perigee, until the centre of the orb OR falls at M; for the orb will be raised above GK, and the star, at R, will again be least distant from the centre F, between K and L. Here, then, 3 mutually equal revolutions coincide: namely that of the Earth relative to the apogee of the orb of Mercury's eccentric; the libration of the centre along the diameter LM; and the motion of the planet on the eccentric from the line FG back to the same FG. From these, however, the motion of the intersections GH and KP from the apsis of the centre differs. And so in the middle spaces of the quadrants GH and KP the star does not pass through without a difference of longitude, and there it would undergo its greatest prosthaphaeresis, if that instability of the centre did not stand in the way; nor does it allow that prosthaphaeresis to occur which the eccentricity FL would otherwise have required, since as the centre of the eccentric approaches F the prosthaphaeresis gradually diminishes, until it vanishes entirely at about the middle intersections HP.

Diagram. Set at the right of the right column, with the text running in a narrow measure beside it. Large circles overlap, meeting near H on the left and P on the right. A vertical line runs from G at the top down to the foot, with R printed below it; a horizontal line runs from H to P, crossing the vertical at F. A small circle stands about F, with M above it and L below it.

- G (top, above the uppermost circle)
- O (on the vertical, below G; a small sort)
- M (on the vertical, top of the small circle)
- F (centre of the small circle)
- H (left end of the horizontal)
- P (right end of the horizontal)
- L (on the vertical, below the small circle)
- K (on the vertical, on a circle near the foot)
- R (below the foot of the lowest circle)

Methodus Calculi Geometrici in Hipotheſi Mercurij.

The method of geometric calculation in the hypothesis of Mercury.

X. As regards the equations of the centre, the method is exactly the same as the one given for Venus in number 4. For the equations of the orb, or of commutation, it is indeed the same, but the semidiameter of the orb of Mercury is needed in the parts in which the radius of the annual orb was assumed; and that semidiameter varies according to the libration made either on the diameter of the epicycle, in Copernicus's earlier form, or on that of the small circle, in his later form. This semidiameter must therefore be equated, and the rest then carried out entirely as we taught there. To equate that semidiameter, however, three elements are needed: namely that there be given the least semidiameter of

the orb of Mercury; the least semidiameter either of the epicycle in Copernicus's first form, or of the already mentioned small circle in the 2nd, in the same kind of parts; and the anomaly of the eccentric. For the second sine of this anomaly doubled must be found; or, if it exceeds a quadrant, the second sine of its complement to the semicircle; or, if it exceeds a semicircle, the second sine of the excess; or finally, if the anomaly exceeds three quadrants, the second sine of its complement to the circle. Then that second sine must be multiplied by the semidiameter of the epicycle or small circle, and the sum divided by the whole sine. If the anomaly belongs to the 1st or 4th quadrant, the quotient must be added to the semidiameter of the small circle or epicycle, for this gives the addition to be made to the least semidiameter of the orb of Mercury, which we need in order to obtain the prosthaphaereses of the orb. But if the anomaly belongs to the 2nd or 3rd quadrant, that quotient must be subtracted from the semidiameter of the epicycle or small circle, and the remainder added to the least semidiameter of the orb. See this procedure illustrated by an example in chapter 10, problem 6.

De Numeris ac Menſuris ad Hypotheſim Veneris ac Mercurij Copernicæam ſpectantib.

On the numbers and measures relating to the Copernican hypothesis of Venus and Mercury.

XI. Although *Copernicus* gives the numbers of the motions in Book 5, chapter 1, and the measures of the orbs of Venus in chapters 21, 22 and 23, and of Mercury in chapters 27 and 30, *Mästlin* nevertheless decided, on Kepler's *Prodromus* or *Mysterium Cosmographicum*, chapter 15, and in his appendix to the same *Prodromus*, to fish them out from the Prutenic prosthaphaereses, so that it might be known whether and where Reinhold had corrected them. For Reinhold admits in the preface to the Prutenic Tables that they need correction, because Copernicus, by then an old man, slipped in calculation more than once; I shall give a sample of such a slip below, chapter 8, number 6. We therefore decided to present these measures from both, that is, from Mästlin, just as if they were Reinhold's, and from Ptolemy, as they stand. But to make their comparison readier, we have added one zero to all the measures of Copernicus, because Copernicus uses a radius of the annual orb of only 10000 parts, whereas Mästlin uses one of 100000 parts. So if one zero is taken away from the Copernican measures set out in the following table, you will at once have them as they stand in Copernicus's book. Recall, besides, that the whole eccentricity of the equant of Venus suited to the time of Ptolemy was found by Copernicus to be 416 parts, of which the radius of the Earth's annual orb is 10000; that it was therefore bisected, as in the superior planets; and that half of it, 208 parts, was given to the least eccentricity of the deferent, and a quarter, 104 parts, to the semidiameter of the small circle producing the anomaly of the eccentricity. Afterwards, however, in his own times, Copernicus found the

whole eccentricity of the equant to be 350 parts. This he did not bisect; rather, keeping the semidiameter of the small circle at 108 and the diameter at 208, he gave the remainder of the 350, that is, 142, to the least eccentricity of the deferent; and the distance of the centre of that small circle from the centre of the annual orb, which he had earlier made 312, he made 246 in his own time. Nor is the least eccentricity of Mercury in this hypothesis exactly half of the whole or greatest eccentricity. For Copernicus the least is 524 and the greatest 948, of which the radius of the annual orb is 10000; for Mästlin, who uses a radius of 100000, the former is 5231 and the latter 9460. Further, neither of them presents these measures in semidiameters of the Earth, so it fell to us to derive them from the semidiameter of the annual orb, that is, from the mean distance of the Sun from the Earth, which Copernicus makes 1142, Mästlin 1160. The greatest, however, Copernicus makes 1179 in his own time, with the Sun's eccentricity almost least, as does Mästlin too; but with the eccentricity greatest, Copernicus makes the greatest 1190, Mästlin 1208½. In this place, however, we shall indicate only some distances of these planets from the Earth, both in Ptolemy's age and in Copernicus's, taking their eccentricity from the centre of the annual orb and considering the Earth on the line of apsides of each planet. Distances from the Sun, because of the many comparisons they involve, we reserve for Section 6, chapter 2, number 3, where we shall give several rules about them; for in considering the idea of this hypothesis, these distances can be viewed by taking the Sun's eccentricity now greatest, now least, and in either case supposing either that the apogees of these planets coincide with the Sun's apogee at the same point of the zodiac, or that they stand diametrically opposite the solar apogee. For these cases we shall give there rules and perfectly complete tables. Meanwhile we shall set below only the solar distances, collected from Mästlin's numbers as given to the radius of the annual orb, and suited to the time of Ptolemy.

[Margin: The authors from whom the following table is taken.]

[Margin: Note on the eccentricity of ♀ and ☿.]

MENSVRÆ AD COPERNICI Hypotheſim ſpectantes

Measures relating to the hypothesis of Copernicus

Where ♀ has M, the measures there agree with its greatest eccentricity, which it had in Ptolemy's time.	Planets	Parts of which the radius of the annual orb is 100000: Copernicus	Semidiameters of the Earth: Copernicus	Semidiameters of the Earth: Mästlin
Whole eccentricity of the centre of the orb from the centre of the annual orb: EN	♀	3500	40	40
	♀ M	4160	48	48
AF	☿	9480	108	110
Least eccentricity of the centre of the orb from the centre of the annual orb: EM	♀	1750	20	20
	♀ M	2080	24	24

Where ☿ has M, the measures there agree with its greatest eccentricity, which it had in Ptolemy's time.	Planets	Parts of which the radius of the annual orb is 100000: Copernicus	Semidiameters of the Earth: Copernicus	Semidiameters of the Earth: Mästlin
AG	☿	5240	60	61
Semidiameter of the small circle of the eccentricity: MA	☿	875	10	10
	☿ M	1040	12	12
GE	☿	2120	24	24½
Semidiameter of the epicycle: DH	☿	1900	21½	22
Semidiameter of the orb of the planet: MV	☿	71930	821	834
Mean	☿	37630	428	436
Greatest: GR	☿	39530	451	459
Least: FL	☿	35730	405	413
Semidiameter of the annual orb: EL	☿	100000	1142	1160
AC	☿	100000	1142	1160
Greatest distance from the centre of the Earth: HQ	☿	173680	1983	2014
HQ	☿ M	174010	1987	2018
BK	☿	145210	1655½	1683
Least distance from the centre of the Earth: GQ	☿	26320	301	306
GQ	☿ M	25990	297	302
BL	☿	54790	629	637
		Mästlin		
Eccentricity from the ☉ of the centre of the small circle of the eccentricity	☿	1262		15
	☿	10270		119
Greatest distance from the Sun, or aphelion	☿	74232		861
	☿	48114½		558
Least distance from the Sun, or perihelion	☿	69628		708
	☿	23345½		271

Mästlin derives from the Prutenic Tables Mercury's whole eccentricity as 9460 parts, its least as 5231, and the semidiameter of the small circle of the eccentricity as 2114. In all the remaining measures of the former column he agrees with Copernicus, but in the third column he disagrees, both because of the disagreement just mentioned and because he makes the semidiameter of the great orb 1160 semidiameters of the Earth.

[Margin: For ☿ see the figure in number 3, and for ☿ number 8.]

[Margin: That is, of the small circle of the anomaly of the eccentricity.]

[Margin: For the time of Ptolemy.]

MOTVS MEDIVS Seu Aequalis.

Mean or equal motion.

Motion		Deg.	′	″	‴	⁗	V	VI
		Of the planet on its orb, or of commutation						
Daily	♀	0	36	59	28	0	7	18
	☿	3	6	24	14	5	35	48
Annual	♀	225	1	45	20	44	46	0
	☿	53	57	25	44	2	46	0
		Motion of the apogee						
Annual	♀	None under the fixed stars						
	☿	0	0	57	50	38	0	0

RADIX MOTVVM ad Primum CHRISTI Annum Ad locum & diei momentum, de quo sect. 2. cap. 2. ad finem.

Root of the motions for the first year of Christ, for the place and moment of the day discussed in Section 2, chapter 2, at the end.

		Sign.	Deg.	′	″	‴	⁗
Of commutation	♀	4	6	46	31	35	37
	☿	1	16	53	1	26	57
Of the apogee	♀	1	18	21	0	0	0
	☿	6	6	34	48	41	0

These motions are counted from the first star of Aries. If you add to them the Copernican precession of the equinoxes, that is, the apparent motion of the fixed stars, you will have the motions counted from the mean vernal equinox. The apogee of Venus, however, perpetually keeps the same distance from the first star of Aries, 48° 20′ or 21′. I shall speak of the periods of the motions at the end of the Section, and of the greatest digressions and the equations in chapter 7.

SCHOLIA.

Scholia.

I. Just as Copernicus preferred to use an epicycle of an epicycle in the Moon, and eccentrepicycles in the three superior planets, so in the two inferior planets he preferred to use eccentrics of the eccentric, as being more convenient for demonstrating the commensuration of the motions; although, because of their mutual equivalence, he could have used any of the three forms set out in Section 2, chapter 2, indifferently. By these he achieved that neither Venus nor Mercury should undergo that insane parallax which they would have to suffer in the Ptolemaic and Alphonsine hypothesis. For in that hypothesis Venus at perigee is sometimes no more than about 200 semidiameters of the Earth away from the Earth, and Mercury no more than 65; whereas in the Copernican hypothesis

Venus is removed by more than 340, and Mercury by more than 383, however near their perigee they may be.

[Margin: The difference between the hypothesis of Ptolemy and that of Copernicus in parallaxes.]

II. Kepler, however, in Book 6 of the *Epitome of Copernican Astronomy*, page 766, believes that the eccentrics of the eccentric were introduced into the theory of Venus and Mercury only to compensate in turn for the amount by which Copernicus varied the distances of the Earth from the Sun beyond what was due, since he did not know the true eccentricity of the great orb, which, as Kepler would have it, is half of what Copernicus, together with the ancients, believed. The evidence is that Copernicus librates the eccentric of the eccentric of Venus through a space as great as that superfluous half in the eccentricity of the great orb, and the eccentric of Mercury through a slightly smaller space, because in Venus the line of apsides coincides very nearly with the line of apsides of the Sun, whereas in Mercury it departs further from it. For that reason the apogee of Venus was raised too high for him because of the excessive eccentricity of the great orb, and that of Mercury was depressed too far. But these conjectures of Kepler, ingenious as they may be, still do not establish what he wants; nor has the bisection of the eccentricity in the Sun yet been shown.

[Margin: Kepler's censure of Copernicus.]

III. Yet it may seem inelegant in this hypothesis that it measures the anomaly of the eccentric from the mean motion of the Sun or Earth rather than from the true motion, since this motion is fictitious and does not actually take place. But this difficulty is general, in all the planets. What seems rather to destroy the physical cause of the motions and the plan of providence is that, because of the doubled motion which Copernicus assigns to the small circle varying the eccentricity, the eccentricity is the same both at the apogee and at the perigee of the eccentric. For what is the purpose of varying the eccentricity if at the greatest and least distance, where the difference ought to be greatest, there is nevertheless no difference at all? Or what probable reason could there be for this motion to differ not at all at the extremes and greatly in the middle? The same seems to be objected to the variation of the epicycle of Mercury, whose semidiameter is put as least at apogee and perigee alike. But if, all things computed, the observations cannot be satisfied without this hypothesis, we shall have to swallow what seems to us this inelegance. Yet Copernicus ought rather to have built this hypothesis of his on more recent observations; but after citing only three of Mercury and one or two of Venus, he builds everything else on observations made or reported by Ptolemy. *Mästlin* raised this complaint against him in the letter to Kepler that stands among the additions to the *Mysterium Cosmographicum*, chapter 19.

[Margin: Three objections to the hypothesis of Copernicus.]

CAPVT III.

Chapter III.

De Hypotheſi Lansbergiana Veneris ac Mercurij.

On the Lansbergian hypothesis of Venus and Mercury.

De Theorica Veneris.

On the theory of Venus.

I. We place this hypothesis next after the Copernican one, because in it *Lansbergen*, in his *Theoricae motuum Caelestium*, chapters 11 and 12, emulated the Copernican hypothesis; and it differs from that one only in the measures of the orbs and the numbers of the motions. So look at the figure added here. In it, near the Sun, A is the centre of the great orb GH; GH is the line of apsides of Venus; on that line C is the centre of the small circle DB of the anomaly of the eccentricity, whose diameter DB is 204 parts, of which AG, the radius of the great orb of the Earth, is 10000; the semidiameter CD is 102; AB, the least eccentricity, is 145; and AD, the greatest, 349. Therefore, with the Earth placed at H, the ☉ is between D and E, because the ☉'s eccentricity is greater.

Diagram. Set at the right of the right column, with the text running in a narrow measure beside it. A vertical line runs from G at the top to H at the foot. Two large circles lie about it, one inside the other, and a small circle stands on the line about C. The labels read down the line are G, E, D, C, B, A, F, H: D stands at the top of the small circle and B at its foot; A is just below B; F is below the foot of the inner large circle.

- G (top of the line): G
- E (on the line, below G): E
- D (top of the small circle): D
- C (centre of the small circle): C
- B (foot of the small circle): B
- A (below B): A
- F (lower part of the line): F
- H (foot of the line): H

Now the centre of the eccentric of Venus is moved on the circumference of the small circle DB in the order of the signs, but with a motion swifter than the mean motion of the Earth in longitude (which is the same as the mean motion of Venus and Mercury), counted from the apogee of ♀, which is called the motion of the anomaly of the centre. Whenever the Earth falls on the line of apsides of Venus, that is, is at G or at H, the centre of the orb of Venus is at B, the point of least eccentricity; but with the Earth 90° distant from G or from H, that centre is at D. Therefore let the eccentric orb of Venus, EF, now be described from B, its perihelion E with respect to the Sun placed near A, and likewise its apogee with respect to the Earth placed at H. Let F be the aphelion,

and indeed also the perigee with respect to the Earth stationed at G. But it is the mean one [?], if it is reckoned from the mean equinox.

[Margin: An explanation of aphelion and apogee, omitted by Lansbergen.]

Methodus Calculi Geometrici in hac ♀ Hypotheſi.

The method of geometric calculation in this hypothesis of ♀.

II. In the following diagram, A is the centre of the great orb PO, and BD is the small circle of the anomaly of the eccentricity, discussed above. Let there be given, for example, a mean motion of the Sun of 196° 6′ 5″, counted from the mean equinox, and let the equal motion of the apogee of Venus from the same point be 46° 14′ 40″. When this is subtracted from the mean motion of the Sun, there remains the anomaly of the centre, 149° 51′ 25″. These degrees, counted from the line PO of the mean apogee in the order of the signs, give at M the mean place of the Sun, opposite which stands I, the place of the Earth (the mean place), on the line MAI. The same anomaly, doubled, gives the motion of the centre of the orb. Let 149° 51′ 25″ doubled, then, that is, 299° 42′ 50″, be counted from the point B, rightward towards D; for the arc corresponding to them will be BDE. So the centre will be at E; from it describe the orb of Venus GKN. Next, through the centre E draw the straight line LN parallel to the line IM of the mean motion of the Sun, and on it let N be the apogee of the orb of ♀ and L the perigee. Now imagine Venus to be at K. We must therefore find the place of the centre, and then its apparent place, that is, as seen from the Earth.

First, for the arc EB, the remainder of the circle DEB, find the versed sine BF (which is obtained by subtracting the second sine FC from the whole sine) and the first sine EF. Multiply EF and FB each separately by the semidiameter CB, which is 102 parts, and divide the total by the whole sine; for you will then have EF and BF known in parts of which the radius AP of the great orb is 10000. Add BF, so known, to the least eccentricity AB, that is, to 145 parts, and AF will be known in the same kind of parts. Now when EH is produced towards P until the perpendicular IH from the Earth I falls on it, GH will be equal to FE.

[Margin: 1st step, for the equation of the centre.]

Diagram. Set in the left column, inside the sentence ending at FE. A large outer circle with a vertical line from P at the top to O at the foot. Inside it, a second circle, passing through K at the upper left, L at the upper right and N at the lower left. On the vertical line, a small circle about C, with D above it and with B at its foot. From I on the outer circle at the upper right, a dotted line runs horizontally to G on the vertical line, and a solid line runs to K; solid lines also run from I to E and from I down through the centre region to M on the outer circle at the lower left, where a rayed Sun stands beside the letter. A straight line runs from L through E to N, and another from K to E. A dotted

line runs down from H to E. A short line runs from E to F. The sign ♀ stands in the lower right of the band between the two large circles.

- P (top of the outer circle): P
- G (on the vertical line, at the top of the second circle): G
- H (left of G, at the head of the dotted line): H
- I (outer circle, upper right): I
- K (second circle, upper left): K
- L (second circle, upper right): L
- D (vertical line, above the small circle): D
- C (centre of the small circle): C
- F (inside the small circle, below C): F
- E (left of the foot of the small circle): E
- B (vertical line, foot of the small circle): B
- A (vertical line, just below B): A
- N (second circle, lower left): N
- ♀ (lower right, between the two large circles): the sign of Venus
- M (outer circle, lower left, beside the rayed Sun): M
- O (foot of the outer circle): O

Second, because the complement MO of the anomaly of the centre, that is, of the arc PM, is given, and PI is equal to it because of the equal angles at the vertex A, find the sine IG of the arc PI and its second sine AG, in parts of which the radius AP of the great orb is 10000. Then add HG to GI, so that the side HI may be known. Conversely, take AF away from the sine AG, and FG will remain, that is, HE, which is equal to it, known in the same kind of parts. Next, when EI is drawn, the triangle EHI will be formed, right-angled at H, in which from the given sides about the right angle, HI and HE, the angle HIE is obtained by means of trigonometry. Likewise in the triangle AGI, right-angled at G, from the given GI and AG you will seek the angle AIG; take EIH from it, and AIE will remain, that is, the prosthaphaeresis of the centre, to be subtracted from the mean motion of the Sun M, because the anomaly of the centre is less than a semicircle, so that the true place of the centre, or the centric longitude of Venus, may be known. In addition, the base EI must be sought in the same triangle.

[Margin: 2nd step.]

Third, from the tables find the mean anomaly of the orb of Venus and count it in the order of the signs from the apogee of the orb N, through L, as far as K, where Venus is; for when the semicircle LN is taken away, the arc KL remains, which the angle KEL measures. The angle IEL, because of the parallelism of the lines LN and IM, is equal to the angle AIE, that is, to the equation of the centre. This, then, added to the angle KEL, gives the angle KEI. With this, with the side EI already found, and with the radius EK of the orb of Venus known in parts of which the radius of the great orb is 10000, the angle EIK is found, that is, the prosthaphaeresis of the orb, subtractive, because the anomaly of the orb ends in the second semicircle. When this is subtracted from the

centric longitude of Venus, there will remain the place of Venus K as seen from the Earth I, from the mean equinox, and, with the prosthaphaeresis of the equinoxes, from the true equinox.

[Margin: 3rd step, for the equation of the orb.]

Finally, fourth, in the same triangle EKI, from the same data you will find, if it is needed, the side KI, that is, the distance of Venus from the centre of the Earth. You have this example worked out in numbers and completed in *Lansbergen's Theoricae*, chapter 12; turn to it, if you wish.

[Margin: 4th step, for the distance of the planet.]

De Theorica Mercurij.

On the theory of Mercury.

III. This, likewise, was taken from Copernicus by *Lansbergen*, chapters 13 and 14 of the *Theoricae*, but with the measures corrected. So in the following little diagram A is the centre of the great orb of the Earth, near the Sun; GRLP is the great orb; and GL is the line of apsides of Mercury, on which C is the

Diagram. Set across the right column, inside the sentence naming C. A large outer circle with a vertical line from G at the top to L at the foot, and R and P inside it at the left and the right. Inside it, a second circle, with M at its left and N at its right. At the top of the vertical line, a small circle about E, with K just below it; at the lower part of the line, where the second circle meets it, a small circle about F, with I just above it. Near the middle of the line, a small circle about C, with D above it and B below it, and with A on the line lower down.

- G (top of the line): G
- E (centre of the small circle at the top): E
- K (below that circle): K
- D (above the middle small circle): D
- C (centre of the middle small circle): C
- B (below the middle small circle): B
- A (below B): A
- M (second circle, left): M
- N (second circle, right): N
- R (left, inside the outer circle): R
- P (right, inside the outer circle): P
- I (above the small circle at the foot of the second circle): I
- F (centre of that small circle): F
- L (foot of the line): L

centre of the small circle of the anomaly of the eccentricity, whose semidiameter CB is 212 parts, of which the radius AG of the great orb is 10000. The greatest eccentricity of the orb of Mercury from the centre of the great orb is AD, 947 parts; the least is AB, 523 parts; BD is 424; and the least semidiameter of the orb of Mercury is 3573 such parts. The Sun, moreover, is between A and

B. Now let the centre of the orb be at D, and from it let the orb EMFN be described, carrying the centre of the epicycle E in the order of the signs, from E to M, which is called the motion of the anomaly of the orb, to be computed from the line of apsides GL. The semidiameter KE is 190, and the diameter GK 380 parts; on this diameter Mercury is moved up and down, its motion beginning from K. The centre of the orb, further, is moved on the small circle BD, beginning from D, in the order of the signs, with a motion twice as swift as the motion of the anomaly of the centre, that is, the mean motion of the Earth, computed from the mean apogee G; and the reciprocal motion on the diameter of the epicycle is also as great. So when the Earth falls on the line of apsides of Mercury, the centre of the orb of Mercury is at D, and then the semidiameter of the orb is least, namely as great as DK.

Methodus Calculi Geometrici pro Mercurio.

The method of geometric calculation for Mercury.

IV. *Lansbergen* himself puts this example before the eyes in other words. On the line of apsides of Mercury, PO, let A be the centre of the world, nearest the Sun, and let C be the centre of the small circle BD of the anomaly of the eccentricity. Let the mean motion of the Sun be 147° 1′ 53″, and the mean motion of the apogee of Mercury 179° 4′ 59″. This, subtracted from the motion of the ☉ (a whole circle being taken in, as is customary), leaves the anomaly of the centre of ☿, 327° 56′ 54″. Counted in the order of the signs from the mean apogee O, through IP, this shows the Sun at M and the Earth at I. Doubled, with a whole circle cast off, it gives the motion of the centre of the orb of ☿, 295° 53′ 48″. These degrees, then, counted in the order of the signs from D through B, show the place of the centre at E, from which the orb of Mercury LQN is to be described. Through that centre the diameter LN must be drawn parallel to the line IM of the mean motion of the Sun or Earth, and on it L will be the apogee of the orb. Further, let Mercury be supposed at K, and let the Earth I be joined to K and E by the straight lines IK and IE; finally, from D draw the perpendicular DF to IM. With these preparations made:

[Margin: Chapter 14 of the *Theoricae*.]

[Margin: These were the motions in the year of Nabonassar 486, on the 30th of Pauni, in the evening.]

First, the arc OM, that is, 32° 3′, is the complement of the arc PM, and its measure is the angle OAM, whose complement to a right angle is FDA, 57° 57′, since AFD is a right angle. Therefore in the triangle ADF, right-angled at F by construction, from the said angle and the base AD of 947 parts, the trigonometer will find the side AF, of 802½ parts, and DF, of 502½.

[Margin: 1st step, for the equation of the centre.]

[Margin: By 32 of Book 1 of Euclid.]

Second, in the triangle BDE, right-angled at E because of the parallelism already mentioned, from the angle DBE, 32° 3′, because

[Margin: By 29 of Book 1 of Euclid.]

it is equal, because of the same parallelism, to the angle DAF, and from the common BDE, 57° 57′, and from BD, 424 parts, you will find DE, 225 parts. When these are taken away from DF, EF remains, 277½.

[Margin: 3rd step.]

Third, in the triangle IEF, right-angled at F, the side EF is now given; and IF, made up of the radius AI of the annual orb and the side AF found above, 802½, is 10802½. You will therefore find the angle FIE, 1° 28′, that is, the equation of the centre, to be

[Margin: In this diagram correct the engraver, by imagining the line NL to be drawn through the points B, E of the circumference of the small circle.]

Diagram. Set across the left column, inside the sentence ending at the base IE. A large outer circle with a vertical line from O at the top to P at the foot. Inside it, a second large circle. At the left, on the second large circle, a small circle about K, with H at its left and G at its right. Near the centre, on the vertical line, a small circle about C, with D above it, E at its right and B [?] below it; A stands on the vertical line lower down. A straight line runs from G through the small circle to E, and a short line from D to E. Straight lines run upward to the right across the centre: one from I at the lower left to the rayed Sun beside M on the outer circle at the upper right, and one from N to L. A further straight line rises from I to the small circle about K. F stands below and to the right of E. Q stands below the small circle about K. The sign ☿ stands in the lower right of the band between the two large circles.

- O (top of the outer circle): O
- D (above the middle small circle): D
- L (upper right, on the second circle): L
- M (outer circle, upper right, beside the rayed Sun): M
- H (left of the small circle at the left): H
- K (centre of that small circle): K
- G (right of that small circle): G
- Q (below that small circle): Q
- C (centre of the middle small circle): C
- E (right of the middle small circle): E
- B [?] (below the middle small circle, with lines crossing it): B
- F (below and to the right of E): F
- N (lower left, on the second circle): N
- A (vertical line, below the middle small circle): A
- I (outer circle, lower left): I
- ☿ (lower right, between the two large circles): the sign of Mercury
- P (foot of the outer circle): P

added to the mean motion of the ☉, so that the centric longitude of Mercury, or the place of the centre E, comes to 148° 29′ 53″. In the same triangle you will find the base IE, 10805 parts.

Fourth, from the tables of *Lansbergen*, for the time noted in the margin, the anomaly of the orb of Mercury was 114° 17′. These degrees, counted in the order of the signs from the apogee L as far as Mercury K, give the arc LK, that is, the angle KEL, and the remainder KEN, 65° 43′. If you add to this IEN, which is equal to the equation of the centre FIE because of the parallelism so often mentioned, the angle IEK will be 67° 11′.

[Margin: 4th step, for the equation of the orb.]

Fifth, on the small circle GH, because the motion of its anomaly is as great as the motion of the centre of the orb on the small circle BD, that is, in our case 295° 54′, to be counted from G, the perigee of the epicycle, this arc will be GHQ, and the remaining arc QG 64° 6′. Its versed sine KG is 5632 parts, of which the whole sine of the epicycle is 10000. So when the sine KG is multiplied by the semidiameter of the epicycle, 190 parts, and the total divided by the whole sine, KG comes out as 107 parts, of which the radius of the great orb is 10000. When these are added to the least semidiameter of the orb of ☿, 3573, the semidiameter KE becomes 3680.

[Margin: 5th step.]

Sixth, then, in the triangle KIE, in which the side KE has just been found, the side IE by the third step and the angle IEK by the fourth, you will seek the angle KIE, that is, the prosthaphaeresis of the orb, 19° 53½′. This, added to the centric longitude of Mercury, will give the place of Mercury, 168° 23′ 23″ from the mean equinox, but, with the prosthaphaeresis of the equinoxes, 169° 11′ 14″ from the true equinox, that is, in ♍ 19° 11′ 14″. Now Hipparchus, in the year of Nabonassar 486, on the 30th of Pauni, saw Mercury in the evening preceding Spica by a little more than 3°. Spica, for *Lansbergen*, was then in ♍ 22° 26′ 4″; so ☿ preceded it by 3° 14′ 50″. If you want the distance of Mercury from the Earth, that is, the side KI, you can hunt it down in the same triangle KIE from the data above.

[Margin: 6th step.]

[Margin: From Ptolemy, Book 9, chapter 7.]

De Numeris ac Menſuris ad Hanc Hypotheſim ſpectantibus.

On the numbers and measures relating to this hypothesis.

V. *Lansbergen* gives the numbers of the motions, with their roots, in his tables, pages 135 and 143, and at the end of the *Thesaurus*, page 185; the measures of the eccentricities and the semidiameter of the orbs he gives in the *Theoricae*, chapters 11 and 13. From these, however, and from the semidiameter of the great orb, that is, the mean distance of the Sun from the Earth, which he holds to be 1498 33/60 semidiameters of the Earth, we shall draw out many more measures, relating to a full knowledge of this hypothesis, both in semidiameters of the Earth and in parts of the radius of the annual orb.

[Margin: The foundations of the following table.]

Do not be surprised that *Lansbergen* here gave measures proportioned to the radius of the great orb, whereas he made the measures of the three superior planets proportioned to the radius of their own orbs; for in the theory of the two inferior planets the radius of the great orb of the moving Earth plays the part of the eccentrics of Venus and Mercury. Now, in the distances from the Sun, we shall suppose that the Sun and these planets have the same line of apsides, and that the apogees of all of them lie under the same point of the zodiac, so that it may appear how far the measures of this hypothesis can be extended or contracted. In that case, because, with the Sun and the Earth placed on the line of apsides, the centre of the orb of ♀ would be at its own least eccentricity but the centre of the orb of ☿ at its greatest, and because the eccentricity of the Sun, however great it may be, is less than any eccentricity of Mercury, Mercury, when at apogee, would therefore also be at aphelion and at its greatest distance from the Sun; when at perigee, however, it can be at perihelion. Not so with Venus. For the greatest eccentricity of Venus, for *Lansbergen*, is almost as great as the least of the Sun; so Venus at apogee would be at perihelion, and at perigee can be at aphelion. Therefore ♀ and ☿ have inverted apsides with respect to the Sun according to the idea of this hypothesis, and indeed of the Copernican too.

[Margin: Note the remarkable difference between ♀ and ☿ in their distance from the Sun.]

Hence, besides the rules for the distances of these planets from the Earth given in chapter 2, number 11, there arise others for the distance from the Sun. For from the radius or semidiameter of the orb of ♀ must be subtracted the difference between the greatest eccentricity of Venus, which for *Lansbergen* is 422, and the least, which is 349, so that in either case the least or perihelial distance of Venus from the Sun may be known. The same difference, added to the semidiameter of the orb of ♀, will give its aphelial distance. The aphelial distance of ☿, on the other hand, is known if you subtract from its apogeal distance the aggregate of the radius of the annual orb and the eccentricity of the Sun. The perihelial distance of ☿, finally, is known if you subtract from the least semidiameter of the orb of Mercury the difference between the eccentricity of the Sun and the whole eccentricity of Mercury.

[Margin: Rules for the distances of ♀ and ☿ from the Sun.]

MENSVRÆ AD LANSBERGII HYPOTHESIM Spectantes.

Measures relating to the hypothesis of Lansbergen.

See the figures of numbers 1 and 3.		Parts of which the radius of the great orb is 10000	Semidiameters of the Earth
Eccentricity, greatest: AD	♀	349	52 3/10
AD	☿	947	142
Eccentricity, least: AB	♀	145	21 7/10
AB	☿	523	78 2/5
Semidiameter of the small circle: BC	♀	102	15 3/10
BC	☿	212	31 7/10
Semidiameter of the epicycle: EG	☿	190	28½

See the figures of numbers 1 and 3.		Parts of which the radius of the great orb is 10000	Semidiameters of the Earth
Semidiameter of the planet's orb: BE	♀	7193	1077
Mean: DE	☿	3763	635 7/10
Least: DK	☿	3573	564
Semidiameter of the great orb: AG	♀	10000	1498½
AG	☿	10000	1498½
Greatest distance from the centre of the Earth: EH	♀	17338	2597
EL	☿	14520	2204
Least distance from the centre of the Earth: FH	♀	2952	399
IL	☿	7374	792
Distance from the Sun, greatest, at the eccentricity of the Sun: greatest	♀	7615	1118
	☿	4099	643
Least	♀	7542	1097
	☿	4171	643
Distance, least, at the eccentricity of the Sun: greatest	♀	6761	992
	☿	3047	359
Least	♀	6844	1057
	☿	2975	485
Eccentricity of the ☉: greatest	☉	421	63
Least	☉	349	41

[Margin: Eccentricity of the centre of the planet's orb.]
[Margin: That is, of the small circle of the eccentricity.]
[Margin: Or of the annual orb.]
[Margin: Apogeal distance.]
[Margin: Perigeal.]
[Margin: Aphelial distance.]
[Margin: Perihelial.]

MOTVS MEDIVS ſeu AEQVALIS.

Mean or equal motion.

Motion		Deg.	′	″	‴	⁗	V	VI
		Of the anomaly of the orb						
Daily	♀	0	36	59	29	29	11	6
	☿	3	6	24	12	1	8	6
Annual	♀	225	1	54	22	33	29	30
	☿	53	57	13	6	54	16	30
		Of the apogee of the eccentric						
Daily	♀	0	0	0	14	5	59	30
	☿	0	0	0	18	51	36	20
Annual	♀	0	1	25	46	26	57	30
	☿	0	1	54	43	56	1	40

[Margin: ♀ after 1, ☿ after 3 periods.]

Radix Motuum ad Annum 1. CHRISTI Goëſæ in Meridie primi diei Ianuarij

Root of the motions for the year 1 of Christ, at Goes, at noon on the first day of January.

		Root
Anomaly of the orb	♀	$4^s\ 3°\ 52'\ 56''$
	☿	$1^s\ 17°\ 24'\ 11''$
Apogee	♀	$1^s\ 22°\ 42'\ 40''$
	☿	$6^s\ 7°\ 25'\ 6''$

I shall speak of the greatest digressions and the equations more opportunely in chapter 7.

SCHOLIA.

Scholia.

I. In the year of the Lord 1491, on September 3, in the morning, Bernhard Walther deduced the place of Mercury, in ♍ 3° 5′, from the distance of Mercury from Aldebaran taken with armillaries, supposing Aldebaran in ♊ 2° 35′. But Lansbergen, in reporting this in the *Treasury of Observations*, page 181, corrects the place of Aldebaran, saying that it was in ♊ 3° 12′, and accordingly that Mercury was in ♍ 3° 42′; and from his own tables he gathers that it was in 3° 39′. Yet I should like to know why he does not likewise correct the place of Venus, which in the same *Treasury*, page 173, he reports from Ptolemy, Book 10, chapter 4, as observed in the year of Christ 138, on December 24, by the same Ptolemy, in ♏ 6° 30′. For Ptolemy asserts that it was deduced from the distance of Venus from Spica; and in his own time he reckoned Spica to be in ♍ 26° 40′, as is clear from his catalogue in Book 7. According to the tables of Lansbergen, however, it was in ♍ 26° 59′; so Venus should have been placed by Lansbergen in ♏ 6° 49′. But he passed over this correction in silence, because it conflicted with his tables and his hypothesis of Venus, from which, in the same place, he gathers that Venus was then in ♏ 6° 31′. In vain, then, does he boast that his tables answer to this observation. And elsewhere we have more than once noticed this dissimulation in Lansbergen, when observations, if they were corrected as he is otherwise accustomed to correct them, contradict his tables.

[Margin: Either forgetfulness or crafty dissimulation on Lansbergen's part in reconciling his tables with observations.]

II. But I make no complaint about the disagreement of the Lansbergian calculation from the Danish or Longomontanian, which in Venus sometimes extends to a degree and a half and more, as may be seen in the year 1643 on February 12 and March 9, and in Mercury to 5°, 6°, 7°. For there will be

someone who would rather shift the blame onto Longomontanus, who, it is agreed, erred in Mercury in the year 1631, on November 6, by more than 7°.

[Margin: The disagreement between Lansbergen and Longomontanus.]

III. The Lansbergian theory of Mercury can be reduced to Lansbergen's hypothesis, as transformed by Hérigone, in volume 5, Book 1 of the *Theoricae*, proposition 12, into the hypothesis of a stationary Earth, with a second epicycle added and the eccentricity taken not from the centre of the annual orb but from the Earth. The hypothesis of Antonín Maria Šírek, in Book 3 of the *Radius Sidereomysticus*, is also akin to the Lansbergian: although he uses a zodiac transferred into the system of each planet, he nevertheless follows the measures of Lansbergen, varies the eccentricity with him, and uses the mean motion of the ☉. Here, then, are the measures that Šírek sets down. Of the parts of which the radius of the annual orb is 10000, the semidiameter of the orb is, for ♀, 7193, and for ☿, 3573; and the eccentricity of ♀ is least 145, mean 247, greatest 350; but of ☿, least 523, mean 784, greatest 947.

[Margin: The Lansbergian hypothesis of ☿ transferred by Hérigone to the hypothesis of a stationary Earth.]

[Margin: Šírek's hypothesis.]

[Margin: And the measure.]

CAPVT IV.

Chapter IV.

De Tychonica ſeu Longomontanica Hypotheſi Veneris ac Mercurij.

On the Tychonic or Longomontanian hypothesis of Venus and Mercury.

Theorica Ptolemaica reformata.

The Ptolemaic theory reformed.

I. In Book 2 of the *Theorica*, from chapter 17, Longomontanus begins the hypotheses of these planets, proposing a threefold form. The first is the Ptolemaic, but reformed, which he calls conformable to geometry; the second is the Copernican, which we have already set out with him in chapter 2, number 4; and the third the Tychonic. First, then, the Ptolemaic. In it AB is the semidiameter of the orb AED of Venus or Mercury, described from the centre B, and C is the centre of the Earth, from which B is distant by the quantity BC, that is, by the eccentricity, not the whole Ptolemaic one but three-quarters of it. The remaining fourth part of the eccentricity is EL, the semidiameter of the lesser epicycle KFL, whose centre is sometimes at the apogee A of the eccentric, sometimes at the perigee D, and sometimes, indeed usually, elsewhere. Let it now be at E, distant from the apogee, for example, by the arc AE of 45°, which is called the anomaly of the eccentric. Let the diameter KL of the epicycle,

however, remain always parallel to the line of apsides AD. The lesser epicycle, moreover, is to carry on its circumference the centre of the greater epicycle HMI, and to revolve it with a revolution double the motion of the anomaly of the centre in ♀ and triple in ☿, but in such a way that when the centre of the lesser is at A, the centre of the greater is at K, the perigee of the lesser epicycle. The greater, finally, is to carry the planet on its circumference, the motion of the anomaly of the orb being computed from the apogee H in the order of the signs.

Diagram. Set at the right of the right column, with the text running in a narrow measure beside it. A straight vertical line runs down the right side from A at the top through B and C to D at the foot. A large arc leaves A leftward, passes over to E, and a large curve runs down the left side to D at the foot, so that the figure stands as a half circle on the line AD. About E stands a small circle, passing through L above and K below and through F at the right. About F stands a larger circle, with G and H at its upper left, N at the right and M at the lower left, with star-shaped marks on the circle at N and M, and P where several lines cross it. Straight lines run down from the region of E and F to C and B, and from M to B and C; further lines run from N and from F down to C, one of them passing P and I. A short dotted line joins E and F.

- G (above the larger circle, at the top): G
- H (larger circle, upper left): H
- L (top of the small circle): L
- A (head of the vertical line): A
- E (centre of the small circle): E
- F (right of E, centre of the larger circle): F
- N (larger circle, right, below a star-shaped mark): N
- K (below the small circle): K
- P (beside a star-shaped mark, lower right): P
- I (right of P): I
- M (larger circle, lower left, below a star-shaped mark): M
- B (vertical line): B
- C (vertical line, below B): C
- D (foot of the vertical line): D

Methodus calculi Geometrici in hac Forma.

The method of geometric calculation in this form.

II. Let the centre of the lesser epicycle be, as I said, at E, distant from A by 45°. Therefore, with twice as many degrees, that is, 90°, counted from the perigee K in the order of the signs, the centre of the greater epicycle HI will be at F, if we are speaking of ♀ (otherwise the arc KF in ☿ ought to be of 135°). The angle KEF is therefore given, which in ♀ is double the angle ABE; and because of the parallelism of the lines AD and KL, the angle KEB is always equal to the alternate angle ABE. When the straight lines CE and CF are

drawn, then, from the centre of the Earth to the centres of the epicycles, and the straight line BE from the centre B to the centre of the lesser, two triangles arise, to be solved for the equation of the centre.

[Margin: By 29 of Book 1 of Euclid.]

First, then, in the triangle BEC, the radius BE and the eccentricity BC are given, as is supposed from the measures of this hypothesis, and the angle CBE, as being the complement of the angle ABE. So the side CE will not be hidden from one skilled in triangles, nor the angle BEC; and this angle, added in this place (for sometimes it is to be subtracted) to the angle BEF, gives the angle CEF.

[Margin: 1st step, for the equation of the centre.]

Second, then, in the triangle CEF, in which the angle CEF is given, and the side CE as found, and, from the measures of the hypothesis,

[Margin: 2nd step.]

the semidiameter EF of the epicycle, find the angle ECF, that is, the equation of the centre, in this first semicircle of the anomaly to be subtracted from the mean motion of the Sun, which is the same as the mean motion of ♀ and ☿, so that the longitude of the centre F from the mean equinox may be known. In the same triangle you will also seek out FC, the distance of the centre of the greater epicycle from the Earth; and when HI is drawn parallel to BE, and CF is produced to G, HFG will be equal to the angles BEC and FCE, already found, taken together.

Third, imagine the planet to be at M, to which draw the straight lines MF to the centre of the greater epicycle and MC to the centre of the Earth. For in the triangle MFC the semidiameter MF is given from the measures of the hypothesis, and the side FC has already been found; and the angle CFM is the complement of the angle MFG, which is as great as the anomaly of the orb counted from the true apogee G. This is known if you add the angle HFG to the mean anomaly of the orb HM. So the angle MCF will not be hidden, that is, the prosthaphaeresis of the orb, in this first semicircle GMP to be added to the centric place of the planet, so that the true or apparent place of the planet from the mean equinox may become known. But if the planet were at N, with FN and CN drawn, the triangle CFN would have to be solved, and then the mean anomaly of the orb would be the arc HMN, whose complement to the circle would be the arc HGN. From this the arc HG would have to be taken away, and the angle GFN so left would reveal the angle CFN. With this, and with the semidiameter FN and the side CF found as above, the angle FCN would have to be sought, to be subtracted from the centric place of the planet to obtain its true place from the mean equinox, and, with the prosthaphaeresis of the equinoxes, from the true equinox. Finally, if the planet is at N or M, as at the points of contact at which the straight lines CN and CM touch the orb, it will be at greatest digression, morning at N, evening at M. These will be known from the triangles FCN and FCM, right-angled at N and M, if the

semidiameter FN or FM is given and the distance CF is sought out as above, as will be said in chapter 7, Problem 2.

[Margin: 3rd step, for the equation of the orb.]

[Margin: The method for the greatest digression of the planet is indicated.]

Tychonica Hypotheſis pro Venere & Mercurio.

The Tychonic hypothesis for Venus and Mercury.

III. In place of the eccentricity, two epicycles are substituted here, proportionally, as we said in the third Copernican form, Section 2, chapter 2, number 5, and chapter 3, number 4. In the following diagram, then, let ABD be the line of apsides, and on it let B be the centre of the Earth, from which the concentric AMD of these planets is to be described, carrying the centre E of the greater epicycle, whose diameter KR is to be always parallel to the line AD. With the centre placed at its apogee K, let the lesser epicycle LC be described, whose diameter CL is to be kept in the same position as KR. Further, while the centre E of the greater moves in the order of the signs from the apogee A, the lesser epicycle meanwhile, with a revolution double in ♀ and triple in ☿ relative to it, that is, twice or three times greater than the anomaly of the centre, is to carry in the order of the signs the centre of the orb of the planet, namely the centre F, beginning its revolution in ♀ from the perigee C of the lesser epicycle towards F, but in ☿ from the apogee L. Next, through F, the centre of the orb, draw BFG, and through the same point draw the diameter HI of the orb, parallel to the straight line BE. For the mean apogee of the orb will be H and the true apogee G, from which the motion of the planet on the

Diagram. Set in the left column, with the text running in a narrow measure at its left. A straight vertical line runs down the right side from A through B to D at the foot. A curve leaves A leftward, passes through the region of E and M, and runs on down to D. A large circle stands about F, passing H and G at its upper left, N near A at the right, I at the lower right and M at the lower left. A middle circle stands about E, with P at its upper left and R inside its foot; a short vertical line runs from K through the small letter c and E to R. A small circle stands about K, with L above it and F at its right. Straight lines: from H through F to I; from G through F to B; from P through E to B; from M to B; from F horizontally to N; from N to B.

- G (large circle, upper left, above H): G
- H (large circle, upper left): H
- L (above the small circle): L
- F (right of the small circle, centre of the large circle): F
- A (head of the vertical line): A
- K (centre of the small circle): K
- N (right, near the vertical line): N
- P (upper left of the middle circle): P
- c (foot of the small circle, set small): C

- E (centre of the middle circle): E
- I (large circle, lower right): I
- R (foot of the middle circle): R
- M (lower left, where the curve meets the large circle): M
- B (vertical line): B
- D (foot of the vertical line): D

Ptolemaic epicycle is counted in the order of the signs, that is, the mean anomaly of the orb if counted from H, the true if from G. Finally, let the planet itself be imagined now in the first semicircle HMI of the anomaly of the orb, say at M, and now in the second, as at N. The anomaly of the eccentric, moreover, is obtained here too by subtracting the mean motion of the apogee of the planet from the mean motion of the Sun. And here too, whether BN and BM touch the orb, and the planet is at N in morning greatest digression or at M in evening, will be distinguished as I taught above at the end of number 2, and below in chapter 7, problem 2.

Methodus calculi Geometrici in hac Hypotheſi.

The method of geometric calculation in this hypothesis.

IV. First, with straight lines drawn to the centre of the orb from the two centres of the epicycles, that is, EF and KF, the side EF is sought in the triangle KEF. For in it are given the semidiameters EK and KF of the epicycles and the angle EKF, which is as great as the arc CF, that is, as great as the doubled anomaly of the eccentric; this is made known by subtracting the motion of the apogee from the mean motion of the Sun. But in ☿, LCF is the tripled anomaly. Moreover, the angle KEF is sought. Next, because by the parallelism of the lines KR and AD the angle KEB is equal to the angle DBE, that is, to the complement of the anomaly of the eccentric, if you take the angle KEF from it, the angle BEF will be known in the triangle BEF, in which the side EF has been found and the radius BE is given. So the side BF will not be hidden, nor the angle EBF, which is the equation of the centre, here to be subtracted from the mean motion of the planet, that is, from the mean motion of the Sun, so that the centric place of the planet may be had.

[Margin: 1st step, for the equation of the centre.]

[Margin: Here Longomontanus is very obscure, and so must be made clear by us.]

Second, if the planet is at M, count from the mean apogee H the mean anomaly of the orb, the arc HM, and add the equation of the centre to it, so that you may have the coequated or true anomaly of the orb, that is, the arc GHM, which the angle MFG measures; its complement will be MFB. With this, and with the semidiameter FM of the orb and the side FB already found, you will obtain the angle MBF, that is, the prosthaphaeresis of the orb, here to be added to the centric place of the planet, so that its true place in the zodiac may be had. But if the planet were at N, the complement to the circle of the

mean anomaly HMN would be HGN; the arc GH, that is, the equation of the centre, taken from it, would leave GN, the complement of the true anomaly; and the complement of this to two right angles would be the angle BFN, with which, and with the semidiameter FN and the side BF, you would find FBN, the prosthaphaeresis of the orb, in such a case to be subtracted from the centric place, etc.

[Margin: 2nd step, for the equation of the orb.]

De Menſuris ad Hypotheſim hanc ſpectantibus.

On the measures relating to this hypothesis.

V. So that the calculation already mentioned can be carried out and the distances of these planets determined, we shall fish out the numbers of the motions from the tables of the *Astronomia Danica*, and the measures of the eccentricities or semidiameters from Longomontanus, Book 2 of the *Theorica*, chapter 18, for ♀, and chapter 20, for ☿. From these, and from the mean distance of the Sun, which is the radius of the annual orb, equivalent to the eccentric or rather the concentric of these planets (from that distance, I say, which Longomontanus puts at 1288 semidiameters of the Earth), and from the greatest, 1334, and the least, 1242, we shall construct other measures, neglected by Longomontanus; since not Tycho but he himself completed this hypothesis. For conceiving these measures on the line of apsides, however, he adds this diagram, especially for Mercury. In it the concentric AFKH, described from the centre of the Earth C, carries A, the centre of the epicycle PQ, whose semidiameter AI in ♀ contains three-quarters of the whole eccentricity and has on its periphery I, the centre of the lesser epicycle DV, whose semidiameter DI is the remaining fourth part

Diagram. Set in the right column, with the text running in a narrow measure at its left. Two large circles stand one above the other and overlap. The upper has E at its top, B at its left and G at its right. The lower has F at its left, H at its right and K at its foot. Where they overlap stands a middle circle about A, with P at its left and Q at its right; above it, and overlapping it, stands a small circle with I inside it and D above it, and with V in the overlap below I. A vertical line runs from the small circle at I through V, A, M and C to K. M stands just below the middle circle. A short curved line runs from Q towards the right, along the edge of the lower circle.

- E (top of the upper circle): E
- B (left, inside the upper circle): B
- D (above the small circle): D
- G (right, inside the upper circle): G
- I (inside the small circle): I
- V (below I, in the overlap): V
- P (left of the middle circle): P
- Q (right of the middle circle): Q

- A (centre of the middle circle): A
- M (on the vertical line, below the middle circle): M
- F (left, inside the lower circle): F
- C (centre of the lower circle): C
- H (right, inside the lower circle): H
- K (foot of the lower circle, on the vertical line): K

[Margin: Longomontanus, 2 *Theorica*, chapter 20.]

of the whole eccentricity. But in ☿, AI has four-fifths, and DI a fifth part, of the whole Ptolemaic eccentricity. And finally, this epicycle carries at its perigee V the centre of the orb, or greatest epicycle, of Venus, but at its apogee D the centre of the orb of Mercury, which orb is to be EBMG. The motion of the epicycle DV, moreover, is in ♀ doubled relative to the motion of the anomaly of the eccentric, counted from A to F; therefore, when the centre A comes to K, the centre of the orb returns to V, the perigee of the lesser epicycle. But in ☿ it is tripled; and so, when the centre A has descended to K, the centre of the orb, which had been at D, will be at V. But because this diagram is mutilated and does not distinctly represent the apogeal and perigeal distances of these planets, we shall substitute another.

[Margin: The difference between ☿ and ♀.]

[Margin: The imperfection of Longomontanus's diagram.]

VI. In the line of apsides AD, then, let the centre of the Earth be C, from which the Sun A or D is at a mean distance, so that the annual orb of the Sun is ABBDEE; and let both A and D be the centre of the greater

Diagram. Set across the left column. A vertical line runs from O at the top to S at the foot. A large circle about C occupies the middle of the line, its top at A and its foot at D. On it, above and below, stand two like groups of circles, each cut by the large circle at B on the left and E on the right. The upper group: an outer circle from O to Q through B and E; within it, a circle with R at its top and T near its foot; within that, a small circle about A, with F at its left and G at its right; and, overlapping that small circle above, a smaller circle about I, with K above it and L below it. The lower group: an outer circle from P to S through B and E; within it, a circle with V at its top and X near its foot; within that, a small circle about D, with F at its left and G at its right, and with Z just below it; and, overlapping that small circle above, a smaller circle about H, with N above it and M below it. The sign ♀ stands on the outer circle of the upper group at the upper left, and the sign ☿, set aslant, on its inner circle at the upper right.

- O (top of the vertical line): O
- ♀ (outer circle of the upper group, upper left): the sign of Venus
- R (top of the circle within): R
- ☿ (that circle, upper right, set aslant): the sign of Mercury
- K (above the smallest circle of the upper group): K
- I (centre of that circle): I
- L (below I): L

- F (upper group, left of the small circle): F
- A (upper group, centre of the small circle): A
- G (upper group, right of the small circle): G
- B (upper group, left, where the circles cross): B
- T (on the line, below A): T
- E (upper group, right, where the circles cross): E
- Q (foot of the outer circle of the upper group): Q
- C (centre of the large circle): C
- P (top of the outer circle of the lower group): P
- V (top of the circle within): V
- N (above the smallest circle of the lower group): N
- H (centre of that circle): H
- M (below H): M
- B (lower group, left, where the circles cross): B
- E (lower group, right, where the circles cross): E
- F (lower group, left of the small circle): F
- D (lower group, centre of the small circle): D
- G (lower group, right of the small circle): G
- Z (on the line, below D): Z
- X (on the line, below Z): X
- S (foot of the vertical line): S

epicycle FG; and let this carry I or H, the centre of the lesser epicycle KL or MN; and let this epicycle finally carry at its perigee L, or apogee M, the centre of the orb that carries Venus, which orb is to be OBQE when Venus is at the apogee O of its orb (otherwise called the Ptolemaic epicycle), but PBSE when it is at the perigee P. But let the lesser epicycle at its apogee K or M carry the centre of the orb that carries Mercury, which orb is to be RT when Mercury is at the apogee R, but VX when it is at the perigee V of its orb. For in this way the differences of the distances will appear better. This too is worth knowing: in this hypothesis, because of the varying annual distance of the Sun from the Earth, or because of the varied eccentricity, the distances of these planets from the Earth change, but not those from the Sun, because the eccentricity is taken from the Sun, as from the centre of this planetary system, and the annual orb of the Sun stands in place of the eccentric. In the Copernican hypothesis, on the other hand, because of the change in the distance of the Earth from the Sun and in the eccentricity of the Earth, it is not the distances from the Earth that vary but the distances from the Sun; because in that case the annual orb of the Earth stands in place of the eccentric of these planets, and from its centre the eccentricities are measured.

[Margin: The difference between the Copernican and Tychonic hypotheses in the distances.]

MENSVRÆ AD HYPOTHESIM TYCHONIS ſeu LONGOMONTANI ſpectantes

Measures relating to the hypothesis of Tycho or Longomontanus.

		In parts of which the radius of the concentric or of the annual orb of the ☉ is 100000	Semidiameters of the Earth
Whole eccentricity from the point of the mean motion of the ☉: AK	☿	3208	41
AK	☿	6822	74
Half of the eccentricity: AL	☿	1604	21
Three-fifths of the eccentricity: AL	☿	3411	45
Semidiameter of the greater epicycle: AI	☿	2406	31
AI	☿	5685	59
Semidiameter of the lesser epicycle: IL	☿	802	10
IL	☿	1137	15
Radius of the concentric, or semidiameter of the annual orb of the ☉: CA	☿	100000	1288
CA	☿	100000	1288
Semidiameter of the greatest epicycle, or orb: LO	☿	72248½	931
KR	☿	38500	496
Distance from the Sun, greatest: DP	☿	73852½	952
AR	☿	45322	570
Distance from the Sun, least: AO [recte: AQ, per the book's errata; corrected by hand to AQ in this copy]	☿	73852½ [recte: 70585, per the book's errata, which quote this row as printed *AO. 73852½. 952.* and correct it to *AQ. 70585. 910.*]	9[?][?] [recte: 910, per the book's errata, which quote the printed figure as 952; in this copy the last two figures are overwritten in pen as 10 and the type beneath cannot be read]
VD [recte: TA, per the book's errata]	☿	31678	422
Greatest distance from the Earth, at the distances of the Sun from the Earth: greatest: CO	☿	177423½	2286
CR	☿	148893	1904
Mean: CO	☿	173852½	2240
CR	☿	145322	1858
Least: CO	☿	170281	2194
CR	☿	141751	1812
Least distance from the Earth, at the distances of the Sun from the Earth: greatest: PC	☿	29718½	382
VC	☿	71893	912

		In parts of which the radius of the concentric or of the annual orb of the ☉ is 100000	Semidiameters of the Earth
Mean: PC	☿	26147	336
VC	☿	68321½	866
Least: PC	☿	22575½	290
VC	☿	64750	820
Distance of the Sun from the Earth, greatest	☿	103571	1334
Least	☿	96428	1242

Anyone who wants the Tychonic measures should use the distances of the Sun established by Tycho himself, which I set down in Book 3, chapter 7, number 15.[62]

[Margin: See the figure of number 6.]
[Margin: Beside the rows AK: Or DN.]
[Margin: Beside the rows AL: Or DM.]
[Margin: Beside the rows AI: Or DH.]
[Margin: Beside the rows IL: Or HN.]
[Margin: Beside the rows CA: Or CD.]
[Margin: Beside the rows LO and KR: Or MP, MV.]
[Margin: Aphelial distance.]
[Margin: Perihelial.]
[Margin: Apogeal distance, threefold, from the centre of the Earth.]
[Margin: Perigeal, threefold.]

MOTVS MEDIVS ſeu ÆQVALIS

Mean or equal motion.

		Deg.	′	″	‴	⁗	V	VI
		Of the anomaly of the orb						
Daily	☿	0	36	59	28	46	17	44
	☿	3	6	24	15	23	4	58
Annual	☿	225	1	50	1	37	52	40
	☿	53	57	33	35	25	12	50
		Of the apogee of the eccentric						
Daily	☿	0	0	0	13	37	41	40
	☿	0	0	0	17	17	56	40
Annual	☿	0	1	22	54	18	28	20
	☿	0	1	45	14	29	43	20

[Margin: ☿ after 1, ☿ after 3 periods.]

[62] In this copy the row Distance from the Sun, least, carries pen corrections. The printed 73852½ on the ☿ row is struck through; a handwritten figure beside the row, in the outer margin, reads 705[?]5; and the last two digits of 910 are overwritten. On the ☿ row the printed letters VD are struck through and TA is written beside them.

RADICES motuum ad Annum 1. CHRISTI Hafniæ in Meridie primi diei Ianuarij

Roots of the motions for the year 1 of Christ, at Copenhagen, at noon on the first day of January.

		Root
Anomaly of the orb	♀	$4^s\ 6^\circ\ 21'\ 20''$
	☿	$1^s\ 18^\circ\ 13'\ 48''$
Apogee	♀	$1^s\ 22^\circ\ 36'\ 26''$
	☿	$6^s\ 13^\circ\ 49'\ 16''$

I shall speak of the greatest digressions and the equations more opportunely below, in chapter 7.

SCHOLIA.

Scholia.

I. Besides what was said about these Tychonic hypotheses in the common discussion in section 2, chapter 3, in its scholia, Longomontanus admits (Book 2 of the *Theorica*, chapter 17) that the superior planets and the Moon look to the true course of the Sun, but these inferior planets to its mean motion. But Kepler, in the *Epitome of Copernican Astronomy*, p. 762, contends that these too depend, in the eccentric anomaly, on the true motion of the Sun, just as the superior planets do in the anomaly of the orb. For this reason Tycho's old hypothesis, which considers here a point other than the centre of the Sun's body, is called uncorrected by Kepler. Added to this is the excessive parallax of the Sun and Venus employed by Tycho, which Kepler criticises in the *Ephemerides*, p. 15.

[Margin: Correction of this hypothesis.]

II. Although the eccentricity of Venus has been found smaller in this age than in Ptolemy's, it does not follow from this that it has become smaller, as Copernicus thought. For, as Longomontanus says (Book 2 of the *Theorica*, chapter 18), this can be ascribed to the defect of the ancient observations: both because of the refractions, in which Venus is generally observed wrapped, which they omitted, and because of the uncorrected motions of the Sun and the fixed stars used by the ancients. So it is no wonder if, for this reason, the Ptolemaic eccentricity has decreased by more than half a degree. As for that remarkable appearance of ♀ seen by Tycho in Denmark, although it was joined to the Sun almost at the very perigee of the epicycle, Longomontanus testifies to it in Book 2, chapter 18, saying that it was seen in 1587 February 21 in the evening, and afterwards in the morning on the 24th, on account of a northern latitude of 9°.

[Margin: Was the eccentricity of ♀ once greater than now?]

[Margin: ♀ seen by Tycho at the perigee of the epicycle.]

III. This is certainly worth noting in this Danish hypothesis: Mercury, because the revolution of the centre of its orb is triple the anomaly of the

eccentric, has its centre nearest the Earth not only at the perigee of the eccentric, and farthest at the apogee, but at other places of the eccentric as well. For Longomontanus (Book 2 of the *Theorica*, chapter 20) does not subscribe to Ptolemy, who thought that Mercury is found nearest the Earth only in the sextants from the perigee of the eccentric, and that there it runs into the greatest digressions of all, in the morning in Aquarius, in the evening in Gemini. He says that Ptolemy's observations, when corrected and weighed against the emended motion of the Sun and the fixed stars, conflict with the Ptolemaic hypothesis and with the equations set at that place of the eccentric. Boulliau, however (Book 10 of the *Astronomia Philolaica*, chapter 1), denies that all the observations of Mercury can be represented by a revolution of the orb repeated three times. Certainly the error that Hortensius assigns to Mercury in the Danish calculation is huge — I reported it in Book 3, chapter 3, at the end of Scholium 3 — since they strayed from the true place of Mercury by 7° 13′, even though Longomontanus (Book 2, chapter 20) says that his hypothesis does not err in ☿ by more than 10′. Indeed, it sometimes errs by 9°.

[Margin: Ptolemy's correction in the hypothesis of ☿.]

[Margin: Longomontanus's error in ☿.]

CAPVT V.

De Kepleriana, & Bullialdica Hypotheſi Veneris, ac Mercurij.

Chapter 5. On the Keplerian and Boulliauan hypothesis of Venus and Mercury.

I. Kepler, in Book 6 of the *Epitome of Copernican Astronomy*, part 3, and Boulliau, in the *Astronomia Philolaica*, Book 9 on Venus and Book 10 on Mercury, touch on this hypothesis in few words, because it is the same as that of the three superior planets, sufficiently explained by us, with its properties, in section 2, chapter 5. Thus, in their opinion, Venus and Mercury move on the circumference of an ellipse, in one of whose foci or umbilici stands the motionless Sun, as at the centre of the universe. The line of apsides is drawn through the centre of the Sun; the motion of the second inequality is reckoned not from the mean but from the true motion of the Sun; and the planet's path runs along an elliptical line, which Boulliau teaches is manifest in Mercury (Book 11, theorem 13). Again, their eccentricity is to be taken from the Sun and bisected, one half of it being given to the centre of the ellipse and the other half to the other of the foci, which is to be the point of equal motion, for the reasons brought forward in that chapter 5, from number 5 to 14.

[Margin: Epilogue of this hypothesis.]

II. Yet the theory of these planets differs from that of the superior planets. First, because those are carried outside the Earth's annual orb and these inside it. Second, because in the three superior planets the eccentric orbs, or their ellipses, differ from the ellipse of the annual orb, and are named eccentrics because they perform the office of eccentrics in representing the inequality of the first motion; while the annual orb of the Earth, by its motion, supplies in them the

place of the Ptolemaic epicycles and displays the commutations of the second motion. But in Venus and Mercury the annual orb of the Earth performs the function of the eccentric, because the mean motion in longitude is the same for the Earth, ♀ and ☿; and conversely the orb proper to these planets, that is, each one's own ellipse, eccentric to the Sun and referred to the motion and place of the Earth, produces the second inequality. Therefore the one great or annual orb has taken from the old astronomy the three epicycles of the superior planets and the three eccentrics of the Sun and the inferior planets. Now the apogee of Venus is at present at about 1° ♋, that of Mercury at about 2° ♐, and that of the Earth at about 7° ♑. On these assumptions, consider the following figure, in which, for ease, let the line of apsides of ☉ and ♀ be imagined as the same.

[Margin: The difference of the theory of the inferior planets from that of the superior.]

III. Let A be the Sun, the centre of the world, from which the ecliptic BXC has been described, whose diameter BC is to be the line of apsides of the Earth; and let the bisected eccentricity of the Earth's annual orb be AI. With centre I let the Earth's ellipse ETG be described, in which the Earth at E, aphelial under Capricorn at B, is to see the Sun A as apogee from itself under Cancer; and the same Earth at G, having become perihelial, is to see the Sun A as perigee to itself under Capricorn.

Diagram. A geometric figure of single-column width in the right column. An outer circle, the ecliptic, carries the twelve zodiacal signs round its rim, read clockwise from the top: ♋ above C at the top, ♊ by F, ♉, ♈, ♓, ♒, ♑ below B at the foot, ♐ by L, ♏, ♎, ♍, ♌. A vertical diameter runs from C at the top through G, O, A, K, I, N, H, E down to B. Three nested curves stand inside the ecliptic. A slanting line runs from F at the upper right down through Y and S to R and L at the lower left. A nearly horizontal line runs from X at the left rim through T to N; other straight lines fan from T towards A, Q and P, and a short line runs from T to V. Lying sideways to the right of N, on the two inner curves, stand the signs ☿ (inner curve) and ♀ (middle curve).

- Rim, top: C, with ♋ above it
- Rim, upper right: F, with ♊ above it
- Rim, foot: B, with ♑ below it
- Rim, lower left: L, with ♐ below it
- Rim, left: X, with Z below it
- Vertical line, top to foot: C, G, O, A, K, I, N, H, E, B
- Slanting line: F, Y, S, R, L
- Inside, left of the vertical line: T, V, Q, P
- Labels read: A, B, C, E, F, G, H, I, K, L, N, O, P, Q, R, S, T, V, X, Y, Z, the signs ☿ and ♀ inside the figure, and the twelve zodiacal signs. Labels not read: none noted.

With the line OH, the line of apsides of Venus, drawn through the Sun, and AK taken in it as half of the whole eccentricity AN, which is to be a little less than AI, and with the centre made at K, let the ellipse of Venus OVH be

described inside the annual orb, and in it let the apogee be O when the Earth is at E, and the perigee H. Then let the line of apsides of Mercury FL be drawn through the Sun, in which its whole eccentricity from the Sun is to be AP, whose half is AQ; and with the centre made at Q, let the ellipse of Mercury RS be drawn, whose apogee is to be R and perigee S, with the Earth placed at Y. Now let the lines of apsides move slowly under the ecliptic in the order of the signs; and in the same direction let the Earth move from G to T with a mean motion, which is to take the place of the mean motion in longitude of these planets. Meanwhile let Venus in its ellipse, and Mercury in its own, move towards the same side with a motion begun from the line of their apsides, as Venus from O to V, and with as great a motion as the anomaly of the orb usually is. Finally, the motion of the Earth, counted from the aphelion of Venus in the order of the signs, as the arc GT if the Earth is at T, or from the apogee of Mercury, as the arc LYT, is and is called the eccentric anomaly.

Methodus Calculi Geometrici in hac forma.

Method of geometrical calculation in this form.

IV. Let the Earth be at T and Venus at V. First, with the straight line NTX drawn from N, the other of the foci, as from a point

[Margin: 1. Step for the equation of the centre.]

of equal motion, and with the straight line ATZ from A, the Sun, the other focus, the angle ATN measures the equation of the centre, to be subtracted in this first semicircle of the anomaly, so that the mean place of the centre, which is Z, is converted into the coequated place, which is X. So when the motion of the planet's apogee is subtracted from the mean motion of the Earth (that is, of the Sun), the arc GT remains, which is the eccentric anomaly. With it, from the given whole eccentricity of Venus AH [recte: AN] and its half AK, hunt out the equation, both the physical, that is, the angle KTN, and the optical, namely the angle ATK, so as to have the whole equation of the centre ATN. Hunt it out, I say, in that proportional way which Kepler teaches for the superior planets, on which see our section 2, chapter 5, numbers 14 and 16. Or, since that way is most troublesome and lacks geometric rigour, take refuge in Boulliau's way and in the ellipse resolved into two circles, on which see the same section, number 17; and do likewise for Mercury.

Second, draw straight lines to Venus at V from the Sun A and from the Earth T. When the eccentric place of the planet, that is, the place equated by the preceding equation, is subtracted from the true place of the Sun (or the place of the Sun from the place of the planet, if the latter is greater), you will have the angle of commutation, or the angle at the Sun (but if it exceeds a semicircle its complement is taken), that is, the angle TAV. With this angle, and with AT, the distance of the Sun from the Earth, known from the Sun's anomaly and the tables containing that distance, and with the semidiameter of the orb of Venus, or rather the interval AV, known from the anomaly of the orb through

the proper tables, you will seek the angle ATV, that is, the equation of the orb, to be subtracted from the eccentric place in the first semicircle of the anomaly of the orb and added in the second, so as to have the true or apparent place of the planet seen from the Earth.

[Margin: 2. Step for the equation of the orb.]

De Menſuris ad hanc Hypotheſim ſpectantibus.

On the measurements pertaining to this hypothesis.

V. Kepler indicates his own in the *Epitome of Astronomy*, p. 765, and in the Rudolphine tables of equations, and from him Hérigone (volume 5, p. 580) and Boulliau (Book 1, chapter 9); while the same Boulliau gives his own in Book 9, chapter 2 for ♀, and in Book 10, chapters 2, 3 and 4 for ☿, and in the tables of equations. But from these we have gathered many other measurements, assuming the distance of the Sun set down by them and reported by us in Book 3, chapter 7, and in Kepler's case a twofold one. The distances from the Earth could be represented for some definite time, at which the lines of apsides do not coincide; yet it seemed best to show them in a possible case, in which the line of apsides of the Sun (or Earth), Venus and Mercury would be the same, and indeed so that the centre of the annual orb lies towards one quarter and the centres of the orbs of ♀ and ☿ towards the other, above the Sun, so that while the Earth saw the Sun at apogee, the planets would attain the greatest distance from the Earth that they can have. Granted that this has never happened, and perhaps never will. So in the following diagram, A is the Sun; I is the centre of the annual orb, whose arcs EF and DG indicate the rest of the annual orb; K is the centre of the orb of Venus, indicated by the arcs LM and NQ; and P is the centre of the orb of Mercury, indicated by the arcs TV and XZ. From this the distances can be derived without difficulty in the case where the centre of the orb of Venus would be between A and I, and so for the others. But take care not to confuse Kepler's measures of the eccentricities: in the *Epitome*, and from it Hérigone, he sets them in comparison with the greater semidiameter of the orb or ellipse which each planet describes about the Sun, taken as 100000, whereas Boulliau, reporting Kepler's, and we below, express them in parts of which the radius of the annual orb is 100000. And join the table written below with the other small table to be placed in chapter 9, number 16.

[Margin: Authors from whom the following table is taken.]

Diagram. A narrow schematic set into the right column. A single vertical line runs from B at the top to C at the foot. Six short arcs cross it, each with a letter at either end: E–F near B, L–M near O, T–V near R, X–Z near S, N–Q near H, and D–G near C. Between the T–V and X–Z arcs the line carries P, K, A, I in that order from the top, each with a short tick.

- On the line, top to foot: B, O, R, P, K, A, I, S, H, C
- Arc ends, left: E, L, T, X, N, D
- Arc ends, right: F, M, V, Z, Q, G

• Labels not read: none noted.

MENSVRÆ ad KEPLERI & BVLLIALDI HYPOTHESIM Spectantes

Measurements pertaining to the hypothesis of Kepler and Boulliau.

[Margin: See the figure, number 5.]

	Planet	Parts of which the radius of the great orb is 100000: Kepler	Parts of which the radius of the great orb is 100000: Boulliau	Semidiameters of the Earth: Boulliau	Semidiameters of the Earth: Kepler from the *Ephemerides* and *Mars*	Semidiameters of the Earth: Kepler from the *Epitome* and the Rudolphine Tables
Eccentricity of the centre of the ellipse, AK	♀	500	575	8	9	17
Eccentricity of the centre of the ellipse, AP	☿	8149	8105	118	144	275
Greater semidiameter of the ellipse, KO	♀	72400	72398	1057	1280	2448
Greater semidiameter of the ellipse, PR	☿	38806	38585	563	691	1313
Lesser semidiameter of the ellipse	♀	72391	72390	1056	1279	2447
Lesser semidiameter of the ellipse	☿	37958	37806	551	681	1284
Greatest distance from ☉, AO	♀	72900	72973	1065	1289	2465
Greatest distance from ☉, AR	☿	46955	46690	681	830	1588
Least distance from ☉, AH	♀	71900	71823	1049	1271	2431
Least distance from ☉, AS	☿	30657	30480	445	542	1037 [recte: 1038, per the book's errata]
Distances from the Earth, at the least distances of the Sun from the Earth: greatest, CO	♀	171073	170700	2500 [recte: 2499, per the book's errata]	3025	5791 [recte: 5789, per the book's errata]
Same: greatest, CR	☿	145128	145516	2115	2566	4914 [recte: 4912, per the book's errata]
Same: least, CH	♀	26273	25403	385	465	895 [recte: 893, per the book's errata]
Same: least, CS	☿	67516	66746	989	1194	2289
Distances from the Earth, at the mean distances of the Sun from the Earth: greatest, CO	♀	172900	172973	2525	3057	5846

	Planet	Parts of which the radius of the great orb is 100000: Kepler	Parts of which the radius of the great orb is 100000: Boulliau	Semidiameters of the Earth: Boulliau	Semidiameters of the Earth: Kepler from the *Ephemerides* and *Mars*	Semidiameters of the Earth: Kepler from the *Epitome* and the Rudolphine Tables
Same: greatest, CR	☿	146955	146690	2141	2598	4969
Same: least, CH	♀	28100	28177	411	497	950
Same: least, CS	☿	69343	69520	1015	1226	2344
Distances from the Earth, at the greatest distances of the Sun from the Earth: greatest, CO	♀	174727	175747	2551	3089	5901 [recte: 5893, per the book's errata]
Same: greatest, CR	☿	148782	149464	2167	2638	5024 [recte: 5026, per the book's errata]
Same: least, CH	♀	29927	30951	437	529	1005 [recte: 1007, per the book's errata]
Same: least, CS	☿	71170	72294	1041	1258	2399 [recte: 2400, per the book's errata]
Distance of the Sun from the Earth: least	☉	98173	97226	1434	1736	3327
Distance of the Sun from the Earth: mean	☉	100000	100000	1460	1768	3381
Distance of the Sun from the Earth: greatest	☉	101827	102774	1486	1800	3438

[Margin: From the Sun.]
[Margin: Aphelial and perihelial distance.]
[Margin: Distance from the Earth.]

Appendix ad præcedentem Tabulam.

Appendix to the preceding table.

In the Rudolphine Tables Kepler has the aphelial distance of Venus as 72914 and the perihelial as 71912, and the aphelial distance of Mercury as 46956 and the perihelial as 30656. But when Boulliau's ellipse is resolved into two circles, his semidiameter of the greater circle will be 72394 parts in Venus and 38196 in Mercury, and that of the lesser circle 8 in Venus and 390 in Mercury, of which parts the radius of the great orb is 100000.

MOTVS MEDIVS ex KEPLERO.

Mean motion, from Kepler.

Motion		Deg.	′	″	‴	⁗	V	VI
		Of the anomaly of the orb						
Daily	♀	0	36	59	29	45	47	49
	☿	3	6	24	15	35	29	37
Annual	♀	225	1	56	3	35	41	55
	☿	53	57	34	50	55	22	5
		Of the apogee of the eccentric						
Daily	♀	0	0	0	12	49	13	55
	☿	0	0	0	17	13	6	46
Annual	♀	0	1	17	59	29	39	35
	☿	0	1	44	44	46	9	50

[Margin: ♀ after 1, ☿ after 3 periods.]

RADIX Motuum ad Annum 1. CHRISTI In Meridie 1. diei Ianuarij Vraniburgi

Root of the motions for the year 1 of Christ, at noon of January 1, at Uraniborg.

		Root
Anomaly of the orb	♀	$4^s\ 3^\circ\ 16'\ 49''$
	☿	$1^s\ 7^\circ\ 40'\ 54''$
Aphelion	♀	$8^s\ 26^\circ\ 33'\ 10''$
	☿	$6^s\ 26^\circ\ 14'\ 39''$

MOTVS MEDIVS, ex BVLLIALDO.

Mean motion, from Boulliau.

Motion		Deg.	′	″	‴	⁗	V	VI
		Of the anomaly of the orb						
Daily	♀	0	36	59	38	56	0	37
	☿	3	6	24	25	8	7	11
Annual	♀	225	1	55	50	43	47	34
	☿	53	57	32	54	23	44	24
		Of the apogee of the eccentric						
Daily	♀	0	0	0	8	19	51	53
	☿	0	0	0	17	7	15	24
Annual	♀	0	0	50	40	50	36	8
	☿	0	1	44	9	8	44	3

[Margin: ♀ after 1, ☿ after 3 periods.]

RADIX Motuum ad Annum 1. CHRISTI In Meridie diei 1. Ianuarij Vraniburgi

Root of the motions for the year 1 of Christ, at noon of January 1, at Uraniborg.

		Root
Anomaly of the orb	♀	$4^s\ 3^\circ\ 46'\ 15''$
	☿	$1^s\ 7^\circ\ 7'\ 59''$
Aphelion	♀	$9^s\ 12^\circ\ 50'\ 44''$
	☿	$6^s\ 25^\circ\ 18'\ 49''$

If you add the mean motion of the Sun to the anomaly of the orb, the result is the mean motion of Venus and Mercury in longitude, as given in the Rudolphine or Philolaic Tables. Conversely, if you subtract the mean motion of the Sun from the mean motion of these planets, there will remain the motion of the mean anomaly of the orb. But if you subtract the true place of the Sun from the true place of the centre of these planets, you will have the true anomaly of the orb. On the difference between the places of the aphelion and the apogee we must speak below, for they differ greatly in Venus.

SCHOLIA.

Scholia.

I. In the old astronomy, given the dimensions of the orbs, there was not sufficient room between the Luminaries for the orbs of Mercury and Venus. Copernicus removed this imperfection, as Kepler notes in the *Epitome*, p. 765. But because he did not bisect the eccentricity, as Kepler would have preferred, the amount by which the distances of these planets varied because of the whole eccentricity had to be compensated by the Copernican eccentric of the eccentric [Eccentri Eccentricum]. And although the Rudolphine Tables seem to surpass the other tables in the motions of Mercury, because that conjunction of Mercury with the Sun, of which I spoke in Book 3, chapter and scholium 3, could be predicted from them so near the truth, Kepler himself nevertheless admits in the *Ephemerides*, p. 14, that in his hypothesis, which rests on Brahean observations, an error of some minutes, up to a quarter of a degree and more, can be feared in Venus; and that in Mercury his calculation can incur an error of a whole day as to its conjunctions with the Sun, that is, of 5° in the commutation, while the apparent place of Mercury can be off by 1½°. He excuses himself on the ground that all the observations of Mercury written down by Tycho over 20 years, if those that are reliable and undoubted are selected, fall within a single third of Mercury's eccentric; the other two thirds remain unobserved, at least in this age and in our hemisphere; and opportunities to detect the error are very rare, because of Mercury's rare appearances. Finally he warns of a greater variation that occurs in these two planets when they are near their stations or are retrograde.

[Margin: Comparison of the old astronomy with the new in these planets.]

[Margin: Kepler's excuse concerning the motions of ♀ and ☿.]

II. Boulliau, in Book 10 of his *Astronomia*, chapter 3, affirms from the observations of Gassendi and Hortensius that the distance of Mercury and the Sun

defined by Kepler is longer than it should be, and indeed that the mean motion of Mercury in the Rudolphine Tables must be increased in the roots by 11′ 29″. He had said, however, in chapter 1, that before Kepler, when all used a perfect circle, it had been impossible to force the motions of Mercury to the laws of numbers: for no hypothesis, not even Longomontanus's tripled revolution of the centre of the orb, satisfies the phenomena or produces the longer stay of Mercury towards the aphelion and the faster passages through the perihelion that recent and accurate observations require. Finally, in the same chapter 1, he does promise a sufficiently exact proportion of the orbs of ☿ and the place of the aphelion and nodes; but the mean motion, which depends on comparing ancient observations with recent ones, he does not promise as equally certain, because he has not found suitable observations among the ancients, not even in the manuscript codex of Ptolemy preserved at Paris in the Royal Library.

[Margin: Praise of Boulliau in the hypothesis of ☿.]

CAPVT VI.

De Noſtris Veneris ac Mercurij Hypotheſibus.

Chapter 6. On our own hypotheses of Venus and Mercury.

I. How arduous it is to bind Venus and Mercury to the laws of a definite hypothesis I have already indicated in the scholia of the preceding chapters. For although the tables of the other astronomers satisfy a few observations within a fifth or sixth part of a degree, they nevertheless stray from many other observations sometimes by a whole degree and more, whether this is through the fault of the hypothesis or, rather, of the observations. For if we choose to accept the observations handed down so far, there are many which seem to preserve no analogy of the motions, so much so that at the same places of the eccentric and the epicycle they undergo very different measures, not subject to the rules of human geometry. These planets, then, but especially Mercury, describe wonderful and almost labyrinthine spires around the Sun, a specimen of which our excellent old man Giuseppe Biancani used to sketch for us with the following diagram, in which the orb of the Sun is OSQ and the Sun S, around which Mercury N flies, now in a very wide circuit ABCDM, now

[Margin: Excuse for the astronomers in the hypothesis of ♀ and ☿.]

in a very narrow one HIFG, now at a certain middle withdrawal along the line MEF or FLM; and the same is to be understood of Venus, for she does not always digress from the Sun by the same measure. So, then, if we are to leave the angelic theory, the one perceived by the moving Intelligences, which perhaps cannot be grasped by any bonds of our geometry, and are at last to fit to human understanding some form on which tables of the motions may rest, we need far more observations than those that now exist, and indeed observations either undertaken by ourselves or of such a kind that the manner of observing lets us be certain of the correction to be applied to the observed place. Moreover, the roughly 70 observations of Venus and 25 of Mercury that

we have already made are not enough for us. We therefore intend, God the governor of our life granting it, to obtain more, and at the same time to await Tycho's, which now lie under the press at Cologne. Meanwhile we shall use some selected from those we shall report in chapter 9, and on them we shall build the following hypotheses, intending to correct their measures in Tome 3 if a multitude of certain phenomena should compel us to do so.

[Margin: The author's apology for putting off a more perfect hypothesis to another time.]

Diagram. The figure of the spires of Mercury and Venus around the Sun, set into the right of the left column with the text running narrow beside it. A large arc rises from O and Q, the ends of a straight base line. Above it stands a circle headed by A, and within and across these run several curved loops meeting at F and M on either side. A rayed star stands just left of S on the arc through S. Straight lines rise from the middle of the base line to the curves.

- A (top, outside the upper circle): point of the wide circuit
- N (upper left within the circle, with a planetary sign directly below it): Mercury
- E (top, right of the vertical line)
- I (on the vertical line, below E)
- F (left, where the curves meet)
- S (on the vertical line, beside the rayed star): the Sun
- H (right, on the inner loop)
- M (right, where the curves meet)
- G (on the vertical line, below S)
- K (left, on the large arc)
- B (left, below K)
- L (on the vertical line, below G)
- D (right, below M)
- C (on the vertical line, below L)
- O (left end of the base line)
- Q (right end of the base line, a swash sort)

Expositio Figuræ Communis Veneri, & Mercurio.

Explanation of the Figure Common to Venus and Mercury.

II. Let A be the centre of the Earth, of the ecliptic and of the world, through which passes CAD, the line of apsides of Mercury and Venus. On this line let I be the centre of the eccentric orb CODX, which carries the epicycles of the planets, whose centres in their various positions are C, F, O, D, X, Q. And let AI be either the eccentricity, if it is unvarying and always the same, or the mean eccentricity, if it varies. In that case, from I describe the small circle BZ containing the anomaly of the eccentricity, and let the variation take place by a libration along the diameter BZ, up and down, but let it take its laws from the circumference of the small circle, according to the analogy of second sines,

as we shall presently say. The radius or semidiameter of the planets' orb is to be as great as the radius of the Sun's annual orb, and the eccentric of these planets is to move with exactly the same mean motion as the Sun, in the order of the signs, from the apogee C toward F, O, etc., and from the perigee D toward X, Q. Finally, let the semidiameter of the epicycle become now larger, now smaller through a certain libration, which takes its laws from a circle, likewise according to the ratio of second sines.

[Margin: Explanation of the former diagram.]

Diagram. The former of the two diagrams common to Venus and Mercury, set across the left column. A large outer circle encloses the eccentric circle, on which six small epicycle circles stand at G–C at the top, F upper left, O left, D at the bottom, X right and Q upper right. A small dashed circle stands about I with B above and Z below. Straight lines run from A, below the small dashed circle, to the epicycles and out to the outer circle; a vertical line runs from G through C, B, I, Z, A, L and D to the bottom of the outer circle.

- G (top, on the outer circle)
- C (centre of the top epicycle)
- E (left of the top epicycle, on the eccentric)
- H (upper left, at the outer circle)
- M (upper left, on the epicycle about F)
- θ (a sort of that shape, inside the epicycle about F, to the right of F)
- F (centre of the upper-left epicycle)
- V (left of the epicycle about F)
- N (below the epicycle about F)
- P (top of the left epicycle)
- O (centre of the left epicycle)
- β (left, on the outer circle)
- B (top of the small dashed circle)
- I (centre of the small dashed circle)
- Z (bottom of the small dashed circle)
- A (below the dashed circle, where the straight lines converge)
- K (left, on the bottom epicycle)
- L (top of the bottom epicycle)
- D (centre of the bottom epicycle)
- π (a sort of that shape, at the foot of the vertical line, on the outer circle)
- S (upper right, at the outer circle)
- T (upper right, on the epicycle about Q)
- δ (left of the epicycle about Q, set with its bowl above and its tail below)
- Q (centre of the upper-right epicycle)
- R (below the epicycle about Q)
- Y (top of the right epicycle)
- X (centre of the right epicycle)
- a swash sort resembling ϙ or a Q (right, on the outer circle) [?]

Now, so that the anomaly of the eccentricity, if it is needed, and of the epicycle's semidiameter may be managed, let A in the small circle added here be the limit of the greatest eccentricity or epicycle-semidiameter, C of the mean, and B of the least. Let the difference AC between the greatest and the mean, or CB between the mean and the least, be known, and let us suppose the greatest to occur when the centre of the epicycle is at the apogee of the eccentric, the least when it is at the perigee, and the mean when it is at the mean longitudes. Therefore, given the anomaly of the eccentric from the tables (it will be given if the motion of the apogee of Venus and Mercury is subtracted from the mean motion of the Sun, or, under another hypothesis to be indicated, from the true motion), transfer it to the small circle added here. If it belongs to the first quadrant AGD or the fourth ALF, that is, if the anomaly of the eccentric is less than 90° or greater than 270°, multiply the second sine of the anomaly just mentioned, or of its complement to the circle, say CI, by the given difference AC, whether of the eccentricity or of the epicycle's semidiameter, and divide the total by the whole sine. For the quotient arising from this, added to the mean eccentricity or semidiameter, will yield the eccentricity or semidiameter to be applied later in finding the equations of the centre and of the orb. But if the anomaly of the eccentric belongs to the second quadrant DHB, that is, if it is greater than 90° and less than 180°, or to the third, BMF, that is, if it is greater than 180° but less than 270°, multiply the second sine, say CK, of the complement to the semicircle or of the excess over the semicircle by the difference CB of the eccentricity or semidiameter. Then, having divided the total by the whole sine, subtract the quotient from the mean eccentricity or semidiameter, so as to leave the desired eccentricity or epicycle-semidiameter. It is not necessary, however, for one and the same small circle to serve both the upper and the lower difference: it may happen that two are needed, that is, that the difference AC is not equal to the difference CB.

[Margin: Explanation of the small circle governing the anomaly of the eccentricity and of the epicycle's semidiameter.]

Diagram. The added small circle, set into the right of the right column with the text running narrow beside it. A vertical diameter AB is crossed by three parallel horizontal chords, GL through I, DF through C, and HM through K.

- A (top)
- G (left, upper chord)
- I (on the diameter, upper chord)
- L (right, upper chord)
- D (left, middle chord)
- C (centre)
- F (right, middle chord)
- H (left, lower chord)
- K (on the diameter, lower chord)
- M (right, lower chord)
- B (bottom)

Proponitur Prior Hypothesis Veneris.

The First Hypothesis of Venus Is Set Out.

III. First, let the eccentricity of Venus either be the same as the eccentricity of the Sun, namely, for us, 3460 parts of which the radius of the eccentric orb is 100000, since many observations favour this position; and accordingly let the measure of the equations of the eccentric be the same at an equal anomaly of the eccentric. Or, if one is willing to undergo a labour that is by no means necessary, let the mean eccentricity AI in the former figure be 3400 parts, the least, AZ, 3300, and the greatest, AB, 3500, so that the difference BI or IZ is 100. And according to the anomaly of the eccentric let the temporary eccentricity be found, by the method just given in the explanation of the latter diagram. With this, with the radius 100000, and with the angle they enclose, which the anomaly of the eccentric itself will reveal, find in the usual way the distance of the epicycle's centre from the centre of the Earth, and the prosthaphaeresis of the eccentric, to be subtracted in the first semicircle of the anomaly and added in the second, so as to obtain the place of the epicycle's centre, which is called the planet's eccentric place, or the centre of the planet. The anomaly of the eccentric itself becomes known by subtracting the motion of Venus's apogee from the mean rather than the true motion of the Sun, for a reason to be indicated in the scholia. The distance of the epicycle's centre, however, is to be kept for use, on which see precept 3 shortly.

[Margin: Method of the eccentric equation.]

Second, for the equations of the orb or epicycle, the epicycle's semidiameter must be found. Attending to many and more correct observations of Venus, this is greatest around the apogee of the eccentric, and nearly as great as far as the mean longitude; it is least around the perigee. Nor, in the meantime, after solving 300 triangles and more, could I discover other, more probable measures than that the greatest semidiameter be 73140 parts and the least 70640, so that the lower difference is 2500 parts, of which the radius of the eccentric is 100000. For the mean between these, 71890, to which geometry otherwise invited us, could not be chosen, since the observations everywhere protest when the epicycle stands near the mean longitudes or above them. Therefore, if the anomaly of the eccentric belongs to the upper semicircle, the same semidiameter of the epicycle, 73140 parts, is always to be applied. But if the anomaly belongs to the lower semicircle, multiply the second sine of the complement to 180° or of the excess over 180° by 2500, divide the total by 100000, and subtract the quotient from the 73140 parts, to obtain the desired semidiameter of the epicycle.

[Margin: Inquiry into the epicycle's semidiameter.]

Third, to the mean anomaly of the orb taken from our tables, add the equation of the eccentric itself if the centre of the epicycle is in the first or left semicircle of the centric anomaly, or subtract it if it is in the second or right semicircle; and the equated anomaly of the orb will result. Its complement to

the semicircle, if it does not exceed 180°, or its excess, if it does, will give the angle enclosed by the distance of the epicycle, found by the first step, and the semidiameter of the epicycle, found by the second. From these three givens, then, you will find trigonometrically the angle set at the centre of the Earth, that is, the angle opposite the epicycle's semidiameter. For this will be the epicyclic equation, or the equation called of the orb, to be added to the eccentric place of Venus if the equated anomaly of the orb is less than 180°, but subtracted if it is greater; and you will have the place of Venus in the zodiac as seen from the Earth. Its later reduction from the ecliptic to the orbit, and conversely, will be discussed in the section on the phenomena of latitude. How many observations of different ages this hypothesis satisfies within a fifth part of a degree I forbear to say here; granted that it may not satisfy some, whether they are suspect because analogy is not preserved or for some other reason. But I am not bound to be much more fortunate in this business than so many and such celebrated astronomers have been. I must, however, bring some light to the precepts by an example.

[Margin: Method of the equation of the orb.]

EXEMPLVM pro ♀.

Example for ♀.

IV. Kepler reports in the *Astronomia Optica*, p. 305, that the Heart of Leo was seen by Mästlin covered by Venus in the morning, at 4^h after midnight of the day 1574 September 16. Now the Heart of Leo was then, according to our tables established in Book 6, last chapter, at ♌ 23° 57′. So Venus was either at that very spot or distant from it by a very few minutes, namely as many as the spread of her rays could hide that star by. At that time the motions of the Sun and Venus from our tables stood as follows.

	Value
☉ Mean motion	6^s 4° 25′
♀ Apogee	2^s 29° 23′
Hence the anomaly of the eccentric	3^s 5° 2′
Mean anomaly of the orb	8^s 21° 8′

With these laid down, in the following diagram let the line of apsides be AD, and on it let the centre of the Earth be C, and let the centre of Venus's eccentric orb be B. Counting from the apogee A, in the order of the signs, 3^s 5° 2′, that is, the arc AK, describe from K the epicycle MHV. Then, drawing from the centre of the eccentric, through the centre of the epicycle, the straight line BK as far as M, M will be the mean apogee of the epicycle; and drawing the straight line CKG through the same centre from the centre of the Earth, G will be the true apogee. Next, counting from the mean apogee in the order of the signs 8^s 21° 8′, that is, the arc MHV, Venus will fall at V. To this point

draw both the semidiameter KV and the straight line CV from the centre of the Earth.

[Margin: Delineation of the diagram for this example.]

Diagram. The diagram for the example, set into the right of the left column with the text running narrow beside it. A vertical straight line runs from A at the top through B and C to D at the foot. A large arc runs from A leftward and down through K to D. About K is drawn the epicycle, with G and M at its left and H at its right; V, marked with a small star, stands at its top. Straight lines run from B through K to M, from C through K to G, from K to V, and from C to V.

- A (top of the vertical line)
- B (on the vertical line, above C)
- C (on the vertical line): centre of the Earth
- D (foot of the vertical line)
- V (top of the epicycle, with a star): Venus
- G (left of the epicycle, on the line from C)
- M (left of the epicycle, below G, on the line from B)
- K (centre of the epicycle)
- H (right of the epicycle, on the line CG)

First, then, since the eccentricity BC is given as 3460 parts of which the radius BK is 100000, and the angle ABK as 95° 2′, and hence its complement KBC as 84° 58′, the angle BKC is found by the doctrine of triangles to be 1° 58′. This prosthaphaeresis, subtracted here from the mean motion of the Sun, that is, from 6^{s} 4° 25′, leaves the centre K, that is, the eccentric place of Venus, at ♎ 2° 27′. In the same triangle BKC, the distance KC is found to be 99780 parts of which the radius BK is 100000.

[Margin: 1st step.]

Second, since the anomaly of the eccentric exceeds a quadrant but not a semicircle, and its complement to the semicircle is 84° 58′, the second sine of this, 8774, multiplied by the lower difference of the semidiameter, 2500, makes 21923000. When this is divided by the whole, 100000, there remain 219 parts to be taken from the greatest semidiameter, 73140, which leaves the epicycle's semidiameter KV as 72921 parts.

[Margin: 2nd step.]

Third, since the subtractive equation of the centre, that is, the angle CKB, was 1° 58′, and this equals the angle GKM, if it is added to the mean anomaly counted from the mean apogee, which was 8^{s} 21° 8′, the equated anomaly counted from the true apogee G, that is, the arc GHV, will be 8^{s} 23° 6′. Therefore, the semicircle GMH being removed, there will be left the arc HV, that is, the angle CKV, of 83° 6′. With this, with the distance CK of 99780 found by the first step, and with the semidiameter KV of 72921 found by the second, the equation of the orb KCV is found to be 38° 30′. When this is subtracted from the centric place of Venus, which was ♎ 2° 27′, the place of Venus is left at ♌ 23° 57′, exactly as Mästlin saw it.

[Margin: 3rd step.]

Indicatur Posterior Hypothesis Communis Veneri ac Mercurio.

The Second Hypothesis, Common to Venus and Mercury, Is Indicated.

V. The second hypothesis is nothing other than that of Kepler and Boulliau, transferred, however, to the hypothesis of a resting Earth: that is, if we use the true place of the Sun, which is to be at one of the foci or umbilici of the ellipse described by Venus and Mercury around the Sun, and the eccentricity of the ellipse's centre from the Sun, and the semidiameter of the ellipse. But since not all observations favour the place of the aphelion established by those authors, it is still in doubt in what position, in such a hypothesis, the ellipse is to be described with respect to the Earth and the Sun together. Indeed, one might doubt whether the axis of the ellipse is to be held always in a position parallel to the line of apsides of the Sun, or whether the minor diameter of the ellipse is to be kept always equidistant from the line of the Sun's true place; and whether the anomaly of the orb is to be counted from the aphelion of the ellipse or rather from the apogee. To determine these things, and the correct measures, more observations are needed. We therefore cannot bring forth the hypothesis we have conceived, and readers should not take it amiss if we have preferred to indicate it rather than suppress it altogether. Certainly, when it has matured, it will represent that apparent diversity of the semidiameter.

Proponitur Mercurij Hypothesis.

The Hypothesis of Mercury Is Set Out.

VI. Let the figure described at number 2 be repeated here, and in it let the greatest eccentricity of Mercury, AB, be 10000 parts of which the radius of the eccentric is 100000, which it is to have when the centre of the epicycle is placed at C, the apogee of the eccentric. Let the mean, AI, be 8000 parts, namely when the centre of the epicycle is at O or X, that is, at the mean longitudes; and let the least, AZ, be 6000 parts, which it is to have when the centre of the epicycle is placed at D, the perigee of the eccentric. Thus the difference of the extreme eccentricity from the mean, that is, the semidiameter of the small circle, BI and IZ, is 2000 parts. And let the eccentricity vary according to the ratio of the second sines corresponding to the anomaly of the eccentric, or to its complement to the semicircle or circle, or to its excess over the semicircle, as I taught at number 2 by means of the second diagram explained there. The semidiameter of the epicycle is likewise to vary according to the same ratio, but so that the semidiameter EC and DK is least when the centre of the epicycle is at C, the apogee of the eccentric, or at D, the perigee, and OP or XY is greatest when that centre is at O or X, that is, at the points of the mean longitudes, which is to say when the centre of the epicycle stands a quadrant from the apogee of the eccentric. Further, let the greatest semidiameter be 45500 parts

of which the radius of the eccentric (which here too is as great as the radius of the annual orb) is 100000, and the least 38500 parts, so that the difference is 7000. Finally, let the anomaly of the eccentric be obtained by subtracting the motion of Mercury's apogee from the mean motion of the Sun, for far more observations favour this

[Margin: Variation of the epicycle's semidiameter.]

than an anomaly derived from the true place of the Sun. In all the remaining matters, as regards the method of finding the prosthaphaereses of the eccentric and of the orb or epicycle, let the procedure be as was said in the hypothesis of Venus, and as will shortly be explained by the example itself.

EXEMPLVM pro ☿.

Example for ☿.

VII. In the year 1631, on the morning of November 7, Pierre Gassendi observed Mercury under the Sun, and from the circumstances of the observation he judiciously determined that the conjunction with it took place at $17^{h}\ 23^{m}$ after noon of the 6th, with respect to Uraniborg, when Mercury was in Scorpio 14° 36′, as he himself reports in the *Institutio Astronomica*, and Boulliau in Book 10, chapter 7. At that moment, from our tables, the motions of the Sun and Mercury stood as follows.

	Value
Mean motion of ☉	$7^{s}\ 16°\ 4'$
Apogee of ☿	$7^{s}\ 29°\ 59'$
Hence the anomaly of the eccentric	$11^{s}\ 16°\ 5'$
Mean anomaly of the orb	$6^{s}\ 6°\ 15'$

Let the former figure of number 2 be repeated here too, and, counting from the apogee C, in the order of the signs, $11^{s}\ 16°\ 5'$, that is, the arc CODQ, let the centre of the epicycle be placed at Q. Since this is a little distant from the apogee, namely by 13° 55′, the eccentricity will therefore be close to greatest, and nearly AB. So, drawing from a point a little below B the straight line BQT, T will be the mean apogee of the epicycle; and drawing AQS from the centre of the Earth, S will be the true apogee. And counting from T through S δ the mean anomaly of the orb, $6^{s}\ 6°\ 15'$, the place of Mercury will fall at R, which is to be joined to the centre of the Earth and of the epicycle by the straight lines AR and QR. With these preparations made:

[Margin: Delineation of the diagram for this case.]

First, in the triangle BAQ, the radius BQ is given as 100000 parts, and the angle ABQ, the complement of the angle CBQ, which we said to be 13° 55′, is 166° 5′. Therefore, if the eccentricity AB is given, there will be enough givens to solve the triangle. Now the second sine of the complement of the anomaly of the eccentric to the circle, that is, of 13° 55′, is 97065; multiplied by

the difference of the eccentricity, 2000, it makes 194130000. This total, divided by 100000, leaves 1941 particles to be added to the mean eccentricity AI of 8000 parts, and the eccentricity AB comes out as 9941 parts. With this, with the radius BQ and with the angle ABQ just mentioned, there is obtained AQ, the distance of the epicycle's centre, as 109680 parts of which the radius BQ is 100000, and the angle AQB as 1° 15′, that is, the prosthaphaeresis of the centre. Added to the mean motion of the Sun, this gives the centric place of Mercury at ♏ 17° 19′.

[Margin: 1st step.]

Second, the complement of the anomaly of the eccentric to the circle was 13° 55′, and its second sine 97065. This, multiplied by the difference of the epicycle's semidiameter, 7000, makes 679455000. When this total is divided by the whole sine, 100000, there remain 6795 parts to be taken from the greatest semidiameter, 45500, and there remains QR, the epicycle's semidiameter, of 38805 parts.

[Margin: 2nd step.]

Third, since the equation of the centre was 1° 15′ and additive, it is therefore applied in the contrary way and subtracted from the mean anomaly of the orb, 6^s 6° 15′, and the equated anomaly of the orb will come out as 6^s 5° 0′. This, counted from the true apogee S, will give the arc SδR; and when 6^s are removed from it, there will be left the angle AQR of 5° 0′. With this, with the distance AQ of 109680 parts obtained by the first step, and with the semidiameter QR of 38805 parts obtained by the second, the angle QAR is found to be 2° 43′, that is, the equation of the orb, to be subtracted from the centric place of Mercury, which was ♏ 17° 19′. And there remains the apparent place of Mercury in the zodiac, ♏ 14° 36′, exactly as Gassendi observed it. How far the tables of others, especially the Danish tables, or those of Longomontanus, have stood from that place, we said in Book 3, chapter 3, scholium 3.

[Margin: 3rd step.]

Would that one could always attain the place of Mercury with such happiness of hypothesis! If we do not attain it, not all the blame is to be thrown on the hypothesis; part of it must also be put on the slippery observations. For it is enough for us to attain by this method, very closely, several places of the more certain observations, until a multitude of clear observations suggests a way of further polishing and perfecting the measures of the orbs of Mercury and Venus. Meanwhile, let the synopsis of our measures and numbers stand in the following table, with the distances, on the supposition that the radius of the annual orb, that is, the mean distance of the Sun from the Earth, is 7300 terrestrial semidiameters, according to what was said in Book 3, chapter 7. In the greatest and least distances, however, we shall assume those which could have been or could be in our hypothesis if the duration of the world had been or were to be so great that the apogees of the eccentrics of the Sun, Venus and Mercury stood under the same or opposite points of the zodiac; even though, in these times, the apogee of Venus is much nearer to the apogee of the Sun

than the apogee of Mercury is, and indeed the perigee of Mercury is much nearer to the apogee of the Sun than the apogee of that same Mercury is.

MENSVRÆ NOSTRÆ Hypothesis Mercurij & Veneris.

Our measures for the hypothesis of Mercury and Venus.

Quantity		Planet	In parts of which the radius of the eccentric is 100000	In semidiameters of the Earth
Eccentricity	Total, AB	♀	3460	263 [recte: 253, per the book's errata]
		☿	10000	730
	Mean, AI	☿	8000	584
	Least, AZ	☿	6000	438
Semidiameter of the small circle	BI	☿	2000	146
Semidiameter of the epicycle	Greatest, OP & EC	♀	73140	5339
	Least, KD	♀	70640	5157
	Greatest, OP	☿	45500	3322
	Least, EC & KD	☿	38500	2811
Difference of the semidiameters		♀	2500	183
		☿	7000	511
Distance of the epicycle's centre from the centre of the Earth	Greatest, AC	♀	103460	7563
		☿	110000	8030
	Mean, AO	♀	100000	7300
		☿	100000	7300
	Least, AD	♀	96540	7037
		☿	94000	6862
Distance of the planet from the centre of the Earth	Greatest, AG	♀	176600	12902
		☿	148500	10841
	Mean	♀	100000	7300
		☿	100000	7300
	Least, AL	♀	23900	1880
		☿	55500	4051
Distance of the planet from the Sun	Greatest	♀	73140	5339
		☿	51960	3794
	Mean	♀	71890	5248
		☿	38400	2803
	Least	♀	70640	5157
		☿	24840	1813

[Margin: See figure 1 of number 2.]
MOTVS MEDII.
Mean motions.

Anomaly of the orb		Signs	Deg.	′	″	‴
Daily	♀	0	0	36	59	30
	☿	0	3	6	24	30
Annual	♀	7	15	1	53	35
	☿	1	23	57	33	32
Apogee of the eccentric		**Signs**	**Deg.**	**′**	**″**	**‴**
Annual	♀	0	0	1	24	0
	☿	0	0	1	50	15

[Margin: After the revolutions.]
RADIX Motuum Ad Annum 1. CHRISTI ineuntem In Meridie vltimo Decembris Bononiæ.
Root of the motions for the beginning of the year 1 of Christ, at noon of the last day of December, at Bologna.

		Value
Anomaly of the orb	♀	1ˢ 13° 17′ 47″
	☿	1ˢ 3° 49′ 24″
Apogee	♀	1ˢ 22° 40′ 0″
	☿	6ˢ 10° 0′ 0″

On the greatest equations and digressions, see Chapter 7.

SCHOLIA.

Scholia.

I. Someone will perhaps wonder why the superior planets, in their epicyclic motion, that is, in the anomaly of the orb, adjust themselves to the Sun's true or apparent motion; and why, accordingly, to obtain the anomaly of the orb, their eccentric place must be subtracted from the Sun's true place according to our opinion, which was in the end that of Tycho and Longomontanus, to say nothing of Kepler and Boulliau; and yet Venus and Mercury, in their eccentric motion, that is, in the anomaly of the eccentric, do not regard the Sun's true motion but rather its mean motion, and, according to our opinion, to obtain their anomaly of the eccentric, the motion of the apogee of these planets must be subtracted from the Sun's mean motion, as all ancient astronomy was accustomed to do. But he will stop wondering if he considers that the superior planets, in their motion around the Sun, do not describe epicycles around the Sun; or that they regard the Sun in such a way that they principally regard the Earth, or the Sun as ordered around the Earth, and therefore follow the motion of the Sun that appears to the Earth; whereas Venus and Mercury, of

themselves and in the first place, regard the Sun, around which they describe their epicycles, and therefore follow the Sun as ordered around the Earth only secondarily, and so regard its motion just as if it were viewed from the centre of the Sun's eccentric. Next, the multitude of observations, which are better represented by this hypothesis, drove us to it. We have, though, also indicated another hypothesis, which can use the Sun's true motion and ellipses described around it.

[Margin: Why in Mercury and Venus we use the mean motion of the ☉.]

II. There will also be those who criticize our hypotheses on the ground that we use the small circle that varies the anomaly of the eccentric in such a way that we assume neither a doubled anomaly nor a motion of the centre made along the circumference of this small circle. But since by this method of ours the geometrical calculation turns out easier, and is supported by no smaller number of observations, it did not seem right to impose that labour on astronomers; and it is expedient to try this method too, and rather to change the measures of the orbs and motions so that they agree with the observations.

[Margin: On the use of the small circle of the anomaly of the eccentric.]

CAPVT VII.

Chapter VII.

De Maximis Veneris ac Mercurij à Sole Digressionibus, & Aequationibus Orbis.

On the greatest digressions of Venus and Mercury from the Sun, and on the equations of the orb.

I. We treat the greatest elongations from the Sun and the prosthaphaereses of the orb together because, in the hypothesis of the Earth going around the Sun and reckoning the anomaly of these planets from the Sun's true place, the greatest equation of the orb, or greatest parallax of the orb, or angle of commutation, and the greatest wandering of Venus and Mercury from the Sun, are one and the same; granted that, in the hypothesis of the Earth at rest, or regarding the Earth's mean motion, they differ. Moreover, the greatest digressions, which the Greeks call [Greek: ἀπόστασεις (apostaseis, "distances")] or [Greek: διάστασεις (diastaseis, "separations")], occur without dispute when Venus or Mercury is at those points of the orb at which a straight line drawn from the centre of the Earth is tangent to their orb, that is, makes a right angle with the semidiameter of the epicycle, or of an orb equivalent to the epicycle. These indeed occur before the first station and after the second, and farther from the stations in ♀ than in ☿. Kepler, in the *Epitome of Copernican Astronomy*, p. 763, lays down a twofold sign of the greatest digressions: the first is if the apparent motion of these planets in an ephemeris is equal to the Sun's true motion; the other, if, observed with a telescope, they appear dichotomous, that is, like a bisected Moon. The size of these digressions and their variety depend

on 3 causes. The *first* is the size of the epicycle or orb and its diameter, which in our hypothesis is not always the same but is variable along a spire; in an elliptical orbit, however, in place of the semidiameter of the orb, the greatest and least distance of the planet from the Sun, one of the foci, is used. The *second* is the distance of the centre of the epicycle or orb from the centre of the Earth, especially if their eccentricity varies, as we more truly judge it does in Mercury; or, in the hypothesis of the Sun at rest, the differing distance of the Sun from the Earth. The *third* is the variation of the Sun's true place. From these three elements, then, let there be

[Margin: Where and when the greatest digressions from the ⊙ occur.]

[Margin: The causes that vary the measure of the digressions.]

Problema 1. Aequationes Orbis Veneris ac Mercurij Maximas Definire.

Problem 1. To define the greatest equations of the orb of Venus and Mercury.

II. In the following diagram let G be the centre of the Earth, and B the centre of the epicycle of Venus or Mercury; and from B as centre let their epicycle, or orb, HDFE be described, which the straight lines GD and GE, drawn from the centre of the Earth, touch at points D and E; for by Euclid III.18 they will make a right angle at the points of contact with the semidiameters of the orb BD and BE. Now let there be given the distance BG of the centre of the orb from the centre of the Earth, both when the centre B is at the apogee of the eccentric and when it is at the perigee; let there also be given the semidiameter of the epicycle or orb in the same kind of parts, or, in the hypothesis of the ellipse, the distance of the planet from the Sun. For in triangle BGD, right-angled at D, or in BGE, right-angled at E, the angle DGB or BGE will be given, measuring the greatest apogeal or perigeal equation of the orb. In brief, let it be: as the distance BG to the whole sine, so the semidiameter BD or BE to the sine of the angle at G; or join the residual of the logarithm of the distance BG to the logarithm of the semidiameter of the epicycle or orb, and the logarithm of the angle measuring the greatest equation of the orb will result. See below the examples and the method of Boulliau to be brought forward at number 9, and another example from Copernicus below, ch. 9, Problems 4 and 6. How, moreover, from the greatest equations of the orb once established at the apogee and perigee of the eccentric the other greatest equations of the orb are derived, or are established by this Problem from other and yet other distances of the centre of the epicycle, is clear from what was said about the equations of the three superior planets in Section 2, ch. 12, to be applied in like manner to these planets.

Diagram. An orb with tangents from the Earth, set into the right of the right column with the text running narrow beside it. A circle with centre B carries a vertical diameter from H at the top through B and F; the line continues down to G, below the circle. Two straight lines descend from the left and right

of the circle to meet at G. An arc runs from A on the left through the region of B to C on the right. Short lines join B to D and to E.

- H (top of the circle)
- B (centre of the circle)
- A (left end of the arc, outside the circle)
- D (left, on the circle, beside A)
- E (right, on the circle)
- C (right end of the arc, outside the circle)
- F (bottom of the circle, on the vertical line)
- G (foot of the figure, where the two straight lines meet)

Problema 2. Digreſſiones Veneris ac Mercurij à Sole Maximas Definire.

Problem 2. To define the greatest digressions of Venus and Mercury from the Sun.

III. In the hypothesis of Kepler and Boulliau, and of others who hold that Venus and Mercury go around the Sun B, unmoved, as one of the foci of an elliptical orbit equivalent to the Ptolemaic epicycle, and who reckon the eccentric anomaly from the Sun's true place, the greatest equations of the orb and the greatest digressions from the Sun of these planets are the same, as I have just said; hence by the first Problem the second has been satisfied. But if you draw from G the arc of the ecliptic ABC, and A is the eastern part and C the western, the angle BGD will measure the greatest evening digression, made at D; and the angle BGE the morning digression, made or to be made at E. In the theory of others, however, which employs an Earth at rest, or subtracts the apogee of these planets from the Sun's mean motion so as to have the eccentric anomaly, one must know the time at which the centre of the epicycle of ♀ or ☿ is at the apogee and at the perigee of the eccentric, and compute for that time the Sun's motion, both mean and true. For the greatest equation of the orb in both places, that is, at apogee and perigee, will be equal to the greatest digression from the line of the Sun's mean motion. Therefore, by adding to it or subtracting from it the equation of the Sun, that is, the difference of the true place from the mean, the greatest digression will be established, according as the Sun is more eastern or more western; and the same is to be done at other places of the eccentric besides the perigee and apogee, since at those places the line drawn through the centre of the epicycle of Venus or Mercury is parallel to the line of the Sun's mean motion. In Mercury, however, which is judged to make the least digressions at the apogee of the eccentric, and greater ones at perigee, but the greatest at the thirds of the circle, what must be done will be clear from what is to be said in ch. 9, Problem 3.

IV. Before we set out in tables a synopsis of opinions on the greatest equations and wanderings, it is worthwhile to indicate in advance what ancient and recent authors have judged about them. Among them, in the first place, Pliny the Elder, Book 2, ch. 8, speaks as follows: "Below, a huge star called that of Venus

circles the Sun, wandering in an alternating course, etc."; and shortly after: "It completes the circuit of the zodiac in 348 days, never standing away from the Sun farther than 46°, as Timaeus holds. In a similar way, but by no means in size or power, the star of Mercury next to it, called by some that of Apollo, is carried in a lower circle" (he would better have said "inner") "with a circuit nine days swifter, shining now before sunrise, now after sunset, never more than 23° removed from the Sun, as this same author and Sosigenes teach." And in ch. 17: "Let it be said first, then, since they are different stars, why the star of Venus never departs from the Sun farther than 46°, and Mercury 23°, and why they often [?] turn back toward the Sun short of those limits: both have their apsides turned the other way, etc." As to Venus's wandering, Martianus Capella supports Pliny the Elder in Book 8 of *De Nuptiis Philologiae et Mercurii*, towards the end, saying: "But Venus, like the Moon, wanders about, departing from the Sun's orb by fifty moments, although it cannot stray more than 46°." But about Mercury he had written a little earlier: "But this same Stilbon, though it accompanies the Sun from different circles, will nevertheless never be able to stray from it beyond 32°, nor to stand off by two signs."

[Margin: Pliny the Elder's opinion, from Timaeus and from Sosigenes.]

[Margin: The opinion of Martianus Capella.]

V. Far more exact is Ptolemy, who, in Book 9, ch. 7, where he treats those of Mercury, and in Book 10, ch. 1, where those of Venus, and in Book 12, ch. 9, where the greatest digressions of both, discusses them first from the Sun's mean place and then from its apparent place, on the basis of the observations of Theon, an astronomer of his own time, and of his own. Ptolemy was more or less followed by al-Farghānī, difference 15; Jābir ibn Aflaḥ, Book 7 of his *Astronomy*, ch. 3, correcting Ptolemy; Regiomontanus, Book 12 of the *Epitome of the Almagest*, propositions 13 and 14; Fernel, Book 1 of the *Cosmotheoria*, ch. 5 and 6; Peuerbach, in his theory of Venus and Mercury, and on that above all Wurstisen, pp. 190 and 218. Ptolemy judged that Mercury makes its least digressions from the Sun in the sign ♎, and greater ones in ♈, yet not the greatest there, but in ♊ and in ♒, as Copernicus gathers in Book 5, ch. 25; although the Ptolemaic table places Mercury's greatest in ♍, and in ♒ the morning ones, in ♌ the evening ones; and he judged that it comes nearer the Earth not at the perigee of the eccentric but at two points a third of the circle distant from the apogee.

[Margin: The opinion of Ptolemy and of the ancients.]

VI. Copernicus himself, in Book 5, ch. 20, subscribing to Ptolemy, teaches that Venus's digressions on both sides are smaller around the apogee and greater around the perigee, but he transfers them to the hypothesis of the moved Earth. In ch. 25, however, he teaches that these digressions vary from the variation of the epicycle, which he admits to be now greater, now smaller; and in ch. 28 he reports that Mercury makes greater digressions around the sides of the hexagon than at perigee, not because the planet there comes nearer the Earth than at the perigee of the eccentric, but because the planet there describes a greater

circle. How large these digressions are can be gathered from the chapters already mentioned, and from what will be said more fully in ch. 9, Problems 4 and 6, and finally from the greatest prosthaphaereses of the orb that Reinhold gives in the Prutenic Tables and Mästlin in the Appendix to Kepler's *Prodromus*, from which Lansbergen in his tables does not depart far; and Magini, of whom presently.

[Margin: The opinion of Copernicus.]

VII. But Giovanni Antonio Magini, in Book 1 of the *Theoricae*, in the theory of Venus, ch. 4, and in the theory of Mercury, ch. 9 and 10, followed the Ptolemaic hypothesis as to the form of the diagram in such a way that he nonetheless corrected the numbers or measures from the Prutenic Tables. He therefore teaches that Venus's elongations from the Sun's mean place are greater at the perigee of the eccentric than at the apogee; that among the perigeal ones the morning ones are greater than the evening ones; but that with the centre of the epicycle at the apogee of the eccentric the morning ones are sensibly equal to the evening ones; and that they are greater than all of these when the centre of the epicycle is 115° from the apogee of the eccentric in the order of the signs and Venus is 7^{s} 13° from the apogee of the epicycle, reckoning in the variation that the true place of the epicycle's centre produces from the mean. If, on the other hand, the elongations of Venus are viewed from the Sun's true place, the morning ones turn out greater than those computed from the mean place when the Sun is in the first semicircle of its eccentric, because the Sun's true motion is less than its mean; and they then become greatest with the Sun holding an anomaly of 92°, and very greatest at the greatest eccentricity. Conversely, the evening ones come out greater from the true than from the mean place of the ☉ when the Sun is in the latter semicircle of its anomaly, and greatest at 268°, and very greatest at its greatest eccentricity. But here Magini errs, putting morning ones for evening ones, and vice versa.

[Margin: The opinion of Magini and Barazanus [recte: Baranzanus].]

[Margin: Magini's slip.]

The greatest digressions of Mercury from the Sun's mean place occur at the points of the eccentric at which the centre of the epicycle is 120° from the apogee on either side, if Mercury is at the points of contact of the epicycle; and both the evening and the morning ones are greater at the western contact of the epicycle than at the eastern. As for the Sun's true motion, they grow in the same way as was said of Venus, according as the Sun is in the first or the latter semicircle of its eccentric, and around 92° and 268° of solar anomaly they become greatest; and still greater at the Sun's greatest eccentricity. Baranzanus follows Magini, in Part 2 of the *Uranoscopia*, from p. 200.

VIII. Longomontanus, in Book 2 of the *Theorica*, ch. 19, corrects the greatest digressions of Venus from the Sun, reported by Ptolemy in Book 10, ch. 1 and 2, according to Tycho's positions of the fixed stars and the corrected mean motion of the Sun, so as to derive from them the motion of the anomaly of the orb. But there and in ch. 20 he uses the Sun's mean motion, which he judges

these planets regard; and in ch. 22 he indicates the structure of the canon of the equations of the orb by proportional minutes, about which we said enough in Section 1, ch. 12, from no. 5. We shall therefore gather the greatest equations from the tables he appends to ch. 22, although they could be gathered from the measures of the orbs he gives and from his diagrams, according to what was said in ch. 4, at the end of numbers 2 and 3. And although Kepler, in the *Epitome of Copernican Astronomy*, p. 763, touches on this matter very lightly, saying that the greatest digression of ♀ is confined within 47° 15′, and that of ☿ at apogee within 29° and at perigee within 18°, yet from his measures, set out in ch. 5, no. 5, and from the Sun's distances in Book 3, ch. 7, we shall gather juster digressions.

[Margin: The opinion of Longomontanus.]

[Margin: The opinion of Kepler.]

IX. Ismaël Boulliau, in Book 9, ch. 4, and Book 10, ch. 6, splendidly defines the greatest equations of the orb of Venus and Mercury at aphelion and perihelion, both theirs and the Earth's, and lays down that they are the same as the greatest digressions from the Sun; for he teaches that the apparent motion of these planets and the line of apsides are referred not to the centre of the annual orb, that is, to the Sun's mean motion, as Longomontanus and Lansbergen held with the ancients, but to the Sun itself, through which the line of apsides passes. In Book 11, theorems 11 and 12, he tries to demonstrate this about Mercury from four of its greatest digressions from the Sun; and he indicates the same demonstration for Venus from what was said in Book 9. For Venus, therefore, he uses this first diagram, in which E is the centre of the annual orb AD, and AED the Earth's line of apsides; the eccentricity from the Sun G is EG; and the Earth is at aphelion at A and at perihelion at D, its distance AG being 101784 parts and DG 98216, of the parts of which EA, the radius of the annual orb, is 100000. With the Earth placed at A, then, and Venus at aphelion at B, the point of contact, and with the tangent AB drawn and Venus's aphelion distance BG of 72973 parts, he gathers the angle BAG, of the greatest digression from the Sun and of the greatest parallax of the orb. Likewise, with Venus placed at H and the tangent AH drawn, from its perihelion distance GH, 71823, he gathers the greatest digression GAH. And with the Earth at perihelion at D, and Venus now at aphelion at C, now at perihelion at I, the tangents being drawn

[Margin: The opinion and method of Boulliau.]

[Margin: The method for the greatest digressions of ♀.]

Diagram. Boulliau's figure for Venus, set into the right of the right column with the text running narrow beside it. A large outer circle has A at its top and D at its foot, joined by a vertical line. Within it stand two smaller circles, a middle and an inner, both off-centre toward the foot and cut by the vertical line. Straight lines run from A to B and to H, and from D to C and to I. Short lines join G to B, H, I and C. The sign ♀ stands in the left of the space between the outer circle and the middle circle.

- A (top of the outer circle)

- B (right, on the middle circle)
- H (right, on the inner circle, below B)
- E (on the vertical line, upper of the two letters above G)
- E (a second E on the vertical line, directly below the first, with a horizontal rule between them)
- G (on the vertical line, below the two letters)
- I (left of G, on the inner circle)
- C (left, on the middle circle)
- D (foot of the outer circle)
- ♀ (left, in the outer space)

DC and DI; and with Venus's aphelion distance CG, 72973, but perihelion distance GI, 71823, he gathers for these positions the angles of the greatest digressions and equations, GDC and GDI, as you will see in the table to be shown presently.

He uses a similar method in Book 10, ch. 6, for Mercury's digressions. The Earth's annual orb ABC is described and the line of apsides AC drawn, on which the Sun is at H; Mercury's orb in aphelion is DEF or IM, and in perihelion KLF; the Earth at aphelion is at A, its distance AH being 101784, and at perihelion at C, its distance being 98216; and Mercury's aphelion distance DH or IH or MH is 46690 parts, its perihelion distance HK or HL 30480 parts. For with the tangents AI, AK, CM, CL drawn, he gathers in the triangles right-angled at the contacts the angles of the digressions HAI, HAK, HCM, HCL, as in the tables below.

[Margin: The method for the greatest digressions of ☿.]

Diagram. Boulliau's figure for Mercury, set into the right of the left column with the text running narrow beside it. A large outer circle has A at its top, B on its left and C at its foot, with a vertical line joining A to C. Within it stand smaller nested circles, off-centre, cut by the vertical line near H. Straight lines run from A down to I and to the inner circle by K, and from C up to M and to the inner circle by L. Short lines radiate from H to D, I, M, K and L. The sign ☿ stands in the upper left of the space within the outer circle.

- A (top of the outer circle)
- ☿ (upper left, in the outer space)
- D (upper right, on an inner circle)
- E (upper left, on an inner circle)
- I (right, on an inner circle)
- L (left, on the smallest circle)
- H (centre, on the vertical line)
- K (right of H, on the smallest circle)
- F (below and left of H)
- B (left, on the outer circle)
- M (left, on an inner circle)
- C (foot of the outer circle)

I. TABVLA DIGRESSIONVM A SOLE MAXIMARVM ♀ & ☿.

Table I. Of the greatest digressions from the Sun of ♀ and ☿.

Authors	♀	☿
Pliny the Elder	46° 0′	23° 0′
Capella	46° 0′	32° 0′
al-Farghānī	48° 0′	28° 0′
Jābir ibn Aflaḥ	47° 45′	28° 35′
Fernel	47° 50′	28° 37′
Biancani	48° 0′	28° 0′

[Margin: Pliny the Elder, from Timaeus and Sosigenes.]
[Margin: Biancani, in the *Sphaera*, Book 11, ch. 2; Book 12, ch. 5.]

II. TABVLA DIGRESSIONVM A SOLE MAXIMARVM ex Ptolemæo

Table II. Of the greatest digressions from the Sun, from Ptolemy.

Beginnings of the signs	♀ Morning	♀ Evening	☿ Morning	☿ Evening
♈	45° 14′	45° 12′	24° 14′	19° 36′
♉	45° 17′	45° 31′	22° 13′	21° 7′
♊	45° 34′	44° 49′	20° 18′	23° 41′
♋	45° 56′	44° 25′	18° 17′	26° 16′
♌	46° 20′	44° 31′	16° 35′	27° 37′
♍	46° 28′	44° 45′	16° 8′	26° 17′
♎	46° 45′	45° 41′	17° 46′	23° 31′
♏	46° 47′	46° 30′	21° 32′	20° 58′
♐	46° 1′	47° 13′	26° 9′	19° 28′
♑	46° 7′	47° 35′	28° 37′	19° 14′
♒	45° 41′	47° 34′	28° 17′	18° 51′
♓	45° 20′	47° 7′	26° 24′	19° 0′

[Margin: Ptolemy, Book 12, ch. 9.]

Before you come to the following Table III, you must be reminded again, dear reader, of what was said above at no. 7: namely, that Magini, and with him Baranzanus, in this treatment of the greatest digressions, used morning ones for evening ones. For, intending to show that the digressions from the true place of the ☉ are sometimes greater than the digressions computed from the mean place of the ☉, he gives an example in the morning ones, which occur with the centre of the epicycle in the first semicircle of the eccentric, where the true motion of the ☉ and of the centre of the inferior planets is less than the mean; and in the evening ones, which in the latter semicircle, etc. Yet it is certain that a morning planet in the first semicircle of the eccentric is less elongated from the true than from the mean place of the ☉, and the reverse in the latter. We, however, have thought it better to show the digressions with their numbers, as they stand in the author.

[Margin: A notice to the reader regarding the following table.]

III. TAB. DIGRESSIONVM à Sole Maximarum Copernicæarum ex Magino, & Baranzano.

Table III. Of the greatest Copernican digressions from the Sun, from Magini and Baranzanus.

Digressions	From the mean place of the ☉	Distance of the epicycle's centre from the apogee of the eccentric	Distance of the planet from the apogee of the epicycle
♀ Morning	48° 8′ 38″	3ˢ 25°	7ˢ 13°
	45° 10′ 19″	0ˢ 0°	7ˢ 14°
♀ Evening	46° 51′ 29″	6ˢ 0°	4ˢ 17°
	45° 10′ 19″	0ˢ 0°	4ˢ 15°
☿ Morning	19° 3′ 6″	0ˢ 0°	3ˢ 24°
	26° 34′ 56″	4ˢ 0°	3ˢ 24°
	21° 8′ 14″	8ˢ 0°	3ˢ 24°
☿ Evening	19° 3′ 6″	0ˢ 0°	3ˢ 24°
	21° 8′ 14″	4ˢ 0°	3ˢ 24°
	26° 34′ 56″	8ˢ 0°	3ˢ 24°

Digressions	From the true place of the ☉	Distance of the epicycle's centre from the apogee of the eccentric	In the eccentricity of the ☉
♀ Morning and evening	47° 1′ 0″	0ˢ 0°	Least
	47° 33′ 43″	0ˢ 0°	Greatest
♀ Morning	49° 59′ 19″	3ˢ 25°	Least
	50° 32′ 2″	3ˢ 25°	Greatest
♀ Evening	48° 42′ 10″	6ˢ 0°	Least
	49° 14′ 53″	6ˢ 0°	Greatest
☿ Evening and morning	20° 53′ 47″	0ˢ 0°	Least
	21° 26′ 30″	0ˢ 0°	Greatest
☿ Morning	28° 25′ 37″	4ˢ 0°	Least
	28° 58′ 20″	4ˢ 0°	Greatest
	22° 58′ 55″	8ˢ 0°	Least
	23° 31′ 38″	8ˢ 0°	Greatest
☿ Evening	22° 58′ 55″	4ˢ 0°	Least
	23° 31′ 38″	4ˢ 0°	Greatest
	28° 25′ 37″	8ˢ 0°	Least
	28° 58′ 20″	8ˢ 0°	Greatest

[Margin: Magini, in the *Theoricae*, ♀ ch. 4, ☿ ch. 9.]
[Margin: Baranzanus, Part 2 of the *Uranoscopia*, pp. 200 and 202.]
[Margin: In the anomaly of the ☉.]
[Margin: 3ˢ and 2°.]
[Margin: 8ˢ and 28°.]

[Margin: 3^s and 2°.]
[Margin: 8^s and 28°.]

IV. TAB. DIGRESSIONVM & ÆQVATIONVM Orbis Maximarum, ex Keplero & Bullialdo

Table IV. Of the greatest digressions and equations of the orb, from Kepler and Boulliau.

♀	Earth at aphelion: ♀ at aphelion	Earth at aphelion: ♀ at perihelion	Earth at perihelion: ♀ at aphelion	Earth at perihelion: ♀ at perihelion
Kepler	45° 48′ 20″	45° 0′ 0″	47° 48′ 34″	46° 56′ 38″
Boulliau	45° 48′ 9″	44° 52′ 51″	47° 59′ 9″	47° 0′ 36″

☿	Earth at aphelion: ☿ at aphelion	Earth at aphelion: ☿ at perihelion	Earth at perihelion: ☿ at aphelion	Earth at perihelion: ☿ at perihelion
Kepler	27° 30′ 30″	17° 33′ 18″	28° 30′ 34″	18° 9′ 40″
Boulliau	27° 18′ 13″	17° 25′ 28″	28° 23′ 2″	18° 4′ 45″

[Margin: Boulliau, Book 9, ch. 4; Book 10, ch. 6. Kepler, from his measures.]

V. TABVLA DIGRESSIONVM A SOLE MAXIMARVM ex noſtra Hypotheſi.

Table V. Of the greatest digressions from the Sun, from our hypothesis.

	♀	☿
At the apogee of the eccentric	46° 58′ 4″	22° 28′ 16″
At mean distance	48° 59′ 6″	26° 10′ 41″
At the perigee of the eccentric	49° 1′ 6″	29° 2′ 24″

VI. TABVLA ÆQVATIONVM MAXIMARVM Cum Gradibus Anomaliæ, in quibus eueniunt

Table VI. Of the greatest equations, with the degrees of anomaly at which they occur.

♀	Greatest equation of the centre: greatest equation	Greatest equation of the centre: anomaly of the eccentric	Greatest equation of the orb, at the apogee of the eccentric: greatest equation	Greatest equation of the orb, at the apogee of the eccentric: anomaly of the orb	Greatest equation of the orb, at the perigee of the eccentric: greatest equation	Greatest equation of the orb, at the perigee of the eccentric: anomaly of the orb
Ptolemy	2° 41′ 0″	102° 0′	45° 57′ 0″	134° 3′ 0″	47° 49′ 0″	132° 11′ 0″
Alfonso	2° 10′ 0″	93° 0′	45° 59′ 0″	134° 1′ 0″	47° 41′ 0″	132° 19′ 0″

♀	Greatest equation of the centre: greatest equation	Greatest equation of the centre: anomaly of the eccentric	Greatest equation of the orb, at the apogee of the eccentric: greatest equation	Greatest equation of the orb, at the apogee of the eccentric: anomaly of the orb	Greatest equation of the orb, at the perigee of the eccentric: greatest equation	Greatest equation of the orb, at the perigee of the eccentric: anomaly of the orb
Coper-nicus	2° 0′ 0″	90° 0′	45° 10′ 0″	134° 50′ 0″	47° 44′ 0″	132° 16′ 0″
Prutenic Ta-bles	2° 0′ 17″	91° 0′	45° 10′ 19″	134° 49′ 41″	47° 35′ 52″	132° 24′ 8″
Mag-ini	2° 0′ 17″	91° 0′	45° 10′ 20″	134° 49′ 40″	46° 51′ 29″	133° 8′ 31″
Lans-ber-gen	2° 0′ 0″	90° 0′	45° 10′ 0″	134° 50′ 0″	47° 36′ 0″	132° 24′ 0″
Lon-gomon-tanus	1° 50′ 16″	61° 0′	45° 19′ 20″	134° 40′ 40″	49° 4′ 31″	130° 55′ 29″
Ke-pler	0° 47′ 36″	91° 0′	45° 48′ 20″	134° 11′ 40″	47° 48′ 34″	132° 11′ 26″
Boul-liau	0° 54′ 36″	91° 0′	45° 48′ 9″	134° 11′ 51″	47° 59′ 9″	132° 0′ 51″
Our-selves	1° 59′ 0″	91° 0′	44° 59′ 4″	135° 0′ 56″	47° 2′ 6″	132° 57′ 54″

☿	Greatest equation of the centre: greatest equation	Greatest equation of the centre: anomaly of the eccentric	Greatest equation of the orb, at the apogee of the eccentric: greatest equation	Greatest equation of the orb, at the apogee of the eccentric: anomaly of the orb	Greatest equation of the orb, at the perigee of the eccentric: greatest equation	Greatest equation of the orb, at the perigee of the eccentric: anomaly of the orb
Ptolemy	2° 52′ 0″	93° 0′	21° 56′ 0″	158° 4′ 0″	23° 57′ 0″	156° 3′ 0″
Al-fonso	3° 2′ 0″	96° 0′	22° 2′ 0″	157° 58′ 0″	24° 3′ 0″	155° 57′ 0″
Coper-nicus	3° 1′ 0″	96° 0′	19° 3′ 0″	160° 57′ 0″	24° 12′ 0″	155° 48′ 0″
Prutenic Ta-bles	3° 0′ 28″	96° 0′	19° 3′ 6″	160° 56′ 54″	24° 12′ 47″	155° 47′ 13″
Mag-ini	3° 0′ 28″	96° 0′	19° 3′ 6″	109° 0′ 0″	23° 14′ 17″	113° 0′ 0″
Lans-ber-gen	3° 0′ 0″	96° 0′	19° 3′ 0″	160° 57′ 0″	24° 5′ 0″	155° 55′ 0″
Lon-gomon-tanus	3° 34′ 0″	108° 0′	21° 7′ 33″	158° 52′ 27″	24° 0′ 33″	155° 59′ 27″
Ke-pler	24° 11′ 17″	105° 0′	18° 9′ 40″	161° 50′ 20″	28° 30′ 34″	151° 29′ 26″

☿	Greatest equation of the centre: greatest equation	Greatest equation of the centre: anomaly of the eccentric	Greatest equation of the orb, at the apogee of the eccentric: greatest equation	Greatest equation of the orb, at the apogee of the eccentric: anomaly of the orb	Greatest equation of the orb, at the perigee of the eccentric: greatest equation	Greatest equation of the orb, at the perigee of the eccentric: anomaly of the orb
Boul-liau	24° 17′ 20″	105° 0′	18° 4′ 45″	161° 55′ 15″	28° 23′ 2″	151° 36′ 58″
Our-selves	4° 34′ 34″	110° 0′	20° 29′ 16″	159° 30′ 44″	24° 11′ 41″	155° 49′ 19″

For us, however, for whom Venus retains and Mercury acquires the greatest semidiameter of the epicycle at the epicycle's mean longitude, the greatest equation of the orb in that position is also, for Venus, 47° 0′ 6″, but for Mercury 27° 3′ 24″.

CAPVT VIII.

Chapter VIII.

De Artificio Inueſtigandi Apogeum & Motum Apogei ac Radicem motus Apogei, Veneris ac Mercurij.

On the technique for investigating the apogee, the motion of the apogee and the root of the motion of the apogee, of Venus and Mercury.

I. Ptolemy (Book 10, ch. 6) and Copernicus (Book 5, ch. 20) judged that, for distinguishing each inequality of the minor planets and the commensuration of the orbs, the method is easier in Venus and Mercury, because of the fixed limits of their greatest digressions from the Sun on either side; and for that reason he dealt with these before the three superior planets. But the opposite seems truer, because the superior planets can be observed in their acronychal oppositions with the Sun, when they are wholly stripped of the second inequality. Venus and Mercury, by contrast, lack the second inequality only in their conjunctions with the Sun, of both kinds, and outside these moments they are always involved in both inequalities, of motion in the eccentric and in the epicycle. In their superior conjunctions, moreover, they hide behind the Sun, and in their inferior conjunctions they are for the most part overwhelmed by the Sun's rays, so that they cannot be seen. In or under the very disc of the Sun they cannot be seen as blackish spots without a telescope, nor except very rarely, and when their apparent latitude is less than the Sun's apparent semidiameter; and it has not so far been established that such a conjunction of Venus was observed, nor will one be easily observable for many centuries. Mercury, however, we know to have been seen once under the Sun by Pierre Gassendi, in accordance with what was said on the spots of the Sun, Book 3, ch. 3, Scholium 3. Venus too very rarely attains so great a latitude in conjunction with the Sun at perigee that it does not fear the Sun's rays and can be observed, though we have shown

that this did happen, ch. 7, Scholium 1. Therefore an aid, necessary indeed but not as certain as in the superior planets, had to be sought from the greatest elongations from the Sun; and how the places of the apogees or aphelia were investigated by means of them must be briefly indicated.

[Margin: Whether the two inequalities are more easily distinguished in ♀ and ☿ than in the three superior planets?]

Problema 1. De Loco & Motu Apogei, & Aphelij Veneris ac Mercurij inueſtigandis, more Ptolemaico.

Problem 1. On investigating the place and motion of the apogee and aphelion of Venus and Mercury, in the manner of Ptolemy.

II. This technique is taught by Ptolemy, Book 9 of the *Magna Constructio*, Book 9 [sic], ch. 7, and Book 10, ch. 1; and there by Regiomontanus in the *Epitome*, Book 9, propositions 11 and 14, and Book 10, propositions 1 and 2; by Copernicus, Book 5, ch. 20 and 26; by Longomontanus, Book 2 of the *Theorica*, ch. 18; by Kepler in the *Epitome of Copernican Astronomy*, Book 6, p. 761; by Hérigone, vol. 5 of the *Cursus Mathematicus*, p. 661, or Book 3 of the *Theoricae*, proposition 31; and by Boulliau, Books 9 and 10 of his *Astronomy*.

[Margin: Authors of this method.]

First, then, you must observe, or select from the observations of others, two greatest digressions of these planets, of equal quantity, one morning, the other evening, either from the Sun's mean place, according to the old astronomy, to which Longomontanus and Lansbergen adhere in this, or from its true and apparent place, according to other more recent authors. We have indicated the signs of the greatest digressions in ch. 7, no. 1; and in those signs their longitude in the zodiac is supposed to be observed, or to have been observed, and likewise the Sun's mean or true place in the zodiac to be known: for from these it is verified whether they stood equally distant from the Sun's place, on this side in the morning digression and on that side in the evening one.

[Margin: Step 1, for the place of the line of apsides.]

Now the greatest digressions of these planets occur at the points of the orb or epicycle to which a straight line drawn from the centre of vision touches the orb and makes a right angle with the semidiameter of the epicycle or orb. So if the angles at the centre of vision, which measure the greatest digressions, are equal, and the semidiameters of the epicycle are equal, the centres of the epicycles will also stand equally distant from the line of apsides and equally distant from the centre of vision, just as the planets too will stand equally distant from that same centre, and the equations of the orb will be equal on both sides. It will therefore be certain that the line of apsides passes through the centre of vision and through the two points of the zodiac at which the arc of the zodiac intercepted between the two places of the Sun, mean or true, where the Sun stood at the time of the already mentioned digressions, is bisected; and that at one of those points of the zodiac is the apogee and at the other the perigee of

the eccentric. The conditions omitted by the ancients, however, which this method, such as it is, needs in order to hold good, we shall add at no. 10 and in the Scholia.

Second, two other greatest digressions of the same planet, equal to each other but unequal to the former, are to be observed or selected, with the places of the planet and of the Sun as above. For if they are individually smaller than the former digressions, they will show that the apogee lies in that arc of the zodiac in which the Sun was at their time; if they are greater, they will show the perigee. The same will be done by two greatest digressions in one of which the Sun's mean or true place stood at one of the already mentioned points through which the line of apsides passes, and in the other at the other; for the greater angle of digression marks out the perigee, and the smaller the apogee.

[Margin: Step 2, for distinguishing the place of the apogee from the place of the perigee.]

Third, then, let the two places of the apogee observed as above be compared, one from antiquity, the other recent, and let the number of degrees, resolved into minutes, be divided by the number of years elapsed from the ancient to the recent observation; for the quotient will show the annual motion of the apogee. But light must be added to these precepts by an example and a figure.

[Margin: Step 3, for the motion of the apogee.]

EXEMPLVM.

Example.

Ex Ptolemæo, & Copernico.

From Ptolemy and Copernicus.

Diagram. Woodcut figure in the left column, standing after the third line of no. III. A large outer circle, the zodiac, centred on A, with G at its top, π at its foot, β at far left and γ at far right. Within it a second circle, the eccentric, on which the centres of six small equal circles (epicycles) stand at C (top), F (upper left), Q (upper right), O (left), X (right) and D (bottom). A vertical line runs from G down through C, B, I and A to D and π. A small dashed circle stands about I, with B above it and a letter below it. Straight lines run from A to the epicycle centres and on to the outer circle, and from A to tangent points on the epicycles.

- G (top of outer circle): zodiac point, the apogee in the text
- π (foot of outer circle): zodiac point, the perigee in the text
- β (far left, outer circle): Sun's mean place
- γ (far right, outer circle): Sun's mean place
- H (upper left, outer circle): Sun's mean place
- S (upper right, outer circle): Sun's mean place
- C (top epicycle, centre); E (at the left end of its horizontal radius)

- F (upper-left epicycle, centre); M, V, N (about its circumference); θ (tangent point)
- Q (upper-right epicycle, centre); δ (tangent point); T, R (about its circumference)
- O (left epicycle, centre); P (tangent point)
- X (right epicycle, centre); Y (tangent point)
- D (bottom epicycle, centre); L (above it, on the vertical line); K (to its left)
- B (top of the dashed circle); I (centre of the dashed circle and of the eccentric); Z (below I, crossed by lines)
- A (centre of the zodiac, where the lines converge)

[Margin: Ptolemy, Book 10, ch. 1; Copernicus, Book 5, ch. 20.]

III. In the attached figure, A is the centre of the zodiac Gβπγ, and I the centre of the eccentric of ♀ or ☿, which is CODX. Theon the mathematician, then, in the 16th year of Hadrian, that is, in the year of Christ 132, on 21 Pharmuthi, namely March 8 [octauo Idus Martij], at the first hour of the following night, observed the evening greatest digression of Venus, which stood at δ and preceded the middle of the Pleiades by as great a longitude as is the longitude of the whole asterism of the Pleiades, and so, according to Ptolemy's table, stood at ♉ 1° 30′, when the Sun's mean place, Q, was at ♓ 14° 15′. The greatest digression of Venus from the Sun's mean place, that is, the angle QAδ, was therefore 47° 15′, as Ptolemy relates. He too, in the 4th year of Antoninus, that is, in the year of Christ 142, on 11 Thoth, as that day was passing and the 12th dawning, namely on the morning of July 30 [3. Kalend. Augusti], observed the greatest digression of Venus, but a morning one, when it stood at θ and was distant from the middle knee of Gemini toward the east by as much as half the Moon; that is, according to Ptolemy, it occupied ♊ 18° 30′, when the Sun's mean place, F, was at ♌ 5° 45′. The greatest digression from the Sun's mean place was therefore at that time also 47° 15′, and so the angle FAθ was equal to the angle QAδ. Now, with the semidiameters Qδ and Fθ taken as equal, and the angles at the contacts δ and θ equal, being right angles, the triangles QAδ and FAθ turn out not only similar but also equal, and the distances AQ and AF equal to each other, just as Aδ and Aθ are equal to each other. From this Ptolemy, in accordance with his own teaching in Book 9, ch. 6, concludes that the line of apsides GAπ passes through the points G and π, at which the arcs HGS and HπS, intercepted between the two mean places of the Sun in the zodiac marked by H and S, are bisected. From S, that is, from ♓ 14° 15′, in the order of the signs, as far as H, ♌ 5° 45′, there are 141° 30′; half of these, the arc SG, 70° 45′, added to S itself, shows the point G at ♉ 25°, and its opposite, π, at ♏ 25°. He therefore concluded that at that period the apogee of Venus lay at one or the other of these points of the zodiac.

[Margin: Per Book 1, proposition 26 of Euclid.]

Second, Ptolemy selected two greatest digressions of Venus. One was a morning digression observed by Theon in the 4th year of Hadrian, at dawn on

the 20th day of the month of Athyr, that is, in the year of Christ 119, on the morning of October 12 [quarto Idus Octobris], when Venus, P, stood at 20′ of the first degree of ♍, and the Sun's mean place at ♎ 17° 52′. The other was an evening digression observed by Ptolemy himself in the 21st year of Hadrian, or the year of Christ 136, on 9 Mechir, which was December 25 [8. Kal. Ianuarij], when Venus, Y, stood at ♒ 19° 36′, and the Sun's mean place, X, at ♑ 2° 4′. Each digression from the Sun's mean place was therefore 47° 32′, and the angle OAP equal to the angle XAY. From this is gathered the arc βGγ, between the two mean places of the Sun in the zodiac, β and γ, of 285° 48′; its half, Gγ, of 142° 54′, added to the Sun's place γ, ♑ 2° 4′, shows the point G at ♉ 24° 58′, that is, very nearly at 25°, as above. But since the angles OAP and XAY were greater than the angles FAθ and QAδ, and the angles at the contacts P, θ, δ, Y are right and equal, and the semidiameters of the epicycles are always assumed equal, it follows that the angle POA was smaller than the angle θFA, just as the angle YXA was smaller than the angle δQA; and so the distances AO and AX of the epicycle's centre were smaller than the distances AF and AQ. Therefore in the two former digressions the centre of the epicycle was nearer the greatest longitude, and the point G was the apogee of Venus, and the point π the perigee. From this, then, he finally inferred that in his time the apogee of Venus was at ♉ 25° and the perigee at ♏ 25°, which he confirmed in Book 10, ch. 2, from two digressions, in one of which the Sun's mean place was now at about ♉ 25° and in the other at about ♏ 25°; for he found the latter greater than the former. By very similar observations of the greatest digressions of Mercury, made by himself, Ptolemy concluded in Book 9, ch. 7, that the apogee of Mercury in his time was at ♎ 10° and the perigee at ♈ 10°; and in ch. 8 he confirmed the same from one greatest digression of Mercury with the Sun's mean place at ♎ 9° 15′, which was the smaller, and from another, with the Sun at ♈ 11° 5′, which was the greater.

[Margin: Example of step 2.]

[Margin: The apogee of ♀ and ☿ in Ptolemy's time.]

Third, Ptolemy, examining in Book 9, ch. 7, equal greatest digressions of Mercury on either side, observed by the ancients 400 years before the time of his own observations and reported by Hipparchus, found that the apogee of Mercury was formerly at ♎ 6°. Because of this he determined that in 400 years it had traversed 4°, namely from ♎ 6° to ♎ 10°, and so advances 1° in 100 years. In the same chapter he affirms that he detected the same motion in the motions of the other minor planets, saying: "From these, then, and from those that we have considered individually in the other planets, we find similarly in the five planets that the diameters which pass through the greatest and least longitudes move about the centre of the zodiac in the order of the signs, and that this motion is equal in speed to the motion of the sphere of the fixed stars. For, as we have demonstrated, that sphere moves very nearly one degree in a hundred years."

[Margin: Example of step 3, for the motion of the apogee.]

IV. Further, the greatest digressions that Ptolemy used to work out both the place and motion of the apogee and other matters concerning the theory of Mercury and Venus are set out below, keeping the order that Ptolemy keeps, for he begins with Mercury. Of these, Ptolemy observed the first four and the last four except the penultimate; Theon the penultimate; Timocharis the fifth and perhaps the 6th, 7th and 8th. Of the observations of Venus, Theon made the first, the second and the fifth; Ptolemy the rest, and he uses the first four of Mercury and the first four of Venus in investigating the apogee.

How Longomontanus corrected some observations of Venus will be clear from no. 7 and from the 3rd table in no. 8; how Boulliau corrected some of Venus and of Mercury I shall say in no. 9; and how, finally, we corrected the observations of each planet I shall say in no. 10. I wish this said here so that no one may think himself compelled to abide by these digressions of Ptolemy, which we set out below in each table, as if they were worthy of unshaken belief and needed no correction whatever.

DIGRESSIONES Mercurij ac Veneris Maximæ à loco Solis Medio Selectæ ex Ptolemæo & ad Vraniburgi seu Bononiæ Meridianum reductæ.

The greatest digressions of Mercury and Venus from the Sun's mean place, selected from Ptolemy and reduced to the meridian of Uraniborg or of Bologna.

Order	Current years: of the emperors	Current years: of Christ	Months with completed days: Egyptian, day	Months with completed days: Julian, day		☿ place observed	☉ mean place, Ptolemy	Greatest digression of ☿ from the mean ☉
1	Hadrian 16	132	Phamenoth 16	Feb. 2	Evening	♓ 1° 0′	♒ 9° 45′	21° 15′
2	Hadrian 18	135	Epiphi 18	June 4	Morning	♉ 18° 45′	♊ 10° 0′	21° 15′
3	Antoninus 1	138	Epiphi 20	May 31	Evening	♋ 7° 0′	♊ 10° 30′	26° 30′
4	Antoninus 4	141	Phamenoth 18	Feb. 1	Morning	♑ 13° 30′	♒ 10° 0′	26° 30′
	Nabonassar	before Christ						
5	486	262	Choeac 17	Feb. 11	Morning	♑ 22° 20′	♒ 18° 10′	25° 50′
6	486	262	Phamenoth 30	May 25	Evening	♓ 23° 40′	♈ 29° 30′	24° 10′
7	486	262	Payni 30	Aug. 23	Evening	♍ 19° 30′	♌ 27° 50′	21° 40′
8	491	257	Pharmuth 5	May 29	Evening	♊ 29° 20′	♊ 2° 50′	26° 30′
9	512	237	Thoth 9	Oct. 29	Morning	♎ 14° 10′	♏ 5° 10′	21° 1′
10	564	185	Thoth 27	Nov. 3	Morning	♏ 2° 20′	♏ 24° 50′	22° 30′
	Emperor	of Christ						
11	Hadrian 19	135	Athyr 14	Dec. 9	Morning	♍ 20° 12′	♎ 9° 15′	19° 3′

Order	Current years: of the emperors	Current years: of Christ	Months with completed days: Egyptian, day	Months with completed days: Julian, day		☿ place observed	☉ mean place, Ptolemy	Greatest digression of ☿ from the mean ☉
12	Hadrian 19	136	Pachon 19	May 21	Evening	♉ 4° 20′	♉ 27° 35′	23° 15′
13	Hadrian 14	130	Mesori 14	July 4	Evening	♌ 6° 20′	♋ 10° 5′	26° 15′
14	Antoninus 1	138	Epiphi 20	May 31	Evening	♋ 7° 0′	♊ 10° 30′	26° 30′

[Margin: Ptolemy, Book 9, ch. 7 (against rows 1–4).]
[Margin: Ptolemy, Book 9, ch. 7 (against rows 5–10).]
[Margin: Ptolemy, Book 9, ch. 8 (against rows 11–12).]
[Margin: Book 9, ch. 9 (against rows 13–14).]

Order	Current years: emperor	Current years: of Christ	Months with completed days: Egyptian, day	Months with completed days: Julian, day		♀ place observed	☉ mean place, Ptolemy	Greatest digression of ♀ from the mean ☉
1	Hadrian 12	127	Athyr 21	Oct. 11	Morning	♍ 0° 20′	♎ 17° 52′	47° 32′
2	Hadrian 16	132	Pharmuth 21	Mar. 8	Evening	♉ 1° 30′	♓ 14° 15′	47° 15′
3	Hadrian 21	136	Mechir 9	Dec. 25	Evening	♒ 19° 36′	♑ 2° 4′	47° 32′
4	Antoninus 4	140	Thoth 12	July 29	Morning	♊ 18° 30′	♌ 5° 45′	47° 15′
5	Hadrian 13	129	Epiphi 2	May 19	Morning	♈ 10° 36′	♉ 25° 24′	44° 48′
6	Hadrian 18	134	Pharmuth 2	Feb. 17	Morning	♑ 11° 55′	♒ 25° 30′	43° 35′
7	Antoninus 4	140	Pharmuth 4	Feb. 17	Evening	♓ 13° 50′	♒ 25° 30′	48° 20′

[Margin: Ptolemy, Book 10, ch. 1 (against rows 1–2).]
[Margin: Book 10, ch. 2 (against rows 4–5).]
[Margin: Book 10, ch. 3 (against row 6).]

V. Regiomontanus subscribed to Ptolemy, *Epitome*, Book 9, propositions 12 and 13, on the basis of the observations made or reported by Ptolemy; so much so that he too concluded in these words: "Therefore, by these and similar indications in the other wandering stars, we are compelled to judge that the longer and nearer longitudes have a connection with the motion of the fixed stars."

[Margin: The opinion of Regiomontanus on the motion of the apogees.]

More universally, al-Farghānī asserted the same of all the planets, difference 17, where he did not hesitate to say: "The revolution of the fixed stars and of the auges of the seven planets, and of their gensahar, traverses the circle of the signs in thirty-six thousand years."

[Margin: And of al-Farghānī.]

Since al-Battānī judged that the fixed stars move 1° in 66 years, he therefore says in ch. 47 that the apogees of the five minor planets also advance by the same amount of motion; and from this, rather than from any observation of his own, he placed in his time the apogee of Venus, as also of the Sun, at ♊ 25° 14′, and that of Mercury at ♎ 21° 58′.

[Margin: The place and motion of the apogee of ♀ and ☿ in al-Battānī's time.]

From this, no doubt, very many Arabs and the Alphonsines thought that the apogee of Venus and that of the Sun are perpetually joined at the same place in the zodiac, and that this moves, together with the auges of the other minor planets, with the motion of the eighth sphere, and completes its period only in 49,000 years. Their error at last became clear, when the apogee of Venus was separated from the apogee of the Sun, and the motion of the latter was detected to be far swifter than that of the apogee of Venus, as we shall presently show. Granted, almost all the followers of Peuerbach subscribed to the Alphonsines in Peuerbach's *Theoricae*; especially Wurstisen, p. 189, and before him Fernel, *Cosmotheoria*, Book 2, ch. 7, and afterwards Clavius, at the end of the *Sphere*, p. 471, assigning the same motion to the auges of these planets as to the fixed stars, and a period of 49,000 years.

[Margin: The opinion of the Alphonsines on these apogees.]

[Margin: The opinion of Peuerbach, Fernel and Clavius.]

VI. The method by which Copernicus investigates, for the times of Ptolemy, the apogee of Venus, Book 5, ch. 20, and of Mercury, Book 5, ch. 26, is exactly the same as Ptolemy's; and from their greatest digressions from the Sun's mean place, observed by Theon and Ptolemy and with nothing corrected, he determines that the apogee of Venus was at ♉ 25°, and gathers that of Mercury to have been at ♎ 10°. But under the sphere of the fixed stars, that is, reckoning the place of the apogee from the first star of Aries, he says that that of Venus was at 48⅓° and that of Mercury at 183⅓°: namely with the precession of the equinoxes subtracted, that is, the recession of the equinoctial points, against the order of the signs, from the first star of Aries, which in Ptolemy's time stood 6° 20′ from the vernal intersection, as Copernicus has it.

[Margin: The opinion of Copernicus on the ancient place of the apogee of ☿ and ♀.]

As for the motion of these apogees and their place in the time of Copernicus, it is gathered from his Book 5, ch. 23, that the apogee of Venus perpetually keeps the same distance, 48⅓°, from the first star of Aries, and so recedes no further from the vernal equinox than the fixed stars themselves recede or seem to recede. For there he adduces two observations of Venus, one of Timocharis, the other his own, made in the year 1529, and examines both by his hypothesis, in the form we gave in ch. 2, no. 4, with the apogee of Venus remaining at 48° 20′ from the first star of Aries; and he reckons it from the equinox for his own time in no other way than by adding to the 48° 20′ the precession of

the equinoxes, of 27° 24′, from which for the year 1529 the apogee of Venus, reckoned from the vernal equinox, comes out at Gemini 15° 44′. Rightly, therefore, do Longomontanus, Book 2 of the *Theorica*, ch. 19, and Magini, in the theory of Venus, ch. 1, and canon 33 of the *Secunda Mobilia*, gather that on Copernicus's hypothesis the apogee of Venus is immobile under the sphere of the fixed stars, and moves away from the equinoxes by no other motion than that by which the fixed stars seem to move away from them, and therefore completes its period just as the fixed stars do.

[Margin: In what way the apogee of ♀ is immobile in the hypothesis of Copernicus.]

As for the motion of the apogee of Mercury, Copernicus, Book 5, ch. 30, brings forward three observations of Mercury, one of the year 1491, made by Bernhard Walther, two of the year 1504, made by Johannes Schöner; and he examines them by the hypothesis and figure that we gave in ch. 2, no. 9, of this section, assuming a more probable place for the apogee, so as to be more certain whether, with that assumed and the rest remaining as they were, namely the commensuration of the orbs, which was in Ptolemy's

[Margin: By what motion the apogee of ☿ moves, in the opinion of Copernicus.]

time, as he had found from Ptolemy's own observations, the observed places of the planets are represented. And so he says: "From these three observations, then, we wish to examine the course of Mercury for the present time. In them we think it must be granted that the commensurations of the circles have remained from Ptolemy even until now, since in the other planets too the good earlier authors are not found to have been deceived in this respect. If along with these we also have the place of the apse of the eccentric, nothing further would be lacking in the apparent motion of this star as well. Now we assumed the place of the highest apse at 211½° [part. CCXI. S.], that is, at 28½° of Scorpio [XXVIII. S. grad. Scorpij]; for we could not accept a smaller one without prejudice to what was observed." When the examination is finished he concludes: "These three places, therefore, agreeing as they do with what was observed, attest beyond doubt that the very place of the highest apse of the eccentric is the one we were assuming, 211½° [part. CCXI. S.] under the sphere of the fixed stars at this our time."

With this laid down, in order to gather the motion of this apogee, he recalls from Ptolemy's doctrines that it stood, in the 21st year of Ptolemy Philadelphus, at 182° 20′ under the sphere of the fixed stars; and from that time to the year 1504 he counts 1768 Egyptian years and 233 days, in which time, he says, the highest apse of Mercury moved 28° 10′ under the sphere of the non-wandering stars. And he closes the chapter with these words: "Now when we compare 28° 10′ [partes XXVIII. Scrup. 10.], through which the apogee of the eccentric has moved, with this time, it will be seen to have moved by one degree in 63 years [LXIII. annis], if indeed it has been uniform."

But a manifold blunder of Copernicus lies hidden here. For with 183° 20′, at which this apogee stood under the fixed stars in the time of Ptolemy the astronomer, subtracted from 211° 30′, those 28° 10′ result; but when the subtraction is made of the 182° 20′ at which he says it stood in the time of Ptolemy Philadelphus, 29° 10′ result. So the occasion of the slip was the ambiguous name of the Ptolemies.

[Margin: Copernicus's blunders and his manifold inconsistency with himself.]

Next, if 1° requires 63 years, then certainly this motion, if indeed it was uniform, as he himself supposes, requires 1774 years for 28° 10′, but 1837 for 29° 10′.

Third, if from the 21st year of Philadelphus to A.D. 1504 there are 1768 Egyptian years and 234 days, that is, 1767 Julian years and 155 days, then the year of Philadelphus fell 264 years before Christ. If you add to these the 136 years after Christ at which Ptolemy the astronomer flourished, that makes 400 years, the very number Ptolemy himself counts. Comparing, then, 182° 20′ and 183° 20′ under the sphere of the fixed stars, it follows that in 400 years the apogee of Mercury completed a single degree under that sphere. How do these things agree with one another?

Fourth, if in Ptolemy's time this apogee was at ♎ 10° and in Copernicus's time at ♏ 28° 30′, then in 1368 years, the number there are from the year 136 to 1504, it completed 48° 30′ from the vernal equinox. From these subtract the difference in the precession of the equinoxes, or in the motion of the fixed stars, due to this interval, namely 21° 4′ (since that precession was 6° 20′ under Ptolemy the astronomer and 27° 24′ in Copernicus's time), and 27° 26′ will remain, which is how far this apogee ought to have moved under the fixed stars. Mästlin saw this inconsistency in Copernicus, but rather confusedly, and therefore reports it differently, just as though Copernicus had set this apogee as immobile from Ptolemy Philadelphus to Ptolemy the astronomer, as I shall say in Scholium 2.

Meanwhile, what is more certain and more worth noting is this: if, from the equinox, or under the zodiac, it completed 48° 30′ from Ptolemy to Copernicus, in 1368 or 1370 years, then it completes one degree in about 28½ years, and in A.D. 1630 it would be at the 2nd degree of Sagittarius, and would complete its period in 10,260 Julian years.

[Margin: The motion of the apogee of ☿ under the zodiac, from the data given by Copernicus.]

Granted, Hérigone, in volume 5 of the *Cursus Mathematicus*, Book 1 of the *Theorica*, proposition 11, says that according to Copernicus's observations this apogee is today at the beginning of Sagittarius, and completes its period under the sphere of the fixed stars in 22,405 Egyptian years and under the zodiac in 11,995 years.

[Margin: The opinion of Hérigone on the Copernican motion of the apogee of ☿.]

Magini, however, says in the theory of Mercury, ch. 2, that the mean apogee completes its period in 22,700 Egyptian years; but in ch. 5 he denies that the true apogee of Mercury can traverse the whole zodiac, since it is in fact confined within fixed limits, because he fuses the Ptolemaic and Alphonsine hypotheses into one, according to the scheme set out in ch. 1, no. 6.

[Margin: The opinion of Magini on this motion.]

Finally, Lansbergen, although a most devoted follower of Copernicus, nevertheless sets the annual motion of the apogee of ☿ at 1′ 55″ and of ♀ at 1′ 26″, and for A.D. 1500 the root of the apogee of ☿ at ♏ 25° 15′ 22″ and of ♀ at ♊ 28° 28′ 29″.

[Margin: The opinion of Lansbergen on the place and motion of the apogee of ♀ and ☿.]

VII. I come to Longomontanus, who in Book 2 of the *Theorica*, ch. 18, excuses the Tychonians on the ground that at other seasons rarely, and in summer scarcely ever, because of the obliquity of the sphere at that altitude of the pole and the perpetual twilights, were they able to continue their observations of Venus and Mercury as they should have done, in order to distinguish, from the immediate increases and decreases of the digressions, the greatest recessions from the Sun, which occur at the contact with the orb, and from them to gather, by the Ptolemaic and Copernican method, the place of the apogee, the greatest equations, and the commensurations of the orbs. He therefore uses another method, a posteriori, and brings forward 9 observations of Venus, seven of Tycho's and two made by himself, which I shall present in the following no. 8.

[Margin: The method of Longomontanus for ♀.]

These observations he examines by geometrical calculation, using both the reformed Ptolemaic scheme and the Copernican, which I have already set out in ch. 2, no. 4, and ch. 4, no. 1. In that examination he takes it as certain that Venus consistently looks to the mean rather than the true motion of the Sun; and applying the Alphonsine and Prutenic tables, he weighs which of them come closer to the observations. From this examination, at first in rough fashion, then with the motions refined and the calculation repeated many times, until the difference of the computed place from the observed stayed within 10′, he gathered the following propositions.

First: from the observations made in December, in which the equality of the digressions on either side of the Sun's mean place shows that they were not far from the perigee of the eccentric, that the apogee of Venus has at this time advanced beyond the place the Prutenic tables fix, and comes closer to the Alphonsine place; and that at the completed year of our Saviour 1585 it stood distant in longitude 89° 15′ from the true vernal equinox, but 89° 8′ from the mean.

[Margin: The apogee in the year 1585.]

Second: from the first and last observations, and then from others, that the semidiameter of the orb of Venus, which for Ptolemy is the epicycle, is greater

than Copernicus set it, by 300 parts, of which the radius of the eccentric orb is 100,000; that is, not 71,930 but 72,248½.

[Margin: The semidiameter of the orb and the eccentricity of ♀.]

Third: that the total eccentricity of Venus is not greater than 3208 parts, and accordingly that in the Tychonic scheme the third part, which belongs to the semidiameter of the lesser epicycle, is 802, and the remaining two-thirds, of the semidiameter of the greater epicycle, are 2406 parts. That this eccentricity is smaller than the Ptolemaic is to be attributed not so much to a change in the eccentricity as to the unreliable and conjectural observations of Theon and Ptolemy.

Fourth: from observations 3 and 4, in which Venus, very close to conjunction with the Sun and almost at the perigee of the epicycle, could be observed by Tycho in Denmark, thanks both to the obliquity of the sphere and to a northern latitude of Venus of 8′ and more [octo & ampliùs minutorum], greater than the tables of Alfonso and of Copernicus allow, because of which it was possible to see Venus in the evening on February 21 and soon afterwards in the morning on the 24th: from these, I say, he gathers that the mean anomaly of the orb, or anomaly of commutation, is greater than the Prutenic by 1° 18′. He confirms this from observation 6, in which Venus was nearer the apogee of the epicycle.

[Margin: The anomaly of the orb of ♀.]

With these four foundations of his hypothesis laid, he compares the observed places of Venus with the places derived by calculation, and shows that they never disagree with one another by more than 12′, as can be seen in table 1 of those that follow, in which the days are given in the style of the old calendar, and the hours as hours after noon at Uraniborg [Vranigurbi].

VIII. Having investigated these matters for his own time, he proceeds to ch. 19, in which he investigates the periodic motions and the roots of the motions of Venus from recent observations, which we shall append in table 1, compared with ancient ones, which Ptolemy thought were made at the greatest digression of Venus from the Sun's mean place. But Longomontanus himself tries to show that they were not the greatest, correcting the places of Venus from his own hypothesis about the places of the Sun and the fixed stars suited to Ptolemy's time; for Ptolemy, Book 10, ch. 1 and 2, recorded both Theon's observations and his own from the distance of Venus from the fixed stars taken by astrolabe. For this correction see the third of the tables that follow, in which the hours are given for the meridian of Alexandria, and the anomaly of the epicycle, or orb, of Venus is added, taken from the Prutenic tables and corrected by a subtraction of 45′, as that period requires, and also the correction of this anomaly, discussed in no. 7 under the Fourth proposition. The Sun's mean place, moreover, is set in the second table from the Ptolemaic tables, and in the third from the Danish tables.

Tab.I. OBSERVATIONES VENERIS Comparatæ cum loco eiusdem ex Longomontani hypothesi supputato.

Table I. Observations of Venus compared with its place computed from the hypothesis of Longomontanus.

Order	In completed years of Christ	Observed longitude	Observed latitude	Computed longitude	Mean longitude of ♀ from the true equinox	Anomaly of the eccentric	Anomaly of the epicycle
1	1585 Sept. 14, $17^h\ 15^m$	♌ 15° 58′	2° 8′ A	♌ 15° 53′	6^s 3° 48′ 20″	3^s 4° 33′ 30″	7^s 7° 10′ 0″
2	1587 Jan. 15, $4^h\ 40^m$	♓ 16° 55′	2° 39′ B	♓ 16° 50′	10^s 4° 17′ 15″	7^s 5° 0′ 45″	5^s 7° 45′ 17″
3	1587 Feb. 21, $6^h\ 5^m$	♓ 16° 1′	8° 56′ B	♓ 15° 49′	11^s 10° 48′ 48″	8^s 11° 32′ 15″	6^s 0° 33′ 7″
4	1587 March 2, $17^h\ 30^m$	♓ 10° 7′	8° 26′ B	♓ 10° 14′	11^s 20° 9′ 10″	8^s 20° 52′ 30″	6^s 6° 23′ 38″
5	1588 Dec. 14, $19^h\ 40^m$	♏ 17° 10′	3° 10′ B	♏ 17° 6′	9^s 3° 52′ 14″	6^s 4° 33′ 15″	7^s 19° 2′ 10″
6	1590 Dec. 17, $20^h\ 0^m$	♐ 20° 0′	0° 27′ B	♐ 20° 4′	9^s 6° 21′ 15″	6^s 6° 59′ 25″	10^s 20° 57′ 9″
7	1594 Dec. 15, $5^h\ 10^m$	♒ 21° 0′	1° 16′ A	♒ 21° 6′	9^s 3° 48′ 5″	6^s 4° 21′ 45″	4^s 20° 4′ 20″
8	1610 Dec. 12, $4^h\ 40^m$	♒ 17° 58′	1° 29′ A	♒ 18° 4′	9^s 0° 57′ 48″	6^s 1° 9′ 30″	4^s 21° 8′ 38″
9	1616 March 9, $17^h\ 0^m$	♒ 15° 24′	not observed	♒ 15° 24′	11^s 28° 0′ 53″	8^s 28° 6′ 15″	8^s 1° 28′ 40″

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Tab.II. OBSERVATIONES VENERIS Digreſſionum à medio Solis loco, factę Alexandriæ, quas Ptolemæus maximas putauit.

Table II. Observations of the digressions of Venus from the Sun's mean place, made at Alexandria, which Ptolemy thought were the greatest.

Order	Observers	Years of the emperors	Egyptian months	Completed days	Mean place of ☉	Observed place of ♀	Digressions of ♀ from the mean ☉
1	Theon	Hadrian 16	Pharmuthi	21, evening	♓ 14° 15′	♉ 1° 30′	47° 15′
2	Ptolemy	Antoninus 4	Thoth	11, the following morning	♌ 5° 45′	♊ 18° 30′	47° 15′
3	Theon	Hadrian 12	Athyr	21, the following morning	♎ 17° 52′	♍ 0° 20′	47° 32′
4	Ptolemy	Hadrian 21	Mechyr	9, evening	♑ 2° 4′	♒ 19° 36′	47° 32′
5	Theon	Hadrian 13	Epiphi	2, the following morning	♉ 25° 24′	♈ 10° 36′	44° 48′
6	Ptolemy	Hadrian 21	Tybi	2, evening	♏ 25° 30′	♑ 12° 50′	47° 20′

[63] A is austral, south; B is boreal, north.

[Margin: From Ptolemy, Book 10, ch. 1 (against rows 1–2).]
[Margin: Book 10, ch. 2 (against rows 4–5).]

Tab.III. OBSERVATIONES Prædictæ à Longomontano Correctæ.

Table III. The observations already mentioned, as corrected by Longomontanus.

Order	Current years of Christ	Completed Julian months and days	After noon at Alexandria	Mean place of ☉	Corrected place of ♀	Digression from the mean ☉	Anomaly of the orb of ♀
1	132	March 8	$7^h\ 15^m$	♓ 14° 49′	♉ 1° 39′	46° 50′	4^s 26° 22′
2	140	July 29	$16^h\ 12^m$	♌ 6° 12′	♊ 19° 1′	47° 11′	7^s 26° 13′
3	127	October 11	$17^h\ 20^m$	♎ 18° 20′	♍ 2° 6′	46° 14′	7^s 24° 2′
4	136	December 25	$6^h\ 0^m$	♑ 2° 36′	♒ 19° 36′	47° 0′	4^s 27° 5′
5	129	May 19	$16^h\ 10^m$	♉ 25° 53′	♈ 10° 57′	45° 4′	7^s 25° 16′
6	136	November 18	$6^h\ 10^m$	♏ 26° 9′	♑ 13° 4′	46° 55′	4^s 4° 17′

But because in observation 3 the corrected place of Venus differs from the Ptolemaic by 1° 46′, Longomontanus attributes this either to an error in the day noted by Ptolemy, or to a deliberate error serving the Ptolemaic hypothesis, especially in view of the place of the star at the tip of the southern wing of Virgo, which Ptolemy puts at ♌ 29°, far from the truth; for Theon observed the place of Venus from the distance of Venus from that star.

[Margin: Ptolemy suspected by Longomontanus of astronomical unfaithfulness.]

Now although Longomontanus judges those observations of the ancients uncertain, yet because, with the apogee of ♀ retained at 25½°, roughly where Ptolemy put it in his time, and with the motion gathered from that and the other things laid down, discussed in no. 7, the more recent observations are well represented, he therefore retains it for A.D. 140 at ♉ 25½° from the true equinox, and in his own time, that is, at the completed year 1585, at ♊ 29¼°. But applying the precession of the equinoxes, 20′ at the one date and 7′ at the other, he deduces that in the 1445 Julian years elapsed the apogee of ♀ completed 33° 18′, and therefore 1′ 22″ 57‴ 30⁗ yearly, and 2° 18′ 16″ 12‴ in 100 years. By subtracting these from the Sun's mean motion he gathers the anomaly of the eccentric that you have in the Danish tables.

[Margin: What the anomaly of the eccentric is, and from what it is to be gathered?]

As for the motion of the anomaly of the orb, or epicycle, he does not use the Ptolemaic observation of A.D. 138 December 15, $15^h\ 10^m$ at Copenhagen, or the 2nd year of Antoninus, on the day Tybi 29, in the morning at $4^h\ 45^m$ at Alexandria, by which Ptolemy observed Venus from the fixed stars

in the forehead of Scorpio; for on various conjectures he quite often finds "greater reliability in the Ptolemaic observations" wanting. He uses rather the observation made by Timocharis, as Ptolemy reports in Book 10, ch. 4, in the 13th year of Philadelphus, at the end of the day Mesore 17, the following morning; that is, 272 before Christ, October 11, at 18^h after noon at Alexandria, but 16^h 15^m at Copenhagen. At that time Venus appeared exactly to have obscured the star opposite Vindemiatrix; and this star at that time, and hence Venus, was for Longomontanus at ♏ 3° 10′, with a northern latitude of 1° 16′.

[Margin: Ptolemy again and often suspected by Longomontanus.]

With the Sun's mean motion from the true equinox set, then, at 6^s 16° 6′ 30″, and so the distance of Venus from it at 42° 56′ 30″, and with the apogee of Venus then at ♉ 16° 30′ and this subtracted from the Sun's mean place, he gathers the anomaly of the eccentric as 4^s 29° 56′ 30″. And using the eccentricity and the semidiameter of the orb discussed in no. 7, by way of the scheme of the reformed Ptolemaic hypothesis whose use we explained in ch. 2, no. 4, he derives that the mean anomaly of the orb, or of commutation, or that computed from the mean apogee of the epicycle, was 250° 59′ in the time of Timocharis, whereas the Prutenic tables put it at that time at 252° 6′, and the Ptolemaic at about the same. But this is no wonder, since Ptolemy and Copernicus here used digressions of Venus reckoned to be the greatest, and accordingly a right angle at the contact with the orb, and in that treatment a small error in observation brings a large error into the anomaly of the orb. For it is better, he says, to use observations made near the apogee or perigee of the epicycle, although he admits that Timocharis's observation was not of this kind; but since no other, better observation of the ancients is available, one must settle for this one.

[Margin: The motion of the anomaly of the orb of ♀.]

Finally, therefore, since the same anomaly was found in A.D. 1586 to be greater than the Prutenic by 1° 18′, as I said in no. 7, and in the year before

Christ 272 smaller by 1° 7′, therefore in the interval of 1857 Julian years the motion of the anomaly of the orb of Venus exceeded the Prutenic motion by 2° 25′, and this excess was 4″ 41‴ each year.

This, in summary, is the method of Longomontanus, certainly full of labour and ingenuity, but one in which we nevertheless miss a comparison of the hypothesis with more than 9 observed places of Venus. For these four elements, namely the place and motion of the apogee, the eccentricity, the semidiameter of the orb, and the motion of the anomaly of the orb, can be reconciled in various ways so as to satisfy either these 9 observations or others no less certain, and yet not satisfy many others; just as in fact the Danish tables do not satisfy many, so that here they sometimes deviate from observation by a whole degree and more. Nor can he be excused on the ground that he did not have more observations of Tycho, for more exist in volume 1 of the *Progymnasmata*, besides those in the Tychonic papers. Nor can he say that he used only seven Tychonic ones because they seemed better suited to the purpose, some because of their nearness to the apogee or perigee of the eccentric or epicycle, some because

of their nearness to the contact with the orb and the greatest digression. For he ought to have examined the motion and the size of the orb further, and to have seen whether they agreed with all the other observations, or at least with very many of the more certain ones.

[Margin: A critique of the method and hypothesis of Longomontanus.]

Yet he himself acknowledged a great difficulty in this, in Book 2, ch. 25, which he closes with these words: "And here, by the grace of God, we put an end to the restoration of astronomy, where, in accordance with our plan, we have emended the hypotheses and tables that are to display the celestial phenomena to the infallible observations that could be had by us; and in such a way that we would wish no one to pass judgment on these persistent labours of ours, which we hope will benefit all fair-minded people and those who love these disciplines, that is, to slight them or perversely despise them, except one who takes the trouble to compare them properly with the celestial appearances, so that he may thereby rather pronounce a just censure on them, or, by at last putting his own hand to the tablet [manum tabulæ admouendo], may himself feel how difficult this task has been." This candid confession we cannot receive without commendation.

[Margin: The difficulty of reconciling the hypothesis with the observations.]

But neither do we approve the investigation of the anomaly of the orb from Venus when nearest the perigee of the epicycle: for there she cannot be seen except subject to great refractions, and a small error then does great harm, because of the very acute angles formed at the centre of the epicycle and at the centre of the Earth.

IX. By exactly the same method the same Longomontanus, in Book 2 of the *Theorica*, ch. 20 and 21, investigates a posteriori, by comparing the Prutenic motion with observed places of Mercury, the apogee, the eccentricity, the semidiameter of the orb, and the motion of the anomaly of the orb, assuming after various trials of calculation the values that seem to agree best with the observations; and from this method he sets down the following propositions.

[Margin: The method of Longomontanus for ☿.]

First: Mercury has the same motion of the eccentric as the mean motion of the Sun; but as regards its motion on the epicycle it adjusts itself to the Sun's mean rather than true motion, as Venus also does.

Second: from the Ptolemaic method applied to the digressions of Mercury on either side of the Sun's mean motion, by moderate conjecture, and then from other, more accurate calculations with somewhat more certainty, it is established that at the beginning of this century the apogee of Mercury was near the beginning of Sagittarius, and indeed at the completed year 1585 at ♐ 0° 30′.

[Margin: The apogee of ☿.]

Third: the small circle, along whose straight diameter Copernicus swings and varies the diameter of the orb, is illegitimate, and errs most near the apogee and perigee of the eccentric, where the celestial phenomena require a breadth

of the orb greater than the Prutenic by two degrees and more, and a swifter motion of Mercury. Accordingly, the truer semidiameter of the epicycle, or orb, of Mercury is 38,500 parts, of which the radius of the eccentric is 100,000.

[Margin: The semidiameter of the orb of ☿.]

Fourth: the eccentricity of Mercury varies, so that the radius of the eccentric is carried on the circumference of a small circle, with a motion not double but triple the motion of the anomaly of the eccentric. For it is from this that Mercury appeared to Ptolemy to make its greatest digressions from the Sun in Aquarius in the morning and in Gemini in the evening, and to Copernicus near the sides of the trigon or hexagon. The semidiameter of this small circle is 1137 parts, namely a fifth part of the whole eccentricity from the Earth (or, in the Copernican scheme, from the centre of the great orb), which is 5685 parts, of which the radius of the eccentric is 100,000.

[Margin: The eccentricity of ☿ and the variation of the eccentricity.]

Fifth: the mean anomaly of the orb of Mercury at the end of the year 1585 is greater than the Prutenic by 2° 10′, and so is 1ˢ 13° 2′ 8″.

[Margin: The root of the anomaly of the orb in the year 1585.]

The first seven Tychonic observations, the third from last by the Landgrave of Hesse, and his own last two, from which Longomontanus deduced the theses already mentioned, comparing his calculation with them and supposing the motions and quantities already mentioned, are set out here before your eyes for the meridian of Copenhagen, with the days given in the style of the old calendar, by means of the following table.

Obſeruationes Mercurij comparatæ cum Loco ſupputato ex Hypotheſi Longomontani.

Observations of Mercury compared with the place computed from the hypothesis of Longomontanus.

	Years of Christ	Old style	After noon	Observed place of ☿	Observed latitude	Computed place of ☿	Mean longitude from the true equinox	Anomaly of the eccentric	Anomaly of the epicycle
1	1585	Nov. 14	19ʰ 0ᵐ	♏ 13° 4′	2° 10′ B	♏ 12° 54′	8ˢ 4° 0′ 15″	0ˢ 3° 30′ 0″	8ˢ 16° 22′ 20″
2	1585	Nov. 23	19ʰ 20ᵐ	♏ 25° 3′	1° 25′ B	♏ 24° 58′	8ˢ 12° 53′ 20″	0ˢ 12° 23′ 0″	9ˢ 14° 22′ 30″
3	1586	Oct. 24	19ʰ 10ᵐ	♌ 22° 35′	☩ ☩ ☩	♌ 22° 28′	7ˢ 13° 4′ 30″	11ˢ 12° 34′ 0″	8ˢ 5° 6′ 30″
4	1586	Oct. 28	18ʰ 30ᵐ	♌ 26° 33′	2° 17′ B	♌ 26° 28′	7ˢ 16° 49′ 30″	11ˢ 16° 29′ 0″	8ˢ 17° 27′ 0″
5	1587	Jan. 9	4ʰ 50ᵐ	♒ 17° 48′	0° 1′ B	♒ 17° 49′	9ˢ 28° 22′ 50″	1ˢ 16° 52′ 0″	4ˢ 2° 28′ 20″
6	1590	March 6	6ʰ 50ᵐ	♉ 13° 44′	1° 42′ B	♉ 13° 36′	11ˢ 23° 55′ 20″	3ˢ 23° 20′ 0″	3ˢ 11° 41′ 10″
7	1592	Feb. 3	5ʰ 40ᵐ	♓ 12° 20′	0° 47′ B	♓ 12° 29′	10ˢ 22° 50′ 45″	2ˢ 22° 15′ 0″	3ˢ 23° 8′ 30″

	Years of Christ	Old style	After noon	Observed place of ☿	Observed latitude	Computed place of ☿	Mean longitude from the true equinox	Anomaly of the eccentric	Anomaly of the epicycle
8	1593	May 11	$9^h\ 30^m$	♊ 23° 16′	2° 0′ B	♊ 23° 4′	11^s 29° 23′ 0″	5^s 28° 51′ 0″	3^s 22° 4′ 0″
9	1607	April 15	$9^h\ 0^m$	♉ 21° 5′	1° 40′ B	♉ 21° 12′	1^s 3° 20′ 50″	5^s 2° 18′ 0″	2^s 15° 50′ 6″
10	1610	Dec. 5	$19^h\ 0^m$	♐ 2° 42′	✢ ✢	♐ 2° 35′	8^s 24° 10′ 0″	0^s 23° 34′ 0″	8^s 10° 11′ 0″

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X. He then passes in ch. 21 to inquiring into the mean motion of both the apogee and the anomaly of the orb of Mercury. And for the motion of the apogee he brings forward no observation of the ancients, because he hopes for nothing better from them than from the observations of Venus made by Ptolemy with the astrolabe, fearing from them an error of 10′ and more. Assuming, then, for the completed year 264 before Christ the apogee of Mercury at ♎ 6° 5′ from the mean equinox, and in A.D. 1586 at ♐ 0° 12′ from the same equinox, and so an advance of 54° 7′ in the interval of 1850 Julian years, and 1′ 45″ 18‴ yearly, he says that this motion satisfies the more recent observations; and therefore he concludes that Ptolemy, who placed the apogee of ☿ in the time of Timocharis at ♎ 6°, did better than in placing it in his own time at ♎ 10°.

[Margin: The motion of the apogee of ☿.]

For the motion of the anomaly of the orb, however, he uses an ancient observation, made, as Ptolemy reports in Book 9, ch. 10, in the year 484 of Nabonassar, on the 18th of Thoth, with the 19th following, on whose morning Mercury appeared to stand off from the northern star in the forehead of Scorpio by one Moon toward the west, but by two Moons toward the north. That was the year 265 before Christ, at 17^h after noon of November 14 at Copenhagen. At that time, from his own hypothesis of the fixed stars, he says that the northern star of the forehead of Scorpio was at ♏ 1° 36′ with a northern latitude of 1° 16′, and Mercury at ♏ 2° 35′ with a northern latitude of 2° 20′.

[Margin: The motion of the anomaly of the orb of ☿.]

Therefore, taking from his own tables the mean motion of the Sun, of the apogee, and of the anomaly of Mercury, and the measures of the eccentricity and of the semidiameter of the orb, discussed in no. 9, and applying the figure of the reformed Ptolemaic hypothesis, on which see our ch. 2, no. 4, he gathers the mean anomaly of the orb as 210°, which at that time Ptolemy's tables give as 211° 45′ and Copernicus's as 212° 34′. He rests, moreover, on this observation, because Mercury stood only a little distance from the Sun and was nearer the perigee of the epicycle. So, comparing this motion with the motion recently

[64] B is boreal, north. Crosses stand in the latitude cells of rows 3 and 10 as printed, three in the one and two in the other, and no figure is set there.

found, he gathers that the mean annual motion of the anomaly of the orb, or of commutation, of Mercury is, besides three whole revolutions, $1^s\ 23^\circ\ 57'\ 33''\ 35'''$.

[Margin: Critique of Longomontanus's hypothesis.]

But in this business likewise more observations are still wanted; nor did the ancient ones seem to deserve scorn. For grant that some error in them is to be feared, yet it is not always so great as the error the Danish tables later incurred in Mercury, up to 7°, as we have already said in Book 3, ch. 3, scholium 3; and elsewhere they often differ by several degrees from the observations of Gassendi and others. Be that as it may, for this labour and for his effort, such as it was, to restore astronomy, we owe this man a great deal.

[Margin: Kepler's view on the aphelion of ♀ and ☿.]

XI. Now Kepler was more concerned with distinguishing the aphelion of these planets from the apogee than with determining the place of the apogee more precisely. And so in ch. 15 of the *Mysterium Cosmographicum*, and in the *Epitome of Copernican Astronomy*, Book 6, part 3, or p. 764, he teaches that the apogee of Venus in Ptolemy's age was at ♉ 25° and that of Mercury at ♎ 10°, and that now the former lies in ♊ and the latter in ♏; but that the aphelion of ♀ in Ptolemy's age was in ♍ and is now at ♒ 2°; and that the aphelion of ☿ is found to have once been at ♏ 4° and now at ♐ 15°.

[Margin: And Mästlin's.]

More precisely, Mästlin, in his notes to ch. 15 of the *Mysterium Cosmographicum*, distinguishes the apogee of each from the aphelion as in the following small table.

	Age of Ptolemy: Apogee	Age of Ptolemy: Aphelion	Age of Copernicus: Apogee	Age of Copernicus: Aphelion
♀	♉ 25° 0′	♍ 4° 39′	♊ 15° 44′	♍ 19° 48′
☿	♎ 10° 0′	♎ 29° 42′	♏ 28° 30′	♐ 13° 40′

Kepler continues in the *Epitome*, and from the motion of the aphelion of Mercury in the order of the signs he gathers, as probable, the same about the aphelion of Venus, whose ancient place, however, he holds can be shown only from a distance, because of the small eccentricity of Venus, to which a moderate error in the Sun's true place, and so in the digressions from the Sun, does much more harm than to Mercury, which has a large eccentricity. He likewise ascribes the reason why the aphelion of Venus lies so far from its own apogee and from the Sun's to the eccentricity of Venus, which is much smaller than the eccentricity of the great orb; from this it comes about that Venus is much further from the Sun, and so aphelial, in ♍, where the Earth comes closest to the Sun, than in ♋, where the Earth is farthest from the Sun. Conversely, because the eccentricity of Mercury is much greater than the eccentricity of the great orb, the apogee of Mercury that the old astronomy established is much closer to the aphelion of Mercury than to the apogee of the Sun. From this arises a further doubt about the way of determining the aphelion of Venus and of Mercury, which we resolved in Section 2, ch. 10, nos. 3 and 4. But so that

no one, running through the subjects of the problems, may think it is missing here, let this stand:

Problema 2. Dato Apogeo Planetarum inuenire Aphelium & è conuerso; dummodo detur ipsorum Eccentricitas, & Distantia Solis à Terra.

Problem 2. Given the apogee of the planets, to find the aphelion, and conversely, provided their eccentricity and the distance of the Sun from the Earth are given.

See the solution of the problem in Section 2, ch. 10, nos. 3 and 4.

[Margin: Wendelen's ingenious fiction on the motion of the apsides.]

XII. In Kepler's place we must put Govaert Wendelen, who, contemplating more deeply the foundations of the Rudolphine Tables in his third *Lampas Arcanorum Caelestium*, expounds by this theorem the little verse set at its head,

Apsidibus ratio dat sextantaria legem ["The sextantal ratio gives the law to the apsides"],

namely: *The speeds of the apsides are to one another in the sextantal ratio of their eccentricities.* He calls a ratio sextantal when, for any number proposed, one takes the root which, multiplied six times, produces that number. For example, if 64 and 729 are proposed, their sextantal roots are 2 and 3, as is clear from the six multiplications by 2 and 3 written below.

2	4	8	16	32	64
3	9	27	81	243	729

So if the eccentricities of two planets were to each other as 64 to 729, he says that the speeds of the apsides, or the motions of the apogees, will be to each other as 2 to 3, that is, as their sextantal roots, and indeed most precisely. And so, from the Rudolphine Tables, he lays it down that to complete one degree the aphelia of the planets require the years written below.

The aphelion traverses 1 degree in

	Years	& Months
♄	47	9
♃	49	2
♂	44	1
♀	69	5
☿	38	3

"All of this," he says, "we shall demonstrate to answer to our own trials too, from very many years back; and that this, hitherto unknown to all mortals, is the third marvel of divine wisdom and a wound to genethliac folly." When

Wendelen has demonstrated this with the phenomena saved, then at last he will earn the laurel.

[Margin: Boulliau's method and view.]

XIII. Ismaël Boulliau, in Book 9, ch. 6, investigating the translation of the aphelion of Venus, uses the aphelion's place and its mean motion as taken from the Rudolphine Tables, together with the semidiameter of the orb and the eccentricity of the centre, which he had previously determined from 4 digressions of Venus that he takes to be the greatest; and on these suppositions he examines two ancient observations of Venus. The earlier, reported by Ptolemy in Book 9, ch. 4, was made by Timocharis in year 476 of Nabonassar, on day 17 of Mesori, at hour 12 of the night, or 17^{h} after noon at Uraniborg, when Venus was seen at ♍ 2° 47′ 20″ with north latitude 1° 25′. But Boulliau's calculation, supposing the aphelion of ♀ at that time at ♌ 9° 1′ 26″, shows Venus then at ♍ 2° 9′ 54″. From various circumstances, however, he judges that the motion of the aphelion should be slowed rather than its place moved back. The later, reported in a manuscript codex of the Royal Library, was made at Athens in A.D. 510, and of Nabonassar 1258, August 20, on the evening of which Venus was seen applied to Jupiter and visibly all but conjoined with him; and he examines this application on the supposition of the Rudolphine aphelion. Since he says that the aphelion of ♀ was found, for the year 1597, or 2345 of Nabonassar, at ♒ 5° 19′ 43″, it follows that in the interval of 1868½ years from Timocharis's observation to the year 1597 it advanced 26° 18′ 17″, and annually 50″ 40‴ 50⁗ 36⁗′ 7⁗′′ 38⁗′′′ 37⁗′′′′ 10⁗′′′′′. In this way he probes the place of the aphelion of ♀ a posteriori, and through the most troublesome windings of calculation.

He does no differently with the aphelion of Mercury, which in Book 10, chs. 2 and 9, he places for the year 1636 at ♐ 12° 25′, after examining places of ☿ observed by the Hessians, by himself or by Gassendi, especially near the greatest digressions from the Sun; although in ch. 1 he says that Gassendi's were not observed precisely at the contact of the orb, warning that an error of 3 or 4′ in the place of ☿ produces an error of 2 or 3° in its aphelion, and that such an error, and a greater one, is easily incurred through refractions, to which the observation of Mercury is liable. Again, in ch. 9, he adduces among others an ancient observation made in year 484 of Nabonassar, on day 18 of Thoth, the 19th following, that is, 265 B.C. November 14, at $16½^{h}$ at Uraniborg, when Mercury was seen at ♏ 1° 57′ 30″ with north latitude 2° 6′. And since he had placed the aphelion of ☿ in A.D. 139 at ♏ 3° and in 1632 at ♐ 12° 31′ 3″, he therefore, keeping the proportion of the motion, places this aphelion for that year 265 at ♎ 22° 23′ 3″. With this laid down, however, Mercury comes out for him at ♏ 7° 32′ 32″, whereas it was observed at 1° 57′ 30″; and Mercury cannot be represented in that place unless, for that year 265 B.C., the aphelion of ☿ is set at ♎ 17° 40′. And so, keeping the aphelion for the year 1632 at ♐ 12° 31′ 13″, he gathers in ch. 10 a progress of 54° 51′ and 13″ in an interval of 1896

Egyptian years, and in each single year 1′ 44″ 9‴ 8⁗ 44⁗′ 3⁗′′ 2⁗′′′ 16⁗′′′′ 42⁗′′′′′. This, in summary, is Boulliau's method.

[Margin: Critique of Boulliau's hypothesis.]

But in Boulliau we find more observations wanting, especially of Venus, because it is easy to reconcile the apogee, the eccentricity, the semidiameter of the orb and the motion of the anomaly in such a way that they satisfy 6 or 8 observations, and yet afterwards do not satisfy many others. Then, in Book 9, ch. 1, from only two observations of Iustus Byrgius [Jost Bürgi] and two of Tycho, he wrongly deduces that the greater digression of these was also the greatest and at the contact of the orb; for to determine that, it is not enough for one digression to have been greater than the preceding one; it must also have become smaller on the following day. So at least three observations are required, so that the greatest increase can be gathered from the immediately following decrease. And yet, supposing those to be greatest, he founded the commensuration of the orbs on them. Now, since the observations of the places of ♀ and ☿ selected by him, and emended from his own and Tycho's principles, could be of great use to us, we decided to enter these too in tables. The observations of ♀ are taken from Boulliau's Book 9, and those of ☿ from Book 10, with two added from Book 11, theorems 11 and 13; and they were reduced by him to the meridian of Uraniborg, as you see noted in the table itself.

OBSERVATIONES ♀ & ☿. Selectæ ex Bullialdo eas corrigente iuxta suas de Sole & Fixis hypotheses, & referente locis in margine adductis. Diebus stylo Gregoriano notatis.

Observations of ♀ and ☿. Selected from Boulliau, who corrects them according to his own hypotheses of the Sun and the fixed stars, and who reports them in the places cited in the margin. Days noted in the Gregorian style.

Observers	Years of Christ current; months and days completed	After noon at Uraniborg	♀ Observed place	⊙ True place	♀ Greatest digression from the ⊙'s true place
Jost Bürgi	1592 March 3	$17^h\ 30^m$	♍ 27° 18′	♓ 13° 59′	46° 41′
Jost Bürgi	1594 December 27	$5^h\ 0^m$	♒ 23° 2′	♍ 5° 49′	47° 13′
Tycho	1600 February 21	$20^h\ 27^m$	♍ 16° 22′	♓ 3° 8′	46° 46′
Tycho	1601 May 9	$8^h\ 30^m$	♋ 4° 28′	♉ 19° 4′	45° 24′
Unknown	510 August 20	$7^h\ 0^m$	♏ 18° 21′	♌ 28° 33′	
Tycho	1590 December 27	$20^h\ 0^m$	♐ 20° 0′	♍ 6° 25′	
Jost Bürgi	1595 August 9	$16^h\ 0^m$	♋ 25° 39′	♌ 25° 17′	

Observers	Years of Christ current; months and days completed	After noon at Uraniborg	☿ Observed place	☉ True place	☿ Greatest digression from the ☉'s true place
Boulliau	1633 May 26	$9^h\ 0^m$	♋ 18° 16′	♊ 5° 40′	

[Margin, set against the rows of this block in order: Boulliau, Book 9 (first row); ch. 1; ch. 1; ch. 1; then ch. 6; ch. 2; ch. 3; ch. 2.]

Observers	Years of Christ current; months and days completed	After noon at Uraniborg	☿ Observed place	☉ True place	☿ Greatest digression from the ☉'s true place
Theon	130 July 4	$6^h\ 40^m$	♌ 7° 18′	♋ 10° 20′	26° 58′
Ptolemy	132 February 2	$4^h\ 40^m$	♓ 2° 8′	♒ 12° 49′	19° 20′
Ptolemy	139 July 7	$15^h\ 0^m$	♊ 21° 16′	♋ 13° 22′	22° 6′
Ptolemy	141 February 1	$16^h\ 40^m$	♍ 14° 42′	♒ 13° 0′	28° 19′
Bernhard Walther	1504 March 18	$7^h\ 21^m$	♈ 26° 35′	♈ 7° 43′	18° 53′
The Landgrave	1593 May 23	$9^h\ 30^m$	♊ 25° 21′	♊ 2° 29′	22° 52′
Boulliau	1625 February 13	$6^h\ 20^m$	♓ 13° 27′	♒ 25° 28′	17° 59′
Gassendi	1634 January 3	$6^h\ 30^m$	♒ 2° 33′	♍ 13° 34′	18° 59′
Gassendi	1634 October 3	$18^h\ 0^m$	♍ 22° 59′	♎ 10° 49′	17° 50′
Gassendi	1635 January 24	$18^h\ 30^m$	♍ 10° 5′	♒ 5° 14′	25° 9′
Gassendi	1635 September 17	$17^h\ 0^m$	♍ 7° 4′	♍ 24° 54′	17° 50′
Gassendi	1636 July 16	$9^h\ 0^m$	♌ 21° 33′	♋ 24° 29′	27° 4′
Ptolemy	139 May 17	$6^h\ 9^m$	♊ 18° 35′	♉ 24° 26′	
Tycho	1587 January 19	$4^h\ 50^m$	♒ 17° 48′	♍ 29° 13′	
Tycho	1592 February 13	$5^h\ 40^m$	♓ 12° 20′	♒ 24° 24′	
Boulliau	1630 December 11	$19^h\ 50^m$	♏ 29° 13′	♐ 20° 22′	
Gassendi	1631 November 6	$15^h\ 14^m$	♏ 14° 38′	♏ 14° 38′	
Gassendi	1632 July 30	$16^h\ 0^m$	♋ 24° 26′	♌ 8° 5′	

Ob-servers	Years of Christ current; months and days completed	After noon at Uraniborg	☿ Ob-served place	☉ True place	☿ Greatest digression from the ☉'s true place
Gassendi	1635 December 1	$5^h\ 53^m$	♑ 0° 26′	♐ 9° 22′	

[Margin, set against the rows of this block in order: Boulliau, Book 10 (first row), ch. 9 (second row), the next two rows without a note; then Book 10, ch. 9; Book 10, ch. 1 (six rows); Book 10, ch. 2; then Book 10, ch. 10; Book 10, ch. 7; Book 10, ch. 7; Book 11, theorem 13; Book 10, ch. 5; Book 10, ch. 3; Book 11, theorem 13.]

Add to these the observation of Venus made by Timocharis in the year 272 before Christ, October 11, after noon at Uraniborg $15^h\ 16^m$, which Boulliau places at 2° 31′ of Virgo, with the Sun at 14° 55′ of Libra, as he himself thinks.

CAPVT IX.

Chapter 9.

Conatus Noster pro inquisitione Apogei Veneris ac Mercurij.

Our effort in the investigation of the apogee of Venus and Mercury.

[Margin: Kepler's critique of the ancient astronomers concerning the apogee of ♀ and ☿.]

I. What Kepler said about the aphelion of these planets in the *Epitome of Copernican Astronomy*, p. 763, can be said of their apogee too: namely, that *in ancient times, where the places of these were could be shown only from a distance*, especially in Venus, because it returns more rarely to its greatest digressions, and to its small eccentricity a very small error in the observation, or in the Sun's place, does great harm. And so he admits that the ancients did indeed scent out the places of these apogees, but in such a way that they could have erred by a whole sign, let alone by a degree.

[Margin: Causes of the uncertainty in this matter.]

The cause of this uncertainty is the multiplicity of conditions required in the greatest digressions, if the place of these apogees is to be drawn from them by Ptolemy's method. *First*, we must be certain of the day of the greatest digression, which requires the observation of several days, so that it can be distinguished from the increase and decrease of the elongation from the Sun. On this ground Longomontanus, Book 2, ch. 18, excuses himself and the Tychonians, as I said in ch. 8, no. 7; and Boulliau, Book 10, ch. 1, denies that some digressions observed by Pierre Gassendi were precisely at the contact of the orb, that is, precisely greatest. *Second*, two greatest digressions of the same planet must occur within a few years that are equal to each other, one of them, however, being a morning and the other an evening elongation. *Third*, they must be at a distance as nearly equal as possible from the apogee of the eccentric, for which,

however, some rough knowledge of the place of the apogee is presupposed; and with this granted, the less they stand on either side from the apogee of the eccentric, the less is the danger of error. *Fourth*, the semidiameter of the orb, or of the epicycle, must be supposed unvaried, or equal; but since Copernicus supposes that in Mercury it varies according to the anomaly of the eccentric, therefore, from the equal greatest digressions taken from Ptolemy, he defines the place of the apogee. Therefore, if this variation depends equally on both sides on the distance from the apogee of the eccentric, this condition is included in the third. But it can hardly come about that within the few years in which the place of the apogee is sought, two digressions should occur with the already mentioned conditions concurring, since within those years their equality alone is rarely found, as anyone can see who surveys the tables set out in ch. 8 at the end of nos. 8, 9 and 12, and those to be given at the end of no. 2. Nor have we, although we have many of Venus and many more of Mercury (since it performs six greatest digressions from the Sun every year, and sometimes seven), yet been able to obtain two perfectly equal to each other, one morning and the other evening; these we shall perhaps acquire, God granting, before the third volume appears. But that Ptolemy handed down his own to us as so precisely equal is not free from the suspicion of a concealed difference, which he may have thought could be disregarded, so that we might have some astronomy rather than none.

Since, however, neither Tycho nor we agree with Ptolemy on the places of the Sun and the fixed stars, the places of Venus and Mercury must first be corrected according to our principles, before we apply the greatest digressions from the Sun to the search for the apogee. Of this correction I shall point out one or two examples.

[Margin: Example of the correction of the place of the fixed stars.]

Ptolemy relates in Book 10, ch. 1 of the *Almagest* that Venus was observed near the Pleiades in the 16th year of the Emperor Hadrian, that is, A.D. 132, on day 21 of Pharmuthi, in the evening, that is, March 8; and that she appeared to him to lead the middle or bright star of the Pleiades by as great a space as the whole asterism of the Pleiades occupies, but to be slightly further south than they, when she had made her furthest digression from the Sun. Then he adds: *Since, therefore, the middle of the Pleiades then occupied, according to our principles, 3° of Taurus, and their extent in longitude is 1° and very nearly 30′, it is evident that the star of Venus then held 1° 30′ of Taurus.* But in Tycho's catalogue, taking the first and last of the Pleiades, that asterism occupies 1° 34′. From my own observations, however, and from the catalogue and motion of the fixed stars established in Book 6, ch. 24, the middle of the Pleiades at that time was at 4° 3′ of Taurus. From this I subtract 1° 34′, and there remains the corrected place of Venus at ♉ 2° 29′. In the same way, then, considering the places of other fixed stars from whose distance the ancients or more recent observers estimated the longitude of Venus or Mercury, we have corrected the places of the planets.

[Margin: Timocharis's observation is weighed.]

II. Now the observation of Timocharis deserves particular consideration, made in year 476 of Nabonassar, on day 17 of Mesori, the 18th following, but in the year before Christ 272, October 11, at $15^{h}\ 17^{m}$ after noon at Uraniborg, when, as Ptolemy relates in Book 10, ch. 4, Venus was seen as a morning star, [Greek: κατειληφὼς τὴν ἀντικείμενον τῶν [?]γοτρυγητῆρι [recte: προτρυγητῆρι] ἀκριβῶς], that is, *comprehending exactly the star set opposite the Previndemiatrix*. So he does not say that she covered or obscured that star with their centres joined in the same line with the centre of Timocharis's sight, as Theon explained and as Longomontanus, Book 2 of the *Theorica*, ch. 19, and Lansbergen in the *Treasury of Observations*, p. 172, set down; but that she comprehended, or reached, that star either with her margin or with her rays, as Kepler in the *Astronomia Optica*, p. 304, and Boulliau in the *Astronomia Philolaica*, Book 9, ch. 6, rightly rendered this passage. Boulliau, however, omits the word [Greek: ἀκριβῶς (akribōs, "exactly")], or *exactly*, because on his hypothesis Venus was at 2° 31′ ♍, and that star (namely the leading one among the four in the southern wing of Virgo, as all those already mentioned hold with Ptolemy) was at ♍ 2° 47′, and so 16′ distant from Venus; which Boulliau attributes to the expansion of Venus's rays, occupying so great a space. But Longomontanus puts both stars then at 3° 10′ ♍, and Lansbergen at 3° 21′. We, however, from the motion and roots treated in Book 6, last chapter, put that fixed star at 3° 24′ ♍. And the word *exactly* seems to us to signify the application of Venus's rays to the margin of the fixed star, within which rays it afterwards lay hidden in what one might call an Aphrodisian setting; and her rays could have occupied about 10′, so that Venus was then at very nearly ♍ 14′[65] — if indeed this is the star that Timocharis meant.

For I do not see how it can be said to be set opposite the Vindemiatrix, since the latitude of the Vindemiatrix is 16° and more to the north, while the leading one of those four in the left or southern wing has a latitude of 1° 25′ and is more westerly than the Vindemiatrix by 5° and more. With better right, however, the star following the leading one of the said four is opposed to the Vindemiatrix; for it has the same magnitude and, to the senses, the same longitude, and crosses the meridian at almost the same moment. In Tycho's catalogue its longitude for the year 1600 is ♎ 4° 35′, and that of the Vindemiatrix ♎ 4° 23′; and for us that star was then at ♍ 8° 40′. So Venus, if she reached it with her margin, was at ♍ 8° 39′; if with her rays expanded, at about ♍ 8° 30′. But Ptolemy substituted a different star, with the other astronomers suspecting nothing; nor would we have come to this suspicion, had Ptolemy's star not been most troublesome to us. For I have never been able to reconcile the place of Venus estimated from it with many other places observed at almost the same location on the eccentric and the epicycle. We leave to each their own judgement, however, provided that from this uncertainty, or from the diversity of the place assigned to that

[65]the page sets Gr. with no degree figure before 14′

very star by the authors already mentioned, it is clear that a hypothesis that does not exhibit this place of Venus is not to be condemned straight away.

Again, when Walther says that in 1494 September 18, in the morning, Venus was seen by him to lead Regulus by about 10′, it does not follow that she was in fact only 10′ west of Regulus. For two stars in the horn of Capricorn seen with the naked eye, because of the expansion of their rays, appear to be about one digit apart, but with the telescope at least 3 digits, magnified though these are according to the quality of the tube. So ♀ must have been much further west of Regulus still.

Now let us set out again, in the following table, selected observations of Venus and Mercury reduced to the meridian of Uraniborg or Bologna, with the mark of a small letter *m* added to those digressions that were, or were thought to be, greatest, so that from these we may obtain, if it can be done, the apogee of these planets, and other matters to be treated in the following chapters.

XXXIV. LOCA VENERIS Obseruata cum Digressione à Sole Matutina littera M, & Vespertina litt. V, indicata, & maxima vbi lit.m, vt vel alij vel nos censemus, correctis tamen locis ex nostra de ☉ & Fixis Hypothesi.

XXXIV. Observed places of Venus, with the morning digression from the Sun indicated by the letter M and the evening by the letter V, and the greatest where the letter m stands, as either others or we judge, the places however being corrected from our hypothesis of the ☉ and the fixed stars.

	Historians, or Reporters	Observers	Years of Christ current; Gregorian style, month and day	After noon	♀ Observed place	☉ Mean place	☉ True place	♀ Digression from the ☉: mean	♀ Digression from the ☉: true	Kind of digression
1	Ptolemy, Book 10, ch. 1	Theon	127 October 11	$16^h\ 20^m$	♍ 2° 10′	♎ 18° 36′	17° 0′	46° 26′	44° 50′	M m
2	Book 10, ch. 2	Theon	129 May 19	$17^h\ 0^m$	♈ 11° 8′	♉ 26° 13′	26° 44′	45° 5′	45° 36′	M m
3	Book 10, ch. 1	Theon	132 March 8	$6^h\ 0^m$	♉ 2° 29′	♓ 15° 4′	17° 2′	47° 25′	45° 27′	V m
4	Book 10, ch. 1	Ptolemy	136 December 25	$6^h\ 0^m$	♒ 20° 42′	♑ 2° 38′	3° 22′	48° 4′	47° 20′	V m
5	Book 10, ch. 3	Ptolemy	140 February 17	$4^h\ 0^m$	♈ 15° 6′	♒ 25° 20′	27° 14′	46° 46′	47° 52′	V m
6	Book 10, ch. 1	Ptolemy	140 July 29	$15^h\ 30^m$	♊ 18° 43′	♌ 6° 27′	4° 52′	47° 44′	46° 9′	M m
7	Book 10, ch. 2	Ptolemy	136 November 18	$6^h\ 10^m$	♑ 12° 49′	♏ 26° 26′	25° 54′	46° 3′	46° 35′	V m
8	Walther, in his observations	Walther	1494 September 18	$17^h\ 0^m$	♌ 22° 20′	♎ 6° 48′	4° 49′	44° 28′	42° 29′	M
9	Kepler, *Optics* 305	Mästlin	1574 September 15	$16^h\ 0^m$	♌ 23° 57′	♎ 4° 25′	2° 27′	40° 28′	38° 30′	M

	Historians, or Reporters	Observers	Years of Christ current; Gregorian style, month and day	After noon	♀ Observed place	☉ Mean place	☉ True place	♀ Digression from the ☉: mean	♀ Digression from the ☉: true	Kind of digression
10	Boulliau	The Hessians	1592 March 3	17h 30m	♍ 27° 20′	♓ 12° 4′	13° 53′	44° 44′	46° 33′	M m
11	Book 9, ch. 1	or Jost	1594 December 27	5h 0m	♒ 23° 4′	♍ 5° 45′	5° 44′	47° 19′	47° 20′	V m
12	& 3	Bürgi	1595 August 19	16h 0m	♌ 25° 44′	♌ 27° 51′	26° 21′	32° 7′	30° 41′	M

Residuum Tabulæ Præcedentis.

The rest of the preceding table.

	Historians, or Reporters	Observers	Years of Christ current; Gregorian style, month and day	After noon	♀ Observed place	☉ Mean place	☉ True place	♀ Digression from the ☉: mean	♀ Digression from the ☉: true	Kind of digression
13		Tycho	1585 September 24	17h 15m	♌ 16° 0′	♎ 3° 49′	1° 51′	47° 49′	45° 51′	M m
14	Longomontanus,	Tycho	1587 January 25	4h 40m	♓ 16° 56′	♒ 4° 15′	5° 13′	42° 41′	44° 38′	V
15	Book 2, ch. 18	Tycho	1587 March 3	6h 5m	♓ 16° 1′	♓ 10° 49′	12° 37′	5° 12′	3° 24′	V
16		Tycho	1587 March 12	17h 30m	♓ 10° 7′	♓ 20° 8′	22° 3′	10° 1′	11° 56′	M
17		Tycho	1588 December 24	19h 40m	♏ 17° 11′	♍ 3° 52′	3° 46′	46° 41′	46° 35′	M m
18		Tycho	1590 December 27	20h 0m	♐ 20° 2′	♍ 6° 22′	6° 22′	16° 20′	16° 20′	M
19		Tycho	1594 December 25	5h 10m	♒ 21° 1′	♍ 3° 47′	3° 41′	47° 48′	47° 42′	V m
20	Tycho, *Epistolae Astronomicae*,	Tycho	1587 January 19	5h 50m	♓ 13° 6′	♍ 28° 36′	29° 26′	44° 30′	43° 40′	V
21	p. 56	Tycho	1587 January 24	4h 20m	♓ 16° 21′	♒ 3° 19′	4° 14′	42° 41′	41° 35′	V
22	Boulliau,	Tycho	1600 February 21	20h 27m	♍ 16° 23′	♓ 1° 24′	3° 3′	45° 1′	46° 40′	M m
23	Book 9, ch. 1	Tycho	1601 May 9	8h 30m	♌ 4° 30′	♉ 17° 34′	19° 1′	46° 56′	45° 29′	V m
24	Longomontanus,	Longomontanus	1610 December 22	4h 40m	♒ 17° 59′	♍ 0° 58′	0° 45′	47° 1′	47° 14′	V m
25	Book 2, ch. 18		1616 March 19	17h 0m	♒ 15° 24′	♓ 28° 0′	29° 59′	42° 36′	44° 34′	M
26	Kepler, *Optics* 305	Kepler	1598 September 24	15h 0m	♌ 24° 17′	♎ 3° 34′	1° 36′	39° 17′	37° 19′	M

	Historians, or Reporters	Observers	Years of Christ current; Gregorian style, month and day	After noon	☿ Observed place	☉ Mean place	☉ True place	☿ Digression from the ☉: mean	☿ Digression from the ☉: true	Kind of digression
27	Boulliau,	Boulliau	1629 December 27	$7^h\ 0^m$	♒ 16° 50′	♑ 6° 23′	6° 21′	40° 27′	40° 29′	V
28	Book 9, ch. 2		1633 May 26	$9^h\ 50^m$	♋ 18° 17′	♊ 4° 39′	5° 40′	43° 38′	42° 37′	V
29	In letters	Father Arzet, S.J.	1636 November 13	$18^h\ 45^m$	♎ 6° 42′	♏ 23° 46′	22° 23′	47° 4′	45° 41′	M m
30	to me	We	1644 June 23	$9^h\ 0^m$	♌ 16° 37′	♋ 2° 31′	2° 41′	44° 6′	43° 56′	V m
31		We	1644 July 10	$8^h\ 24^m$	♍ 4° 15′	♋ 19° 16′	18° 54′	44° 59′	45° 11′	V m
32		We	1644 November 28	$18^h\ 0^m$	♎ 20° 50′	♐ 8° 37′	7° 35′	47° 47′	46° 45′	M m
33		We	1646 July 7	$15^h\ 20^m$	♊ 0° 18′	♋ 16° 6′	15° 48′	45° 48′	44° 32′	M m
34		We	1649 July 29	$16^h\ 10^m$	♋ 7° 15′	♌ 8° 5′	7° 7′	30° 20′	29° 22′	M

XXXVIII. LOCA MERCVRII Obſeruata, cum Digreſſione à Sole M, ideſt Matutina, vel V, ideſt Veſpertina, ſiue fuerit putata m, ideſt maxima, ſiue non; correctis locis ex noſtra de Sole ac Fixis Hypotheſi.

XXXVIII. Observed places of Mercury, with the digression from the Sun marked M, that is, morning, or V, that is, evening, whether or not it was thought to be m, that is, greatest; the places being corrected from our hypothesis of the Sun and the fixed stars.

Order	Historians, or Reporters	Observers	Years current before Christ; Gregorian style, month and day	After noon	☿ Observed place	☉ Mean place	☉ True place	☿ Digression from the ☉: mean	☿ Digression from the ☉: true	Kind of digression
1	Ptolemy, Book 9, ch. 7	Timocharis	265 November 14	$16^h\ 30^m$	♏ 1° 16′					
2	Book 9, ch. 7	Timocharis	262 February 11	$17^h\ 0^m$	♑ 22° 24′	♒ 17° 6′	19° 0′	24° 42′	26° 36′	M m
3	Book 9, ch. 7	Timocharis	262 August 23	$6^h\ 0^m$	♍ 19° 20′	♌ 26° 52′	24° 55′	22° 28′	24° 25′	V m
4	Book 9, ch. 7	Timocharis	257 May 29	$6^h\ 0^m$	♊ 28° 51′	♊ 2° 55′	3° 1′	25° 56′	25° 50′	V m
5	Book 9, ch. 7	Timocharis	237 October 29	$18^h\ 0^m$	♎ 13° 46′	♏ 4° 20′	3° 17′	20° 34′	19° 31′	M m
		Years of	Christ							

Order	Historians, or Reporters	Observers	Years current before Christ; Gregorian style, month and day	After noon	☿ Observed place	☉ Mean place	☉ True place	☿ Digression from the ☉: mean	☿ Digression from the ☉: true	Kind of digression
6	Ptolemy, Book 9, ch. 9	Theon	130 July 4	6h 40m	♌ 7° 44′	♋ 10° 54′	9° 58′	26° 50′	27° 46′	V m
7	Book 9, ch. 7	Ptolemy	132 February 2	4h 40m	♓ 2° 9′	♒ 10° 30′	12° 14′	21° 39′	19° 55′	V m
8	Book 9, ch. 7	Ptolemy	135 June 4	17h 0m	♉ 19° 56′	♊ 11° 31′	11° 32′	21° 35′	21° 36′	M m
9	Book 9, ch. 7	Ptolemy	136 May 21	6h 0m	♉ 5° 34′	♉ 28° 1′	28° 29′	22° 27′	22° 55′	V m
10	Book 9, ch. 7	Ptolemy	138 May 31	6h 0m	♋ 8° 26′	♊ 7° 25′	7° 33′	31° 1′	30° 53′	V m
11	Book 9, ch. 10	Ptolemy	139 May 17	6h 9m	♊ 19° 2′	♉ 23° 55′	24° 30′			
12	Book 9, ch. 8	Ptolemy	139 July 7	15h 0m	♊ 21° 22′	♋ 14° 1′	13° 0′	22° 39′	21° 38′	M m
13	Book 9, ch. 8	Ptolemy	141 February 1	16h 40m	♍ 14° 48′	♒ 10° 50′	12° 34′	26° 2′	27° 46′	M m
14	Copernicus,	Walther	1491 September 3	17h 10m	♍ 3° 9′	♍ 21° 45′	19° 51′	18° 42′	16° 42′	M
15	Book 5, ch. 30	Walther	1491 September 8	17h 0m	♍ 13° 34′	♍ 26° 41′	24° 44′	13° 7′	11° 10′	M
16	Copernicus, ibid.	Schöner	1504 March 18	7h 21m	♈ 26° 35′	♈ 5° 36′	7° 34′	20° 59′	19° 1′	V m
17	Longomontanus, Book 2, ch. 20	The Hessians	1593 May 23	9h 30m	♊ 25° 23′	♊ 1° 21′	2° 27′	24° 2′	22° 56′	V m
18	Tycho, in	Tycho	1587 January 19	4h 50m	♒ 17° 50′	♍ 28° 23′	29° 9′	19° 27′	18° 41′	V
19	*Epistolae Astronomicae*, p. 56	Tycho	1587 January 24	5h 15m	♒ 21° 9′	♒ 3° 19′	4° 14′	17° 50′	16° 55′	V
20	—	Tycho	1587 January 25	5h 21m	♒ 21° 22′	♒ 4° 18′	5° 13′	17° 4′	16° 9′	V
21	Longomontanus,	Tycho	1590 March 16	6h 50m	♈ 13° 46′	♈ 3° 57′	5° 56′	9° 49′	7° 50′	V
22	Book 2, ch. 20	Tycho	1592 February 13	5h 40m	♓ 12° 22′	♒ 22° 52′	24° 19′	19° 30′	18° 3′	V
23		Tycho	1593 May 21	9h 30m	♊ 23° 18′	♉ 29° 22′	31° 31′	24° 56′	21° 47′	V m
24	Longomontanus,	Longomontanus	1607 April 25	9h 0m	♉ 21° 8′	♉ 3° 21′	5° 5′	27° 47′	16° 3′	V
25	Book 2, ch. 20		1610 December 15	19h 0m	♐ 2° 44′	♐ 24° 39′	24° 14′	21° 55′	21° 30′	M m

Order	Historians, or Reporters	Observers	Years current before Christ; Gregorian style, month and day	After noon	☿ Observed place	☉ Mean place	☉ True place	☿ Digression from the ☉: mean	☿ Digression from the ☉: true	Kind of digression
26	Lansbergen, *Treasury of Observations*	Hortensius	1626 May 22	10h 0m	♊ 23° 52′	♊ 0° 25′	1° 34′	23° 27′	22° 18′	V
27	Boulliau, Book 10, ch. 1	Gassendi	1625 February 13	6h 20m	♓ 13° 30′	♒ 23° 38′	25° 23′	18° 7′	17° 52′	V m
28	Book 11, th. 13	Boulliau	1630 December 11	19h 50m	♏ 29° 14′	♐ 20° 55′	20° 21′	21° 41′	21° 7′	M
29	Book 10, ch. 5	Gassendi	1631 November 6	17h 24m	♏ 14° 36′	♏ 16° 4′	14° 30′	1° 28′	☉ 6′	M
30	Book 10, ch. 3	Gassendi	1632 July 30	16h 0m	♋ 24° 26′	♌ 9° 11′	8° 10′	14° 55′	13° 54′	M
31	Book 10, ch. 1	Gassendi	1635 September 17	17h 0m	♍ 7° 4′	♍ 26° 48′	24° 52′	19° 44′	17° 48′	M m
32	Book 11, th. 13	Gassendi	1635 December 1	5h 54m	♑ 0° 36′	♐ 10° 14′	9° 19′			V
33	Book 11, th. 13	Gassendi	1636 July 16	9h 0m	♌ 21° 35′	♋ 25° 7′	24° 32′	26° 28′	27° 3′	V m
34	Book 10, ch. 1	Gassendi	1634 October 3	18h 0m	♍ 23° 1′	♎ 12° 51′	10° 53′	18° 50′	17° 52′	M m
35	Book 10, ch. 1	Gassendi	1634 January 3	6h 30m	♒ 2° 35′	♑ 13° 18′	13° 31′	19° 17′	19° 4′	M m
36	Book 10, ch. 1	Gassendi	1635 January 24	18h 30m	♑ 10° 7′	♒ 4° 13′	5° 9′	24° 6′	25° 2′	M m
37		We	1643 March 12	6h 30m	♈ 9° 52′	♓ 21° 8′	23° 3′	18° 44′	16° 49′	V m
38		We	1644 November 29	18h 20m	♏ 17° 47′	♐ 9° 36′	8° 37′	21° 49′	20° 50′	M m

Inquisitio Apogei Veneris Ad Ptolemæi tempus.

Inquiry into the apogee of Venus at the time of Ptolemy.

III. Although Ptolemy's method is not all that exact, yet since we have scarcely a better one a priori, we must try to find, roughly, the place of the line of apsides by bisecting the lesser arc of the zodiac intercepted between two places of the Sun, with Venus or Mercury digressing equally from it on either side. Thus, for example, from what was said in ch. 8, no. 8, the greatest distance of Venus from the Sun's mean place, as corrected by Longomontanus, was 47° 11′ in the morning and 47° 0′ in the evening, and none among them are more nearly equal. Suppose, then, that we bisect the error, which can easily lie hidden, so that each was 47° 5′, and use the Sun's mean places as set down by Longomontanus, which were ♌ 6° 12′ and ♑ 2° 36′. The lesser arc between these is 146° 24′, half of which is 73° 12′. This, then, added to ♌ 6° 12′, gives the line of apsides of Venus in ♎ 19° 24′, though Ptolemy would

have it pass through ♏ 25°. That place is favoured by digressions 2 and 3 of Venus, corrected by us from the Sun's true place, which surpass all the others in nearness to equality: the morning one was 45° 36′ and the evening one 45° 27′, and each may easily be 45° 32′. The Sun's true places, however, were ♉ 26° 44′ and ♓ 17° 2′, and the lesser arc between them was 69° 42′. Half of this, 34° 51′, added to ♓ 17° 2′, gives the line of apsides in ♈ 21° 53′. If the middle value is chosen, therefore, the apogee of Venus seems at that time to have been at 20° 38′ of Aries and the perigee at 20° 38′ of Libra.

[Margin: The apogee of ♀ very different from Ptolemy's.]

It is clear, then, from how far off, as Kepler truly said, this apogee could be discerned from ancient observations, since those observations, not corrected without cause, show it a whole degree and more before Ptolemy's. Granted that Copernicus and others after him, not daring to depart further from Ptolemy, kept it for that time around ♉ 25°, and with that set so tempered the eccentricity, the semidiameter of the orb and the anomaly that this hypothesis satisfied some observed places of Venus. Yet if someone were to achieve the same or a better agreement of calculation by changing these elements and setting the apogee of Venus around ♈ 20° at Ptolemy's time, he would not seem to deserve being hissed off at once, since it was found rather there, by the same method as Ptolemy's and with more correct digressions. Nor is there any doubt that if Ptolemy had used more correct digressions and had established this apogee far elsewhere, his followers would have tried to keep it in that same place as nearly as possible, and would have changed the other elements of the hypothesis as often as it took for the hypothesis to satisfy some selected observations.

Inquisitio Apogei Veneris Ad noſtra tempora.

Inquiry into the apogee of Venus for our own times.

IV. Among the more recent greatest digressions of Venus from the Sun reported in our table, no evening ones can be compared so equally with morning ones as those written below, from which a far different place of the perigee is found, as for brevity's sake you see in the following synopsis; its opposite is the place of the apogee.

[Margin: Evidence for the apogee of ♀ in this century.]

Order of observations	Digressions of ♀	☉ Mean place	Place of the perigee of ♀
13	47° 49′	♎ 3° 49′	♏ 18° 48′
19	47° 48′	♍ 3° 47′	
19	47° 48′	♍ 3° 47′	♐ 21° 12′
32	47° 47′	♐ 8° 37′	
24	47° 1′	♍ 0° 58′	♐ 14° 22′
29	47° 4′	♏ 23° 46′	

We are therefore compelled to wait for other observations, whether ours or others', until two greatest digressions, especially from the Sun's true place,

perfectly equal, one of them morning and the other evening, are made known to us, so that from them a greater light may rise for us in this darkness; and in the meantime to explore which place of this apogee established by other astronomers fits more, and more certain, phenomena.

Inquiſitio Apogei Mercurij Ad Ptolemæi tempus.

Inquiry into the apogee of Mercury at the time of Ptolemy.

V. Among the greatest digressions of Mercury from the Sun, morning on one side and evening on the other, none are more equal to one another than those written below, as will be clear to anyone surveying our preceding table of observations of Mercury. And so, when the lesser arc of the zodiac between the Sun's places is bisected, an apogee of Mercury emerges very different from the one recent astronomers set, as may be seen in the following list, in which, however, we set down the perigee, indicated by the greater digressions; but its opposite indicates the apogee.

[Margin: ☿. The perigee, and from it the apogee, at the time of Ptolemy.]

Order of observations	Digressions of ☿	☉ Mean place	Perigee
7	21° 39′	♒ 10° 30′	♈ 10° 0′
8	21° 35′	♊ 11° 31′	
		☉ True place	
2	26° 36′	♒ 17° 6′	♈ 9° 31′
4	26° 50′	♊ 1° 55′	
6	27° 46′	♌ 10° 54′	♈ 25° 52′
13	27° 46′	♒ 10° 50′	

Therefore, if you prefer the number of testimonies, the first two comparisons favour Ptolemy, who sets the apogee of Mercury in ♎ 10° and the perigee in ♈ 10°. But if you prefer the Sun's true place and the exact equality of the digressions, it seems it must be advanced as far as nearly ♎ 26°.

Inquiſitio Apogei Mercurij Ad noſtra tempora.

Inquiry into the apogee of Mercury for our own times.

VI. Using the same method, you will not find among the preceding greatest digressions of Mercury more suitable comparisons for scenting out its apogee as it fits this century than those written below.

Order of observations	Digressions of ☿	☉ Mean place	Apogee of ☿
34	18° 50′	♎ 12° 51′	♍ 2° 0′
37	18° 44′	♓ 21° 8′	
		☉ True	
27	17° 52′	♒ 23° 38′	♐ 18° 17′
34	17° 52′	♎ 12° 51′	

Order of observations	Digressions of ☿	☉ Mean place	Apogee of ☿
27	17° 52′	♒ 23° 38′	♐ 10° 38′
31	17° 48′	♍ 26° 48′	

Insinuatur triplex ratio aliquid de his Apogeis decernendi.

A threefold way of deciding something about these apogees is suggested.

VII. So far we have hunted the signs and places of these apogees with a looser search, and are not equipped with sufficient observations of whose evidence one can be sure; so we point out a threefold way out of this labyrinth. *The first* way, one well trodden by Longomontanus, Lansbergen, Boulliau, and, as

I think, Kepler, is that we should not, without some enormous reason, depart far from the place of the apogee and the motion established by Ptolemy and Copernicus, but should examine and perfect it a posteriori, from the comparison of observations supporting it in greater or better number. When we have done this, we shall surely find that there are no tables by these authors which yield all the observations, or such and so great a number of them that they must without fear be preferred to the others; nor, likewise, any which cannot defend themselves by some great number of observations, say 10 or 12; and that sometimes the Tychonic or Danish tables come closer to the truth, sometimes the Lansbergian, sometimes the Rudolphine, sometimes Boulliau's.

[Margin: 1. Choice of the apogee of ♀ and ☿.]

Therefore, if someone should put his tables to the test by choosing a mean between these, he will not in the meantime have acted unwisely, roughly in this manner.

Year of Christ		Place of the apogee
1, at the beginning	♀	1ˢ 22° 40′
	☿	6ˢ 10° 0′
1600, completed	♀	3ˢ 0° 0′
	☿	7ˢ 29° 0′
Hence the annual motion of the apogee		
	♀	1′ 24″ 0‴
	☿	1′ 50″ 15‴

[Margin: 2. Relation of the apogee to the beginning of the world.]

VIII. The second way is similar to the one we gave on the occasion of the solar apogee in Book 3, ch. 25, nos. 5 and 6: namely, if we so distribute the motion and the roots of the motion of these apogees that at the beginning of the world they fall on some notable point of the zodiac. There, moreover, we said that the most probable beginning of the world was the year 4063 before Christ. So, multiplying the annual motion of the apogee of ♀ and ☿ chosen here a little earlier by 4063 years, and subtracting the sum from the beginning of the year of Christ, the apogee of ♀ at the beginning of the world would fall at ♒ 17° 35′ 48″ and that of ☿ at ♌ 6° 36′ 14″ 15‴, much too far from the

cardinal points of the zodiac. They could therefore be drawn back, at most, so that the apogee of ♀ would be at the start of ♒ and that of ☿ at the start of ♌; they would then have wholly reversed apsides, and the apogee of one would coincide with the perigee of the other. From this and from other indications arises the third way, to be set out shortly.

IX. Pliny the Elder notes, Book 2, ch. 17, that Mercury and Venus have reversed apsides, but he spoke in another sense; for the apogee of ♀ and the perigee of ☿ are nearer to the solar apogee, as Kepler too noted, Book 6 of the *Epitome of Copernican Astronomy*, Part 3. We have already seen that from the beginning of the world this antithesis of the apogees was not far off, if we embrace the motion and the roots of the more celebrated astronomers. Moreover, from the places of the apogees in nos. 3, 4, 5 and 6 it is evident that some digressions yield, for Ptolemy's century, the apogee of Venus around ♈ 22° and the apogee of Mercury then around ♎ 25°; and now the apogee of Mercury around the start of ♍, where astronomers set the perigee of Venus. What, then, if someone should affirm that these planets have always had apsides plainly and diametrically opposed, and that the apogee of Venus always has been and always will be where the perigee of Mercury was and will be, especially since they have many other opposite properties? With this granted, it would follow that the motion of each apogee is equally great, and the eccentricity, the semidiameter and the anomaly of the orb would have to be so arranged that, taken together, they agreed with all observations, or with the most select. I have not been ashamed to point this out to students of astronomy.

SCHOLIA.

Scholia.

[Margin: Equality of the diameter of the epicycle required.]

I. I said in no. 1 that, to discern the places of the zodiac through which the line of apsides passes, it is required that the semidiameter of the orb or epicycle be the same either always, or at least at the same distance from the apogee of the eccentric. Otherwise the inequality of the diameter in another position could present us with equal digressions, so that the line bisecting the arc between the Sun's places would not be the line of apsides. And indeed Regiomontanus, when in Book 9 of the *Epitome of the Almagest*, props. 8 and 9, he shows that the greatest and contrary digressions from the Sun's mean place are also equal to one another, supposes that the epicycle there, at least, has an equal diameter. Moreover, two greatest digressions, one morning and the other evening, could be equal to each other and yet the line bisecting the arc between the Sun's places not be the line of apsides, if either the eccentricity were unequal or the equations of the eccentric unequal. For those digressions must also be contrary in this respect: that is, the additive equation of the eccentric place in one must be as great as the subtractive equation in the other digression.

[Margin: Geber's censure of Ptolemy.]

Otherwise, as Jābir ibn Aflaḥ shows acutely but at length in Book 7 of the *Astronomy*, from p. 110 in my copy, at almost infinitely many places of the eccentric there can occur two equal digressions from the Sun, one morning and the other evening, in which the centre of the epicycle is not equally distant on either side from the line of apsides. And so he sharply rebukes Ptolemy for thinking that two greatest digressions, one evening and the other morning, suffice to discern the place of the zodiac through which the line of apsides passes. He adds, however, that Ptolemy erred little, not of himself but by accident, in setting the apogee of Mercury at 10° of Libra and that of Venus at 25° of Taurus; I believe because in those digressions he met with an arc between the Sun's places much smaller than a semicircle. For the smaller that arc is, the smaller are the equations of the eccentric and the differences of the equations, and so the less is the danger of straying from the place of the apogee or perigee. And since the equality of these equations cannot be known unless the apogee is known beforehand, which is, however, what is being sought here, no other way remains except through smaller and smaller arcs between two places of the Sun at the time of the digressions of Venus and Mercury; or else an assumed place of the apogee must be confirmed a posteriori by the agreement of more observations.

[Margin: Copernicus's inconsistency on the apogee of ☿ refuted by Mästlin.]

II. There survives in Kepler, in the *Mysterium Cosmographicum*, ch. 19, part of a letter written by Mästlin to Kepler, in which he complains of Copernicus both that he used none of his own observations of Mercury, and that in deducing its apogee he is not consistent with himself. For in Book 5, ch. 26, for around the year of Christ 140, Copernicus sets the apogee of Mercury at ♎ 10°, but at 183° 20′ from the first star of Aries; and in that same 183° 20′ he sets it in ch. 29 for year 21 of Ptolemy Philadelphus, as though it had rested motionless under the sphere of the fixed stars for 400 years, although in ch. 30 it seems to him to have completed one degree in 63 years, provided that motion is taken as equal. Mästlin adds that Reinhold too stuck at this point in the Prutenic Tables: he fitted the place of the apogee at ♎ 10° and at 1[?]3° 20′ from the first star of ♈ to the time of Ptolemy Philadelphus, but for the time of Ptolemy the astronomer placed it at ♎ 15° and 188° 50′ under the orb of the fixed stars, against Ptolemy's observations. Therefore, unless the fixed stars are admitted to move, truly from the equinox or apparently because of the recession of the equinoxes against the order of the signs, these tenets of Copernicus can scarcely be reconciled.

CAPVT X.

Chapter 10.

De Artificio Eccentricitatis, & Commenſurationis orbium Veneris ac Mercurij inueſtigandæ; vbi occaſio iterum de Mercurij Digreſſionibus Maximis.

On the technique for investigating the eccentricity and the commensuration of the orbs of Venus and Mercury; where there is occasion again to speak of the greatest digressions of Mercury.

[Margin: Authors on this technique.]

I. The following treat this matter: Ptolemy, Book 9, ch. 9, on Mercury, and Book 10, chs. 2 and 3, on Venus; Regiomontanus, in the *Epitome of the Almagest*, Book 9, props. 15, 16, 17, 18, 19, 20, and Book 10, props. 1 and 3; Jābir ibn Aflaḥ, Book 7 of the *Astronomy*; Pedro Nunes, on Peuerbach's *Theoricae*, p. 247; Copernicus, Book 5, chs. 21 and 27; Longomontanus, Book 2 of the *Theorica*, chs. 18 and 20; Boulliau, Book 9, ch. 1, and Book 10, ch. 1; and Hérigone, vol. 5 of the *Cursus Mathematicus* book 3 of the *Theorica*, prop. 31. From these, but in a better order and more clearly, we shall carry out our purpose through the following problems.

Problema 1. Proportionem inuenire quam habent Semidiameter Epicycli & Eccentricitas Deferentis ad Radium Eccentrici.

Problem 1. To find the proportion that the semidiameter of the epicycle and the eccentricity of the deferent bear to the radius of the eccentric.

II. What must be given here are the places of the apogee, and so also of the perigee, of the eccentric; and then two greatest digressions of the planet from the Sun, one with the Sun standing in the apogee of Venus or of Mercury, the other with it standing in the perigee, or as near to it as possible. This is taken according to the mean motion in the old astronomy, or according to the apparent motion in the new; and the quantity of the semidiameter of the epicycle is supposed to be always the same. On these assumptions, let BE be the line of apsides, on which let A be the centre of the Earth and of the world, and O the centre of the eccentric, whose diameter is BE, whose semidiameter or radius is OB or OE, and whose eccentricity is AO. Let the centre of epicycle DK be under the apogee, and E the centre of epicycle GM under the perigee. Then draw from A to the circumferences of the epicycles the tangents AD and AG, and join these to the centres of the epicycles by the semidiameters BD and EG. This being laid down:

Diagram. Set into the left column, at the right of the text. A vertical line of apsides runs down the figure. At its head stands a circle with an inner arc about a centre on the line (the epicycle under the apogee); at its foot, a smaller circle with an inner circle about a centre on the line (the epicycle under the perigee). Two straight lines from a point on the axis, A, run up to touch the upper circle at D on the right and down to touch the lower circle at G on the

right. Radii are drawn from the centres out to the tangent points and to one further point in each circle.

- B (on the axis, in the upper circle): centre of the upper epicycle, as the text uses it
- C (upper circle, to the right of and above B): at the end of a line from B
- H (on the axis, just below B): unnamed in the text of this page
- D (right side of the upper circle): tangent point
- K (on the axis, at the foot of the upper circle)
- O (on the axis, midway down, to the left of the line): centre of the eccentric
- A (on the axis, where the lines cross): centre of the Earth and the world
- M (on the axis, at the head of the lower circle)
- L (on the axis, just below M)
- G (right side of the lower circle): tangent point
- F (lower circle, right of E)
- E (on the axis, in the lower circle): centre of the lower epicycle

[Margin: First step of the problem.]

First, in triangle ABD, the angle at D is right because of the contact, by 18 of Book 3 of the *Elements*; and the angle BAD is known from the greatest digression observed when the Sun is under the apogee of the planet in question, under which B lies. Therefore the third angle ABD, which measures the arc DK, is also known, by 32 of Book 1 of Euclid. So from the tables of sines the proportion of the three sides of this triangle to one another will be known, if BD is taken as the whole sine, and AD is sought as the tangent of the angle ABD, that is, KBD, and AB as the secant. Similarly, in triangle AEG, right-angled at G, the angle EAG is known from the other greatest digression; and from that the remaining acute angle GEM, and so the proportion of its secant AE to the whole sine EG. Since, then, BD is supposed to be as great as EG (granted that for another purpose we have drawn them unequal), if the secants AB and AE are joined, the proportion of the whole diameter BE to BD or to EG, as to the whole sine, will be known. And by bisecting BE at O, the proportion of the semidiameter OB to BD will also be known; and the remainder or difference AO will likewise be known in parts of which BD is the whole sine.

Second, then, let the semidiameter OB be taken in another kind of parts, say either as 60 degrees or as a radius of 100000 parts. So let it be made: as OB in the former kind of parts is to OB taken as 60 or as 100000, so BD, the whole sine, is to a fourth term; and the semidiameter BD of the epicycle will be known in parts of which the radius of the eccentric is either 60 or 100000. Again, let it be made: as BO in the former kind of parts is to BO as 60 or 100000, so AO in the former kind is to a fourth term; and the eccentricity will become known in parts of which the radius of the eccentric is 60 or 100000.

EXEMPLVM.

Example.

Ex Ptolemæo in ☿.

From Ptolemy, on ☿.

[Margin: First digression of ☿, with ☉ placed in its apogee.]

III. Ptolemy, Book 9, ch. 8, in year 19 of Hadrian, observed two greatest digressions of Mercury from the mean place of the Sun. One was in the morning, on the day Athyr 14 with the 15th following; and from its distance from Regulus, taken with the astrolabe, he reckoned Mercury to be in ♍ 20° 12′. The mean Sun, for him, was in 9° 15′ ♎, that is, very nearly under the apogee of Mercury. So the angle of digression BAD was 19° 3′, and KBD 70° 57′, whose secant AB is 306379 of the parts of which BD is 100000.

[Margin: Second digression of ☿, with ☉ placed in its perigee.]

The other was in the evening of Pachon 19, when, from the distance of Mercury from Aldebaran, or the southern eye of Taurus, he reckoned Mercury to be in 4° 20′ ♉, and the mean place of the Sun in ♈ 11° 5′, that is, very near the perigee of Mercury. So the greatest digression of Mercury and the angle EAG was 23° 15′, and the angle MEG 66° 45′, whose secant AE is 253229 parts, of which EG is 100000. The whole BE, therefore, is 559608 of such parts, and its half OB 279804, which, subtracted from AB, leaves AO, 26575. Now, as 279804 is to 100000, so BD, 100000, is to 35758. Again, as 279804 is to 100000, so AO, 26575, is to 9498. Therefore, from what Ptolemy gives, the semidiameter of Mercury's epicycle is 35758 parts, and the simple or whole eccentricity 9498, of which the radius of the eccentric is 100000.

[Margin: So also Regiomontanus, Book 9, prop. 15.]

Ptolemy, however, using another kind of parts and a more troublesome trigonometry, in Book 9, ch. 9, concludes that, in parts of which AB is 120, BD is 39° 9′ and AE 99° 9′, the whole BE 219° 9′, and therefore BO 109° 35′ and AO 10° 25′.

EXEMPLVM.

Example.

Ex Ptolemæo in ♀.

From Ptolemy, on ♀.

[Margin: First digression of ♀, with ☉ placed in its apogee.]

IV. The same Ptolemy, Book 10, ch. 2, reports that Theon observed, in year 13 of Hadrian, on Epiphi 2 in the morning, Venus at its greatest digression from the mean place of the Sun, placed in ♉ 25° 24′, that is, very nearly under the apogee B of Venus. From the distances from fixed stars taken by Theon, he concludes that Venus was in ♈ 10° 36′. So the angle of digression BAD was

44° 48′, and the remaining angle KBD 45° 12′, whose secant AB is 141918 of the parts of which BD is 100000.

[Margin: Second digression of ♀, with ☉ placed in its perigee.]

He himself, in year 21 of Hadrian, on Tybi 2, in the evening, observed Venus at its greatest elongation from the mean place of the Sun, placed in ♏ 25° 30′, that is, almost in the perigee of Venus; and from the distance of Venus from the Horns of Capricorn he reckoned Venus to be in ♑ 12° 50′. So the angle of greatest digression EAG was 47° 20′ and MEG 42° 40′, whose secant AE is 135274 of the parts of which the whole sine EG is 100000. The whole BE, therefore, is 277192, and its half OB 138596, which, subtracted from AB, leaves AO, 3322. Let it be made, then: as 138596 is to 100000, so BD, 100000, is to 72094. Again let it be made: as 138596 is to 100000, so 3322 is to 2339. For in this way the semidiameter of Venus's epicycle will come out as 72094 parts, and the eccentricity 2339, of which the radius of the eccentric is 100000.

[Margin: So also Regiomontanus, Book 10, prop. 3.]

But Ptolemy, Book 10, ch. 2, using another kind of parts and his usual roundabout calculations, concludes that, in parts of which AB is 120, the semidiameter of the epicycle of ♀ is 84° 33′ and AE 115° 1′, the whole BE 235° 1′, and the half OB 117° 30′; and that, in parts of which OB is 60, the semidiameter BD is 43° 10′ and the eccentricity AO 1° 15′. But since it was not yet established for Ptolemy from these figures whether the eccentricity so found was the eccentricity of the point about which the planet's motion is uniform, that is, whether it was the eccentricity of the equant or rather of the deferent, another inquiry was needed, of which I shall speak in Problem 4.

[Margin: The author's excuse for neglecting this method.]

We, for our part, from the observations set out above in ch. 9, no. 2, using digressions 24 and 30, in one of which the Sun was at almost the beginning of Capricorn and in the other of Cancer (that is, there in the perigee, here in the apogee of ♀), could have fished out the eccentricity and the semidiameter of the orb. But we were not certain whether 24 was the greatest; indeed, from the motion of the anomaly we found that Venus was not then at the very contact of the orb. So we are waiting, in the hope that, God favouring, as it was possible at the beginning of Cancer, so it may be possible at the beginning of Capricorn to observe the greatest digression of Venus. And we wish for the same in Mercury, with the Sun around the beginnings of Sagittarius and Gemini. Meanwhile, the measure of the orbs will have to be tracked down a posteriori.

Problema 2. Eccentricitatem Mercurij ac Veneris à Sole, & Semidiametrum Orbis eorumdem, Inueſtigare.

Problem 2. To investigate the eccentricity of Mercury and Venus from the Sun, and the semidiameter of their orb.

[Margin: For the hypothesis of the moved Earth.]

V. On the hypothesis of the Earth circling the unmoving Sun, one must investigate the eccentricity of these planets from the Sun, and the semidiameter of the eccentric orb, which is equivalent to the Ptolemaic epicycle, since the Earth's annual orb takes the place of the Ptolemaic eccentric. Let AB, then, be the diameter of the annual orb and at the same time the line of apsides, not of the Earth but of these planets, and let the annual orb AB be described from the centre C; and let D be the centre of the orb of ♀ or ☿. While by mean motion the Earth is at A, under the apogee of ♀ or ☿, so that the centre D is then at its greatest distance from the Earth, let the planet be observed at E, at its greatest digression from the mean place of ☉, or of the Earth. Then let the Earth be at B, under the perigee of the planets, and let the planet then be observed at F, at its greatest digression from the mean place of the Sun or of the Earth.

Diagram. Set into the left column, at the right of the text. A large circle (the annual orb) with a vertical diameter from A at the top to B at the foot. On the diameter, just above the middle, stand C and, below it, D, with a small circle about D. From A a straight line runs down to touch the small circle at E on its left; from B a straight line runs up to touch it at F on its left. Radii join D to E and D to F.

- A (top of the large circle)
- B (foot of the large circle)
- C (on the diameter, at the head of the small circle's area): centre of the annual orb
- D (on the diameter, centre of the small circle)
- E (left side of the small circle, upper): tangent point
- F (left side of the small circle, lower): tangent point

[Margin: First step.]

For when the tangents AE and BF and the radii ED and FD are drawn, the triangles will be right-angled at E and F, by 18 of Book 3 of the *Elements*. So once the angles A and B are known from the digression, the remaining acute angles, namely ADE and BDF, will be known, each being the complement of its own digression to a right angle; and the secants AD and BD of these angles will be known in parts of which the radius ED or FD of the orb has been assumed. When these are added, the whole AB will be known, and, when it is bisected, the half AC, which is the semidiameter of the annual orb; and when this is taken from the secant AD, the difference CD will be known. Now, then, if AC is taken as 100000 parts, or in semidiameters of the Earth corresponding to the Earth's mean distance from the Sun, let it be made: as CA is to the 100000 parts, or to the semidiameters of the Earth just mentioned, so the whole sine DE, say 100000, is to another term. And again let it be made: as CA is to the 100000 parts, or to the semidiameters of the Earth contained in the mean distance of ☉ from the Earth, so DC, known in the former kind of parts, is to another term. The semidiameter DE of the orb will then be known, and its eccentricity from the point C about which the Earth moves uniformly; known,

I say, in parts of which the semidiameter of the annual orb, or the Earth's mean distance, is 100000, or contains so many semidiameters of the Earth.

[Margin: Second step.]

Second, then, with the said eccentricity known, and the eccentricity of the Sun from the Earth, or of the Earth from the Sun, and the angle they contain, which is known from the difference between the places of the Sun's apogee and the apogee of these planets, or from its complement to two right angles, the base subtending this angle will also be known, namely the eccentricity of the centre of the orbs of ♀ or ☿ from the Sun, as will be clear from the figure set out in Section 2, ch. 10, no. 3.

[Margin: Copernicus's conclusion from this problem.]

Further, Copernicus, Book 5, ch. 20, sets out two observations of ♀ taken from Ptolemy, which I have already reported at no. 4. Then, in ch. 21, supposing the Earth at A, under about 25° of Scorpio, so that B is under the apogee of Venus, which was in 25° ♉, and then at B, so that A is the perigee, as regards the mean motion, he concludes, but by another and less easy calculation, that in parts of which AC is 100000, the semidiameter DE of the orb of Venus is 7193, and DC about 208. Similarly, in Book 5, ch. 26, he sets out the two digressions of Mercury that I reported at no. 3, and concludes in ch. 27 that, in parts of which AC is 100000, the semidiameter of the orb of Mercury is 35731, and the eccentricity CD 9479, not without some error of calculation.

But Mästlin, in the notes to ch. 15 of the *Mysterium Cosmographicum*, sets out from Copernicus's data the eccentricities both from the Sun and from the centre of the great orb, which we have reported in the table at the end of ch. 2.

Problema 3. Diſcernere ac determinare Quibus in locis Eccentrici contingant Veneris ac Mercurij Maximæ Maximarum Digreſſionum à Sole.

Problem 3. To discern and determine at which places of the eccentric the greatest of the greatest digressions of Venus and Mercury from the Sun occur.

VI. Supposing, as Ptolemy and the whole of the old astronomy suppose, that the quantity of the epicycle is the same at any place of the eccentric, the greatest digressions from the Sun, that is, those occurring at the contacts of the epicycle, become smaller with the centre of the epicycle standing in the apogee of the eccentric than in the perigee. Even so, it could be doubted whether the greatest of the greatest occur in the perigee or elsewhere. Considering, then, the greatest digressions of Venus, Ptolemy finds that the greatest of all always occur at the perigee of the eccentric, and the least of the greatest at the apogee. But in Book 9, chs. 7 and 8, comparing the greatest digressions of Mercury, he finds that the greatest of the greatest do not occur when the centre of the epicycle is in 10° ♈, where the perigee of Mercury then was, but with the centre placed near 10° ♒, and again near 10° ♊. For the greatest digression near 12° ♈ was found by him to be 23° 15′, whose double is 46° 30′. But with

the centre of the epicycle placed near 10° of Aquarius, he obtained a greatest evening digression of Mercury of 21° 15′ and a morning one of 26° 30′; when these are added, the arc of the epicycle comprehended by the tangents drawn from the Earth comes to 47° 45′. Conversely, at 10° of Gemini, he finds the greatest evening digression of Mercury to be 26° 30′ and the morning one 21° 15′; when these are likewise added, the arc of the epicycle is 47° 45′, certainly greater than the arc with the centre of the epicycle placed in the perigee of the eccentric.

[Margin: Corollaries on ☿ being nearest to the Earth twice a year.]

From these observations, Ptolemy there, and also Jābir ibn Aflaḥ, Book 7 of the *Astronomy*; Regiomontanus, Book 9 of the *Epitome*, props. 16 and 21; Pedro Nunes, on Peuerbach's *Theoricae*, from p. 247 to p. 255; Magini, in the theory of Mercury, ch. 9; and most expositors of this old theory, draw these corollaries. First, that the centre of Mercury's epicycle comes nearer to the Earth at the places of the eccentric distant on either side by a third part of the circle, or by 4ˢ, from the apogee of the eccentric, than in what is commonly called the perigee, that is, than at the point opposite the apogee itself. Second, that Mercury, or at least the centre of the epicycle, comes nearest to the Earth twice a year: formerly, namely, in 10° ♒ and ♊, where it stood a third part from the apogee; but now (if Ptolemy's hypothesis were true) in about 1° ♈ and ♎, where it stands a third part from the apogee. Third, that the eccentricity of the deferent of the epicycle is not the same as the eccentricity of the equant, and that the centre of the deferent is joined with the centre of the equant only once in a year, but at other times moves against the succession of the signs and comes to be more distant from the centre of the world than the centre of the equant. For in this way it can happen that, at the thirds from the apogee, the centre of the epicycle approaches nearer to the Earth than in the perigee; and the rest, of which we spoke in this Section, ch. 1, from no. 4 to no. 8.

[Margin: Copernicus's opinion on the cause of the greater digressions of ☿ away from the perigee.]

VII. I said at no. 6, supposing the same quantity of the semidiameter of the epicycle, in the old astronomy, or of the orb equivalent to it in the new one, which moves the Earth. For if the semidiameter of this epicycle or orb is supposed to vary, so that around the trine from the apogee of the eccentric, or around the sides of the hexagon, it becomes greater than in the perigee, the greatest of the greatest digressions can occur at those places of the eccentric, even if the centre of the epicycle does not approach nearer to the Earth at them than in the perigee. And thus Copernicus, Book 5, ch. 28, attributes that excess of the digressions at those places away from the perigee, not to an approach of the epicycle to the Earth, but to an excess of the semidiameter of the orb, as we shall say at no. 14. For already in ch. 27, as I shall show in Problem 5, he had set the semidiameter of the orb of ☿ at least 3573 parts, mean 3763, greatest 3953, of which the radius of the annual orb is 10000, and had given the law of its variation. Of this I have said enough in ch. 2, no. 10, and ch. 3, no. 4, step

5, where I speak of Lansbergen imitating Copernicus in this, and I shall speak of it again shortly in Problem 6.

[Margin: Kepler's opinion on the same.]

Kepler, however, attributes the reason why Mercury seemed to Ptolemy to approach nearest to the Earth at two places of the eccentric each year to the elliptical figure of the orb, as we shall show in Scholium 2. Supposing Ptolemy's opinion, however, we must give the method of investigating the eccentricity of the deferent and of the equant and of distinguishing them.

Problema 4. Dıſcernere Eccentricitatem Aequantis ab Eccentricitate Deferentis, & Semidiametrum circelli eam in Mercurio ac Venere variantis.

Problem 4. To distinguish the eccentricity of the equant from the eccentricity of the deferent, and the semidiameter of the small circle that varies it in Mercury and Venus.

VIII. In the figure below, let AP be the line of apsides of Mercury or Venus, and on it the centre of the Earth T;

and the centre of the equant, or of the mean motion, C. From C, therefore, let CD be erected perpendicular to AP, so that the centre of the epicycle is a quadrant distant from the apogee at the unequal greatest digressions; for although this is not necessary, it is easier, as Regiomontanus notes, Book 9, prop. 17, and Book 10, prop. 4. On that line, from E, describe the epicycle, and to it draw from T the tangents TV and TM, and to the points of contact draw the radii EV and EM. Join the centres of the Earth and of the epicycle by the straight line TE, and apply, in this position of the eccentric, two greatest digressions of the planet from the mean place of the Sun according to Ptolemy's teachings, one in the evening, of the planet at V, the other in the morning at M.

Diagram. Set into the left column, at the right of the text. A horizontal line of apsides runs along the foot, with A at the left and P at the right. A perpendicular rises from it at C to D at the top. A circle (the epicycle) stands about E on that perpendicular, with D at its head. From T on the base line two tangents rise to the circle, to V on the left and to M on the right. A line rises from T, parallel to CD, to F, on the horizontal chord through E that ends at M.

- D (top of the perpendicular, at the head of the circle)
- E (on the perpendicular, centre of the circle)
- F (right of E, on the chord)
- M (right end of the chord, on the circle): tangent point
- V (left side of the circle): tangent point
- A (left end of the base line)
- K (on the base line, between A and C)
- C (on the base line, at the foot of the perpendicular)
- T (on the base line, right of C)
- P (right end of the base line)

For in the triangles ETV and ETM, right-angled at the contacts V and M, each of the acute angles will be known. When the two greatest and unequal digressions are added, the angle VTM will be known; TE, the common base of the similar triangles, bisects it. So ETM will be known, whose complement to a right angle is TEM, and from this the secant of TEM will be known in parts of which EM is the whole sine. And since EM is known by Problem 1 in parts of which EC, the radius of the eccentric, is 100000, ET will therefore be known in that kind of parts; and in the same parts, in the right-angled triangle ECT, CT will become known.

[Margin: A more universal solution of the problem.]

But if the centre E is not 90° from the apogee, then draw TF parallel to CD, the line of mean motion drawn from the centre of the ecliptic. So if the morning digression is less than the evening one, as is supposed in the figure, then when it is taken from the angle ETM, or from the half of the whole MTV, the angle ETF will be known, since it is known from elsewhere that CT, or EF, is far less than FM, and that the said remainder therefore subtends EF rather than FM. Since, then, EC and FT are parallel, CET will now be known; and ECT is supposed, being the distance of the centre E from the apogee. Therefore in triangle ECT, in parts of which EC is 100000, CT also will be known in the same parts: the eccentricity of the centre of the equant, or of the point about which the motion is uniform. When this is compared with the whole eccentricity treated in Problem 1, it will be established whether and by how much it differs from it. But in this treatise the same semidiameter of the epicycle is supposed at every place of the eccentric; and since we do not suppose that, as was established above, we shall have to proceed by another way.

[Margin: The opinion of Ptolemy, Regiomontanus and Copernicus on the difference between the eccentricities of the equant and the deferent.]

IX. In roughly this way, but with a more troublesome calculation than by secants, Ptolemy, Book 10, ch. 3, and with him Regiomontanus, Book 10 of the *Epitome*, prop. 4, finds the eccentricity of the equant of ♀, or TC, to be about 2° 30′ in parts of which the radius of the eccentric is 60. So, since by Problem 1 he had found the eccentricity of the deferent to be 1° 15′, he concluded that this is about half of that, and that the whole eccentricity of the equant is to be bisected to obtain the eccentricity of the deferent. By a somewhat different reasoning, Ptolemy, Book 9, ch. 9, and Regiomontanus, prop. 18, found that the semidiameter of the small circle varying the eccentricity of Mercury is about a third of the greatest eccentricity of the deferent, that is, 3 parts of which the radius of the eccentric is 60; that the eccentricity of the equant is the same number of parts; and that the centre of the said small circle has an eccentricity of 6 parts, and so has double the eccentricity of the equant, as I have already said in ch. 1, no. 4. But whether this itself is true, Regiomontanus, Book 9, prop. 20, urges us to test by examining whether this hypothesis agrees with many observations of Mercury, especially at the greatest digressions from the Sun; if, however, he says, *you wish to amuse yourself in play.*

[Margin: Copernicus's method for the eccentricity of ☿ and ♀.]

X. But Copernicus, Book 5, ch. 27, says that the whole and greatest eccentricity of the deferent of Mercury's epicycle is 948 parts of which the radius of the eccentric is 10000, but that the diameter of the small circle varying the eccentricity is 424 parts, and its semidiameter 212. To demonstrate these he uses the right-hand part of the following diagram, in which AB is the line of apsides of Mercury, and on it C is the centre of the Earth's annual orb, and I the centre of the small circle that varies the eccentricity of Mercury's orb and places its centre now at D, the point of greatest eccentricity, now at F, the point of least eccentricity, the one due to the equant itself, so that FD is the diameter of the small circle varying the eccentricity. Further, with the perpendicular CE drawn from C, which is to be the semidiameter of the annual orb, with the Earth placed at E, a quadrant distant from Mercury's apogee A, and with the semi-orb OHGQ of Mercury described from F, he draws to it the tangents EH and EG with the radii FH and FG, and finally he draws the straight line EF from the centre of the Earth to the centre of the orb.

Diagram. Set across the right column. A vertical line runs from A at the top to B at the foot, with O, C, F, I, D and Q on it in that order downward. A circle about F passes through O at its head and Q at its foot, with P on it at the upper left and H and G on its right; its right half is the semi-orb OHGQ. A small circle about I stands between F and D. From E, far to the right on the level of C, straight lines run to H and G and to F and C. From M, far to the lower left, straight lines run towards the upper arc and across the figure, with L near F; a dotted line runs from M to N, near the foot of the vertical.

- A (top of the vertical)
- O (on the vertical, top of the arc)
- H (upper right of the arc): tangent point
- C (on the vertical): centre of the annual orb
- K (right of C)
- P (upper left of the arc)
- F (on the vertical, below C)
- E (far right): the Earth
- L (left of F)
- I (on the vertical, in the small circle)
- D (on the vertical, below I)
- G (lower right of the arc): tangent point
- Q (on the vertical, below D)
- N (lower left, at the end of the dotted line)
- M (far lower left)
- B (foot of the vertical)

Further, taking from Ptolemy two greatest but unequal digressions of Mercury, observed when the Sun, and accordingly the Earth, stood a quadrant from the apogee of Mercury as regards mean motion, of which the morning one was 20° 15′ and the evening one 26° 15′, he concludes from them the whole

angle GEH, 46° 30′, and the half HEF, 23° 15′. And taking the smaller angle, that of the lesser digression, from the angle HEF, he concludes the angle CEF, 3°. Therefore in triangle CEF, right-angled at C, the remaining acute angle CFE is given, 87°, whose secant is FE, tangent CE, and whole sine CF. So, with CE, the radius of the annual orb, set at 10000 parts, he concludes FE as 10014 such parts, and from the proportion of the whole sine to the tangent CE he concludes CF as 524 parts of which the radius CE is 10000. So when CF is taken from the whole eccentricity of the deferent, which, as I said, in Problem 2 he finds to be 948 parts, there is left the diameter FD of the small circle, 424 parts, the semidiameter FI, 212, and the whole CFI, 736.

[Margin: Copernicus's view on the eccentricity of ♀.]

The eccentricity of Venus, moreover, he derives in Book 5, ch. 22, by a similar method from the greatest digressions at the mean distance from the apogee, but Ptolemaic ones; and he concludes that the whole eccentricity, or that of the equant, namely CD, is double the eccentricity of the deferent he had concluded in ch. 20, which here shall be CF. For he found this to be 208 parts, but CD to be 416 parts, and therefore the diameter FD of the small circle 208 parts, the semidiameter FI 104, and the distance of the centre of the small circle from the centre of the great orb, that is, CI, 312 parts, of which the radius of the annual orb is 10000. He ends chapter 22, however, by saying: *All these things that have been demonstrated about Venus are found to agree also with our times, so that what was formerly 416 parts in all is now 350, as many observations teach us.* Hence Kepler, ch. 15 of the *Mysterium Cosmographicum*, and Rheticus, in his first *Narratio*, infer that Copernicus believed that the eccentricity of Venus had decreased, as had the Sun's.

Finally, Copernicus holds that the motion of the centre of the eccentric on the small circle FD, in both ♀ and ☿, is double the mean motion of the Earth.

Problema 5. Inueſtigare an & quantum Semidiameter orbis Mercurij in Apogeo & Perigeo Eccentrici, differat à Semidiametro in Medijs diſtantijs.

Problem 5. To investigate whether and by how much the semidiameter of Mercury's orb in the apogee and perigee of the eccentric differs from the semidiameter at the mean distances.

XI. I said at no. 7 that Copernicus, and with him Lansbergen, the variation in the digressions of Mercury that are greater outside the perigee than at the perigee, explain away by a varied size of the epicycle. Copernicus investigated this variation in Book 5, ch. 27, in the immediately preceding diagram. For in triangle EHF, right-angled at the contact H, with angle HEF of 23° 15′, as we said at no. 10, he gathers by Problem 2 that the semidiameter of the orb FH, in this position of the Earth, namely when it is at mean longitude, or distant from the apogee of Mercury by a quadrant, is 3947 parts of which EF is 10000; but of which EF is 10014 and CE 10000, FH is 3953. Above, however, by Problem

2, when the Earth was at the apogee or perigee of Mercury, he had gathered the semidiameter of this orb to be 3573 parts of which the semidiameter of the annual orb CE is 10000. He therefore sets the difference of these semidiameters, shown by the line HK, at 380 parts, and accordingly the mean semidiameter at 3763. But although at the mean distances of the centre of the epicycle Mercury describes its largest orb, it does not for that reason undergo there the greatest of its greatest digressions, but lower down, around the thirds [trientes], because of its greater nearness to the Earth. At the perigee, on the other hand, where it ought to undergo the greatest, it does not undergo them, because it describes its smallest orb there.

[Margin: The threefold size of the semidiameter of the orb of ☿.]

Problema 6. Digreſſiones Mercurij maiores extra Perigeum, ex varia Orbis quantitate cum Copernico demonſtrare.

Problem 6. To demonstrate with Copernicus, from the varying size of the orb, that the digressions of Mercury are greater outside the perigee.

[Margin: Copernicus's three suppositions for the hypothesis of ☿.]

XII. Copernicus states in Book 5, ch. 28, that around the sides of the hexagon Mercury makes greater digressions from the mean place of the Sun than at the perigee of the eccentric, that is, at the places of the eccentric orb where it is distant from its apogee by 60° or 120° on one side or the other; indeed, greater than at the mean distances. This holds provided that three things are assumed. First, that the centre of the orb is carried round on the small circle of the anomaly of the eccentricity, with a double revolution to one revolution of the Earth itself (which for others is the mean motion of the Sun), beginning from the apogee of the small circle. Next, that Mercury librates along a straight line in the semidiameter of the orb, but according to the laws of another, imaginary small circle, whose anomaly likewise is double the mean motion of the Earth, and that it therefore describes now a larger, now a smaller orb around the Sun. Third, that the centre of this small circle is carried by the orb of Mercury having the mean semidiameter. With these things assumed, look at the left-hand part of the preceding diagram, in which the line of apsides of ☿ is AB, and on it the apogee A and the perigee B; C is the centre of the annual orb, and the Earth is at M, making angle MCB of 60°, so that it thus sees, on the opposite side along the line MC, the Sun distant in its mean place from A, the apogee of Mercury, by the same number of degrees; for in this way ACM will be 120°. Meanwhile the centre of the orb on the small circle FD, with its double revolution, that is, of 120° counted from D, will have completed the arc DL, subtended by angle DIL, and let the centre of the orb be at L. From L let the half-orb OPNQ be described, and to it from M let the tangents MP and MN be drawn, together with the radii LN and LP to the points of contact, at which Mercury is supposed to be when it digresses furthest from the mean place of the Sun. Finally, from M let two straight lines be drawn, one, ML, to the centre of the orb, the other,

MI, to the centre of the small circle FD. With these things assumed, it will be established that in this position, that is, at a hexagonal distance of the Sun from the apogee of Mercury and of the Earth from the perigee, the digressions are greater than at the perigee, as will appear presently in the Example.

EXEMPLVM Demonſtrationis Copernicanæ.

Example of the Copernican demonstration.

[Margin: 1. Step of the demonstration.]

XIII. First, since CI was shown by Copernicus to be 736 parts of which MC, the radius of the orb of the Earth, is 10000, as I said at the end of Problem 4, and angle MCB, that is MCI, has been set at 60°, from this angle and the sides CM and CI there follows, by the analysis of triangles, the third side MI of 9655 parts, and angle CMI of 3° 47′. By this amount the interior angle CIM, by 32 of Book 1 of Euclid, is less than the exterior angle MCA, now set at 120°. Therefore CIM is 116° 13′. But angle LID is also given as 120°, and accordingly FIL, or CIL, by 13 of Book 1 of Euclid, is 60°. When this is subtracted from angle MIC, there remains angle MIL of 56° 13′. With this, and with MI, already found to be 9655 parts, and IL, the semidiameter of the small circle, shown from what was said at the end of Problem 4 to be 212 parts, angle LMI of 1° 4′ is gathered trigonometrically. When this is taken from angle CMI, found above to be 3° 47′, there remains angle CML of 2° 43′. By this angle the centre of the orb of Mercury is separated from the mean motion of the Sun, indicated or seen from the Earth along the line MC; and from the same data side ML is gathered, of 9540 parts. Therefore in triangle MLP, side ML is known, and angle P at the contact is right, by 18 of Book 3 of Euclid. So, in order that we may know how large the angle PMI of greatest digression is in this hexagonal position, we must inquire into the size of the semidiameter of the orb, that is, LP.

[Margin: 2. Step.]

Second, then, let the star of Mercury be understood to librate on the small circle ADBF drawn alongside, whose diameter AB, from what was said in Problem 5, is 380 parts and its semidiameter CB 190, of which the semidiameter of the annual orb of the Earth is 10000. Let this diameter be understood to be a portion of the line PL of the preceding figure, whose anomaly is according to the laws of this small circle, just as if it took place on its circumference, and with as much motion as is the motion of the small circle performing the anomaly of the eccentricity, that is, double the mean motion of the Sun, counted from A. So in our case let the arc ADH of 120° be counted. When HK, the sine of arc HB, which is 60°, has been drawn, BK will be cut off, a versed sine of 95 parts, as is clear from 15 of Book 5 and 12 of Book 13 of Euclid's *Elements*, or from the tables of sines. For CK, the second sine of arc HB of 60°, is 5000 of which CB is 10000; therefore KB, the remainder of the radius, is 5000. So it will come out that as CB, 10000, is to 190, so KB, 5000, is to 95. Accordingly

the remainder AK will be 285 parts, namely the portion to be added to the smallest semidiameter of the orb, 3573, on which see Problem 5. And thus in the earlier figure the semidiameter PL comes out at 3858 parts, of which CE, or CM, is 10000.

Diagram. A small circle set into the right column, with a vertical diameter from A at the top to B at the bottom, and three horizontal chords crossing it: an upper one through I, the horizontal diameter through the centre C, and a lower one through K.

- A (top of the circle): end of the diameter
- G (left end of the upper chord)
- I (on the vertical diameter, at the upper chord)
- L (right end of the upper chord)
- D (left end of the horizontal diameter)
- C (centre)
- F (right end of the horizontal diameter)
- H (left end of the lower chord)
- K (on the vertical diameter, at the lower chord)
- M (right end of the lower chord)
- B (bottom of the circle): end of the diameter

[Margin: 3. Step.]

Third and finally, in triangle MPL, right-angled at P, from the sides ML, 9540 by step 1, and PL, 3858 by step 2, the angle PML of the greatest digression of Mercury from the mean place of the Sun is gathered as 23° 52′, and accordingly the whole angle PMN, from the morning and the evening digression, as 47° 44′, when the mean place of the Sun is distant from the apogee of Mercury by 60° and the Earth by 120°, that is, there in a trigonal, here in a hexagonal position. But when the Sun or the Earth is at the apogee or perigee of Mercury, the angle of digression consisting of the morning and the evening, by the Ptolemaic observations, on which see us in Problem 1, no. 3, and Copernicus Book 5, ch. 26, was only 42° 18′, because the evening one was 23° 15′ and the morning one 19° 3′. And when the Sun was at the mean distance from the apogee, or distant by a quadrant from the apogee of Mercury, the aggregate of the greatest morning and evening digression was 46° 30′, as is clear from the observations of Ptolemy, on which see no. 10, at the end of Problem 4. Therefore the greatest digressions of Mercury from the Sun are greater in the hexagonal position than at the perigee, and indeed than at the mean distance from the apogee, which was to be shown.

[Margin: Conclusion of the demonstration.]

[Margin: Copernicus's obscurity made clear.]

XIV. What has been said throws light on the beginning and the end of ch. 28 of Book 5 of Copernicus, which otherwise seemed obscure. For he begins thus: *From this, too, it will seem less surprising that around the sides of the hexagon of the circle Mercury makes greater digressions than at the perigee, since they are even greater than those we have just demonstrated, so that in one revolution of the Earth its*

orb was believed by the ancients to become nearest to the Earth twice; that is, because it makes greater digressions than those which in ch. 27 he had taught occur at the mean distances. And he concludes ch. 28 by saying: *So here it has become greater than both by one degree and 14′* [parte vna & ſcrup. XIIII.], *not because the orb of the planet is nearer to the Earth than it was at the perigee, but because the planet describes a larger circle here than there. All of this agrees with present as well as past observations, and flows from equal motions.* He said *than both* [vtroque] because by a slip of the pen he had written the sum of the perigee digressions as 46½° [Gr.XLVI. S.] in place of 44½° [XLIV. S.]; for that is how this copy is to be emended from ch. 26. So the aggregate of the hexagonal digressions is greater than the quadrantal ones by 1° 14′, but than the perigee ones by 3° 25′. It is also clear why in Book 5,

[Margin: *Metodum* [?] in Copernicus, and from it a slip of Copernicus, is corrected.]

ch. 25, he said that Mercury, especially in the time of Antoninus, used to make its smallest digressions from the Sun in Libra, and greater ones in Aries, yet not the greatest, which it makes in ♒ and ♊.

[Margin: Longomontanus's opinion on the eccentricity of ♀ and the semidiameter of the epicycle.]

XV. Let us now move on to Longomontanus. We do not repeat his diagrams here, which are sufficiently set out in ch. 4, nos. 2 and 3, because he shows the eccentricity of these planets, or the semidiameter of their orbs or epicycles, with no demonstrations offered a priori. Instead he works only a posteriori, testing which values best fit certain observations, and finally adopts those so tested, as if they would satisfy all observations. Since we have already set out this method of his, with our critique of it, on the occasion of the apogee, together with the observations he used, in ch. 8, from no. 7 to no. 10, there is no reason to repeat it, and we therefore send the reader there. It is enough to repeat that the semidiameter of the orb of Venus is set by him at 72248½, and the whole eccentricity at 3208, of which the radius of the eccentric is 100000; but the semidiameter of the orb of Mercury at 38500, and the whole eccentricity at 5685 parts.

[Margin: Kepler's measures and foundations.]

XVI. What Kepler determined on this point is gathered from the *Mysterium Cosmographicum*, the Rudolphine Tables and the *Epitome of Copernican Astronomy*. At p. 761 of that work he indicates the method of investigating the eccentricity that we set out beforehand in Problem 2. But at pp. 732 and 765 of the same work, and Hérigone after him in vol. 5, p. 765, he expresses the eccentricity of the centre of the ellipse from the Sun in parts of which the semidiameter of the planet's orb is 100000. In the same parts he sets the distances of the planet from the Sun, the other focus of the ellipse; among these the mean distance from the Sun is as large as the greater semidiameter of the ellipse. Boulliau, on the other hand, in Book 1 of the *Astronomia Philolaica*, expressed the Keplerian values in parts of which the semidiameter of the annual,

or great, orb of the Earth itself is 100000. He sets them, I say, as fitted to the harmonic proportions and intervals, and to the regular solids of which he had spoken in the preface to the *Mysterium Cosmographicum*: *The Earth is the circle that measures all; circumscribe a dodecahedron about it, and the circle enclosing this will be Mars. Circumscribe a tetrahedron about Mars, and the circle enclosing this will be Jupiter. Circumscribe a cube about Jupiter, and the circle enclosing this will be Saturn. Now inscribe an icosahedron in the Earth, and the circle inscribed in it will be Venus. Inscribe an octahedron in Venus, and the circle inscribed in it will be Mercury. You have the reason for the number of the planets.* It seems good, then, to gather here into a table, from Kepler, the eccentricities and the distances from ☉ of all these.

Planet	Eccentricity from ☉, in parts of which the radius of the planet's orb is 100000	Eccentricity from ☉, in parts of which the radius of the annual orb is 100000		Distance from ☉, in parts of which the radius of the annual orb is 100000
			Aphelion	1005207
♄	5700	54207	Mean	951000
			Perihelion	896793
			Aphelion	544708
♃	4822	25058	Mean	519650
			Perihelion	494592
			Aphelion	166465
♂	9263	14115	Mean	152350
			Perihelion	138235
			Aphelion	101800
♀	694	500	Mean	100000
			Perihelion	98200
			Aphelion	72900
☿	21000	8149	Mean	72400
			Perihelion	71900
			Aphelion	46955
Earth	1800	1800	Mean	38806
			Perihelion	30657

66

Kepler himself openly teaches, at p. 732 of the *Epitome of Copernican Astronomy*, that the measures of these orbs which we have set out in the little table were taken from Copernicus, and corrected and worked through by him to the harmonic intervals of the world, and at the same time to distances from the Sun that fit the five regular solids. He says: *These intervals have been built up from observation of the parallaxes of the orb, on which see below; but I have explained their archetypal causes in Book 4, fol. 454, from the* Mysterium Cosmographicum*, and fol. 471, from Book 5 of the* Harmonics. But more on these intervals in Book 9, where the harmonic system is treated. Meanwhile, however, I have chosen to give together the Keplerian eccentricities and distances from the Sun of the 5 planets, and the eccentricities in parts of which 100000 is both the radius of the planet's orb, as Kepler has it in the *Epitome* and Hérigone has it, and the radius of the annual orb of the Earth, as Boulliau has it and as Kepler himself assumes in the tables of equations of the Rudolphine Tables, though not without some disagreement with himself.

[Margin: Boulliau's inquiry into the eccentricities of ♀ and ☿.]

XVII. Boulliau, in Book 9, ch. 1, of the *Astronomia Philolaica*, chooses four greatest digressions of Venus from the Sun, two observed by Tycho and two by Iustus Byrgius [Jost Bürgi], instrument-maker to the Landgrave of Hesse. I have already reported these at ch. 8, no. 13, in the earlier little table along with the others, but they must now be separated from them, and there must be added from Boulliau both the equated anomalies of the Sun, and the distances of the Earth from the Sun corresponding to those anomalies, and the remaining things required for this business. The sequence of the observations, in the style of the new calendar, with the hours after noon reduced to the meridian of Uraniborg, is therefore as follows.

	Year	Month	Day	Time
I.	1592	March	2	$17^h\ 30^m$
II.	1594	December	27	$5^h\ 0^m$
III.	1600	February	21	$20^h\ 27^m$
IV.	1601	May	9	$8^h\ 30^m$

Look now at the following figure, in which the annual orb of the Earth, described from the centre O, is BCAD, and the line of apsides of the Sun and of the Earth is ♈ ♋. The orb of Venus, on the other hand, is ST, described from the centre I; F is the Sun; the eccentricity of the annual orb from the Sun

[66] The table is given as printed. In its last three rows the distance groups stand one row out of step with the eccentricities beside them: in the other rows each group differs from its mean by the second eccentricity (54207, 25058, 14115), whereas the group printed against ♀ differs by 1800, the Earth's figure; that against ☿ by 500, Venus's; and that against the Earth by 8149, Mercury's.

is OF, and that of the orb of Venus IF; and the line of apsides of Venus is ST, whose aphelion is S and perihelion T.

Diagram. Set in the right column. A large circle with a smaller circle inside it, off-centre towards the lower left of the large circle's interior. A vertical line runs from the top of the large circle to B at its foot. From the point at F straight lines radiate to points on both circles, including the tangents the text names.

- sign (top of the large circle, just right of the vertical line): two stacked loops on an upright stem; not identified [?]
- D (upper right, on the large circle)
- A (right, on the large circle)
- C (lower right, on the large circle)
- B ♋ (foot of the large circle)
- V (lower left, at the small circle)
- S (upper left, on the small circle)
- T (foot of the small circle)
- G (top of the small circle, right of the vertical line)
- *a* (italic, right side of the small circle)
- *d* (italic, right side of the small circle, below *a*)
- O (just left of the vertical line's lower half, above F)
- I (left of O)
- F (below the point from which the lines radiate)
- ♀ (large sign, left of I, inside the small circle)

Now Venus at the first digression was at G; at the 2nd, at V; at the 3rd, at *a*; at the 4th, at *d*. And the Earth at the 1st was at A, seeing Venus in ♍ along the tangent AG; at the 2nd it was at B, seeing Venus in ♒ along the tangent BV; at the 3rd it was at C, seeing Venus along the tangent C*a*, in ♍; finally, at the 4th the Earth was at D, seeing Venus along the tangent D*d*, in ♋. So when the already mentioned tangents have been drawn from the Earth to the places of Venus, and these places have been joined to F, which is barely perceptibly distant from the centre I of the orb of Venus, the angles at the points of contact come out, by 18 of Book 3 of Euclid, imperceptibly different from right angles. Therefore, given the true places of the Sun seen from the Earth along the lines AF, BF, CF and DF, and compared with the places of Venus, the angles of the greatest digressions of ♀ will be given, whose complements to the remaining right angle are the angles at the Sun; and from this the places of ♀ seen from the Sun will be established. And given the equated anomalies of the Sun, the distances of the Sun from the Earth will be given from the Philolaic table of the Sun, in parts of which the radius of the annual orb is 100000.

All of these stood as follows, in the order in which I listed the observations.

[Margin: 1. Step of the demonstration.]

True places of the Sun	Places of ♀ seen from the Earth
♓ 13° 58′ 37″	♍ 27° 17′ 36″
♍ 5° 48′ 51″	♒ 23° 1′ 36″

True places of the Sun	Places of ♀ seen from the Earth
♓ 3° 8′ 12″	♍ 16° 22′ 12″
♉ 19° 4′ 17″	♋ 4° 27′ 50″

Angles of digression of ♀ from ☉	Places of ♀ seen from ☉
FAG 46° 41′ 1″	♎ 27° 17′ 33″
FBV 47° 12′ 45″	♉ 23° 1′ 36″
FC*a* 46° 46′ 0″	♎ 16° 20′ 32″
FD*d* 45° 23′ 53″	♎ 4° 27′ 50″

Equated anomaly of ☉	Distances of the Earth from ☉
8^s 7° 34′ 24″	AF 99328
6^s 0° 18′ 48″	BF 99216
7^s 26° 40′ 44″	CF 99027
10^s 12° 44′ 33″	DF 101205

[Margin: 2. Step.]

Second, since in the already mentioned triangles, right-angled at the contacts, the angles of digression are given, and the bases, which are the distances of the Earth from the Sun, the distances of Venus from the Sun, in parts of which the radius of the annual orb is 100000, will not be hidden from a trigonometer. They stand in order as follows.

In the triangles	Distances of ♀ from the Sun
AFG	FG 72267
BFV	FV 72077
CF*a*	F*a* 72148
DF*d*	F*d* 72052

[Margin: 3. Step.]

Third, given the places of ♀ seen from ☉, and compared with the apogee of ☉ and its distance, the arcs on the circle of Venus corresponding to the coequated anomaly will be given. These arcs become known according to the technique indicated in Book 3, ch. 24, in method 4, and in this Book 7, section 2, ch. 8, no. 9, on the circle of the diagram below, in which let the line of apsides be MV and the aphelion V, and let the first observation be at A, the second at B, the third at Y and the fourth at D. For if the arcs AD, DB and BA are known from the differences of the arcs corresponding to the anomaly, the angles at the circumference will also be known, since they are half the angles at the centre, by 20 of Book 3 of Euclid's *Elements*; and the chords of those arcs, which are the sides of the triangles below, will not be hidden, from the tables

of sines. According to Boulliau the arcs, angles and chords or sides are as you see in the following synopsis.

Diagram. Set into the left column with the text running down its left side. A circle with a vertical diameter from V at the top to M at the foot and a horizontal diameter from C at the left to T at the right, crossing at I. Chords join B, D, A and others, and several short horizontal and vertical lines are ruled inside the upper right quadrant.

- V (top of the circle)
- O (on the vertical diameter, just below V)
- D (upper right of V, on the circle)
- B (upper left, on the circle)
- N (on the vertical diameter, level with B)
- E (right of N)
- X (upper right, on the circle)
- s (small italic letter, inside, below E and X)
- Y (right, on the circle, below X)
- H (on the vertical diameter, below N)
- F (inside, below s)
- G (on the vertical diameter, below H)
- A (right, on the circle, level with G)
- C (left end of the horizontal diameter)
- I (centre)
- L (on the horizontal diameter, right of I)
- K (on the horizontal diameter, right of L)
- T (right end of the horizontal diameter)
- ♀ (below the horizontal diameter, left of the vertical)
- M (foot of the circle)

Arcs	Angles	Sides
AD 22° 49′ 46″	ABD 11° 24′ 53″	AD 19792
DB 131° 26′ 14″	BAD 65° 43′ 7″	DB 91152
BA 205° 15′ 40″	BDA 102° 37′ 50″	BA 97489

[Margin: 4. Step.]

Fourth, as Boulliau taught in Book 1 of his *Astronomia*, the lines drawn from the umbilicus or focus of the ellipse, which here is the Sun, to the periphery increase from the point of perihelion to the quadrant in the ratio of the versed sines, and from the quadrant to the aphelion in the ratio of the right sines, and they decrease from the aphelion V to the focus in the ratio of the versed sines. Therefore the versed sines will be VG for the first observation, VN for the second, VH for the third and VO for the fourth, and the differences will be OG 216 for the first and fourth and ON 25 for the second and fourth. So, recalling the method given in Book 3, ch. 24, method 4, and Book 7, Section 2, ch. 8, no. 9, the ratio of the eccentricity of Venus and of the semidiameter of each

ellipse to the radius of the annual orb, and the aphelion, will be found in like manner through the following proportions.

[Margin: 5. Step.]

Fifth, then, as OG, or DF, is to DA, so ON, or DE, is to DS, of 2300 parts. From this angle DBX is gathered as 1° 24′ 3″ and arc DX as 2° 48′ 6″, being double it by 20 of Book 3 of Euclid; therefore BDX will be 134° 14′ 20″, and BV 67° 7′ 10″. But point B is the place of Venus seen from the Sun, which was ♉ 23° 1′ 36″. From this Boulliau gathers the aphelion in ♒ 0° 8′ 46″.

[Margin: 6. Step.]

Sixth, since arc DV is 64° 19′ 4″ and DT 25° 40′ 56″, with right sine DL 43358, and AT is 2° 51′ 10″, with right sine FL 4977, the remainder DF is therefore 38361. So let it be that as DF, or OG equal to it, 38361, is to the difference in the other kind of the parts OG, which we said is 216, so the radius 100000 is to 566; and that would be the size of the eccentricity of the orb of Venus. From these he gathers the perihelial distance of Venus as 71738, the mean as 72304 and the aphelial as 72870, of which the radius of the annual orb is 100000. But in ch. 2, from other observations of Venus, with similar obscurity and method, he gathers the aphelion of ♀ in ♒ 5° 19′ 43″, the eccentricity VI as 575 parts, the mean distance of Venus from the Sun as 72398 parts, the perihelial as 71823 and the aphelial as 72973, which he retains in ch. 4. By not dissimilar reasoning in Book 10, using, however, confirmations a posteriori through the agreement of the assumed hypothesis with the observations, he draws out the eccentricity of Mercury from the Sun as 8105 parts, the aphelial distance as 46690 parts, the mean as 38585 and the perihelial as 30480, of which the radius of the annual orb is 100000.

[Margin: Boulliau's conclusion on the eccentricity and the measures of the orb of ♀ and ☿.]

XVIII. Among other diagrams he uses this following one, which it may suffice to indicate, from his Book 10, ch. 1. In it the line of apsides of Mercury is PF, on which I is the centre of the annual orb PBF, and D the Sun, the focus of the ellipse; LEN is the orb of Mercury, DN the perihelial or least distance of Mercury from the Sun, and DL the aphelial and greatest. The half-difference of these reveals the eccentricity of the centre of the ellipse from the Sun, while their sum, halved, reveals the greater semidiameter of the ellipse and the mean distance from the Sun. The Earth is placed at three places: namely at A, from which it sees the Sun along AD, and along the tangent AE sees Mercury at E, there digressed furthest from the Sun; the difference of the true place of the Sun and the true place of Mercury, that is, the greatest digression, is angle DAE; and angle ADE is the difference of the place of the Earth seen along DA, and of Mercury along DE, seen from the Sun. Next, the Earth at B sees Mercury at K, and sees the same from C. But having sampled this indication, let us hurry on to other things.

Diagram. Set across the right column. A large circle with a vertical diameter from P at the top to F at the foot, and a smaller circle inside it, off-centre to the

right. From the point at D straight lines run out to C, A, E, B and other points; one line, from D through N towards L, is dotted.

- P (top of the large circle)
- A (upper right, on the large circle)
- L (on the smaller circle, on the dotted line)
- E (right, on the smaller circle)
- ☿ (large sign, inside the large circle, left of the smaller circle)
- I (on the vertical diameter)
- D (below I, where the lines meet)
- C (left, on the large circle)
- N (lower left, on the smaller circle, on the dotted line)
- G (below N)
- K (lower right of G)
- B (lower right, on the large circle)
- F (foot of the large circle)

XIX. But we, using observations selected chiefly from the preceding chapter, were never able to exhibit all of them by any change of a new hypothesis, much less by the hypotheses of others, although that labour wore us out with 300 analyses of triangles for Venus and nearly 500 for Mercury. So, compelled by necessity, we chose those measures which, of the more certain observations, all or most would yield, leaving the final refinement to the third volume, in case an abundance of the most certain observations should compel us to change the measures and numbers. Meanwhile, using the method a posteriori, we have judged that the eccentricity of Venus is by no necessity different from the solar eccentricity, and is therefore always 3460 parts of which the radius of the eccentric is 100000; and that the whole eccentricity of Mercury is 10000 parts, the mean 8000 and the least 6000, provided that it varies by libration on the diameter of the small circle, according to what was said in ch. 6.

[Margin: Our opinion on the eccentricity of ♀ and ☿.]

We noticed that the semidiameter of the epicycle of Venus varies not at all, or not very perceptibly, from the apogee of the eccentric down to the mean longitudes, and that as long as the centre of the epicycle is in the upper part of the eccentric, it is 73140 parts or as close to that as possible; but that when the centre of the epicycle is below the mean longitudes, it varies and shrinks more and more until it becomes least at the perigee, about 70140 parts; and that this variation takes place in the ratio of the second sines corresponding to the complement of the anomaly of the eccentric to the semicircle, or to its excess above the semicircle.

[Margin: And on the semidiameter of the epicycle of ♀.]

Finally, by the same inquiry the semidiameter of the epicycle of Mercury was found to be least at the apogee and at the perigee, 38500 parts, but greatest at the mean longitudes, about 45500 parts; and this variation takes place in the ratio of the second sines corresponding to the anomaly of the eccentric, or to its complement to the circle, if the centre of the epicycle is in the upper

part of the eccentric; or corresponding to the complement of that anomaly to the semicircle, or to its excess above the semicircle, if it is in the lower part. Granted, we know that some few observations resist this hypothesis; but they either reject all analogy with the many others, or require a far larger number of observations than those that exist so far, in order that the hypothesis may be fitted to them too.

[Margin: And on the semidiameter of the epicycle of ☿.]

SCHOLIA.

Scholia.

[Margin: Copernicus's excessive confidence in Ptolemy.]

I. Certainly trusting too much in Ptolemy's observations, Copernicus judged that the measures and the hypothesis, of Mercury especially, could satisfy all ages; and in Book 5, ch. 30, he did not hesitate to say: "We think it must be granted that the commensurations of the circles have remained from Ptolemy even now, since in other matters too they are not found to have misled the good earlier authors." And Mästlin, in a letter to Kepler printed after ch. 19 of the *Prodromus*, says that this is a monstrous assumption, since the eccentricity whether of the Sun or of the Earth, and indeed of Mars and of Venus too, having varied in Copernicus's opinion, gives rise to a suspicion of variation in the others. Kepler, however, who denies that the Sun's eccentricity has varied, is free of this suspicion as to the remaining planets.

[Margin: Cause of the digressions of ☿ in Aquarius and Gemini, according to Kepler.]

II. Kepler refers the reason why Ptolemy posited two places of the eccentric in Mercury, at which the centre of the epicycle seems to come closer to the Earth than at the perigee, to the elliptical figure that ☿ describes around the motionless Sun, and to the motion of the Earth. That is, when the Earth was in the 10th degree of Aries, formerly the perigee of Mercury, and the Sun at 10° of Libra, the shortest part of the line of apsides was turned toward the Earth, namely the perihelion of Mercury, situated in the 4th degree of ♉. So the Earth viewed the eccentric ellipse, which for Ptolemy was the epicycle, as trimmed at the sides with its lunules cut off, and therefore the epicycle appeared very small and more elevated. Likewise, when the Sun was seen to pass to Aries, or the Earth had crossed to Libra, the epicycle was again seen with its sides narrowed, the aphelion of Mercury tending toward the Earth, since the Earth was in ♎ and the aphelion of ☿ at about 4° of ♏. Therefore one cause of the epicycle's apparent smallness at the apogee remained at the perigee as well, namely the shortness of the transverse diameter of the ellipse. But conversely, when the Sun appeared in ♒ or in ♊, and the Earth had been carried to the 10th degree of ♌ or ♐, the greater diameter of the ellipse was presented to sight less obliquely, just as if the epicycle had become larger.

[Margin: Kepler's censure of the observations of Ptolemy, as begging them and dragging them to his own opinion.]

So Kepler, at any rate; yet he adds that the calculated numbers do not represent this exactly on both sides, and that Mercury does not wander in ♒ to the same extent as in ♊. He therefore asserts that the observations were sought out by Ptolemy and, as it were, begged from the ancients; and that, since they had been recorded crudely, they were easily fitted so that what happened in one third from the apogee of the eccentric would happen in the other third also. In Venus, the cause Kepler adduces ought to have presented the same thing; but because of its very small eccentricity from the Sun, the ellipse of Venus falls imperceptibly short of a perfect circle.

[Margin: The motions of ☿ and ♀ are neither circular nor uniform in actual fact, even in the hypothesis of Copernicus.]

III. From what has been said, especially about Mercury, it can be established that the celestial motions which Copernicus and his attendant Lansbergen so doggedly maintain to be both circular and uniform are not in fact such. For that libration of Mercury along the diameter of the small circle, by which the diameter of the orb varies, certainly takes place along straight lines, according to the increments and decrements of the second and versed sines, which are unequal in equal degrees of the periphery. And from that motion, together with the motion of the orb carrying that small circle or its diameter, or rather carrying Mercury as it nods away from or toward the Sun, Mercury must describe a kind of spire with a remarkable inequality of real motion. Copernicus seems tacitly to have met this objection in Book 5, ch. 28, with the words "all of which flow together from uniform motions". He did not say "come about", since no real part of the orb of Mercury moves along the periphery of that small circle, by which the recession from and approach to the Sun occurs; at most the motive force of Mercury, an Intelligence, say, looks to that small circle shining back in its own mind, just as we contemplate it on our sheets of paper, so as to receive from it the law for moving the star of Mercury away from the Sun or toward the Sun. Similarly, in Boulliau's hypothesis, rules are drawn out by means of a circle feigned in the imagination, for increasing or diminishing the lines of the ellipse according to the ratio of the sines of a circle in which there is nevertheless no real motion. And the same holds proportionally in Venus.

CAPVT XI.

Chapter XI.

De Motu & Epocha Anomaliæ mediæ Orbis ſeu Epicycli Mercurij & Veneris conſtituendis.

On establishing the motion and epoch of the mean anomaly of the orb, or epicycle, of Mercury and Venus.

[Margin: How many kinds of motion ♀ and ☿ have.]

I. It must be recalled that Venus and Mercury have a twofold principal motion (for the motion of the centre of the eccentric on the small circle, the motion of libration on the other small circle of Mercury, and the motion of the apogee of the eccentric are less principal than these). The first is that of the eccentric, by which, in the hypothesis of the Earth at rest, the centre of the epicycle of these planets moves; and this is as great as the mean motion of the Sun, and is begun from the same starting point and root. But in the hypothesis of a moving Earth it is the mean annual motion of the Earth itself, for in that hypothesis the annual orb plays the part of the ancient eccentric. If this motion is reduced by equations to the true or apparent motion, it then becomes irregular and anomalous. Therefore, by subtracting the motion of the apogee of these planets from the true motion of the Sun or Earth, the residual motion is truly called the anomaly of the eccentric, because it is irregular owing to the irregularity of the true motion of the Sun or Earth. But if the motion of the apogee is uniform in itself and is subtracted from the mean motion of the Sun or Earth, as is done in the older astronomy, it can be called the anomaly of the eccentric only by a kind of imperfect resemblance, since it has in it no anomaly or irregularity. The second motion is that by which the circumference of the Ptolemaic epicycle, or of the Copernican eccentric orb that takes the place of that epicycle, moves carrying the planet with it, or by which the planet moves on the epicycle or orb; and since this motion is irregular it is called the motion of the anomaly of the orb. Insofar as this motion exceeds the mean motion of the Earth, Copernicus, in Book 5, ch. 1, calls it the motion of commutation and the anomaly of commutation. The sum of this motion and of the mean motion of the Sun or Earth makes up the mean motion of longitude of these planets, and it is this that our discussion and our last labour concern in determining the second inequality of the motion in longitude of these planets.

[Margin: Authors on the investigation of this motion.]

How this is done is taught above all by Ptolemy, Book 9, ch. 10 and 13, on Mercury, and Book 10, ch. 4 and 5, on ♀, although he had proposed such motions earlier in Book 9, ch. 3; likewise by Regiomontanus, Book 9, from prop. 2 to 5 and props. 20, 22 and 23, on Mercury, and Book 10, prop. 6, on Venus; Copernicus, Book 5, ch. 22, 23, 29 and 30; Longomontanus, Book 2 of the *Theorica*, ch. 18, 19, 20, 21; Boulliau, Book 9 of his

Astronomia, ch. 2 and 8, on ♀, and Book 10, ch. 9 and 10, on Mercury. From these I shall briefly indicate the technique of collecting this motion, first crudely, then more finely.

[Margin: 1st precept, for finding the motion of the anomaly of the orb.]

II. First, count the greatest digressions from the Sun that immediately succeed one another and are of the same kind, that is, all morning or all evening, so that it may be known how much time passes from one morning digression to the next morning one made immediately after it, or from one evening digression to the next evening one. These times will indeed be found unequal; but from many revolutions of this kind (and the more of them you obtain from your

own and others' observations, the better), by choosing a middling one between the fastest and the slowest, you will be certain that within the given number of completed years only so many whole revolutions can be accomplished. For example, in 8 years no more epicyclic revolutions of Venus than 5, and in 46 years no more of Mercury than 145.

[Margin: 2nd precept.]

III. Second, from digressions observed in antiquity and from those observed recently, but with many centuries between the former and the latter, choose two of the same size and of the same kind, made at the same place of the eccentric, or at the same anomaly of the eccentric, with the Sun placed at the same anomaly of its own, one ancient and the other recent. Of this sort would be two morning digressions, each of 46° in ♀, or of 26° in ☿, if in both cases the Sun or the Earth were then distant from the apogee of these planets by 30° in the order of the signs, or were at the apogee of the eccentric of these planets, or at the perigee. For by Precept 1 it is established how many whole revolutions were made in so great an interval of years, but more crudely; in this way, by the rule of proportions, the same thing can be known more finely. For with these things set down, the planet has returned to the same point of the epicycle and the same distance from the apogee of the epicycle, and nothing remains over and above the whole revolutions. But if it is difficult to obtain two such digressions with the conditions already mentioned, use the following precept.

[Margin: 3rd precept.]

IV. Third, then, choose two very accurate observations of the true place of the same planet, especially when it has digressed furthest from the Sun, one very ancient and distant by a long interval of years, the other new, near the apogee or perigee of the eccentric. From the given commensurations of the orbs, and from the mean or even the true motion of the Sun or Earth, and from the observed true place of the planet, you will be able, according to whichever form of hypothesis you have chosen for the purpose (say the Ptolemaic, or the Copernican, or the Tychonic, etc.), to collect the angle of the prosthaphaeresis of the orb, and from it the arc of the epicycle by which the planet stood distant in both observations from the mean apogee of the orb or epicycle. That arc will be the very mean anomaly of the orb which you seek. This will also help in establishing the epoch of this anomaly. By distributing, then, the number of whole revolutions with its remainder, if there is any beyond the whole ones, over the number of years, days and hours by which the recent observation is distant from the ancient one, according to the rule of proportions, you will learn both how great the mean or uniform motion on the epicycle is, appropriate to any given time, and conversely how much time is required for completing a given arc of the epicycle by uniform motion.

[Margin: 4th precept.]

V. Weigh one pair or triple after another of observations of the place of Venus or Mercury, according to the form of hypothesis you have chosen, applying the motion of the anomaly of the orb found so far by the preceding precepts,

and see whether the difference of anomaly, that is, between the anomalies of the orb applied to the individual observations, agrees with the observations, or whether something should be added to or subtracted from them. The better observations for this will be those of pairs of greatest digressions, one evening and the other morning, taking their sum; and of these, better still are those made at the apogee or at the perigee of the eccentric, or at the mean longitudes, or pairs of digressions for each of the three places mentioned.

[Margin: An objection is dispelled, with a particular warning.]

For although the disagreement with the observations may be attributed not wholly to a badly established anomaly of the orb, but to the eccentricity, or the semidiameter of the orb, or the motion of the Sun, all of which require some filing, yet no other way remains for us except to examine, by more and more observations, the lines and numbers of the motions assumed in this way a posteriori. But if you see that some observations can be reconciled by no change with others more certain, and keep no analogy with them, you will rightly suspect the observations of being deceptive. Now it seems that some light should be added to the precepts by an example.

VI. But an example of the first precept is not needed, for Ptolemy, Book 9, ch. 3, from Hipparchus, and Copernicus, Book 5, ch. 1, have already handed down to us both the time owed to many whole revolutions and the revolutions corresponding to definite sums of days and hours, as I reported in Section 2, ch. 9, in connection with the three superior planets. The mean motions then derived by them and corrected by others I have indicated sufficiently in the small tables placed at the ends of the chapters, after the explanations of each of the hypotheses of the distinguished astronomers. Whatever else you may want for this I shall give at the end of this chapter. Let us therefore give an example for the other precepts, and in the form of the Copernican hypothesis, to which many are less accustomed; for from it you will easily gather by what reckoning and method to proceed in the others.

EXEMPLVM

3. Præcepti pro ♀.

In Forma Copernicæa, præmiſſa figuræ delineatione.

Example of Precept 3 for ♀, in the Copernican form, with the delineation of the figure set out first.

[Margin: Delineation of the Copernican figure.]

VII. Suppose one or even two greatest digressions of Venus from the mean place of the Sun to have been observed, one morning and the other evening, when the Earth by its mean motion was distant by 45° from the apogee of the eccentric of Venus; and let the morning digression be 47° 20′ and the evening one 45° 40′. You will then draw the figure thus. Bisect at B the line of apsides of Venus AD, whose apogee is A and perigee D; from B describe the Earth's

annual orb AED; and having counted from A, in the order of the signs, 45° of the anomaly of the eccentric, that is, the arc AE, place the Earth at E, and join it to the centre of the annual orb, or of the mean motion of the Earth, by the straight line BE, which will be the radius of the annual orb, taken by Copernicus to be 10000 parts. Next, taking BK of 312 parts, by which, in Copernicus's opinion, the centre of the small circle producing the anomaly of the eccentricity is distant from the centre of the Earth's annual orb, from centre K, at the interval CK of 104 parts, describe the small circle CFL, if the observation is ancient and close to the times of Ptolemy; but if it is recent and nearer to the time of Copernicus, let BK be 246 parts and CK, as above, 104. Since the centre of the orb of Venus moves on the periphery of this small circle from its perigee C toward the same parts toward which the Earth moves, but with a doubled motion, and the mean motion of the Earth has been set at 45°, you will count twice as many degrees, that is 90, from C to F, that is, the arc CF; and from F as centre you will describe the orb of Venus INHM, whose diameter IH you will draw parallel to the line of the Earth's mean motion, that is, to line BE. And with the star of Venus set at N, the point of morning digression, and at M, the point of evening digression, you will draw to

Diagram. The Copernican delineation for Venus, set in the right column. A large circle, the annual orb, has A at the top and D at the bottom, joined by the vertical line of apsides, with B near the middle of that line. E stands on the upper left of the large circle, with lines running from it down to the inner figures. A smaller circle, the orb of Venus, lies within, carrying I at upper left, N at the left and M at the right, each with a star-shaped mark, H at the lower right, and G just below its lower edge beside the line of apsides. A much smaller circle lies around K, with F on its left and L below it, and a small C at the top of the small circle, just below B. A dashed line runs from P on the upper right of the large circle, through the inner figures, to Q at the lower left of the large circle.

- A (top of large circle): A
- D (bottom of large circle): D
- E (upper left of large circle): E
- P (upper right of large circle): P
- I (upper left of orb of Venus): I
- N (left of orb of Venus, star-shaped mark): N
- M (right of orb of Venus, star-shaped mark): M
- H (lower right of orb of Venus): H
- G (just below the lower edge of orb of Venus, beside the line of apsides): G
- B (on the line of apsides): B
- K (centre of the small circle): K
- F (left of the small circle): F
- L (below the small circle): L
- C (top of the small circle, just below and left of B; set small): C

• lower end of the dashed line: Q it from the Earth the tangents EN and EM; and you will join the points of contact by the straight lines FN and FM with the centre of the orb of Venus, so that angles N and M become right, by III.18 of Euclid's *Elements*. Therefore, with EF drawn from the Earth to F, the centre of the orb, triangles ENF and EMF will be right-angled at N and M. Finally, from the Earth to the centre of the small circle you will draw the straight line EF [recte: EK], and through the centres of the orb of Venus and of the annual orb the straight line PBFQ, and KF, the semidiameter of the small circle. The portion BF will be the eccentricity of the centre of the orb of Venus.

[Margin: Distinguishing the true and the mean apogee in the hypothesis of Copernicus and of Ptolemy.]

With this done, G will be the true apogee of the orb of Venus, at which Venus, if she were there, would be furthest from the Earth placed in such a position; for this apogee, like the true apogee of the epicycle in the Ptolemaic hypothesis, is marked by the straight line EFG, drawn from the centre of the Earth through the centre of the orb or epicycle to the opposite point of its periphery. But the mean apogee will be H, marked by the line IH, as being parallel to line BE of the Earth's mean motion, drawn through the centre of the mean motion of the planet, which in this hypothesis is the motion of the Earth, whose annual orb supplies the place of the Ptolemaic eccentric. In the same way, in the Ptolemaic hypothesis the mean apogee is marked by a line drawn from the centre of the equant eccentric, that is, from the centre of mean motion, through the centre of the epicycle to the opposite point of the periphery of the epicycle. Now suppose, around the times of Ptolemy, the apogee of Venus A at 25° of Taurus; for thus the Earth E will have been, from the observations set down, at ♋ 10°, as being 45° distant from A, and from there it will see along line EB the mean place of the Sun under the 10th degree of ♑. Therefore Venus at N, in the morning, distant from that place by 47° 20′ against the order of the signs, and seen from the Earth along line EN, was at ♏ 12° 50′. But Venus at M, in the evening, distant from the same place by 45° 40′ in the order of the signs, and seen along line EM, was at ♒ 25° 40′. With these things properly understood, let the places of Venus be examined by the following method.

EXEMPLVM.

Calculi Geometrici.

Example of the geometrical calculation.

[Margin: Step 1.]

VIII. First, in triangle KEB, the radius BE, 10000, is given, and BK of 246 parts, and angle KBE of 135°, as the complement to two right angles of angle ABE, set at 45° as the anomaly of the eccentric. So by the laws of triangles KEB will be 1° 18′ [recte: 1° 8′, per the book's errata], and therefore BKE 43° 52′, by I.32 of the *Elements*, and the base KE 10203 parts.

[Margin: Step 2.]

Second, in triangle KEF, side KE of 10203 parts is now given, and FK, the semidiameter of the small circle, of 104 parts; and angle FKE is known by subtracting BKE, found to be 43° 52′, from the doubled anomaly of the centre, that is, from angle CKF of 90°. So FKE is 46° 8′. Therefore, by trigonometry, angle KEF is 0° 25′ and side EF 10133.

[Margin: Step 3.]

Third, adding angles BEK and KEF gives angle BEF of 1° 33′, which is the prosthaphaeresis of the centre, to be subtracted (since the Earth is within the first semicircle of the anomaly of the eccentric) from the mean place of the Sun, which was ♍ 10°; and there remains the eccentric place of Venus, that is, the place of the centre F, seen along line EF, at ♍ 8° 27′. But Venus at M, in the evening, is taken as observed at ♒ 25° 40′. So the difference of the true place of Venus from the place of the centre, that is, the equation of the orb, was 47° 13′, namely angle MEF. Conversely, Venus in the morning at N was observed at ♏ 12° 50′. So the difference between the apparent place of Venus and the place of the centre was 55° 37′, namely angle FNE, which follows from the data of the fictitious example, even though in fact the equation of the orb of Venus can never be 55° 37′.

[Margin: Step 4.]

Fourth, in triangle MEF, angle MEF has now been found, 47° 13′, by the third step (or, from Copernicus, the semidiameter FM of the orb of Venus is 7193), and the angle at M is right, because this was a greatest digression. So, without side EF, angle EFM can be found at once, as the complement of angle MEF to a right angle; EFM is therefore 42° 47′. But if the observed digression were not greatest, or you were unsure of it, use side EF, found by step 2 to be 10133 parts, and with it and the semidiameter FM and angle FEM, find angle EFM; add to it angle EFI, which is as great as BEF, that is, 1° 33′, because of the parallelism of lines BE and HI, which makes the alternate angles equal, by Euclid I.29. For the whole angle IFM results, which here is 44° 20′, and so great is the arc MI; when it is subtracted from the semicircle IMH, there remains the arc of the mean anomaly of the orb MH, 45° 40′, as counted from the mean apogee of the orb in the order of the signs. But if you wished to consider Venus placed at N, with angle NFE found as above, angle EFI would have to be subtracted from it, and the remaining angle IFN, that is, arc IN, would have to be increased by the semicircle HMI, so that the arc of mean anomaly HMIN, computed from the mean apogee H in the order of the signs, might become known.

[Margin: Use of the already mentioned calculation, and conclusion of the procedure.]

IX. It is clear, then, by what device, even from a single place of Venus properly observed, whether at a greatest or at a not-greatest digression, the mean anomaly corresponding to the time of observation is to be gathered. So in this way let two places of Venus, one very ancient, the other new, be

observed after a long stretch of years, and from them let the mean anomaly be found; from the number of years you will gather by the first precept the whole revolutions of this anomaly, and besides them the remainder, if there is any. Thus if in the second observation the anomaly HMN was found to be 140°, and in the earlier one HM 45° 40′, apart from the whole revolutions, the remaining arc MIN will be 94° 20′. You will then reason like this: if in so much time, or so many years, days, hours and minutes (the time, that is, being resolved into minutes), Venus beyond the whole revolutions completed 94° 20′, then in one year or one day (first resolved into minutes) how many degrees and minutes will it complete on its orb by uniform or mean motion? And you will proceed similarly with Mercury.

[Margin: A universal rule for any hypothesis using circles.]

In other hypotheses too, after finding the equation of the centre, and from it the place of the centre and the distance of the centre of the orb or epicycle from the Earth, what has to be sought is not the equation of the orb, since that is known from the true observed place of Venus and from the place of the centre. Rather, from that equation, the semidiameter of the epicycle and a right angle, if it is a greatest digression, or from the already mentioned distance, the arc of the epicycle must be sought, subtended by the angle made by that distance and by the semidiameter of the epicycle. To this arc the equation of the centre must be added, if it was subtractive with respect to the centric place; and then the complement of the arc so increased to the semicircle will be the desired mean anomaly of the orb. But if the equation of the centre was additive, it must be subtracted from the already mentioned arc of the orb or epicycle, and to the remainder of the arc a semicircle must be added, to obtain the mean anomaly of the orb.

[Margin: Mean motion of ♀ and ☿.]

Moreover, when the digressions reported in ch. 9, no. 2, are examined by this technique, the mean annual motion of Venus is found to be 225° 1′ 53″ 35‴, and that of Mercury 53° 57′ 33″ 32‴, so that in Venus Lansbergen, and in Mercury Kepler and Longomontanus, are in this respect nearer the truth.

De Epocha motus huius.

On the epoch of this motion.

X. On establishing the epochs of this motion there is no need to say more. For once the mean anomaly has been found from some exact observation made at a definite time, and from its mean motion corresponding to the given interval of time, the place of the planet on the epicycle, computed from the mean apogee, that is, the anomaly of the orb, is easily carried over to any other time. And we have set our own roots at the end of ch. 6.

De Periodis motuum Veneris ac Mercurij.

On the periods of the motions of Venus and Mercury.

[Margin: Authors from whom the following table comes.]

XI. This section must be closed with a periodic table of the motions, which I promised here at no. 6 and in Sect. 2, ch. 9, no. 1. It was supplied to us, according to the mind of Ptolemy and of the Alphonsines, by al-Farghānī, diff. 17; Reinhold on Peuerbach's *Theoricae*, pp. 105 and 130; Wurstisen in the same place, p. 198; Maurolico at the end of the *Cosmographical Dialogues*; Clavius at the end of the *Sphere*; Anton Deusing and Gassendi in the *Institutio Astronomica*, Book 2, ch. 7; and Hérigone in vol. 5 of the *Cursus Mathematicus*, pp. 534 and 541. But either from their own opinion or according to the mind of Copernicus, it was supplied by Magini, Canon 33 of the *Secunda Mobilia*, and from him by Baranzanus, part 2 of the *Uranoscopia*, p. 158; by Kepler, ch. 1 of the *Mysterium Cosmographicum* and in the *Epitome of Copernican Astronomy*, p. 760; by Langius in the *Theoricae Planetarum*, ch. 12 and 13; and by Antonín Maria Šírek in the *Radius Sidereomysticus*, Book 1, ch. 2, and Book 3, ch. 1. The motion of these planets on the epicycle can, moreover, be considered together with the motion of the Sun, whose mean motion concerns their system and is common to them, as something intrinsic to their whole motion, as happens in the Ptolemaic hypothesis. On this reckoning the motion of Venus on the epicycle takes about 19 months, or 584 days etc., and that of Mercury about 4 months, or 115 days. For in that much time, by their mean motion on the epicycle, but with the Sun's motion supposed to advance meanwhile, they revolve from the mean apogee of their epicycle to the same mean apogee, from one mean and apogeal conjunction to the next apogeal one, and from one greatest digression to another of the same kind, for example from one evening digression to another evening one. But if their motion is considered around the immobile ☉, according to Copernicus, and the motion of the Earth is not their motion but an extrinsic parallax, or commutation of sight, then Venus runs through its epicycle, or rather its orb, by mean motion in 224 days etc., and Mercury in about 88 days; that is, the former in 7 ½ months, the latter in about three months. In that time Venus, it is true, appears to the moving Earth to complete the whole zodiac, whereas Mercury, because of its speed and narrow orb, does not appear to the Earth to have completed it, except in so far as it appears direct. But here are the tables promised more than once above.

Revolution on the epicycle with the Sun, or around the Sun as mobile, considering the mean motion [REVOLVTIO in Epicyclo cum Sole vel circa Solem Mobilem, ſpectato motu medio]

According to	♀ Days	♀ Hours	♀ ′	♀ ″	♀ ‴	☿ Days	☿ Hours	☿ ′	☿ ″	☿ ‴
al-Farghānī	584	about				116	about			
Reinhold, Wurstisen	583	22	15	0	0	115	21	5	0	0
Maurolico, Clavius	583	22	12	0	0	115	21	5	0	0
Magini, Baranzanus						115	21	5	0	0
Langius	583	22	10	38	51	115	21	3	20	54
Antonín Maria Šírek	583	22	0	0	0	115	21	12	0	0

According to	♀ Days	♀ Hours	♀ ′	♀ ″	♀ ‴	☿ Days	☿ Hours	☿ ′	☿ ″	☿ ‴
Ourselves	583	22	10	0	0	115	21	3	30	0

Revolution in its own orb around the Sun taken as immobile [REVOLVTIO in Orbe proprio circa Solem ſumptum vt Immobilem]

According to	♀ Days	♀ Hours	♀ ′	♀ ″	♀ ‴	☿ Days	☿ Hours	☿ ′	☿ ″	☿ ‴
Gassendi	225	about				88	about			
Magini and Baranzanus	218	22	15	0	0					
Antonín Maria Šírek	224	17	26	0	0	87	23	45	0	0
Kepler, relative to the fixed stars	224	17	53	2	14	87	23	15	36	0
Kepler, relative to the ecliptic	224	17	44	55	14	87	23	14	24	0

SECTIO QVARTA

Section Four

DE PASSIONIBVS QVINQVE PLANETARVM MINORVM SECVNDVM LATITVDINEM.

On the affections of the five minor planets according to latitude.

CAPVT I.

Chapter 1

Obſeruationes, & Hypotheſes Veterum Aſtronomorum de Latitudine Quinq. Planetarum.

Observations and hypotheses of the ancient astronomers on the latitude of the five planets.

[Margin: Three kinds of passions in the planets.]

I. The affections, or affections and properties, of these planets belong to them in some cases according to latitude, that is, their recession from the ecliptic; in others according to longitude; and in others, finally, according to altitude. I have therefore decided to set out the doctrine on them separately, in the three sections that follow.

[Margin: The reason for the order in explaining these passions.]

We begin with latitude, both because, as Copernicus observes in the preface to Book 6, the digressions of these stars from the ecliptic make no small difference to their appearances, their occultations at rising and setting, and the other things to be discussed next; and because their true places are said to be known only when their longitude has been established together with their latitude. Latitude therefore had to be joined to longitude at once. Moreover,

the aids and instruments by which their longitude is observed will also serve to observe their latitude, whether by astrolabes or zodiacal armillaries, by the Arabic torquetum, by azimuthal quadrants, by Tychonic sextants, with which the distance of a planet from two fixed stars of known longitude and latitude is taken, or by whatever other instrument among those I have set out in my *Liber Organicus*, which, God willing, will be the second book of the second volume of my *Almagest*. What, then, the ancients, and Ptolemy above all, observed by these means concerning the latitude of these stars and the inclinations of the orbit to the ecliptic, or also of the epicycle to the plane of the orbit; and by what theory they themselves, or the Alphonsines and Peuerbachians, used to show these alternations, I shall gather in this one chapter, from what has been handed down to us by Ptolemy, Book 13, ch. 1 to 6; al-Battānī, ch. 47; al-Farghānī, diff. 18; Jābir ibn Aflaḥ, Book 9 of the *Astronomy*; Proclus in the *Hypotyposes*, ch. 3, 4, etc.; Peuerbach in the theory of the latitude of the planets, and there above all Nonius, annotation 9; Wurstisen from p. 373; Reinhold from p. 215; Franciscus Sarzosus [Francisco Sarzosa] in the *Aequator Planetarum*, Book 2, ch. 21; Regiomontanus in the *Epitome of the Almagest*, Book 13; Copernicus, Book 6; and Magini, Book 2 of the *Theoricae* from ch. 3 to 11, and Canons 49 and 50 of the *Secunda Mobilia*.

De Latitudine Obſeruata, eiuſq. conditionibus in tribus Superioribus Planetis.

On the observed latitude and its conditions in the three superior planets.

II. When the latitude of these planets had been observed at various times, partly by his predecessors and partly by himself, Ptolemy detected the following properties.

[Margin: 1st property of the latitude of ♄, ♃, ♂.]

First, the latitude of the three superior planets is northern whenever the centre of the epicycle is in that segment of the eccentric in which the apogee of the eccentric lies, and southern when it is in the segment in which the perigee lies.

[Margin: 2nd property.]

Second, they stay longer in northern latitude than in southern.

[Margin: 3rd property.]

Third, the greatest northern latitudes occur around the apogee of the eccentric, and the southern around the perigee; yet the southern are greater than the northern, that is, the perigeal than the apogeal.

[Margin: 4th property.]

Fourth, other things being equal as regards position on the eccentric, the latitudes, both northern and southern, are greater when the planet is at the perigee of the epicycle, and so near opposition with the Sun and in acronychal position, than at the apogee of the epicycle, or near conjunction with the Sun. From this and the preceding it follows that the greatest latitudes of all are those

southern ones which occur when the centre of the epicycle is placed in the lower or perigeal segment of the eccentric, and the planet at the perigee of the epicycle.

[Margin: 5th property.]

Fifth, Ptolemy found in his time the limit of greatest northern latitude: that of Mars near the end of Cancer, where he had also found its apogee, very nearly; that of Jupiter 20° from the apogee of the eccentric in the order of the signs, or around the beginning of Libra; and that of Saturn 50° away, but in præcedentia of the apogee of the eccentric, which is likewise around the beginning of Libra, since he believed the apogee of Mars to lie at the end of Cancer, that of Jupiter at about the 10th degree of Virgo, and that of Saturn at the 20th of Scorpio. From this it is inferred that Mars is at the apogee of the eccentric and at greatest northern latitude at the same time, and conversely undergoes its greatest southern latitude at the perigee of the eccentric; but that Jupiter's epicycle reaches the apogee of the eccentric before it reaches the limit of greatest northern latitude, and likewise reaches the perigee of the eccentric before the limit of greatest southern latitude; and conversely that Saturn's epicycle reaches the limit of greatest and northern latitude much sooner than the apogee, and by as much sooner the limit of greatest southern latitude than the perigee of the eccentric.

[Margin: 6th property.]

Sixth and last, comparing the places of the most ancient limits with the places found in his own age, Ptolemy recognized that the places of the limits, and those a quadrant away from them, the places of the nodes, move with a very slow motion in the order of the signs, and indeed with a motion imperceptibly different from the motion of the fixed stars and apogees, that is, 1° in 100 years. The Alphonsines accepted this property, but adapted it to their own motion of the fixed stars.

Theorica Ptolemaica & Alphonſina Latitudinis Planetarum Superiorum.

The Ptolemaic and Alphonsine theory of the latitude of the superior planets.

III. From the properties already mentioned, with the eccentric and epicycle now taken as assumed for demonstrating the motion in longitude, Ptolemy and all the Arabs after him, with the Alphonsines and Peuerbachians, laid down the following hypothesis on the motion in latitude, which I shall set out point by point.

[Margin: 1st part of the hypothesis.]

First, the planes of the eccentrics that carry the epicycles cut the plane of the ecliptic at the centre of the world, so that the greater segment of the eccentric is inclined towards the north and the lesser towards the south. The axis of the eccentric of Mars, lying very nearly in the same plane as the axis of the ecliptic, nearly intersects the axis of the ecliptic, or rather is inclined to it in as nearly

the same plane as possible; but the axis of the eccentric of Saturn and of Jupiter does not lie in the same plane as the axis of the ecliptic but in a very different one, because the points of greatest latitude do not lie at the apsides themselves, not even very nearly, as in Mars. Moreover the two diametrically opposite points at which the plane of the ecliptic is cut by the eccentrics are called nodes (as was said of the Moon). At them no latitude occurs, and they are always 90° on either side from the limits, or termini, of the greatest latitudes.

[Margin: 2nd part.]

Second, the angle of inclination by which the eccentric of these planets is obliquely inclined to the ecliptic is constant and of one and the same quantity. This angle arises at the centre of the world and is gathered from the greatest latitude, the semidiameter of the deferent, and the semidiameter of the Sun's annual orb, as I shall show below. By his calculation Ptolemy found it to be 2° 27′ in Saturn, 1° 24′ in Jupiter, 1° 0′ in Mars. This inclination of the eccentric is called, to distinguish it from the inclination of the epicycle, [Greek: ἔγκλισις τοῦ ἐκκέντρου (enklisis tou ekkentrou, "inclination of the eccentric")].

[Margin: 3rd part.]

Third, since at the same places of the eccentric a differing greatest latitude was found, none at the nodes, and a greater one at the true perigee of the epicycle than at its apogee, this phenomenon could not be satisfied by the inclination of the eccentric alone. Recourse had to be had to an inclination of the plane of the epicycle to the plane of the eccentric, one not fixed or constant, however, but variable. The Ptolemaics and Alphonsines therefore laid down that the epicycle librates on its axis, that is, on a straight line drawn through the centre of the epicycle to the points on the epicycle's circumference a quadrant distant from the true apogee of the epicycle, at which the mean longitudes from the aux of the epicycle occur. For this reason they call this axis the diameter of the mean longitudes, while the straight line bisecting it at right angles is called the diameter of the apsides of the epicycle. This libration is carried out as follows. When the centre of the epicycle is placed at the nodes, the plane of the epicycle coincides with the plane of the ecliptic, and so no latitude arises for the planet at any point of the epicycle; yet the diameter of the mean longitudes then declines from the plane of the deferent. As the centre of the epicycle recedes from the northern node, the plane of the epicycle gradually recedes more and more from the plane of the eccentric, towards the north as regards the perigee and towards the south as regards the apogee of the epicycle, until at the northern limit the inclination becomes greatest. After that, the recession of the plane of the epicycle from the plane of the deferent diminishes, until at the other node it becomes least. Then, as the centre of the epicycle recedes from the southern node and traverses the semicircle, or nearly the semicircle, of the southern segment, the perigee of the epicycle bends towards the south and the apogee towards the north, until at the limit of greatest southern latitude the inclination becomes greatest. In both places the quantity of the inclination was found by Ptolemy to be 4° 30′ in Saturn, 2° 26′ or 30′ in Jupiter, 2° 15′ in Mars.

[Margin: 4th part.]

Fourth, then, to bring about this libration of the epicycle, those who have embodied the Ptolemaic circles in matter, as Peuerbach and Magini did, add a certain small orb containing the epicycle within itself. This orb is to move on four equal small circles, two of which adhere to the eccentric, one opposite the apogee and the other opposite the perigee of the epicycle; and on the circumference of these stand the other two, on whose circumference the apogee and perigee of that small orb are fixed, and which are to move with a motion double the motion of the first small circles and in the contrary direction.

From all this the following corollaries are deduced.

[Margin: 1st corollary.]

First, the axis of the epicycle is never parallel to the axis of the deferent, nor to the axis of the ecliptic outside the nodes; at the nodes, however, it is always parallel to the axis of the ecliptic.

[Margin: 2nd corollary.]

Second, the body of the planet, outside the nodes, in the upper part of the epicycle, lies within the ecliptic and the orbit of the eccentric; but in the lower part of the epicycle it lies outside both, and farther from the ecliptic than the plane of the eccentric.

[Margin: 3rd corollary.]

Third, because of the motion of the epicycle in latitude, the apogee of the epicycle, whether mean or true, does not fall on the line drawn from the centre of the Earth or of the equant through the centre of the epicycle, although they are usually determined by such lines when the discussion concerns longitude alone, abstracting from latitude.

This is the sum of the old theory of the latitude of these three planets, which cannot be represented in a plane but instead instruments skilfully built for the purpose are needed. For those endowed with the power of imagination, however, it will not be difficult.

De Latitudine, Inclinatione, & Reflexione Veneris ac Mercurij.

On the latitude, inclination and reflection of Venus and Mercury.

[Margin: Threefold difference of latitude in ♀ and ☿.]

IV. According to Ptolemy's observation and opinion, Venus and Mercury have a threefold variation in latitude. The first of these is called [Greek: ἔγκλισις ἐκκέντρου (enklisis ekkentrou, "inclination of the eccentric")], that is, the inclination of the eccentric, or the Deviation of the plane of the eccentric from the plane of the ecliptic. The second is called [Greek: ἔγκλισις ἐπικύκλου (enklisis epikyklou, "inclination of the epicycle")], namely the Inclination or Declination of the plane of the epicycle to the plane of the eccentric, by which the diameter of the apsides of the epicycle librates this way and that, towards north and south. The third is called in these planets simply [Greek: λόξωσις

(loxōsis, "obliquation")], or Obliquation [Obliquatio] or Reflection [Reflexio], by which the diameter of the mean longitudes of the epicycle is bent transversely from west or east towards north or south. One node Ptolemy, Book 13, ch. 1 and 2, calls the node of the subtracting semicircle, because it lies in that half of the eccentric's semicircle in which the equation of the centre is subtracted from the mean centre, or from the anomaly of the centre; the other node he calls the node of the adding semicircle, because it lies in that half of the eccentric's semicircle in which the equation of the centre is added. So much for the terms.

Deuiationis Leges.

Laws of the Deviation.

V. First, when the centre of the epicycle precedes the apogee of the equant by 90°, it is at the node of the Dragon's Head, or ascending node, and then there is no Deviation of the plane of the eccentric from the plane of the ecliptic; the two lie folded together in one common plane.

[Margin: Deviation in the 1st quadrant.]

Second, when the centre of the epicycle ascends from the node of the Head towards the apogee of the equant eccentric, the Deviation gradually increases until, when the centre of the epicycle occupies the apogee, it becomes greatest: in Venus 10′, and to the north; in Mercury 45′, and to the south.

[Margin: 2nd quadrant.]

Third, when the centre of the epicycle begins to descend from the apogee of the equant, the Deviation begins to diminish, and it decreases continuously until at the node of the Tail it vanishes altogether and the planes are again united in one and the same plane.

[Margin: 3rd quadrant.]

Fourth, when the centre of the epicycle has receded from the node of the Tail towards the perigee of the equant, the Deviation of the eccentric begins to grow, and it increases continuously until at the perigee itself it becomes greatest: in Venus 10′ to the north, in Mercury 45′ to the south. For as soon as the centre of the epicycle passes the descending node, the part of the eccentric in which the epicycle lies begins to verge north in Venus and south in Mercury.

[Margin: 4th quadrant.]

Fifth, once the perigee of the equant has been passed, the centre of the epicycle begins to ascend towards the apogee, and the Deviation to diminish, until it again becomes nil when the centre of the epicycle holds the ascending node. Thus, together with the eccentric revolution, one period of the Deviation is completed.

[Margin: 1st corollary. Situation of the nodes on the eccentric.]

From this you will gather, first, that the nodes of the eccentric and the ecliptic (which we shall call the nodes of the Deviation) lie at the mean longitudes of the eccentric, 90° distant on one side from the apogee and on the other from the perigee of the eccentric; and that the limits of greatest Deviation lie on the

very line of apsides of the eccentric, although these are not the limits of greatest latitude of all.

[Margin: 2nd corollary.]

Second, gather that the centre of the epicycle of Venus is always in that segment of the eccentric which verges to the north, and the centre of the epicycle of Mercury always in that which verges to the south, even though the other part of the eccentric is meanwhile bent to the opposite quarter.

[Margin: 3rd corollary.]

Third, that the axes of the deferents of Venus and Mercury nod this way and that, approaching and receding from the poles of the zodiac, and that the inclination called Deviation, or the inclination of the eccentric to the ecliptic, is not fixed, as in the three superior planets, but inconstant. Regiomontanus adds, Book 13, prop. 5, that while the latitude of Venus is greatest, that of Mercury is least or nil, and vice versa.

Inclinationis & Reflexionis Epicycli leges.

Laws of the Inclination and Reflection of the epicycle.

VI. The Inclination too, by which the plane of the epicycle with its diameter of apsides is inclined to the plane of the eccentric, is wonderfully attuned to the motion in longitude, but in a way exactly opposite to the Deviation.

[Margin: Inclination and Reflection.]

For first, when the centre of the epicycle occupies the apogee of the equant, the diameter of the apsides of the epicycle does not decline at all from the plane of the eccentric; but the diameter of the mean longitudes of the epicycle is twisted transversely, so that its left, or eastern, half is bent with the greatest Reflection, 3° 30′ to the north in Venus and 7° to the south in Mercury, the other half being turned transversely to the opposite quarter.

[Margin: 1st quadrant.]

Second, as the centre of the epicycle recedes from the apogee of the equant, the Reflection begins to diminish and the Inclination of the epicycle to arise, the apogee of the epicycle of Venus being inclined to the north and that of Mercury to the south, and the perigees to the opposite sides. From there the Reflection continuously decreases and the Inclination increases, until the centre of the epicycle comes to the descending node. For there all Reflection vanishes, and the Inclination of the apogee of the epicycle becomes greatest: of Venus 2° 30′ to the north, of Mercury 6° 15′ to the south.

[Margin: 2nd quadrant.]

Third, as the centre of the epicycle descends from the southern node towards the perigee of the equant, the already mentioned Inclination stays on the same sides but gradually diminishes, until it vanishes at the perigee; while the Reflection begins to arise and increase, yet so that the left, or eastern, semidiameter of the mean longitudes is bent transversely to the south in Venus and to the north in Mercury, the other part being bent to the opposite quarter,

until at the perigee itself the Reflection again becomes greatest, and as great as it was at the apogee.

[Margin: 3rd quadrant.]

Fourth, as soon as the centre of the epicycle leaves the perigee of the equant, the Inclination and the diminution of the Reflection begin, but in such a way that the apogee of the epicycle of Venus inclines towards the south and that of Mercury towards the north, with the perigees tilted to the contrary sides, until, when the centre of the epicycle occupies the northern node, the transverse Reflection ceases entirely and the Inclination again becomes greatest, as great as it had been at the southern node.

[Margin: 4th quadrant.]

Fifth and last, as the centre of the epicycle ascends from the northern node towards the apogee of the equant, the diminution of the preceding Inclination begins, and the Reflection, so that the left, or eastern, half of the diameter of the mean longitudes of the epicycle is again twisted to the north in Venus and to the south in Mercury. This Reflection grows until at the apogee of the equant it becomes greatest, but at that same place the whole Inclination of the epicycle ceases.

[Margin: 1st corollary.]

From this is gathered, first, the perpetual accord of the Deviation with the Reflection, and the opposition of both to the Inclination: for where both Deviation and Reflection increase, the Inclination diminishes, and vice versa; and where the Deviation is greatest and the Reflection greatest, there the Inclination is nil, and vice versa.

[Margin: 2nd corollary.]

Second, that the properties of the latitudes of Venus are exactly opposite to the properties or affections of Mercury, in so far as they arise from the position of the centre of the epicycle on the eccentric.

[Margin: 3rd corollary.]

Third, that in the whole upper semicircle of the eccentric, which runs from the ascending node through the apogee of the eccentric to the descending node, the Deviation of the eccentric and the Reflection of the left, or eastern, part of the epicycle are northern in Venus and southern in Mercury; so that if the planet is in the eastern part of the epicycle, the latitude of Venus is necessarily northern and that of Mercury southern. But in the lower semicircle of the eccentric, the opposite of these happens as regards the Reflection.

Speculum varietatis Latitudinum Veneris ac Mercurij.

Mirror of the variation of the latitudes of Venus and Mercury.

VII. Ptolemy, Book 13, ch. 1, al-Farghānī, diff. 18, Regiomontanus, Book 13 of the *Epitome of the Almagest*, prop. 2, and Copernicus, Book 6, ch. 1, did try to distinguish for us the variation of the latitudes of Venus and Mercury; and Reinhold did so somewhat more clearly on Peuerbach, p. 225, adding also a

diagram showing eight principal positions of the eccentric. But it has seemed best to me to serve the imagination and the memory still better, by adding the epicycles, with their semicircles divided into morning and evening, and marking by the letters A the southern latitude and B the northern, with the planet placed at the four cardinal points of the epicycle, so that from these the others may be more easily understood. Consider, then, the following figure, in which T is the centre of the Earth; R the centre of the eccentric, whose line of apsides is GTP, on which the apogee of the eccentric is G and the perigee P; the line ETK is the line of the mean longitudes of the eccentric, passing through the nodes; E is the place of the southern node, 90° distant from the apogee in the order of the signs; while K is the northern node, 270° distant from the apogee in the order of the signs. Now D is the midpoint of the first quadrant of the eccentric; F, of the second; H, of the third; L, of the fourth. From the eight points of the eccentric, namely G, D, E, F, P, H, K, L, as many epicycles have been described, the larger of which is the epicycle of Venus, and the smaller that of Mercury. In the same order, the diameters of the apsides of the epicycle, on which the Inclination depends, are BGB, BDA, BEA, BFA, BPB, AHB, AKB and ALB. These divide each epicycle into a right semicircle, which is called western and preceding, and that of the morning planet, and is therefore marked with a double MM; and a left one, which is called eastern and succeeding, and that of the evening planet, and is therefore marked with a double VV. The diameters of the mean longitudes of the epicycle, to which the Reflexion relates, and which ought to bisect the diameters of the apsides at right angles, are likewise, in the same order, BGA, BDA, OEO, BFA, BPA, BHA, OKO, ALB. Keeping this same order for them, in eight propositions to be set beneath the figure we shall cover every variety of latitude that (unless I am mistaken) belongs to these planets according to Ptolemy, and we shall present it afresh to be contemplated as if at a single glance.

Diagram. A woodcut set in the left column, between the text and proposition 1. A large circle, the eccentric, carries eight double epicycles (an outer and an inner circle on one centre) at eight points spaced around it: G at the top, then L, K, H on the right, P at the bottom, and F, E, D on the left. Inside, a vertical line runs down from the epicycle at G through T to the epicycle at P, and a horizontal line runs from the epicycle at E through R to the epicycle at K; R stands above that horizontal line and T just below it on the vertical. Each epicycle is crossed by two diameters. The letters A, B, M and V stand on or around every epicycle, on both the outer and the inner circle and at the ends of its diameters.

- Top epicycle: centre G.
- Upper right epicycle: centre L.
- Right epicycle: centre K, with the sign ☊ beside it.
- Lower right epicycle: centre H.
- Bottom epicycle: centre P.
- Lower left epicycle: centre F.

- Left epicycle: centre E, with the sign ☋ below it.
- Upper left epicycle: centre D.
- Centre of the figure: R, T.
- Above and below the left epicycle (E), and below the right epicycle (K): O. Above the right epicycle: C [recte: O; the text names this diameter OKO].
- Around each epicycle: A, B, M, V.

[Margin: At the apogee of the eccentric.]

1. Let the centre of the epicycle be at G, the apogee of the eccentric. The Inclination of the diameter BGB is nil. The Deviation of the eccentric is at its greatest: for Venus 10′ northern, but for Mercury 45′ southern; and the latitude too is of this kind and this size, if the planet is at the apogee or perigee of the epicycle. The Reflexion of the diameter BGA, however, is at its greatest: for Venus to the north and for Mercury to the south as regards the left semicircle of the epicycle; but as regards the right, it is southern for Venus and northern for Mercury. And so evening Venus is northern and morning Venus southern, especially around the mean longitudes of the epicycle; but evening Mercury is southern and morning Mercury northern.

[Margin: At the middle of the 1st quadrant.]

2. Let the centre of the epicycle be at D, the middle of the first quadrant. The Inclination grows, the Deviation and Reflexion diminish; and the Inclination and Deviation of Venus are northern, those of Mercury southern. The Reflexion of Venus is likewise northern for the left semicircle, southern for the right; but the Reflexion of Mercury is southern in the left semicircle, northern in the right. Therefore the latitude of Venus, at that time, at the apogee of the epicycle and at the middle of the evening semicircle is northern; at the perigee and at the middle of the morning semicircle it is southern. By contrast, the latitude of Mercury at apogee and at the middle of the evening semicircle is southern, but at perigee and at the middle of the morning semicircle northern.

[Margin: At the end of the 1st quadrant.]

3. Let the centre of the epicycle be at E, the end of the 1st quadrant, or at the node ☋. The Deviation and Reflexion are nil, and the plane of the eccentric lies in the plane of the ecliptic, while the diameter OEO lies in the plane of the eccentric and of the ecliptic. Therefore, if the planet is at O, the mean longitudes of the epicycle, it has no latitude at all. The Inclination of the apogee, however, is at its greatest, for Venus to the north and for Mercury to the south. Therefore Venus in the upper part of the epicycle is northern, and in the lower southern; but Mercury in the upper part is southern, and in the lower northern.

[Margin: At the middle of the 2nd quadrant.]

4. Let the centre of the epicycle be at F, the middle of the 2nd quadrant. The Inclination diminishes, both that of the apogee, northern for Venus

and southern for Mercury, and that of the perigee, southern for Venus and northern for Mercury. The Deviation grows, northern in Venus and southern in Mercury. The Reflexion of the left semicircle is likewise southern for Venus and growing, and northern for Mercury and growing; that of the right semicircle is northern for Venus and southern for Mercury, and growing. And so the latitude of Venus at apogee is northern, at perigee southern; at the mean longitude of the epicycle, morning Venus is northern and evening Venus southern. By contrast, the latitude of Mercury at apogee is southern and at perigee northern; at the mean longitude, morning Mercury is southern and evening Mercury northern.

[Margin: At the perigee of the eccentric.]

5. Let the centre of the epicycle be at P, the perigee of the eccentric. The Inclination of the diameter BPB is nil. The Deviation of the eccentric is at its greatest, for Venus 10′ to the north, for Mercury 45′ to the south; and their latitude too is of this kind and this size, if they are at the apogee or perigee of the epicycle. The Reflexion, however, is at its greatest: that of the semicircle which is left with respect to the Earth looking towards P is southern in Venus and northern in Mercury, but that of the right is northern in Venus and southern in Mercury. Therefore morning Venus is northern and evening Venus southern, but morning Mercury is southern and evening Mercury northern.

[Margin: At the middle of the 3rd quadrant.]

6. Let the centre of the epicycle be at H, the middle of the third quadrant. The Deviation is northern in Venus and southern in Mercury, diminishing in both. The Inclination of the apogee of the epicycle grows, and is southern for Venus and northern for Mercury. The Reflexion of the left semicircle diminishes, and is southern for Venus and northern for Mercury; that of the right is northern for Venus and southern for Mercury. As a result, the latitude of Venus at apogee is southern and at perigee northern, while that of Mercury at apogee is northern and at perigee southern. At the mean longitudes of the epicycle, however, the latitude of evening Venus is southern and of morning Venus northern, whereas that of evening Mercury is northern and of morning Mercury southern.

[Margin: At the end of the 3rd quadrant.]

7. Let the centre of the epicycle be at K, or at the northern node. The Deviation and Reflexion are nil. The Inclination of the apogee is at its greatest, for Venus to the south and for Mercury to the north. For that reason, if they are at the mean longitudes O, they have no latitude at all; but if they are in the upper part of the epicycle, Venus is southern and Mercury northern, and in the lower part Venus is northern and Mercury southern.

[Margin: At the middle of the 4th quadrant.]

8. Let the centre of the epicycle be at L, the middle of the fourth quadrant. The Inclination diminishes, but that of the apogee is still southern for Venus and northern for Mercury. The Deviation grows, northern for Venus and southern for Mercury. The Reflexion also grows: as regards the left semicircle it is northern in Venus and southern in Mercury; as regards the right, southern in Venus and northern in Mercury. Accordingly, the latitude of Venus at apogee is southern and at perigee northern; and at the mean longitudes of the epicycle morning Venus is southern and evening Venus northern. By contrast, the latitude of Mercury at apogee is northern and at perigee southern; at the mean longitudes of the epicycle morning Mercury is northern, but evening Mercury southern.

De Motu Nodorum.

On the Motion of the Nodes.

VIII. According to Ptolemy, Book 13, ch. 6, the motion of the nodes and limits is inferred to be no other than that of the apogees. For the northern limit of Mars is always joined to the apogee of the eccentric, while the northern limit of Saturn always stands 50° from the apogee against the order of the signs, and that of Jupiter 20° in the order of the signs. And so, to obtain the argument of latitude of the planet, or its distance from the limit, for Mars one must use the equated anomaly of the eccentric itself, or the longitude from the apogee; but for Saturn the 50° must be added to the distance from the apogee, and for Jupiter the 20° subtracted. The nodes of Venus and Mercury, however, always stand from the line of apsides, the one 90° and the other 270°, as regards the latitude arising from the Deviation and Reflexion; although as regards the latitude arising from the inclination of the epicycle they lie on the line of apsides. Now follows a synopsis displaying the size of the latitudes and the places of the nodes. But note, from Ptolemy, Book 13, ch. 3, the inclination of the eccentric of Saturn found by him, 2° 26′, and that of Jupiter, 1° 24′;

he adds, however: *In place of these, for convenience's sake, we have used 2° 30′ and 1° 30′*; and therefore we have placed these values in the table along with him. We have followed Ptolemy's text and source, rather than the figures of Reinhold, Wurstisen, Magini and others, which either have been corrupted by printers, or were taken from a faulty text of Ptolemy, or were perhaps adapted by Reinhold and Magini to the Alphonsine or Peuerbachian hypotheses.

Synopſis Latitudinum ex Ptolemæo, & Alijs

Synopsis of the latitudes from Ptolemy and others

Planet	Deviation of the eccentric, or Inclination	Greatest Inclination of the epicycle	Greatest Reflexion of the epicycle
♄	Fixed, 2° 30′	4° 30′	0° 0′
♃	Fixed, 1° 30′	2° 30′	0° 0′
♂	Fixed, 1° 0′	2° 15′	0° 0′
♀	Greatest, 0° 10′	2° 30′	3° 30′

Planet	Deviation of the eccentric, or Inclination	Greatest Inclination of the epicycle	Greatest Reflexion of the epicycle
☿	Greatest, 0° 45′	6° 15′	7° 0′

Latitudo Maxima Trium Superiorum
Greatest latitude of the three superior planets

Planet	Au-thor-ities	Northern, at the apogee of the eccentric: at the apogee of the epicycle	Northern, at the apogee of the eccentric: at the perigee of the epicycle	Southern, at the perigee of the eccentric: at the apogee of the epicycle	Southern, at the perigee of the eccentric: at the perigee of the epicycle
♄	Ptolemy	2° 4′	3° 2′	2° 2′	3° 5′
♄	Rein-hold	2° 3′	3° 3′	2° 1′	3° 5′
♄	Mag-ini	2° 3′	3° 2′	2° 2′	3° 5′
♃	Ptolemy	1° 7′	2° 4′	1° 5′	2° 8′
♃	Rein-hold	1° 6′	2° 5′	1° 4′	2° 8′
♃	Mag-ini	1° 6′	2° 4′	1° 5′	2° 7′
♂	Ptolemy	0° 8′	4° 21′	0° 4′	7° 7′
♂	Rein-hold	0° 5′	4° 21′	0° 2′	7° 30′
♂	Mag-ini	0° 5′	4° 30′	0° 4′	6° 50′

Latitudo Maxima Duorum Inferiorum
Greatest latitude of the two inferior planets

Planet	On the epicycle	Latitude from the Inclination	At the degree of the anomaly of the eccentric
♀	Apogee	1° 2′	90 northern; 270 southern
♀	Perigee	6° 22′	90 southern; 270 northern
☿	Apogee	1° 45′	90 southern; 270 northern
☿	Perigee	4° 5′	90 northern; 270 southern

Latitudo ex Deuiatione Maxima in vtriuſque circuli Abſidibus
Greatest latitude from the Deviation, at the apsides of each circle

Planet	Position	Latitude	Direction
♀	Apogee and perigee	0° 10′	Northern
☿	Apogee and perigee	0° 45′	Southern

Latitudo ex Reflexione maxima
Greatest latitude from the Reflexion

Planet	Latitude	On the eccentric	And at the contact of the epicycle
♀	2° 30′	Apogee	Morning southern; evening northern
♀	2° 30′	Perigee	Morning northern; evening southern
☿	2° 30′	Apogee	Morning northern; evening southern
☿	2° 30′	Perigee	Morning southern; evening northern

Nodorum ac Limitum loca ad Annum CHRISTI 140.
Places of the nodes and limits for the year of Christ 140

Planet	Northern node	Southern node	Northern limit	Southern limit
♄	♋ 3°	♑ 3°	♎ 3°	♈ 3°
♃	♋ 1°	♑ 1°	♎ 1°	♈ 1°
♂	♈ 29½°	♎ 29½°	♋ 29½°	♑ 29½°
♀	♒ 25°	♌ 25°	♉ 25°	♏ 25°
☿	♑ 10°	♋ 10°	♎ 10°	♈ 10°

On the nodes of Venus and Mercury, Kepler and Longomontanus hold otherwise; on this, read Scholium 4. The motion of the nodes is the same as that of the apogees.

SCHOLIA.

Scholia.

[Margin: Whether there is Reflexion in the superior planets?]

I. You will ask whether, in the three superior planets, besides the Deviation of the eccentric from the ecliptic and the Inclination of the epicycle from the eccentric, there is also some Reflexion, as in the inferior ones. Magini will answer, in Book 2 of the *Theoricae*, ch. 4, that it must be admitted, because without it, it cannot come about that, with the centre of the epicycle placed at the nodes, the plane of the epicycle lies in the plane of the ecliptic, and that outside the nodes the axis of the mean longitudes is parallel to the plane of the ecliptic, as is supposed in the preceding hypothesis. However, since that inflexion makes a very slight and scarcely sensible difference in the latitude of the superior planets, it has so far been safely neglected by astronomers. I later found the same thing in Francisco Sarzosa, who in Book 2 of the *Aequator Planetarum*, ch. 21, admits Reflexion in the three superior planets and shows that it was admitted by Ptolemy, Book 13, ch. 1, where he says: "But the diameters of the orbs of revolution, etc., always remain equidistant from the surface of the circle of the signs; and although they are reflected, the reflexion has no quantity worth troubling about." Pedro Nunes remarks the same in annotation 9 on the affections of the planets explained by Peuerbach. He further adds that only twice in one revolution of the centre of the epicycle is the axis on which the epicycle moves in longitude equidistant from the axis of the ecliptic, namely when the centre of the epicycle is at the nodes; and that it is never equidistant from the axis of the eccentric.

[Margin: The Ptolemaic figures here vary from one author to another.]

II. If anyone consults the figures for the Reflexion or the latitude given by al-Battānī, al-Farghānī and Jābir ibn Aflaḥ in the places cited at the beginning of the chapter, he will be amazed at how very different they are, whether this has arisen from a printing fault, or from a difference in the calculation, or in the manuscript of Ptolemy from which they derived them. Therefore our figures are not to be condemned out of hand for disagreeing with those already mentioned; rather, the Ptolemaic text itself should be consulted and attentively considered.

[Margin: Ptolemy's errors concerning latitude.]

III. Ptolemy went less astray regarding latitude in the three superior planets than in the inferior ones, because he set the inclination of the superior planets as fixed and kept their epicycles very nearly parallel to the ecliptic. But he made the inclination of the inferior planets variable, whereas, as will be said in chs. 3, 4 and 7, it is more probably invariable; granted that, since he placed the eccentrics of both and their epicycles below the Sun, he was almost forced to devise so many variations. How far he went astray, however, in equating the motion of the nodes with the motion of the fixed stars and of the apogees was revealed after many centuries by the place of the nodes, which shows that motion to be very different from the motion of the fixed stars and of the apogees, as will be said in ch. 4, no. 6, and in the following chapters. But Ptolemy did not have sufficient observations by the ancients with which to distinguish these motions; and since he saw from rough observations that they were very slow, he judged that no evidently sensible error could be committed if he supposed these three to move with equal slowness, namely the sphere of the fixed stars, the auges of six planets (for he excepts the Moon), and the nodes of the five minor planets. Finally, he went astray in the points of the greatest latitudes, as will be clear from what is to be said in ch. 4, schol. 2.

[Margin: Kepler's apparent contradiction in assessing Ptolemy's opinion.]

IV. But Kepler appears to have erred against Ptolemy when he says, in the *Epitome of Copernican Astronomy*, Book 6, part 3, or p. 770: "Ptolemy set the nodes of Venus in the apsides as he called them; of Venus, that is, the ascending node at 25° ♉, the descending at 25° ♏; so that the northern limit (with a line drawn from the Sun through it) was at 25° ♌.

[Margin: Places of the nodes and limits of ♀ and ☿ according to Ptolemy.]

of Mercury, the nodes likewise in the apsides, the ascending at 10° ♎, the descending at 10° ♈." Longomontanus too, in Book 2 of the *Theorica*, ch. 24, puts their nodes on the line of apsides. But Ptolemy, Book 13, chs. 1 and 2, says explicitly that the nodes of Venus and Mercury stand a quarter of a circle away, on one side from the apogee and on the other from the perigee of the equant eccentric. He places them at the mean longitudes, the one at the middle of the semicircle of the eccentric in which the equation of the centre is subtracted, the other at the middle of the semicircle in which it is added. For that reason he calls the former the node of the subtracting semicircle and

the latter that of the adding one, as Wurstisen noted on Peuerbach's *Theoricae*; and Wurstisen himself also recognises the limits of the greatest latitudes at the apogee and perigee of the equant, along with Peuerbach and the other followers or expounders of the old astronomy. That same point is included in Copernicus's statement, though it is perhaps ambiguous to some, in Book 6, ch. 1, where he says of Venus and Mercury: "At the mean longitudes, namely when the line of the mean motion of the Sun stands a quadrant away from their highest or lowest apse, and the stars themselves are a quadrant of their orbs, evening or morning, away from the same line of the mean motion of the Sun, they found no departure in them from the circle of the signs, by which they understood that they were then at the common section of their orbs and of the zodiac; and this section passes through their apogees and perigees." But from the words that follow it is clear that he is speaking of the apogees and perigees of the epicycle, whereas in the earlier words he speaks of the apse of the eccentric. Magini takes it no differently in the *Theoricae Planetarum*, Book 2, ch. 6, and in Canon 50 of the *Secunda Mobilia*, recognising in both places, as the mind of Ptolemy and of Copernicus, the limits of Venus and Mercury at the apogee and perigee of the equant, and the nodes at the mean longitudes, or a quadrant away from the apogee. But the conflict is removed, because for Kepler and Longomontanus, with the Deviation and reflexion taken away (whose nodes Ptolemy put at the mean longitudes of the eccentric, and its limits at the apsides) and with this attributed to the Earth's approach or to other equivalent causes, there remained only the latitude coming from the orb of the epicycle and its inclination; and the nodes of this Ptolemy would have placed at the apsides of the eccentric. We, however, put the nodes of the Deviation in the little table, as we also indicated above, no. 8.

[Margin: The varied naming of latitude.]

V. Although the latitude of these planets, in the first semicircle counted from the ascending node in the order of the signs, is northern, growing in the first quadrant and diminishing in the second, and in the other semicircle, from the descending node, is southern, growing in its first quadrant and diminishing in the second, Magini nevertheless notes, in Canon 49 of the *Secunda Mobilia*, that latitudes are called ascending as long as the planet is ascending on the epicycle, which for the three superior planets happens from opposition with the Sun until conjunction; and descending as long as the planet is descending on the epicycle, which happens from conjunction to opposition. The inferior planets, however, ascend from the inferior conjunction with the Sun until the superior, and from there onward descend.

VI. To complete the theory of latitude in the Ptolemaic hypothesis, it must be recalled that the figure set out above at no. 7 serves, as it lies, chiefly for Venus, as regards the naming of the nodes; but it is easily carried over to the opposite quarter, towards which the apogee of Mercury, almost opposite the apogee of Venus, was directed. Moreover, the motion of latitude is counted in Venus from the northern node, but in Mercury from the southern. Lastly, various

propositions in Regiomontanus, Book 13 of the *Epitome of the Almagest*, should be read. For he shows in props. 9 and 17 that the Inclination of the epicycle of the inferior planets brings no sensible error into the motion in longitude, since the greatest difference from this does not exceed 2′ in Venus and 3′ in Mercury; and in props. 13, 14, 15, 16 that the greatest angle of this diversity occurs near the points of contact of the epicycle, in Mercury at the very point of contact, but in Venus usually outside the point of contact. Copernicus teaches similar things in Book 6, ch. 6, adding from Ptolemy that the greatest Reflexions occur at the contacts, or around the greatest digressions from the Sun, and that the latitudes arising jointly from the Reflexion and Deviation are in Venus greater in the north than in the south by 20″, but in Mercury greater in the south than in the north by about 1° 30′.

CAPVT II.

Chapter II.

De Copernici & Sectatorum eius Hypotheſi circa Latitudinem Planetarum Minorum.

On the hypothesis of Copernicus and his followers concerning the latitude of the minor planets.

[Margin: Followers or expounders of Copernicus.]

I. Copernicus himself sets out his hypothesis on latitude in Book 6. Rheticus followed it in his first *Narratio*, usually appended to the works of Copernicus, from p. 138 in my copy; Reinhold in the Prutenic Tables, precepts 71 and 72; as regards the figures or measures of latitude, and in part as regards the explanation of the libration, Magini, *Theoricae*, Book 2, especially ch. 7; and Lansbergen in his *Theoricae*, from ch. 15 to ch. 19, although he corrects Copernicus as regards the figures, the measures and the motion of the nodes. Finally, some knowledge of this hypothesis is provided by Longomontanus, Book 2 of the *Theorica*, chs. 14 and 24, and by Hérigone, vol. 5, Book 2 of the *Theorica*, ch. 5.

Copernici Placita de Latitudine trium Superiorum.

Copernicus's tenets on the latitude of the three superior planets.

[Margin: 1st part of the Copernican hypothesis.]

II. First, Copernicus, Book 6, ch. 1, takes as given the alternations of latitude observed by Ptolemy, which we set out in ch. 1, and that the nodes and limits move with the same motion as their apogees. Adducing no observation of his own thereafter, he is wholly occupied in representing Ptolemy's observations in his own hypothesis on the motion of the Earth. Not, he says, that the Earth's great orb, remaining always the same in the plane of the ecliptic, induces any latitude in them of itself, but because their apparent latitude changes with the approach and recession of the Earth. For the three superior planets, at their

acronychal shinings, or when they are seen from the Earth opposite the Sun, appear to recede from the ecliptic by a departure always greater than in any other situation of the Earth: in the northern semicircle of the eccentric they digress to the north, and in the southern to the south.

[Margin: 2nd part.]

Secondly, because the greatest latitudes, even at the acronychal oppositions, let alone around the conjunctions with the Sun, appear different and undergo a greater variation than the Earth's approach or recession requires; and because the three superior planets have no epicycles in the Copernican hypothesis, but in their place there is the great or annual orb of the Earth, whose ecliptic nevertheless always lies in the plane of the celestial ecliptic: for these reasons he lays it down that the inclination of the eccentrics of Saturn, Jupiter and Mars to the ecliptic is not fixed, but movable by a certain libration commensurable with the revolutions of the Earth's great orb. So he says in that ch. 1; and he sets out this libration more fully in ch. 2, with very nearly the following figure. Let A be the centre of the celestial ecliptic, and at the same time of the annual mean motion of the Earth. From it let there be described both the great orb itself, HIKL, and, in the same plane, the celestial ecliptic FCGN, to which let the eccentric BCDM of some superior planet be inclined. Let the already mentioned inclination of the eccentric be midway between the greatest and the least: BF at the northern limit, and GD at the southern limit. Let the ascending or northern node be E, and the descending or southern node C; and let the common section of the ecliptic and of the orbit of the planets be represented by the straight line CAE. And let these nodes and limits shift by no other motion than that by which the apogees of these planets move.

Diagram. A woodcut set into the foot of the right column, with the text running down its left side. An outer circle and an eccentric circle meet at C on the left and E on the right, joined by the straight line CAE through A. A small circle about A stands within them, and a vertical line runs from F at the top through A to D at the foot. Further lines run from the point on the small circle marked O up to the upper part of the circles, and from the rayed point marked P to the right side at m and n. The signs of three planets stand outside the circle at the foot.

- Top, on the outer circle: F.
- Upper left: a ring with a stem and crossbar beneath it, perhaps the sign ♀ [?], and R below it.
- Upper right: N, and M below it.
- On the vertical, below F: B, then H.
- On the small circle, to the right of the vertical: O.
- Left end of the horizontal: C; right end: E.
- On the small circle, on the horizontal: I (left), L (right).
- Centre: A.
- Lower left of the small circle: a rayed star, with P below it.
- On the vertical, below A: K, then G; at the foot: D.

- Right side, lower: m, then n.
- Below the circle, from left to right: ♄, ☋, ♂ (with D standing between ☋ and ♂).

Further, let the angles of inclination, say BEF and GED, vary by a certain libration similar to the libration of the precession of the equinoxes that we explained in Book 3, ch. 29, according to the simple motion of commutation, or according to the mean anomaly of the orb. Thus, whenever the Earth is nearest to the planet — as if the Earth is at H and the planet at B, or the Earth at K and the planet at D, which happens at acronychal oppositions — the apparent latitude of the planet itself is greatest, and the angle of inclination is greatest. But at conjunctions with the Sun, when the Earth at K is most remote from the planet at B, or the Earth at H from the planet at D, the planet's latitude is smallest, and the inclination of the eccentric to the ecliptic is smallest. The measure of the libration is gathered by subtracting the smallest inclination from the greatest. From Copernicus, Book 6, ch. 3, in Saturn the greatest is 2° 44′ and the smallest 2° 16′, and so the whole libration is 28′. In Jupiter the greatest inclination is 1° 42′ and the smallest 1° 18′, and the whole libration 24′. And in Mars the greatest inclination is 1° 51′, the smallest 9′, and the whole libration 1° 42′. Half of the libration serves as the semidiameter of the small circle by which this libration is accounted for.

[Margin: 3rd part.] Third, with these things presupposed, two causes of the varying latitude are now to hand. One is the libration through BF and GD, which, according to the motion of the mean anomaly of the orb, varies the inclination and so the latitude, even if the Earth were always in one place, say at H. The other is the motion of the Earth itself. For since by the laws of optics nearer things look larger, the angle of inclination and the latitude arising from it will certainly appear greater to the Earth when near at H than when remote at I, and smallest to the Earth when most remote at K. So, with the planet placed at B, with the greatest latitude that the libration of the inclination can produce, that greatest northern latitude will nevertheless decrease continuously from there as the Earth goes through the semicircle HIK; then, once K has been passed, that greatest northern latitude will grow continuously as the Earth ascends through the semicircle KLH. Conversely, with the planet placed at D, the limit of greatest southern latitude, this same latitude will diminish continuously while the Earth is carried through the semicircle KLH, and will increase apparently while the Earth goes through the semicircle HIK. Finally, if the planet is at the nodes C or E, then although it is acronychal, with the Earth placed at I or at L, no latitude will appear; for since there is no angle of inclination, it cannot look greater or smaller through the Earth's approach or withdrawal. We shall set out another figure from Copernicus bearing on this in ch. 8, no. 19.

Copernici placita de Latitudine duorum Inferiorum.

Copernicus's tenets on the latitude of the two inferior planets.

[Margin: 4th part, for Venus and Mercury.] III. To demonstrate the varying latitude of Venus and Mercury, in the same ch. 2 he uses the following diagram. In it E is the centre of the Earth's great orb ABCD, set in the plane of the ecliptic, to which let the orb of Venus and of Mercury (which in this hypothesis serves in place of the epicycle) be inclined — the eccentric orb GKF, I say — at an inclination midway between the greatest and the smallest. Let the common section of the annual orb and of the planet's eccentric orb be G and F, and let the line AEC of this section pass through the planet's apogee F and perigee G; and let the nodes of this section move in the order of the signs by the same motion by which the planet's apogee and perigee move. For when both the planet and the Earth are on the same line AEC, there will be no latitude. But when they are at the mean distance from the apogee, the inclination will be greatest; and this inclination is varied by a double libration, commensurable with the Earth's revolution.

Diagram. A woodcut set in the left column, interrupting the sentence at *seruit* | *pro Epicyclo*. A large circle carries a vertical diameter and a horizontal diameter crossing at the centre E. Inside it stand two further circles, shifted against one another so that they overlap and cross: one passes through the points on the vertical above and below the centre (G, F) and the points on the horizontal (K, L); the other passes through T above and H below. A straight line runs obliquely through E from R at upper left to S at lower right.

- Top of the large circle, on the vertical: A.
- On the vertical, above the centre, where the inner circles meet: G.
- Upper left, at the end of the oblique line: R.
- Upper right, on the inner circle: T.
- Left end of the horizontal: B, with O beside it inside the large circle.
- On the horizontal, left of the centre: K.
- Centre: E.
- On the horizontal, right of the centre: L.
- Right end of the horizontal: D.
- Lower left, on the inner circle: H.
- Lower right, at the end of the oblique line: S.
- On the vertical, below the centre, where the inner circles meet: F.
- Foot of the large circle, on the vertical: C.

By the force of the first libration, which occurs twice in one revolution of the Earth with respect to the apsides, the axis, or line of section, is held on the line passing through the apogee and perigee, so that whenever the line of the Sun's mean motion is at the apogee or perigee of these planets, the angle of section becomes greatest, and at the mean longitudes, smallest. By the force of the second libration, a second circle (homocentric in Venus, but eccentreccentric [Eccentrieccentricus] in Mercury), oblique to the first circle GKFL and having its common section with it at R and S, has a movable axis RS, on the condition that while the Earth is at A, the planet is at the extreme limit of the deviation, say at T; and however far the Earth has advanced from A, the planet recedes

that far from the limit T. So if, for example, the Earth, having completed a quadrant of the great orb, comes from the apogee A to B, the mean longitude, the planet too, having completed a quadrant of its orb, comes from the limit T to the node R — the node, I say, of this deviation. When the Earth is at C, the planet is at the limit H; and when the Earth is at D, the planet is at the other node S. Now when the planes of the two orbs coincide at the middle of this libration and at the nodes R and S, and thereafter strain in different directions, the former semicircle of the deviation, STG, which was southern, passes into the north; and on it Venus, leaving the south, makes again for the north, never by this libration alone to enjoy the south. But Mercury, following the contrary parts, remains southern. So this second libration induces the greatest deviation in the planet when by force of the first it ought to have no latitude. The way of demonstrating the species of these latitudes in particular I have decided to reserve for chapter 8, where, especially from Problem 15 to 19, many things bearing on the understanding of this hypothesis can be read; there we shall treat the way of finding the latitudes arising from the Inclination, from the Reflexion and from the Deviation, according to the mind of Copernicus.

Georgij Ioachimi Rhetici Explicatio Copernicanæ hypothesis de Latitudine Veneris ac Mercurij.

The explanation by Georg Joachim Rheticus of the Copernican hypothesis on the latitude of Venus and Mercury.

IV. To the obscurity of Copernicus in this part, however, it has seemed good to add light from the very words of Rheticus, who argues as follows in the *Narratio prima*:

[Margin: Copernicus, the teacher of Rheticus.] *Within the great orb, the sphere of Venus comes first. The Lord Teacher [D. Præceptor] therefore posits that the plane in which Venus moves declines from the plane of the ecliptic, or of the great orb, on a diameter through the proper apsides of the first deferent, so that the eastern half is raised from the plane of the ecliptic's surface to the north, by the angle of inclination that in Ptolemy's hypotheses the plane of the epicycle would make with the plane of the deferent; and the western half to the south. By "eastern" one must understand that half which runs from the place of the highest apsis* in the order of the signs, *etc. By this hypothesis alone, a simple one, it will be easy to see through all the rules of declinations and reflexions, with their causes, from the relation of the Earth's place to the planet's plane. For when by the Earth's annual motion we have arrived at the parts opposite to the highest apsis of the first deferent, where we suppose the orb of Venus to be as an epicycle, and at the apogee of its deferent, then the plane in which Venus is carried will seem to us reflected from the plane of the ecliptic: for in such a position we view it crosswise. And since we observe the same plane from a lower place, the part that projects to the north will be on the left for us, who turn our eyes to the south, and the remaining part, to the south, on the right. But as the Earth proceeds upward toward the highest apsis of the planet, the orb of Venus is believed to descend from the apogee*

of its own orb, and so we begin to look down, as if from a higher place, on the inclined plane itself of the deferent carrying Venus. Therefore the reflexion changes successively into declination, so that at a quadrant's distance from the former place, wherever the planet is seen in the elevated parts, it has declination only from the ecliptic. In such a position, since we, clinging to the Earth, are opposite the half of the deferent that runs from the highest apsis in the order of the signs *and is raised from the plane of the ecliptic to the north, the Ancients said that the epicycle of Venus is at the descending node, and that the apogee of the epicycle declines most to the north, and the perigee to the south. Next, as the annual motion of the Earth carries us aloft toward the place of the highest apsis of Venus, its orb, as if an epicycle, will seem to make for the lowest apsis of its deferent. And the plane of the epicycle (for us, the plane in which the star of Venus is), which before was inclined for us to the plane of the ecliptic, will appear to reflect itself again toward us, and the northern half of the deferent, projecting out of the plane of the ecliptic, will become the right-hand one, because we look on the orb of Venus from above. But when the centre of the Earth has arrived at the place of the highest apsis of Venus, no declination and only reflexion will be seen; and yet the orb of Venus would be believed, in the opinion of the Ancients, to be at the lowest apsis of its deferent. And this is the order of the phenomena* [Greek: τῶν φαινομένων (tōn phainomenōn, "of the phenomena")] *while the centre of the Earth completes a semicircuit, ascending from the place of the lowest apsis of Venus, in the order of the signs, to the place of the highest apsis of Venus. In the same way, as the Earth descends, the reflexion will gradually change into declination in our view. And since the half of the plane of the deferent running from the highest apsis against the order of the signs [in antecedentia] comes opposite to us by such a progress of the Earth, the apogee of the deferent of Venus begins to decline to the south from the plane of the ecliptic; until, with the Earth set at the ninetieth degree from the place of the apsis, both halves are seen declined to the plane of the ecliptic, and the orb of Venus, as an epicycle, is thought to be at the node ascending to the highest apsis. As the Earth withdraws from that place, the declination changes again into reflexion; and, having reached the place of the lowest apsis of Venus, the Earth begins again to produce the same appearances of latitude in Venus. From this it is clear that with the Earth placed at the line of the apsides of Venus, the plane of the deferent carrying the planet appears reflected; at the quadrants from these, declined; and in the intermediate places, mixed latitudes are seen.*

[Margin: The latitude of the deviation of Venus.] V. *But since, besides these latitudes, which the Ancients attributed to the epicycle of Venus, another deviation also, so called by the Ancients and called by Ptolemy the inclination of the eccentric circles* [Greek: τῶν ἐκκέντρων κυκλῶν ἡ ἔγκλισις (tōn ekkentrōn kyklōn hē enklisis, "the inclination of the eccentric circles")], *mingles with them, and since they demonstrated it through the centre of the deferent of the epicycle (which has now been removed), the Lord Teacher [D. Præceptor] has indicated that another method, one more in accord with the observations, is to be entered on. To grasp more easily, no differently from the rest proposed so far, this method of my Lord Doctor Teacher [D. Doctoris Præceptoris mei] for saving the deviation, let us take the plane we have*

just mentioned to be the mean plane and so fixed, from which the true plane wanders now here, now there, by a certain rule. But since all motions are grasped with less labour and expense by reference to the poles, it must be held at the outset that one of the poles of the mean plane is raised to the north from the plane of the ecliptic by the quantity of the angle of inclination, and the other, opposite, is depressed by the same amount to the south; and that what we are going to show happens with the northern pole, or with those things too around it, must be understood of the southern pole in a similar but opposite way. Accordingly, let us take it that around the northern pole of the mean plane there is a movable circle, whose line from the centre corresponds to the greatest obliquities of the mean plane from the true plane; and let the northern pole of the true plane itself describe, by the motion of libration, the diameter of the circle just mentioned. Further, let the movable circle follow the motion of the planet, so that Venus, advancing by its own motion, leaves behind the intersection of any two that follow one another, and on this condition: that when a year is complete it returns at last to the one it left. When a great circle is drawn through the poles of both planes, then, counting 90° on either side from its common intersection with the true plane, when the poles of the true and mean planes differ, the nodes or intersections just mentioned are determined. Meanwhile, while the period of Venus to one or other of the nodes is completed, let the diameter of the movable circle just mentioned be described twice by the pole of the true plane, by the motion of libration. Let these things come about so, that the planet may appear to have made such a pact with the centre of the Earth that whenever the Earth is at the apsides of the deferent, Venus, wherever it is on the true deferent, deviates most to the north from the mean plane, that is, stands furthest outside the mean path. Further, with the Earth a quadrant distant from the apsides of the deferent, let the planet itself, with its whole true plane, lie in the plane of the mean deferent. But while the Earth traverses the remaining intermediate places, let the planet too keep its course in intermediate deviations. That this pact of the Earth and the planet might be perpetual, God ordained that the first small circle of libration (so to speak) should revolve once in the same time in which one return of Venus to one or other of the nodes takes place. To make these things clearer by an example: if, at some beginning of the motion of deviation, the northern pole of the true plane is furthest south of the adjacent pole of the mean plane, and Venus just stands at the greatest limit of the deviation, which is northern, with the centre of the Earth also lingering at one of the apsides of Venus, then in a quarter of a year the Earth by its annual motion will come to the place midway between the apsides, and at the same time the planet to its intersection or movable node. And since the motion of libration is commensurate with the planet's motion to the nodes or intersections, the first small circle of libration will also complete a quadrant, and through the remaining small circle, which is twice as fast as the other, the pole of the true plane will be set beneath the pole of the mean plane; and so the two planes will be joined. But as the planet withdraws from this node, the Earth will proceed to the other apsis of the first eccentric, and the pole of the true plane will be moved forward by the libration from the pole of the mean plane to the north. Thus it will come about that even if Venus is southern, as in our example, the southern latitude nevertheless diminishes; if northern, the same latitude grows.

And when that place has been reached, the pole of the true plane will by the motion of libration touch its greatest limit to the north, and the planet, by its annual motion to the nodes, midway between the two sections, will again have its greatest deviation to the north. So it appears that the motion of the assumed circle has this use: that in a year the revolution of Venus to the nodes takes place; that always, with the Earth placed on the line of the apsides, the planet, wherever it is on its true plane, has its greatest deviation from the mean plane; and that with the Earth set midway between the two apsides, it is at the nodes. Further, by the motion of libration it comes about that when Venus is at one of the nodes the two planes are joined, and that the part of the true plane it enters always departs to the north from the mean plane; so that, as is fitting, this latitude remains perpetually northern.

[Margin: On the latitude of Mercury.] VI. *Just as the plane of Venus that it has pleased us to call mean is intersected by the ecliptic on the line of the apsides of the first eccentric, and the half of that plane running from the highest apsis* in the order of the signs *projects to the north, the remainder, by the law of opposition, tending to the south: so in Mercury, in a similar way, there is a mean plane, which is inclined on either side from the plane of the ecliptic on the line of its apsides, as was fitting, so that conversely the half of the mean plane running from the highest apsis against the order of the signs is northern. Therefore in the annual revolution of the Earth's centre, the declinations and reflexions in Mercury will be found interchanged, namely with respect to those of Venus. But so that this variety might be the more conspicuous, God also arranged the deviation of the true plane of Mercury from the mean so that the half it enters would always depart from the mean plane to the south, and that with the Earth standing at the apsides themselves it would lie, with its true plane, in the mean plane. So it comes about, finally, that in latitude it has no differences from Venus besides those stated, except that this deviation too is greater in Mercury than in Venus; just as it also has a greater angle of inclination. For the rest, the remaining varieties of the latitudes of Mercury will be gathered very easily, not otherwise than in Venus.* Thus far Rheticus, without the aid of any figure.

De Deuiatione Eccentricorum Veneris & Mercurij Copernicana, ex Io. Antonio Magino.

On the Copernican deviation of the eccentrics of Venus and Mercury, from Giovanni Antonio Magini.

VII. To represent this deviation according to the calculation of Copernicus, Magini, *Theoricae* Book 2, ch. 7, assigns it to two small circles placed outside the plane of the ecliptic opposite the apogee and perigee, to the north for Venus and to the south for Mercury; their diameter is 10′ in ♀ and 45′ in ☿. To their circumferences is fixed the orb of deviation, which contains in its own plane the eccentric deferent and takes on, at the motion of those small circles, a libration that is completed twice while the centre of the epicycle completes one period once on the eccentric; and this on the following condition. [Margin: The libration of the deviation, from Magini.] First, when the centre of the epicycle

is 90° from the apogee of the equant, but against the order of the signs, there is no deviation, and the plane of the eccentric coincides with the plane of the ecliptic. Second, when the centre of the epicycle recedes from the place already mentioned, as if from a node, and ascends toward the apogee, the whole plane of the deferent recedes from the plane of the ecliptic, in Venus toward the north, in Mercury to the south, until at the apogee itself it is parallel to the ecliptic and stands furthest from it, namely 10′ in Venus and 45′ in Mercury; and there the deviation becomes greatest. Third, when the centre of the epicycle recedes from the apogee, the deviation decreases, until at the other quasi-node, which is 90° from the apogee in the order of the signs, there is no deviation and the plane of the eccentric lies in the plane of the ecliptic; and thus one entire revolution of the small circles has been completed. Fourth, when the centre of the epicycle recedes from the other quasi-node and passes to the perigee half of the eccentric, the second libration of the small circles begins, and the plane of the eccentric is again separated from the plane of the ecliptic, toward the north in Venus and toward the south in Mercury; and the deviation grows until the centre of the epicycle is at the perigee of the equant, and there the deviation becomes greatest and the plane of the eccentric is parallel to the plane of the ecliptic. Fifth, as the centre of the epicycle recedes from here, the deviation diminishes, until, with the centre of the epicycle 90° distant from the perigee, it comes to nothing; which happens at the place equivalent to the northern node. For I have called the places where there is no deviation quasi-nodes, because in this hypothesis they are not truly nodes, since the plane of the eccentric nowhere intersects the ecliptic, but either coincides with it there or, receding from them, stays wholly outside the plane of the ecliptic, toward the north in Venus and toward the south in Mercury; and to those quarters, outside these quasi-nodes, the centre of the epicycle itself always belongs. But since they are equivalent to nodes in this, that the whole latitude of the deviation ceases there, they can be called quasi-nodes. [Margin: Problems on the Deviation, Inclination and Reflexion indicated.] See now, if you like, the two Problems bearing on this matter below, ch. 8, nos. 23 and 24. There you will also have figures and problems for the latitude arising from the Inclination and the Reflexion, from no. 20 to 23.

De Motu Nodorum.

On the motion of the nodes.

VIII. The nodes and limits of these five planets move by no other motion than the motion of the apsides, that is, the motion by which their apogees move, according to the mind of Copernicus. So it comes about that the nodes and limits of Venus, just as we said of the apogee in Sect. 3, ch. 2, are immobile under the sphere of the fixed stars and are perpetually 48° 21′ distant from the first star of Aries; but they move with an apparent motion, the same motion by which the first star of Aries seems to recede from the section of the vernal

equinox (for in reality Copernicus attributes this motion not to the fixed stars themselves, but to the equinoctial points receding against the order of the signs from the fixed stars). The nodes of the three superior planets and of Mercury, on the other hand, are changeable, following the motion of their apogees. It remains for us to gather into a synopsis the numbers and measures bearing on this hypothesis, which we have taken from Copernicus himself: Book 6, ch. 3, where he treats of the latitude and inclination of the three superior planets; ch. 5, where he treats of the declinations of Venus and Mercury; chs 6 and 7, where he treats of the obliquation or inflexion of their planes; and ch. 8, where he treats of the deviation; and from his tables and the Prutenic Tables.

SYNOPSIS Latitudinum ex Copernico. Latitudo Maxima trium Superiorum

Synopsis of the latitudes from Copernicus. Greatest latitude of the three superior planets

Planet	Latitude	At ♂ with ☉, or at apogee	At ☍ with ☉, or at perigee
♄	North	2° 3′	3° 2′
	South	2° 2′	3° 5′
♃	North	1° 6′	2° 4′
	South	1° 5′	2° 7′
♂	North	0° 6′	4° 30′
	South	0° 45′	6° 50′

Planet	Greatest inclination	Smallest inclination	Whole libration
♄	2° 44′	2° 16′	0° 18′ [recte: 0° 28′ — the row's own figures, 2° 44′ and 2° 16′, differ by 28′, and the text of p. 613 prints 28′]
♃	1° 42′	1° 18′	0° 24′
♂	1° 51′	0° 9′	1° 42′

Planet	Greatest declination	Greatest reflexion	Greatest deviation
♀	2° 30′	3° 29′	0° 10′
☿	6° 15′	7° 0′	0° 45′

Latitudo Maxima conueniens

Greatest latitude corresponding

Planet	Distance	To the greatest inclination	To the greatest reflexion	To the greatest deviation
♀	Apogee	1° 2′	2° 27′	0° 6′
	Mean	4° 30′	2° 30′	0° 10′
	Perigee	6° 22′	2° 34′	0° 15′
☿	Apogee	1° 45′	2° 17′	0° 33′

Planet	Distance	To the greatest inclination	To the greatest reflexion	To the greatest deviation
	Mean	2° 55′	2° 30′	0° 45′
	Perigee	4° 5′	2° 45′	1° 10′

This speaks of the apogee, perigee or mean distance of the centre of the orb from the Earth.

Radix Nodorum ac Limitum ex Copernico ad Annum CHRISTI circiter 1530.

Root of the nodes and limits, from Copernicus, for about the year of Christ 1530.

Planet	Node, north	Node, south	Limit of greatest lat., north	Limit of greatest lat., south
♄	♌ 7°	♒ 7°	♏ 7°	♉ 7°
♃	♋ 27°	♑ 27°	♎ 27°	♈ 27°
♂	♉ 27°	♏ 27°	♌ 27°	♒ 27°
♀	♊ 16°	♐ 16°	♍ 16°	♓ 16°
☿	♏ 28°	♉ 28°	♒ 28°	♌ 28°

The motion of the nodes is the same as the motion of the apogees.

Species Latitudinum Veneris ac Mercurij

The species of the latitudes of Venus and Mercury

[Margin: From Copernicus, Book 6, ch. 9.]

Species	Quadrants of the anomaly: of the eccentric	Quadrants of the anomaly: of commutation	♀	☿
Declination	1 & 2	1 & 4	North	South
	1 & 2	2 & 3	South	North
	3 & 4	1 & 4	South	North
	3 & 4	2 & 3	North	South
Reflexion	1 & 4	1 & 2	North	South
	1 & 4	3 & 4	South	North
	2 & 3	3 & 4	North	South
	2 & 3	1 & 2	South	North

But the latitude of the deviation in Venus is always northern; in Mercury it is always southern. The anomaly of commutation is the motion by which the Earth's mean motion is exceeded by the proper motion of Venus and Mercury, or by which it exceeds the motion of Saturn, Jupiter and Mars. The anomaly of the eccentric orb is that which bears on the Ptolemaic epicycle.

SCHOLIA.

Scholia.

I. *What has been said about the orbs of Venus and Mercury according to the mind of Copernicus can be applied to the epicycles employed by Magini, in whose place Copernicus has the orbs of these planets, which they describe around the Sun, taking their eccentricity from the centre of the Earth's great orb. Likewise, since the Earth's great orb in Copernicus takes the place of the epicycles of the three superior planets, what I said about the latitude fitting them at the Earth's greatest or smallest distance from them holds equally in Magini's hypothesis of the latitude due to them at the apogee or perigee of the epicycle. For although Magini used numbers taken from Copernicus or the Prutenic Tables, he nevertheless transferred them into the schemes of Ptolemy and Peuerbach and into the hypothesis of a resting Earth, constructing for us a kind of Ptolemaic-Copernican hypothesis:*

and adding certain small orbs and small circles which he judged suitable for completing the librations and alternations of the latitudes.

[Margin: On the geometric calculation of latitude.] II. *We have put off the form of the geometric calculation for finding the latitude of these planets at any given time to ch. 8, from no. 16, though we shall give a taste of it in ch. 3, where we shall set out Lansbergen's calculation, which is Copernican in substance and differs only in its numbers and measures, and in ch. 4, where, with Longomontanus, we shall show how to fit the Danish measures and numbers to the Copernican hypothesis as well.* [Margin: The error of Copernicus in the motion of the nodes.] *From that same chapter, moreover [au-dem; recte: autem], it will become clear at no. 6 how far Copernicus erred in assigning to the nodes of these planets the motion of the apogees, and so advancing the places of the nodes too far, especially in Jupiter, where he overshot by a full 20°. And from ch. 5, no. 4, and its Scholia 3, 4 and 5, it will become clear whether and why he went wrong in making the inclinations of Venus and Mercury librate and vary.*

CAPVT III.

Chapter 3.

De Hypothesi Lansbergiana circa Latitudinem Quinque minorum Planetarum.

On Lansbergen's hypothesis concerning the latitude of the five minor planets.

I. Lansbergen's hypothesis of the latitude of the planets differs from the Copernican both in measures and numbers and in the places and motion of the limits and nodes. [Margin: Whether Lansbergen's hypothesis is Copernican.] He thinks that these are in no way moved with the motion of the apogees, as Copernicus, too much attached to Ptolemy, thought, but (except for Jupiter, whose node he makes fixed) by a motion proper and peculiar to them, though very slow, just as he holds the apogees too are driven. For the rest, as regards the theory of the Earth carried around the annual orb, and as regards the inclinations of the orbs to the plane of the ecliptic, in which the Earth's annual orb always lies, it is the very image and offspring of the Copernican hypothesis, as is clear

from Lansbergen's *Theorica motuum cælestium*, from chapter 15 to 19, from which

De Latitudine Trium Superiorum.

On the latitude of the three superior planets.

II. In the following diagram, let A be the centre of the ecliptic FCGE and of the Earth's great orb HIKL, which lies in the same plane of the ecliptic. [Margin: Beware of the error in the first figure of Lansbergen, ch. 15.] To this plane, as also to the ecliptic, let the orb BCDE of Saturn, Jupiter or Mars be inclined, with a fixed inclination and a constant angle BEF and DEG. Let the limit, or end of greatest latitude, be at B on the northern side and at D on the southern; let the common section of the planetary orbit and the ecliptic be CAE; and let the northern or ascending node be E, and the southern or descending node C. But let the nodes and limits of Jupiter not be changed, since they are fixed in the same place: for Jupiter's northern node remains perpetually in Cancer 5° 30′. Those of Saturn and Mars, however, shift in the order of the signs, by a motion to be given in small tables at the end of the chapter. And let the straight line FAD be drawn, cutting the line of sections CAE at right angles and in two equal parts at A. On these assumptions there arises a twofold variety of latitude in these three planets, answering to the twofold anomaly of the motion in longitude. The first is called *Centric*: it is the one that has its angle at A, the centre of the Earth's great orb, and it would be seen by an eye placed there, near the Sun. The second is the one seen from the globe of the Earth, and it has its angle at the centre of the terrestrial globe: for example the angle MON, if the Earth is at O and the planet at M; or the angle QOR, if the planet is at R; or *m*P*n*, if the Earth is at P and the planet at *m*. This latitude is called *True* or *Apparent*, and it differs from the latitude seen from the Earth's surface only as far as parallax in latitude requires. The first depends on the planet's distance from either of the nodes and on the inclination of the orb to the ecliptic. The other depends on the centric latitude and on the planet's distance from the centre of the Earth; and they are worked out by solving a double triangle. Thus if the planet is at M and the Earth at O, one seeks FB, the right sine of the inclination FEB, and the sine of the arc EM by which the planet is distant from the nearer node, according to the motion taken from the tables. When these are found, make: as sine EB, that is, the whole sine, is to sine EM, so sine BF is to sine MN, namely the sine of the planet's centric latitude, which throughout the semicircle EBC is northern. Next, having found the planet's distance from the centre of the Earth, as is done in the geometric calculation of longitude (in our case MO), make: as MO, the distance, is to sine MN of the centric latitude, so OM, taken as radius, is to MN, the sine of the true latitude, or of the angle MON. And the Earth's mean motion, which is always diametrically opposite the Sun's mean motion, will give the Earth's place on the annual orb, on which the angle of true latitude is to be based.

Diagram. Set into the lower left column, with the text running narrowly beside it. Two large tilted curves cross at the two ends of a horizontal line C–A–E: one passes through F at the top and G at the foot, the other through B and D, so that F stands outside B at the top and D outside G at the foot. A vertical line F–B–H–A–K–G–D passes through the centre A. A smaller circle about A carries H at its top, I at its left, L at its right and K at its foot. On it stand O, upper right, and P, lower left, P marked with a small rayed sign. From O straight lines run to M and N at the upper right and to R and Q at the upper left; from P lines run to *m* and *n* at the lower right. Below the lower curve stand three planetary signs, either side of D.

- F (top, on the outer curve): point of the ecliptic
- Q (upper left, outer curve) and N (upper right, outer curve)
- R (upper left, inner curve), B (top, inner curve), M (upper right, inner curve)
- H, I, L, K (top, left, right and foot of the small circle)
- O (upper right of the small circle), P (lower left of the small circle)
- C (left end), A (centre), E (right end)
- *m* and *n* (lower right)
- G (foot, inner curve), D (foot, outer curve)
- ♄ and ♃ (below the curve, left of D); ♂ (below the curve, right of D)

EXEMPLVM.

Example.

Latitudinis ♄, ♃, ac ♂ Geometricè inuestigatæ ex Lansbergio.

Of the latitude of ♄, ♃ and ♂ investigated geometrically, from Lansbergen.

III. *In the preceding diagram, let Saturn be at M and the Earth at O; Jupiter at* m *and the Earth at P; and Mars at R and the Earth at O.* [Margin: The latitudes of the three superior planets, one observed in antiquity for each.] *And let the year of Nabonassar be 519, Tybi 22, on the evening of which, as Ptolemy reports in Book 11, ch. 7, Saturn's northern latitude was observed at Alexandria, as large as the latitude of the star in the southern shoulder of Virgo, that is, 2° 43′. Let the year of Nabonassar be 507, day 17 of Epiphi, 16⅔^h after whose noon Jupiter's southern latitude was observed at Alexandria, according to Ptolemy, Book 11, ch. 3, the same as the latitude of the southern Ass, namely 10′. Finally, let the year of Nabonassar be 476, day 19 of Athyr, 18^h after noon, at which hour, according to Ptolemy, Book 10, ch. 9, the northern latitude of Mars was observed at Alexandria, very nearly the same as the latitude of the uppermost star in the forehead of Scorpius, 1° 15′. For these times, from the calculation of longitude undertaken by Lansbergen in his* Theoricae⋆, ch. 10, their centric longitude is given and their distance from the centre of the Earth, and the other motions from Lansbergen's tables, as below.⋆

Quantity	Planet	Line	Value
Centric longitude [Longitudo centrica]	♄		98° 51′ 57″
	♃		57° 35′ 35″
	♂		111° 57′ 18″
Mean place of ☉ for [Medius Locus ☉ pro]	♄		343° 19′ 16″
	♃		99° 6′ 50″
	♂		292° 58′ 29″
Equal motion of the node E ☊ [Motus æqualis Nodi E ☊]	♄		81° 0′ 0″
	♃		95° 30′ 0″
	♂		26° 28′ 35″
Distance of the planet from the Earth [Distantia Planetæ à Terra], in parts of which the radius AB is 10000 [Qualium Radius AB est 10000.]	♄	OM	9105
	♃	Pm	10916
	♂	RO	8970

[Margin: Step 1.] *Given these, first subtract the motion of the northern node from the planet's centric longitude, and you will have the planet's distance from the northern node* in the order of the signs. In Saturn this will be EM, 77° 51′ 57″, whose sine is 9776. In Jupiter it will be EBCD*m*, 352° 6′, and its complement to the circle, the arc* m*E, 7° 54′, whose sine is 1374. And in Mars it will be EBR, 145° 28′ 43″, whose complement to the semicircle will be the distance RC from the nearer node, 34° 31′ 17″, whose sine is 5666.*[67]

[Margin: Step 2, for the centric latitude.] Second, *find the sine of the given inclination for each planet; once found it remains useful for good, because the inclination is fixed, and it is as follows.*

Planet	Inclination [Inclinatio]	Inclination, value	Sine of the inclination [Sin^9 Inclinationis]	Sine, value
♄	BEF	2° 31′	BF	439
♃	DEG	1° 20′	DG	233
♂	BCF	1° 50′	BF	320

*So make: as the whole sine of quadrant EB, 10000, is to sine BF, 439, so the sine of arc EM, 9776, is to NM, sine 429, of Saturn's centric latitude. And as the whole sine of quadrant ED, 10000, is to sine DG, 233, so sine E*m*, 1374, is to sine* mn*, 32, of Jupiter's centric latitude. And as the whole sine of quadrant CB, 10000, is to sine BF, 320, so sine CR, 5666, is to sine QR, 181, of the centric latitude of Mars.*

[Margin: Step 3, for the true latitude.] Third, *in triangle MON the distance MO, 9105, is given, and sine MN, 429. So make: as MO, 9105, is to MN, 429, so*

[67]As printed, the degrees of these three distances do not follow from the table above, though the minutes and seconds do, within a minute in Jupiter. By subtraction the table gives 17° 51′ 57″ for Saturn, where the text has 77° 51′ 57″; 322° 5′ 35″ for Jupiter, where the text has 352° 6′; and 85° 28′ 43″ for Mars, where the text has 145° 28′ 43″. The printed figures stand in both places.

OM, of 10000 parts, is to MN, 471, the sine of angle MON, namely 2° 42′, the true northern latitude for Saturn. [Margin: Saturn.] *Again, in triangle* mPn *the distance* mP⋆, 10916, is given, and sine⋆ mn⋆, 32. So make: as⋆ mP⋆, 10916, is to⋆ mn⋆, 32, so⋆ mP⋆, 10000, is to⋆ mn⋆, 29, the sine of angle⋆ mPn⋆, 0° 10′, that is, of the true latitude of Jupiter.⋆ [Margin: Jupiter.] *Finally, in triangle QOR the distance RO, 8970, is given, and sine QR, 181. So make: as RO, 8970, is to QR, 181, so RO, the whole sine 10000, is to QR, 202, the sine of angle QOR, 1° 10′, namely the northern latitude of Mars.* [Margin: Mars.]

The true latitude was therefore [Fuit ergo Latitudo Vera]:

Planet	Observed [Obseruata]	Computed [Supputata]
♄	2° 43′ B	2° 42′
♃	0° 10′ A	0° 10′
♂	1° 15′ B	1° 10′

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[Margin: Drawing the figure for the true latitude.] IV. To draw a suitable figure, moreover: when the motion of Saturn's node was taken from the Sun's mean motion, the remainder was the Sun's distance from the northern node, 262° 19″ [recte: 19′] 16″. Counted on the great orb from L through H and I, these brought the Sun to P, and so the Earth to O, where the visual angle of the true latitude is based. This is not because the Sun in this hypothesis is on the circumference of the great orb, but because it stands over against the Earth, diametrically opposite it; for in fact it is near A, the centre of the great orb. In the same way, when the place of Jupiter's northern node was taken from the Sun's mean motion, there remained the Sun's distance from the northern node, 63° 37′. Counted from L in the order of the signs, these bring the Sun's place to O, and over against it the Earth to P, where the visual angle of the true latitude is. Finally, from the Sun's mean motion and the place of the node of Mars, the Sun's distance from the northern node is gathered as 266° 29′ 54″. Counted from L through H, these bring the Sun's place to P and that of the opposite Earth to O.[69]

De Latitudine Veneris ac Mercurij.

On the latitude of Venus and Mercury.

V. For the theory of the latitudes of Venus and Mercury, let the Earth's great orb HIKL be described from A. In its plane let there be the orb FCGE, and to this let the orb BCDE of Venus or of Mercury be inclined. Let the nodes

[68] B and A stand for Boreal, northern, and Austral, southern; the Computed column prints no letter.

[69] The Saturn and Mars distances here follow from the table on the facing page. Jupiter's does not: 99° 6′ 50″ less the printed 95° 30′ is 3° 36′ 50″, where the text has 63° 37′. The printed figures stand.

be the northern at E and the southern at C, and through them let the straight line of common section CAE pass; produced as far as the great orb, let it be IAL, which HAK cuts at right angles at A, the centre of the great orb. Let the limits of greatest latitude be the northern at B and the southern at D. Let the angles of the inclinations, BEF and DEG, be fixed and unchangeable: in Venus 3° 30′, in Mercury 6° 16′. And let the nodes and limits be carried in the order of the signs by a very slow motion, on which see the tables below. Now imagine the planet at M and the Earth at O, or the planet at S and the Earth at Q. The planet's centric latitude, viewed from A, will then be the arc MN or ST, and the true latitude, viewed from the Earth, will be the angle MON or SQT. Their variation and the finding of them depend, as in the superior planets, on the planet's centric longitude, on the motion of the node and on the planet's distance from the Earth, as will shortly be made clear by examples.

Diagram. Set into the lower left column, with the text running narrowly beside it. A large circle HIKL has H at the top, I at the left, L at the right and K at the foot, with a vertical diameter HAK and a horizontal diameter IAL crossing at the centre A. Rayed solar signs stand on the circle at R, upper left, and at P, lower left. Within the circle, curves cross at C on the left and at a point right of A on the horizontal line, one passing through F at the top and G below, the other through B and D. Two diagonals pass through A: R–A–Q, and O–A–P with Z lying on the lower-left part of it. From O straight lines run in to N and M; from Q straight lines run up to T and S.

- H (top), I (left), L (right), K (foot): the large circle
- R (upper left, with rayed sign), O (upper right), P (lower left, with rayed sign), Q (lower right)
- F, N (upper part, within the circle), B, M, T
- Y (between M and S); S
- C (left), A (centre), E (right of A, on the horizontal line, its middle arm merged with the line)
- G, D (below the centre), Z (lower left), X (lower right)

EXEMPLVM.

Example.

Latitudinis ♀ ac ☿ Geometricè inuestigatæ ex Lansbergio.

Of the latitude of ♀ and ☿ investigated geometrically, from Lansbergen.

VI. *In the preceding figure, let Venus be placed at S and the Earth at Q, but Mercury at M and the Earth at O. For in the year of Nabonassar 476, Mesori 17, Venus as a morning star was seen at Alexandria to cover the leading star of the four that are in the southern wing of Virgo, whose latitude was then 1° 21′ north. And in the year of Nabonassar 484, day 18 of Thoth, Mercury as a morning star was seen in the same place more northerly than the uppermost star of the forehead of Scorpius, which has a latitude of 2° 15′, by two Moons, or about one degree. For these times, from*

Lansbergen's calculation of longitude and from his tables, the following motions are given.

Quantity	Planet	Line	Value
Centric longitude [Longitudo centrica]	♀		195° 5′ 5″
	☿		227° 44′ 15″
Equated anomaly of the orb [Anomalia Orbis æquata]	♀		249° 11′ 32″
	☿		213° 36′ 46″
Motion of the node [Motus Nodi] E ☊	♀		50° 55′ 16″
Motion of the node [Motus Nodi] C ☋	☿		217° 0′ 2″
Distance of the planet from the Earth [Distantia Planetæ à terra]	♀	SQ	9943
	☿	MO	7506

But the distance of ♀ is in parts of which the radius of the great orb is 10000, while that of ☿ is in parts of which the radius of Mercury's orb is 3818.

[Margin: Step 1, for the centric latitude.] *Given these, first, when the motion of the northern node has been taken from the centric longitude of Venus, there remains the Sun's distance from the northern node, for Venus 144° 10′. Counted from L through H, these give the Sun's place at R and the Earth's at Q. For Mercury, when the motion of the southern node has been taken from Mercury's centric longitude, there remains the distance from the southern node, 10° 44′. Counted from I to P, these give the Sun's place at P and the Earth's at O. So we are given arc OL, the complement of arc LHO, and the arc EX equal to it, 35° 50′; and the equated anomaly of the orb, counted from the true apogee V through DS, 249° 12′. When the semicircle is taken away there remains XS, 69° 12′, and when EX is subtracted from this there remains ES, 33° 22′, namely the distance of Venus from the northern node, whose sine is 5500. Similarly for Mercury we are given LO, 10° 44′, to which the arc EY is equal; and the equated anomaly of the orb, 213° 37′, counted from Z through D as far as M, will be the arc ZDM. When the semicircle ZDY is taken away there remains YM, 33° 37′. When EY is added to this, the whole arc EM becomes 44° 21′, which is the amount of Mercury's distance from the northern node; and its sine is 6990.*

[Margin: Step 2, for the latitude of ♀.] Second, *find the sine of the given inclination, which is as follows.*

Planet	Inclination BEF [Inclinatio BEF]	Sine of the inclination [Sinus Inclinationis]	Sine, value
♀	3° 30′	BF	610
☿	6° 16′	BF	1092

So make: as the whole sine of quadrant EB, 10000, is to sine BF, 610, so the sine of arc ES, 5500, is to ST, sine 335, in parts of which AE is 10000. But in parts of which AE, the radius of Venus's orb according to Lansbergen, is 7193, ST is 241 for the centric latitude, since as 10000 is to 335, so 7193 is to 241. Therefore in triangle QST, right-angled at T because the arc ST and its sine are perpendicular, from the base QS, namely the distance of Venus from the Earth, 9943, and the side ST, 241, ST is found to be 242, namely the sine of the angle SQT, 1° 23′, of the northern latitude of Venus.

[Margin: Step 3, for the latitude of ☿.] Third, *make: as the whole sine of quadrant EB, 10000, is to sine BF, 1092, so sine EM, 6990, is to MN, 763, in parts of which radius AE is 10000. But in parts of which radius AE is 3818, MN will be 291. For as 10000 is to 763, so*

3818 is to 291. Therefore in triangle MON, right-angled at N, we are given the base MO, the distance of Mercury from the Earth, 7506, and the side MN, 291. From these the angle MON is gathered trigonometrically as 2° 13′, namely of the northern latitude of Mercury. In this treatment, moreover, the semidiameter of the orb of ☿, already found in the calculation of longitude, is presupposed, according to what was said in Sect. 3, ch. 3, no. 4, since in this hypothesis it is variable.

The latitude *was therefore* [Fuit ergo Latitudo]:

Planet	Observed [Obseruata]	Computed [Supputata]
♀	1° 21′ B	1° 23′
☿	2° 15′ B	2° 13′

[Margin: Šírek, a follower of Lansbergen.] It remains to gather Lansbergen's numbers and measures into a synopsis, supplying the greatest latitudes, which he himself omitted, and which are gathered from the sine of the inclination of the orbs and the greatest or least distance of the planet from the Earth. Lansbergen's measures were adopted by Antonín Maria Šírek in his *Radius Sidereomysticus*, Book 3, ch. 1, as was the motion of the nodes.

Synopsis of the latitudes from Lansbergen [Synopsis Latitudinum ex Lansbergio]

Greatest inclination of the orb [Inclinatio Orbis Maxima]

Planet	Inclination
♄	2° 31′
♃	1° 20′
♂	1° 50′
♀	3° 30′
☿	6° 16′

Greatest latitude of the superior planets [Latitudo Maxima Superiorum]

Planet	Side	At the apogee of the orb [In Apogeo orbis]	At the perigee of the orb [In Perigeo orbis]
♄	South and north [A. & Bor.]	2° 17′	2° 48′
♃	South and north [A. & Bor.]	1° 7′	1° 38′
♂	North [Borealis]	1° 9′	4° 34′
♂	South [Australis]	1° 4′	6° 45′

Greatest latitude of the inferior planets from the declination [Latitudo Maxima Inferiorum ex Declinatione]

Planet	Side	At the apogee of the orb [In Apogeo orbis]	At the perigee of the orb [In Perigeo orbis]
♀	North [Bor.]	1° 25′	8° 54′
♀	South [Austr.]	1° 28′	8° 36′
☿	North [Bor.]	1° 32′	4° 5′
☿	South [Austr.]	1° 32′	4° 5′

Planet	Latitude from the reflexion [Latitudo ex Reflexione]	At anomaly of the orb [In Anomaliæ orbis]
♀	2° 31′	4ˢ 16°
♀	2° 31′	7ˢ 14°
☿	2° 14′	3ˢ 22°
☿	2° 14′	8ˢ 8°

Places of the nodes and limits at the beginning of the year 1 of Christ [Nodorum ac Limitum Loca ad Annum 1. CHRISTI ineuntem]

Planet	Northern node E [Nodus Boreus E]	Southern node C [Nodus Austr. C]	Northern limit of greatest latitude B [Limes max. Latitud. Boreus B]	Southern limit of greatest latitude D [Limes max. Latitud. Austr. D]
♄	♊ 25° 16′	♐ 25° 16′	♍ 25° 16′	♓ 25° 16′
♃	♋ 5° 30′	♑ 5° 30′	♎ 5° 30′	♈ 5° 30′
♂	♈ 29° 30′	♎ 29° 30′	♋ 29° 30′	♑ 29° 30′
♀	♉ 23° 53′	♏ 23° 53′	♌ 23° 53′	♒ 23° 53′
☿	♉ 8° 0′	♏ 8° 0′	♌ 8° 0′	♒ 8° 0′

Daily motion of the nodes [Motus Diurnus Nodorum]

Planet	‴	⁗	V.	VI.
♄	11	0	24	20
♃	0	0	0	0
♂	6	34	31	14
♀	6	26	28	28
☿	2	14	16	39

SCHOLIA.

Scholia.

I. *It must be noted that the motion of Jupiter's node is so slow that, if the Ptolemaic and Tychonic observations are true, about which we shall speak in ch. 4, no. 6, in the 1456 years that elapsed between Ptolemy and Tycho it has not crept forward* in the order of the signs *even by 6°. It is so far from having completed 26° since then, as*

Copernicus believed, assigning it the same motion as the apogee, in the manner of the ancients. [Margin: Criticism of Lansbergen on the immobility of Jupiter's node.] *So the smallness of this motion gave Lansbergen the handle for imagining Jupiter's node perpetually fixed in Cancer 5° 30′, where it was found toward the end of the last century. But if a motion must be eliminated altogether because it is small, made over a great interval of time, how many other motions smaller than the motion of this node would have to be abolished, which Lansbergen nevertheless accepts: for example, that of the obliquity of the ecliptic? Then, if the place of this node had to be set as immovable, it seems a place should have been chosen midway between the place observed by Ptolemy, which was Cancer 1°, and the place observed by Tycho in Cancer 7°, and so it should have been fixed in Cancer 3° or 4°, so that even in this way the observations of both periods would be better satisfied.*

II. *Lansbergen's excessive brevity and the neatness he strove for in his* Theoricae *were the reason that he did not set out the theory of the reflexion of Venus and Mercury, though he nevertheless accepts it in his tables.*

CAPVT IV.

Chapter 4.

De Hypothesi Latitudinis Quinque Minorum Planetarum Danica, seu Longomontanica.

On the Danish, or Longomontanan, hypothesis of the latitude of the five minor planets.

I. Longomontanus sets out the theory of the three superior planets concerning the motion in latitude in the *Astronomia Danica*, Book 2 of the *Theorica*, ch. 14. He first remarks that from the varying latitude, which other things being equal is always greater at a smaller distance of the planet from the Earth, the varying elevation of these planets above the Earth is confirmed a posteriori, since optical reasoning requires that nearer arcs appear larger. [Margin: An argument against the homocentrics from the latitude of the planets.] This the defenders of the homocentrics certainly did not see. He shows it by a figure in which the angle at the Earth, now nearest to the planet and now farthest, subtends the arc of greatest latitude. But one grounded in the elements of optics has no need of another figure, and the matter will become quite plain from this other figure, fitted to Copernicus's hypothesis. In it let E be the centre of the annual orb FHG, which takes the place of the Ptolemaic epicycles. Let two straight lines be drawn: one, AEB, in the plane of the ecliptic and of the annual orb; the other, CED, in the plane of the eccentric of these planets, cutting AB at the centre E. Let C be the northern limit of latitude, and let the greatest northern latitude be measured by the angle CFA, when in the planet's acronychal oppositions with the Sun at G the Earth is at F; but by the angle CGA, when in its conjunctions with the Sun, towards F, the Earth is at G. But let the southern limit be D, and let the greatest southern latitude be measured by the an-

Diagram. Set into the lower right column, with the text running narrowly beside it. A circle about E, with F at its top, H at its left and G at its foot, is crossed by a near-vertical straight line A–F–E–G–B. A second straight line runs through E from C, at the upper left above the circle, to D, at the lower right below it. Further straight lines join C to F and to G, and D to F and to G. The signs ♄, ♃ and ♂ stand one below another inside the circle on its right side.

- C (top left), A (top right)
- F (top of the circle), H (left, inside the circle), E (centre), G (foot of the circle)
- B (foot, left), D (foot, right)
- ♄, ♃, ♂ (right side, inside the circle) gle BGD if the Earth is at G, in the acronychal positions, etc., or by the angle BFD if the Earth is at F. For it is clear that the exterior angle AFC is greater than the interior angle AGC, and the exterior angle BGD greater than the interior angle BFD, by Euclid I.32; and therefore that the same greatest latitude will nevertheless appear greater to an Earth nearby than to one far off.

II. With these things assumed, he presents the places of the limits and nodes observed by Tycho in the acronychal oppositions, in which the latitude appears greatest because of the planet's nearness to the Earth, and the greatest latitude observed on that occasion, which we shall give in our usual way at the end of this chapter. He teaches that to investigate the simple or mean inclination of the orbs to the ecliptic one needs the greatest latitude: in the preceding figure, the angle AFC, measuring the greatest northern latitude, if the Earth moves and is at F; or rather the angle BGD, measuring the greatest southern latitude, which, as I said, is greater than the northern, if the Earth is at G. Next one needs the semidiameter of the annual orb, that is, the mean distance of the Sun from the Earth, namely the side EG; and finally the planet's distance from the centre of the eccentric orb, namely the side DE, or DG, the distance from the Earth if the planet in the acronychal position is at D. For in the triangle DGE, given the sides EG and DE or DG, and the angle EGD which they contain, which is the complement to two right angles of the angle BGD measuring the greatest southern latitude, a trigonometer will easily obtain the angle GED, the inclination of the plane CED to the plane of the ecliptic AEB. [Margin: Longomontanus's foundations for the inclination.] From his own hypothesis on the longitude of the planets, therefore, he sets down, in units of which the radius of the eccentric is 100000, both the semidiameter of the annual orb and the least distance of the three planets from the centre of their own orb when they are in the acronychal position and have the greatest southern latitude, as you see in the following little table.

	Greatest southern latitude	Semidiameter of the annual orb	Distance of the planet from the centre of the orb
♄	2° 49′	10426	97593
♃	1° 40′	19349	95230

	Greatest southern latitude	Semidiameter of the annual orb	Distance of the planet from the centre of the orb
♂	6° 42′	65095	89090

[Margin: The inclination of the three superior planets found.] From these he derives the mean and simple inclination of the plane of the eccentric to the plane of the ecliptic: in Saturn 2° 31′, in Jupiter 1° 19′ 15″, in Mars 1° 48′ 40″. He also confirmed this from the greatest northern latitude, using the triangle CEF and the sides EC and EF. In Saturn and Jupiter he found the same inclination, that is, the angle CEA, equal to the angle BED. But in Mars, using the side CE as 110910 and EF as 65895, according to the correction given in the theory of the longitude of Mars, he found from the greatest northern latitude observed by Tycho, 4° 33″ [recte: 4° 33′], the inclination of the plane of Mars, that is, the angle CEA, to be 1° 50′ 55″. From this he came to suspect that the greatest southern latitude observed by Tycho ought to be corrected somewhat, and increased, because of refractions not sufficiently eliminated in the calculation. So he took the inclination handed down by the ancients, 1° 50′, as a mean between 1° 48′ 40″ and 1° 50′ 55″, and, reversing the demonstration, found the greatest northern latitude of Mars to be 4° 31′ and the southern 6° 47′. [Margin: The inclination of the orbs of the three superior planets invariable.] And so he established that the inclination of the orbs of these three planets to the annual orb, which serves in place of the Ptolemaic epicycles, and hence to the ecliptic, is invariable, contrary to what the ancients held about the epicycles of these planets, and fixed at a constant angle. With this hypothesis made, he gives a method of investigating the latitude of the planets for a given time in the threefold hypothesis, but principally in the Copernican; we shall divide this method into the following problems.

Problema 1. Data Inclinatione &c. Latitudinem Planetæ Superioris ad datum tempus Geometricè inquirere, in Forma Copernicea.

Problem 1. Given the inclination, etc., to investigate geometrically the latitude of a superior planet for a given time, in the Copernican form.

III. With centre A let the annual orb FE, etc., be described, and let the Earth be at F, near the acronychal position E, viewing the planet C, of whose eccentric BC is a portion, and to the planet let the straight line AC be drawn from A. Let BD be a portion of the ecliptic, and let the inclination of the planet's orb to it be the angle CBD. Then let the perpendicular CD be drawn to the ecliptic, and let C and D be joined with the Earth by the straight lines CF and DF, and D with A by the straight line AD. Now let BC be given, the planet's distance from the nearer node B. For from the whole sine, the sine of the arc BC and the angle B, the sine CD will be found, in units of which the radius of the eccentric is, for example, 100000, and also its arc DC. Next, DC

must be reduced to proportion with the planet's given distance CA, previously found from the calculation of the longitude; and also AD, in the triangle ACD, right-angled at D, in which the angle DAC is given, equal to the arc DC that was found, and the side CA. Afterwards, in the triangle DAF, in which AD is now given, the semidiameter AF of the annual orb is taken as given in units of which the planet's radius is known; and so is the angle DAF, because it is the complement to a semicircle of the anomaly of the annual orb, that is, of the distance of the Earth F from the planet's centric place in the ecliptic, which is D. In that triangle, I say, the side DF is derived. Finally, in the triangle CDF, from CD and DF now found and the right angle D, the angle CFD is derived, which determines the latitude of the planet C as seen from the Earth F. In the acronychal position itself, however, the calculation becomes easier.

Diagram. Set into the right column, with the text running narrowly beside it on the left. A circle about A carries E at the top of its circumference and F just to the right of it. From B, at the left outside the circle, two arcs rise to the upper right, one to C and one to D; C and D stand close together above the circle. Straight lines join C to D, C and D to F, and A to C, to D (through E) and to F. A straight line continues from A down to the foot of the circle. The sign ♄ stands inside the circle, left of that line.

- C (top, the upper end of the upper arc), D (top right of C)
- B (left, the lower end of the arcs)
- E (top of the circle), F (circumference, upper right)
- A (centre)
- ♄ (inside the circle, below A, left of the vertical)

Problema 2. Idem Inueſtigare in Tychonica Forma.

Problem 2. To investigate the same in the Tychonic form.

IV. [Margin: Delineation of the scheme in the Tychonic form.] Longomontanus adapts this problem to Mars, because of the particular difficulty in that planet, in roughly this way. From E, the centre of the unmoved Earth, describe the Sun's annual orb AKF, whose diameter is KA. Let the Sun be at F, distant from A, the acronychal position, by the arc AF. Then on the spherical surface let the arcs BC, a portion of the planet's eccentric, and BD, a portion of the ecliptic, be described, whose nearer node is B. Let Mars, for example, be at C; let the perpendicular CD be drawn to the ecliptic; and let D and C be joined with the Sun F by the straight lines CF and DF, so that the line FD, away from the planet's opposition, is parallel to the diameter AK. From the Earth let the straight line EF be drawn to the Sun, the straight line EC to the planet, and the straight line ED to the ecliptic. The measure of the latitude to be investigated will then be the angle CED. [Margin: Step 1.] First, therefore, in the spherical triangle CBD, from the whole sine and the sine of the arc BC, by which the planet is distant from the nearer node B, the right sine CD is found, with its arc, the angle of inclination B being assumed; and to this arc the angle CFD is equal.

But because DC has been found in units of which the radius of the sphere is 100000, it must be reduced to proportion with FC, the planet's distance from the Sun, as to a radius; this distance is here taken as known from the calculation of the planet's longitude. [Margin: Step 2.] Secondly, in the triangle CFD, from the angle at F, or rather from the right angle at D, and the sine DC, obtained in units of the side CF, and the side

Diagram. Set into the lower right column, with the text running narrowly beside it on the left. A circle about E, with K at its left and A at its right joined by a diameter through E, and F on the circumference below A. D and C stand inside the circle below E, to its left. Straight lines join E to A and to K (the diameter), E to F, E to D, E to C, D to F, C to F and D to C. From B, below and left of the circle outside it, one line rises to D, and another rises through C and runs on to E.

- K (left of the circle), A (right of the circle), F (lower right, on the circumference)
- E (centre)
- D (left, inside the circle), C (below D, to its right, inside the circle)
- B (foot, outside the circle)

CF itself, DF is sought. [Margin: Step 3.] Thirdly, in the triangle DFE, ED is sought from the semidiameter EF of the annual orb, in units in which FC, the radius of the planet's orb, was known; from DF, now found; and from the angle EFD, which, because of the parallels AE and FD that the Tychonic hypothesis requires, is equal to the angle AEF, measuring the distance of the Sun F from the planet's opposition with the Sun. From these three givens, I say, ED is sought. [Margin: Step 4.] Finally, in the triangle CDE, right-angled at D, from DC and DE now found, the angle DEC is found, which determines the apparent and true latitude of the planet as seen from the Earth E.

Problema 3. Data inclinatione vniformi Epicycli ad Eclipticam; inuenire Latitudinem Planetæ in forma Ptolemaica.

Problem 3. Given the uniform inclination of the epicycle to the ecliptic, to find the latitude of the planet in the Ptolemaic form.

V. I said "uniform", as the more recent astronomers, with Longomontanus, would have it, asserting that this uniformity of the inclination satisfies the phenomena more correctly. On this assumption, in the accompanying figure, let MAL be the diameter of the ecliptic, and let it be intersected at the centre of the world A by CAB, the diameter of the eccentric orb of some superior planet. From the ends of this diameter, B and C, as centres, let the epicycles FH, to the north, and KG, to the south, be described, whose planes always stand uniformly and equidistantly to the plane of the ecliptic, so that their diameters FH and KG are parallel to the diameter of the ecliptic MAL. Next let the centre A be joined with the point F by the straight line AF, and with H by the straight line AH; then with G and K by the straight lines AG and AK. From all these points

and from B and C let perpendiculars be drawn to ML, namely HL, BN, FQ, GO, CP and KM. And when HF has been produced to E and KG to D, let the straight line ED be drawn through A to their ends, cutting ML at A at right angles.

Diagram. Set across the left column, breaking the paragraph. A long horizontal line M–P–O–A–Q–N–L runs through A. To the upper left, a circle with the diameter K–C–G above that line; to the lower right, a circle with the diameter F–B–H below it. From A straight lines run to K, C and G on the left and to F, B and H on the right. Short perpendiculars drop from K, C and G to M, P and O, and rise from F, B and H to Q, N and L. A dotted line runs from G to D and from F to E, and a dotted vertical joins D and E through A. The signs ♄ ♃ ♂ stand together above the right-hand circle.

- K (left), C (centre of the left circle), G (right end of the left diameter), D (above A, at the top of the dotted vertical)
- M (far left, on the horizontal), P (below C), O (below G)
- A (centre, on the horizontal)
- Q (above F, on the horizontal), N (above B), L (far right, on the horizontal)
- E (below A, at the foot of the dotted vertical), F (left end of the right diameter), B (centre of the right circle), H (right end of the right diameter)
- ♄ ♃ ♂ (above the right-hand circle)

With these designated: if in its conjunctions with the Sun the planet is to the north, at the epicycle's apogee H, the angle of latitude will be HAL; if to the south, at K, it will be KAM. But if in the acronychal oppositions it is at the perigee F, to the north, the angle of latitude will be FAQ; if to the south, at the perigee G, it will be GAO. These angles indeed vary as the sides AB and AH, and likewise AC and AK, will vary, with the planet's distance from the node on the eccentric kept the same, and its arc towards the ecliptic equal to the angles of inclination NAB and PAC. But their right sine, BN and CP, must be reduced to proportion with the planet's distances on the eccentric, AB or AC, just as AN and AP must. So, for example, in the triangle BAN, right-angled at N, from the given AB, the distance of the centre of the epicycle from the centre of the Earth, obtained by the calculation of the longitude, and from the inclination BAN, the sine BN is sought, to be reduced to units of AB, and the side AN. When NQ, equal to the semidiameter FB of the epicycle, is taken from AN, AQ remains in the triangle AFQ, right-angled at Q, in which FQ is given, equal to BN itself. So the angle QAF will not remain hidden, which determines the latitude in such a position. Let it be enough to have indicated this for the other cases.

De Motu Nodorum ac Limitum trium Superiorum Planetarum.

On the motion of the nodes and limits of the three superior planets.

VI. [Margin: Copernicus and Ptolemy criticised by Longomontanus.] About to treat this in Book 2, ch. 15, Longomontanus criticises Copernicus, and indeed Peuerbach and Regiomontanus, because, without consulting the heavens, they affirmed with Ptolemy that the nodes of these planets are moved with the same motion as the apogees, and so assigned them too fast a motion. And although he does not trust Ptolemy much, yet because older foundations are not available, he takes from Ptolemy that the limit of Saturn is 50° short of the apogee, that of Jupiter 20° beyond the apogee, and that of Mars at the apogee itself, about the year of the Lord 136. But he corrects the apogee of Mars from agreement with the greatest northern latitude, and resets it at Cancer 25½°. The places of the apogees of Jupiter and Saturn he corrects by removing 21′ of the precession of the equinoxes, so that they are reckoned from the mean equinox; just as the places of the nodes and apogees recently observed by Tycho about the year 1600 he corrects by adding 7′, to reduce them to the mean equinox. For in this way the mean or equal motion of the nodes is better derived from the interval of time and the motion made in the meantime. Finally, he appends this synopsis.

Years of Christ	Saturn: corrected apogee	Saturn: north node	Jupiter: corrected apogee	Jupiter: north node	Mars: corrected apogee	Mars: north node
136	♏ 23° 50′	♋ 3° 21′	♍ 14° 0′	♋ 1° 21′	♋ 28° 15′	♈ 25° 51′
1600	♐ 26° 27′	♋ 20° 23′	♎ 7° 32′	♋ 6° 53′	♌ 28° 42′	♉ 16° 53′
Difference	32° 37′	17° 2′	23° 32′	5° 32′	30° 27′	21° 2′

Therefore it is clear from these how plainly the followers of Ptolemy and Peuerbach, and Copernicus himself, erred [Margin: The error of the ancient astronomers in the motion of the nodes.] in setting the motion of the nodes and of the apogees to be the same, since in an interval of 1464 years the apogee of Jupiter, for example, has been advanced 23° 32′ and the north node hardly 5° 32′, and so with the rest. Further, from this it is derived by the rule of proportion: if the nodes have advanced so many degrees and minutes in 1464 years, how much they advance with equal motion in single years, or in 20, or in 100, or in 1000, namely:

Equal motion of the nodes [Motus Nodorum Æqualis]

Years	♄ Deg.	′	″	‴	♃ Deg.	′	″	‴	♂ Deg.	′	″	‴
1	0	0	41	53	0	0	13	36	0	0	51	43
20	0	13	57	42	0	4	32	8	0	17	14	26
100	1	9	48	31	0	22	40	39	1	26	12	8
1000	11	38	5	15	3	46	46	33	14	22	1	18

De Latitudine Veneris ac Mercurij.

On the latitude of Venus and Mercury.

VII. [Margin: Proposition 1, on the motion and place of the nodes.] Having completed the latitudes of the three superior planets, Longomontanus treats the latitude of Venus and Mercury in Book 2 of the *Theorica*, chs. 24 and 25. First, he affirms that the ancients wrongly thought the place of the nodes and of the apogees to be the same. For the south node of Mercury does cling perpetually to the apogee; but the north node of Venus, which once was at its apogee, precedes the apogee in this century by 16°, so that the apogee in this century is at the end of ♊ while the north node of Venus is at ♊ 13°. From this he derives that the motion of this node is slower than that of the apogee, and that it completes one degree in about 100 years. [Margin: Proposition 2, on the immutability of the inclination. Fixed in these planets also.] Secondly, he affirms that in the intersections of these nodes the inclination of the latitude is perpetually stable, and that it is invariable in every revolution of the centre of the planet's orb, according to the hypothesis of the unmoved Earth, or of the Earth itself in the hypothesis of the moved Earth; but with the roles exchanged in the first departures from the ecliptic, so that what applies to Venus towards the north applies to Mercury towards the south. And because the angles of the apparent inclinations change regularly with the change in the distance of these planets from the Earth, he affirms that their eccentric is not different from the plane of the ecliptic. [Margin: Assertion 3, on the identity of the inclination and the reflexion.] Thirdly, he asserts that Ptolemy wrongly separated the inclination from the reflexion, since in reality they are the same thing, and that he set it greatest at the point of contact of the epicycle. But this arose from the fact that around the point of contact of the orb, on the occasion of the greatest digressions from the Sun, he always made the angle of inclination smaller by one degree than the inclination observed near the perigee of the epicycle and the quadrants of the eccentric requires; which Ptolemy, however, could scarcely observe in a less oblique sphere. Moreover, in all five planets every variation of latitude depends on the planet's distance from the nodes, at which the angle of perpetual inclination lies, on the planet's motion in the epicycle or its own orb, and on the planet's distance from the Earth. [Margin: Assertion 4, on the superfluity of the Deviation.] Fourthly, he says that the Deviation introduced by Ptolemy, and indeed at its greatest 10′ to the north in Venus and 45′ to the south in Mercury, is superfluous, and has no sufficient foundation in observations properly examined. [Margin: Assertion 5, on the quantity of the inclination.] Fifthly, having compared the observations with one another, he says that he has found the angle of the mean inclination to be 3° 30′ in Venus and 5° 40′ in Mercury, and that by these values the phenomena are satisfied.

Diagramma Latitudinum Veneris ac Mercurij in forma Ptolemaica & Tychonica.

Diagram of the latitudes of Venus and Mercury in the Ptolemaic and Tychonic form.

VIII. From A, the centre of the world, let the eccentric BDFH of Venus or Mercury be described, whose diameters DH and BF intersect each other at right angles at A; and BF is the line of the nodes of the eccentric.

Diagram. Set across the left column, breaking the paragraph. A large circle about A, the eccentric, carries eight small circles, the epicycles, whose centres stand on it. A vertical line runs from the top small circle B through A to the bottom small circle F, and a horizontal line from the left small circle D through A to the right small circle H. In each small circle a vertical diameter RS and a horizontal diameter LK cross at the centre, and a further slanting diameter NM passes through it. The words *Orien* and *Occid* stand on either side of A, read as printed, the first with a stroke over its end; below them, on either side of the vertical, stand the signs ♀ and ☿.

- A (centre), with *Orien* to its left and *Occid* to its right; ♀ and ☿ below
- B (top small circle): R, N, L, K, M, S
- C (upper left small circle): R, N, L, K, M, S (this S stands below the circle)
- D (left small circle): R, N, L, K, M, S
- E (lower left small circle): R, N, L, K, M, S
- F (bottom small circle): R, N, L, K, M, S
- G (lower right small circle): R, N, L, K, M, S
- H (right small circle): R, N, L, K, M, S
- I (upper right small circle): R, N, L, K, M, S

For the present, neglecting the eccentricity, let the centres of the epicycles, with their epicycles, be at the points B, C, D, E, F, G, H and I. In any place on the eccentric the epicycles retain sections similar to the eccentric's, and their parallelism, so that their diameters RS are parallel to the diameter BF itself, and the diameters LK parallel to the diameter DH, at whatever position on the eccentric the epicycle may be. Now, since the orb or epicycle is inclined to the ecliptic, let diameters MN be drawn in every position of the epicycle through the centres of the epicycles, at which there is a common intersection with the diameters LK; and let the angle of inclination NBL or KBM be perpetually 3° 30′ in Venus and 5° 40′ in Mercury. Let the north node be at B, which now coincides with the apogee of Mercury but in Venus precedes the apogee by 16°, and the south node at F. With these things set down, as long as Venus is in the eastern semicircle of the epicycle RNS, it will be in northern latitude, and in the western semicircle of the epicycle SMR it will be in southern latitude. These roles are exchanged in Mercury, so that the eastern semicircle of the epicycle belongs to southern latitude and the western to northern. The transitions from the eastern semicircle to the western and back take place at the epicycle's apogee R and perigee S; and the beginning of the revolution is taken in Venus from the section B, in Mercury from the section F. Notwithstanding this, at the end of ch. 4 he accepts the reflexion of the epicycle at the apogee and perigee of the eccentric, and, imitating Copernicus, carries it into the tables.

Diagramma Latitudinum Veneris ac Mercurij in forma Copernicæa.

Diagram of the latitudes of Venus and Mercury in the Copernican form.

IX. From A, the centre of the annual orb and of the world, let the Earth's annual orb be described, BD, substituted in place of the eccentrics of Venus and Mercury. For the sake of ease, neglecting the eccentricity of the orbs, let the inner orb of Venus or Mercury, KFCG, be concentric with it, substituted for the Ptolemaic epicycles. Although Mercury's large eccentricity does much to vary the latitude, in Venus it makes scarcely a perceptible difference, because it is far smaller and its inclination is small. Now, drawing the line of common section BAD, let it be the same in Mercury as the line of apsides, but in Venus let it precede the apogee by 16°. At right angles to it let FE be drawn, and across FE let HG run, making the angles FAH and GAE equal to the perpetual inclination of these orbs to the annual orb or ecliptic, which angle is 3° 30′ in Venus and 5° 40′ in Mercury. And let there be supposed, as above, an entirely contrary alternation of the latitudes: wherever the Earth may be, the latitude of Venus is northern in the eastern hemicycle KHC and southern in the western CGK; but, on the contrary, the latitude of Mercury is southern in the rising semicircle and northern in the setting one.

Diagram. Set into the right column, with the text running narrowly beside it on the left. Two concentric circles about A. A vertical line runs from B at the top of the outer circle through K, at the top of the inner circle, and A to C, at the foot of the inner circle, and D, at the foot of the outer. A horizontal line runs from F to E on the inner circle through A, and a slightly inclined line from H to G on the inner circle through A. The signs ♀ and ☿ stand in the lower ring between the two circles, on either side of the vertical.

- B (top, outer circle), K (top, inner circle)
- F (left, inner circle), H (just below F), E (right, inner circle), G (just above E)
- A (centre), C (foot, inner circle), D (foot, outer circle)
- ♀ (lower left, between the circles), ☿ (lower right, between the circles)

Latitudinem Veneris Geometricè inquirere in forma Tychonica.

To investigate the latitude of Venus geometrically in the Tychonic form.

X. Let the latitude of Venus be investigated for the year 1585 September 15, Old Style, but 25 New Style, in the morning, at $5\frac{1}{4}^{h}$ after midnight, which was observed by Tycho, after the removal of parallax, as 2° 8′ to the south, with longitude in ♌ 15° 58′. From the Danish tables, moreover, the equated anomaly of the eccentric was 92° 43′ and the equated anomaly of the orb 229° 0′, although in this matter the mean anomalies could be used. [Margin: Delineation of the Tychonic figure.] With these set down, and given the north

node of Venus for that time at 13° of Gemini, that is, anticipating the apogee by 16°, let there be described from A, the centre of the world and of the Earth, at least a semicircle of the concentric BCD, on which is the centre of the great epicycle or orb of Venus. Let the section of the eccentric and the ecliptic be BAD, and let the node ☊ be B, from which the arc BC is counted in the order of the signs; this arc consists of the anomaly of the eccentric and the distance of the node from the apogee, and so is made up of 92° 43′ and 16°, and is therefore 108° 43′. From the centre C let the orb of Venus, EGKN, be described, whose diameter KE is parallel to the line of nodes BD. When ACH is drawn, the arc EH will be, by the law of the motions of Venus, similar to the arc BC, that is, 108° 43′, and the centric place of Venus will be designated by the line ACH. But because the true place of Venus, from the observed or calculated longitude, is found at M, and the arc HKM at 219°,

Diagram. Set into the lower right column, with the text running narrowly beside it on the left. A horizontal base line D–A–B carries a large semicircle above it, the concentric, on which C, the centre of the orb, stands. About C is drawn a circle in a solid line, and a dotted circle of about the same size set a little lower, which runs inside the solid circle at the top and outside it at the foot. A horizontal diameter runs from K through C to E, and a vertical from G through C to F. From A straight lines run through C to H, and to M. C is joined to M; a short vertical drops from the line KE, near E, to M, with P beside it; and a second line, partly dashed, rises from A to I beside M. The sign ♀ and the word *Tycho* stand inside the semicircle at the lower right.

- H and G (top, side by side, H to the left), K (left), E (right)
- C (centre of the circles), P (below E), M and I (lower right, side by side, M to the left), N (lower left, on the solid circle), F (below the circle)
- D (base, left), A (base, centre), B (base, right)
- ♀ and *Tycho* (inside the semicircle, lower right) if to these the arc EH is added, the whole EHKM becomes 327° 43′. Next let the straight line GCF be drawn, perpendicular to KE. The angle MCN, or the arc NM, will be 57° 43′, and EM 32° 17′, provided, that is, that from the arc EHM there is subtracted the arc EKN, or three-quarters of the circle, which is 270°. Since, therefore, Venus is in the western semicircle of the epicycle, its latitude will be southern, according to the alternations set out above. To investigate that latitude, let the orb EKF be described in the plane of the ecliptic, inclined to the orb of Venus, and let the straight line CMI be drawn; and from M, the place of Venus, the perpendicular MP to EK; and from A, the two straight lines AM and AI. And let the distance MA of Venus from the Earth be assumed, from the calculation of longitude, to be 63120 parts, of which the radius of the eccentric is 100000. Let the rest be done as below.

EXEMPLVM. Calculi Geometrici.

Example of the geometrical calculation.

XI. [Margin: Step 1.] First, let it be: as the whole sine 100000 is to PM, the right sine of the arc EM, 32° 17′, which sine is 53411, so is the sine of the angle of inclination MEI, that is, of 3° 30′, which is 6105, to the sine MI, 3261. Next, in parts of which the radius of the orb of Venus is, according to Longomontanus, 72248 of the 100000 of the radius AC, MI will be 2356. [Margin: Step 2.] Secondly, in the triangle MAI, with the side MI now found, the distance MA of Venus from the Earth set at 63120, and the angle at I right, because the arc of latitude is drawn perpendicular to the ecliptic, you will find the angle MAI to be 2° 8′: that is, the apparent latitude of Venus, agreeing entirely with the observed latitude and close to the one the Alphonsine calculation gives, even though the Prutenic calculation falls short at this place by half a degree. [Margin: Error of the Prutenic Tables.]

In his tables, however, he recognized that the reflexion must be applied, just as in Mercury, as we shall soon say, although Longomontanus has not set out the theory of this reflexion in Venus.

De Calculo Latitudinis pro Mercurio.

On the calculation of the latitude for Mercury.

XII. In the same Chapter 24 of Book 2 of the *Theorica*, Longomontanus affirms that the calculation of the latitude of Mercury is similar to the calculation for Venus, provided that the latitude is recognized as southern in the eastern semicircle and as northern in the western, and that nothing is added to the anomaly of the eccentric, but the node is kept on the line of apsides. But because of the great eccentricity and inclination of Mercury, he says there is some difference between its latitude when it is at the apogee and the latitude it has at the perigee of the orb, as was gathered from observations of the years 1585 November 15, in the morning, and 1593 May 11, in the evening. Because of this difference he adopts the Copernican small circle, taken up by him to represent the Ptolemaic deviation, and uses it to vary the inclination, so that the variation of the mean inclination of Mercury, set above at 5° 40′, is 15′ at the apogee and 18′ at the perigee. With these set down, and with the distance of Mercury at the apogee of the eccentric as 106822 and at the perigee as 95452, in parts of which the radius of the eccentric is 100000, and with the semidiameter of the epicycle as 38500 of the same parts, he gathers that the inclination of Mercury near the quadrants of the epicycle is 1° 34′ at the apogee of the eccentric and 3° 33′ at the perigee; and that the obliquation, or greatest reflexion, is 2° 3′ at the apogee of the eccentric and 2° 19′ at the perigee. These are so limited with respect to the inclination of the little small circle [parui circelli] that 2° 18′ remain at the apogee and 2° 1′ at the perigee.

Synopsis of latitudes from Longomontanus [Synopsis Latitudinum ex Longomontano]

Greatest inclination to the ecliptic [Inclinatio ad Eclipticam maxima]

Planet	Greatest inclination
♄	2° 31′ 0″
♃	1° 19′ 15″
♂	1° 50′ 0″
♀	3° 30′ 0″
☿	5° 40′ 0″

Greatest latitude [Latitudo Maxima]

Planet		At the apogee of the epicycle	At the perigee of the epicycle
♄	South & North	2° 17′	2° 49′
♃	South & North	1° 7′	1° 39′
♂	North	1° 9′	4° 31′
♂	South	1° 4′	6° 47′
♀		1° 28′	9° 2′
☿		1° 34′	3° 33′

[Margin: From the declination.]

Greatest latitude from the reflexion near contact, or at the anomaly of the orb written below [Latitudo Maxima ex Reflexione circa contactum seu ad Anomaliam orbis infraſcriptam]

Planet	Greatest latitude from reflexion	Anomaly of the orb
♀	2° 33′	4^{s} 16°
♀	2° 33′	7^{s} 14°
☿	2° 36′	8^{s} 10°

Places of the nodes and limits at the beginning of the year of Christ 1 and at the completion of 1600 [Nodorum ac Limitum Loca ad Annum CHRISTI 1. ineuntem & 1600. completum]

Planet	Nodes & limits	Year 1	Year 1600
♄	Node B	♋ 1° 46′ 4″	♋ 20° 23′ 0″
♄	Node A	♑ 1° 46′ 4″	♑ 20° 23′ 0″
♄	Limit B	♎ 1° 46′ 4″	♎ 20° 23′ 0″
♄	Limit A	♈ 1° 46′ 4″	♈ 20° 23′ 0″
♃	Node B	♋ 0° 50′ 10″	♋ 6° 52′ 0″
♃	Node A	♑ 0° 50′ 10″	♑ 6° 52′ 0″
♃	Limit B	♎ 0° 50′ 10″	♎ 6° 52′ 0″
♃	Limit A	♈ 0° 50′ 10″	♈ 6° 52′ 0″
♂	Node B	♈ 23° 53′ 46″	♉ 16° 53′ 10″
♂	Node A	♎ 23° 53′ 46″	♏ 16° 53′ 10″
♂	Limit B	♋ 23° 53′ 46″	♌ 16° 53′ 10″

Planet	Nodes & limits	Year 1	Year 1600
♂	Limit A	♍ 23° 53′ 46″	♒ 16° 53′ 10″
♀	Node B	♉ 6° 36′ 25″	♊ 13° 28′ 41″
♀	Node A	♏ 6° 36′ 25″	♐ 13° 28′ 41″
♀	Limit B	♌ 6° 36′ 25″	♍ 13° 28′ 41″
♀	Limit A	♒ 6° 36′ 25″	♓ 13° 28′ 41″
☿	Node A	♎ 13° 48′ 21″	♐ 0° 36′ 31″
☿	Node B	♈ 13° 48′ 21″	♊ 0° 36′ 31″
☿	Limit A	♍ 13° 48′ 21″	♓ 0° 36′ 31″
☿	Limit B	♋ 13° 48′ 21″	♍ 0° 36′ 31″

Annual motion of the nodes [Motus Nodorum Annnus]

Planet	′	″	‴
♄	0	41	53
♃	0	13	36
♂	0	51	43
♀	0	36	0
☿	1	45	18

SCHOLIA.

Scholia.

1. Longomontanus reports (Book 2 of the *Theorica*, Chapter 14) that in the year 1593 the latitude and longitude of Mars were observed by Tycho on August 10, Old Style, when Mars appeared to have its greatest southern latitude, 6° 19′, [Margin: On the diminished latitude of Mars as it tends towards opposition with the Sun.]

and that afterwards, on August 25 of the same year, at 17^{h} 30^{m}, when the true opposition of Mars with the Sun took place, the latitude of Mars was observed by the same man to be 6° 5′; and that therefore the latitude decreased while Mars was tending towards acronychal opposition, contrary to the estimation of the ancients and the canons. Because of this diminution a great scruple lodged in Tycho's mind, which was still in suspense. [Margin: The scruple of Tycho and Kepler on this.] Accordingly the following words were written in the book of observations of that same year, and Kepler repeats them in his commentaries on the motion of Mars, Chapter 66: "It is worth considering that Mars, about August 10, had its greatest southern latitude, and afterwards decreased, so that on the 24th, at opposition, it was brought about a quarter of a degree nearer the ecliptic. The canons, however, even with the place of the greatest latitude corrected to the 18th degree of Aquarius, by no means show this, however the greatest latitude is assumed there. The cause of this must be diligently investigated." Therefore Longomontanus, to remove this scruple from himself and the Tychonians, drew the north node of Mars, which he had earlier placed with Tycho at 18° of Taurus, back to the 17th degree, and

likewise the south node to the 17th degree of Scorpio, as we have set it in the table at the end of the chapter. Kepler, moreover, says in that Chapter 66 that he came to Tycho, who was by then living in Bohemia, and often asked him about the theory of the latitudes; and that Tycho answered that the nodes are in opposite places, but that the sections pass through the points of the Sun's mean place, or through the centre of its epicycle, and added: "This is a wonder, that the greatest latitudes occur before or after the oppositions with the Sun, that is, the true ones." [Margin: Tycho's answer on this.]

II. But Kepler, who in Chapter 15 gives the observed southern latitude of Mars for August 25 as 6° 2′, affirms in that Chapter 66 of the Commentaries on Mars that Ptolemy went wrong because of the most intricate motions of the inclinations, deviations and reflexions, and because he was fixed on his notion of the epicycle. [Margin: Kepler's opinion of this diminution.] As soon as he saw the planet at opposition with the Sun digress from the ecliptic towards one quarter, he asserted that at conjunction with the Sun it digresses towards the opposite quarter, so that there would be some compensation, and an equality of restitution and a coherence with the Sun; and therefore he judged that the greatest latitude occurs at the moments of opposition, and the contrary latitude at the moments of conjunction. But the true cause why the greatest latitude occurs now at the moment of true opposition, now before it, now after it, Kepler says, is the proportion of the inclinations to the distance of Mars from the Earth. "For," he says, "the latitude is greatest when the distance of Mars from the Earth grows or shrinks in the same proportion in which the lines of the inclinations of Mars grow or shrink; and the latitude increases when the proportion of the distance decreases more than the proportion of the lines of inclination, or when, as the former decreases, the latter on the contrary grows. Conversely, the latitude diminishes either when the distance of Mars from the Earth grows more than the lines of inclination in their own proportion, whatever it may be, or when, as the distance grows, those lines diminish. These things happen now at opposition, now before it, now after it, according as the opposition falls on the limit, or before it, or after it." What the inclination of the lines of Mars is will be told in the next chapter. Meanwhile Kepler confirms this alternation from the *Ephemerides* of the year 1604, in which the greatest northern latitude of Mars occurred around March 6, although the true opposition followed a whole month later; and on October 7, when its greatest southern latitude occurred, Mars was between quintile and sextile aspect with the Sun. So too at the end of the year 1605 the greatest northern latitude of Mars occurred as the Sun was passing from the quintile to the quadrature of Mars; but in the year 1606, at the end of July, the greatest southern latitude occurred with the Sun placed in trine to Mars; and in the year 1607 its greatest northern latitude came a little after the conjunction of Mars with the Sun. Further, by his own special calculation Kepler, in the same Chapter 66, derives the latitude of Mars observed in the year 1593 on August 10 as 6° 19′ 10″, as nearly as possible the same as the observed one.

III. The latitude of Venus was observed by Tycho in the year 1587, on the evening of February 21, Julian style, and a little later on the morning of the 24th, at 8° 56′ north; that is, much greater than the tables of Ptolemy, Alfonso and Copernicus would allow. [Margin: The latitude of Venus greater than the ancients thought.] So, partly from so great a latitude and partly from the obliquity of the Danish horizon, it came about that Venus could be observed almost at inferior conjunction itself with the Sun and at the perigee of the orb or epicycle, despite the rays of the nearer Sun. Longomontanus recalls having seen this, as a prodigy of observation, among Tycho's records (Book 2 of the *Theorica*, Chapter 18). But Mästlin testifies, in his additions to Rheticus's *Narratio prima*, p. 136, that this same thing was observed by him from the 21st and 24th of February of the year 1587. [Margin: Venus seen almost at the perigee of the epicycle because of its latitude.] That, I believe, is why Kepler states on p. 71 of the Rudolphine Tables that Venus is sometimes seen for two or three days together both morning and evening, emerging in the morning before it is hidden in the evening. This is confirmed even by the fact that in the year 1549[70] we saw Venus through the tube, appearing with the form and size that we showed in this Book 7, Section 1, Chapter 2, Number 4, on June 29, that is, on the 8th day before the perigee conjunction with the Sun, although it had a latitude of only 2°; and we would certainly have seen it nearer the conjunction if it had had the greatest latitude.

CAPVT V.

Chapter 5.

De Hypotheſi Ioannis Kepleri circa Latitudinem Quinque Minorum Planetarum.

On the hypothesis of Johannes Kepler concerning the latitude of the five minor planets.

I. We shall gather Kepler's whole theory of the latitude of these five planets from what he himself says in the *Ephemerides*, p. 14; in the *Epitome of Copernican Astronomy*, Book 4, p. 599, Book 5, p. 674, and Book 6, pp. 753 and 770; in the Rudolphine Tables, Chapters 21 and 24, Precept 108; but above all in the *Commentaries on Mars*, Chapters 12, 13, 14, and the whole of the fifth part, or from Chapter 61 to 70. In the foundations of the *Ephemerides*, however, he has only this: "Each orbit is inclined to the ecliptic at its own constant angle: here there are no circles of deviation, none of obliquation, none of reflexion." In the Rudolphine Tables, besides the tables from which the places and motions of the nodes are gathered, he gives precepts of calculation for the latitude and for telling when the latitude will be greater than the inclination. These precepts are founded on what is said in the *Epitome* and in the *Commentaries on Mars*; and from these Magini too, in his *Supplement to the Ephemerides*, extracted a special

[70] so printed

Canon 11 for finding the true latitude of Mars. Let us see, then, what he teaches in the *Commentaries on Mars* and in the *Epitome*, reducing his whole doctrine on this matter to definite and distinct propositions, and bringing some light to it by the very order and connection of the propositions and by explaining the doubts that arise among them.

II. First, latitude, by Kepler's definition and the common one, is the arc of the circle of latitudes described beneath the fixed stars and intercepted between the ecliptic and the place of the planet as seen from the Earth. [Margin: Definition of latitude.] Its measure is the angle made at the centre of the Earth by two straight lines drawn from this centre, one to the centre of the planet's body, the other to the planet's place in the ecliptic; and it is called the true latitude. For if this angle is considered at an eye placed on the surface of the Earth, it is called the apparent latitude, and its difference from the true is called the parallax in latitude. The inclination, on the other hand, is the angle made at the centre of the planetary system (which for Kepler is the Sun) by two straight lines drawn out from this centre, one to the centre of the planet's body, the other to the place in the ecliptic corresponding to it. [Margin: What the inclination is for Kepler, what for Ptolemy?] But for Ptolemy the inclination is the angle made at the centre of the Earth by two straight lines drawn from it, one through the centre of the epicycle, the other to the centric place of the planet in the ecliptic, speaking of the inclination of the eccentric to the ecliptic. For, speaking of the inclination of the epicycle to the eccentric, it is the greatest angle made at the centre of the epicycle by straight lines drawn from there, one to the centre of the eccentric, the other through the plane of the epicycle towards the centre of the eccentric. From what has been said it follows that the latitude is the inclination of the eccentric, or its arc, as seen from the Earth.

III. Secondly, the latitude that appears to us has two causes, one physical, the other optical. [Margin: Twofold cause of latitude.] The physical cause is the inclination of the planet's orb to the plane of the ecliptic; the optical cause is the withdrawal or approach of the Earth to the planet, or of the planet to the Earth, since by Proposition 56 of Euclid's *Optics* things that are nearer appear larger. Therefore the arc corresponding to the angle of inclination must appear larger when the planet is nearer the Earth, and smaller when it is farther off, whether the Earth by its annual motion approaches it, or the planet itself, because of the motion of its eccentric, approaches the Earth. [Margin: Where, other things being equal, the greatest latitude occurs.] Hence it happens that, other things being equal, the greatest latitudes occur in those acronychal oppositions of the three superior planets in which the centre of the Ptolemaic epicycle is at the perigee of the eccentric, because the planet is also at the perigee of the epicycle and so nearest the Earth; and this nearness would be still greater if the perigee of the Sun lay under the same place of the zodiac. And since the perigees of the three superior planets are in the southern segment of the eccentric, among their greatest latitudes those towards the south are greater than those towards the north. In Venus and Mercury, on the other hand, other

things being equal, the greatest latitudes occur at the perigee of the orb or epicycle, that is, at inferior conjunction with the Sun, because of the same nearness to the Earth, which is greater at that time. I said "other things being equal" because, owing to the differing proportion of the inclinations and the distances, it can happen, and often does, that the greatest latitudes occur away from the moment of opposition of the three superior planets with the Sun, or of the lower conjunction of the inferior planets with it, as will be said in Number 9, and as was already touched on concerning Mars in Scholium 2 of the preceding chapter.

IV. Thirdly, the inclination, which is the physical cause of the latitude of these five planets, is for Kepler always the same, fixed and invariable in itself and with respect to the mean or royal ecliptic [Eclipticæ mediæ ſeu Regiæ]. [Margin: The constancy or immutability of the inclination.] So in one whole revolution of the planet the angle by which the plane of the planetary orb is inclined to the plane of the ecliptic remains of exactly the same size; you will see that size in the little tables at the foot of the chapter. Therefore the planets run out from the ecliptic to the side of the world, towards the north or towards the south, in such a way that the plane they trace with the centre of their body is straight, and inclined to the plane of the ecliptic at a constant inclination. And if the hypothesis of the Keplerian ellipse is transferred to the Ptolemaic eccentrepicycle, the plane of the epicycle, because of the constancy of the inclination already mentioned, would remain perpetually parallel to the plane of the ecliptic. So Kepler teaches in the *Epitome of Astronomy*, pp. 599 and 753, and in the *Commentaries on Mars*, Chapters 13 and 14; and in Chapter 14 he shows that the planes of the eccentrics are [Greek: ἀτάλαντα (atalanta, "not librating")]. [Margin: The planes of the eccentrics free from libration.] He is speaking of the plane of the eccentric ellipse, which for Kepler, single and simple, is equivalent to the Ptolemaic aggregate of eccentric and epicycle. The inclination of this epicycle (which for Copernicus is the orb) Copernicus, to say nothing of Ptolemy, made librating, or variable by libration, with the plane of the epicycle nodding this way and that; even though Ptolemy did make the inclination of the eccentric of the superior planets constant. Speaking, therefore, of the total effect of that Ptolemaic aggregate, just as if it were one eccentric orb, or of the Copernican orb, Kepler affirms that its plane is [Greek: ἀτάλαντον (atalanton)], that is, unlibrating, or immune from the variation of libration.

[Margin: Arguments for the immutability already mentioned.] He shows this from the fact that in Chapter 13, from three positions of observations of Mars in different aspects with the Sun, the greatest inclination was always found to be the same, 1° 50′. How the greatest inclination is gathered from observations we shall teach in the problems of Chapter 8. In the first manner of observation the Sun was about sextile or quintile to Mars, that is, so near conjunction that it cannot be observed any nearer by us; in the second it was in quadrature with Mars; and in the third in opposition itself with Mars; and yet in all these places

the greatest inclination, derived from the latitude, was without perceptible difference an inclination of the plane to the ecliptic of 1° 50′. What he finds in Mars, therefore, he confidently affirms of the remaining minor planets in that Chapter 14, saying: "Let us therefore conclude most firmly (for why should I not conclude in general what has no cause for belonging to one single planet, although I have the same thing demonstrated from observations in Venus and in Mercury too) that the inclination of the planes to the ecliptic does not vary at all; and let him who follows Ptolemy learn from this that the plane of the epicycle is perpetually parallel to the plane of the ecliptic." Understand this of the superior planets. Hence, then, he censures Ptolemy and Copernicus for not having seen that by the parallelism already mentioned and the constant inclination, together with the approach to and withdrawal from the Earth, all the phenomena of the latitudes can be saved. [Margin: Ptolemy, Copernicus and Apian censured by Kepler.] He censures Copernicus the more, because, clinging too closely to the Ptolemaic measures, when he saw that the latitudes increase with the Earth's approach to the planets, but not precisely as much as Ptolemy's measures require, he did not dare to reject these increases, and to furnish them he introduced a libration, not indeed of the epicycles, which Copernicus abolished, but of the eccentric orbs, by which their inclination would vary. From this it has come about that in the calculation of the old astronomy there is no conjunction of Mars and the Sun that is not, as the astrologers call it, corporeal, or through the body, contrary to the end sought by God and Nature in the latitudes, namely that there should not be excessive agitations of the sublunary powers from corporeal conjunctions occurring frequently. [Margin: The end of latitude, introduced by God.] For a conjunction is corporeal when one and the same straight line drawn from the centre of the Earth passes through the centre of the body of each planet as they come together in the same place of the zodiac. He also pities Peter Apian, who in his imperial work [opere Cæſareo] laboured with most ingenious spires, little crowns, helices and volutes to present to us the Ptolemaic fictions of librations for representing the latitudes.

V. Fourthly, however, the inclination of the eccentrics already mentioned to the temporary ecliptic [Eclipticam temporaneā], if the obliquity of the ecliptic varies with the succession of the ages, is different in different ages, as Kepler judges more probable in the *Epitome of Astronomy*, p. 754, and in the *Commentaries on Mars*, Chapter 68. [Margin: The accidental mutability of the inclination, in line with the change of the obliquity of the ecliptic.] For although this variety cannot be shown from the latitudes observed by Ptolemy compared with the observations of our time, because this variety is of a few minutes, which cannot be derived from Ptolemy's coarse observations, since he usually used an instrument having no division of degrees except into tens of minutes, it is nevertheless more probable that the ecliptic to which the planes of the eccentrics keep a constant inclination, or to which, in the reformed Ptolemaic form, the planes of the epicycles remain parallel, is that mean or royal ecliptic along which the Sun or the Earth travels at the mean obliquity

of the ecliptic. And so, when the obliquity of the ecliptic changes, it is more likely that the eccentrics or epicycles of the planets are variously inclined to the temporary ecliptic, so as to follow the Sun, or the Earth travelling along it, with some very small variation; just as the fixed stars appeared to Tycho and others to change their latitude accidentally, because of the change of the temporary ecliptic, about which I spoke in Book 6, Chapter 15. What sort of circle that royal one is, however, to which both the latitudes of the fixed stars and the inclinations of the planetary orbs keep a constant relation, is not yet certainly clear; and so Kepler calls it that blind path in Chapter 68, although he speaks of it more clearly in Book 7 of the *Epitome*, and says that its withdrawal from the equator is more truly called the latitude of the equator from the ecliptic. The variation of this inclination presupposes, in this hypothesis, that the nodes of the planets and of the Earth, that is, the sections which the orbits of the planets make with that blind circle along which the plane of the Earth travels, move with an unequal motion. This accidental variation, then, is the one I indicated when in Number 4, near the beginning, I said that the inclination, in itself and considered over one period, is invariable, but not when the periods of all the ages are considered. This variation seems to be confirmed from the greatest northern latitude of Mars observed with Regulus; for the latitude of Regulus, for Tycho, increased accidentally in line with the change of the obliquity of the ecliptic, and the greatest northern latitude of ♂ also increased, since it was observed by Ptolemy as 4° 20′, but by Tycho as very nearly 4° 32′. But against this stands the greatest southern latitude of Mars, observed with the southern stars of Capricorn, which for Ptolemy was almost 7° and today is 6° 52′. So we are still left in suspense by Ptolemy's observations.

VI. Fifthly, in Kepler's hypothesis the planes of the eccentrics and of the ecliptic are so inclined that they cut one another in one straight line, which passes through the centre of the body of the Sun. [Margin: What the common line of section of the planes is.] In each planet this line extends to places of its own ecliptic that are opposite one another from the centre of the Sun; since in two different revolutions of the same planet, at the places where the planet lacks latitude, those places have been observed to be diametrically opposite with respect to the centre of the world; and Mars, for example, being placed on the ecliptic, or lacking latitude, at 17° of Taurus, returns to the ecliptic and is free of latitude again at 17° of Scorpio. In the Ptolemaic hypothesis, however, that line passes through the centre of the Earth. So Kepler teaches in the *Epitome*, pp. 599 and 753, and in the *Commentaries on Mars*, Chapter 12, where he brings forward observations bearing on this. From these, counting the 687 days of the eccentric revolution of Mars, he finds that Mars was at the north node first in the year 1590 on March 7, then in the year 1592 on January 23, then in the year 1594 on October 28, exactly as the observations of latitude made by Tycho on those or the nearest days demanded. From this, and from the inclination of the eccentric of Mars to the ecliptic, he shows in Chapter 67 that the line of apsides passes through the centre of the Sun as regards its true place, but not through

the centre of the Sun's mean motion, which for Tycho is the centre of the solar epicycle.

VII. *Sixth,* the physical cause of the inclination of the eccentric to the ecliptic and of the latitude itself is, for Kepler, twofold. [Margin: The physical cause of latitude, the final one.] One is final: that the oppositions and corporeal conjunctions of the planets with the Sun should not occur too often, since they stir up the power of sublunary things too much, as I have already indicated in Number 4, from Kepler, *Commentaries on Mars*, Chapter 14; just as the latitude of the Moon also ensures that central eclipses of the Sun and Moon are not frequent. The other, whether efficient or subjective, is the very formation of the planet's body: namely magnetic fibres, rectilinear, whether internal or in the outer crust, or the rotation of the planet about its axis. By the force of these the parallel position is retained for the whole time of the circuit, and the planet is so directed that, when it is itself under the ecliptic, a fibre or arc of gyration touches the orbit, and inclines somewhat to the north with one of its extremities and to the south with the other, and therefore the planet is driven through the other semicircle towards opposite quarters. For once the inclination to the mean or royal ecliptic had been acquired for the purpose already mentioned, it was fitting that the fibres of latitude should always retain the same position with respect to the sides of the world, as was said of the fibres of longitude when dealing with the Moon, in Book 4, Chapter 27, in the Scholia, according to Kepler's thinking. For his notions on this as regards latitude, see the *Epitome* from p. 602 on, and the *Commentaries on Mars*, Chapter 63.

VIII. *Seventh,* the laws of latitude and inclination according to Kepler are these. [Margin: Rules of latitude, and of inclination compared with it.] Latitude always comes with some inclination; for where there is no inclination, as happens at the nodes, there is also no latitude. Whenever the distance between the planet and the Earth is the same as that between the planet and the Sun, and the triangle having those distances as its legs is therefore isosceles, the inclination is equal to the latitude. This happens when the angle of elongation of the planet from the Sun is half the angle of commutation, or when there is less than a quadrant of a circle between the apparent place of the planet and that of the Sun; yet this arc, required for the equality just mentioned, is nearly a quadrant in Saturn, less in Jupiter, and least in Mars. But when the elongation of the planet from the Sun is greater than this arc, or the distance of the planet from the Earth smaller, the latitude exceeds the inclination; conversely, when the elongation of the planet from the Sun is smaller than that arc, or the distance of the planet from the Earth greater, the latitude is exceeded by the inclination. In Venus, however, whose greatest latitude is very great and whose inclination is small, the latitude equals the inclination whenever the angle of elongation together with the angle of commutation makes up two right angles; but whenever the angle of elongation is smaller than the angle of commutation, the latitude is smaller than the inclination; and whenever it is greater, the latitude is also greater. [Margin: The difference between the superior planets and between ♀

and ☿.] But in Mercury, which has a great inclination and a small latitude, the latitude is always smaller than the inclination. So Kepler teaches, partly in the *Epitome*, pp. 755 and 771, partly in the Rudolphine Tables, precept 108.

IX. *Eighth,* the latitude decreases when the distance of the planet from the Earth grows more than the lines of inclination do, each in its own proportion, whatever that may be; or when, as the distance grows, those lines diminish. [Margin: On the decrease and increase of latitude, especially the greatest.] The latitude increases, however, when the proportion of the distance decreases more than the proportion of the lines of inclination, or when, as the former decreases, the latter grows. But the latitude becomes greatest when the distance of the planet from the Earth grows or decreases in the same proportion in which the lines of the planet's inclinations grow or decrease. This happens sometimes at the moments of true opposition with the Sun, sometimes before or after; and indeed in Mars sometimes by a whole month, as we showed in Scholium 2 of Chapter 4. Granted that in Saturn and Jupiter, because of the very slow change of inclination from day to day, the greatest latitude very nearly coincides with the true opposition of the Sun, it nevertheless recedes further from conjunction with the Sun. So in Venus, as also in Mars, around the stations and retrogradations the latitude shows a great variety, one that can scarcely be bound by fixed rules; but in Mercury, because of the very rapid separation of the orbs, the variety there is not so great. This is roughly what Kepler says in the *Epitome*, p. 756, in the *Commentaries on Mars*, Chapter 66, and in the Rudolphine Tables, precept 109.

X. *Ninth,* how great the inclination is can be gathered from the *Epitome of Copernican Astronomy*: for the superior planets on p. 753, [Margin: How great the inclination of the eccentrics is.] for the inferior planets on p. 771; and it will be set out shortly in small tables at the end of the chapter. But Kepler gives the method of deriving it from the latitude in the *Commentaries on Mars*, Chapter 13, where he investigates it for Mars, from afar and close to the truth, in three ways, and finds it to be approximately 1° 50′; and in Chapter 62, bringing it under the file, he finds 1° 50′ 25″ or 1° 50′ 30″. But we reserve those methods for the Problems to be given in Chapter 8, so as not to break the thread of Kepler's teaching too much. How great the greatest latitude of Mars is, however, he investigates in the *Commentaries on Mars*, Chapter 65. [Margin: Likewise how great the greatest latitude is.] The northern one he investigates at the oppositions from the greatest distance of Mars from the Sun, of 166465 parts, and the least distance of the Sun from the Earth, 98200, and from the inclination now established as 1° 50′ 30″; at conjunction, however, from what has been said and the greatest distance of the Sun, 101800, in parts of which the radius of the annual orb is 100000. The southern one he investigates at opposition from the least distance of Mars from the Sun, and that of 138234 parts, and the greatest distance of the Sun from the Earth, 101800; but at conjunction from the least distance of the Sun, 98200. And this first on the supposition that, as was once the case, the apogee of Mars and the aphelion of the Earth

are joined in one line and the ecliptic retains its position among the fixed stars; then on the supposition that the apogee of Mars is joined to the perihelion of the Earth. But because in this century neither conjunction obtains, and the limits of Mars are roughly midway between the apsides of Mars and those of the Sun or Earth, he therefore investigates both latitudes of Mars, at ♂ and ☍ with the Sun, from the distances set out above; all these latitudes I shall give in one view, with the latitudes of the other planets, at the end of the chapter. In the *Epitome*, moreover, pp. 756 and 771, he gives the greatest latitudes of the other planets, noting that the northern latitude of Venus when retrograde runs out in this century to nearly 9°, about as great as the southern will be in other centuries; but that the northern latitude of Mercury is about 2° 30′ and the southern 5°, always smaller than the inclination of Mercury's orbit.

XI. *Tenth,* he gives the places of the nodes in the *Epitome of Copernican Astronomy*, pp. 753 and 770, and in the Rudolphine Tables, in the Epochs of the Nodes; [Margin: On the place and motion of the nodes.] but he teaches the method of finding the places of the nodes and limits in the *Commentaries on Mars*, Chapters 12 and 61, and we shall deal with these more deliberately in Chapter 8, among the common Problems. In those same places of the *Epitome* he notes that the nodes and limits of the three superior planets move with a slower motion than the fixed stars move in the hypothesis of a resting Earth (for in the hypothesis of an Earth moved with annual and diurnal motion, the fixed stars stand truly fixed), and therefore, although they move in the order of the signs with respect to the vernal equinox, with respect to the fixed stars they nevertheless move against the order of the signs. Since this holds true in the four planets already mentioned, judging by the motion of the nodes, which is gathered from their places as handed down by Ptolemy and compared with their places as found by Tycho, it seems likely that it happens also in the motion of the node of Mercury. He therefore holds Ptolemy suspect, who placed the north node of Mercury, together with the apogee, at ♎ 10°; and today it is at 13° 30′ of Scorpio. For it would follow from this that in the 1460 years after Ptolemy this node advanced 33° 30′, that is, much more than the fixed stars, which are found to have advanced by about 27°; so this node would be going in the order of the signs even with respect to the fixed stars. However, because Ptolemy found the node of Venus at the apogee, and the node of Mercury not much farther from the apogee of Mercury, Kepler believes that, lured by the neatness of his hypothesis, Ptolemy drew the node back from the end of Libra to 10° of Libra. And so at last he asserts that the nodes of all five minor planets move more sluggishly than the apparent motion of the fixed stars, and with respect to the fixed stars move against the order of the signs. Further, that the motion of the nodes is unequal, if the obliquity of the ecliptic changes and the eccentrics of the planets vary their inclination with respect to the temporary ecliptic, he shows in the *Commentaries on Mars*, Chapter 68, and in Book 7 of the *Epitome*, p. 931. [Margin: The inequality of the motion of the nodes.] But he shows also that neither this variety nor the most exact inclinations of

the planets can be known, unless posterity, while the world lasts, comes to be more certain whether the obliquity of the ecliptic really changes. Therefore Kepler concludes, in Chapter 68 on Mars, in these words: "Since, therefore, we lack suitable observations from antiquity, the very condition of the matter compels us to leave this discussion of the motion of the nodes, like many other things, to posterity: if, that is, it shall please God to grant the human race a just span of time in this world for learning these remaining things thoroughly." [Margin: Some imperfection of present astronomy still remains.] And on the same occasion and in the same sense he closes Book

7 of the *Epitome of Copernican Astronomy*, saying: "The truest inclinations of the planets to the royal way, therefore, and the causes, quantities and quarters of the motions of the limits and nodes: these, I say, and other such things lie hidden in the Pandects of the age to come; not to be learned before God, the judge of the ages, himself immortal, has opened this book to mortals. To him be praise, honour and glory for ever and ever, Amen." Now follow the small tables so often promised.

Inclination of the plane of the ellipses, eccentric from the Sun, to the ecliptic, at the centre of the Sun, fixed in itself and invariable [Inclinatio Plani Ellipſium à Sole Eccentricarum ad Eclipticam in centro Solis per ſe Fixa & iuuariabilis.]

Planet	Inclination
♄	2° 32′ 0″
♃	1° 20′ 0″
♂	1° 50′ 30″
♀	3° 22′ 0″
☿	6° 54′ 0″

[Margin: From the *Epitome*, pp. 753 and 771, and from Hérigone, volume 5, p. 591.]

Greatest latitude in this century [Latitudo Maxima Hoc Seçulo]

Planet	Northern	Southern
♄	2° 49′ 0″	2° 50′ 0″
♃	1° 48′ 0″	1° 50′ 0″
♂	1° 8′ 30″	2° 4′ 20″
♂	4° 31′ 45″	6° 52′ 20″
♀	8° 47′ 0″	8° 41′ 0″
☿	2° 30′ 0″	5° 0′ 0″

[Margin: From the *Epitome*, pp. 756 and 771, and from the *Commentaries*, Chapter 65.]

[Margin: At ♂ ☉ (beside the first ♂ row).] [Margin: At ☍ ☉ (beside the second ♂ row).]

Greatest latitude of Mars in other centuries [Latitudo Martis Maxima alijs Sæculis]

If the apogee of ♂		Northern	Southern
is joined to the apogee of ☉	at ♂ ☉	1° 8′ 34″	1° 4′ 36″
	at ☍ ☉	4° 29′ 10″	6° 58′ 24″
is joined to the perigee of ☉	at ♂ ☉	1° 9′ 32″	1° 3′ 32″
	at ☍ ☉	4° 44′ 12″	6° 20′ 50″

Places of the nodes and limits about the year 1620, indicated by the letter N or L [Loca Nodorum & Limitum Anno circiter 1620. Literâ N, aut L, indicata.]

Planet		Northern	Southern
♄	N	♋ 21°	♑ 21°
	L	♎ 21°	♈ 21°
♃	N	♋ 7°	♑ 7°
	L	♎ 7°	♈ 7°
♂	N	♉ 19°	♏ 19°
	L	♌ 19°	♒ 19°
♀	N	♊ 12½°	♐ 12½°
	L	♍ 12½°	♓ 12½°
☿	N	♏ 13½°	♉ 13½°
	L	♒ 13½°	♌ 13½°

[Margin: From the *Epitome*, pp. 753 and 770.]

Place of the north node at the years of Christ [Locus Nodi Borei Ad Annos CHRISTI]

Planet	1, beginning	1600, completed
♄	♊ 19° 14′ 33″	♋ 20° 59′ 59″
♃	♋ 3° 52′ 43″	♋ 5° 25′ 58″
♂	♈ 29° 4′ 40″	♉ 16° 44′ 32″
♀	♉ 22° 7′ 25″	♊ 13° 0′ 45″
☿	♈ 4° 32′ 11″	♉ 12° 25′ 22″

[Margin: From the Rudolphine Tables.]

[Margin: The south node is opposite the north; each limit stands 90° in consequentia from its own node.]

Hence the annual motion of the nodes and limits [Hinc Motus Annuus Nodorum ac Limitum]

Planet	′	″	‴	⁗	V	VI
♄	1	11	24	15	55	51
♃	0	3	29	40	35	12
♂	0	39	43	3	40	50
♀	0	46	58	4	13	20
☿	1	25	11	7	55	50

SCHOLIA.

Scholia.

I. I did not wish to be more long-winded in explaining the variety of the inclinations and the inequality of the motion of the nodes that would arise if the obliquity of the ecliptic changed. For although Kepler was copious on this in the *Commentaries on Mars*, Chapter 68, and in Book 7 of the *Epitome*, he later taught outright that it is more probable that the obliquity of the ecliptic does not change, as we have pursued more fully in Book 3, Chapter 27. [Margin: The constancy of the inclinations and the equality of the motion of the nodes is more probable.] Accordingly, on his view, not merely on ours, it is more probable that not even the latitudes or inclinations of the planets change accidentally upon a change of the ecliptic. Nor does the latitude of the fixed stars stand in our way, which for the same cause seemed mutable, and indeed for Tycho was changed; for see what we said in Book 6, Chapter 15.

II. A geometrical calculation for the latitude, without any diagram, is given by Kepler in the *Epitome of Copernican Astronomy*, p. 757, in this truly compendious form. [Margin: A compendium of the geometrical calculation for latitude.] For he says: "As the sine of the angle of elongation is to the sine of the angle of commutation, so the tangent of the complement of the inclination is to the tangent of the complement of the latitude." Therefore, he says, let the sine of commutation, increased by 5 ciphers, be divided by the sine of the elongation of the planet from the Sun; let the quotient be multiplied into the tangent of the complement of the inclination; and when the last 5 digits are thrown away, the tangent of the complement of the latitude will come out. Or more expeditiously still: from the logarithm of commutation subtract the logarithm of the elongation of the planet from the Sun, and add the remainder to the mesologarithm of the complement of the inclination; for the sum will be the mesologarithm of the complement of the latitude. Remember, however, that the anomaly of commutation, or the angle at the Sun, is gathered by counting from the place of the Sun to the centric place of the planet, either in the order of the signs or against the order of the signs, so that the arc is always smaller than a semicircle. The elongation of the planet from the Sun, on the other hand, is counted from the place of the Sun to the place of the planet as seen from the Earth, in the order of the signs if it is smaller than 180°, against the order of the signs if greater. So by subtracting the apparent place of the Sun from the eccentric place of the planet, or the latter from the former, so that less than a semicircle remains, you will have the angle of the anomaly of commutation in the first case, but in the second its complement, according to precept 93 of the Rudolphine Tables and p. 750 of the *Epitome*. But by subtracting the true place of the Sun from the true place of the planet, or the latter from the former, you will have the angle of elongation, or the angle at the Earth. Kepler gives a somewhat different rule in the *Commentaries on Mars*, Chapter 62, and Magini gives it after him; I shall speak of it in Chapter 8.

III. Kepler asks, in the *Epitome*, p. 771, why, if the inclination of Venus and Mercury to the ecliptic is fixed, Ptolemy nevertheless introduced two librations of the epicycle; and he answers that this was done because the annual motion of the Earth was unknown and was ascribed to the very centre of the epicycle, carried annually with the Sun. But Ptolemy seems to have done this with little consistency, since he should have set the inclinations of Venus and Mercury to the ecliptic as fixed, just like those of the three superior planets, since every libration of the orbits is absurd in itself, because it produces a tortuous path for the planet; and whatever Ptolemy achieved by those librations is achieved by a fixed inclination and the approach or recession of the Earth, if corrected observations are employed. Therefore, as much probability as the fixed inclination has, so much has the motion of the Earth. [Margin: What the argument for the annual motion of the Earth from the inclination of the planets is worth.] And yet this argument, resting on mere probability, Kepler calls Most Evident for the annual motion of the Earth, there and on p. 543. But is it not enough if, in the Ptolemaic hypothesis, the epicycles are so moved that their planes are parallel to the ecliptic, as it is clear from what was said in this Chapter 5, Number 4, that Kepler taught; or that we remedy the anomaly of latitude, if there is any, in some other way with the Earth at rest, for example by a variation of the semidiameter or of the eccentricity?

IV. Another cause on account of which Ptolemy set two librations in the inferior planets is devised by Kepler [Margin: A second cause of the Ptolemaic libration.]

in the *Epitome*, p. 775: namely, that Venus at perigee, in conjunction with the Sun, should never, thanks to a latitude increased by the libration, be interposed between the Sun and our sight; and that it would thus answer to the older writers who, because Venus was never seen under the Sun, placed it above the Sun. I have already spoken of this argument in this Book, Section 1, Chapter 4. But the more recent astronomers obtain that very thing without that libration.

V. Moreover, Kepler adduces a third cause of the libration of the eccentric introduced by Ptolemy in Venus and Mercury, on p. 773 of the *Epitome*: namely, that in Mercury the southern latitudes should turn out greater than the northern. [Margin: A third cause of the libration.] But he adds that if Ptolemy had set his epicycles unequally disposed around the Sun, for instance by an elliptical figure, or, as we do, by a variation of the semidiameter, he would have had no need of that libration. For the south limit of Mercury is nearer to its aphelion than the north limit is; and the farther it recedes from the Sun, the nearer it comes to the Earth, when Mercury is retrograde, than the north limit does; and therefore the southern latitude of Mercury turns out greater than the northern. The southern latitude of Mars also is greater than the northern, because its south limit is near the perihelion, as Hérigone noted, volume 5, p. 613. In Venus, however, although the north limit is in Virgo, close to the perihelion, the northern latitudes are nevertheless greater than the southern,

because the eccentricity of Venus, much smaller than the eccentricity of the Sun and of Mercury, does in this what the inclination alone cannot do, and does do in Mercury, which has an eccentricity greater than the Sun's. [Margin: What does the small eccentricity of Venus contribute to the latitudes?] This argument seems to undermine our opinion that the eccentricity of Venus is no other than the Sun's. But we do not much fear on this account, because either by a variation of the semidiameter of the epicycle in the first form, or by an elliptical epicycle in the second form of our hypothesis, indicated in its place, we can find a cause for this variety; and otherwise the difference between the northern and southern latitudes of Venus is very small, as Kepler himself grants on p. 775 of the *Epitome*, so that the probability of our hypothesis is not removed on its account.

CAPVT VI.

Chapter 6.

De Iſmaelis Bullialdi Hypotheſi circa Latitudinem Quinque minorum Planetarum.

On the hypothesis of Ismaël Boulliau concerning the latitude of the five minor planets.

I. This most profound astronomer is, as to the substance of the hypothesis, a most faithful follower of Kepler: namely in the elliptical orbit of the planets around the motionless Sun as one umbilicus of the ellipse, the Sun leading around the Earth and these five planets; in the immutability of the inclination of the elliptical orbit to the ecliptic and to the plane of the Earth's annual orb; and in the line of section, by which the planet's orbit cuts the ecliptic, which is to be carried through the centre of the solar globe. Nevertheless, in the places and motion of the nodes, as well as in the quantity of the latitude and inclination, he relies on his own findings; and in each of his books on these planets, after the treatment of longitude, he treats the inclination of the orb and the places and motion of the nodes. Accordingly we too, following the order of his books in the *Astronomia Philolaica*, shall set out distinctly his doctrine concerning the latitude of the planets, while bringing together at the end of the chapter the numbers and measures in a single synopsis, after our custom.

De Saturni Latitudine.

On the latitude of Saturn.

II. *First,* in Book 6, Chapter 3, by a technique not unlike the one by which he usually seeks the place of the aphelion, Boulliau seeks the places of the nodes and the inclination of Saturn's orb to the plane of the Earth's annual orb, or of the ecliptic, an inclination he considers fixed in all the planets. Because this technique is common to all 5 planets, we shall give it in Chapter 8, among the

common Problems; for now we shall present only its foundations in observations, and its conclusions. He therefore assumes three Tychonic observations of the longitude and latitude of Saturn outside the acronychal position, and to these he adds, from the calculation of longitude, the distances of Saturn from the Sun and from the Earth. From these and from the semidiameter of the annual orb he gathers the angles of the latitude to be seen, which he compares with the observed latitudes. [Margin: Boulliau's method indicated.] And first he gathers from afar the place of the ascending node at ♋ 24° 26′ 15″, and from it the inclination, now 2° 11′ 28″, now 2° 18′ 14″, now 2° 29′ 8″; and from this variety he infers some defect in the observation. Therefore, so that those three observations can be reconciled, while the other measures of Saturn's orb and the places of its longitude and of the Earth remain intact, and so that the latitude sought geometrically may be very close to the observed latitude, he assumes the place of the north node at 20° 25′ 24″ of Cancer for the year 1590, and the inclination of Saturn's orbit to the ecliptic as 2° 30′. [Margin: The inclination of ♄.] With these assumed, on going through the calculation again he finds latitudes differing from the observed ones by the smallest difference. He confirms this also by a fourth observation of latitude made by himself, exceeding this hypothesis by scarcely a minute and a half; and therefore he rests content with the assumed place of the node and the inclination. The foundations of the calculation are these, from 3 observations made at Uraniborg and a fourth at Paris, the hours being reckoned from noon in both places.

Of Christ: year, month, day, hour	♄ place as seen from ☉	Observed southern latitude of ♄	Distance of ♄ from the Sun, in parts of the radius 100000	Distance of ♄ from the Earth, in parts of the radius 100000	Latitude of ♄ to be gathered from the given measures, etc.	Latitude gathered from the assumed place of the node and inclination
1587 January 9, 10h	♉ 2° 18′ 51″	2° 28′ A	95596	95628	2° 28′ 4″	2° 26′ 46″
1590 February 8, 8h	♊ 13° 49′ 18″	1° 30′ A	94338	92409	1° 28′ 8″	1° 29′ 12″
1590 September 7, 12h	♊ 21° 46′ 2″	1° 11′ A	94239	94317	1° 11′ 2″	1° 11′ 42″
1640 October 12, 8h	♒ 28° 50′ 2♑″	1° 35½′				1° 36′ 40″

III. *Second,* in Book 6, Chapter 4, from the inclination of Saturn's orbit to the ecliptic already found above, 2° 30′, and from the aphelion and perihelion distances of the Earth and of Saturn, he gathers, by a method to be given in Chapter 8, the greatest possible latitude at oppositions and conjunctions, whether the Earth is at aphelion and Saturn then at aphelion or at perihelion, or

the Earth is at perihelion and Saturn then at aphelion or at perihelion. [Margin: The variety of the greatest latitudes of ♄.] I shall give these latitudes at the end of the chapter in a single synopsis.

IV. *Third,* in the same place, Chapter 6, when about to treat the translation or motion of the nodes of Saturn, which Kepler had gathered from the places of the node observed by Ptolemy and compared with the places observed by Tycho, he tries to refute Kepler from other observations of Walther, from which he says the numbers established in the Rudolphine Tables diverge; and he contends that the motion of Saturn's node is much slower than Kepler supposed. [Margin: The motion of the node of ♄.] The first observation of Bernhard Walther at Nuremberg was made with the armillary spheres, as he himself testifies, in the year 1504, in which, he says, from the third to the sixth day of February he could not see the eighth star of Gemini, since it was covered by the stationary Saturn; but on the 8th Saturn was observed at ♋ 11° 18′, with southern latitude 2′, and the eighth of Gemini appeared at a distance of 4 digits from Saturn, as if in the same latitude, most carefully: so Walther in his observations. Boulliau, however, affirms that that star is not the 8th but the 12th of Gemini, in the groin or belly of the southern Twin, having a latitude of 13′ 30″ south, and he assumes that there took place on February 4 the conjunction of Saturn with [Margin: Boulliau's criticism of the Rudolphine Tables.]

this star, and he gathers from the tables of longitude the place of Saturn at Cancer 10° 49′, whereas, from the observation of the 8th day, it should have been at 11° 33′ on the 4th; but as seen from the Sun, at Cancer 15° 10′ 28″. He suspects, however, a mistake in the noting of the day, and an error of about 10′. Notwithstanding these things, he thinks the place of the north node can be gathered from this ὁλοσχερέστερος [Greek: holoscherésteros, "more in the gross"], that is, approximately, but sufficiently as far as the motion to be derived from it is concerned. He therefore gathers it from the distance of the planet from the Sun and from the Earth, and from the observed latitude of 13′ 30″, and he finds Saturn distant from the north node by 4° 44′ 2″. When these are added to the place of Saturn as seen from the Sun, the place of the north node in the year 1504 comes out at Cancer 19° 54′. But in the years 1587 and 1590 it was found, above in number 2, at Cancer 20° 25′ 24″. Therefore in 86 years the node advanced at most 31′; and yet the Rudolphine Tables put the motion of this node in 86 years at 1° 42′ 24″. [Margin: Error of the Rudolphine Tables.]

The second observation of the same Walther was made in 1504 March 28, at 7^h after noon, when Saturn was seen further west and further north than the eighth star of Gemini, by as much as the space of 2 or 3 fingerbreadths; from which he concludes that the conjunction of Saturn with that star was on March 29 at 7^h, since on the night following the 28th it was seen much nearer that star. From this Boulliau gathers the place of Saturn's north node in that year at ♋ 19° 30′ 30″. He then adds a third observation, made at Athens in the year of Christ 503 February 21, at 9^h 45^m, when Saturn was seen to be occulted by the edge of the Moon, and at 11^h 55^m was seen to be freed from the Moon.

From this, by the motions and place of the Moon, he gathers Saturn's southern latitude of 3′, and from this the place of the north node at ♋ 12° 36′ 21″. Since, then, in the month of May of the year 1588 this node, by the observations of the years 1587 and 1590, was at ♋ 20° 25′ 24″, in 1085 years and 3 months it certainly advanced only 7° 49′ 3″. From this proportion he derives the annual motion of the node of ♄ as 25″ 54‴ 51⁗ 42^{v} 45^{vi} 42^{vii} 5^{viii}. Finally, in Chapter 6, he examines the already mentioned Athenian observation by the Rudolphine Tables, and shows that, with the Rudolphine motion of the node, it would have been impossible for the Moon to occult Saturn. [Margin: Likewise an error of the Rudolphine Tables.] But this could have come about not from the motion of Saturn's node alone, but from many other partial causes.

De Iouis Latitudine.

On the latitude of Jupiter.

V. *First,* the same author, in Book 7, Chapter 4, by the same method he had used for Saturn, derives from three observations, but his own, a fixed inclination of the orbit of Jupiter to the ecliptic. First, from the distances of the planet from the Sun and from the Earth, he finds the north node of Jupiter at 10° 5′ 22″ ♋, and the inclination 1° 20′ 10″. [Margin: Inclination of ♃.] Having assumed these, he tests whether the observed latitudes of Jupiter follow from them; and on finding some disagreement, he corrects the place of the node and the inclination so that latitudes as little removed from the observed ones as possible follow from them at last, and there he rests, namely on an inclination of 1° 21′ 48″ and a node at ♋ 8° 52′ 10″. The observations and the foundations of the calculations are written below; they were made at Paris, with the hours counted from noon. But he omits the latitudes gathered from the repeated calculation.

Year of Christ	Month	Day	Hour	Place of ♃ as seen from ☉	Observed latitude of ♃	Distance of ♃ from the Sun (radius 100000)	Distance of ♃ from the Earth
1634	April	12	8½	♋ 5° 54′ 10″	0° 4′ A	99605	100481
1637	July	16	9	♎ 9° 13′ 25″	1° 18′ B	104854	101685
1639	August	31	8	♐ 8° 55′ 3″	0° 42′ B	102161	100812

VI. After this, in Chapter 5, he places the north limit of Jupiter at ♎ 10° 13′ 25″, and from this he derives the greatest northern and southern latitudes of Jupiter, both at the conjunction of Jupiter with the Sun and at opposition; and he follows through the aphelial and perihelial variation of both in Chapter 6. [Margin: Variety of the latitudes of ♃.] But the limit ought to have been placed at ♎ 8° 52′ 10″, so as to be a quadrant distant from the node, whose place was chosen at ♋ 8° 52′ 10″. Finally, in Chapter 8, in order to find the motion of this node, he takes a latitude of Jupiter observed in the year of Christ 508 by a certain anonymous person and reported in a manuscript of the royal library of

Paris, in which it is said that on the morning of the 30th day of Thoth, that is, September 27, Jupiter was seen so near the Heart of Leo that it stood 3 digits from it toward the north. Boulliau, however, says that the spreading of the light narrowed the true distance for the observer by one digit, and that therefore Jupiter was further north than Regulus by 4 digits, or 10′; that Regulus then had a latitude of 26′ 30″ north; and that therefore Jupiter had 36′ 30″. [Margin: Motion of the node of ♃.] From this, and from the longitude of Jupiter and of the Sun, and the distances of Jupiter from the Sun and from the Earth, he gathers the place of the north node at Cancer 1° 8′ 49″ at the beginning of the year of Christ 509. But at the beginning of the year 1637 it was found at Cancer 8° 52′ 10″. Therefore in 1129 years it advanced 7° 43′ 21″, and in one year 24″ $37'''$ $28''''$ 0^{v} 25^{vi} 30^{vii} 33^{viii}.

De Latitudine Martis.

On the latitude of Mars.

VII. *First,* then, Boulliau, in Book 8, Chapter 7, investigates ὁλοσχερέστερος the place of the node of Mars from a single observation of a northern latitude of Mars of only 45″, made in 1593 December 10, in the evening, when the place of Mars as seen from the Sun was at Taurus 15° 57′ 2″. From this it is gathered that the north node, at which there was no latitude, was at Taurus 15° 30′. Then, from an observation of a northern latitude of 2° 35′, and from the place of Mars as seen from the Sun in 1586 December 15, at 18^{h} 30^{m}, and from the distances of Mars from the Earth and from the Sun, he derives the inclination of the eccentric of Mars to the ecliptic as 1° 53′ 4″. But in Chapter 10 he inquires into these things more accurately from three Tychonic observations, in his usual way, and in the year 1585 he obtains the north node at Taurus 15° 22′ 8″ and the inclination 1° 51′ 4″. [Margin: Inclination of ♂.] The observations, made at Uraniborg, are these.

Year of Christ	Month	Day	Hour	Place of ♂ as seen from ☉	Observed latitude of ♂	Distance of ♂ from the Sun	Distance of ♂ from the Earth
1593	December	10	5	♉ 15° 57′ 2″	0° 0′ 45″ B		
1586	December	15	18	♌ 20° 18′ 5″	2° 35′ 0″ B	109118	64495
1585	January	24	9	♌ 18° 47′ 42″	4° 31′ 0″ B	109069	98627
1585	March	12	10	♍ 9° 25′ 5″	3° 22′ 0″ B	109054	99873
1585	April	26	10	♍ 29° 21′ 51″	1° 49′ 45″ B	107746	101140

VIII. *Second,* in Chapters 11 and 12, he investigates the greatest apparent latitudes of Mars and all their variety at aphelion and perihelion, in conjunctions and oppositions with the Sun, from the inclination found above and from the

greatest and least distances of Mars; these I shall present at the end of the chapter. [Margin: Variety of the latitudes of Mars.] Lastly, in Chapter 14, from a latitude of Mars of 1° 13′, observed in the year of Christ 498, May 1, at 9^{h}, and reported in a manuscript of the Royal Library, and also from the distance at that time of Mars from the Sun, 103492, and of the Earth from the Sun, 101465, he derives the place of the north node at Taurus 0° 48′ 36″, which in the year 1585 was found above to be at Taurus 15° 22′ 8″. [Margin: Motion of the node of ♂.] Therefore in 1087 years, etc., the node of Mars advanced 14° 33′ 32″, and each year 48″ 11‴ 55⁗ 17^{v} 22^{vi} 41^{vii} 35^{viii}.

De Latitudine Veneris.

On the latitude of Venus.

IX. *First,* in his usual way, Boulliau, in Book 9, Chapter 3, inquires into the place of the node of Venus and the inclination of its fixed orbit to the ecliptic, chiefly from two observations, and at last he concludes the place of the north node at ♊ 14° 5′ 39″ and the inclination 3° 22′ 52″. [Margin: Inclination of ♀.] The principal observations were these, which we present in the following table: the former of them, for the inclination, made at Uraniborg by the Tychonians, the latter at Kassel by the Landgrave's people of Hesse, with the hours after noon.

Year of Christ	Month	Day	Hour	Place as seen from ☉	Observed latitude of ♀	Distance of ♀ from the Sun	Distance of the Earth from ☉
1594	December	17	5	♑ 5° 48′ 51″	1° 6′ 35″ A	72077	98216
1595	August	8	16	♊ 11° 41′ 3″	0° 4′ 25″ A	72055	101122

X. *Second,* he examines the variation of the latitudes of Venus at aphelion and perihelion, etc., in Chapter 5; of these below, in the synopsis. [Margin: Variation of the latitude of ♀.] As for the motion of the north node, in Chapter 7, from an observation of the year of Christ 510, August 20, on which, at 7^{h} after noon, at Uraniborg, Venus was seen joined with Jupiter: from this conjunction he gathers the north node of Venus at ♊ 4° 59′ 59″, which in the year 1596 was found at ♊ 14° 5′ 39″. [Margin: Motion of the node of ♀.] Therefore in 1086 years, etc., it completed 9° 5′ 40″, and so each year it completed 30″ 8‴ 37⁗ 29^{v} 40^{vi} 23^{vii}.

De Latitudine Mercurij.

On the latitude of Mercury.

XI. *First,* in this planet too, Boulliau, in Book 10, Chapter 7, hunts for the fixed inclination of the eccentric ellipse to the ecliptic and the place of the nodes, using three observations of Tycho. Applying to them his own measures

of the distances, and arranging the three places of Mercury on one circle, but keeping the proportion and the other things that are to be preserved, from the Rudolphine Tables themselves, which rest on a Tychonic foundation, he finds an error in two Tychonic observations, committed either by the observer or by the printer. For from them the place of the north limit is gathered at ♌ 18° 38′ 15″, and of the north node at ♉ 18° 38′ 15″. From this there follows a northern latitude of Mercury much greater than 47′; and yet, as Longomontanus reports in Book 2 of the *Theorica*, Chapter 20, Mercury is said to have been observed by Tycho in 1592 February 3 with a northern latitude of 47′. Indeed, from those three places compared together, with the measures and the proportion of the Rudolphine motions kept, it follows that Mercury should have been observed as southern, though it is put as northern. For that reason, setting aside the Tychonic observations for the time being, he turns to two observations of Pierre Gassendi, already examined above by him on the occasion of investigating the motion in longitude. [Margin: An error in the Tychonic observations found by Boulliau.] The first was in 1634 January 6, at $6^{h}\ 30^{m}$, when Gassendi observed Mercury as northern with a latitude of 1′ 52″. At that time Mercury as seen from the Sun was at ♉ 12° 42′ 26″ and already northern; therefore the node of Mercury, which is determined by the straight line of the common section of the planes of this orbit and the ecliptic, drawn through the Sun, preceded the 18th degree of Taurus. The same is confirmed by the observation of the conjunction of Mercury with the Sun in 1631 November 7, observed through the telescope by Pierre Gassendi in the morning. For in it the place of Mercury as seen from the Earth was at Scorpio 14° 37′ 42″, and as seen from the Sun at 14° 37′ 33″; but the opposite place of the Earth as seen from the Sun was at Taurus 14° 37′ 42″, and the opposite place of Mercury as seen from the Sun at Taurus 14° 37′ 33″. And when Mercury left from beneath the disc of the Sun, it appeared already northern, by 6′ 20″ from the Earth and by 13′ 50″ from the Sun, having been southern a little before. Therefore the node preceded the 18th degree, of Taurus on that side and of Scorpio on this. With these things set out first, he corrects the second observation of Tycho and draws the north node back to Taurus 11° 20′, so that the proportion may be kept; and from this, in Tycho's first observation, he determines the distance of Mercury from the south node, against the order of the signs, as 66° 37′ 6″, with the latitude of that place seen from the Sun 6° 47′ 42″ (according to the rule often given, especially with the figure in Chapter 5, number 2). So if one takes, as the sine of the distance from the south node is to the given latitude, so the radius to the angle of inclination, the inclination of the orbit of Mercury is found as 7° 24′ 21″; and using the same proportion in the third Tychonic observation, he finds the inclination 6° 21′ 33″. Accordingly, choosing a mean, he sets with Kepler the inclination of Mercury at 6° 54′. [Margin: The Keplerian inclination of ☿ established by Boulliau.] With this assumed, he investigates the node by the following technique, from the conjunction of Mercury with the Sun observed by Gassendi, in which, as above, the latitude seen from the Sun was 0° 13′ 50″.

For he takes: as the tangent of the whole inclination is to the whole sine, so the tangent of the given latitude to the sine of the arc of the distance of Mercury from the north node, which arc is 1° 54′ 12″. Now when the latitude was noted, Mercury was leaving the disc of the Sun at 11^h 16^m Uraniborg time, and its place as seen from the Sun was Taurus 15° 15′ 4″. When therefore the arc of the distance of Mercury from the node in the order of the signs is taken away, there remains the north node of Mercury at Taurus 13° 20′ 52″ in 1631 November 7. [Margin: Place of the node ☊ of ☿.] So at the beginning of the year 1632 it was at ♉ 13° 21′ 5″. The Tychonic observations mentioned above are written below, in Old Style.

Year of Christ	Month	Day	Hour	Place of ☿ from ☉	Observed latitude	Distance of ☿ from the Sun	Distance of ☿ from the Earth
1585	November	14	19	♏ 4° 42′ 53″	2° 18′ B	35912	105881
1587	January	9	5	♉ 11° 47′ 29″	0° 1′ B	31142	96459
1592	February	3	6	♊ 4° 42′ 20″	0° 47′ B	30513	98027

XII. *Second,* Boulliau continues in Book 10, Chapter 8, and from the inclination and the greatest and least distances of Mercury from the Sun at apogee or perigee, that is, with the Earth placed now at aphelion, now at perihelion, he determines the whole possible variety of the greatest latitudes in conjunctions and oppositions with the Sun, which we shall give presently in the synopsis. [Margin: Variety of the greatest latitude of Mercury.] Afterwards, in Chapter 11, for the translation or motion of the node, he uses a very ancient observation, which he had discussed in Chapter 9 and which is reported by Ptolemy, Book 9, Chapter 7. Ptolemy says that in the year of Nabonassar 484, on the day of Thoth following the 19th (this was the year before Christ 265, November 19, with the 20th following), in the morning, Mercury was seen to stand in the order of the signs, by the interval of one Moon, from the line passing through the northern and the middle star of the forehead of Scorpio, and was seen to stand toward the north by two Moons from the northern star of the forehead of Scorpio. By Boulliau's calculation the true place of the Sun at that time was at ♏ 19° 46′ 32″, and that of Mercury as seen from the Sun ♋ 1° 59′ 5″; the distance of the Sun from the Earth was 98292, and that of Mercury from the Earth 76271 and from the Sun 34768, in parts of which the radius of the annual orb is 100000. From these and the fixed inclination of 6° 54′ he gathers the northern latitude of Mercury as 4° 35′ 42″. Then he uses another observation reported by Ptolemy, in the year of Nabonassar

486, Choeac 17, with the 18th following, in the morning, at 16^h 10^m Uraniborg time, when Mercury was observed to stand off from the bright star of the tail of Capricorn, toward the north, by three Moons. That star, by Tycho's reckoning, was at ♑ 21° 40′, with a southern latitude of 2° 29′. At that time,

by Boulliau's calculation, the place of Mercury as seen from the Sun was ♎ 6° 41′ 31″, but from the Earth ♑ 22° 51′ 24″, that is, further advanced than that star by 1° 11″, although Ptolemy says it was at the same longitude, perhaps because that star and Mercury were seen in the same almucantar. For that reason, although Boulliau admits that he is uncertain about the place of Mercury and is at a loss [hærere sibi aquam][71], and that this observation was made only πλατικῶς [Greek: platikōs, "broadly, in general terms"], so that the motion of the nodes cannot be precisely defined from it, he nevertheless, because of the longer longitude of Mercury than of the fixed star, assumes that the latitude of Mercury then was 1° south as seen from the Earth. From this, and from the distance of Mercury from the Sun, 46489, and the place of the Earth at ♌ 19° 32′ 24″, he gathers trigonometrically the latitude of Mercury as seen from the Sun as 1° 37′ 47″ south; from this and from the northern latitude acquired by the preceding observation, the south node is found to have been between Virgo and Libra. When the second observation is used, however, if one takes, as the sine of the angle of inclination of 6° 54′ is to the whole sine, so the sine of the latitude of 1° 37′ 47″ to the sine 23510, there will come out a distance of Mercury, against the order of the signs, of 13° 35′ 52″ from the place of Mercury as seen from the Sun, which we said was at ♎ 6° 41′ 31″. The place of the south node of Mercury therefore falls at ♍ 23° 5′ 41″, and the north node at ♓ 23° 5′ 41″ (which in the year of Christ 1631, November 7, was found above at ♉ 13° 20′ 52″), namely in the year of Nabonassar 2380, Choeac 19. So in 1894 Egyptian years the node advanced 50° 15′ 11″, and each year 1′ 35″ 31‴ 4⁗ 37^{v} 30^{vi} 28^{vii} 30^{viii} 46^{ix}. [Margin: Motion of the node of ☿.]

Greatest latitude of the three superior planets in ♂ *with* ☉ *and in* ♌ [Latitudo Maxima trium Superiorum in ♂ cum ☉ & in ♌]

Planet		Planet at aphelion or perihelion	Earth at aphelion	Earth at perihelion
♄	in ♂	Aphelion	2° 16′ 12″	2° 16′ 40″
		Perihelion	2° 14′ 44″	2° 15′ 12″
	in ♌	Aphelion	2° 46′ 48″	2° 46′ 8″
		Perihelion	2° 49′ 18″	2° 48′ 4″
♃	in ♂	Aphelion	1° 8′ 30″	1° 9′ 20″
		Perihelion	1° 7′ 52″	1° 8′ 16″
	in ♌	Aphelion	1° 40′ 26″	1° 39′ 40″
		Perihelion	1° 42′ 48″	1° 41′ 54″
♂	in ♂	Aphelion	1° 4′ 8″	1° 5′ 6″
		Perihelion	1° 8′ 45″	1° 9′ 40″
	in ♌	Aphelion	4° 45′ 31″	4° 30′ 44″
		Perihelion	7° 3′ 2″	6° 25′ 10″

[Margin: Boulliau. ♄: Book 6, Chapter 4. ♃: Book 7, Chapter 6. ♂: Book 8, Chapter 12.]

[71]literally, that the water sticks for him

Greatest latitude of the inferior planets. The marks M and V indicate morning and evening conjunction; and D and R, the planet direct and retrograde [Latitudo Maxima Inferiorum. Notæ M, matutinam, & V, veſpertinam coniunctionem; & D, directum, R, retrogradum Planetam indicant.]

Planet		Planet at aphelion or perihelion	Earth at aphelion	Earth at perihelion
♀	in ☌ V.D.	Aphelion	1° 24′ 28″	1° 26′ 14″
		Perihelion	1° 24′ 5″	1° 25′ 40″
	in ☌ M.R.	Aphelion	8° 34′ 16″	9° 47′ 40″
		Perihelion	8° 8′ 49″	9° 15′ 29″
☿	in ☌ V.D.	Aphelion	2° 10′ 6″	2° 13′ 16″
		Perihelion	1° 35′ 16″	1° 37′ 56″
	in ☌ M.R.	Aphelion	5° 46′ 2″	6° 10′ 18″
		Perihelion	2° 55′ 48″	3° 5′ 2″

[Margin: Boulliau. ♀: Book 9, Chapter 5. ☿: Book 10, Chapter 8.]

Inclination of the orb to the ecliptic, fixed and constant [Inclinatio orbis ad Eclipticam Fixa & conſtans.]

Planet	Inclination
♄	2° 30′ 0″
♃	1° 21′ 48″
♂	1° 51′ 4″
♀	3° 22′ 52″
☿	6° 54′ 0″

[Margin: Boulliau. ♄: Book 6, Chapter 3. ♃: Book 7, Chapter 4. ♂: Book 8, Chapter 7. ♀: Book 9, Chapter 3. ☿: Book 10, Chapter 7.]

Annual motion of the north node [Motus Nodi Borealis Annuus]

Planet	′	″	‴	⁗	v	vi	vii	viii	ix
♄	0	25	54	51	42	45	42	5	0
♃	0	24	37	28	0	25	30	33	0
♂	0	48	11	55	17	22	41	35	0
♀	0	30	8	37	29	40	23	0	0
☿	1	35	31	4	37	30	28	30	46

[Margin: Boulliau. ♄: Book 6, Chapter 6. ♃: Book 7, Chapter 8. ♂: Book 8, Chapter 14. ♀: Book 9, Chapter 7. ☿: Book 10, Chapter 11.]

Root of the motion of the north node at the beginnings of the years of Christ 1 and 1601 [Radix motûs Nodi Borei Ad Annum CHRISTI 1. & 1601. ineuntes]

Planet	Year 1	Year 1601
♄	♋ 8° 59′ 19″	♋ 20° 30′ 49″

Planet	Year 1	Year 1601
♃	♊ 27° 40′ 38″	♋ 8° 37′ 23″
♂	♈ 25° 18′ 21″	♉ 16° 44′ 32″
♀	♊ 0° 43′ 16″	♊ 14° 7′ 38″
☿	♈ 0° 1′ 48″	♉ 12° 30′ 39″

[Margin: From Boulliau's *Philolaic Tables*.]

SCHOLIA.

Scholia.

I. If anyone wants a compendium of the geometrical calculation, let him know that it is the same as the one we gave from Kepler in Chapter 5, scholium 2, although certain figures to be placed in Chapter 8 may throw light on this business. [Margin: On the geometrical calculation.] And though Boulliau, with wonderful candour, as is his way, investigates astronomical truth most acutely, he will not, however, deny that a universal law of motions, especially of the smallest ones, of which kind is the motion of the nodes, is not so certainly gathered from only three observations at a time; and he himself surely knows that, if more observations are used, there will be not a few that resist the measures established above. [Margin: A judgment on Boulliau's method.]

II. Boulliau, in Book 6, Chapter 6, might seem to have acted rashly in deriving, from so short an interval as that between Walther and Tycho, indeed from less than a whole century, the difference in the motion of the nodes of Saturn, and from it so slow a motion. [Margin: On the slowness of the motion of the node of Saturn.] But since he found the same slowness in that same place from an interval of 1085 years, and it agrees closely with the sluggishness of the other motions of Saturn and with the comparison with the motion of the node of Jupiter, it might seem probable. Yet because it follows from it that Ptolemy erred in the place of this node by almost 10°, it seems rather harsh, unless it is softened by the mean of fairness.

CAPVT VII.

Chapter 7.

Noſtra Hypotheſis De Latitudine Quinque minorum Planetarum.

Our hypothesis on the latitude of the five minor planets.

I. Since our hypothesis is the Ptolemaic, but reformed, and supposes many of the things that Ptolemy, or Longomontanus correcting him in this place, posit, and these have already been explained in Chapter 1, number 7, and Chapter 4, number 8, there is no need to repeat them here, but only to indicate in what it differs. *First,* then, we accept, with Longomontanus, Kepler, Lansbergen

and Boulliau, that the inclination of all these planets to the ecliptic is stable and constant; the inclination, I mean, of the eccentric orb in the three superior planets, but of the epicycle in the two remaining ones. [Margin: The constancy of the inclination.] We accept it with that parallelism which we set out above in Chapter 4, numbers 5 and 8, and with the same alternations of the latitude as it changes toward the north or toward the south, that of Venus into the quarter opposite to that of Mercury; and in such a way that, with the centre of the epicycle placed at the apogee of the eccentric, the latitudes are not always northern, and at the perigee southern, as in the three superior planets, but in turn, both in Venus and in Mercury, sometimes northern, sometimes southern. [Margin: The difference between the superior and inferior planets.] Nor also do they remain longer in latitude northern than in southern, as the three superior planets do. For Venus and Mercury allot half of their revolution around the Sun, which in Venus is of 224^{d} and almost 18^{h}, and in Mercury of almost 88^{d}, to the northern semicircle, and the other half to the southern. That is, Venus spends almost 112^{d} in northern latitude, and Mercury 44^{d}, and each spends as many in southern latitude. [Margin: The difference between the superior and inferior planets.] Let the diagram set out in Chapter 1, number 7, but corrected in Chapter 4, number 8, be brought back here.

As for the remaining variations, which Ptolemy sought to defend by means of Deviation or Obliquation, we consider them either as still undiscovered and not resting on clear observations, or, if there are any, as capable of being sufficiently explained away: both from the varying distance of the planets from the Earth, and from the variation of the eccentricity and the change in the semidiameter of the epicycle, and above all from a more correct inclination of the epicycle of Venus and of Mercury to the orbit lying perpetually in the plane of the ecliptic. And this holds until a multitude of observations, to be brought forward and discussed in Volume 3, God willing, either compels us to think otherwise or confirms us in this opinion.

II. As regards the quantity of the inclination: after examining the latitudes dealt with in Chapters 4, 5 and 6, and some others, I judged that the inclination to choose was, for Saturn, $2^{\circ}\ 31'$; for Jupiter, $1^{\circ}\ 25'$; for Mars, $1^{\circ}\ 50'\ 30''$; for Venus, $3^{\circ}\ 24'$; and for Mercury, $6^{\circ}\ 54'$. [Margin: The quantity of the inclinations.] From these, by the method to be given in the Problems of Chapter 8, which follows shortly, the variations of the greatest latitudes are easily derived, provided that our distances of the centre of the epicycle from the centre of the Earth are used, and the semidiameters of the epicycles, variable with that variation which we taught in Sections 2 and 3.

III. As to the ancient and recent places of the nodes, we have no early observations made at the nodes themselves or as close to them as possible, and we are compelled either to believe Ptolemy or not to depart greatly from him on the place of the nodes in that age. For as to the observations that Boulliau brought forward in Chapter 6, number 4, to advance the node of Saturn and to reduce its motion: who, weighing them attentively, would not recognise

either that they are not recorded with such clear marks, or that those of Walther are separated by too short an interval from those of Tycho, that is, not even by a whole century, so that they are on that account unsuited to deriving so slow a motion? Yet, so as not to depart from the third observation, which he adduces in the same place, as far as Ptolemy would wish, we shall choose a kind of middle way, until it becomes possible to seek out firmer foundations for these matters from the observations of Volume 3. In the remaining planets, moreover, by a not dissimilar reasoning we shall choose places and motions of the nodes which seemed to satisfy most of the selected observations, although not all. So, not to detain the reader longer on these matters, we shall set out a synopsis of our hypothesis about them in the small tables below. From the place of the north node, moreover, the Limit of the greatest northern latitude is easily derived, because it stands 90° from the node in the order of the signs. The south node, however, is diametrically opposite the north node, and the southern Limit is diametrically opposite the northern Limit. Further, I considered it pointless to cut up the annual motion as far as the sixth or tenth order of sexagesimal fractions [ſexta vel decima ſcrupula ſcrupulorum], since the whole dispute lies in the minutes and seconds, and it is more than enough to have come closer to the truth in these.

The inclination here, moreover, for the three superior planets, is both that of the eccentric to the ecliptic and that of the epicycle parallel to it, to the eccentric. But in the inferior planets it is that of the epicycle to the ecliptic and to the orbit lying in the plane of the ecliptic, in accordance with what was said in number 1.

Synopsis of our latitudes [SYNOPSIS Noſtra Latitudinum]

Planet	Inclination to the ecliptic	Annual motion of the node ☊
♄	2° 31′ 0″	0′ 50″
♃	1° 25′ 0″	0′ 15″
♂	1° 50′ 30″	0′ 40″
♀	3° 24′ 0″	0′ 40″
☿	6° 54′ 0″	1′ 35″

Greatest latitude [Latitudo Maxima]

Planet	In the apogee of the epicycle: northern	In the apogee of the epicycle: southern	In the perigee of the epicycle: northern	In the perigee of the epicycle: southern
♄	2° 18′	2° 14′	2° 48′	2° 53′
♃	1° 12′	1° 10′	1° 44′	1° 47′
♂	1° 9′	1° 4′	4° 32′	7° 9′
	In the apogee of the eccentric	In the perigee of the eccentric	In the apogee of the eccentric	In the perigee of the eccentric
♀	1° 24′	1° 26′	8° 18′	9° 9′
☿	1° 47′	1° 59′	3° 41′	4° 44′

Places of the nodes and limits at the beginning of the year of Christ 1 and the completion of 1600. N signifies node, L limit; B northern, A southern [Nodorum ac Limitum loca ad Annum CHRISTI 1. ineuntem & 1600. completum. N. Nodum ſignificat, L. Limitem; B. Bor. A. Auſtr.]

Planet	N / L	B / A	Year 1	Year 1600
♄	N	B	♊ 28° 26′ 40″	♋ 20° 40′ 0″
	N	A	♐ 28° 26′ 40″	♍ 20° 40′ 0″
	L	B	♍ 28° 26′ 40″	♎ 20° 40′ 0″
	L	A	♓ 28° 26′ 40″	♈ 20° 40′ 0″
♃	N	B	♋ 0° 20′ 0″	♋ 7° 0′ 0″
	N	A	♍ 0° 20′ 0″	♍ 7° 0′ 0″
	L	B	♎ 0° 20′ 0″	♎ 7° 0′ 0″
	L	A	♈ 0° 20′ 0″	♈ 7° 0′ 0″
♂	N	B	♈ 28° 57′ 50″	♉ 16° 44′ 30″
	N	A	♎ 28° 57′ 50″	♏ 16° 44′ 30″
	L	B	♋ 28° 57′ 50″	♌ 16° 44′ 30″
	L	A	♍ 28° 57′ 50″	♒ 16° 44′ 30″
♀	N	B	♉ 25° 43′ 20″	♊ 13° 30′ 0″
	N	A	♏ 25° 43′ 20″	♐ 13° 30′ 0″
	L	B	♌ 25° 43′ 20″	♍ 13° 30′ 0″
	L	A	♒ 25° 43′ 20″	♓ 13° 30′ 0″
☿	N	B	♈ 0° 17′ 20″	♉ 12° 30′ 40″
	N	A	♎ 0° 17′ 20″	♏ 12° 30′ 40″
	L	B	♋ 0° 17′ 20″	♌ 12° 30′ 40″
	L	A	♍ 0° 17′ 20″	♒ 12° 30′ 40″

CAPVT VIII.

Chapter 8.

De Artificio Inueſtigandi Nodum, Inclinationes Orbitæ, & Latitudines Quinque Planetarum.

On the technique of investigating the node, the inclinations of the orbit, and the latitudes of the five planets.

I. Although in Chapters 3, 4 and 6 we have indicated some problems bearing on this, and have indeed worked some out completely, there still remain others, often promised but deliberately passed over at that point, so that the reader should not be too much distracted, especially in the new and in itself arduous hypothesis of Kepler and Boulliau. I have resolved to set these out collectively, without the lengthiness of diagrams and problems.

Problema 1. Locum Nodi obſeruando inueſtigare.

Problem 1. To investigate the place of the node by observing.

II. This problem is solved by no other technique than observation of the latitude of the planets, made with fine instruments on those days on which, from an ephemeris, it is in some way evident that the planet will lack latitude. The observations are continued on those days, so that from the increments or decrements of the latitude, and from their proportion, the time at which the planet was in the ecliptic can be gathered, with account taken, however, of the correction for parallax and refraction, if any should intervene. Some examples of this kind may be sought from Kepler, in the *Commentaries on Mars*, Chapter 12. Reduced to a synopsis, these are written below, from Tychonic observations made and corrected at Uraniborg.

	In hours after noon: year	Month	Day	Hours after noon	♂ Latitude observed	
1	1590	March	4	$7^h\ 10^m$	0° 3′ 12″	A
2	1592	January	23	$10^h\ 25^m$	0° 0′ 0″	
3	1592	February	6	$10^h\ 25^m$	0° 7′ 0″	B
4	1593	December	10	evening	0° 0′ 45″	B
5	1595	October	27	$12^h\ 22^m$	0° 2′ 20″	A
6	1595	October	28	at night	0° 0′ 25″	B
7	1595	January	4	morning	0° 3′ 0″	A
8	1589	April	15	at night	1° 7′ 0″	B
9	1589	May	9	....	0° 0′ 0″	

Since, then, by the 5th and 6th observations Mars was at the north node, if one counts back from about noon of October 28 the 687 days in which Mars completes its eccentric revolution, it will be found to have been at the north node in 1593 December 10, as observation 4 indicates. Counting another 687 days back again from there, Mars will be found at the north node in 1592 January 23, exactly as observation 2 shows. From this you will gather that, once a planet has been observed at either node, the time at which it will be at the opposite node, or will return to the same node, is easily established without any other ephemeris, so that, if you wish, you can confirm that same time by observation. Therefore the following small table should be consulted for the revolution to the nodes.

Return from node to node [Reditus à Nodo ad Nodum]

Planet	From the node ☊ to ☋	From ☊ to ☊, or from ☋ to ☋
♄	$5379^d\ 14^h\ 29^m$	$10759^d\ 4^h\ 58^m$
♃	$2166^d\ 7^h\ 25^m$	$4332^d\ 14^h\ 50^m$
♂	$343^d\ 12^h\ 0^m$	$687^d\ 0^h\ 0^m$
♀	$112^d\ 8^h\ 53^m$	$224^d\ 17^h\ 46^m$
☿	$43^d\ 41^h\ 37^m$	$87^d\ 23^h\ 14^m$

Having therefore obtained the moment at which the planet was at either node, if the place of the planet in longitude is not established from observation,

it must be derived from the tables, by computing both its mean and its true place in longitude, so that it may appear whether in this return to the nodes, which are diametrically opposite, it follows the mean or the true motion. As for how the time at which the planet falls into the node is gathered from a latitude observed near the nodes, Kepler teaches this by examples on Mars, in Chapter 61, namely from the proportion of the motions. For, as he himself relates there, in 1595 October 28, at 12^h, the southern latitude of Mars was observed as 4′ 30″, and on November 3, at the same hour, a northern latitude of 19′ 45″ was observed by Tycho. [Margin: Place of the node of ♂ in the years 1595 and 1600.] For in 6^d the latitude changed by 24′, and by 4′ daily; therefore it completed 4′ 30″ in 1^d and 4^h, and it was at the node on October 30, at 8^h before noon. At that time, by Kepler's calculation, Mars, and so the place of the north node, was at 16° 45′ 24″ of Taurus. From other observations, however, compared with this one and with one another, he chose for the year 1600 the place of this node at ♉ 16° 44′ 32″, as is clear from the Rudolphine Tables.

Problema 2. Latitudinem Maximam obſeruando inueſtigare.

Problem 2. To investigate the greatest latitude by observing.

III. This too is done by continuing the observations of the latitude on those days on which, from an ephemeris or from elsewhere, there is a suspicion that the planet is approaching the limit of greatest latitude, and by considering the proportion of the increment, and afterwards of the decrement. Further, from the time at which it was at the node, and from a quadrant of the eccentric revolution of the three superior planets, or of the epicyclic revolution of the two inferior planets, a conjecture is formed, though from a distance, about the time of its arrival at the limits. [Margin: 1. A rule for the time of observing the limits.] Thus, for example, if Mars was at the north node in 1595 October 30, then, because a fourth part of its revolution, which is of 687 days, is 172 days, you will gather that it was near the north limit around April 20 of the year 1596. This rule, however, is liable to a difference of some days, but even so it is not wholly useless for scenting out the time. Another rule, for Saturn and Jupiter, is that the observation of the latitude be made around their oppositions with the Sun, near the acronychal shinings; for then their latitude is usually greatest, both northern and southern, but the southern is the greater. In Mars, however, although it often reaches its greatest latitude near opposition with the Sun, it nevertheless sometimes undergoes it many days, and indeed a whole month, before or after the moment of opposition, as we said in Chapter 4, Scholium 2. Venus, when it is retrograde, experiences in this century its greatest northern latitude in Pisces, and its almost greatest southern latitude in Virgo, which is nevertheless less than the northern. In Mercury, by contrast, other things being equal, the southern is greater than the northern, and in this century it usually reaches the former around the middle of Aquarius and the latter around the middle of Leo, if it is retrograde there. On this occasion I point out that

perhaps the greatest southern latitude of Mars observed by Tycho, 6° 42′, as Longomontanus reports in Book 2, Chapter 15, was not in fact yet the greatest, because Tycho believed that it occurs at the moments of opposition. [Margin: The greatest southern latitude of Mars.] It is no wonder, then, if for us it runs out as far as 7⅙°.

Problema 3. Inclinationem totam ſeu maximam Orbitæ ad Eclipticam, Data aut non data Latitudine maxima inquirere, pro tribus Superioribus Planetis in forma Ptolemaica.

Problem 3. To inquire into the whole or greatest inclination of the orbit to the ecliptic, the greatest latitude being given or not given, for the three superior planets in the Ptolemaic form.

IV. In the Ptolemaic hypothesis this procedure is taught by Ptolemy himself, in Book 13, Chapter 3, of the *Almagest*, and on that passage by Regiomontanus in the *Epitome*, Book 13, Propositions 5 and 6; and we showed in Chapter 4, number 3, how it was indicated by Longomontanus. The operation turns out simpler and easier, however, if the epicycles are supposed to be parallel to the ecliptic, or if the greatest latitude is known beforehand. For in the following figure let CBD represent the plane of the ecliptic, and in it let B be its centre and that of the Earth. Through B let MBF be drawn, representing the plane of the orbit of the planet inclined to the ecliptic, with the angle of inclination MBD on one side and FBC on the other. On the same line let BA be the greatest distance, or longer longitude of the epicycle, and BP the least. With centres taken at the apogee A and the perigee P of the eccentric, let the epicycles EFG and HKM be described, whose planes, as well as the diameters of the apsides of the epicycle, namely EG and HK, are to be parallel to the ecliptic CD. For by 29 of Book 1 of the *Elements*, the angle BAG, by which the epicycle is inclined to the eccentric, will be equal to the angle ABC, by which the eccentric is inclined to the ecliptic; just as HPB, too, is equal to MBD. Now let the apogees E and K of the epicycles be joined with the centre B by the straight lines BE and BK, and likewise the perigees G and H by the straight lines BG and BH. And let there be given, as I suppose, the greatest and least distances of the centre of the epicycle, together with the semidiame- ters of the epicycles in the same kind of parts, and the distance of the planet from the centre of the Earth. With these things posited, *first,* let the planet be at the perigee H of the epicycle, when its centre is at the perigee of the eccentric. [Margin: Case 1 of the problem.] In the triangle PBH, in which are given the distance BP, the semidiameter PH, and the distance BH of the planet, let the angle HPB be investigated, which, because of the parallelism supposed, is equal to the angle of inclination MBD. But if HK is not supposed parallel to BD, then the greatest latitude, that is, the angle HBD, must be known beforehand from observation; and in the triangle PHB the angle HBP must be inquired into and subtracted from the latitude HBD, so that the inclination MBD is left known. *Second,* if the planet is at the

apogee K, and the centre of the epicycle at P: from the semidiameter PK and the distances BP and BK, inquire in the triangle BPK into the angle KPB, whose complement to two right angles will be the angle HPB, equal to the inclination MBD, if HK is supposed parallel to BD. [Margin: Case 2.] Otherwise the angle KBP must be investigated and added to the observed greatest latitude KBD, so that the inclination MBD results. *Third,* if the centre of the epicycle is at the apogee A, and the planet at the apogee E: in this case too, in the triangle BAE, from the given distances BA and BE and the semidiameter AE, inquire into the angle BAE. [Margin: Case 3.] For it will be the complement to two right angles of the angle BAG, which is equal to the inclination CBF, if EG is supposed parallel to the ecliptic. Otherwise inquire into the angle EBA, and add it to the observed greatest latitude CBE, so that the inclination CBF results. *Fourth,* if the planet is at G: then in the triangle BAG, with the distances AB and GB and the semidiameter AG, investigate the angle BAG, equal to the inclination CBF, if EG is parallel to CD. [Margin: Case 4.] Otherwise the angle ABG must be investigated and subtracted from the observed greatest latitude CBG, so that what remains is the desired angle of inclination CBF. Therefore either the already mentioned parallelism must be supposed, or the greatest latitude, besides the semidiameter of the epicycle and the distance of the centre of the epicycle and of the planet from the Earth, found by the calculation of longitude, as we taught in the geometrical calculation of longitude in its proper places.

Diagram. Set into the left column, with the text wrapped around its left side. A long vertical line runs from C at the top to D at the foot, and passes through B, at which the other lines of the figure meet. Three circles stand around B. Above and to the right of the vertical line is a circle with centre A, with E at its top, F at its upper right and G at its foot. Below and to the left is a circle with centre P, with H at its top, and M and K at its foot. To the right of B is a smaller circle with centre N, with O at its upper left, L at the right and R at the lower right; Z stands inside it, near the upper edge. Straight lines run from B to the points of each circle, and a line passes from F through B to M. A line joins E and G through the upper circle, and another joins H and K through the lower one.

- C (top of the vertical line): C
- D (foot of the vertical line): D
- B (junction of the lines): B
- A (centre of the upper circle): A
- E (top of the upper circle): E
- F (upper right of the upper circle): F
- G (foot of the upper circle): G
- P (centre of the lower circle): P
- H (top of the lower circle): H
- M (lower left of the lower circle): M
- K (foot of the lower circle): K
- N (centre of the right-hand circle): N

- O (upper left of the right-hand circle): O
- L (right of the right-hand circle): L
- R (lower right of the right-hand circle): R
- Z (inside the right-hand circle): Z

Problema 4 Inclinatiōnem Epicycli ad Eccētricum pro Venere ac Mercurio, Data latitudine maxima, inquirere in forma Ptolemaica.

Problem 4. To inquire into the inclination of the epicycle to the eccentric for Venus and Mercury, the greatest latitude being given, in the Ptolemaic form.

V. According to the teaching of Regiomontanus in Book 13, Proposition 4, of the *Epitome of the Almagest*, let CD in the preceding figure be the line of apsides of the eccentric, and in it the centre of the Earth, B. Let a plane surface be understood, passing through the nodes and falling perpendicularly on the ecliptic, which is to cut the sphere of the epicycle so that the common section is OLR; and let the circle OR be described from N. From B let the line BNL flow, and from the same point the lines BO, to the perigee, and BR, to the apogee of the epicycle. But let us suppose the planet then to be at O, and on BO produced let the perpendicular NZ fall from N. Therefore in the triangle BZN, right-angled at Z, since the greatest latitude, which here is OBN, is given by observation according to what was said in Problem 2, the proportion of the distance BN of the epicycle to NZ will be given. But with the semidiameter NO of the epicycle given, its proportion to the distance BN is also known; therefore the proportion of the same NO to NZ will become known. From these known quantities NO and NZ, the angle NOZ will become known in the right-angled triangle ONZ. By 32 of Book 1 of the *Elements* this angle is equal to the interior angles OBN and ONB. So, with OBN taken away, there will remain the known angle ONB, which is the very inclination sought.

Problema 5. Data Inclinatione Orbitæ ad Eclipticam Inueſtigare latitudines maximas Planetarum; in forma Ptolemaica.

Problem 5. The inclination of the orbit to the ecliptic being given, to investigate the greatest latitudes of the planets, in the Ptolemaic form.

VI. In the same figure of Problems 3 and 4: if the centre of the epicycle is at the apogee A of the eccentric, and the planet at E, the apogee of the epicycle, then the inclination CBF, equal to the angle BAG (the parallelism of the lines EG and CD being supposed), reveals the angle BAE, as being the complement to two right angles of the angle BAG. [Margin: Case 1.] Therefore in the triangle BAE, from the already mentioned angle, from the semidiameter AE of the epicycle, and from the greatest distance AB of the centre of the epicycle, you will investigate the angle EBA, which, taken from the inclination CBF, will leave the greatest latitude of the planet, CBE, in such a position. But if

the planet is at G, then from the distance BA, the semidiameter AG, and the angle BAG, which is equal to the angle of inclination CBF, with the already mentioned parallelism posited, you will obtain the angle ABG, which, added to the inclination, will make up the angle of greatest latitude CBG. [Margin: Case 2.] Now let the centre of the epicycle be at P, the perigee of the eccentric, and the planet at K, the apogee of the epicycle. [Margin: Case 3.] In this case too, in the triangle BPK, the angle KPB will be known, for it is the complement of the angle HPB, which is equal to the inclination MBD. So from the least distance BP of the epicycle and the semidiameter PK of the epicycle you will inquire into the angle PBK, which you will take from the inclination, and the greatest latitude KBD will be left. Finally, if the planet is at the perigee H, then in the triangle HPB, from the semidiameter PH, the distance BP and the angle of inclination HPB, gather the angle PBH. [Margin: Case 4.] Add this to the inclination MBD, and the greatest latitude HBD will result, provided HK and CD are parallel, as is supposed in the reformed hypothesis of Ptolemy. Otherwise one must use the distance of the planet, together with the distance and semidiameter of the epicycle, and inquire into the angle opposite the semidiameter of the epicycle; and this angle, added to the inclination if the planet is in the perigee of the epicycle, or taken away if in the apogee, will make the greatest latitude known, etc.

For ♀ and ☿, however, from the given inclination BNO of the epicycle to the eccentric, from the distance BN, and from the semidiameter NO, you will obtain the latitude NBO, or NBR, etc. [Margin: The problem is applied to ♀ and ☿.]

Problema 6. Data Latitudine maxima Inclinationem Orbis ad Eclipticam, & viciſſim data Inclinatione Latitudinem maximam inquirere in forma Copernicæa.

Problem 6. To inquire, the greatest latitude being given, into the inclination of the orb to the ecliptic, and conversely, the inclination being given, into the greatest latitude, in the Copernican form.

VII. From E, the centre of the annual orb, describe the annual orb FHG. Let AEB be the index of the plane of the ecliptic, and CED that of the plane of the eccentric, inclined to it at the angle AEC or BED. Let the Earth be at F, seeing by the line FC the planet C, at the limit of greatest latitude, opposite the Sun placed near E; or by the line FD let it see the planet D, conjoined with the Sun at the other limit. Or let the Earth be at G, and let it see by the line GD the planet D opposite the Sun, but by the line GC the planet C joined to the Sun.

Diagram. Set into the right column, with the text wrapped around its left side. A circle has its centre at E, marked in a small box, with F at its top, H at its left and G at its foot. Two long straight lines cross at E: one runs from A above the circle to B below it, and the other from C above the circle to D

below it. Lines run from F and G to the points C and D. Inside the circle, to the right of the crossing lines, the signs ♄, ♃ and ♂ are set one below another.

- C (above the circle, upper left): C
- A (above the circle, upper right): A
- F (top of the circle): F
- H (left of the circle): H
- E (centre, in a small box): E
- G (foot of the circle): G
- B (below the circle, left): B
- D (below the circle, right): D
- ♄, ♃, ♂ (inside the circle, on the right, one below another): planetary signs

In the first case, therefore, the triangle EFC must be solved, in which EF, the semidiameter of the annual orb, is supposed known, and the distance of the planet, either EC from the centre E, or CF from the Earth. [Margin: Case 1.] So if the greatest latitude CFA is given, CFE, its complement, will be given; and from these three you will gather the angle of inclination AEC. Conversely, if the inclination CEF is given, you will gather the angle CFE, which is the complement of the greatest latitude CFA. [Margin: Case 2.] In the second case, however, the triangle FED must be solved, in which are given EF, the semidiameter of the annual orb, and, of the planet D, the di- stance, either DE or DF. So if the greatest latitude BFD is given, you will gather the angle FED, which is the complement of the inclination AEC. Conversely, if the inclination is given, and hence its complement, you will inquire into the angle of greatest latitude EFD, that is BFD.

[Margin: Case 3.] In the third case, however, there is the triangle GED, whose sides are, as I suppose, known: EG, the semidiameter of the annual orb, and GD, the planet's distance from the Earth, or ED, its distance from the centre of the annual orb. Therefore, if the greatest latitude BGD is given, and from it its complement, you will find the inclination GED; but if the inclination is given, you will find EGD, which is the complement of the angle BGD, that is, of the greatest latitude.

[Margin: Case 4.] Finally, in the fourth case, look at the triangle GEC, whose sides are likewise known: EG, the semidiameter of the annual orb, that is, the mean distance of the Earth from the Sun, and either EC or GC, the distance of the planet from E or from the Earth at G. So suppose the greatest latitude CGA, that is CGE, is given: you will then inquire into GEC, the complement of the inclination CEA; and, that being known, conversely into the greatest latitude CGA. Therefore, whether the planet attains its greatest latitude in opposition to the Sun or in conjunction with it, the known latitude will show the inclination, or the known inclination the greatest latitude.

Problema 7. Latitudinum Maximarum Viciſſitudines omnes, data Inclinatione orbis Planetæ alicuius ſuperioris ad Eclipticam, & diſtantijs Planetæ à Sole & à terra, inquirere in forma Copernicæa & Philolaica.

Problem 7. To inquire into all the changes of the greatest latitudes, given the inclination of the orb of any superior planet to the ecliptic and the planet's distances from the Sun and from the Earth, in the Copernican and Philolaic form.

VIII. This question could be answered by the preceding problem, by changing the Earth's mean distance from the Sun into its distance, known beforehand, now at aphelion and now at perihelion. Still, to exercise the wits of beginners, I choose to set out and explain more clearly two diagrams of Ismaël Boulliau. He himself presents the first of them in Book 6, chapter 4, for the variation of the greatest latitudes of Saturn; the other in Book 7, chapter 6, and Book 8, chapter 12, for the variation of the latitudes of Jupiter and Mars. But each serves for all three superior planets. So in the first diagram let DA be the diameter of the planet's orb, an orb that it is enough to have indicated by the arcs DR and AV, etc. Let the Sun be at O, and the planet's aphelion at A,

Diagram. A geometric figure set into the left column. A horizontal straight line runs from D at the left to A at the right, passing through O near the centre. Arcs of the planet's orb stand at both ends of it (R below D, V above A). About O are drawn a larger circle, partly dotted, and a smaller dotted circle, with T, I, Y and Z marked on or near the horizontal line. Straight lines run from C at the upper left and from B at the lower right to those points and to D and A; a long line joins C and B through O. Below the figure stand three planetary signs, those of Saturn, Jupiter and Mars.

- C (upper left): point of the perpendicular from D
- V (upper right, on the arc): point of the planet's orb
- D (left end of the horizontal line): the planet's perihelion
- Z (on the line, left of Y): the Earth's position
- Y (on the line, left of O): the Earth's position
- O (centre): the Sun
- I (on the line, right of O): the Earth's position
- T (right of I, at the upper right of the circle): the Earth's position
- A (right end of the horizontal line): the planet's aphelion
- R (lower left, on the arc): point of the planet's orb
- B (lower right): foot of the perpendicular from A
- ♄ ♃ ♂ (below the figure): Saturn, Jupiter, Mars

its perihelion at D; and let its greatest distance from the Sun, AO, and its least, DO, be given. Now let the straight line COB represent the plane of the ecliptic, inclined at the angle COD and AOB. To it from the planet at A or D let a perpendicular be drawn, on one side AB, on the other DC. Next let the Earth at aphelion be at T, seeing the planet at aphelion at A, opposite the Sun, or the planet at perihelion at D, joined to the Sun. Again let the Earth

at aphelion be at Z, seeing the planet at A, at aphelion, in conjunction with the Sun, or the planet at D, at perihelion, opposite the Sun. Conversely, let the Earth at perihelion be at I, and let it see from there the planet at aphelion at A, opposite the Sun, or at D, at perihelion, in conjunction with the Sun. And again let the Earth at perihelion be at Y, seeing the planet at aphelion at A, in conjunction, or at perihelion at D, opposite the Sun. And let these four positions of the Earth be connected by straight lines with the points B and C of the ecliptic. With this laid down:

[Margin: Case 1: if the Earth is at aphelion and the planet is at aphelion and opposite the Sun.] In the first position of the Earth, T, since in the triangle AOB, right-angled at B, there are given AO, the planet's greatest distance from the Sun, and the angle AOB, of the inclination of the orbit to the ecliptic, inquire by trigonometry into the side AB and the angle OAB. Then, when the Earth's greatest distance from the Sun, TO, has been subtracted from AO, just as if OT and TA lay on the same line, AT remains. Therefore in the triangle ABT, with AT and AB and the angle BAT known, you will find ATB, the angle of the greatest latitude seen from the Earth at aphelion when the planet is at aphelion and opposite the Sun.

[Margin: Case 2: if the Earth is at aphelion and the planet is at perihelion and joined to the Sun.] But if the Earth is in the same place, and the planet, at perihelion, is in conjunction with the Sun, then in the triangle DOC, right-angled at C, from the planet's distance from the Sun, DO, and the inclination DOC, you will obtain DC and the angle CDT. Then, on adding together the distances, the planet's at perihelion, DO, and the Earth's at aphelion, TO, DT will be given in the triangle DTC. In that triangle, from the known DC and DT and the angle CDT, you will obtain the angle DTC, which is the measure of the greatest latitude seen from the Earth at aphelion when the planet, at perihelion, is in conjunction with the Sun.

[Margin: Case 3: if the Earth is at aphelion and the planet is at perihelion, opposite the Sun.] Now let the Earth, at aphelion, be at Z, seeing the planet at D. Then in the triangle DCO, right-angled at C, you will obtain, as in the second case, DC and CDO. But when the Earth's greatest distance from the Sun, ZO, has then been subtracted from the planet's least distance from the Sun, DO, DZ will remain known. With it, in the triangle CDZ, and with the DC and CDZ already found, you will find the angle DZC, that is, the greatest latitude of the planet at perihelion and opposite the Sun, seen from the Earth at aphelion.

[Margin: Case 4: if the Earth is at aphelion and the planet is at aphelion and joined to the Sun.] But if the Earth at Z sees the planet at A, then in the triangle AOB you will obtain, as in the first case, AB and the angle OAB. To the planet's greatest distance from the Sun, AO, however, you will add the Earth's greatest distance from the Sun, OZ, so that AZ becomes known in the triangle AZB. And from AB and ZAB you can find the angle AZB, namely

the greatest latitude of the planet at aphelion and joined to the Sun, seen from the Earth at aphelion.

[Margin: Case 5: if the Earth is at perihelion and the planet is at aphelion and opposite the Sun.] Now let the Earth be placed at I and the planet at A. Then in the triangle AOB, right-angled at B, the distance AO and the inclination AOB are given; so seek AB and OAB. Then take the Earth's least distance from the Sun, OI, from the planet's distance at aphelion, AO, and AI will be left, in the triangle AIB, in which AB and the angle at A have been found. So the angle of greatest latitude AIB, seen from the Earth at perihelion with the planet at aphelion and opposite the Sun, will not stay hidden.

[Margin: Case 6: if the Earth is at perihelion and the planet is at perihelion and in conjunction with the Sun.] But if at that time the planet is at D, seek as above, in the right-angled triangle OCD, the side DC and the angle CDO. Next join together the perihelion distances, OI of the Earth and OD of the planet, and in the triangle CDI, with DI, DC and CDI now known, you will obtain the angle of greatest latitude CID, seen from the Earth at perihelion when the planet is in conjunction with the Sun and at perihelion.

[Margin: Case 7: if the Earth is at perihelion and the planet is at aphelion and in conjunction with the Sun.] Now with the Earth placed at its perihelion, Y, and the planet at A, first, in the triangle OAB, right-angled at B, with the planet's distance at aphelion and the inclination O, seek AB and OAB. Then, on joining the distances AO, of the planet at aphelion, and OY, of the Earth at perihelion, the side AY will be known in the triangle AYB. Since AB and the angle at A are given in it, the angle AYB, of greatest latitude, will also become known, when the Earth at perihelion sees the planet at aphelion in conjunction with the Sun.

[Margin: Case 8: if the Earth is at perihelion and the planet is at perihelion and opposite the Sun.] Lastly, with the Earth staying in the same place but seeing the planet at D, first obtain, as above, in the triangle COD, the side DC and the angle CDO, so that with these and with DY (which becomes known by subtracting the Earth's perihelion distance YO from the planet's perihelion distance DO) CYD may become known, the angle of greatest latitude when the Earth at perihelion sees the planet at perihelion opposite the Sun.

The diagram, however, could have been made more easily, and these problems solved more expeditiously, if AD had been placed as the ecliptic and CB in the planet's orbit. For in that way the angle ATB, for example, the measure of the greatest latitude, would become known at once from the adjacent angle OTB, known from the solution of the single triangle TOB, etc.

Secundum Schema Bullialdi.

Boulliau's second diagram.

IX. The other diagram is almost the same, but it has the planet's orb marked MTY, in which the aphelion is M, the perihelion Y, and the mean distance

TS; and the Earth's annual orb GHF, with its aphelion G and perihelion F; the Sun is at S. The perpendiculars drawn to the ecliptic are LN, AO and VZ. The angles of the given inclination are LSM, VST and YSO; but the angles measuring the latitude, for the different positions of the Earth, are LHM, if the Earth is at H; or LKM, if it is at K; or ODY, if at D; or OBY, if at B; or VPT, if at P; or VRT, if it is at R. These vary with the variation in the distances of the Earth and the planet from the Sun, and are inquired into in the same way as in the first diagram; granted that in this one the Earth's line of apsides does not coincide in longitude with the planet's line of apsides, and that for the Earth's aphelion or perihelion distances those must be used which correspond to the Sun's equated anomaly at the time when the Earth is in the place in which it sees the planet joined to the Sun, or opposite it, or at the mean distance T.

Diagram. Boulliau's second diagram, set across the left column at its head. A large outer circle is crossed by a horizontal straight line from Y at the left to M at the right and by a vertical line down to T at the foot. Within it stand a second circle and dotted circles about S, the centre. Straight lines run from L at the upper right and from O at the upper left to points on the horizontal line, and from V at the foot to points near the centre. Beside R, inside the outer circle at the lower right, stand three planetary signs, those of Saturn, Jupiter and Mars.

- G (upper left, on an inner circle): the Earth's aphelion
- O (upper left): a position of the planet
- Y (left end of the horizontal line): the planet's perihelion
- A (on the line, under O): foot of the perpendicular from O
- B (on the line, next to A): a position of the Earth
- D (on the line, right of B): a position of the Earth
- E (above S): not cited in the text of this page
- S (centre): the Sun
- X (on the line, right of S): not cited in the text of this page
- H (on the line, right of X): a position of the Earth
- K (on the line, right of H): a position of the Earth
- N (on the line, right of K): foot of the perpendicular from L
- M (right end of the horizontal line): the planet's aphelion
- L (upper right): a position of the planet
- P (below S): a position of the Earth
- F (lower right of P, on an inner circle): the Earth's perihelion
- R (below P): a position of the Earth
- V (lower left of the vertical line): a position of the planet
- Z (next to V): foot of the perpendicular from V
- T (foot of the outer circle): the mean distance TS is measured to it
- ♄ ♃ ♂ (beside R): Saturn, Jupiter, Mars

Problema 8. Data Inclinatione Orbitæ ad Eclipticam, Inueſtigare viciſſitudines Latitudinum maximarum Veneris ac Mercurij, in forma Copernicæa & Philolaica.

Problem 8. Given the inclination of the orbit to the ecliptic, to investigate the changes of the greatest latitudes of Venus and Mercury, in the Copernican and Philolaic form.

X. From Boulliau, Book 9, chapter 5, let the following diagram serve for the latitudes of Venus. In it let the Sun be at D, and the diameter of the annual orb AI, with the Earth at aphelion at A, its greatest distance from the Sun being AD, but its least ID. Let the plane of Venus's orb be BDH, inclined to the plane of the ecliptic GDK at the angle BDK and HDG. Let Venus at aphelion be now at B, now at H, with the aphelion distances BD and HD; and at perihelion let it be at C or at F, with the perihelion distance FD or CD. From these places of Venus draw to the plane of the ecliptic the perpendiculars BK, CL, GH and FE, and let the four points K, L, E, G of the plane of the ecliptic be connected by their straight lines with the Earth at aphelion and at perihelion. For in this problem there will be eight cases, as in the preceding one.

Diagram. The diagram for Venus, set into the right half of the left column with the text running down beside it. A long spindle of straight lines runs from A at the top, under a short arc, to I at the foot, above a short arc. About D, near its middle, stand a larger circle and a smaller circle, with further straight lines crossing between them. The sign of Venus stands to the right, between the two circles' right edges and outside the spindle.

- A (top): the Earth at aphelion
- B (on the large circle, upper part): Venus at aphelion
- K (right of B): foot of the perpendicular from B
- C (on the small circle, upper part): Venus at perihelion
- L (right of C): foot of the perpendicular from C
- D (centre): the Sun
- E (below D): foot of the perpendicular from F
- F (right of E, on the small circle): Venus at perihelion
- G (left, on the large circle): foot of the perpendicular from H
- H (on the large circle, lower part): Venus at aphelion
- I (foot): the Earth at perihelion
- ♀ (right): Venus

For these: [Margin: Case 1.] First, in the triangle DBK, right-angled at K, from the given inclination BDK and Venus's aphelion distance BD, you will obtain BK and the angle DBK. Its complement ABK must be used in the triangle ABK, in which, besides BK, AB will be known if Venus's aphelion distance is subtracted from the Earth's aphelion distance; and so you will obtain the angle BAK, which measures the greatest latitude of Venus at B, seen from the Earth at aphelion, in the conjunction with the Sun of Venus at perigee and at aphelion. [Margin: Case 2.] Second, in the triangle CDL, right-angled at L, from the inclination CDL and Venus's perihelion distance CD, you will gain

CL and the angle DCL. Using its complement and the sides CL and AC, which is known by subtracting CD from AD, you will find in the triangle ACL the angle CAL, the measure of the greatest latitude when the Earth at aphelion sees Venus, at perihelion, in perigee, joined to the Sun. [Margin: Case 3.] Third, in the triangle DHG, right-angled at G, with DH, Venus's aphelion distance, and the inclination GDH, seek GH and the angle GHD; and with these in the triangle AGH, HD and AD having been joined, gather the angle GAH, that is, the measure of the greatest latitude when the Earth is at aphelion and Venus, at aphelion, in the apogee of its orb, is in conjunction with the Sun. [Margin: Case 4.] Fourth, in the triangle DEF, right-angled at E, with Venus's perihelion distance DF and the inclination FDE, inquire into EF and the angle DFE; and with these in the triangle AEF, AD and DF having first been joined into one line, investigate the angle EAF, namely the greatest latitude of Venus at perihelion and in conjunction with the Sun above it, when the Earth is at aphelion.

From these you readily understand how to proceed in the remaining four cases, with the Earth placed at perihelion at I. [Margin: Case 5.] For Venus is then either joined to the Sun below it, at its aphelion H; and then in the triangle DGH, GH and the angle DHG must be sought, with whose complement GHI, the side GH, and the remainder HI, after DH has been subtracted from ID, you will seek the angle of latitude GIH. [Margin: Case 6.] Or Venus is joined to the Sun below it at its perihelion F, and one must use first the triangle EDF and then the triangle EIF. [Margin: Case 7.] Or Venus, joined to the Sun above it, is at aphelion B, and one must use first the triangle BDK and then the triangle BIK. [Margin: Case 8.] Or finally Venus, joined to the Sun above it, is at perihelion C, and then one must use the triangle CDL and after it the triangle CIL. And in every case the angles at I, subtending the greatest latitudes, must be investigated. In this hypothesis, however, the line of apsides of the Earth and of Venus is supposed to be the same, so that the greatest latitude among those possible may be determined; granted that in this century of ours these lines are not joined.

Schema pro Mercurio.

Diagram for Mercury.

XI. You have this diagram in Boulliau, Book 10, chapter 8. Nor does it in fact differ from the diagram for Venus, except that in this one the angles of inclination of the orbit to the ecliptic are greater, as is fitting. The method of gathering the greatest latitudes of Mercury, in eight cases, is exactly the same as for Venus. So it will be enough to have set out the diagram and indicated those cases. In it, then, there is the annual orb, or great orb of the Earth, AF, with its diameter AF, and the Earth at aphelion at A and at perihelion at F, and the Sun at P.

Diagram. The diagram for Mercury, set into the right half of the right column at its middle, with the text running down beside it. A large outer circle has A at its top and F at its foot; a long spindle of straight lines joins A and F. Within the outer circle stand a second circle and two smaller circles about the region of P. Straight lines join the marked points. The sign of Mercury stands at the upper left, inside the outer circle.

- A (top of the outer circle): the Earth at aphelion
- B (below A): Mercury at aphelion
- M (right of B): not cited in the text of this page
- C (right, on the second circle): foot of the perpendicular from B
- Z (centre line, above D): Mercury at perihelion
- D (right of Z): foot of the perpendicular from Z
- O (left of P): a position of Mercury
- P (centre): the Sun
- Y (below P): Mercury at perihelion, or foot of the perpendicular from O; the text uses Y in both ways
- S (lower left of Y): not cited in the text of this page
- E (left, on the second circle): foot of the perpendicular from G
- G (lower centre): Mercury at aphelion
- F (foot of the outer circle): the Earth at perihelion
- ☿ (upper left): Mercury

The remaining orbs, larger or smaller, within the annual orb, show the path of Mercury, whose aphelion is at B and G and whose perihelion is at Z or Y. The line of apsides BG, which coincides, as is imagined here, with the Earth's line of apsides, also represents the plane of Mercury's orb, inclined to the plane of the ecliptic EPC at angles, on one side EPF, on the other BPC. To that plane perpendiculars are drawn from the four places of Mercury, namely BC, ZD, OY and GE; and the four points of this plane on which those perpendiculars fall are connected by straight lines with the Earth at aphelion, A, and at perihelion, F. With this laid down, by means of the triangles right-angled at C, D, Y, E, with the inclination and Mercury's distances from the Sun, the sizes of those perpendiculars and the angles at Mercury are inquired into; and with these, and the distances of Mercury and the Earth from the Sun, or the remainders of the Earth's distance, the angles at A or I [recte: F] measuring the greatest latitudes are sought. [Margin: 8 cases of the latitudes of Mercury.] For here too there are 8 cases. Either the Earth at aphelion, A, sees Mercury joined to the Sun below it at aphelion B or at perihelion Z, or placed above it at aphelion G or perihelion Y. Or the Earth at perihelion, F, sees Mercury joined to the Sun below it at aphelion G or at perihelion Y, or above it at aphelion B or at perihelion Z. But, as we noted above at the end of no. 8, in these two diagrams too it would be easier to conceive the line AF as the ecliptic, and EC as the line of apsides in the plane of the planetary eccentric; and then the perpendiculars to the ecliptic would have to be drawn from the points C, D, Y, E. But since we are with Boulliau, I choose to indicate the following problem from him.

Problema 9. Inclinationem Orbitæ ad Eclipticam, & ſimul Nodi locum inueſtigare, ex Latitudinibus Planetæ quamuis non maximis, & Diſtantijs ipſius ac terræ, in forma Copernicæa & Philolaica.

Problem 9. To investigate the inclination of the orbit to the ecliptic, and at the same time the place of the node, from latitudes of the planet, even though not the greatest, and from the distances of the planet and the Earth, in the Copernican and Philolaic form.

XII. In the first of the following diagrams let A be the Sun and C the centre of the annual orb LP. Let B be the centre of the orb of a superior planet, its orb being DFKH; the eccentricity from the Sun is BA, and the Earth's eccentricity AC. Let the orb of the ecliptic, lying in the plane of the annual orb, be DIKE, and the nodes D and K. And let the planet have been observed from the Earth at L now at O, now at F, or in any number of other positions, say at G from the Earth at P, etc.

Diagram. The first of the two diagrams for Problem 9, set into the right part of the left column with the text running down beside it. A large circle and a second circle almost coincident with it are cut by a vertical line from D at the top to K at the foot. A smaller circle stands about the region of A and C. Many straight lines run from O, R, I and F at the upper right to L and A, and from G and S at the right to A and P. Three planetary signs, those of Saturn, Jupiter and Mars, stand at the top, just right of the vertical line.

- D (top): a node
- O (upper right, outside the circle): a place of the planet
- R (below O): foot of the perpendicular from O
- I (below R): foot of the perpendicular from F
- F (right): a place of the planet
- L (left of centre, on the small circle): the Earth
- E (far left): on the orb of the ecliptic
- B (centre, upper): centre of the planet's orb
- C (centre, left): centre of the annual orb
- A (centre): the Sun
- H (lower left): on the planet's orb
- S (right, lower): foot of the perpendicular from G
- G (right, lower, outside S): a place of the planet
- P (below A, on the small circle): the Earth
- K (foot): a node
- ♄ ♃ ♂ (top, right of the vertical line): Saturn, Jupiter, Mars

From those places of the eccentric let perpendiculars OR, FI, GS, etc. be drawn to the plane of the ecliptic. Let these points of the plane of the ecliptic, as well as the places of the planet, be connected by straight lines both with the Sun at A and with the Earth at L or P. Let the angles of latitude seen from the Earth be given, such as the angles OLR, ILF, SPG, etc.; and let the planet's

distances also be given, from the Earth, namely OL, FL, GP, and from the Sun, namely OA, FA, GA, etc., together with the places of longitude seen from the Sun and from the Earth. For first, in the right-angled triangles already mentioned, from the given angle of latitude at L or P and the planet's distances OL or PG, the straight lines OR, IF, SG, etc. are given. These are reduced to proportion with the distances from the Sun, so that it may be known how large those perpendiculars are as seen from the Sun at A. But Boulliau inquires into them in a circle concentric with the Sun, so that the diminution of those same lines OR, IF, SG may be established at equal distances from it, and they may be taken as right sines. Next he inquires into the greatest of those that can be set as ordinates at right angles between the planes of the planet's orb and of the ecliptic. To do this he transfers the places of the planet seen from the Sun into the circle IDT, whose diameters LD and IT cut each other at right angles; and on its circumference he considers the arcs intercepted between the places of the planet seen from the Sun, say the arc BC and AB, from the first to the second and from the second to the third observation. From these he drops the perpendiculars AG, BF, CE.

Diagram. The second of the two diagrams for Problem 9, set into the right part of the left column with the text running down beside it. A circle is cut by a vertical diameter from I to T and a horizontal diameter from L to D, crossing at H. Horizontal lines run from the vertical diameter to A, B and C on the circumference at the upper right, and vertical lines fall from A, B and C to the horizontal diameter at G, F and E. Short lines join A, B and C.

- I (top): end of the diameter IT
- Q (below I, on the diameter): level of A; not cited in the text of this page
- S (below that, on the diameter): level of B; not cited in the text of this page
- V (below S, on the diameter): level of C; VC is equal to HE
- A (upper right, on the circumference): a place of the planet
- B (below A, on the circumference): a place of the planet
- R (inside, below B): in the triangle RCB
- C (below B, on the circumference): a place of the planet
- L (left): end of the diameter LD
- H (centre): the centre of the circle
- G, F, E (on the horizontal diameter, right of H): feet of the perpendiculars from A, B and C
- D (right): end of the diameter LD
- P (lower right, on the circumference): end of the arc BCP
- T (foot): end of the diameter IT

When this is done, the ordinates corresponding to the perpendiculars gathered from the observation of the latitudes and seen from the Sun are HE, HF, HG. From the given arcs, namely CB, BA, AC, he inquires into their sines, and from them the angles ACB and BAC, and also the differences of the ordinates GE, GF, FE; and, setting up a proportion, he makes as GE to AC, so FE to

RC. So then, in the triangle RCB, with the sides BC and RC known together with the angle BCR, he investigates the angle CBR and the arc CP; CB added to this makes the arc BCP, and half of it is BD. So if the planet's place in the observation at B is known, going back from there against the order of the signs will give the place D, through which the line of nodes HD passes.

Second, to obtain the greatest of the ordinates between the planes of the ecliptic and of the planet's orbit, which reveals the angle of inclination: from the arc CD given above, and from it CI, and from HE known above, to which VC is equal, he makes, as the sine VC of the arc CI to the radius HD, so HE, or VC, to HD, in parts of which HE was known, in the circle concentric with the Sun. But with the planet's distance from the Sun taken as the radius, if one makes, as the radius of the circle concentric with the Sun, 100000, to the planet's distance, so HD to another quantity, the greatest of the ordinates will be found; it is, namely, the sine of the angle measuring the greatest inclination.

You see, reader, that this technique is like the one by which this author usually investigates the eccentricity and the place of the aphelion, which we have indicated elsewhere. But I did not choose to illustrate it with examples, both because excessive length would too much disturb the series of problems of this chapter, and because it is better to use an observation of the greatest latitude and through it to inquire into the inclination, just as also to deduce the place of the node from an observation of the least latitudes. If anyone, however, wants examples of this treatment [pragmatia], he should consult Boulliau himself, Book 6, chapter 3; Book 7, chapter 4; Book 8, chapter 10; Book 9, chapter 3; and Book 10, chapter 7.

Problema 10. Nodi locum methodo Tychonica inquirere.

Problem 10. To inquire into the place of the node by the Tychonic method.

XIII. Kepler, attributing this method to Tycho, proposes it in the *Commentaries on Mars*, chapter 12, in roughly this way. Let A be the place of the node under the sphere of the fixed stars, and G the Sun, at which, in the Copernican as much as in the Tychonic manner, the orbs of the planets cut the ecliptic. And let E be the ecliptic place of the planet, that is, the point of the ecliptic to which, from the place C of the planet I, seen under the fixed stars from the Earth at H, the perpendicular arc CE is drawn to the ecliptic AB. This is the arc of latitude, but at the same time it determines the planet's longitude, considered not in its orbit but in the ecliptic. Let the Earth be at H when the planet I is in opposition to the Sun, and from the Earth there let one greatest latitude be observed, that is, the arc CE, seen under the angle EHC. For in that case AC will be a quadrant of a great circle, and the latitude, since it is the greatest, will measure the angle CAE. Now let another latitude EC have been observed, but not the greatest, from which nevertheless the place of the node is to be gathered; and at the same moment let the longitude of the planet I, seen from H under the point C of the fixed stars, have been observed. For when the arc CE

is drawn, as before, perpendicular to the ecliptic AB, there arises the spherical triangle CAE, right-angled at E, in which are given the arc of the observed latitude CE and the angle of greatest latitude CAE. So the arc AE, which is the distance of the planet's ecliptic place from the nearer node, will not stay hidden.

Diagram. Set into the right part of the right column with the text running down beside it. Long straight lines fall from A at the upper left, and from C at the right, down to G at the foot, where the Sun's rayed sign stands. An arc runs from A across to C, and an arc from A curves down through E and B; two further arcs cross the lines lower down, one through B and I, the other through H. Short straight lines join E, F, D and C.

- A (upper left): the place of the node
- C (right, end of the arc from A): the planet's place under the fixed stars
- F (on the arc, below C and left): on the inner arc AF
- E (left, below F): the planet's ecliptic place
- D (right of E): not cited in the text of this page
- B (left, below E): end of the ecliptic AB
- I (right of B, at the crossing of lines, marked with a small star): the planet
- H (below B, with a small dark disc): the Earth
- G (foot, with the rayed disc): the Sun

Therefore, since the point E of the ecliptic is known, the place of the node A will also be known; and the practice will be the better, the smaller the arc EC is, and so the nearer the place E is to the node A. So, as Kepler relates, Tycho, from the greatest north latitude CAE of acronychal Mars, 4° 34½′, observed first in the year 1585, and then from the arc EC, of a latitude of 5′ 15″, observed in the year 1595, investigated the distance from the node A of the place E, which was ♉ 17° 56′ 5″. Kepler points out, however, that CAE, strictly speaking, is not as large as the greatest latitude is, because AC is a bent arc; nor is AC the path of the planet as it would be seen from the centre of the Sun, but some inner arc, say AF; and therefore that A is not the very place of the node in this treatment either.

Problema 11. Inclinationem Planorum Eclipticæ & Orbitæ Planetæ inueſtigare tribus modis à Keplero excogitatis.

Problem 11. To investigate the inclination of the planes of the ecliptic and of the planet's orbit in three ways devised by Kepler.

Primus Modus.

First way.

XIV. These ways, not indeed all of them exact, Kepler, in the *Commentaries on Mars*, chapter 13, admits; yet it is useful that posterity should not be cheated of knowing them. The places of the limits and of the nodes must be known beforehand. One must suppose, for example, that in the year 1600 the northern limit of Mars is at ♌ 17° and the southern at ♒ 17°, and one must obtain an

observation of the planet in which it is equally distant from the Sun and from the Earth, and in which a line drawn from the Sun falls on the places of the limits; or, as in the Ptolemaic form, the centre of the epicycle is at the nodes, and the planet and the centre of the epicycle are equally distant from the Earth.

Diagram. Set into the right part of the left column with the text running down beside it. A tall isosceles triangle stands on a horizontal base from B at the left, where the Sun's rayed sign stands, to A at the right, where a small dark disc stands. Its two sides meet at the apex C, and a vertical line falls from C to D on the base. A short line runs up and to the left from C to a star-shaped sign, with E just below it.

- E (top, left of C, under the star-shaped sign): the planet, placed on its orbit
- C (apex): the planet's place in a point of the plane of the ecliptic
- B (lower left, with the rayed disc): the Sun
- D (foot of the vertical line, on the base): the right angle of triangle CDA
- A (lower right, with the dark disc): the Earth

These things supposed, let the Sun be B and the Earth A, and let the straight line BA be drawn, on which there stands the isosceles triangle BCA. Let C be the place of the planet in a point of the plane of the ecliptic, from which the perpendicular CE has been drawn to the orbit of the planet, placed at E. For in this way EC will appear equal whether it is viewed from B or from A. But if it is viewed from the Sun, which is the centre in which, according to Copernicus and Tycho, the orbits of the ecliptic and of the planet are inclined, and through which the line of nodes passes, then EC is the measure of the inclination; whereas if it is viewed from the Earth at A, it is the measure of the latitude. So if the planet is in such a situation that this isosceles triangle is formed, the observed latitude will equal the inclination sought.

For the triangle to be isosceles, however, the place of the Sun seen from the Earth and the eccentric place of the planet must be less than 90° apart; otherwise, if the angle BAC is right or obtuse, ACB will not be equal-legged. To know how great the angle CAB, or the angle of the planet's elongation from the Sun, ought to be, one must know beforehand the distance AB, that is, the semidiameter of the annual orb, and CA, the distance of the planet from the Earth when it is at the place of the limits, and this in parts of which AB is known. For in the triangle CDA, right-angled at D, from AC and half of AB, that is AD, one gathers the angle CAD, the planet's elongation from the Sun, and hence the time at which to observe the latitude of the planet standing at the limits, and in such a way that the apparent latitude equals the inclination. But how difficult it is to have observations endowed with these conditions Kepler shows, with many examples, in the same place.

Secundus Modus.

Second way.

XV. The second way requires an observation that is indeed select and rare, but if one is to be had, it shows the inclination at once, without any labour of calculation. The observation of the latitude must be such that the line drawn through the Sun and the Earth falls on the planet's nodes, already known beforehand, and that at that moment there occurs a perfect quadrature of the planet with the Sun; for then the observed latitude itself will be as great as the greatest inclination of the planet's orbit to the ecliptic. To make this understood, let the Sun be at A and the Earth at B.

Diagram. Set across the left column. A horizontal straight line runs from I at the left to O at the right, passing through A, where the Sun's rayed sign stands, and B, where a small dark disc stands. Above it stand two arcs from I to O, the outer rising to C and D, the inner lying a little below. From A two straight lines rise, one to E and one, vertical, to C, and a short line joins E to C; from B a single vertical line rises to D, with F lying on it. Two sloping lines run up and to the left, from A to H and from B to G. Star-shaped signs stand at E, F, G and H, all four on the inner arc.

- I (far left, on the horizontal line): a node
- H (left, on the inner arc): the point in the plane of the planet's orbit
- G (right of H, on the inner arc): the planet
- C (top of the outer arc): the place of the limit
- E (below C): the planet, at the greatest latitude
- D (right of C, on the outer arc): top of the line from B
- F (below D): the planet in quadrature
- A (on the horizontal line, with the rayed disc): the Sun
- B (right of A, with the dark disc): the Earth
- O (far right, on the horizontal line): a node

Let IEO be the semicircle of the planet's orbit, and ICO that of the plane of the ecliptic. Let the line of nodes IO pass through A and B, and let the place of the limit be at C, so that, with the planet set at E, the angle EAC subtends both the greatest latitude and the inclination. At that moment it is distant from the nodes by the quadrants EAO and EAI, and so AE is perpendicular to IO. Now let another perpendicular BF be erected from the Earth at B. When the planet is at F, it will be in quadrate aspect with the Sun, since from B the Sun and the planet are seen a quadrant apart, that is, at the right angle ABF. In such a case, therefore, with the line AB passing through the nodes, the angle FBD of the observed latitude will be as great as EAC, that is, as great as the greatest inclination. The reason is one that is almost self-evident, and which Kepler therefore does not prove there but assumes:

[Margin: Theorem or axiom of Kepler.] *when two planes cut each other, whichever two lines are drawn in each plane to the same point of the line of section, at right angles to that line, they always enclose one and the same angle.* The occasion for such an observation had arisen on 1584 October 30, but the Tychonians made no observation then, only on the 12th of the following November. On 1585 April 26, however, at $9^{h}\ 42^{m}$, Mars was observed at 21° 26′ of Leo, with a

north latitude of 1° 49′ 15″, the Sun being at 16° of Taurus, that is, very near the north node of Mars. So Mars was a little more than 90° from the place of the Sun and had passed quadrature by a little; therefore the latitude was a little less than the inclination, and this confirms an inclination in Mars of about 1° 50′.

[Margin: Extension of the demonstration to every latitude.] Kepler extends this demonstration to other cases, in which the planet's latitude has been observed even though it is outside quadrature. For the latitude seen from the Earth is as great as the inclination of the point of the orbit that is as far from the node, seen from the Sun, as the planet appears to be from the same node, seen from the Earth. For example, if the planet is at G, seen along BG, and the line AH is drawn parallel to this, then H and G will be seen at the same place among the fixed stars, because of the parallelism of the said straight lines and the immense distance. And so the point H, in the plane of the planet's orbit, will appear as far from the node I, seen from the Sun at A, where the angles of inclination stand, as the planet G appears from the same node I, seen from the Earth at B. Therefore the latitude seen from B and the inclination seen from A will appear equal. [Margin: The same demonstration in the Ptolemaic hypothesis.] Kepler shows in the same place that the first part holds in the Ptolemaic hypothesis, provided that the plane of the epicycle, for the equipollence of the hypotheses, is set always parallel to the plane of the ecliptic.

Tertius Modus.

Third way.

XVI. Kepler uses this way in the *Commentaries on Mars*, chapters 13 and 62. According to it, let the Sun be at A, the Earth at B, and C the place of the planet in its eccentric orbit. For the straight line ABE, passing through the Earth, will fall on the ecliptic, and the straight line from the Sun to C, that is AC, will fall on the planet's orbit, since in this Keplerian hypothesis the planes of the orbs of the five minor planets are taken to cut the ecliptic at the centre of the Sun; hence the inclination of the orbit will be the angle EAC.

Diagram. Set into the right part of the right column with the text running down beside it. A very tall, narrow figure. Straight lines rise from A at the foot to E and to C at the top, E to the left of C; B stands on the line AE, about a third of the way down from the top, and a straight line joins B and C. A further straight line rises from A to the right of AC, passes D, and stops well short of the top; and a straight line falls from C to D.

- E (top left): on the ecliptic, on the line ABE
- C (top right): the planet in the eccentric orbit, or the centre of the epicycle in the Ptolemaic form
- B (left, on the line AE): the Earth in the Keplerian hypothesis
- D (right, on the short line from A): the planet at the perigee of the epicycle
- A (foot): the Sun, or the Earth in the Ptolemaic form

So let the greatest latitude of the planet be observed from the Earth, that is, the angle EBC, so that EAC is the greatest inclination, and let the distance of the planet from the Sun, that is AC, be given in parts of which AB, the distance namely of the Earth from the Sun, is 100000. Then let it be as AC to the angle EBC, so BA to the angle BCA. This, taken from the external angle EBC, by 32 of the first book of the *Elements*, will give the angle of inclination EAC.

In the reformed Ptolemaic form, however, A will be the Earth and C the centre of the planet's epicycle, whose plane CD must be parallel to the ecliptic AE itself. Let the planet be acronychal at the perigee of the epicycle, D, that is, next to the limit. The inclination of the epicycle to the orbit, that is ACD, will be equal to the inclination EAC of the orbit to the ecliptic. And the semidiameter of the epicycle CD is as great as the semidiameter of the annual orb in the other hypothesis, if the equipollence of the hypotheses is to be perfect. Hence the latitude BAD, apparent to the Earth at A, is as great as EBC, apparent to the Earth at B, and the angle of inclination of the epicycle is as great as the inclination of the orbit sought above. But if you cannot observe the greatest latitude, at least observe the one nearest the greatest, around the planet's oppositions to the Sun, so that from its distance from the limit you may scent out the greatest inclination.

Problema 12. Data Inclinatione maxima, & diſtantia Planetæ aut puncti in Eccentrico à propiore Nodo, inuenire Inclinationem tali puncto conuenientem.

Problem 12. Given the greatest inclination and the distance of the planet, or of a point on the eccentric, from the nearer node, to find the inclination belonging to such a point.

XVII. Kepler points out this problem in the *Commentaries on Mars*, chapter 63, and in the *Epitome of Copernican Astronomy*, page 699, from which: let it be as the whole sine to the sine of the greatest inclination, so the sine of the distance of the point on the eccentric from the nearer node to the sine of the inclination sought. So the inclination is sought in the same way as the declination of points of the ecliptic from the equator.

Problema 13. Data Inclinatione, & reliquis mox ponendis; Latitudinem Planetæ ad quoduis tempus determinare.

Problem 13. Given the inclination, and the other things to be set out shortly, to determine the planet's latitude for any time.

XVIII. [Margin: Copernican form, from Lansbergen.] First, in the Copernican form, we gave the solution of this problem, with Lansbergen, in chapter 3, from number 2 to 6. The sum of the rule is this: subtract the motion of the north node from the centric longitude, or centric place, of the planet, to get the planet's distance from the north node, to be counted in the order of the signs; find the sine of this distance or of its complement, and likewise the sine of the

inclination; and let it be as the whole sine to the sine of the inclination, so the sine of the distance from the node, or of its complement, to the sine of the centric latitude. Next, as I taught when discussing the calculation of longitude, the planet's distance from the Earth must be known beforehand; and then let it be as the planet's distance from the Earth to the sine of the centric latitude, so the whole sine to the sine of the latitude sought. For Mercury and Venus, however, see number 6 in the same place.

[Margin: Form of Copernicus or Tycho, from Longomontanus.] Second, likewise in the form of Copernicus or of Tycho, we gave the way of this problem from Longomontanus in chapter 4, numbers 3, 4 and 10; you should reread these if you wish to use that form.

[Margin: In the Copernican form, from Kepler.] Third, from Kepler, in the *Epitome of Copernican Astronomy*, Book 6, at the end of part 2, or page 756, together with what he had taught in the same work from page 688 to 750, the following rule is gathered. First, subtract the planet's true place in longitude from the true place of the Sun, and the elongation from the Sun will be left. The angle of this arc, if the arc is less than a semicircle, is called the angle at the Earth; for if it is greater than a semicircle, the true place of the Sun must be subtracted from the true place of the planet, so that an arc less than a semicircle is left, fit to subtend the angle at the Earth. Further, subtract the true place of the Sun from the centric place of the planet, or the latter from the former, so that the remainder is always less than a semicircle, and you will have the anomaly of commutation, whose angle is called the angle at the Sun. With this prepared, let it be as the sine of the angle of elongation to the sine of the angle of commutation, so the tangent of the complement of the inclination (which is here supposed known) to the tangent of the complement of the latitude desired. So if the sine of the angle of commutation, first increased by five ciphers, is divided by the sine of the elongation, and the quotient is multiplied by the tangent of the complement of the inclination, then, with 5 ciphers thrown away from the total, the tangent of the complement of the latitude will come out. Or, by the shortest method of all, as Kepler says in the same place, subtract the logarithm of the elongation from the logarithm of the commutation, and to the remainder add the mesologarithm of the complement of the inclination; for the total will be the mesologarithm of the complement of the latitude.

[Margin: In the same form, from Magini.] Fourth, from the same Kepler, in the *Commentaries on Mars*, chapters 62 and 66, and from there from Magini in the *Supplement to the Ephemerides*, Canon 11. First, subtract the true place of the Sun from the true place of the planet in the zodiac, and there will remain the angle at the Earth, or the angle of the planet's elongation from the Sun. If this exceeds 90°, you must take its complement to the semicircle; if it has exceeded the semicircle but not 270°, you must take that excess; but if it exceeds 270°, you must take the complement to the circle. For in this way you must use the angle at the Earth in order to have its sine. And when the place of the Sun, true, is subtracted from the eccentric place of the planet, as above, the angle

of commutation will remain. Second, subtract the place of the north node from the eccentric place of the planet, and there will remain the argument of latitude: north, if the argument does not exceed 6^s, but south if it exceeds 6^s. With this argument seek in the Rudolphine Tables the inclination of the orbit to the ecliptic. Third, multiply the sine of the commutation by the sine of the inclination, and divide the total by the sine of the elongation. For the quotient will be the sine of the latitude sought. For Kepler has shown, in the *Commentaries on Mars*, chapter 62, that as the sine of the planet's elongation from the Sun is to the sine of the angle at the Sun, or of commutation, so the sine of the inclination is to the sine of the latitude. Granted, Magini uses this other analogy, namely: as the sine of commutation to the sine of the elongation, or of the angle at the Earth, so the sine of the inclination to the sine of the latitude. On this basis he teaches, for the sake of brevity, that the sine of the elongation is to be multiplied by the sine of the inclination, and the total multiplied by the second secant of the commutation; for the total, he says, is the sine of the latitude sought.

Problema 14. Latitudines trium Superiorum & Inclinationes non maximas Copernicæa methodo inquirere.

Problem 14. To inquire by the Copernican method into the latitudes of the three superior planets, and inclinations that are not the greatest.

XIX. Here one must remember that libration by which, according to what was said in chapter 2, number 2, the inclination of the orbs of Saturn, Jupiter and Mars to the ecliptic varies, by a variation analogous to the simple motion of the anomaly of the orb. With this laid down, let there be understood a great circle of inclination, cutting the plane of the ecliptic at right angles with its own plane, and let the common section be the line AD, etc., passing through the limits of the greatest latitudes, of which let A be the northern limit. Let the straight line CD be the common section by which the planet's orbit cuts the already mentioned circle of inclination, and let it pass through D, the centre of the great orb EF; this orb, however, is to be conceived here as having a common section with DC at D alone. Let the Earth, in the acronychal position, that is, at the planet's mean opposition to the Sun, be at E, and the planet at C; and outside the acronychal position let it be at F; and let the arc FE be known, by which it is distant from the acronychal position.

Diagram. Set into the right part of the right column with the text running down beside it. A circle stands at the foot of the figure, with three planetary signs, those of Saturn, Jupiter and Mars, inside it at the bottom. A horizontal line joins A and C at the top. A vertical line falls from A through E and G to D, the centre of the circle, and on to the bottom of the circle. A straight line runs from C down to D; a line runs from A to F and a line from C to F; a short horizontal line runs from F to G.

- A (top left): the northern limit, on the common section AD

- C (top right): the planet
- E (on the vertical line, at the top of the circle): the Earth in the acronychal position
- F (left, on the circle): the Earth outside the acronychal position
- G (on the vertical line, right of F): foot of the perpendicular from F
- D (on the vertical line, inside the circle): the centre of the great orb

[Margin: From Copernicus, Book 6, chapter 4.] Now from F draw the perpendicular FG to AD, and from C the perpendicular CA; and let A and C be joined to F by the straight lines AF and CF. First, the angle of inclination ADC is sought, which is greatest when the Earth is at E. But since, with the Earth set at F, the inclination decreases, and its libration is commensurate with the mean anomaly, which is known from the given arc EF and its versed sine EG, then if it is made as the whole sine to half the libration, so EG to the difference to be subtracted from the greatest inclination, or to GD, to be added to the mean, the inclination will be known, and its difference from the greatest inclination; and so the angle ADC, in the triangle ACD right-angled at A, will be known for the present. Now let the planet's distance CD from the centre of the great orb be given by the calculation of longitude, and in parts of which the radius DE of the great orb is known or assumed. For from this, by the rules of triangles, AC and AD will be given, and from this the ratio of CD to GD, the second sine of the arc EF. So, when GD is taken away, AG will be known, and at the same time the sine FG, because of the ratio it has with ED and GD. Therefore in the triangle AFG, right-angled at G, the base AF will not stay hidden. Finally, in the triangle CAF, right-angled at A, you will inquire into the angle AFC, which measures the latitude of the planet C, apparent to the Earth placed at F.

Problema 15. Veneris ac Mercurij Latitudines ex Inclinatione ortas inueſtigare ex Copernico, qua occaſione attingitur Reductio.

Problem 15. To investigate, from Copernicus, the latitudes of Venus and Mercury arising from the inclination, on which occasion the reduction is touched on.

XX. [Margin: Latitude of ♀ and ☿ arising from the inclination.] Although Ptolemy teaches this problem in Book 13, chapter 4, and Regiomontanus at the same place in the *Epitome*, proposition 8, it has nevertheless pleased me to take it rather from Copernicus. Copernicus, in Book 6, chapter 5, supposes that the latitude of Venus and Mercury is so entangled in three kinds of wandering from the ecliptic that the latitude of inclination alone is at times separated from the others; and that this greatest latitude was observed by Ptolemy, whether to the south or to the north: in Venus at apogee 1° 2′, at perigee 6° 22′; in Mercury at apogee 1° 45′, at perigee 4° 5′. From this, he says, one gathers the greatest inclination of Venus to be 2° 30′, and of Mercury 6° 15′. [Margin: Explanation of the scheme.] From these he teaches how to demonstrate other, non-greatest latitudes of this kind by the following diagram, in which let there

be understood the plane of the circle of inclinations at right angles to the plane of the ecliptic, and cutting the ecliptic in a common line of section ABC. Let DBE be the common section of the orb of Venus or of Mercury, whose centre is B, and from which the orb CHF has been described; and let the centre of the Earth be A. Let ABE, or CBD, be the angle of inclination of the planetary orbit to the ecliptic. To the diameter of the apsides DBE let the diameter OBF stand at right angles, which he calls the diameter of mean longitudes. In the plane of the ecliptic, of the lines of the plane of the planetary orb, let there be only the diameter OBF. The other straight lines, drawn at right angles to DE, are to be parallel to the plane of the ecliptic, and are to meet at right angles the plane that is at right angles to the ecliptic, that is, the plane of the circle of inclination, to be conceived here in the imagination. For now we are considering only the inclination of the diameter DBE, without any transverse reflexion of the diameter OBF.

Diagram. Set into the right part of the left column at its head, with the text running down beside it. A circle stands at the top. Its diameters cross at B. Inside the circle, below and to the left of B, a small figure of straight lines joins H at the left edge to M, L and K. Long straight lines run down to A at the foot: one from H, at the left edge of the circle, one from C through B, and at least one more, converging on A from the region of E and K. Between them stand two planetary signs, that of Mercury higher and that of Venus lower.

- C (top of the circle): on the line ABC
- D (right of C, on the circle): apogee of the orb
- O (left, on the circle): an end of the diameter of mean longitudes
- H (far left, on the circle): the planet
- B (centre of the circle): centre of the orb
- M (left of B, below it): foot of the perpendicular from H to the plane of the ecliptic
- L (below B, to the right of M): foot of the perpendicular from K to the plane of the ecliptic
- F (right, on the circle): the other end of the diameter of mean longitudes
- K (below M and L): foot of the perpendicular from H to DE
- E (bottom left, on the circle): perigee of the orb
- A (foot): the centre of the Earth

Now let AB be given, the distance of the centre of the epicycle, or rather of the orb, from the Earth; and BC, the semidiameter of the planet's orb, according to what was said in Section 3 when discussing the geometrical calculation of longitude. Let there also be given the angle ABE of the whole inclination, and the planet's distance from the apogee D or the perigee E of the orb, namely the arc HE, if the planet is at H; and let the task be to investigate its latitude in this situation.

[Margin: Step 1.] First, then, let the perpendicular HK be drawn from H to DE, and from these ends let the perpendiculars HM and KL be drawn to the plane of the ecliptic, to which the planet's orbit is inclined; and let them be

joined by the straight line ML. For there will arise the rectangular parallelogram HMLK, and the angle that measures the latitude will be HAM, while the angle LAM will be the measure of the prosthaphaeresis of the orb in longitude. Now since from the arc HE the angle HBE is given (for DBE is to be conceived in the plane of the circle of inclination, represented by CBA, and K and L in the same plane; just as there is to be conceived in the plane of the ecliptic not only CBA but also AM, though not E; and F and O, though not HA), HK will be known as the right sine, and from that BK as the second sine, in parts of which the semidiameter of the orb, EB, is the whole sine. And in the triangle BKL the angle KBL is given, because it is the greatest inclination of the orb, and BLK is right by construction; so the sides KL and BL will not stay hidden.

[Margin: Step 2.] Second, since AB is given in parts of which BE, the semidiameter of the orb, is known, HK will therefore become known in the same parts, and ML, which is equal to it, and KL, to which HM is equal; and when BL is subtracted from AB, AL will be known.

[Margin: Step 3.] Third, in the triangle AML, right-angled at L, since the sides ML and AL are known, the base AM will become known by trigonometry, and the angle MAL, which is the prosthaphaeresis of the orb in longitude.

[Margin: Step 4.] Fourth, in the triangle HAM, right-angled at M, with the sides AM and HM now found, one will find the angle HAM, which is the measure of the latitude sought, and, if one pleases, the planet's distance HA from the Earth.

[Margin: Step 5, for the reduction to the orbit.] Fifth, if it is wished on this occasion to distinguish the difference of the longitude in the orbit from the longitude in the ecliptic, arising from the inclination, let the diameter HL of the parallelogram HMLK be conceived, and let its quantity be investigated from the sides HM and HK, now known. Then in the triangle HAL, in which besides HL there are now known AL and AH from what was said above, let the angle HAL be sought; for this will be the measure of the prosthaphaeresis of the orb in longitude, viewed in the orbit. So this, compared with the prosthaphaeresis MAL found above, will give the difference sought. This difference is usually greatest in the middle of the quadrants OE and FE, that is, at a distance of 45° from the perigee of the orb. And so Copernicus chose his examples of this problem in Venus and Mercury set at H, with HE supposed to be 45°. From his measures of the orbs he found the latitude arising from the inclination alone in this situation to be 1° 47′ for Venus, with a difference of the prosthaphaereses of the orb of 2′; and 2° 16′ for Mercury, with a difference of the prosthaphaereses of the orb of 7′.

So far on the latitudes arising from the inclination alone, when it is without any deviation or reflexion, which happens when the centre of the epicycle (speaking in the Ptolemaic manner) is set at the mean longitude of the eccentric.

Problema 16. In quo loco Epicycli ſeu Orbis contingant maximæ Latitudines Veneris & Mercurij ortæ ex Reflexione, vnde maxima Reductio.

Problem 16. In what place of the epicycle or orb the greatest latitudes of Venus and Mercury arising from reflexion occur, from which comes the greatest reduction.

XXI. This problem too is an old one, found in Ptolemy, Book 13, chapter 4, and in Regiomontanus in his *Epitome*, Book 13, proposition 11, where he teaches that the greatest latitude of reflexion occurs when the planet is at the contact of the epicycle, that is, at the point where the epicycle is touched by a straight line drawn from the centre of the Earth; and in propositions 13, 14, 15 and 16, in which he shows that the greatest difference between the longitude of these planets in their orbit and their longitude viewed in the ecliptic, a difference arising from the latitude of reflexion, comes about near the contact: in Mercury, at that contact itself; in Venus, near the contact indeed, but mostly outside the contact itself. [Margin: Note on the reduction.] But I have preferred to set out this problem from Copernicus, Book 6, chapter 6, with whom we suppose that the greatest latitude arising from reflexion comes about when the centre of the Ptolemaic epicycle is at the apogee or perigee of the eccentric. And indeed Ptolemy, Book 13, chapter 4, says that the greatest latitude of this kind was observed by him at the contacts, that is, where the greatest digressions of Venus and Mercury from the Sun also occur; and he says that at that time the northern latitudes of Venus were found greater than the southern by 20′, and the southern latitudes of Mercury greater than the northern by 1° 30′.

[Margin: Exposition of the scheme.] So first it must be shown that these greatest latitudes occur at the contact of the orb. For this purpose, in the figure added here, let ARC be the line of the common section of the plane of the ecliptic and of the plane of the orb of Venus or Mercury, the orb, I mean, CEB, described from the centre P. Let this line pass through the apogee of the orb, C, and its perigee, G; and let the centre of the orb be at the perigee or apogee with respect to the Earth placed at A. Let the plane of the orb CDB be oblique, or bent crosswise, to the plane of the ecliptic, so that any straight lines drawn perpendicular to the diameter CG, such as DH, ER, FL, enclose angles equal to the whole obliquation, or reflexion; such are the angles DHM, ERN and FLO.

Diagram. Set into the right part of the right column with the text running down beside it. A circle stands at the top, with a vertical diameter from C to G. Inside it a framework of straight lines joins D, M, H, N, E, R, F, O and L. Below the circle, straight lines converge from the lower part of the circle down to A at the foot, with a line of dots running from B to A.

- C (top of the circle): apogee of the orb
- M (below C and left of the diameter): foot of the perpendicular from D
- H (right of M, on the diameter): where DH meets CG
- D (upper left, on the circle): a point of the orb
- N (below D and M): foot of the perpendicular from E

- E (left, on the circle): the contact
- P (right of N, on the diameter): centre of the orb
- K (right, inside the circle): not cited in the text of this page
- R (below P, on the diameter): where ER meets CG
- O (left of R): foot of the perpendicular from F
- F (lower left, on the circle): a point of the orb
- L (below R, on the diameter): where FL meets CG
- B (right, on the circle): on the orb CEB
- G (bottom of the circle): perigee of the orb
- A (foot): the Earth

Let there be understood, drawn from A, the line AE (which the engraver has, however, left out here), touching the orb at E, from which the perpendicular ER was drawn; and AD and AF, cutting the orb. From the points D, E, F let the perpendicular straight lines DM, EN, FO be drawn to the plane of the ecliptic beneath, and let them be joined to the points of the diameter CG by the straight lines MH, NR and OL; and from the angles M, N, O let the straight lines MA, NA, OA be let down to the centre of the Earth, A. For the angles HAM, RAN and LAO will be the measure of the prosthaphaeresis of the orb as to longitude; but the angles of latitude arising from reflexion will be DAM, EAN, FAO.

With this laid down, I say that the angle of latitude EAN, at the contact, is the greatest of all. For the angle EAR is the greatest of all, since it is made by the outermost line, or tangent, AE, and since the side ALRH lies in the common line. And so ER has a greater ratio to EA than HD has to DA, and than LF has to FA. But as ER is to EN, so HD is to DM, and LF to FO; for the angles of reflexion that they subtend are equal, and the angles M, N, O, because of the perpendiculars drawn, FO, EN, DM, are right. So NE has a greater ratio to EA than MD has to DA, and than OF has to FA. Moreover the angles DMA, ENA and FOA are right, since A is in the plane of the ecliptic, to which the perpendiculars were drawn. Therefore the angle EAN, belonging to the contact, is greater than the angle DAM, and than FAO, and than all others whatever constituted in this way.

Problema 17. Inuenire angulum Reflexionis maximæ, & inde maximas Latitudines ad Apogeum & Perigeum Eccentrici ortas ex Reflexione; vnde Reductio ab Ecliptica ad Orbitam colligetur.

Problem 17. To find the angle of greatest reflexion, and from it the greatest latitudes at the apogee and perigee of the eccentric arising from reflexion; from which the reduction from the ecliptic to the orbit will be gathered.

XXII. In the figure immediately preceding, let KAB be the observed angle of greatest latitude, with the planet placed at the contact B, to which let the semidiameter of the orb, PB, be drawn. Let the perpendiculars BL to CG and

BK to the plane of the ecliptic be drawn, and let K and B be joined by the straight line KB; and let the angle of reflexion KLB be the one to be found.

[Margin: Step 1.] First, since the distance of the centre P from the Earth, that is AP, is supposed given in parts of which PB, the semidiameter of the orb, is known, and the angle ABP at the contact is right, the side AB cannot be unknown.

[Margin: Step 2.] Second, since as the base AP is to AB, so the base PB of the triangle PBL is to the side LB, by 8 of the sixth book of the *Elements*, BL will therefore be known. And since the observed or assumed latitude KAB is given, together with AB as found, therefore in the triangle ABK, right-angled at B [recte: K], the side KB can be found.

[Margin: Step 3.] Third, therefore, in the triangle KLB, right-angled at K because of the perpendicular BK drawn to the plane of the ecliptic, since LB and KB are now known, the angle of reflexion KLB becomes known.

See the rest in Copernicus, if you like, Book 6, chapter 7. From this it will not be hard to find the prosthaphaereses of the orb, that is, the angles LAB and KAL, so that their difference may be known and from it the reduction from the ecliptic to the orbit, if the sides AK and AL are found first.

Moreover, Copernicus, Book 6, chapter 7, taking from Ptolemy the latitudes on either side, mean ones, or at the mean longitude of the eccentric, which arise from reflexion, of 2° 30′, gathers the obliquation KLB in Venus to be 3° 29′, and the difference of the equation of the orb only 3′; in Mercury, the obliquation about 7°, and the difference of the equations of the orb about 8′. But with the centre of the epicycle or orb placed at apogee, he gathers the greatest latitude arising from reflexion: of Venus 2° 27′, of Mercury 2° 17′. With it placed at perigee, however, the latitude of Venus is 2° 34′, of Mercury 2° 45′.

[Margin: Part 2 of the Problem.] For once the greatest obliquation has been found, and the distance AP of the centre P, both at apogee and at perigee, he seeks the already mentioned latitudes in this way.

[Margin: Step 1.] First, with the distance AP, at apogee or perigee, given, and the semidiameter PB in parts of the same kind, the side AB is given in the triangle APB, right-angled at the contact B. Hence it comes about that as AP is to AB, so PB is to LB, by 8 of the sixth book of the *Elements*. Therefore in the triangle LKB, right-angled at K, with the side LB now found and the angle of reflexion KLB, the side KB can be found.

[Margin: Step 2.] Second, in the triangle AKB, right-angled at K, with KB and AB now known, the latitude KAB will be found, both at apogee and at perigee.

Problema 18. Inuenire an & quanta ſit Variatio Deuiationis ſecundùm Copernicum.

Problem 18. To find whether there is a variation of the deviation, and how great it is, according to Copernicus.

XXIII. Ptolemy set the deviation of Venus as always to the north and 10′, and that of Mercury as always to the south and 45′. Copernicus, Book 6, chapter 8, raises the doubt whether this is always the same at apogee and at perigee, or whether it undergoes a change, and a sensible one. Afterwards he affirms that it is indeed changeable, but not so that this change produces a sensible error, and this he demonstrates in his hypothesis as follows.

[Margin: Delineation of the scheme.] In the following diagram, let ABC be the line of the common section of a plane at right angles to the ecliptic, and A the centre of the Earth; and let CFE be the orb of the planet, whose centre B is now at its greatest, now at its least distance from the Earth, or, as Ptolemy would say, now at the apogee, now at the perigee of the eccentric, namely when the deviation turns out greatest and the plane of the orb CFE, according to Copernicus, is parallel to the plane of the ecliptic. So let DF be conceived as the diameter of the orb deviating from the plane of the ecliptic, and parallel to the diameter CE; and CDF in the plane CBA, under the common section. Now let DF be bisected at G, so that G is the centre of a circle parallel to the plane of the ecliptic, and let the centres G and B be joined by the perpendicular GB; and let the three points, namely the apogee D, the perigee F and the centre G of that parallel, be joined to A by straight lines.

Next, let the angle of mean deviation BAG be 10′ in Venus and 45′ in Mercury. First, then, in the triangle ABG, right-angled at B, with the mean distance AB given and the angle at A, BG will be given, in parts of which AB is given; and GB is equal to the sine of the arc DC and FE.

Diagram. Set into the right part of the right column with the text running down beside it. A circle stands at the top. A vertical line falls through it from C at the top of the circle, through B, to E at the bottom of the circle, and continues down to A at the foot. A straight line runs from D down through G to F, and a short horizontal line joins G and B inside the circle. D stands at the upper left of the circle and F at its lower left. Straight lines converge from D, from G and from F down to A. Below the circle two planetary signs stand one above the other to the right of the lines, that of Mercury higher and that of Venus lower. A small dark disc stands on the converging lines a little above A.

Second, therefore, conceiving the triangle DAC, right-angled at the point nearest to C, on which the sine drawn from D would fall, from that given sine and the distance AC, nearest to the apogee distance itself, the angle of greatest deviation at apogee is gathered: in Venus 6′, and in Mercury 33′.

[Margin: Greatest and least deviation of ♀ and ☿.] Similarly, conceiving the triangle FAE, right-angled however at a point a little above E, on which the sine drawn from F would fall, from the said sine and the perigee distance

AE, or one a little greater, the angle of greatest deviation at perigee, FAE, is gathered: in Venus about 15′, in Mercury 1° 10′.

Copernicus adds, however, that these differences were unobservable to the ancients, because, since it comes about at the apogee and perigee of the orb, at which the conjunction of these planets with the Sun takes place, the planets lie hidden under the rays of the Sun; and that they therefore followed a deviation observed elsewhere, as simple and unchanging, but that it is in truth variable.

[Margin: Scheme for the libration of the deviation.] This variation he accounts for by a libration through the small circle DF, on whose diameter DF the plane of the orb CBA is carried back and forth, so that it is now in the plane of the ecliptic under F, now at its greatest distance under D, now at the distance of the mean deviation under B; and the libration takes its laws from a motion made along the circumference of the small circle, double the motion of revolution of the centre of the planet's orb, laws that we have already set out in chapter 2, number 7. From the numbers just gathered, of greatest and least deviation, the diameter DF of the small circle is set at 14′ in Venus and 70′ in Mercury.

Diagram. Set into the right part of the right column with the text running down beside it. A small circle stands on two low arcs, which cross each other at C at the lower left and at A at the lower right, each running a little beyond. The upper arc passes through L and crosses the circle between B and E; the lower, flatter arc crosses the lower part of the circle. A short line drops from L to the lower arc. D stands at the top of the circle and F below it, with a vertical line between them on which B and E stand inside the circle, B above E. The planetary sign of Venus stands above and to the left of the circle, and that of Mercury above and to the right. The sign of the ascending node (☊) stands above C and that of the descending node (☋) above A.

Problema 19. Latitudinem Mercurij ac Veneris ex Deuiatione ortam determinare ſecundùm Copernicum.

Problem 19. To determine the latitude of Mercury and Venus arising from the deviation, according to Copernicus.

XXIV. Following Copernicus, Book 6, chapter 8, let ABI be the line in the common section of the planet's orb and the ecliptic, with the Earth A placed at the planet's apogee or perigee; and let us use AB, the quasi-mean distance, of 10000 parts. From the centre C let the orb of the planet DEF be described, parallel to the plane of the ecliptic and at its greatest distance from the ecliptic. Let its diameter DF (which we cannot represent in a plane) be understood under ABI, and its centre C connected with the plane of the ecliptic by the perpendicular line CB.

[Margin: Delineation of the scheme.] Let the planet be at E, distant from F, the perigee of the orb, by the arc EF of 45°. Let perpendiculars be drawn: EG to CF, and, from the ecliptic to the plane of the orb lying beneath it, KE and

HG. Let K and H be joined by the straight line KH, so that the right-angled parallelogram EGHK arises; and finally let E and K be joined to the Earth by the straight lines AE and AK.

With these laid out: [Margin: Step 1.] first, from the greatest deviation the sine CB is given, by the preceding Problem, in parts of which AB is 10000; and in the same kind of parts the semidiameter of the orb, CE, is supposed known. Therefore the second sine BH will be given too, equal to CG; and in the right-angled triangle CEG, EG will be given, equal to KH.

[Margin: Step 2.] Second, with BH subtracted from AB, AH will be given; therefore in the trian- gle KAH, right-angled at H and with the sides AH and KH given, AK will be given.

[Margin: Step 3.] Third, in the triangle AKE, right-angled at K, in which AK is given, and EK, equal to GH, that is, to CB, the angle EAK will be given, which measures the latitude arising from the greatest deviation of the orb, when the planet is at E. From this you will understand how, with the deviation found according to the laws of the libration itself, and given the distance AB of the centre of the orb and the arc by which the planet is distant from the perigee of its orb, you may hunt out the latitude arising from the deviation, according to the mind of Copernicus.

Diagram. Set into the right part of the left column with the text running down beside it. A circle stands at the top, with D and I side by side at its top and F at its foot. A vertical line falls from D through C and G to F, and a second from I through B and H, continuing below the circle down to A. A short horizontal line joins C and B in the upper half of the circle, and a line runs from C down and to the left to E, at the left edge of the circle. In the lower half a small figure of straight lines joins E, K, G and H, H standing at the right edge of the circle. Straight lines run from E and from K down to A at the foot. To the right of them stand two planetary signs one above the other, that of Mercury higher and that of Venus lower.

CAPVT IX.

Chapter 9.

De Reductione loci Planetæ ab Ecliptica ad Orbitam ipſius, & ab Orbita ad Eclipticam, deque Aequatione orta ex Curtatione Interualli.

On the reduction of a planet's place from the ecliptic to its orbit, and from the orbit to the ecliptic, and on the equation arising from the curtation of the interval.

I. *This care*, says Kepler in the *Commentaries on Mars*, chapter 9, *Ptolemy did not think necessary with regard to the planets; Copernicus did not neglect it in the Moon; Tycho Brahe diligently embraced it for the sake of subtlety.* But his memory fails him. [Margin: Authors who have concerned themselves with this reduction.] For Copernicus too, in Book 6, chapters 5 and 7, did not neglect

it in Venus and Mercury, and he taught *the way of weighing what diversity the latitude brings to the longitude*, to use his own words; and so, before him, did Regiomontanus in the *Epitome*, Book 13, propositions 13, 14, 15, 16, 17, 19. Kepler himself, however, was the most diligent in this: in the *Commentaries on Mars*, chapter 9, in the *Epitome of Copernican Astronomy*, Book 5, page 700, and in the Rudolphine Tables, chapter 21. Boulliau, in Book 11 of the *Astronomia Philolaica*, theorem 15, and Book 12, precept 9; and Renieri, in the *Tabulae Mediceae* as reissued, from page 32. Magini, for his part, touches on this lightly in the *Supplement to the Ephemerides*, Canon 10, and Hérigone in volume 5 of the *Cursus Mathematicus*, page 559.

[Margin: The cause of the reduction and the curtation.] It must therefore be understood that when we gather the places of the planets from observation, we estimate their ecliptic place, that is, their place at the point of the ecliptic on which the arc of the circle of latitude drawn through the body of the planet falls. So if we wish to define their place in their own orbit, in which they move, it must be reduced to the orbit, since outside the nodes and the limits of the latitudes it is different. Conversely, when we compute their places from the scheme of the hypothesis and from tables, we consider them in their own orbit; so if we want the ecliptic place, the other reduction is needed in turn. And since the equation of the orb depends on the distance of the planet from the centre of the ecliptic, which distance, with respect to the ecliptic place, is smaller than the distance of the planet itself, that distance must therefore be curtated, and from this curtation the equation of the orb must be corrected, so that the most exact place in longitude may be determined, above all in Mars, Venus and Mercury, in which both the reduction and the curtation would be neglected at greater risk of error. But, beginning from the reduction, I choose to set out first certain definitions and propositions, as foundations of the problems to be added on this subject.

Definitiones ad Reductionem ſpectantes.

Definitions concerning the reduction.

[Margin: What is the ecliptic place?] 1. The place of a planet in the ecliptic *is the point of the ecliptic at which a circle of latitude, at right angles to the ecliptic and passing through the place of the planet's body under the sphere of the fixed stars, cuts the ecliptic.*

[Margin: What is inclination?] 2. The inclination of a planet, or of a point in the planet's orbit, *is the angle at the centre of the ecliptic made by two straight lines, one drawn in the plane of the planet's orbit to the planet, or to a point existing in that orbit, the other in the plane of the ecliptic to the planet's ecliptic place.* For although inclination belongs rather to lines or planes, it is nevertheless attributed to the planets and points that are on those lines. Further, the centre of the ecliptic in the hypothesis of Kepler and of Boulliau is the Sun; for most it is the centre of the Earth.

[Margin: What is the circle of inclination?] 3. The circle of inclination *is a great circle in the sphere of the fixed stars, at right angles to the ecliptic, and described from the centre of the ecliptic.*

[Margin: What is the arc of inclination?] 4. The arc of inclination *is the arc of the circle of inclination intercepted between the eccentric place of the planet and the ecliptic place of the same.*

[Margin: What is the argument of inclination?] 5. The argument of inclination *is the arc of the planet's orbit under the fixed stars, intercepted between the north node and the eccentric place of the planet, counted in the order of the signs.*

[Margin: What is the reduction?] 6. The reduction to the ecliptic *is the small arc by which the argument of inclination and the arc of the ecliptic counted from the common node to the planet's ecliptic place differ from each other.*

[Margin: Why and when the reduction is to be applied.] For by the demonstrations of Theodosius in the *Sphaerics* it is gathered that when the circle of latitude passes through the poles of the ecliptic and of the planetary orbit, the arcs counted from the common section to the circle of latitude are equal, and so no reduction is needed. But when it passes not through the poles of the orbit but only through the poles of the ecliptic, it is indeed at right angles to the ecliptic but oblique to the planet's orbit; and so the arc between the planet's place in its own orbit and the nearer node is greater than that between the planet's ecliptic place and the same node; and therefore the place in the orbit must be reduced to the place in the ecliptic, if the question concerns the planet's place in the ecliptic.

[Margin: Where is the greatest reduction?] 7. The greatest reduction to the ecliptic occurs at about the middle of the quadrants counted from the nodes, just like the greatest difference between the right ascension and the arc of the ecliptic corresponding to it, as Kepler teaches in the *Epitome*, page 700, and Boulliau in Book II, Theorem 15.

Problema 1. Reductionem Loci Planetæ in ſua Orbita ad Eclipticam inueſtigare, & adhibere.

Problem 1. To investigate the reduction of a planet's place in its orbit to the ecliptic, and to apply it.

II. In the spherical triangle DAC, let AD be a quadrant of the ecliptic and AC a quadrant of the planetary orbit, and A their common node. If the planet is at C, distant from the node A by a quadrant, or 90°, it will be at the limit of greatest latitude, and the arc AD will be as great as AC, and therefore no reduction will be needed, just as none is at A either.

Diagram. Set into the right part of the right column with the text running down beside it. A three-sided figure with curved sides. An arc runs from D at the left through E, at its top, to A at the right. A second arc runs from A at the right, through B, down to C at the lower left. A straight side joins D and C, and a further line joins E and B.

Now let the planet be at B, through which let BE be drawn, an arc of latitude at right angles to the ecliptic, so that the spherical triangle AEB arises, right-angled at E. And let the greatest inclination of the planetary orbit to the ecliptic be given, which the angle A measures; let there further be given the argument of inclination, or the arc AB, counted from the nearer node to the place of the planet in the eccentric. For from these three givens, namely the right angle E, the angle A of greatest inclination and the base AB, you will seek, by the doctrine of right-angled spherical triangles, the arc AE, whose difference from the arc AB will be the reduction to the ecliptic.

So multiply the second sine of the greatest inclination by the tangent of the argument of inclination, and divide the total by the whole sine; for there will remain the tangent of the reduced argument, that is, of the arc AE. [Margin: Abridgement of the problem.] Or add the residual of the second logarithm of the greatest inclination to the mesologarithm of the planet's distance from the nearer node; for there will result the mesologa- rithm, the second one, of the distance of the planet's ecliptic place from the same node, whose difference from the argument of inclination is the reduction. From tables, moreover, the reduction is readily gathered, with the argument of latitude, or distance from the node.

[Margin: Use of the reduction.] The use of the reduction is as follows. If the planet is heading from the nodes to the limits, the reduction must be subtracted from the argument of inclination; but if from the limits to the nodes, it must be added. And the remainder in the one case, the sum in the other, added to the place of the preceding node, gives the planet's place in the ecliptic. Or, if the planet is heading to the limits, it must be subtracted from the place computed in the orbit; if to the nodes, it must be added; and the planet's place in the ecliptic will result. Let the contrary be done in the reduction from the ecliptic to the orbit.

Problema 2. Reductionem Planetæ ab Ecliptica ad Orbitam inuenire & adhibere.

Problem 2. To find the reduction of a planet from the ecliptic to the orbit, and to apply it.

III. In the preceding triangle AEB, given the angle A of greatest inclination and the side AE, namely the distance of the ecliptic place from the nearer node, you will seek the base AB by the rules of right-angled spherical triangles; for it will be the distance from the node of the planet considered in its orbit. Its difference from the arc AE will be the reduction sought, but to be applied in the contrary way, namely to be added to the distance of the ecliptic place if the planet is heading from the nodes to the limits, and subtracted if from the limits to the nodes. [Margin: Abridgement of the problem.] Now the second logarithm of the angle A, added to the second mesologarithm of the side AE, will give the second mesologarithm of the base AB.

Problema 3. Reductionem Maximam inuenire.

Problem 3. To find the greatest reduction.

IV. Use the arc AB, taken as about 45°, if you seek the reduction to the ecliptic; or AE, likewise about 45°, if you seek the reduction to the orbit; and do the rest as in problem 1 or 2. I said *about*, because although the greatest reduction occurs around the semiquadrant AB or AE, it does not always occur at the exact limit of 45°.

[Margin: Where the greatest reduction is, from Boulliau.] For Boulliau, in Book II, theorem 15, has demonstrated not only that the reductions viewed from the Earth are greater than those viewed from the Sun, if the Earth is nearer to the planet than the Sun is, as it is at the oppositions of the superior planets to the Sun; but also that they are greater in Mars beyond 45°, namely at 55° of distance from the node, because when it is in opposition the Earth is nearest to it, and the planet's place in its orbit is turned face-on toward us; that they diminish, however, at the quadratures because of the oblique view, and at conjunction with the Sun; but that the greatest reduction at the oppositions exceeds by a few seconds the greatest reduction about 45°, and is found to be unusual [?] [recte: *solitam* for *insolitam*, per the book's errata, giving "the greatest reduction usually found about 45°"]. In Mercury, moreover, the reduction seen from the Sun is always greater than that viewed from the Earth, because it is always nearer to the Sun than to the Earth. In Venus it is sometimes greater, as also in Mars, because of the greater nearness of acronychal Mars to the Earth, and of retrograde Venus likewise to the Earth, when it is about 30° past the node.

De Curtatione Interualli.

On the curtation of the interval.

V. On this, Kepler and Boulliau in the places cited at number 1. From these, note the following. Since the plane of the planetary orbit is inclined to the plane of the ecliptic, and to investigate the planet's latitude a straight line perpendicular to the ecliptic is drawn from the planet's orbit, subtending the angle of inclination, it is necessary that the point of the ecliptic on which that perpendicular falls be nearer to the centre of the world, where the angle of inclination is, than the point of the planetary orbit from which that perpendicular is drawn. And so that ecliptic interval, or the distance of the point in the plane of the ecliptic that terminates that perpendicular, is smaller than the distance of the point or planet set in the planetary orbit. So that shorter distance is called by Kepler and his followers the *curtate distance*; and the excess by which it is surpassed by the greater interval, or the small portion that the curtate distance lacks, is called the *curtation*.

To put this before the eyes, for the superior planets recall the figure set out in chapter 8, number 8. For in it the straight line AD represents the plane of the planetary orbit, and the straight line COB the plane of the ecliptic, on which the perpendiculars AB and DC fall from the planets A or D. So the angles at

B and C are right, and greater than either angle at the base, by 32 of the first book. Therefore the base AO, subtending the angle ABO, that is, the distance of the planet A from O, the centre of the world, is greater, by 19 of the first book of the *Elements*, than the side OB, which is the distance of the point in the ecliptic on which the perpendicular AB falls; and by the same reasoning the distance DO is greater than CO.

For the inferior planets, however, look at the figure placed in chapter 8, number 11, in which BG indicates the plane of the planet's orbit and CE the plane of the ecliptic, to which the perpendiculars BC and ZD are let fall from the planet at B or Z; or, with the planet placed at G or Y, the perpendiculars GE or YO are drawn. So the distance CP from P, the centre of the ecliptic, is shorter than BP, etc. The definitions relevant here are therefore these.

[Margin: What is the curtate distance?] 1. The curtate distance *is the straight line in the plane of the ecliptic intercepted between the centre of the ecliptic (which for Kepler and Boulliau is the Sun) and the perpendicular drawn to the ecliptic from the centre of the planet's body.*

[Margin: What is curtation?] 2. Curtation *is the small portion of the planet's distance from the centre of the world corresponding to the sagitta, or versed sine, of the planet's inclination, in the proportion in which the whole interval corresponds to the whole sine.* So indeed Kepler in the *Epitome of Copernican Astronomy*, page 701. Or, from Hérigone, volume 5, page 559: *it is the excess by which the planet's distance from the centre of the world surpasses its curtate distance.*

In place of the world, however, these authors put the Sun, which they consider to be the centre of the ecliptic, and the point at which the angle of inclination of the orbit to the ecliptic is formed. From the definitions stated there arises this theorem, the foundation of the following problem.

3. *As the whole sine is to the planet's distance from the centre of the ecliptic, so is the second sine of the inclination to the curtate distance.* So let it stand.

Problema 4. Data diſtantia Planetæ à centro Eclipticæ, inuenire diſtantiam Curtatam.

Problem 4. Given the planet's distance from the centre of the ecliptic, to find the curtate distance.

VI. Let it be made: as the radius, or whole sine, to the planet's distance from the centre of the ecliptic, so the second sine of the inclination to the curtate distance. How the planet's distance from the centre of the world is gathered is clear from what was said about the geometrical calculation of longitude. By what method the inclination becomes known, so that its second sine may be known from it, we said in chapter 8, problem 12, or number 17.

[Margin: Where reduction and curtation are nil, where greatest.] Kepler, moreover, teaches on page 701 of the *Epitome* that the distance of the ecliptic point is curtated most around the limits, and still more if the planet is at the limit nearer to the planet's aphelion; understand this of the distance taken from

the centre of the Sun. But in the hypothesis that sets the centre of the ecliptic at the centre of the Earth, it must be understood of the limit nearer to the apogee itself.

But Boulliau, in Book 11, theorem 15, and Book 12, precept 9, teaches that at the oppositions of the three superior planets to the Sun the reduction is greatest and the curtation nil, because the planet's place in the orbit is turned face-on toward us. Conversely, at the quadratures with the Sun the reduction is diminished, because the planet's place in the orbit is viewed obliquely, and the curtation then turns out greatest, and the greater the nearer the planet comes to the limit. He teaches that the curtation of the superior planets is never turned face-on toward us; but that in Venus and Mercury it is turned face-on at the contacts of the orb, where the greatest digressions occur, because the perpendicular falls from the eye on the line drawn from the Sun to Venus or Mercury; and so at the contacts there are no reductions with respect to the Earth, but greatest curtations; conversely, at the conjunctions with the Sun there are no curtations, but greatest reductions.

VII. [Margin: Use of the curtation.] But you will ask what the use of the curtation is. I answer that it serves to correct the equation of the orb, which is usually obtained by means of the planet's distance from the Sun or from the centre of the world. So since the curtation always diminishes the planet's distance from the centre of the world, by its force something must always be subtracted from the equation of the orb. Hence if the equation of the orb was additive, it will add less to the planet's first-equated motion; and if subtractive, it will subtract less. The rules of this practice are of the following kind, supposing tables in which the reduction and the curtation are set out. With the place of the north node subtracted from the place of the planet (the centric place), there remains its distance from the node, with which the greatest prosthaphaeresis of the orb is gathered from the Rudolphine or Philolaic tables or the like, and the curtation in the other small table. Next, in the table of equations of the orb, the equation of the orb belonging to the anomaly of the orb is gathered, as was said in connection with the calculation of longitude. With these prepared, let this proportion be made: as the greatest parallax, or greatest equation of the orb, is to the curtation found in the tables, so is the equation of the orb belonging to the anomaly of the orb to the curtation, which is always to be subtracted from the equation of the orb first found, so that the most exact equation of the orb may be had. The precept of the curtation in the Rudolphine Tables stands somewhat differently; on its use see their precepts 90, 91 and 92, which will be set out more clearly in the precepts of our own tables. [Margin: For which planets the curtation and the reduction are necessary.] The reduction, let alone the curtation, can nevertheless safely be neglected in Saturn and Jupiter; but in Mars, Venus and Mercury it must not be neglected, because neglecting it brings in an error sometimes of 3′ in Mars; just as in Venus and Mercury too, in conjunctions with the Sun, it produces an error not to be despised. According

to Kepler and Boulliau, the greatest reduction and curtation are as you see in the following small table.

Planet	Authority	Greatest reduction	Greatest curtation
♄	Kepler	1′ 41″	
♄	Boulliau	1′ 27″	0′ 20″
♃	Kepler	0′ 29″	
♃	Boulliau	0′ 30″	0′ 11″
♂	·····	0′ 53″	1′ 10″
♀	·····	2′ 59″	5′ 56″
☿	Kepler	12′ 30″	
☿	Boulliau	12′ 45″	10′ 22″

For Copernicus, however, the greatest diversity in longitude, arising from the mere latitude of inclination, does not exceed 2′ in Venus and 7′ in Mercury; and the one arising from the greatest reflexion, when the planet is at the contact of the orb, 3′ in Venus and 8′ in Mercury, as is clear from what we said at chapter 8, Problems 15 and 17.

SCHOLIA.

Scholia.

I. In the reduction of the planets to the ecliptic, or from the ecliptic to the orbit, which Ptolemy neglected, which Copernicus, Lansbergen and Longomontanus applied in the Moon, and which Tycho aimed at in the other planets nearer the Sun, Kepler found two things wanting, in the *Commentaries on Mars*, chapter 9. To understand them, let the scheme set out at chapter 8, number 13 be repeated here. In it A is the place of the node among the fixed stars and AB an arc of the ecliptic, to which let arc AC be equal; let the planet I be seen from the Earth H under the point C of the fixed stars; and let the perpendicular arc CE be drawn from C to the ecliptic, so that the planet's place reduced to the ecliptic is at E.

Diagram. A geometric figure set into the left column of the first scholium, the text running down its left side. From A at the top, a curved arc runs right to C, and further lines descend from A, one of them a curve through E to B, and long straight lines converge downward on a rayed disc at the foot. Lines from C run down and left to E and through F and D towards B and I; arcs cross the figure at B and I and again at H, extending to the right. The planet I is marked by a star-shaped point, H by a dark round point, and G stands beside the rayed disc.

- A (top left, apex): place of the node among the fixed stars
- C (right, end of the upper arc): point of the fixed stars under which the planet is seen
- F (centre, below the upper arc): place where the line from the Sun meets arc CE

- E (left of centre): the planet's place reduced to the ecliptic
- D (centre, just below and right of F): point where the arc from A through F cuts CB
- B (below E): place opposite the Sun in the case the text sets out
- I (below D, at the star-shaped point): the planet
- H (lower, beside the dark round point): the Earth
- G (foot, beside the rayed disc): the Sun

With this set down, some have thought that the planet is not precisely opposite the Sun G except when arc AC, the visible distance of the planet from the node, equals arc AB; or only when the place opposite the Sun is B, namely the point to which arc CB is drawn from C, cutting off arc AB. At that time the planet is indeed seen exactly opposite the Sun, but in reality it is not. For let the straight line GIF be drawn from the Sun G through the planet I as far as arc CE, and let F be the place where it falls; through F let an arc be drawn from A as far as CB, and let CB be cut by it at D. For it is evident that the plane of the eccentric of the planet, seen from H under C, is aligned not under AC but under AF, since these three points G, I, A lie in the same plane; and that when the true opposition with the Sun occurs, the planet will truly be under F, but when the place opposite the Sun is B, the planet will be under D, since in both cases it appears under C. Now AD is shorter than the legs of the isosceles BAC. Therefore B, the place opposite the Sun, is farther removed from A than the place D, under which the planet stands at that moment which some used for obtaining the opposition with the Sun, in order to find the planet stripped of the second inequality; although at that time it was in fact more involved in the second inequality. Since, then, the orbit of the planet free of the second inequality lies under AD, the visible latitude is EAC, and the true inclination of the line GF to the ecliptic is FGE, it comes about that at the opposition the second inequality of latitude is almost at its greatest, and its measure is the angle HIG. And just as the whole latitude EC makes AC longer than arc AE by an excess as great as arc EB, so part of this visible latitude makes that same AC longer than AF, and indeed sometimes by 9′, by which one can go wrong in examining acronychal oppositions. The true method of reducing a planet from the ecliptic to the orbit, therefore, is this: with E known, the planet's place in the ecliptic from observation, let the inclination FGE be sought by the problems given in chapter 8; and in the triangle AFE, right-angled at E, with the distance AE from the node known, and the arc FE, which is the measure of the inclination EGF, let arc AF be sought. Its difference from arc AE will be the difference between the planet's ecliptic place E and its place F in the orbit.

II. If anyone wishes to find by a geometrical route the difference of the equations arising from the mere inclination of the plane of Venus and of Mercury to the ecliptic, let him consult the Copernican problem and its fifth step, which we have already given on another occasion at chapter 8, number 20, which is problem 15; and the one arising from reflexion we taught in the same place, at problem 17. There are, moreover, some notable propositions

in Regiomontanus, Book 13 of the *Epitome of the Almagest*, concerning the reduction and the curtation together: namely prop. 13, which shows that in the Ptolemaic hypothesis of Venus and Mercury the diversity of the longitude reckoned in the ecliptic from the true longitude in the orbit becomes greatest near the point of contact of the epicycle, though not always at the very point of contact; and prop. 14, which shows that in Mercury it occurs when Mercury is at the contact itself; and prop. 15, where he shows that in Venus it mostly occurs outside the contact.

III. [Margin: Reduction of aspects.] What might be wanted here on the reduction of aspects I have preferred to move to the following section, where aspects will be discussed, that is, to section 5, chapter 8, from number 5.

CAPVT X.

Chapter 10.

De Tabulis Latitudinis, earumq. vſu.

On the tables of latitude and their use.

I. By what technique the tables of latitudes are constructed and to be applied is made clear enough by Ptolemy, Book 13, chapters 4, 5 and 6; Regiomontanus in the *Epitome of the Almagest*, prop. 21; Copernicus, Book 6, chapter 9; Wurstisen on Peuerbach, p. 385; Reinhold in the Prutenic Tables, precepts 71 and 72; Magini in the *Secunda Mobilia*, canons 49 and 50, and in the *Supplement to the Ephemerides*, canon 11, and in the *Theoricae*, chapters 5 and 10; Longomontanus, Book 2 of the *Sphaerics*, chapters 15 and 25; Kepler in the Rudolphine Tables, chapter 21, with precepts 108 and 109 added; Lansbergen, precepts 16 and 17; Boulliau, Book 12, precepts 8 and 9; Renieri in the *Tabulae Mediceae*. But the manner of using them belongs to the precepts of the tables, to be given in the practical part of this first volume. The technique of composing them reduces to proportional minutes, the greatest latitudes from apogee to perigee being presupposed; it is like the one by which the tables of equations are made up through proportional minutes, and there is no need for us to dwell on these things any further.

SECTIO QVINTA

Fifth Section

DE PASSIONIBVS QVINQVE PLANETARVM MINORVM SECVNDVM LONGITVDINEM, NON EXCLVSA LATITVDINE.

On the affections of the five minor planets in longitude, latitude not excluded.

PROHEMIVM.

Preface.

[Margin: Enumeration of the passions in longitude.] Now that the affections or affections and accidents that belong to these five planets from their motion in latitude have been dealt with, it follows that we should treat those that befall them from the motion of longitude. Either the planets are compared with themselves, in which way they are called slow and swift, and increased or diminished in number; or direct, as far as speed is concerned, or stationary or retrograde, as far as slowness is concerned. Or they are compared with the quarters of the world, in which way they are called eastern or western, morning or evening. Or they are compared with the light of the Sun and with its motion together, on which account they are called increased or diminished in light, or hypaugi, or rising or setting heliacally. Or, finally, any two planets are compared with each other either as to position, from which come the various aspects, chiefly the great or greatest conjunctions and the revolutions of conjunctions, or as to the harmonic configuration arising from the speed and slowness of motion. We shall have to discuss these in roughly this order. [Margin: Why the digressions of ♀ and ☿ were treated in Section 3.] The greatest digressions of Venus and Mercury from the Sun, which certainly belong to motion in longitude, were more conveniently spoken of in Section 3, on the necessary occasion of investigating their own proper motion. And I said in the title of the section "latitude not excluded" because some variety does nevertheless befall these very affections of longitude from the varying latitude of the planets.

CAPVT I.

Chapter 1.

Planetæ Tardi aut Veloces; Aucti aut Minuti numero quando dicantur?

When are planets called slow or swift, increased or diminished in number?

I. These affections are mentioned by Peuerbach in the *Theoricae*, and there by Reinhold, p. 197, Wurstisen, pp. 251 and 255, and Nonius, p. 278; by Fernel, Book 2 of the *Cosmotheoria*, chapter 9; by Magini, Canon 97, and Book 2 of the *Theoricae*, chapter 13; by Langius in the *Theoricae*, chapter 14; and by Kepler in the *Epitome of Copernican Astronomy*, p. 822. From these:

[Margin: The swift planet.] A planet is called swift, or increased in course, when the line of the planet's true motion moves according to the succession of the signs more quickly than the line of mean motion; or when the apparent motion under the zodiac in the order of the signs is faster from day to day than the mean diurnal motion. This happens to the five minor planets in the upper half of the epicycle, the more so as the superior planets approach conjunction,

or the inferior planets the superior conjunction, or the apogee of the epicycle itself.

[Margin: Slow.] A planet is called slow, or diminished in course, when the line of true motion moves according to the series of the signs more slowly than the line of mean motion, or even moves against the series of the signs, or seems to stand still; or when the apparent daily motion is either nothing, or against the order of the signs, or less than the mean motion. This happens to these five planets in the lower part of the epicycle, especially near the perigee of the epicycle; whereas the Moon, on the contrary, is slower in the upper part of the epicycle and swifter in the lower part.

[Margin: Moderate.] A planet is called moderate in course when the lines of true and of mean motion proceed equally; or when the apparent diurnal motion is equal to the mean motion, which happens to the planets about their passage through the mean longitudes.

[Margin: Increased in number.] A planet is called increased in number, in Greek [Greek: προσθετικὸς (prosthetikos, "additive")] or [Greek: προσθέτης (prosthetēs, "one who adds")], or, as Fernel puts it, increasing the number, when the equation of the argument or of the epicycle is added to the mean motion. This happens to these planets in the first semicircle of the epicycle, that is, from the apogee of the epicycle in the order of the signs as far as the perigee of the epicycle. Or, as Kepler says, when both equations, of the eccentric and of the orb, or at least the greater of them, are additive, which can happen to these planets whether direct or retrograde.

[Margin: Diminished in number.] A planet is called diminished in number, in Greek [Greek: ἀφαιρετικός (aphairetikos, "subtractive")] or [Greek: ἀφαίρετης (aphairetēs, "one who subtracts")], or diminishing the number, when the equation of the argument, or epicyclic equation, is subtracted from the mean motion, which happens to these 5 planets in the latter semicircle of the epicycle. Or a planet is called diminished in number, as Kepler says, when both equations, of the eccentric and of the epicycle, or the greater of them, are subtractive.

[Margin: Corollary.] From this it is clear that a planet can be swift in course and yet diminished in number, as when it is in the fourth quadrant of the epicycle; and on the other hand slow in course and yet increased in number, as when it is towards the end of the second quadrant of the epicycle, whether it is stationary there or retrograde. Granted, in the first quadrant it is at once swift and increased in number, and in that part of the third quadrant which is nearer the perigee it is at once slow in course and diminished in number. It is further clear that the planets are swifter at the apogee of the epicycle than at the perigee, speaking even of the speed of retrogression, for they advance more there than they recede here.

II. [Margin: On tables of the diurnal motion of the planets.] Their apparent or true diurnal motion helps in distinguishing the speed of the planets, whether gathered from ephemerides or from the special tables which Giovanni Antonio

Magini published in Volume 2 of the *Secunda Mobilia*, from page 465 for the true diurnal motion of the Sun and Moon, and from p. 510 to 548 for the other planets. He teaches their construction in Volume 1, Canon 47, and their use in Canon 48, where he corrects Canon 36 of the Prutenic Tables, in which the daily motion near the stations in ♂ ♀ ☿ differs by no negligible difference from the motion deduced from the tables of mean motions and the equations. [Margin: Error of the Prutenic Tables in the daily motion.] In Canon 47 he gives a twofold way of constructing tables of diurnal motion, with examples, but that does not belong to this place, and it is enough to have pointed a finger to the source. From those tables, however, though they are derived from Copernican motions, we shall give a specimen of the diurnal motion of the planets at the principal places of the eccentric and of the epicycle, so that from it the evidence of speed and slowness and the more notable marks may be taken somewhat more accurately than from the small table which Origanus composed from them or from the Prutenic tables, in Volume 1 of the *Ephemerides*, part 2, chapter 2. In this table, then, the greater the direct motion, the greater the speed; but the greater the retrograde motion, the greater the slowness. The greatest speed is at the perigee of the eccentric and the apogee of the epicycle; the greatest slowness, at the perigee of the eccentric and of the epicycle together.

True diurnal motion [DIVRNVS MOTVS VERVS]

Planet	In the epicycle	At the apogee of the eccentric	At the mean longitude of the eccentric	At the perigee of the eccentric	
♄	Apogee	0° 7′ 7″	0° 7′ 35″	0° 8′ 9″	Swiftest
♄	Perigee	0° 4′ 45″	0° 4′ 58″	0° 5′ 13″	Slowest
♃	Apogee	0° 12′ 58″	0° 13′ 38″	0° 14′ 24″	Swiftest
♃	Perigee	0° 7′ 36″	0° 7′ 44″	0° 7′ 57″	Slowest
♂	Apogee	0° 38′ 21″	0° 42′ 3″	0° 47′ 2″	Swiftest
♂	Perigee	0° 24′ 11″	0° 13′ 26″	0° 17′ 32″	Slowest
♀	Apogee	1° 13′ 0″	1° 15′ 0″	1° 16′ 0″	Swiftest
♀	Perigee	0° 38′ 0″	0° 34′ 0″	0° 33′ 0″	Slowest
☿	Apogee	1° 43′ 0″	1° 52′ 0″	1° 54′ 0″	Swiftest
☿	Perigee	0° 35′ 0″	1° 2′ 0″	1° 2′ 0″	Slowest

CAPVT II.

Chapter 2.

De Planetis Directis, ac Retrogradis eorumque Stationibus.

On direct and retrograde planets and their stations.

I. Nothing in the whole planetary system has so strengthened and enlarged, for the five minor planets, the name of wanderers, which the Moon and the Sun share with them, as the heteroclite, so to speak, or at any rate anomalous, not so much variety as a kind of inconstancy and almost self-contradiction in their motion in longitude. For now they hurry in a direct course from west to east, yet striving against the diurnal motion of the primum mobile, as it is considered to be. Now, as if with a hesitant and doubtful step, they stand still and cling for a while to the same place; until, as if they had changed their minds and repented of their former course in the order of the signs, they seem to go back by their own motion from east to west and to retrace their steps against the order of the signs, as though returning to favour with the diurnal motion of the primum mobile and choosing a palinode or palinchorea. But not long after, checking their step again, they halt for a little, and soon, breaking their compact with the diurnal motion, they direct their course once more towards the eastern quarter. This truly wonderful exchange of alternations gave Copernicus his handle for freeing those heavenly lights from these motions, absurd as he at any rate judged them, and for transferring them, from the annual motion of the Earth, to a mere deception of our sight.

[Margin: Authors who treat of station, retrogradation, etc.] The chief authors, moreover, who discuss this subject are Apollonius of Perga; Pliny the Elder, Book 2, chapters 15 and 17; Vitruvius, Book 9, chapter 4; Ptolemy, Book 12 of the *Magna Constructio* or *Almagest*; al-Battānī, chapter 46; al-Farghānī, Difference 15; Jābir ibn Aflaḥ, Book 8 of the *Astronomy*; Proclus in the *Hypotyposes*, chapter 4; Peuerbach in the *Theoricae*, and there Osualdus, Reinhold from p. 135, Wurstisen from p. 231, and Pedro Nunes, annotations 1, 2 and 3 on Peuerbach's theory of the affections of the planets; Regiomontanus in the *Epitome of the Almagest*, Book 12, from prop. 4 to 12; Piccolomini in the *Theoricae*, Book 2, chapter 4; Maurolico, *Cosmographia*, Dialogue 3, p. 95; Barozzi, *Cosmographia*, Book 3, pp. 267 and 274; Magini, Book 2 of the *Theoricae*, chapters 11 and 12, and in the *Secunda Mobilia*, canons 94, 95, 96; Langius in the *Theoricae*, chapter 14; and Longomontanus, Book 1 of the *Theorica*, chapter 1. Of those who ascribe these phenomena to the motion of the Earth: Copernicus, Book 5, chapters 3, 35 and 36; Reinhold in the Prutenic Tables, precepts 68, 69 and 70; Rheticus in his *Narratio prima*, towards the end; Lansbergen, precepts 39 and 40, and in the *Theoricae*, chapter 21; Galileo, Dialogue 3 on the system of the world, p. 253 in my copy; Kepler in the *Mysterium Cosmographicum*, chapter 1, in the *Epitome of Copernican Astronomy*, Book 6, from p. 735 to 750 and 767, and in the Rudolphine Tables, precepts 106, 107; Hérigone, Volume 5 of the *Cursus Mathematicus*, Book 2, *Theorica*, chapter 7, from phenomenon 10 to 17; and Boulliau in the *Philolaus*. From these we shall derive the following theorems and problems in order and distinctly.

Nomina & Definitiones ad Directionem, Stationem, ac Retrogreſſum ſpectantia.

Names and definitions concerning direction, station and retrogression.

II. [Margin: What is a direct planet?] A planet is called direct, or progressing, and in Greek [Greek: ὑπολειπτικὸς (hypoleiptikos, "following behind")], that is, following or proceeding in the order of the signs, when the line of the planet's true or apparent motion (the one, namely, drawn from the centre of the Earth through its body) proceeds in the zodiac according to the succession of the signs; or succeeding to one place of the zodiac after another, the preceding parts being left behind. [Margin: What is direction?] Direction or progression is called in Greek [Greek: ὑπόλειψις (hypoleipsis, "a falling behind")] or motion [Greek: εἰς τὰ ἑπόμενα (eis ta hepomena, "towards what follows")], namely a continuous motion in the consequentia of the signs. This happens to these five planets in the upper part of the epicycle, or in the whole upper semicircle of the epicycle, and besides in that part of the lower semicircle in which the motion in the epicycle seconds the motion of the centre of the epicycle; and that centre perpetually progresses according to the succession of the signs, if it is not burdensome meanwhile to use the Ptolemaic hypothesis for the sake of learning.

[Margin: What is a retrograde planet?] A planet is called retrograde or retroceding, and in Greek [Greek: προηγούμενος (proēgoumenos, "going before")] or [Greek: προηγητικὸς (proēgētikos, "preceding")], that is, receding or preceding against the order of the signs, when the line of the planet's true motion goes back in the zodiac against the order of the signs. [Margin: What is retrogradation?] Retrogradation, or retrocession or regress, is called [Greek: προηγήσις (proēgēsis, "a going before")]. This happens to these five planets in that part of the lower semicircle of the epicycle in which the line of the planet's true motion, or the one drawn from the centre of the Earth through the centre of the planet's body, goes back day by day westward, or against the order of the signs, even if the line of the true motion of the centre of the epicycle proceeds eastward.

[Margin: What is a stationary planet?] A planet is called stationary or stational, and in Greek [Greek: στηρίζων (stērizōn, "standing fixed")], that is, standing or halting, when it seems to stand for some time in the same place of the zodiac, neither proceeding nor retroceding. [Margin: What is a station?] And that station, or in Cicero's word *institio*, is called in Greek [Greek: στήριγμος (stērigmos, "a fixing, a station")]. This happens to the 5 minor planets when the line of the planet's true motion shows the planet on the following day in the same place of the zodiac where it was on the preceding day. This occurs because the progress made by the force of the centre of the epicycle is equally retarded by the beginning and the end of the regress. This happens, however, twice in one revolution of the epicycle to each of these planets, namely at

the boundary of both motions, direct and retrograde; and these boundaries lie below the points of contact of the epicycle, as will be shown later.

[Margin: First station.] The *first station*, in Greek [Greek: στήριγμος πρώτος (stērigmos prōtos, "first station")], is the one that occurs in the left, or eastern, or earlier semicircle of the epicycle, reckoned in the order of the signs from the apogee of the epicycle; or the one that occurs in the passage from direct to retrograde motion; or the planet's stay between the last moment of progress and the first moment of regress. [Margin: Morning and evening station.] In the three superior planets this station is called *morning*, because it follows the planet's morning rising; but in the inferior planets *evening*, because they have the line of mean motion in common with the Sun, and it therefore occurs when they are evening planets.

[Margin: Second station.] The *second station*, in Greek [Greek: στήριγμος δεύτερος (stērigmos deuteros, "second station")], is the name given to the one that occurs in the right, or later, or western semicircle of the epicycle; or the one that occurs in the passage from retrograde to direct motion; or that stay of the planet which falls between the last moment of retrogression and the first moment of progress. In the three superior planets it is called evening, but in the inferior planets morning. So the first station of the superior planets occurs when, after conjunction with the Sun, they are making for opposition, and the second when, after opposition, they are making for conjunction. In the inferior planets, however, the first occurs when, from the upper or apogeal conjunction, they are making for the lower or perigeal one.

[Margin: Points of the stations.] The *points of the stations* are two arcs, both in the epicycle and in the zodiac, in which the planets seem to linger, and which therefore appear to us as if they were points, because the planet seems to remain motionless, with the centre of its body, in the same place of the zodiac and of the epicycle. For an equal position of the epicycle's centre in the eccentric, that is, at an equal distance of the epicycle's centre from the apogee of the eccentric, these points stand equally far on either side from the apogee and the perigee of the epicycle.

[Margin: Arc of direction.] The *arc of direction*, Ptolemy's [Greek: περιφέρεια ὑπολειπτικὴ (periphereia hypoleiptikē, "arc following behind")], is the arc of the epicycle reckoned from the point of the second station through the apogee to the point of the first station.

[Margin: Arc of retrogradation.] The *arc of regression*, Ptolemy's [Greek: περιφέρεια προηγητικὴ (periphereia proēgētikē, "preceding arc")], is the arc of the epicycle continued from the point of the first station, through the perigee, to the point of the second station. This arc is always smaller than the arc of direction, because both stations occur within and below the points of contact of the epicycle.

[Margin: Arcs of the 1st and 2nd station.] The *arc of the first station* is the arc of the epicycle reckoned from the apogee of the epicycle in the order of the signs to the point of the first station; the *arc of the second station* is the one

counted from the apogee, through the 1st station and the perigee of the epicycle, to the point of the second station.

III. [Margin: Definitions in the hypothesis of the moved Earth.] The definitions already mentioned also hold in the hypothesis of the Earth moving with an annual motion, once mention is dropped of the epicycle in the superior planets, in place of which the great orb is to be substituted, and of the Ptolemaic eccentric in the inferior planets, in place of which the great orb is likewise to be substituted. So in this Copernican and Philolaic hypothesis a planet that is superior and slower than the Earth is [Margin: Direct.] *direct* when the visual lines, or lines of apparent motion, of the planet and the Earth on neighbouring days intersect one another between the Earth and the planet, or at the centre of the Earth, or behind the Earth in the quarter away from the planet; [Margin: Stationary, retrograde.] it is *stationary* when the two lines just mentioned are parallel; and it is *retrograde*, finally, when their intersection falls beyond the planet. The planets that are inferior, or interior to the annual orb, are *direct* when the lines already mentioned, which join to each other the termini of the diurnal motions of the planet and the Earth, bend in the order of the signs; and *retrograde* when those lines bend back against the order of the signs and intersect one another behind the Earth, in the quarter turned away from the Sun. The points of the stations are determined in this hypothesis from a distance and somewhat crudely, by drawing from the superior planet to the Earth's great orb, or from the Earth to the inferior or interior orbs of the planets, two straight lines tangent, in the former case to the annual orb, in the latter to the orb of Venus or of Mercury; for the points of the stations lie below the points of contact, though not far from them, except in Mars. [Margin: Arc of retrogradation.] The *arc of retrogradation*, finally, is either the arc of the ecliptic included between two neighbouring apparent stations of the planet; or the arc of the great orb included within the two pairs of parallels that determine the stations; or else the arc of the planet's eccentric which the planet runs through from one station to the next, determined by the two pairs of parallels, etc. So, more or less, Kepler in the *Epitome of Copernican Astronomy*, pp. 743 and 767. For the figure for this hypothesis see below, chapter 4.

Loca in Epicyclo, in quibus fiunt Stationes, seu Quantitas arcuum ad eas spectantium, cum numero & tempore tum earum, tum Directionum ac Retrogressuum.

The places in the epicycle at which the stations occur, or the size of the arcs relating to them, with the number and time both of the stations and of the directions and retrogressions.

IV. [Margin: Aspects with the Sun at which the stations occur.] First, if we want to designate the places of the stations from a distance, Saturn becomes stationary between the square and the trine aspect with the Sun, Jupiter roughly at the trine, and Mars beyond the trine, both before and behind; Venus after

five signs of the argument on one side and after six on the other; Mercury after four on one side and seven signs of the argument, or of the anomaly of the orb, on the other. But if we want to know these places more accurately, then either from the Ptolemaic canon, Book 12, chapter 8, or from the Prutenic tables, which in this matter Magini followed in Book 2 of the *Theoricae*, chapter 12, and Lansbergen in his tables, p. 158, we shall take the arcs of the first and second station. When the arc of the first station is subtracted from the arc of the second, the arc of retrogradation will remain; and this, subtracted from the whole circle, will leave the arc of direction. So, according to the authors mentioned, the size of these arcs is as written below.

Arcs of the stations, etc., from Ptolemy [ARCVS STATIONVM &c. EX PTOLEMÆO.]

Planet	In the eccentric	Arc of station 1	Arc of station 2	Arc of retrogradation	Arc of direction
♄	Apogee	112° 45′	247° 15′	134° 30′	225° 30′
♄	Mean long.	114° 1′	245° 59′	131° 58′	228° 2′
♄	Perigee	115° 29′	244° 31′	129° 2′	230° 58′
♃	Apogee	124° 5′	235° 55′	111° 50′	248° 10′
♃	Mean long.	125° 32′	234° 28′	108° 56′	251° 4′
♃	Perigee	127° 11′	232° 49′	105° 38′	254° 22′
♂	Apogee	157° 28′	202° 32′	45° 4′	314° 56′
♂	Mean long.	162° 18′	197° 42′	35° 24′	324° 36′
♂	Perigee	169° 9′	190° 51′	21° 42′	338° 18′
♀	Apogee	165° 51′	194° 9′	28° 18′	131° 42′ [recte: 331° 42′, per the book's errata; corrected by hand to 331 in this copy]
♀	Mean long.	167° 7′	192° 53′	25° 45′	134° 15′ [recte: 334° 15′, per the book's errata; corrected by hand to 334 in this copy]
♀	Perigee	168° 21′	191° 39′	23° 18′	336° 42′
☿	Apogee	147° 14′	212° 46′	65° 32′	294° 28′
☿	Mean long.	144° 40′	215° 20′	70° 40′	289° 20′
☿	Perigee	144° 40′	215° 20′	70° 40′	289° 20′

Arcs of the stations, etc., from the Prutenic tables, Magini and Lansbergen [ARCVS STATIONVM &c. Ex Prutenicis, Magino & Lansbergio.]

Planet	In the eccentric	Arc of station 1	Arc of station 2	Arc of retrogradation	Arc of direction
♄	Apogee	112° 38′	247° 22′	134° 44′	225° 16′
♄	Mean long.	113° 58′	246° 2′	132° 4′	227° 56′
♄	Perigee	115° 21′	244° 39′	129° 18′	230° 42′

Planet	In the eccentric	Arc of station 1	Arc of station 2	Arc of retrogradation	Arc of direction
☿ [recte: ♃, per the book's errata; corrected by hand to ♃ in this copy]	Apogee	124° 8′	235° 52′	111° 44′	248° 16′
♃	Mean long.	125° 40′	234° 20′	108° 40′	251° 20′
♃	Perigee	127° 19′	232° 41′	105° 22′	254° 36′
♂	Apogee	157° 33′	202° 27′	44° 54′	315° 6′
♂	Mean long.	162° 51′	197° 9′	34° 18′	325° 42′
♂	Perigee	168° 56′	191° 4′	22° 8′	337° 52′
♀	Apogee	166° 1′	193° 59′	27° 58′	332° 2′
♀	Mean long.	167° 7′	192° 53′	25° 46′	334° 14′
♀	Perigee	168° 15′	191° 45′	23° 30′	336° 30′
☿	Apogee	146° 50′	213° 10′	66° 20′	293° 40′
☿	Mean long.	143° 55′	216° 5′	72° 10′	287° 50′
☿	Perigee	146° 0′	214° 0′	68° 0′	292° 0′

On the boundaries of the stations according to Kepler, I shall speak more opportunely at the end of chapter 4.

V. The time, moreover, in which these planets linger in the stations, or run through the arcs already mentioned, varies and differs not only as between the planets but also for the same planet when it is at one time swifter and at another slower. Nevertheless, for anyone who wants to know it roughly, we say that the mean time of a station is about 8 days for Saturn, 4 for Jupiter, 2 for Mars, 1 ½ for Venus and 0 ½ for Mercury. Again, the mean time of the direct course is about 244 days for Saturn, 284 for Jupiter, 705 for Mars, 542 for Venus and 93 for Mercury. The mean time of retrogradation, likewise, is 136 or 140 days for Saturn, 120 or 122 for Jupiter, 73 for Mars, 42 for Venus and 22 for Mercury. Or, if we follow the opinion of Ptolemy in Book 12, from chapter 2 to 6, the mean times of the regressions stand as follows, as also reported by Jābir ibn Aflaḥ in Book 8.

Mean time of the regressions, from Ptolemy [Tempus Mediocre Regressuum ex Ptolemæo]

Planet	At the apogee of the eccentric	At the perigee of the eccentric
♄	$140^{d}\ 16^{h}$	$136^{d}\ 0^{h}$
♃	$122^{d}\ 12^{h}$	$118^{d}\ 0^{h}$
♂	$80^{d}\ 0^{h}$	$64^{d}\ 12^{h}$
♀	$43^{d}\ 0^{h}$	$40^{d}\ 16^{h}$
☿	$22^{d}\ 12^{h}$	$21^{d}\ 0^{h}$

But if you want the true time for each single year, consult the ephemerides, especially if they note these times separately, as Origanus did at the end of each year. The inequality in these is, however, much greater in Mars, Venus and

Mercury. From these and other phenomena, before we investigate their causes from the theory of the Ptolemaic or Copernican hypothesis, we may now take a foretaste of the following propositions, accepted by astronomers as almost axioms.

Proposition 1. *The higher any planet is, or the slower in its own proper motion, the longer it lingers, other things being equal, in either of its stations.*

[Margin: Frequency of stations, progressions and regressions.] Proposition 2. *The higher any of the three superior planets is, the more stations it has; among the inferior planets, however, the nearer it is to the Sun, the more stations it has.* Therefore Saturn makes more stations than Jupiter, and Jupiter more than Mars, just as Mercury makes more than Venus, because the Sun by its motion, or the Earth, overtakes the slower Saturn sooner and therefore more often, and overtakes Jupiter sooner than Mars. Conversely, Mercury completes its period around the Sun in a shorter time, and overtakes the Sun's place or the Earth sooner than Venus does. In this respect, then, the superior and inferior planets behave in opposite ways: the slowest of the superior planets, but the swiftest of the inferior, make the more frequent stations. The same is to be understood of the directions and retrogradations, for they recur as often as both stations do.

[Margin: Duration of direction and of retrogression.] Proposition 3. *The higher any one of the superior planets is, whether than another planet or than itself, or the farther any of the inferior planets is from the Sun, the longer it remains retrograde, but the shorter direct.*

[Margin: Ratio of the intervals and of the time.] Proposition 4. *There is not, however, the same ratio between the durations of the progressions or regressions as between the intervals of the planets.* This is clear to anyone who compares the times of these arcs, given a little earlier, with the distances of the planets from the Earth, set out at the end of the chapters of Sections 2 and 3. For Saturn is nearly twice as high as Jupiter, and Jupiter about three times higher than Mars; and yet the time of Saturn's retrogression is only slightly longer than Jupiter's, and the time of Jupiter's retrogression is no longer than about one and a third times that of Mars.

[Margin: Arc of the ecliptic in the regress.] Proposition 5. *The higher any one of the superior planets is, the smaller the arc of the ecliptic it runs through in its regress.* This proposition seems contrary to the third; but there the discussion concerns the time in which the planet frees itself from the regress.

Proposition 6. *The higher any one of the superior planets is, whether than another or than itself, the greater the arc of retrogradation and the smaller the arc of direction it runs through.*

Proposition 7. *In Saturn, Jupiter, Mars and Venus, the closer the centre of the epicycle is to the perigee of the equant, or to the centre of the world, the nearer the points of the stations are to the perigee of the epicycle.*

Some of the propositions already mentioned are held by Nonius, Magini, Kepler, Hérigone and others listed in chapter 1; on these, however, read the scholia of the following chapter 3.

CAPVT III.

Chapter 3.

Theorica & Causæ Stationum, Directionum, & Retrogradationum ex sententia Antiquorum ac Hypothesi præsertim Ptolemaica.

The theory and causes of the stations, directions and retrogradations, according to the opinion of the ancients and the hypothesis, especially the Ptolemaic.

I. [Margin: Opinion of the Latins on the cause of the stations and retrogradations.] Some of the Latins, not very much skilled in mathematical matters, conjectured that the cause of slowing the motion of these planets, and of quickening, halting or drawing them back, lay in the aspects of the Sun, and judged that the square of the Sun harms Saturn, the trine Jupiter, and Mars something more. *They do not know*, says Kepler in the *Epitome of Copernican Astronomy*, p. 744, *that these aspects, insofar as they can indeed become effective on these lower things, are terrestrial rather than celestial beings: for the aspects are not in the planets themselves, but only here on the Earth. Nor did they consider that aspects are formally beings of reason*, etc.

[Margin: Opinion of Pliny the Elder.] On these matters, however, Pliny the Elder spoke crudely or obscurely enough in Book 2, chapter 15, saying of the three superior planets: *In the triangle, at 120°, they make morning stations, which are also called first; and likewise at 120° from the other side, on its approach, evening stations, which they also call second.* And in chapter 16: *There is no doubt that at their morning risings the number also increases, and that from the first stations the three superior planets decrease down to the second stations. This being so, it will be evident that from the morning rising they climb in latitude, since on that departure the motions first begin to be added more sparingly; and that at the first stations they also go up in altitude, since then the numbers first begin to be subtracted and the stars to go back. The reason for this must be given separately. Struck, in the part we have named and in the triangle, by the ray of the Sun, they are prevented from keeping a straight course, and they are lifted on high by a fiery force. This cannot be perceived at once by our sight, and so they are thought to stand still: from this the station also took its name. The violence of that same ray then advances, and the vapour, beaten back, forces them to go back. This happens much more at their evening rising, with the whole Sun opposite, when they are driven out to the highest apsides and are least seen, because they are at their farthest height and are carried with the least motion, all the smaller when this happens in the highest signs of the apsides. From the morning rising the latitude is descended, with the motion now diminishing itself more sparingly, yet not increasing before the second stations, when the altitude too is descended, as the ray comes over from the other side and by the same force presses back down to the Earth what it raised into the heaven from the former triangle; so much does it matter whether the rays come from below or come over from above. And the same things*

happen much more at the evening setting. This is the account of the superior stars, more difficult than that of the others, and given by no one before us. How much rind there is in these words, and how little kernel? In summary, however, Pliny the Elder judges that from the triangle, or first trine aspect, to the second, the ray of the Sun comes upon the three superior planets from above, and by the force of its fiery vapour draws them back upward as if with a hook; but that in the remaining portion of the orb the ray comes from below and urges the planets on. Not much better is what he adds in chapter 17, as when he says that Mercury stands nearly motionless for four days, and refers its stations or regressions and those of Venus to the Sun. *For those that strain towards the vapour of the Sun also descend with difficulty.* And shortly after: *In the star of Mars, whose course is most unobservable, it never makes a station with the star of Jupiter in the triangle, and very rarely when it is 60° apart, a number that produces the six-angled forms of the world.*

II. Almost the same opinion was held, before Pliny the Elder, by Vitruvius, in Book 9, chapter 4, from whose very many words we have selected these: *The stars of Mercury and Venus, crowning the Sun itself as a centre with their paths around the rays of the Sun, make their regressions backward and their retardations, and by reason of that circling they also linger with stations in the spaces of the signs.* And shortly after: *The star of Mars, in whichever signs it runs through more swiftly when it has made a station, fills out the reckoning of the number of its days.* And, after mentioning the revolution of Jupiter and Saturn: *These, however, which complete their circlings above the path of the Sun, especially when they are in the trine that it has entered, then do not advance, but linger, making regressions, until the same Sun has made its passage from that trine into another sign.* [Margin: A trifling opinion on the cause of the stations.] *Some hold that this comes about because, they say, when the Sun is farther off at a certain distance, the stars wandering through those parts on paths that are not lit are impeded by obscured lingerings. That does not seem so to us. For the splendour of the Sun is visible and open, without any obscurations, throughout the whole world, as it appears even to us when these stars make their regressions and lingerings. So if across such great intervals our sight can perceive this, why do we judge that obscurities can be cast upon the divinities and splendours of the stars?* After this he at once adds his own opinion in these words: [Margin: Opinion of Vitruvius on their cause.] *So this account will rather stand firm for us: that just as heat calls forth all things and draws them to itself (as we also see fruits rising from the ground to a height through warmth, and no less the vapours of water being raised from springs to the clouds through arcs), in the same way the vehement force of the Sun, its rays stretched out in the form of a triangle, draws to itself the stars that follow it, and, as though reining in and holding back those running ahead, does not let them advance but compels them to go back towards itself and to be in the sign of another triangle. Perhaps it will be asked why the Sun, in the fifth sign from itself rather than the second or third, which are nearer, produces retentions in these heats. I shall explain how this seems to happen. Its rays in the world extend in lines, as in the form of a triangle with equal sides, and that is neither more nor less than to the fifth sign from*

it. So if the rays, poured through the whole world, wandered in circlings, and were not extended and drawn out into lines in the form of a triangle, they would scorch nearer things, etc. For the rest, which is certainly obscure amid such frequent mention of rays, see Vitruvius, reader, if you have the time and the inclination.

III. [Margin: Opinion of Apollonius of Perga and of Ptolemy on the cause of the stations and regressions.] But Apollonius of Perga, and Ptolemy following him in Book 12, chapter 1 and those after it, and the whole school of astronomers down to Copernicus, attributed the cause of the stations and retrocessions apparent to us to the motion of the centre of the epicycle and of the planet in the epicycle, both made in consequentia of its own orb, and to a certain commensuration of the epicycle with the deferent. For Apollonius showed that a regress cannot befall any star unless its epicycle is so arranged that its semidiameter has to the straight line drawn from the centre of the world to the perigee of the epicycle a greater ratio than the motion of the centre of the epicycle has to the planet's motion in the epicycle; and that if it has this ratio, the point of the station is bounded by the line which, drawn from the centre of the world, cuts the epicycle so that half of the portion enclosed within the epicycle keeps to the outer portion the ratio that the motion of the centre of the epicycle has to the motion of the planet in the epicycle. And although Apollonius taught this of a concentric deferent, Ptolemy nevertheless adapted it to the eccentric.

Theorem 1. *If the ratio of the semidiameter of the epicycle to the distance of the perigee of the epicycle from the centre of the Earth is not greater than the ratio of the speed of the motion of the centre of the epicycle to the speed of the motion of the planet in the epicycle, the star cannot appear retrograde to us. But if it is greater, then, whether the epicycle moves in a concentric or in an eccentric, and even if both motions, of the epicycle and of the deferent, are according to the succession of the signs, an apparent station and retrogradation about the perigee of the epicycle will be possible, indeed necessary.*

IV. [Margin: Description of the figure for this and certain other propositions.] This theorem is demonstrated, after Apollonius, by Ptolemy in Book 12, chapter 1; Jābir ibn Aflaḥ, Book 8 of the *Astronomy*; Regiomontanus in the *Epitome of the Almagest*, Book 12, propositions 1, 2 and 4 taken together; Nonius, annotation 1 on the Peuerbachian affections of the planets; and Copernicus, Book 5, chapter 35. For this purpose, in the following diagram let E be the centre of the Earth, and ED the distance of the centre D of the epicycle AC from the centre of the world; and, ED being produced to A, let A be the true apogee of the epicycle and C the perigee. With tangents EK and EL drawn from E to the periphery of the epicycle, let the eastern and left contact be at the point L, and the right and western at the point K.

Then, from E, let two lines EGB and EMH be drawn cutting the epicycle, in any way at all, but so that the arcs intercepted by them and the perigee, namely CG and CM, are equal; and let the lower points of intersection be joined to

the centre of the epicycle by the straight lines DG and DM. We shall explain the remaining lines and letters below.

[Margin: 1st part of the theorem.] With these things set down, I say *first* that if the ratio of the semidiameter DC to the distance CE is not greater than the ratio of the speed of the centre D to the speed of the planet in the epicycle, it cannot come about that the planet appears retrograde. For if it can, it can certainly do so most of all near the perigee C, where the motion of the planet in the epicycle, or of the argument, most diminishes the motion in longitude of the centre D. But it cannot, however close the planet G or M may be to the perigee C. For let the arc GC be as small as possible. Because the base of the triangle DGE is divided into two portions, EC and DC, and one of them, DC, is not smaller than the side adjoining it, DG, but equal to it, since each is a semidiameter of the same epicycle, the ratio of the line DC to CE will be greater than that of the angle GED to the angle EDG (by the demonstration of Apollonius, of Ptolemy in Book 12, chapter 1, and of Regiomontanus in Book 12 of the *Epitome*, proposition 3; we assume this demonstration for the moment, and shall bring it in below, so that the order of the present one is not disturbed). Therefore the ratio of the angle DEG to the angle EDG will be smaller than that of the line DC to CE, by Euclid, Book 5 of the *Elements*. But the ratio of DC to CE was laid down as not greater than the ratio of the speed of the centre D to the speed of the planet G in the epicycle. Much smaller, therefore, will be the ratio of the angle DEG to the angle EDG than the ratio of the speed of the centre D to the speed of the planet G. Now the speed of the planet G is determined by the angle CDG; therefore the angle of the speed of the epicycle is greater than the angle CEG. Now let the angle that must be greater than DEG be, I say, DEL. So if D, the centre of the epicycle, were at rest while the planet describes the arc GC, which subtends the visual angle GEC, the planet would seem to go back against the succession of the signs. But because in the same time the centre D has described the arc DL, which subtends the greater angle DEL, in consequentia of the signs, the planet will therefore appear to move directly and in the order of the signs, by a motion as great as the difference, the angle LEG, by which the angle LED exceeds the angle GED. Given, then, what was laid down before, it is not possible for the planet to appear retrograde, however small the arc GC may be. And the same will be shown in like manner by taking the arc CM on the other side. For while, because of the motion of the planet M in the epicycle, the arc CM will seem, under the angle CEM, to have been passed through against the series of the signs, yet in the same time the centre D, from the outer point K, has described the arc DK and the greater angle DEK. And so, since the motion of the centre D in the succession of the signs overcomes the motion of the planet M against the succession, the planet will appear to have advanced directly.

Diagram. Set into the left column, with the text running down its left side. A circle, the epicycle, with a long narrow fan of straight lines running from the single point E at the foot up to the circle and through it. A vertical line runs

from E through C and the centre D to the top of the circle. A nearly horizontal line through the centre joins L at the left to K at the right, the two outermost lines of the fan meeting the circle at those points. Shorter lines join the centre to G and M and to points on the lower arc between them.

- R and A (top of the circle, set side by side and separated by a point): R.A
- F (upper left, on the circle)
- B (upper left, below F, on the circle)
- H (upper right, on the circle)
- S and D (at the centre, S left of the vertical line, D right of it)
- L (far left, at the horizontal line's end)
- P (inside the circle, on the horizontal line, right of L)
- Q (inside the circle, on the horizontal line, immediately right of P)
- O (inside the circle, on the horizontal line, right of centre)
- K (far right, at the horizontal line's end)
- G (lower left, on the circle)
- V (bottom of the circle, right of G)
- T and I (bottom of the circle, right of V, set close together)
- C (bottom of the circle, on the vertical line)
- M (lower right, on the circle)
- E (foot of the figure, where all the lines meet)

[Margin: 2nd part of the theorem.] *Second*, I say that if the ratio of the line DC to CE is greater than the ratio of the speed of the centre D to the speed of the planet in the epicycle, it must be that the planet sometimes appears retrograde. This can be gathered both from the converse method of the preceding demonstration and from what is to be said below. For through the planet, as it approaches the perigee of the epicycle and has passed the point of contact, say through the planet G, a secant EGB can be drawn, such that, with its portion BG inside the epicycle bisected at P, the ratio of the half PG to the outer portion GE is as the ratio of the speed of the centre D to the speed of the planet G in the epicycle; and say the same of the half OM to ME. From this it follows that the planet at G or M is stationary, and within these points, that is, in the arc GCM, retrograde, as will be demonstrated below. For in receding from the line EA, the lines of the secants enclosed within the epicycle gradually diminish, and their outer portion increases. Now what was assumed earlier, in point 1 above, in the parenthesis, must be shown.

Theorem 2. *If the base of a rectilinear triangle is cut into two portions, one of which is not smaller than the side adjoining it, the ratio of that portion to the rest of the base will be greater than the ratio of the angles at the base, with their order interchanged.*

V. [Margin: Lemma of Apollonius.] Apollonius used this as a lemma, as Ptolemy attests in Book 12, chapter 1; Regiomontanus uses the same in Book 12 of the *Epitome*, proposition 3, and Copernicus in Book 5, chapter 35. For this, in the following figure let there be a triangle DAB, whose base AB is cut at I so that one portion, say BI, is not smaller than the side BD adjoining it; for the

ratio of the line BI to IA will be greater than that of the angle DAB to the angle DBA. First let BI be equal to BD; and let a line BG be drawn from B parallel to the dividing line DI, as far as the point E, at which the other side AD, when produced, meets it. Likewise, let a line DG be drawn from D parallel to the line AB. For the two sides BI and GD of the parallelogram IBGD will be equal, just as DI and BG are. Therefore from D, with the semidiameter DG, let an arc of a semicircle be described; for it will pass through B, since DG is equal to BI and so to BD. Therefore the ratio of the triangle EGD to the triangle DGB is greater than the ratio of the sector CDG to the triangle DGB, since the sector CGD is part of the triangle EDG. But the ratio of the sector CDG to the triangle GDB is greater than its ratio to the sector GDB; much greater, therefore, is the ratio of the triangle EDG to the triangle GDB than that of the sector CDG to the sector GDB. Moreover, the ratio of the triangle EDG to the triangle GDB is as the ratio of the line EG to the line GB, since these are bases of triangles of the same height; and EG is to GB as ED to DA, and therefore as BI to IA. Therefore the ratio of the triangle EDG to the triangle GDB is as that of the line BI to IA; and likewise the ratio of the sector CDG to the sector GDB is as the ratio of the angle CDG to the angle GDB; and the two angles DAB and DBA are equal to these angles of the sectors. So the ratio of the sector CDG to the sector GDB is as that of the angle DAB to the angle DBA. But the ratio of the triangle EDG to the triangle GDB was shown to be greater than the ratio of the sector CDG to the sector GDB; therefore also the ratio of the line BI to IA is greater than the ratio of the angle DAB to the angle DBA, which was to be shown. But if BI is greater than DB, it will be possible, with straight lines drawn as before and the arc described, to argue as above.

Diagram. Set into the right column, with the text running down its left side. A tall narrow triangle with E at the apex and A at the foot, the vertical side EA passing through C and D. The side from E runs down through G to B. An arc about D runs from C through G and B towards I. D is joined to G and to B; G is joined to B; B runs to A through I; and D is joined to I.

- E (apex)
- C (on the vertical side, where the arc begins)
- G (on the slanting side, where the arc meets it)
- D (on the vertical side, centre of the arc)
- B (far left of the figure)
- I (on the line from B to A)
- A (foot)

Applying this lemma, therefore, to the preceding theorem: since the triangle EGD is cut at C so that the portion DC is not smaller than, but equal to, the side DG adjoining it, the ratio of the line DC to CE will be greater than the ratio of the angle GED to the angle GDE, which is what was assumed there.

Theorem 3. *Neither the Sun, even if it is supposed to move in a concentrepicycle, nor the Moon is capable of station and retrogradation.*

VI. It is common among astronomers, and above all to Regiomontanus, Book 12 of the *Epitome*, proposition 4, to Maurolico in dialogue 3 of the *Cosmographia*, p. 95, to Wurstisen on the *Theoricae*, p. 248, to Reinhold on the *Theoricae*, p. 143, and to Kepler in the *Epitome of Copernican Astronomy*, p. 749; and it is shown, *first*, of the Sun. [Margin: Why the Sun is incapable of retrogradation.] For if the Sun moves, as we have supposed elsewhere along with most authors, in a simple eccentric, it moves around the centre of the Earth always in consequentia of the signs, and in that hypothesis there is no meeting with another motion that at some time draws it back against the succession of the signs. But if it moves in a concentrepicycle, as Longomontanus would have it, then, to save the phenomena of the Sun, it must move in the epicycle against the order of the signs at the top, and the centre of the epicycle must move in the order of the signs around the centre of the world with equal speed of motion, so that it completes each day as many degrees on the periphery of the concentric as the Sun completes on the circumference of the epicycle; and the semidiameter of the Sun's epicycle must be not sensibly greater than the eccentricity of the Sun usually employed in the other hypothesis, that of the eccentric, as is clear from what was said in Book 3, chapter 19. Now the eccentricity, and hence the semidiameter of the epicycle, does not have to the radius a ratio greater than about 4000 to 100000, as is clear from what was said in Book 3, chapter 24, or as 4 to 100. Therefore in the figure of Theorem 1, DC to the outer portion CE is only as 4 to 96, or as 1 to 24, and so it is much smaller than the ratio of the speed of the centre D to the speed of this planet in the epicycle, since the latter is as 1 to 1, the ratio of equality. Therefore, by Theorem 1, the Sun, with the phenomena saved, cannot appear stationary or retrograde.

[Margin: Why the Moon is incapable of station and regress.] *Second*, the Moon is indeed carried by an eccentrepicycle, so that the epicycle is borne in the order of the signs in its lower part, but against the order of the signs in its upper; yet the speed of the centre of the epicycle is nearly equal to the speed of the Moon in the epicycle. For the mean daily motion of the centre of the Moon's epicycle is $13^\circ\ 10'\ 35''\ 1'''\ 18''''\ 16^{v}$, while the daily motion of the Moon in the epicycle, or of the simple anomaly, is $13^\circ\ 3'\ 53''\ 56'''\ 30''''$, as we said in Book 4, chapter 23. And the semidiameter of the Moon's epicycle is to the radius of the eccentric as 10576 to 100000, or very nearly so, according to what was said in Book 4, chapter 25. Therefore the ratio of DC, the semidiameter of the epicycle, to the outer portion CE of the radius (it is as 10½ to 89½) is much smaller than the ratio of the speed of the centre D to the speed of the Moon in the epicycle, which is almost the ratio of equality. Therefore not even the Moon is capable of station or retrogression.

Theorem 4. *If the luminaries were capable of station and retrogression, this would happen to them in the upper part of the epicycle, that is, at the apogee of the epicycle.*

VII. [Margin: Opinion of Maurolico on the stations of the luminaries.] Maurolico pointed this out to us in dialogue 3 of the *Cosmographia*, p. 93,

saying: *As to the luminaries, whose epicycles are carried in the contrary sense: the semidiameter of the epicycle is to be compared with the whole secant. So if a regress befell the luminaries, the points of the stations would fall in the upper arc of the epicycle; and in place of the outer portion of the line bounding the stationary point, the whole bounding line would have to be taken. We have carried out the demonstration of this matter elsewhere, although it is rather a matter of curiosity than of any usefulness, and was therefore omitted by Apollonius.* But this can easily be made clear, because station and retrogression occur where two motions come together, of which the one, towards one quarter, either equally blunts the other, towards the other quarter, or overcomes it. Since, therefore, the luminaries in the epicycle, with the other phenomena saved, cannot move towards the quarter contrary to the motion of the centre of the epicycle, that is, against the order of the signs, except in the upper part of the epicycle, only in that part would the points of the stations and the arc of regression lie, if these affections were not otherwise incompatible with them.

Theorem 5. *All five minor planets necessarily become stationary and retrograde at some time.*

VIII. This too is accepted by everyone. And even if the experience of observations were lacking, from which it is evident a posteriori that these planets have quite often appeared retrograde, and hence stationary, yet this same thing was demonstrated, from the hypothesis of motion in longitude that uses an eccentrepicycle, by Ptolemy, with Apollonius, in Book 12, chapter 1; Regiomontanus in the *Epitome*, Book 12, proposition 4; Jābir ibn Aflaḥ, Book 8; Pedro Nunes on Peuerbach's theory of the affections, annotation 1; and on the same affections Reinhold, p. 145. The sum of the demonstration is this: those planets necessarily become at some time stationary and retrograde whose semidiameter of the epicycle has to the distance of the perigee of the epicycle from the centre of the Earth a greater ratio than the ratio of the speed of the centre of the epicycle to the speed of the planet in the epicycle, as is clear from Theorem 1. But all five minor planets, at every distance of the centre of the epicycle from the Earth, are of this kind, as is clear from their motions and from the ratios of the orbs set out and proved in this Book 7, Sections 2 and 3. Therefore all are necessarily subject to stations and retrogressions. However, to make the subsumed proposition of the preceding syllogism more readily evident, Reinhold adds two small tables. In one of them, according to the Ptolemaic and almost the Alphonsine and Peuerbachian hypothesis, is contained on the one side the ratio of the semidiameter of the epicycle DC, as in the figure of number 4, to the distance CE of the perigee C from the centre of the Earth E; on the other, the ratio of the speed of the centre of the epicycle to the speed of the planet in the epicycle, the size of the epicycle and the speed of its centre being reduced to the same number 390. In the other is contained the ratio that remains after the ratio of the speed of the centre to the speed of the planet has been subtracted from the greater ratio, that of the semidiameter

DC to the distance CE, the former number being reduced to the same 3367. It seemed good to copy them out here.

[Margin: See the figure of Theorem 1.]

	In the eccentric	Ratio of the line DC to CE, as 390 to	Ratio of the speed of D to the speed of the planet, as 390 to	Residual ratio, after the smaller is subtracted from the greater, is as 3367 to
♄	At apogee	3415	12482	921
	Mean long.	3220	11087	978
	At perigee	3025	9837	1028
♃	At apogee	1738	4665	2156
	Mean long.	1645	4235	1308
	At perigee	1552	3836	1361
♂	At apogee	262	505	1748
	Mean long.	202	343	1980
	At perigee	146	213	2307
♀	At apogee	163	269	2034
	Mean long.	152	244	2097
	At perigee	139	220	2127
☿	At apogee	794	1293	2067
	Mean long.	650	1229	1779
	Nearest the Earth	575	1190	1627
	At perigee	598	1152	1748

Remember that in Ptolemy's hypothesis Mercury comes nearest the Earth not opposite the aux of the epicycle, but about the thirds.

Theorem 6. From this comes a problem for determining the points of the stations. *The planet makes a station at that point of the periphery of the epicycle through which a straight line drawn from the centre of the Earth as far as the concave side of the same periphery has its half-chord in the same ratio to the outer portion as the speed of the centre of the epicycle has to the speed of the planet in the epicycle.*

IX. This is Reinhold's proposition on Peuerbach's *Theoricae*, p. 146, and that of Pedro Nunes in the same place, annotation 1; in the form of a problem

for finding the points of the stations it is demonstrated, with Apollonius and Ptolemy, by Regiomontanus in the *Epitome*, Book 12, proposition 5. For both, therefore, let the figure of the first theorem be repeated here, and besides the lines drawn and set out there, let there also be drawn from the centre of the Earth let two other straight lines be drawn cutting the epicycle and produced as far as the concave periphery; let these be EVF and EIR. And let the points of intersection V and I be joined to the centre of the epicycle by the straight lines DV and DI, and the point F to G and I by the straight lines FG and FI. And of the former secants EB and EH, let the inner portions, or chords, GB and MH be bisected at P and O by the perpendiculars DP and DO drawn from D, just as the chords FV and RI are to be understood as cut in two at Q and S by their respective perpendiculars. With these things set down, I say that if PG, which is half of the inner portion BG, has to the outer portion EG of the secant EB the same ratio that the speed of the centre D has to the speed of the planet in the epicycle, the point of the first station is at the point G; just as the point of the second station is at M, if the half OM of the chord HM has to the outer portion EM of the whole secant EMH the same ratio that the speed of the centre D has to the speed of the planet in the epicycle. We suppose, moreover, that DC has a greater ratio to CE than the speed of D to the speed of the planet, so that the planet is capable of standing and going back. It is further supposed that such a secant can be drawn between EA and the tangents EL and EK; and that the farther the secants stand from the straight line drawn through the centre of the epicycle, the smaller the ratio their inner portion has to their outer.

Diagram. Set into the left column, with the text running down its left side. The figure the text calls the figure of the first theorem, repeated: a circle, the epicycle, with a long narrow fan of straight lines running from the single point E at the foot up to the circle and through it. A vertical line runs from E through C and the centre D to the top of the circle. A nearly horizontal line through the centre joins L at the left to K at the right. Lines join the centre to G and M and to points on the lower arc between them, and lines run from F down to points on the same arc.

- R and A (top of the circle, set side by side and separated by a point): R.A
- F (upper left, on the circle)
- B (upper left, below F, on the circle)
- H (upper right, on the circle)
- S and D (at the centre, S left of the vertical line, D right of it)
- L (far left, at the horizontal line's end)
- P (inside the circle, on the horizontal line, right of L)
- Q (inside the circle, on the horizontal line, immediately right of P)
- O (inside the circle, on the horizontal line, right of centre)
- K (far right, at the horizontal line's end)
- G (lower left, on the circle)
- V (bottom of the circle, right of G)
- T and I (bottom of the circle, right of V, set close together)

- C (bottom of the circle, on the vertical line)
- M (lower right, on the circle)
- E (foot of the figure, where all the lines meet)

[Margin: Lemma of Nunes and Reinhold for the points of the stations.] This, however, Pedro Nunes and Reinhold demonstrate in the places above. For in this same figure the chord RI is smaller than AC, the greatest of the chords; FV is smaller than RI; and BG smaller than FV, by 15 of the third book of the *Elements*; and by 8 of the same, EC is smaller than EI; EI than EV; and EV than EG. Therefore the half DC is greater than the half-chord SI, and this is greater than QV, and this than PG. And therefore, by 8 of the fifth book of the *Elements*, the ratio of DC to CE is greater than that of SI to CE; the ratio of SI to IE greater than that of QV to IE; and finally the ratio of QV to VE greater than that of PG to VE; and by the same proposition SI to CE is greater than to IE. Therefore the ratio of DC to CE is much greater than the ratio of SI to IE; and in the same way it will be proved that the ratio of SI to IE is greater than that of QV to VE; and of QV to VE greater than that of PG to GE; and that the greatest of all is that of DC to CE. With this lemma assumed, let us come to the main demonstration.

X. [Margin: Demonstration of Theorem 6.] So let the portion QV of the secant EF be supposed to have to VE the same ratio that the speed of the centre D has to the speed of the planet; and, taking an arc VG, as small as you please, towards the apogee, and VI towards the perigee, let the remaining lines be drawn as above. For because the base FE of the triangle FGE is divided into two portions, of which VF is greater than the side FG adjoining it, then by Theorem 2 of this chapter the ratio of the line FV to VE will be greater than the ratio of the angle FEG to the angle GFE, and therefore greater than the ratio of double the angle FEG to double the angle GFE. Therefore the ratio of half the line FV, that is, of the line QV, to the line VE is greater than that of the angle FEG to double the angle GFE, that is, to the angle at the centre GDV, which is double the angle GFV or GFE, by 20 of the third book of the *Elements*. But the ratio of the half QV to VE was laid down as equal to the ratio of the speed of D to the speed of the planet in the epicycle; therefore the ratio of the speed of the centre D to the speed of the planet is greater than that of the angle FEG to the same angle GDV. Therefore the angle of the speed of the centre D, corresponding to the angle GDV of the speed of the planet, is greater than the angle FEG, or equal to the angle of the speed of the centre D. For while the planet in the epicycle describes the arc GV, that is, the angle GDV, it will seem at the centre of the world to have described against the succession of the signs, as far as the epicycle's part goes, the angle GEV; but in the same time the centre of the epicycle describes the arc QL in the order of the signs, and the whole epicycle advances by as much as the angle LEV subtends. The epicycle, or the centre D, has therefore advanced more than the planet has gone back, by an excess as great as the angle GEL. Therefore in the whole arc GV the planet appears direct.

[Margin: 2nd part of the theorem.] Now let the arc VI be taken towards the perigee of the epicycle, however slight, and let the lines ordered above be drawn. For the base FVE will be so cut at V that the portion EV is greater than the side EI adjoining it. Therefore, by Theorem 2 of this chapter, the ratio of EV to VF will be greater than that of the angle IFE to the angle IEF; and so, conversely, the ratio of FV to VE will be smaller than that of FEI to the angle IFE, and therefore smaller than that of double the angle FEI to double the angle EFI. From this it comes about that the ratio of the half QV to the line VE is smaller than that of the angle FEI to double the angle IFE, namely to the angle at the centre VDI. But the ratio of the half QV to VE was laid down as equal to the ratio of the speed of the centre D to the speed of the planet. Therefore the ratio of the angle of the speed of the centre D to the angle of the speed of the planet is smaller than the ratio of the angle VEI to the angle VDI. And since the angle VDI is the one that measures the speed of the planet in the epicycle, the angle of the speed of the centre D will be smaller than the angle VDI. So let such a smaller angle be VET. While, then, the planet in the epicycle describes the arc VI, that is, the angle VDI, it seems from the centre of the Earth to have described the angle VEI, against the order of the signs, as far as the part of the motion in the epicycle goes; but in the same time the centre D, according to the succession of the signs, has described an arc subtending the angle VET. Therefore the retrocession of the planet is greater than the progress of the centre D, by an excess as great as TEI, through which the planet will seem to have gone back. [Margin: Conclusion of the theorem.] Now, therefore, since the planet has been shown to be direct in the whole arc GV and retrograde in the whole arc VI, it follows that the point V is the end of direction and the beginning of retrogradation, and therefore is the point of the first station. And the same will be shown in like manner of the point of the second station.

Theorem 7. *The points of the stations are nearer the perigee of the epicycle than the points of contact, that is, they lie within the points of contact.*

XI. If the planet moved with no other motion than its motion in the epicycle, in the order of the signs at the top and against the order of the signs at the bottom, as Capuanus [?] supposes in the *Theoricae*, the first station and the beginning of retrogradation would occur at the left contact of the epicycle, as in the figure of Theorems 1 and 6, at the point L, and the second at the right contact K. But because it moves with the motion of the centre of the epicycle, when it is at the left contact it is therefore still being carried forward towards the east and is still direct; and because of the same motion of the centre D it begins to be direct at M before it has reached the contact K. Then, since to make a station, by Theorem 6, it is required that the inner half-portion of a secant of the epicycle have to the outer portion a ratio equal to the ratio of the speed of the centre of the epicycle to the speed of the planet, whereas a tangent has no portion inside the epicycle, it is clear that the station cannot be at the contacts themselves; nor yet before the contact L or after the contact

K, because there the motion of the planet goes along with the motion of the centre D towards the east. It remains, therefore, that the stations occur within the points of contact, at points nearer the perigee C of the epicycle; and this opinion is indeed common to astronomers.

Theorem 8. *The points of the stations stand equally far from the perigee of the epicycle at an equal distance of the centre of the epicycle from the centre of the Earth.*

XII. This is demonstrated by Pedro Nunes in annotation 2 on the Peuerbachian affections of the planets. For in the figure of Theorem 6 let the point of the first station be at

G, and of the second at M, and let it be supposed that the motion of the centre D keeps the same ratio to the motion of the planet in the epicycle at both places, since the same distance DE of the centre D from the centre of the Earth is posited. Therefore by Theorem 6, as half of the straight line BG is to GE, so will half of the straight line HM be to ME, by 11 of the fifth book of the Elements; and therefore half of the straight line BG will be equal to half of the straight line HM. For if it were greater, then EG would be greater than EM, by 14 of the fifth book, and BG greater than MH; and so the rectangle contained by the whole EB and by EG would be greater than the rectangle contained by EH and by EM, which is impossible and contrary to the corollary of proposition 35 of Book 3 of Euclid. For the same reason half of BG cannot be less than half of HM. Now, as half of BG is to half of HM, so is EG to EM, by alternate proportion; therefore EG and EM will be equal. So when the equal lines DG and DM are drawn, the angles EDG and EDM will be equal by 8 of the first book, and thus by 26 of the third book the two arcs GC and MC, that is, the distances of the stationary points from the perigee C, will be equal.

Theorem 9. *The greater the epicycle, the nearer to the perigee of the epicycle are the points of the stations, if the stations occur at the contact itself.*

XIII. Nonius also touches on this theorem, in annotation 3 on the affections of the planets. In the adjoined diagram, let the greater epicycle KD and the lesser HC be described from the common centre B, and let their common distance from the centre of the Earth A be AB. From A let the tangents AC and AD be drawn, and let the points of contact be joined to B by the straight lines BC and BD; for by 18 of the third book of the Elements the angles at C and D will be right. Finally, let H and K be the perigees of the epicycles. With this laid down, the tangent AD of the greater epicycle cannot fall between AB and the tangent AC of the lesser, or coincide with it. For the triangles ADB and ACB, right-angled at D and C, stand on a common base subtending the right angle, and each singly has its three angles equal to two right angles, by 32 of the first book of the Elements; yet in such a way that the one which lies next to the same end of the base subtends a greater side than the other, namely the angle BAD subtends the side BD, which is the semidiameter of the greater epicycle. Therefore by the scholia to 26 of the first book of the Elements the angle BAD will be greater than the angle BAC; and so by 32 of the first book of the Elements the remaining angle ABD will be less than the remaining angle

ABC. Consequently, if BC is understood to be produced to the periphery of the greater epicycle, it will fall above D and will enclose with BK an arc greater than the arc DK. Now, as the angle ABC is to a right angle, so are the arc HC and the arc KD, etc., bounded by the higher point on which BC produced would fall, to the quadrants of their circles, by the last of the sixth book of the Elements; and so the arcs already mentioned will be proportional, or similar. Accordingly the arc KD will be less in proportion to its circle than the arc HC in proportion to its own, and accordingly the contact D will be nearer to the perigee K of the greater epicycle than the contact C to the perigee H of the lesser. But because the station does not occur at the contact itself, it does not immediately follow that in a greater epicycle the station is nearer the true perigee of the epicycle, unless other causes assist, as they do in fact assist in Saturn, Jupiter, Mars and Venus. Now let us come to the usefulness of selected problems.

Diagram. Set into the left column, with the text running down its left side. A long vertical axis joins two pairs of concentric circles, a larger pair at the top about B and a smaller pair at the foot about E, with A on the axis between them. From A straight lines rise to C on the inner upper circle and D on the outer upper circle; B is joined to C and to D. H stands on the axis at the foot of the inner upper circle and K at the foot of the outer one, with O lower on the axis, above A. In the lower pair, M and L stand on the axis at the top of the outer and inner circles; lines from A reach F on the inner circle and G on the outer, and E is joined to F and to G.

- Labels read, top to bottom: B, C, D, H, K, O, A, M, L, G, F, E. None unread.

Problem 1. *Given the distance of the centre of the epicycle from the apogee of the eccentric, to investigate the speed of the planet and of the centre of the epicycle corresponding to the mean motion.*

XIV. [Margin: Step 1.] First, with the mean distance of the centre of the epicycle from the aux of the eccentric, or with the mean centre, that is, with the mean anomaly of the eccentric, take the equation of the centre from the tables. To that same distance add the arc of mean motion in longitude, and with the sum look up a second equation of the centre; you will then have the difference of the equations, if there is any, which is to be called the centric difference. [Margin: Step 2.] Second, if the centre of the epicycle is between the mean longitudes and the perigee of the eccentric, subtract the centric difference from the mean motion of longitude; if otherwise, add it, provided that both the mean centre and the sum just mentioned place the epicycle in the same quadrant of the eccentric. Otherwise, if by the one it is in the first quadrant and by the other in the fourth quadrant of the eccentric, subtract the sum of the centric equations from the mean motion; but if by the one it is in the 2nd and by the other in the 3rd quadrant, add it. And so you will have the true motion of the centre of the epicycle and its speed. [Margin: Step 3.] Third, for the speed of the planet in the epicycle, take the mean argument or mean anomaly of the

orb corresponding to the proposed mean motion; this will be done easily if it is known to what time the given mean motion corresponds. Then to the mean argument add what you subtracted in order to obtain the speed of the centre of the epicycle (namely either the difference or the sum of the centric equations), or if you added it, subtract it; and you will have the true motion of the planet in the epicycle, and so its speed. So, very nearly, Regiomontanus in Book 12 of the *Epitome of the Almagest*, proposition 7.

Problem 2. *To determine the distance of the points of each station from the true apogee of the epicycle, given the distance of the centre of the epicycle from the apogee of the eccentric.*

Diagram. Set into the right column at its foot, with the text running down its left side. The figure the text calls the figure of the first theorem: a circle, the epicycle, with a long narrow fan of straight lines running up from the single point E at the foot, through the circle, to its upper arc. A vertical line runs from E through C and the centre D to the top of the circle at A. A nearly horizontal line through the centre joins L at the left to K at the right. Lines join the centre to G, M and points on the lower arc between them, and a line drops from F through P towards G.

- Labels read: R and A at the top; F, B and H on the upper arc; S and D at the centre; L, P, Q and K with O on the horizontal line; G, V, T and I set close beside it, C and M on the lower arc; E at the foot.
- Not settled: the second of the two letters set together between V and C, which is I or T; its head is lost in the drawn arc. [?]

XV. Taking up again the figure of the first theorem, let the distance of the centre of the epicycle from the apogee of the eccentric be given, and so, by the preceding problem, let the ratio of its speed to the speed of the planet be known. Let EGB be drawn so that PG, half of the chord GB, has the same ratio to GE as the known speed of the epicycle has to the known speed of the planet; and let the perpendicular DP be drawn from D to BG, and the straight line DG from D to G. For what is sought is the arc ABG, since G is the place of the first station, given the equality of ratios already mentioned, by Theorem 1; just as, if you draw another similar secant EMH, M will be the point of the second station. First, then, since the ratio of PG to GE is given (for it is as great as that of the speed of the epicycle to the speed of the planet), the double ratio of the chord BG to PG will be known, and by composition the ratio of the secant BE to the portion GE will be known; and from the measures of the orbs and the anomaly of the eccentric, the ratio of the semidiameter DC of the epicycle to the distance DE will be known, and therefore also the ratio of the line AC to CE and of AE to CE. So the rectangle contained by the whole AE and EC can also be known, and that contained by the whole EB and the outer part EG, in the same square parts, since they are equal by the corollary to 36 of Book 3 of Euclid. [Margin: By the first of the sixth book of Euclid.] Since the ratio of BE to GE is known, let it be made that as BE is to GE, so is their rectangle to the square on GE, whose root EG will become known in the parts in which the

semidiameter DC, or DG, is given; therefore EB too will be known in the same parts, as will BG and its half PG. Therefore in the triangle PDG, right-angled at P, with the sides PG and DG known, the remaining side DP will become known, and also the angle PDG. Second, because the line PE is also known, in the triangle EPD, right-angled at P, the angle DEP will become known from the known PD and PE; its complement to a right angle is the other angle, EDP. If you take away from this the angle PDG, there will remain the angle GDE, measuring the arc GC; and its remainder GBA, to the semicircle, will be the distance of the planet G, staying in the first station, from the true apogee of the epicycle. Similarly the arc AHM will be found for the planet M, occupying the point of the second station. See Regiomontanus, Book 12, propositions 6 and 8.

Problem 3. *To put the arcs of the stations into tables.*

XVI. Ptolemy gives this problem piecemeal, in Book 12: chapter 2 for ♄, chapter 3 for ♃, chapter 4 for ♂, chapter 5 for ♀ and chapter 6 for ☿, investigating the points of the stations at the greatest, mean and least distance of the centre of the epicycle from the centre of the Earth. Then, in chapter 7, he sets in first place in the Rule of Proportions the difference of the greatest distance from the mean one; in second place the difference of the mean distance from the distance belonging to the centre of the epicycle at the time when the station occurs; and in third place the excess of two stations, of which one belongs to the greatest distance and the other to the mean. Multiplying the third number by the second, he divides the product by the first, and subtracts the quotient from the arc of the station which the mean longitude gives, or adds it, as the case itself requires. He uses the same method for those positions of the epicycle that are between the mean longitude and the perigee of the eccentric, and in this way he extracts the arcs of the stations for every position of the epicycle on the eccentric. In this method, however, it is supposed that the arcs of the stations increase or decrease proportionally by as much as the centre of the epicycle, in receding from the mean longitude towards the apogee or perigee of the eccentric, recedes from the centre of the world or approaches it. As Jābir ibn Aflaḥ notes in Book 8, p. 129, and Regiomontanus in Book 12, proposition 11, this is not necessarily true, if we wish to keep geometric rigour; and so he himself adds a more precise method, at the end of which, however, he admits as follows: *But the neglect of this precision will introduce no perceptible error; and so I judge that Ptolemy's procedure, which, although it is not worked out to the kernel, is nevertheless as convenient as need be, should be followed.* For this reason I have not thought that the reader's application should be burdened with the labour of that precision.

Problem 4. *Given the arc of the first station, to derive from it the arc of the second station, of retrogradation, and of direction.*

XVII. First, subtract the arc of the first station from 360°, and the arc of the second station will be left. Second, subtract the arc of the first station from the arc of the second station, and the arc of retrogradation will be left. Third, subtract the arc of retrogradation from the whole circle, and the arc of direction

will be left. These rules follow from the definitions of the arcs given in chapter 2, and they are used by Peuerbach, Magini, and Reinhold on p. 140.

Problem 5. *Given their arc, to investigate the time of retrogradation and of direction.*

XVIII. In the *Theoricae*, Peuerbach orders these arcs to be divided by the daily motion of the argument, or of the anomaly of the epicycle. Reinhold illustrates this precept there, on p. 140, with the example of Mars. Let, he says, the equated centre of Mars, or its centric place, be 66° 40′; for to this Ptolemy's tables give an arc of the first station of 161°. Therefore if the true argument of Mars at that position of the centre of the epicycle is 161°, Mars will be in its first station, which he says happened about 1542 March 17. So when this is subtracted from the whole circle there remains the arc of the second station, 199°; when the arc of the first station is subtracted from the arc of the second there is left the arc of regression, 38°; and when this is taken from 360° there remains the arc of direction, 322°. Now let the arc of regression be resolved into seconds, 136800″, and let these be divided by the daily motion of the argument, which is 27′ 42″, that is, 1662″. They give 82 days. But this rule, based on the mean motion of the argument, gives only the mean time, which in Mars often departs from the true time by several days; and Reinhold admits in the same place that, from the ephemerides for the year 1542, the true time of retrogradation was only 72 days. He therefore teaches that a correction is to be applied, of the following kind, so that we may come closer to the true time. Since the centre of the epicycle of Mars advances under the zodiac each day by almost half a degree, in 82 days it advances nearly 41°. Add half of these, 20½°, to the equated centre of Mars used before, which was 66° 40′, and the sum is 87° 10′. For this, look up again from the tables the arc of the first station, for it will be 163° 6′. From this, by the rules of Problem 4, the arc of regression is gathered, 33° 48′, which when resolved makes 121680″. If you divide these again by the daily motion of the mean argument, 73 days come out in the quotient, agreeing with the true time of retrogradation or close to it. This rule, says Reinhold, comes closer to the truth because arcs of regression taken for the opposition of Mars to the Sun, or for the middle of the retrogradation, are less deceptive than those taken for its beginning or end.

Problem 6. *To investigate the arc of the zodiac which a planet traverses while it is retrograde.*

XIX. We owe this too to the diligence of Reinhold, on Peuerbach's *Theoricae*, p. 142. According to him, first see what equation of the argument belongs to the arc of retrogress in the epicycle found by Problem 4, and at the same time how far the centre of the epicycle advances in the time in which it traverses the arc of regress. Then subtract this latter arc from the former; the remainder will be the arc of the zodiac which the planet traverses while retrograde. As in the preceding example, in which the centre of Mars was 87° 10′ and the arc of the first station 163° 6′, to which corresponds an equation of the argument of 27° 18′; doubled, this makes the arc of the zodiac corresponding to the arc of regress in the epicycle, namely 54° 36′. The planet would recede through this

whole arc if the centre of the epicycle were not meanwhile advancing in the order of the signs. Further, in the 73 days found above it advances by mean motion very nearly 38° 15′; hence the motion of regress exceeds the motion of progression by 16° 20′. Granted, by the ephemerides for the year 1542 Mars regressed only 16°.

SCHOLIA.

Scholia.

I. [Margin: 1. Cause varying the arcs of the stations.] *The variation of the arcs of the stations and of the retrogressions, etc., depends on three causes, in the opinion of* Peuerbach*, to which* Magini *adds a fourth in his* Theoricae*, Book 2, chapter 12. But these are to be weighed and limited by annotations 2 and 3 of* Pedro Nunes *on the Peuerbachian affections of the planets, by the scholia of* Reinhold *on the same, p. 138, and by* Magini*. The first is the differing distance of the centre of the epicycle from the centre of the Earth; for, other things being equal, the nearer the centre of the epicycle is to the Earth, the nearer the point of the station is to the perigee of the epicycle, as* Reinhold *shows there, and* Nonius *in annotation 2. But this cause does not hold in Mercury, for a reason to be touched on below: for in Mercury, when the centre of the epicycle is placed at the mean longitude of the eccentric, the points of the stations are further from the perigee of the epicycle, but when it is placed at the apogee of the eccentric, less so; and they are furthest from it when the centre of its epicycle is distant from the apogee of the equant by a third. For that distance is, by Ptolemy's reckoning, 32° 56′ at the apogee, and around the mean longitudes*

34° 56′, and at the third 35° 31′, and at the perigee 35° 20′. So if the planet is nearer the Earth anywhere else than at the perigee of the eccentric, the proposition is not verified, as Nonius *shows in that annotation 3.*

[Margin: 2. Cause.] *The second cause is the size of the epicycles in comparison with the eccentric: for the greater the epicycle, the less the points of the stations are distant from the perigee of the epicycle, unless the speed of the planet stands in the way. The greatest of all is the epicycle of Venus; then that of Mars, after that of Mercury, next that of Jupiter, and the least of all that of Saturn, with respect to its own eccentric. And so the points of the stations of Venus and Mars are least of all distant from the perigee of the epicycle; then those of Mercury, after that those of Jupiter, and most of all those of Saturn. In absolute terms, however, the greatest of all is the epicycle of Mars, and the least of all that of Mercury.*

[Margin: 3. Cause.] *The third cause is the varying slowness of the motion of the argument, or of the planet in the epicycle. This is most effective in Mars and in Venus, the centre of whose epicycle traverses the whole zodiac sooner than the planet completes one revolution in the epicycle. This cause, taken not singly but together with the size of the epicycle, brings it about that the points of the stations come nearer to the perigee of the epicycle. But if these two causes are not joined, a planet in an epicycle of any size whatever could complete its period so quickly that the points of the stations*

would come no nearer to the perigee of the epicycle than in another, small epicycle in which the planet's motion was slower. Just as, conversely, unless Venus and Mars had very large epicycles, neither could be retrograde, because of its slower revolution in the epicycle as compared with the motion of the eccentric.

[Margin: 4. Cause.] *The fourth cause is the variation of the eccentricity; but this is reduced to the first, because it varies the distance.*

II. [Margin: Was Pliny the Elder rightly understood by Reinhold on the stations of Mars?] *Reinhold, on Peuerbach, p. 141, thinks it was said by Pliny the Elder in Book 2, chapter 17, that Mars never makes a station. But Pliny the Elder said that it never does so* with the star of Jupiter in trine; *otherwise he would contradict himself in chapter 15, where he says:* The star of Mars, as being nearer, feels the rays even from the quadrate, etc.; the same, when stationary, lingers six months in the signs, otherwise two months, while the others do not fill four months at both stations. *Granted that its station can be observed only with difficulty and scarcely, because, as Reinhold notes in the same place, the periodic time of Mars on the eccentric and in the epicycle is almost the same in quantity, since the daily motion of the epicycle is 31′ 26″ and that of the argument 27′ 42″. So, because of this near equality, the motion of the planet in the epicycle against the order of the signs does not quickly or easily equal or exceed the motion of the centre of the epicycle, and only near the perigee of the epicycle; and once it has equalled it, it is carried against the order of the signs very quickly.* [Margin: Notable points on the stations of Mars and Venus.] *That around the apogee and the mean longitudes of the deferent the arc of the first station is smaller in Mars than in Venus, but on the contrary around the perigee it is somewhat greater in Mars than in Venus, arises from the eccentricity of Mars, which is greater not only in absolute quantity but in proportion to the orbs, as the same Reinhold holds.*

III. [Margin: Stations and regressions in the Danish hypothesis.] *How Tycho and Longomontanus represented the stations* [Sationes] [recte: Stationes] *and retrogressions of the planets in their hypothesis is not clear from their own writings. Tycho indicates, however, in volume 1 of the* Progymnasmata*, p. 478, that these depend on the annual revolution of the centre of the orbs with the Sun, and on two epicycles; but certainly in this hypothesis the demonstration of them is not so simple as in the Ptolemaic one.*

CAPVT IV.

Chapter 4.

Theorica & cauſæ Stationum ac Retrogreſſuum in Hypotheſi Terræ motæ.

The theory and causes of stations and retrogressions in the hypothesis of the moved Earth.

I. In the Copernican hypothesis, that whole alternation of directions, stations and retrogressions has a purely optical cause, in the eyes of the Earth as it goes

round through the annual orb and views these planets from a changing position. We must set this out from what is taught by Copernicus, Book 5, chapters 3, 35 and 36; Rheticus at the end of the *Narratio prima*; Galileo in dialogue 3 of *De Mundi systemate*; Kepler in the *Mysterium Cosmographicum*, chapter 1, in the *Astronomia Optica*, p. 331, and in the *Epitome of Copernican Astronomy*, pp. 735 and 767; Lansbergen in the *Theoricae*, chapter 21; Hérigone in volume 5 of the *Cursus Mathematicus*, p. 604; and Longomontanus, who sets out the same hypothesis but does not follow it, in Book 1 of the *Theorica*, chapter 1.

In brief, then, the cause is the annual motion of the Earth, which overtakes the superior planets more quickly, or moves more slowly than the inferior or inner planets are moved, and so, by parallax or commutation of view, looks up to them or down on them now ahead of itself on the left, now behind it on the right. [Margin: Causes of stations and regressions in the three superior planets.] For in the adjoined scheme let C be the centre of the annual orb, in the Sun or near the Sun, and let the annual orb itself be DE, in which the Earth moves from E to K, etc.; and let the orb of Saturn, Jupiter or Mars be AB, in which the planet moves with a slower motion than the Earth, from A to L, etc. Now let the planet be supposed at A, from which AKL and AHM are drawn, tangent to the annual orb at H and K, and let the common diameter be ACB. For if the Earth is at E, the true motion of the planet A will be in the same line as the Sun C, or as the centre of the mean motion of the Earth, and the planet A will be acronychal and nearest to the Earth. But with the Earth placed at D and the planet at A, it should indeed be seen in the same line ABC, in conjunction with the Sun or the centre of the mean motion of the Earth; but because it then becomes hypaugus on account of the brightness of the Sun, by which it is overwhelmed, it cannot be observed. Outside these cases, because the Earth traverses its great orb in the space of a year, from E through KDH, returning to E, while the superior planet, being slower, meanwhile passes through some part of its orb, the Earth, while traversing the greater arc KDH, approaching through KFD to the point D placed under the apogee B, and receding from there through the arc DGH, will appear, I say, to add to the motion of the planet the angle HAK; but from there through E, which lies under the perigee A of the planet, as far as K, to take the same angle away from the motion of the planet. Accordingly, as long as the subtractive motion of the Earth exceeds the additive motion of the planet, the planet will appear to be abandoned by the Earth and, left behind, to go back against the order of the signs, namely while the Earth is occupied in the whole arc HEK. But when the subtractive motion of the Earth is equal to the additive motion of the planet, the planet will appear neither to advance nor to regress but to stand, which occurs at the point H in the first station and at K in the second. In the remaining arc KDH it will appear to advance directly; and it will appear to retrogress most of all to the Earth arriving at E and viewing the planet acronychal.

Diagram. Set into the right column, with the text running down its left side. A large circle, the orb the text calls AB, with a smaller circle inside it

about the centre C. A vertical diameter runs from B at the top through D, C and E to A at the foot. Straight lines from A pass through H to M at the left and through K to L at the right, H and K standing on the inner circle and M and L at the large circle. Two further lines run down from B on either side, towards G and F on the upper part of the inner circle.

• Labels read: B, A, M, L, D, F, G, C, H, E, K. None unread.

II. [Margin: The same causes for the inferior planets.] For Venus and Mercury, however, whose orbs are like Ptolemaic epicycles but described about the Sun C, and for which the annual orb of the Earth by its motion serves as the Ptolemaic eccentric, let the orbs in the preceding figure be exchanged: let the annual orb of the Earth be BMAL, and the orb of Venus or Mercury the inner one, DHK. Let the Earth be at A, from which two visual lines AKL and AHM are drawn, tangent to the orb of the planet at K and H; and let the motion of the Earth and of the planet be in the order of the signs, towards the same parts, but let the motion of the planet be swifter than that of the Earth. With this supposed, the planet will appear to the Earth A to move in its orb, as in an epicycle, in the order of the signs, through the whole arc KDH, in the middle of which, D, is the apogee; but in the remaining arc HEK it will appear to move against the order of the signs, and in the first case to add the angle HAK to the mean motion of the Earth, in the second to take it away. Accordingly, as soon as the subtractive motion of the planet is equal to the additive motion of the Earth, the planet will appear stationary at H and then at K; but when the subtractive motion of the planet is greater than the additive motion of the Earth, the planet will appear to recede, namely in the arc HEK, but principally at E, the place of perigean conjunction.

III. So far we have outlined the Copernican hypothesis of stations and regressions by means of customary schemes; but some light must be added to these shadows from a new scheme, which we have put together partly from Kepler's *Epitome of Copernican Astronomy*, partly from Galileo, and it is of this kind. From O, the centre of the world, that is, from the Sun, let the annual orb of the Earth BGM be described, and the orb of some superior planet, say Jupiter, which

is to be *bgm*; and let SVY be the arc of the zodiac in the sphere of the fixed stars. Draw the line of opposition OGV: that is, for when, with the Earth at G, Jupiter at *g* appears to us acronychal and opposite the Sun O. On either side of it take any number of equal arcs of the annual orb, for instance BC, DE, FG, GH, HI, IK, KL, LM. To these there correspond in Jupiter's orb dissimilar arcs, smaller in the proportion of the Jovial orb to the annual orb, since while the Earth traverses its whole annual orb once, Jupiter completes only about a twelfth part under the fixed stars. Let those arcs, then, be *bc, cd, de, ef, fg, gh, hi, ik; kl, lm*. And let both the Earth move in the order of the signs from B to D, G, etc., and Jupiter from *b* to *d, g*, etc. With the Earth placed at B, then, and Jupiter at *b*, and the line B*b*P drawn, Jupiter will appear to us under the zodiac at P. But when the Earth has been carried forward to C and Jupiter to *c*, and

the line C*c*Q has been drawn, the planet will appear under Q, having gone direct in the zodiac from P to Q.

Diagram. Set across the left column, interrupting its text. At the top a shallow arc, the zodiac, carries above it, from left to right, the letters S R T, then Q A V P Z, then X N Y. Below stands a large circle, Jupiter's orb in the text, whose upper arc carries the italic letters, from left to right, *m l* K *i h g f e d c b*. Inside it a smaller circle, the annual orb in the text, carries from the lower right round its upper arc to the lower left the letters B, C, D, E, F, G, H, I, K, L, M, with an italic *a* below its foot. A rayed disc with a ringed dot at its centre stands within the smaller circle. A vertical line rises from the rayed disc through G, the large circle and the crossing lines to the zodiac. Straight lines run from the points of the smaller circle through the upper arc of the large circle, cross one another in a narrow waist above it, and fan out to the letters on the zodiac.

- Top arc: S, R, T, Q, A, V, P, Z, X, N, Y
- Large circle, upper arc: *m*, *l*, K, *i*, *h*, *g*, *f*, *e*, *d*, *c*, *b*
- Smaller circle: B, C, D, E, F, G, H, I, K, L, M; *a* at the foot
- Centre: a rayed disc, no letter seen beside it

Now let the Earth advance to D and Jupiter to *d*: for along the visual line D*d*R it will appear still to have gone forward in a direct course from Q to R, following the series of the signs. [Margin: New scheme for the stations and retrogradations of the superior planets, from Galileo and Kepler.] But if the Earth should begin to interpose itself more directly between the Sun and Jupiter, or to approach the opposition G, and, having advanced to E, with Jupiter at *e*, should be so placed that the line D*d*R of the preceding day is almost parallel to the line E*e*S of the following day, Jupiter will appear to have lingered almost under S or the next point R. That is, it will be either stationary [Sationarius; recte: Stationarius] or retrograde from S to R only barely perceptibly, on account, namely, of the immense distance of the fixed stars and the parallelism, or virtual parallelism, of the two neighbouring visual lines. But as the Earth proceeds to F and Jupiter to *f*, Jupiter will appear along the line F*f*T under the point T of the zodiac, and so will appear to have gone back from S to T by the sensible arc ST; granted that earlier, in the time in which the Earth completed the arc EF, Jupiter will have appeared to linger between the points S and T, as if motionless and stationary. Now let the Earth have come from F to G, the place of Jupiter *g*'s true opposition with the Sun O. For along the line OG*g*V Jupiter will appear to us under the point V of the zodiac, and so to have gone back very greatly against the order of the signs from T to V; although in reality, in this hypothesis, Jupiter and the Earth have always proceeded with uniform motion in their orbs and under the zodiac. Now let the Earth have passed the moment of opposition and gone forward from G to H; and similarly, though not by a similar arc, let Jupiter have gone forward from *g* to *h*. For along the visual line H*h*X Jupiter will appear, likewise by a very large arc VX nearly equal to the arc TV, to have gone retrograde from V to X, against the series of the signs.

And when the Earth is at I and Jupiter at *i*, it will appear along the line I*i*Y under the point Y, having regressed a little further from X to Y; and presently, when the visual lines of consecutive days, drawn from the Earth at K through Jupiter at *k*, are parallel or nearly parallel, Jupiter will appear along the line KN either to have lingered motionless between X and Y, or to have begun, barely sensibly, to advance direct from Y to N, towards X, following the series of the signs. Afterwards, as the Earth proceeds to L and Jupiter to *l*, Jupiter along the line L*l*Z will appear under Z, having, that is, directed its course from N to Z with an advance greater as far as appearance goes; and with one greater still each day, when the Earth is at M and Jupiter at *m*; and along the line M*m*A it will appear under A, having advanced, namely, from Z to A. [Margin: Arc of retrogradation.] The whole apparent regression of Jupiter, then, will fill the arc SY, or RN. The same is to be understood likewise of Saturn and Mars; granted that in Saturn the stations and regressions are more frequent, because the Earth catches up with Saturn, which is slower than Jupiter, sooner and more often at the point of opposition than it does Jupiter; and that, conversely, they are rarer in Mars than in Jupiter, for the contrary reasons.

[Margin: On the stations and regression of Venus and Mercury.] IV. Now let the orb of Venus or Mercury be BGM, and let the annual orb of the Earth be *bgm*, and let these planets move in the order of the signs faster than the Earth moves in the order of the signs; for it will be shown proportionally that the appearance of stations and regressions arises not from any real halt or retreat of theirs under the zodiac, but from the motion of the Earth and the optical commutation of the visual lines. For since their orbs lie within the annual orb, and they revolve faster than the Earth, it comes about first, in the part of the eccentric M*a*B in which the apogee *a* lies, with respect to the Earth at *g*, that they appear to go in the order of the signs even if they are supposed to be at rest, and far more so if they move, and that faster than the Earth. But in the part of the eccentric nearer the Earth, such as EGI, they will appear to go retrograde, because opposite parts of circles, viewed from outside, will appear to have opposite motions; and the lines of vision that connect the endpoints of the daily motions will be bent back against the order of the signs, and will cut one another behind the Earth, in the quarter turned away from the Sun. And since the Earth is in the position in which, by the scheme of the preceding number, Jupiter or any other of the superior planets stood, while Venus or Mercury is in the position in which the Earth stood there, it follows that, although they are always in reality advancing, they nevertheless appear at times to stand still and then to go retrograde: to stand still, I say, when the visual lines of consecutive days, drawn from the planet through the Earth, are parallel.

[Margin: Cause of the apparent stations, from the parallelism of the visual lines.] V. From what has been said you will now more easily understand the definitions belonging to this hypothesis that we set out in advance in chapter 2, number 2, among which it is said that planets appear stationary when two visual lines of consecutive days, drawn through the Earth and the planet as far as the

sphere of the fixed stars, are parallel. And the reason is that in this Copernican hypothesis the distance of the fixed stars is so great that, in relation to it, the diameter of the Earth's annual orb vanishes, or produces no sensible parallax. Much less sensible, therefore, will be the distance between two parallels, say DR and ES, since they enclose not the diameter of the annual orb but a far smaller particle, corresponding to the arc of the Earth's daily motion, as the arc ED nearly is. Those parallels, therefore, produced to the sphere of the fixed stars, will appear to fall on the same point of the zodiac, and so the planet will appear, for that day or those consecutive days, to stand in the same place. As for directions and retrogradations: given unequal revolutions of the Earth and the planet in the order of the signs, the intersections of the visual lines of consecutive days must occur in every variety of way, as Kepler shows in the *Epitome of Copernican Astronomy*, p. 740: that is, now between the superior planets and the Earth, now at the centre of the Earth, now behind the Earth in the quarter turned away from the Sun, in which cases they are direct; now beyond the planets, in which case they are retrograde. As regards the inferior planets, however, the lines already mentioned must now bend in the order of the signs, now bend back against the order of the signs, and thus [the planets must be] now direct, now retrograde in appearance. Now some theorems and problems peculiar to this hypothesis are to be indicated; the rest, which it has in common with the Ptolemaic, are not repeated.

Theorem 1. *If the portion of a straight line drawn from our eye and cutting the orb of Venus or Mercury, the portion enclosed by that orb, is bisected in such a way that its half has to the outer portion of the secant the same ratio as the motion of the Earth has to the motion of the planet, the planet will appear to stand still at those points of the planet's circumference which are nearer the perigee and through which the section already mentioned passes; but in the arc intercepted by them, in which the perigee lies, it will appear retrograde, and in the remainder progressive.*

VI. Copernicus demonstrates this in Book 5, chapter 35, having first assumed the lemma of Apollonius, about which we spoke in chapter 3, number 5. To understand this theorem, use our figure set out in chapter 3, number 4; in it, however, the Earth or our eye is to be supposed at E, and the orb of Mercury or Venus to be AGC, described from D. The semidiameter CD of the orb is to be taken as having a greater ratio to CE than the velocity of the Earth has to the velocity of the planet, so that the planet may be capable of station and retrogression. On these assumptions it will be possible to find some line from E cutting the orb AC, say the line EF, whose inner portion VF is so bisected at Q that its half, that is QV, has to VE the same ratio as the motion of the Earth E has to the motion of the planet. For all lines from E cutting the orb AC are so related that the greatest of the inner portions, or chords, is ADC, and the others, not drawn through the centre, become smaller and smaller until they reach the tangent EL, which has no inner portion, and so no ratio of inner portion to outer. Therefore, between a portion having a greater ratio to the outer one and a portion of no ratio, there must be found some portion that has an equal

ratio. If, then, there is such a secant EF on one side, and EH on the other, the planet will appear to stand still at V and at M; to go retrograde in the arc VCM, containing the perigee M, and to go forward in the remainder MAV, as far as the judgement of the eye goes. The demonstration, as to the rest, is plainly the same as those brought forward above in chapter 3, number 10.

Theorem 2. *If a straight line drawn through our eye so cuts the annual orb of the Earth that half of the section enclosed in the annual orb has to the outer portion of the secant lying between our eye, when the Earth is nearer the superior planet, and the planet, the same ratio as the motion of the planet has to the motion of the Earth, the planet will appear to stand still at those points through which the section already mentioned passes; and to go retrograde in the whole arc of the annual orb intercepted within those points and containing the perigee; but to go forward in the remaining circumference of the annual orb.*

VII. This likewise is from Copernicus, Book 5, chapter 35, and it is demonstrated in the same way as the preceding, provided that the annual orb is conceived to be AGC, and one of the superior planets to be at E, and the line EVF, or EMH, to be so cut that the half QV, or OM, has to VE, or ME, the same ratio as the velocity of the planet has to the velocity of the Earth. For on these assumptions the planet will appear to stand still to the Earth placed at V or at M; and in the whole arc MCV, containing the point of opposition C, at which the planet appears nearest the Earth, it will appear to go retrograde, but in the remainder MAV to go forward.

Theorem 3. *If half of the section enclosed by the orb, of a straight line drawn from our eye and cutting the orb of Venus and Mercury, has to the remaining outer segment a greater ratio than the velocity of the Earth has to the velocity of the planet, the planet will appear to advance further in the order of the signs; but if a smaller one, it will appear to go back against the order of the signs.*

Theorem 4. *If half of the section enclosed by the annual orb, of a straight line drawn from our eye and cutting the annual orb of the Earth, has to the remaining outer segment a greater ratio than the velocity of any of the three superior planets has to the velocity of the Earth, the planet will appear to advance in the order of the signs; but if a smaller one, to go back against the order of the signs.*

VIII. Copernicus has both theorems gathered into a single proposition, Book 5, chapter 35, whose demonstration follows from the demonstration of Theorems 1 and 2.

Theorem 5. *If two visual lines of consecutive days, led out from our eye through the planet as far as the sphere of the fixed stars, are parallel, the planet will appear to stand still under the same place of the zodiac; but in the arc that includes the planet's perigee and is included within the two pairs of the parallels already mentioned, it will appear to go retrograde, and to go forward in all the rest.*

IX. This theorem is gathered from what Kepler demonstrated in the *Epitome of Copernican Astronomy*, p. 736 and p. 768, and we have already indicated the reason in number 5.

Problem 1. *To investigate the points of the stations and the arcs of regressions and progressions.*

X. Copernicus teaches this practice in Book 5, chapter 36: that is, by what method, in the figure set out in chapter 3, number 4, and given the half of the line VF, that is QV, having to VE the same ratio as the motion of the Earth has to Venus or Mercury, if their orb is AC and the Earth or our eye at E; or the ratio which the motion of Saturn, Jupiter or Mars has to the annual motion of the Earth, if the annual orb is AC and the planet at E: by what method, I say, from the motions of the Earth and the planet, and from the semidiameter DV (in the one case of the orb of the inferior planets, in the other of the annual orb), and from the distance of the centres ED and the perigeal distance CE, the angle EDV may be found, which measures the arc VC of half the retrogradation. For when this is subtracted from the semicircle AVC, there remains the place of the station computed from the apogee A, or the arc of the first station; adding to this double the arc VC, that is VM, gives the arc ACM of the second station. And when the arc VCM, which is the arc of retrogradation, is taken from the whole circle, there remains the arc MAV of the apparent progression or direction. The demonstration is the same as the one we gave in chapter 3, number 15, with the epicycle of Venus and Mercury changed into an orb about the Sun, and the epicycle of Saturn, Jupiter and Mars into the annual orb, etc.

[Margin: Difficulty of this practice.] Copernicus nevertheless admits in the same chapter 36 that, because the orbs of the planets are not concentric with the great orb of the Earth, and their motions are not equal but anomalous as far as appearance goes, the ratio of their speed to the velocity of our eye with the Earth, which is likewise carried round in an unequal motion, therefore varies from time to time and that this practice is difficult, if we wish to follow geometric rigour. For that reason he closes chapter 35 and Book 5 itself with these words, which Lansbergen almost transcribed in the *Theoricae*, chapter 21: *Nevertheless, since that variable motion of the star according to sight, and the ambiguity of the stations, from which Apollonius's assumption in no way relieves us, bring no small difficulty: I do not know whether someone might not do better by inquiring into the stations simply and from a nearby place, in the way in which we inquire into the conjunction of an acronychal star with the line of the Sun's mean motion, or the meeting of any stars whatever, combining them from the known numbers of their motions; which we leave to each person's pleasure.* Hence Lansbergen concludes above that the surest way is this: having first sought the places of the Earth and the planet in longitude for those times at which we know beforehand, approximately, that there is a risk of stations, we determine by comparing them the time of the stations and of the regressions intercepted between them, as the computers of ephemerides usually do. Let us now hear Kepler.

Problem 2. *To assign to any anomaly of the planet its angles of commutation and its prosthaphaeresis of the orb; or, in the inferior planets, the elongation, at which it becomes stationary.*

XI. Kepler, in the Rudolphine Tables, chapter 24, precept 104, does not indeed claim in this matter geometric [Greek: ἀκριβείαν (akribeian, exactness)], but the nearest approach to it, and a greater one than Apollonius of Perga attained, who touched on very few of the many causes that produce the variety of the stations. Three things, therefore, he presupposes as prepared and known. The *first* is the ratio of the Sun's temporary distance, on the one hand from the planet at the curtate interval, on the other from the Earth. The *second* is the ratio of the daily arcs of the eccentric, both of the planet, which the anomaly itself will indicate, and of the Sun, likewise known approximately from the anomaly. The *third* is that the rays drawn from the Sun to the planet and to the Earth form, with the subtenses of their daily arcs of the eccentric, outward angles greater than a right angle by the amount of each one's semidiurnal arcs; which, however, is not exactly true except in the apsides themselves.

On these assumptions, let S in this small scheme be the Sun, O the Earth, and A one of the superior planets; or, conversely, let A be the Earth and O one of the inferior planets. Let AD be the daily arc of the eccentric of one superior planet, but OP that of the inferior one, and greater; and to the visual line drawn AO let a neighbouring parallel CP be drawn. Now, the ratio is given of the distance AS of the Sun from the superior planet to SO, its distance from the Earth O; or of the Sun's distance SA from the Earth to SO, its distance from the inferior planet. The ratio of the daily arc AD to OP is given besides. So, drawing from A the line AE parallel to SO and AB parallel to OP, as AS is to SO, so will AC be to AE, that is, to OI, which is equal to it; and therefore the ratio of these will be known. In the same way, since AB is equal to OP, therefore the ratio of DA to AB will be known. Finally, since the triangles ASD and OSP are supposed isosceles (since AD is to be conceived as the chord of the arc AD described from S, just as OP is the chord of the arc described from S), and the angle OSP is greater than ASD, therefore the angle SOP is smaller than SAD, and the angle CAD is greater than IOP, or EAB; and each is greater than a right angle by the amount of the semidiurnal arcs. Now, since the angle of commutation is to be sought, through which the planet becomes stationary, and its complement to two right angles is the angle ASO, or CAE, seek therefore from the givens the angle CAE, starting from the position of ACB itself as known.

Diagram. Set into the left column, with the text running down its left side. A long vertical line runs from C at the top through A to S at the foot, ending a little below S. Short lines near the top join A to E, D and B. Two long lines slant down to the right, as the text's parallels AO and CP: one starts above and to the left of A, passes by A and O and runs on past them; the other runs from C through E, D, B and I to P. A line rises from S through O to I, and a short line joins O to P.

- On the vertical line: C (top), A (left of the line), S (foot)
- Near the top of the slanting line from C: E, D, B
- Lower group: I, O, P

Kepler gives an example of this calculation in the same Rudolphine precept 104. To it he adds precept 105, for finding more easily, but approximately, from small tables, both angles of commutation at which, for any one ratio of the distances, the first and second stations occur; and precept 106, for distinguishing the first station from the second, and for recognising whether the daily arc and the distance of the Sun from the Earth have been taken correctly. These precepts bear on the use of the tables. It is now convenient to append from the Rudolphine Tables the angle of commutation determining the points of the stations on either side, at four notable places of the eccentric.

Angle of commutation for the arcs of the stations, from the Rudolphine Tables. Four positions of the eccentric anomaly.

	Stations	0	90	180	270
♄	1	113° 48′	115° 27′	116° 53′	114° 37′
♄	2	113° 57′	114° 47′	116° 50′	115° 24′
♃	1	123° 57′	126° 22′	128° 15′	125° 38′
♃	2	124° 54′	126° 24′	127° 15′	125° 41′
♂	1	157° 40′	164° 0′	170° 8′	162° 46′
♂	2	158° 13′	164° 23′	169° 49′	162° 27′
♀	1	167° 47′	167° 58′	166° 55′	166° 43′
♀	2	167° 58′	167° 35′	166° 43′	167° 7′
☿	1	153° 48′	144° 58′	136° 46′	145° 19′
☿	2	154° 10′	145° 9′	136° 23′	146° 28′

SCHOLIA.

Scholia.

[Margin: Kepler's encomium of Copernican astronomy in the stations, etc.] I. *On the occasion of the stations and regressions, Kepler extols Copernican astronomy, both in chapter 1 of the* Mysterium Cosmographicum *and in the* Epitome Astronomiæ, *p. 734, because, while the old astronomy multiplies epicycles but is silent, or merely mutters and stands astonished, and does not explain the causes of the stations and regressions, the far simpler Copernican hypothesis so saves all this by the single motion of the Earth about the Sun, added to the eccentric motions, as to state the causes of these variations. But, leaving simplicity aside for now, this way of commending from here the hypothesis of the moved Earth does not seem to me so solid. For either it is judged absurd that these five planets appear stationary and retrograde, and this by no means, since they appear so in any hypothesis, and the undoubted phenomena demand it; or that they so appear as to be so in reality in the heaven, and not only optically, that is, as to our judgement resting on the eyes, and this does not happen even in the Ptolemaic hypothesis, for in it both the centre of the epicycle in the circumference of the eccentric orb and the planet in the circumference of the epicycle move perpetually in the order of the signs. Or, finally, it is absurd that these planets are at times carried in quarters contrary to and different from those towards which the centres of the epicycles tend, and that it comes about from this that, if the lines*

of apparent motion under the zodiac are considered, they at times appear to stand still, at times to go retrograde: this very thing necessarily happens, in its own way, in the Copernican hypothesis, in which whatever is said of the Ptolemaic epicycles is transferred to the annual orb. For Kepler admits in the same Epitome, *p. 738, that the Earth in its annual orb runs with the planet now from the same quarter, now from the opposite one; and that, although both it in its orb and the planet in its own always go on in the order of the signs, nevertheless it is at times carried towards a quarter from which the planets appear to recede; as in the figure of number 3: if the Earth goes from M through a to B, and the superior planet from the point b to m, it will appear to go in the opposite quarter. I would not deny, however, that this discrepancy, such as it is, of the motion of the epicycle from the motion of the planet in the epicycle arises in the Ptolemaic hypothesis from the planet's intrinsic system, but in the Copernican from an extrinsic one, since the annual orb is extrinsic to the system of the other planets.*

[Margin: Euclid's optical theorem for stations and retrogradations.] II. *Since Euclid demonstrated in proposition 54 of the* Optics *that if some things are carried at unequal speed, and among them also the eye, those that are carried with the eye at equal speed appear to sta-*

nd; those that go more slowly than the eye, to fall back in the opposite direction; those that go faster, to run ahead of the eye. This is what happens with ships under way, which appear to stand still to an eye travelling along with them, to fall back from an eye going faster, or to fly past an eye going more slowly. Nevertheless Kepler admits in the *Epitome of Copernican Astronomy*, p. 739, that this cannot be applied so conveniently to setting out the optical causes of the stations and retrogressions that occur on circular courses, on which the planet and the eye sometimes do not go the same way, as Euclid supposes in his theorem.

III. [Margin: Aspects at which stations occur.] To someone asking why Saturn, between the quadrate and the trine aspect of the Sun but nearer the quadrate, Jupiter at about the trine, and Mars beyond the trine, become stationary both before and after, Kepler replies in the *Epitome of Copernican Astronomy*, p. 744. This happens, he says, because if a superior planet made no progress in its eccentric, it would appear to stand at the points of contact with the annual orb, at which a right angle is formed by two lines, one drawn from our sight, or from the centre of the Earth, to the Sun, the other to the planet, and so very nearly at the quadrate aspect itself. But since in its eccentric Saturn nevertheless progresses less than Jupiter, and Jupiter less than Mars, the two parallel visual lines that produce the appearance of a station recede from the points of contact toward the perigee, that is, from the quadrate aspect: less in Saturn, more in Jupiter, and more still in Mars. He adds, however, that if the planet's daily arc were as great as that of the Earth, or of the annual orb, no station would appear except at the very opposition to the Sun.

IV. [Margin: Comparison of the arcs of regression.] But how is it that Saturn runs through the smallest arc of the ecliptic in its regression, Jupiter a moderate one, and Mars the greatest? Copernicus alone (says Kepler in the

Epitome of Copernican Astronomy, p. 747) uncovers the cause of this, while the old astronomy mutters and introduces unequal epicycles with no cause indicated for their size. In Copernicus's hypothesis, however, this comes about because Saturn is very far from the annual orb, Jupiter nearer, and Mars moves closest to it. So if someone on a higher planet were to look down on the arc of the annual orb that bears on the regression, it will seem very small to him, even if the planet were at rest. But into this cause there enters the far swifter motion of the Earth, beyond the motion of the higher planet.

V. [Margin: How conjunctions of the three planets, direct or retrograde, are possible.] Truly astonishing, and most shrewdly tracked down from a comparison of the motions and asserted by Kepler in the *Epitome of Copernican Astronomy*, p. 749, is this: that with Saturn direct, Jupiter and Mars cannot be conjoined to it unless they too are direct; nor can Saturn or Jupiter be conjoined to Mars when it is retrograde, unless they too are retrograde. But with Saturn retrograde, Jupiter and Mars can be conjoined to it while direct; and with Jupiter retrograde, Mars can be conjoined to it while direct.

CAPVT V.

Chapter 5.

De Planetis Orientalibus Occidentalibuſue, Auctis Minutiſue lumine, Hypaugis ſeu Combuſtis, Emerſionibus & Occultationibus, Arcuque viſionis ad has requiſito.

On eastern and western planets, planets increased or diminished in light, hypaugi or combust planets, emersions and occultations, and the arc of vision required for them.

I. Planets are called eastern [ORIENTALES] not because they stand away from the Sun in the order of the signs, but on the contrary when they precede it in the zodiac, because they rise above the horizon at night, that is, between sunset and sunrise. Their setting, however, the day conceals, since the Sun rises before they set. And because they can usually be seen in the morning before the Sun has risen, they are called morning planets [*Matutini*], and in Greek Ἠῷοι or ἑῷοι [Greek: ēōioi, heōioi, "of the dawn, of the morning"]. Conversely, they are called western [*Occidentales*] when they stand away from the Sun in the order of the signs by less than a semicircle, because they set at night and the day hides their rising. At that time they are usually seen above the horizon after the setting of the Sun, [Margin: Which planets are eastern and morning, which western and evening.] and for that reason they are called evening planets [*Vespertini*], and in Greek ἑσπέριοι [Greek: hesperioi, "of the evening"]. More briefly, from Magini, book 2 of the *Theoricae*, ch. 14, and Canon 98 of the *Secunda Mobilia*: they are called eastern and morning when they rise above the horizon in the morning before the Sun, and western and evening when they set or go down below the horizon in the evening after the setting of the Sun. And so all, together with Peuerbach on the affections of the

planets, and especially Magini in the places cited and Kepler in the *Epitome of Copernican Astronomy*, book 6, p. 823.

II. [Margin: Order of these phases in the three superior planets.] The three superior planets, however, like the fixed stars too, because they are slower than the Sun, turn at the middle of direction, that is, at the moment of conjunction with the Sun, from western into eastern, or from evening into morning. They remain so from that moment until the diametrical opposition to the Sun, whether or not they are visible to us because of the nearby brilliance of the Sun; that is, they are such from the time they descend from the apogee of the epicycle to the mean longitudes, and from there to the perigee. By contrast, from the moment of opposition to the Sun and the middle of retrogradation, at which they become ἀκρονύκτοι [Greek: akronyktoi, "acronychal"], that is, shining at both ends of the night and so lasting the whole night, they turn from eastern and morning into western and evening as far as the conjunction, whether or not they appear to us near the conjunction; and this happens while they climb from the perigee of the epicycle to the apogee. [Margin: Order in the inferior planets.] But Venus and Mercury, at the moment of perigee conjunction with the Sun, that is, at the middle of retrogradation, become morning and eastern, and they remain so until the following conjunction with the Sun, which is however an apogee one, that is, until the middle of direction. Then, descending from the apogee of the epicycle, they become western and evening, as far as the perigee of the epicycle. See a synopsis of these in the following little table.

Eastern, morning, and preceding the Sun are:

♄ ♃ ♂ – from ♂ to ☍, with ☉, in the left semicircle of the epicycle.

☿ and ♀ – from perigee ♂ to apogee ♂, with ☉, in the right semicircle of the epicycle.

Western, evening, and following the Sun are called:

♄ ♃ ♂ from ☍ to ♂, with ☉, in the right semicircle of the epicycle.

♀ and ☿ from apogee ♂ to perigee ♂, with ☉, in the left semicircle of the epicycle.

III. [Margin: Increased and diminished in light.] Planets are called increased in light [*Aucti lumine*] when they recede from the Sun, if they are swifter than it, or the Sun from them, if they are slower, by as great a recession as suffices for them to be visible in spite of the Sun's brilliance. They are called diminished in light [*Minuti lumine*] when they approach the Sun, or the Sun approaches them. For this reason Venus and Mercury are at their greatest increase of light when they are at their greatest digressions from the Sun, that is, at the contacts of the visual lines with the epicycle, insofar as this increase depends on remoteness from the Sun; granted that at that time they appear only dichotomous, and that nearer the perigee of the epicycle they show a greater apparent semidiameter but a crescent narrower in breadth, and shine with greater light because of their closer approach to the Earth. But in this increase it is intensity rather than extent that is considered; otherwise the extensive increase of Venus at perigee

is by far the greatest. The three superior planets, however, are most increased in light at their oppositions, both because of their greatest recession from the Sun and because of their nearness to the Earth, since they are at the perigee of the epicycle. And if they are at the perigee of the eccentric and of the epicycle together, which in Mars happens at about the beginning of Aquarius or of Pisces, they become most greatly increased in light, [Margin: Mars taken for a comet or a new star.] so much so that Mars there has more than once been taken for a comet or a new star, as in 1529 in July and August, and in 1561, 1593 and 1608 in the same months. By contrast, at the apogee of the eccentric and of the epicycle together they become most diminished, both because of the brilliance of the Sun, with which they are conjoined beneath the zodiac, and because of their greatest distance from the Earth.

IV. [Margin: Hypaugi.] Planets are hypaugi, in Greek ὑπάυγοι [Greek: hypaugoi, "under the sun's rays"], when they are under the rays of the Sun, that is, under the brightness arising from the Sun's rays, for that is what ἀυγαὶ [Greek: augai, "rays, brightness"] means, with the Sun's brilliance overpowering our eyes so that they do not perceive the light of nearby stars. In this case they are also called combust [*Combusti*], as if the Sun were burning them up; but astronomers use the word in one sense and judicial astrologers in another. [Margin: When planets are combust.] For as Magini teaches in Canon 98 of the *Secunda Mobilia* and in the *Isagoge* to the *Ephemerides*, ch. 28, astronomers call them combust for the whole time they lie hidden under the rays of the Sun, from any occultation whatever, whether morning or evening, until their emersion from under the Sun's rays; astrologers, on the other hand, call them combust when they stand less than half of their orbs away from the Sun, before or after. But hy- paugi are so called when they are elongated from the Sun beyond half of their orbs, yet in such a way that they are enclosed within the space made up of the sum of the two orbs, namely of the Sun and of the planet. [Margin: Conjunction of the body with the ☉.] Finally, they are said to be in the heart of the Sun, or conjoined with the Sun by body, when the centre of their body is less distant from the centre of the body of the Sun than the sum of the semidiameters of the Sun and the planet; which sum does not usually exceed 19′ for the ancients, but more truly 17′. We shall set out the sizes of the orbs, or of the light, below in ch. 7, no. 4.

V. [Margin: Cause of the varying distance between emersion and occultation.] The distance of an emersion from under the Sun's rays from the adjoining occultation varies, moreover, even when the planets are compared with themselves. Kepler enumerates chiefly six causes of this variety in the *Epitome of Copernican Astronomy*, p. 836. The first is the obliquity of the zodiac. For the same star, all else being equal, if it rises in the morning in the signs that ascend straight, that is, those from the beginning of Cancer to the beginning of Capricorn through Libra, can stand much higher above the horizon, or above the circle of position of the Sun, than if it does so in the signs of oblique ascension, which are those from the beginning of Capricorn through Aries

to the beginning of Cancer. So too it stands higher, and is seen longer and better, if it sets in the evening in the signs of right descension, which are those from the beginning of Capricorn through Aries to the beginning of Cancer, than if it sets in the signs descending obliquely. The second is the obliquity of the horizon, with the addition especially of the differing thickness of the air and of refractions. The third is the difference in the motion in longitude both of the Sun and of the planets. From this it comes about that the stay of the fixed stars under the rays of the Sun, all else being equal, is shorter than that of Saturn, and Saturn's than that of Jupiter, and Jupiter's than that of Mars, because the slower a star is in its direct motion, the faster the Sun leaves it. But since Venus and Mercury are swifter than the Sun, their stay under the rays of the Sun is shorter than that of the fixed stars themselves. The Moon too, at its evening emersion, frees itself very quickly from the Sun's rays, Mercury quickly, but Venus more slowly. The fourth cause is the differing latitude of different planets, and indeed of the same planet; for since the variety of the declinations is increased by this, the first and second causes vary sensibly. The fifth is the difference in the apparent magnitude of the stars, which the differing distance from the Earth varies. The sixth is the difference of face in Mercury and Venus, proportionately as in the Moon, since they appear now horned, now gibbous, etc.

De Arcu Viſionis alijſque arcubus ad Emerſionem & Occultationem Planetarum ſpectantibus.

On the arc of vision and the other arcs bearing on the emersion and occultation of the planets.

VI. [Margin: Authors who treat of the occultation and emersion of the planets.] These arcs are treated by Ptolemy in book 13 of the *Almagest*, from ch. 7 to the end of the work, and there by Regiomontanus in the *Epitome*, book 8 from prop. 11 and book 13 from prop. 23; al-Battānī, ch. 48; al-Farghānī, differentia 26; Jābir ibn Aflaḥ, book 9 of the *Astronomia*, at the end; Wurstisen on Peuerbach's *Theoricae*, p. 288; Osualdus in the *Problemata*, prop. 68; Erasmus Reinhold in the Prutenic Tables, precept 73; Copernicus, book 2, ch. 13; Giovanni Antonio Magini in the *Theoricae*, ch. 15, and in the *Secunda Mobilia*, canon 99, and in book 8 of the *Primum Mobile*, problem 22, and in the Tables of Directions of the primum mobile, canons 20, 21 and 22; Kepler in the *Epitome of Copernican Astronomy*, book 3 from p. 369, and in the Rudolphine Tables, precept 101, or p. 75; Lansbergen in his tables, p. 160, where there is a canon of emersion and occultation without any precept; Longomontanus, book 2 of the *Sphaerics*, ch. 4, problem 3; and Denis Pétau, book 1 of the dissertations of the *Uranologion*, ch. 14. From these, after the definitions, we shall select the following problems and precepts.

1. [Margin: What is the arc of vision?] The arc of vision of a star is the arc of the vertical circle intercepted between the horizon and the centre of

the Sun at the time when the star first appears, or ceases to appear, on the horizon.

2. [Margin: Interval of emersion.] The interval of emersion of a star, with respect to the Sun and the horizon, is the arc of the ecliptic lying between the centre of the Sun and the horizon at the moment when the star, touching the horizon, first begins to appear.
3. [Margin: Interval of occultation.] The interval of occultation is the arc of the ecliptic enclosed between the Sun and the horizon at the moment when the star, touching the horizon, begins to be hidden.

In these definitions it is not so much the ascent above the horizon, or the descent below it, that is considered (for those pertain to true rising and setting), as the first and last appearance owing to recession from, or approach to, the rays of the Sun. Now, to determine the emersions and occultations of the planets, we chiefly need the arc of the ecliptic by which a star placed on the ecliptic, or reduced to the ecliptic, is distant from the Sun at the moment when it first begins to appear, or ceases to appear. But this distance varies for the 6 causes enumerated in no. 5; granted that Ptolemy, in book 13, ch. 7, reviews only three as the chief ones, namely the differing apparent magnitude of the star, the differing obliquity of the ecliptic to the horizon, and the differing latitude. These produce an incredible variety in the arcs and times of emersions and occultations in the same star, let alone in different ones. Very wisely, therefore, Ptolemy sought out a kind of mean that would always be the same with respect to the same star and in the same climate. This mean is the arc of vision. Yet to find it he used a certain definite interval of emersion and occultation, so that afterwards, by a demonstrative regress, through the arc of vision once established for each star, he could bring the remaining variety of emersions and occultations back to a geometrical rule. So let this be:

Problema 1. Arcum Viſionis in vnoquoque Climate inueſtigare.

Problem 1. To find the arc of vision in each climate.

VII. [Margin: 1. Precept.] First, following Ptolemy, book 13, ch. 7, and Regiomontanus, book 13, prop. 23, observe the moment of time at which each star first appears on the horizon, when before it did not appear although the sky was clear; or when it first begins to be hidden on the horizon, when before it was visible. For that moment of time compute the true places, both of the Sun and of the star, in longitude and latitude, and collect the distance of the star from the Sun on the ecliptic, that is, the arc of the ecliptic lying between the Sun and the ecliptic place of the star. Keep this, together with the latitude, if there is any. According to the authors already mentioned, it is best to make these observations when the Sun is in the beginning of Cancer, because at that time the air in the northern climates seems thinner and more purged, the vapours having by then been burned off, and the inclinations of

the zodiac to the horizon are moderate. Otherwise, with the horizon beset by thicker vapours, as happens at other times, especially in climates very far from the equator, it is almost impossible to see any star on the horizon itself; rather, when it first appears it is always higher than the physical or sensible horizon itself. And so as not to prepare yourself for observation in vain, consult tables or ephemerides and see whether, with the Sun about the beginning of Cancer, a conjunction of the star with the Sun is impending, at least in that month.

Further, Ptolemy, from his own observations and those of the ancient Chaldeans made in Syria and Phoenicia, that is, at the beginning of climate 4, where the longest day is $14^h\ 15^m$, and in which he says the more certain observations were made by the Chaldeans, Egyptians and Greeks; in that climate, I say, while the Sun was about the beginning of Cancer, he found the arcs of the ecliptic, or intervals, that you see in the little table. Thus with the Sun placed at the beginning of ♋, Saturn would emerge there in the morning from the rays of the Sun when it was distant from the Sun against the order of the signs by 14°, Jupiter by 12° 45′, and Mars by 14° 30′; Venus, however, in the evening by 5° 40′, and Mercury in the evening by 11° 30′.

Distances of the planets from the ☉ placed at the beginning of ♋, for the emersions of the planets in climate 4.

Planet	Distance
♄ morning	14° 0′
♃ morning	12° 45′
♂ morning	14° 30′
♀ evening	5° 40′
☿ evening	11° 30′

[Margin: 2. Precept, in the first case.] Second, look at the following figure, in which let the meridian be HVL, the horizon HCR, and the ecliptic ECL; the Sun below the horizon be at A, to which let the portion VA of a vertical circle be drawn from the vertex V; and let the planet, when it first appears on the horizon lacking all latitude, be at C. In the spherical triangle CAB, then, right-angled at B (although Ptolemy uses rectilinear triangles because of the smallness of the arcs), the arc of vision AB is sought. In that triangle the arc or base CA is given, namely the distance of the planet from the Sun, by the first precept; and from tables of the primum mobile, the angle ACB of the ecliptic with the horizon, corresponding to the place of the planet C. Therefore the side AB, which was being sought, cannot remain unknown.

Diagram. Set in the right part of the left column, with short lines of text running down its left side. A large circle carries V at the top; a small circle at its upper right, with P inside it and K just outside it to the right; E at the upper left; R at the right above the middle; L at the right just below the middle; H at the left; and at its foot a second small circle with M inside it and N just outside it to the left. A line runs from the left-hand H through C to R; a line runs from

E through C to L. An arc falls from V past B to A. From the small circle about P two lines come down close together to the inner letter H, one going on to D and the other to I. From the small circle about M two lines rise close together to G, going on to O and to F. Two letter H stand on the block, one on the large circle at the left and one inside it near B.

- Large circle: V (top), E (upper left), H (left), R (right, upper), L (right)
- Small circle, upper right: P (inside), K (outside)
- Small circle, foot: M (inside), N (outside)
- Interior: O and F (side by side, left of centre), G (below them), C (centre), H (right of centre), B (beside H), I and D (side by side, below H), A (below B)

[Margin: 2. Case.] But if the ecliptic place of the star is D, and the star is at H, having the northern latitude DH, and it is at the beginning of Cancer or Capricorn, then the arc PH, drawn from the arctic pole of the world P, will be an arc set in a straight line with the arc of latitude HD. Then in the spherical triangle PHR, right-angled at R, in which PR is given, the altitude of the pole, as I suppose, and PH, which becomes known by subtracting the obliquity of the ecliptic from the complement of the latitude if the star is at the beginning of ♋, but by adding it to the same complement if the star is at the beginning of ♑, seek the angle PHR, to which the angle CHD is equal, in the small triangle CDH, right-angled at D. In this triangle, from the given latitude DH and the angle CHD just found, seek the side CD, and add it to the observed distance DA; and so in the triangle ACB seek the arc of vision AB as before. The same procedure must be followed if the latitude of the planet appearing at G is southern, say OG, and its ecliptic place is O, but with MGO drawn from the antarctic pole. For from MG, known by subtracting the obliquity of the ecliptic from the complement of the southern latitude, etc., and from the altitude of the southern pole, one must find the angle HGM, equal to the angle CGO; with it, and with the latitude OG, in the triangle GOC, right-angled at O, you will seek the arc of the ecliptic OC, and you will take it away from the distance OCA of the planet from the Sun, so that there remain both the arc CA and the point C of the ecliptic, and from that the angle of the ecliptic and the horizon (which, however, you can gain in solving the said triangle COG), so that with its help and that of the remaining distance CA you may obtain the side AB.

[Margin: 3. Case.] And if the star H, endowed with a northern latitude, is not at the beginning of Cancer or Capricorn, draw from the pole of the ecliptic K the arc KHI through the star, for it will be an arc of a circle of latitude; and connect the poles of the world and of the ecliptic by the arc PK, which is as great as the obliquity of the ecliptic. Further, let the declination of the star be supposed known or found, which in this case will differ from the latitude. With these things laid down, in the triangle PHK, PK is given, as I said, and PH, the complement of the declination of the star, and the angle KPH, which the distance of the true place of the star from the beginning of Cancer or Capricorn

makes plain. Therefore, by the science of oblique spherical triangles, the angle PHK will not remain hidden. Next, in the triangle PHR, right-angled at R, from the given altitude of the pole PR and the complement of the declination PH, you will find the angle PHR; subtract from it the angle PHK, and there remains the known angle KHR, to which the angle IHC is equal, in the small triangle CIH, right-angled at I, in which the latitude IH is given. Therefore the arc CI will not remain hidden; added to the observed distance IA between the star and the Sun, it will make up the arc CA. In this way the point C of the ecliptic, set on the horizon, will be known, and from that the angle RCL of the ecliptic and the horizon; with this and with the arc CA, in the triangle CAB, right-angled at B, seek the arc of vision AB. From this you see how to proceed if the latitude is southern and the star is away from the beginning of Capricorn or Cancer. For in the triangle NGM, from the complement of the declination MG, the arc MN, as great as the obliquity of the ecliptic, and the angle NMG, known from the distance of the star from the beginning of Cancer or Capricorn, you will find the angle NGM. And in the triangle MHG, right-angled at H, from the altitude of the southern pole HM and the complement of the declination MG, you will find the angle MGH, from which you will take away the angle MGN; and there will remain the angle NGH, that is FGC. With this and with the latitude GF, in the triangle GFC, right-angled at F, you will seek the arc FC, which you will subtract from the distance FA, and you will carry out the rest as above. Remember, however, that in all these cases, for the morning emersions of Saturn, Jupiter and Mars, the point C of the ecliptic is rising, but for the evening emersion of Venus and Mercury it is setting, etc.

VIII. In this way Ptolemy, in book 8, ch. 6, from observed emersions of the fixed stars, and in book 13, ch. 7, derived the arcs of vision written below for climate 4, permanently serving for the same star.

Arcs of vision in climate 4, from Ptolemy.

Fixed stars (magnitude of the fixed stars)	Arc	Planets	Arc of vision
1	12° 0′	♄	11° 0′
2	13° 0′	♃	10° 0′
3	14° 0′	♂	11° 30′
4	15° 0′	♀	5° 0′
5	16° 0′	☿	10° 0′
6	17° 0′	☩	
7	18° 0′	☩	

[Margin: Whether the arcs of vision are correct for any climate.] Although the already mentioned arcs hold for the end of the third and the beginning of the fourth climate, Erasmus Reinhold used them at the end of the Prutenic Tables as if they held in every climate, though promising in precept 73 fuller canons for other climates. Magini employed the same arcs both at the end of the *Secunda Mobilia* and in the Tables of Directions from p. 195, for altitudes of

the pole of 43°, 44° and 45°; granted that, by astronomical rigour, the greater the declinations of the fixed stars, the smaller the arc of vision required for them to appear. But planets too, at a greater altitude of the pole, require a smaller arc of vision in order to emerge, because the higher the pole, the greater the ortive amplitude of the stars, and the farther they stand from the Sun while emerging, and so the less the brightness of the horizon harms them. So Kepler observes in the Rudolphine Tables, p. 76, adding that for the morning emersion or evening occultation of Venus and Mercury a far smaller arc of vision is required, because they are then at perigee and appear larger, than for the evening emersion or morning occultation, because they are then at about the apogee of the orb or epicycle and diminished in light. [Margin: Venus and Mercury demand smaller arcs of vision.] So it comes about that Venus is often visible by day too, with the Sun above, let alone below, the horizon. See also Scholium 1. In this part, therefore, astronomy is still defective, because suitable observations of emersions and occultations, made through all climates, are lacking. In the meantime we are forced to use the arcs found by Ptolemy, as not introducing a very sensible difference in other climates.

Problema 2. Dato Arcu Viſionis &c. Inuenire Diſtantiam Stellæ à Sole in Ecliptica requiſitam ad Emerſionem aut Occultationem, ſeu Interuallum emerſionis & occultationis.

Problem 2. Given the arc of vision, etc., to find the distance of a star from the Sun on the ecliptic required for emersion or occultation, that is, the interval of emersion and of occultation.

IX. This problem likewise has been pointed out to us by Ptolemy, book 13, ch. 7; Regiomontanus in the *Epitome of the Almagest*, book 8, props. 13, 14 and 15; Jābir ibn Aflaḥ, book 9; Copernicus, book 2, ch. 13; Magini, book 2 of the *Theoricae*, ch. 15, and in the *Directions*, canon 20; and Daniel Santbech in *On the Observations of Phenomena*, prop. 21. Although we touched on this problem in book 6, ch. 20, at the end of no. 15, on the occasion of the rising and setting of the fixed stars, it still should not have been missing here; indeed, it did not seem good to restrict it to the planets alone. Further, the arc of vision is supposed known, with the Sun placed at the beginning of Cancer, as Ptolemy did, or elsewhere once and definitely, from the then-observed inter-

vals of emersions and occultations: here, however, the same intervals are sought for whatever place of the zodiac the Sun and the other stars occupy. Such a place is supposed given or assumed, and known likewise is the latitude, if there is any, in kind and in quantity, together with the altitude of the pole. On these assumptions, let everything in the preceding figure be as set out there, and [Margin: Case 1.] let the planet first be at C, with no latitude at all: for since its place on the ecliptic is given, there will also be given from the tables of the primum mobile, for the given altitude of the pole, the angle ECH or RCL, namely that of the ecliptic and the horizon. Therefore in triangle ABC,

right-angled at B, from the angle at C and the given arc of vision AB, you will gather by means of trigonometry the base AC, which will be the interval of emersion or occultation, according as the star C is taken to be ascending or descending.

[Margin: Case 2.] Second, let the star be at H, having northern latitude, and let its place on the ecliptic be D or I; for if it is at the beginning of Cancer or Capricorn, the arc of declination will coincide with the arc of latitude, namely the arc PHD. Accordingly, in triangle PHR, right-angled at R, from the given altitude of the pole PR and the complement of declination PH, seek the angle PHR, that is, CHD; with this, and with the latitude DH, in triangle CHD, seek the side CD and the angle HCD. Then with this angle C and with the arc of vision AB, in triangle CAB, right-angled at B, seek the base CA, and subtract from it the arc CD; for the arc DA will be left as the interval of emersion or occultation that was wanted.

[Margin: Case 3.] Third, let the star be at H, possessed of northern latitude, and [?] far from the beginning of Cancer or Capricorn. With the arc PHD of declination and the arc KHI of latitude drawn, first in triangle PHR, right-angled at R, from the pole's altitude PR and the complement PH of the star's declination, find the angle PHR. Then in triangle PHK, in which PK is the distance of the poles, known from the obliquity of the ecliptic, PH is known from the complement of declination, and the angle HPK from the distance of the star's true place from the beginning of Cancer or Capricorn, seek the angle PHK, and subtract it from the angle PHR, so that what remains is KHR, that is, CHI. With this, in triangle CHI, right-angled at I, and with the latitude HI, determine both the side CI and the angle at C. Finally, with this angle and with the arc of vision AB, in triangle CAB, right-angled at B, seek the base AC; take from it the arc CI, and the interval of emersion or occultation IA will be left. You will proceed in the same way if the star's latitude is southern, namely GO or GF, with the arcs MGO drawn from the south pole of the world and NGF from the south pole of the ecliptic.

Problema 3. Idem ope Tabularum adipiſci.

Problem 3. To obtain the same with the aid of tables.

X. By this technique Ptolemy, book 13, ch. 8 and 9, showed us the intervals of emersions and occultations for the five planets, placed at the beginning of the signs, for a pole altitude of about 34°. In imitation of him Reinhold, at the end of the Prutenic Tables, gave a similar small table for the beginnings of the signs, for what may be, I do not know, the pole altitude of Frombork, 54° 22′. Magini transcribed it without any correction in the second volume of the *Secunda Mobilia*, p. 727, but supposing it to hold for the 3rd climate; and so did Lansbergen in his tables, p. 160, although it does not fit so exactly the pole altitude of Goes, which is 51° 31′; and so did Daniel Santbech, prop. 21. The use of these tables is taught by Reinhold in precept 73 and by Magini in the *Secunda*

Mobilia, canon 99. But the same Magini built another table, for pole elevations of 43°, 44° and 45° and for all the degrees of the signs, which is found near the end of the tables they call those of the Directions of the primum mobile, and in the same volume, canons 21 and 22, he taught its notable use. Kepler, however, left out tables of this kind, which take almost no account of latitude and are adapted to only one climate or another. Yet so as not to seem to have despised this precept, he adopts the arc of vision handed down by Ptolemy, and teaches how to find the arcs and times of emersions or occultations through the table of the rising angle [tabulam anguli orientis], in precept III of the Rudolphine Tables. But we reserve these precepts for the practical part of this first volume, in which the already mentioned tables will have to be set out.

Problema 4. Dato Arcu Viſionis Tabulas Interualli emerſionis atque occultationis ad quamcumque Poli eleuationem conſtruere.

Problem 4. Given the arc of vision, to construct tables of the interval of emersion and occultation for any elevation of the pole.

XI. What Reinhold promised at the end of the precepts, and Magini delivered for three pole altitudes, 43°, 44° and 45°, almost at the end of the tables of the primum mobile, you will be able to carry out for any pole altitude, not only at the beginnings of the signs but also at any degrees of the signs, by a very short precept which Regiomontanus (book 8 of the *Epitome of the Almagest*) and Daniel Santbech (*On the Observation of Phenomena*, prop. 21) have pointed out to us. It runs as follows. Multiply the right sine of the arc of vision by the whole sine, and divide the total by the sine of the angle of the ecliptic and the horizon that belongs to the beginning of the given sign or to any degree of it. The quotient will be the sine of the arc of apparition or occultation.

EXEMPLVM.

Example.

Let the Sun be in ♊ 2° 26′. To these, under a pole altitude of 48°, there corresponds an angle of the ecliptic and the horizon of 22° 41′, whose right sine is 38564. Now let the arc of vision of Mercury be 10°, whose sine is 17365. Multiplied by the whole sine, 100000, it makes 1736500000. When this total is divided by 38564, there remains 45028, that is, the sine whose arc, 26° 46′, shows the distance of this planet from the Sun on the ecliptic required for it to be able to appear at rising. But in this precept the planet is supposed to have almost no latitude.

Problema 5. Dato interuallo Emerſionis aut Occultationis, Diem emerſionis aut occultationis coniÿcere, dummodo Digreſſio Planetæ à Sole non ſit minor illo interuallo.

Problem 5. Given the interval of emersion or occultation, to infer the day of emersion or occultation, provided the planet's digression from the Sun is not smaller than that interval.

XII. First, from the true argument, or equated anomaly of the orb, that is, the distance of the planet from the true apogee of the epicycle, gather the kind of emersion or occultation: for if a superior planet is a little after the apogee, a morning emersion is near; if a little before the apogee, an evening occultation. An inferior planet, however, a little after the apogee is nearer to an evening emersion, and a little before the apogee to a morning immersion; but a little before the perigee it is nearer to an evening occultation, and a little after the perigee to a morning emersion. [Margin: Mercury not visible even at its greatest digressions.] Second, from the place of the Sun and of the planet in the zodiac, which the ephemerides supply, draw out the distance between them that answers to the interval of emersion or occultation; for on the day when it is equal to the interval, the planet will first begin to emerge or to be hidden; but as long as it is smaller than that interval, the planet will lie hidden under the Sun's rays. It happens not rarely, moreover, that Mercury's greatest digression from the Sun is smaller than that interval, and for that reason it cannot appear.

For the remaining problems bearing on risings and settings, go back to book 6, ch. 21.

SCHOLIA.

Scholia.

I. It was said above, toward the end of no. 8, that Venus often needs no arc of vision, or depth of the Sun below the horizon, in order to be seen, since she is visible in the daytime, and has rather often been observed by us, even with the naked eye, from about her rising to her setting, especially around her greatest digressions from the Sun; but at those times she is very far from the Sun's rays. We now put forward another marvel peculiar to her, most worthy of note, which befalls her from great latitude, especially northern. [Margin: Venus appearing in the evening and in the morning on the same day.] It is this: on the same natural day she is seen in the evening shortly after sunset, and is Hesperus, and on the following morning she appears in the morning shortly before sunrise, and is soon Lucifer. That this comes about by favour of northern latitude, nearly at the perigee of the epicycle, is taught by Maurolico, dial. 3 of the *Cosmographia*, p. 101, and, following Regiomontanus, by Reinhold on Peuerbach, p. 153, who adds that she is of this kind around the beginning of Aries at the perigee of the epicycle, and awaits an example of this phase from the ephemeris for the year 1547, about the beginning of March. This double

phase is helped, however, by the size of the apparent diameter in altitude, of which we gave a specimen in this book, sect. 1, ch. 2, no. 4. It is helped too by the outstanding brilliance of Venus, on which there is this from Pliny the Elder, book 2, ch. 8: "Now in size it stands beyond all other stars, and it is of such brightness that shadows are cast by the rays of this one star." [Margin: The brilliance of Venus producing a shadow.] From this, I would believe, Martianus Capella, book 8 of the *De Nuptiis Philologiae*, took this encomium of Venus: "Alone of the five stars she casts a shadow, as the Moon does; and alone, long visible, she does not yield to the brightness of the emerging Sun." Granted that Jupiter too may throw some shadow behind opaque bodies. It is no wonder, then, if, with a northern latitude of almost 9°, the greatest apparent size at the perigee of the epicycle, and the brilliance of this star all working together, Venus in the year 1587 was seen from February 21, on which she was an evening star, and on the following days, while on February 24 she was by then a morning star: seen, I say, both by Tycho, as Longomontanus testifies in book 2 of the *Theorica*, ch. 18, and by Mästlin, as he himself states in his additions to the *Narratio prima* of Rheticus, p. 136 in my copy. Kepler had this in view in the Rudolphine Tables, p. 71, where, at the foot of the latitudes of Venus, he says as follows: "At times ♀ is seen in the morning and in the evening together for two or three days, emerging in the morning before she is hidden in the evening." And in the *Epitome of Copernican Astronomy*, p. 838: "Venus, which often lies hidden for a very long time, on other occasions, namely when retrograde in Pisces, does not set in the evening until after conjunction with the Sun in longitude, and rises in the morning before conjunction, and so emerges in the morning before she is hidden in the evening, so that on the same day she is visible in the evening and in the morning, notwithstanding that she is thinned to a horn."

II. [Margin: The twofold emersion and occultation of Mars.] The same author, on the same page of the *Epitome*, observes that Mars sometimes rises twice as a morning star out of the Sun's rays in the intervening time, though after conjunction with the Sun it is hidden again for a while; and that sometimes too it is hidden twice in the rays before conjunction with the Sun, an irregular rising having occurred in between. [Margin: The emersion of Mercury in ♉ or ♏: when and where is it impossible?] He observes also that Mercury in our hemisphere, as a morning star in ♉ and as an evening star in ♏, although digressed very far from the Sun, nevertheless does not emerge from the Sun's rays, and so offers itself to our sight more rarely, while in the southern hemisphere it is then seen very clearly; which Ptolemy too had noted long before, book 13, ch. 8.

III. [Margin: On the arc of vision for the smallest stars, and on refraction.] The arc of vision for the smallest stars is about as great as the depth of the Sun at the beginning of morning twilight and the end of evening twilight, which is not everywhere or always 18°, as is clear from what was said in book 1, ch. 31.

And since the horizontal refraction is about as great in the stars as in the Sun, there is no need to correct the already mentioned arcs on its account.

CAPVT VI.

Chapter 6.

De Ortu & Occaſu Quinque Minorum Planetarum.

On the rising and setting of the five minor planets.

I. We have set out abundantly the Latin and Greek names and the kinds of risings and settings, and of the various configurations pertaining to them, in book 1, ch. 32, but more accurately in book 6, ch. 21, on the occasion given by the fixed stars; though schemes placing those kinds before the eyes are found in book 1, ch. 32. What concerns heliacal risings and settings, that is, emersions from the Sun's rays and occultations made at the horizon, we have already pursued in the preceding chapter, though referring the reader to book 6, ch. 20, for many problems. [Margin: Authors who treat of the rising and setting of the planets.] About the remaining kinds of rising and setting, namely the cosmical and the acronyctal or acronychal, the authors have not been as careful as about occultations and apparitions; those authors I listed in ch. 5, no. 6. The same authors or others who also treat of these kinds of risings and settings belonging to the planets are to be listed, namely: Ptolemy, book 8, ch. 4, and there Regiomontanus, applying that passage to the planets, especially the superior ones; al-Battānī, ch. 48 and 49; al-Farghānī, differentia 24 and 26; Jābir ibn Aflaḥ, book 9; Peuerbach in the *Theoricae Planetarum*, on the affections, and there Reinhold from p. 149 and Wurstisen from p. 282; again Reinhold in the Prutenic Tables, precept 73; Copernicus, book 2, ch. 13; Alessandro Piccolomini in the *Theoricae*, book 2, ch. 6; Magini in the *Theoricae Planetarum*, book 2, ch. 14, and in the *Secunda Mobilia*, canons 98 and 99, and in the Tables of Directions, canons 19, 20, 21 and 22; Kepler in the *Epitome of Copernican Astronomy*, book 6, p. 834; Barozzi in the *Cosmographia*, p. 133; Origanus, part 2 of volume 1 of the *Ephemerides*, ch. 8. Nor are the things Pliny the Elder touches on obscurely, book 2, ch. 15 and 17, without relevance here. From these authors we shall set out briefly which kinds of rising and setting belong to these five planets, in what order, and at what places on the epicycle, and we shall not be reluctant to repeat here some things handed down in book 6, ch. 21.

Species & Ordo Occaſuum & Ortuum, tribus Superioribus Planetis conuenientium.

Kinds and order of the settings and risings belonging to the three superior planets.

For the Greek names see book 6, ch. 21, p. 467; for the schemes see book 1, ch. 32.

II. [Margin: True morning rising and true evening setting.] First. They rise in the morning together with the Sun, by true or cosmical rising, and together with it they set in the evening, likewise by true setting: that is, at the same moment as the Sun they ascend above the horizon in the morning and descend below it in the evening, since they are conjoined with it in the same place of the zodiac. This happens to them at the apogee of the epicycle and in the middle of the course of direction, where they are swiftest, but at that time they lie hidden under the Sun's rays, and so neither of the two is apparent. Once the ♂ with ☉ is complete, they become morning and eastern stars.

[Margin: Heliacal morning rising.] Second. They rise heliacally in the morning, or emerge from the rays of the Sun as it leaves them behind, since they are slower than it. This rising is called the apparent morning or heliacal rising: namely when they first begin to appear on the horizon in the morning before sunrise, which happens a little after the apogee of the epicycle. After this, every day at dawn they appear earlier and earlier, that is, by more moments or even hours before sunrise; and when the Sun rises they stand higher and higher above the horizon, but almost at once they vanish, although Jupiter around the quadrate aspect is visible for nearly a whole quarter of an hour after sunrise. They continue as morning and eastern stars through the whole left semicircle of the epicycle, and appear at each dawn, thereafter higher and higher above the eastern horizon, and finally close to the western horizon.

[Margin: True evening rising and true morning setting.] Third. They rise in the evening acronychally, by true evening rising: that is, they ascend above the horizon as the Sun descends, and on the same natural day they set cosmically, or by true setting in the morning, namely descending below the horizon at the same moment at which the Sun ascends. They are then called [Greek: παννύχιοι (pannychioi, "all-night")] or night-long, and [Greek: ἀκρονύκτοι (akronyktoi, "acronychal")], that is, shining at both ends of the night, or rather from one end to the other, as happens to the full Moon; and Ptolemy in book 10 calls this phase [Greek: ἀκρόνυκτον διάμετρον (akronykton diametron, "acronychal diameter")]. This happens to them in opposition with the Sun, namely when they are at the perigee of the epicycle and in the middle of retrogradation, that is, slowest as to motion in the order of the signs, though swiftest against the order of the signs. Once the ☍ with ☉ is complete, they become evening and western stars, and remain so through the whole right semicircle of the epicycle; every night they appear after sunset in the east, raised higher and higher above the horizon, then on the meridian, and finally in the west.

[Margin: Heliacal evening setting.] Fourth. They set in the evening by apparent or heliacal setting, hiding themselves under the rays of the Sun, which, as they are slower than it, overtakes and forestalls them; and whereas on the preceding days they appeared after sunset on the western horizon, they cease to appear there and lie hidden under its blaze until their morning emersion. This occultation happens a little before conjunction with the Sun and the apogee, as

the emersion does a little after. And at emersion they [Greek: ἐπιτέλλουσιν (epitellousin, "rise")], at occultation they [Greek: κρύπτονται (kryptontai, "are hidden")], as the Greeks say.

They do not have the remaining kinds of apparent rising and setting, that is, the evening heliacal rising and the morning heliacal setting, because they are slower than the Sun and neither free themselves from it nor overtake it, but are rather forestalled by it and uncovered by its departure.

Specis & Ordo Ortuum & Occaſuum Mercurio & Veneri conuenientium.

Kinds and order of the risings and settings belonging to Mercury and Venus.

III. [Margin: The earlier true morning rising and true evening setting.] First. They rise cosmically, or, by true rising in the morning, ascending above the horizon together with the Sun; and on the same day they set with the same Sun in the evening, by true setting, descending below the horizon together with the Sun, since they are conjoined with it above, at the apogee of their epicycle, and so in the middle of their direction and swiftest. But neither their rising nor their setting is apparent, because they lie hidden under the Sun's blaze. Once the apogee ☌ with ☉ is complete, they become evening and western stars.

[Margin: Heliacal evening rising.] Second. They rise heliacally in the evening, or by apparent evening rising, emerging by their own speed, being direct, and a little after the apogee of the epicycle, from the Sun's rays; after its setting they begin to appear on the western horizon. After that they continue as evening and western stars through the whole left semicircle of the epicycle, and as the Sun sets, or when it has set a little before, they are seen each evening higher above the western horizon, up to their greatest digression from the Sun. After this they approach the first station, and then, lower and slower, having become retrograde, they go to meet the Sun, which will at last overtake them.

[Margin: Heliacal evening setting.] Third. They set heliacally in the evening, or by apparent evening setting hide themselves under the Sun's rays, and in turn are hidden by it, which happens to them a little before the perigee of the epicycle, and when retrograde.

[Margin: The later true morning rising and true evening setting.] Fourth. They again rise cosmically in the morning, and by true setting set in the evening, since they are conjoined with the Sun again, but below, at the perigee of the epicycle and in the middle of retrogradation, in which they are slowest as to motion in the order of the signs but swiftest as to motion against the order of the signs. Once the perigee ☌ with ☉ is complete, they at once become morning and eastern stars.

[Margin: Heliacal morning rising.] Fifth. They rise heliacally in the morning, that is, by apparent morning rising, emerging from the Sun's rays, or rather left behind by the Sun as the swifter, since they are themselves retrograde. After that they continue as morning and eastern stars through the whole right

semicircle of the epicycle, and as the Sun rises or is soon to rise they become higher every day, up to the greatest digression from the Sun, which follows the second station. From there, with hurried course, they become every day nearer the horizon, but swifter, and hastening toward the apogee.

[Margin: Heliacal morning setting.] Sixth. They set heliacally in the morning, or by apparent morning setting hide themselves under the rays, a little before the apogee ☌ with ☉; and whereas on the preceding days they appeared at dawn on the horizon, they cease to be seen there, and lie hidden under its splendour until their evening emersion.

[Margin: Corollary.] They do not have the remaining kinds of true rising and setting, that is, the true evening rising and the true morning setting, because they cannot digress from the Sun by a whole semicircle of the heaven, nor come into diametrical opposition with it. Therefore the three superior planets have all four kinds of true rising and setting, but lack two kinds of the apparent; the two inferior planets, by contrast, have all four kinds of apparent rising and setting, but lack two kinds of true rising and setting.

But so that you may have a shorter summary, look at the following small table, in which the letter D signifies a direct planet and R a retrograde one.

Synopsis of the order of the already mentioned phases.

Order	♄ ♃ & ♂	On the epicycle	
1	True morning rising	At apogee and in	D
2	True evening setting	☌ with ☉	D
3	Heliacal morning rising	A little after ☌ ☉	D
4	True evening rising	At perigee and in	R
5	True morning setting	☍ with ☉	R
6	Heliacal evening setting	A little before ☌	D
	They lack the heliacal evening rising and the heliacal morning setting		

Order	♀ & ☿	On the epicycle	
1	True morning rising	At apogee and in	D
2	True evening setting	☌ ☉ above	D
3	Heliacal evening rising	After ☌ ☉ above	D
4	Heliacal evening setting	Before ☌ ☉ below	R
5	Second true morning rising	At perigee and	R
6	Second true evening setting	☌ ☉ below	R
7	Heliacal morning rising	After ☌ ☉ below	R
8	Heliacal morning setting	Before ☌ ☉ above	D

Order	♀ & ☿	On the epicycle
	They lack the true evening rising and the true morning or cosmical setting, regularly and in most climates	

SCHOLIA.

Scholia.

I. [Margin: Notable points on the oppositions [recte: apparitions, per the book's errata] of ♂, ♀ and ☿.] It should be noted with Kepler, in the *Epitome of Copernican Astronomy*, p. 835, that sometimes in Venus when retrograde the order of the apparitions is interchanged, and other changes occur, on which book 6, ch. 21, in the Scholium, where we showed that a true evening rising and a cosmical setting can belong to Venus and Mercury. And in Mars too the order of apparitions can be interchanged. As for Ptolemy's teaching, book 13, ch. 8, that Mercury at perigee in Taurus and at apogee in Scorpio cannot be seen in the fourth climate, this does not hold for much more northerly climates, as Reinhold rightly teaches in the *Theoricae*, p. 153.

II. [Margin: Variation of the seasons from the aspects and apparitions of the planets.] Also from the same Reinhold, in the same place, p. 151, and from Ptolemy, book 1 of the *Apotelesmatica*: just as the Sun makes the four seasons of the year, so proportionally Saturn, Jupiter and Mars moisten notably from the heliacal morning rising to the morning station; from there to opposition with the Sun and the acronychal rising they heat; from there to the second station they parch; and from there to the heliacal evening setting they chill. Similarly Venus and Mercury moisten from the morning rising to the nearest station, from there to the morning setting they heat, and again from the evening rising to the other station they dry, and from there to the evening occultation they rather chill. To this opinion subscribe the author who wrote the isagoge to Abū Maʿshar, and also Pontanus [Giovanni Pontano], *De rebus cælestibus*, although the commentator of the *Quadripartitum* and Guido Bonatus [Guido Bonatti] think otherwise.

III. We gave another small table and an epigram, for distinguishing the kinds of risings and settings belonging to the planets and the fixed stars, in book 1, ch. 32.

CAPVT VII.

Chapter 7.

De Aſpectibus Planetarum cum Sole & inter ſe ſe.

On the aspects of the planets with the Sun and among themselves.

I. Two very notable things happen to the configuration of planets taken two at a time, with one another or with the Sun: one concerning the rays sent down

to the Earth, from which arise aspects or radiations; the other concerning speed or slowness in apparent motion, from which harmony arises. We have already said no little about aspects in book 1, ch. 33, where with two schematisms we instructed the beginner sufficiently. Now, however, we must take up the treatment of them again more accurately. [Margin: Authors who treat of the aspects.] They are dealt with especially by Ptolemy, book 3 of the *Harmonics*, ch. 8, and book 1 of *De Iudiciis Astrorum*, ch. 12; Firmicus, book 2, ch. 29; al-Battānī, ch. 49 and 54; Jābir ibn Aflaḥ, book 6; the author of the isagoge to the *Astrologia* of Abū Maʿshar, book 6, ch. 8; Valentinus Naiboda [Valentin Naboth] on Alcabitius [al-Qabīṣī], that is, in the *Elementa Astrologica*, p. 52 and 324; Reinhold on Peuerbach's *Theoricae* from p. 179; Wurstisen in the same from p. 299; Maurolico in the *Cosmographia*, p. 46; Magini, book 2 of the *Theoricae*, ch. 17, and in the *Isagoge* to the ephemerides, ch. 30 and 31, and in the *Secunda Mobilia*, canon 52, and in the Directions of the primum mobile, from canon 39 to 41; Langius in the *Theoricae*, ch. 15;

Origanus in volume 1 of the *Ephemerides*, part 3, ch. 6; Kepler in the *Ephemerides* from p. 27 to 38, and in the book *De Stella Nova* from ch. 7 to 10, and in the *Epitome of Copernican Astronomy*, p. 840, and in the *Harmonics*, book 4, ch. 6, and in the *Mysterium Cosmographicum*, ch. 12. From these we shall select the things more worth knowing.

Quid ſit Aſpectus, & Quæ illius conditiones.

What an aspect is, and what its conditions are.

II. [Margin: The ancient definition of aspect.] An aspect, as Naiboda defines it, is an arc of the zodiac, or an interval reckoned in the zodiac, from which the stars communicate the greatest force to one another. But by the definition of Magini in the *Directions*, canon 39, an aspect is the radiant mingling of planets and stars with one another, from certain determinate places of the circle or sky, by which their united power flows in and is applied to a certain determinate part of the Earth. But Kepler, in the *Epitome of Copernican Astronomy*, because he denies that rays sent from star to star communicate their power to it, and admits only a union of radiant power made on the Earth, defines aspect in these words: *An aspect is an angle at the Earth formed by the luminous rays of two planets, effective in stimulating sublunary nature.*

[Margin: Kepler's definition corrected.] But neither conjunction nor opposition, which he nevertheless accepts among the principal aspects, forms an angle at the Earth of itself when the conjunctions or oppositions are central, let alone when they are made σωματικῶς [Greek: σωματικῶς (sōmatikōs, "bodily")], that is, by body. Therefore it seems to me that aspect should be defined thus: *An aspect is the concurrence on the Earth of rays sent down straight from two stars, effective in stimulating sublunary nature.*

Now this effectiveness arises from the perfect proportion of the already mentioned angle to the four right angles standing around one point of the

Earth. The perfection of this proportion requires two things. 1. That the terms of the proportions be commensurable; that is, that the angle making the aspect be an aliquot part, or consist of aliquot parts, measuring the whole circle or four right angles. 2. That the arc which is the measure of this angle be cut from its circle geometrically, namely by a straight line that is expressible [effabilem], or at least knowable [scibilem].

Moreover, the regular figures that can be inscribed in a circle and are knowable are infinite, if one considers the continued division of the arcs and the doubling of the sides or chords; yet the radical ones are three, namely the trigon, the tetragon and the pentagon. For the hexagon is the offspring of the trigon with its sides bisected, and the dodecagon of the hexagon, etc.; so too the octagon is the offspring of the tetragon, and the decagon, icosagon [Icoſigonum] etc. of the pentagon. And although the already mentioned perfection is required on the side of geometry, yet physically a certain magnitude of the angle, or incidence of the rays, is required, so that from this and from the already mentioned perfection of proportion there may arise a force effective in stimulating the nature of sublunary things. For in truth the efficacy of aspects is seen on the Earth, not in the stars themselves passively; granted that, by a certain semi-poetic manner of speaking, astrologers attribute many things to the planets, just as if they were all on the very surface of the fixed stars and gazed at one another and by mutual irradiations struck, helped, weakened and fostered one another, etc.

Quot ſint Aſpectus antiqui & Noui, & Quæ illorum ſubdiuiſio.

How many aspects there are, old and new, and what their subdivision is.

III. Old astrology recognised only five, namely conjunction, opposition, the tetragon or quadrate, the trigon or trine, and the hexagon or sextile, for a reason to be stated in ch. 8, no. 6. But Kepler, partly from Mästlin, Fabricius and certain physicians, partly from his own observation and from stimuli found in sublunary nature, added 8 others, whose selection will be discussed in Chapter 8, from number 2 to 5. Their names and quantities are shown in the following table, the new being distinguished from the old by a difference of type. I have added the Greek names for those aspects whose arcs subtend the sides of regular figures inscribed in a circle.

Names of the aspects	Signs	Quantity of the angles	Greek names
1 ☌ Conjunction [Coniunctio]	0	0°	σύνοδος (synodos),
2 *Semisextile* [Semiſextus]	1	30°	δωδεκάγωνος (dōdekagōnos)
3 *Decile* [Decilis]	.	36°	δεκάγωνος (dekagōnos)
4 *Octile* [Octilis]	.	45°	ογδόγωνος (ogdogōnos)
5 Sextile [Sextilis]	2	60°	ἑξάγωνος (hexagōnos)
6 *Quintile* [Quintilis]	.	72°	πεντάγωνος (pentagōnos)
7 Quadrate [Quadratus]	3	90°	τετράγωνος (tetragōnos)

Names of the aspects	Signs	Quantity of the angles	Greek names
8 *Tridecile* [Tridecilis]	.	108°	
9 Trine [Trinus]	4	120°	τριγώνος (trigōnos)
10 *Sesquiquadrate* [Seſquadrus]	.	135°	
11 *Biquintile* [Biquintilis]	.	144°	
12 *Quincunx*	5	150°	
13 Opposition [Oppoſitio]	6	180°	διάμετρος (diametros)

Of these, no other applies to Mercury besides ☌ with ☉, unless perhaps the semisextile, in some hypothesis granting it a digression from the Sun of 30°. To Venus with the Sun, however, only the first four aspects apply; but between Venus and Mercury nearly the first seven can occur; and to the other planets, any two of them, all the aspects apply without distinction.

[Margin: Subdivision of aspect.] Any aspect is further subdivided into true or apparent, and mean, according as the true or mean place or motion is considered in it; but commonly, with the exception of ☌ and ☍, the true ones are considered. Again it is divided into partile and platic. A partile aspect, or perfect and exact one, is when two stars are exactly as many degrees apart as are required for that aspect. A platic aspect, πλάτικος [Greek: πλάτικος (platikos, "broad")], or by orb, or loose, is when between two stars there is not the exact quantity of arc required for that aspect, but it is nevertheless found if the amplitude or light of their orb is considered. But if a planet is so joined to another that it is less distant than the sum of the apparent semidiameters of both, this is called a *bodily conjunction*, or conjunction by body, σύνοδος σωματικὴ [Greek: σύνοδος σωματικὴ (synodos sōmatikē, "bodily conjunction")]. [Margin: Bodily ☌, or by body.] Moreover, the quantity or light of the orbs to which they extend their powers, as determined by the astrologers together with Cardano on book 3 of the *Quadripartitum*, text 63, is set out below.

Quantity or light of the orbs [Quantitas ſeu Lux orbium]

Fixed stars, magnitude	Light of the orbs	Planets	Light of the orbs
1	7° 30′	♄	10° 0′
2	5° 30′	♃	12° 0′
3	3° 30′	♂	7° 30′
4	1° 30′	☉	17° 0′
5	1° 0′	♀	8° 0′
6	0° 30′	☿	7° 0′
		☾	12° 30′

But Origanus holds that the orbs of the planets are spaces of the zodiac within which planets joined to the Sun are hidden for some time; these he confuses with the arcs of vision, although from 3 *Aphorisms* 29 he determines the quantity of the orbs thus: ♄ 9°, ♃ 10°, ♂ 8°, ☉ and ☾ 15°, ♀ 8°, ☿ 7°.

[Margin: Simple and composite aspect.] Next, just as the partile is subdivided into simple and composite, according as longitude alone is considered in the

former, but longitude and latitude in the latter, so the platic is subdivided into application and defluxion [Defluxum] or separation. [Margin: Application and defluxion.] Application occurs when a swifter planet approaches a slower one; when it recedes from it, this is called defluxion, provided the distance is not greater than their rays. And it can happen that a lower planet is heavier, or slower, than some upper one, if one of the lower planets is retrograde or diminished in number.

[Margin: Right and left aspect.] Furthermore, every aspect is divided into left and right. A left aspect is one reckoned, with respect to some determinate point in the sky, according to the succession of the signs; a right aspect against the order of the signs. Of these the right is stronger when compared with the angles of the sky, because the stars are borne towards it by the force of the primum mobile; but the left is judged stronger when compared with other stars, because the planets approach it by the force of their own motion. But the astrologers say more about these things in their introductions to ephemerides.

De Reductione Aſpectuum ad Eclipticam.

On the reduction of aspects to the ecliptic.

IV. [Margin: Authors who treat of this reduction.] On this reduction one should consult Regiomontanus in the Tables of Directions, problem 31; Valentinus Naiboda on Alcabitius, from p. 324; Cyprián Lvovický in the *Method of Directions*, p. 13 before the end; Magini in the Tables of Directions of the primum mobile, canon 39; and Kepler in the *Ephemerides*, p. 37, and in the *Epitome of Copernican Astronomy*, p. 845.

Whether, then, you consider rays sent from one star to the other and from there to the Earth, or rays sent upward from a star to the zodiac (for we suppose that any star diffuses rays of light and of an occult quality spherically, or in every direction), the centres of the stars must nevertheless be regarded just as if they were in the concave of the primum mobile, since it is in that concave that we consider their places; and straight lines are drawn from the points of the highest sky through the centres of the two stars. These lines either meet one another at the centre of the Earth, as happens in opposition, or coincide and stretch towards one another, as in conjunction, or make at the centre of the Earth an angle subtending an arc of a great circle in the highest sky. That great circle will be the ecliptic if neither of the stars has latitude; but it will cut the ecliptic if they have latitude.

Now if they lack latitude, there is no need of reduction. But if one or both of the stars has a rather large latitude, not less than a degree and a half, reduction will be needed, because latitude alters both the strength of the radiations and the intervals of the aspects, since radiations or aspects are judged most effective in the ecliptic itself. It is indeed true that Ptolemy, in book 1 of the *Quadripartitum*, ch. 21, neglected this reduction in the remaining aspects on account of the slight difference, because it rarely happens that two planets placed in sextile or

trine aspect have the greatest or a very great latitude, and because the rays of the stars, from wherever they are sent, make an angle at the centre of the Earth; and Naiboda and the older authors subscribe to Ptolemy.

But in judging the meetings of applications and defluxions, whether or not they are bodily, latitude makes a great difference. For then neither do the lines of conjunctions, to be drawn from the Earth through the centres or bodies of the two stars, coincide, nor do the lines of opposition, drawn from one star through the centre of the Earth to the other, meet one another in a straight line; rather they are inclined to one another and make an angle at the centre of the Earth, unless the south latitude of the one is as great as the north latitude of the other. Therefore account must be taken of latitude, in the mind of Ptolemy. Most of the more recent authors, however, accept the reduction for other aspects, especially the sextile and trine. So let us set out its rules.

[Margin: General rules of the reduction.] 1. Rule. *If the stars lack latitude, or do not have a latitude greater than 1° 30′, the reduction can safely be neglected.*

2. Rule. *The quadrate aspect of any planet with the Sun or with another star devoid of latitude does not need reduction, however great a latitude the planet itself has.*
3. Rule. *The quadrate aspect of two stars, each of which has a great latitude, needs reduction. But if only one has latitude, the place of the quadrate radiation is 90° distant on either side from the star's true longitude and does not need reduction.*
4. Rule. *To an aspect less than a quadrant, if the latitude of the stars making the aspect is of a different species, or notably different in quantity, there corresponds in the ecliptic a smaller arc.*
5. Rule. *To an aspect greater than a quadrant, if the latitude of the stars making the aspect is of a different species, or notably different in quantity, there corresponds in the ecliptic a greater arc.*
6. Rule. *If the latitudes of the configured stars are nearly equal and of the same quarter or species, then greater arcs of the ecliptic correspond to all aspects without distinction.*
7. Rule. *An opposite radiation falls on the point of longitude diametrically opposite, with a latitude of the same quantity but of contrary denomination. Therefore the longitude indeed does not need correction or reduction; but the latitude of the radiation is north if the latitude of the radiating star is south, or south if the latter is north.*

Note that for the fixed stars the reduction arises from latitude alone.

Problema 1. Aſpectum ✶ & △ ad Eclipticam Reducere methodo ac tabulis Regiomontani & Magini.

Problem 1. To reduce the ✶ and △ aspect to the ecliptic by the method and tables of Regiomontanus and Magini.

V. Regiomontanus, in the *Directions* published at Wittenberg in the year 1606, p. 231, gave a small table for equating radiations, but Magini has a larger

table in canon 39 of the *Directions*. It is set out below, and it is founded on the solution of a right-angled spherical triangle whose base is 60° and whose side is the given latitude of the star in the circle of latitude. With the help of these givens, one seeks the arc of the ecliptic that is the longitude due to the sextile aspect reduced to the ecliptic; and the same longitude, subtracted from 180°, will give the longitude due to the trine aspect.

Equation of the aspects or radiations of the stars [Æquatio Aſpectuum ſeu Radiationum Stellarum.]

Latitude or declination	Arc ✶	Arc △	Latitude or declination	Arc ✶	Arc △
1	60° 0′	120° 0′	31	54° 19′	125° 41′
2	59° 59′	120° 1′	32	53° 52′	126° 8′
3	59° 57′	120° 3′	33	53° 24′	126° 36′
4	59° 55′	120° 5′	34	52° 55′	127° 5′
5	59° 52′	120° 8′	35	52° 23′	127° 37′
6	59° 49′	120° 11′	36	51° 50′	128° 10′
7	59° 45′	120° 15′	37	51° 14′	128° 46′
8	59° 40′	120° 20′	38	50° 37′	129° 23′
9	59° 35′	120° 15′	39	49° 57′	130° 3′
10	59° 29′	120° 31′	40	49° 15′	130° 45′
11	59° 22′	120° 38′	41	48° 31′	131° 29′
12	59° 15′	120° 45′	42	47° 43′	132° 17′
13	59° 7′	120° 53′	43	46° 52′	133° 8′
14	58° 59′	121° 1′	44	45° 58′	134° 2′
15	58° 50′	121° 10′	45	45° 0′	135° 0′
16	58° 40′	121° 20′	46	43° 58′	136° 2′
17	58° 29′	121° 31′	47	42° 51′	137° 9′
18	58° 17′	121° 43′	48	41° 39′	138° 21′
19	58° 4′	121° 56′	49	40° 21′	139° 39′
20	57° 51′	122° 9′	50	38° 56′	141° 4′
21	57° 37′	122° 23′	51	37° 23′	142° 37′
22	57° 22′	122° 38′	52	35° 42′	144° 18′
23	57° 6′	122° 54′	53	33° 49′	146° 11′
24	56° 49′	123° 11′	54	31° 43′	148° 17′
25	56° 31′	123° 29′	55	29° 20′	150° 40′
26	56° 12′	123° 48′	56	26° 36′	153° 24′
27	55° 52′	124° 8′	57	23° 22′	156° 28′
28	55° 31′	124° 29′	58	19° 21′	160° 39′
29	55° 8′	124° 52′	59	13° 53′	166° 7′
30	54° 44′	125° 16′	60	0° 0′	180° 0′

The use of the table is as follows. Given the latitude of the star, with it in the first column look in the second for the arc of the sextile aspect, or in the third for the arc of the trine aspect. Reckon this arc according to the order of the signs from the place of the planet's longitude, and you will have the left aspect; but if you count the already mentioned arc from the place of the planet's longitude against the order of the signs, you will have the right aspect. Know, however, that to the left sextile aspect there corresponds, in the diametrically

opposite place, the right trine aspect, and to the right sextile, the left trine; so that when one is known, the other becomes known.

EXEMPLVM.

Example.

Let Venus be at 20° of Gemini with a north latitude of 6°. To this there corresponds in the preceding table, under the ✶ aspect, an arc of 59° 49′, and under the △ aspect, 120° 11′. So add 59° 49′ to the 20th degree of Gemini, and you will find 19° 49′ of Leo, which Venus regards with a left sextile aspect; but on the contrary subtract, or count against the order of the signs, and you will land at 20° 11′ of Aries, a place which Venus regards with a right aspect. Diametrically opposite these stand the trine aspects: to the left one the right trine, at 19° 49′ of Aquarius, and to the right one the left trine, at 20° 11′ of Libra. You would also find these trine aspects if you added 120° 11′ to the 20 degrees of Gemini for the left, and subtracted them for the right. Furthermore, the left quadrate aspect will end at 20° of Virgo and the right at 20° of Pisces, for each is 90° distant from the longitude of Venus; just as the opposite aspect will be at ☍ 20°, but the latitude of the aspect will be 6° south. Thus, then, will the determination of these aspects stand.

Aspects of a planet placed at 20° ♊ with north latitude 6° [Aſpectus Planetæ in Gr. 20. ♊ poſiti cum Latitud. Bor. Gr. 6.]

	Left aspect: sign	Left aspect	Right aspect: sign	Right aspect
✶	♌	19° 49′	♈	20° 11′
□	♍	20° 0′	♓	20° 0′
△	♎	20° 11′	♒	19° 49′
☍	♐	20° 0′	♐	20° 0′
	But to the ☍ aspect assign latitude 6° south.			

Problema 2. Reducere Aſpectus ad Eclipticam priori methodo Kepleri, per tabulam Primi Mobilis.

Problem 2. To reduce aspects to the ecliptic by Kepler's earlier method, by means of the table of the primum mobile.

VI. I call the earlier method the one Kepler gives in the foundations of the *Ephemerides*, p. 37. For it one needs the first table of the primum mobile, or of the first analogy, which he transcribed from Regiomontanus into his Tables of Directions. One enters that table with the complement of the star's latitude at the head of the table, and in the area with the complement of the aspect if it is less than a quadrant, or with its excess over a quadrant if the aspect is greater than a quadrant. For on the same line you will have in the left margin a degree whose difference from the area number will be the equation, to be subtracted from an aspect less than a quadrant but added to an aspect greater than a quadrant. The arc of the aspect thus corrected is to be added to the

planet's longitude for a left aspect, and subtracted for a right one, etc., [Margin: 1. Case, if only one star has latitude.] if only one star has latitude.

EXEMPLVM.

Example.

Let the latitude of Venus on 1617 August 8 be 6° 38′, and suppose you wish to know how far the place of Venus should be from the Sun in the ecliptic for there to be a semisextile aspect, or one of 30°, between these two bodies. Since the aspect is less than a quadrant, apply its complement, which is 60°. Look for this in the area of the already mentioned table, under the complement of the latitude, 83° 22′, found at the head of the table; for on the same line as 60 degrees you will see corresponding in the left margin 60° 41′, if you apply the proportional part when the degrees are not found exactly in the table. So subtract from the semisextile aspect, or 30 degrees, the difference of the marginal number from the area number, which is 41′, and there will remain 29° 19′. So when the Sun is 29° 19′ distant from Venus, the aspect of Venus with the Sun will be semisextile and left.

VII. [Margin: 2. Case for the latitude of both stars, but different in species.] But if each star has latitude of a different species, the already mentioned table is not useful in geometrical rigour, because it is founded on a right-angled triangle, whereas in this case we need an oblique-angled triangle. Yet for the planets, which are always within the zodiac, you will be able to use it with a double entry (overlooking the small errors that lie hidden in this method) and using a combined difference, corresponding to the complement of the arc that arises from the sum of the aspect multiplied by the latitude of each and divided by the sum of the latitudes.

EXEMPLVM.

Example.

Let the latitude of Venus be 10° and of Mars 7°, one north, the other south, and let the distance required between these planets in the ecliptic be sought, so that they regard each other with a semisextile aspect, or one of 30°. The sum of the latitudes is 17°. Let it be, then: as 17 to 30, so the north latitude 10 to 18, whose complement is 72°. Again, let it be: as 17 to 30, so the south latitude 7 to 12, whose complement is 78. Therefore, in the first table of the primum mobile of Regiomontanus or Magini, or of the first analogy, look under 80°, which is the complement of the latitude 10°, and in the area for the number 72, and you will have in the margin 74° 58′, whose difference from the area number is 2° 58′. Again, under 83°, which is the complement of the latitude 7°, look in the area for the number 78; for in the margin you will have 80° 13′, and so the difference from the area number will be 2° 13′. Combining the differences, then, the sum of the differences is 5° 11′. This, taken from the aspect of 30° (because it is less than a quadrant), leaves the corrected aspect 24° 49′. So if Mars and Venus, or

any two planets, endowed with so great and such a latitude, are 24° 49′ apart in the ecliptic, they will regard each other with a semisextile aspect. If, however, you gathered this difference from an oblique-angled triangle, the arc of the ecliptic would be 25° 6′.

VIII. [Margin: 3. Case for the latitude of both planets, but of the same species.] But if the latitudes are of the same species, for either planet take the point opposite it, with a latitude of the contrary quarter; and for the aspect, its complement to a semicircle. Divide this complement according to the proportion of the latitudes, and, having sought the complement of the quotient in the area under the complement of the latitude, find the difference of the marginal number from the area number. Do the same for the other planet. Finally, add the sum of the differences to the aspect if its complement is less than a quadrant; subtract it if greater.

EXEMPLVM.

Example.

Let the biquintile aspect, or 144°, be sought between two planets whose latitude, for each, is 5° north. The complement of the aspect 144 to a semicircle is 36°, and the sum of the latitudes is 10°. Let it be, then: as 10 to 5, so 36 to 18, whose complement is 72. Look for this number in the area of the table, under the complement of the latitude, which is 85, and you will find in the margin 72° 41′. So the difference of the marginal number is 41′. Double this (because to the latitude of the other planet, being equal, there will likewise correspond a difference of 41′), and the sum is 1° 22′, which must be added to the aspect 144. So these planets will not regard each other with a biquintile aspect unless they are 145° 22′ apart in the ecliptic.

Problema 3. Reducere Aſpectus ad Eclipticam poſteriore methodo Kepleri per Antilogarithmos Rudolphinarum.

Problem 3. To reduce aspects to the ecliptic by Kepler's later method, by means of the antilogarithms of the Rudolphine Tables.

IX. This later procedure is given by Kepler in the *Epitome of Copernican Astronomy*, p. 845, for three cases, through the use of the table of antilogarithms in the Rudolphine Tables. [Margin: 1. Case for a single latitude.] The first case is when only one of the planets has latitude. Then from the antilogarithm of the latitude subtract the antilogarithm of the aspect if it is less than a quadrant, or, if it is greater than a quadrant, the antilogarithm of the complement of the aspect to a semicircle; for the remainder will be the antilogarithm of the arc of the ecliptic corresponding in the first case to the aspect, and in the second to the complement of the aspect to a semicircle.

EXEMPLVM.

Example.

Let one of the planets have a latitude of 10°, and let the semisextile aspect be sought in the ecliptic, that is, 30°. From the antilogarithm of 30°, which is 14384, subtract the antilogarithm of 10°, which is 1531; the remainder will be 12853, namely the antilogarithm to which 28° 26′ corresponds. So if the planets are 28° 26′ apart in the ecliptic, then in their own orbit they will regard each other with a semisextile aspect of 30°. But if the biquintile aspect 144 is sought, its complement is 36, from whose antilogarithm 21193 the antilogarithm of 10°, subtracted, leaves 19662, the antilogarithm of 34° 46′. When these are taken from 180°, there remains an arc of the ecliptic of 145° 14′. This distance of the planets shows that the planets in their orbit regard each other with a biquintile aspect of 144°.

X. [Margin: 2. Case of equal latitudes.] The second case is when both planets have equal latitudes; and then, the aspect being bisected, one seeks, as in the first case, the reduction suited to half the aspect, if la- titudes are of different species; or to the complement of half the aspect to a quadrant, if the latitudes are of the same species. The doubled reduction, or arc of the ecliptic, will then be the arc sought. [Margin: 3. Case of unequal latitudes.] The third case is when the latitudes differ in quantity, in the same or in different species; and the method is too laborious, as Kepler admits, who nevertheless gives it somewhat obscurely, by a bare example without rules.

Problema 4. Reducere Aſpectus ad Aequatorem.

Problem 4. To reduce aspects to the equator.

XI. In the *Directions of the Primum Mobile*, canon 40, Magini shows that the aspects of any stars whatever have great force when their distance is considered not only in the ecliptic but also in the equator; and he relies on the authority of Ptolemy, book 3 of *De Astrorum Iudiciis*, ch. 11, and of Cardano on the same book, ch. 14. The method of reducing aspects to the equator is exactly the same as that of reducing them to the ecliptic, given in Problems 1, 2 and 3, provided that the declination of the star or stars is used in place of the latitude, and an arc of the equator in place of the arc of the ecliptic.

Problema 5. Aſpectus Planetarum ad Aequatorem reductos, referre ad Circulos Poſitionum, vel ad circulos Horarios, vel ad Cuſpides Domorum.

Problem 5. To refer the aspects of the planets, once reduced to the equator, to the circles of position, or to the hour circles, or to the cusps of the houses.

XII. In the *Directions of the Primum Mobile*, canon 41, Magini thinks this reduction useful, and thinks that the force of planets is greatly altered by the rays of planets extended on the equator, when those rays fall either on the same

circle of position or on the same hour circle, etc. But since this reduction is less commonly used, I forgo the prolixity of rules and examples. As for you, if you have the leisure to lose better hours, consult Magini there.

Problema 6. Aſpectus Planetarum reducere ad Eclipticam, vel ad Aequatorem ex opinione noſtra.

Problem 6. To reduce the aspects of the planets to the ecliptic, or to the equator, according to our own opinion.

XIII. It certainly seems to me that the aspects of planets on different orbits inclined to the ecliptic are not to be determined with respect to the ecliptic or the equator until they have been reduced to the ecliptic by means of the given latitude, or to the equator by means of the declination, in the way we usually reduce the Moon's place from the lunar orbit to the ecliptic; since they exert their energy on the Earth in the ecliptic, or relatively to the ecliptic or the equator. And so if the places of two stars in the ecliptic are, for example, 60° apart, only then shall I pronounce that to be their ecliptic sextile aspect; or, if they are so apart in the equator, their equatorial one. Therefore, for each planet having latitude or declination, a single right triangle must be solved, in which the latitude or declination is assumed as given, as one side about the right angle, and also either the angle opposite it, that is, the inclination of the orbit to the ecliptic or to the equator, or at any rate the base itself, that is, the distance of the star from the nearer node, according to what will be said in the Problems of the primum mobile.

An & qui ſint Aſpectus Benigni, qui Maligni, Aut validiores, & cur?

Whether there are benign aspects, and which they are; which are malign, or which stronger; and why?

XIV. Among the cardinal or principal aspects, the trine and the sextile are judged by Ptolemy, *De Astrorum Iudiciis* book 1, ch. 12, to be agreeable, or fortunate and benign; but the opposition and the quadrate aspect disagreeable, and unfortunate or malign, because the former embrace like places of the signs, that is, all masculine or all feminine, and the latter unlike ones. Cardano on that passage, and Magini in the *Directions*, canon 39, deny that this reason is sufficient. And so Pontanus on that passage of Ptolemy, and Lucius Bellantius [Lucio Bellanti], question 9, art. 4, say that the opposition is malign because it is made by rays that oppose one another and collide, in nearly the way two contrary winds stir up great discords of storms. The quadrate aspect, on the other hand, is made by rays that do not indeed oppose one another head-on, yet strike each other at the sides; and because they stand at a point equidistant from the most conflicting termini, that is, from the termini of the opposition, they are therefore judged to be of middling enmity. Magini adds that a ray from a star or sign to another sign or star from which it is 90° distant falls in such a

way that it is reflected at an equal angle to the place diametrically opposite the first star or radiating sign, if that on which the radiation falls is taken as a solid body fit for reflecting; but that from the Earth it is reflected back on itself, since it falls perpendicularly, and is thus disturbed; and that for both these reasons it takes on a certain malignity. The trine aspect, however, both through the perfection of a figure that can be inscribed in a circle and does not presuppose others, and because it consists of rays that do not oppose one another but rather assist one another and conspire into one, is judged fortunate and beneficent, as is the sextile also. But Pico della Mirandola, and our own Alessandro de Angelis, and Kepler, *De Stella Nova* ch. 9, do not accept this, but either ridicule and attack it or, like Kepler, limit it. For he denies that any aspect is of itself fortunate or unfortunate; yet he maintains that they are distinguished only by their vehemence in stirring up sublunary nature. [Margin: The force of aspects asserted by experiments.] For it is established by experience that when there are no configurations of aspects, it is then above all that there is great calm in the air; but when the planets are configured in aspects, it is then above all that varieties of meteors are observed, each in its own kind. Kepler thinks, moreover, that what the aspects stir up is a reason [rationem] most natural not to bodies but to spiritual faculties, so that it perceives some harmonic agreement or other. But so far as these matters contain a certain physical speculation, they do not concern this work of ours; so far as that harmonic configuration touches astronomical geometry, however, it must be explained by us in the following chapter, though briefly. From that chapter, at number 6, it will also be clear why among the aspects the most powerful is ☌, next ☍, and after these □ ✶ △.

CAPVT VIII.

Chapter 8.

De Configurationibus Harmonicis, ac de Aſpectuum fundamento in Harmonicis proportionibus.

On harmonic configurations, and on the foundation of the aspects in harmonic proportions.

I. On the connection of the aspects with harmonic proportions, or on the harmony itself among the motions of the planets, one should read first of all Ptolemy, *Harmonics* book 3, ch. 8, and *Quadripartitum* book 1, ch. 12; Firmicus, book 2, ch. 29; Reinhold on Peuerbach's *Theoricae*, p. 182; Magini in the *Directions*, canon 39; but, most accurately and most learnedly of all, Kepler in the *Harmonics*, book 4, ch. 6, and in the *Mysterium Cosmographicum*, ch. 12, and in the book *De Stella Nova*, from ch. 7 to 10, and in the *Dissertatio cum Nuncio Sidereo*, and in the foundations of his *Ephemerides*, from p. 33, and finally in the *Epitome of Copernican Astronomy*, p. 900.

Origo Aſpectuum ex Harmonicis proportionibus, aut cognatio cum illis.

The origin of the aspects from harmonic proportions, or their kinship with them.

II. Sound indeed seems to have nothing in common with light and the rays of the stars; yet the consonances of sounds have in common with the aspects of the heavenly lights their numeration, and a geometrical and cosmopoetic origin. For just as, when the string of a monochord is struck at certain intervals, our ears are roused and struck, so the faculties of sublunary nature, by a certain configuration of the aspects imitating harmonic proportions, are stirred and provoked to exert their powers; so much so that Kepler, in the *Ephemerides*, suspected that God, Best and Greatest [DEVM O. M.], either from harmonic ratios derived the laws for arranging the aspects, or attuned human ears, the judges of concords, to the heavenly aspects. Moreover, out of the seven harmonic divisions of the monochord, made by means of a magadium or saddle [ephippij] placed under the string that is stretched and to be plucked, the older astronomers considered four principal ones in determining the aspects, whether the circle is divided into 360° or the zodiac into 12 signs, as if it were one string; and they assigned to them their harmonic proportions in this way.

[Margin: Proportion of ☍ to □, to △ and to ✶.] The opposition has to the quadrate, that is, 180° to 90°, or 6^s to 3^s, the double proportion, which they call the diapason; to the trine, that is, to 120° or to 4^s, it has the sesquialter proportion, which they call the diapente; and finally to the sextile, that is, to 60° or to 2^s, it has the triple proportion, which is either simple or composed of the diapason and the diapente.

[Margin: Proportion of □ to ✶.] The quadrate aspect has to the sextile, that is, 90° to 60°, or 3^s to 2^s, the sesquialter proportion, which the Greeks call ἡμιόλιον [Greek: ἡμιόλιον (hēmiolion, "one and a half")], and musicians the diapente.

[Margin: Proportion of △ to □.] The trine has to the quadrate the ratio ἐπίτριτον [Greek: ἐπίτριτον (epitriton, "one and a third")] or sesquitertian, as 120 to 90, or 4^s to 3^s, which they call the diatessaron.

III. But Kepler thought that, besides the unison, which corresponds to the conjunction, seven divisions of the circle should be considered, corresponding to as many harmonic divisions of the string. To these divisions of the circle correspond the chords fitted to the circle, or the sides of the notable regular figures inscribed in a circle: namely the side of the sexangle, the side of the quincangle, the side of the quadrangle, the side of the triangle; the chord subtending three eighths, and the chord subtending two fifths, and the diameter. From these arise eight aspects, namely conjunction, sextile, quintile, quadrate, trine, sesquiquadrate [Seſquadrus], biquintile and opposition; just as in music there are, besides the unison, seven other strings concordant with the lowest, and indeed with one another and with the whole string, each arising from one of the divisions of the whole monochord. And since in the following years he

had observed, at the three new aspects of the planets with the Sun, very frequent and notable changes of the air, excepting the sesquiquadrate, which seemed effective more rarely, he therefore, having detected their efficacy, decided that they too should be entered on the roll of aspects; and he says that from that time the prognosticators had vied with one another in using them. That the division of the quintile is harmonic is clear, because it cuts off from the whole circle a fifth part, or 72° of 360°. [Margin: The harmony of the new aspects is shown.] But if you divide the whole string into 5 parts and put the magadium under the first fifth, both the first fifth with the remaining four and either of them with the whole string will be concordant. So too for the harmony of the sesquiquadrate [Seſquiquadri], or sesquadrus, or trioctile (for they are the same): since it cuts off from the circle 135°, that is, three times 45°, which are three parts of eight, since 45 taken three times makes 135, and eight times, 360, let the monochord therefore be divided into 8 parts and the magadium be applied at three eighths. For both the upper part, or segment of 3, with the lower of 5 (which is the major hexachord), and the lower of 5 with the whole string of 8 (which is the minor sixth [ſexta mollis]), will give a harmonious sound. Finally, for the biquintile, which cuts off from the circle twice 72°, that is, 144°, or two fifths: divide the string into 5 parts and put the magadium under at two fifths; for both the segment of 2 with the remaining segment of 3, and the whole string of 5 with the 2 and with the 3, will be in consonance.

IV. [Margin: The origin of the other aspects.] Later on, however, some astrologers, whether by mistake taking in place of the sesquadrus a *Sequadrus* [Sequadrum], that is, half of a quadrate, or an octile of 45°, or otherwise from experiments, were issuing prognostications from this aspect; and Mästlin [Mœſtlinus] had pointed out in a certain disputation that, besides the triangle, the quadrangle and the sexangle, there are other regular and beautiful figures fit for forming aspects, namely the quincangle or quintile of 72°, the octangle or octile of 45°, the decangle or decile or semiquintile of 36°, the sesquiquadrate or trioctile of 135°, the biquintile of 144°, and the sesquiquintile or tridecile of 108°. So the astrologers began to put out prognostications to the public from all of these. Kepler too began to doubt whether other aspects should be introduced besides those seven. The octile indeed did not seem admissible, because it is not perfectly harmonic: it cuts off from the whole circle one eighth part; and although, when the string is divided into 8 parts and the magadium applied at 1, the 1 is in consonance with the 8, yet the 1 is not in consonance with the 7, nor the 7 with the 8. And the same must be said of many other divisions that might seem fit for forming an aspect; as if a semisextile of 30° were formed, a vigintile of 18°, a quindecile of 24°, and a quincunx of 150°, and certain others akin to those already mentioned, which cut off 48°, 54°, 96°, 126°, 162°, 168°. But neither does experiment give clear testimony of efficacy to all the harmonic ones, since rare and not very manifest experiments favour the sesquiquadrate, commended by music. [Margin: Music is not always to be followed in aspects.] On the contrary, although no harmonic division

of the monochord corresponds to either the semisextile or the quincunx, yet a frequent and clear force in stirring up storms has been observed, as Kepler asserts. Accordingly, in the *Ephemerides* and in the book *De Stella Nova*, he concludes that in determining aspects music must sometimes be abandoned, since it draws certain principles of its septenary from the very straightness of the string, and these are not at all suited to a circle returning on itself; nor, as from a division of a string there arises another string, is another circle made from the divisions of the zodiac. For although the kinship between aspects and harmonies is great, and both kinds have the same primary origin, from the notable figures that can be inscribed in a circle, yet the number of the seven divisions of the monochord is formed by certain laws, and the number of the aspects by others: from concurring physical causes not yet fully known a priori, and indeed also from a concurrence of geometrical causes, which make the aspect stronger the more of them there are. Those should be chosen, moreover, which have more prerogatives, and attention must be paid partly to the harmonic ratio and partly to the physical force for changing sublunary things. But when one aspect is too near to another that is more powerful and manifest, as the decile of 36° is very close to the very strong semisextile of 30°, then observation is more difficult because of the nearness of the days, and the observer must be more sharp-eyed. Finally, when Kepler in the *Ephemerides* had pronounced further [amplius] and deferred his last judgement to another time, he brought it out in the *Epitome of Copernican Astronomy*, and from mixed principles of physico-harmony he fixed the number of 13 aspects which we listed in the preceding chapter.

V. So then, to take in all the aspects that deserve consideration according to Kepler, and to give them in one view, we shall distribute them into two classes. One will be called that of possible aspects, though not all of these are either harmonic or proved by physical experiment. The other will be called that of the physico-harmonic [Phyſicharmonicorum], which harmonic division alone, or physical efficacy alone, or both, favour; with the letter H added to those that harmonic division supports together with musical proportion and consonance, and with names attached that are wanting in Kepler.

21 possible aspects according to Kepler [Aſpectus 21. Poſſibiles ſecundùm Keplerum]

Order	Names of the aspects	Degrees of the circle
1	Conjunction [Coniunctio]	0°
2	Vigintile [Vigintilis] / vigintangle [Vigintangulus]	18°
3	Quindecile [Quindecilis] / quindecangle [Quindecangulus]	24°
4	Semisextile [Semiſextus] / dodecangle [Dodecangulus]	30°
5	Decile [Decilis] or semiquintile [Semiquintilis]	36°
6	Octile [Octilis] / semiquadrate [Semiquadratus]	45°
7	Hyperoctile [Hyperoctilis]	48°
8	Subsextile [Subſextilis]	54°

Order	Names of the aspects	Degrees of the circle
9	Sextile [Sextilis] / hexagon [Hexagonus]	60°
10	Quintile [Quintilis] / pentagon [Pentagonus]	72°
11	Quadrate [Quadratus] / tetragon [Tetragonus]	90°
12	Hyperquadrate [Hyperquadrus]	96°
13	Tridecile [Tridecilis] / sesquiquintile [Seſquiquintilis]	108°
14	Trine [Trinus] / trigon [Trigonus]	120°
15	Hypertrine [Hypertrinus]	126°
16	Sesquiquadrate [Seſquadrus] / trioctile [Trioctilis]	135°
17	Biquintile [Biquintilis] / decangle [Decangulus]	144°
18	Quincunx	150°
19	Hyperquincunx	162°
20	Hypodiameter	168°
21	Opposition [Oppoſitio] / diameter [Diameter]	180°

Physico-harmonic aspects chosen by Kepler [Aſpectus Phyſicharmonici à Keplero electi]

Order	Names	Degrees of the circle	
1	☌ Conjunction [Coniunctio]	0°	Uniſon [Vniſonus]
2	Semisextile [Semiſextus]	30°	
3	Decile [Decilis]	36°	
4	Octile [Octilis]	45°	
5	✶ Sextile [Sextilis]	60°	H
6	Quintile [Quintilis]	72°	H
7	□ Quadrate [Quadratus]	90°	H
8	Tridecile [Tridecilis]	108°	
9	△ Trine [Trinus]	120°	H
10	Sesquiquadrate [Seſquadrus]	135°	H

Order	Names	Degrees of the circle	
11	Biquintile [Biquintilis]	144°	H
12	Quincunx	150°	
13	☍ Opposition [Oppoſitus]	180°	H

The letter H marks the harmonic aspects, that is, those concordant with their complement to the circle, as with the remaining segment of the string, and at the same time with the whole circle, as with the whole string.

Proportions of the harmonic aspects to the whole circle, to the remaining segment or complement of the circle, or to one another [Proportiones Harmonicorum Aſpectuum cum toto circulo, cum reliquo ſegmento ſeu Complemento circuli, vel inter ſe ſe]

Aspect	To	As	To	Names of the proportions
☌ to	itself [Seipſum]	1	1	Unison. Equality [Æqualitas]
✶ Sextile	Whole circle	1	6	
	Compl. of circle	1	5	
	Semisextile	2	1	Double. Diapason
	Quintile	5	6	
	Quadrate	2	3	
	Trine	1	2	
	Opposition	1	3	
Quintile	Whole circle	1	5	
	Compl. of circle	1	4	
	Sextile	6	5	Sesquiquintan. Semiditone
	Biquintile	1	2	
	Decile	2	1	Double. Diapason
□ Quadrate	Whole circle	1	4	
	Compl. of circle	1	3	
	Semisextile	3	1	Triple. Diapason diapente
	Sextile	3	2	Sesquialter. Diapente
	Octile	2	1	Double. Diapason
	Trine	3	4	
	Opposition	1	2	
△ Trine	Whole circle	1	3	
	Compl. of circle	1	2	
	Semisextile	4	1	Quadruple. Disdiapason
	Sextile	2	1	Double. Diapason
	Quadrate	4	3	Sesquitertian. Diatessaron
	Quincunx	4	5	

Aspect	To	As	To	Names of the proportions
	Opposition	2	3	
Sesquiquadrate	Whole circle	3	8	
	Compl. of circle	3	5	
	Octile	3	1	Triple. Diapason diapente
	Quadrate	3	2	Sesquialter. Diapente
	Opposition	3	4	
Biquintile	Whole circle	2	5	
	Compl. of circle	2	3	
	Trine	6	5	Sesquiquintan. Semiditone
	Decile	4	1	Quadruple. Disdiapason
	Quintile	2	1	Double. Diapason
☍ Opposition	Whole circle	1	2	
	Compl. of circle	1	1	Unison
	Semisextile	6	1	
	Octile	4	1	Quadruple. Disdiapason
	Sextile	3	1	Triple. Diapason diapente
	Quadrate	2	1	Double. Diapason
	Trine	3	2	Sesquialter. Diapente
	Quincunx	6	5	Sesquiquintan. Semiditone

VI. You now see why, after the conjunction, the opposition has the greatest force of all the aspects: because the conjunction, which contains the revolution of the whole circle, or the whole string, contains all the aspects within itself; next, the opposition is connected with very many, namely with eight, if you count in the whole, or ☌; but the quadrate with six; the sextile with six; the trine with six. The remaining three are connected with fewer. This, then, is the genuine reason why ☌ ☍ □ ✶ △ were once the principal and most accepted aspects, and why ☌ and ☍ prevail over the others. But if anyone preferred to multiply the aspects according to the divisions of the whole circle into whole parts, there would be 16 aspects, and with ☌, 17. For it admits 16 divisions of this kind, which you may see in the following line, with the degrees of the quotient corresponding to each division.

Diuiſores Numeri 360.

Divisors of the number 360.

D.	2	3	4	5	6	8	9	10	12	15	18	20	24	30	36	72
Deg.	180	120	99 [recte: 90]	72	60	45	40	36	30	24	20	18	15	12	10	5

De Harmonicis Proportionibus Motuum.

On the harmonic proportions of motions.

VII. Compound harmonic proportions are infinite, but the simple ones are eight, on whose causes and origin Zerlinus [Gioseffo Zarlino] and other

musicians have much to say, as does Kepler in *Harmonics*, book 3. They are those in the table written below. For although the diapason can be resolved into a third and a sixth, or into a diapente and a diatessaron, and a sixth into a third and a diatessaron, and a diapente into thirds, yet they are not properly called compound, because proportions are properly compound when one or more diapasons are added to one simple proportion. Of this kind is the trisdiapason between 1 and 8, for it is composed of a diapason between 1 and 2, one diapason between 2 and 4, and another diapason between 4 and 8. So too the diapason epidiapente between 3 and 1 consists of a diapente, 3 to 2, and a diapason, 2 to 1. Among the simple proportions, moreover, the first four are judged more perfect, just as among the compound ones the most perfect of all are the triple, between 3 and 1, called the diapason diapente or twelfth, and the quadruple, between 4 and 1, called the disdiapason or fifteenth.

8 simple harmonic proportions [8. Simplices Proportiones Harmonicæ]

As	To	Polyonymy or nomenclature
1	1	Proportion of equality. Unison
2	1	Double. Octave. Diapason Διαπασῶν [Greek: Διαπασῶν (Diapasōn, "through all")]
3	2	Sesquialter. Fifth. Diapente ἡμιόλιος Δια-πέντε [Greek: ἡμιόλιος (hēmiolios, "one and a half"); Διαπέντε (Diapente, "through five")]
4	3	Sesquitertian. Fourth. Diatessaron ἐπίτριτος Διατεσσάρων [Greek: ἐπίτριτος (epitritos, "one and a third"); Διατεσσάρων (Diatessarōn, "through four")]
5	4	Sesquiquartan. Hard or major third [Tertia dura ſeu maior]. Ditone Δίτονος [Greek: Δίτονος (Ditonos, "of two tones")]
6	5	Sesquiquintan. Soft or minor third [Tertia mollis ſeu minor]. Semiditone ἡμιδίτονος [Greek: ἡμιδίτονος (hēmiditonos, "half-ditone")]
5	3	Superbipartient thirds [Superbipartiens tertias]. Hard sixth [Sexta dura]. Major hexachord [Hexachordum maius]
8	5	Supertripartient fifths [Supertripartiēs quintas]. Soft sixth [Sexta mollis]. Minor hexachord [Hexachordum minus]

[Margin: The motions of the planets are suited to these proportions.] Now then, these proportions can be seen in the velocities of the planets, as regards their proper daily or hourly motion, true or apparent. But the proper motion of these five planets is involved in a double inequality: one is that which appears to an eye placed, by a fiction, in the Sun; the other, that which appears to an eye placed on the Earth, but which involves the first. [Margin: Proportions fitting the planets viewed from the Sun.] According to the former consideration, or as they are viewed from the Sun, definite harmonic proportions are assigned to definite planets, but they are merely intellectual and not perceptible by any experience of effects; for the two terms of each are not joined at the same time. Thus the variety of the motions of Saturn reaches the terms of the hard third; of Jupiter, of the soft third; of Mars, roughly of the diapente; of Mercury, nearly of the diapason with a soft third. But Venus viewed from there has no harmonic intervals, since it keeps within the comma of 81 to 80. Nor yet the Earth in motion, since it con- tains itself within the semitone of 16 to 15. And

this holds if one considers the proportions assigned to each planet with respect to the different motions of that same planet. But if one considers the harmonies assigned to pairs of planets next to one another, viewed likewise from the Sun, or to any pairs whatever, then on their variety see Kepler, *Harmonics*, book 4, and in the *Epitome*, p. 902. For we ought rather to speak of those that suit the planets viewed from the Earth, although the following problem also teaches how to compute the harmonies from the Sun.

VIII. [Margin: Proportions suited to the planets viewed from the Earth.] With the planets viewed from the Earth, then, the harmonies of the apparent motions can be perceived not by the intellect only but also by sense, inasmuch as astrologers can attend to the times at which the motions of the planets are harmonically tempered together, and to whether some effect of that tempering appears in the meteors; just as, by attending beforehand to these effects, they have learned in what way and at what time the planets are configured harmonically, or quasi-harmonically, by reason of position or aspects, by which the nature of sublunary things seems to be stirred most of all. And to all the planets so viewed all the harmonic proportions belong, since they become stationary and at times lack all apparent motion. This being laid down, let the following problem be taken from Kepler's *Epitome of Copernican Astronomy*, p. 904.

Problema 1. Inuenire Qualis ſit Harmonia inter Diurnos Planetarum motus veros, ſpectatos ſiue ex Sole, ſiue ex Terra.

Problem 1. To find what harmony there is between the true daily motions of the planets, viewed either from the Sun or from the Earth.

As the greater of the daily motions is to 240, so let the lesser of the daily motions be to something else, and you will have the harmony sought, or the argument of the harmonic proportion, simple or compound, in the following little table, exactly or as nearly as possible. If from the Sun, take the daily motions seen from the Sun; but if from the Earth, take the daily motions viewed from the Earth.

Table indexing the harmonic proportions, with the monochord taken as 240 parts [Tabella Index Proportionum Harmonicarum poſito Monochordo partium 240.]

[Margin: Beware of the errors in Kepler's table.]

Simple proportion [Proport. Simplex.]		Compounded with the diapason [Compoſita cum Diapaſon]						
Diapason	120	60	30	15	7½	3¾	1⅞	15/16
Major sixth [Sexta dura]	144	72	36	18	9	4½	2¼	1⅛
Minor sixth [Sexta mollis]	150	75	37½	18¾	9⅜	4 11/16	2 11/32	1 11/64
Diapente	160	80	40	20	10	5	2½	1¼
Diatessaron	180	90	45	22½	11¼	5⅝	2 13/16	1 13/32

Simple proportion [Proport. Simplex.]		Compounded with the diapason [Compoſita cum Diapaſon]						
Major third [Tertia dura]	192	96	48	24	12	6	3	1½
Minor third [Tertia mollis]	200	100	50	25	12½	6¼	3⅛	1 9/16
Unison [Vniſonus]	240							

EXEMPLVM.

Example.

Let the Sun's true daily motion be 60′ and that of Mars 48′. Multiplying 48′ by 240 gives 11520; dividing these by 60, the quotient is 192. In the preceding table this number indicates the simple harmonic proportion of the major third [Tertiæ duræ], or ditone. Again, let the daily motion of Jupiter be 13′ and that of Venus 1° 42′, or 102′. Multiplying the lesser motion, 13, by 240 then gives 3120. Divide this sum by 102, and the quotient is 30 30/51. This indicates, in the first line, a proportion compounded roughly of the diapason and the disdiapason, that is, from the trisdiapason: for since 120 with respect to 240 is the diapason, and 60 the disdiapason, 30 will be the trisdiapason. So if 80 should come out in the quotient, the proportion would be compounded of the diapente and the diapason; if 40, it would be compounded of the diapente and the disdiapason, and so on.

SCHOLION.

Scholium.

I. [Margin: The speed of the planets in their proper motion.] Aristotle believed that all the planets move of themselves with the same speed, their own orbs being considered. If that were true, the times of the mean motions would be as the orbs and their diameters. But this is utterly false; for the higher each planet is and the more remote from the Sun, the less space it covers in its mean motion in one hour; so much so that, if you assume the Ptolemaic distance of the Sun and the others that depend on it, Saturn in its own mean motion about the Sun, toward the east, covers 960 Italian miles in one hour, Jupiter 1280, Mars 2400, the Earth, if it is supposed to move about the Sun, 2960, Venus 3200, Mercury 4800; although according to Kepler's distance of the Sun from the Earth these numbers are to be roughly tripled throughout, and according to us and certain others are to be increased by much more still, as we shall say in Section 6. For this is an affection arising from altitude, the motions in longitude being assumed. [Margin: The proportion of the motions to the orbs and intervals.] There too we shall explain that notable proposition by which, in book 5 of the *Harmonics*, and book 4 of the *Epitome*, p. 501, and in the Rudolphine Tables, ch. 24, Kepler affirms that in the primary planets the mean distance of one planet from the common centre of the world, which for

him is the Sun, is to the greater distance of another planet in the sesquialter ratio of their periodic times; or that the proportion of the periodic times is sesquialter of the proportion of the orbs, or of the converse proportion of their whole degrees, because whatever has a small number has a large degree. Wendelen extends this proportion also to the periodic motions of Jupiter's satellites about Jupiter, as we shall see in its place. I wanted to point this out here so that no one, on the occasion of the harmonic proportions, should look here for this one as well, which holds between motions and intervals. But how to investigate the absolute proportion of the eccentric daily motions of the Sun and a planet, Kepler teaches in the Rudolphine Tables, precept 103.

CAPVT IX.

Chapter 9.

De Reuolutionibus Planetarum & Coniunctionibus Magnis & Maximis, earumque effectibus.

On the revolutions of the planets and on the Great and Greatest conjunctions, and their effects.

I. Among all the aspects already treated in chapters 7 and 8, the most excellent is the synod or conjunction, for the reason already given in chapter 8, number 6. Among the conjunctions of the planets, moreover, the most celebrated among astronomers and astrologers alike are those called the Greatest and the Great, namely of two or three of the superior planets. For the conjunctions of all the planets, though by far the greatest, or the Greatest of the greatest, have been passed over by the astrologers, because they do not recur exactly within the brief duration of this world. [Margin: Authors who [have written] on the Great conjunctions, etc.] But we must discuss all of these in this place, from what has been taught or pointed out to us by Ptolemy in the *Centiloquium*, aphorisms 50 and 63, and there by Cardano, Pontanus, Georgios Trapezountios and other commentators; Abū Maʿshar in the book *De magnis coniunctionibus*, with an *Isagoge* to it by an unnamed author, and the commentaries on him by Henricus Mechliniensis [Hendrik Bate of Mechelen]; Alcabitius in the treatise *De coniunctionibus planetarum ac revolutionibus*, and in the *Isagogicus ad magisteria stellarum*, differentiae 1 and 4, and there Leopoldus de Austria [Leopold of Austria], tracts 4 and 5, and Valentinus Naiboda, or in the *Enarratio elementorum astrologiae*, from p. 39 and again from p. 352; Māshāʾallāh the Arab, in the book *De coniunctionibus*; Alliacensis [Pierre d'Ailly] in the treatise *De concordia historiae et astrologiae*; Giovanni Pico della Mirandola, *Contra astrologos*, book 5; Lucio Bellanti, book 5 against Pico, and question 11 of *De astrologica veritate*; Reinhold on Peuerbach, from p. 283; Giuntini in the *Speculum Astrologiae*, ch. 8; Origanus in volume 1 of the *Ephemerides*, or in their introduction, part 3, member 2, ch. 1; Cyprianus Leovitius [Cyprián Lvovický], *De magnis coniunctionibus*; Io. Franciscus Spina [?] in two books *De maximis coniunctionibus*; Andrea Argoli

in the *Pandosion Sphaericum*, ch. 73; Kepler in the book *De Stella Nova*, from ch. 6 to 11, and in the *Mysterium Cosmographicum*, the preface and ch. 12, in the notes, and in the *Epitome of Copernican Astronomy*, book 2, p. 189, and book 6, pp. 846 and 854; Gassendi in the *Institutio Astronomica*, book 2, ch. 15.

Quid ſit Apocataſtaſis ſeu Reuolutio, & Quotuplex.

What apocatastasis or revolution is, and of how many kinds.

II. Revolution, in Greek [Greek: ἀποκατάστασις (apokatastasis, "restoration")] or [Greek: ἐξελιγμὸς (exeligmos, "a rolling out")], that is, restitution or reversion, is the return of some star from one and the same term back to the same term to which it was previously joined, whether that term is another star or some notable point of the heaven. This return is the more notable the more conditions concur toward both conjunctions: as when the Sun and the Moon are conjoined again in the same place of the zodiac, or the Sun returns to the same point in which it stood at a person's natal beginning, a revolution customary among astrologers; or when the Moon returns from one node to the same node in the same place of the zodiac; or indeed to ☌ or ☍ with the Sun in the same place of the zodiac and at the same node in which it had stood at the other ☌ or ☍; or when not two only, but three or more stars come together again at once, especially if at the same point of the zodiac. [Margin: Two kinds of revolutions.] The chief kinds of apocatastases [Apocataſtaſeων] are two. One is that of the planets referred to the Sun and to the same place of the zodiac; the other is that of the other planets referred to one another and to the same places of the zodiac. It is of the first kind when one seeks the time in which some planet and the Sun run through the whole zodiac, or are restored to the same place of the zodiac; or within which a planet carries its stations and retrogradations round the whole zodiac; or the Moon likewise leads her phases round the whole zodiac, and so on.

Quantum ſit tempus Reuolutionum cuiuſque Planetæ ad Solem in eodem loco Zodiaci.

How long the time is of the revolutions of each planet with respect to the Sun in the same place of the zodiac.

III. According to Kepler's opinion in the *Epitome of Copernican Astronomy*, p. 848, from which the hypotheses of other astronomers do not notably depart on this point, Saturn and the Sun return together to the same place of the zodiac very nearly within 59 years, so that while the Sun measures out the zodiac 59 times, Saturn measures it out twice. [Margin: ♄ Revolutions to ☉.] Thus, since in 1590 June 9 there was a ☌ of ♄ and ☉ in 18° ♊, we are certain, counting 59 years from then, that Saturn was joined with the Sun in the year 1649 on June 9 in 18° ♊, or at most 19°; just as, since in 1590 on December 17 it was opposed to the Sun standing in 25° ♐, one could declare that in 1649 on December 17 it would return to the same opposition in 25° ♐. [Margin: ♃

Revolutions.] Jupiter, on the other hand, walks round the zodiac seven times within 83 years of the Sun. [Margin: ♂ Revolutions.] Mars, however, keeps its revolutions with the Sun at great intervals. But the longer its apocatastases are taken, the more accurate they are; and the most accurate is held to be the period of 79 years, in which Mars completes 42 revolutions with respect to the Sun to the same place of the zodiac. [Margin: ♀ Revolutions.] Venus returns to the same place fairly exactly within 8 solar years, and is thus joined to the Sun in the upper part of her orb in five places, each two of the five being 72° apart. From these five places the following unions move off at a slow pace, by eights of years, so that those intervening 72° are hardly used up within two half-centuries, and the whole zodiac is frequented by conjunctions of this kind. [Margin: ☿ Revolutions and ☾.] By contrast, Mercury goes round the zodiac 54 times within 13 solar years. Finally the Moon, since she runs through the zodiac somewhat faster than the Sun, needs a greater number of years in order to be restored more accurately with the Sun to the same place of the zodiac, according to the various lunisolar cycles about which enough was said in book 4, ch. 19. The most accurate of these cycles is the one of 3400 years, in which the Moon is restored to the Sun in the same place of the zodiac, and at the same hour, indeed at very nearly the same minute, after 42053 completed synodic months. But here for you is a synopsis of the revolutions of the planets with respect to ☉.

Revolutions of the planets with respect to the Sun in very nearly the same place of the zodiac [Reuolutiones Planetarum ad Solem in eodem proximè loco Zodiaci]

	Solar years [Anni Solares]	Revolutions [Reuolutiones]		Solar years [Anni Solares]	Revolutions [Reuolutiones]
♄	59	2	♀	8	5
♃	83	7	☿	13	54
	15	8		19	235
	17	9		72	940
♂	32	17	☉	257	3055
	47	25		304	3760
	79	42		3400	42053

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One thing further, I think, should not be passed over: namely that the mean conjunction of Mars with ☉ can differ from the true one by no more than 20 days, and the mean opposition with it by no more than 25 days; but for Saturn and Jupiter the distance of the mean from the true ☌ or ☍ with the Sun is much smaller.

[72] On the page a rule divides the rows under ♃ (and under ☿) from those that follow; ♂ stands centred on the five rows 15 to 79, and ☉ on the five rows 19 to 3400, so each sign governs its whole block.

Quæ ſit Coniunctio Magna & Maxima.

What the Great and the Greatest conjunction is.

IV. Māshāʾallāh the Arab, in the book *De coniunctionibus*, calls it the Greatest when Saturn, Jupiter and Mars are conjoined on the same day or as nearly as possible, wherever in the zodiac they finally come together; and he says that from this conjunction a presumption is to be drawn about the transfer of kingdoms, about the coming of a great prophet, and so on. He calls it the Greater, however, when Saturn and Jupiter are joined; the Middling or Mean when Saturn and Mars; the Great when Jupiter and Mars meet. But Abū Maʿshar, with the whole school of astrologers and almost of astronomers, calls it the Greatest conjunction when Saturn and Jupiter are conjoined at the beginning of Aries, which is the beginning of the fiery triplicity, or in the passage from the end of the watery triplicity, that is, from the end of Pisces, to the fiery triplicity, which begins at the beginning of Aries, especially if it occurs after a revolution of about 800 or 900 years. For they hold that the greatest changes of empires and of sublunary nature are either brought about or foreshadowed from this. The other conjunctions of Saturn and Jupiter they commonly call Great, whether they occur after every 20 years or after 200, when a revolution has been made from the beginning of one triplicity to the beginning of another, namely from the beginning of ♈ to the beginning of ♌, or from the beginning of the latter to the beginning of ♎, or from the beginning of the latter to the beginning of ♒. These two-hundred-year conjunctions some call Greater, others Middle; those of Saturn and Mars, however, they call Lesser. Yet the generic name for all of these is great conjunction, that is, of two superior planets. [Margin: Why they are called Great.] This is because those planets are the heaviest or slowest, and their stay in bodily or platic conjunction lasts much longer than that of the other planets, and the effects or the significations of such conjunctions extend over many years. Those, on the other hand, who with Cardano call the eight-hundred-year conjunctions Great call the twenty-year ones Lesser and the two-hundred-year ones Middle. In this controversy, just as it is easy to discern from the attendant circumstances which conjunction authors are speaking of, so we shall act prudently if we always add some marks by which it is distinctly clear which species of conjunctions we are speaking of. No one, however, would deny that this greatness of conjunctions can be taken either from the dignity and efficacy of the planets, or from the concurrence of several of them in one place of the zodiac on the same or on neighbouring days, or from the greater interval at which they revolve to the same place of the zodiac, or from the dignity of the place of the zodiac itself; and that the more conditions of this kind have marked out some conjunction, the greater the prerogative by which it deserves the name of Great or Greatest conjunction. [Margin: A new ordering of conjunctions.] But speaking of the conjunctions of the three superior planets, it seems to me that three classes should be established, and three degrees in each one. Let the

first class be that of the conjunctions of ♄ and ♃, which may be called in Greek [Greek: κρονοζευτικὰς (kronozeutikas, "Saturn-yoked")], that is, Saturnijovial; the second is the ☌ of ♄ and ♂, which let us name [Greek: κροναρητικὰς (kronarētikas, "Saturn-Martial")], namely Saturnimartial; the third is the ☌ of ♃ and ♂, which we shall call [Greek: ἀρηζευτικὰς (arēzeutikas, "Mars-yoked")], that is, Martijovial. Let the first degree in each class be when the conjunction is periodic, the whole zodiac having been unrolled, but with the revolution made from one cardinal point to the same point, as from the beginning of Aries back to the beginning of Aries; and these we shall call Greatest or Great, each in its own class. Let the second degree be when the places of the conjunctions make a passage from one cardinal point to another immediately following or preceding, or at least to the opposite one, as from the beginning of Aries to the beginning of Capricorn; these we shall call Lesser. This degree shows up more clearly in the Cronozeutic conjunctions; in the Arizeutic, however, the passage to the opposite is to be looked for, as from the beginning of ♈ to the beginning of ♎, as we shall say at number II. Let the third degree be that of conjunctions following one another most nearly and immediately, which we shall call Least. Unless we should prefer to call those of the first class Great, those of the 2nd Middle, those of the 3rd Lesser, and within each one to distinguish not the ☌ themselves but the intervals into least, middle and greatest.

Medianè an Veræ Coniunctiones ſint Conſiderandæ cùm de Magnis aut Maximis agitur?

Whether mean or true conjunctions are to be considered when the Great or Greatest are in question?

V. [Margin: Votes for mean ☌.] The mean conjunction, that is, the one occurring along the lines of mean motion, or, according to Ptolemy's hypothesis, the one by which the centres of the epicycles are conjoined under the same place of the zodiac: this, I say, is what Abū Maʿshar and almost all astrologers considered, as Giovanni Pico affirms, *Contra astrologos*, book 5, ch. 6; and the more recent authors, with Origanus and Kepler, do so too. The reasoning for them seems to be this. First, the ease of defining equal intervals of conjunctions, that is, by equal or mean motions, whereas the intervals of true conjunctions are unequal because of the double inequality, namely of the centre and of the planet on the epicycle. Second, because the mean conjunction is made with respect to the centre of the epicycle, and the conjunction of the epicycle seems more powerful than the conjunction of the planet alone, since the whole epicycle with the planet is nobler than the planet alone, and contains the power of the planet and something more. Third, because there is no point to which the planet is always indifferent except the centre of the epicycle. Fourth, because when conjunctions of this kind are considered in order that a judgement may be formed from them about the great changes of the world, which do not occur

except with some latitude of years, it does not seem that one should proceed more scrupulously in investigating the true conjunctions of these planets.

VI. [Margin: Votes for true ☌.] Bellanti, however, defends the contrary opinion, question 11, art. 3, and book 5 against Pico: namely that true conjunctions are to be preferred to mean ones, though if epicycles really are granted in the heaven, something is to be allowed to the mean ones as well. His first reason is that mean motion is imperfect and needs the perfecting of the equation, whereas true motion is perfect. The second is that the motion of the epicycle on the eccentric exists for the sake of the planet's motion on the epicycle, as a means for the sake of an end. The third is that no diversity is discerned in the orbs of the planets except for the star carried to other parts; and the star is nobler than any unknown part, and therefore the true motion, which belongs to the star, is to be preferred. The fourth is that epicycles and mean motions either have been invented by astronomers for ease of calculation and to help the imagination, or at least it is uncertain whether they really exist in the heaven; and figments or uncertainties ought not to be put before things certain and true and evidently appearing. And although the whole prevails over the part, yet the planet on the epicycle is like a diamond in a ring, which intensively is worth more than the ring alone; or, if you take the whole aggregate, you must conclude that the motion of both is to be attended to, but principally the true motion of the planet.

Giovanni Pico, however, *Contra astrologos*, book 5, ch. 6, teaches that neither mean nor true conjunctions of the planets have force for bringing about or signifying great changes; but if they did, attention would rather have had to be paid to the true ones than to the mean, the latter being fabricated and resting on a fictitious motion, for the convenience of carrying out calculation by mean and equal motions. He confirms this from the conflict with the terms or effects that astrologers have laid down from mean conjunctions. For in the year of grace 1226, by the Alphonsine Tables, Jupiter and Saturn migrated, in their conjunction made in Aquarius, from the earthy triplicity to the airy. Therefore, according to mean motions and Abū Maʿshar's opinion, they were going to remain in that same triplicity for 240 years; and yet after about 140 years, namely in the year of grace 1365, they migrated from the airy triplicity to the watery, and met in Scorpio. Since, however, not all mean conjunctions of Saturn and Jupiter have fallen at the times the astrologers most wished for, and they therefore fled not to the true conjunctions but to another figment, namely by saying that their effects follow neither at once nor quickly, but after many years, or even centuries of years, [Margin: Giovanni Pico's acumen against the slowness of mean ☌.] Pico sharply attacks them with this dilemma: *It is not for a powerful and efficacious cause to begin late what it is going to do, but rather to stop late, and to begin either at once or quickly. Therefore the power of these conjunctions is either efficacious or weak. If efficacious, it cannot long cease so as not to act; if weak, it cannot last long enough to act late indeed, yet at some time. A great force tames what hinders and resists it; a weak one vanishes, and over a longer stretch of*

time dissolves itself by its own thinness. What of the fact that in this interval of time new conjunctions are always supervening, whose nature, as they themselves say, is to erase the earlier ones, since a supervening constellation crushes the preceding one?

VII. [Margin: Our opinion.] But I judge, not absolutely, but on the hypothesis that meetings of this kind are effective in bringing about or signifying notable changes in sublunary things, that attention should be paid principally to the true conjunctions, no differently than to the true synods of the luminaries in eclipses. For if there is any greater force in them than in other lesser conjunctions, it is surely on account of the ponderosity, as they call it, of these two planets, that is, their gravity and slowness, because of which, once conjoined, they last for three or four days, whereas the conjunction of the centres of their two epicycles is momentary. Yet I think that mean motions and mean conjunctions should be employed, not only so that by an easier first computation we may scent out from afar the times of the true conjunctions, but also because, if they precede the true ones, they begin the influx of those by a certain application, and are as it were dispositions toward the impression to be made through the true ones; and if they follow the true ones, by a certain defluxion they retain something of the impression already made, or the planets, thus neighbouring, still persist in the influx once impressed, as long as the mean conjunction lasts.

Quæ ſint loca Zodiaci, in quibus Magnæ aut Maximæ Coniunctiones fiunt, & quid, quot, & quales Triplicitates Signorum, quiſue illarum ordo.

Which are the places of the zodiac in which Great or Greatest conjunctions occur; and what, how many and of what kind the triplicities of the signs are, and what their order is.

VIII. First, the most ancient astrologers before Abū Maʿshar and Alcabitius divided the signs of the zodiac into four classes endowed with the qualities of the elements. The first contains three fiery signs, namely ♈ ♌ ♐; the second three earthy, ♉ ♍ ♑; the third three airy, ♊ ♎ ♒; the fourth three watery, ♋ ♏ ♓. And accordingly they called the first triplicity, that is, one composed of three signs, the Fiery Triplicity; the 2nd the Earthy Triplicity; the 3rd the Airy Triplicity; the 4th the Watery Triplicity. Why they also called the same ones the Fiery, Earthy, etc. triangles or trigons, understand as follows. If Saturn and Jupiter meet at the beginning of Aries, or of the fiery triplicity, after about 20 years another conjunction of theirs will occur, but far from the beginning of Aries against the order of the signs, or backward by about a third of the zodiac, or by nearly 120°; and so it will fall at the beginning of Sagittarius, which is likewise a fiery sign. And the third conjunction, after about another 20 years, will go back through another four signs to the beginning of Leo, a sign equally fiery. Thus, since the places of these nearest conjunctions are about a third of the circle distant from one another, it comes about that in 60 years the places

of three conjunctions form a nearly equilateral triangle inscribed in the zodiac, and regard one another by a quasi-trine or trigon aspect, as can be seen in the figure below: in which are placed the mean synod of Saturn and Jupiter that occurred in the year 1603 in Sagittarius (the true one, however, was on December 24, in 9° 36′ ♐) and the one that occurred in the year 1623

Diagram. A zodiac ring in the right column, a band between two concentric circles divided by radial strokes into twelve compartments, each holding the sign of its zodiac constellation. Within the inner circle, bundles of straight lines, some solid and some dotted, run between three regions of the ring, near ♐, near ♌ and near ♈, so as to form nearly equilateral triangles; dotted lines lead from the dated labels out toward the ring. No lettered points. The sentence the figure illustrates is interrupted by it on the page.

- Ring, top going clockwise: ♎ (top, left of centre); ♍ (top, right of centre); ♌ (upper right); ♋ (right); ♊ (lower right); ♉ (lower right, below ♊); ♈ (bottom, right of centre); ♓ (bottom, left of centre); ♒ (lower left); ♑ (left); ♐ (upper left); ♏ (upper left, near the top).
- Near ♐, set diagonally inside the circle: 1603, 1663, 1723.
- Near ♌, set upside-down inside the circle: [?]743 (the first figure, if one is printed, falls on a line of the figure), 1683, 1623.
- Near ♈, set vertically inside the circle: 1583, 1643, 1703, 1763.

in Leo, and the one that occurred in the year 1643 in Aries. [Margin: Examples of the triplicities.] Since, however, the places of these conjunctions are not exactly a third of a circle apart, but somewhat less, namely 117°, in præcedentia of the signs, it comes about that at every sixtieth year the conjunction passes its initial place by about 9°, and in 180 years by 27°. Therefore in about 200 years, in which 10 conjunctions occur, the vertices of the triangles, having completed each sign of one class, pass over to the signs of another triplicity; so that if the first was Fiery, the second is Earthy, and the second decade of conjunctions stays in these three signs. Hence in about 800 years, that is, 794 years and 332 days, the four triplicities and the whole zodiac are completed, and the conjunction of ♄ and ♃ returns to the beginning of Aries and becomes a Greatest conjunction. You will see, moreover, in the scheme set above, three conjunctions assigned to Sagittarius and three to Leo, each 60 years apart in its own sign, but around the beginning of Aries and the end of Pisces four, 60 years apart from one another, together with the years in which they occurred or will occur, so that by this example one may carry on such transmigrations of the Great conjunctions.

IX. But so that beginners may understand without confusion the backward leap of the mean conjunctions of ♄ and ♃, I have decided to construct for them the following table containing eight great periods, each of which begins from a Greatest conjunction, that is, one occurring in the passage from the end of ♓ to the beginning of ♈; for the explanation of this table, see what follows.

TABVLA MAGNARVM & MAXIMARVM CONIVNCTIONVM Saturni, & Iouis, quarum Maximæ sunt in prima serie sub annorum titulis

Table of the Great and Greatest conjunctions of Saturn and Jupiter, of which the Greatest stand in the first row under the headings of the years

Triplicities	Periods of Great ☌: order	Periods of Great ☌: places	I (years current before Christ)	II (years current before Christ)	III (years current before Christ)	IV (years current before Christ)	V (years current before Christ)	VI (before Christ)	VII (years of Christ)	VIII (years of Christ)	Periods of Great ☌: places	Periods of Great ☌: order	Triplicities
	1 or	♓ 29°; ♈ 0°	3980	3185	2390	1595	801	6 An. Chr.	789	1583	♓ 29°; ♈ 0°	1 or	
Fiery	2	♐ 2°	3960	3165	2370	1575	781	15	809	1603	♐ 2°	2	Fiery
	3	♌ 5°	3940	3145	2350	1556	761	35	829	1623	♌ 5°	3	
	4	♈ 8°	3920	3125	2330	1536	741	55	849	1643	♈ 8°	4	
	5	♐ 11°	3900	3105	2310	1516	721	75	869	1663	♐ 11°	5	
	6	♌ 14°	3880	3086	2291	1496	701	95	889	1683	♌ 14°	6	
	7	♈ 17°	3861	3066	2271	1476	681	115	908	1703	♈ 17°	7	
	8	♐ 20°	3841	3046	2251	1456	661	135	928	1723	♐ 20°	8	
	9	♌ 24°	3821	3026	2231	1436	641	154	948	1743	♌ 24°	9	
	10	♈ 27°	3801	3006	2211	1417	622	174	968	1763	♈ 27°	10	
Earthy	11	♍ 0°	3781	2986	2191	1397	602	194	988	1782	♍ 0°	11	Earthy
	12	♏ 2°	3761	2966	2171	1377	582	214	1008	1802	♏ 2°	12	
	13	♉ 5°	3741	2947	2151	1357	562	234	1028	1822	♉ 5°	13	
	14	♍ 8°	3722	2927	2131	1337	542	253	1047	1842	♍ 8°	14	
	15	♏ 11°	3702	2907	2112	1317	522	273	1067	1862	♏ 11°	15	
	16	♉ 14°	3682	2887	2092	1298	502	293	1087	1882	♉ 14°	16	
	17	♍ 18°	3662	2867	2072	1278	482	313	1107	1902	♍ 18°	17	
	18	♏ 21°	3642	2847	2052	1258	463	333	1127	1921	♏ 21°	18	
	19	♉ 24°	3622	2827	2032	1238	443	353	1147	1941	♉ 24°	19	
	20	♍ 27°	3602	2807	2012	1218	423	373	1167	1961	♍ 27°	20	
Airy	21	♎ 0°	3582	2787	1992	1198	403	393	1186	1981	♎ 0°	21	Airy
	22	♊ 2°	3563	2768	1972	1178	383	412	1206	2001	♊ 2°	22	
	23	♒ 5°	3543	2748	1953	1158	363	432	1226	2021	♒ 5°	23	
	24	♎ 8°	3523	2728	1933	1139	343	452	1246	2040	♎ 8°	24	
	25	♊ 11°	3503	2708	1913	1119	323	472	1266	2060	♊ 11°	25	
	26	♒ 14°	3483	2688	1893	1099	304	492	1286	2080	[?] 14°	26	
	27	♎ 18°	3463	2668	1873	1079	284	512	1306	2100	♎ 18°	27	

Triplicities	Periods of Great ☌: order	Periods of Great ☌: places	I (years current before Christ)	II (years current before Christ)	III (years current before Christ)	IV (years current before Christ)	V (years current before Christ)	VI (before Christ)	VII (years of Christ)	VIII (years of Christ)	Periods of Great ☌: places	Periods of Great ☌: order	Triplicities
	28	♊ 21°	3443	2648	1853	1059	264	532	1325	2120	♊ 21°	28	
	29	♒ 24°	3424	2628	1833	1039	244	551	1345	2140	♒ 24°	29	
	30	♎ 27°	3304	2608	1814	1019	224	571	1365	2160	♎ 27°	30	
Watery	31	♋ 0°	3384	2589	1794	999	204	591	1385	2179	♋ 0°	31	Watery
	32	♓ 2°	3364	2569	1774	980	184	611	1405	2199	♓ 2°	32	
	33	♏ 5°	3344	2549	1754	960	165	631	1425	2219	♏ 5°	33	
	34	♋ 8°	3324	2529	1734	940	145	651	1444	2239	♋ 8°	34	
	35	♓ 11°	3204	2509	1714	920	125	670	1464	2259	♓ 11°	35	
	36	♏ 14°	3284	2489	1694	900	105	690	1484	2279	♏ 14°	36	
	37	♋ 18°	3265	2469	1675	880	85	710	1504	2299	♋ 18°	37	
	38	♓ 21°	3245	2450	1655	860	66	730	1524	2318	♓ 21°	38	
	39	♏ 24°	3225	2430	1635	840	46	750	1544	2338	♏ 24°	39	
	40	♋ 27°	3205	2410	1615	820	26	770	1564	2358	♋ 27°	40	

You should not, however, be scrupulous with this table set out in this way; rather, as Kepler says in the book *De Stella Nova*, ch. 7, all these things are to be taken broadly, especially since there is great diversity among the astronomical tables in the collection of the mean motions, and the mean conjunction often belongs to the class of one triplicity, while the true one belongs to that of another. For example, in the year 7 before the common epoch of Christ the true conjunction of Saturn and Jupiter was in 23° ♓, around June 22; but in the following year, 6, on January 22, the mean one was in 29° 26′ ♓, and, by the more correct tables, readily at the beginning of Aries. So too in the year 1583 their true conjunction was at the end of Pisces; indeed even the mean one, by the Prutenic Tables, was in 28° 14′ ♓, which Cardano nevertheless puts, from the Alphonsine Tables I believe, at the beginning of Aries. So finally in the year 1543 the true union of Saturn and Jupiter was in 26° of Pisces, which Francesco Montebruni nevertheless calls a Greatest conjunction in his *Ephemeris*, because a few months later the mean one followed, in 8° or 9° of Aries. [Margin: 1. A point to note in the table above.] First, then, in the table set out above you will see that in each period, begun from a Greatest conjunction, there are forty Great conjunctions, 20 or 19 years apart from one another, of which the first decade always belongs to the Fiery triplicity, the second to the Earthy, the third

to the Airy and the fourth to the Watery. [Margin: 2. A point to note.] Second, at the head of the table you will see the Greatest conjunctions set opposite the marginal number 1, the oldest of which, near the rising of the world, was in the year 3980 before Christ; the second in the year 3185; the third in the year 2390; the fourth in the year 1595; the fifth in the year 801; the sixth in the year 6 before the birth of the Saviour; the seventh in the year of Christ 789; the eighth in the year 1583, and it will last until the year 2378, if this world lasts so long. [Margin: 3. A point to note.] Third, you will see that the places of the Great conjunctions go back in the zodiac through nearly four signs, or through 117°, so that if the first was in 28° or 29° of Pisces, the second falls in 1° or 2° of Sagittarius, the third in 4° or 5° of Leo, and so on. These places are indicated, not only in the first or eighth period but in the others as well, by the numbers of the left and right margins, set opposite those years. For example, since ♐ 2° corresponds to conjunction 2, and the years 3960, 3165, 2370, 1575, 781, 16[73], 809 and 1603 correspond to the same conjunction, the mean conjunction of Saturn and Jupiter is therefore understood to have occurred in each of those years in 2° ♐, usually without an error of a whole degree; although, if we listen to Andrea Argoli, *Pandosion Sphaericum*, ch. 73, the platic conjunction, or by orb, suffices to name the species of a conjunction. But he is speaking of true conjunctions, for he says that on 1643 January 26 the true meeting of Saturn and Jupiter was in 26° of Pisces, but that since the light of the orb of Saturn is 10° and that of Jupiter 12°, they can be said to have been conjoined at the beginning of Aries, just as also in the year 1583, although the partile conjunction was in 21° of Pisces.

Quanta ſint Interualla Coniunctionum Magnarum ac Maximarum quoad motus medios.

How great the intervals of the Great and Greatest conjunctions are with respect to the mean motions.

X. [Margin: 1. Opinion of the ancients.] The first opinion was that of Aboassar or Abū Maʿshar, *De magnis coniunctionibus*, tract 1, differentia 2, followed by Alcabitius, differentia 4, Alliacensis, Leopoldus and most of the older astrologers: namely, that the interval from one Greatest conjunction to the next is 960 solar years; from the beginning of one triplicity to the beginning of another, 240 years, so that in each of the triplicities 12 Great conjunctions occur, and sometimes 13 or 11; from one great conjunction to the next, 20 years; and from the conjunction of ♄ and ♂, which they call Lesser but restored in Cancer, 30 years. For they suppose that every 20 years the conjunctions of Jupiter and Saturn recur in such a way that they stand 242½°, or 8ˢ 2½°, in the order of the signs from the initial place, so that if the first was at the beginning of Aries, the second, after 20 years, falls in 2° 30′ of Sagittarius, and so on.

[73]the table above prints 15 at this place

[Margin: 2. Opinion of Cardanus and Naiboda.] The second opinion is that of Cardano, *Aphorisms* 1, aph. 65, and of Valentinus Naiboda on Alcabitius, differentia 4, who asserts that from one Greatest conjunction to the next there lie only 794 years, 214^{d}; from the beginning of one trigon to the beginning of another, 198 years and nearly 8 months, or 236^{d}; and from one of the lesser of the Great conjunctions to the next, not 20 years but 19 years, $315^{d}\ 19^{h}$. For the place of the following conjunction, he says, stands 242° 59′ 9″ in the order of the signs from the place of the preceding one. Naiboda adds, however, that a lengthier inquiry into this matter does not pertain to the substance of the true art, since he thinks the force of the Great conjunctions partly erroneous and partly of less moment than some reckon. For Cardano, however, the three already mentioned are called Great, Middle and Lesser, but for Naiboda, Greatest, Great and Lesser.

[Margin: 3. Opinion of Reinhold, Magini, Origanus.] The third opinion is that of Reinhold in the Prutenic Tables, precept 37; of Magini, canon 53 of the *Secunda Mobilia*, and in the tables of the same work, p. 549; and of Origanus, vol. 1, p. 448: that the time from one Lesser conjunction of ♄ and ♃ to the next is 19 Egyptian years and $318^{d}\ 12^{h}\ 59^{m}\ 23^{s}$, in which time the place of the following conjunction stands 242° 58′ 42″ 44‴ in the order of the signs from the place of the preceding one; that from the beginning of one triplicity to the beginning of another it is 198 years, $265^{d}\ 9^{h}\ 53^{m}\ 52^{s}$, on completion of which period the place of the conjunction has advanced 279° 47′ 7″ 25‴; and finally that from one Greatest conjunction to the next there lie 794 years, $331^{d}\ 15^{h}\ 35^{m}\ 28^{s}$, in which almost the whole zodiac is completed and the passage is made from the end of the fourth triplicity to the beginning of the first; that is, the place of the synod comes to stand at 359° 8′ 29″ 41‴. But to reduce these years to Julian solar years, they must be divided by 4, and the quotient subtracted from the number of days.

[Margin: 4. Opinion of Kepler.] The fourth opinion is that of Kepler, *Epitome of Copernican Astronomy*, pp. 189 and 855, who puts 794 years from Greatest to Greatest, nearly 200 years from the beginning of one trigon to the beginning of another, and nearly 20 years from one ☌ to the next nearest. For it is enough for him in that place to use round numbers; in the Rudolphine Tables, however, ch. 33, pr. 180, he teaches a way of defining these times more accurately, should anyone wish to examine them more scrupulously. For us it will be enough to have both the synopsis of the following small table and the problem that will come after it.

☌ ♄ & ♃. **Interualla in Annis ſed Iulianis**

☌ *of* ♄ *and* ♃. *Intervals in years, but Julian years*

☌	Authorities	Interval
Lesser	Abū Maʿshar, Alcabitius, etc.	20 years $0^{d}\ 0^{h}\ 0^{m}\ 0^{s}$
	Cardano, Naiboda	19 years $315^{d}\ 19^{h}\ 0^{m}\ 0^{s}$

☌	Authorities	Interval
	Reinhold, Magini	19 years 314^{d} 12^{h} 59^{m} 23^{s}
	Origanus	
	Kepler	about 20 years
Middle	Abū Maʿshar, Alcabitius	240 years 0^{d} 0^{h} 0^{m} 0^{s}
	Cardano, Naiboda	198 years 236^{d} 0^{h} 0^{m} 0^{s}
	Reinhold, Magini	198 years 216^{d} 9^{h} 53^{m} 52^{s}
	Origanus	
	Kepler	about 200 years
Greatest	Abū Maʿshar, Alcabitius	960 years 0^{d} 0^{h} 0^{m} 0^{s}
	Cardano, Naiboda	794 years 214^{d} 0^{h} 0^{m} 0^{s}
	Reinhold, Magini	794 years 133^{d} 15^{h} 35^{m} 28^{s}
	Origanus	
	Kepler	about 794 years
But according to Egyptian years		
Lesser	Reinhold, Magini and Origanus	19 years 318^{d} 12^{h} 59^{m} 23^{s}
Middle		198 years 265^{d} 9^{h} 53^{m} 52^{s}
Greatest		794 years 331^{d} 15^{h} 35^{m} 28^{s}

Problema 1. Inuenire Interuallum inter duas Medias & proximas Coniunctiones, aut Syzygias binorum Planetarum, ſeu Tempus periodicæ Syzygiæ mediæ binorum quorumuis Planetarum, ſecundùm quaſcumq. tabulas.

Problem 1. To find the interval between two mean and nearest conjunctions or syzygies of two planets, or the time of the periodic mean syzygy of any two planets, according to any tables whatever.

XI. Reinhold, precept 37, and Magini, canon 53, teach this as follows. Subtract the lesser daily mean motion of the centre of the epicycle, or of longitude, that is, that of the slower planet, from the greater daily mean motion, or that of the faster planet; and by the remainder, or excess of the greater, divide the whole circle, first resolved into fractions of the same order as the daily motion. For the quotient will show the time between two mean syzygies, for instance between two ☌ or between two ☍; and the mean motion of the slower planet made in that time will show the place of the syzygy in the zodiac.

EXEMPLVM.

Example.

In the Prutenic Tables the daily motion of Jupiter exceeds the daily motion of Saturn by 2′ 58″ 40‴ 16⁗ 51′′′′′ 25′′′′′′ 2′′′′′′′. When the whole circle, which consists of six sixties of degrees, is reduced to sevenths and divided by the excess already mentioned, likewise reduced to sevenths, there remain 2 second sixties of days, 0 first sixties, 53 days and about 32′ 28″ 28‴ of a day, according to the form of Arabic logistic; but according to the usual form, 7253 days, or 19 Egyptian years, 318^{d} 12^{h} 59^{m} 23^{s} 12‴.

This is the time between one mean ☌ of ♄ and ♃ and the next. If you want Julian years, divide the Egyptian years by 4, and subtract the quotient from the days.

Quæ ſint Interualla & loca mediarum Coniunctionum Maxima Minora, & Minima in tribus ſuperiorib. Planetis.

What the Greatest, Lesser and Least intervals and places of the mean conjunctions are in the three superior planets.

XII. I answer, following the preceding problem and without departing far from the Prutenic Tables, that they are those which the table set below will show. From it you will see that the Greatest or Periodic conjunctions are completed with a more exact revolution in the unions of Saturn and Jupiter than in the unions of Jupiter and Mars, since the period of the latter, when the circle is finished, passes the initial place by 6° 9′; or than in the conjunctions of Saturn and Mars, since, when the circle is completed, the last ☌ occurs 8° 24′ beyond the former place. You will see further that the Middle intervals of the conjunctions of Saturn with Jupiter occur with a passage from the beginning of one triplicity to the beginning of another, as nearly as possible; but that in the conjunctions of Mars and Saturn the nearest cardinal point is passed by 8° 24′; while in the Middle intervals of Jupiter and Mars this passage cannot be considered. For if Jupiter and Mars have been joined at the beginning of Aries, they will be joined in the following conjunction at the beginning of Gemini, then at the beginning of Leo, after that at the beginning of Libra, etc. Therefore the Middle interval of these must be determined from the passage from one cardinal point to the opposite one, namely from the beginning of ♈ to the beginning of ♎, or from the beginning of ♋ to the beginning of ♑. Finally you will observe that in one period of the conjunctions of ♄ and ♃, 40 ☌ occur; but in one of ♄ and ♂, 15 ☌; and in one of ♃ and ♂, 6.

Interualla ☌ Trium Superiorum Planetarum

Intervals of the ☌ of the three superior planets

Planets	Intervals	Julian years	Arc of the zodiac	Number of ☌
♄ & ♃	Min.	19 years 314^{d} 12^{h} 59^{m}	8^{s} 2° 59′	1
	Med.	198 years 216^{d} 9^{h} 54^{m}	8^{s} 29° 47′	10
	Max.	794 years 133^{d} 15^{h} 35^{m}	11^{s} 29° 8′	40
♄ & ♂	Min.	2 years 3^{d} 20^{h} 52^{m}	0^{s} 24° 36′	1
	Med.	6 years 15^{d} 11^{h} 29^{m}	3^{s} 8° 24′	4
	Max.	24 years 68^{d} 1^{h} 4^{m}	12^{s} 9° 0′	15
♃ & ♂	Min.	2 years 86^{d} 10^{h} 26^{m}	2^{s} 0° 41′	1
	Med.	6 years 259^{d} 7^{h} 18^{m}	6^{s} 2° 3′	3
	Max.	13 years 153^{d} 14^{h} 36^{m}	12^{s} 6° 9′	6

[Margin: Time of the revolution of a ☌, roughly.] More roughly, however, Argoli, *Pandosion Sphaericum*, ch. 73, says that the conjunction of ♄ and ♃ occurs in the 20th year; of ♄ and ♂ every 25th month; of ♃ and ♂ every 27th

month; of ♄ and ☉ every 13th month; of ♃ and ☉ every 14th month; of ♂ and ☉ every 27th month; of ☉ and ♀ in the 10th month; of ☿ and ☉ every two months.

Quæ ſint Interualla Verarum ſeu apparentium Coniunctionum, & quæ diſtantia illarum à medijs.

What the intervals of the true or apparent conjunctions are, and what their distance from the mean ones is.

XIII. This question, indeed, asks too much, because of the excessive inequality of the motions of these planets, which does not allow a certain and precise rule to be determined. Between the true conjunctions themselves, however, so short a time can sometimes intervene, because of the condition of direct and retrograde motion and the position of the apsides, that three conjunctions of Saturn and Jupiter take place within one year, as happened in the year 5 before the birth of Christ the Lord and in the years 1563 and 1564, as will be clear from the history of notable conjunctions to be given in the next chapter.

CAPVT X.

Chapter 10.

Hiſtoria Coniunctionum Inſignium, & Mutationum Generalium in Mundo cum eis connexarum. Qua occaſione Diſceptatur an Coniunctiones huiuſmodi, aut aliæ magnæ Reuolutiones ſint cauſæ vel Signa magnarum mutationum in Mundo ſublunari.

History of notable conjunctions and of the general changes in the world connected with them. On which occasion it is debated whether conjunctions of this kind, or other great revolutions, are causes or signs of great changes in the sublunary world.

I. To proceed in order in this celebrated controversy, we shall first set out a history of certain more notable conjunctions of the three superior planets, drawn from the astronomical tables; and we shall add a synopsis of the mean conjunctions occurring at the cardinal points of each triplicity, with the more notable effects or events that stand at a nearer interval of time from them, so that it may better appear from this whether, and how great, a connection there can be between these conjunctions and those changes.

Coniunctiones Corporales quinque Planetarum.

Bodily conjunctions of the five planets.

II. [Margin: Almost bodily ♂ of ♄ and ♃.] That Jupiter should cover Saturn wholly or in part, and that their conjunction should occur [Greek: σωματικῶς (sōmatikōs, "bodily")], or by body, can hardly happen once within many

centuries, says Kepler in the *Optics*, p. 305. It seems to Kepler, however, to have happened in the year 1563 (for so the passage of the *Optics* is to be emended from Kepler himself, *De Stella Nova Serpentarii*, p. 184), since it is established from the ephemerides of that year that on September 27 Jupiter, direct, was conjoined in ♌ 4° 12′ to Saturn, direct, while each had ascending north latitude, the former 10′, the latter 14′. But since the sum of the visual semidiameters is not 4′ in these planets, either the latitude must be corrected, or, because of the expansion of the rays, it can be reduced to the bodily conjunctions. But from the observation of Giuntini, of which I shall speak shortly at no. 4, it appears that the latitude must be corrected, and that Saturn was truly covered by Jupiter. This conjunction was, however, notable, because in the following year, 1564, Jupiter, retrograde, was conjoined to Saturn, likewise retrograde, in ♌ 0° 56′ on January 26. Kepler points out, moreover, in the book *De Stella Nova*, p. 184, that the conjunctions of these planets that occur in Leo and Cancer are the more important, because the raising nodes [nodi euehentes] of both are in Leo, while in Cancer they ascend toward the apogee, and the altitude of this sign in our hemisphere favours a closer conjunction, that is, a bodily or quasi-bodily one. [Margin: Bodily ☌ of ♃ and ♂, of ♀ and ♂.] Mästlin saw Jupiter covered and wholly eclipsed by Mars at Tübingen on 1591 January 9, for the ruddy and fiery colour of the covering planet showed that Mars was the lower; and he saw Mars covered by the whiter Venus on 1590 October 3, at 5^{h} in the morning, as Kepler reports in the *Optics*, p. 305, [Margin: Mutual eclipses of ♀ and ☿.] adding that mutual occultations of Venus and Mercury can occur; and he suspects from his own observation that Mercury was occulted by Venus on 1599 June 8. [Margin: ♀ under the Sun.] He says further that no part of the Sun can be covered by Venus in this century, but that it was possible, and will be in other centuries. [Margin: ☿ under the Sun.] For about Mercury seen under the Sun enough has been said in book 3, ch. 3, scholium 3.

Coniunctiones ♄ ♃ & ♂ Insignes.

Notable conjunctions of ♄, ♃ *and* ♂.

III. [Margin: Greatest conjunction of ♄ ♃ ♂ shortly before the birth of Christ.] Although the mean conjunction of Saturn and Jupiter usually lasts until Mars overtakes them and completes their conjunction with its own true or mean one, we shall nevertheless report here three above the others. The first was in the Julian year 40, or 6 before the common epoch of Christ, in which, on January 22, in

Pisces at 29° 26′, Saturn and Jupiter came together in mean motion; and shortly afterwards, on February 25, there followed the mean conjunction of Mars and Saturn in 3° of Aries, and on March 1 the mean conjunction of Mars and Jupiter in 5° of Aries. But the true conjunction was more remarkable. For in the Julian year 39, or 7 before Christ, around June 22, there had been a true conjunction of Saturn and Jupiter in 23° of Pisces; and when, shortly

afterwards, each had become stationary and soon retrograde, in the month of August Jupiter at last overtook Saturn again as it crept against the order of the signs, at about 21° of Pisces. Finally, in the month of December, Jupiter overtook Saturn for the third time in 17° of Pisces. But Mars had joined them in the Julian year 40, in February, at the end of Pisces or the beginning of Aries. Therefore within six or seven months three true Greatest conjunctions of Saturn and Jupiter took place at the end of the watery triplicity and toward the beginning of the fiery; and about four months afterwards they were followed by a twofold conjunction of Mars, one with Saturn, the other with Jupiter.

Since this conjunction, then, was close to the birth of our Saviour and to the Star that announced him to the Magi, it gave Kepler occasion to believe, in the book *De Stella Nova in Serpentario*, ch. 26, and in the book *De Iesu Christi Servatoris nostri vero anno natalitio*, that the opinion of Laurentius Suslyga [Wawrzyniec Suślyga] the Pole was most probable. Suslyga adds four years to our customary epoch and asserts that Christ was in truth born not in the Julian year 45, as the common epoch of the Christian world and the Roman calendar suppose, but in the Julian year 41, that is, two years after the true and Greatest conjunction of the superior planets; and that the star which shone on the Magi began to shine in the very place of the Greatest conjunction, [Margin: The Star of the Magi arose under a Greatest conjunction.] in which respect it was like the new star that arose in Serpentarius in the year 1603 under the great conjunction of the three superior planets. Kepler therefore says that God himself, to accommodate himself for the time being to the rules of the Chaldeans, who at the passage of the conjunctions from the watery to the fiery triplicity expected the greatest changes in the world and universal renewals, kindled that star at the very moment of that conjunction; indeed, perhaps in the same place, so that the Magi, who were watching that conjunction, might more easily catch the new star with their eyes; and that he so ordered the times of that conjunction that it would serve to announce the coming of his Son, who was soon to appear in the flesh. He adds, moreover, that two years passed between the first shining of the star and the arrival of the Magi and the cradle of Christ. He therefore concludes that the year in which he was writing, if truth be regarded, is to be counted not 1606 but 1610, if not 1611; although he admits that the epoch in use for so many centuries is not to be changed, and that it is enough to know that its beginning is taken from the year 5 of Christ. I, however, in my chronological book have shown by solid arguments, unless I am mistaken, that our common era lacks either no year at all or at most one only.

As for this conjunction: although I do not deny that God could have used it to announce the birth of Christ that was to follow shortly after, just as he used the rainbow, a natural sign of fair weather, to signify his perpetual covenant with Noah and his pact not to destroy the human race any more by a universal flood, I nevertheless judge it safer and more pious to hold that the Star had no dependence on, or subordination to, that conjunction in what it signified, as

the Holy Fathers everywhere teach in their homilies and treatises on the Most Holy Epiphany. Indeed, I judge that God deliberately kindled it some years afterwards, so that this independence might be recognised, and so that by the miracle and by his illumination, inner and outer, the Magi might discern the renewal of the world and the birth of the great Prophet and King of Kings, which that Greatest conjunction could only foreshadow and stammer out.

IV. [Margin: ♂ ♄ ♃ ♂ ☉ ☾ and ☿ in the year 1524.] In the year of Christ 1524, in the month of February, Saturn, Jupiter and Mars appeared conjoined on almost the same day in 10° of Pisces, with Venus preceding by a few degrees, and with a conjunction of the Sun and Mercury going before them a little earlier at the end of Aquarius, and with the Moon passing through all of them in the space of thirty hours; so that, given the meeting of so many planets, perhaps no conjunction since the founding of the world has been more celebrated, as Kepler judges in the book *De Stella Nova*, ch. 29.

Again, on 1563 September 27 Saturn and Jupiter were conjoined in ♌ 4° 8′ by the Alphonsine calculation, but on August 26 at 10^{h} in 29° of Cancer according to the Prutenic Tables, [Margin: Three ♂ of ♄ and ♃ in 1563 and 1564.] with very nearly the same latitude toward the same side, that is, in an almost bodily conjunction; and when they had become retrograde, ♄ on November 6 and ♃ on the 14th, they were again truly conjoined on 1564 January 26 in 0° 56′ of Leo. Afterwards, in the same year, they became direct again, ♃ on the 14th and ♄ on the 21st of March. For the third time they appeared joined on April 14 in ♋ 28° 56′; and this conjunction was followed in the same year 1564 by a true conjunction of Mars and Saturn on June 6 in ♌ 3° 15′, and of Mars with Jupiter on June 15 in ♌ 9°. The conjunction was therefore notable, both because it was in a sign of the fiery triplicity, and because it was tripled within seven months, and because it was of the three superior planets very close together. So, at any rate, the ephemerides of Carelli have it; although Kepler, in the book *De Stella Nova*, ch. 29, calls the conjunction of 1563 conspicuous only in such a way as to assert that what was granted in the age of Christ has been wholly denied to these times of ours and to the 250 years around us: namely, that around opposition to the Sun, Saturn and Mars, having met and being retrograde, can be joined twice or three times in the same year. As for the day, however, all the tables erred. For Francesco Giuntini, in the preface to his astronomical tables, says that he observed the conjunction of Saturn and Jupiter at Orange [Aurangæ] on August 24 at 14^{h} 30^{m} after noon, when Jupiter, more to the north, covered almost the whole of Saturn. To this seems to belong the Great or Greatest conjunction of the three superior planets at the end of Cancer, from which, Henricus Mechliniensis [Hendrik Bate of Mechelen] taught in his commentary on Abū Maʿshar, the Flood issued; and Petrus Alliacensis [Pierre d'Ailly] followed him in his questions on Genesis, interpreting these planets conjoined in the watery signs as the cataracts of heaven. But to the pious ears of the sacred interpreters this origin of the Flood cannot but seem absurd.

Coniunctiones aliæ ♄ & ♃ inſignes, & de Cauſa nouæ Stellæ Anni 1603.

Other notable conjunctions of ♄ and ♃, and on the cause of the new star of the year 1603.

V. [Margin: ☌ of ♄ and ♃ in the year 1603.] Besides the mean conjunctions that took place at the beginning of the fiery triplicity, or the end of the watery, which we have already listed, namely in the years before Christ 3980, 3185, 2390, 1595, 801, 6, and in the years of Christ 789 and 1583, the most celebrated proved to be the one that occurred in the year 1603, when, after the mean conjunction held in 1° 7′ of Sagittarius, the true conjunction of Saturn and Jupiter took place in 8½° of Sagittarius on December 17, though with Mars absent in Libra; Mars afterwards overtook Saturn in 10° of Sagittarius on 1604 September 29, and Jupiter on October 8 or 9. Magini and Origanus, however, place the true conjunction of Saturn and Jupiter on December 24 in 9° 36′ ♐. This synod, moreover, was held to be notable by Kepler in the book *De Stella Nova in Serpentario*, and by Io. Francisco Spina [?], book 2 of *De maximis coniunctionibus*, ch. 2, because of that very new star which appeared in Serpentarius in the year 1604, on October 8 or 9, in ♐ 17° 43′ with north latitude 1° 56′; and because it fell shortly after the beginning of the fiery triplicity, namely in 8½° ♐ as to the true conjunction but in 1° as to the mean, although, more exactly, this triplicity began from the immediately preceding mean conjunction of Saturn and Jupiter, which took place in the year 1583, either in 29° of Pisces, as the Prutenic Tables will have it, or in 1° of Aries, as the Alphonsine do, and was the eighth since the founding of the world; the true conjunction, however, was in 21° of Pisces. But Kepler admits there, in ch. 7, that all these beginnings must be taken in a broad sense. [Margin: Kepler's opinion on the efficient cause of new stars.] Finally, in ch. 27, he concludes that this star was created anew by God, whether using nature as his minister or not, and associated with a great conjunction, and by special providence, so that, marked by so many prerogatives, it might the more clearly rouse mortals to understand the things of which he wished them to be forewarned. The prerogatives were these: that the rising of the Star fell in the year [recte: after the year, per the book's errata] of a Greatest conjunction of Saturn and Jupiter; that it fell on the very day of the conjunction of Mars with Jupiter, and 8 or 10 days after the conjunction of Mars with Saturn; and that it stood close to the very place of those conjunctions, namely in 18° of Sagittarius. If, however, this prodigy had to be attributed to any natural cause, he thinks it should be attributed to an animal faculty, which he makes similar to the animal faculty of sublunary nature, which forms little animals out of rotting matter, such as bees, flies and butterflies, or certain shapes of plants, flowers, etc.; a faculty which he says is endowed with a certain sense, by which it perceives the place and time for doing these things; and which is roused and moved by the rays of the stars and their aspects, not passively, by receiving something that compels it to act so, but objectively, that is, it is stimulated in

the manner of an object, here and now, to act accordingly. In this way, then, that animal faculty which presides over the heavens, and whose task it is to preserve the transparency of the ether, therefore forms anew, out of impure and superfluous bodies diffused through the ethereal air and gathered into one, new stars in the immobile heaven of the fixed stars, and mobile comets in the mobile heaven of the planets; especially if it senses that this is fitting to be done at such a conjunction of the planets, or because of some other aspect by which it is objectively moved. But, as I said, he thinks outright that this star was kindled by God himself to signify something great to the inhabitants of the terrestrial globe, to whom the conjunction of the planets appeared. For although in the same book, p. 195, he grants other inhabitants, of a species different from men, to other celestial globes, nevertheless the conjoined superior planets appeared not to them but to the dwellers on Earth.

Auctoritates ac Rationes pro vi Maximarum Coniunctionum ad excitandas vel ſignificandas inſignes ac generales mutationes in Mundo.

Authorities and reasons for the power of the Greatest conjunctions to rouse or to signify notable and general changes in the world.

VI. There is no need to list the authors of this opinion, who after Abū Maʿshar and Alcabitius have so multiplied that very few astrologers have not stood by it. Among them, however, the chief have seemed to be Alliacensis, who for that reason wrote a book on the concord of history and astrology, confirming from great events the great force of these Great conjunctions, although later, in the *Elucidarium*, he corrected himself in large part; and Cyprián Lvovický, Io. Franciscus Spina [?], Origanus, Argoli and the others adduced at the beginning of ch. 9; and countless others who, either foreseeing the great conjunction of the year 1603 or on seeing the new star then kindled, spread innumerable prognostications of great future changes in pamphlets issued throughout Europe. Kepler, in the book on that new star, is to be counted among them. [Margin: Kepler's view on the vanity of astrology.] For although he says in ch. 2: *Although astrology has much vanity in it, and is for the greater part unworthy that good hours should be spent on it; yet because this disease belongs to not a few, but pervades the greatest part of the human race, so that whoever teaches the mathematical arts without mentioning it seems to speak too little to common use, I cannot refrain from beginning the explanation of the history of the new star, already given, with a mention of the astrological traditions.* Nevertheless, in ch. 7, considering the ages of the world and the principal epochs, which fell almost at the very moments of the Greatest conjunctions at the beginnings of the fiery triplicity, he says that *he takes very great delight in this series of trigons, and in what is, so to speak, a compendium most useful for helping the memory.* And in ch. 8, 9 and 10 he teaches that by the aspects of the planets sublunary nature is not indeed compelled but is roused, in the way the senses are roused by objects, to set new things in motion,

above all by the Greatest conjunctions; and that around the time of the Great conjunctions there were such great commotions of natures, and of their natural effects and instincts, that they gave astrologers a handle for attributing them to the conjunctions themselves, or to the planets so conjoined, as to efficient causes. Finally, in ch. 26, 27 and 39 he teaches that God, not by chance but by special providence, at the moment of the Greatest conjunction that took place in the Julian year 40, kindled the star for the Magi, and in the year 1603 the new star in Serpentarius for us, in order to support the power of signifying that astrologers believed the conjunctions to have, and to illustrate it by a new portent. This is, in the end, to concede that they are taken up by God as a sign at will, or, as they say, *ad placitum*, at least so that he may accommodate himself to the opinions of the astrologers, who at any rate use them as signs of great changes.

VII. But the strongest and most particular reason, passing over those usually brought forward in general for the efficacy of any aspect and syzygy of the planets, is the connection of notable vicissitudes and changes on the globe of the Earth with these Greatest conjunctions. Alliacensis draws this connection from chronology; so do Argoli, *Pandosion Sphaericum*, ch. 73; Io. Franc. Spina [?], book 2, ch. 2; Origanus, vol. 1, p. 449; and Kepler in the *Epitome of Copernican Astronomy*, p. 855, and in the book *De Stella Nova*, ch. 7, though using round numbers of 800 years for the revolution of the Greatest conjunction. To these we shall add more exact numbers from what was said in ch. 9, no. 9. Here, then, is a chronological table from these authors, who however leave out the reform of the calendar. Argoli adds that the law of idolatry began from the earthly trigon under Belus, about 1420 years before Christ; the Jewish law from the airy trigon, the Christian from the fiery, the Mahometan from the watery, whose triplicity, he says, began in the year of the Lord 591 and lasted until 789.

Order of the triplicities	Greatest ☌, years before CHRSTVM [recte: CHRISTVM]: Kepler	Greatest ☌, years before CHRSTVM [recte: CHRISTVM]: ours	Notable events and changes
1	4000	3980 Adam	The beginning of the world or the fall of Adam.
2	3200	3185 Enoch	Brigandage, then the building of cities. The invention of the arts.
3	2400	2390 Noah	The universal Flood, the restoration of the world.
4	1600	1595 Moses	The plagues of Egypt. The Exodus. The written Law.
5	800 Isaiah	801 Romulus	The beginning of the Olympiads; Rome founded. The new era of Nabonassar.
6	Christ	6 Augustus	The monarchy of Augustus flourishing. The description of the world. The birth of Christ.
7	800 Charlemagne	789	The transfer of the Roman Empire to the Franks.

Order of the triplicities	Greatest ☌, years before CHRSTVM [recte: CHRISTVM]: Kepler	Greatest ☌, years before CHRSTVM [recte: CHRISTVM]: ours	Notable events and changes
8	1600 or Gregory XIII	1583, 1603	The reform of the calendar; the embassy of the kings of Japan to the supreme pontiff. Three new stars in the heavens.

Since, then, by using round numbers of 800 years the times of the events already mentioned are easily committed to memory, as Kepler pointed out, I could not restrain myself, or hold back our Urania (who is, after all, the highest of the Muses), from pouring out the following verses, stirred by a kind of poetic surge rather than by the frenzy of empty prophecy.

The FIERY TRIPLICITY, called the Greatest conjunction of Saturn and Jupiter, returns every eight hundred years. The FIRST holds the fall of Adam and the beginnings of the world; savage brigandage, then cities and arts, the SECOND. The THIRD, the Flood, and the seeds of a renewed race. The FOURTH calls Moses, and writes the Law in marble; and the FIFTH, the Olympiads, Rome, and the era of Babylon. The SIXTH stands astonished at CHRIST GOD, and at Augustus the monarch. The SIXTH triumphs for CHRIST GOD, more than you, Augustus. With the Empire transferred, the SEVENTH gives Charles the Great. The EIGHTH reforms the sacred rites and the cycles of the calendar, subjecting to Gregory the Japanese, the furthest kingdoms of the earth, and three times renews portents with new stars on Olympus. What if the threatening NINTH, thrice threefold, should blaze for the world extinguishing the kindled supreme lamp of life, after seven decades of ages and the Sabbaths of the great Week, and should call back the beginnings of a new Kingdom, so that the pious ages may merit a perpetual Sun, and the impious ages be damned to eternal night?

[Margin: Mark 13.] However, lest, with the Gospel protesting and Christ sounding in our ears that saying, *But of that day or hour no one knows, neither the angels in heaven, nor the Son, but the Father alone*, we should seem to want to prophesy, since this saying, as Augustine acutely said, loosens the fingers of all who calculate: we reject and condemn all dependence of that last day on the influences of the stars, and all force of certain signification. And if it should be thought best that not even a suspicion of doubt be raised, let the censure of the sacred sponge, with my free consent, delete the last six verses that my pen poured out.

Auctoritates ac Rationes contra Maximarum Coniunctionum vim significandi, nedum efficiendi prædictos euentus & mutationes.

Authorities and reasons against the power of the Greatest conjunctions to signify, let alone to bring about, the events and changes already mentioned.

VIII. This is not the place for refuting the rashness of judicial astrology, long since proscribed and infamous for its very frequent lies among sensible men learned in astronomy itself, let alone among those learned in the sacred codes and canons. For although that class of men will always be forbidden in the City, and indeed in the world, and yet will always be retained, nevertheless, once the *Almagest* is finished, if it so please God, I will, so far as in me lies, shake and undermine the foundations of this Babylonian citadel from the innermost recesses of astronomy. But now I must indicate what tells against the excessive power, urged by so many, of the Greatest conjunctions in changing sublunary nature and human polities, or in certainly signifying their changes.

[Margin: 1st argument, and the opinion of Ptolemy discussed.] *First.* Ptolemy says in the *Centiloquium*, aphorism 50: *Do not overlook the one hundred and nineteen conjunctions: for in them lies the knowledge of the things that come to be in the world, both of generation and of corruption*; and in aphorism 63: *When Saturn and Jupiter are joined, see which of them is higher, and pronounce according to its nature. Do the same also with the other stars.* In the former he speaks of the 119 comparisons of conjunctions that can occur among the seven planets, and in the latter of the height of a planet at the apogee of the epicycle, as if reigning on a highest throne, as Pontanus [Giovanni Pontano] explains there. Yet notwithstanding these passages, Valentinus Naiboda, otherwise a great teacher and patron of astrology, on Alcabitius, differentia 4, and Argoli, *Pandosion Sphaericum*, ch. 74, maintain that Ptolemy attributed the power of signifying great events either solely or chiefly to eclipses of the luminaries, not neglecting the stations and heliacal risings of the three superior planets, or even their conjunctions, provided these fall at the time of an eclipse. He tries to show this from book 2 of the *Apotelesmatica* and from other passages and arguments, [Margin: The opinion of Valentinus Naiboda.] adding: *And if you should wish above all to pursue the conjunctions of the heavy planets in the way in which Ptolemy treats eclipses of the luminaries, the very difficulty of the matter will easily drive you from your purpose, since human understanding does not grasp the exact time of a conjunction of the superior planets.* From this my second argument now arises.

[Margin: 2nd argument.] *Second*, then. Either a Greatest conjunction has the power of certainly signifying the greatest events insofar as it is a conjunction of Saturn and Jupiter, whenever and wherever at length it happens; and this the astrologers neither say nor can say, because the events signified would always be of the same kind. Or it has that power insofar as it happens in such a degree of the zodiac and at such a moment; and this no tables have yet delivered with all misgiving excluded, [Margin: Note the disagreement of the Alphonsine and Prutenic tables.] since the ☌ of ♄ and ♃ of the year 1563 was carried forward by the Alphonsine tables to September 27 and to the 4th degree of Leo, whereas the Prutenic tables drew it back to August 26 and the 29th degree of Cancer. So one would have had to judge in one way from the nature of Leo, a fiery sign, as they will have it, and in another from that of Cancer, a watery sign: that is, in an entirely opposite manner. In reality it took place on August 24 at

14^h 30^m after noon at Orange, as Giuntini observed and reported in the preface to his astronomical tables, adding: *Jupiter, on the north side, as it were covered Saturn, which was on the south side, and both of these stars were found at the end of the 28th degree of Cancer.* We have reported similar errors in the conjunction of Saturn and Mars in this Book 7, sect. 1, ch. 6, no. 5; they arise from the slowness of the motions, especially of Saturn, which does not make its progress manifest except after many days.

[Margin: 3rd argument.] *Third.* It is more just to use true rather than mean conjunctions in this judgement, for the reasons brought forward in ch. 9, no. 5; and yet Abū Maʿshar and almost all his followers used mean ones, which not rarely occurred in signs of the opposite triplicity, as we have shown above more than once. It is therefore plain that, if the properties of events depended on the conjunctions, at least as to signification, they should have been assigned differently from these men's decrees; and that what was ascribed, for example, to the watery triplicity belonged to the fiery. That art is therefore uncertain and vain which rests on false or inconstant principles; indeed, it does not even deserve the name of art.

[Margin: 4th argument.] *Fourth.* That very division of the signs into fiery, earthly, airy and watery, and the naming of the trigons after the elements, does not come from the nature of things but from the mere choice of men, as Kepler shows in *De Stella Nova*, ch. 6; and it is founded on a mere analogy with the images of the signs. For because Cancer, Scorpio and Pisces are aquatic animals, the signs in which such stars once stood have been called watery. But why should Aquarius not rather be called watery, and Taurus and Scorpio fiery, since the largest stars in them, Aldebaran [Palilicium] and Antares, rival the fires of Mars with their redness? That is, if the naming was indeed taken from the stars once conjoined with those signs.

[Margin: 5th argument.] *Fifth.* The fictions of Abū Maʿshar and of many Arabs founded on these and similar conjunctions are so ridiculous, and so much at odds with the events that actually followed, that they surpass cabalistic trifles and the fables of the Koran in levity and futility; and I almost regret having turned my eyes to them, however much with the intention of attacking them. So when he says, in book 1, differentia 4, that if at the time the conjunction changes some other one of the planets is lord of the fourth of the ascendant, that planet is the significator of the mounts and the garments that the greatest part of the new sect will use. *If it is* ♄, he says, *it signifies that the greatest part of the mounts of the citizens of that sect will be nags; and if it is* ♂*, horses; and if it is* ☿*, asses; and if it is* ♀*, camels; and if it is* ☾*, oxen or cows.* But what has slowest Saturn to do with nags? What has most learned and most shrewd Mercury to do with the braying and stupidity of asses? What has loveliest Venus to do with the hump and the foul gape of camels? And if a young bull or heifer imitates the horns of the Moon with its own horns, Venus seen through the telescope appears no less horned all the same. But away from our pages with the other ravings with which this author teems.

[Margin: 6th argument.] *Sixth.* The chronology of events, on which our opponents have built many dogmas in order to win great dignity for the Greatest or Great conjunctions, is either false or not rarely so disputed that it cannot suffice for founding an art. So when Abū Maʿshar says, tract. 1, diff. 1, that between the creation of Adam and the night of Friday on which the Flood occurred there were 2226 years, 1 month, 24^{d} 4^{h}, whereas by our chronology, confirmed by clear proofs, there were only 1685. But it would be endless to pursue these things, and I see that they can defend themselves by the credit of the historians they have followed. We shall, however, shortly sample some other things in the following argument.

[Margin: 7th argument.] *Seventh.* The deceptiveness of the predictions about the rise or fall of sects, deduced from these conjunctions or from the position of the stars, shows their shameful vanity. [Margin: False predictions of the astrologers about the end of the Christian religion, etc.] Once the mathematicians, says St Augustine, *De Civitate Dei*, book 18, ch. 53, and Io. Picus [Giovanni Pico della Mirandola], book 5 of *Adversus Astrologos*, ch. 1, when the faith of Christ began to sprout, kept saying that it would last not more than 365 years; and yet the one thousand six hundred and almost fiftieth year now rolls by since his Church flourished like a lily among thorns, and it will flourish until the consummation of the age. That same Abū Maʿshar, tr. 2, last differentia, said that this same religion of ours would last only 1460 years, because that, of course, is the number of years they call the greatest years of the Sun, or the great canicular year, on which see what was said in book 3, ch. 13, scholium 4; and that the sect of the Arabs would last 544 years. Those 1460 years have passed, and not yet has our religion passed away, because heaven and earth will pass away, but the words of our Lord Jesus Christ, who promises it perpetuity, will not pass away. Nor was Abū Maʿshar more truthful about the duration of his own sect: since it began in the year 622, it should have perished in the year 1166. Abū Maʿshar was still living in that century, and he likewise added that the time of that change was near.

Abraham the Jew had predicted that the Messiah would come in the year of Christ 1464, from a position of the stars like the one that obtained when Moses, having led the people out of bondage, gave the law he had received from God. "I," says Pico in the place cited above, "born in the year just before it, and now in my thirtieth year, have learned the falsity of that prediction by experience." Arnaldus Hispanus [?], a distinguished physician but far too steeped in this superstition, threatened that the Antichrist would come in the year of grace 1345, because of the conjunction of the three superior planets in Aquarius. More than 300 years have flowed by since that time, and no other Antichrists have come except such as there were in apostolic times too, that is, heresiarchs.

Cardinal Petrus Alliacensis, whose purple was, alas, all too deeply stained with this Arabic ink, believed the astrologers in the year the General Council of Constance was held, when they were predicting the ruin or monstrous

destruction of the Catholic religion, and he feared that after that schism, which the council nevertheless removed, the Antichrist was at hand. Nicolaus Cusanus, in his book *De diebus nouiss.*, judged that the end of the world would come in the year 1700 or 1734. Cardano (*De varietate rerum*, book 2, ch. 2) and Iacobus Naclantus [Giacomo Nacchianti] (in the *Prælud.* of his *Medulla*, ch. 4) held that it would come in the year 1800, from the position of the stars. Others, according to Nausea [Friedrich Nausea] in his book *De consumatione sæculi*, predicted from the conjunction of so many planets in the year 1524, discussed above, a flood over the whole globe or the greatest part of it, and they struck such fear into some people that these withdrew to live in the mountains, or raised up buildings, or made ready enormous ships. Others predicted the end of the law of Christ for the year 1048, from the conjunction of ♄ and ♃ in July of the year 1044, as Abulensis [Alonso Tostado] has it on Matthew ch. 22, question 231. Ridiculous too is that astrologer of Toulouse who, fearing from a similar aspect a flood on a fixed day, built himself a citadel; yet Florimundus [?], in his book *De extremo Iudicio*, reports that the day was perfectly clear. Mercurius historicus [?] relates that Regiomontanus asserted the end of the world for the year 1588. But Petrus Alliacensis, in his book *De concordia Astr. & Histor.*, ch. 60 and 61, extends it to the year 1789.

For other opinions or conjectures about the end of the world, but taken from elsewhere than the stars, see St Irenaeus, *Adversus Haereses*, book 5, ch. 28; St Ambrose on Matthew ch. 17; St Augustine, *De Civitate Dei*, book 20, ch. 7, and Vives [Juan Luis Vives] on that place; Georgius Venetus [Francesco Giorgi], *De harmonia mundi*, Canticle 3, tome 7, ch. 7; Hadrianus Finus, *Flagellum contra Iudæos*, book 4, ch. 20; Galatinus [Pietro Galatino], book 9, ch. 11; Sixtus Senensis [Sisto da Siena], book 5, annotation 190; Hieronymus Maggius [Girolamo Magi], in his book *De exustione Mundi*; Henricus Emstius [?], *De fine mundi*; and Florimundus, *De extremo Iudicio*.

The astrologer Valens too, as Scaliger reports in book 5 of *De emendatione temporum*, had predicted that Constantinople, restored by Constantine, would last only 696 years. Would that the prophecy of Io. Franciscus Spina (book 2, ch. 3) were true, though it is built on an empty foundation. From the great conjunction of Saturn and Jupiter of the year 1603 he gathers that the sect of the Turks is to be exterminated in this century or at the beginning of the next. For the origin of this extermination, he says, sprouted in that conjunction, flowered in the second, of the year 1623, will yield fruit in the third, of the year 1643, and the fourth or the fifth will complete the fruit; and so the superstition of Muhammad will perish in the year 1663 or 1683, or at the very latest in the year 1702. On this occasion I go on to the 8th argument.

[Margin: 8th argument.] *Eighth.* Either a greatest or a great conjunction generates or signifies events that are to follow immediately, and this the astrologers neither dare to say nor can, with history protesting on every side; or it does so only after some notable time, so much so that Abū Maʿshar (tr. 1, differ. 1) grants that 279 years intervened between the conjunction that

signified the Flood and the Flood itself. If that is so, then, as even Bellanti himself teaches (*Against Pico*, book 5), those effects can no more be attributed to great conjunctions than to other syzygies of the stars formed at a nearer time: say, the eclipses of that year, and six hundred other configurations of the wandering and unwandering stars. Bellanti does add that in this matter the opinion of Alliacensis and of others, who in such a case attribute those effects to great conjunctions, can be neither proved nor disproved. For he says: "It seems to me most evident that it cannot be proved or disproved that these or those changes in the affairs of the world come from these or those constellations, unless we diligently keep before our eyes the great conjunctions and the others we have called necessary, and the effects corresponding to those times. For in the first place the astrologers would be proving lightly that these or those effects proceeded from great conjunctions, on the ground that they occurred in the intervening times, etc." But because he considers that an effect which a great conjunction has begun grows little by little, in the course of time, through other syzygies of the planets, and at last matures, he therefore judges that this opinion cannot even be disproved.

[Margin: 9th argument.] *Ninth* and finally: if only the greatest conjunctions are considered, those formed at the beginning of the fiery triplicity, or at least at the beginning of any triplicity, many events of the greatest moment and notable changes in affairs nevertheless happened before them, so that they could not have been foreshadowed by conjunctions that came after. But if appeal is made to the conjunction immediately preceding, it often belongs to a triplicity of a different kind, and one less suited to those effects than the great or greatest conjunction that followed. Consult our *Chronology*, or any other you reckon more exact, and compare the times of the changes and more notable events with the great or greatest conjunctions entered in the table at ch. 9, no. 9, and you will see a very great discrepancy throughout.

Finally, there has been no century, indeed scarcely a half-century, without some notable events in the world: for instance wars between nations, or new monarchies, or barbarian invasions, or persecutions of the Church, or schisms, or heresiarchs, or founders of new sects, or pestilences, famines, earthquakes, floods. So there is no reason why these should be foreshadowed to us from heaven, from afar and confusedly, by the conjunction of two planets; for we know from another source, from the history of all the centuries, that no century will be immune from vicissitudes of this kind. But God granted the particular and distinct signification of events to his prophets, not to astrologers.

SCHOLIA.

Scholia.

I. [Margin: Changes in the world from the change of the apsides.] With the same vanity with which astrologers derive, from great conjunctions, general changes and catastrophes of affairs in this world, they suppose that these

same things depend on the change of the obliquity of the ecliptic and of the eccentricity, and also of the apogees, a change which, because their motion is slow, works or signifies effects that are slow but notable. Foremost among such people is Cardano, in book 1 of the Ptolemaic *Centiloquium*, aph. 37, who says: "Changes of the apsides change kingdoms, regions and religions"; and in book 5, aph. 129, speaking of the change of the solar apogee, he dares to say: "The apsis of the ☉, travelling from ♈ to ♋, renders the southern part uninhabitable, but makes the northern habitable; from ♋ to ♎, it makes the northern well habitable and dominant over the southern; from ♎ to ♑, it makes the southern habitable, but by no means ruling over the northern part; from ♑ to ♈, it makes the southern ruling over a desolate northern part."

[Margin: Cardanus's opinion is empty.] Yet since the motion of the Sun's apogee, from the beginning of the world, has barely completed the first quadrant of the ecliptic, and even now is around the beginning of the second, as is clear from what was said in book 3, ch. 25, by what experience did Cardano learn what is to happen in later times? In past times, that is, from the founding of the world down to the present time, during which the Sun's apogee has passed from the beginning of ♈ to the beginning of ♋, the southern part of the globe has not been rendered more and more uninhabitable. Granted that, with the Sun at apogee in Cancer and at perigee in Capricorn, winter and summer are more moderate than if it were at apogee in Capricorn and at perigee in Cancer, while conversely both seasons of the year are less moderate for those in the south; still, other causes concur that diminish that excess. So farewell to Cardano, and to Origanus, who subscribes to his view (vol. 1, part 3, member 2, ch. 1). Nor is the foundation much greater in the apogees of the other planets: how great the uncertainty is about their place and motion is clear from what was said in Sections 2 and 3. On the change of monarchies according to the change of the solar eccentricity and of the obliquity of the ecliptic, against Rheticus, [Margin: Change of the obliquity and of the eccentricity of the ☉.] I said enough in book 3, ch. 26, schol. 1, especially since it is more probable that the Sun's eccentricity and the obliquity of the zodiac are always the same, from what was said in book 3, ch. 27; and Argoli is with us on this in the *Pandosion*, ch. 73.

II. [Margin: The Platonic Great Year.] The Platonic Great Year is very celebrated among writers: when it has come round, the planets and stars are all believed to return to the same point and the same goal from which, as if from the starting gates, they once began to run. Hence that passage of Cicero in the *Dream of Scipio*, or rather of Scipio in the dream of Cicero: "For people commonly measure the year only by the return of the Sun, that is, of one star. But when all the stars have returned to the same place from which they once set out, and after long intervals have brought back the same configuration of the whole heaven, then that can truly be called a revolving year, etc."

Yet this year seems fictitious rather than real. For even the revolution of the Moon to the Sun alone, with all the circumstances that attended some one

syzygy with it in the past, has not yet been demonstrated to be possible: for who has ever demonstrated, says Boulliau (book 3, ch. 3), that the motions of the Moon are commensurable with the motions of the Sun? [Margin: The incommensurability of the celestial motions.] As for the rest, hear Kepler in the *Mysterium Cosmographicum*, who from one postulate shows that there will be no Platonic year. "For let it be granted," he says, "that the eccentricity has a rational proportion to the orb; for the radii of the orbs will be irrational, because they are related as those inscribed in and circumscribed about the bodies, which are irrational, because they follow from the ratio of the subtense in a square and of the section according to extreme and mean ratio, which are two examples of irrational proportions in geometry. But the motions have a proportion to the radii; therefore the motions are irrational to one another. And so, even if they should last for infinite ages, they will never return to the same beginning and goal of the Platonic year, because not even in an infinite division of time can a common measure be found which, repeated often enough, would equal the whole."

SECTIO SEXTA

Section Six

DE PASSIONIBVS PLANETARVM SECVNDVM ALTITVDINEM.

On the affections of the planets according to altitude.

PROHEMIVM.

Preface.

The affections that befall the planets according to altitude are: 1. The variation of their distance from the centre of the Earth or of the world. 2. The proportion of these intervals among themselves and with the motions, whether it is harmonic or not. 3. Their order in the system of the world. 4. Parallax, or variation of view. 5. Speed in motion, whether common or proper, insofar as it varies with greater or lesser distance from the centre of the universe. 6. The variation of apparent magnitude, if they are viewed from the Earth. 7. The diversity of apparent magnitude, if they are imagined to be viewed from some other place in the world. 8. Their true magnitude in three dimensions. 9. The length and thickness of the shadows they cast. 10. Eclipses, by which, either with their shadow or by the interposition of their body, they can hide the other planets or the fixed stars from us. All these things must therefore be discussed, with God's help, in the same order, and an end at last put to this seventh book.

CAPVT I.

Chapter 1.

An varietur Diſtantia Planetarum à centro Terræ vel Mundi.

Whether the distance of the planets from the centre of the Earth or of the world varies.

I. [Margin: On the varying distance of the ☉.] I taught in book 3, ch. 7, from eclipses and parallaxes, and in ch. 10, from the diversity of the apparent semidiameter, that the distance of the Sun from the centre of the Earth varies, as being much more probable; although in ch. 8, schol. 2, and more expressly in ch. 10, schol. 2, I taught that the variation of the Sun's distance cannot be demonstrated. [Margin: On the varying distance of the ☾.] That the Moon's distance is variable we showed against the Homocentrists, not only from eclipses but much more evidently from the diversity of parallaxes and of the apparent semidiameter, in book 4, ch. 13 and 14 (reread the scholia of the latter) and ch. 16.

What is now to be said of the other five planets could be called into doubt, especially since the defenders of the homocentrics, namely Girolamo Fracastoro, Giovanni Battista Amico and Io. Antonius Delphinus [Giovanni Antonio Delfino], reported in the scholia already mentioned, try to refer all the apparent diversity in their diameters to various vapours, or to parts of the heaven of different density and transparency; when these come between, varied refraction of the rays is supposed to produce that diversity in apparent magnitude. These things could indeed seem probable at first sight, especially if the diminution of the diameter or disc near the Sun were attributed to the Sun's brilliance overwhelming the smaller lights of the minor planets, so that accordingly at their greatest distances from the Sun they could appear larger, as is actually observed, than when they come nearer to heliacal setting. And to me, formerly, when I was younger and not yet admitted to the inner sanctum of astronomy, there seemed no necessity to introduce a differing distance for these planets. But much stronger reasons for this diversity of intervals stand in the way, reasons which, taken together, come as close as possible to evidence.

II. [Margin: 1st argument for the change of distance of the 5 planets from the Earth.] For first, they sometimes appear very tiny and sometimes much larger than themselves, not only when one compares their visible magnitude at the horizon, or as seen among vapours or clouds, with that seen farthest from the horizon or in a clear and cloudless sky; not only in different parts of the heaven; and not only with the Sun's light or the brightness of twilight close by or present; but even when they are viewed in the same part of the heaven, in the same degree of serenity of the sky or clearness of the air, and at the same depth of the absent Sun below the horizon. For let Jupiter or Mars, for example, be observed at mid-heaven when, having become acronychal, it stands diametrically opposite the Sun, especially when the equation of the centre, which a few days before had been subtractive, has turned additive; then let it be observed around the quadrate aspect with the Sun; and finally let it be observed when it is so far from conjunction with the Sun that the Sun is nevertheless below the horizon, and morning or evening twilight is not present,

that is, before the first or after the second sextile aspect, or even around the semisextile. For it will appear much larger in opposition than in quadrature, and in quadrature larger than in the sextile or semisextile aspect; and this not only to the naked eye but also through the telescope, so much so that the bright disc of Mars is seen larger in opposition than near conjunction with the Sun: five times larger to the naked eye, but through the telescope sixty or fifty times as regards area, and seven times larger as regards diameter, as Galileo also notes in dialogue 3 of *De Mundi systemate* and in the *Trutinator* [*Il Saggiatore*], p. 217. And more than once, when near opposition it became [Greek: περιγειότατος (perigeiotatos, "nearest to the Earth")] in Aquarius and Pisces, it was believed to be a comet or a new star, so increased in light did it seem. Nor can the whole of so great a decrease around the sextile or semisextile be attributed to the brilliance of the absent Sun, since fixed stars viewed in the same aspect with the Sun do not appear smaller than themselves as seen around opposition to it.

III. [Margin: 2nd argument.] *Second.* Venus most manifestly, and also Mercury and Mars, are carried around the Sun in such a way that they are illuminated by it in the manner of the Moon, as the spyglass [Tubospicillum] attests; so that this diversity of phases and faces cannot be fittingly explained except by asserting that they run around now above and now below the Sun, which is itself an argument for a varied distance. Then Venus, especially when retrograde in the middle of its regression, or around conjunction with the Sun, when from an evening star it is about to become a morning star, or when it has recently become a morning star, appears with a horn very narrow in breadth but very long in height. Observed with the best telescopes, it thus seems forty times larger than itself as regards area than when, direct at the other conjunction, it offers itself to be viewed with a disc that is round indeed but very slender, as Galileo notes in the *Trutinator*, p. 217, and in Dialogue 3 of *De Systemate Mundi*. And again, when it is retrograde and stands about 15 degrees from the nearer conjunction with the Sun, it shines with a far ampler disc and a greater lamp than when, direct, it stands 15 degrees from the other conjunction. That diversity therefore arises not from the vicinity of the Sun, which is equally absent in both cases, but from the differing interval between Venus and our view; for by prop. 56 of Euclid's *Optics* it is necessary that the same true magnitudes appear larger when nearer the eye and smaller when more remote. See, please, a specimen of these phases in Section 1 of this book, ch. 2.

IV. [Margin: 3rd argument.] *Third.* The latitude of the planets, especially of the superior ones, that is, their departure from the ecliptic, appears, other things being equal, much greater when they are around the middle of their retrograde course than when they are around the middle of their direction; and greater still if, when retrograde, they pass from the subtractive equation of the centre to the additive. Their latitude is also greater in opposition to the Sun than around conjunction, and, other things being equal, the southern is greater than the northern. By these arguments it was almost evident to astronomers

that these planets sometimes become perigean and approach closer to the Earth, and sometimes recede from it and become apogean; and accordingly that the same arc of the circle of latitude, or of inclination to the ecliptic, is nevertheless seen now under a greater angle and now under a smaller, so much so that the southern latitude of Mars when acronychal and retrograde is 7° 9′, but the northern only 4° 32′; and so for the rest, according to what was said in sect. 3, ch. 7. Practitioners have therefore rightly established that the apogees of the eccentrics of Saturn, Jupiter and Mars always incline to the north, and the perigees always to the south.

V. [Margin: 4th argument.] *Fourth.* Neither anomaly, or inequality, in the motion of longitude, which is most manifest in these five planets, can be accounted for and represented more conveniently and simply than by eccentric circles, or else eccentrepicycles or epicepicycles, as is clear from the hypotheses of the most famous astronomers set out in sections 2 and 3. But with these posited, or with spirals and whorls, or ellipses that yield the equipollence of the motions and prosthaphaereses, the planets must necessarily be now raised on high and now pressed down toward the centre of the world; and this diversity is far more simply explained by one circle or another than by the innumerable *circitores*, *anticircitores*, *vecti* and *contravecti* by which the Homocentrists, and the haters of a varied distance, are compelled to defend the diversity of the motions. It is accordingly much more likely than concentric orbs and the immutability of the intervals. But no one who has attentively considered the arguments already mentioned, and the harmony of the motions, will be of so childish a mind as to say that the planets, or the parts of the heaven between them and the ecliptic, now swell and expand, and now subside and contract.

CAPVT II.

Chapter 2.

Quanta & Quàm varia ſit Diſtantia Planetarum Minorum à centro Terræ, & à Sole.

How great and how varying is the distance of the minor planets from the centre of the Earth and from the Sun.

I. [Margin: The distance of the ☉ and the ☾.] Enough has been said about the distances of the Sun in book 3, ch. 7, at the end of which there is a table showing the threefold distance, that is, the greatest, mean and least, according to the opinion of 26 authors. About the Moon's distances from the Earth too, it was said at length in book 4, ch. 14; at the end of it we likewise gave a table of these from the opinion of nearly 15 different authors. The scholia of these chapters, full of uncommon learning, should be reread at this point. We therefore want those tables to be consulted just as if they were added to the following distances of the five minor planets. For these too I could have sent the reader off to the distances placed at the end of the chapters of sections 2

and 3 of this book. But those are bound up with the other proportions of the orbs, for instance the eccentricities and the semidiameters of orbs, epicycles, etc., and they are moreover given in parts of which the radius of the annual orb of the Sun or Earth, or the radius of the planet's orb, is 100000, let alone in semidiameters of the Earth. So it seemed best in this place to separate the distances of these planets from the Earth, to examine them more carefully, and to represent them only in semidiameters of the Earth: both because I know that such distances are generally what is sought, and because these will be of more use to us for determining the other properties of the planets according to altitude, since the other proportions of the orbs to the radius serve rather for the geometrical calculation of the motion in longitude and its theory. Whoever desires those proportions is free to fetch them from there: the solar ones from book 3, ch. 7, and its scholium; the lunar from book 4, ch. 14, and its scholium; those of the three superior planets from this book 7, sect. 2; and those of the two inferior from book 7 and sect. 3, by consulting the tables placed at the end of each chapter.

If anyone wishes to look at these distances in larger figures and put them before his eyes, perhaps at the end of this section or in book 9 I will supply them or the method of drawing them.

De Planetarum Diſtantijs Ptolemaicis & Alphonſinis à Sole & à Terra.

On the Ptolemaic and Alphonsine distances of the planets from the Sun and from the Earth.

II. Apart from the distances of the luminaries, neither Ptolemy nor Alfonso expressed for us the distances of the other planets. From their hypothesis, however, various authors derived them, in various ways, though omitting the distances from the Sun that we have had to add. The chief of these were: al-Battānī, ch. 50; al-Farghānī, diff. 16 and 21; Jean Fernel, *Cosmotheoria*, book 1; Maurolico, at the end of the dialogues of his *Cosmographia*; Clavius on the *Sphere*, p. 215, and at the end of the *Sphere*, nearly following Maurolico; Mauro Fiorentino and Francesco Giuntini in their *Spheres*, from which Baranzanus drew in part 1 of the *Uranoscopia*; Barozzi, *Cosmographia*, book 1, p. 48, following al-Farghānī; Copernicus, as given by Kepler in the *Epitome of Copernican Astronomy*, p. 494, correcting the Ptolemaic ones; Mästlin, on Kepler's *Prodromus*, p. 92, gathering Saturn's greatest distance from data given by Reinhold; Tycho, *Progymnasmata*, vol. 1, p. 468, reporting those of al-Battānī from his own manuscript; and Lansbergen, *Uranometria*, book 3. Among these Maurolico, a truly diligent man, declares that in the Sun's eccentricity and the semidiameter of the lunar epicycle he adhered to Alfonso, as being more truthful in those matters; but in deriving the distances of Venus and Mercury from the solar ones, he chose the Ptolemaic ones, as being more convenient; in the rest he kept to al-Farghānī, al-Battānī and Thābit.

Now the universal rules for gathering or understanding the distances in this hypothesis must be given, though only after first setting out some measures that bear on the matter.

Measures in semidiameters of the Earth, Ptolemaic or Alphonsine

From		al-Farghānī, Maurolico, Clavius	Jean Fernel
Eccentricity of the deferent	♄	981	1284
	♃	532	599
	♂	503	611
	♀	12	14
	☿	17	18
Eccentricity of the equant	♄	1972	2568
	♃	1064	1199
	♂	1006	1222
	♀	24	29
	☿	6	6
Semidiameter of the epicycle	♄	1866	1922
	♃	2226	2507
	♂	3313	3714
	♀	462	496
	☿	44	44
Radius of the orb of the eccentric, or mean distance from the centre of the Earth	♄	17225	19403
	♃	11612	13079
	♂	5032	5642
	♀	642	689
	☿	116	117
	☉	1168	1256
Eccentricity	☉	42	39

Regulæ Pro Diſtantijs Ptolemaicis vel Alphonſinis Planetarum à Terra & à Sole.

Rules for the Ptolemaic or Alphonsine distances of the planets from the Earth and from the Sun.

[Margin: Mean distance.] 1. Rule. The mean distance of a planet, whether from the Earth (always understand: from the centre of the Earth) or from the Sun (that is, from the centre of the Sun's body), is of the same quantity as the radius of the eccentric, or the semidiameter of the deferent. The mean distance occurs a little below the mean longitudes of the eccentric and the epicycle.

[Margin: Greatest distance from the Earth.] 2. Rule. The greatest distance of a planet from the Earth is made up of the eccentricity of the deferent and the semidiameters of the eccentric and the epicycle. It occurs when the planet, at apogee in both eccentric and epicycle, is joined to the Sun.

[Margin: Least distance from the Earth.] 3. Rule. The least distance of a planet from the Earth is obtained by subtracting from the semidiameter of the

eccentric the sum made of the semidiameter of the epicycle and the eccentricity of the deferent. It comes out least when a superior planet, at perigee in eccentric and epicycle, is opposed to the Sun, while an inferior planet, at perigee in eccentric and epicycle, is joined to the Sun, if these distances are regarded in the line of apsides. Otherwise Mercury, with the centre of its epicycle at a third of the eccentric [in triente Eccentrici] on either side of the eccentric's apogee, can come nearer to the Earth, at the perigee of the epicycle, than anywhere else.

[Margin: Greatest distance of ♄ ♃ ♂ from the ☉.] 4. Rule. The greatest distance of a superior planet from the Sun, among all the great or greatest ones, occurs if the apsides are turned about, that is, if the planet's apogee lies under the place of the zodiac opposite to the one under which the Sun's apogee lies: namely when the planet, at apogee in both places, is joined to the Sun at perigee. It is therefore found by subtracting the Sun's least distance from the Earth from the planet's greatest distance from the Earth; and this is the greatest of the greatest. But if the apogees coincide, the greatest is then obtained by subtracting the Sun's greatest distance from the Earth from the superior planet's greatest distance from the Earth.

[Margin: Least distance of ♄ ♃ ♂ from the ☉.] 5. Rule. The least distance of a superior planet from the Sun occurs at its opposition with the Sun if the apogees are set opposite to each other, that is, it occurs when the planet, at perigee in both places, is opposed to the Sun at perigee. It will therefore be found by adding the Sun's least distance from the Earth to the planet's least distance from the Earth; and this is the least of the least. But if the apogees coincide, the least is obtained by adding the Sun's greatest distance to the planet's least distance from the Earth. But this will not be the least of the least.

[Margin: Greatest distance of ♀ and ☿ from the ☉.] 6. Rule. The greatest distance of an inferior planet from the Sun occurs if the apogees of these planets are opposed to the Sun's apogee, and the planet is then joined to the Sun at apogee in the lower position, that is, at the perigee of its epicycle. It is therefore found by subtracting the inferior planet's least distance from the centre of the Earth from the Sun's greatest distance from the Earth. And this will be the greatest of the greatest. But if the apogees coincide, the greatest distance from the Sun will be obtained by subtracting the planet's greatest distance from the centre of the Earth from the Sun's greatest distance from the Earth; but to the remainder must be added the diameter of the inferior planet's epicycle.

[Margin: Least distance of ♀ and ☿ from the ☉.] 7. Rule. The least distance of an inferior planet from the Sun occurs if the apogees of these planets are opposed to the Sun's apogee, and the inferior planet is then joined to the Sun at perigee in the upper position, that is, at the apogee of its epicycle. It is therefore found by subtracting the inferior planet's greatest distance from the Earth from the Sun's least distance from the Earth. But if the apogees coincide, it will be obtained by subtracting the planet's greatest distance from the Earth from the Sun's greatest distance from the Earth. But this will not be the least of the least.

These already-mentioned distances, however, do not in fact occur, nor can they occur in these ages, because the Sun's line of apsides does not coincide with the line of apsides of these planets. But we are seeking those that are possible in this hypothesis, if the world had lasted, or were going to last, long enough for the lines of apsides of the Sun and of each of the planets to coincide in the same line.

[Margin: Extent of the planetary heaven.] 8. Rule. Add the true semidiameter of the planet's body to the planet's greatest distance from the Earth, or subtract it from the least, and you will have, in the former case, the semidiameter of the convexity and, in the latter, that of the concavity; that is, the radius of the convex and of the concave surface of the heaven within which the planet ascends and descends. The difference of the semidiameters will be the thickness of that planet's heaven.

De Diſtantijs Planetarum Minorum à Sole & à Terra ex Copernici, Mæſtlini, & Lansbergij Hypotheſi.

On the distances of the minor planets from the Sun and from the Earth, from the hypothesis of Copernicus, Mästlin and Lansbergen.

III. From what sources I elsewhere derived the Copernican, Mästlinian and Lansbergian distances and measures of the orbs is clear from sect. 2, ch. 2, no. 7, and ch. 4, no. 4, and from sect. 3, ch. 2, no. 11, and ch. 3, no. 5. But there, either following the reckoning in parts of the radius of the planet's orb or of the annual orb, taken from Mästlin or Copernicus, and sometimes disregarding the eccentricity of the Earth, we indicated the distances of the planets from the Sun or from the Earth only from afar; and those distances also suffer from several errors, whether of the pen or of the printer. In this place we shall make good those slips with interest. And we shall consider the distances of the planets both in the case in which (if the world lasted so long) the apogees of these planets would coincide with the Sun's apogee under the same point of the zodiac, and in the case in which they would be diametrically opposed to it. Moreover, in the distances from the Sun, we shall take account of both the greatest and the least eccentricity of the Sun, so that the greatest differences of the intervals possible in this hypothesis may appear.

[Margin: Summary of the Copernican hypothesis for the distances.] We shall set out the rules for deriving these distances using, of the three forms of the Copernican hypothesis, the one that consists of an eccentre of the eccentric [Eccentro Eccentrici], that is, in which the eccentric varies its eccentricity, taken from the centre of the annual orb, by means of a small circle, whose schemes we set out in the chapters already mentioned. That was the reason why we have joined Lansbergen, who uses the same form here, with Copernicus and Mästlin. We therefore suppose that the superior planets, whenever they are at the apogee or perigee of their eccentric, have the least eccentricity, that is, half of the whole eccentricity, which Ptolemy would call the eccentricity of

the deferent. So too, whenever the Earth falls on the line of apsides of Venus and Mercury, Venus has its least but Mercury its greatest eccentricity, and then too Mercury has the least semidiameter of its orb, by force of the librating small circle. Further, the annual orb of the Earth is equivalent to the Ptolemaic epicycles of the three superior planets, and to the eccentrics of the two inferior ones. Finally, from the chapters already mentioned, I take as a foundation the measures written below, with the letter M set against the least eccentricity of Venus, which is to be used with the Sun's greatest eccentricity. Although it seems this should be done for Mars as well, there is no danger of great error because of its larger eccentricity, and so we have chosen the middle one among its least eccentricities.

Measures in semidiameters of the Earth

		From	Copernicus	Mästlin	Lansbergen
Eccentricity	Least	♄	697	605	849
		♃	271	278	371
		♂	170	176	221
		♀	20	20	21
	M	♀	24	24	22
		☉	37	37	41
	Greatest	☉	48	48	63
	Total	☿	108	110	142
Semidiameter	or radius	♄	10517	10933	14880
	of the orb	♃	5951	6085	8091
		♂	1733	1764	2275
		♀	821	834	1077
	annual	☉	1142	1160	1498½
	Least	☿	405	413	564

Regulæ Pro Diſtantijs Planetarum à Sole & à Terra Determinandis in prædicta Hypotheſi.

Rules for determining the distances of the planets from the Sun and from the Earth in the hypothesis already mentioned.

[Margin: Mean distance.] IV. First rule. The mean distance from the centre of the Earth, in any one of the minor planets, is always as great as the radius or semidiameter of the eccentric. In Venus and Mercury, therefore, it is as great as the radius of the annual orb, that is, as the Sun's mean distance from the Earth. The mean distance from the Sun, by contrast, is as great as the radius of the eccentric of the three superior planets and of the lesser orb of the two inferior ones.

[Margin: Greatest distance of ♄ ♃ ♂ from the Earth.] 2. Rule. The greatest distance of the three superior planets from the centre of the Earth occurs only at their conjunction with the Sun. For it, join together the semidiameters of the annual orb and of the planet's orb, and the planet's least eccentricity.

[Margin: Least distance of the same from the Earth.] 3. Rule. The least distance of the three superior planets from the centre of the Earth occurs only at opposition with the Sun. For it, subtract from the semidiameter of the planet's orb the sum made of the semidiameter of the annual orb and of the planet's least eccentricity.

[Margin: Greatest distance of the same from the ☉.] 4. Rule. The greatest distance of the three superior planets from the Sun occurs only at conjunction with the Sun. For it, from the sum made of the semidiameter of the orb and the planet's least eccentricity subtract, if the apogees coincide with the Sun's apogee, but add, if they are opposed to it, whichever eccentricity of the Sun you prefer: namely either the greatest, of Ptolemy's time, or the least, of our times. This is to be understood below, whenever the Sun's eccentricity is mentioned. But if you want the greatest of the greatest possible, subtract the Sun's least distance from the Earth from the greatest distance of the superior planets from the Earth.

[Margin: Least distance of the same from the ☉.] 5. Rule. The least distance of the three superior planets from the Sun occurs only at opposition with the Sun. For it, subtract from the semidiameter of the planet's orb, if the apogees coincide, the difference between the solar eccentricity and the planet's least eccentricity; but if the apogees are opposed, the sum of those eccentricities. But if you want absolutely the least of the least possible, add to the Sun's least distance from the Earth the least distance of the superior planets from the Earth.

[Margin: Greatest distance of ♀ and ☿ from the Earth.] 6. Rule. The greatest distance of Venus and Mercury from the centre of the Earth occurs only at their conjunction in the highest part of the orb. For it, add to the semidiameter of the annual orb the semidiameter and the least eccentricity of the orb of Venus; but the least semidiameter and the total eccentricity of the orb of Mercury.

[Margin: Least distance of the same from the Earth.] 7. Rule. The least distance of Venus and Mercury from the centre of the Earth occurs only at their conjunction with the Sun in the lowest part of the orb. For it, subtract from the semidiameter of the annual orb the sum made of the semidiameter and the least eccentricity of the orb of Venus, or of the least semidiameter and the total eccentricity of the orb of Mercury.

[Margin: Greatest distance of the same from the ☉.] 8. Rule. The greatest distance of Venus and Mercury from the Sun occurs if their apogees are opposed to the Sun's apogee, and these planets are joined either in the lower position to the Sun at apogee, or in the upper position to the Sun at perigee. Therefore add to the semidiameter of the orb of Venus the sum made of the total eccentricity of the Sun and the least eccentricity of Venus; but to the least semidiameter of the orb of Mercury add the sum made of the total eccentricity of both the Sun and Mercury. In this way you will have their greatest distance from the Sun possible in this hypothesis. But if their apogees coincide with the Sun's apogee, then to the least semidiameter of the orb of Mercury, or the ordinary semidiameter of Venus, add the difference between the total eccentricity of

Mercury or of Venus and the total eccentricity of the Sun. But this will not be the greatest of the greatest, as the former was.

[Margin: Least distance of the same from the ☉.] 9. Rule. The least distance of Venus and Mercury from the Sun occurs if their apogees are opposed to the Sun's apogee, and these planets are joined either in the upper position to the Sun at apogee, or in the lower position to the Sun at perigee. Therefore subtract from the semidiameter of the orb of Venus the aggregate made of the total eccentricity of the Sun and the least eccentricity of Venus; but from the least semidiameter of Mercury subtract the aggregate made of the total eccentricity of both the Sun and Mercury. And this will be the least of the least. But if their apogees coincide with the Sun's apogee, the least will be obtained by subtracting from the semidiameter of the orb of Venus the difference between the Sun's total eccentricity and Venus's least eccentricity; and from the least semidiameter of the orb of Mercury subtract the difference between the total eccentricity of Mercury and that of the Sun. But this will not be the least of the least.

De Diſtantijs Planetarum Minorum à Sole & à Terra ex Tychonis & Longomontani Hypotheſi.

On the distances of the minor planets from the Sun and from the Earth, from the hypothesis of Tycho and Longomontanus.

V. We have already reviewed the places from which we derive these distances, at sect. 2, ch. 3, no. 8, and sect. 3, ch. 4, no. 5, and in the scholia of those chapters, where we also exposed the errors of Baranzanus and others in reviewing the Tychonic distances. Now, as is our way, these must be treated more accurately. For there we did not cover all the cases, and, with Tycho (*Progymnasmata*, vol. 1, p. 476), we assumed the Sun's mean distance from the Earth in determining the greatest and least distances from the Earth, neglecting the Sun's eccentricity, which we shall not neglect now.

[Margin: Why the variation of the eccentricity is neglected here.] Granted, it may have seemed right to disregard the variation possible from the variation of this eccentricity, because Tycho (*Progymnasmata*, vol. 1, pp. 32 and 33), although he admitted it, did not determine how great it is in any given age, on account of the disagreement between the observations of al-Zarqālī and al-Battānī on the change of the solar apogee, and their agreement on the eccentricity; while Longomontanus (*Theorica*, book 1, ch. 2, in the commentary on the Sun) denies that it varies, as many others also do, as I said in book 3, ch. 26. Moreover, since the eccentricities of the planets in this hypothesis have been transformed into semidiameters of epicycles, keeping the same quantity everywhere, there is no reason to vary the distances from the Sun because of the concurrence or opposition of the apogees. We must, however, change the distances from the Earth, because of the Sun's eccentricity; for they stand one way if the apogees of these planets coincide with the Sun's apogee, and another if they are opposed

to it. Meanwhile, look at the foundations or elements of these distances in the following small table. In it we have, however, supplied the Tychonic ones for Jupiter and Mars, since Tycho left us only their mean distance from the Earth (*Progymnasmata*, vol. 1, p. 476). But since at p. 479 of the same place he nearly follows Copernicus, applying a moderate increase and dividing the whole Copernican eccentricity into 4 equal parts, three parts to the semidiameter of the greater, and

[Margin: See the table at sect. 2, ch. 2, no. 7.] the fourth to the semidiameter of the lesser epicycle in his distribution, we have, by a conjecture that is surely probable, determined the epicycles of Jupiter and Mars as well, according to Tycho's mind. Therefore, as the whole Copernican eccentricity of ♄, of 1394 semidiameters of the Earth, is to the whole Tychonic, 1200 (of which three parts are 900 and the fourth 300), so the whole Copernican eccentricity of Mars, 334, is to the whole Tychonic eccentricity of Mars, about 290; and the whole Copernican eccentricity of Jupiter, 542, to the whole Tychonic eccentricity of Jupiter, 472. Therefore, by dividing the whole eccentricity of ♃ and ♂ into 4 portions, and giving one to the lesser and the rest to the greater epicycle, the semidiameter of the lesser and of the greater epicycle comes out as large as you see in the small table.

Measures in semidiameters of the Earth

			From Tycho	From Longomontanus
Eccentricity		☉	40	46
	Total	☿	☩	74
	Half	♀	☩	21
Semidiameter of the epicycle	Lesser	♂	72½	74¼
		♃	118	158
		♄	300	358
	Greater	♂	217½	297
		♃	354	473
		♄	900	1075
	Greatest	♀	☩	931
		☿	☩	496
Radius of the orb		♂	1745	1967
		♃	3990	6605
		♄	10550	12354
		☉	1150	1288

Regulæ pro determinandis Diſtantijs in Hypotheſi Danica à Sole & à Terra.

Rules for determining the distances from the Sun and from the Earth in the Danish hypothesis.

[Margin: Mean distance.] VI. First rule. The mean distance of ♄ ♃ and ♂ from the Earth or from the Sun is as great as the radius or semidiameter of the

orb concentric with the Sun. But in ♀ and ☿ it is obtained by adding to the least distance half the difference between the least and the greatest.

[Margin: Greatest distance of ♄ ♃ ♂ from the ☉.] 2. Rule. The greatest distance of the three superior planets from the Sun occurs if you add to the radius of the planet's orb the semidiameter of the greater epicycle, and take away from the sum the semidiameter of the lesser epicycle.

[Margin: Least distance of the same from the ☉.] 3. Rule. The least distance of the three superior planets from the Sun is obtained if you subtract from the radius of the planet's orb the semidiameter of the greater epicycle, and add to the remainder the semidiameter of the lesser.

[Margin: Greatest distance of the same from the Earth.] 4. Rule. The greatest distance of the three superior planets from the centre of the Earth, if their apogees coincide with the Sun's apogee, would occur when a conjunction came about between those planets, placed at apogee, and the Sun, which is of course at apogee; and it is obtained if you add the Sun's greatest distance from the Earth to their greatest distance from the Sun. But if the apogees are opposed, it would occur when they, at apogee, were joined with the Sun at perigee; and it is obtained by adding the least distance of the ☉ from the Earth to their greatest distance from the ☉. But this is not the greatest of all, as the former was.

[Margin: Least distance of the same from the Earth.] 5. Rule. The least distance of the three superior planets from the centre of the Earth, when the apogees coincide, would occur if they, at perigee, were opposed to the Sun at apogee; and it is obtained if the Sun's greatest distance from the Earth is subtracted from their least distance from the Sun. But if the apogees are opposed, it would occur when they, at perigee, were opposed to the Sun at perigee; and then it would be obtained by subtracting the Sun's least distance from the Earth from their least distance from the Sun. But this is not the least of all.

[Margin: Greatest distance of ♀ ☿ from the ☉.] 6. Rule. The greatest distance of Venus and Mercury from the Sun is made by adding to the semidiameter of the planet's greatest epicycle half the eccentricity of Venus, or the total eccentricity of Mercury.

[Margin: Least from the ☉.] 7. Rule. The least distance of Venus and Mercury from the Sun is obtained by subtracting from the semidiameter of the planet's greatest epicycle the half-eccentricity of Venus or the total eccentricity of Mercury, when the Sun is at its greatest distance from the Earth; for if it is at its least, then only the least eccentricity of Mercury is to be taken away from the semidiameter of the epicycle.

[Margin: Greatest distance of the same from the Earth.] 8. Rule. The greatest distance of Venus and Mercury from the Earth, when their apogee coincides with the Sun's apogee: if the Sun is at its greatest distance from the Earth, it is made by adding the Sun's greatest distance from the Earth to their greatest distance from the Sun; but if the Sun is at its least distance from the Earth, their least eccentricity must be taken from the semidiameter of the epicycle, and the

remainder added to the Sun's least distance from the Earth. When, however, the apogees are opposed: if the Sun is at its greatest distance from the Earth, it is made by taking their least eccentricity from the semidiameter of the epicycle and adding the remainder to the Sun's greatest distance from the Earth. But if the Sun is at its least distance from the Earth, there must be added to the Sun's least distance from the Earth the semidiameter of the epicycle and, besides that, the half eccentricity of Venus, but the total eccentricity of Mercury.

[Margin: Least distance of the same from the Earth.] 9. Rule. The least distance of Venus and Mercury from the Earth, when the apogees coincide: if the Sun is at its greatest distance from the Earth, it is made by subtracting the semidiameter of the greatest epicycle from the Sun's greatest distance from the Earth; but to the remainder must be added the half eccentricity of Venus, but the total eccentricity of Mercury. But if the Sun is at its least distance from the Earth, then from the Sun's least distance from the Earth must be subtracted the sum made of the semidiameter of the epicycle and the least eccentricity of either planet. When, however, their apogees are opposed to the Sun's apogee: if the Sun is at its greatest distance from the Earth, from the Sun's greatest distance from the Earth must be subtracted the sum made of the semidiameter of the epicycle and the least eccentricity of either star. But if the Sun is at its least distance from the Earth, then there must be subtracted from the semidiameter of the epicycle the half eccentricity of Venus, but the total eccentricity of Mercury, and the remainder must be subtracted from the Sun's least distance from the Earth.

10. Rule. In all five minor planets, Mercury excepted, the difference between the greatest and the least distance in the same kind of comparison is always of the same quantity; thus, for example, in Venus it is always 931 semidiameters of the Earth.

De Diſtantijs Planetarum Minorum à Sole & à Terra ex Kepleri & Bullialdi Hypotheſi.

On the distances of the minor planets from the Sun and from the Earth, from the hypothesis of Kepler and Boulliau.

VII. We gave the distances of the planets that fit this hypothesis at sect. 2, ch. 5, no. 20, and sect. 3, ch. 5, no. 5. There, in the measures that depend on the distance of the Sun, we showed a double column in the table: one relying on the solar distances that Kepler set down in the *Commentaries on Mars* and in the *Ephemerides*, the other on the distances given in the *Epitome of Copernican Astronomy* and in the Rudolphine Tables. Since these latter are of later care, they will be chosen by us in this place; and they will be carried over to other cases possible in this hypothesis, different from those we devised there for the distances from the Earth, so that we may learn the possible extremes.

Measures in semidiameters of the Earth

From		Kepler	Boulliau
Eccentricity of the centre of the ellipse from the ☉	Annual orb	52	26
	♀	17	8
	☿	275	118
	♂	327	206
	♃	847	370
	♄	1805	804
Total eccentricity, or distance of the foci from each other	Annual orb	104	52
	♀	34	16
	☿	550	236
	♂	654	412
	♃	1694	740
	♄	3610	1608
Major semidiameter of the ellipse, or mean distance	Annual orb	3381	1460
	☿	1313	563
	♀	2448	1057
	♂	5150	2224
	♃	17569	7628
	♄	32156	13931
Greatest distance	☉	3469	1486

In this hypothesis there is no variation of the Sun's eccentricity, and the greatest and least distances from the Sun are the same whether the aphelia coincide with the Earth's aphelion or are opposed to it. The distances from the Earth, however, vary in many ways. The foundations of the possible distances in both cases, namely the concurrence or the opposition of the aphelia, are in the table given above. Kepler's foundations, taken from the analogy of the intervals and of the bodies of the world, will be explained below in Chapter 7, from number 11.

Regulæ pro Diſtantijs à Sole & à Terra in hac Hypotheſi

Rules for the distances from the Sun and from the Earth in this hypothesis

[Margin: Distance from the ☉, greatest and least.] VIII. First Rule. The greatest distance of the five minor planets from the Sun is made up of the greater semidiameter of the ellipse and the eccentricity of the centre of the ellipse from the Sun, which is one of the foci. The least is obtained by subtracting, from the greater semidiameter of the ellipse assigned to any planet, the eccentricity of the centre of the ellipse from the Sun.

[Margin: Greatest distance of ♄ ♃ ♂ from the Earth.] 2. Rule. The greatest distance of the three planets from the centre of the Earth, if their aphelia coincide with the Earth's aphelion, is made up of their greatest distance from the Sun and the Earth's least distance from the Sun. But if the apsides are turned about, that is, if their aphelia are opposed to the Earth's aphelion, it is made up of their greatest distance and the Earth's greatest distance from the Sun. It therefore becomes known when these are joined.

[Margin: Least distance of the same from the Earth.] 3. Rule. The least distance of the three superior planets from the centre of the Earth is obtained by subtracting, from the greater diameter of the planet's ellipse, the sum made of the planet's greatest distance from the Sun and the Earth's least, if the aphelia coincide, or the Earth's greatest distance from the Sun, if they are opposed.

[Margin: Greatest distance of ♀ and ☿ from the Earth.] 4. Rule. The greatest distance of Venus and Mercury from the centre of the Earth. When their aphelion coincides with the Earth's aphelion: if the Earth is at its greatest distance from the Sun, it is obtained by adding the greater semidiameter of the ellipse to the Earth's greatest distance from the Sun, but subtracting from the sum the eccentricity of the centre of the ellipse, of Venus and of Mercury alike. But if the Earth is least distant from the Sun, it is obtained by adding to the Earth's least distance from the Sun the sum made of the semidiameter of the ellipse and the eccentricity of the centre of the ellipse. When, however, the aphelia are opposed: [Margin: In 4 cases.] if the Earth is most distant from the Sun, it is obtained by adding together the Earth's greatest distance from the Sun, the semidiameter of the ellipse and the eccentricity of the centre of the ellipse; but if the Earth is least distant from the Sun, it is obtained by adding the semidiameter of the ellipse to the Earth's least distance from the Sun, and subtracting from the sum the eccentricity of the centre of the ellipse.

[Margin: Least distance of the same from the Earth.] 5. Rule. The least distance of Venus and Mercury from the centre of the Earth. When the aphelia coincide: if the Earth is most distant from the Sun, it is obtained by subtracting from the Earth's greatest distance from the Sun the sum made of the semidiameter and the eccentricity of the centre of the ellipse. But if the Earth is least distant from the Sun, it is obtained by subtracting the semidiameter of the ellipse from the Earth's least distance from the Sun, but the eccentricity of the centre of the ellipse must be added to the remainder. When, however, the aphelia are opposed: [Margin: In 4 cases.] if the Earth is most distant from the Sun, it is obtained by subtracting the semidiameter of the ellipse from the Earth's greatest distance from the Sun, but the eccentricity of the centre of the ellipse must be added to the remainder. But if the Earth is least distant from the Sun, the sum made of the semidiameter and the eccentricity of the centre of the ellipse must be subtracted from the Earth's least distance from the Sun.

De Diſtantijs Quinque Minorum Planetarum à Terra & à Sole ex noſtra Hypotheſi.

On the distances of the five minor planets from the Earth and from the Sun according to our hypothesis.

IX. We come at last to the intervals of these planets according to our own hypothesis, derived partly in Section 2, Chapter 6, number 13, and Section 3, Chapter 6, number 7, and partly to be derived in this place, by considering both the conjunction of the apogee of these planets with the Sun's apogee and the

diametrical opposition of the apogees. Let no one be surprised if the intervals that depend on the Sun's distance from the Earth are much greater with us than with many others; for they have as much probability as the distances of the Sun from the Earth established in Book 3, Chapter 7, and the proportions of the intervals to that distance, which are to be indicated in Scholium 1. The remaining foundations for the rules and distances about to be set down may be seen in the following table, the Sun's mean distance being not the round 7300 but, more precisely, 7327.

Measures in semidiameters of the Earth

		Greatest	Least
Eccentricity	☉	253	253
	♀	253	253
	☿	730	584
	♂	2200	1900
	♃	3635	3321
	♄	8906	7884
Semidiameter of the epicycle	♀	5339	5157
	☿	3322	2811
	♂	7805	6727
	♃	7417	6738
	♄	8249	7373
Radius of the eccentric	☿ ♀ ☉	7327	
	♂	11000	
	♃	36500	
	♄	73000	

Regulæ pro Diſtantijs Planetarum à Sole & à Terra in Noſtra Hypotheſi.

Rules for the distances of the planets from the Sun and from the Earth in our hypothesis.

X. The rules for deriving the distances of the three superior planets from the Earth and from the Sun are the same as the rules for the Ptolemaic hypothesis given at number 2. For the inferior planets, which we arrange not below the Sun but around it, they are the following. In these we suppose that Venus has the same eccentricity as the Sun, and the greatest semidiameter of the epicycle at the apogee of the eccentric and the least at the perigee; [Margin: Summary of the hypothesis for the distances.] that Mercury, on the other hand, has the greatest eccentricity at the apogee of the eccentric and the least at the perigee, but in both places the least semidiameter of the epicycle; and that the semidiameter of the eccentric of each is the same as the radius of the Sun's annual orb. In the three superior planets, however, we set the greatest eccentricity and semidiameter of the epicycle at apogee, and both of them least at perigee.

[Margin: ♀ and ☿: greatest distance from the Earth.] 1. Rule for the greatest distance of Venus and Mercury from the Earth. When their apogees

coincide with the Sun's apogee: if the centres of the epicycles are at the apogee of the eccentric, add to the Sun's mean distance from the Earth the greatest semidiameter of the epicycle of ♀ and the least of ☿, and in addition the whole eccentricity for each. [Margin: In 4 cases.] But if the centres of the epicycles are at the perigee of the eccentric, add to the Sun's mean distance from the Earth the least semidiameter of the epicycle, but from the sum subtract the whole eccentricity of Venus and the least of Mercury. Likewise, when the apogees are opposed to the Sun's apogee: if the centre of the epicycle is at the apogee of the eccentric, add to the Sun's mean distance from the Earth the greatest semidiameter of the epicycle of ♀ and the least of ☿, and the whole eccentricity of the planet. But if the centre of the epicycle is at the perigee of the eccentric, add to the Sun's mean distance from the Earth the least semidiameter of the epicycle, but from the sum subtract the whole eccentricity of Venus and the least of Mercury.

[Margin: Least distance of the same from the Earth.] 2. Rule for the least distance of Venus and Mercury from the Earth. When the apogees coincide: if the centre of the epicycle is at the apogee of the eccentric, subtract from the Sun's mean distance from the Earth the greatest semidiameter of the epicycle of ♀ and the least of ☿, but to the remainder add the whole eccentricity of ♀ or ☿. But if the centre of the epicycle is at the perigee of the eccentric, [Margin: In 4 cases.] subtract from the Sun's mean distance from the Earth the least semidiameter of the epicycle, and in addition the whole eccentricity of ♀ but the least of ☿. Similarly, when the apogees are opposed: if the centre of the epicycle is at the apogee of the eccentric, subtract from the Sun's mean distance from the Earth the greatest semidiameter of the epicycle of ♀ and the least of ☿, but to the remainder add the whole eccentricity for each. But if it is at perigee, subtract from the Sun's mean distance from the Earth the least semidiameter of the epicycle, but from the remainder subtract further the whole eccentricity of Venus and the least of Mercury.

3. Rule for the greatest distance of Venus and Mercury from the

[Margin: ♀ and ☿: greatest distance from the ☉.] Sun. When the apogees coincide: if the centre of the epicycle is at the apogee of the eccentric, use the greatest semidiameter of the epicycle of Venus; but to the least semidiameter of the epicycle of Mercury add the difference between its whole eccentricity and that of the Sun. But if the centre of the epicycle is at the perigee of the eccentric, use the least semidiameter of the epicycle of Venus; [Margin: In 4 cases.] but to the least semidiameter of the epicycle of Mercury add the difference between its least eccentricity and the Sun's whole eccentricity. When, however, the apogees are opposed: if the centre of the epicycle is at the apogee of the eccentric, add to the greatest semidiameter of the epicycle of ♀ and the least of ☿ the sum made of the whole eccentricity of the planet and of the Sun. But if the centre of the epicycle is at the perigee of the eccentric, add to the least semidiameter of the epicycle the sum made of the eccentricity of the Sun and the whole eccentricity of Venus, but the least of Mercury.

[Margin: Least distance of the same from the ☉.] 4. Rule for the least distance of Venus and Mercury from the Sun. When the apogees coincide: if the centre of the epicycle is at the apogee of the eccentric, use the greatest semidiameter of the epicycle of Venus; but from the least semidiameter of the epicycle of Mercury subtract the difference between its whole eccentricity and that of the Sun. But if the centre of the epicycle is at the perigee of the eccentric, use the least semidiameter of the epicycle of Venus, but from the least semidiameter of the epicycle of Mercury subtract the difference between its least eccentricity and the Sun's whole eccentricity. When, however, the apogees are opposed: [Margin: In 4 cases.] if the centre of the epicycle is at the apogee of the eccentric, from the greatest semidiameter of the epicycle of ♀ and the least of ☿ subtract the sum made of the eccentricity of the Sun and the whole eccentricity of the planet. But if the centre of the epicycle is at the perigee of the eccentric, from the least semidiameter of the epicycle subtract the sum made of the eccentricity of the Sun and the whole eccentricity of Venus, but the least of Mercury. Which distances are the greatest of all, and so on, the following tables will show.

[Margin: Quantity of the greatest digressions from the ☉.] As for the semidiameters of the Earth contained in the greatest digression of these planets from the Sun, we shall speak of them in Chapter 12, number 4.

Monitum Pro Sequentibus Tabulis.

Notice concerning the following tables.

XI. From the authors already listed above, or from others soon to be reviewed, we have collected the following tables, or have constructed some of them from the foundations and rules already set out. In these we decided to include the greatest and least distances of these planets from the Earth and from the Sun, not as large as they actually turn out at this time, but as large as each author's hypothesis can allow, if the apogees of the minor planets either coincided under the same place of the zodiac with the Sun's apogee or, conversely, were diametrically opposed to it; so that, when all have been computed, the absolutely greatest and least distance can be chosen, if anyone wishes to use it. Besides these we decided to add the distances given by Giuseppe Biancani at the end of his *Sphaera* (but he gives only the mean ones, from Tycho, as does Andrea Argoli in the *Pandosion Sphaericum* from Chapter 52 on), by Athanasius Kircher in the *Ars Magna Lucis et Umbrae*, p. 762, by Antonín Maria Šírek in his *Radius Sidereomysticus*, Book 4, Chapter 2, member 5, and by Gassendi in the *Institutio Astronomica*, Book 3, Chapter 11, where he gives the Tychonic and Lansbergian ones, but middling ones [mediocres]. I have also added the distances of Govaert Wendelen, from his own letter written to me, and those of Posidonius, from the distance of the Sun from the Earth that he set (Book 3, Chapter 7), on the supposition, as the prosthaphaereses require, that Saturn's mean distance from the Earth is ten times the Sun's distance from the Earth, Jupiter's five times,

Mars's one and a half times, Venus's three-quarters, and Mercury's one-third, approximately.

Anyone who wants the following distances in miles or leagues should consult the foot of Table 4 of Chapter 4, or multiply them by the miles or leagues contained in the semidiameter of the Earth, according to whichever opinion he prefers to follow among those reported in Book 2, Chapter 7. [Margin: On reducing the distances to miles.]

Table I. Distances of Saturn, Jupiter and Mars from the centre of the Earth. In semidiameters of the Earth.

Authors	♄ Saturn: greatest	mean	least	♃ Jupiter: greatest	mean	least	♂ Mars: greatest	mean	least
Ptolemy or Alfonso, by Jean Fernel	22612	19403 1/5	16194 1/5	16185 2/3	13079 1/3	9972 1/2	9967 1/3	5641 5/6	1316 2/5
Ptolemy, by Copernicus and Kepler	25737	22000	18263	18253	14750	11248	11241	6393	1545
al-Battānī, by Francesco Giuntini and Mauro Fiorentino	17571	15373	13180	13171	10704	8237	8232	4704	1176
al-Battānī, by Tycho	18500	15800	13100	☩	10423	☩	☩	4584	☩
al-Battānī, by me	18094	15257	12420	12411	10217	8024	8018	4597	1177
al-Farghānī and Thābit, by Maurolico and Clavius	20072	17225 1/4	14378 1/3	14369 1/4	11611 1/2	8853 4/5	8848	5032	1216
al-Farghānī, by Francesco Barozzi	20215	17312	14409	14400	11638	8876	8870	5045	1220
Copernicus, by me	12356	10517	8678	7364	5951	4538	3045	1733	421
Michael Mästlin, by me	12698	10933	9168	7523	6085	4647	3100	1764	428
Tycho, by me, at the ☉'s greatest distance	12340	10550	8760	5416	3990	2564	3080	1745	410
At the ☉'s least distance from the Earth	12260	10550	8840	5336	3990	2644	3001	1745	490
Longomontanus, at the ☉'s greatest distance	14405	12354	10303	8255	6605	4956	3524	1967	410
At the ☉'s least distance from the Earth	14313	12354	10395	8163	6605	5048	3432	1967	502
Kepler, if the aphelia coincide	37285	32156	27027	21740	17569	13398	8801	5150	1499
If the aphelia are opposed to the Earth's aphelion	37399	32156	26913	21854	17569	13284	8915	5150	1385
Lansbergen, by Gassendi and by me	17227	14880	12534	9960	8091	6222	3994	2275	556
Boulliau, if the aphelia coincide	16169	13931	11693	9432	7628	5824	3864	2224	584
If the aphelia are opposed to the Earth's aphelion	16221	13931	11641	9484	7628	5772	3916	2224	532
Govaert Wendelen	☩	139677	☩	☩	75039	☩	☩	22257	☩
Posidonius, by our calculation	☩	13000	☩	☩	65000	☩	☩	19500	☩
Antonín Maria Šírek	☩	20000	☩	☩	10799	☩	☩	3047	☩
Giuseppe Biancani and Kircher, from Tycho	☩	10550	☩	☩	3990	☩	☩	1745	☩

Authors	♄ Saturn: greatest	mean	least	♃ Jupiter: greatest	mean	least	♂ Mars: greatest	mean	least
We, for the present	90155	73000	57743	47552	36500	26441	21005	11000	2373

Table II. Distances of Venus and Mercury from the centre of the Earth. In semidiameters of the Earth.

Authors	If their apogees, with the Sun's apogee	♀ Venus: greatest	mean	least	☿ Mercury: greatest	mean	least
Ptolemy or Alfonso, by Jean Fernel		1199 2′	689	179 2′	178 28′	117 2′	65 36′
Ptolemy, by Copernicus and Kepler		1407	808	210	209	137	65
al-Battānī, by Francesco Giuntini and Mauro Fiorentino		1078	622	166	165	114 35′	64 10′
al-Battānī, by Tycho		☩	618	☩	☩	115	☩
al-Battānī, by me		1070	618	166	165	115	65
al-Farghānī and Thābit, by Maurolico and Clavius		1115 33′	641 45′	167 57′	167 37′	116 3′	64 29′
al-Farghānī, by Francesco Barozzi		1120	643 30′	167	166 40′	115 25′	64 10′
Copernicus, by me		1987	1142	297	1655	1142	629
Michael Mästlin, by me		2018	1160	302	1683	1160	637
Tycho, by Giuseppe Biancani		☩	1150	☩	☩	1150	☩
Lansbergen himself		2598	1498 ½	399	2176	1498 ½	821
Lansbergen, by me and Gassendi		2597	1498	399	2204	1498 ½	792
Antonín Maria Šírek		☩	1438	☩	☩	764	☩
Govaert Wendelen		☩	10552	☩	☩	5713	☩
Posidonius, by me		☩	9360	☩	☩	5070	☩
Longomontanus, at the ☉'s distance from the Earth: greatest	coincide	2286	1355	424	1904	1403	912
	are opposed	2244	1313	382	1785	1274	764
Least	coincide	2152	1221	290	1693	1160	627
	are opposed	2194	1263	332	1812	1311	820
Kepler, at the ☉'s distance from the Earth: greatest	coincide	5869	3421	973	4476	3163	1850
	are opposed	5893	3450	1007	5026	3713	2400
Least	coincide	5789	3341	893	4912	3899	2886
	are opposed	5555	3207	859	4362	3049	1736
Boulliau, at the ☉'s distance from the Earth: greatest	coincide	2535	1478	421	1931	1368	805
	are opposed	2551	1494	437	2167	1604	1041
Least	coincide	2499	1442	385	2115	1554	989
	are opposed	2483	1421	369	1879	1316	753
We, for the present, at the ☉'s distance from the Earth: greatest	coincide	12919	7580	2241	10868	8057	5246
	are opposed	12231	7074	1917	9700	6889	4078

Authors	If their apogees, with the Sun's apogee	♀ Venus: greatest	mean	least	☿ Mercury: greatest	mean	least
Least	coincide	12231	7074	1917	9700	6889	4078
	are opposed	12919	7580	2241	10868	8057	5246

Table III. Distances of Saturn, Jupiter and Mars from the Sun, if they have the same line of apsides as the Sun. In semidiameters of the Earth.

Authors	If the apogees, with the ☉'s apogee	♄ Saturn: greatest	least	♃ Jupiter: greatest	least	♂ Mars: greatest	least
Ptolemy or Alfonso, by Jean Fernel	coincide	21303	17493	14877	11282	8658	2625
	are opposed	21408	17398	14982	11177	8763	2520
Ptolemy, by Copernicus and Kepler	coincide	24200	19800	16716	12785	9704	3082
	are opposed	24323	19677	16839	12662	9827	2959
al-Battānī, by me	coincide	16948	13566	11265	9170	6872	2323
	are opposed	17026	13488	11343	9092	6950	2245
al-Farghānī and Thābit, by Maurolico and Clavius	coincide	18863	15588	13160	10063	7639	2426
	are opposed	18951	15500	13248	9975	7727	2337
Copernicus, with the Sun's eccentricity greatest	coincide	11166	9868	6174	5728	1855	1611
	are opposed	11262	9772	6270	5632	1951	1515
Least	coincide	11177	9857	6185	5717	1866	1600
	are opposed	11251	9783	6229	5643	1940	1526
Michael Mästlin, with the Sun's eccentricity greatest	coincide	11490	10376	6315	5855	1892	1656
	are opposed	11586	10280	6411	5760	1988	1540
Least	coincide	11501	10365	6326	5844	1903	1625
	are opposed	11575	10291	6400	5770	1977	1551
Tycho, by me		11150	9950	4226	3754	1890	1600
Longomontanus, by me		13071	11637	6921	6290	2190	1744
Kepler, by me		33961	30351	18416	16722	5477	4823
Boulliau, by me		14735	13127	7998	7258	2430	2018
Lansbergen, with the Sun's eccentricity greatest	coincide	15666	14094	8399	7783	2433	2117
	are opposed	15792	13968	8525	7657	2559	1991
Least	coincide	15688	14072	8421	7761	2455	2095
	are opposed	15770	13990	8503	7679	2537	2013
Giuseppe Biancani, from the ☉'s apogee		10588	✢	3652	✢	1628	✢
We, for the present	coincide	82575	65323	39972	34021	13425	9953
	are opposed	83081	64817	40478	33515	13931	9447

Table IV. Distances of Venus and Mercury from the Sun, if they have the same line of apsides as the Sun.

Authors	If the apogees, with the ☉'s apogee	♀ Venus: greatest	least	☿ Mercury: greatest	least
Ptolemy or Alfonso, by Jean Fernel	coincide	1101	34	1219	1061
	are opposed	1130	6	1230	1055
al-Farghānī and Thābit, by Maurolico and Clavius	coincide	1017	30	1129	979
	are opposed	1041	6	1145	963
Copernicus, with the ☉'s eccentricity greatest	coincide	845	797	465	345
	are opposed	893	749	561	249
Least	coincide	838	804	476	334
	are opposed	878	764	550	260
Michael Mästlin, with the ☉'s eccentricity greatest	coincide	858	810	475	351
	are opposed	906	762	571	255
Least	coincide	851	817	486	340
	are opposed	891	777	560	266
Longomontanus		951 ½	910	570	422 or 525
Kepler, by me		2465	2431	1588	1038
Boulliau, by me		1065	1049	681	445
Giuseppe Biancani, from the ☉'s apogee		855	☩	572	☩
Lansbergen, with the ☉'s eccentricity greatest	coincide	1118	1036	643	485
	are opposed	1163	992	769	359
Least	coincide	1097	1057	643	485
	are opposed	1139	1015	747	381
We, at the ☉'s distance from the Earth: greatest	coincide	5339	5339	3288	2334
	are opposed	5663	4651	3502	1828
Least	coincide	5157	5157	2996	2626
	are opposed	5845	4833	3794	2120

If you want the measures set out in the preceding tables in Italian miles, see the rule to be given at the foot of the last table of Chapter 4; or, for Kepler's, consult our Athanasius Kircher in *Magnes*, p. 561.

SCHOLIA.

Scholia.

[Margin: On the size of our distances.] I. If the distance of these planets from the Earth or from the Sun that arises from our hypothesis seems excessive to anyone, let him call two foundations to mind. One is the distance of the Sun itself from the Earth, which, from what was said in Book 3, Chapter 7, is for us nearly midway between the extremes asserted by astronomers, since there has been no lack of those who extended it to about 15000 semidiameters of the Earth. The other is the proportion of the mean distance of these planets to the

distance of the Sun from the Earth. This proportion is gathered from the commensurations of their orbs with the annual orb, through the prosthaphaereses of the orb, since the motion of these planets is tied to the motion of the Sun, as all astronomy agrees. Now the distance of Venus and Mercury is smaller among the more ancient authors, because they placed those planets below the Sun; but all after Copernicus arrange them around the Sun, so that their mean distance from the Earth is about as great as the Sun's as well. The proportion of the remaining planets, derived from the distance of the Sun (on which see Book 3, Chapter 7) and from the mean distances just reported in Tables 1 and 2, stands as you see in the following small table, the fractions in Saturn and Jupiter having been reduced to the nearest round numbers. Speaking, moreover, of the distance from the Sun, that too, according to Copernicus, Kepler in the *Epitome of Copernican Astronomy*, p. 455, and many others, is about tenfold in Saturn, fivefold in Jupiter and one-and-a-half-fold in Mars, relative to the distance of the Earth from the Sun. Therefore we have done nothing incongruous if, following in the footsteps of more recent astronomy, we have made the distance of ♄ from the Earth tenfold, Jupiter's fivefold and Mars's one-and-a-half-fold of the distance of the Sun. From this there results for us a mean distance of Saturn of 73000 terrestrial semidiameters, that is, almost twice as great as Kepler's, just as Kepler's is almost twice as great as Tycho's; for if we had listened to Posidonius and Wendelen, it would be twice as great again as ours.

The proportion of the mean distance from the Earth of the superior planets to the mean distance of the Sun from the Earth, taken as 1, is as below.

Authors	♄	♃	♂
Ptolemy, by Jean Fernel	15	10	4
Ptolemy, by Copernicus and Kepler	18	12	5
al-Battānī, by Tycho and me	14	9	4
al-Farghānī, by Maurolico, Clavius and Barozzi	14	10	4
Copernicus and Michael Mästlin, by me	10	5	1 ½
Tycho and Giuseppe Biancani, by me	9	4	1 ½
Longomontanus, by me	10	5	1 ½
Kepler, by me	10	5	1 ½
Lansbergen, by me	10	5	1 ½
Boulliau, by me	10	5	1 ½
Antonín Maria Šírek, by me	10	5	1 ½
We	10	5	1 ½

[Margin: On harmonic intervals, etc.] II. The question of the archetypal and harmonic intervals of the planets would certainly seem to belong here: namely, whether they should be so limited as to keep harmonic proportions among themselves, and with the whole distance of the fixed stars or of the outermost heaven, or with the diameters of the regular bodies that can be inscribed in a sphere. But that question concerns the general constitution of the world-system, and it also takes in the intervals of the fixed stars and of the

luminaries; a fuller exposition and examination of it is therefore reserved by us for Book 9, where, among other matters, the harmonic system will be treated. Meanwhile one may consult both what we have already indicated in Book 3, Chapter 7, in the scholia, and in Book 4, Chapter 14, likewise in the scholia, and in Book 6, Chapter 7, and in this Book 7, Chapter 1, and what will be said below in Chapter 8, from number 11; and also what Kepler teaches in the *Mysterium Cosmographicum*, Chapters 14, 20 and 21, in *De Stella Nova*, Chapter 16, in the *Harmonics*, Book 5, Chapter 4, and in the *Epitome of Copernican Astronomy*, Book 4, from p. 452 to 483, and 501, and in the Rudolphine Tables, p. 71; Pierre Hérigone in Volume 5 of the *Cursus Mathematicus*, from p. 570; and Wendelen in a most learned letter written to me, if he allows me to publish it. From these I shall indicate here some selected propositions, to be examined elsewhere.

[Margin: Proportion of the intervals from the 5 regular bodies.] The first is that the intervals of the five minor planets among themselves were defined by God according to the ratio of the five regular bodies. For, as Kepler says in the preface to the *Mysterium Cosmographicum*, the Earth is the circle that measures all the others, that is, the Earth's annual orb, because it lies midway between the convexities of the three superior spheres and of the two inferior ones, which he thinks are so proportioned that the five regular bodies can be inscribed in them in this order: the cube or hexahedron, the tetrahedron, the dodecahedron, the icosahedron and the octahedron. For around the Earth's annual orb circumscribe a dodecahedron, and around this dodecahedron circumscribe an orb: it will be the concave of the sphere of Mars. Around the convex of Mars circumscribe a tetrahedron, and around this circumscribe an orb: it will be the concave of the sphere of Jupiter. Around its convex circumscribe a cube, and around the cube an orb: it will be the concave of the sphere of Saturn. Then in the concave of the annual orb inscribe an icosahedron, and in this inscribe an orb: it will be the convex of the sphere of Venus. In its concave inscribe an octahedron, and in the octahedron inscribe an orb: it will be the convex of the sphere of Mercury. To fill out the interval of Jupiter, however, the circuits of the four satellites around Jupiter contribute, just as the Moon's circuit around the Earth contributes to the size of the annual orb. From this Kepler thinks it is shown why there are six orbs in the Copernican system, namely those of the five minor planets, and a sixth of the Earth with the Moon: because, just as the four intervals of the hand require 5 fingers, so the intervals of the five regular bodies require 6 orbs. For the Sun has no heaven of its own in that system, since it rests at the centre of the world.

[Margin: Harmonic intervals and their proportion with the periodic times.] The second proposition is that the intervals of the primary planets approach the proportions of the five regular bodies in such a way that they still differ from them somewhat, on account of the harmonic proportions which GOD, as Regulator [Harmoſtam] of this music of the world, was obliged, or it was fitting for him, to keep with the periodic motions of the same planets. Accordingly, the already mentioned authors say that the distances of the planets and of the

Earth from the Sun, the centre of the world, are in the sesquialteral ratio of their periodic times. So, for example, if the periodic time of Saturn is 30 years and that of Jupiter 12, and you take the cube roots of these and multiply them by themselves, then whatever the proportion between the square numbers, such will it be between the distance of Saturn and that of Jupiter from the Sun. They say this holds even if you do not compare neighbouring planets with each other. Thus, since the time of Saturn is 30 years, whose cube root is 3 11/100, while the periodic time of the Earth's annual motion is 1, whose cube root is 1, then multiplying the numbers 3 11/100 by themselves gives 9 672/1000, and 1 multiplied by 1 gives 1. Therefore the distance of Saturn from the Sun is to the distance of the Earth from the Sun as 9 672/1000 to 1, or as 9672 to 1000; and this ratio will come out the more exactly, the more precise and accurate the times assumed. By this method it was found that, relative to the distance of the Earth from the Sun, the distance of Saturn is nearly tenfold, Jupiter's fivefold, Mars's one-and-a-half-fold, Venus's in the ratio of 3 to 4, and Mercury's one third, approximately. [Margin: Proportion of motions and distances, from Wendelen.] Or, if you want finer numbers, and reduce the periodic times of the planets to small fractions of hours, and take the distance of the Earth from the Sun as 100000, the proportion of the intervals was found by Wendelen as you see in the present small table.

♄	953153
♃	519966
♂	152366
The Earth	100000
☿	72336
♀	38710

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The third proposition is Kepler's in the *Epitome of Copernican Astronomy*, p. 154, and that of Wendelen himself in the letter to me already mentioned. [Margin: Proportion between the motions and intervals of the satellites of Jupiter.] They observe that the distances of Jupiter's satellites from Jupiter are in the sesquialteral ratio of the periodic times in which they circle Jupiter. I gave a foretaste of this proportion in Section 1, Chapter 3, Scholium 3. On the proportion of the spheres and of the planetary bodies there will be discussion below in Chapter 8, from number 11, where certain matters relating to the intervals as well will be explained from Kepler's analogies.

CAPVT III.

Chapter 3.

[74] the signs of the last two rows stand as the leaf sets them; the text just above gives Venus about 3 to 4 and Mercury about one third of the Earth's distance.

De Planetarum Diſtantijs inter ſeſe, vel à Fixis ſtellis, Maximis ac Minimis.

On the greatest and least distances of the planets from one another, or from the fixed stars.

I. There is no less danger of error here than in the greatest and least distances from the Earth; indeed the danger is greater, unless certain rules are set out first for beginners, and on these we have built the double table that follows. So let this be

[Margin: 1. Rule, for planets and fixed stars.] First Rule. From the distance of the lowest fixed stars from the centre of the world subtract the planet's least distance from the centre of the world, and you will have the Greatest Distance of the planet from fixed stars placed on the same side. But if you subtract the planet's greatest distance from the distance of the fixed stars, you will have the Least Distance of the planet from the fixed stars. I said *lowest* because it is about these that the question is usually raised, whether Saturn's shadow reaches them, and whether the space between them and Saturn is sufficient for the ascent and descent of comets or new stars; I am not speaking of those supposed to be higher, in the depth of the Firmament.

[Margin: 2. Rule, for the greatest distance of ♄ ♃ ♂ from one another.] Second Rule. You will obtain the greatest distance of the three superior planets from one another as follows. In the hypothesis of Copernicus, Lansbergen, Kepler and Boulliau, add together the greatest distance from the Earth of the higher planet, the least distance from the Earth of the lower, and also twice the least eccentricity of the lower. In the hypothesis of Tycho and Longomontanus, on the other hand, join the greatest distances from the Earth of each planet, and take away the diameter of the Sun's annual orb and also twice the Sun's eccentricity. We are speaking here, moreover, of the greatest distance of the greatest, and of the least of the least. In the Ptolemaic hypothesis and in ours, however, join the greatest distances of the two planets from the Earth, but from the sum subtract the diameter of the epicycle of the lower planet, the greatest in ours, the ordinary one in Ptolemy's. But take care not to think that the greatest can arise from the sum of two greatest, say those of Saturn and Jupiter: they cannot each attain their own greatest from the centre of the Earth except in conjunction with the Sun, and then any two of the superior planets tend toward the same quarter and rather approach one another. One must therefore use, on one side, the greatest of the higher planet conjoined with the Sun, and on the other the distance of the lower, not indeed the absolutely greatest, but the greatest compatible with its opposition to the Sun.

[Margin: 3. Rule, for the least distance of ♄ ♃ ♂ from one another.] Third Rule. You will obtain the least distance of the three superior planets from one another as follows. To the least distance from the Earth of the higher planet add, in the hypothesis of Ptolemy and in ours, the diameter of the epicycle of that same higher planet, which for us is then at its least; in the

hypothesis of Copernicus, Lansbergen, Kepler and Boulliau, add the diameter of the annual orb; but in the hypothesis of Tycho and Longomontanus, add, besides the diameter of the annual orb, twice the Sun's eccentricity. Then from this aggregate subtract the greatest distance from the Earth of the lower planet in each hypothesis.

[Margin: 4. Rule, for the greatest distance of the superior planets from the inferior.] Fourth Rule. To the greatest distance from the Sun of any of the three superior planets add the greatest distance from the Sun of either of the two inferior planets, and the greatest distance between a superior and an inferior planet will be made up, in any hypothesis. Conversely, join the least distances from the Earth, of the superior planet on one side and of the inferior on the other, but in the hypothesis of Longomontanus add, further, to the aggregate of the already mentioned least planetary distances, twice the Sun's eccentricity, and the least distance between them will result. [Margin: For the least.] For neither the greatest nor the least distance of the inferior planet should be subtracted from the least of the superior: when the superior planets attain their least distance they are opposed to the Sun, and then Venus and Mercury are toward the opposite quarter, and so are more distant from any of the superior planets than when each of the planets attains its least distance (in such a case) from the Earth, but with the two opposed to each other.

[Margin: 5. Rule, for the greatest distance of ♀ and ☿ from one another.] Fourth Rule [recte: Fifth]. You will obtain the greatest distance between Venus and Mercury as follows. In the Ptolemaic hypothesis of al-Farghānī, and in those of Copernicus, Longomontanus and Boulliau, from the greatest distance of Venus from the Earth subtract the least distance of Mercury from the Earth; but in the hypothesis of Kepler, of Lansbergen and our own, from the greatest distance of Mercury from the Earth subtract the least of Venus. For we are speaking of the greatest possible distance, deduced from the measures of these hypotheses. [Margin: For the least.] The least distance between Venus and Mercury, on the other hand, you will obtain thus. In the Ptolemaic hypothesis of al-Farghānī, etc., subtract the greatest of Mercury from the least distance of Venus from the Earth; but in the Danish hypothesis of Longomontanus, the least of Venus from the least of Mercury; and in ours, subtract the greatest of Mercury from the greatest distance of Venus from the Earth. But in the hypothesis of Copernicus, Lansbergen, Kepler and Boulliau, add to the least distance of Venus from the Earth twice the greatest eccentricity of Venus, and subtract this aggregate from the least distance of Mercury from the Earth.

[Margin: 6. Rule, for the distance of ☾ from the others.] Sixth Rule. The greatest distance of the Moon from any planet whatever is made up of the greatest distance of the Moon on one side, the Moon however being opposed to the Sun, and the greatest distance of any planet from the Earth on the other. The least, however, is obtained by subtracting the greatest distance from the Earth of the Moon opposed to the Sun from the least distance of whichever superior planet the comparison is made with; but by subtracting the greatest of

the Moon conjoined with the Sun, if it is compared with one of the inferior planets.

[Margin: 7. Rule, for the distance of the planets from the ☉.] We have already given a Seventh Rule, for the distance from the Sun, adapted to each hypothesis, in Chapter 2.

There now follows the table of such distances, in which for the Ptolemaic ones we have used those of al-Farghānī, which Maurolico, Clavius and Barozzi generally chose. Where little crosses stand, it is a sign that the distances of the fixed stars from the Earth, or the least or greatest distances of Venus and Mercury, were not determined by the authors. Remember, moreover, that this table gives the extreme distances among those possible, if the world had lasted so long or were going to last so long; and that in looking them up, the planet that is higher is always to be sought in the former column, and in the latter the fixed stars or the lower planet which is being compared with the higher.

Table I. Greatest distance of the planets from one another, or from the fixed stars. In semidiameters of the Earth.

Comparison		Ptolemy, al-Farghānī	Copernicus	Lansbergen	Tycho	Longomontanus	Kepler	Boulliau	We
♄ with	✶	5842	✝	41945466	5160	✝	59973087	✝	152257
	♃	29990	17436	24190	15376	19991	52377	22733	122873
	♂	22295	13117	18224	13040	15260	39438	17165	95550
	☉	18951	11262	15792	11150	13071	33961	14735	83081
	♀	19992	12155	16955	✝	14023	36426	15800	88926
	☿	20096	11823	16561	✝	13641	35549	15426	86875
	☾	20039	12424	17294	12405	14465	26854	16285	90222
♃ with	✶	11366	✝	41951778	1143	✝	59986716	✝	183559
	♂	16592	8125	10957	6116	9110	23993	10428	52947
	☉	13248	6270	8525	4226	6921	18416	7998	40478
	♀	14289	7163	9688	✝	7873	20881	9063	46323
	☿	14393	6831	9294	✝	7491	20004	8689	44272
	☾	14336	7432	10027	5477	8315	21913	9548	47619
♂ with	✶	18861	✝	41957444	13590	✝	59998615	✝	207627
	☉	7727	1951	2559	1890	2190	5477	2430	13931
	♀	8768	2844	3722	✝	3142	7942	3495	19776
	☿	8872	2512	3328	✝	2760	7065	3121	17725
	☾	8815	3113	4061	3141	3584	8974	3980	21079
☉ with	✶	18867	✝	41956554	12880	✝	59996640	✝	202926
	♀	1041	893	1163	✝	952	2465	1065	5845
	☿	1145	561	769	✝	570	1588	681	3794
	☾	1273	1244	1614	1238	1392	3528	1548	7644
♀ with	✶	19913	✝	41957601	✝	✝	59999141	✝	207830
	☿	1052	1358	1805	✝	1659	4167	1798	8951
	☾	1083	2055	2665	✝	2346	5952	3615	12986
☿ with	✶	20044	✝	41957207	✝	✝	59998264	✝	205961
	☾	135	1723	2271	✝	1964	5085	2231	10935
☾ with	✶	20013	✝	41957949	13948	✝	59999946	✝	209949

Vſus Tabulæ præcedentis & ſequentis.

Use of the preceding and following table.

Suppose the first thing to be looked up is the greatest distance of Venus from Jupiter. If you look for ♀ in the first column, you will seek ♃ in the second column in vain, and so the greatest distance between these planets will seem to have been omitted. But it is not so: in order not to multiply the series of numbers needlessly, it seemed best to give these distances once only, in such a way that the higher star is first to be looked up in the former column and compared with the lower star, which is to be noted in the second column. Since ♃, then, is higher than Venus, look for ♃ in the first column and ♀ in the second: for opposite Venus ♀ you will find the greatest distance between these planets, according to the Ptolemaic hypothesis as found in al-Farghānī, to be 14289 terrestrial semidiameters. According to Copernicus, however, it is 7163; according to Lansbergen, 9688; according to Tycho, not indeed none, as if Tycho had thought there was none, but not yet determined by Tycho, who was forestalled by death. In its place there follows the distance according to the view of Longomontanus, 7873 semidiameters of the Earth. For Kepler, however, it is 20881; for Boulliau, 9063; and for us, 46323. In the same way the other greatest distances in the preceding table, or least distances in the following one, will be looked up.

Table II. Least distance of the planets from one another, or from the fixed stars. In semidiameters of the Earth.

Comparison		Ptolemy, al-Farghānī	Copernicus	Lansbergen	Tycho	Longomontanus	Kepler	Boulliau	We
♄ with	✶	148	☩	41940773	1660	☩	59962601	☩	119845
	♃	3741	3598	5571	5724	4717	11821	5077	24937
	♂	9262	7917	11537	8060	9448	24760	10645	51484
	☉	15500	9772	13968	9950	11637	30351	13127	64817
	♀	14546	8975	12933	☩	10685	27317	12010	59913
	☿	14442	9307	13355	☩	11022	27644	12394	61782
	☾	14411	8610	12467	8699	10243	26854	11577	57676
♃ with	✶	5851	☩	4194804	5160	☩	59978146	☩	162448
	♂	4457	3777	5225	1864	4101	11131	4776	18912
	☉	9975	5632	7657	3754	6290	16722	7258	33515
	♀	9022	4035	6621	☩	5338	13574	6141	28511
	☿	8918	5167	7043	☩	5675	13911	6525	30481
	☾	8887	4470	6155	2503	4896	13343	5708	26374
♂ with	✶	11229	☩	41954006	10920	☩	59991085	☩	108995
	☉	2337	1515	1991	1600	1744	4823	2018	9447
	♀	1384	718	955	☩	792	1675	901	4543
	☿	1280	1050	1377	☩	1129	2012	1285	6412
	☾	1249	353	489	349	350	1326	468	2306
☉ with	✶	18857	☩	41956449	12820	☩	59996531	☩	202420
	♀	6	749	992	☩	910	2431	1049	4651

Comparison		Ptolemy, al-Farghānī	Copernicus	Lansbergen	Tycho	Longomontanus	Kepler	Boulliau	We
	☿	963	249	359	✢	422	1038	445	1828
	☾	1057	1040	1382	1238	1184	3349	1372	7010
♀ with	✶	18962	✢	41955413	✢	✢	59994107	✢	196856
	☿	1	284	236	✢	337	843	368	2051
	☾	104	229	332	✢	230	800	305	2103
☿ with	✶	1994	✢	41955796	✢	✢	59994974	✢	198654
	☾	1	561	725	✢	567	1677	689	3972
☾ with	✶	20013	✢	41957933	13936	✢	59999941	✢	209933

Expoſitio Tabulæ III. Sequentis.

Explanation of Table III, which follows.

However, for recognising or rejecting eclipses of the planets by means of their shadows, besides their least distances from one another, for which Table II is to be consulted, certain other distances are useful. These are not, to be sure, least absolutely, but least nevertheless in the case in which an eclipse can be feared for one planet from the shadow of another, because the shadow of the planet casting its shadow toward the other also comes out both broader and longer, owing to the caster's greatest distance from the Sun. It therefore seemed best to append a third small table. In its first column is set the planet from whose shadow a shadow is feared, and in the second the planet for which it is feared from the other's shadow, with the conditions required for the position that comes closest to such eclipses. In the remaining columns is set the least distance of those two planets under such conditions. Where a little cross stands, it is a sign either that the comparison is impossible or inept in that hypothesis, or that those distances are lacking, as in Tycho's, who did not leave us the extreme distances of Venus and Mercury.

Table III. Least distance of the planets from one another, in the cases indicated in columns 1 and 2, for distinguishing eclipses from the shadow of the planets, etc. In semidiameters of the Earth.

Planet whose shadow is feared	Planet for which it is feared from the shadow (above the ♂ signs: in ☉)	al-Farghānī	Copernicus	Lansbergen	Tycho	Longomontanus	Kepler	Boulliau	We
☿ at perigee	☾ at perigee ♂	10	7	28	✢	573	1682	697	4024
♀ at perigee	☾ at perigee ♂	114	242	345	✢	236	805	312	1863
♀ at perigee	☿ at perigee ♂	103	✢	✢	✢	✢	✢	✢	✢
♀ at apogee	☿ at apogee ♂	948	✢	✢	✢	✢	✢	✢	✢
☿ at perigee	♀ at perigee ♂	✢	332	422	✢	337	877	384	2161
♀ [recte: ☿] at apogee	♀ at apogee ♂	✢	332	422	✢	382	867	384	2051
☿ at apogee	♂ at perigee ♂	✢	1050	1376	✢	1460	3235	1337	7505
♀ at apogee	♂ at perigee ♂	✢	718	955	✢	1001	2358	953	4974

Planet whose shadow is feared	Planet for which it is feared from the shadow (above the ♂ signs: in ☉)	al-Farghānī	Coper-nicus	Lans-ber-gen	Ty-cho	Lon-gomon-tanus	Ke-pler	Boul-liau	We
♂ at apogee ♂	♃ at the perigee of the eccentric ♂	4457	3777	2996	1864	4100	11245	4828	19905
♃ at apogee ♂	♄ at the perigee of the eccentric ♂	3135	3598	5569	5724	4717	11935	5129	26835

CAPVT IV.

Chapter 4.

De Cælorum, intra quos Planetæ vagantur, Altitudine, Craſſitie, & Ambitu; cum Milliaribus debitis Circumferentiæ toti, aut vni Gradui.

On the height, thickness and circuit of the heavens within which the planets wander, with the miles due to the whole circumference or to one degree.

I. The thickness of each heaven, that is, the space within which the planet's whole excursion with its orbs is contained, is easily found if the semidiameter of the concave surface is subtracted from the semidiameter of the convex surface: the remainder will be the thickness. But how large the semidiameter of each surface is cannot be found precisely from the greatest and least distance of the planet itself; different rules must be followed according to the difference of hypotheses. Always, however, the greatest of the greatest and the least of the least distances possible in each hypothesis are to be used.

[Margin: 1. Rule, for the Ptolemaic hypothesis and our own.] *First Rule.* In the hypothesis of Ptolemy, al-Battānī, al-Farghānī and our own, the semidiameter of the convexity of any heaven is made up of the planet's greatest distance from the centre of the Earth and the planet's true semidiameter. But in our hypothesis, for Jupiter, there must be added the semidiameter of the epicycle which the outermost satellite describes around Jupiter, increased by one unit, so that the body of the satellite itself may amply have its place in the heaven of Jupiter; of this semidiameter we shall speak below, in Chapter 12. The semidiameter of the concavity of the heaven, on the other hand, is found by subtracting the true semidiameter of the planet's body from its least distance from the Earth; and besides, in our hypothesis, for Jupiter, there must further be subtracted the semidiameter of the greatest epicycle, that of the outermost of the satellites, and one unit more. And where fractions of numbers occur here and in what follows, a whole unit can be put in their place; for it is not necessary that the planet's heaven be bounded exactly by the planet itself, but some little space above and below can be admitted. The true semidiameter, moreover, we shall take from Table 2 of Chapter 9 for the 5 planets, for we gave those of the ☉ and ☾ in Books 3 and 4.

[Margin: 2. Rule, for the Danish hypothesis.] *Second Rule.* In the hypothesis of Tycho and Longomontanus, the semidiameter of the convexity of the heaven of the three superior planets and of the Moon is made up of the planet's greatest distance from the centre of the Earth and the diameter of the lesser epicycle. The semidiameter of the concavity is found by taking the diameter of the lesser epicycle from the planet's least distance from the Earth. When these are observed, sufficient room is left for the epicycles of Jupiter's satellites, and indeed for the satellites themselves. But the semidiameter of the convexity of the heaven of the Sun, of Venus and of Mercury is made up of their greatest distance from the Earth and the true semidiameter of the planet's body; that semidiameter, subtracted from the least distance, will leave the semidiameter of the concavity.

[Margin: 3. Rule, for the hypothesis of the moving Earth.] *Third Rule.* In the hypothesis of the Earth going annually around the Sun at the centre of the world, with the Moon as its attendant, and using simple orbs without epicycles, of which kind is one of the three forms of Copernicus, and those of Lansbergen, Kepler and Boulliau, the distances from the centre of the Sun are to be used, and the heaven of the Moon is not distinguished from the heaven of the Earth; rather, the Moon's distance is to be reckoned with the Sun's distance from the Earth, since the Moon is a companion, as it were, of the Earth, just as the Medicean Stars are of Jupiter. Therefore the semidiameter of the convexity of the Earth-Moon heaven [cæli Terrilunaris] is made up of the Earth's greatest distance from the centre of the Sun, the greatest distance from the centre of the Earth of the Moon opposed to the Sun, and the true semidiameter of the Moon's body. The semidiameter of the concavity of this heaven, on the other hand, is found by subtracting from the Earth's least distance from the centre of the Sun the sum of the greatest distance from the Earth of the Moon joined to the Sun and the true semidiameter of the Moon's body. But for the remaining planetary heavens, add to the planet's greatest distance from the centre of the Sun the planet's true semidiameter, and besides, for Jupiter, the semidiameter of the greatest epicycle of the satellites, increased by one unit for the body of the satellite itself, and you will have the semidiameter of the convexity of the heaven. Subtract, on the other hand, from the planet's least distance from the centre of the Sun the true semidiameter of the planet's body, and for Jupiter further the semidiameter of the greatest epicycle of the satellites increased by one unit, and you will have the semidiameter of the concavity of the heaven. And yet these authors recognise a single planetary heaven, liquid and permeable, as do the Tychonians too. Indeed, among them neither the Sun nor Mercury nor Venus is in the heaven above, but below.

[Margin: Authors of the following tables.] II. With these things laid down, here, reader, are the tables which were compiled or handed down to us by Fernel in Book 1 of the *Cosmotheoria*, and following him Egnazio Danti, from Table 22 of the *Mathematical Tables*; Maurolico at the end of the *Cosmographia*; Clavius on Chapter 1 of the *Sphere*, p. 214 in my copy; and Barozzi in Book 1 of

the *Cosmographia*, p. 49. Although these follow al-Farghānī in the number of terrestrial semidiameters, nevertheless in the number of miles corresponding to one semidiameter of the Earth, Maurolico follows al-Farghānī, while Clavius and Barozzi follow Ptolemy, as does Baranzano in Part 1 of the *Uranoscopia*, although in the number of semidiameters he reports the opinion of Giuntini and Mauro Fiorentino derived from al-Battānī, and then that of Tycho. The remaining tables I constructed myself, and Table 4 in particular with the diameter of the convexity made to the circumference as 100 to 314, according to the teaching of Book 1, Chapter 4. Where the small spaces are empty, it is a sign that the measures from which the height of these heavens could be gathered were not handed down by the authors.

Table I. Semidiameter of the convexity and concavity of the heavens, with their thickness, expressed in semidiameters of the Earth. In the hypothesis of the moving Earth.

Authors	Semidiameters of the heaven	Earth with ☾	☿	♀	♂	♃	♄
Copernicus, by me	of convexity	1248	561	893	1951	6270	11262
	of concavity	1036	249	749	1515	5632	9772
	thickness	212	312	144	436	638	1490
Lansbergen, by me	of convexity	1627	770	1163	2560	8555	15796
	of concavity	1370	484	991	1990	7627	13964
	thickness	257	286	172	570	928	1832
Kepler, by me	of convexity	3544	1589	2466	5479	8060	33963
	of concavity	3257	1037	2430	4821	7196	30349
	thickness	287	552	36	658	864	4614
Boulliau, by me	of convexity	1541					
	of concavity	1368	✢	✢	✢	✢	✢
	thickness	173					

Remainder of Table I. Semidiameter of the convexity and concavity of the heavens, with their thickness, expressed in semidiameters of the Earth. In the hypothesis of the stationary Earth.

Authors	Semidiameters of the heaven	☾	☉	☿	♀	♂	♃	♄
Ptolemy, by Fernel	of convexity	64	1315	178	1199	9968	16190	22612
	of concavity	33	1199	64	178	1315	9968	16190
	thickness	31	116	114	1021	8653	6222	6422
Ptolemy, by Copernicus and Kepler	of convexity	64	1216	209	1408	9968	18257	25741
	of concavity	33	1120	65	209	1315	11244	18259
	thickness	31	96	144	1198	8653	7013	7482
al-Battānī, by Giuntini and Mauro Fiorentino	of convexity	64	1176	166	1079	8232	13171	17571

Authors	Semidiameters of the heaven	☾	☉	☿	♀	♂	♃	♄
	of concavity	33	1079	64	166	1176	8232	13171
	thickness	31	97	102	913	7056	4939	4400
al-Farghānī and Thābit, by Maurolico	of convexity	64	1215	168	1116	8849	14374	20077
	of concavity	33	1116	64	168	1215	8849	14374
	thickness	31	99	104	948	7634	5525	5703
al-Farghānī, by Clavius	of convexity	64	1216	168	1121	8854	14378	22612
	of concavity	33	1121	64	168	1216	8854	14378
	thickness	31	95	104	953	7638	5524	8234
al-Farghānī, by Barozzi	of convexity	64	1220	167	1120	8876	14405	20220
	of concavity	33	1120	64	167	1220	8876	14405
	thickness	31	100	103	953	7656	5529	5815
Tycho, by me	of convexity	63	1186					12940
	of concavity	52	1114	✢	✢	✢	✢	8160
	thickness	11	72					4780
Longomontanus, by me	of convexity	63	1340	1905	2287	3450	8435	14763
	of concavity	52	1236	626	289	336	4776	9945
	thickness	11	104	1279	199	3114	3659	4818
We	of convexity	67	7614	10869	12921	21006	47677	90176
	of concavity	51	7040	4077	1917	2372	26316	57722
	thickness	16	574	6792	11006	18634	21361	32454

Table II. Semidiameters of the convexity and concavity of the heavens, and of their thickness, in Italian miles, according to the opinion on the miles contained in a semidiameter of the Earth that is indicated in column 2.

Authors: of the semidiameters of the Earth	of the miles	Semidiameter	♄ of Saturn	♃ of Jupiter	♂ of Mars	☉ of the Sun	♀ of Venus	☿ of Mercury	☾ of the Moon
Ptolemy, by Fernel and Danti	Fernel	Convexity	881868006	3141953	38876865	1129529	4677983	699159	251390
		Concavity	3141953	38876864	5129529	4677983	696159	251440	129705
		Thickness	2504668	424265089	33746734	451906	3981824	447719	121685
al-Battānī, by Giuntini and Baranzano	Ptolemy	Convexity	6036535	24527487	22829722	4042960	3708954	570608	220566
		Concavity	4527487	22829722	4042960	3708954	570608	220566	113436
		Thickness	1509048	16977648	2425426	334006	2238346	350042	107130
al-Farghānī, by Clavius	Ptolemy	Convexity	8094247	31467897	31692400	4353025	4013923	600167	229687
		Concavity	5146789	73169240	04353025	4013923	600167	229687	120531
		Thickness	2947457	19775497	27339373	339102	3413756	370480	109056

Authors: of the semidiameters of the Earth	of the miles	Semidiameter	♄ of Saturn	♃ of Jupiter	♂ of Mars	☉ of the Sun	♀ of Venus	☿ of Mercury	☾ of the Moon
al-Farghānī, by Barozzi	Ptolemy	Convexity	72378409	31563352	31772045	4367045	4009091	597784	229687
		Concavity	51563352	31772045	43670454	4009091	597784	229687	119199
		Thickness	20815057	19791307	27405009	357954	3411307	368097	110488

Table III. Circuit or circumference of the great circle of the heavens, in Italian miles, or the miles due to one degree of it.

Authors		♄	♃	♂	☉	♀	☿	☾
Ptolemy, by Fernel	1°	1536600	1099800	678200	89700	81900	12133	4352
	Circuit	553811700	396524700	244136100	32209200	28667000	4360200	156700
al-Farghānī, by Maurolico and Clavius	Circuit	508781250	323612500	199209375	27361875	25230375	3772500	1443750
al-Farghānī, by Barozzi	Circuit	454950000	324112500	199710000	27450000	25200000	3757500	1443751
We	1°	6510647	3443648	1516943	550487	932654	783650	4828
	Circuit	2343936393	1239473218	546033436	197910424	335855016	28210182	1742519

Table IV. Circuit of the heavens in semidiameters of the Earth, or those due to one degree of the circumference.

Authors	Heaven	☾	☉	☿	♀	♂	♃	♄	✶ of the fixed stars
Ptolemy, by Fernel	1°	1 1/9	23	3 1/9	21	174	282	394	789
	Circuit	402	8258	1118	7530	62599	101673	142003	284013
Ptolemy, by Copernicus and Kepler	1°	1 1/9	21 2/9	4	24 25/36	157	318 1/2	449	☩
	Circuit	402	7636	1312	8842	62599	114654	161653	
al-Battānī, by Giuntini and Mauro	1°	1 1/9	20 1/2	3	19	143 1/3	229 3/4	306 1/2	331 1/3
	Circuit	402	7385	1042	6776	51697	82714	110346	119320
al-Farghānī, by Maurolico	1°	1 1/9	21	3	19 1/2	154 1/3	250 2/3	350 2/9	349 1/3
	Circuit	402	7530	1056	7008	55572	90269	126083	125756
al-Farghānī, by Clavius	1°	1 1/9	21	3	19 1/2	154 1/3	250 3/5	394 4/9	789
	Circuit	402	7636	1056	7041	55603	90294	142003	284013

Authors	Heaven	☾	☉	☿	♀	♂	♃	♄	✶ of the fixed stars
al-Farghānī, by Barozzi	1°	1 1/9	21 10/36	3	19 1/2	154 2/3	250 2/3	352 2/3	705 4/9
	Circuit	402	7662	1049	7034	55741	90463	126980	253963
Tycho, by me	1°	1 1/12	27	✢	✢	✢	✢	226	244 2/9
	Circuit	394	7448					81263	87920
Longomontanus	1°	1 1/12	23 7/18	33 8/36	40	60 1/6	158 1/4	258	✢
	Circuit	394	8415	11964	14362	21666	52972	92712	
We	1°	1 1/6	133	189 1/3	225 7/18	366 4/9	831 31/36	1573	36633 1/3
	Circuit	421	47816	68157	81144	131924	299462	566305	31 188 000

Authors	Heaven	Earth with ☾	☿	♀	♂	♃	♄	✶ fixed stars
Copernicus	1°	21 7/9	✢	✢	✢	✢	✢	✢
	Circuit	7837						
Lansbergen	1°	28 1/3	13 4/9	20 5/18	44 2/3	154	275 1/2	365967
	Circuit	10218	4836	7304	16077	55468	99193	131748 120
Kepler	1°	61 5/6	27 2/3	43	95 5/9	140 11/18	592 1/2	523333
	Circuit	22256	9979	15486	34408	50617	213319	188 400 000
Boulliau	1°	27	✢	✢	✢	✢	✢	✢
	Circuit	9676						

One semidiameter of the Earth contains, in Italian miles: according to Ptolemy, Clavius, Barozzi and Giuntini, 3580; according to al-Battānī, 4868; according to al-Farghānī and Maurolico, 3894; according to Copernicus, Tycho, Longomontanus and Kepler, 3440; according to Lansbergen, perhaps following Snel, 3919; according to Fernel, 3900; according to Boulliau, perhaps following Gassendi, 4177; and according to ourselves, 4139, in keeping with what was said in Book 2, Chapter 7. Choose, therefore, whichever opinion you like, and multiply the semidiameters of the Earth set out in the preceding small tables, and you will have the same measures in Italian miles.

SCHOLIA.

Scholia.

In Quibus præcipuè attinguntur digniſſima ſcitu pro Syſtemate Mundi.

In which the matters most worth knowing for the system of the world are chiefly touched on.

I. If anyone wants the semidiameters of the epicycles, or of the deferents, or of the equants, or even the eccentricities, in semidiameters of the Earth, he has them partly in Sections 2 and 3, in the tables at the end of each chapter, and partly in Chapter 2 of this section. If anyone wants the same reduced to miles, we have indeed judged that superfluous; he may nevertheless consult Fernel in Book 1 of the *Cosmotheoria*, and Egnazio Danti, who follows him, in his *Mathematical Tables* from Table 22. From these, however, it is our pleasure, in order to illustrate the Ptolemaic hypothesis that they follow, to add the Fernelian miles due to the semidiameter both of the deferent and of the epicycle of the planets.

Italian miles corresponding to the semidiameter of the Ptolemaic deferent and epicycle, according to Fernel and Egnazio Danti.

	Semidiameter of the deferent	Semidiameter of the epicycle
☾	190515	20150
☿	456430	171145
♀	2687071	1933199
☉	4903756	has no epicycle
♂	22003197	14485437
♃	51009409	9788504
♄	75665295	8197074

[Margin: On the height and circuit of the elementary sphere and of the eighth.] II. There will also be someone who might more reasonably want, at this place too, for conceiving the extreme measures of the world-system, the diameter and circuit of the elementary sphere, and the semidiameter of the convexity of the firmament, and the circuit of each. But he has the semidiameter of the convexity of the elementary sphere in Tables 1 and 2 of the preceding chapter, for it is the same as the semidiameter of the concavity of the heaven of the Moon. So if that is doubled, the diameter will be had; and if one makes, as 7 to 22, or rather as 100 to 314, so the already mentioned diameter to another quantity, the circumference or circuit of the elementary sphere will be known. For the sphere of the firmament, or the eighth, which is that of the fixed stars, either the table we gave in Book 6, Chapter 7, no. 14 must be consulted, or the greatest distance of the fixed stars must be doubled to obtain the diameter, and one must make, as 100 to 314, so that diameter to the circumference of the fixed stars.

[Margin: The imperfection of the Ptolemaic heavens.] III. In the Ptolemaic intervals of the heavens, this above all is to be noted: there is a danger that a superior planet at perigee may touch or even strike an inferior one at apogee,

especially Mercury against the Moon; further, that the heaven of Jupiter lacks a space able to hold the satellites of Jupiter; and that Venus and Mercury are always below the Sun, while Mars is always farther from the Earth than the Sun. [Margin: Mercury, Venus and Mars pass through the heaven of the Sun.] These things are contrary to the evident phenomena of recent astronomy. But some natural philosophers or theologians, who have never put hand or eye to celestial observations, fearing from this for the solidity of the heavens or for the older system, peddle the claim that the discoveries and observations of recent astronomers are uncertain. Let them therefore go back over what I said in Section 2, Chapter 3, Scholium 4, and what is to be said in Chapter 7, no. 3, on the parallaxes of Mars. [Margin: Arguments for which ♂ in ☍ with ☉ becomes nearer the Earth than the Sun.] For the prosthaphaereses, or equations of the motions, that are necessary if the mean motion of Mars is to be brought to the apparent motion observed in the acronychal oppositions with the Sun, and to agree with it, establish that a parallax somewhat greater than the parallax of the Sun is required. Those observations, carefully taken with Tycho's instruments, cannot be brought to rule unless the radius of the eccentric of Mars, that is, its mean distance from the Sun, is one and a half times [sesquipla] the radius of the annual orb, that is, the mean distance of the Sun from the Earth, and unless the radius of the epicycle of Mars is equal to, or very slightly less than, the radius of the annual orb. From this it follows by geometric necessity that Mars in opposition with the Sun, when it is found at the perigee of the epicycle, especially if it is then also at the perigee of the eccentric, becomes much nearer to the Earth than the Sun itself at perigee. And this very thing is confirmed by the unusual increase of the apparent diameter in that position, on which see what is to be said in Chapter 10, Scholium 1. Thus Kepler, a most diligent examiner of Tycho's observations of acronychal Mars, was at last compelled to acknowledge that the parallax of Mars was then somewhat greater than the parallax of the Sun. [Margin: Testimonies of astronomers for the nearness of Mars.] So long, then, as the opposite is not established from other, more certain observations, we must stand in this with Tycho and Kepler, and indeed with Copernicus, from whose intervals the same thing follows, and with Lansbergen and Boulliau and Wendelen and Galileo, that is, with the most illustrious lights of astronomy. I add also Giovanni Antonio Magini. For although in a letter to Tycho dated 1590 September 13 [Idibus Septembris], which is found towards the end of Tycho's *Mechanica Astronomica*, he had said: "I cannot but greatly approve the system of the universe you have devised, although I should wish that the orb of the Sun and that of Mars did not intersect at all", he nevertheless at once added: "But if, as I have learned from that student, you have observed that acronychal Mars comes nearer to the Earth than the Sun does, an intersection of this kind must certainly be admitted." To this doubt and to the whole letter Tycho replied in a most learned letter on 1590 December 1 [Calendis Decembribus], which I have in Magini's *Tables of Directions of the Primum Mobile*, p. 80, affirming among other things the following: "That the

orb of Mars does not wholly encircle the Sun's, but crosses it in two places, affects you somewhat, and perhaps others also, and keeps you in two minds; with me no scruple at all remains on this. [Margin: Fragments of letters of Magini and Tycho on the approach of Mars to the Earth.] For what has been believed until now, with Aristotle's authority thus persuading in vain, that the heaven consists of an impenetrable and adamantine material and is packed with real and solid orbs that turn the stars fixed in them, has no foundation and is utterly foreign to the truth of the matter, as is sufficiently clear from the comets themselves at least, if not from elsewhere too." And shortly after: "Indeed, the point of which our often-mentioned Gellius [Gellius Sascerides] reminded you, that Mars in its acronychal position is brought nearer to the Earth than the Sun itself, has been continually investigated by us with close inquiry and found to be so, especially about the end of 1582, when, high in ♋, with its north latitude too raising it further, it was in opposition to the Sun. For I discovered at that time, from many accurate observations made in various positions and carefully compared with one another, that it admitted a parallax somewhat greater than the Sun's, which could by no means happen if its whole revolution were completed in orbs standing above the sphere of the Sun, as the Ptolemaic arrangement holds. I also often attended to its retrograde passages when it was acronychal, and after an interval of some days found them to become somewhat swifter than the Ptolemaic reckoning allows; the Copernican calculation agrees better here, especially if the application is made of the truer apogee and of the opposition with the Sun's mean motion. And from this it was plain, not obscurely and with sufficient certainty, that Mars, when passing through the part of the heaven turned away from the Sun, is brought nearer to the Earth than the Sun itself." In these things, and in Kepler's commentaries on the motion of Mars, Magini so far acquiesced in the *Supplement to the Ephemerides* that on these foundations he composed new tables for finding the place of Mars. Nor is it an obstacle that Kepler examined the opposition of Mars with reference to the true rather than the mean place of the Sun: for through these too he found a parallax of Mars that was indeed smaller than Tycho's, but still greater than the Sun's.

[Margin: Authors opposed to Tycho on this.] IV. This being so, I could indeed wish that certain members of our Society, too much attached to the solidity of the heavens, had not taken a stand against this Tychonic doctrine with arguments that are truly weak. Among them Charles Malapert, in his short work on the *Austrian Stars*, said that Tycho seemed to have done good work, if he had not made his violent Mars, breaking through the orbit of the Sun against all right and law. So too Adam Tanner, in the *Dissertation on the Heavens*, q. 7, paragraph 6, and, subscribing to him, Bartholomæus Amicus [Bartolomeo Amico], Treatise 5, *De caelo*, q. 5, art. 5. Their reasons reduce to this single one: that Tycho at times erred, either in observing or in the conclusions to be drawn from observation, or was not so exact, as he himself admits somewhere. But this is to abuse his admission. For if he is to be abandoned on that account even

when he affirms, as happens in our case, that he worked most diligently and most subtly, neither he nor any of the astronomers would have to be believed in anything; which not even Tanner would dare to assert. Tanner nevertheless indicates that, by multiplying epicycles in a single heaven common to the Sun, Mercury, Venus and Mars, these new phenomena can be defended with the solidity of the heavens intact; of this we shall have to speak in Book 9. Further, the fact that Rothmann long disagreed with Tycho, and wrote in a letter to Tycho, which stands among Tycho's letters on p. 129, that the Tychonic system introduces confusion into the spheres of the planets, and that one planet enters the heaven of another, is not of such weight that it can prevail with us against the already mentioned observations. Lastly, Mercury and Venus must necessarily pass through various places of the heaven of the Sun, through which the Sun itself at times travels. If, therefore, it is not absurd for Mercury and Venus to intersect parts of the Sun's heaven, it ought not to seem absurd either that Mars in its body should pass through a place through which the Sun at times passes, and be, as it were, a satellite of the Sun; and so that all these three planets, Mars, Venus and Mercury, belong with the Sun to a single heaven.

[Margin: Our new system of the world.] V. Hence, then, there arose for me the occasion of confirming my old opinion on the new semi-Tychonic system, which I indicated with its own scheme in Book 3, Chapter 6, no. 5, and shall explain more fully in Book 9. For in the Ptolemaic system there are those imperfections and absurdities that I indicated at the beginning of Scholium 3. In the Tychonic system, that confusion of the heavens seems to offend many. In the Copernican, the Sun has no heaven but rests at the centre of the world, and with respect to it the Earth, with the Moon as companion, is not down but up in the heaven, having below and beneath itself the heaven of Venus and Mercury, and the Sun; and above itself Saturn, Jupiter, and Mars, which in its acronychal shinings is nearer to it than the Sun: a system that we cannot uphold, for many reasons. So, in order that all things may be reconciled, and that we may lay down a system of the world complete in every respect and without any absurdity, we say that seven huge and major bodies of it were divinely founded. The first is the Earth at the centre with the elementary sphere, and the Moon, most closely joined to it by kinship of nature and, as it were, the immediate vehicle of the influences that it receives from the superior bodies and pours back into the elementary region. The second is the heaven assigned to the Sun and its companions, namely Mercury, Venus and Mars; and Mars, as the outermost of the Sun's satellites, shares in the properties of Venus and Mercury on one side and of Saturn and Jupiter on the other, and goes round the Sun in such a way as to embrace the Earth too within its circuit. The third is the heaven allotted to Jupiter and its satellites. The fourth is the heaven serving Saturn and its circumambulators. The fifth is the heaven set apart for the new stars and for very many comets going up and down, as intermediate between the wandering and the unwandering stars. The sixth is the huge sphere of the fixed stars. The seventh, finally, is the Empyrean, the seat of the blessed

minds. Hence it has come about that no satellites have been assigned to Mars, or Venus, or Mercury, since these are not primary lights of the world, nor were they to exercise their own monarchy in a heaven of their own; whereas the Sun, as by far the greatest of all the planets and the source of planetary light, was given larger satellites of wider circuit; and that the heaven of Saturn and of Jupiter is to be ordered not around the Sun, as Tycho does, but around the Earth, as you see in our scheme. Granted that all the planets depend both on the Sun, as the efficient cause of their light and the ideal cause of one of their two motions, and on the Earth as their final cause, the orbs of all the planets had to be eccentric to both. But more of this, God willing, in Book 9.

[Margin: A correction to be applied to what was said in Book 3, Chapter 6, no. 5.] VI. I said in Book 3, Chapter 6, no. 5, that if we saw Jupiter from as great a distance as that from which we see the Sun, Jupiter would appear to us a little smaller than the Sun now appears. For at that time, when I was teaching philosophy at Parma, I followed the Tychonic distances with Biancani; so that if Jupiter were seen from 1150 terrestrial semidiameters, which is the Tychonic mean distance of the Sun, and has 4 semidiameters of the Earth in its own semidiameter, as Kepler holds, Jupiter would appear to us under an angle of about 26′, while the Sun appears under an angle of about 30′. Afterwards, however, with the distances and the true semidiameters of the planets corrected, one will have to speak differently, in keeping with what is to be said in Chapter 13, Table 5.

CAPVT V.

Chapter 5.

De Tempore, quo Mobile vniformi aut difformi motu deſcenderet ex ſupremo cuiuſque Planetæ Cælo, aut ex Fixis.

On the time in which a moving body, in uniform or difform motion, would descend from the highest heaven of each planet, or from the fixed stars.

I. I have elsewhere read in treatises on the heaven, and have heard from some people, that in order to explain the almost immeasurable height of the heavens they usually employ the time in which some moving body, travelling through the heaven, would traverse the whole height of the heaven, if nothing obstructed its descent and it were preserved in its integrity from the beginning to the end of the motion. Some, indeed, apply uniform motion to this computation, others uniformly difform motion, and in either case they require a great number of years. A specimen of this in uniform motion comes from Simon Maiolus [Simone Maioli] in the *Dies Caniculares*, Colloquy 1, p. 63 in my copy, where he imagines a man riding at top speed and covering 60 Italian miles every day. On this assumption he assigns to each heaven the years and days required to traverse its height down to the Earth, as can be seen in the following small table, which agrees with the Ptolemaic hypothesis.

If a uniform moving body covers 60 Italian miles daily, it will reach the Earth from the top of the heaven in years and days:

	Years	Days		Years	Days
☾	5	2	♂	184	5
☿	10	0	♃	1291	2
♀	26	0	♄	2065	11
☉	160	9	✶	2754	6

II. But someone might want a specimen of this height for some much swifter moving body, for instance couriers riding the fastest horses, or ships driven by the strongest wind, or birds, or even lightning, and so the question would become endless. Yet so that we may not seem to have done nothing here for the curious, let us imagine some moving body covering 1000 Italian miles every day, which is about as many as there are in the long stretch of Italy from the Alps to the far end of Sicily, and let us use the height of the heavens from our own hypothesis. On these assumptions, here is a second small table.

If a uniform moving body covers 1000 Italian miles daily, it will reach the Earth from the top of the heaven in years and days:

	Years	Days		Years	Days
☾	0	265	♂	238	10
☉	87	231	♃	539	130
☿	128	141	♄	1050	0
♀	149	18	✶	23819	70

De Tempore, quo Graue aliquod deſcenderet à maxima Lunæ, Solis, Saturni, ac Fixarum altitudine ad Terram.

On the time in which some heavy body would descend from the greatest height of the Moon, the Sun, Saturn and the fixed stars to the Earth.

II. In this controversy too, extreme opinions, but born of inexperience, are read or heard again and again. Some, to exalt the height of the heavens, boast that a millstone would not descend from the eighth sphere all the way to the Earth even after many thousands of years. Others, by contrast, could not bear to wait without anxiety even for the space of a single hour for such a stone, let fall perpendicularly from there onto their own heads, without dreading the fate of Aeschylus; for they report that the bald head of Aeschylus looked to an eagle like a rock, onto which it dropped a tortoise, to break it and devour it. [Margin: Galileo's opinion on the descent of heavy bodies from the Moon to the Earth.] Nearer to the truth came both that author who, in Galileo's Dialogue 2 *De Mundi systemate*, said that a cannonball would not reach the Earth from the Moon except in 6^h, and Galileo himself, who says that an iron ball of 100 pounds would fall from the Moon to the Earth within 4^h. Read again

what I taught more fully on this matter in the whole of Book 2, Chapter 21. From it I assume, first, from the most certain experiments of ours and of Father Francesco Maria Grimaldi cited there, that the spaces which a heavy moving body traverses in natural descent are to one another as the squares of the times, which Galileo above and Giovanni Battista Baliani, in his short work on the natural motion of heavy bodies, also asserted. Second, from many experiments of ours reported there, a ball of chalk of 8 ounces, let alone one of iron of one pound (for we used very many chalk balls of equal weight, so that by often repeated drops from many towers the proportion of the increase of velocity, and the time required to traverse a determinate space, might be more evident), I assume, I say, that a ball of this kind, in 2^s of time, measured with a most exact pendulum applied for the purpose, traversed 60 Roman feet, and in 4^s, 240 feet. If, therefore, one makes, as the square of the time of 4^s, which is 16, to the square of a given greater time, so the space of 240 feet to another space, one will have the space that a ball of this kind can traverse in the given time, if it were not worn away, or dispersed by the heat it had taken on; but if it descended naturally, under the impulse of its own weight, through a medium of equal density and fluidity such as the air around the Earth. In the space of one hour, then, it would traverse 38880000 Roman paces of 5 feet each, that is, 7 39/50 terrestrial semidiameters, and in the space of 2^h, 31 terrestrial semidiameters. If therefore one makes, as the space of 31 terrestrial semidiameters to another given space, say to the distance of the Moon, the Sun, Saturn or the fixed stars, so the square number of 2^h, which is 4, to another number, one will have a square number whose square root will be the time in which a chalk ball of 8 ounces, and much more an iron ball of one pound, would pass through the given distance in its descent. Take, for example, the Sun's greatest distance as 7580 terrestrial semidiameters, as I judge it to be. If one makes, as 31 to 7580, so 4 to another number, the number 978 2/31 will result, whose nearest square root, choosing the nearer round number, is 31. So in 31^h, that is, in $1^d\ 7^h$, that ball will pass through the interval of 7580 or 7600 terrestrial semidiameters.

By this method, then, as a courtesy to our readers, I have decided to set out in the following table the time in which an iron ball of this kind would most certainly traverse the intervals of the Moon, the Sun, Saturn and the fixed stars. I have, however, taken round numbers for the intervals and added the authors who assert that interval or one very close to it, so that in a problem of such moment we may not bind our readers to our own opinion on these intervals. Since, however, the substance of the ether and of the most liquid heaven is much thinner than the thick air around the Earth, there is no doubt that that iron ball, unless it melted and were dispersed by fire caught from the friction, would fall to the Earth much faster still. Further, in reducing semidiameters of the Earth to German miles, each of which contains four Italian miles, we shall use the round number 1000, which is very nearly how many are contained in one semidiameter of the Earth. For there is no need to proceed more scrupulously in this place: to estimate the time already mentioned it is enough to have noted

the intervals close to the truth, and likewise the days and hours, with no hunting after the minutest fractions.

The approximately true time for an iron ball of one pound to descend by natural descent to the Earth from the Moon, or the Sun, or Saturn, or the sphere of the fixed stars.

Distance of the star	Authors or foundations or calculations of the distances	Distance of the stars, in the nearer round numbers of the distance from the Earth asserted by the authors listed below: semidiameters of the Earth	The same: German miles	Time of descent: days and hours
☾ Of the Moon	Most astronomers	64	64000	0^{d} 2 7/9^{h}
	Hipparchus, formerly	72	72000	0^{d} 3^{h}
☉ Of the Sun	Ptolemy, al-Battānī, Copernicus, Tycho	1200	1200000	0^{d} 12^{h}
	Longomontanus	1300	1300000	0^{d} 13^{h}
	Lansbergen and Boulliau	1500	1500000	0^{d} 14½^{h}
	Kircher and Antonín Maria Šírek	2000	2000000	0^{d} 16^{h}
	We	7580	7580000	1^{d} 7^{h}
	Posidonius	13000	13000000	1^{d} 17^{h}
	Govaert Wendelen	15000	15000000	1^{d} 20^{h}
♄ Of Saturn	Tycho and Copernicus	12000	12000000	1^{d} 15½^{h}
	Longomontanus	14000	14000000	1^{d} 18½^{h}
	Boulliau	16000	16000000	1^{d} 21½^{h}
	Lansbergen	17000	17000000	1^{d} 23^{h}
	al-Battānī	18000	18000000	2^{d} 0¼^{h}
	al-Farghānī, Francesco Maurolico	20000	20000000	2^{d} 2⅘^{h}
	Clavius, Barozzi	20000	20000000	2^{d} 2⅘^{h}
	Ptolemy, Jean Fernel, Egnazio Danti	23000	23000000	2^{d} 3½^{h}
	Ptolemy, for Copernicus and Kepler	26000	26000000	2^{d} 10^{h}
	Kepler himself	37000	37000000	2^{d} 21^{h}
	We	90000	90000000	4^{d} 12^{h}
✶ Of the fixed stars	Tycho	14000	14000000	1^{d} 18½^{h}
	al-Battānī	19000	19000000	2^{d} 1½^{h}
	Francesco Maurolico	20000	20000000	2^{d} 2⅘^{h}

Distance of the star	Authors or foundations or calculations of the distances	Distance of the stars, in the nearer round numbers of the distance from the Earth asserted by the authors listed below: semidiameters of the Earth	The same: German miles	Time of descent: days and hours
	al-Farghānī, Magini	40000	40000000	3d 0h
	Jean Fernel, Clavius	45000	45000000	3d 4h
	We, for the present	100000	100000000	4d 18h
	or	125000	125000000	5d 8h
✶ Of the fixed stars in the Copernican hypothesis, from calculation	Of Tycho and Longomontanus	7900000	790000000	22d 1h
	Of Galileo	12000000	1200000000	31d 20h
	Of Scheiner	13000000	1300000000	39d 23h
	Of Kepler	34000000	3400000000	87d 6h
	Of Antonín Maria Šírek	42000000	4200000000	97d 0h
	Our own	47000000	4700000000	122d 18h
	Of Martinus Hortensius	68000000	6800000000	177d 23h
	Of Lansbergen	14000000000	1400000000000	17666d 16h

Note that the time recorded here is not entirely required for the descent of an iron ball of one pound. For it would suffice for a chalk ball of 8 ounces, and so much more for an iron ball of 8 ounces; and therefore much more still it suffices for an iron ball of one pound, which we chose for good measure.

CAPVT VI.

Chapter 6.

De Velocitate Quinque minorum Planetarum.

On the speed of the five minor planets.

I. We have, if I am not mistaken, set out well enough the speed of the Sun's motion in Book 3, Chapter 12, that of the Moon in Book 4, Chapter 18, and finally that of the fixed stars in Book 6, Chapter 7, number 18. It now remains to examine the speed of the five minor planets. I do not mean their speed in degrees and minutes, which each of them runs through daily in the zodiac by its own proper motion; of this enough has been said in Section 5, Chapter 1, and it is not an affection following from height or distance from the Earth. I mean their speed as regards the part of a determinate space, say the leagues or miles that a planet traverses in a given time, or as regards the time that would be required for some moving body of known speed to cover as much space as any of these planets covers in so much time. Very few treat of this speed;

yet it is set before us by Fernel in Book 1 of the *Cosmotheoria*, by Egnazio Danti, following him, from Mathematical Table 22, by Kepler in the book *De Stella Nova*, Chapter 16, and by Pierre Hérigone in volume 5 of the *Cursus Mathematicus*, pp. 581 and 583. These authors, however, consider the speed of these planets only in their proper motion from west towards east, although the speed of the diurnal motion, by which they complete the whole circumference of their heaven in 24^h, is far greater. But Kepler and Hérigone abstained from this speed, because in their hypothesis there is no motion of the primum mobile in these planets; rather, its appearance arises from the libration and rotation of the Earth about its own centre and axis. Lansbergen, for his part, grants the fixed stars a proper motion but denies them the common motion of the primum mobile.

Further, the speed of either motion depends on the arc of a great circle that the planets describe as they move, either with the topmost point of their body or with its centre. The determinate size of this arc depends on the varying circuit, that is, on the varying size of the circumference, which takes its variety from the differing distance of the planet at apogee. On the size of the whole circuit we have already set out Table 4 above, in Chapter 3. It serves for the highest point of the planet's body, or, in the hypothesis of Tycho and Longomontanus, for the apogee of the smaller epicycle, where we can imagine the planet to be without any great difference as regards the business at hand, just as, in the case of Jupiter, we can assign to Jupiter the path of its outermost satellite as regards the motion of the primum mobile. On these suppositions we shall construct a table for the diurnal motion of the primum mobile in the hypothesis of a resting Earth. As a more common and more convenient measure we shall use semidiameters of the Earth, which the planets or the highest points of the heavens traverse in one hour, or in one minute or second of time, rather than miles, paces or feet, which vary too much among different authors or peoples. If anyone nevertheless wants Italian miles, let him use the rule given at the end of the last table of Chapter 4. For the speed of the planets' proper motion towards the east, we shall give a few things in the following Scholia.

The speed of the minor planets in the motion of the primum mobile, expressed in semidiameters of the Earth, those which their highest points, at both apogees, cover in one hour, or in one minute, or in one second of time.

From the data of the authors	Parts of time	☿ (semid. of the Earth)	♀ (semid. of the Earth)	♂ (semid. of the Earth)	♃ with its companions (semid. of the Earth)	♄ with its companions (semid. of the Earth)
Of Ptolemy, for Jean Fernel	1^h	46 7/12	312	2608	4236	5917
	1^m	0 47/60	5 1/5	43 2/3	70 3/5	98 3/5
	1^s	0	0	0 44/60	1 11/60	1 39/60
Of Ptolemy, for Copernicus and Kepler	1^h	54 2/3	368 1/6	2608 1/3	4777 1/4	6735 1/2

From the data of the authors	Parts of time	☿ (semid. of the Earth)	♀ (semid. of the Earth)	♂ (semid. of the Earth)	♃ with its companions (semid. of the Earth)	♄ with its companions (semid. of the Earth)
	1^m	0 55/60	6 1/8	43 1/2	79 3/5	112 1/4
	1^s	0	0	0 43/60	1 19/60	1 52/60
Of al-Battānī, for Francesco Giuntini and Mauro [?]	1^h	43 5/12	282 1/3	2154	3446 5/12	4597 3/4
	1^m	0 43/60	4 2/3	35 54/60	57 1/3	76 2/3
	1^s	0	0 5/60	0 3/5	0 57/60	1 17/60
Of al-Farghānī, for Francesco Maurolico	1^h	44	292	2315 1/2	3761 5/24	5253 1/2
	1^m	0 44/60	4 52/60	38 3/5	62 2/3	87 1/2
	1^s	0	0 5/60	0 39/60	1 3/60	0 27/60
Of al-Farghānī, for Clavius	1^h	44	293 3/8	2316 19/24	3762 1/4	5916 19/24
	1^m	0 44/60	4 53/60	38 1/2	62 2/3	98 3/5
	1^s	0	0 5/60	0 38/60	1 3/60	1 39/60
Of al-Farghānī, for Barozzi	1^h	43 17/24	293 1/12	2322 1/2	3769 7/24	5290 5/6
	1^m	0 44/60	4 53/60	38 2/3	62 49/60	88 1/6
	1^s	0	0 5/60	0 39/60	1 3/60	1 28/60
Of Longomontanus	1^h	494 1/3	598 5/12	902 3/4	2207 1/6	3863
	1^m	6 1/4	9 58/60	15	36 47/60	64 23/60
	1^s	0 1/10	0 1/6	0 1/4	0 37/60	1 4/60
For us	1^h	2839 7/8	3381 1/4	5496 5/6	12477 7/12	23595 5/24
	1^m	47 1/3	56 1/3	91 3/5	207 57/60	393 1/4
	1^s	0 47/60	0 56/60	1 32/60	3 1/2	6 33/60

For the speed of the Sun see Book 3, Chapter 12; for that of the Moon, Book 4, Chapter 18; for that of the fixed stars, Book 6, Chapter 7, number 18.

SCHOLIA.

Scholia.

I. The speed set out in the tables above is that of the motion of the planet itself when placed at the apogee of its eccentric and, at the same time [?], of its epicycle, or of a circle or ellipse equivalent to them. But Fernel, in Book 1 of the *Cosmotheoria*, and Egnazio Danti, from Mathematical Table 22, have set it out for us both as regards the motion of the centre of the epicycle towards the east and as regards the motion of the planet in the epicycle, taken separately, as can be seen in the following small table.

[Margin: The speed of the motion of the centre of the epicycle, and of the planet in the epicycle.]

The Ptolemaic speed of motion of the 6 minor planets, from Jean Fernel and Egnazio Danti, in 1 hour.

Motion	Of the centre of the epicycle: Italian miles	Of the planet in the epicycle: Italian miles
♄	1826 and 910 paces	5678 and 124 paces
♃	3084 and 704	6425 and 747
♂	8388 and 283	4863 and 860
♀	1926 and 772	867 and 75
☿	327 and 386	9283 and 816
☾	1834 and 609	191 and 525

[Margin: The speed of the planets' proper motion, from Hérigone.]

II. Pierre Hérigone, in volume 5 of the *Cursus Mathematicus*, p. 582, investigates the speed of the planets' proper motion by this method. First he supposes that one degree of a great circle of the Earth contains 60 Italian miles, and the whole circumference 21600 miles. The 24th part of these, which belongs to one hour, is 900, and this divided by 60 gives 15 miles, which a point of the Earth placed on the equator traverses in one minute of time, while the Earth, as he himself posits, turns about its own centre daily.

[Margin: The speed of the Earth's motion according to Hérigone.]

Since, however, the ratio of the Earth's semidiameter to the mean distance of the Earth's centre from the Sun is, for him, as 1 to 1142, he says that the motion of the Earth about its own centre is to the motion of the Earth's centre about the Sun as 365 1/4 to 1142, or as 15 to 47; and that therefore the motion by which the centre of the Earth advances eastward in one minute of time is 47 Italian miles. From this it follows that the motion, in a minute of time, of the Earth's surface about the Earth's centre is exceeded by the motion of the centre; and that the part of the surface illuminated by the noon Sun covers 32 Italian miles, 15 being subtracted from 47 because of the motion of rotation in the opposite direction, while the part opposite to it covers 62 Italian miles, 47 and 15 being added because that motion seconds the other. From this difference of motions Galileo judged that the flux and reflux of the sea arises, like water shaken to and fro in a vessel.

Secondly he says that, because the Earth's semidiameter is to the mean distance of the Moon as 1 to 56, and the Moon completes its period around the Earth in about 28 days, the ratio of the motion of the Earth about its own centre to the Moon's motion is therefore as 28 to 56, that is, as 1 to 2, or as 15 to 30; and so the Moon advances in the order of the signs around the Earth 30 Italian miles in one minute of an hour, and 1800 in one hour, as regards the motion of the centre of the epicycle. Hence, because this motion is exceeded by the motion of the Earth's centre, the Moon too, with respect to one looking from the Sun, proceeds according to the succession of the signs. But at full Moons, he says, it covers 77 Italian miles in one minute of time, and at new Moons only 17. These agree with Kepler, according to what we said in Book 3, Chapter 12, in the Scholium on the speed of the Earth, and in Book 4, Chapter 18, in the Scholia on the speed of the Moon. I have decided to repeat them here, since by the same method, from the intervals of the planets and their periodic

motions around the Sun, he gathers the Italian miles that each planet traverses in its orb around the Sun in the mean motion of one minute of time, as is clear from the following list. In it the first column of numbers contains the ratio of the paths covered in the same time, so that if the Earth covers 60 spaces in one hour, Saturn covers 19 200/10000 such spaces, Jupiter 25 9825/1000 [recte: 9825/10000, as in the table that follows], and so on; the second contains the Italian miles covered in one minute of time.

The speed of the planets' proper motion according to Pierre Hérigone.

	Ratio of the motions	Italian miles in one minute of time: min.	Med.	Max.
♄	19 200/10000		15	
♃	25 9825/10000		20	
♂	45 7050/10000		36	
The Earth	60 0	32	47	62
♀	65 1600/10000		51	
☿	93 1344/10000		72	
☾	38 2978/10000	17 ♂	30	☍ 77

III. But Kepler, in the book *De Stella Nova*, Chapter 16, first gives the speed of the hourly proper motion of the planets in German miles, but according to the intervals and periodic motions of Copernicus; then he gives the speed of the motion of the Earth around the Sun and of the Moon around the Earth. Although I indicated this in Book 4, Chapter 18, it is worth while here to give it in a single view with the speed of the other planets, in the following small table, with our addition of the motion they complete in one minute of an hour, in both German and Italian miles.

[Margin: The speed of the planets' proper motion according to Kepler.]

The speed of the proper motion of the planets around the Sun, or of the Moon around the Earth, in the Copernican hypothesis, from Kepler.

	1 hour: German miles	1 hour: Italian miles	1 minute: German miles	1 minute: Italian miles
♄	300	1200	5	20
♃	400	1600	6 2/3	27
♂	600	2400	10	40
The Earth	740	2960	12 1/3	49
♀	800	3200	13 1/3	53
☿	1000	4000	16 2/3	67
☾ in ♂	264	956	4 2/5	18
in □	476	1904	7 11/12	32
in ☍	1216	4864	20 1/4	81

CAPVT VII.

Chapter 7.

De Parallaxi Quinque Minorum Planetarum.

On the parallax of the five minor planets.

I. The skill of the astronomers could not investigate the heights of the planets by any more certain method than by means of the lunar parallaxes. For once these had first been found in the way I taught in Book 4, Chapters 13 and 14, the distances of the Moon from the centre of the Earth were sought; from there a step was made to all but demonstrating the distances of the Sun, in the manner I taught at length in Book 3, Chapter 7; and from these the Sun's parallaxes at once became clear, through the problems given in the same place, Chapter 8.

[Margin: The astronomers' order in investigating parallaxes and distances.]

Since, however, the other five planets have the measures of their intervals bound and connected, by a wonderful tempering and harmony, no less to the interval between the Sun and the Earth than their motions to the motion of the Sun or of the Earth, a more certain method therefore first seeks their distances from the Earth from the commensuration of the orbs that compose the planetary system with the radius of the annual orb of the Sun or the Earth, as we have done so far; and then from those distances it gathers the parallaxes by the following problem. For although some conjecture about the parallaxes of the planets, especially of Mars and Venus when they come closest to the Earth, can be drawn from their apparent motions and the analogy of these, that conjecture is nevertheless very weak, and always carries the suspicion that the variation arises rather from some small deception in the observations, or from uncertainty in the things assumed in deducing the place from observations, than from a variation of view arising from sheer distance.

II. Since, therefore, we are now clear about the opinion of the distances from what was said in Chapter 2, it will not be difficult to deduce the horizontal parallaxes of these planets from the mean, or least, or greatest distance. These are the foundation of the others, by the problem and scheme set out in Book 3, Chapter 8, which you should reread here just as if it were inserted. The sum of the rule is this: if the distance of the planet is known in parts of which the Earth's semidiameter is one, you are to find the residual of the logarithm of the distance [Reſiduum logarithmi]; for that, when looked up and used as a logarithm, will show the angle of horizontal parallax.

[Margin: Summary of the rule.]

Or let one make, as the given distance to the radius or whole sine, so one semidiameter of the Earth to the sine of the angle measuring the horizontal parallax. By this technique, then, the following table of parallaxes has been constructed for the greatest and least distances; from these it will be easy to judge the parallaxes belonging to the mean distance, although in Venus and Mercury the mean parallaxes are equal to the parallaxes of the Sun placed at its mean distance from the Earth.

Table of the horizontal parallaxes for the five minor planets, at greatest and least distance from the Earth.

In the distance: / Authors of the distance	♄ great- est	♄ least	♃ great- est	♃ least	♂ great- est	♂ least	♀ great- est	♀ least	☿ great- est	☿ least
Ptolemy	7″ 50‴	11″ 40‴	11″ 10‴	21″ 15‴	0′ 19″	2′ 30″	2′ 25″	18′ 40″	17′ 15″	62′ 0″
al-Battānī	11″ 30‴	18″ 0‴	16″ 30‴	25″ 20‴	0′ 25″	2′ 52″	3′ 20″	24′ 0″	34′ 0″	53′ 30″
al-Farghānī	10″ 30‴	13″ 40‴	13″ 20‴	23″ 15‴	0′ 23″	2′ 50″	2′ 58″	24′ 0″	33′ 50″	53′ 0″
Copernicus	16″ 30‴	22″ 55‴	27″ 40‴	44″ 55‴	1′ 8″	8′ 40″	1′ 43″	11′ 30″	2′ 5″	5′ 40″
Tycho	17″ 0‴	28″ 20‴	33″ 10‴	60″ 19‴	1′ 9″	8′ 35″	1′ 40″	11′ 25″	2′ 4″	5′ 35″
Longomontanus	14″ 0‴	21″ 0‴	24″ 0‴	41″ 45‴	0′ 59″	8′ 35″	1′ 35″	11′ 25″	1′ 50″	5′ 50″
Kepler	5″ 50‴	7″ 45‴	9″ 30‴	15″ 40‴	0′ 23″	2′ 27″	0′ 36″	3′ 45″	0′ 42″	2′ 0″
Lansbergen	12″ 0‴	16″ 10‴	21″ 10‴	34″ 0‴	0′ 46″	6′ 55″	1′ 19″	8′ 35″	1′ 35″	4′ 18″
Ismaël Boulliau	12″ 50‴	17″ 45‴	22″ 5‴	36″ 10‴	0′ 47″	6′ 20″	1′ 15″	12′ 30″	1′ 50″	4′ 50″
Ourselves	2″ 20‴	3″ 30‴	4″ 20‴	7″ 45‴	0′ 10″	1′ 26″	0′ 16″	1′ 48″	0′ 19″	0′ 50″

III. Much would have to be said here about the parallaxes of Mars, especially of acronychal Mars, that the learning of astronomers requires; but we have set this out generously in Section 2, Chapter 3, Scholium 4, all of which we wish to apply to this chapter as well. From what was said there, I recall to memory two points especially worth noting. The first is that the parallax of acronychal Mars, that is, in its oppositions with the Sun, is greater than the solar parallax, and that Mars then, especially in Aquarius in these centuries, comes nearly twice as close to the Earth as the Sun does. This is established from the commensuration of its orbs and the Sun's, deduced through the prosthaphaereses, and from the striking increase in its visible diameter. The second is that this same parallax is hardly 2′; indeed, Kepler says in the *Epitome of Copernican Astronomy*, p. 479: "But I have observed that the parallax of Mars is by no means sensible." From this Kepler and we have drawn a powerful argument for diminishing the solar parallax and increasing the Sun's distance from the Earth. So let Tycho take his leave here, who forced on us an excessive parallax of Mars, just as he did of the Sun, and too small a distance for each; granted that he boasts, in volume 1 of the *Progymnasmata*, p. 414, that he has "precisely measured" the parallaxes of acronychal Mars.

[Margin: The parallax of Mars hardly sensible, though greater than the solar.]

In the parallaxes of Venus too he went wrong, though by an error consistent with those in the solar parallaxes. Kepler said, no less candidly than correctly, in the foundations of his *Ephemerides*, p. 15: "In Venus there occur Brahean observations which the nearness of the Sun, the inequality of the air, and the star's position nearest the Earth, that is, its parallax, bring under suspicion of error, so that they sustain credit by nothing except the authority of a most diligent craftsman. Relying on these observations (because they seemed very opportune in establishing the annual commutation), I fear that, through the votes of many others collated, I may be found guilty of some minutes. This most laborious examination of the witnesses, and perilous sifting of the reliable from the unreliable, requires another time."

[Margin: The parallaxes of Venus.]

And he immediately adds these further words: "No small labour also remains in separating the observations of Venus by day, very frequently made by Brahe, from those by night, and those demonstrated from both, which I had unwittingly mixed together, at no small risk of error. For there remains, undecided between the Landgrave and Brahe, the dispute about 6′ of the Sun's motion from the fixed stars, and the testimonies waver this way and that.

[Margin: Tycho's excess in the parallaxes of Venus.]

For the parallax of Venus cannot be as great as the one Brahe assumed in the *Progymnasmata*, fitted to the parallax of the Sun handed down by the Ancients, which is without doubt smaller."

[Margin: Vol. 1 of the *Progymnasmata*, from p. 157.]

IV. As regards Saturn and Jupiter, Tycho admits that their parallaxes are hardly sensible with any instrument, for in volume 1 of the *Progymnasmata*, p. 414, he says: "Although this measurement of parallaxes and distances finds hardly any place in the three superior planets, because of their huge remoteness from the Earth, which makes this variation very small, especially in Saturn, where it scarcely exceeds a third of 1′, nevertheless, by applying a most exact instrument which finely divides aliquot parts of 1′, it will be possible to try these parallaxes too, as far as can be done; especially in Mars when it is acronychal, whose parallaxes I have several times, around that time, carefully and precisely measured, so as to establish whether, having become all-night, it was brought nearer the Earth than the Sun itself, as the Copernican and also our own hypothesis demands."

[Margin: Tycho's opinion on the parallaxes of Saturn and Jupiter.]

Then, on p. 661, he criticises Thomas Digges, who had said: "If the parallaxes of the three most remote planets can truly be detected by any art, they can be found either by his own Problems or by no geometrical precepts at all." In these Problems, however, he assumed that the phenomenon moved by no other motion than the motion of the universe. But since these planets move by a proper motion, "which even in slowest Saturn" (the words are Tycho's) "readily exhausts the smallest parallax, how, I ask, will he unravel the parallaxes of these planets from them? If he hopes to accomplish this at their stations, those do not last long, especially in Mars; and neither are their times known so precisely that the parallaxes can then be examined accurately enough. I pass over the fact that the variation of latitude presents some impediment, etc." And finally Tycho concludes thus: "Intermediate examinations are less discriminable, so that perceiving the parallaxes of the three superior planets seems all too inaccessible. We, however, have not left it untried to test this in acronychal Mars by some other method and, as we hope, to attain it with sufficient certainty." Rashly, therefore, do some who only sip astronomy with the very tips of their lips boast that the parallaxes of Saturn and Jupiter are sensible.

[Margin: Thomas Digges's excessive confidence regarding the parallaxes of the three superior planets.]

V. Further, Tycho must be forgiven those words in volume 1 of the *Progymnasmata*, p. 415: "Nor would it be altogether superfluous to attempt this in Mercury and Venus when they approach the Earth, if, being then very close to the Sun, they did not close off the opportunity for this observation." He is speaking there of parallaxes to be investigated by observation. But Venus at perigee, when it has a great latitude, can be observed, and was in fact observed by Tycho, as we said in Section 3, Chapter 4, Scholium 2, and Section 5, Chapter 5; and we ourselves observed it around perigee, as we said in Section 1, Chapter 2, number 4. At that time, therefore, its parallax can also be observed, by comparing the visible place with the true place.

[Margin: Tycho's error about the parallax of Venus at perigee as unobservable.]

SCHOLIA.

Scholia.

I. From what was said in the preceding chapter it is gathered that, in computing motions, the parallaxes of Mars, Venus and Mercury cannot be disregarded without risk of sensible error when, having passed the mean distance, they approach perigee. When they are above the Sun, however, the risk of erring is not so great, and much less in Saturn and Jupiter. This risk vanishes altogether in our hypothesis, and far more so in that of Posidonius and Wendelen, who place the Sun, and consequently these planets, still twice as high above the Earth as we do.

[Margin: In which planets and when parallax may be disregarded.]

II. That excess in the parallaxes of Venus which Tycho adopted in volume 1 of the *Progymnasmata* was the reason why the places of the fixed stars too, determined from their distance from Venus and from Venus's distance from the Sun, contain some fault. This is what Kepler meant to indicate in the words quoted above, in number 3. Let no one wonder, therefore, if the distances of the fixed stars from one another, and the Tychonic places in longitude and latitude, sometimes differ from ours by 1′, or even by 2′.

[Margin: The Tychonic places of the fixed stars vitiated by the parallax of Venus.]

CAPVT VIII.

Chapter 8.

Præmittuntur Problemata ad Apparentem ac Veram Magnitudinem Quinque Minorum Planetarum pertinentia, & ea occaſione Explicantur Analogiæ interuallorum & Corporum cæleſtium Keplerianæ.

Problems relating to the apparent and true size of the five minor planets are set out first, and on that occasion Kepler's analogies of the intervals and of the celestial bodies are explained.

It is evident that the variation in the apparent and true diameter of the planets depends on their distance from the Earth. Before we report the opinions on these matters, both those of others and our own, I think that general problems concerning them should be set out first, so that recourse to them may afterwards be readier. Let there be, therefore:

Problema 1. Data diſtantia Planetæ à terra, & ſemidiametro ipſius apparenti in illa diſtantia, Inquirere veram eiuſdem Semidiametrum ac Diametrum.

Problem 1. Given the distance of a planet from the Earth and its apparent semidiameter at that distance, to find its true semidiameter and diameter.

I. In the attached diagram, let the centre of the Earth be at C and the distance of the planet's centre AC or PC. From the centre of the planet's spherical body let a circle be described with equal semidiameter AB and DP, one from A, the other from P, representing a great circle on the globe of the planet. To this circle let two straight lines be drawn from the centre of the Earth: one, CD, touching the planet at D, when it is nearer the Earth; the other, CB, touching it at B, when it is more remote. For two triangles will arise, PCD and ACB, right-angled at the contacts D and B, by 18 of Book 3 of Euclid. Therefore, if the planet is at A, and the distance AC and the angle ACB are given, namely the angle measuring its apparent semidiameter at the time when the planet is at that distance, then by the first rule of our rectilinear right-angled triangles the side AB will easily be gathered, that is, the true semidiameter, in the kind of parts in which the distance AC was given. In the same way, if the planet is at P, then with the distance PC and the apparent semidiameter, that is, the angle PCD, the true semidiameter DP will be gathered. And when the semidiameter is doubled, the diameter will be known. In short, then, let it be made: as the whole sine or radius to the given distance, so the sine of the given angle, or of the apparent semidiameter, to something else, and the true semidiameter will come out. Or, more briefly, add to the logarithm of the distance the logarithm of the apparent semidiameter; the sum will be the logarithm of the true semidiameter.

[Margin: Summary of the problem.]

Diagram. A small geometric figure set into the right side of the left column. Two equal circles stand one above the other, with a point C well below them. Straight lines run up from C past the lower circle to the upper circle. In the

upper circle a short radius joins its centre A to the point B on its left edge; in the lower circle a short radius joins its centre P to the point D on its left edge. Labels read off the block, all five: A, B, P, D, C. None unread.

- A (centre of the upper circle)
- B (left edge of the upper circle)
- P (centre of the lower circle)
- D (left edge of the lower circle)
- C (foot of the figure, where the lines meet)

Problema 2. Data Vera ſemidiametro Planetæ, inuenire Apparentem ſemidiametrum ipſius, cuiuis datæ diſtantiæ conuenientem.

Problem 2. Given the true semidiameter of a planet, to find its apparent semidiameter, suited to any given distance.

II. In the preceding scheme already set out, let the planet be at A, holding its greatest distance from the Earth, now known, and let the side AB be given, that is, the true semidiameter in the same kind of parts. Then by Rule 2 of our plane and right-angled triangles the angle ACB, opposite the given side, can be found; and this is the angle under which the apparent semidiameter is seen and which measures it. Again, let the planet be at P, that is, at a lesser distance, whether that be the mean, or the least, or any other, which distance is known either in itself or by subtracting the difference AP from the greatest distance AC. For since the true semidiameter DP is given, by the same rule the angle DCP will certainly become known, measuring the planet's apparent semidiameter in that position. In short, then, let it be made: as the given distance to the radius, so the true semidiameter to the sine of the angle determining the apparent semidiameter. Or add to the logarithm of the true semidiameter the residual of the logarithm of the given distance: for the sum will be the logarithm of the angle measuring the planet's semidiameter as it appears, or as it would appear at that distance of the planet if its appearance were not otherwise prevented, for example by the rays of the Sun. By doubling the visual [?] semidiameter, the whole diameter will be known.

[Margin: Summary of the problem.]

Gather from what has been said that the distances required by the hypothesis of any astronomer must agree with the variation or difference of the apparent diameter, and vice versa; otherwise, if they do not agree, the hypothesis will be convicted of falsity in one or the other.

[Margin: Corollary.]

Problema 3. Aliter ſed Mechanicè abſoluere 1. & 2. Problema.

Problem 3. To solve Problems 1 and 2 in another way, but mechanically.

III. Our Giuseppe Biancani, not so much to avoid the labour of solving triangles by sines as to entice the young to astronomy by an easy and more

evident method of measuring on Earth stars that exist in the heavens, directed that on some plane, say on a huge sheet or board, a straight line be drawn and divided into as many small equal parts as there are in the distance of the planet from the Earth; indeed, that one of those small parts be subdivided into some smaller equal parts, say into 10 or 100. Then, from one end of the line, as from a centre, an arc of 60° was to be described with the help of a compass, or of a ruler equivalent to a compass, at an interval matching the distance, and it was to be divided into degrees, and 1° into 60′. In this way, either from the given arc of the apparent diameter the ratio of the chord, that is, of the planet's true diameter, to the distance would be established with the help of the compass; or from the true diameter, taken as a chord, the doubled sine and the arc determining the visual angle, that is, the angle measuring the apparent diameter. But at great distances this rather crude procedure certainly proves inconvenient.

Problema 4. Data Diametro Vera, Planetæ, Inquirere Circumferentiam circuli in ipſius corpore maximi, & è conuerſo.

Problem 4. Given the true diameter of a planet, to find the circumference of the greatest circle in its body, and conversely.

IV. Let it be made: as 7 to 22, so the given or found diameter to something else; or (more exactly) let it be made: as 100 to 314, so the diameter to something else, and the circumference will come out in the same kind of parts in which the diameter was known. You have the basis of this precept in Book 1, Chapter 4. Conversely, if it is made: as 314 to 100, so the circumference to something else, the diameter will become known.

Problema 5. Datis Diametro & Circumferentia Planetæ inuenire, tum Aream ſeu capacitatem circuli in eo maximi, tum ſuperficiem ipſius in menſuris quadratis.

Problem 5. Given the diameter and circumference of a planet, to find both the area or capacity of the greatest circle in it, and its surface, in square measures.

V. First, take, that is, multiply, the semicircumference by the semidiameter, or a fourth part of the circumference by the diameter, and the area of the greatest circle will result, known in square measures: so that if the measure of the diameter was in feet as to length, the total made by the already mentioned multiplication will be in square feet, that is, small areas each of whose sides equals 1 foot.

[Margin: Part 1 of the problem.]

Second, multiply the diameter by the circumference, or quadruple the area found by the first precept; for either way the number of square measures will result which equal the convex surface of the spherical planet, as is supposed here. I laid the foundations of these precepts in Book 1, Chapters 4 and 6.

[Margin: Part 2 of the problem.]

Problema 6. Datis Diametris veris ducrum [recte: duorum] Planetarum, & vnius Circumferentia vel Area, Inuenire alterius Circumferentiam vel Aream.

Problem 6. Given the true diameters of two planets and the circumference or area of one, to find the circumference or area of the other.

VI. First, if the circumference of one is given, let it be made: as the diameter of the planet whose circumference is known to the diameter of the other, so the known circumference to the one sought. For the latter will likewise turn out known, because the diameters of different circles are to one another as their circumferences, and conversely, as Pappus of Alexandria demonstrated in Book 11 of the *Collection*, Theorem 5.

[Margin: Part 1 of the problem.]

Second, if the area of one is given, let it be made: as the square number of the diameter of the planet whose area is known to the square of the diameter of the other planet, so the given area to the area sought. For circles are to one another as the squares of their diameters, by 2 of the twelfth book of Euclid's *Elements*.

[Margin: Part 2 of the problem.]

Problema 7. Datis Diametro vera Planetæ & ſuperficie conuexa; Inquirere ſoliditatem corporis Planetæ in menſuris cubicis.

Problem 7. Given the true diameter of a planet and its convex surface, to find the solidity of the planet's body in cubic measures.

VII. Multiply a sixth part of the surface by the diameter, or a third part of the surface by the semidiameter; either way a number will result containing as many cubes as equal the whole solidity of the planet's spherical body.

Problema 8. Data proportione inter veras Diametros binorum Planetarum, vel Planetæ & Terræ; Inuenire proportionem corporis Planetæ ad corpus Planetæ, vel ad ſoliditatem Terræ; ſeu quot vicibus maior minorem contineat.

Problem 8. Given the proportion between the true diameters of two planets, or of a planet and the Earth, to find the proportion of the body of one planet to the body of another, or to the solidity of the Earth; that is, how many times the greater contains the lesser.

VIII. It is certain that the globe or sphere with the greater diameter is the greater. To find, therefore, the proportion of the lesser sphere to the greater, one must seek the triplicate ratio of the lesser diameter to the greater diameter;

or each diameter must be multiplied cubically, and the proportion of those cubes sought. "For cubes are in the triplicate ratio of their sides," by 12 of Book 8 of Euclid's *Elements*; and "spheres are to one another in the triplicate ratio of their diameters," by 18 of Book 12 of Euclid's *Elements*, "or of their semidiameters," by 19 of the Fifth. Therefore, taking the diameters as the sides of cubes, spheres are to one another as the cubes of their diameters. This is indeed a most beautiful analogy: just as circles are to one another as the squares of their diameters, by 2 of the twelfth book of the *Elements*, so spheres are to one another as the cubes of their diameters.

[Margin: The proportion of sphere to sphere.]

Now by Definition 10 of Book 5 of the *Elements*, if there are several quantities in continued proportion, the first has to the second a simple ratio, but the first to the third a duplicate, the first to the fourth a triplicate, the first to the fifth a quadruplicate, and so on to infinity. Thus, if the proportion of doubleness is taken and continued in these numbers, 2, 4, 8, 16, 32, between 2 and 4 there will be a simple proportion of doubleness, but between 2 and 8 a duplicate, between 2 and 16 a triplicate, and between 2 and 32 a quadruplicate. A square number is one generated by multiplying a given number into itself; a cubic number, one generated by multiplying a square number by the root of the square. Thus, if 2 is given and multiplied by 2, 4 results, which is a square number whose side or root is 2. If 4 is multiplied again, by 2, 8 will result, which will be the cube of the number 2, and 2 will be the side of the cube, or the cube root, of the number 8. On these premises two rules arise for solving the problem.

[Margin: What is a triplicate ratio?]

The first rests on Proposition 18 of Book 12 of Euclid, in which it is said that "spheres are to one another in the triplicate ratio of their diameters." Therefore, if the proportion between the lesser and the greater diameter is continued as far as the fourth term, the triplicate ratio of the first to the fourth will be obtained, and just such and so great will be the proportion of that lesser sphere to the greater.

[Margin: Rule 1 for solving the problem.]

EXEMPLVM.

Example.

Let there be two spheres, one having a diameter of 2 palms, the other of 4 palms, that is, as 2 to 4. Let it be made, then: as 2 to 4, so 4 to 8; then, as 4 to 8, so 8 to 16. For the lesser sphere will be to the greater as 2 to 16; that is, the sphere of 2-palm diameter will be contained, as to solidity, 8 times in the sphere of 4-palm diameter.

The second rule rests on Proposition 12 of Book 8 of the *Elements*, in which it is said that "spheres are to one another as the cubes of their diameters." So let the diameters be multiplied cubically, each separately; for whatever proportion

holds between the cubic numbers of the diameters, the same will hold between those spheres.

[Margin: Rule 2 of the same problem.]

EXEMPLVM.

Example.

Let the spheres be the same as above, one of 2 palms, the other of 4 palms as to diameter. For when 2 is multiplied by 2, 4 results, and again, when 4 is multiplied by 2, 8 results; and this is the cube of the number 2. On the other side, when 4 is multiplied by 4, 16 results; and when 16 is multiplied by 4, 64 results, which is the cube of the number 4. Therefore, as 8 to 64, so the 2-palm sphere to the 4-palm sphere; and 8 is contained in 64 8 times, therefore that sphere too is contained in this one 8 times.

I have chosen to give these two rules separately, so that they do not seem to conflict if you should happen to find one used by some authors and the other by others, and at the same time for testing one by the other, because of their equivalence. Since the cube of unity is unity itself, if some sphere and its diameter are taken as 1, as the common measure of the proportion of the others, and if its proportion to the diameter of another is continued to the fourth term, it will at once be had how many times the lesser sphere is contained in the greater. But when the true diameter of a star does not contain one whole diameter of the Earth, but a fraction of unity, it is expedient to reduce the denominator of the fraction to a hundred or a thousand and to find the numerator proportional to it; and then the Earth's diameter is to be taken as of 100 or 1000 parts. For in this way the fourth term of the triplicate ratio, or the cubic numbers both of the terrestrial diameter and of the number that was the numerator of the fraction, now taken as whole, will be found more easily.

[Margin: A caution on the use of this rule.]

Problema 9. Data Semidiametro, atque adeò Diametro conuexitatis cuiuſuis cæli, inuenire tum Circumferentiam ſeu ambitum eius, tum Aream circuli maximi, tum ſuperficiem Conuexam, tum Soliditatem illius Cæli.

Problem 9. Given the semidiameter, and therefore the diameter, of the convexity of any heaven, to find its circumference or circuit, the area of its greatest circle, its convex surface, and the solidity of that heaven.

IX. This problem is contained in the problems set out above, from 4 to 8, if what was said there about the globes of the planets is applied to the globes of the heavens. But if the interior and lower heavens are wholly distinct from the upper ones, yet from those that immediately contain them, the solidity of the lesser must be taken away from the solidity of the greater, so that the solidity of the greater alone remains, not as a sphere but as an orb or spherical vessel; just

as from the least of the heavens, that is, from the lunar solidity, the elementary sphere will have to be subtracted.

Problema 10. Data Diſtantia duorum quorumuis Planetarum à Sole, vel periodo motuum, & vnius ſoliditate; inuenire alterius ſoliditatem, ex hypotheſi Kepleriana.

Problem 10. Given the distance of any two planets from the Sun, or the period of their motions, and the solidity of one, to find the solidity of the other, from the Keplerian hypothesis.

X. I have not proposed this problem absolutely, because it is not established that it is absolutely true, or equally applicable to every hypothesis, especially since in this hypothesis the Earth is supposed to be one of the planets going round the Sun, as the Moon is a planet going round the Earth. The solution of this problem, however, rests on certain propositions which, because of their beauty and their great probability in that hypothesis, as if axioms

Kepler calls them in the *Epitome of Copernican Astronomy*, Book 4; and he explains their foundations from p. 474 to p. 499. But from among them Pierre Hérigone chose three, in volume 5 of his *Cursus Mathematicus*, and stated them absolutely in Book 2, Chapter 3, Axioms 18, 19 and 20. They are the following; of these, however, Kepler abandoned the second and held to the first.

[Margin: 1st Keplerian axiom.]

Axiom 1. *The bodies of the planets have among themselves the same proportion as the semidiameters of their orbs, or their mean distances from the Sun.*

[Margin: 2nd axiom.]

Axiom 2. *The times of the periodic motions are in the sesquialteral proportion of the proportion of their bodies and of their mean distances from the Sun.*

[Margin: 3rd axiom.]

Axiom 3. *The proportion between the bodies of the planets is the duplicate of the proportion of their weights, or of the quantity of matter enclosed in the bodies.*

From these axioms, if they are accepted, the rules written below follow.

[Margin: 1st rule.]

First Rule: Let it be made, as the distance of the planet whose solidity is known is to the distance of the other planet whose solidity is sought, so the known solidity to the solidity sought. Thus, since the mean distance of the Earth from the Sun according to Kepler is 3469 semidiameters of the Earth (this is what he chose in the *Epitome*, so that these proportions might be preserved, although he had earlier made it 1800 in the *Commentaries on Mars* [in Marte] and in the *Ephemerides*), and that of Saturn from the Sun is 32256, if the solidity of the Earth is taken as 1, and it is made as 3469 to 32256, so 1 to another number, there will come out in the quotient approximately the number 9. Therefore the solidity of the Earth will be to the solidity of Saturn as 1 to 9.

[Margin: 2nd rule.]

Second Rule, from Axiom 3: Given the periodic times of two planets, seek two numbers that are mean proportionals between those two times. For the smallest term, taken to the next greater, will give the ratio of the matter they possess; but compared to the third, it will give the ratio both of the solidity of the bodies and of the mean distances from the Sun. For example, since the periodic time of the Earth round the ☉ is 1 year, and that of Saturn about 29, and the mean proportionals between these numbers are 3 and 9, fractions being left aside, the matter of the Earth will be to the matter of Saturn as 1 to 3, but the solidity of the Earth to the solidity, or distance to distance, as 1 to 9. Therefore, when the Earth's mean distance from the Sun, which for Kepler is 3469, is multiplied by 9, the result will be approximately the mean distance of Saturn from the Sun, namely 31221. We do not set down a third rule, because Kepler did not afterwards accept Axiom 2.

Doctrina Kepleri de Præcipuorum Mundi Corporum Proportionibus.

Kepler's doctrine of the proportions of the principal bodies of the world.

XI. Let us consult Kepler, then, and see by what indications he hunted down the analogies already mentioned. In Book 4 of the *Epitome of Copernican Astronomy*, Part 1, no. 4, he reasons as follows. The globe of the Sun, since it is the chief among the bodies of the world (and perhaps the first in the order of creation, if by the name of the light made on the first day one understands the Sun), ought not to receive its proportion from other bodies, but rather the others from it; and first of all the Earth, since it was to be the home of the creature that contemplates the works of GOD and measures them by means of sight. And the first ratio of measuring ought to be taken from the first visible thing, that is, from the visible quantity of the Sun. Now the visible quantity of the Sun, or its apparent diameter, is according to Aristarchus and all astronomers the seven hundred and twentieth part of the circle described from the Earth through the Sun, that is, it is 30′.

[Margin: Kepler's most ingenious reasoning on the proportion of the bodies of the world.]

This number 720, moreover, arises from harmonic ratios; for the smallest number that offers itself for determining all the parts of the monochord, so as to constitute the system of the double diapason, or of the soft and hard *cantus* [mollis & duri cantûs], is 720, as the same Kepler shows in the *Harmonics*, Book 3, Chapter 6. Since, then, the motions of the planets were to be accommodated to the harmonic system of the soft and hard *cantus*, as Kepler thinks he has demonstrated in Book 5 of the *Harmonics*, it was fitting that the first body, which was to lead the chorus of this harmony, should, by the appearance of its diameter with respect to the Earth that was to perceive this harmony, divide the circle for the Earth-dwellers in such a way that this division would be the index and measure of the appearance of the harmonic motions, by the very

division of the monochord, that is, into 720 parts. These contain 360 twice, 240 three times, 180 four times, 124 [recte: 144] five times, 120 six times, 90 eight times, 80 nine times, 72 ten times, 60 twelve times, 48 fifteen times, 45 sixteen times, 40 eighteen times, 36 twenty times, 30 twenty-four times; and this is the most fertile form of division into aliquot parts.

Now, if the Sun had to appear to the Earth at its greatest distance under an angle of 30′, that is, under the 720th part of the circle, the Earth must necessarily then have been a little more than 229 semidiameters of the body of the Sun distant from the Sun, as geometry teaches. But if the Earth, being the home of the contemplating creature, had to be the first to perceive these proportions and to measure with a measure best known to itself, then it too had to be, by its own semidiameter, as by the line best known to it of all lines, the measure of the intervals, and by its own body the measure of the principal bodies of the world. And so the first proportion had to be between the semidiameter of the Earth and its interval from the Sun, just as the first also between the body of the Earth and the body of the Sun. And no simpler, better known or more suitable proportion could be set up than an equality of proportions, that is, that the semidiameter of the Earth should be to the distance of the Sun, or of itself from the Sun, as the body of the Earth is to the body of the Sun.

These foundations laid, he sets the semidiameter of the Sun at 100000 parts, so that the interval between Sun and Earth is 22918166 parts; and he divides the cube of the number 100000, which is 100000,00000,00000, by 22918166. Of the quotient (which he says is, that is, approximately, the sine of 15′, or of the apparent semidiameter of the Sun), namely of the number 43631764, he says that the nearest square root, 6606, is the semidiameter of the Earth. For as 6606 is contained in 22918166, the interval of Sun and Earth, 3469 ⅓ times, so the cube of the number 6606 is contained in the cube of the number 100000, of which numbers the former has been assigned to the semidiameter of the Earth, the latter to that of the Sun. For we have already said that sphere is to sphere as the cubes of the diameters are, that is, the cubes in which the globes are inscribed. Hence it comes about that the semidiameter of the Sun contains the semidiameter of the Earth fifteen times, or a little more; but the body of the Sun contains the body of the Earth about 3469 times, that is, as many times as the interval of the Sun contains the semidiameter of the Earth; in strictness, however, the triplicate ratio of the diameters requires 3375. For as 1 to 15, so 15 to 225, and as 15 to 225, so 225 to 3375.

[Margin: Determination of the true distance of the ☉ from the Earth and of the Sun's semidiameter.]

That this distance of the Sun from the Earth is nearly three times greater than the one set by the ancient astronomers he confirms, first, from the parallax of Mars at perigee, which in the acronychal position ought to be greater than the parallax of the Sun; and yet Kepler says: *"But I have observed that the parallax of Mars is by no means sensible."* Therefore, if the parallax of Mars, even when it is nearest the Earth, is not sensible, the parallax of the Sun is much less sensible,

since the Sun is further from the Earth than acronychal Mars. But if the Sun were distant from the Earth only 1200, as much as the ancients supposed, the horizontal parallax of the Sun would be sensible, namely 3′; therefore the Sun is much further from the Earth than 1200 semidiameters of the Earth.

[Margin: 1st confirmation of the great distance of the Sun, from the insensible parallax.]

Second, because otherwise Mars would come too close to the Earth, and from its apparent diameter observed through the Belgian telescope it would be concluded that Mars is smaller than the Earth. This conflicts with the analogy due between the size of the bodies of the world and the order of their position, for that analogy requires that a higher planet also be larger; and in the Copernican system Mars is higher than the Earth.

[Margin: 2nd confirmation, from the size due to Mars.]

Third, as the parallax of the Sun decreases, and so as the distance increases, the parallax of the Moon from the Sun increases, which serves outstandingly for correcting the calculation of eclipses.

[Margin: 3rd confirmation, from eclipses.]

Fourth, if the distance of the Sun from the Earth is increased at least in this way, the body of the Sun turns out larger than all the bodies of the planets fused into one, which is very congruous with the physical cause of the celestial motions: namely, that the Sun, which according to Kepler's opinion leads all the planets round and imparts motion to them, should also be larger than all the moving bodies taken together.

[Margin: 4th confirmation, from the size due to the Sun.]

And this is Kepler's first step for determining the proportion between the body of the Earth and the body of the Sun, and between the distance of the eye contemplating the stars from the centre of the Earth and the distance of the Earth from the centre of the world, which is the Sun. But since Father Paulus Casatus [Paolo Casati], while reading these things, was in doubt how the square root 6606 rather than the cube root was to be applied according to Kepler, he himself later resolved his own doubt by a theorem and a problem, which I have chosen to insert here; and they are the following.

Theorema. Si numerus Cubus ad alium Cubum eam habeat rationem, quam datus quipiam numerus habet ad ſecundi Cubi radicem cubicam; Radix quadrata primi cubi ſumpti vt quadrati, eſt media proportionalis inter datum numerum & datæ radicis cubicæ quadratum.

Theorem. If a cube number has to another cube the same ratio that some given number has to the cube root of the second cube, then the square root of the first cube, taken as a square, is the mean proportional between the given number and the square of the given cube root.

Let the given first cube be S, the second cube A, and let its cube root be C. Let there be given a number D that has to C the same ratio as the cube S to the cube A. And let B be taken, the square of the root C. I say that the square root of S itself is the mean proportional between D and B.

S.	A.	D.	C.
		B.	

For since, as S to A, so D to C, therefore S times C is equal to A times D. But the cube A is equal to C times B; therefore S times C is equal to D times B times C. So, when each term of the equation is divided by C, there remains S equal to D times B. Therefore D, and the square root of S itself, and B, the square of the given root C, are continued proportionals. From this arises

Problema. Dato numero Cubo, & alio quopiam non cubo, inuenire cubum, ad quem datus cubus eam habeat rationem, quam datus non cubus habet ad quæſiti cubi radicem cubicam.

Problem. Given a cube number and some other number that is not a cube, to find a cube to which the given cube has the same ratio that the given non-cube has to the cube root of the cube sought.

Let the given cube be S, the given non-cube D. The cube A is sought, and its cube root C, such that the ratio S has to A is the ratio D has to C. Since, then, by the theorem above, the square root of S itself is the mean proportional between the given D and B, the square of the root C, divide the number S by the given D, and B, the square of the root C, will result. When B has thus been found, the square root C extracted from it will be the cube root sought; and if this is multiplied by B, the square that was found, A will be generated, the cube sought. Which was to be done.

De Proportione Interuallorum & Corporum Lunæ ac Telluris iuxta Keplerum.

On the proportion of the intervals and bodies of the Moon and the Earth according to Kepler.

XII. Kepler thinks that the second place among the bodies of the world is owed to the Moon, as a star owed to the Earth, to assist the growth of terrestrial things; and to the contemplating creature, so that it might begin its observations of the other stars from the Moon, and especially of eclipses, by which we are instructed first and most evidently about the course of the stars. It is plain that this comes about most correctly if, at the greatest remoteness from the Earth, the apparent diameter of each is equal, and the Moon can exactly hide the Sun. So the diameter of the Moon, like that of the Sun, had to occupy the 720th

part of the circle, that is, half a degree. And since the Moon is an earthly and secondary planet, and made for obscuring the Sun, it had also to follow the pattern of the proportions of the Sun and the Earth, both as to the interval and as to the mass of the body. It was fitting, then, that just as the body of the Earth is to the body of the Sun as the semidiameter of the Earth is to the interval of the Earth from the centre of the world, so likewise the body of the Earth should be to the body of the Moon as the distance of the Moon from the Earth is to the semidiameter of the Earth. Nor does this conflict with observations; for Tycho found the greatest distance of the Moon in the quadratures to be greater than 59 and a little less than 60 semidiameters of the Earth. From what has been set out, then, it follows that the semidiameter of the lunar orbit, or the distance of the Moon from the Earth, is the mean proportional between the semidiameter of the Earth and the semidiameter of the annual orb, or the distance of the Earth from the Sun. For as 1 to 59, so 59 to 3481, that is, very nearly 3469, which is the distance of the Earth from the Sun. So by seeking the square root of the Earth's distance from the Sun, we have the distance of the Moon from the Earth. And when the cube of the Earth's semidiameter is divided by 59, and the cube root of the quotient is sought, the semidiameter of the lunar body results. Accordingly, the semidiameter of the Earth contains the semidiameter of the Moon a little less than four times, and the body of the Earth contains the body of the Moon a little less than 59 times. Therefore the body of the Sun, which contains the body of the Earth 3469 times, will contain the body of the Moon, when 3469 is multiplied by 59, 204671 times. Kepler therefore says that these proportions, by a wonderful agreement of likelihoods, have their own dignity, and are so worthy of belief in themselves that they can be used in the place of an axiom.

De Proportione Interuallorum & Corporum Quinque reliquorum Planetarum, iuxta Keplerum.

On the proportion of the intervals and bodies of the five remaining planets, according to Kepler.

XIII. Nothing, says Kepler in the *Epitome of Copernican Astronomy*, Book 4, p. 484, is more in keeping with nature than that the order of sizes should be the same as that of the spheres. And so, of the six primary planets (for he makes the Moon a secondary, and a planet of the Earth rather than of the world), the one that is nearer and innermost of all and describes the smallest orb round the Sun, namely Mercury, should be the smallest of the planets; next larger Venus, then the Earth, then Mars, then Jupiter; and largest of all Saturn. Nor was it right that the bodies of the planets should be adjusted to the body of the Sun by the same laws as the Earth; otherwise size would be contrary to the order of the spheres, and Mercury would turn out largest and Saturn smallest, which conflicts with archetypal reasons and with the observation of the apparent diameters. Further, the dimension of bodies is threefold: one according to diameters, the second

by surfaces, the third according to bulk [corpulentiam]. Spherical surfaces are in the duplicate ratio of their diameters, but bulks in the triplicate.

[Margin: A threefold ratio in the proportion of the intervals and bodies, from a threefold disagreement [trina diſſenſione].]

Either, then, the proportion of the intervals and bodies had to be taken according to the diameters, so that, as the interval of Saturn from the Sun is ten times greater than the interval of the Earth, so the diameter of Saturn would be ten times greater than the diameter of the Earth; and this conflicts with the observations of the diameters made with the Belgian telescope. Or it had to be taken according to the surfaces: that is, as the interval of Saturn is tenfold of the interval of the Earth from the Sun, so the surface of the body of Saturn would be tenfold of the surface of the Earth. This once seemed more probable to Kepler, because from it there follows a proportion of the bodies that is the sesquialteral of the proportion of the intervals, and Saturn is thirty times greater than the Earth, as it is thirty times slower; while the proportion of the diameters is at least half the proportion of the intervals: analogies that favour archetypal reasons. Or finally it had to be taken according to the bulks directly, so that Saturn is ten times greater than the Earth, as it is ten times higher from the Sun than the Earth, and so with the others. This Johannes Remus Quietanus [Ioannes Remus Quietanus] chose, because, as Kepler admits above, on p. 485, the observations of the apparent diameters favour it, and so do the true sizes collected from those observations and the distances. And so Kepler yields to Remus and to the observations, especially since it is not at odds with archetypal reasons that, just as earlier the proportion of the bodies of the Sun and the Earth, and of the Earth and the Moon, was made the same as that between the semidiameter of the Earth and the semidiameters of the lunar orbit and of the Earth's annual orb, so the proportion between the bodies of the planets should be the same as that between the semidiameters of their orbs, or between their mean distances from the Sun.

So Saturn will be nearly ten times greater than the Earth, Jupiter five times, Mars one and a half times; but Venus will be a little greater than three-quarters [dodrante], or 9 parts of 12, of the Earth's body, and Mercury a little greater than a third of the same body; that is, it will contain nearly a third part of the Earth.

[Margin: The proportion between the bodies and the intervals of the 5 planets is established.]

A further reason for yielding to the observations was that Kepler admits that acronychal Jupiter has often been seen by him, at the perigee of the eccentric, to occupy about 50″, where Saturn, by the testimony of Remus, occupies 30″. Acronychal Mars at perigee in Aquarius, moreover, is somewhat larger than Jupiter, but not by much. But about these appearances the reader will judge more surely from our observations, to be presented below in Chapter 10.

De Proportione inter Molem & Denſitatem Corporum Planetariorum, iuxta Keplerum.

On the proportion between the mass and the density of the planetary bodies, according to Kepler.

XIV. On p. 487 of the *Epitome*, Kepler lays down the following about this proportion. First, that the same density, or quantity of matter, does not suit these 6 principal bodies of the world, since the dissimilarity of the periodic times is an argument for dissimilar density.

[Margin: 1st proposition, on dissimilar density.]

Second, that the Sun is the densest of all the bodies of the whole world, because its immense light and active force cannot exist without a proportionate subject, that is, a very great quantity of matter compressed into a small mass. Thus in the *Astronomia Optica*, Chapter 6, he had said that as much matter was once heaped together by God into the one Sun as there is in the rest of the universe. It is fitting, then, that the closer a body is to the Sun, the denser it should be.

[Margin: 2nd proposition: the Sun, for Kepler, the densest of all bodies.]

Third, that the proportion between the densities of these bodies is not the same as that between their intervals or masses; for this would offend against the law of variety, and the quantity of matter would be the same in all the planets. For if, for example, the density of Saturn were to the density of Jupiter as 10 to 5, as its distance and mass are to the distance and mass of Jupiter, then, when the mass of Saturn, 10, is multiplied by the density of Jupiter, 5, a density of 50 would come out, as much as would result if the mass of Jupiter, 5, were multiplied by the density of Saturn, 10. But it seems more congruous that the density should be different, as I said in the first place, and that in the same order in which they recede from the densest centre of the world, in that same order, but not in the same proportion, they should vary in density.

[Margin: 3rd proposition, on the proportion of densities and masses, to be rejected.]

Fourth, therefore, he affirms that the proportion of density is exactly half the proportion both of the intervals and of the masses, and one and a half times that of the diameters, which are in the globes, and three-quarters that of the surfaces. For in this way it comes about that Saturn, which is twice as high as Jupiter, is one and a half times heavier or slower, and one and a half times rarer than Jupiter. There is added another geometrical elegance, namely that the density or quantity of matter is the mean proportional between the mean distances of two planets from the Sun, or the semidiameters of their orbits, or is taken from the mean proportional.

[Margin: 4th proposition, on the proportion of densities and masses, to be chosen.]

On this basis, both here, on p. 490 of the *Epitome*, and in his book in the German language on weights and measures, he presents the following small

table, as an index of the proportion of density among the principal bodies of the world, with metals or stones added that are similar to them in density.

Proportion of density among the bodies, from Kepler.

♄	324	*Hardest gem*
♃	438	*Lodestone*
♂	810	*Iron*
Earth	1000	*Silver*
♀	1175	*Lead*
☿	1605	*Mercury* [Hydrargyrum]
☉	1800	*Gold*
or	1900	*Gold*

De Proportione Magnitudinis & Interualli inter tres præcipuas Mundi Regiones.

On the proportion of size and interval among the three principal regions of the world.

XV. At the centre of the world, as Kepler would have it, is the Sun; and the space occupied by the body of the Sun Kepler calls the first region of the world. Around this is the space extending to the greatest distance of Saturn from the Sun, which is the second region, called the region of the moving bodies [Mobilium]; so the sphere of the whole planetary system embraces the Sun and this space. The remaining space, as far as the outermost surface of the fixed stars, is the third region, containing in itself the two preceding, and it is called the sphere of the fixed stars. That space, however, which extends from the Sun, the Sun excluded, to the concave surface of the sphere of the fixed stars, he calls the ethereal air [Auram ętheriam]; the remaining space, which lies between the concave and the convex surface of the sphere of the fixed stars, he calls the walls of the world, or its skin, or its tunic, or the crystalline orb.

He distributes the intervals of these from p. 491 on, as follows. From the centre to the surface of the body of the Sun there are 15 semidiameters of the Earth; from the surface of the Sun to the centre of the Earth, 229 semidiameters of the body of the Sun; and from the centre of the Sun to the distance of Saturn from the Sun, nearly ten times this interval, that is, 2290 semidiameters of the Sun, or at any rate 2000, because the mean distance of Saturn is not exactly tenfold, but nearly tenfold, the interval of the Earth from the Sun. From the Sun to the fixed stars, moreover, he sets 4000,000 semidiameters of the Sun, or forty times a hundred thousand semidiameters of the Sun. So, when 229 is multiplied by 15, there result from the Sun to the Earth 3454 terrestrial semidiameters; if you add to these 15 for the semidiameter of the Sun, there result from the centre of the Sun to the Earth 3469 semidiameters of the Earth. And when 2000 is multiplied by 15, there result 30000 semidiameters of the Earth from the Sun to Saturn. Finally, when 4000,000 is multiplied by 15, there result 60,000,000 terrestrial semidiameters from the Sun to the fixed stars, that is, six hundred

times a hundred thousand terrestrial semidiameters. And the same reasoning holds for whole diameters of the Earth with respect to the diameter of the sphere of the moving bodies, or of Saturn, or to the diameter of the sphere of the immovable bodies, or of the fixed stars.

The foundation of these intervals is that he thinks it fitting that, just as the distance of the Moon from the Earth is the mean proportional between the semidiameter of the Earth and the semidiameter of the Earth's annual orb, and the orb of the Moon is the mean proportional between the orb or body of the Earth and the annual orb of the Earth itself, so the semidiameter of the sphere of the moving bodies, or the distance of Saturn from the Sun, should be the mean proportional between the semidiameter of the body of the Sun, which is the centre of the world and the source of motion, and the semidiameter of the sphere of the immovable fixed stars, which is the place containing all things; and by the same reasoning, the sphere of the moving bodies, or of the planetary system, should be a mean proportional between the body of the Sun and the body of the sphere of the fixed stars.

[Margin: The foundation of these intervals.]

He had said above, moreover, that the distance of Saturn from the Sun is nearly tenfold the distance of the Earth from the Sun, and so contains the Earth's distance, which is 229 solar semidiameters, ten times, that is, nearly 2290, or in round and more exact terms 2000. So, when the proportion is made, as 1 semidiameter of the Sun to the 2000 semidiameters of the Sun contained in the semidiameter of the sphere of Saturn, so 2000 to another number, there result 4000,000 semidiameters of the Sun for the semidiameter of the sphere of the fixed stars. These, multiplied as I said by 15, give 60,000,000 semidiameters of the Earth. Thus it comes about that the semidiameter of the sphere of the fixed stars contains the semidiameter of the orb of Saturn two thousand times, and this is its square root. But since spheres are to one another in the triplicate ratio of their diameters, it follows that the body of the Sun is to the body of the sphere of the fixed stars as 1 Sun to 64000,000,000,000,000,000 bodies equal to the Sun, and to the sphere of Saturn as 1 to 8000,000,000. Further, from this distance of the fixed stars and the whole diameter of the Earth's annual orb, it comes about that the parallax of the annual orb turns out for him to be 12″. But we have shown in Book 6, Chapter 9, no. 11, that he erred in this very thing.

De Proportione Denſitatis ſeu Copiæ Materiæ cum Interuallo & Magnitudine Trium præcipuarum Mundi Regionum, & de Craſſitie tum Sphæræ ſolius Fixarum, tum Cæli Cryſtallini, iuxta Keplerum

On the proportion of density, or quantity of matter, with the interval and size of the three principal regions of the world, and on the thickness both of the sphere of the fixed stars alone and of the crystalline heaven, according to Kepler.

XVI. Since for Kepler the Sun is the centre of the world, he therefore thinks that from the founding of the world the whole matter of the world's machine was divided into three parts. One of them was compacted into the body of the Sun, so that there should be as much matter in it alone as there is in all the rest of the ethereal air. Another third part was likewise thinned out and spread through the immense space of the world, which extends from the Sun to the concave surface of the sphere of the fixed stars. The remaining third part was expanded into an orb and placed around the world on the outside like walls, or a skin; its thickness runs from the concave to the convex sphere of the fixed stars, and this he calls the crystalline heaven. He assigns it no greater thickness than about two German miles, or 8 Italian miles. Its density he makes a mean proportional between the density of the body of the Sun and the density of the whole ethereal air; and so also the space occupied by it, or the mass of the body contained between the concave and convex sphere of the fixed stars, he makes a mean proportional between the space occupied by the body of the Sun and the space occupied by the ethereal air. Above, moreover, the proportion of the body of the Sun, as regards space or mass, to the body of the ethereal air, or of the sphere of the fixed stars up to its concave surface, was set out as the proportion between 1 and 64000,000,000,000,000,000. Between these numbers or terms the mean proportional is 8000,000,000. That, therefore, is how great the mass of that crystalline heaven is, or the thickness of the heaven containing the fixed stars; and so from it there could be made 8000,000,000 bodies equal to the Sun in mass. If this were reduced and coagulated into one spherical lump, it would make a sphere whose semidiameter would be greater than the Sun's semidiameter 2000 times, that is, greater than 30000 semidiameters of the Earth, granted that, spread out as it is, it is no thicker than one six-thousandth part of one solar semidiameter, or about two German miles. For the cube root of the compound number 64000,000,008,000,000,001 is 4000,000 1/6000.

Summa Propoſitionum & quaſi Axiomatum Kepleri.

Summary of Kepler's propositions and, as it were, axioms.

1. Proposition. *As one semidiameter of the Earth is to the interval of the Earth from the Sun, so is the one mass or body of the Earth to the body of the Sun; and conversely: or, as many times as the Earth's semidiameter is contained in the Earth's distance from the Sun, so many times is the body of the Earth contained in the body of the Sun.* Kepler, p. 478 of the *Epitome*.
2. Proposition. *As the Moon's distance from the Earth is to the Earth's semidiameter, so is the body of the Earth to the body of the Moon.* Kepler, p. 478 of the *Epitome*.
3. Proposition. *The Moon's distance from the Earth, or the semidiameter of the Moon's orbit, is a mean proportional between the semidiameter of the body of the Sun and the semidiameter of the annual orb, or the distance of the Earth from*

the Sun. Kepler, p. 482 of the *Epitome*. Therefore the Moon's distance from the Earth is the square root of the Earth's distance from the Sun.

4. Proposition. *The Moon's orbit, in its mass, is a mean proportional between the body of the Earth and the body of the Earth's annual orb.* Kepler, p. 491 of the *Epitome*.
5. Proposition. *If the surfaces of the bodies of two primary planets were to one another as their intervals from the Sun are to one another, the ratio of the bodies would turn out sesquialteral to the ratio of the intervals.* And so Saturn would be thirty times greater than the Earth and Jupiter twelve times, just as it is also slower. Kepler, p. 484 of the *Epitome*. Outright, however, he chose the following analogy.
6. Proposition. *As the intervals of two primary planets from the Sun are to one another, or as the semidiameters of their orbits are to one another, so is the body or mass of the one to the body or mass of the other.* And so Saturn is nearly ten times greater than the Earth, Jupiter five times, Mars one and a half times; and Venus has 3/4, Mercury 1/3, of the Earth. Kepler, p. 485 of the *Epitome*.
7. Proposition. *The semidiameter of the sphere of the moving bodies, or Saturn's distance from the Sun, is a mean proportional between the semidiameter of the body of the Sun, taken as the centre, and the semidiameter of the sphere of the immovable bodies, or fixed stars; therefore Saturn's distance from the Sun is the square root of the distance of the fixed stars from the Sun.* Kepler, p. 491 of the *Epitome*.
8. Proposition. *The body of the region of the moving bodies, or of the planetary system, is a mean proportional between the body of the Sun and the body of the sphere of the fixed stars, or of the whole world.* Kepler, p. 491 of the *Epitome*.
9. Proposition. *The nearer each of the planets is to the Sun, the densest of all bodies, the denser it is, in the same order but not in the same proportion of the interval; and the farther from it, the rarer.* Kepler, p. 487 of the *Epitome*.
10. Proposition. *The ratio of density between the bodies of two planets is half the ratio of their masses, and one and a half times the ratio of their diameters, and three-quarters of the ratio of their surfaces.* Kepler, p. 488 of the *Epitome*.
11. Proposition. *There is as much of the world's matter in the Sun alone as in the whole ethereal air, and just as much again in the thickness of the crystalline heaven containing the fixed stars.* Kepler, p. 495 of the *Epitome*.
12. Proposition. *Just as the density of the crystalline, or star-fixing [ſtellifixeri], heaven is a mean proportional between the density of the Sun and the density of the ethereal air, so the thickness or mass of the crystalline is a mean proportional between the mass of the Sun and the mass of the ethereal air.* Kepler, p. 496 of the *Epitome*.

Keplerian numbers, rounded however, or derived from Kepler's, and bearing on the intervals or the mass of the bodies of the world.

Contains	Semidiameters of the Sun		Semidiameters of the Earth	
☾ semidiameter	0	1/60	0	1/4
☉ semidiameter	1	0	15	0
Semidiameter of the Moon's orbit	4	0	59	0
Semidiameter of the annual orb	229	0	3469	0
Semidiameter of the orbit of ♄	2290	0	34690	0
But in round figures, etc.	2000	0	30000	0
Semidiameter of the sphere of the fixed stars	4000,000	0	60,000,000.	0
Interval from Saturn to the fixed stars	3998000	0	59970000.	0
From the concave to the convex of the crystalline	0	1/6450	0	1/430

The bodies listed below contain	The Sun, times		The Earth, times		The Moon, times
☾ Moon	0	1/204671	0	1/59	1
☿ Mercury	0	1/10407	0	1/3	20
♀ Venus	0	1/4836	0	3/4	45
Earth	0	1/3469	1	0	59
♂ Mars	0	1/2647	1	1/2	89
♃ Jupiter	0	1/690	5	0	295
♄ Saturn	0	1/345	10	0	590
☉ Sun	1	0	3469	0	204671

Contains	The Sun, times	The Earth, times
The Moon's orbit	64	222016
Saturn's orbit	8000,000,000	27752,000,000,000.
The crystalline heaven	8000,000,000	27752,000,000,000.
The space between the convex of ♄ and the concave of the fixed stars	63999,999,984,000,000,000.	2469999,944,496,000,000,000.
The whole sphere of the world	64000,000,000,000,000,000.	2220000,000,000,000,000,000.

SCHOLIA.

Scholia.

[Margin: Our distance is confirmed from Kepler's doctrine.]

I. *The arguments of Kepler, discussed in the preceding chapter toward the end of number 11, for raising the Sun three times more than the ancients did, tell more in favour of our distance of the Sun and of the planets that depend on it in this. For if the Sun is distant from the Earth by only 3469 semidiameters, and acronychal Mars by about 1400 or 1500, as follows from his hypothesis, the Sun will certainly have a horizontal parallax of 1′ and Mars of 2′, each of which is perceptible. But if they are removed from the Earth twice as far and more, as we do, it is much truer that the parallax of the Sun and of Mars is imperceptible, or scarcely perceptible. Nor on that*

account will that property collapse, that the Sun is distant from the Earth by about 229 semidiameters of its own body because it is seen by us under an angle of about 30′, or occupying an arc that is the 720th part of the circle of the Sun's orb. For as the distance grows, with that angle preserved, the quantity of the Sun's true semidiameter grows in like manner. So from that reasoning the distance of the Sun from the Earth does not yet come out determined, since with that reasoning preserved the distance could be increased indefinitely. But that distance was chosen by Kepler so that the Moon's distance should be a mean proportional between the semidiameter of the Earth and the distance of the Sun from the Earth.

[Margin: Critique of the Keplerian analogies.]

II. *Although Kepler's analogies display their own beauty and their author's ingenuity, yet they rest for the most part on the hypothesis of the Sun at rest in the centre of the world, of the Earth going round the Sun as a primary planet, and of the Moon relegated among the secondary planets; and they savour of nothing but a certain material elegance of bare geometry and arithmetic, with no mention or account taken of the end and the effects that divine Providence chiefly laid down in determining the mass and the intervals of the planets. They therefore lose a great part of their likelihood, at least among those who both hold to the hypothesis of the Earth at rest and measure the proportion of intervals and sizes chiefly from the end. And if the mass of the planets constructed from these analogies conflicts with the more certain observations of the planets' apparent diameter, to be presented in Chapter 10, those otherwise beautiful analogies will collapse much more. That they do conflict we shall show in that chapter, scholia 1 and 2.*

[Margin: The proportion of the planetary masses, from Šírek.]

III. *Antonín Maria Šírek, in his* Radius Sidereomysticus*, gathers by another analogy the proportion that the masses of the planets have to one another. For in Book 4, chapter 2, member 4, he affirms that the masses of the planets other than the Sun are to one another as the square roots of their revolutions. Accordingly, having resolved the periodic times into hours, he seeks their square roots and compares them with one another. Thus, because Saturn's periodic time is 10747 hours and Jupiter's 4330, and the square root of the first is 107 and of the second 66, he says that the mass of Saturn is to the mass of Jupiter's body as 107 to 66; and so of the others, as you see in the following small table. From elsewhere, however, he takes the proportion of the mass of the Sun's body, which he makes 190 to the mass of the Earth; and so, since he compares the Sun with the remaining planets in that ratio, the result is that the mass itself of the planets is obtained if the square root of the periodic time is sought.*

Proportion of mass to mass	
☉	190
♄	107
♃	66
♂	26

Proportion of mass to mass	
♀	19
☿	9
☾	5

CAPVT IX.

Chapter 9.

Indicantur Modi, quibus obſeruatæ ſunt aut obſeruabiles Planetarum Minorum Diametri, & afferuntur Auctorum loca, & opiniones ipſorum de Apparenti ſimul ac Vera horum Magnitudine.

The methods are set out by which the diameters of the minor planets have been observed or can be observed, and passages of the authors are brought forward, with their opinions on both the apparent and the true size of these planets.

I. In Book 6, chapter 9, we have already set out eight methods used by others or by us for observing the apparent diameters of the smaller stars. After first giving six general causes of the eyes' deception in this matter, we then taught one by one how much deceptiveness or clarity there is in each of those methods. You must therefore read that whole chapter again, just as if it had been inserted here as well. On that assumption we shall only indicate those methods, and which of those 8 methods were used by the authors to be reviewed in the following tables.

[Margin: Method 1, deceptive.]

The *first* method, then, works by estimation with the eye, comparing the star's visible diameter with a portion of the Moon or with the distance of two fixed stars close to each other, for instance two of the Pleiades or of the Hyades. It is deceptive above all because of the adventitious rays that the naked eye, by itself or through the interposition of the medium, places around the stars, increasing their discs by a certain spurious surround of rays and taking just as much away from their distances from one another.

[Margin: Method 2, deceptive.]

The *second* method works by the slits or holes of sights and their distance from the eye. It is deceptive both for other reasons and chiefly because the visual angle is formed not at the surface of the eye, from which the distance to the sights is taken, but at the bottom of the eye. And so both the diameters of stars taken in this way and distances taken with a radius, whether Greek or Latin, are foisted on us larger than is right. All astronomers to whom the use of the telescope was unknown, that is, down to Tycho inclusive, are reckoned to have used one or other of these two methods; so it is no wonder if they erred so greatly, increasing the diameters of these planets.

[Margin: Method 3, deceptive.]

The *third* method, by a thread hung perpendicularly whose thickness is known and by the withdrawal of the eye from it until the star is exactly covered by the thread, which Galileo used on occasion, has already been rejected or corrected by us in Book 6, chapter 9, number 3.

[Margin: Method 4, good.]

The *fourth* method works with the help of the telescope, which strips the stars of their adventitious hair-like fringe [capillamentum]. The capacity of the tube must first have been determined, whether by the method that *Galileo* gives in the *Sidereus Nuncius*, p. 7, though it is little suited to celestial objects, or from what fraction of the Moon it takes in, or from a known distance of two fixed stars taken in by it. One then observes what part of that capacity the apparent diameter of the fixed stars occupies. This method was used by *Martinus Hortensius*, *Galileo*, *Gassendi*, *Scheiner*, *Kepler*, and in his work *Io. Remus Quietanus* [Johannes Remus Quietanus]; and by us too, especially on Venus and Saturn, but after first dividing the diameter of the tube's capacity into 4 parts and comparing the planet's diameter with that, for in this way, as *Father Grimaldi* advises, the estimate is clearer. Our telescope, moreover, using a lens convex on both faces at each end, when tested on the Moon, takes in 30′.

[Margin: Method 5, suspected of deceptiveness.]

The *fifth* method works by receiving the bright image of Jupiter or Venus through one or more glasses of the telescope on clean paper, measuring the diameter of that image and comparing it with the diameter of the Sun's image, received with the same glasses and at the same distance of the paper from the glass and divided into very small particles. Hortensius used this method on Jupiter and Scheiner on Venus. But *Pierre Gassendi*, together with the image of the Sun, caught on paper the black image, or little shadow, of Mercury seen under the Sun, and from comparing it with the Sun's diameter he defined its apparent quantity: see the examples below in chapter 10, scholia 4 and 5.

[Margin: Method 6, ours.]

The *sixth* method is carried out with our two radii, the triangular and the scalar, on which see Book 6, chapter 9, number 6; and it serves at least for judging within what nearest limits the quantity of the apparent diameter of stars smaller than the luminaries lies.

[Margin: Method 7, ours, the best.]

The *seventh* method requires the approach of Saturn or Jupiter to some fixed star with so small a difference of latitude that not only can the planet be taken in with the fixed star in the same view through the spyglass, but it also comes as close to the star as possible. How the quantity of the apparent diameter is deduced with the greatest certainty from the planet's true daily or hourly motion, and from its distance from the fixed star on the day immediately before or after its conjunction with that star, we shall teach better by example than by a longer precept in the next chapter, numbers 2 and 8. We used this method on ♄ and ♃, and D. Vicenç Mut on ♃.

[Margin: Method 8, very good.]

The *eighth* method, by various little circles prepared on paper and the comparison with them of the discs of stars viewed through the telescope, so that the little circle is chosen that of all of them appears to fit the star's disc most closely, and is compared by compasses with the diameter of Jupiter or Saturn already known by method 4 or 7, has been used by us, with the attestation and help of the eyes of several people, to whom the same circle appeared, among the others, most nearly equal to the star's disc. The planets' apparent diameters established by these methods we have therefore put into tables, deducing from them the true size by the problems of chapter 8.

To this method or to the fifth can be reduced the comparison of a star's disc copied onto paper and then compared with one of the smaller spots of the Moon, the apparent quantity of which spot is known from our *Selenographia*, Book 4, chapter 7, and from the Appendix bearing on that chapter, from its proportion to the Moon's apparent diameter. But if we want to rely on spots of the Moon already drawn elsewhere, the observation turns out more deceptive, since those spots vary in apparent size with the varying libration, latitude and altitude of the Moon and with the varying illumination from the Sun according to the Moon's various phases. And if, at the same time as we compare the star with the spots, the proportion of the spot to the Moon's diameter has to be determined exactly anew, too much labour is laid on astronomers and the opportunity will be rarer. For these reasons, when I had tried some observations of this kind, I set them below the others, which are much more certain. But just as this leaf was to go under the press, there at last reached our hands the *Selenographia* of Johannes Hevelius, who in chapter 55, p. 447, prefers this method to all others, but older ones (for our others he has not yet seen), and from it brings out the diameters of certain stars. On this occasion he advises that a perforated metal leaf, whose hole is as wide as a pea, be placed over the lens farther from the eye, so that the stars may in this way be better stripped of their adventitious rays and appear rounder or bounded by a more certain edge, especially those that radiate a more vivid light, like Venus, Mercury and Sirius.

The observations of Gassendi, Hortensius and Scheiner, moreover, we shall report in chapter 10, scholia 4 and 5, and our own in chapter 10.

Auctores ex quibus ſequentes Tabulæ collectæ ſunt.

The authors from whom the following tables have been gathered.

II. The authors from whom we have gathered the following tables, with the utmost good faith, are: *al-Battānī*, chapter 50; *al-Farghānī*, section [differentia] 22; *Jean Fernel*, Book 1 of the *Cosmotheoria*, and, following him closely, *Egnazio Danti* in his mathematical tables, from table 22; *Wurstisen* on Peuerbach's *Theoricae*, p. 355, and, reporting the same numbers, *Giovanni Antonio Magini* in the *Theoricae Planetarum*, Book 3, chapter 19; *Francesco Maurolico*, in the small tables added to dialogue 3 of his *Cosmographia*, following al-Farghānī closely; *Clavius* on chapter 1 of Sacrobosco's *Sphere*, p. 186 in my copy, mostly following

Maurolico and adding some things of his own; *Francesco Barozzi*, Book 1 of the *Cosmographia*, p. 50, and Book 4, p. 337; *Tycho*, volume 1 of the *Progymnasmata*, from p. 468 to 475; *Redento Baranzano*, *Uranoscopia*, part 1, p. 123, and part 2, p. 116, following now Clavius, now Tycho; *Giuseppe Biancani* in his *Sphaera*, in the several books on the planets and in the tables at the end of the *Sphaera*, following Tycho; *Scheiner* in the *Sol Ellipticus*, p. 25, in the *Disquisitiones Mathematicae*, number 42, and in Letter 1 to Markus Welser; *Galileo*, dialogue 3 of the *De Mundi systemate* and *De Maculis solaribus*, p. 110; *Kepler* in the *Epitome of Copernican Astronomy*, volume 2, p. 485, reporting the opinion of *Johannes Remus*, in the *Optics*, p. 333, and in the Rudolphine Tables, precept 110; *Lansbergen*, Book 3 of the *Uranometria*; *Martinus Hortensius* in the *Dissertatio* with Pierre Gassendi, *De Mercurio sub Sole viso & Venere inuisa*; *Pierre Gassendi* in the *Life of Nicolas-Claude Fabri de Peiresc*, and in the *Institutio Astronomica*, Book 2, chapter 14, where he also reports the opinions of the ancients, and Book 3, chapter 11, where he brings forward those of Lansbergen and Hortensius; *Andrea Argoli* in the *Pandosion Sphaericum*, from chapter 52 to 59, following Tycho and Biancani; *Pierre Hérigone*, volume 4 of the *Cursus Mathematicus*, pp. 60 and 62, where, though borrowing the apparent semidiameters from Hortensius, he took the true sizes from elsewhere, and volume 5, p. 573, where he follows the Keplerian proportions; *Antonín Maria Šírek* in his *Radius Sidereomysticus*, Book 4, chapter 2, members 4 and 5; *Athanasius Kircher* in the *Ars Magna Lucis et Umbrae*, volume 2, after p. 761; and *D. Vicenç Mut* in a letter written to me on the diameter of Jupiter, a fragment of which I shall quote in the next chapter.

[Margin: Praise of Hortensius and Mut, and Lansbergen's boasting.]

Among these no one came closer to the truth than *Hortensius* and *Mut*. Yet Hortensius erred greatly on Saturn at perigee and likewise on Venus at perigee, perhaps because he did not have the chance of observing both at perigee, as we did, according to what is to be said in chapter 10. And that *Lansbergen* too pronounces in this place that there is no doubting his numbers arose from excessive confidence, which he did not think could be blunted by plain observations.

[Margin: On the tables of the sizes of the ☾, the ☉ and the fixed stars.]

Further, I have given tables and authors for the size of the Sun in Book 3, chapter 10, number 12, and chapter 11, number 1; for that of the Moon in Book 4, chapters 16 and 17; and for the size of the fixed stars in Book 6, chapter 9. From these, assuming with al-Battānī and al-Farghānī that the Sun's apparent diameter at mean distance is 31′ 20″, it is easy to gather the apparent diameters of the five minor planets. For they say that the diameter of Saturn, as regards appearance, is the eighteenth part of the Sun's diameter, Jupiter's the twelfth and Mars's the twentieth. But on Venus and Mercury they disagree: al-Battānī makes Venus's diameter the tenth part and Mercury's the fifteenth part of the Sun's diameter, whereas al-Farghānī makes Venus's the twentieth and Mercury's the twenty-fifth.

Table 1. Apparent diameters of the minor planets: mean, at the planet's mean distance from the Earth; or minimum at apogee and maximum at perigee.

Authors	Apparent diameter	♄ ′	♄ ″	♄ ‴	♃ ′	♃ ″	♃ ‴	♂ ′	♂ ″	♂ ‴	♀ ′	♀ ″	♀ ‴	☿ ′	☿ ″	☿ ‴
al-Battānī	Mean	1	44	28	2	36	40	1	34	0	3	8	0	2	5	20
al-Farghānī	Mean	1	44	28	2	36	40	1	34	0	1	34	0	1	15	12
Jean Fernel	Mean	1	40	6	2	30	50	1	29	0	5	0	0	2	12	0
Egnazio Danti	Mean	1	40	0	2	32	0	1	28	0	4	0	0	2	12	0
Wurstisen & Magini	Minimum	5	0	0	8	0	0	6	0	0	9	0	0	5	0	0
	Maximum	7	0	0	11	0	0	10	0	0	12	0	0	9	0	0
Tycho, Clavius, Biancani, Argoli	Mean	1	50	0	2	45	0	1	40	0	3	15	0	2	10	0
Lansbergen	Mean	0	50	0	1	15	0	0	45	0	1	30	0	1	0	0
Martinus Hortensius, and with him *Hérigone* and *Gassendi*	Minimum	0	31	0	0	38	30	0	9	0	0	15	20	0	10	0
	Mean	0	37	0	0	50	0	0	36	0	0	53	0	0	19	0
	Maximum	0	42	40	1	1	40	1	4	0	1	40	0	0	28	0
Hortensius on ♄ with its companions	Maximum	1	0	0												
Kepler	Maximum	0	30	0	0	50	0	6	30	0	7	6	0		✢	
Galileo	Minimum		✢			✢			✢		0	10	0		✢	
Scheiner in Hortensius	Minimum										0	34	0			
	Mean		✢			✢			✢		2	40	0		✢	
	Maximum										3	42	0			
Antonín Maria Šírek	Mean	2	0	0	3	0	0					✢				
Father Grimaldi and we; but ♄ with its companions	Minimum	0	46	0	0	38	18	0	10	6	0	33	30	0	9	20
	Mean	0	57	0	0	49	46	0	22	0	1	4	12	0	13	48
	Maximum	1	12	0	1	8	46	1	32	0	4	8	0	0	25	12

Table 2. True diameters of the minor planets, expressed by the times, that is, by the whole or fractional numbers, by which the true diameter of each planet contains the Earth's diameter.

Authors	♄		♃		♂		♀		☿	
Ptolemaics: *al-Battānī*	4	1/14	4	1/3	1	1/9	0	3/10	0	1/20
Ptolemaics: *al-Farghānī*	4	1/2	4	4/7	1	1/6	0	4/9	0	1/10

Authors	♄		♃		♂		♀		☿	
Ptolemaics: *Jean Fernel*	2	3/4	9	1/8	2	1/3	0	53/60	0	4/60
Francesco Maurolico, Clavius, Barozzi	4	1/2	4	4/7	1	1/6	0	3/10	0	1/28
Tycho, Biancani, Kircher	2	9/11	2	2/5	0	5/12	0	6/11	0	3/8
Kepler	2	1/5	1	7/10	1	15/100	0	30/33	0	5/7
Baranzano	4	1/2	4	1/3	1	1/6	0	3/10	0	1/26
Lansbergen	3	6/16	2	94/100	0	496/10000		653/1000	0	435/1000
Pierre Hérigone	2	82/100	2	40/100	0	42/100	0	55/100	0	37/100
We, but ♄ with its companions	20	1/6	8	4/5	0	52/100	1	15/100	0	1/4

Antonín Maria Šírek, in Book 4, chapter 2, gives the diameters of the planets in hourly leagues [Leucis horarijs], of which the Earth's diameter is 2000: namely ♄ 11262, ♃ 6946, ♂ 2736, ♀ 1578, ☿ 946, ☾ 490, ☉ 20000.

Table 3. Solidity or bulk of the minor planets, expressed by the times, that is, by the whole or fractional numbers, by which the body of each planet contains the body of the terrestrial globe, or a tiny portion of it.

Authors	♄		♃		♂		♀		☿	
al-Battānī	79	0	81	0	1	1/3	0	1/36	0	1/19000
al-Farghānī	91	0	95	0	1	5/8	0	1/28	0	1/22000
Jean Fernel	91	1/8	94	3[?]/39	1	5/9	0	1/11	0	1/21952
Francesco Maurolico	91	1/8	95	1/2	1	1/2	0	27/1000	0	1/21952
Clavius, and formerly *Baranzano*	91	1/8	95	1/2	1	1/2	0	1/37	0	1/21952
Barozzi	91	0	95	0	1	5/8	0	1/28	0	1/21952
Kepler	10	0	5	0	1	1/2	0	9/12	0	1/3
Tycho, Biancani, Andrea Argoli	22	0	14	0	0	1/13	0	1/6	0	1/19
Hence *Baranzano* and *Kircher*	22	0	14	0	0	1/13	0	1/6	0	1/19
Lansbergen	46	2/3	25	2/5	0	1/8	0	1/3	0	1/12
Hortensius	2	1/5	0	4/5	0	1/1534	0	1/1109	0	1/6510
Pierre Hérigone	22	0	14	0	0	8/100	0	17/100	0	5/100
Antonín Maria Šírek	165	0	42	0	3	0	0	1/2	0	1/8
We, but ♄ without its companions	891	0	685	0	0	14/100	1	1/2	0	1/256

In using the preceding table, when only fractional numbers are given, the numerator of the fraction indicates what part the planet holds in the body of the Earth; the denominator of the fraction, that is, the number below the little line, signifies the times that the Earth contains the planet. For example, according to *al-Battānī* the body of Mercury holds only one single part of 19000 parts of the Earth; and conversely the Earth contains 19000 bodies equal to Mercury.

Table 4. Proportion of the diameter of the planets to the diameter of the Earth.

Authors	♄ as	to	♃ as	to	♂ as	to	♀ as	to	☿ as	to
Clavius and *Barozzi*	9	2	32	7	7	6	3	10	1	28
Pierre Hérigone	31	11	12	5	5	12	6	11	3	8
Athanasius Kircher	31	11	12	5	6	11	6	11	3	8

Table 5. Proportion of the solidity of the planets to the solidity of the Earth.

Authors	♄ as	to	♃ as	to	♂ as	to	♀ as	to	☿ as	to
Clavius	729	8	32768	643	343	216	127	1000	1	21952
Baranzano	729	8	32768	343	343	216	27	2000	1	21952
Pierre Hérigone	22	1	14	1	1	13	1	6	1	19

You see opinions mixed together, and Clavius's figures corrupted by Baranzano.

Table 6. Proportion of the solidity of the planets to the solidity of the Sun or ☾.

Authors	To	♄ as	to	♃ as	to	♂ as	to	♀ as	to	☿ as	to
Biancani	☉	1	6 4/21	1	10	1	1820	1	840	1	2918
Biancani	☾	180	1	560	1	40	13	7	1	40	19
Šírek	☉	107	190	66	190	26	190	19	190	9	190

III. Add to the preceding tables the small Keplerian tables set at the end of chapter 7. And if you wish to have the solidity of the planets in cubic miles, see in Book 2, chapter 7, how many cubic miles one single Earth contains, and multiply that number by the number of times by which the planet contains the Earth; or, if the planet is smaller than the Earth, divide the Earth's cubic miles by the number by which the Earth contains the planet. If you wish to have the true diameter of the planets, see likewise in Book 2, chapter 7, how many miles are contained in the Earth's diameter, etc. I am pleased, however, to set here the Italian miles to which Father Athanasius Kircher reduced Kepler's measures, in his *Magnes*, p. 561.

The Keplerian size of the bodies of the world in Italian miles, from Father Athanasius Kircher.

	Diameter in Italian miles	Body in cubic Italian miles
Earth	7000	343,000,000,000
☾	2281	11867,954,041
☿	377	21253,933
♀	3766	41637,598,696
☉	42900	78,953,589,000,000
♂	9100	753,571,000,000
♃	35620	45194,100,328,000
♄	15100	43243,551,000,000

CAPVT X.

Chapter 10.

Noſtræ & Aliorum Obſeruationes Diametrorum Apparentium Quinque Minorum Planetarum cum vera Magnitudine, tam ſecundùm noſtras, quàm ſecundùm aliorum Diſtantias.

Observations, our own and others', of the apparent diameters of the five minor planets, with their true size, according both to our distances and to those of others.

I. Just as I once lamented that recent astronomers had hunted the Sun's distance from the Earth in no other way than through conjectures from eclipses, a method I showed in Book 3, chapter 7, to be most uncertain; and just as, therefore, still unaware of Wendelen's attempt, I searched by another, more certain inquiry for the solar distance, on which the distance of the five minor planets depends, and found it proportional, at almost the middle position, between the distance that Kepler finally raised in the *Epitome of Copernican Astronomy* to 3469 semidiameters of the Earth (driven to it by the parallax of Mars at its acronychal position, scarcely perceptible and yet then exceeding the solar parallax) and the one that Wendelen carried up to 14905, for the reasons adduced in that same Book 3, chapter 7; and just as I was no less concerned to investigate the apparent diameter of the luminaries in several ways, as is clear from Book 3, chapter 9, and Book 4, chapter 16: so too I was long concerned about the diameters of the remaining planets and of the fixed stars. For I saw that the ancients, with great and gross errors, had obscured astronomy on this matter rather than illuminated it, while more recent writers, who were not ignorant of the use of the telescope, do come closer to the truth, yet still disagree among themselves; and I did not want to depart from anyone's opinion without grounds. Yet I could not rest content with the observations taken with our radii, set out in Book 6, chapter 9, number 6, nor with others made by the telescope and by projecting the images of each planet onto paper, because no occasion had yet presented itself of determining without misgiving the diameter of one of these planets, and by its standard the diameters of the rest. Later, however, GOD granted that occasion to *Father Francesco Maria Grimaldi* and to us; and so we now go on with a livelier spirit to advance astronomy in this area, making our start from Jupiter, and having selected from among a good many observations the clearest of all.

Obſeruatio Diametri Iouis.

Observation of the diameter of Jupiter.

II. In the year of Christ 1649, while Father Francesco Maria Grimaldi of our Society was observing the zones of Jupiter with a truly excellent telescope 11 Roman feet long, he noticed, when June had already begun, that Jupiter was drawing daily closer to that star of the fourth magnitude which is the leading

one of the four stars in the left wing of Virgo, and the sixth among the stars of the constellation of the Virgin in the Ptolemaic, Tychonic and Rudolphine catalogues. According to the same catalogue, that star has a northern latitude of 1° 25′ and at that time a longitude, that is, an ecliptic place, in ♍ 29° 56′ 25″. Jupiter too was at that time, according to Montebruni's Lansbergian ephemeris, at the end of Virgo, with a northern latitude of about 1° 25′. He therefore had no doubt that Jupiter, in conjunction with that star, or on the day immediately before and the day after that conjunction, could be seen together with that star at a single glance within the compass of the telescope, and that from its distance from the star and its daily motion one could pronounce with the greatest certainty on its apparent diameter. When he had told me of this opportunity, so that I might be present to assist in and test the observation, he was thereafter, on those days, wholly intent on this observation, which we shall set out more clearly with the schematism attached.

Diagram. Two equal circles, each a disc of Jupiter, stand at left and right in the right column, with a single sign for Jupiter above each. A horizontal straight line runs from K on the left circle through its centre C, then through D at the mid-point of the figure, to R on the right circle, passing through that circle's centre B. In each circle a hatched band lies above the horizontal line and parallel to it: NL in the left circle, PM in the right. A star-shaped point A stands at the foot of the figure; a vertical line rises from A to D, a straight line runs from A to C, and a straight line runs from A through G, H and B to F at the top right of the right circle. The four quarters of the sky are engraved in italic: Septentrio (North) above, Meridies (South) below A, Oriens (East) at lower left, Occidens (West) at lower right.

- N, L (left circle, ends of the upper edge of the band): point labels
- K (left circle, left end of the horizontal line): point label
- C (left circle, centre): point label
- D (on the horizontal line, above A): point label
- P, M (right circle, ends of the band): point labels
- R (right circle, right end of the horizontal line): point label
- F (right circle, upper right, end of the line from A): point label
- B (right circle, centre, on the line from A): point label
- H (right circle, on the line from A between G and B): point label
- G (right circle, lower left limb, on the line from A): point label
- A (foot of the figure, at the star): point label
- ♃ (above each circle): the sign of Jupiter
- Septentrio, Meridies, Oriens, Occidens: North, South, East, West

All labels read; none unread.

[Margin: The scheme of the observation is set out.]

Let A be the leading star of the four in the left wing of Virgo. Let B be the centre of Jupiter when it was more westerly than that star, that is, the centre of the disc or circle of apparent Jupiter, FPG; and let C be the centre of the same when it had become more easterly. Let PM and NL be its bands, parallel to the

line RBCK drawn through the centres BC; these bands, as is established from several other observations, are sensibly parallel to the ecliptic, as is also the path of the Jovian satellites, and so RDC too was parallel to the ecliptic. Then from A, a star more southerly than Jupiter was, let a perpendicular arc AD be drawn to RK; and let the centres of both Jovian discs be joined to A by the straight lines AC and AB, with AB produced as far as F, so that GBF is the diameter of Jupiter. Let this diameter be understood as divided not only into two at B, but also into four at H, so that GH is a fourth part of it. This is how Father Grimaldi chose to have it, for a finer measure of the interval between the limb G and the star A.

[Margin: The observation itself is set out.]

III. On June 9, then, in the first hour after sunset, Jupiter appeared at B, more westerly and more northerly than the star A. The distance of the limb G from the star A, that is, the straight line AG, was plainly 7 parts of which GH was one, or 3½ of which GB, that is, Jupiter's semidiameter, was one; and the whole AB was 9 parts of which GH was one, or 4½ of which GB will be one. For by all these comparisons the size of the interval AB was confirmed. And since the spyglass showed Jupiter most brightly round and stripped of all adventitious hair of rays, and represented it far larger than you see it in this schematism (for there the proportion of the lines is represented, not the size of the true image), and since it was very close to the star A, the eye measured that space with as much clearness as if a hand were defining it with compasses on a paper laid beneath. Straightway on June 10, in the same first hour of the night, Jupiter appeared at C, more easterly and more northerly than the star A, but somewhat less distant from it than on the 9th: for the distance AC was exactly 8 parts of which GH was one, or 4 parts of which GB is one. Moreover, with the straight line BC drawn by estimation on each of the two days, which both to the bands of Jupiter and to the path of the four little Jovian planets and finally to the ecliptic was sensibly parallel, and with the perpendicular AD drawn to it, Jupiter, when with its centre it was at D, in conjunction with the star A, stood away from it towards the north, that is, was more northerly than it, by the line AD, which, all things considered, was estimated with certainty at 4⅓ parts of which GH was one.

[Margin: 1. Step for the size of the chord BC.]

First, then, by the doctrine of triangles, in the triangle ABD, right-angled at D, given the side AD of 4⅓ parts and the base AB of 9 parts, the side BD is gathered to be very nearly 8 parts, an imperceptible difference being disregarded. Again, in the triangle ACD, right-angled at D, given the side AD of 4⅓ parts and the base AC of 8 parts, CD is deduced to be as nearly as possible 6 parts. And so the chord, that is, the whole daily path of Jupiter, BC, was 14 parts of which GH, a fourth part of Jupiter's diameter, was one; or 7 parts of which the semidiameter BG is one; or equal to 3½ apparent diameters of Jupiter's disc.

[Margin: 2. Step for the apparent diameter of ♃.]

Second, the true motion of Jupiter from noon of the 9th to noon of the 10th was 2′ 30″; but from the first hour of the night of the 9th to the first hour of the night of the 10th it was 2′ 40″, that is, 160″. Let it be, then, as 3½ to 160″, so 1 to 46″. Therefore Jupiter's apparent diameter in that position was exactly 46″, and its semidiameter 23″. Again, let it be as CB of 14 parts to CB of 160″, so AD of 4⅓ parts to 49″. Jupiter was therefore more northerly than that star by 49″; and so, if the star's northern latitude is 1° 25′, Jupiter's northern latitude was 1° 25′ 49″ at the moment of conjunction, which fell on the 9th at 21^h after noon, and Jupiter's true longitude then was ♍ 29° 56′ 25″.

[Margin: The disagreement of the tables with the sky.]

Granted that the Prutenic tables then set the longitude of ♃ in ♍ 29° 40′, with a latitude of 1° 36′; the Danish tables in ♎ 0° 1′, with a latitude of 1° 33′; and the Lansbergian tables in ♍ 30°, that is, ♎ 0° 0′, with a latitude of 1° 24′; yet all of them set Jupiter's daily motion at 2′ 40″, or 2′ 42″, or 2′ 38″.

[Margin: 3. Step for the true diameter of Jupiter.]

Third, because the anomaly of the eccentric of Jupiter was 1° 55′ and the equated anomaly of the orb very nearly 248½°, Jupiter's distance from the Earth, by our hypothesis, was 39500 semidiameters of the Earth; and it was very nearly at the apogee of the eccentric and a little below the mean longitude of the epicycle. So from that distance, and from the angle of the apparent semidiameter, 23″, there is gathered by Problem 1 of chapter 8 a true semidiameter of Jupiter of 4 42/100 of which the Earth's semidiameter is one.

[Margin: The apparent semidiameter of Jupiter at the greatest, mean and least distances.]

And from this, by Problem 2 of the same chapter, from Jupiter's distances, the greatest 47552, the mean 36500 and the least 26441 semidiameters of the Earth, there is deduced an apparent semidiameter at the apogee of the eccentric and of the epicycle of 19″ 9‴, at the mean distance of 24″ 53‴, and at both perigees of 34″ 23‴. These numbers will result from any other hypothesis that preserves the proportion of the intervals to the Sun's distance, that is, to the radius of the annual orb.

[Margin: Observation of the diameter of Jupiter by D. Vicenç Mut.]

IV. When these things had already been most certainly established among us, there was brought to me, behold, a letter from the most illustrious D. Vicenç Mut, a truly outstanding astronomer, celebrated too for the *Sol Alphonsinus* he recently published, dated 1649 June 30, from Majorca. In it he reported to me, in exactly these words, that on the very same days as we, and yet suspecting nothing of our observation, he had observed the diameter of Jupiter. *I observed Jupiter with the leading star of the left 4 in the wing of* ♍*, on the 9th of this month, at* 10^h*. Jupiter was more westerly than the said leading star of the 4 in the left wing, and this star stood from its limb by, most exactly, one diameter of Jupiter itself. Moreover the star was more southerly than the line of the satellites by scarcely more than Jupiter's semidiameter, and seemed about to meet it, grazing the limb. On the following day at the same hour* ♃ *was already more easterly, and between those 24 hours Jupiter,*

by comparison with the star, was seen to have run through three diameters of its body, weighed with the highest precision. So Martinus Hortensius is not far wrong on Jupiter's diameter in this position, 48″. This I observed on the 9th and 10th of this month of June; but on May 11 Jupiter had been more easterly than the said star by almost 2′ or 1′ 30″, and they stood from each other a little less than the two contiguous stars in the second of the Hyades. For it had then been retrograde. So he wrote; I wrote back to him about our observation, and it is not easy to say with what joy we were filled at so happy an accord and an agreement so very close, down to the seconds of this diameter, although for just reasons I was compelled to follow our own observation instead.

[Margin: Šírek's error on the diameter of Jupiter.]

Compare also with ours the observations of Hortensius and Gassendi, on which see scholium 4 below, and you will see how false is the opinion of Antonín Maria Šírek, who says in Book 4 of his *Radius Sidereomysticus* that the diameter of Jupiter, from his own observation, is 3′.

Obſeruatio Diametri Martis.

Observation of the diameter of Mars.

V. Of all the observations of this diameter, the clearest of all seemed to us the one made in 1649 on July 22, when Mars, viewed with our very large telescope, appeared with a diameter exactly half as great as Jupiter's. This was evident when the image of each was at once copied onto paper, each planet was examined again and often, and the diameter of each circle was divided into equal small parts. Jupiter was then, as to the anomaly of the eccentric, at 5°, and as to the anomaly of the epicycle at 287°; so its distance from the Earth, by our hypothesis, was 43100 semidiameters of the Earth. From this, and from its true semidiameter found in number 3 above, 4 42/100 semidiameters of the Earth, there is deduced by Problem 2 of chapter 8 an apparent semidiameter of Jupiter of 21″ 5‴. Half of that, 10″ 32‴, was therefore the apparent semidiameter of Mars in that position.

[Margin: The true diameter of Mars.]

At the same time Mars stood 86° in the order of the signs from the apogee of the eccentric, and 255° from the apogee of the epicycle. Its distance from the Earth, according to us, was accordingly 11200 semidiameters of the Earth. From this, with the angle 10″ 32‴, there is gathered by Problem 1 of chapter 8 its true semidiameter, 0 516/1000 of which the Earth's semidiameter is one.

[Margin: And the apparent one at three distances.]

From this, by Problem 2 of the same chapter, and from our distances of Mars from the Earth, the greatest 21005, the mean 11000 and the least 2373 semidiameters of the Earth, is derived its apparent semidiameter at both apogees, 5″ 3‴; at the mean distance, 11″; and at both perigees, 46″.

Obſeruatio Diametri Veneris.

Observation of the diameter of Venus.

VI. In that same year 1649, on July 26, the day dedicated to St Anne, in the morning, with the telescope turned on the Moon, it was confirmed that its field, from the eastern limb near the Lake of Posidonius as far as the eastern shore of the Caspian Sea, took in 30′, a crystal convex on both sides being used near the eye. The telescope was then turned at once on sickle-shaped Venus; and when a fourth part of the diameter of the circle taken in by the telescope was considered, it appeared again and often that the interval from horn to horn of Venus equalled a fourth part of that fourth, that is, was contained 18 times in the telescope's whole field, and so was 1′ 40″. Comparing its image with the image of the same planet seen on June 29 of the same year, whose scheme we gave in Section 1 of this book, chapter 2, it seemed half as great as on June 29, namely 3′ 20″; for on June 29 it had been close to the perigee of the epicycle. Also on June 24, Venus, compared with Jupiter by means of the telescope, contained Jupiter 3⅔ times; Jupiter then, by the principles established in number 3, occupied 42″, and so Venus 2′ 34″; and with the image of Venus set against the image close to perigee of which I spoke above, it seemed as 3 to 4. From all these things, then, it is confirmed that at the perigee of the epicycle, on July 7, it was 3′ 20″. Finally, on 1650 January 2, when Jupiter by our reckonings was 43″ in apparent diameter, Venus, seen through the telescope, was about as 4 to 5 to Jupiter, that is, 34″ 24‴.

[Margin: The true semidiameter of Venus.]

First, then, using the diameter of 3′ 20″, or the semidiameter of 1′ 40″, which was very nearly its size on July 7, when it was at the perigee of the epicycle but at an anomaly of the eccentric of 15°, and so stood only 2400 semidiameters of the Earth from the Earth by our hypothesis, there follows by Problem 1 of chapter 8 a true semidiameter of Venus of 1 15/100 of which the Earth's semidiameter is one.

[Margin: And the apparent one at three distances.]

Second, from this true semidiameter and from our distances of Venus from the Earth, the greatest 12919, the mean 7418 and the least among the possible ones 1917 semidiameters of the Earth, we shall gather the apparent semidiameter of Venus at both apogees, 16″ 45‴; at the mean distance, 32″ 6‴; and at both perigees, 2′ 4″. See now the observations of Scheiner in scholium 5 below, and compare them with ours.

Obſeruatio Diametri Mercurij.

Observation of the diameter of Mercury.

VII. In 1650, on January 2, with the horizon perfectly clear, Mercury was seen close to its greatest digression from the Sun, and with its image copied onto paper, after it had first been examined long and much through the telescope, and the same done with Jupiter's image, and in this way Mercury's diameter was found to be to Jupiter's diameter as 2 to 5. Now Jupiter's apparent diameter at that time, from the principles established above, was 44″, as we shall show

shortly in number 8. Mercury's diameter in that position, therefore, was 17″ 36‴, or at most 18″.

[Margin: True diameter of Mercury.]

First, the anomaly of Mercury's centre was then 42° and the equated anomaly of the orb 149°; hence its distance from the centre of the Earth, by our reckoning, was 5800 semidiameters of the Earth. From this, and from the apparent semidiameter of 9″, by Problem 1 of chapter 8, Mercury's true semidiameter is derived as 0 25/100, in parts of which the Earth's semidiameter is one.

[Margin: And its apparent diameter at three distances.]

Second, from our greatest distance of Mercury, 10868, the mean, 7473, and the least, 4078 semidiameters of the Earth, there is deduced by Problem 2 of chapter 8 its apparent semidiameter at both apogees, 4″ 40‴; at the mean distance, 6″ 54‴; and at both perigees, 12″ 36‴. I pass over the remaining observations, which are less certain than this one. See also the observations of Gassendi and Hortensius below, in scholium 4.

Obſeruationes Diametri Saturni.

Observations of the diameter of Saturn.

VIII. We have deliberately put off the highest of the planets to this last place, because its diameter is entangled in several inequalities, owing to the varied figure that it presents to our gaze together with its flanking attendants [Collateronibus], a figure that quite often departs from the circular or spherical. Among the other times at which we observed it, the most opportune was that very recent one when, having become acronychal at both perigees, namely of the eccentric and of the epicycle, it came down as far as it could toward the Earth and, spending the night with us, granted the fullest view of itself for contemplation, at our complete leisure. For in 1649 it was at the perigee of the epicycle about December 18, and around the 27th at the perigee of the eccentric; on those days, and indeed from December 12 until January 10 of the year 1650, its diameter was observed often and in many ways, both when it was taken alone and when taken with its attendants [lateronibus].

[Margin: 1. Observation of the diameter of Saturn.]

First, Saturn at perigee was viewed together with Jupiter through the same very large telescope of ours, both by direct and careful inspection of each and by copying the image of each onto paper at once; the visible diameters of each were divided, in the inspection by estimation, in the image by compasses. Saturn's whole composite diameter AB was to Jupiter's diameter BZ as 13 to 8. But the interior or simple diameter, that of Saturn taken alone, namely FG, was to Jupiter's diameter as 6¼ to 8, and the remaining diameter CD as 5¾ to 8. At that time the anomaly of Jupiter's eccentric was 18° and the equated anomaly of its epicycle 65°; and so by our hypotheses its distance from the Earth was about 44000 semidiameters of the Earth. From this, and from its true semidiameter found in number 3, 4 42/100 semidiameters of the Earth, Jupiter's apparent semidiameter at this time is gathered, by Problem 2 of chapter 8, as 22″, and its

whole diameter as 44″. If, therefore, one makes as 8 to 13, so 44″ to another, the diameter AB will come out as 71″ 30‴. If as 8 to 6¼, so 44″ to another, FG will result as 34″ 22‴. Finally, if as 8 to 5¾, so 44″ to another, CD will be 31″ 38‴.

Diagram. A full-width figure across both columns. At left, a large horizontal ellipse represents Saturn with its companions, with a dark crescent inside it toward each end, the crescents' hollow sides facing the centre, and the ellipse's interior between them left white. At right, immediately beyond the ellipse, stands a circle, Jupiter, crossed by a slanting hatched band that runs down from its upper left limb to its right limb. To the left of the ellipse, a vertical dotted line descends from the star at H to L, and a horizontal dotted line runs from L to A at the ellipse's left end. A second vertical dotted line descends from the star at P to the top of the ellipse, and resumes below it to R.

- H (upper left, at a star): point label
- L (left, foot of the dotted line from H): point label
- A (left end of the ellipse): point label
- K (inside the ellipse, at the outer edge of the left crescent): point label
- F (inside the ellipse, at the inner edge of the left crescent): point label
- O (centre of the ellipse): point label
- G (inside the ellipse, at the inner edge of the right crescent): point label
- M (inside the ellipse, at the outer edge of the right crescent): point label
- B (right end of the ellipse, between it and the circle): point label
- C (above the ellipse, at its top): point label
- D (below the ellipse, at its foot): point label
- P (above the ellipse, at a star): point label
- R (below the ellipse, foot of the dotted line from P): point label
- ♄ (above the ellipse, between P and the middle): the sign of Saturn
- ♃ (above the circle): the sign of Jupiter
- Z (right limb of the circle, at the end of the hatched band): point label

[Margin: 2. Observation of the diameter of Saturn.]

Second, over the same days, two stars observed among the Hyades occupied almost a fourth part of the circle that equalled the telescope's field, that is, 6½′ or a little more, while Saturn's diameter AB seemed to be contained 5½ times in that space between the stars. Again, from Saturn's more distant end to a certain star of Gemini near Propus (a star that is not in the catalogues of the fixed stars and is visible only with the telescope, but was chosen by us because it lay on the straight line drawn through AB, which is usually parallel to the equator), the interval was equal to 5½ Saturnian diameters AB. The space from the end B to the same star, however, was of 4½ such diameters, most evidently to me. Now the interval from the end A to that star, compared often with the already mentioned interval in the Hyades, seemed entirely equal to it; and so the diameter AB was, by every method, 71″ 30‴ or 72″.

[Margin: 3. Observation of the diameter of Saturn.]

IX. Third, in 1650, on the night following January 2, at hour 1 after sunset, Saturn stood, with its diameter AB, very nearly on the same straight line as the

already mentioned star lying west of it; and from other observations, as we said in Section 1, chapter 2, number 7, the line AB, that is, the line passing through the centres of Saturn and of the Saturnian companions, is parallel to the equator. But on the night following January 3, at the 2nd hour of the night, because Saturn was still moving retrograde along its orbit, which was not parallel to the equator, it had already become both more southerly and more westerly than the already mentioned star. And, as Father Francesco Maria Grimaldi observed most accurately, a straight line drawn perpendicularly from the star P to the line AB fell as nearly as possible through F, at R, cutting off exactly a fourth part of the diameter AB. Then, at 5¼ hours of the night, with Saturn moving back toward the west and leaving that little star behind, the star appeared on the perpendicular HL, so that the interval AL was exactly equal to the space AF, that is, to a fourth part of Saturn. (Here, however, F is taken for a point scarcely distant from Saturn's central disc, toward K, and placed in that dark interval, whether of aether or of some other body, which divides the companion AK from the body of Saturn itself, FG.) That moment of time was awaited and chosen because of the equality just mentioned, since before hour 5¼ LA was not yet equal to AF, and afterwards it was manifestly greater. Now the true daily motion of Saturn on that day was 4′ 30″, from the more accurate tables; therefore the hourly motion was 11″ 15‴; therefore from hour 2 to hour 5¼ the centre of Saturn, O, completed 36″ 34‴, that is, a space equal to its semidiameter AO. For when the perpendicular from the star P fell at R, AF was a fourth part of the whole AB; and when the perpendicular fell from H at L, LA was equal to AF; therefore LF was equal to the semidiameter AO. So the whole AB, from this observation, was 73″ 8‴. But above, from the earlier observations, it was 71″ 30‴ or 72″: the difference between such different ways of observing is plainly negligible, or rather the agreement is altogether marvellous, and it is reported in scrupulous good faith.

[Margin: The true diameter of Saturn is established.]

We shall therefore take Saturn's perigee diameter as 72″, or 1′ 12″, and FG as 34″ 30‴. From these, by Problem 1 of chapter 8, with Saturn's least distance from the Earth of 57743 semidiameters of the Earth, we shall gather its true semidiameter: the simple one, FO, 4⅘, and the composite one, AO, 10 8/100, in parts of which the Earth's semidiameter is one.

[Margin: And its apparent diameter at three distances.]

From the mean distance, 73000, and from the greatest, 90155, by Problem 2 of chapter 8, we shall deduce the apparent semidiameter: at the mean distance, the simple one FO, 13″ 20‴, and the composite one AO, 28″ 30‴; but at the greatest distance, FO, 11″, and AO, 23″; provided it is supposed that Saturn with its companions in its various phases keep the proportion of apparent size that arises from the variation of distance. Compare now with our observations the observation of Hortensius, to be given below in scholium 4.

Monitum Pro tabulis ſequentibus.

Notice for the following tables.

X. In the first table I have set out the apparent sizes of these five planets, and the true sizes derived from our distances by the geometrical problems of chapter 8. If, however, there is anyone who wishes to use the apparent diameters observed by us but the distances of others, and to gather the true size from these, so as to know within what limits we are certain of the size of these planets, according to the extreme opinions of the most celebrated astronomers, then to satisfy that person I have also constructed a second table. In it I have taken four of the more notable classes of distances, namely: that of Wendelen, and nearly that of Posidonius, derived from their distance of the Sun from the Earth; our own; the Keplerian; and the Tychonic, to which the Ptolemaic, al-Battānī's, the Copernican and that of a good many more recent writers are reduced with a small difference (as far as this business goes). Here, then, in the first table is collected what we think about these matters from our own observations.

I. TAB. MAGNITVDO APPARENS, ET VERA QVINQVE PLANETARVM Minorum ex obſeruatione noſtra Diametrorum, noſtriſque Diſtantijs.

Table 1. The apparent and true size of the five minor planets, from the diameters as we observed them and from our distances.

	At a distance from the Earth	♄ with companions			♄ alone			♃			♂			♀			☿		
		′	″	‴	′	″	‴	′	″	‴	′	″	‴	′	″	‴	′	″	‴
Apparent diameter	Greatest	0	46	0	0	22	0	0	38	18	0	10	6	0	33	30	0	9	20
	Mean	0	57	0	0	26	40	0	49	46	0	22	0	1	4	12	0	13	48
	Least	1	12	0	0	34	30	1	8	46	1	32	0	4	8	0	0	25	12

		♄ alone		♃		♂	♀		☿		
The true diameter has diameters of the Earth		9	3/5	8	4/5	0	52/1001	15/100	0	1/4	
The circumference has diameters of the Earth		28	1/3	27	63/1001		63/1003	3/5	0	78/100	
The area of a great circle, in square diameters of the Earth		70	0	64	0	0	21/1001	4/100	0	1/25	
The surface, in square diameters of the Earth		272	0	242	0	0	85/1004	15/100	0	1/5	
The planet's body contains	Earths	891		685		0	14/1001	1/2	0	1/256	
The planet's body contains	Moons	49005		37675		7	7/10 82	1/2	0	1/5	

		♄ alone		♃		♂	♀	☿
The planet's body is contained	By the Sun	43	1/3	56	3/7	275714	25733	9881600

But the true diameter of Saturn with its companions contains, on our hypothesis, 20⅙ diameters of the Earth. We have not given their other measures, because they do not make up a single globe or a constant figure.

II. TAB. MAGNITVDO VERA QVINQVE PLANETARVM MINORVM Poſita Diametro Apparenti ex noſtra obſeruatione, & Diſtantijs medijs à terra infraſcriptis.

Table 2. The true size of the five minor planets, with the apparent diameter taken from our observation and with the mean distances from the Earth written below.

	Authors	♄ alone		♃		♂	♀		☿	
Mean distances of the planets from the Earth, in semidiameters of the Earth	Tycho	10550		3990		1745	1150		1150	
	Kepler	32156		17569		5150	3328		3474	
	We	73000		36500		11000	7418		7473	
	Wendelen	139677		75039		22257	14600		14600	
The planet's diameter would have diameters of the Earth	For Tycho	1	37/1000		96/1000		18/1000	36/1000		4/100
	For Kepler	4	18/1004		1/4	0	54/1001	0	0	22/100
	For us	9	3/5	8	4/5	0	52/1001	15/1000		1/4
	For Wendelen	18	36/1001	8	16/1002		36/1004	2/5	0	96/100
The planet's body would have the Earth's body so many times	For Tycho	2	4/7	0	3/10	0	1/130 0	1/40	0	1/1562
	For Kepler	72	0	71	4/5	0	1/8 1	0	0	1/16
	For us	891	0	685	0	0	14/1001	1/2	0	1/256
	For Wendelen	6087	0	6080	0	13	14/10085	2/5	0	9/10

SCHOLIA.

Scholia.

I. There is a great dispute, or rumour, about the variation in the apparent size of Mars, since the prosthaphaereses necessary to recent astronomy, in its acronychal position, prove that it comes much closer to our Earth than the Sun does, as I said in Section 2, chapter 3, scholium 4, and in this Section 6, chapter 4, scholia 3 and 4, and chapter 7, number 3. Granting this, it seems necessary that it should appear very notably greater in its acronychal position, especially when it is at perigee in Aquarius in respect of both the eccentric and the epicycle, than near its apogeal conjunction with the Sun.

[Margin: The size of Mars as seen at perigee.]

As to the proportion, Galileo, in dialogue 3 on the World System, introduces Salviati inveighing against the Copernican system and objecting that, if Copernicus's hypothesis were true, Mars in opposition to the Sun would be seen with a disc sixty times greater than when it is most remote, whereas it does not turn out even four or five times greater than itself as it hastens to conjunction with the Sun. He then has the same man reply to himself and to Sagredo and Simplicio: namely, that if Mars is viewed with the naked eyes, near conjunction it appears larger than it is because of the adventitious rays that either the deception of our eye or that of the medium places around it, like a mane or curls; and so it then appears not much smaller than when it is nearest the Earth. But if it is viewed in both positions through a large telescope, which strips Mars of that adventitious hair, it is seen in opposition sixty or fifty times greater than near conjunction with the Sun: understand this of the disc, or area of the apparent circle; and so as to the diameter it is seen eight or seven times greater in ☍ than in ☌ with ☉, for circles are to one another as the squares of their diameters, by 2 of Book 12 of the *Elements*. But by our reckonings Mars at its least distance from the Earth appears 92″ in diameter, and at its greatest 10″ 6‴; that is, nine times greater. And so, as to the circle of its disc, it appears in perigee ☍, compared with itself in ☌, as 8464 to 101, that is, greater as to area by 84⅗ times.

[Margin: Mars believed to be a new star.]

Nor is support from experience lacking. For Kepler reports in the *Astronomia Optica*, p. 333, from Philipp Melanchthon, that in 1529, in the months of July and August, Mars was seen with so prodigious an appearance that it was believed to be a new star; and the same, he says, must have happened in 1561 in August, in 1593 likewise in August, and in 1608 in July. As for Gassendi's assertion, however, in Book 2 of the *Institutio Astronomica*, chapter 9, that Mars at apogee scarcely equals stars of the second magnitude, and at perigee rivals the size of Venus and Jupiter, that must be taken to mean that it then surpasses Jupiter but is surpassed by Venus. But what are you to do with Kepler? In the *Epitome of Copernican Astronomy*, p. 485, he had said: "Mars, acronychal and at perigee in Aquarius, does appear greater than Jupiter, yet not by much", asserting in the same place that Jupiter at perigee had often seemed to him to be 50″. Afterwards,

in the Rudolphine Tables, precept 110, he errs first (if he is speaking of ♄ with its companions) in saying that ♄ never appears greater than about 30″; and presently he says that through the telescope the semidiameter of Mars, when it approaches the Earth, ought to be seen as 3′ 15″, and therefore its diameter as 6′ 30″, which he says is grounded on the equal proportion between the intervals from the Sun and the bodies of the planets, treated in the *Epitome*.

[Margin: The apparent size of Venus at perigee.]

II. Venus too is remarkable in this variation. For it Kepler, in the same place of the Rudolphine Tables, gathers from his own hypotheses a semidiameter of 3′ 33″ at perigee, and so a diameter of 7′ 6″, and that as seen through the telescope.

[Margin: Venus believed to be a new star.]

For although it is true, as he says in the *Optics*, p. 333, that Venus has quite often been believed to be a new star, as in 1603 in February and March, because, as is said in the same place, it was seen through clouds and horizontal refractions, and other optical causes concur toward this sort of quasi-prodigy (so much so that once at Parma Doctor Claudius Achillinus [Claudio Achillini], and later at Bologna others on three occasions, likewise appealed to me to indicate whether a star of unusual size, seen by chance at that time in the public square, was one of the comets; and I answered them, and not wrongly, that it was Venus), yet it is plainly false, what he says in the Rudolphine Tables, that it ought to be seen through the telescope as 7′ 6″ in diameter, since we, with an excellent spyglass, saw it close to the perigee of the epicycle as no greater than 3′ 30″. Truer, however, is what Galileo asserts in dialogue 3 on the World System: namely, that Venus to the naked eye does not appear much smaller at apogee than itself at perigee, but that if it is viewed with the telescope, and its adventitious hair is taken away, it appears forty times smaller at the apogee of its epicycle or orb than at perigee. For by our reckoning it would appear there 33″ 30‴ and here 4′ 8″; therefore, with the diameters squared, the disc there is 860″ and here 48325; therefore the area of the apogeal circle is smaller than the perigeal by 56 times, granted that as to the diameter it may appear only seven times smaller there than here.

[Margin: Kepler's analogy between the intervals and masses of the planets is false.]

III. From our observations and those of Hortensius concerning the diameters of the planets, those Keplerian analogies between the masses of the planets and their intervals, otherwise sought out with so much ingenuity and expounded by us in chapter 7 from number 7, now manifestly collapse. He had set up an analogy between the surfaces of the bodies and the intervals, judging that the surfaces of the planets are to the surface of the Earth as their interval from the Sun to the Earth's interval from the Sun, from which it followed that the proportion of the bodies is sesquialteral of the proportion of the intervals. Just as he abandoned that analogy, and on seeing a few observations of Johannes Remus Quietanus on the apparent diameters of the planets said on p. 485 of the *Epitome*:

"I yield place, however, to Remus, I yield to the observations"; and just as in the Rudolphine Tables, at the end of precept 110, which concerns the investigation of the planets' apparent semidiameters, he added: "I assume, however, in the foundations of this precept, from the *Epitome of Copernican Astronomy*, that the bodies of all the planets are in the proportion of the intervals between them and the Sun, and therefore their diameters in the subtriplicate. Let judgement and censure rest with whoever shows a more probable proportion": so, without doubt, if he had seen the observations of Hortensius, Gassendi, Vicenç Mut, Father Grimaldi and our own, he would with the same candour and love of truth have yielded place to them and have abandoned that very analogy. He would indeed have looked for another, so that the Copernican hypothesis and the *Mysterium Cosmographicum* might not be undermined, but in vain. For I shall teach more lucidly, God granting, in Book 9, where the system of the world is treated, that GOD did not geometrize so materially that he begged the proportions of bodies from the mere intervals themselves, or sought no other praise for his works than from geometry of this purely mathematical kind.

[Margin: A specimen of the observations of Hortensius and Gassendi.]

IV. Further, let us draw a specimen of Hortensius's observations from his dissertation with Gassendi. He had found the field of his tube to be 42′, because it took in exactly both the space between the two Kids [Hœdos], which for Tycho are 42′ apart, and one and a quarter times the Moon when the Moon was 33′ 30″. He saw two contiguous stars in the following horn of Capricorn distant from each other by an eighth part of the tube's field, that is, 5¼′, and two contiguous stars of the Hyades distant from each other by very nearly the same amount; and a fifth part of this small space was occupied, as he saw in 1632, by the diameter of acronychal Jupiter, that is, 1′, the size at which Gassendi too estimated the diameter of Jupiter, inspected through a tube, in 1630.

[Margin: The diameter of Jupiter observed by Hortensius and Gassendi.]

Again, in the same year 1632, on November 13, the day on which Jupiter was opposite the Sun, he received Jupiter's image on paper through the glasses of the telescope. Though it appeared with faint edges, it yet seemed round to a keen effort of the eyes, and its diameter was of 4 parts of which the Sun's diameter, taken the day before with the same telescope on a small board, had been 130. And again on another day he found that the diameter of the Jovian image equalled 6 parts of which the solar diameter was 195. So, since the Sun's diameter was then 34′, he concluded that Jupiter's diameter was 1′ in its acronychal position.

[Margin: The diameter of Venus and of Mercury observed by Hortensius.]

He determined the diameter of the remaining planets by observing them by direct sight through the telescope and comparing them both with the interval of the contiguous Hyades and with Jupiter's diameter as received on the board, when it was of the size A———B and, compared with the Sun's diameter, was 48″. At that time Venus's diameter was seen to be to AB as ⅓ to 1, and Mercury's to Venus's diameter as ⅔ to 1, when both were approaching their

apogeal conjunction with the Sun. Hence he inferred that the apogeal diameter of Venus is 15″ 20‴ and that of Mercury 10″. From Lansbergen's distances he deduced the perigeal diameter of Venus as 1′ 40″ and of Mercury as 28″; but he erred, because he did not see Venus at perigee in the appearance in which we did.

[Margin: The diameter of Mars observed by Hortensius and Gassendi.]

Mars too, in 1632 on February 9, in its acronychal position and joined to the Moon, he observed in some fashion through the tube, and he saw that its distance from the Moon's limb did not take up one lunar digit, and that Mars's diameter was contained about four times in that distance. The lunar digit was then 2′ 42″; so for him the diameter of Mars was then not much greater than 36″. This agrees with Gassendi, for whom it then stood between 30″ and 40″. Mars was then near the apogee of its eccentric. Accordingly, from Lansbergen's distances he concluded that the diameter of Mars at both apogees is 9″ and at perigee 1′ 4″: an error not indeed to be wondered at, from so slippery an observation.

[Margin: The diameter of Saturn observed by Hortensius.]

Finally he relates that in 1625, in February, Saturn at perigee in ♌ and round was seen very nearly equal to Jupiter, which had 40″ in diameter; but that in 1632 Saturn alone was seen as 36″ of the measure in which Jupiter was 40″, while with its appendages of little globes, which did not wholly stand out, it was seen as one minute, 1′. Accordingly, from the distances he gathers Saturn's apogeal diameter without companions as 31″ and the perigeal as 42″ 40‴. But if he had seen Saturn at both perigees, and with our larger telescope, he would have come closer to the truth.

[Margin: The diameter of Mercury observed by Gassendi.]

As for Gassendi, as he himself reports in Book 2 of the *Institutio Astronomica*, chapter 14, he observed Mercury under the Sun in 1631, on the morning of November 7, when, he says, it painted itself on the disc of the Sun with its own little shadow, on paper on this side of the telescope, and its diameter could be compared with the Sun's diameter, and it was found to be 20″ or at most 23″. This wonderfully favours our observations, by which Mercury at both perigees is 25″ 36‴; for it was then indeed at the perigee of the epicycle, but near the apogee of the eccentric.

[Margin: A dispute between Scheiner and Galileo about the diameter of Venus.]

V. Our Scheiner had said, in his letters to Markus Welser of the year 1611, that Venus at perigee, unless latitude prevents it, can be seen when it is under the Sun; but that on December 11, from Magini's Prutenic tables and ephemeris, Venus was in conjunction with the Sun with a latitude of 9′ and yet was by no means seen, although it should have been seen like a large spot, since its apparent diameter, by the assertion of Tycho and Clavius, is to the solar in subdecuple proportion, and so of at least 3′, and it lacks its own light. Therefore, since it was not seen by him when he observed this most attentively with a Dutch tube [tubo

Belgico], he concluded that Venus, then at apogee, had been above the Sun. But Galileo, in his letters to the same Welser, especially p. 110 and following, reproves these authors for having observed Venus by night, gleaming all round with adventitious hair, rather than by day or with the telescope, when its genuine size appears, and for having pronounced indifferently that the diameter of Venus is a tenth part of the solar, since at apogee and in morning conjunction, in which Venus was on that December 11, it is not a two-hundredth part of the solar diameter, that is, it is scarcely 10″; and he says that those opposed to the motion of Venus around the Sun can therefore say that it was not seen under the Sun either because of the smallness of its diameter, or because it is always above the Sun, or because it shines with some light of its own. Afterwards Scheiner, in the *Sol Ellipticus*, p. 25, reports that the diameter of Venus was observed by him in 1612, its radiant image being received on paper through the glasses of an optical tube, with the same separation of the lenses and distance of the paper by which it held the Sun's image, then 34′. And compared with the Sun's diameter with the help of a compass, the diameter of Venus was found on October 23 to be 2′ 40″, but on November 27 and 29 to be 2′ 12″, and on December 26 to be 2′ 0″. He admits, however, that that observation was a shadowy one, and reserves a complete treatment of this matter for another time. Venus had by then risen far from the lowest apsis of the epicycle. He therefore concludes, and confirms what he had said in the *Apelles post tabulam*: namely, that it would have come about that Venus was seen under the Sun because of the size of its diameter.

[Margin: Criticism of Scheiner.]

But in truth this is concluded only on the hypothesis that the opponents should grant that the conjunction took place at the perigee of the epicycle. For though Scheiner is a friend to me, truth is a greater friend. Hortensius, moreover, from this observation of Scheiner's and from Lansbergen's distances, concludes, not absolutely but hypothetically, that Venus's apogee diameter is 34″ and its perigee diameter 3′ 42″. This supports our observations wonderfully. And Scheiner rightly refutes Tycho, who assigns Venus a diameter of 3′ 15″ at mean distance.

[Margin: Earlier and rougher observations of Johannes Hevelius.]

VI. It remains for us, in the last place, to point out the observations of Johannes Hevelius, the last to become known to us, from his *Selenographia*. In chapter 3 of that work he rightly advises that, to strip the discs of stars, especially those of livelier light, of their adventitious rays, the convex lens of the telescope that is farther from the eye should be covered with a small perforated metal leaf [bracteola], whose hole should be about the size of a pea. In this way, he says, Arcturus appeared to him larger than Aldebaran [Palilicium], and Aldebaran larger than Regulus.

[Margin: Diameter of Venus and Jupiter.]

And on 1644 March 22, at 7^{h} in the evening, Venus, very close to Jupiter, was seen to be smaller than Jupiter, although to the naked eye it had appeared

much larger; we too have observed this about Venus more than once. And in chapter 4 he affirms that on 1645 January 13 Venus appeared to him to be of about 10 digits, but on 1646 April 21 of less than one digit. Then he relates that on 1644 November 22, at 7^h in the morning, Mercury appeared to him smaller than Mars and than 1½′ [ſeſquiminuto].

[Margin: Diameter of Mercury.]

And on 1645 May 16, at $8½^h$ in the evening, Mercury, still sickle-shaped, appeared to him larger than fixed stars of the first magnitude through a tube fitted with the leaf mentioned above. He admits, however, that even in this way Mercury is not stripped of all its adventitious rays, because of its very close neighbourhood to the Sun, which prevents it from being seen by day; whereas Venus, visible by day at its great digressions from the Sun, is denuded of that borrowed head of hair by the prevailing light of day itself, and much more so if the already mentioned leaf is applied.

[Margin: Somewhat better observations by the same man.]

So much for Hevelius's rougher observations. For later, in chapter 55, he teaches a way of estimating the diameters of stars by comparison with the spots of the Moon; on this see our judgement above, chapter 9, at the end of Method 8.

[Margin: Diameter of Jupiter and Venus.]

Yet by that method Jupiter appeared to him, on 1646 April 22, to have a diameter of 1′ 2″. And in chapter 56 he reports that on 1647 January 20 the diameter of Jupiter, compared with the spots of the perigee Moon, appeared to him to be 1′, or 55″, if the Moon's diameter was then 34′ or 31′. Earlier, however, in chapter 55, he had said that on 1646 April 22 Venus, observed by him in the same way, at the perigee of the epicycle, was 1′ 22″, and that in 1644, about the morning □, it appeared to him to be 46″: understand quadrature, when it appears dichotomous. Therefore the 10 digits mentioned above are to be understood of the phase of Venus, showing 10 of its 12 parts illuminated.

CAPVT XI.

Chapter 11.

Noſtræ & Aliorum Obſeruationes circa Diametros Apparentes Fixarum, ex Planetarum Diametris deductas, cum Vera Fixarum Magnitudine, tam ſecundùm noſtras, quàm ſecundùm aliorum diſtantiam Fixarum à terra vel aſſertam vel aſſerendam.

Observations, our own and others', on the apparent diameters of the fixed stars, derived from the diameters of the planets, with the true size of the fixed stars, according both to our distance of the fixed stars from the Earth and to the distance that others assert or must assert.

I. In Book 6, chapter 9, because of the dependence that the observation of the diameter of the fixed stars has on the observation of the diameter of the planets, and because of the affinity of the evidence, we put off to this place the observations of the fixed stars made on their apparent diameters and then finally confirmed from here, after the firm determination of the diameters of Jupiter and Saturn. The principal method, and the most evident of all, was this. On white paper we drew various images, circular ones for the disc of Jupiter and egg-shaped ones for Saturn with its companions. And having inspected both planets long and often through a large telescope, we chose from the many that figure which, as the eye was brought at once and repeatedly from the telescope to the paper, seemed of all the most closely matching the size of the planet's disc as it then appeared. But so as not to trust our own eyes alone, Father Grimaldi and I, both together and separately, brought in the eyes of several others as co-witnesses, who, forewarned of nothing about the figure chosen by us, were themselves to choose the one that seemed the closest match.

[Margin: See the figures in chapter 10, number 8.]

And so at the end of 1649 and the beginning of 1650 that figure of Saturn and of Jupiter was chosen which we showed in chapter 10, number 8. At that time, from what was said there, the apparent diameter of Saturn was 34″ 30‴ and that of Jupiter 44″. We divided the diameter BZ of Jupiter's disc into 200 small parts, of which an uncia of the Roman foot is 100; and of such parts the inner diameter, that of Saturn alone, FG, was 160. With these prepared, we again drew on paper a number of small circles of larger and smaller diameter; and having inspected through the same telescope the fixed stars discussed below, and frequently turning the eye back to those small circles, we chose the one that seemed most nearly equal of all to the size of the fixed star. Then others were brought in separately, above all Father Paulo Casato [Paolo Casati] and Father Matthęo Tauerna [?], outstanding professors, the former of philosophy and the latter of sacred oratory, and most fit to discern such things for the sharpness of their eye and mind and the keenness of their judgement. Using the same telescope, each of them, with a truly wonderful agreement, chose those very figures that Father Grimaldi and I had chosen. And so, with all the anxiety and suspense that had held us until then removed, and rejoicing, as it were, over stars caught in so noble a fishing, we took the diameters of those small circles with a very sharp compass and applied them to the diameter of Jupiter and of Saturn; and from the proportion they had to these we drew out the apparent size of the diameter of the fixed stars listed below, and rested in it with complete confidence. Nor do we doubt that, if all the experiments mentioned are repeated by skilled men who are not steeped in others' opinions on the size of the planets or the distance of the fixed stars, and who use a suitable spyglass and suitable witnesses, they will agree with us without any perceptible difference.

[Margin: Why Hortensius's opinion on the diameters of the fixed stars is suspect.]

II. For we know, certainly, that Hortensius, in his dissertation with Gassendi, greatly reduced the diameters of the fixed stars, so far as to allow only 10″ to Sirius and 8″ to the other stars of the first rank. But he did so for two reasons. First, because in comparing their diameter with Jupiter's he used a telescope showing too small an image of Jupiter, as is clear from the schematism of the image he gave, which we discussed in scholium 4 of chapter 10. For its diameter was only 60 parts, of which the diameter of our Jupiter, close to apogee, was 200; so that in working out the proportion itself the danger of error lay more readily in the smaller measure than in the larger. Second, because he was wholly given over to defending the Copernican hypothesis as regards the distance of the fixed stars required in it, so that the parallax of the fixed stars arising from the Earth's annual motion should not prove perceptible. But since from so great a distance there follows a size of the fixed stars that is excessive and has seemed monstrous to most people, he thought, in order to water down that charge, that the diameters and sizes of the fixed stars should be watered down. I gather this both because he concludes: "It does not follow, then, from the Copernican motion of the Earth that the size of the fixed stars is increased beyond measure, since we see that they are much smaller than the Earth's annual orb, against certain people"; and from that very quasi-cunning by which both he and Lansbergen expressed the size of the fixed stars compared not with the globe of the Earth, lest it seem excessive, but with the sphere of the Earth's annual orb.

[Margin: Lansbergen's false opinion on the same diameters.]

And yet Lansbergen, in the *Uranometria*, Book 3, element 20, following neither Hortensius nor others, sets a rather larger diameter of the fixed stars at his mere discretion, in these words: "Al-Battānī defines the apparent semidiameter of the non-wandering stars of the first magnitude as 45″, and Tycho Brahe as 1′; we, as ½′. For through the optical tube it appears much smaller still." As if, indeed, more trust were not to be placed in the tube, which strips the discs of the stars bare and scrapes away their adventitious curls of rays, than in the naked eye or the mere judgement of an arbitrary estimate.

[Margin: Galileo's false opinion on the same diameters.]

III. Galileo and Kepler labour, in our view, under a suspicion similar to that under which Hortensius labours. For Galileo, in Dialogue 3 on the world system [de ſyſtemate Coſmico], replying to those who had objected to the Copernican hypothesis the immensity of the fixed stars that arises from it, does rightly teach that before the use of the Dutch glass [ſpecilli Belgici] astronomers went much too far in the size of the apparent diameter of the minor planets and of the fixed stars. Yet he himself falls too far short of it when he denies that stars of the first magnitude exceed 5″, and says that it therefore does not follow that they are larger than the Earth's annual orb. For he can easily be convicted of falsehood by anyone who uses a telescope larger and more perfect than Galileo's tubes. And how deceptive the method of observing these diameters

by a suspended thread is, we have shown sufficiently in Book 6, chapter 9, number 3.

[Margin: Kepler's twofold opinion on the diameter of the fixed stars.]

Kepler too, in the book *De Stella Nova*, chapters 16 and 21, before the use of the telescope, had allowed 4′ to Sirius and at least 2′ to any star of Orion's belt. Afterwards, in the *Epitome of Copernican Astronomy*, p. 498, to the small question "How large do the fixed stars appear, seen from the Earth?" he answers: "Skilled craftsmen deny that any size, as of a round body, is detected by inspection through the telescope, because rather, the more perfect the instrument, the more the fixed stars are represented as mere points, from which bright rays go out in the form of hair and are scattered." In the same place he also judges that the Sun is larger in bulk of body than the fixed stars. But leaving these matters aside, we shall follow the evidence of the observations carried out by us with a large and very powerful telescope, and we shall append the following tables, with the true size derived from various distances, so that they may be ready for every outcome of the dispute, especially on behalf of the Copernican hypothesis.

[Margin: How the stars may appear round and without rays.]

That the skilled craftsmen of whom Kepler has just spoken saw the fixed stars as radiant points must be thought to have come about, if not from imperfection of the eyes, then at least from the way the telescope was used. For if they had laid over the lens farther from the eye a leaf or plate [bracteam ſeu laminam] pierced in the middle, with a hole whose diameter equals about a quarter of an uncia of the Roman foot, they would have seen the fixed stars so shorn of adventitious rays that they both appeared perfectly round, and, not as points but as small discs so enlarged by the power of the telescope, showed a definite size to one comparing them with the disc of Jupiter or Saturn, whose size is known in the manner we stated above. I said "about" because that hole should be somewhat larger for the smaller stars and smaller for the larger ones, as will be clear to anyone who tries it.

I. TABVLA. OBSERVATIONES DIAMETRI Apparentis Fixarum, comparatæ cum Diſco Saturni ſolius quando erat 35″. & Iouis quando 44″. ſed illius diametro diuiſâ in 160. huius in 200. particulas

Table 1. Observations of the apparent diameter of the fixed stars, compared with the disc of Saturn alone when it was 35″, and of Jupiter when it was 44″, the diameter of the former being divided into 160 small parts and that of the latter into 200.

Fixed stars	In parts of which the diameter of ♄ is 160, of ♃ 200	Hence the apparent diameter is	Old order of magnitude
Sirius	82	18″ 0‴	1
Bright star of Lyra	79	17″ 24‴	1
Arcturus	76	16″ 42‴	1
Capella	73	16″ 8‴	1

Fixed stars	In parts of which the diameter of ♄ is 160, of ♃ 200	Hence the apparent diameter is	Old order of magnitude
Aldebaran	70	15″ 24‴	1
Spica	68	15″ 5‴	1
Regulus	64	14″ 5‴	1
Regel	62	13″ 40‴	1
Fomahant	61	13″ 25‴	1
Antares	60	13″ 12‴	1
Hydra	58	12″ 45‴	1
Tail of ♌	57	12″ 30‴	1
Procyon	56	12″ 20‴	2
Aquila	50	11″ 0‴	2
Orion's belt	40	8″ 50‴	2
Bright star of the Crown	38	8″ 21‴	2
Polaris	36	7″ 54‴	2
Head of Medusa	32	7″ 3‴	3
Propus	28	6″ 10‴	4
Brighter of the Pleiades	24	5″ 16‴	5
Alcor	20	4″ 24‴	6

II. TAB. MAGNITVDO VERA FIXARVM MAXIMÆ ideſt SIRII, & MINIMÆ, cuiuſmodi eſt ferè Alcor: Poſita Diametro obſeruata à nobis in Sirio 18″. in Alcor 4″. 24‴. & Diſtantia Fixarum aſſerta in Hypotheſi TERRÆ QVIESCENTIS.

Table 2. The true size of the largest fixed star, that is, of Sirius, and of the smallest, of which kind Alcor nearly is, with the diameter observed by us set at 18″ for Sirius and 4″ 24‴ for Alcor, and with the distance of the fixed stars asserted on the hypothesis of a resting Earth.

Authors	Distance of the fixed stars from the Earth	True size of Sirius or Canis Major		True size of Alcor, which is near the middle of the tail of Ursa Major	
Distances	In semidiameters of the Earth, in round figures	The true diameter holds diameters of the Earth	The body contains the Earth, times	The diameter holds diameters of the Earth	The body contains the Earth, times
Tycho	14000	0 61/100	0 1/5	0 15/100	0 3/10000
Ptolemaics	Max. 40000	3 1/2	42 0	0 86/100	0 7/10
We	210000	17 1/2	5355 0	4 0	64 0

III. TAB. MAGNITVDO VERA FIXARVM MAXIMÆ & MINIMÆ Ideſt Sirij & Alcor: Poſita diametro obſeruatâ à nobis in Sirio 18″. & in Alcor 4″. 24‴. ac Diſtantia Fixarum aſſerta à Sectatoribus HYPOTHESIS COPERNICANÆ

Table 3. The true size of the largest and smallest fixed stars, that is, of Sirius and Alcor, with the diameter observed by us set at 18″ for Sirius and at 4″ 24‴ for Alcor, and with the distance of the fixed stars asserted by the followers of the Copernican hypothesis.

Authors	Distance of the fixed stars from the Earth in	True size of Sirius or Canis Major		True size of Alcor, in the tail of Ursa Major	
Distances	Semidiameters of the Earth	The diameter holds diameters of the Earth	The body contains the Earth, times	The diameter holds diameters of the Earth	The body contains the Earth, times
Hortensius	10,312,227	899	726,572,600	442	86,355,888
Galileo	13,046,400	1138	1,473,760,072	558	173,741,112
Lansbergen	41,958,000	3658	48,947,466,312	1796	5,793,206,336
Kepler	60,000,000	5232	143,219,847,228	2568	16,933,994,432

IV. TAB. MAGNITVDO VERA FIXARVM MAXIMÆ & MINIMÆ Ideſt Sirij & Alcor: Poſita diametro apparenti obſeruatâ à nobis in Sirio 18″. in Alcor 4″. 24‴. & Diſtantia aſſerendâ à Copernicanis, ſi velint Parallaxim Fixarum factam à motu Terræ annuo non excedere 10″. & tueri Diametrum Orbis Annui ab ipſis poſitam.

Table 4. The true size of the largest and smallest fixed stars, that is, of Sirius and Alcor, with the apparent diameter observed by us set at 18″ for Sirius and 4″ 24‴ for Alcor, and with the distance that must be asserted by the Copernicans, if they wish the parallax of the fixed stars produced by the Earth's annual motion not to exceed 10″ and wish to maintain the diameter of the annual orb that they have laid down.

Authors	Distance of the fixed stars to be asserted	True size of Sirius or Canis Major		True size of Alcor, in the tail of Ursa Major	
Distances	Semidiameters of the Earth	The diameter holds diameters of the Earth	The body contains the Earth, times	The diameter holds diameters of the Earth	The body contains the Earth, times
Copernicus	47,439,800	4170	71,677,713,000	1992	4,378,454,048
Hérigone	49,502,400	4350	82,312,875,000	2068	8,844,058,432
Galileo	49,832,416	4380	8,427,672,000	2092	9,155,562,688

Authors	Distance of the fixed stars to be asserted	True size of Sirius or Canis Major		True size of Alcor, in the tail of Ursa Major	
Boulliau	60,227,920	5300	148,877,000,000	2530	15,941,277,000
Lansbergen	61,616,122	5424	159,371,956,024	2588	17,333,761,472
Kepler	142,746,428	12550	1,976,656,375,000	6000	216,000,000,000
Wendelen	604,589,312	53200	15,056,882,800,000	25380	1,767,384,872,000

For the foundations of these distances see Book 6, chapter 7, number 15.

CAPVT XII.

Chapter 12.

De Quantitate Maximarum Digreſſionum tum Iouialium Satellitum à Ioue; tum Veneris ac Mercurij à Sole.

On the amount of the greatest digressions, both of Jupiter's satellites from Jupiter and of Venus and Mercury from the Sun.

I. If you remember, in this book, section 1, chapter 3, number 5, we promised an exact determination of the greatest digression of the Medicean planets from Jupiter in minutes and seconds, once Jupiter's diameter had been established, since nearly all who have measured those digressions with the telescope use as their measure the diameter of Jupiter itself. So Galileo in the *Sidereus Nuncius*; Simon Marius in his *Mundus Iovialis*; Kepler in the *Dissertatio cum Nuncio Sidereo* and in the *Epitome of Copernican Astronomy*, p. 554; Scheiner in the *Disquisitiones Mathematicae*, number 39; Hérigone in volume 5 of the *Cursus Mathematicus*, p. 580; and Antonín Maria Šírek in the *Radius Sidereomysticus*, Book 4, chapter 4, who adds their greatest distance from Jupiter in semidiameters of the Earth, but supposing Jupiter's diameter to be 3′. Further, Kepler, and Wendelen in a letter given to me, subscribe to the digressions determined by Galileo and Marius, because it follows from them that the periodic motions around Jupiter are in the sesquialteral ratio of the ratio of their intervals from Jupiter. Hérigone, moreover, judges that these digressions were determined from Jupiter's mean distance from the Earth. Yet Hevelius, in chapter 4 of the *Selenographia*, assigns to the innermost satellite 3′, to the next-to-innermost 5′, to the next-to-outermost 8′, and to the outermost 13′ or 14′, as regards digression from Jupiter: as if, when Galileo and Marius observed these digressions and rated them at just as many Jupiters, Jupiter appeared to be of 1′. Their opinions, then, are in the following small table.

Digreſſio Maxima Satellitum ♃ à Ioue

Greatest digression of the satellites of ♃ from Jupiter.

	Diameters of Jupiter			Semidiameters of the Earth
Satellites	Galileo and Marius	Scheiner	Šírek	Šírek
1	3	3	3	20¾
2	5	4	4	27
3	8	5	6	41
4	Mar. 13; Galil. 14	10	10	69½

Kepler, Hérigone, Wendelen and Hevelius side with Galileo and Marius.

II. But to determine this distance we need, besides the number of Jupiter diameters by which they have been seen to digress most from Jupiter (a number that is perhaps variable because of the narrower or wider circuits that the same satellite describes around Jupiter), both the apparent diameter, as observed by us, and Jupiter's mean distance from the Earth. For from these, by the problem to be brought forward in number 5, we shall easily determine that digression in semidiameters of the Earth, not only from our distance of Jupiter from the Earth but also from that of others, so that readers may be better satisfied. Here, then, is the table we constructed for this purpose.

DIGRESSIONES MAXIMÆ SATELLITVM IOVIS A IOVE In Minutijs reſpondentibus Iouis diametro in Media ipſius Diſtantia per nos obſeruatæ, iuxta numerum diametrorum Iouis, digreſſionibus attributum, & in Semidiametris Terræ.

Greatest digressions of Jupiter's satellites from Jupiter, in minutes corresponding to Jupiter's diameter as observed by us at its mean distance, according to the number of Jupiter diameters assigned to the digressions, and in semidiameters of the Earth.

Diameters of Jupiter		3	4	5	6	8	10	13 [?]	14
Minutes for the diameters of ♃		2′ 30″	3′ 20″	4′ 10″	5′ 0″	6′ 40″	8′ 20″	10′ 50″	11′ 40″[75]
Authors of the distance of ♃	Mean distance of ♃, in semidiameters of the Earth	Semid. of the Earth	Semid. of the Earth	Semid. of the Earth	Semid. of the Earth	Semid. of the Earth	Semid. of the Earth	Semid. of the Earth	Semid. of the Earth
Ptolemy	14750	11	14	18	22	29	36	47	50
Copernicus	5951	4	6	7	9	12	14	18	20
Tycho	3990	3	4	5	6	8	10	13	14

[75]the page prints 40′, with a single stroke

Diameters of Jupiter		3	4	5	6	8	10	13 [?]	14
Longomontanus	6605	5	6	8	10	12	16	20	22
Kepler	17569	13	17	21	26	34	43	55	60
Lansbergen	8091	6	8	10	12	16	20	25	27
Boulliau	7628	6	8	10	12	15	19	24	26
We	36500	24	36	44	53	71	89	115	124

[Margin: Use of the preceding table.]

III. The table is used as follows. First choose, in the earlier small table, the opinion you prefer on the number of Jupiter's diameters contained in the greatest digression of any one of the satellites from Jupiter, and look for that number at the head of this table. Under it you will have how many minutes and seconds are comprised, in the heaven of Jupiter, by the angle made by lines drawn from our eye, one to the centre of Jupiter's body, the other to the centre of the satellite at its greatest digression from Jupiter. In the same column you will also have the semidiameters of the Earth corresponding to that digression, opposite the mean distance of Jupiter from the Earth according to whichever author you have chosen. For example, if you think with Galileo that the remotest or outermost satellite digresses from Jupiter as far as 14 diameters of Jupiter, you will see under 14, in the last column, 11′ 40″ set below, and at the foot of the column 124 semidiameters of the Earth corresponding to our distance of Jupiter, which we judge the truer. And this same value is the semidiameter of the epicycle the satellites are thought to describe around Jupiter.

[Margin: How large Jupiter appears when viewed from the satellites.]

Anyone who wishes to know how large Jupiter appears when viewed from its satellites should consult small table 3 of chapter 13 below.

De Quantitate Digreſſionum Veneris ac Mercurij à Sole ipſo vel à linea medij loci Solis.

On the size of the digressions of Venus and Mercury from the Sun itself or from the line of the Sun's mean place.

IV. Venus and Mercury too are, as it were, satellites of the Sun. On Tychonic principles Biancani, in his *Sphaera*, Books 11 and 12, and after him Andrea Argoli in the *Pandosion Sphaericum*, chapters 58 and 59, define their greatest elongations from the Sun as 855 semidiameters of the Earth for Venus and 585 for Mercury, and they think that the semidiameter of the epicycle these planets describe around the Sun is just as large. In the opinion of other astronomers, however, this measure varies, both because of the varying distance of the Sun from the Earth that each posits from his own hypothesis, and because of the varying angle they attribute to the greatest digression, and the distance of these planets

from the Earth, which varies by reason of the eccentric. To resolve the small astronomical questions that could be raised on this point, it will be enough for us to take the mean distance of the Sun and of these planets as the same, and the angle of greatest digression, already known from Section 3, chapter 7; and from these data, according to the better-known opinions, to reduce the size of these elongations to semidiameters of the Earth. By the old astronomy these are to be understood with respect to the line of the Sun's mean place; by the recent astronomy, with respect to the centre of the Sun's body. Let this therefore be the

Problema. Data Solis Diſtantia mediocri à Terra, & arcu vel angulo maximæ Digreſſionis à Sole ♀, ac ☿, vel Satellitum Iouis à Ioue, Inuenire quantitatem Digreſſionis in ſemidiametris terræ.

Problem. Given the Sun's mean distance from the Earth, and the arc or angle of greatest digression of ♀ and ☿ from the Sun, or of the satellites of Jupiter from Jupiter, to find the size of the digression in semidiameters of the Earth.

V. In their greatest digressions from the Sun, just like the satellites in theirs from Jupiter, these planets stand (if you consult the figure given in Book 3, chapter 11, number 3) at M or F, the point of their epicycle at which the straight line BM or BF, drawn from our eye B, touches the epicycle QDF. By Euclid 3.18 it therefore makes a right angle at M and F with the epicycle's semidiameter AM or AF, and the right triangle AFB or AMB arises. If the angle ABM or ABF is given, solving this triangle readily gives the size of the side AM or AF, in measures of the same kind as those in which the distance of the centre A, of the Sun or of Jupiter, from the Earth B is known. In short, then, make: as the radius is to the distance of the Sun or Jupiter from the Earth, so the sine of the angle measuring the greatest digression is to the side AM or AF. Or more briefly: to the logarithm of the distance AB add the logarithm of the angle just mentioned, and the result will be the logarithm of the side AM or AF, that is, of the distance sought.

But if, from the measures of the hypothesis itself, the size of the greatest semidiameter of the epicycle, or of the orb equivalent to an epicycle, is already known in semidiameters of the Earth, that value, with no further computation, will be the distance we are seeking here. Consult, therefore, the following small table.

MAXIMÆ DIGRESSIONES ♀ ac ☿ à ☉ vel à linea medij loci Solis; in ſemidiametris Terræ.

Greatest digressions of ♀ and ☿ from the ☉ or from the line of the Sun's mean place, in semidiameters of the Earth.

Measured	According to the authors	♀, semid. of the Earth	☿, semid. of the Earth
From the line of the mean ☉	Ptol. Maur. Clau.	461 41′	43 31′
From the line of the mean ☉	Jean Fernel	495 42′	43 53′
From the line of the mean ☉	Copernicus	821 0′	451 0′
From the line of the mean ☉	Michael Mästlin	834 0′	459 0′
From the line of the mean ☉	Lansbergen	1077 0′	708 0′
From the ☉ itself	Tycho and Biancani	855 0′	585 0′
From the ☉ itself	Longomontanus	931 0′	496 0′
From the ☉ itself	Kepler	2448 0′	1313 0′
From the ☉ itself	Boulliau	1057 0′	563 0′
From the ☉ itself	Ourselves	5339 0′	3322 0′

CAPVT XIII.

Chapter 13.

De Magnitudine Apparenti Planetarum, Fixarum, Telluris, ac Orbium aliquorum, ſi aliunde ſpectentur, quàm ex Terra.

On the apparent size of the planets, the fixed stars, the Earth and certain orbs, if they are viewed from somewhere other than the Earth.

I. Long ago, in Book 2, chapter 6, scholia 4 and 5, we inquired whether this Earth of ours, which seems and is so large to us, would look like a point when viewed from some heaven, and we brought forward no contemptible learning from Cleomedes and Maurolico, even though we did not complete the whole problem there but offered only a specimen of it. Then in Book 3, chapter 10, scholium 4, we showed how large the Sun would appear to a spectator placed on each other planet, from the calculation of Boulliau and of Antonín Maria Šírek, assuming Kepler's intervals. Now, however, a more universal treatment must be given, since in the preceding chapter we have completed our discussion of the intervals of the planets and the fixed stars from each other, and the true diameter of the stars, according to the opinions of the more distinguished astronomers, is already established. We need these two elements to solve the following problems. But before them I will give a foretaste of what certain others, besides the four authors already mentioned and just listed, have indicated to us on this matter.

[Margin: Scheiner's statement on various moons.]

Scheiner, then, in the *Disquisitiones Mathematicae*, towards the end, number 43, says: *But if a person were stationed on Saturn, he would see absolutely all the planets like moons; and from their number the sunspots are not to be excepted either, if they are stars or enduring bodies.* For it is assumed that they and the planets have

their primary light from the Sun. Therefore, with the eye placed on Saturn, all the other planets would go around the Sun, with respect to that eye, in the way Venus and Mercury now do; and similar phases would also appear on Saturn if the eye were above Saturn at some moderate distance, and on the little planets of Jupiter with the eye outside their epicycles but placed not far away.

[Margin: Kepler's teaching on the Moon and on the Moon's digressions from the Earth when viewed from the heaven.]

So Kepler too observes in the *Dissertatio cum Nuncio Sidereo*, p. 11 ½, where he also says that the diameter of the Moon's orbit, since it is scarcely the thirtieth part of the diameter of the Earth's own great orbit around the Sun, seems less than 2° when viewed from the Sun; and since the height of Saturn is about tenfold, and that of Jupiter about fivefold, the distance of the Earth from the Sun, the Moon viewed from Saturn cannot therefore digress from the Earth beyond 18′ or 12′, and viewed from Jupiter not beyond 36′ or 24′. He then adds: *Let the Sun's parallax be 3′, even though I think it much smaller. The Earth viewed from the Sun will then have 6′, the Moon 1½′. Indeed a much smaller Earth will also leave less to the Moon, namely not 1′. And this, viewed from Saturn, perhaps 6″ secon- ds will be seen; and from Jupiter, 12″. It is plainly so, then: what the Moon is to us on our Earth, it is not to the other globes,* that is, it does not appear either so large or with so evident a change of phases. *And what those four little moons are to Jupiter, they are not to us; and in turn each of the globes of the planets, and its inhabitants, are served by circulators of their own.*

[Margin: Inhabitants on Jupiter, posited by Kepler.]

From this consideration we conclude about inhabitants of Jupiter with the highest probability; and this indeed seemed equally so to Tycho, from the consideration of the vastness of those globes alone.

[Margin: How large the Sun or the annual orbit appears from Saturn, or the orb of Saturn and of the Earth viewed from the fixed stars, according to Kepler?]

The same author, in the book *De Stella Nova Serpentarii*, chapter 16, assumes that, since the Sun's semidiameter seen from the Earth is 15′ and Saturn is ten times higher, the Sun's semidiameter viewed from Saturn seems ten times smaller, namely 1′ ½. And because he posits the semidiameter of Saturn's sphere as a mean proportional between the semidiameter of the Sun and the distance of the fixed stars, the semidiameter of Saturn's sphere viewed from the Sun therefore appears as 1′ ½. Hence, since the semidiameter of the annual orb is the tenth part of the diameter of the orb of ♄, he infers that the semidiameter of the Earth's annual orb seen from the fixed stars appears as no more than a tenth part of 1½′, that is, 9″, and that the parallax of the fixed stars produced by the semidiameter of the annual orb is that large. On this, however, see what I said in Book 6, chapter 7, number 11.

[Margin: How large some fixed stars would be if viewed from other fixed stars?]

II. In chapter 21, moreover, he asserts that the three stars in the belt of Orion, of the second magnitude, have at least 2′ in diameter and are 81′ apart from each other. Hence, if they lie on the same spherical surface, of which we are the centre, then with the eye placed on one of those stars, another will appear under an angle of about 2 ¾°, an amount that five Suns set in a row and touching one another would not occupy for us on the Earth. Finally, in the *Epitome of Copernican Astronomy*, p. 497, he asks how large the Sun appears when seen from the fixed stars, and answers that it is seen under an angle of 6‴, which is the six-hundredth part of 1′: for he had already posited that they are 60,000,000 semidiameters of the Earth distant.

[Margin: The Earth viewed from Jupiter.]

Hérigone, however, in volume 5 of the *Cursus Mathematicus*, p. 665, affirms that the apparent conjunctions of the satellites of Jupiter occur at almost the same moment everywhere on the Earth, because the Earth viewed from Jupiter is smaller than 1′, since Jupiter, which is larger than the Earth, does not appear larger than 1′ when seen from the Earth. If you join these points with what we said in Book 2, chapter 6, scholium 5, and Book 3, chapter 10, scholium 4, then besides the varied learning that will delight you, you will see how important are the problems we are going to give in this chapter.

Problema 1. Quanta ſit apparitura Tellus viſa ex Planetis vel Fixis Stellis?

Problem 1. How large will the Earth appear when seen from the planets or the fixed stars?

[Margin: First way of solving the problem.]

III. First, take the horizontal parallax of the Sun from Book 3, chapter 8, and of the Moon from Book 4, chapter 14; that of the other planets from chapter 7 of this Section, and that of the fixed stars from Book 6, chapter 7; and double it. In this way you will obtain the apparent diameter of the Earth viewed from the planets or from the fixed stars.

[Margin: Second way.]

Second, another way: in the figure added here, solve triangle CAB, right-angled at the contact B, or triangle CDP, right-angled at the contact D, if the base CA or CP is given, as is assumed, that is, the distance of the planet or fixed star C from the centre of the Earth placed at A or at P, in parts of which the Earth's semidiameter AB or PD is one; and find the angle ACB or DCP. Its double will be the measure of the diameter of the Earth as it will appear to the eye placed at C.

Diagram. Woodcut set in the left column beside number III. Two circles of equal size stand one above the other, with a point below them from which three lines rise: one to the upper circle's centre through the lower circle's centre, and one to each circle's left edge, where a radius meets it.

- A (centre of upper circle, right of the vertical line): centre of the Earth, as the text says
- B (left edge of upper circle): contact of the line from C; AB is the Earth's semidiameter
- P (centre of lower circle): centre of the Earth in the second position
- D (left edge of lower circle): contact of the line from C; PD is the Earth's semidiameter
- C (foot, where the lines meet): the planet or fixed star; the eye

Problema 2. Quanta ſit apparitura Stella quæuis ſiue Errans ſiue Inerrans, ſpectata ex aliquo Planetarum, aut ex Fixis in maxima aut minima ab ipſis diſtantia.

Problem 2. How large will any star, wandering or non-wandering, appear when viewed from one of the planets or from the fixed stars, at its greatest or least distance from them?

IV. Take from table 1 or 2 of chapter 3 the greatest or least distance, as needed, of the star from the planet or from the fixed stars in question, and from table 2 of chapter 9, or from tables 2, 3 and 4 of chapter 11 of this Section, the true semidiameter of the planet or fixed star to be viewed. For in the figure just given that semidiameter will be AB and the distance AC, or the semidiameter DP and the distance PC. Therefore, by solving triangle ACB or DCP, right-angled at the contacts B and D, you will be able to find the angle C, whose double is the measure of the apparent diameter under which the planet or fixed star A or P would appear to the eye C placed on another star.

Problema 3. Quanta diſtantia requiratur, vt Stella videatur ſub dato angulo, ſeu datæ quantitatis apparentis.

Problem 3. What distance is required for a star to be seen under a given angle, or at a given apparent size?

V. You will solve the present problem by the same right triangles we have been discussing, if the true semidiameter of the star in question is known from the tables mentioned in Problem 2. From this, as the side AB or DP, and from the angle C, you will find by trigonometry the required distance AC or PC.

Problema 4. Quantus appareat Orbis Annuus Solis aut Terræ, vel quiuis orbis Planetarius, viſus ex Saturno, vel ex Fixis.

Problem 4. How large the annual orb of the Sun or the Earth, or any planetary orb, appears when seen from Saturn or from the fixed stars.

VI. You will carry out this problem by the same scheme, given, as I assume, the semidiameter of the sphere of the annual orb of the Sun or the Earth, which semidiameter is the mean distance of the Sun from the Earth; or, if the discussion concerns another sphere, given its semidiameter, or the mean distance of the

planet from the Earth, from table 1 of chapter 2 of this Section; and given besides the distance of Saturn or of the fixed stars from which the annual orb, or any other, is supposed to be viewable. For in the figure above, the semidiameter of the sphere to be viewed will be either AB or DP, and the distance from which the orb is imagined to be viewed either AC or PC. Therefore, by analysing the right triangle ABC or PDC, you will learn the angle ACB or DCP, whose double will be the measure of the diameter of the sphere as it appears to the eye C placed on Saturn or on the fixed stars. We have, however, said much that bears on this about the parallax of the annual orb in Book 6, chapter 7. On the hypothesis of the Copernicans it is useful to reduce to unity the Earth's semidiameters contained in the semidiameter of the sphere to be viewed, and to use the distance AC or PC in parts of which the semidiameter of the sphere to be viewed is one.

[Margin: Šírek's advice on the planets viewed from Saturn, etc.]

VII. It is fitting to add here what Antonín Maria Šírek observes in his *Radius Sidereomysticus*, Book 3, chapter 2, member 3. There he says that, with the eye supposed to be on the planets, it will be seen as a most delightful spectacle how far one of them, with respect to another, is going to retrograde, stand still and digress from the Sun. *For*, he says from his own measures, *Saturn would see Jupiter never straying beyond about 75° from the Sun; Mars about 24°; Venus 12; Mercury scarcely 5°.* These and similar things will not be difficult to determine from the mutual distances of the planets from one another and from the Sun, according to the hypothesis of each astronomer. Accordingly, an eye placed on Saturn, and much more on one of the fixed stars, would judge not only Mercury and Venus, as we now judge from the Earth, but also Mars and Jupiter to be satellites of the Sun, since they do not digress from it by a whole semicircle.

VIII. On these foundations, then, we have constructed the following tables, restricting however a question in itself almost infinite to the more notable cases, which will serve us later, in Book 9, for a fuller knowledge of the system of the world. Granted, the horizontal parallax itself, doubled, can satisfy the question.

I. TABVLA. DIAMETER APPARENS TERRESTRIS GLOBI Spectati ex Fixis aut Planetis in Maxima aut Minima ipſorum à Terra diſtantia.

Table 1. Apparent diameter of the terrestrial globe, viewed from the fixed stars or the planets at their greatest or least distance from the Earth.

Au- thors	✶ Fixed, Min.	✶ Fixed, Max.	♄ Min.	♄ Max.	♃ Min.	♃ Max.	♂ Min.	♂ Max.	☉ Min.	☉ Max.	♀ Min.	♀ Max.	☿ Min.	☿ Max.	☾ Min.	☾ Max.
Ptolemy	✢	✢	23″ 20‴	5″ 40‴	0′ 43″	0′ 22″	5′ 0″	0′ 38″	6′ 8″	5′ 54″	37′ 20″	4′ 50″	124′ 0″	34′ 30″	3° 0′	1° 47′
al-Battānī	20″ 56‴	✢ ✢	36″ 0‴	23″ 0‴	0′ 51″	0′ 33″	5′ 44″	0′ 50″	6′ 28″	6′ 14″	48′ 0″	6′ 40″	107′ 0″	68′ 0″	3° 0′	1° 45′
al-Farghānī	20″ 28‴	10″ 0‴	27″ 20‴	1″ 0‴	0′ 47″	0′ 27″	5′ 40″	0′ 46″	6′ 6″	5′ 52″	48′ 0″	5′ 56″	106′ 0″	67′ 40″	3° 0′	1° 46′

Authors	✶ Fixed, Min.	✶ Fixed, Max.	♄ Min.	♄ Max.	♃ Min.	♃ Max.	♂ Min.	♂ Max.	☉ Min.	☉ Max.	♀ Min.	♀ Max.	☿ Min.	☿ Max.	☾ Min.	☾ Max.
Copernicus	✢	✢	45″	50‴33″	0‴1′	30″	0′	55″	17′	20″2′	16″	6′	12″	6′	0″	23′ 0″	3′	26″	11′	20″4′	10″	2°	11′	1°	46′
Tycho	30″ 12‴	✢	56″	40‴34″	0‴2′	1″	1′	6″	17′	10″2′	18″	6′	14″	6′	0″	22′ 50‴3′	20″	11′	10″4′	8″	2°	11′	1°	58′
Longomontanus	✢	✢	42″	0‴28″	0‴1′	24″	0′	48″	17′	10″1′	58″	5′	30″	5′	20″	22′ 50‴3′	10″	11′	40″3′	40″	2°	12′	1°	59′
Kepler	12‴‴	✢	15″	30‴11″	40‴0′	32″	0′	19″	4′	54″	0′	46″	2′	4″	2′	0″	7′	30″	1′	12″	4′	0″	1′	24″	2°	7′	1°	56′
Lansbergen	17‴‴	✢	32″	20‴4″	0‴1′	8″	0′	42″	13′	50″1′	32″	4′	46″	4′	36″	17′ 10″	2′	38″	8′	36″	3′	10″	2°	14′	1°	57′
Boulliau	✢	✢	35″	30‴25″	40‴‴12″		0′	44″	12′	40‴1′	34″	4′	48″	4′	42″	25′ 0″	2′	30″	9′	40″	3′	40″	2°	8′	1°	52′
Ourselves	4″ 14‴	2″ 0‴	7″ 0‴	4″	40‴0′	16″	0′	9″	2′	52″	0′	20″	0′	58″	0′	56″	3′	36″	0′	32″	1′	40″	0′	38″	2°	14′	1°	57′

II. TABVLA. MAGNITVDO SOLIS APPARENS Spectati ex Stellis Fixis, aut alijs Planetis, in Minima ipſorum à Sole diſtantia.

Table 2. Apparent size of the Sun, viewed from the fixed stars or the other planets at their least distance from the Sun.

Authors	✶ Fixed	♄	♃	♂	♀	☿	☾
Ptolemaics	1′ 50″ 0‴	0° 2′ 10″	0° 3′ 20″	0° 14′ 22″	18° 34′ 0″	0° 35′ 32″	0° 35′ 30″
Copernicus	✢	0° 3′ 52″	0° 6′ 44″	0° 24′ 58″	0° 50′ 20″	2° 31′ 30″	0° 36′ 20″
Tycho	2′ 28″ 0‴	0° 3′ 48″	0° 10′ 0″	0° 23′ 30″	✢	✢	0° 35′ 40″
Longomontanus	✢	0° 3′ 12″	0° 6′ 2″	0° 21′ 42″	0° 41′ 42″	1° 30′ 0″	0° 32′ 48″
Kepler	0′ 0″ 6‴	0° 3′ 24″	0° 6′ 10″	0° 22′ 5″	0° 42′ 28″	1° 39′ 37″	0° 32′ 6″
Lansbergen	0′ 0″ 9‴	0° 3′ 40″	0° 6′ 50″	0° 23′ 46″	0° 51′ 55″	2° 22′ 20″	0° 37′ 15″
Boulliau	✢	0° 3′ 40″	0° 6′ 38″	0° 23′ 52″	0° 45′ 56″	1° 48′ 0″	0° 35′ 10″
Ourselves	2′ 20″ 0‴	0° 3′ 30″	0° 7′ 0″	0° 24′ 46″	0° 50′ 18″	2° 8′ 0″	0° 33′ 20″

III. TAB. MAGNITVDO APPARENS IOVIS Spectati ex ipſius Satellitibus: poſita ipſorum ab Ioue diſtantia ex Galilæo, & diametro vera Iouis ex Auctoribus infraſcriptis

Table 3. Apparent size of Jupiter viewed from its own satellites, their distance from Jupiter being taken from Galileo and the true diameter of Jupiter from the authors named below.

Authors	From satellite 1	From satellite 2	From satellite 3	From satellite 4
Ptolemy	42° 40′	24° 40′	15° 50′	9° 10′
Tycho	103° 6′	56° 2′	34° 10′	19° 18′
Lansbergen	60° 0′	34° 56′	21° 40′	12° 44′

Authors	From satellite 1	From satellite 2	From satellite 3	From satellite 4
Ourselves	42° 34′	22° 50′	14° 6′	8° 4′

IV. TAB. MAGNITVDO APPARENS IOVIS, SATVRNI, SIRII, ALCOR, Si ſpectētur ex tanta diſtantia, ex quanta ſpectamus Solem. Poſita vera Diametro eorum per Nos deducta, & Noſtra Solis Diſtantia mediocri 7300.

Table 4. Apparent size of Jupiter, Saturn, Sirius and Alcor, if they are viewed from as great a distance as that from which we view the Sun, their true diameter being that deduced by us, and our mean distance of the Sun being 7300.

For us it is	True diameter, in diameters of the Earth	Apparent diameter
♃	8 ⅘	8′ 15″
♄ alone	9 ⅗	9′ 5″
♄ with its companions	20 ⅘	19′ 30″
Sirius	17 ½	15′ 0″
Alcor	4 0	3′ 50″

V. TAB. MAGNITVDO APPARENS SIRII Fixarum maximæ, & ALCOR Fixarum minimæ, ſpectatę ex Saturno. Sumpta vera Diametro ex Nobis, ſed Diſtantia ex infraſcriptis Auctoribus.

Table 5. Apparent size of Sirius, the largest of the fixed stars, and of Alcor, the smallest of the fixed stars, viewed from Saturn, the true diameter being taken from us but the distance from the authors named below.

Authors, distances	Sirius	Alcor
Ptolemaics	13° 34′ 0″	3° 6′ 0″
Tycho	1° 12′ 20″	0° 16′ 30″
Ourselves	0° 12′ 18″	0° 2′ 50″
Lansbergen	0° 0′ 17″	0° 0′ 4″
Kepler	0° 0′ 16″	0° 0′ 4″

VI. TAB. MAGNITVDO APPARENS ORBIS ANNVI TELLVRIS & Orbis Saturni, ſpectati ex Fixis. Poſitis Diſtantijs Copernicanorum.

Table 6. Apparent size of the Earth's annual orb and of the orb of Saturn, viewed from the fixed stars, the distances of the Copernicans being assumed.

Authors (Copernicans)	Annual orbit of the Earth	Orb of Saturn
Galileo	0′ 40″	6′ 30″
Kepler	0′ 17″	4′ 18″
Lansbergen	0′ 15″	3′ 0″
Hortensius	1′ 0″	11′ 20″

Authors (Copernicans)	Annual orbit of the Earth	Orb of Saturn
Hérigone	6′ 0″	9° 30′

CAPVT XIV.

Chapter 14.

De Mutuis vel non mutuis Eclipſib. Planetarum vel interpoſitu Corporum, vel Vmbrar: vbi de Prolixitate harum Vmbrarū.

On the mutual or non-mutual eclipses of the planets, by the interposition of bodies or of shadows; where also on the length of these shadows.

I. As for the first kind of eclipse, which arises from the interposition of an opaque body between our sight and a star otherwise visible, there is no doubt that the lower planets can hide from us any star higher than themselves, if they are in conjunction at the same place of the zodiac and a large latitude does not prevent it, that is, a latitude greater than the sum of the semidiameters of the occulting star and the star to be occulted. Kepler discusses these occultations and gives examples in the *Astronomia Optica*, p. 304. He notes, moreover, that part of the Sun can be hidden by Mercury in this century and at almost all times, but not by Venus in this century, although it could once have been and at some time will be, as its latitude permits. But I wish to set down here some examples of occultations of this kind, beginning with the Moon.

Catalogus Stellarum à Stellis occultatarum.

Catalogue of stars occulted by stars.

[Margin: ☾ covers ♂.] II. The Moon covered Mars, as Aristotle observed and attests in *De caelo*, Book 2, chapter 12; and Kepler says, in the *Astronomia Optica*, p. 305, that this happened only in 357 BC, April 4. [Margin: ☾ covers ♃.] The Moon also covered Jupiter in 807, in January, with a certain monk watching, about whom see Kepler in the same place; and in 1646, on December 24, in the evening before 8^{h}, it covered Jupiter, which emerged at Danzig at 8^{h} $39½^{m}$, with Hevelius observing. [Margin: ☾ covers ♄.] And in 1482, January 12, in the morning, it occulted Saturn, as Bernhard Walther reports in his observations and as Kepler weighs in the *Optics*, p. 409. [Margin: ☾ covers ♀.] It also occulted Venus, in 1529, March 12, in the evening, as Copernicus saw and reports in Book 5, chapter 23. [Margin: ☾ covers the Heart of Leo and of Scorpius.] More often, however, it has covered fixed stars that lie within the zodiac: as when, in 1486, October 21, in the morning, it covered Regulus, with Bernhard Walther observing, in his observations; and in 1637, September 23, on the following night, it occulted the Heart of Scorpius, as D. Vincenzo Renieri observed and reports near the beginning of the *Tabulae Mediceae*.

[Margin: ☾ covers Aldebaran.] In 1644, moreover, on November 15, in the morning, the Moon began to occult Aldebaran at 3^h 5^m, with Hevelius observing at Danzig. And in 1645, on October 8, it began to obscure Aldebaran at 1^h 43^m after midnight, again with Hevelius observing at Danzig. He says that this star emerged from the Moon at 2^h 57^m, and from this he gathers that the middle of the obscuration was at 2^h 20^m and that the duration of this eclipse was 1^h 14^m.

[Margin: ♃ covers ♄.] Among the minor planets said to have occulted one another, the first was Jupiter, which in 1563 occulted Saturn, as Kepler reports in the *Optics*, p. 305. [Margin: ♂ covers ♃.] Then Mars, which in 1591, January 9, covered Jupiter while Mästlin watched, as Kepler says in the same place. [Margin: ♀ covers ♂.] And Venus, which in 1590, October 3, at 5^h in the morning, occulted Mars; [Margin: ♀ covers ☿.] and in 1599, June 8, covered Mercury, on the testimony of Kepler in the same place. [Margin: ☿ covers ☉.] I say nothing of Mercury occulting a small part of the Sun, for enough has been said on this in Book 3, chapter 3, scholium 3.

[Margin: ♃ covers fixed stars.] Finally, the minor planets have not rarely covered some one of the fixed stars. For Jupiter occulted the Southern Ass in 241 BC, September 4, in the morning, according to Ptolemy, Book 11, chapter 3; twice it occulted another star, which is in the feet of Gemini, on the testimony of Aristotle, *De caelo*, Book 2, chapter 12; and it occulted Propus, the star before the feet of Gemini, in the year of Christ 1633, December 19, in the morning, as Pierre Gassendi, who observed it, reports in his *Life of Peiresc*.

[Margin: ♂ covers fixed stars.] Mars, moreover, was seen to occult the northern star in the forehead of Scorpius, in the year 272 before our Saviour, January 18, in the morning, according to Ptolemy, Book 10, chapter 9; and the outermost star, which is in the wing of Virgo, as Gassendi observed and recounted in the *Life of Peiresc*. [Margin: Whether ♀ covered Regulus.] But whether Venus covered Regulus in 1574, September 16, at 4^h in the morning, with Mästlin observing, and in 1598, September 25, at 3^h in the morning, on the testimony of Kepler in the *Optics*, p. 305, some affirm and others deny.

De Vmbris Planetarum.

On the shadows of the planets.

[Margin: Authors who have written on these shadows.] III. No astronomer, so far as I know, has discussed the shadows of the minor planets except our own Biancani, in his *Sphaera*, both towards the end and in the individual treatises on these planets. From him Andrea Argoli took the same measures in the *Pandosion Sphaericum*, from chapter 51 to 59, though he suppressed his name; and Father Athanasius Kircher, at the end of volume 2 of the *Ars Magna Lucis et Umbrae*, did not depart far from Biancani. The shadow of Jupiter darkening its satellites, however, has been treated by nearly all who have treated its four companions, as I reported in Section 1, chapter 3 of this Book. The Moon's shadow has been

dealt with sufficiently in Book 4, chapter 17, from Problem 5 on, and in Book 5, chapters 8 and 9, where the Moon's shadow is treated as bounded by the Sun's rays, both unrefracted and refracted in the air round the Moon, if such air is admitted.

[Margin: 3. Assumptions for measuring the shadows.] To come now to the measure of the shadows cast by the opaque bodies of the remaining planets, we first assume three things. First, that the bodies of the planets are opaque and spherical or globular, which many think of Saturn too when taken alone; though for the shadow it casts together with its attendants [lateronibus], let it suffice to use the composite diameter of Saturn, whether it is then spherical or not. Second, that all the planets are in reality smaller than the Sun, as is now clear from Table 3 of chapter 9 and Table 2 of chapter 10 of this Section; from which it follows that if they cast a shadow, it is not calathoid or cylindroid but conoid or quasi-pyramidal, from what was said in Book 5, chapter 3. One thing, however, must be noted: the shadow that Saturn casts together with its companions is not always of the same shape, just as it does not always turn its face toward us with the same appearance, according to what has been said elsewhere and will be said in the following Scholia. Hence it also follows that a diameter of one constant size is not to be used to find the measure of the shadow that Saturn with its companions extends onto another star. Third, that all the planets receive either all or nearly all of their light from the Sun, according to what was said in Section 1, chapter 5. From this it follows that they cast, into the part turned away from the Sun, a pointed shadow that finally ends in some point. With these laid down, let this be

Problema 1. Inuenire Axem ſeu altitudinem Vmbræ Planetæ, tam maximæ, data maxima; quam minimæ, data minima ipſius à Sole diſtantia; & differentia veræ ſemidiametri Solis ac Planetæ à Sole illuſtrati.

Problem 1. To find the axis or height of a planet's shadow, both the greatest, given the planet's greatest distance from the Sun, and the least, given its least distance, and given the difference between the true semidiameters of the Sun and of the planet lit by the Sun.

IV. We gave the greatest and least distances of the planets from the Sun in chapter 2, Tables 3 and 4, and in chapter 3, Tables 1 and 2; their true diameter in chapter 9, Table 2, and in chapter 10, Table 2; and finally the true diameter of the Sun in Book 3, chapter 11. Now since as diameter is to diameter, so is semidiameter to semidiameter, if you subtract the semidiameter of the minor planet from the semidiameter of the Sun, there will remain the difference of the semidiameters that we need here. This Problem, moreover, is solved in four ways.

[Margin: 1. Method.] The first method, used by Boulliau in Book 4, chapter 3, is this. In this first scheme, let AB be the known semidiameter of the Sun and

CD that of the minor planet; and from the Sun's centre A, through the centre C of the minor planet, let the straight line AC be drawn, to be produced until at E it meets the straight line BDE, which touches the Sun at B and the minor planet at D. For the height of the shadow sought will be CE. To find it, cut off from AB a portion AF equal to the semidiameter CD of the minor planet; for FB will be the difference or excess by which the semidiameter exceeds the semidiameter of the other planet. And finally, from the point of section F, draw a parallel to the end D of the diameter, which, with an imperceptible difference, coincides with the point of contact of the line BDE. This produces two similar triangles, FDB and AEB, and the parallel FD cuts the sides AB and BE proportionally, by proposition 2 of the sixth book of the *Elements*. Therefore, since AF is known, being equal to CD, and FB, the difference of the semidiameters, and FD, which is equal to AC, the distance of the planets, by proposition 33 of the first book of the *Elements*: let it be made, as the difference BF is to the whole BA, so FD to the whole AE. In this way the whole AE will be known, in the parts in which the distance AC was known; and when the distance AC is subtracted from the whole AE, there will remain the known height CE of the shadow. Because, however, the point of contact D lies somewhat lower than the end of the semidiameter CD, that is why this comes about.

Diagram. The first scheme, a small woodcut set into the right column. A straight line along the top is lettered A F B, with a large semicircle hanging below it, the Sun's hemisphere. Two lines run from its ends down to a vertex at the foot, forming a tall narrow cone. A vertical line runs from A to the vertex, and a second vertical runs from F to the right-hand tangent at D. On the axis a small circle stands inside the cone, its upper half white and its lower half solid black, the diameter between the halves ending at D on the right. Below it a short cross-line is lettered L N M, and beneath this the point of the cone is hatched.

- A (top, on the line, at the foot of the axis): centre of the Sun
- F (top, between A and B): the point of section
- B (top right): the Sun's tangent point
- C (centre of the small circle): centre of the minor planet
- D (right end of the small circle's diameter): point of contact
- L (left end of the cross-line), N (on the axis), M (right end of the cross-line): not cited in the text of this page
- [?] (a small mark inside the hatched cone, just right of the axis below N, perhaps a letter T; not resolved)
- E (below the vertex): the point of the shadow

[Margin: 2. Method] The second method is the one familiar to us. For it, in this second scheme, let EMF be the hemisphere of the Sun and ABCD the globe of a planet smaller than it; and from the Sun's centre K, through the planet's centre H, let the straight line KH be drawn and extended until at G it meets the straight lines EAG and FCG, which touch the Sun at the points E and F and the minor planet at A and C. To these points of contact let the radii KE,

KF and HA, HC be drawn from the centre of each planet. By proposition 18 of the third book of the *Elements*, these will make right angles with the tangents at the points of contact E, A, etc.

Finally, from EK let EL be cut off, equal to the semidiameter AH, and from H let the straight line HL be drawn. For it will be perpendicular to EK, making right angles at L on both sides, and at the same time parallel to EAG, by proposition 33 of the first book of the *Elements*. Hence the triangle KHL, right-angled at L, will be similar to the triangle KGE, right-angled at E, by proposition 4 of the sixth book; and by proposition 2 of the sixth book of the *Elements*, as the side KL is to KE, so is the side KH to KG. Now the side KL is known, because it is the excess of the Sun's semidiameter over the semidiameter of the minor planet; the distance KH of the planets is also known. Therefore, if it is made, as KL is to the whole semidiameter KE, so the distance KH to KG, the straight line KG will be known in the same kind of parts; and when the distance KH is subtracted from it, there will remain the known HG, which is the height or axis of the shadow AGC that was sought. Or, since the triangles KHL and AHG are similar, let it be made, as KL is to KH, so AH to HG.

Diagram. The second scheme, a small woodcut set into the left column. A straight line along the top carries K at its centre and E and F at its ends, with a large semicircle hanging below it, the Sun's hemisphere. Two straight lines descend from E and F to a vertex G at the foot, forming a tall narrow cone. A vertical axis runs from K to G. A small circle stands on the axis inside the cone, touching the two lines at A and C, with radii drawn from its centre H to A and C. A slanting line runs from L, near the top on the left, down to H.

- K (top centre): centre of the Sun
- E (top left), F (top right): the points where the lines touch the Sun
- L (inside the cone, near the top left): foot of the perpendicular from H
- M (on the axis, just below the semicircle)
- I (left of B, where the slanting line meets the small circle): not cited in the text of this page
- B (on the axis, top of the small circle)
- H (centre of the small circle)
- A (left), C (right): the points of contact on the small circle
- D (on the axis, bottom of the small circle)
- G (below the vertex): the point of the shadow

[Margin: 3. Method] The third method is carried out by means of the same second figure, but not without trigonometric labour. For first, in the triangle KHL, right-angled at L, the angle KHL is found from the given distance KH and the difference KL of the semidiameters. Because of the parallelism of the lines HL and EG, on which the straight line KG falls, this angle is equal to the interior angle AGH, by proposition 29 of the first book of the *Elements*. Then, in the triangle AGH, right-angled at the contact A, the base GH, namely the height of the shadow, is found from the angle AGH just obtained and the semidiameter AH.

[Margin: 4. Method.] The fourth method, which Biancani suggests after the second in Book 4 of his *Sphaera*, chapter 3, is mechanical and is carried out with the help of compasses. On some very large plane you draw the preceding figure, keeping the proportions that have to be kept, so that on KH you can mark off by division the individual equal parts, each representing one semidiameter of the Earth, with the semidiameters KE and HA proportional to it. For compasses spanning HG and laid along KH will show the size of HG itself. But the operation is a slippery one.

Problema 2. Diſcernere an & cuius Planetæ Vmbra poſſit, aut non poſſit alterum Planetam inumbrare, ſi Latitudo hanc Eclipſim permittat.

Problem 2. To determine whether, and whose, planet's shadow can or cannot darken another planet, if the latitude allows this eclipse.

[Margin: 3. Conditions required for an eclipse of the planets by a shadow.] V. Three things are required for this test: namely, the order itself of position among the planets, the distance between them, and the size of shadow belonging to them at such a distance. As regards position, no planet can darken another unless it can be interposed between that other and the Sun. And no higher planet can be interposed before a lower one, that is, one nearer the Earth, except ♀ and ☿, which can be interposed between the Sun and the Moon, and Mercury, which can pass between the Sun and Venus even when Venus comes nearer to the Earth than Mercury does. Conversely, every lower planet can be interposed before a higher planet, except Venus compared with Mercury: even when perigean, Venus cannot pass between Mercury and the Sun, for it is certain that both are carried around the Sun in such a way that Venus surrounds it in a wider circle than Mercury does.

[Margin: Mercury immune from the shadow of the others by reason of position.] From what has been said it follows that Mercury alone, by reason of position, cannot be darkened by any planet, being the innermost of all to the Sun, and that Hermes forever enjoys the light of Apollo's face. As regards the mutual distance of the planets, it is certain that, other things being equal, a greater risk of being darkened by another threatens a planet when they are least or less distant from one another than when they are most or more distant. As regards the size of the shadow: the conical shadow of globes is both longer and wider, or both more drawn out and thicker, when they are farther from the greater illuminating body (in our case, from the Sun) than when they are less far from it. Therefore there is also a greater risk of darkening from that lower planet which, and when it, is interposed at its greatest or a greater distance from the Sun, than from the one which, or when it, is at its least or a lesser distance from the Sun. With these things indicated beforehand, the distances and the shadows must be compared, but only those that can meet in the same quarter; for if the shadow exceeds the distance, one planet can be darkened by

the other, but it cannot if the distance exceeds the shadow. We shall therefore first set out a table of shadows, and then settle this matter very quickly with a second small table.

I. TABVLA. VMBRÆ ALTITVDO MAXIMA ET MINIMA In Semidiametris Terræ.

Table 1. Greatest and least height of the shadow, in semidiameters of the Earth.

From the data of the authors	♄ Max.	♄ Min.	♃ Max.	♃ Min.	♂ Max.	♂ Min.	♀ Max.	♀ Min.	☿ Max.	☿ Min.	☾ Max.	☾ Min.	Earth Max.	Earth Min.
al-Battānī	34052	26976	27492	23639	1390	449	58	2	212	198	71	54	245	229
al-Farghānī	83270	69750	34445	25935	2080	629	104	3	254	214	72	55	271	252
Tycho	14700	13116	3622	3218	157	133	✢	✢	✢	✢	73	63	282	268
Kepler	5226	4669	2830	2572	451	400	176	172	111	74	66	57	246	238
Lansbergen	13160	11640	5683	5104	183	143	129	102	22	10	62	57	239	222
Ourselves	11868	09259	14572	12145	423	302	177	141	28	14	60	53	231	213

II. TABVLA. VMBRÆ ALTITVDO MAXIMA ex Tychonicis principijs.

Table 2. Greatest height of the shadow, from Tychonic principles.

Authors	♄	♃	♂	♀	☿	☾	Earth
Biancani and Argoli	12527	859	147	102	43	67	254
Athanasius Kircher	12532	869	147	102	43	68	264

VI. Our Biancani rightly concluded, at the end of his *Sphaera*, from Tychonic principles, that apart from the eclipse of the Moon by the Earth's shadow, the eclipse of the Earth by the Moon's shadow and the eclipse of some fixed stars by Saturn's shadow, no other eclipse by the shadow of the planets can occur (the Jovian satellites excepted), because their greatest shadows are shorter than the distances between the nearer planets. But what is to be said under other hypotheses will be clear from the following propositions, which are proved from the size of the shadows, given in the preceding small table, and from the distances of the planets from the Earth or from one another, already set out in the tables of chapters 2 and 3. Because of the affinity of the hypotheses and the proportion of the measures, one can safely decide about the Copernican hypothesis from that of Lansbergen, about the Danish hypothesis of Longomontanus from the Tychonic, and about the Philolaic hypothesis of Boulliau from the Keplerian: granted that neither Copernicus nor Longomontanus nor Boulliau has left us the apparent or true size of the five minor planets, on which, however, the determination of the size of the shadow depends.

Theoremata de Eclipſibus Poſſibilibus, vel non poſſibilibus ab Vmbra Terræ vel Planetarum.

Theorems on eclipses that are possible or not possible by the shadow of the Earth or of the planets.

1. Theorem. *In any hypothesis whatever, the Earth's shadow can obscure the Moon, and the Moon's shadow the Earth.* This is clear from what was said in Book 5, chapters 3, 8 and 9.
2. Theorem. *In the Ptolemaic hypothesis, especially that of al-Farghānī, Mercury can darken the Moon, and Venus Mercury, and Jupiter Saturn and the fixed stars, and Saturn the fixed stars; but the remaining eclipses of planets by the shadows of the others are impossible.*
3. Theorem. *In every other hypothesis of a more celebrated astronomer, neither can the Earth darken any planet except the Moon, nor can any planet darken any planet that is not its own companion or satellite.*
4. Theorem. *In every other hypothesis, neither the Earth nor any planet can darken the fixed stars, except in the Tychonic, in which Saturn alone can darken some fixed stars; yet it cannot obscure them, if the fixed stars have their own light, but can only, by a kind of penumbra, deprive them of that secondary light which they would otherwise have from the Sun.* The same would have had to be said in our own hypothesis, had we not, in Book 6, chapter 7, raised the fixed stars as far as 210000 semidiameters of the Earth precisely in order to avoid this shadow. For although, as we said in the same place, there is no danger of an absolute eclipse of the fixed stars by the shadow of the planets, since they have their primary light from themselves, yet because the Sun, by the magnitude of its brilliance, does in the end add some light to them, then if they were not at a distance exceeding the length of the shadow of Saturn taken with its attendants, some of those in the zodiac could be deprived of that secondary light and suffer a semi-eclipse.

But it is more probable that, just as GOD has allowed only the Moon, as a terrestrial planet, to be subject to the alternations of eclipses that arise from the Earth's shadow, and the satellites of Jupiter to the shadow of Jupiter, because they are its companions, while in every hypothesis of the more recent astronomers He has exempted all the remaining planets from these shadowings, so too, and much more, He willed the non-wandering stars to be exempt. Thus from this too they would earn the name of fixed stars, being unchangeable even with respect to that very addition of light received from outside, which they receive from the Sun. And in this too they would be a symbol, either of the Blessed, who, walking in the light of the divine countenance, will shine like the stars of the firmament for perpetual eternities, or else some sort of trace of the divine immutability itself, with which there is no shadow of change. Besides which, a great space between the fixed stars and Saturn suits both comets and new stars and the symmetry of the world system, so that the interval from Saturn and

the last sphere of the planets to the fixed stars is at least as great as that from the centre of the world to Saturn.

SCHOLIA.

Scholia.

De Vmbra & Figura Saturni & Comitum.

On the shadow and figure of Saturn and of its companions.

[Margin: Examination of Saturn's shadow and of the spots appearing on it.] I. In Section 1, chapter 2, number 7, above, I said it is almost established that both Saturn's companions and Saturn itself likewise lack light of their own, from the shadow Saturn throws onto them; and in chapter 5, number 5, I hinted at this same opinion of mine, which I used to hear commonly asserted on all sides. For I had not yet noticed that, with our eye on this side of Saturn, the caster of that shadow, and lying almost between Saturn and the Sun, it is impossible for that shadow to spread out to the sides, and so to show us its companions darkened on both sides, unless it were calathoid, and Saturn were therefore not a little but much larger than the body of the Sun that illuminates it. I came to this suspicion while debating the matter with the Lord Antonio Rocca, my Uranian friend, a man most learned above all and most worthy of the friendship of any of the wise. At that time, since we had not yet found a way of determining Saturn's apparent diameter evidently, I kept turning over in my mind, and was in doubt, whether, with other evident phenomena preserved, a Saturn larger than the Sun could be upheld, so that with its shadow calathoid the phenomena of its attendants, which we depicted in Section 1, chapter 2, in figures 3, 4 and 5, could be more easily explained.

But when the observation of the diameter had been made, on which see chapter 10 of this Section, that whole suspicion vanished; and we were made certain that Saturn is much smaller than the Sun, and therefore that, if it has its primary light from the Sun, its shadow is conical and does not suffice to explain the cause of both those obscurities or blacknesses that are seen equally, at the same time, between Saturn and its companions. It may be, then, that Saturn, shining of itself, is prevented from being able to illuminate its attendants entirely by some other body distinct from it, or by some part at its back protruding in the form of a mountain; or that it has caverns devoid of light, which, by the varied libration or rotation of its body, it displays to us at different times in different figures; or rather that it is itself lit by its attendants; or that, as in the Moon, however full, there are certain larger spots, that is, parts not so suited as the others to reflect the Sun's light toward us, and these therefore appear to us darker, especially at so great a distance; or that it is something else, which will be made manifest to us or to our posterity after many experiments with the phenomena.

[Margin: The new and multiform figure of Saturn.] Meanwhile nothing occurs to me that satisfies altogether, especially since not only Fontana, in 1644 and 1645, but also some of the Fathers of our Society, and above all Father Mattheus Tauerna [?], attest that they saw through the telescope Saturn having the appearance of the figure set down at the end of these scholia, and showing several bodies, partly spherical, partly egg-shaped. This, however, it has not yet been granted us to see, but only under the figures shown in Section 1, chapter 2, and in this Section, chapter 10, number 8. To this last figure seen by us, the one most closely similar is that which I later saw in Hevelius's *Selenographia*, chapter 4, in figure G and letter C, seen by him in 1645, in September, October and November.

II. Hevelius adds, moreover, that in 1642, in September and October, Saturn was seen by him on separate occasions as quite round. Gassendi attests that it was seen so by him in the same year, in the month of August, in the *Judgement on the 9 Stars Seen around Jupiter*, p. 14, affirming that Saturn was likewise seen round by Galileo 30 years before. From this Hevelius gathers that Saturn's companions lie hidden behind it at definite times. But what if either both of them or one of them, being no larger than Saturn, passes between it and our eye on the same line at the time when Saturn appears round, and, in receding from it on both sides, casts a shadow onto it? But more observations and periods are needed for the causes of this phenomenon to become known.

[Margin: Hevelius's error on the line of the companions of ♄.] Meanwhile it is certain that in these years the line drawn through the centres of Saturn and the satellites lengthwise is not parallel to the ecliptic, as Hevelius thinks in chapter 4 of the *Selenographia*, I believe from an argument drawn from the line of Jupiter's satellites, but parallel to the equator. This is clear from what was said in Section 1, chapter 2, number 7, and in this Section 6, chapter 10, numbers 8 and 9, and it is confirmed by the very recent observation of Father Francesco Maria Grimaldi of 1650 March 18. For on the same straight line there appeared the centres of Saturn and its companions and a certain small star (unnamed, because it can be observed only with the telescope), distant from the nearest attendant toward the east by about half the Moon. On the night following the 23rd, however, Saturn overtook the small star, leaving it lower down toward the south, at an interval as large as about 4 greater diameters of Saturn would fill. From this it was plain that the line of Saturn and its companions is neither in the ecliptic nor parallel to the ecliptic; otherwise, by its motion in longitude, it would on the 23rd have covered that small star or grazed it, whatever variation by reason of latitude it had undergone in the space of 5 days.

[Margin: Opinions of Antonín Maria Šírek on the figure, light and shadow of Saturn and its companions.] III. But to return to the figure and shadow of Saturn: we must not pass over what Antonín Maria Šírek devised in his *Radius Sidereomysticus*, Book 4, chapter 4. For he says [?] that certain people think that the two handle-stars revolve around Saturn and are therefore hidden in the evening or in the morning; yet he denies that this has ever been observed

by him. But if this should happen, he says that this motion is very slow and observable only after many years; and that at their conjunction with Saturn, evening for the one and morning for the other, Saturn, being alone, must necessarily appear round, and at their quadratures three-bodied. Šírek goes on to say: if it is granted that the attendants already mentioned circle Saturn, and that because of their motion it sometimes appears solitary and round, it must also be granted that it can at times be hidden and eclipsed by the attendants; or, since this has never been seen, one of two things must necessarily be granted;

that is, that the attendants just mentioned revolve on the near side of Saturn; or, if this is so, that they are necessarily endowed with their own light and are to Saturn as luminaries, by whose light, being destitute of its own, it is refreshed and illuminated. He then at once teaches that Saturn cannot be sufficiently illuminated by the Sun because of its huge distance, so great that Saturn does not see the Sun's disc except under an angle of 3′ 11″; and that the Earth is therefore illuminated at least 100 times more strongly and intensely than Saturn. From this he draws the following conclusion: "We therefore think it most probable, indeed necessary, that Saturn obtains light sufficient for itself from somewhere altogether other than the very remote Sun; and since its two attendant stars are seen to be very bright, it surely seems necessary that Saturn borrows from them, on either side, the chief rays of its light."

From this, and from the rotation of Saturn about its own axis, which he sets as completed in $29^{d}\ 10^{h}\ 1^{m}$ (we have already set out the basis of this motion in Book 3, chapter 4), and having assumed, following the opinion of Cardinal Cusanus, inhabitants on Saturn, he reasons that such inhabitants will have most delightful and almost perpetual days, and will experience the alternations of summer, winter, spring and autumn, according as the hemispheres of the attendants turn toward Saturn a part more or less hot or bright. For he had invented something similar about two hemispheres of the Sun's body, as we said in the same chapter 4. He then adds: "One might also perhaps suspect whether the middle globe, luminous by its nature, does not perhaps illuminate the two opaque attendants, all other things, however, being understood to be equal, as has been said." So much for this author's inventions.

[Margin: Šírek's statements are weighed.] IV. For the rest, although much of what has been said displays the man's talent and acumen, certain things cannot at all win my approval. First, from the fact that the attendants, when in conjunction with Saturn, bring no eclipse, it does not necessarily follow either that they revolve on the near side of Saturn or that Saturn, lacking its own light, is illuminated by them. For it can fail to be eclipsed for this reason: that they revolve beyond Saturn; or, if they revolve between us and Saturn, that both they and Saturn have their light from the Sun; or that they have their own light, but Saturn has its light from the Sun. For although Saturn is ten times further from the Sun than the Sun is from the Earth, Saturn can still be illuminated by the Sun with that languid and leaden light which we see in it, especially since the quality of the surface, and the very matter of the body of

Saturn and its companions, assist the reception and reflexion of the light. And certainly that a privation of its own light in Saturn does not necessarily follow from what he said, Šírek himself has admitted, either contradicting himself or correcting himself, when he says that one may suspect that Saturn, luminous by its nature, illuminates its attendants.

I would have preferred him to give a reason why that twin blackness or darkness on either side, which very frequently appears around Saturn, does not come from its companions (for he says that Saturn has never been seen hidden by them), or else where it does come from. Moreover, from the fact that Saturn sometimes appears solitary, it is not a necessary consequence that it appears round: for it can appear egg-shaped, as it sometimes does appear, even when the attendants are visible on both sides, as has been noticed more than once with the telescope, and as may be seen from the figure shown at chapter 10 of this Section.

We therefore do not reject the probability of Šírek's arguments, but the necessity of the consequences, which he draws without evident reason. In the same way we also do not approve that what the same man ingeniously invented about the rotation of Saturn and the other minor planets about their own axes, in his Book 4, chapter 2, member 3, should be thrust upon us as secrets communicated to him almost by divine means. For he says there: "Now as to the time of the rotation of the planets, hear an astounding secret, known until now, so far as I know, to no one": namely, that each of the planets named turns 365 times on its own axis within one of its periodic revolutions, just as the Sun rolls itself 365 times yearly over the Earth. And meeting an unspoken question, he says: "You will ask from where we have taken these secrets, and on what foundations they rest. I answer with the Archangel Raphael, Tobit 12: It is good for now to hide the secret and sacrament of the King, but to reveal and confess the works of God is honourable. Let this answer suffice for the wise reader for now." Relying on his authority, however, our own Father Athanasius Kircher, not absolutely but on a hypothesis, has ingeniously carried certain things over to the harmony of the world and of the planets, in volume 2 of his *Musurgia*, Book 10, p. 380; on these, see below, Book 9, in the discussion of the harmonic system.

It remains for us to close this seventh Book with the scheme, already promised, of the seventh of the planets, as with a seal.

Diagram. Saturn as seen, set across both columns: a large central circle with a smaller circle standing clear of it on either side along the horizontal. Between the central circle and each small circle lies a dark, closely hatched region, broad against the central circle and narrowing to a point against the small circle. A single curved outline passes over the top and under the bottom of the central circle and meets each small circle, enclosing the whole in an elongated oval. Caption: *Saturn as seen in the year 1645, in December, and sometimes also in the year 1644, at Rome and Naples.*

- Labels read: none. The figure carries no letters.

• Labels not read: none.
End of the Seventh Book. [FINIS LIBRI SEPTIMI.]

APPENDIX AD I. PARTEM TOMI I.

Appendix to Part I of Volume I.

PROHOEMIVM.

Preface.

[Margin: 3 reasons for adding this Appendix.] I had three reasons for this Appendix. The first: to add in their proper places certain things that became known to me afterwards, either from books not read before or from letters of astronomers or of others sent to me, or that had slipped away while I was writing. The second: that the teaching on one and the same subject, given for just reasons in different places of this work, may be gathered into one by those who perhaps would not easily manage this from the indexes of both volumes. The third: to explain and complete certain things said rather obscurely or imperfectly and, if anything seemed to need correcting, to correct and retract it myself, of my own accord. In this I believe that no one will be of so severe a brow as to abuse the sincerity of my confession and want me reproved twice: because I have erred somewhere, namely, or nodded when I ought not to have done, and because I was not ashamed to disclose errors or blemishes. The former of these belongs to man; the latter to one who candidly prefers truth and the benefit of others to his own reputation. If, however, any censor of this kind should arise, I shall answer him with no other words than those of St Augustine, from the prologue to Book 1 of the *Retractationes*. [Margin: St Augustine's sincerity and modesty.] "For no one but an imprudent man will dare to reprove me because I reprove my own errors. But if he says that those things which afterwards displeased even me ought not to have been said by me, he speaks the truth and is on my side: for he reproves the same things as I do. For I ought not to reprove them if I ought to have said them. But let him who could not hold the first part, that of wisdom, hold the second, that of modesty: so that he who was not able to say everything without cause for repentance may at least repent of what he has come to know should not have been said."

Ad Præfationem.

On the Preface.

[Margin: Miracles for the Roman calendar.] Article 2, p. 5, where the miracles done for the Roman calendar or for the correct celebration of Easter are treated: add what Paschasius relates in the Letter to St Leo the Pope, which is number 65 among the letters of St Leo; and Cassiodorus, *Variae* Book 8, chapter 33; and Baronius [Cesare Baronio] at the year 417, and from him Odoricus Rinaldus [Odorico Rinaldi] in the *Annales*. [Margin: Roundness of the heavens.]

IN CHRONICO.

In the Chronicle.

Add Ecphantus the Pythagorean, who held, as Philolaus also did, that the Earth moves; although his period is not yet established. To what has been said of Girolamo Cardano, add that he was born with hair, already covered with black, very long and thick hair; that he was pulled by force from his mother's womb and revived in a bath of wine; some say he was born 1501 August 24 [9. Kal. Sept.]. Add likewise Johannes Stadius, born in Loehnoutio, a village of the Brabant country, author of the Bergen tables, and very dear, because of his astrological divinations, to the prince of the Allobroges and to Henry III, King of the French, by whom he was made Royal Professor of Mathematics. He died in the year 1579, June 17 [15. Kalend. Iulij], aged 52 years and almost 2 months. Add also Valentinus Naiboda, born at Cologne, and dead at Padua in his house and found torn with wounds, having once predicted for himself death by iron.

AD LIBRVM I.

On Book I.

Chapter I, column 2. I promised a trigonometry, that is, a fuller one; yet it seemed good to give its Epitome beforehand in Book 10, Section 1. Anyone who does not have from elsewhere the rules for solving triangles by logarithms may have recourse to it meanwhile.

Chapter II, Scholium 7. What I touched on concerning the opinion of the Fathers on the roundness of the heavens must be understood and illustrated from what is said in Book 9, Section 4, chapter 5, number 2.

[Margin: Solidity of spheres.] Chapter VI, Problem 3. Join to it the related problems on the solidity of spheres compared with one another, given later in Book 7, Section 6, chapter 8, from Problem 8.

[Margin: Finding the meridian.] Chapter X. I have added other ways of finding the meridian line, though more recondite ones, in Book 10, Section 4, from Problem 49 to 66.

[Margin: Days of the Sun and of the primum mobile.] Chapter XI. I give a specimen of the double table named here in Book 10, Section 3, Problem 1, and earlier in Book 3, chapter 31; these are to be reconciled together. You have the authors of the tables converting hours into degrees, and conversely, in Book 10, Section 3, Problem 3.

[Margin: Finding the altitude of the pole.] Chapter XII. The remaining ways of either finding or confirming the altitude of the pole, which I judged out of place here, I teach more suitably in Book 10, Section 4, from Problem 67 to 103.

[Margin: Difference of geographical longitude.] Chapter XIII. On finding the difference of geographical longitude, or the distance of the meridians, I propose several other methods to the now better-instructed reader in Book 10,

Section 4, from Problem 104 to 120. What I advised in Scholium 3 about the start of longitude differing among different authors, and from where it is better taken, I discuss more fully in my *Geographical Book*. Meanwhile it is enough to have added the chief opinions. The start of the meridians, then, was made as follows.

[Margin: The start of the meridians varies.] Eratosthenes from the Pillars of Hercules, namely from Abyla and Calpe, as is found in Strabo, Book 1.

Ptolemy and Marinus of Tyre, and the greatest part of the older geographers, from the westernmost of the Fortunate Isles, although many enter on it from the Fortunate Isles indiscriminately. See Ptolemy, *Geography* Book 1, chapter 11, and there his interpreters Werner, Magini, Moletti, Bencius [?].

Ismaël Abilfedea [Abū al-Fidā], the most celebrated geographer of the Arabs, from Cadiz [Gadibus], namely from the island of the Strait of Hercules.

Gerard Mercator and his son Rumoldus [Rumold Mercator] from the island of Palma, which is now the westernmost of the Fortunate Isles; although in the year 1634 the King of France ordered the cosmographers of his kingdom to establish the first meridian on the island of Ferro, as

Fournier relates in Book 12 of the *Hydrographie*, chapter 2.

Gulielmus Iansonius [?] in the 4 maps of the world published in the year 1624; Iosephus Rosaccius [Giuseppe Rosaccio], Simon Stevin, and Willem and Joan Blaeu in the new Atlas: from the island of Tenerife, about the middle of the Fortunate Isles, because of the height of Mount Pico, of which we shall speak in Book 10, Section 4, Problem 30 and Section 6, Problem 50.

Gulielmus Iansonius in the plane hemispheres, Gerard Mercator the younger, Ortelius in the world map, Magini in the table of Africa, Petrus Bercius [Pieter de Bert] in the abridged Europe: from the Island of Fire, which is one of the Hesperides, because on it the magnetic needle declines in neither direction from the meridian line, and the meridian of that island passes through Thule, the furthest of the lands known to the ancients.

The same Iansonius in the world map of the years 1604 and 1607, Gerard Mercator in the description of the polar land, Nicolaus Piscatorius [?] and Franciscus Hoeius [?]: from the island of Corvo [Corui], which is one of the Azores, where most people have thought that the magnetic needle does not decline at all, although Robert Dudley asserts that it declines a little.

Arnoldus de Arnoldis [?] in the world map and Jodocus Hondius in his globe: from the island of St Vincent, both because the magnet does not decline at all there, and because that island and Thule have a common meridian.

Astronomers: from the place to which they fix the roots of the astronomical tables, on which see Book 3, chapter 33, the last table. But Govaert Wendelen begins the Atlantic tables from that meridian of the Atlantic Ocean which passes through Hola and Schaltola, towns of Iceland, formerly Thule.

What if someone were to begin from the meridian from which the first day of the world began, according to the differing opinions on that first day that we have given in Book 9, Section 1, chapter 1, question 8 and chapter 4,

question 6? Or rather from the line which Alexander VI, Supreme Pontiff, in the year 1493 ordered drawn 100 leagues beyond the Hesperides, as the limit of Portuguese navigation from there eastward and of Castilian navigation from there westward, the parties themselves thus seeking his arbitration? Yet in the year 1524 John, King of Portugal, obtained that this line be moved forward by 300 leagues beyond the Canaries, so that it would then pass through the Land of Smokes and over the Cape of Good Appearance; and so it came about that the Moluccas turned out to be western, as Don Francisco López de Gómara and Don Jerónimo Girava relate in the description of the West Indies. Maffeius [Giovanni Pietro Maffei], however, in Book 1 of the *History of the Indies*, and Giovanni Botero in the *Relations*, say that this line was once drawn 140 leagues beyond the Hesperides and afterwards 580; and Fournier, in Book 12 of the *Hydrographie*, chapter 2, says it was drawn 370 leagues beyond the island of St Anthony, the westernmost of the Hesperides.

[Margin: Our opinion.] I, however, began my Table of Longitudes from the island of Palma, whose meridian passes through the eastern part of Iceland or Thule, and very close to Cape Verde, and through the island of St Mary. I did so both in order not to depart without great cause (and none yet exists) from the chief and more common opinion of geographers, and because nearly everyone grants that whatever lies on the near side of that meridian, and within the equator, belongs to the old world, and whatever lies beyond it to the new.

[Margin: Two zodiacs, and the varied naming of the signs.] Chapter XIV. You have the Arabic, Chaldean, Hebrew and Egyptian names of the signs of the zodiac in Book 6, chapter 3, in the Scholia. There I also computed the year before Christ in which the first star of the constellation naming each Sign stood at the beginning of the dodecatemorion [Dodecatemorij], and the longitude and latitude of these constellations. See also what I insisted upon in Book 3, chapter 14, in the Scholia, about distinguishing two zodiacs. [Margin: Greatest latitude of the zodiac and of the planets.] What I said in Scholium 1 of this chapter about determining the latitude of the zodiac from the greatest latitude of the planets must be illustrated or even corrected from what is said later in Book 7, Section 3, chapters 5, 6 and 7, according to the differing opinions on this latitude. Yet, when other ages are considered, the latitude of Mars can be 7° and that of Venus 10°, as Kepler admits in the *Epitome of Copernican Astronomy*, Book 2, p. 150, where he adds that therefore the latitude of the zodiac must be extended not only to 14° but also to 20°. At the end of Scholium 5 I said, following Pliny the Elder, that the obliquity of the ecliptic was discovered by Anaximander; but Plutarch, *De placitis philosophorum* Book 2, chapter 12, attributes this to Pythagoras, while Theon ascribes it to Oenopides.

Chapter XVI. What I said in chapter 14, number 6, and in this chapter on divine Providence in distinguishing the five zones by the obliquity of the ecliptic, Kepler pursues broadly and learnedly in the *Epitome of Copernican Astronomy*, Book 3, from p. 330. [Margin: Reason for the number and latitude of the zones.] For if the ecliptic were not inclined to the equator, but the Sun

ran perpetually along the equator, there would be no variety of seasons and no alternation of the quarters of the year. Conversely, if it were inclined to the equator at a right angle, and the Sun always travelled along some fixed meridian of the movable heaven, the properties of all the zones that now exist would be confounded. For all climes would share in the conditions of the torrid, the temperate and the frigid; but in the zones that are now frigid and temperate, that is, in the greatest part of the globe, there would be excessive cold because of the total recession of the Sun, and excessive heat because of its approach to the zenith [verticem]. As it is, the alternation of these qualities is so distributed that the variety in the nature of living things and animals, suited to their heterogeneous parts and humours, is preserved, and the unbearable long duration of cold and heat is removed; and the extreme regions, torrid and frigid, are joined by the temperate ones between them, and not immediately.

[Margin: Proportion of the parallels to the equator.] Chapter XVIII. On the proportion that the parallels and their degrees have to the equator and to a degree of the equator, see Book 10, Section 4, Problem 1.

[Margin: The mediation of the heaven, taken equivocally among astronomers.] Chapter XXII. In Definition 35, with Copernicus, Book 2, chapter 3, and most others, I defined the mediation of the heaven [Mediationem cęli] as the point of the ecliptic that crosses the meridian at the moment some star crosses it; and with this meaning Magini assigned mediations of the heaven to the chief fixed stars in the introduction to the Ephemerides, part 4, chapter 5, as Montebruni also did later at the end of his Ephemerides. Yet the same Magini, in the Directions of the primum mobile, Canon 8, and in their tables, p. 54, just as Regiomontanus once did in the Directions, Problems 3 and 4 and pp. 16 and 40, used the mediation of the heaven for the point of the equator crossing the meridian together with the point of the ecliptic or the star; that is, for right ascension.

[Margin: Increase of the artificial day from refractions and from the rising of the Sun's limb.] Chapter XXVI. On artificial days lengthened by the refraction of the ray, which apparently raises the Sun, see Book 10, Section 3, Problems 12 and 13, if anyone is hurrying to that learning. But the day is also lengthened, if you reckon it from the rising of the upper limb to its setting, by as much as the time required for the whole disc of the Sun to rise; on this see the small table in the Appendix to Book 3, chapter 10.

Chapter XXVII. If anyone wants the authors of tables of the semidiurnal arc or time, let him consult Book 10, Section 2, Problem 10, where I also teach other ways of composing them, from Problem 7.

[Margin: Altitude of the twilight exhalations.] Chapter XXXI. To complete the teaching of this chapter, two problems are needed first, to be sought from Book 10: one in Section 3, Problem 25, for deducing the moment of time from the fixed stars; the other in Section 2, Problem 221, for finding geometrically the depth of the Sun at that moment. What I promised in this chapter 31, number 3, about correcting this altitude by refracted rays, I chose to perform in Book

8, chapter 14, from Problem 4, where I refuted the opinion of Gassendi, who thinks that it comes out twice as small because of refractions, and I recast a new table for this altitude. Nor did I follow Kepler in the *Epitome of Copernican Astronomy*, Book 1, p. 73, where he thinks that a ray of the Sun, refracted at its first entry, before it goes out of the refracting air and is refracted, is thrown back inward repeatedly within this air, from its surface as from a mirror; for I do not think this air is so bounded that it has the smoothness necessary for a mirror.

[Margin: Kinds of risings and settings, and their definition.] Chapter XXXII. On the kinds of rising and setting of the stars, I have set out many things for the now better-instructed reader in Book 6, chapters 20, 21 and 22, from which very many examples can be sought, and from chapter 21 several kinds as well as definitions and rules; and anything said obscurely or less exactly in this chapter 32 can be illustrated or corrected from there. But let him who instructs beginners beware lest with too abstruse learning he deter them from the study of astronomy rather than attract them.

[Margin: Aspects of the stars.] Chapter XXXIII. Some things about the aspects had to be given as a foretaste in this place; but a fuller treatment was to be put off to Book 7, Section 5, chapter 9, as has been done. Let the reader fly ahead to it, however, if, impatient of delay, he wants to survey together all the material on the same subject.

[Margin: Parallaxes.] Chapter XXXV. A few things about parallaxes are indicated here for novices; more are said elsewhere: namely, on the parallaxes of the Sun in Book 3, chapter 8; of the Moon in Book 4, chapters 13 and 14; of both, for use in eclipses of the Sun, in Book 5, chapter 10; and of the remaining planets in Book 7, Section 6, chapter 7. The complete treatise, however, and all the problems of parallaxes I deliberately reserved for Book 10, Section 5; nor do I yet regret having discussed them in different places, as the proper subject of each book required.

[Margin: Refractions.] Chapter XXXVI. Likewise the candidate in astronomy had to be advised about refractions in this place; but their effects had to be treated afterwards, as the occasion varied, in Book 3, chapter 2, number 10, and chapter 9, and in Book 4, chapter 15, and Book 5, chapter 5. The truly subtler problems, however, had to be reserved for an entire section on them, which is the sixth of Book 10.

AD LIBRVM II.

To Book II.

[Margin: The height of mountains does no harm to the roundness of the Earth.] Chapter I. If you are loath to await the edition of volume 2, in which my *Geographical Book* stands, you may still take the argument from this Book 2 of the work now appearing, chapter 7, where the Earth's diameter is treated, and from Book 10, Section 4, Problem 30, and Section 6, Problem 50, where the

height of mountains is treated. For we know that the highest of the mountains on which history and evidence give any certainty is the Caucasus, and it does not exceed 50 Italian miles, of which there are 8278 in the diameter of the Earth and 4139 in the semidiameter; and in this figure the number 50 is contained 83 times. Take, then, even a wooden sphere whose semidiameter is 83 equal small parts, and place on it some grains or little mounds of wax, their height equal to a single one of the 83 parts; and you will see by experiment with your own eyes that their tiny size does no harm to the physical roundness of the sphere, especially when it is viewed from a distance.

[Margin: The Earth's position in the middle of the world.] Chapter II. In Number 1, where it is said that according to the Copernicans the Earth moves in that annual orb in which the others hold that the Sun moves, this must be understood of that orbit as regards the orbit's size and the manner and speed of the motion, not as though the Earth stood in the same position in the space of the world. To argument 6 of the same chapter a seventh can be added, from Kepler's *Epitome of Copernican Astronomy* [Epitome Aſtronomiæ], Book 1, p. 38, taken from the appearance of the Milky Way, which the Germans call St James's path. For it divides the sphere of the fixed stars into two apparent hemispheres in such a way that, although it is not of great breadth, nowhere on Earth is anything but half of it seen, that is, an arc of nearly 180°. But if the Earth were outside the middle of the world, withdrawn to one side, it would see the whole milky circle at once, as a circle of the sphere that is not a great circle but a small one. If it lay indeed in the plane of the milky circle, but came nearer to it on one side, a smaller part of the Milky Way than 180° would be seen by it, while the antipodes would see many more degrees of it than 180.

Chapter III. In Scholium 1 of this chapter, the arguments from the parallax of the annual orb and from refractions that I brought against the Copernicans, partly in Book 6, chapter 7, and partly in Book 9, Section 4, have force only against the measures of some Copernicans, not absolutely against Copernicus. Against him, however, there are other arguments, physico-mathematically evident, on which see the same Section 4 of Book 9.

[Margin: Whether the centre of heavy bodies is the same as that of the universe.] Chapter IV. In this place I discussed the common centre of gravity of all bodies according to the opinion more received in the schools, that of the Earth's rest. But whether, with the Earth moved through the annual orb, its centre turns out to be the centre of all heavy bodies, which heavy bodies seek in order to have and keep a fixed place in the world, or whether they tend to it so that parts may be joined to their whole, must be discussed in Book 9, Section 4, chapters 8 and 25. The error of the geometers in measuring mountains, indicated in this same chapter at number 8, I repeat in Book 10, Section 4, Problem 32, but there I bring the remedy.

[Margin: The size of the Earth compared with the heaven.] Chapter VI. Scholium 2. If you are in a great hurry here and are eager to know how large the Earth is relative to the Sun, Saturn, etc., see Book 3, chapter 11, Problem 1,

and Book 7, Section 6, chapters 9 and 10. But if you want to know how large it would appear if viewed from the planets or fixed stars, see Book 7, Section 6, chapter 13.

[Margin: The Earth's diameter and circumference.] Chapter VII. I admit that in this chapter, on account of the beauty and necessity of the problem, I reported the opinions of others in such a way that I did not then intend to set them out geometrically in this first volume with figures of their own. But in Book 10, Section 4, from Problem 2, I chose to carry this out, lest, if volume 2 should perhaps never see the light, cosmography should be left incomplete in this part. Let the reader therefore join together, if he pleases, what is said there and here, so as to have a fuller learning on these matters. Moreover, compare the former method, which I reviewed from Father Casati in Number 4, with the one I brought from Clavius in Book 10, Section 4, Problems 5, 6 and 7: for they seem the same, but they are different. Add to these, besides, the opinion of Bartolomeo Crescenzio in Book 2 of the *Nautica Mediterranea*, where he says that ship-masters [Naucleris] have found 87½ miles in each degree of the meridian circle. This is nearly true of the older Roman miles, and Alessandro Piccolomini had said it before him in his work on the sphere. I also remember having read somewhere of 100 English miles attributed to one degree by Francis Drake. Marin Mersenne, on Genesis, p. 111, gives the Earth's semidiameter 3436 Italian miles and the circumference 21600, or 7200 leagues.

Chapter X. Besides the table placed here, see another, constructed by me, in Book 10, Section 4, Problem 39, running up to an eye height of 50 Italian miles of Bologna, or 63 ancient Roman miles, which I take to be the greatest height of mountains. Beneath that one I have placed a third as well, from Kepler. [Margin: Distance of sight.] From all of these, however, Crescenzio stands refuted. [Margin: Crescenzio's error.] In Book 1 of the *Nautica Mediterranea* he had said that the greatest distance from which lookouts on the ship's top lookout post see other ships and their sails is 20 miles, a distance which the eyes of those looking from the bottom of the ship nevertheless do not reach; and he had added that according to most ship-masters the cause of this difference is the roundness of the sea's surface. Yet he dares to pronounce this openly false, because at a distance of 18 miles the sea does not fall away from the straightness of the horizontal line by more than two paces. He himself therefore puts the cause in the thickness of the vapours that lie between the object and the eyes of those looking from the bottom of the ship, whereas those looking from the masthead enjoy purer air. But he errs badly in geometry: for at a distance of 18 Italian miles the sea, because of its roundness, falls away from the horizontal straight line by more than 30 paces, as you see in the table of this chapter. Therefore, if the top of a ship's mast is seen from the top of another's 20 miles away, either both masts must be 15 paces high, so that a height of 30 paces rises from the two together, or whatever one lacks of 15 the other must have to spare. That is, unless perhaps refractions of the denser air help somewhat in seeing

an object otherwise not to be seen from so great a distance, according to what will be said in Book 10, Section 6, Problem 50.

Our Georges Fournier treats this problem learnedly in Book 19 of the *Hydrographie*, chapter 8. Taking from Tycho a diameter of the Earth of 1718 German miles, he shows that a man 6 feet tall cannot see from the surface of the sea beyond a French league, that is, beyond 15000 feet or 3000 paces. He attributes to Eratosthenes the same opinion that Macrobius held, namely that of 180 stadia; but if this is true, he teaches that the place from which the sensible horizon was being viewed was 368 ⅓ feet high, or 73 ⅗ [?] paces; that the place from which Proclus and Geminus estimated the distance of sight was 2274 paces high, if that distance was 125000 paces or 1000 stadia; and that Lilybaeum too is 7358 paces high, if, as Cicero relates in Book 4 of the *Academic Questions*, Strabo from its summit counted ships still lying in the harbour of Carthage, namely from a distance of 1800 stadia or 225 miles, although Pliny the Elder, Book 7, chapter 21, restricts that distance to 125 [CXXV] miles, unless 225 [CCXXV] is to be read. Finally, if what Patrizi relates and the French ship-masters assert is true, [Margin: Fournier's opinion on the distance of sight.] that the mountains of the island of Corsica are seen from as far away as Toulon, namely from a distance of 100 leagues or 300 miles, they must rise 5807 geometric paces. He does not, however, think that any mountain is really found 7358 paces high in the perpendicular; on this we are compelled to disagree with him, because of what was said in Book 1, chapter 11. Yet he rightly concludes that the distance of sight varies with the height of the eye, or of the visible object, or of both.

[Margin: Kepler's opinion on the view of the sea from mountains.] But Kepler too, in the *Epitome of Copernican Astronomy*, p. 26, says that the reason why intervening seas and the circles of visible land look flat is that the highest mountains are no taller than half a German mile, and from that height an object does not appear to be depressed below the horizontal line by more than two degrees. Yet, he says, there is scarcely any mountain so great from which a free view over the sea lies open and is not obstructed by other mountains, forests and so on, unless perhaps the mountains of Chile are of that kind. But on the height of mountains he is mistaken, as I said in chapter 11.

[Margin: Whether the sea is higher than the land.] Chapter XIV. The question could have been added whether the sea is higher than the land of its shores, as it commonly appears to be and as seems to be gathered from Sacred Scripture; but Ascanio Martinengo has already treated this learnedly in the *Glossa Magna*, p. 736. Kepler touched on the optical cause in Book 1 of the *Epitome of Copernican Astronomy*: the eye sees the more remote parts of a floor by a more elevated ray.

[Margin: The depth of the sea.] Georges Fournier, in Book 18 of the *Hydrographie*, chapter 13, lists various depths of the seas. He says that near the Cimbric Chersonese, or Jutland, the bottom of the water has been found with the sounding-lead [bolide] at 24 ells, each of which consists of 6 feet; near Frisia at 18; near Zeeland at 26. Those going from Cape Finisterre toward Ireland on

a north-north-east course [per Nord Nordest], at a distance of 24 leagues from the shore, find the water 60 ells deep; toward Picardy 20 or 24; near the shores of Bordeaux, at the distance at which the eye discovers land, the depth is found to be 40 ells; 22 leagues south of Belle-Île [Inſula Bella], 70 ells; and between England and the Surlingas [the Scilly Isles], 70. He adds, moreover, that unless the lead of the sounding line is proportioned to the length and weight of the rope, it floats and does not descend to the bottom, and that this is why the sea seems in some places to have no bottom. Posidonius, in Strabo, Book 1 of the *Geography*, asserted that the sea had been found 1000 ells deep; although ship-masters, where they have reached a depth of 100 or 120 ells, do not care about the rest, being confident that they are then far from the shore and from shoals. Finally he censures the method of Leon Battista Alberti for measuring the depth of the sea, as I also did in Book 2, chapter 5, number 10. Robert Dudley [Robertus Dudlæus] lists several other depths in the *Arcana of the Sea*; and in Book 2, chapter 8, Rule 34, he concludes that a whitish colour of the sea is a sign of a nearby shore and little depth, a bluish colour of moderate depth, and a deep blue of great depth.

[Margin: The equilibrium of seas.] I return to Fournier, who in the same Book 18, chapter 9, asserts that it is impossible for seas that communicate with one another by their waters, at least through some strait, to be higher one than another. He therefore considers groundless the reason for the fear of the kings of Egypt, that if they joined the Arabian Gulf to the Mediterranean by removing the isthmus, the gulf would overwhelm Egypt. Nor does he believe, as some allege, that the sea at the port of Nombre de Dios [Nominis Dei] is higher than the sea at the port of Panama, or that because of this height there would have been anything to fear if that isthmus of 7 leagues were dug through, as certain Spaniards wished.

[Margin: The tide of the sea.] Chapter XV. Below, in Book 9, Section 4, chapter 14, there will be occasion to go over the history and rules of the tide of the sea again, more accurately and at greater length, against Galileo's fictions.

As for the currents [Currentes] of the waters of the ocean, on which see here at number 1, consult Fournier, Book 9 of the *Hydrographie*, chapters 22, 23 and 31, and Robert Dudley, Book 2, chapter 21, of the *Arcana of the Sea*, where there is very much learning about them; the same author, p. 66, adds rules of flow and ebb. See also what I shall add in Book 9, Section 4, chapter 14, number 13, and the history of the tides in the Cimbric Sea in Heinrich Rantzau, in the *Calendar* [Kalendario], from p. 489.

Chapter XVI. On ships, their kinds and parts, Fournier should be read in the whole of Book 1 of the *Hydrographie*. On the speed of sailing westward I shall speak in the Appendix at chapter 17, and in Book 9, Section 4, chapter 14, number 13.

Chapter XVII. On the winds, for nautical use, no one I have read treats more accurately than our Fournier in the whole of Book 15 of the *Hydrographie*;

yet Robert Dudley adds many things in Book 2 of the *Arcana of the Sea*, from chapter 20, from which I have chosen to indicate the choicer points.

[Margin: Winds of the sea and of the land.] First, winds risen from the vapours of the sea are moister, warmer, more even, more easily raised by the heat of the stars, purer of the fumes of exhalations, and for the most part more violent than those of the land, if you believe Fournier; and the branches of trees near the sea-coasts are for the most part bent by the breeze toward the mainland, which is a sign that winds from the sea are more frequent and stronger. Second, east winds blow chiefly in the morning and west winds in the evening, and east winds are usually stronger and drier. North winds in Europe, on the other hand, blow violently downward from on high and are cold and dry, the opposite of south winds; yet not rarely the same wind is moist and unhealthy in one region and dry and healthy in another, of which Fournier brings examples in chapter 16.

[Margin: General and fixed winds.] Third, general and fixed winds, or those that are lasting, regular and tempered, are those that in the cold zones and the torrid, within the tropics, are carried from rising to setting, following the daily motion of the Sun. Sailing westward is therefore, other things being equal, healthier and faster, so much so that on the return from the East Indies toward the island of St Helena one comes back almost twice as fast as one went. For this reason the Spaniards, to sail more quickly through the Pacific to the Philippines, China and Japan, set out from Acapulco and go down as far as the 10th parallel; there, having caught the east winds, they cover 3000 miles in 85 days. When they have reached the Islands of Thieves [Insulas latronum], they go up to the 13th degree to find the mouths of the Archipelago and from there reach Manila. But on the return from there to Acapulco, to avoid the force of the east winds opposing them, they must turn north as far as the 40th degree. In the same way, those heading from Seville to New Spain sail nearer the Tropic of Cancer; but on the return, after the passage near the Bahama, which is at 18° of latitude, they withdraw from the equator toward Florida and Virginia, as far as the 40th degree, to avoid contrary winds and gain favourable ones, so as to return to Seville by way of the region of the Azores. Granted, the currents of the sea-water toward the west are no slight cause of this difference. Besides these there are other, regular winds, those which ordinarily prevail in some regions at fixed times of the year; Fournier lists them in chapter 13.

[Margin: Free and variable winds.] Fourth, other winds are called free and variable. Most of those that prevail in the temperate zones are of this kind, if they are compared with those that prevail in the cold or torrid zones; they blow more often in the morning or evening than at noon, are less healthy, more violent, and contend with one another, especially beyond the Tropic of Capricorn. [Margin: Stormy winds.] Among these the stormiest of all, called hurricanes [Oricani] or typhoons [Typhones], arise near the tropics when the Sun reaches the tropics, but outside the torrid zone; and they are more violent near the shores or among islands than on the high sea, or where continents

run out crosswise toward the poles and struggle with their own winds against the fixed winds. In the West Indies, in the months of July and August and part of September, the fixed winds cease and hurricanes and the most violent storms rage, called *Spouff* by the English, especially on the Mexican coast and in the Gulf of Bahama. In those months, therefore, few larger vessels trust themselves to the ocean; these storms also breed most pestilent fevers, which kill within 24 hours. In the same months, in the Gulf of the Ganges, on the coast of Natal and along the Cape of Good Hope, typhoons prevail, no milder than hurricanes. [Margin: The violence of typhoons.] But a storm becomes more dangerous when currents of water are carried against the wind, for from this conflict the waves swell excessively. The whirlwinds of typhoons are dangerous, especially to those sailing toward Japan in the Chinese Ocean, so much so that they sometimes snatch away yards and sails and dash them through the air against crags and cliffs, in the way that griffins in Muscovy and Tartary carry off armoured horsemen with their horses. [Margin: Siphons.] In the Mediterranean, more often than in the ocean, there arise Trumpets [Tubæ] or Siphons [Syphones], namely winds enclosed in a cloud, which suck water back from the sea, discharge it onto ships, and sink them. Storms from subterranean winds that swell the waters must also sometimes be feared, even if the air is calm and the sky clear, as Fournier teaches by examples in chapter 18. He also treats most learnedly of St Elmo's Fire, or the flames of the Tyndarids, in chapter 20; of the cause of winds in chapters 22 and 23; and of the signs of storms in chapters 31 and 32; as does Dudley in Book 2 of the *Arcana of the Sea*. Let it be enough to have indicated these.

[Margin: Magnetic declination.] Chapter XVIII. I promised a longer and better table for magnetic declination; but if anyone does not expect it, or cannot wait, let him consult for these matters Robert Dudley, Book 2 of the *Arcana of the Sea*, chapter 8; Fournier, Book 11 of the *Hydrographie*; Pierre Hérigone, volume 4 of the *Cursus Mathematicus*, from p. 426; and Crescenzio, Book 2 of the *Nautica Mediterranea*, chapters 8 and 9.

De Altitudine Regionum Aeris.

On the Height of the Regions of the Air.

Chapter XIX. Number 1. On the height of the refractive air I have given more, but for those further advanced in astronomy, in Book 10, Section 6, from Problem 43 to 51, where I established that it is in some places, namely above the Euxine and Caspian, no less than 18 or 20 miles.

[Margin: The third region of the air.] Number 2. What I said about the height of the twilight exhalations will be treated again, on the occasion of comets, in Book 8, Section 1, chapter 14, where I shall confirm my opinion on this height. I said, moreover, that the uppermost region of the air is as great as that of the exhalations [halituum seu exhalationum]. Understand this to mean that it is not smaller, but not that it is precisely as great as that of the

twilight exhalations; for other exhalations, subtler than these and less opaque, can rise higher than the twilight fumes. And certainly it is fair to leave some place for the purest air, as Cabeo observes on Book 1 of the *Meteorologica*, text 10, question 1. In addition, the height of some sublunary comets urges the same. Besides, this was said according to the opinion of the Peripatetics, who place fire above the air; for those who deny an element of fire distinct from the heaven and the remaining three elements, whom I shall list in Book 9, Section 1, chapter 1, questions 4 and 5, are divided into four classes. The first teaches that fire does indeed exist, but that it extends from the air all the way to the firmament and does not differ from the ethereal or celestial substance; I shall list these in Book 9, Section 1, chapter 1, question 4, and chapter 5, question 3; and add Paracelsus to them. The second, akin to this, places nothing between the air and the firmament but an ethereal breeze, though they do not say that it is fire: so Kepler, here and there in his works. The third places air from the Earth all the way to the firmament: so Froidmont, Book 1 of the *Meteorologica*, chapter 2, with Thomas Fienus [Thomas Fyens] in *The Comet of the Year 1618*, and before him Rothmann, in Tycho's *Letters*, from p. 106 to 112, from 121 to 124, and from 137 to 139, contending from the likeness of their transparency that pure air and ether are one substance and of the same species, while impure air, which produces the refractions, is watery. The fourth places air up to the Moon, and that substantially distinct from the ether or heaven. I shall list its authors in Book 9, Section 1, chapter 1, question 4, and above all Tycho in the places cited, and in volume 1 of the *Progymnasmata*, p. 92; add to them Cabeo on Book 1 of the *Meteorologica*, text 15, question 1, and Franciscus Resta [Francesco Resta], Book 2 of the *Meteorologica*, chapter 2. Let this be said on the occasion of the third region of the air.

[Margin: The second region of the air.] Number 3. What I said about the second or middle region of the air depends on a double controversy, one about the height of clouds, the other about the height of mountains. On both there is great diversity of opinion and uncertainty, and far greater if anyone should wish to determine one from the other, namely the height of clouds from the greatest height of mountains, or the height of mountains from the height of clouds. On the height of clouds I have said enough here; I add, however, the following. Suppose that, as Pliny the Elder, Book 2, chapter 23, reports from the opinion of many, some clouds rise to 900 stadia, or 112 Italian miles; and what Father Emmanuel Maignanus [Emmanuel Maignan] reports in his *Perspectiva Horaria* as observed by himself and others supports this: small clouds around midnight at the interlunar time lit by the Sun, and therefore lying outside the shadow of the Earth at a great height. If these things, I say, are true, the height of mountains is badly estimated from the height of clouds, for I said in chapter 11 that mountains scarcely reach 51 Italian miles in perpendicular height. [Margin: The fallacy of the argument for the height of the air from mountains.] But the argument Cabeo touches on, Book 1 of the *Meteorologica*, text 15, question 2, about Mount Olympus, once immune from winds, clouds and rain and yet not

higher than 10 or 11 stadia, is most fallacious and most unsuited to determining the height of the second region of the air. [Margin: First reason for the fallacy.] First, because the height of Olympus was not taken as a perpendicular above the sea, but in the way I have taught to be fallacious in Book 2, chapter 4, number 8, and Book 10, Section 4, Problem 32. For Plutarch, in the *Aemilius Paulus* [in Paulo Æmylio], relates that Xenagoras measured it from the Temple of Pythian Apollo; so Olympus was much higher above the surface of the sea than 10 stadia, as Maioli too judges in Colloquy 16. [Margin: Second reason.] Second, what St Augustine, Book 1 of *On Genesis against the Manichaeans* and *On Genesis Literally*, Book 1, chapter 14, and Book 3, chapter 2; Solinus, chapter 13; and St Isidore, Book 14 of the *Origins*, chapter 4, relate about the ashes of the sacrifices left untouched by wind and rain on the summit of Olympus: [Margin: The excess of Olympus above the clouds and winds called into doubt.] granted that this is not to be referred to the power divinely permitted to demons, who, to entice the heathen and to win authority for those sacrifices, kept those ashes intact, as Aversa reports, Question 42 of the *Philosophy*, Section 9, it can still be traced to a peculiar property of that region. Because of the quality of its soil, no doubt, it would not supply vapours and exhalations fit for so great an elevation or for strong winds, but only for light breezes. Cardano held this in Book 2 of *De subtilitate*, and held that for the contrary reason some mountains higher than Olympus can yet be subject to winds, snows and rains and have clouds above them, especially if there are lakes not far from the peak from which vapours can rise, and if they are not so rocky, pumiceous or ashy that exhalations can hardly be drawn from them.

[Margin: Third reason for the fallacy.] Third, many call the excess of Olympus above the clouds into doubt for this very reason, that with Aristotle they think the middle region of the air is bounded by the tops of the highest mountains, and that therefore there is no mountain that exceeds the clouds. Among them are Vives and Coquæus [Leonard Coqueau] on St Augustine, *De Civitate Dei* Book 15, chapter 27; Pererius on Genesis, Book 12, disputation [d.] 9; Fernandez on Genesis chapter 7, section 4; and Salianus in volume 1 of the *Annals*, at the year of the world 1656, numbers 34 and 35. The reason being, of course, that Francesco Filelfo [Franciscus Philelphus], as Mazzoni reports on Dante, Book 3, chapter 12, and Maioli in Colloquy 16, climbed Olympus and yet saw clouds not below him but above (which, however, one could attribute, with Francesco Resta, Book 2, chapter 2 of the *Meteorologica*, to the height of Olympus now diminished by the floods of so many centuries, floods being sufficient for this around the middle of the mountain and undermining the foundations of the summit). Or at any rate the Olympus Filelfo explored was a different one, as Mazzoni and Maioli think. But Genesis 7 says of the Flood: *And all the high mountains under the whole heaven were covered. The water was 15 cubits higher over all the mountains it had covered*; and at the end of the Flood it rained from the clouds. Much more, then, did the clouds from which those rains came exceed Olympus and all the mountains; which, however, one might

attribute, not without a miracle, to an extraordinary elevation of the clouds, rising with the increase of the waters that burst from the abyss. And Froidmont rightly denies, Book 2 of the *Meteorologica*, chapter 6, article 5, that the water of the Flood, or its clouds, should be judged by the common laws of meteorology. [Margin: St Augustine's excellent opinion on the height of mountains.] And so St Augustine, *De Civitate Dei* Book 15, last chapter, and in the *Questions on Genesis*, question 10, said shrewdly: *On the height of the mountains, all of which, it is written, the water overtopped by 15 cubits, some wrongly have doubts because of the story of Mount Olympus. For if earth could invade the space of that calm air, where it is said that neither clouds are seen nor winds felt, why not water too, by rising?* By the name of earth he means the summit of Olympus.

Now who would not hiss off Caietanus, who, on Genesis chapter 7, preferred to deny, against the plain and universal sense of Holy Scripture, that absolutely all mountains were covered by the waters of the Flood? For he says that only those were covered which are under the airy heaven, or under the second region of the air, in order, of course, to except those mountains that are said to lift their peak above the clouds. But Sacred Scripture says that all the high mountains under the whole heaven, etc., were covered: what more universal could be said? Or if Holy Scripture is not to be taken so universally, why does Caietanus take with such rigour the Apostle's saying about Adam, *In whom all have sinned*, so that by no reason and no authority of the Doctors [Margin: Caietanus is censured in his exposition of Holy Scripture.] can it be removed from including the Blessed Virgin MARY under the common stain of original sin? Must Olympus, then, on the authority of two or three men, be exempted from the waters of the universal Flood, while the Most Holy MOTHER OF GOD [DEIPARA], exempted by GOD with so many other privileges from the common condition of mankind, could not by the sublimity of her dignity and merits, and by the prerogative of divine munificence toward her, rise above the flood of original guilt as if by the loftiness of a summit: she, I say, of whom it is said in Psalm 86, *Its foundations are in the holy mountains*? But this in passing. Salianus's other reason, for which he thinks fabulous what is told of Olympus, and which Pererius too had adduced, is that on the one hand Olympus, by Xenagoras's measurement, was no higher than 10 ⅙ stadia, and on the other vapours rise to 52 or at least 43 miles, as from the twilights de- monstrate Nonius and Clavius; at which point Salianus and Pererius confuse twilight exhalations with rain clouds, but wrongly, and so they are rightly censured by Francesco Resta in Book 2 of his *Meteorologica*, chapter 2.

[Margin: Pererius and Salianus confuses [confundit] twilight exhalations with clouds.]

[Margin: 4. Reason.]

Fourth. From history it seems certain that there are several mountains which are higher than the clouds and beyond the common hazard of rainstorms, winds and snows. For Geminus, in his *Elements of Astronomy*, chapter 14, affirms of Mount Cyllene too what is said of the untouched ashes of Olympus, and

adds that Satabyrium rises above all clouds. Aristotle, section 26, problem 38, affirms the same of Mount Athos, as do Solinus, chapter 16, and Pomponius Mela, Book 2, chapter 2, who add as evidence the untouched ashes; and for that reason, they say, there was on its summit a town called Acroton, and later Uranopolis and Apollonia, so healthful that people there lived twice as long, and were therefore called Macrobii; and Pliny the Elder, Book 4, chapter 10, subscribes to these authors. And Aelian, in the *Varia Historia*, relates that when physicians advised Plato to leave the unhealthy site of the Academy and move to the Lyceum for his health, he replied: *I would not move even to the upper parts of Athos, which are most healthful, in order to live longer.* On Atlas lifting its head above the clouds, see Solinus, chapter 27; Mela, Book 3, at the end; and Martianus Capella, Book 6, the chapter on Africa, even though Pliny the Elder, Book 5, chapter 22, says that its summit is covered with deep snows in summer too, and Virgil sings of it in *Aeneid* 4:

whose pine-bearing head, with black clouds, is beaten both by winds and by rain.

Franciscus Aluarez [Francisco Álvares], chapter 35 of his *Ethiopian Affairs*, relates that in a province of Ethiopia called Temei there are mountains like pyramids, of no great circuit at the base but so steep that they are higher than the clouds. And Father Alphonsus Oualius [Alonso de Ovalle], chapter 5 of his *Account of Chile*, asserts that the mountains of Chile are so high that the air is scarcely fit for breathing because of its thinness, and that frequent panting has to be tempered with small linen cloths held to the mouth; that not rarely falling stars have clung to the feet and ears of many people and frightened them; that perpetual clouds are looked down on below, the sky in the upper parts being always clear, and that the rainbow is seen underneath like a footstool; and that while it rains and hails below, travellers meanwhile tread those rocks dry and parched, and that the rocks are white and very bright, and the rays of the Sun are thrown back from them as from mirrors; yet in winter they are covered with snows; so that even from this you may notice that the domain of the snows is much higher than that of rainstorms and rains,

[Margin: 5. Reason.]

Fifth. The reason why Aristotle drew the boundary of the second region of the air, assigned to snows and clouds, a little below the summits of the highest mountains was that clouds are not carried along by the motion of the primum mobile, whereas he thought that comets are carried along by that motion and come to be in the highest region of the air. But since that carrying-along [raptus] does not reach below the Moon, the heaven of the planets being of course fluid, as we shall say in Book 9, section 1, chapter 7, he feared in vain for clouds carried above the mountains. Then too, even if that carrying-along were granted, it could not be impeded by the summits of the highest mountains, which are very rare. So the division of the air handed down by Plato in the *Timaeus* seemed sufficient, to which St Damascene [John of Damascus] later subscribed in his *Physics*, chapter 13: namely, into a purest aether, dry and hot, and into a turbulent air, more humid and colder. Pererius, however, in Book

12 on Genesis, disputation 10, places the middle region of the air so far above all the mountains that the Flood either did not reach it, or reached only its lower part.

CONCLVSIO.

Conclusion.

It is therefore established from what has been said that the height of the whole air, or of its regions, is uncertain, since it is uncertain whether below the Moon there is an element of Fire distinct from the aether; and, if there is, how far the air reaches, both the air which is the seat only of the subtler and hotter exhalations, and the air which is the seat of snows, winds, clouds and rains: for twilight exhalations do not exceed 100 Italian miles, as I taught in Book 1, chapter 31, and shall teach in Book 8, section 1, chapter 14; and yet some very light small clouds may have risen beyond this limit. In some places, moreover, and at some times, clouds and winds stay below mountains that are not the highest; and conversely, in other places and at other times, much higher mountains have their heads exposed to snows and clouds, especially if there are lakes around their middle parts, from which vapours can be raised above their summits. Finally, it is wrong to infer either the greatest height of mountains from the greatest height of clouds, or the greatest height of clouds from the greatest height of mountains; or the greatest height of some clouds or some mountains from the greatest height of other clouds or other mountains. In general, however, it seems certain that no mountains are much higher than 50 miles, that no small clouds or twilight exhalations are much higher than a hundred or a hundred and twenty Italian miles, and that air dense enough to be suited to sensible refractions of the rays of the stars is not raised much beyond 20 Italian miles. The first is clear from what was said in Book 2, chapter 11; the second from what was said in Book 2, chapter 19, and in this appendix to that chapter; the third from what will be said in Book 10, section 6, problem 51.

[Margin: 3. Certain things about the height of these.]

CAP. XX. Gassendi, in Letter 1 *De motu impresso*, p. 64, holds with Galileo in Dialogue 2 *On the Systems* that the reason why the arcs of vibration of one and the same pendulum become smaller and smaller, and why the pendulum finally comes to rest, is twofold: namely, the resistance of the air, and the weight of the cord or chain and of all its parts. For the parts nearer the vertex of the motion strive to swing back and forth more often, just as if they were partial and separate pendulums, while the more remote parts do so less often; and from this a certain resistance arises. From this Gassendi infers that if this motion took place in a vacuum, and the hanging weight were suspended from a radius of no heaviness, the motion of the pendulum to and fro would be perpetual.

[Margin: Cause of the pendulum's rest.]

Hydrargyric or mercurial clocks, which Tycho finally abandoned in astronomy and found unreliable (volume 1 of the *Progymnasmata*, p. 149), were taken

up by Robert Dudley, Book 1 of the *Arcana of the Sea*, for measuring a sea voyage, and he teaches their construction and use. He recommends using six clocks full of quicksilver, and gives their measure as set out below.

[Margin: Clocks of water or of quicksilver.]

For the time of the primum mobile (hours, minutes)	Pounds of quicksilver
0^h 1^m	1
0^h 5^m	3
0^h 30^m	4
1^h 0^m	6
6^h 0^m	12
24^h 0^m	24

And he rightly warns that a whole clock, and not a part of it, contains a correct time; and that if the liquid of one entire clock is allowed to run out, sea water can be used in place of quicksilver. For he says he found by experience that from one sunrise to the next 1800 pounds of water often flowed out of his glass vessel, and that he never found a greater difference in weight than half an ounce, to which 30″ correspond, since 1800 pounds, that is, 21600 ounces, correspond to 24 hours, that is, to 1296000 seconds of the hours, and so one ounce corresponds to 60″. But in astronomical observations we cannot use only whole clocks: the count of time has to be ended the moment a star has arrived at the meridian or at another vertical circle, or an eclipse has ended, or the whole diameter of a luminary has passed, and so on. The use of the pendulum is therefore better suited to astronomical observations.

What I said about the pulses of the artery, from the observation of Cardano, of Wendelen and our own, stands somewhat differently in Fromondus, from his own observation, Book 2 of the *Meteorologica*, article 10. For he says that his own healthy artery beats 4450 times in one hour; and that thunder, after the flash of lightning, is heard scarcely one beat later, whereas between the sound of a larger firearm and its flash 10 beats of the artery intervene, which I think too many.

[Margin: How many beats does the human artery make in one hour?]

CAP. XXI. The experiments on the speed of naturally descending heavy bodies will have to be illustrated by many others in Book 9, section 4, chapter 16, since arguments against the Copernicans will be drawn from them. Having considered these, I hope that D. Vicenç Mut will not assert that all heavy bodies, even of unequal weight, fall in equal time, as he did in his *Sol Alphonsinus*, no. 50.

AD LIBRVM III.

On Book 3.

[Margin: The substance of the Sun, and encomia of it.]

CAP. I. Auersa [Raffaele Aversa] thinks the Sun is liquid, and so does Aristarchus of Samos brought back to life, in the *Systema mundi*, p. 23, as Hevelius reports without refuting it, chapter 5 of the *Selenographia*. Moreover, I have not seen fit to gather more encomia of the Sun from elsewhere. Yet on this subject St Gregory Nazianzen, Oration 2 on Theology, St Cyril of Jerusalem, Catechesis 9, and St Ambrose, Book 4 of the *Hexaemeron*, chapter 1, stand in place of all the Fathers; add to these, if you please, Julian's oration on the Sun. On the substance and production of the Sun, there is more from Sacred Scripture and the Fathers in Book 9, section 1, chapters 4 and 5; to these add Francesco Resta, Book 1 of his *Meteorologica*, chapter 3, who suspects that the Sun is an aggregate of many stars, and Caussinus [Nicolas Caussin], Book 2 of *De Domo Dei*, from chapter 2.

[Margin: The magnetism of the Sun rejected.]

CAP. II. In no. 6 a few things are touched on here about the motive force of the Sun; they are treated more fully in their own place, on the movers, namely Book 9, section 2, chapter 1. What is touched on here in no. 7 about the magnetic force of the Sun, Father Niccolò Zucchi rejects in his *New Philosophy of Machines*, part 5, section 12, especially as regards the Moon, whose monthly or half-monthly libration about its own centre does not follow the laws of the annual or half-yearly libration of the Sun about its centre, detected from its spots by Scheiner, and to be explained in Book 9, section 4, chapter 12.

To Scholium 1 of this chapter add what we shall say about the ellipse of the Sun, from Diodorus Siculus, Book 17, in the appendix to chapter 10, near the end.

CAP. III. Do not be surprised that little has been said here about the spots of the Sun. For since the argument taken up here from Galileo, and boasted as evident against the rest of the Earth and for the motion of the Sun, had to be refuted in Book 9, section 4, chapter 12, it seemed best to discuss this matter there more carefully than elsewhere, so that the subtlety of this argument and the solidity of the reply might be more manifest. To the opinions on the substance of sunspots indicated at no. 10, add the opinion of our Adam Tanner, in his dissertation on the heavens, question 7, paragraph 5. There he says that in 1611, from October 21 to the end of the year, and through almost the whole of the following year, he observed spots on the Sun with a spyglass and pointed them out to others; that they are above the Moon and very close to the Sun; but that their phenomena can be defended without corruptibility and without fluidity of the solar heaven, if they are said to be moved like planets on their own eccentrics and epicycles around the Sun. Lessius also taught this, Book 1 of *De Providentia*, chapter 2, and Tanner repeats the same in volume 1 of his *Theological Summa*, disputation 6, question 5 on the heaven, assertion 2. But Ouiedus [Francisco de Oviedo] too, in the single controversy on the heaven, point 2, no. 20, or p. 468, and Arriaga, in the single disputation on the heaven, section 5, no. 42, or p. 443, think that these spots can be explained by local motion, without generation and corruption. But against these authors see

what will be said in Book 9, section 4, chapter 12, nos. 19 and 20. Further, Johannes Hevelius, chapter 5 of the *Selenographia*, judges that these spots are soots expelled from the mass of the Sun by the force of its heat, and faculae brighter parts of the Sun; and he says that the Sun was without spots in 1642 on October 26 and November 18, and in 1642 on October 12. Caussinus, however, Book 2 of *De Domo Dei*, chapter 8, thinks they are circumsolar stars.

[Margin: The substance of sunspots.]

[Margin: The Sun without spots.]

[Margin: Our System.]

[Margin: And D. Baliani's.]

CAP. VI. Our System, proposed here in no. 6, is put forward only as probable, provided that the eccentricities of the planets from the Earth and from the Sun are made so great that the phenomena of the motions are saved without evident error: we shall speak of it in Book 7, section 6, chapter 4, scholium 4, and in Book 9, section 3, chapter 9. What I likewise suspected here about Baliani's system, I explain later, from a letter the author sent me, in Book 9, section 4, chapter 15.

[Margin: Tycho's error, *facundus*, in the parallax of the Sun.]

CAP. VII. No. 3. It seemed that the reader should be forewarned here that Tycho, because he assumed too great a parallax of the Sun, also made the horizontal refraction too great by about 2′, and the greatest declination of the Sun, or the obliquity of the ecliptic, greater by 1½′ than it should have been; and that this error overflowed into the whole hypothesis of the Sun, and indeed into the distances and parallaxes of Venus and Mercury, as well as of Mars, and also into the right ascensions of the fixed stars, which he measured from Venus. But it would have been untimely to press this on a reader not yet instructed in these matters, so we have reserved these things for their own places, where they are more timely. Moreover, that Tycho did not investigate the parallax and distance of the Sun from any other source, but took them over from Ptolemy and Copernicus, is clear from volume 1 of his *Progymnasmata*, p. 97, where he says that parallaxes are sought from the distances, and adds: *We shall therefore assume here the semidiameter of the eccentric of the Sun, as it was traced out by Copernicus from eclipses; namely, of 1142 semidiameters of the Earth.* Then, on p. 99, from the remaining distances of the Sun, investigated through the eccentricity, he gathers the parallaxes for each degree of altitude.

[Margin: D. Vicenç Mut's foundations for the parallax of the Sun.]

But what shall I set against my very dear friend, the most learned D. Vicenç Mut? In a most erudite little work entitled *De Sole Alphonsino Restituto*, published in 1649 and very generously sent to me, he sets, in marginal number 30, the horizontal parallax of the Sun at apogee at 4′ 37″, and at mean distance 4′ 46″; and in no. 45 he sets the distance of the Sun at apogee at only 744 terrestrial semidiameters. But his foundations are 5. The *first* is that the apparent semidiameter of the Sun is equal to the half-angle of the cone of the Earth's shadow and the horizontal parallax of the Sun taken together, which differs

insensibly from the truth. The *second* is that the semidiameter of the Sun at apogee, as observed by himself, is 15′ 39″. The *third* is that the Moon at apogee, at full moons, is 62 terrestrial semidiameters from the centre of the Earth. The *fourth* is that the semidiameter of the Earth's shadow, at the place where the Moon passes at apogee, is 44′ 22″ from Kepler's observations. The *fifth*, which he demonstrates from 3 and 4 by a geometrical figure and proof, is that the half-angle of the cone of the Earth's shadow, with the Sun at apogee, is 11′ 2″; and when these are subtracted from the apparent semidiameter of the Sun, there remains the horizontal parallax of the Sun at apogee, of 4′ 37″.

[Margin: The falsity of the foundations already mentioned indicated.]

But in truth foundations 2, 3 and 4 are uncertain and, from my observations, false. On the second, that is, on the semidiameter of the Sun, see what was said in Book 3, chapter 10, and what will be said in this Appendix on chapter 10 of this book. On the third, see what was said in Book 4, chapter 14, and what will be said in the appendix on chapter 14 of Book 4. Finally, on the fourth, see what was said in Book 5, chapter 4, and you will see how slippery the observation of the semidiameter of the Earth's shadow is, and that it is gathered more surely either from the parallax of the luminaries and the semidiameter of the Sun, or from the distance of the Sun and its semidiameter and the half-angle of the cone of the Earth's shadow, than the parallax is from the semidiameter of the shadow. Besides, the solar distance that D. Mut deduced from this parallax is manifestly contradicted by the distance deduced from the lunar dichotomy, even if the smallest distance compatible with such a phase is granted: as I said in Book 3, chapter 7, Problem 3, and shall soon teach in the next part of the Appendix to this chapter. It is also contradicted by the parallax of acronychal Mars, which ought at times to be 9′ or 10′ if the parallax of the Sun were nearly 5′, as D. Giovanni Antonio Rocca also wrote to me in his judgement on D. Mut's *Sol Alphonsinus*.

De Distantia Solis per Dichotomiam Lunæ Inuestiganda.

On Investigating the Distance of the Sun by the Dichotomy of the Moon.

[Margin: D. Vicenç Mut's doubts about the dichotomy of ☾.]

In 1650, when I had asked D. Vicenç Mut by letter to turn his attention to the lunar dichotomy and, by the Aristarchean Problem, to deduce the distance of the Sun and from it the parallax, he replied in a letter dated from Majorca on March 5 of the same year that this method seemed to him as doubtful as the one which relies for this purpose on eclipses, from the Earth's shadow: especially since Reinhold and Athanasius Kircher, in the *Ars Magna Lucis et Umbrae*, Book 9, fol. 751, put the digression of the dichotomized Moon from the Sun at 87°, whereas I put it at about 89° 30′, and Wendelen at 89° 45′ ½. Yet at that time he declared nothing certain. In a letter dated August 20, however, he advises that he could not determine the dichotomy of the Moon exactly, because to the naked eye it still appeared concave at 9^{h} 30^{m} after noon, but when seen through

the tube it was less concave, and dichotomized around its middle parts; and that the Moon cannot always show the same face in the quadratures, because of the reciprocal motion of the Moon about its own centre, which the variable distance of the spots from the limb shows. But how much this Problem is to be valued, and by how much more surely the distance and parallax of the Sun are gathered from this than from the shadow of the Earth and eclipses, I taught in Book 3, chapter 7, both by comparing the two and by the authority of Kepler, Wendelen and van Langren, cited in this chapter. Please read again Kepler's words cited there at no. 10, where he says that the distance and parallax of the Sun have until now *depended on the unreliable shadow of the Earth*; and in the *Astronomia Optica*, p. 237, he says: *But when the Moon is dichotomos, from a careful view of the section of the Moon's face you may within a few hours pass judgement on the true quadrature.* Yet he adds that on 1602 December 21, in the evening, at the half-sixth hour after noon [hora dimidia sexta post merid.], the Moon was seen by him still concave, when it had already come to the limit [Greek: τῆς διχατομίας [recte: διχοτομίας] (tēs dichotomias, "of the dichotomy")].

[Margin: On the use of dichotomy, the judgement of Kepler, Wendelen and D. Giovanni Antonio Rocca.]

But D. Giovanni Antonio Rocca too, most expert in astronomy, and indeed in the whole of mathematics, when asked by me in a letter to observe the time of the dichotomy with his own tube as well, replied at last after several observations, on 1651 May 19. He says that scarcely any sensible time intervenes between the first quadrature and the appearance of the dichotomy, and that the distance of the Sun from the Earth must therefore be much greater than most astronomers have so far put it; and he urged me earnestly to advance this problem, from which the distances of the Sun and of the five other planets, and many other things, can be determined better than from parallaxes that are in any case unobservable. And he concludes that this is the most excellent among the physico-astronomical Problems, and that even if all the labours of one astronomer and the cares of a whole life had to be spent on it, his effort would be excellently invested. So he writes. But for the observation of the dichotomy to be made properly, the following Rules must be kept.

[Margin: Rule 1.]

The *first* is that the observation be made not with the naked eye only, but with telescopes, and those of the best and very large. For to the naked eye the Moon generally still appears falcate when it is in itself dichotomized, because the parts lying between the horns are illuminated not continuously but discretely, and among them are many lower parts, like valleys or seas, not yet lit by the Sun; while those peaks of which only the tips are illuminated do not appear to the naked eye, because of their smallness. So in the present figure (which indicates the intended point after a fashion) there are the peaks of rocks or of mountains C, D, E, F, G, H, and between them and the curve A Q L M N P B lie unilluminated parts. Therefore, while the telescope sees the section of illumination along the straight line A E F B, the naked eye sees only the curve

A Q M N B. Furthermore, the telescope should be used in such a way, however, that the whole horns are not compared with the intermediate parts: for the horns AQ and BP, since they are parts of the Moon resembling continents and are moreover full of more prominent parts, are lit by a greater and more continuous light than the lower and interrupted parts; so that if the former were compared with the latter, the intermediate parts would still look too concave beside them. And so the time to observe is that at which the intermediate parts C, D, E, F, G, H, set in one and the same straight line, begin to be illuminated together in such a way that this line bisects the disc of the Moon, and the line EA and EK appear equal, cutting off the quadrant AEK. If this cannot be discerned at once, a time must be chosen between two doubtful moments: namely, one at which the Moon begins to appear bisected, but with hesitation in the estimate; the other at which it now appears plainly bisected, with gibbosity almost beginning if it is the first quadrature, or concavity if the second. If you use an excellent and very large telescope, this time will never exceed half an hour, and will indeed stay well within it, provided the Moon is far from the horizon. And if you use the naked eye, for this reason it will not rarely seem concave to you even after the first and before the second quadrature, which Kepler too admits happened to him, in the *Astronomia Optica*, p. 237. It also helps to apply to the eye a lens convex on both sides, which makes the telescope, however long and powerful, still take in the whole Moon, visible in one view, as is usual with us. But then great care must be taken that the line of dichotomy not be viewed outside the middle of the already mentioned lens; otherwise it would appear by no means straight, even when it is so, but curved toward one part or another, according as the eye is applied to this or that part of the lens.

Diagram. Woodcut of the half-illuminated Moon, set in the left column with the text running beside it. The right edge is the smooth convex limb, from A at the top to B at the bottom; the left edge is an irregular terminator with small detached peaks standing beyond it. Hatched dark patches and small round craters mark the face. A straight line runs across the disc from E on the left to K on the right limb.

- A (top of the limb): label
- B (bottom of the limb): label
- C, D, E, F, G, H (down the left side, each beside a small detached peak): labels
- M (just inside the terminator, below the line from E): label
- N (at the terminator, below G): label
- P (at the terminator, near B): label
- K (right limb, at the end of the line from E): label
- Q (just below A, inside the edge): label
- L (at the terminator, just below C): label

[Margin: Rule 2.]

Secondly. Let those lunar quadratures be chosen whose dichotomy can appear with the Moon not only above the horizon, but also placed in the nonagesimal

degree of the ecliptic, or as close to it as possible. For if it is in the nonagesimal, the same plane which divides on the Moon the shining hemisphere from the dark, and which passes through the horns and centre of the Moon, passes at the same time through the centre of the Earth and through the centre of an eye placed on the surface of the Earth; and so there will be no difference between the time of the dichotomy occurring with respect to an eye imagined at the centre of the Earth and the time of the dichotomy observed on the surface of the Earth, whether the Moon has no latitude or not. And the less it is distant from the nonagesimal, the smaller will be the difference of those times; and moreover the diversity which varied refractions could produce will be avoided. But since this will be very rare, of all the dichotomies occurring outside the nonagesimal, the best for our business is the one in which the Moon has no latitude or the greatest; for then the circle of latitude, drawn through the centre of the Moon, will pass through both horns of the Moon, and not so many things will be needed to find the time of the true dichotomy, as we shall soon teach. The altitude of the Moon, however, must have been taken at the moment of the dichotomy, although it can be found by calculation from the given time and place of the Moon.

[Margin: Rule 3.]

Thirdly. If the dichotomy has been observed outside the nonagesimal degree of the ecliptic, it will be necessary to seek the difference of time between the true dichotomy, that is, as reckoned from the centre of the Earth, and the one observed from the surface of the Earth; for which two or three of the following Problems are needed.

1. Probl. Angulum circuli Verticalis, & Latitudinis in centro Lunæ factum inuestigare, Datis Lunæ longitudine ac Latitudine, Nonagesimo Eclipticæ, & vel Altitudine Lunæ, vel Altitudine Nonagesimi, seu Angulo puncti Eclipticæ Orientis cum Horizonte.

Problem 1. To find the angle of the vertical circle and of the circle of latitude made at the centre of the Moon, given the longitude and latitude of the Moon, the nonagesimal of the ecliptic, and either the altitude of the Moon, or the altitude of the nonagesimal, or the angle of the rising point of the ecliptic with the horizon.

[Margin: Case 1, if the Moon has no latitude.]

In the following figure, let ABC be the semicircle of the meridian and AFC that of the horizon; and let DIE be the portion of the ecliptic in which the nonagesimal degree of the ecliptic is N; and let the arc NE of the ecliptic be a quadrant, and the Moon, with no latitude, be at I, through which let the quadrant BIF of the vertical circle be drawn; and let FI be the altitude of the Moon. Then for the moment of the observed dichotomy, let there be sought, by the Problems of the primum mobile to be given in Book 10, section 2, the point E of the ecliptic, ascending or descending, and from it the point N,

which is the nonagesimal, and the angle of the ecliptic and the horizon, namely IEF. For the same moment, let I, the place of the Moon in the ecliptic, also be known; for from it and from the nonagesimal point N the difference NI will be known, and from that the remainder to the quadrant NE, that is, the base IE. Therefore in the triangle FIE, right-angled at F, with the base IE and either the altitude FI, observed but corrected in the usual way by adding the parallax and subtracting the refraction, or, if you have not observed it, with the angle FEI of the rising or setting point, find the angle FIE. Join it to the right angle EIO, which the circle of latitude LI makes with the ecliptic, and subtract the sum from two right angles; for there will remain the angle BIO that was sought.

Diagram. Geometric figure set in the right column with the text running beside it: a semicircular arc from A over B to C, with a shallow arc along the base from A through H, F, K and E to C. Straight lines run from B down to the base at or near H, F and K; a line runs from D through N and I to E; a line runs from L through O toward G; further lines join G and O to E; the line from B to K passes through O.

- A (left end of the base): label
- B (top of the arc): label
- C (right end of the base): label
- D (upper left on the arc): label
- L (upper right on the arc): label
- N (inside, below D, beside a dot): label
- O (inside, below L): label
- I (centre, where the lines cross): label
- G (inside, left of I): label
- H, F, K, E (along the base, left to right): labels

[Margin: Case 2, for northern latitude.]

Now let the Moon be at O, having northern latitude IO, and let the vertical BOK be drawn through it: and first, in the triangle IOE, right-angled at I, from the given IE, as above, and the latitude IO computed for the time of the observed dichotomy, find the base EO and the angle IOE. Next, in the triangle KOE, right-angled at K, with the altitude KO and the base OE, find the angle KOE; subtract it from the angle IOE already found, and there will remain the angle KOI, equal to the angle sought, BOL. But if the altitude KO is unknown, it will be necessary in the triangle IOE to find also the angle IEO, and to add it to the angle DEA of the rising or setting point, so that the angle KEO becomes known; with this and with OE the angle KOE is to be found.

[Margin: Case 3, for southern latitude.]

Finally, let the Moon be at G, with southern latitude IG, and with the vertical BGH drawn. In the triangle GIE, right-angled at I, with IE known as before and the latitude GI, find the base GE and the angle IGE. Next, in the triangle HGE, right-angled at H, with the base GE and the altitude HG of the Moon, find the angle HGE; add it to the angle IGE and subtract the sum from two right angles, for the angle sought, BGL, will remain. But if the altitude HG is

not known, in the triangle GIE find also the angle GEI; subtract it from the angle DEA of the rising or setting point of the ecliptic, and there will remain the angle GEH; with this and with the base GE you will then find the angle HGE.

2. Probl. Inclinationem Cornuum Lunarium ad Verticalem circulum per Lunæ centrum ductum, inuestigare: Dato Angulo Verticalis huius, & circuli Latitudinis per centrū Lunæ ducti; datisq. Latitudine Lunæ, & Distantia loci Ecliptici Lunæ à loco Solis.

Problem 2. To find the inclination of the horns of the Moon to the vertical circle drawn through the centre of the Moon, given the angle of this vertical and of the circle of latitude drawn through the centre of the Moon; and given the latitude of the Moon and the distance of the Moon's place in the ecliptic from the place of the Sun.

If the Moon has no latitude, or its greatest, the plane of the ecliptic or of the lunar orbit will make right angles at the centre of the Moon with the plane of the circle of latitude, drawn likewise through the centre of the Moon; and a straight line drawn from the centre of the solar globe to the centre of the lunar globe will lie very nearly in the plane of the ecliptic or of the lunar orbit. Therefore, since this line makes right angles with the plane passing through the horns of the Moon and separating the illuminated part of the Moon from the dark, the circle of latitude will pass through the horns of the Moon. And so the angle of the vertical circle and of the circle of latitude, given or found by Problem 1, will itself be the inclination of the horns of the Moon to the vertical.

But if the Moon had a sensible latitude, though not its greatest, someone used to proceed to the following scheme. In it, let AS be the arc of the ecliptic between the Sun, S, and the Moon's place in the ecliptic, A, intercepted and known, the places of the Sun and of the Moon being of course known for the moment of the observed dichotomy. And let the Moon's circle of latitude be PL, making right angles with the ecliptic at A, and passing through the centre of the Moon, O if it has northern latitude, or G if southern. To these points let the straight lines SO and SG be drawn from the Sun, and to those lines, through the centres O and G, let the perpendiculars or orthogonals COK and FGM be drawn, representing the plane drawn through the horns of the Moon and separating the illuminated part, CEK or FHM, from the dark.

Diagram. Geometric figure set into the foot of the left column, with narrow text running beside it. A horizontal line runs from P to L through G, M, A, C and O. A vertical line falls from A to S at the foot, and two straight lines run from S up to G and to O, passing through H and E respectively. At G a half-circle hangs below a straight side F–G–M, with its lowest point near H; at O a half-circle hangs below a straight side C–O–K, with its lowest point near E. From G a curved line rises to R; from O a curved line rises to B.

- P (left end of the horizontal line): label

- G, M, A, C, O (along the horizontal line, left to right): labels
- L (right end of the horizontal line): label
- R (above, from G): label
- B (upper right, from O): label
- F (left end of the left half-circle's side): label
- K (right end of the right half-circle's side): label
- H (bottom of the left half-circle): label
- E (bottom of the right half-circle): label
- S (foot, meeting point of the lines): label

[Margin: Case 1, for northern latitude.]

With this laid down, if the Moon is at O, he used the triangle ASO, treated however as spherical and right-angled at A; in it, with the distance AS of the luminaries along the ecliptic and the latitude AO, he sought the angle AOS. For its complement to a right angle would be KOL, since SOK is a right angle and the three angles AOS, SOK and KOL equal two right angles. Further, adding the angle KOL to the angle BOL, given or found by Problem 1, he obtained the angle BOK, by which he would measure the inclination of the Moon's horns to the vertical circle.

[Margin: Case 2, for southern latitude.]

But if the Moon is at G, he used the triangle AGS, right-angled at A, and with the distance AS along the ecliptic and the latitude AG he sought the angle AGS. Subtracting its complement MGA to the right angle SGM from the angle RGA, given or found by Problem 1, he obtained the angle RGM, which measures the inclination of the horns of the Moon to the vertical.

But there is no need of this scheme or of its use. For even if the Moon has a latitude that is not its greatest, the angle of inclination of the circle of latitude to the vertical at the time of the dichotomy is nevertheless the same as when it has either no latitude or its greatest; and what is said of the orbit of the Moon having its greatest latitude must be said likewise of any great circle in which the centre of the Sun and of the Moon lies, and consequently the line drawn from one centre to the other. Here, however, that insensible difference is disregarded which could arise from the Moon's being less than a quadrant distant from ⊙.

3. Probl. Data Inclinatione Cornuum Lunariū ad Verticalem circulum, & Parallaxi altitudinis Lunaris, ac mensis Lunaris veri Duratione; Inuenire Tēpus addendum aut demendum tempori Dichotomiæ obseruatæ, vt notum sit momentum veræ dichotomiæ.

Problem 3. Given the inclination of the horns of the Moon to the vertical circle, the parallax of the Moon's altitude, and the duration of the true lunar month, to find the time to be added to or subtracted from the time of the observed dichotomy, so that the moment of the true dichotomy may be known.

In the third scheme, appended here, let the Moon be beyond the nonagesimal at O, in the western quadrant. Let AOB be the plane of the circle drawn through the horns of the Moon and separating the illuminated part from the dark; let B be the horn that is northern for us in the north, A the southern, and O the centre of the Moon, through which let the portion POK of a vertical circle be drawn. Let ADB be the illuminated limb of the Moon, and let there be drawn the curve AEB through the tips of the horns, and the straight line CED, making right angles with AOB at O and representing a quadrant of the circle of monthly illumination, that is, of the circle which is illuminated successively as a whole from one new moon to the next; for in this way, in the same surface, OE will be the arc and portion of the illumination already mentioned.

Diagram. Two schemes side by side, set into the right column with narrow text running beside them. Each is a semicircle with its round side outward. The left scheme has its arc to the left, from B round through F to A, the arc broken near B; a vertical line runs from R down through G and S and past the arc; a line runs from B to G and on toward A, and a curved line runs from B to A through Q; a straight line runs from G to F. The right scheme has its arc to the right, from B round through D to A; a vertical line runs from P down through O and K and ends in a short foot stroke; a straight line runs from A through O to B, and a curved line runs from B to A through E and K; a straight line runs from O to D.

- B, R (top of the left scheme, left to right): labels
- G (left scheme, centre): label
- Q (left scheme, below G): label
- F (left scheme, left end of the line from G): label
- S (left scheme, below Q, on the vertical): label
- A (left scheme, foot): label
- P, B (top of the right scheme, left to right): labels
- O (right scheme, centre): label
- E (right scheme, right of O): label
- K (right scheme, below O, on the vertical): label
- D (right scheme, right end of the line from O): label
- A (right scheme, foot, left): label

Now when the first and true dichotomy takes place with respect to the centre of the Earth, and an eye placed there would see the half Moon AOBDA shining, and the section AOB as a straight line, yet to an eye on the surface of the Earth all points of the straight line AOB are apparently depressed because of the parallax of altitude, except the points of the horns. They are depressed, I say, comparatively, with respect to the Moon's straight line AOB; for with respect to the fixed stars and the highest heaven we do not deny that all points, even those of the horns, are depressed, but we have no need of this depression. And so the point O is depressed to K, and other points likewise, but always less and less the nearer the points of the straight line AOB are to the tips of the horns, which on the face of the Moon differ insensibly from those seen from the centre

of the Earth. Hence it comes about that, because of the parallax of altitude, in the true dichotomy of the first quadrature the straight line AOB appears to us curved and concave, even when we use a very large telescope; and conversely, when the Moon has been seen as dichotomous on the surface of the Earth, it would already appear gibbous to an eye placed at the centre of the Earth. And the parallax of longitude, considered in the circle of illumination, becomes the arc OE. Therefore, from the time of a dichotomy observed beyond the nonagesimal in the western quadrant, the time corresponding to the arc OE must be subtracted. Thus in the dichotomy of the second quadrature, beyond the nonagesimal in the eastern quadrant, or in the left scheme, when from the centre of the Earth there would be seen the straight line AGB of the true dichotomy, dividing the bright from the dark, then, because the points within AB are apparently depressed for an eye placed on the surface of the Earth, as the point G is to S, and so on, the Moon already appears sickle-shaped along the arc BQA, and so the time of the dichotomy appears already to have passed, although it has not passed. When therefore it previously appeared dichotomous to us, it was not yet so, and therefore the time corresponding to the arc GQ is to be added to the time from noon at which the dichotomy was observed.

Therefore, with the altitude of the Moon given, or found by Problem 1, let there be sought the parallax of the Moon's altitude suited to that altitude at the known position in the anomaly; for as great as that is, so great will the arc OK, or GS, be, in the triangle OKE, or GQS, to be treated as spherical and right-angled at E, or at Q. In this triangle the angle KOE is also known, in the right-hand figure, being the complement to the right angle AOE of the angle AOK, or POB, which is known by Problem 2, since it is the western inclination of the Moon's horns. Likewise the angle SGQ, in the left-hand figure, is the complement to the right angle AGQ of the angle AGS, or BGR, known from the eastern inclination of the horns; therefore the arc OE, or GQ, will not remain hidden. Next, from the ephemeris let there be sought the days and hours of the true lunation within which the dichotomy falls, and let the whole lunation be resolved into hours; and let it be that as the 360° of the whole circle of illumination are to the hours of the whole lunation, so is the arc OE, or GQ, to the hours and fractions of hours that correspond to the arc OE, or GQ. Let this time be subtracted from the time of the first observed dichotomy, or added to the time of the second, and the time of the true dichotomy, as reckoned from the centre of the Earth, will be had.

[Margin: Rule 4.]

Fourth and last, from the ephemeris or astronomical tables let there be sought the moment of the quadrature of ☾ with the Sun; for the difference between it and the moment of the true dichotomy will be the difference of the quadrature and the dichotomy. Let there be sought therefore the true hourly motion of the Moon from the Sun, and let it be that as 1 hour is to the hourly motion of the Moon from the Sun, so is the time of the difference between the quadrature and the dichotomy to the arc of the ecliptic by which the Moon is distant from

the place of the quadrature. This arc is equal to the angle TSL (in the figure placed in Book 5 [recte: Book 3], chapter 7, no. 9, or p. 108), which we need in the Aristarchean problem, set out in this Book, chapter 7, for finding the distance of the Sun from the Earth.

Nor should anyone have doubts about this method because we have transferred the angles made at the centre of the Moon, and found in Problems 1 and 2, to the surface of the lunar globe in Problem 3: for it is enough that they are made by planes intersecting one another in a common section through the centre of the Moon. And since the globe of the Moon appears to us as a disc, if anyone wishes to examine their tiny difference, including that which arises in the angle of inclination of the horns to the same vertical if the disc of the Moon is viewed now from the centre of the Earth, now from its surface, he will find it insensible, and will recognize that he has wasted his time on empty subtlety. Finally, in this way the angle of inclination already mentioned will be found more surely than if it were observed directly with some instrument: we too have devised such an instrument, but it is not worth the effort to describe it here. But the Rules and Problems will be illustrated by an example, and indeed by the most outstanding of 15 dichotomies, more select than the others, observed here at Bologna.

EXEMPLVM.

Example.

On the evening of 1651 April 27, the Moon before the apparent setting of the Sun was, even when viewed with the largest telescope, too concave; but at sunset or a little after, some doubt began as to whether it was still concave. At 34^m after sunset, however, it already appeared dichotomous in the same telescope, and 6^m later, that is, 40^m after sunset, it was so without any further misgiving. At that time it stood 65° high and had passed the nonagesimal by a few degrees, with the northern horn now inclined from the vertical toward the west. Then, 1½ hours after sunset, it plainly appeared gibbous in the largest telescope, although in a small telescope it still seemed dichotomous, and to the naked eye still sickle-shaped and concave. The Sun's place at noon had been ♉ 6° 59′. Therefore at Bologna, where the pole is elevated 44° 29½′, the semidiurnal time and true setting of the Sun was $6^h\ 56^m$ after noon, but because of refractions $6^h\ 58^m$. Adding therefore 40^m, the moment of the dichotomy first observed becomes $7^h\ 38^m$ after noon; but with only 34^m added, the first moment of the dichotomy was at $7^h\ 32^m$ after noon. Since, however, the distance was taken at $7^h\ 38^m$, let us examine the Moon for this moment. Now for that moment the prerequisites for our Problems stood as follows.

[Margin: By Book 5, chapter 10, Problem 1.]

[Margin: By Problem 1 here.]

[Margin: By Problem 3.]

At Bologna, at pole altitude 44° 30′, at 7^h 38^m after noon	Sign, degrees, minutes, seconds
True place of ☉ in the ecliptic	♉ 7° 16′ 0″
Ascending point of the ecliptic	♏ 15° 0′ 0″
Descending	♉ 15° 0′ 0″
Nonagesimal degree of the ecliptic from the horizon	♌ 15° 0′ 0″
Place of the Moon in the ecliptic	♌ 6° 40′ 0″
IE: distance of the Moon from the descending point of the ecliptic	81° 40′ 0″
IO: northern latitude of ☾	5° 13′ 0″
Therefore in the right-angled triangle IOE, the base OE	81° 42′ 9″
And the angle IOE	89° 13′ 30″
And in the right-angled triangle KOE, the altitude KO, true	65° 24′ 10″
And the angle KOE	71° 35′ 21″
Therefore KOI, or BOL	17° 38′ 9″
—	—
Anomaly of ☾ at that time	2^s 10° 0′ 0″
Parallax suited [?] to the altitude of 65°, or OK in the third scheme	0° 24′ 10″
The angle POB, that is, AOK	17° 38′ 9″
Therefore in the right-angled triangle KOE — the angle KOE	72° 21′ 51″
Therefore the arc OE	0° 7′ 19″

[Margin: By Rule 4.]

That lunation was of 29^d 49^m [Dier.29.49′.], or 745 hours. Therefore, if the 360° of the whole circle of monthly illumination take 745 hours, then surely 7′ 19″ of that circle require 14^m 58^s of time. Subtracting these from 7^h 38^m after noon, the hour of the observed dichotomy, the true dichotomy was at 7^h 23^m 2^s. But the true time of the quadrature was 8^h 15^m; therefore the interval between the true dichotomy and the quadrature was very nearly 52^m of time, to which, at the Moon's true hourly motion from the Sun, which was then 29′ 2″, there correspond 25′ 10″.

[Margin: By Book 3, chapter 7, Problem 3.]

And the angle TSL is of this size (in the figure placed in Book 3, chapter 7, no. 9, or p. 108). In this triangle the distance of the Moon from the centre of the Earth, or TL, was for us 61 semidiameters of the Earth; therefore TS, the distance of the Sun, was about 8333 semidiameters of the Earth. But from the hour 7^h 32^m after noon, the distance of the Sun would come out as 7480.

But if, as that someone supposed in the second part of Problem 2, you were to correct the angle BOL by his scheme, which in the third problem is POB, it would come out here as 17° 41′ 28″, from which the arc OE would follow as 7′ 20″, and hence the remaining quantities we found without it, with a difference of one or another second, surely rightly negligible in this subtlety, however great it may be.

COROLLARIVM De Distantia Solis.

Corollary: on the distance of the Sun.

From this dichotomy, and from several others observed with a very large and excellent telescope, it is certain to us that between the true dichotomy and the quadrature there never intervenes a time notably greater than one hour, and so that the angle at the Sun is greater or less than 30′ by a very few minutes; from which it follows that the distance of the Sun is never less than 7000 terrestrial semidiameters. But if opponents want us to abide by an observation of the dichotomy made with the naked eye, or observed indeed with a telescope, but when the lower-lying parts on the Moon are also filled with light, so that half of the Moon's disc appears exactly and continuously illuminated, either course will carry the Sun much higher. For to the naked eye the Moon for the most part appears concave right at the very point of the first quadrature, and sometimes even after; and in a telescope that continuous filling takes place after the first quadrature has already been passed. So that angle would vanish, or come out so slender that it would carry the Sun much higher than Posidonius and Wendelen did, who advanced it beyond 14000 semidiameters of the Earth. The remaining observations of very many dichotomies we shall, God willing, bring forward in volume 3, and we shall confirm this distance of the Sun.

In this same chapter I set out the Tychonic distances of the Sun, such as he himself finally chose; but how from this there arises an eccentricity other than the one he himself laid down, I shall say in the appendix to Chapter 24; and in the same place see the remaining distances of D. Vicenç Mut.

[Margin: Disagreement of the distances and eccentricity of the Sun in Tycho.]

[Margin: Refractions of the Sun.]

CAP. IX. If anyone wants other ways of gathering refractions, and an examination of solar refractions, let him consult Book 10, section 6, from Problem 20 to 30, and see the tables to be placed at the end of the same section. And what is said here in scholium 8 about the Dutch is to be understood according to what will be said in Book 10, section 3, Problem 14.

[Margin: The diameter of the Sun observed by telescope: how certain it is.]

CAP. X. In no. 3, among the ways of observing the diameter of the Sun, I did not list the one done by means of a telescope, receiving the bright image of the Sun on paper or on a white board, because, with the intervening refraction of the crystal lenses, it was not evident to me that this image was correct, and not too contracted or enlarged. Later there appeared the *Sol Alphonsinus* of D. Vicenç Mut, in which this method is handed down. Let the telescope, he says, be of the best lenses, and let its machine, especially the reeds, be immune from every alteration of the weather, and let the rings of the lenses be exactly round, and so fitted that the image of the Sun admitted through the convex lens is not larger than the image projected onto the paper through the concave lens; or let the board and paper EG be at such a distance from the concave lens that the bright image of the Sun is received through a clean circle with a very sharp edge, and let it be orthogonal to the axis HV of the tube. Thus in a very dark room, with a very serene sky, and at the very time of noon, when, that is,

the image of the Sun runs across the paper not obliquely but straight, let the diameter CF of the solar image be taken off with a compass, in the vertical line AB (which you will learn by means of a plumb line), and let it be transferred to another paper KD, and be divided into equal parts as small as possible; and let the extremes C and F be marked. Then, with the instrument left entirely unmoved,

Diagram. Woodcut set across the left column, between the lines ending *immo-* and beginning *to prorsus*. At the upper left, three overlapping circles lie along an oblique straight line, marked P, M and N from lower left to upper right. Dotted rays run from the circles down to the end of a tube marked H. The tube is held by two cross brackets on a long inclined board, shaded on its under side. From the lower end of the tube dotted rays continue to a shaded rectangular board at the foot of the inclined board, carrying an oval image with overlapping outlines. To the left of the lower part stand two separate short scale lines, O–T above and K–D below.

- P, M, N (along the oblique line, lower left to upper right): labels
- H (at the upper end of the tube): label
- G (top corner of the receiving board): label
- B (right edge of the receiving board): label
- R (on the board, within or beside the oval image, upper right): label
- F (within the oval image): label
- V (within the oval image): label
- C (at the lower left of the oval image): label
- A (on the left edge of the receiving board): label
- E (bottom corner, where the two boards meet): label
- O, T (ends of the upper scale line): labels
- K, D (ends of the lower scale line): labels

on the following days, if the Sun's meridian altitude decreases, the bright image will be raised and will be, for example, in the position VR. Let its extremity V therefore be noted, and let the interval CV be measured in parts of the line KD, and let its portion be OT. And from the table of the Sun's declination let there be taken the declination suited both to the noon at which the extremity C was noted and CF taken, and to the noon at which the extremity V was noted; for when the smaller declination is subtracted from the greater, the difference of the declinations will be known. Now let it be that as the difference or interval OT, that is, CV, is to the whole diameter KD of the image, that is, CF, so is the difference of the declinations PM to the whole diameter PN of the Sun; for these are analogous, as is clear. It is useful, moreover, that the difference of the declinations be not less than 15′, or about the semidiameter of the Sun.

[Margin: How great is the diameter of the Sun for D. Mut?]

By this reasoning D. Mut affirms that the diameter of the Sun at apogee was found by him to be 31′ 18″. For on 1648 June 11 and 20 (the difference of declination of those days being 18′ 46″) he noted the portion OT as 2395

particles, of which the whole KD had been 4000. Therefore as 2395 is to 4000, so is 18′ 46″ to 31′ 21″. But between the 21st and the 30th of the same month the difference of declination was 18′ 18″, and OT was 2338 particles; therefore as 2338 is to 4000, so is 18′ 18″ to 31′ 18″. Moreover in the following year too he found the diameter of the Sun at apogee to be 31′ 18″, and for that reason chose this value.

[Margin: How, and how great, was the diameter of the Moon found by D. Mut?]

He likewise took the diameter of the Moon on paper by comparison with the image of the Sun, and found it at an apogee full moon to be 29′ 40″: for he found its image to be of 3791 particles, of which the image of the Sun had been 4000, and he made it that as 4000 is to 31′ 18″, so is 3791 to 29′ 40″. And in no. 51 he says that the diameter of the Sun at perigee was found by him to be 32′ 46″, and that of the Moon at perigee 33′ 10″.

Iudicium de Diametro Luminarium inuenta à D. Vincentio Muto, & de ipsius in hoc artificio.

Judgement on the diameter of the luminaries found by D. Vicenç Mut, and on his [?] in this technique.

The technique just mentioned is without doubt outstanding, and worthy of the genius of this most ingenious astronomer, but very difficult in practice, because it requires observation over several days, during which we cannot have evident assurance that no slight disturbance has occurred in the instrument; and so small a one is enough to introduce into the observation an error greater than 1′, as he himself admits. So I should believe it better, on the same day, to count the transit of the bright image received by such a telescope, with a pendulum showing the time exactly, and to carry out the rest as I taught in this Book 3, chapter 10, provided we were certain that the refraction of the Sun's rays, which takes place from the perpendicular in the lens nearer the paper, does not dilate the image of the Sun more than would be proper for this practice. Because this is uncertain, I therefore praise the diligence of so great a man, but hold as suspect the semidiameter of the Sun derived from it.

For the rest, although these methods show a diameter close to the true one, they are not sufficient for their exact measurement unless those things are reconciled which I said here, in scholium 1, are to be reconciled, and above all the total eclipses of the Moon and of the Sun and the annular eclipses of the Sun, on which see our careful work in Book 4, chapter 16, from no. 15 to 18. For on account of eclipses I was compelled to extend the diameter of the Sun at apogee, which I had obtained from direct observations of it at about 30′ 30″, to 30′ 50″, and that at perigee to 33′ 8″. Otherwise there will be as many opinions about these minutes as there are observers and observations.

[Margin: In how much time does the Sun rise?]

Likewise in this chapter, at no. 6, on the time in which the whole disc of the Sun rises or sets, I said certain things from the method of the ancients, but I did not include our observations, made with the aid of a pendulum (on which see Book 2, chapter 20), which shows very exactly with each vibration one second of time of the primum mobile. It seemed good, therefore, to insert some selected ones here for the reader's instruction. The first small table contains those made with a sky not only cloudless but also with a very bright and very serene horizon; the second small table contains those made with a horizon that was indeed cloudless, but full of reddish vapours and showing an ellipse of the solar image far larger than at other times.

Of the time from the rising of the Sun's upper limb to the rising of its lower limb, observed at Bologna.

Table I. With a very serene horizon.

Year	Month	Day	Vibrations of the pendulum	Time
1641	September	24	184	$3^m\ 4^s$
		28	186	$3^m\ 6^s$
1642	March	27	185	$3^m\ 5^s$
	June	21	321	$5^m\ 21^s$
		22	320	$5^m\ 20^s$
		30	300	$5^m\ 0^s$
	July	16	310	$5^m\ 10^s$
	September	23	186	$3^m\ 6^s$
	October	12	246	$4^m\ 6^s$
		19	247	$4^m\ 7^s$

Remainder of the Preceding Table.

Part	Year	Month	Day	Vibrations of the pendulum	Time
Remainder of Table I	1643	May	20	195	$3^m\ 15^s$
		June	22	328	$5^m\ 28^s$
			26	330	$5^m\ 30^s$
		December	20	380	$6^m\ 20^s$
	1644	March	20	186	$3^m\ 6^s$
Table II	1642	September	14	250	$4^m\ 10^s$
		October	21	350	$5^m\ 50^s$
	1643	January	25	390	$6^m\ 30^s$
	1646	July	4	365	$6^m\ 5^s$

From these it is gathered that the variation of the air and of the refractions, together with the obliquity of the ascensions and with the variation of the arc of the altitude circle intercepted between the Sun about to rise and the horizon, is the cause of the variation in the time in which the Sun's whole disc, or transverse diameter, rises. For the arc already mentioned is smaller at the solstices than at the equinoxes, but the obliquity of the ascensions etc. is

greater; and the refractions, speaking of them in themselves, are greater at the winter solstice than at the equinoxes, and at these than at the summer solstice. Yet when the horizon is most serene and cloudless, we can state generally that at Bologna the Sun's whole diameter rises at the equinoxes in a time of 3^m and 5^s or 6^s, at the summer solstice in 5^m 20^s, at the winter solstice in 6^m 20^s. So if you begin the day from the rising of the Sun's limb and run it to the setting, and add what refraction adds, the artificial day at Bologna comes out at the summer solstice as 15^h 29^m, at midwinter as 8^h 52^m, at the equinoxes as 12^h 9^m. For refraction adds to the semidiurnal arc about 2^m at the solstice, 4^m at midwinter, and 3^m at the equinox.

[Margin: The apparent size of the Sun at the horizon.]

[Margin: The height of Mount Ida.]

What I said about the apparent size of the Sun at the horizon is to be understood, or even corrected, from what was said in Book 10, section 6, chapter 1, question 13. Add to these what historians relate of Mount Ida: that it does not wholly hide the rising Sun from the eyes it stands in front of, because of the vastness of the transverse diameter of the ellipse. But Diodorus Siculus ascribes this to the Ida that is in Phrygia, in Book 17, saying: *Mount Ida is the most eminent of all the mountains around the Hellespont; for on Ida something very rare, and beyond human belief, is found. About the rising of the Dog Star, on the very summit of the mountain, the air, utterly still on every side, ensures that no blast of the winds is felt there; and there, while night still lasts, the Sun is seen to send out its rays, not indeed so that its circular form appears, but rather so that its brightness is seen far and wide, as if a fire had seized the horizon over a great stretch of land.* Here he indicates the elliptical vastness of the Sun rising prematurely for the summit of Ida. But Pliny the Elder does so more clearly, in Book 2, chapter 11, though he ascribes it to the Ida of the island of Crete, and concludes that the Sun is larger than the Earth *because in rising it exceeds Mount Ida in breadth, embracing it amply to right and left, especially at so great a distance.* Strabo relates in Book 10 that this is the highest of the mountains of the Cretans and takes in 600 stadia in circumference; and Solinus says, in chapter 16, that the Sun is seen from it before its rising.

[Margin: The size of the Sun as seen from the planets.]

In Scholium 4 a small table is set out from the opinion and calculations of others, but I gave another, more accurate one in Book 7, section 6, chapter 13.

[Margin: The semiangle of the cone of the Earth's shadow.]

CAP. XI. In Problem 5 of this chapter I derived the semiangle of the cone of the Earth's shadow, with the Sun at apogee, as $14' 52''$, from an apparent semidiameter of the Sun of $15' 19''$. But since I was later forced, because of eclipses, to make the semidiameter of the Sun at apogee $15' 25''$, if you subtract from this my parallax, which is then $27''$, this semiangle would come out as $14' 58''$, that is, larger by $6''$. But in the same Book 3, chapter 11, no. 7, I taught that it is not necessary for the sum of the Sun's parallax and the semiangle of the cone

of the Earth's shadow to be exactly equal to the Sun's apparent semidiameter, and that it suffices if there is no perceptible difference.

[Margin: Where does the increase of the artificial day come from?]

Further, from the fact that the Sun illuminates more than a hemisphere of the Earth, it does not follow that the artificial day comes out longer on this account, for its quantity does not depend on the illuminated terrestrial arc, but on the motion of the Sun's centre on the equator or on a parallel cut by the horizon more or less obliquely. The day is, however, increased if you begin it from the rising not of the Sun's centre but of its limb.

[Margin: Anaximander's error on the size of the Sun.]

[Margin: And Lucretius's.]

Anaximander, in Plutarch, Book 2 of *De placitis philosophorum*, chapter 2, and Theodoret, in Book 4 of *De Græc.*, taught that the Sun is larger than the Earth by only 27 times; so crude was astronomy then. Add this, however, to the scholia of this chapter. But in Scholium 1, among the Epicureans who affirm that the Sun and Moon are truly a foot across, just as they appear, include Lucretius, and copy out his verses from Book 5 of *De rerum natura*, beginning from the line:

Nor can the Sun's wheel be much larger, nor its heat much smaller, than it seems to our senses to be.

For at last, after 24 lines, he concludes universally:

Lastly, whatever fires of the ether you see from here, you may know that they can be smaller by only a very little, or larger by a tiny and brief part.

[Margin: Look at the figure placed in Book 3, chapter 14, no. 3, or p. 132.]

CAP. XIV. No. 1. I described the great Bologna gnomon here in very few words, reserving the rest for the *Liber Organicus*. Yet if anyone wishes to know the other distances on the line AN corresponding to the middle of the ellipse which the Sun's image makes on it at the beginnings of the signs, here they are for him, from measurements taken not only by us but also by DD. Carlo Antonio Manzini and Bartholomæus Provalius [?]. Let him read the letters in the figure set there, and then compare those lines with the following small table. Now, however, the angle XAN is not right, but because of the wear and constant treading of those walking over the threshold A, it is acute: not indeed like XAV, but still so that XA has come out longer by 3⅔ inches, with respect to the eye N looking along the horizontal.

Beginnings of the signs	Line and source	Bolognese feet	Inches	Fraction
♑	AN: Manzini	170	0	0
	Provalius	169	1	20/24
	Myself	169	2	½
♎ ♈	AG: Manzini	65	1	0
	Provalius	64	7	0
	Myself	64	7	22/24
♋	AC: Manzini	26	0	0
	Provalius	25	6	0

Beginnings of the signs	Line and source	Bolognese feet	Inches	Fraction
	Myself	25	6	17/24
♊ ♌	AD: Manzini	30	8	0
	Myself	30	8	3/5
♉ ♍	AE: Manzini	43	10	0
	Myself	43	10	4/5
♓ ♏	AK: Manzini	100	7	0
	Myself	100	8	½
♒ ♐	AI: Manzini	141	7	0
	Myself	141	8	3/5
	AM: Manzini	167	7	0
	Myself	166	8	7/12
	AL: Myself	164	3	0
	For LN	4	11	½

[Margin: Whether the length of the year has been adequately confirmed by D. Mut.]

CAP. XV. No. 6. D. Vicenç Mut, in his *Sol Alphonsinus*, no. 38, tries to confirm the Alphonsine quantity of the equinoctial year, of $365^{d}\ 5^{h}\ 49^{m}\ 16^{s}$. But in this he uses a solar parallax of 4′ 37″ at apogee, which I showed to be false in the appendix to chapter 8. He supposes, besides, that Alexandria is further east than Toledo by $2^{h}\ 54^{m}$, whereas from our *Geographical Book* it is more truly $2^{h}\ 58⅓^{m}$. Finally, he uses the morning equinox of Hipparchus, which is faulty because of refractions, according to what was said in Book 3, chapter 14, no. 9.

CAP. XVIII. D. Vicenç Mut, in the *Sol Alphonsinus*, nos. 32 and 33, affirms that from the vernal equinox to the autumnal equinox the interval is $186^{d}\ 23^{h}\ 54^{m}$, and from the autumnal to the vernal $178^{d}\ 5^{h}\ 55^{m}\ 16^{s}$. But at the equinoxes he uses an excessive parallax, about which I spoke in the appendix to chapter 8.

[Margin: Boulliau's solar hypothesis.]

CAP. XXIII. In no. 10 I indicated too sparingly Boulliau's hypothesis and his way of resolving the ellipse into two circles, but at this place one can see what I later taught in Book 7, section 2, chapter 5, no. 13.

[Margin: Inconsistency between the distances of the Sun and the Tychonic eccentricity.]

CAP. XXIV. No. 8. In Tycho, vol. 1 of the *Progymnasmata*, pp. 30, 31, 60 and 98, the Sun's eccentricity is 3584 parts, of which the radius of the orb, or semidiameter of the Sun's eccentric, is 100000. But on pp. 97, 98 and 423 he puts the Sun's greatest distance at 1182 56/60 semidiameters of the Earth, the mean at 1142, and the least at 1101 4/60; so the eccentricity is 40 55/60, or 41, semidiameters of the Earth. Later, however, on p. 472, he chose a mean distance of the Sun of 1150 semidiameters of the Earth. Therefore if he wanted to maintain the same eccentricity, he ought to have made the Sun's greatest distance 1192 and the least 1108 semidiameters of the Earth, as is clear from adding the eccentricity in the one case and subtracting it in the other; or if he wanted to keep the greatest distance of the Sun at 1182 56/60, or 1183, and the

mean at 1150, he ought to have made the least 1117 and reduced the eccentricity to 33 semidiameters of the Earth. Yet I should think that, had he lived longer, he would rather have increased the Sun's distances than diminished the eccentricity. For that reason I kept the Tychonic eccentricity here, although in Book 3, chapter 7 I did not dare to vary the Tychonic distances on its account. But in this table too I placed the mean distance later chosen by Tycho, so that from this at least there would be an indication that if the eccentricity of 41 were added to it the greatest distance would come out as 1191, and if subtracted from it the least would be 1109.

[Margin: The Sun's eccentricity, apogee and greatest equation, according to D. Mut.]

But D. Vicenç Mut, in the *Sol Alphonsinus*, nos. 32, 33 and 34, puts the Sun's eccentricity at 378122 parts, of which the radius of the orb is 10000000, that is, 29⅕ semidiameters of the Earth; the Sun's greatest prosthaphaeresis at 2° 10′; and the Sun's apogee at 5° 53′ 5″ of Cancer, about the year of Christ 1648. But since he says that he used an interval between the vernal and autumnal equinoxes of $186^{d}\ 23^{h}\ 54^{m}$, and a stay of the Sun in Aries of $30^{d}\ 14^{h}\ 48^{m}$, adding to the Sun's meridian altitudes his own parallax, which I showed in the appendix to Book 3, chapter 8 to be 4′ greater than the true parallax of the Sun, there is no reason for us to discuss further his disagreement with others and with us on this. Moreover, since he puts the Sun's least distance at 744 semidiameters of the Earth, if you subtract the eccentricity 378122 from the radius 10000000, the least distance remaining will be 9621878; but if you add it, the greatest will be 10378122. So in semidiameters of the Earth the mean distance of the ☉ will be 773⅕ and the greatest 802⅖.

CAP. XXV. What I said in no. 4 about the fabrication of those Jews who presided over the arranging of the Alphonsine Tables, I drove home again on another occasion in chapter 28, no. 16, and in Book 6, chapter 16, nos. 3 and 4. This fabrication about the motion of the auxes and of the fixed stars, derived by Kabbalists from the period of the Jubilee, is excellently refuted, and from the same sources as ours, by D. Vicenç Mut in his *Sol Alphonsinus*, nos. 39 and 40. He adds that the Alphonsine Tables published by the Jews in the year 1252 are judged spurious by Scaliger and Mulerius, and also by Stadius in the Bergen tables and by Arguello [?] in his *Ephemerides*, and that in their place are to be used those which Alfonso himself published in the year 1256.

[Margin: The time of the least eccentricity, for Tycho.]

[Margin: The vanity of judicial astrology.]

[Margin: The error of Melanchthon and Cardano about the Sun's approach to the Earth.]

CAP. XXVI. Tycho too rejects the vanity of Rheticus, which we mocked here in Scholium 1, in a letter to Rothmann dated 1588 September 19 [13. Cal. Octobris], p. 131, where this is found: *As to your view, toward the end of the Röslin refutation, that the Sun's least eccentricity has passed, I hold the same myself; for I refer the least eccentricity and obliquity of the Sun to Peuerbach. Nor is Rheticus*

to be listened to either, who, while he writes that al-Battānī abuses the mysteries of astrology too freely, himself falls into that same fault here. For how can a change in the Sun's eccentricity bring about a change of empires? Nor do you unjustly censure this too, which Philipp Melanchthon first introduced, namely that the Sun has now come closer to the Earth; although Cardano too, in Book 3 of the Subtleties, mistakenly states the same. For they have not grasped Copernicus's meaning on this matter. Yet Cardano, in Book 3 of *De Subtilitate*, p. 144 in my copy, reports in Melanchthon's opinion that in the age of Ptolemy or Hipparchus the centre of the Sun's orb stood from the centre of the Earth at 24⅕ diameters of the Earth, that is, 48⅖ semidiameters; but in his own century at only 18⅖ diameters, or 36⅘ semidiameters. So you now see which of the two have erred.

[Margin: The obliquity of the ecliptic, 23° 30′, confirmed from Kepler's thinking.]

CAP. XXVII. Kepler admits, in the Rudolphine Tables, chapter 34, that the Tychonic obliquity of the ecliptic, 23° 31′ 30″, is faulty because of the excessive parallax Tycho assumed. For at the summer solstices Tycho used a solar parallax of 1′ 33″; adding this to the Sun's visible meridian altitude of 57° 35′ 27″, he made the true altitude 57° 37′; and subtracting from this the equator's altitude at Uraniborg, 34° 5′ 30″, he held the obliquity to be 23° 31′ 30″. But Kepler there, since he assumes a much smaller parallax for the Sun at apogee, makes the obliquity of the ecliptic 23° 30′ 30″. So if he had applied our parallax, which is still twice as small, not of one minute but of about half a minute, or 27″, he would surely have acknowledged and accepted an obliquity of only 23° 30′. Further, we found the Sun's parallax and distance, twice as small as Kepler's, by means of the lunar dichotomy, according to what was said in Book 3, chapter 7, with Kepler himself urging us to it.

AD LIBRVM IV.

On Book IV.

[Margin: The tide of the sea from the Moon.]

CAP. I. I said more about the Moon's dominion over the tide of the sea in Book 9, section 4, chapter 14; see also Caussin, Book 2 of *De Domo Dei*, chapter 15.

[Margin: Antichthones.]

CAP. II. Those who in no. 4 are called Antichthones can also be called Antipodes on our own Earth; and I see the word so used for the same people by not a few.

[Margin: The Laconians called lunar, the Arcadians antelunar.]

CAP. III. In no. 4, or at the end of the chapter, add from Pindar that the Olympic contest used in former times to be held at the full Moon, and that the Laconians were called [Greek: σελήνας (selēnas, "moons")], as if lunatics, because they held it a religious scruple to begin anything great before an impending new or full Moon. By contrast the Arcadians, who used to begin

wars and great undertakings before the imminent new or full Moon, were called by their neighbours [Greek: προσελήνοι (proselēnoi, "before the Moon")], as if antelunar; but they themselves turned this to the praise of their antiquity, as if they were older than the Moon herself. On the phases of the Moon see Caussin, Book 2 of *De Domo Dei*, chapter 12.

[Margin: The time of the dichotomy.]

CAP. IV. Problem 7 is to be understood of the dichotomy reckoned from the centre of the Earth; for from the Earth's surface the dichotomy of the first quadrature appears noticeably later, and that of the second earlier, than it should, because of the Moon's parallax, unless the Moon is then in the nonagesimal. On the difference between the dichotomy reckoned from the centre of the Earth and that observed from its surface, see what was said in the appendix to Book 3, chapter 7.

[Margin: The Moon deprived of light in a total eclipse.]

CAP. VI. That the Moon in its total eclipses is sometimes wholly deprived of light is confirmed by Johannes Hevelius, *Selenographia*, chapter 6, from Vitruvius, Book 9, chapter 4, and from an observation of the eclipse of 1642 April 14 made by himself. Johannes Phocylides Holwarda affirms the same about the same eclipse in the *Epitome Astronomiae reformatae*, p. 65, and Kepler about the eclipse of 1620 June 15 in the *Epitome of Copernican Astronomy*, Book 6, p. 825.

[Margin: An inconsistency of Tycho's in the hypothesis of the Moon with the Sun, shown by D. Vicenç Mut.]

CAP. XIV. D. Vicenç Mut, in his *Sol Alphonsinus*, no. 16, shows that it follows from the Tychonic observations and hypotheses that the distance of the Moon, at apogee and full, from the centre of the Earth is 62 23′ semidiameters of the Earth, not 58 8′ as Tycho has it. He shows this, I say, from the data given by Tycho: the semidiameter of the Sun at apogee, 15′; its horizontal parallax at that time, 2′ 54″; and the semidiameter of the Earth's shadow at the place where the Moon at apogee passes through, 43′. Now recall here the figure placed in Book 3, chapter 11, Problem 6, or p. 124. For in it the Sun's apparent semidiameter, that is, the angle BND (as great, because of the Sun's enormous distance, as BAD), is equal to the two interior angles ABN, that is, the Sun's horizontal parallax, and AEN, that is, the semiangle of the cone of the Earth's shadow, taken together, by Euclid I.32. So with the parallax of 2′ 54″ subtracted from the semidiameter of 15′, the angle AEN is left as 12′ 6″, to which because of the parallelism of the lines AG and LK, NLK is equal; but because of the same parallelism the angle GAL, which measures the shadow's semidiameter, is equal to the angle ALK. Therefore, when this, that is 43′, is joined with NLK, 12′ 6″, the whole ALN comes to 55′ 6″. And LAN, the complement to a right angle of the angle GAL, is 89° 17′; so the third angle, ANL, is 89° 47′ 54″. Therefore, with all the angles given in the triangle ALN, and the side AN, which is one semidiameter of the Earth, LN is found to be 62 39/100 semidiameters of the Earth; and with this and the angle LNK, in the right-angled triangle LNK, LK,

that is AG, is found, namely the Moon's distance, of 62 38/100 semidiameters of the Earth.

[Margin: The Moon's parallax and distance as observed by D. Vicenç Mut.]

Further, Mut himself relates in no. 20 that in 1646 June 27, about midnight, he observed at Majorca, where the elevation of the equator is 50° 25′, the meridian altitude of the Moon, full and at apogee, as 28° 27′. He says that it then underwent no refraction, and that it was close to the Tropic of Capricorn, namely according to the Danish tables at 5° 21′ 46″ of Capricorn, with north latitude 2° 15′ 18″ and declination 21° 9′ 12″; but according to Lansbergen at 5° 7′ 20″ of ♑, with latitude 2° 17′ and declination 21° 8′ 30″; and according to Kepler at 5° 21′ 35″, with latitude 2° 15′ and declination 21° 9′ 30″; and finally by Boulliau at 5° 12′ 44″, with latitude 2° 15′ 20″ and declination 21° 9′ 30″. When these declinations are subtracted from the altitude of the equator, there remain the true altitudes of the Moon, and the differences from the visible altitude, that is, the parallaxes, and the lunar distances, as you see in the following small table.

Visible altitude of ☾, 28° 27′.

	True altitude	Parallax	Distance of ☾ in semidiameters
By the Danish tables	29° 15′ 48″	48′ 48″	62
By Lansbergen	29° 16′ 30″	49′ 30″	61
By Kepler	29° 15′ 30″	48′ 30″	about 62
By Boulliau	29° 15′ 30″	48′ 30″	about 62

Yet Tycho gives the lunar altitude of 28° 27′ a refraction of 2′ 11″, and it seems astonishing that at Majorca, which is surrounded on all sides by so great a sea, there are no refractions at that altitude. Certainly, as I observed, the parallax came out for me much smaller and the distance greater because of refractions, as can be seen in the observations reviewed in this chapter; which, however, favours the Alphonsine hypothesis more, as D. Mut himself admits.

Hevelius, moreover, on p. 97 of the *Selenographia*, gives the parallaxes of the Moon written below.

☾ in	Aspect	Parallax, apogee	Parallax, perigee	Distance in semidiameters of the Earth, apogee	Distance in semidiameters of the Earth, perigee
	☌ & ☍	59′ 9″	61′ 39″	58 8′	54 52′
	□	56′ 44″	65′ 36″	60 36′	52 14′

[Margin: Whether the Moon's refraction is less than the Sun's.]

CAP. XV. The reason why Tycho in his tables showed the Moon's refraction as smaller than the Sun's was the excessive parallax of the Sun added to the solar refraction; but more on this in Book 10, section 6, Problems 24 and 27. For it is certain that, with equal density of the air and equal inclination of the star, or visible altitude, the star that is less distant from the Earth has the greater refraction, as will be shown in the same section, chapter 2; and in section 5,

from Problem 73, we shall show how great the greatest refraction and parallax of the Moon and Sun can be supposed to be, and at the end of the section we shall give the refractions of the Moon. Further, Hevelius attests on p. 198 of the *Selenographia* that by the force of refraction the full Moon was seen above the horizon together with the Sun in 1642 at Stettin by Laurentius Eichstadius [Lorenz Eichstaedt].

[Margin: The Moon's diameter according to D. Vicenç Mut.]

CAP. XVI. How D. Vicenç Mut used to observe the Sun's apparent diameter through its bright image received by telescope on paper, I set out from his own *Sol Alphonsinus* in the appendix to Book 3, chapter 10. In nos. 15 and 51 he says, further, that he compared the bright image of the Moon at apogee, received by telescope, with the image of the Sun at apogee, and found it to be of 3791 parts, and that of the Moon at perigee of 4239, of which the Sun's had been 4000. And he had defined the Sun's diameter as 31′ 18″; therefore that of the full Moon at apogee is 29′ 40″ and at perigee 33′ 10″. But I think that for a twofold reason he gathered it larger than he should have: first, because the Sun's diameter in itself was shown to him somewhat larger by the telescope, which dilates the image; then, because the image of the Moon too is larger from the same cause. But one or another observation does not suffice, unless it satisfies all the eclipses.

What I said in no. 8 of this chapter, against the equation of the lunar diameter introduced by Longomontanus, has seemed to some too far-fetched; but I have neither the time nor the wish here to haul on a contentious rope.

CAP. XVII. To Problem 7 another can be added, about which Kepler inquires in the *Dissertatio cum Nuncio Sidereo*, p. 11½: for he asks how large the Moon appears as seen from the Sun and from the other planets. And because he supposes the Sun's distance to be thirtyfold the Moon's distance, Saturn's to be tenfold the Sun's, and Jupiter's about fivefold, he therefore says that the Moon seen from the Sun subtends an angle of about 6′, from Jupiter 12″, from Saturn 6″.

[Margin: The Moon seen from the planets.]

Similar to this table are those we set out in Book 7, section 6, chapter 13, on the apparent diameter of the Earth, the Sun, Saturn, Jupiter and Sirius seen from the planets.

[Margin: The Earth seen from the Moon.]

The table here in Problem 7, set out only from our own opinion, can be illustrated from the opinion of others and from Table 1 of Book 7, section 6, chapter 13, expounded and composed by us. Further, the table of this problem is more accurate, being made with geometric rigour, than the one set out by us from Kepler's method in Book 5, chapter 12, Problem 3. For there the disc of the Earth seen from the Moon at apogee is 53′ 20″ taken twice, that is, 1° 46′ 40″, and at perigee 63′ 45″ twice, that is, 2° 7′ 30″; but here it is 1° 47′ 0″ and 2° 7′ 50″.

[Margin: The Alphonsine size of the Moon's orbs, according to Mut.]

CAP. XXV. No. 3. I supposed the Alphonsine lunar hypothesis to be similar to the Ptolemaic in the form of its theory; as for the numbers of the motions, their difference is easily gathered from what was said in chapter 23. As for the measures, D. Vicenç Mut, in his *Sol Alphonsinus*, no. 52, puts the whole diameter of the orb of the ☾ at 117 28′ semidiameters of the Earth, the semidiameter or radius at 58 44′, and the lunar eccentricity at 3 16′ semidiameters of the Earth; and so the least distance of the Moon in syzygies at 55 28′ semidiameters of the Earth and the greatest at 62, which he had also given in no. 16.

[Margin: Proportional minutes.]

[Margin: How the measures of the Moon's orbs are investigated.]

What I said here on p. 260, very sparingly, about proportional minutes can be illustrated from the figure and from what was said in Book 7, section 2, last chapter. Moreover, the method of investigating geometrically, from three eclipses, the ratio of the semidiameter of the deferent carrying the Moon to the semidiameter of the epicycle, and also the eccentricity, is the same as for the three superior planets, whose three places each have been observed in the zodiac; consult, therefore, Book 7, section 2, chapter 8, and supply from there what is lacking here. I now wish to set out at least one figure for the geometric calculation of the Moon, from Ptolemy, Book 5 of the *Almagest*, and from Regiomontanus in his *Epitome*, from Proposition 8 to 11, which I omitted through a [Greek: μνημόνικον ἀμάρθημα (mnēmonikon amarthēma, "slip of memory")]. From the centre D let there be the semicircle ABG of the lunar deferent, and on it the centre of the epicycle B, and the epicycle PLN, on which let the Moon's globe be at L. Let the eccentricity of the deferent be DE, and let the centre of the Earth and of the ecliptic be E, and let EZ be equal to that DE. And let the centre B be joined by straight lines with the points D, E, Z, these being BD, BE, BZ, and also with the Moon by the straight line BL, which is the semidiameter of the epicycle. Let BE be produced as far as M, until it meets the perpendicular DM drawn to it from D; and with BZ produced to O, let O be the mean apogee of the epicycle, and N the perigee. Further, from Z let another perpendicular ZT be drawn to BM; and finally from the point E let the straight line EL be drawn to the Moon. With these set, you will complete or understand the following propositions,

Diagram. A geometric figure at the foot of the right column, set into the text at the right of the lines from "Ex centro D" downward. A large arc, part of a semicircle, runs from the top of a vertical straight line down the left side to the foot of that line; the line carries, from top to bottom, A, D, E, Z, G. A small circle, the epicycle, stands on the arc at the lower left. Straight lines run from a point at the epicycle's centre up to D and to E, and on past E to a point at the upper right, joined to D by a short line; another line runs to Z. Further lines run from that centre to the circumference at the lower left and on through the circle to the right.

- A (top of the vertical line): A
- D (on the vertical line, below A): D

- M (right of D): M
- T (on the line rising to the right, left of E): T
- E (on the vertical line, below D): E
- Z (below E; the sort stands turned): Z
- G (foot of the vertical line): G
- N (right side of the small circle): N
- L (foot of the small circle): L
- P (lower left of the small circle): P
- C (left of the small circle, outside it): C
- B (centre of the small circle, crossed by a line): B
- Small round letter, lower-case size, on the line at the small circle's upper right; its right side is lost in the line crossing it: c or o [?]

1. Proposition. *The diameter of the Moon's epicycle passing through the mean aux of the epicycle and through its opposite looks toward a point opposite the centre of the eccentric, as far from the centre of the Earth as the centre of the eccentric stands from that same centre of the Earth.*

Regiomontanus demonstrated this in the *Epitome of the Almagest*, Book 5, Proposition 8. Now that point is Z, through which passes the epicycle's diameter OBN, drawn through the mean apogee O and the perigee of the epicycle N; passes through it, I say, if it is produced outside the epicycle as far as the diameter of the eccentric.

2. Proposition. *Given the elongation of the epicycle from the aux of the eccentric, the eccentricity of the Moon and the semidiameter of the eccentric, to find how large the arc of the epicycle is between its two auxes.*

In the preceding figure the line AEG is the line of the aux of the eccentric, and EDA is the longer longitude [longitudo longior]. So if the elongation of the centre B from the apogee, or from the aux, of the eccentric is given, the arc AB will be given, and so the angle ADB, and therefore also BDE. With this, and with the radius BD and the eccentricity DE, *first* let BED be sought, so that DEM may be known. Now the angle EBZ is sought, equal to the angle PBO, which measures the arc PO by which the mean apogee of the epicycle, O, stands from the true apogee P. *Second*, therefore, in the triangle EMD, right-angled at M, the angle DEM is given, and the side DE, which is the lunar eccentricity; so EM will become known, and also the side DM, in the right-angled triangle BDM, in which another side, BD, is given, which is the semidiameter of the eccentric; so the side BM will not remain hidden. Similarly, in the triangle TEZ, right-angled at T, EZ is given, because it is equal to the eccentricity DE, and the angle TEZ, because it is equal to the angle DEM, known as above; so the sides TZ and TE will become known. TE, together with EM, taken away from the side BM already found, will make known the side BT; with this, and with TZ, in the triangle BTZ, right-angled at T, you will find the angle TBZ that was sought.

3. Proposition. *Given the mean motion of the Moon both in longitude and in anomaly, and its mean distance from the Sun, to demonstrate its true place.*

In the preceding figure let the Moon be at L, and let the line of its true place be EL; let the given mean distance of the Moon from the Sun, doubled, be AEB, and let the given longitude of the Moon be the arc AB, or the angle ADB; and finally let the given anomaly be the arc OPL. Therefore by Proposition 2 the line BE is known, in the parts in which the semidiameter of the epicycle BL and that of the eccentric BD are known, and the arc OP is known too. OP, taken from the anomalistic arc OPL, leaves the arc PL, that is, the angle PBL. Subtract this from a semicircle, or from two right angles, for the angle LBE will then be left, in the triangle LBE. Since in this triangle BL and BE are given, as above, you will seek out the angle LEB, that is, the equation to be added to the mean longitude in this position of the eccentric and epicycle (for in the other it would have to be subtracted), so that you may have the true place of the Moon. Thus by Proposition 2 you will obtain the equations of the centre, and by this Proposition 3, though dependently on the second, you will obtain the equations of the epicycle.

CAP. XXVI. Besides the scheme and paradigm of the Tychonic lunar theory, I wish to set out yet a twin to it, from Longomontanus's *Astronomia Danica*, Book 1 of the *Theorica*, chapter 6, and it is as follows. In the following figure let A be the centre of the small circle CQB, which carries C, the centre of the eccentric IDM, whose diameter MI is to be always parallel to the diameter BF of the small circle. Let the eccentric carry D, the centre of the greater epicycle GL, and let this greater epicycle bear along K, the centre of the lesser epicycle HN; and let us suppose the Moon to be at E. To E, from the centres of the epicycles, let the straight lines DE and KE be drawn; and from the same E let the straight line EC be drawn to the centre of the deferent, and EB to the perigee of the small circle FB; and from the same centre C let the straight line CDG be drawn through the centre of the greater epicycle, and from C to B another straight line, which AQ is to cut orthogonally.

Diagram. A geometric figure at the head of the right column, set into the text at the right of the lines from "K, fingamusque" downward. A vertical straight line runs from I at the top to M at the foot, with a large arc, the eccentric, running from its top round the left side to its foot, bowing past D. On the arc at the upper left stands the circle of the greater epicycle about D, and within its upper part a smaller circle about K. A small circle about A stands low at the right of the vertical line, which passes through C on its rim. Straight lines run from C, at the top of the small circle, up to R, I, E and D, and from E down to B at the foot of the small circle.

- N (top of the smallest circle): N
- K (inside the smallest circle): K
- H (foot of the smallest circle, below K): H
- E (right of the smallest circle): E
- R (upper right, on the arc rising to the vertical line): R
- I (head of the vertical line): I
- G (left edge of the larger circle): G

- D (centre of the larger circle): D
- L (lower right of the larger circle): L
- F (top of the small circle, at the head of its diameter BF): F
- C (inside the small circle, at its top): C
- A (right of centre in the small circle): A
- Q (left of the small circle, at the end of a short line): Q
- B (foot of the small circle): B
- M (foot of the vertical line): M
- P (just left of A, inside the small circle; not cited in the prose on this page): P

Methodus Calculi Lunæ Geometrici ex Longomontano, exemplo illustrata anni 1587. Augusti 17. H.17. 25′. post merid.

The method of the Moon's geometric calculation from Longomontanus, illustrated by an example of the year 1587 August 17, 17^h 25^m after noon.

[Margin: Step 1.]

With these set, *first*, from the Moon's mean anomaly let the arc ID be given, 45° 37′ 21″; similar to it, or of as many degrees, is the arc GK. This anomaly doubled will give the arc HE, 91° 14′ 42″. Thus in the triangle DKE, besides the sides KD, of 5800 parts, and KE, of 2900 parts, of which the radius CD is 100,000, these being the semidiameters of the epicycles, the included angle DKE will be given, which measures the arc HE. By means of trigonometry, therefore, the angle KDE becomes known, 26° 18′ 43″, and the side DE, 6541.

[Margin: Step 2.]

Second, in the triangle DCE there are given the side DE, just found; CD, the semidiameter of the eccentric, of 100000 parts; and the angle EDC. For when the arc GK, known above from the simple anomaly, is taken from the semicircle GKL, there remains KL, that is, the angle KDL; and if from this you again take the KDE already found, the angle EDC is left, 108° 3′ 56″. So the angle DCE will become known, 3° 29′ 16″, which is the epicyclic prosthaphaeresis, and the side CE, 102217, which is the distance of the Moon from the centre of the deferent. Now if to the mean motion of the Sun, which was 5^s 5° 42′ 16″, there is added the mean longitude of the Moon from the Sun, which was 9^s 25° 12′ 6″, then with a whole circle cast off there remains the Moon's simple motion from the mean vernal equinox, 3^s 0° 54′ 22″. If from this you subtract the epicyclic equation just found, there remains the equated longitude of the Moon, 2^s 27° 25′ 6″; and if the same equation is taken from the Moon's anomaly, which was 1^s 15° 37′ 21″, the equated anomaly will remain, 1^s 12° 8′ 5″, which, that is to say, the angle ICE measures. Next, from the equated longitude of the Moon subtract the true place of the Sun from the mean equinox, which was 5^s 3° 57′ 30″, and there remains the distance of the luminaries, 9^s 23° 27′ 36″. Its double, with a whole circle cast off from the sum, is 7^s 16° 55′ 12″, and it measures, on

the small circle of the eccentricity, the motion of the centre C from the Earth B, in the order of the signs, that is, the arc BFC.

[Margin: Step 3.]

Third, with the semidiameter AC of the small circle drawn, and the straight line BC, an isosceles triangle BAC will arise, whose angle BAC is the complement, to two right angles, of the angle CAF, known from the excess of the arc CF above a semicircle; and so in our case BAC is 133° 4′ 48″. The semidiameters AB and AC are each of 2174 parts, so the side BC will be of 3988 parts. The angle FBC, moreover, by *Elements* III.20, is half the angle FAC, and so is 23° 27′ 36″. Equal to it is the angle ICR, which, subtracted from the Moon's anomaly once equated, leaves it equated a second time, 18° 40′ 29″, namely the angle RCE.

[Margin: Step 4.]

Fourth, therefore, in the triangle ECB, in which there are given CE, by Step 2, BC, by Step 3, and the angle BCE, which is the complement to two right angles of the anomaly equated a second time, and so is 161° 19′ 31″, you will find the angle BEC, 0° 41′ 25″. Add this to the earlier equation of the epicycle, found above as 3° 29′ 16″, and the whole epicyclic equation becomes 4° 10′ 41″, in this case subtractive.

[Margin: Step 5.]

Fifth, for the libration and variation of the Moon one proceeds thus. The doubled longitude of the luminaries, with a circle cast off, was 7^{s} 16° 55′ 12″, and its excess above a semicircle is 46° 55′ 12″, whose sine is 7304007; and the greatest equation of this Tychonic variation is 40′ 30″. Let it be, therefore: as the radius of the small circle of libration, 10000000, is to 40′ 30″, so is the sine 7304007 to another; and there will come out the equation of 29′ 35″, subtractive in this case. Added to the epicyclic equation found the second time, it makes up the whole equation, 4° 40′ 16″. So subtract this from the simple mean longitude from the mean equinox, which was 3^{s} 0° 54′ 22″, and there remains the equated longitude, 2^{s} 26° 14′ 6″. If to these you add the prosthaphaeresis of the equinoxes, 7′ 13″, the Moon's true longitude from the true vernal equinox comes out at Gemini ♊ 26° 21′ 20″. But Tycho observed it at Uraniborg at Gemini 26° 21′ 30″, namely in the year 1587 August 17, at 19^{h} 25^{m} after noon.

Altera Lunæ Hypothesis à Longomontano inuenta, & dicta Secundaria.

A second hypothesis of the Moon, devised by Longomontanus and called the Secondary.

On the straight line FM conceive two points, A and B, of which let B be the centre of the Earth, and A the centre of the small circle LK, which varies the eccentricity, and on which, as in the earlier example, is C, the centre of the deferent FDM. Through C draw the straight line ICO, parallel to the straight line FM, for at I will be the mean apogee of the eccentric orb. Further, from the centre D describe the equant, or epicycle, NH, on which the Moon is at E;

since by the law of the revolutions, when D is conjoined with the apogee I, the Moon occupies the lowest part of the epicycle, H. And because the motion HE is as great as ID, the angles ICD and HDE are therefore always equal; and indeed ICD measures the Moon's simple anomaly from the mean apogee, while HDE measures the Moon's motion on the epicycle, made from the perigee in the order of the signs. Now AB is the mean eccentricity; and the centre of the orb moves on the small circle KLC in the order of the signs, from K through L to C, with a motion as great as twice the distance of the Moon from the Sun. So at the direct syzygies of the luminaries, that is, at true ☌ and ☍, the centre of the lunar orbit is at the perigee K and nearest the centre of the Earth B; at the quadratures, however, it is found at the apogee L. With the straight line BG drawn, therefore, through C, in our case the straight line BC will be the true eccentricity, G the place of the true apogee, and the angle ABC, or arc IG, the subtractive equation of the anomaly; for the external angle GCI is equal to the internal CBA, because of the parallelism of the lines FM and IO, on which the straight line GCB falls. Now the Moon in its motions is configured to the true opposition or conjunction of the Sun; so to save its phenomena one prosthaphaeresis suffices, gathered from two angles, EKD (to which ECD is always equal) and KEB. Supply here in the figure the lines EB and EC, omitted by the engraver. Besides the third equation, of the variation, about which enough was said above. It suffices to have indicated this secondary theory, which Longomontanus illustrates by an example in Book 1 of the *Theorica*, chapter 6, before the tables of the lunar prosthaphaereses.

Diagram. A geometric figure in the left column, set into the text at the right of the lines from "tur in the order of the signs" down to "tur in apogeo". A vertical straight line runs from F at the top to M at the foot; a second vertical line, parallel to it, runs from I at the top to O at the foot. A large arc runs from the top round the left side to the foot, with a small circle standing on it at the upper left. A small circle stands low at the right, on the line FM. Straight lines run from G at the top down through C, on the left rim of the right-hand small circle, to B; from D to C; and from C to A, the centre of that small circle. Dotted lines run from E to L, and from H and from D to K.

- G (top, left of I): G
- I (top of the second vertical line): I
- F (top of the first vertical line): F
- N (left edge of the small circle at the upper left): N
- E (inside the small circle, upper right): E
- D (inside the small circle, centre): D
- H (inside the small circle, lower right): H
- L (head of the small circle at the right): L
- C (on the left rim of the small circle at the right, on the line IO): C
- A (centre of the small circle at the right): A
- K (foot of the small circle at the right): K
- B (below K, on the first vertical line): B

- O (foot of the second vertical line): O
- M (foot of the first vertical line): M

CAP. XXVIII. In no. 2 I rejected the other focus in the ellipse as idle. For although its authors use it as the point of mean motion, they do not in fact use it as the centre of a circle described around the ellipse, but use a circle described from the centre of the ellipse, etc.

[Margin: The limitation of our hypothesis of the Moon.]

My hypothesis of the Moon, however, I proposed neither as containing in itself the physical cause of the motions nor as perfect. I was content to teach an easier way by which the monthly equation, or second, proceeding from the eccentricity, could be found; for there is no such great difficulty about the calculation of the epicyclic equation. But when the manner of the variation, or third equation, has been established through observations, then, as I hope, I shall present the hypothesis perfect. Among the epicyclic hypotheses, moreover, I see that one comes very close to the truth, which Giovanni Antonio Magini, in the *Supplement to the Ephemerides*, reduced to a new form from the observations of Tycho; provided, however, that it is proportionally corrected so that the greatest equation at the conjunctions [in Copulis] is 5° 1′, and not merely 4° 58′ 27″.

AD LIBRVM V.

To Book V.

CAP. II. To this chapter or to its scholia can be attached rules for the effects of eclipses, which Proclus gives, and from him Pierio Valeriano at the end of his *Sphere*.

CAP. IV. No. 12. You will correct the scheme, badly cut by the engraver, by producing the little straight line KI to the left as far as the line AV, placing at its end the letter N, and drawing to it the line TN, so that the triangle NTK comes out isosceles and resting on the base KN.

[Margin: The quantity of the Earth's shadow.]

As to what I said here in Scholium 1 about the altitude of the refracting air being very small in respect of the Earth's semidiameter, it is confirmed from what is said in Book 10, section 6, Problem 51, where I teach that in the temperate zones it is not much greater than 20 Italian miles; so that the quantity of the shadow is not, for this reason, sensibly varied by the Sun's rays tangent to this air.

[Margin: The portion of the Earth illuminated by the Sun.]

CAP. V. No. 8. To enlarge the arc of the Earth illuminated by the Sun's refracted rays, I used the Sun's Tychonic horizontal refraction of 34′. But it is not so great everywhere or always, as can be seen from my table of refractions, to be placed at the end of Book 10, section 6. Indeed, even for Tycho it was not so great, except on the supposition of a parallax of the Sun at apogee of

2′ 54″, which however is smaller; so from this increase must be taken away as much as the Sun's horizontal parallax is smaller.

[Margin: The Moon's light extinguished in an eclipse.]

That the Moon is wholly extinguished in a total eclipse, as I said here in no. 13, Hevelius confirms in chapter 6 of the *Selenographia*, from Vitruvius, Book 9, chapter 4, and from the eclipse of 1642 April 14 observed by himself and by Phocylides; and Kepler in the *Epitome of Copernican Astronomy*, p. 825, from the eclipse of 1620 June 15.

CAP. X. No. 15. On other ways of gathering the parallax of longitude and of latitude, see Book 10, section 5, chapter 7.

[Margin: Are the extremes of eclipses better observed with less powerful telescopes?]

CAP. XV. In nos. 3, 7 and 14, on observing the beginning and end of an eclipse, especially a lunar one, I recommended the use of the telescope, sometimes a larger one, sometimes a smaller. For smaller telescopes, by taking in the whole luminary and not so rarefying the species, help in discerning whether anything of the disc's roundness has been nibbled away. Larger ones, however, separate the shade [vmbraginem] from the penumbra and the shadow, and by enlarging the black lentil of the shadow at the edges of eclipses they make it possible to estimate its quantity better, by comparison with the Moon's nearer spots of known quantity; so that from this and from the Moon's hourly motion the time to be added or subtracted can be gathered, and the beginning of the eclipse be had more justly.

[Margin: Scheiner's opinion on the telescope in observing eclipses.]

Yet our Scheiner, in the *Rosa Ursina*, Book 4, Part 2, chapter 16, teaches that, absolutely speaking, both the black spots of the Moon and the lentils of the Earth's shadow or penumbra at the extremes of eclipses are discerned with more evident distinction if you use telescopes of less power, that is, shorter, but clear nevertheless. He says: *For if anyone scans the Moon with a shorter tube, he will distinguish its ancient spots well; but if you attack it with a longer and more powerful telescope, that appearance of the spots, through refraction by the lenses, being enlarged and apparently rarefied, will not be seen so easily distinct, but will tire the sight. In lunar eclipses the edges of the Earth's shadow suffer the same; for with more powerful telescopes they are not discerned precisely from the illuminated parts; they are discerned best with weak or short ones.*

But Wendelen says still more, in his preface to the *Eclipses*. For although on p. 4 he said that the durations of eclipses were shown too great by Tycho and the ancients, because they counted the shade and smoke preceding the legitimate beginning of eclipses as part of the legitimate eclipse, since the use of the telescope had not yet become known, yet on p. 8 he warns astronomers:

[Margin: Wendelen's opinion on the use of the telescope in observing eclipses.]

Let them not apply themselves to eclipse observations gapingly or perfunctorily, and let them not allow themselves to be deceived, whether by the Earth's penumbra,

or by the Moon's spurious light, or finally also by telescopes, which need caution no less than those. For the eclipse that has already been noted by the naked eyes, and pronounced by everyone's judgement, is, if a telescope is applied, judged to begin at about the tenth minute after: since, that is, the telescope sees through, even within the darkness, the limb of the Moon still glowing red with some light, and so much the later decides the beginnings of eclipses. So that I do not at all know whether this too lynx-eyed tube has brought us more advantage than danger. Certainly I have always found all the phases of eclipses, the beginnings, digits, immersions, emersions and ends, no less certainly and definitely indicated by the naked eyes than if they were helped by a telescope; nor, if several spectators were present, do I remember the votes ever differing even within the narrow limits of one, or at most a second, minute of an hour. So whenever I prepare to observe, I take diligent care to have with me several youths, or young men of keen sight, and while some handle the telescopes, I attend to what others pronounce on the faith and confidence of their own eyes too. And I would advise all who are curious about these things to pursue the same, since I know that I have never been deceived by it even in the least.

But with all due respect to this most learned and most friendly man, how do these statements of his cohere? For if, through lack of the use of the telescope, Tycho and the older astronomers pronounced on the beginning of an eclipse too soon, how is it that those using the telescope pronounce about ten minutes later than they should on a beginning already made? Or if this second is true, how is it all the same to use it and not to use it, and how was he equally certain in either way about the beginning of an eclipse, within the narrow limits of one or another minute of an hour? Finally, from what source was he certain of the true beginning, so as to be able to judge from it whether in this the naked eye or rather the telescope should be trusted?

[Margin: D. Vicenç Mut's opinion on the same.]

Yet D. Vicenç Mut, in his *Sol Alphonsinus*, nos. 27 and 55, subscribed to Wendelen and Scheiner, asserting that when tubes are used, especially the more powerful, the beginnings of eclipses are observed later and the ends earlier than they should be, and so the durations smaller than is right; and Hevelius holds the same in chapter 56 of the *Selenographia*, p. 451. But a distinction had to be drawn, as we have already indicated. For with the naked eye or with short tubes the small portion of the Earth's shadow or penumbra making the extremity of the eclipse is more easily and evidently distinguished, and by a survey of the whole disc of the luminaries, seen at once, it is more easily recognized whether anything has been nibbled away from its perfect roundness. But that very small portion of shadow has some quantity; and so that it can be known how great a portion of the disc or diameter of the luminary is darkened, and how much time of the Moon's hourly motion from the Sun answers to it, this is better discerned with a strong telescope, if that small portion is compared with some small spot on the Moon.

[Margin: Our opinion on the telescope in observing eclipses of the Moon.]

And besides, with such a telescope the danger of confusing the shade with the legitimate eclipse is better avoided. And if, as is our practice, a lens convex on both sides, of almost a finger's thickness, excellently made, is applied to the eye, or four convex lenses arranged at set intervals, then the telescope, however powerful, takes in the whole disc of the Moon or Sun.

[Margin: The eclipse at the death of Christ.]

CAP. XVIII. On this eclipse, Georgios Kedrenos, in the *Annals of Constantinople*, under the year 19 of the Emperor Tiberius, p. 155: *There was an eclipse of the Sun, the greatest of all that had occurred before, and night fell at the sixth hour of the day, so that stars appeared in the sky. A huge earthquake too occurred around Bithynia, which laid low a great part of Nicaea. So Phlegon relates, and Africanus also.* He goes on, moreover, to teach that this was not a natural eclipse of the Sun, because it was full Moon. Add to these Origen, Book 2 of *Contra Celsum*, arguing from this eclipse; and our Jacob Gretser, Book 1 of *De Cruce*, chapter 31, page 102; and Iansenius [Cornelius Jansen the Elder] in the *Concordia Evangelica*, chapter 143. Scheiner, moreover, in the *Rosa Ursina*, Book 4, Part 2, chapter 13, since he thinks that the lunar shadow covers with its diameter only 70 great German miles, that is, as he himself explains, 350 Italian, therefore thinks it more probable that the miracle lay in this: *that so much and so dense a matter of spots was massed together as to snatch the whole Sun from the sight of mortals, like one spot.* From this he infers that the Moon too altogether, and all the other stars that were accustomed to receive light from the Sun, were then deprived of light, and that this is the darkness which St Dionysius the Areopagite indicates. Georgius Polaccus [Giorgio Polacco], moreover, subscribes to Scheiner in the *Anticopernicus Catholicus*, assertion 108. Add also our Pétau, Book 12 of *De doctrina temporum*, chapter 21. Now Giovanni Battista Capuano, at the end of his exposition of the *Sphere* of Sacrobosco, says that St Dionysius the Areopagite, after the portent of this eclipse, set up in the temple of Pallas an altar to the suffering God unknown to him, about which it is said in Acts 17, and that as a result he more easily believed the Apostle preaching the Gospel, etc.

Ad CAPVT XIX. libri 5. Seu ad Catalogum Eclipsium Appendix.

Appendix to Chapter XIX of Book 5, or to the catalogue of eclipses.

[Margin: Years before Christ]

Denis Pétau of the Society of Jesus, a man learned in every way, in Books 8, 11 and 12 of *De doctrina temporum* reports about 116 eclipses, and enters upon the calculation of all of them from principles partly Tychonic, partly Prutenic or Lansbergian. Since I had once gone through this work, but when chapter 19 of our Book 5 was soon to go to press had at hand only the catalogue of these eclipses once made by me, and since I later had the opportunity of reading that work again, I decided to reread my catalogue and, if anything was to be added

or corrected, to indicate it in this place. This I shall do following the order of time, as I did in that catalogue.

[Margin: 772]

Pétau, Book 8 of *De doctrina temporum*, p. 771: because he enters upon the calculation for July 24 of this year, he finds no eclipse of the Sun, from his Parisian tables. [Margin: 720] The eclipse between the 15th and 16th of the month Phamenoth, however, was on September 1, at 9^h 35^m after noon at Alexandria.

[Margin: 715]

The eclipse of the Sun at the death of Romulus was, for Pétau, of 8 digits 38′, and its duration 1^h 54^m; although later, in Book 10, chapter 21, he puts 9 3/11 digits and a duration of 1^h 46^m, and in the same place adds much to the learning of this chronology.

[Margin: 597 & 585]

The eclipse of the Sun in the year of the Julian Period 4117, or 597 before Christ, if you believe Pétau, *De doctrina temporum*, Book 10, chapter 1, is the one of which Herodotus speaks in Book 1, whose words I reported in Book 5, chapter 2, no. 8; where he sets out and pursues at length the opinions of others on the eclipse predicted by Thales. But why it seems to me it should be brought down to the year 585, I said in the catalogue of eclipses.

[Margin: 582]

[Margin: 581]

Pétau, Book 8, pp. 782 and 793, weighs two other eclipses of the Sun from calculation, not from ancient observation: one in the year 4132 of the Julian Period, that is, 582 before Christ, on October 2 at 5^h 57^m at Rome, of 6¾ digits, whose beginning could be seen at Sardis in Lydia, and it lasted 1^h 34^m. The other he puts on September 21 around noon at Sardis, but he shows from the parallax that there was none, namely in the year 581, in order to refute Scaliger.

[Margin: 580]

Pétau, Book 8 of *De doctrina temporum*, p. 788, computes from his tables an eclipse of the Sun in the year 580 before Christ, or 4234 of the Julian Period, which he says was of 7 digits 14′ and lasted 2^h 51^m, and that the apparent new Moon was at 1^h 50^m after noon at Athens. But it cannot be the one which terrified Xerxes as he moved from Sardis into Greece; for that one occurred when the Olympic contest was imminent, which was celebrated at the summer solstice. Yet what I said on p. 363, at the end, must be corrected, that in this year there was no eclipse of the Sun visible in western Asia; for this is to be understood of the time of this year at which Xerxes set out into Greece.

[Margin: 394]

This eclipse, as Pétau says in Book 8, p. 798, Buntingus [Heinrich Bünting] assigns to the year of the world 3574, and he makes it of only 2 digits.

[Margin: YEARS Before Christ.]

[Margin: 383]

Pétau, in Book 8, p. 800, computes the lunar eclipse of this year from various tables, so as to show how far they disagree with the observation. You may examine this in the small table set out below.

The eclipsing full Moon occurred at Alexandria after midnight at $6^h\ 30^m$.

But from the tables	**It was at**	**Excess**
Alphonsine	$8^h\ 26^m$	$1^h\ 56^m$
Prutenic	$7^h\ 19^m$	$0^h\ 49^m$
Danish	$7^h\ 12^m$	$0^h\ 42^m$
Parisian	$7^h\ 3^m$	$0^h\ 33^m$

Pétau says a great deal against Herwart in *De doctrina temporum*, Book 12, Ch. 37, denying that an eclipse can be inferred from the words of Livy in Book 7 of Decade 1.

[Margin: 343]

[Margin: 337]

Insert here, at this year, the eclipse of the Sun which took place in the year of the Julian Period 4377, or 337 before Christ, on July 14 near sunset. Herwart writes of it in his *Nova Chronologia*, and Pétau in *De doctrina temporum*, Book 12, Ch. 37. This is the one which Herwart thinks Livy described, in Book 7 of Decade 1, in these words: *night seemed to be falling during the day* [noctem interdiu visam intendi]. But, says Pétau in Book 12, Ch. 37, that prodigy, as Livy adds, was soon followed by the dedication of the temple of Moneta, which on Macrobius's testimony took place on June 1 [Kalendis Iunij]. So Herwart is wrong when, in order to show a metachronism of 8 years in the Fasti, he takes Livy to mean the eclipse of the Sun that took place in the year 337 before Christ, on July 14.

[Margin: 331]

The eclipse made famous by the victory of Alexander the Great began, Bünting says in his *Chronicle*, at Arbela at $8^h\ 45^m$ after noon, and in Sicily at $5^h\ 29^m$. But that would have been 31^m before sunset, which agrees less well with Pliny the Elder's account. On this eclipse, however, Pétau, in *De doctrina temporum*, Book 10, Ch. 36, defends Plutarch against Scaliger's slander: Plutarch did not say that this eclipse fell on day 20 of the month of Boëdromion, for his words in the *Alexander* are these: *And the Moon was eclipsed in the month of Boëdromion about the time when they enter on the Mysteries at Athens. On the eleventh night after the eclipse, with the two camps set in sight of each other*, etc.

[Margin: The year Alexandria was founded.]

Moreover, since the foundations of Alexandria in Egypt were laid by Alexander as this same year was running out, as is clear from Diodorus, Plutarch, Curtius and Arrian, it is plain that three authors are wrong: Livy (Decade 1, Book 8), Solinus (Ch. 38) and Cassiodorus in his *Chronicle*. They say Alexandria was founded in the year when C. Poetelius and L. Papirius Mugilanus were consuls; for according to Varro this is the year of the City 428, which falls

in the year 326 before Christ, and so they are out by 5 years. Worse still is Eutropius in Book 2, who says it was founded in the Varronian year of the City 416, anticipating by 7 years. Eusebius, finally, puts its founding one year later. See Pétau, Book 10, Ch. 36.

On the eclipse of the year 219 and of 217, against Herwart, see Pétau, Book 12, Ch. 37.

[Margin: 219]

[Margin: 217]

[Margin: 202]

Bünting makes the eclipse of the year 202 a solar one, small indeed but of 2 digits 38′, in the year of the World 3767. Pétau, however, in Book 8, Ch. 13, p. 810, admits that if he had followed the lunar diameter assigned to full Moons, the eclipse would have been of 2 digits 25′. But he says: *It is better to follow Tycho*.

[Margin: Pétau's error on the lunar diameter.]

Tycho, of course, wrongly believed that at new Moons the lunar diameter is always smaller than the solar, and therefore admitted no total eclipse of the Sun. Enough was said of his error in Book 5, Ch. 20, so that on these matters Pétau too must be corrected.

[Margin: 201]

[Margin: 200]

An uncorrected codex of Ptolemy puts this eclipse in year 52 of the Callippic period; but it must be corrected and read as 54, as Georgios Trapezountios translates and as our Pétau has it in Book 10, Ch. 37. The following eclipse, of the year 200 before Christ, was according to Pétau at $0^h\ 42^m$ after midnight of March 20, a day he begins from that midnight, and of 17 digits 20′.

[Margin: 190]

[Margin: &]

[Margin: 188]

[Margin: &]

[Margin: 168]

Against Herwart's misuse of these eclipses, see Pétau, Book 12, Ch. 37. But Herwart erred above all on the eclipse of the year 618 [recte: 168], which he draws back to the year 160, with the Sun in Aquarius. For that eclipse took place in the year of the City 586, with the Sun not in Aquarius but in Gemini, since Livy, in Book 4 of Decade 5, says that it came *after the solstice had passed, in the most blazing heat*, and Plutarch in the *Paulus* says it was *when the summer was already waning*: evidently because he had read that it fell on September 4 [pridie nonas Septembris], but of the Roman year, which was then out of joint.

[Margin: YEARS Before Christ.]

On the eclipse of the year 188, moreover, Bünting also erred, in the year of the World 3781, making it of only 3 digits and recording it at 3^h after noon at Rome, although Livy, in Book 8 of Decade 4, says that *in daylight, between the*

third and fourth hour of the day, darkness arose. So Pétau does better; see him also at Book 12, Ch. 37.

[Margin: 51]

Before the eclipse of the year 44, put the eclipse of the Sun of the year 51 before Christ, from the calculation of Pétau in Book 8, Ch. 13, p. 819. He says that it took place on March 7, a Wednesday [feria 4], that the apparent new Moon was at Rome at $12^h\ 21^m$ after midnight, and that the eclipse was of 10 ½ digits and lasted $2^h\ 42^m$. In Book 10, Ch. 58, moreover, he contends that this was the eclipse mentioned by Cassius Dio in Book 41, which occurred shortly before the beginning of the Civil War, which fell in the year of the City 705. Read him there.

Eclipses in Annis CHRISTI.

Eclipses in the Years of Christ.

[Margin: YEARS of Christ.]

[Margin: 291]

[Margin: &]

[Margin: 316]

In recording these eclipses an error crept into Pétau's numbers, in *De doctrina temporum*, Book 8, Ch. 13, pp. 831 and 833, where 391 is put for the year 291, and 318 for 316. But they must be corrected from the same Pétau's *Rationarium temporum*, as we have done in the catalogue.

[Margin: 334]

The words of Julius Firmicus must be continued and this added: *That in the consulship of Optatus and Paulinus (to speak of more recent events) the keen application of the mathematicians foretold what would come upon all men.* Further, Mulerius (Ch. 11 of the *Isagoge*) and Pétau (*De doctrina temporum*, Book 8, Ch. 13, p. 836) point out that this eclipse cannot be worked out from the Prutenic Tables.

[Margin: A defect of the Prutenic Tables.]

For by Reinhold's Canon 10 the time of emersion would have been $0^h\ 40^m$ after midnight, when in fact the eclipse was seen from $10^h\ 19^m$ to $13^h\ 14^m$ after midnight. And perhaps for that reason neither Bünting nor Mercator mentions this eclipse.

[Margin: 1573]

[Margin: to]

[Margin: 1612]

Denis Pétau, in *De doctrina temporum*, Book 8, Ch. 13, from p. 753 to p. 769, gives the calculation from his Parisian tables of nearly all the eclipses observed from 1573 to 1612. At the eclipse of 1596, moreover, he notes that the calculation from the Parisian tables differs from the observation by 13^m of time, but from the Danish by 5^m and from the Frisian by 6^m.

[Margin: 1596]

[Margin: 1612]

[Margin: &]

[Margin: 1616]

Father Ioannes Vremanus [?], in a letter to Father Grienberger written from the East Indies, concludes, from eclipses observed there and compared with those that Origanus computed in 1612 and 1616, that Goa lies further east than Frankfurt on the Oder by $4^h\ 2^m$, Macau by $6^h\ 48^m$, Beijing [Pachinum] by $7^h\ 5^m$ and Nanjing [Nanchinum] by $7^h\ 11^m$; our Georges Fournier also reports this, in Book 13 of the *Hydrographie*, Ch. 14.

[Margin: 1621]

On the eclipse of the Sun of May 21 observed at Aix by Gassendi, I took the observation from his letters sent to Rome. But Father Georges Fournier, in Book 12 of the *Hydrographie*, Ch. 14, p. 578, relates that it began there with the Sun at an altitude of 35° 30′, and at $19^h\ 5^m\ 20^s$ after noon of the preceding day; the end after $2^h\ 3^m\ 12^s$, with the Sun at an altitude of 51° 17′; and that therefore the time midway between these was $20^h\ 18^m\ 20^s$, the incidence was of $1^h\ 12^m\ 52^s$, the duration of $2^h\ 25^m\ 14^s$, and the digits 9 23′.

[Margin: 1621]

In the same place, on the eclipse of the Moon of November 28 of the same year, he agrees with us about Gassendi's observation, but adds that the incidence was 59^m and the middle at $14^h\ 53^m$.

[Margin: 1625]

I recorded the eclipse of the Moon under March 24, but I put the hours after noon of the 23rd. Granted, because the eclipse had passed midnight, it may seem to belong to the 24th.

[Margin: 1628]

The observation of the eclipse made by Gassendi, Mydorge [Middorgio] and Bainbridge [Baindbrigio] is also related by Fournier, in the same way as by me, in Book 12 of the *Hydrographie*, Ch. 6. But in Ch. 13, p. 579, he adds another eclipse, of the Moon, on June 14 of the same year, observed at Aix, whose beginning, he says, was observed by Gassendi at $11^h\ 15^m$, the total obscuration at $12^h\ 17^m$ and the emersion at $14^h\ 0^m$.

[Margin: 1629]

In 1629, on June 21, an eclipse of the Sun was observed by Father Rhodes [Alexandre de Rhodes] of the Society of Jesus in the Kingdom of Tonkin, in the city of Rum, which is about 80 leagues, or 240 Italian miles, from the royal seat of Tonkin, has a pole altitude of 18°, and is

[Margin: YEARS of Christ.]

from the Fortunate Isles by about 133°. There, then, he says, the eclipse began about two hours before noon, was of about six digits, and ended about one hour after noon.

[Margin: 1631]

In 1631, on November 8, the eclipse of the Moon at Oxford, with Bainbridge observing, began with Aldebaran at an altitude of 28° 50′, or at 9^h 4^m. It emerged from the shadow with the right shoulder of Orion at an altitude of 34° 30′, that is, at 11^h 58^m, and ended with the same shoulder at an altitude of 41° 20′, or at 13^h 4^m. So Fournier relates, in Book 12 of the *Hydrographie*, Ch. 6.

[Margin: 1632]

In 1632, on October 27, the eclipse of the Moon which I said was observed by four astronomers was also observed by Bainbridge at Oxford, at a pole altitude of 51° 46′, with the Moon at an altitude of 44° 50′. So says Fournier, in Book 12 of the *Hydrographie*, Ch. 6, deducing from this and Gassendi's observation a difference of $31\frac{1}{2}^m$ of time, or 7° 52′, between Oxford and Lyon in France.

[Margin: 1633]

In 1633, on April 8, an eclipse of the Sun was observed at Paris by Master Beaugrand; there it began with the Sun at an altitude of 31° 27′, that is, at 3^h 18^m 8^s after noon, and it ended with the Sun at an altitude of 12° 6½′, that is, at 5^h 23^m 12^s. The digits, so far as could be seen through the clouds, were more than 6. At Digne, however, with Gassendi observing, it was of 8 digits 18′; it began at 4^h $42\frac{1}{2}^m$ and ended at 5^h 45^m. At Aix, with Claudius Fabricius [Nicolas-Claude Fabri de Peiresc] and Garterius [?] observing, it was of 8 digits 48′, starting with the Sun at an altitude of 32°, or at 3^h 29^m, and ending with the Sun at an altitude of 8° 40′, or at 5^h 41^m. But at Oxford Bainbridge observed it entering with the Sun at an altitude of 31° 10′, namely at 3^h 4^m, and ending with the Sun at an altitude of 14° 55′, or at 5^h 58^m, or about to end a little later. So Fournier relates, in Book 12 of the *Hydrographie*, Ch. 17.

[Margin: 1634]

In 1634, on March 14, the beginning of a lunar eclipse was observed by Master Toriduto [?] at Avignon at 9^h $7\frac{1}{2}^m$ after noon, the end at 11^h. But at Tübingen Schikardus [Wilhelm Schickard] observed the beginning at 7^h 40^m, the middle at 9^h 17^m 45^s and the end at 10^h $56\frac{1}{2}^m$, as Fournier reports in Book 12 of the *Hydrographie*, Ch. 7.

[Margin: 1635]

In 1635, on March 3 new style, the eclipse of the Moon is said to have been observed by Gassendi, not at Marseille, as had been written to me, but at Aix. There, the beginning of the eclipse was with Regulus at an altitude of 50° 25′, or at 6^h 47^m; the total obscuration with Regulus at an altitude of 42° 10′, or at 7^h $55\frac{1}{2}^m$; the emersion at 9^h $32\frac{1}{2}^m$; and the end at 10^h 38^m. But the umbrago lasted until 11^h 2^m, and the semidiameter of the shadow appeared three times greater than the lunar semidiameter. So Georges Fournier has it, in Book 12 of the *Hydrographie*, Ch. 8.

He adds that at Paris Boulliau observed the beginning of the total obscuration at 7^h 37^m, the emersion at 9^h 9^m, and therefore the middle at 8^h 23^m. At Digne, D. Bollon Chatoini [?] observed the total obscuration at 8^h 2^m and the end at

10^h $40½^m$. At Amsterdam Hortensius observed the beginning with Regulus at an altitude of 27° 8′, or at 6^h 46^m, the total obscuration at 7^h 51^m, the emersion at 9^h 30^m and the end at 10^h $34½^m$. But at Danzig, with Crüger observing, the beginning was at 7^h 39^m, the total obscuration at 8^h 47^m, the emersion at 10^h 30^m and the end at 11^h 39^m, unless there is some defect in the calculation of the last time. At London, again, Master Gellibrandus [Henry Gellibrand] observed the beginning at 6^h 20^m, the total obscuration at 7^h 37^m, the emersion at 9^h 9^m and the end at 10^h 11^m. At Tübingen, further, Schickard observed the middle at 8^h 43^m. So Fournier, in the place cited above.

Albert Linemann himself, in the short work entitled *Memoria Secularis*, relates what he observed at Königsberg in Prussia. He did not observe the beginning, because clouds were in the way; but when nearly six digits were obscured, Jupiter stood at an altitude of 56° 50′ in the east, that is, at 8^h 19^m. The beginning of the stay was with Procyon at an altitude of 41° 0′ in the west, or at 8^h 54^m; half a digit was recovered with Jupiter at an altitude of 51° 40′ in the west, namely at 10^h 34^m; and the end of the eclipse was with Jupiter at an altitude of 44° 40′, or at 11^h 35^m. But he assumes the places of the Sun, Jupiter and Procyon from the Rudolphine Tables. He adds, from Crüger's letters to him, and confirms, what we gave above from Fournier; and he adds that at Stettin in Pomerania D. Eichstaedt, Doctor of Medicine, observed the beginning of the total obscuration with Regulus at an altitude of 41° 13′, or at 8^h 27^m, as that author himself has it in his *Pædia Astron.*, p. 128. At Franeker, however, Metius observed the middle of the eclipse, from the extremes, at 8^h 41^m, as Phocylides [Johannes Phocylides Holwarda] reports in his *Examen Astronomiæ*, p. 108.

[Margin: 1635]

In the same year, on August 28, Albert Linemann, as he himself reports in his second *Memoria Secularis*, observed the lunar eclipse at Königsberg in Prussia as follows.

[Margin: YEARS of Christ.]

	Altitude of Aldebaran in the east	After midnight
BEGINNING	35° 40′	2^h 33^m
3 digits obscured	37° 25′	2^h 46^m
5 digits obscured	38° 25′	2^h 54^m
6 digits obscured	39° 20′	3^h 2^m
9½ digits obscured	40° 40′	3^h 13^m
Total obscuration	42° 30′	3^h 30^m

He adds that the Moon, while still high above the horizon, vanished entirely around the middle of the total obscuration, both to the naked eye and to the telescope.

[Margin: 1636]

In 1636, on February 20, Albert Linemann, as he himself relates in his third *Memoria Secularis*, observed at Königsberg in Prussia the exact beginning of the lunar eclipse, through the tube, at $10^h\ 45^m$, when Arcturus was at an altitude of 26° in the east. With Arcturus at an altitude of 30° 20′, or at $11^h\ 15^m$, fewer than 3 digits and more than 2½ were obscured, and that was then the middle of the eclipse.

Fournier, however, in Book 12 of the *Hydrographie*, Ch. 10, relates that at Traona [Trahone] in the Valtellina Monsieur Petit, Professor of Mathematics, observed the beginning at $10^h\ 8^m$, the middle at $11^h\ 1^m$ and the end at $11^h\ 54^m$, using the altitude of Aldebaran and of Sirius, and the pole altitude there, observed exactly from the altitude of the Sun at the summer solstice, 46° 10′. He adds an observation of the same eclipse made at Aix by Master Gerardo [?], as in the following small table.

	Altitude of Arcturus	After noon
Beginning	15° 30′	$9^h\ 49½^m$
Digit obscured	16° 30′	$9^h\ 57^m$
Two digits	18° 30′	$10^h\ 6½^m$
2 and ½ digits	19° 20′	$11^h\ 9^m$
End	21° 0′	$11^h\ 33½^m$

At Amsterdam, however, Hortensius observed the end with Arcturus at an altitude of 33° 7′, or at $11^h\ 32^m$.

[Margin: 1637]

In 1637, near the Hurons in New France, Father Castelanus [?] of the Society of Jesus observed, on December 31, the middle of a lunar eclipse at $6^h\ 15^m$, as Fournier has it in Book 12 of the *Hydrographie*, Ch. 14.

[Margin: 1638]

In 1638, on December 20, Gassendi observed the phases of a lunar eclipse, as Fournier reports in Book 12 of the *Hydrographie*, Ch. 11, as you see in the following small table.

Phases		Altitude of Aldebaran in the west	After noon
Umbrago or smoke		52° 46′	$12^h\ 12½^m$
Beginning of the true eclipse		49° 18′	$12^h\ 26½^m$
	1	48° 29′	$12^h\ 42^m$
	2	47° 32′	$12^h\ 48^m$
	3	46° 46′	$12^h\ 53^m$
	4	46° 1′	$12^h\ 58^m$
	5	45° 17′	$13^h\ 2^m$
	6	44° 25′	$13^h\ 7½^m$
Digits obscured	7	43° 41′	$13^h\ 12^m$
	8	42° 51′	$13^h\ 17^m$
	9	41° 59′	$13^h\ 22^m$
	10	41° 1′	$13^h\ 28^m$

Phases		Altitude of Aldebaran in the west	After noon
	11	40° 12′	13^h 33^m
Total obscuration	12	39° 21′	13^h 38^m
		Altitude of the right shoulder of Orion	
Emersion		30° 32′	15^h $13½^m$
	1	29° 41′	15^h 17^m
	2	28° 49′	15^h 24^m
Digits recovered	3	28° 9′	15^h 28^m
	4	27° 15′	15^h 33^m

[Margin: YEARS of Christ.]
The remainder of the preceding observations.

Phases		Altitude of the right shoulder of Orion	Time
	5	26° 20′	15^h $37½^m$
Digits recovered	6	25° 24′	15^h 44^m
	7	24° 42′	15^h 48^m
	8	23° 59′	15^h 52^m
	9	23° 2′	15^h 57^m
	10	22° 3′	16^h 3^m
	11	21° 3′	16^h $8½^m$
End	12	20° 3′	16^h 14^m
End of the umbrago		15° 54′	16^h 37^m

In the same chapter Fournier relates an observation made at Aix by Master Garratus [?], of which you may take this summary.

		Altitude of the Tail of the Lion in the east	After noon at Aix
Beginning of the eclipse		19° 55′	12^h 22^m
True beginning		23° 0′	12^h 39^m
	2	24° 20′	12^h $46½^m$
	5	27° 0′	13^h 0^m
Digits obscured	6	28° 0′	13^h $6½^m$
	8	29° 20′	13^h 14^m
	9	30° 25′	13^h 20^m
	10	31° 45′	13^h $27½^m$
	11	32° 35′	13^h 32^m
		Bright star of ♈	
Total obscuration		17° 10′	13^h 36^m
		Altitude of Arcturus	
	1	26° 50′	15^h 12^m
Digits recovered	2	28° 35′	15^h $21½^m$
	3	28° 55′	15^h 23^m
	4	29° 30′	15^h 26^m
	5	30° 35′	15^h 34^m
		Spica ♍	
	7	19° 40′	15^h 43^m

		Altitude of the Tail of the Lion in the east	After noon at Aix
	8	21° 55′	$15^h\ 59^m$
	10½	22° 40′	$15^h\ 5½^m$
End			$16^h\ 9½^m$

[Margin: 1639]

In 1639, on June 1, an eclipse of the Sun was observed at Danzig by Johannes Hevelius through a bare aperture, with an hour-striking clock corrected to an accurate sundial. Albert Linemann reports on it as follows in his fourth *Memoria Secularis*.

	Time
Beginning of the eclipse	$5^h\ 20^m\ 42^s$
6 digits obscured	$5^h\ 58^m\ 8^s$
Middle	$6^h\ 21^m\ 31^s$
6 digits again	$6^h\ 44^m\ 50^s$
End	$7^h\ 14^m\ 55^s$
Greatest quantity of digits	10 digits 48′
Duration	$1^h\ 54^m\ 13^s$

At Stettin in Pomerania, however, as the same Linemann relates, Master Lorenz Eichstaedt observed the beginning at $4^h\ 58^m$ and the end at $7^h\ 7^m$. At Leiden, moreover, Master Kechelius [Samuel Kechel], using Snel's quadrant, observed the beginning with the Sun at an altitude of 30° 30′, that is, at $4^h\ 33^m$, the end with the ☉ at an altitude of 13° 21′, or at $6^h\ 27^m$, and at the maximum 10 digits 20′. So Linemann reports.

But Father Fournier, in Book 12 of the *Hydrographie*, Ch. 17, relates that at Paris the most celebrated mathematicians, having divided the places among themselves, observed this eclipse. Thus in the monastery of the Discalced Carmelite Fathers, the western window was taken by Master Roberualius [Gilles Personne de Roberval], Master Mydorge the Treasurer and Father Denis Pétau. They found the beginning of the eclipse with the Sun at an altitude of 32° 50′ and the end with it at an altitude of 13° 43′. From this Fournier infers, for the beginning, the hour $4^h\ 19^m\ 28^s$ if Gassendi's pole altitude of 48° 52′ is assumed, but $4^h\ 19^m\ 30^s$ if 48° 45′, with parallaxes and refractions computed.

But Master Beaugrand the Secretary, President Pasqualis [Étienne Pascal] and Master Hardi [Claude Hardy] the Counsellor, near the Châtelet, in a library, with a clock set exactly to the Sun, observed the beginning at $4^h\ 25^m$, the middle at $5^h\ 23^m\ 30^s$ and the end at $6^h\ 34^m$. They also found the ratio of the Sun's apparent diameter to the Moon's apparent diameter to be as 155 to 163, and 9 digits 34′ at the height of the obscuration.

At Blois, Master Beaune [Florimond de Beaune], Royal Counsellor and a most ingenious and excellent mathematician, observed in our college the beginning at $4^h\ 17^m$, the end at $6^h\ 17^m$, and 8½ digits. At La Flèche [Flexijs],

moreover, Fathers of our Society Vatierus [?], Deriennes [?] and Fournier, each a professor of mathematics, observed in the Royal College the beginning with the Sun at an altitude of 34° 30′, or at 4^h 6^m; the middle with the Sun at an altitude of 23° 0′, at 5^h $18½^m$; and the end with the Sun at an altitude of 13°, at 6^h 20^m.

At Aix, further, Gassendi observed the beginning at 4^h 44^m, with the Sun at an altitude of 28° 30′, and the middle at 5^h 45^m, with the Sun at an altitude of 18° 53′ and 8 digits 10′ obscured. At Rome, Master Bertus [?] observed the beginning at 5^h 15^m, with the Sun at an altitude of 23°, the greatest obscuration at 6^h 16^m, with the Sun at an altitude of 12°, and 8½ digits; the Moon's diameter appeared to him smaller than the Sun's by a 34th part. But Kircher says that at Rome the beginning was observed at 5^h 33^m, or with the Sun at an altitude of 38° 24′, and that 8 digits were obscured with the Sun at an altitude of 9° 43′, or at 6^h 10^m. But the altitudes of the ☉, 23° and 38°, disagree too much; yet that is how I read it. In Canada, Father Castelanus [?], at the Residence of the Conception, observed the middle of the eclipse, of 4 digits, at 10^h 15^m in the morning. Father Agathangelus Britannus, a Capuchin, at Cairo in Egypt, says that no part of the Sun was obscured. So Fournier reports, in the place cited above.

[Margin: YEARS of Christ.]

[Margin: 1641]

In 1641, on the night following October 18, Albert Linemann, as he himself reports in his fifth *Memoria Secularis*, observed the following phases of a lunar eclipse at Königsberg in Prussia.

Phases	Altitude of Aquila in the west	After noon
Penumbra	41° 50′	☩ ☩
True beginning of the eclipse	41° 20′	7^h 17^m
3½ digits obscured	40° 20′	7^h 22^m
5 digits obscured	37° 0′	8^h 4^m
6¼ digits, the maximum	33° 36′	8^h 37^m
5 digits, decreasing	30° 0′	9^h 8^m
Exact end	23° 45′	9^h 57^m

He adds that at Danzig Hevelius observed the beginning at 7^h 1^m 15^s, the middle at 8^h 26^m 15^s, the end at 9^h 51^m 0^s, and 7 digits at the maximum; that at Paris Gassendi observed the end at 7^h 50^m; and that at Franeker Phocylides Holanda observed the end at 9^h $3½^m$, as he himself relates in the *Epitome Astronomiæ reformatæ*.

[Margin: 1642]

On the eclipse of 1642, which took place on April 14, which we observed at Bologna and then communicated with others to Govaert Wendelen in 1646, it will not be useless to indicate here what Wendelen thought, from his letter to me dated at Brussels on September 5 [Nonis Septembris] of the same year. He

writes that our observation gives the strongest support to his hypotheses. He regrets that at Paris Boulliau observed the beginning at $12^h\ 10^m$, when it should have begun there at $11^h\ 45^m$, since Herk is east of Paris by 18^m, and at Herk the beginning was observed accurately at $12^h\ 3^m$. He regrets, I say, a disagreement of 25^m in the time; but how justly I fear that he makes Herk too far east, I shall show in my *Geographical Book*.

[Margin: 1643]

The eclipse of the ☾ of September 27, 1643, was observed by Antonius Garratus [?] near Senlis, 2′ east of Paris; its end, says Boulliau in the prolegomena to the *Astronomia Philolaica*, agrees with Boulliau's to within 1′.

[Margin: 1645]

On August 21, Johannes Hevelius says in Ch. 5 of the *Selenographia*, he observed at Danzig the following phases of a solar eclipse, from corrected altitudes of the Sun.

[Margin: YEARS of Christ.]

Phases	After the preceding midnight: time
Beginning	$11^h\ 23^m\ 45^s$
Digits obscured: 2	$11^h\ 38^m\ 0^s$
5	$12^h\ 1^m\ 30^s$
6	$12^h\ 11^m\ 30^s$
7	$12^h\ 22^m\ 0^s$
Middle: 7¾	$12^h\ 41^m\ 20^s$
Digits obscured, decreasing: 7	$12^h\ 54^m\ 45^s$
6	$13^h\ 6^m\ 0^s$
5	$13^h\ 15^m\ 30^s$
4	$13^h\ 23^m\ 45^s$
3	$13^h\ 31^m\ 30^s$
End of the eclipse	$13^h\ 53^m\ 0^s$

But Father Iacobus Billy [Jacques de Billy] of the Society of Jesus, in a short but learned letter written to me, affirms that the beginning of this eclipse was accurately observed at Reims on the 21st at $1^h\ 53^m$ before noon, and the middle at $0^h\ 58^m$ before noon, but that the end could not be observed. He adds that Reims and Châlons lie under the same meridian.

[Margin: 1646]

In 1646, on January 31, Johannes Hevelius observed at Gedanum, or Danzig, in the morning before sunrise, the beginning of the lunar inumbration at $5^h\ 21^m\ 4^s$ after midnight. He says that this beginning was about degree 118 of the Moon's limb and the mountain of the Arcadians (for he is alluding to the names in his *Selenographia*); that 4 digits were obscured at $5^h\ 33^m\ 36^s$ and 11 digits at $6^h\ 12^m\ 52^s$; that the semidiameter of the shadow was estimated at 46′ 30″; and that clouds prevented the rest.

In the same year, on July 27, the eclipse of the Moon could not be observed at Herk by Wendelen. He writes to me, however, in a letter of this year that remnants of the penumbra were visible to him until 7^h 55^m.

[Margin: 1647]

To what I related of the eclipse of the year 1647 observed by Hevelius it must be added that the beginning of the eclipse was about degree 170 of the Moon's limb and the end about degree 247. For in his *Selenographia* he divides the circumference of the Moon's disc into 360 degrees, as though libration allowed those degrees always the same position. We, however, think it better to name certain small spots across which the boundary of the shadow passes.

[Margin: 1649]

In 1649, on May 25, D. Vicenç Mut observed the beginning of a lunar eclipse at Majorca at 13^h 16^m after noon, with Arcturus at an altitude of 42° 10′ towards the west. He rightly wonders that by Wendelen's *Atlantic Tables* it should have begun at Majorca at 12^h 44^m 41^s; he admits, however, that to the naked eye the umbrago began at 13^h 8^m. So he says in a letter to me. But the equation of natural days, which Wendelen neglected, not rarely produces these disagreements.

[Margin: Wendelen's disagreement with the observations.]

[Margin: 1649]

In 1649, on November 4, there was an eclipse of the Sun, which Father Theodorus Moretus [Theodoor Moretus] of the Society of Jesus observed at Kostelec in Bohemia: only the end, that is, at 5^h 15^m after noon, with the image of the Sun brought in through an optical tube; for the clouds denied him the middle and the beginning. He adds that the town lies towards the east of Prague, on the same parallel, at almost 15 great German miles, or 90 or 100 Italian. When he took the altitude of the Sun, the perpendicular was to its shadow as 57 to 546, the shadow observed being the one that indicated the altitude of the Sun's centre, in the middle of the doubtful edge of the solar shadow. Then, having found the meridian line at the same place by means of the Sun's double shadow, he found on November 7 at noon that the height of the perpendicular was to the length of the Sun's shadow as 57 to 108. The most courteous Father wrote this to me at once, on the 8th.

[Margin: D. Vicenç Mut's diligence in astronomical observations.]

Not very long after I had received this, there came to me a letter from Majorca from D. Vicenç Mut, dated the 16th, in which he set out in full his observation of this eclipse, made by him at Majorca accurately, as he usually does; and it went as follows. He received the image of the Sun on paper through the telescope described above in this appendix, at Book 3, Ch. 10. The image of the Sun not yet eclipsed was of 4150 parts, of which the image of the Sun at apogee had been found to be 4000. Now, of the parts of which the whole image was 4150, the obscured part observed in the eclipse was as below.

Parts of the ☉ obscured	Altitude of the ☉	After noon
Beginning: 1	32° 32′	1^h 22½^m
332	31° 20′	
518	30° 33′	
830	29° 56′	
955	29° 27′	
1100	28° 25′	
Middle: 1205	27° 47′	2^h 3⅛^m
830	23° 23′	
518	22° 19′	
332	21° 27′	
End: 1	20° 21′	3^h 2½^m

[Margin: YEARS of Christ.]

[Margin: The Moon's diameter.]

The greatest obscuration was 9′ 25″, of which the diameter was 32′ 28″, or 3 digits 33′. The Moon's diameter, moreover, was of 4230 parts, of which the Sun's was 4150. Looking at the Sun also through a tube, but one fitted with coloured glasses, he clearly saw the Moon somewhat larger than the Sun, and by a little more than on the paper. The Moon was at perigee, and trigonometric exactness gathers its hourly motion as 36′ 43″, but in doing so supposes the sum of the semidiameters of the luminaries to be 32′ 47″, the obscured part of the Sun at the height of the eclipse 9′ 25″, and the true latitude of the Moon 4′ 40″.

[Margin: Praises of the pendulum in the measurement of time.]

Johannes Hevelius observed the same eclipse at Danzig, using a clock that showed every single second, checked against a sundial, which gave 54^m after noon when the striking clock gave 55^m 35^s. As soon as one o'clock after noon was complete, the reciprocal vibrations of the pendulum began to be counted, of which 43¼ made up one minute and 2596 a whole hour. *For these*, says Hevelius, *if rightly counted against a sundial accurately divided into minutes, snatch the palm from all clockwork in showing the time (as experience bears witness)*. What would Hevelius say now of our pendulums, adjusted to the motion of the fixed stars? But let us go on, and set out the phases that the clouds and the towers in the way did not deny him, from his letter to Eichstaedt, written on 1650 January 5 and afterwards forwarded to me when it was made public.

Digits eclipsed	Time by the striking clock	Vibrations of the pendulum, begun from 1 o'clock after noon	Time after noon from the pendulum
Beginning	2^h 27^m 30^s	3807	2^h 28^m 0^s
5¾	3^h 15^m 0^s	5819	3^h 14^m 30^s
5½	3^h 17^m 0^s	5910	3^h 16^m 36^s
5⅜	3^h 19^m 45^s	6029	3^h 19^m 22^s
5	3^h 22^m 30^s	6151	3^h 22^m 10^s
4⅞	3^h 24^m 0^s	6216	3^h 23^m 41^s
4¾	3^h 27^m 0^s	6326	3^h 26^m 19^s
4⅝	3^h 28^m 0^s	6392	3^h 27^m 45^s

Digits eclipsed	Time by the striking clock	Vibrations of the pendulum, begun from 1 o'clock after noon	Time after noon from the pendulum
4¼	$3^h\ 32^m\ 40^s$	6598	$3^h\ 32^m\ 31^s$
3½	$3^h\ 39^m\ 0^s$	6873	$3^h\ 38^m\ 50^s$
1⅛	$3^h\ 56^m\ 0^s$	7611	$3^h\ 55^m\ 56^s$
0½	$3^h\ 59^m\ 0^s$	7741	$3^h\ 58^m\ 57^s$
End	$4^h\ 1^m\ 0^s$	7831	$4^h\ 1^m\ 0^s$

[Margin: Disagreement of the astronomical tables with the sky.]

He adds the position of the horns relative to the ecliptic. The right cusp, inclining towards Corus, stood 53½° from the zenith and 45° from the ecliptic, on the small circle of phases of the computer; the left cusp stood 68° from the zenith and 14° from the ecliptic, when the eclipse was at its height. When a quarter of the Sun's diameter was dark, the right cusp held exactly the point of the zenith. Moreover, the eclipse ended towards the Hellespontius, 55° from the point of the zenith and 27 from the ecliptic; from which he gathers that it began towards Corus at about 60° from the point of the zenith. As for the calculation from the astronomical tables straying so far from the truth, he suspects that something lies hidden in the equations of the times.

[Margin: YEARS of Christ.]

[Margin: 1649]

In the same year 1649, on the night between November 18 and 19, we observed here at Bologna the total obscuration, or immersion, of the Moon, when Procyon was 55° 35′ from the vertex, that is, at $18^h\ 45^m\ 50^s$ after noon of the 18th. We were not able to observe the rest. Father Nicolaus Mascardus [Nicolò Mascardi] of the Society of Jesus, however, wrote to me from the West Indies, namely from the port of Panama, that the beginning of this eclipse was observed by Father Franciscus Ruggius [?], a man highly skilled in mathematics, at Panama at the eleventh hour after noon, and that its duration was 4 hours. From this we are certain that the interval between Bologna and Panama is not $7^h\ 46^m$. For if the total obscuration at Bologna was at $18^h\ 46^m$, then the beginning of the eclipse was before this time, and indeed by very nearly one hour according to astronomical calculation, that is, at $17^h\ 46^m$. If you subtract 11 hours from these, $6^h\ 46^m$ remain, to which there answers a difference of meridians of 101½°. But since neither we in any way, nor perhaps Father Ruggius, observed the beginning with the greatest exactness, we may not conclude from this a difference of longitude that is exact in every respect.

[Margin: The difference of meridians of Bologna and Panama.]

[Margin: 1650]

In 1650, on May 15, the Moon rose above the horizon at Majorca already eclipsed. At the greatest obscuration the illuminated part of the Moon was 9′ or 10′. The end of the eclipse, caught with a telescope, was with Regulus at an altitude of 40° 29′ towards the west, that is, at $9^h\ 32^m\ 24^s$. So the observer himself, D. Vicenç Mut, wrote in full to me in his letter sent to Bologna on

June 24; in it he wonders, indeed, that Wendelen's calculation falls short of this observation by 32′. I judge that this arose chiefly from the fact that he disregarded the equation of natural days.

[Margin: Disagreement of Wendelen's tables with the sky.]

AD LIBRVM VI.

To Book VI.

CAP. III. No. 1. To the authors who present and explain the constellations must be added Caussinus [Nicolas Caussin] in *De domo Dei*, Book 2, Ch. 22, and Iulius Schillerus [Julius Schiller] of Augsburg, Doctor of both Laws, who, with Ioannes Bayerus [Johann Bayer] supplying a new and more accurate *Uranometria*, made the starry heaven Christian, sacred names and figures being put in the place of the ancient figures. In the preface to the work, published in the year 1627, he gives the reason for his undertaking, and he cites in his support St Justin Martyr in the *Oration to Antoninus Pius*; Lactantius in Book 5 of the *Divine Institutes*, Ch. 10; and Blessed Isidore in Book 3 of the *Origins*, the chapter on astronomy, who execrate the Gentiles because they brought into the heavens the names of beasts or of the most wicked men. He also replies to objections, especially to the one in which it was said to be absurd for anyone to say that Saints regard one another in trine or quadrate aspect, as is said of the planets and the other stars.

[Margin: Whether the names of the stars are to be taken from the Saints.]

CAP. VI. No. 2. I have already mentioned Julius Schiller, who made the starry heaven Christian. In this I certainly praise the man's piety, but I do not see that he obtained what he wanted. If anyone nevertheless wishes to know into what persons or sacred things he changed the figures and names of the stars, and does not have that book, let him run through the following catalogue, in which we have also set down the Arabic names corrected by Vuilhelmus Schikardus [Wilhelm Schickard], professor of languages at Tübingen, and the number of stars attributed to each constellation by Schiller, from the *Uranometria* enlarged and reformed by Bayer. Moreover, since he transformed the twelve signs of the zodiac into the 12 Apostles, he also took their characters and marks mostly from the instruments with which they suffered martyrdom. For he assigns to St Peter the key, to St Andrew the cross, to St James the Greater the pilgrims' staff, to St John the chalice, to St Thomas the spear, to St James the Less the fuller's club, to St Philip the small cross, to St Bartholomew the knife, to St Matthew the two-edged axe, to St Simon the saw, to St Thaddaeus the mace, and to St Matthias the axe. And in place of that distich,

Sunt Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libraq. Scorpius, Arcitenens, Caper, Amphora, Pisces.

(They are Aries, Taurus, Gemini, Cancer, Leo, Virgo, / and Libra, Scorpius, the Archer, the Goat, the Amphora, Pisces.)

he appends this:

Clauis, Crux maior, Baculus, Scyphus, Hasta, Sudisque, Crux, Culterque, Bipennis, Serra, & Claua, Securis.

(Key, Greater Cross, Staff, Cup, Spear and Club, / Cross and Knife, Two-edged Axe, Saw and Mace, Axe.)

[Margin: The instruments and characteristic marks of the Apostles.]

Ancient constellations	Schillerian constellations	Arabic names from Schickard	Number of stars
Ursa Minor	St Michael the Archangel	*Arrucabatho*, that is, the wagon	42
Ursa Maior	The Little Ship of St Peter	*Dubol achbaro*	51
Draco	The Holy Innocents	*Attanino*, the Dragon	52
Cepheus	St Stephen	*Chiphus*	43
Bootes	St Sylvester	*Arramichus*, that is, the spear-bearer	38
Coma Berenices	The Scourge of Christ	*Huzimethon*, the Handful	14
Corona Borea	The Crown of Thorns of Christ	*Alphankacho*, the open flower	17
Hercules	The Three Kings, or Magi	*Algathio*, falling forward	57
Lyra	The Manger of the Saviour	*Vueka*, flying by falling	14
Cygnus	The Cross with St Helena	*Addigagato*, the Hen	60
Cassiopea	St Mary Magdalene	*Dhat Alchursi*	34
Perseus	St Paul the Apostle	*Chamilras Algol*	51
Auriga	St Jerome	*Memassich alharan*	33
Serpentarius	St Benedict among the thorns	*Alhague*, standing upon	34
Sagitta	The Nails and Lance of the Saviour	*Alchanzato*, the javelin	13
Aquila	St Catherine, Martyr	*Attayro*, the bird	21
Antinous	Part of St Catherine	.....	19
Delphin	The Water-jar [Hydria Chananeæ]	*Aldelphin*	10
Equulus	The Mystic Rose	*Kataat, alfaras*	4
Pegasus	St Gabriel the Archangel	*Alpharaso*	34
Andromeda	The Sepulchre of Christ	*Almara*	39
Triangulum	The Mitre of St Peter	*Almatlato*, the tripled	6
		Sum	686
Aries	St Peter	*Alchamalo*, Aries	31
Taurus	St Andrew	*Attauuro*, Taurus	50
Gemini	St James the Greater	*Aphellar*	30
Cancer	St John the Evangelist	*Assartano*, Cancer	43
Leo	St Thomas	*Asedaton*, the Lioness	43
Virgo	St James the Less	*Sumbalon*, the Ear of Corn	50
Libra	St Philip	*Midsanor*, Libra	14
Scorpius	St Bartholomew	*Alacrabo*, Scorpius	37
Sagittarius	St Matthew	*Alkauuso*, the bow of Sagittarius	34
Capricornus	St Simon	*Algedio*, the Goat	28
Aquarius	St Thaddaeus	*Addelvvu*, the Bucket	44
Pisces	St Matthias	*Elhaut*, Pisces	37
		Sum	451

RESIDVVM TABVLÆ PRÆCEDENTIS.

The rest of the preceding table.

Ancient constellations	Schillerian constellations	Arabic names from Schickard	Number of stars
Cetus	Sts Joachim and Anne	*Elketos*, Cetus	33
Eridanus	The Crossing of Israel through the Red Sea	*Nahron*, the river	45
Orion	St Joseph, Spouse of the Blessed Virgin	*Algebaro*, the Hero, or *Asschagio*, the raging one	71
Lepus	The Fleece of Gideon	*Alarnebo*, Lepus	13
Columba	Noah's Dove	....	11
Canis Maior	St David	*Aliaminio*, the right of the dogs	20
Canis Minor	The Paschal Lamb	*Algomyso*, of the sycamores	11
Argo Navis	Noah's Ark	*Sephina*, the ship	58
Hydra	The Jordan	*Assugia*, raging	61
Crater	The Ark of the Covenant	*Alkaso*, the cup	11
Corvus	Part of the Ark of the Covenant	*Algorabo*, Corvus	7
Centaurus	Abraham and Isaac	*Asmeat*, the monster	38
Lupus	St Jacob the Patriarch	*Alesid*, the Lioness	30
Ara	The Altar of Incense	*Almugamra*	15
Corona Australis	The Diadem of Solomon	*Aladil Algenubi*	13
Piscis Notius	The Flour-jar of the Widow of Sarepta	*Albaut algenudi*	12
Grus	Aaron the Priest		13
Phoenix	The rest of Aaron		15
Indus	St Job		12
Pavo	The rest of St Job		23
Apus, *Manucodiata*	Eve		11
and *Chamaeleon*	Eve		9
Triangulum Australe	The Sign of Tau		5
Musca	Part of Eve		4
Piscis volans	Abel the Just		10
Dorado with the Cloud	Part of Abel		6
Toucan and	St Raphael the Archangel		8
Hydrus with the Cloud	Part of St Raphael		20
Sum of all the stars: 1692		Sum	555

[Margin: The number of the stars is incomprehensible.]

On the number of the stars being incomprehensible to us, on account of the authority of Holy Scripture, see the opinion of St Augustine, *De Civitate Dei*, Book 16, Ch. 23; St Thomas on Genesis, Ch. 15; and Alonso Tostado on Deuteronomy, Ch. 1, Question 6. Add also Aristotle, *Meteorologica*, Book 1, Ch. 8, where he speaks of the sporades; Seneca, *Naturales Quaestiones*, Book 2, Ch. 32; and finally the Coimbra commentators on *De caelo* 2, Ch. 12, Question 1.

[Margin: Some stars of the firmament are higher than others.]

CAP. VII. No. 2. That some fixed stars are higher than others was also the opinion of Geminus in the *Elements of Astronomy*, Ch. 1, where he says: *It must not be thought that all the stars are placed under the same surface, but that some are higher, others lower.* Gilbertus [William Gilbert] stated the very same thing without hesitation in *De Magnete*, Book 6, Ch. 3, saying: *Nor is there any doubt that, just as the planets are distant from the Earth by unlike intervals, so those huge and very numerous lights are separated by various and very remote altitudes from the Earth.* Johannes Hevelius supports them in his *Selenographia*, Ch. 7.

[Margin: The Copernican distance of the fixed stars.]

No. 15. To the Copernican distances of the fixed stars add the opinion of René Descartes in the *Principles of Philosophy*, Part 3, no. 20. He affirms there that if this distance is compared with distances known to us, the one commonly attributed to the fixed stars is no less incredible than any greater one; but if it is compared with the omnipotence of God the creator, none can be conceived so great that it is for that reason less credible than any smaller one. To explain the phenomena of comets conveniently, however, he thinks that a very great space must be set between the fixed stars and Saturn. But no one has so far demonstrated absolutely that comets are above Saturn, as I show in Book 8, Section 1, Ch. 23.

At the end of no. 17, in the last table, add the distance of the fixed stars of 60 000 000 terrestrial semidiameters, which Kepler later chose in the *Epitome*, as I said here in no. 11; and the distances of Aristarchus, Mästlin and Gassendi, which I report in Book 9, Section 4, Ch. 28, no. 2. Besides these, Peter Crüger, as cited by Hevelius in the *Selenographia*, p. 162, thinks that the sphere of Saturn is to the sphere of the fixed stars, as regards diameter, as a pea to 15 feet, in which 900 peas are contained; and so the diameter of Saturn is contained in the diameter of the firmament 900 times.

[Margin: The position of the Pleiades observed by D. Vicenç Mut.]

CAP. X. On the position of the Pleiades and their distance from one another, observed with the most exact instruments and with the tube, D. Vicenç Mut wrote to me from Majorca in a letter of 1650 March 5, and he says that the distances were those written below.

The middle and bright star of the Pleiades, from the following more northerly star	31′
The following more northerly star, from the lowest, the one nearest the western star	30′
The following more northerly star, from the brighter of the western stars	22′
The following more northerly star, from the more northerly western star	11′
The brighter of the western stars, from the more northerly western star	22′
The middle and bright star of the Pleiades, from the lowest western star	22′
The brighter of the western stars, from the lowest western star	24′
The middle and bright star of the Pleiades, from the upper eastern star	27′
The upper of the eastern stars, from the one at the point towards the east	4′

He adds that the brighter of the western stars crossed the meridian at exactly the same altitude as the bright star of the Pleiades, and the upper of the eastern stars 1′ higher. From these, with the Tychonic longitude and latitude of the bright star of the Pleiades set down, he gathers the following longitudes and latitudes for the end of the year 1649.

	Longitude in ♉	North latitude	Magnitude
The brighter western star of the Pleiades	24° 28′ 11″	4° 9′ 40″	5
The more northerly western star	24° 38′ 30″	4° 31′ 36″	6
The lowest, the one nearest the western star	24° 45′ 0″	3° 53′ 30″	6
The following more northerly star	24° 45′ 30″	4° 23′ 56″	6
The middle and bright star of the Pleiades	25° 6′ 0″	4° 0′ 0″	3
The one at the point towards the east	25° 31′ 30″	3° 51′ 20″	5
The upper of the eastern stars	25° 32′ 30″	3° 54′ 0″	7

From these he gathers that the position set down by Tycho and Lansbergen must be corrected, but above all Lansbergen's, which puts the distance between the middle star of the Pleiades and the western one at 1° 18′, as great as that between the first and middle stars of Orion's Belt, although it is plain even to the naked eye that it is much less. Besides these he sets down six other small stars in this asterism, smaller ones. Van Langren, however, who also observed them himself with a telescope, sets down ten in all, counting the ancient ones, while in position and figure he comes as near as possible to Mut, as I gather from van Langren's scheme sent to me from Brussels in 1647 on July 6.

[Margin: The error of Tycho and Lansbergen in the position of the Pleiades.]

CAP. XVI. I do not know from where Bartholomæus Kechermannus [Bartholomäus Keckermann], in the *Systema Astronomiae*, Book 1, Sect. 1, Ch. 6, has dared to say that according to the common opinion the period of the fixed stars requires 49000 years, unless perhaps he takes for the common opinion the fictions of the Kabbalists, undeservedly attributed to Alfonso.

[Margin: The period of the fixed stars.]

CAP. XVII. I report and refute the fiction of Ioannes Pena [Jean Pena] about the unequal motion of the fixed stars in Book 9, Sect. 4, Ch. 3, in the scholia. This inequality, moreover, Giovanni Antonio Delfino seems to presuppose in his book on the celestial globes and motions, Ch. 12, but he does not establish it on sufficiently firm reasons, since he merely reports the opinions of others. See also Caussinus, *De domo Dei*, Book 2, Ch. 24.

[Margin: The inequality of the motion of the fixed stars is false.]

CAP. XXI and XXII. See also Caussinus, *De domo Dei*, Book 2, Ch. 25.

CAP. XXIII. To the opinion of Aristotle, who thought that the Milky Way is in the highest part of the air, as a great and perpetual comet arising from a continuous supply of exhalations, St Damascene subscribes in his *Physics*, Ch. 14. Among those who have imagined a double Milky Way, one ethereal and the other aerial, is Franciscus Piccolomineus [Francesco Piccolomini], on the

Meteorologica, Ch. 9. But those of Leuven and of Paris, and some Thomists, and most recently Cæsar Raus [?], Tract 4 on the *Meteorologica*, Ch. 1, and Hieronymus Trimarchus [?], on Book 1 of the *Meteorologica*, Disputation 4, Sect. 8, say that Aristotle's text is corrupt. Trimarchus also adduces Maurolico and Father Aegidius Camart [Gilles Camart], a Frenchman, General of the Minims, men most skilled in the Greek language, who say that the Greek text of Aristotle preserved from ancient times in the treasury at Messina does not contain that opinion. Tanner, however, on that passage, says that the Philosopher was speaking only by way of disputation.

That the Milky Way consists of a heap of very small stars is taught, besides those I named in this chapter, by Albertus Magnus, Scaliger, Cardano, Goclenius, Trimarchus above, Francesco Resta in Tract 1 of the *Meteorologica*, Book 1, Ch. 9, Isaach Habrecthus [Isaac Habrecht] in the *Comet of the year 1618*, and Longomontanus in the appendix to the *Astronomia Danica*, Ch. 5. Longomontanus adds, however, that a multitude of small stars is not sufficient for that whiteness unless a denser matter of the heaven lies beneath it in that part; because in a clear and moonless sky innumerable stars set close together are seen everywhere, and yet that tract of the sky does not grow white from them, for lack of density. From this he confirms Tycho's view of new stars and of certain comets as compacted from this denser matter of the heaven. But those stars that are seen outside the Milky Way are neither near one another nor so great in number as those seen in the Milky Way with the help of the telescope, and so that greater density does not seem necessary. Much less is the opinion of Gemma Frisius to be accepted, who, in Book 1 of *De divinis Naturae Characterismis*, Ch. 6, suspected that the Galaxy is in the ethereal or fiery region beneath the lunar heaven, since it neither hides the Moon or the other planets from us nor blunts their light.

[Margin: Opinions on the Milky Way.]

AD LIBRVM VII.

To Book VII.

SECTIO I. See Caussinus, *De domo Dei*, Book 2, from Ch. 16 to Ch. 20.

CAP. II. That the companions of Saturn move along a plane parallel to the equator was observed by Galileo, as he relates in Dialogue 2 of *De Mundi systemate*. What I have indicated about the shadow and the diameter of Saturn is set out in Book 7, Sect. 6, Ch. 14.

[Margin: The companions and shadow of ♄.]

CAP. III. On a fifth satellite of Jupiter, see Scheiner's suspicion in his letters to Markus Welser, p. 38. On the digressions of the satellites from Jupiter, reckoned in the measure of terrestrial semidiameters according to the various opinions on the distance of Jupiter from the Earth, see Sect. 6, Ch. 12 of this Book. Further, Hevelius says in Ch. 4 of the *Selenographia* that the 5 small stars which Šírek took to be Urbanoctavian and Jovial were observed by him in 1642

on August 28 and September 4, and in the years 1643, 1644 and 1645, in the same place, around the 13th degree of Aquarius, and at an unchanged distance. They are therefore fixed stars.

[Margin: The number and digression of the satellites of ♃.]

CAP. IV. The order of the planets is confirmed from the occultations of the superior planets by the inferior; we review some of these in this Book, Sect. 6, Ch. 14. What I said here, however, in No. 4 about Aristotle and in No. 5 about Plato is to be understood according to the probable opinion of some, which will be discussed in Book 9, Sect. 3, Ch. 3 and 4.

[Margin: The order of the planets.]

SECT. IV. CAP. V. The argument from the latitude and inclination of the planets for the motion of the Earth is treated in Book 9, Sect. 4, Ch. 9, No. 13.

SECT. V. CAP. IX. At No. 3 add here, from Kepler's *Harmonics*, Book 5, Ch. 4, the revolutions of the planets about the Sun written below, gathered from the delays of all the degrees of the whole circuit: long, middling and short.

Reuolutiones Planetarum circa Solem

Revolutions of the planets about the Sun

	Days	Minutes of a day	Daily motion, ′	″	‴
Saturn	18769	12	2	0	27
Jupiter	4332	37	4	59	8
Mars	686	59	31	26	31
The Earth with the Moon	365	15	59	8	11
Venus	224	42	96	7	39
Mercury	87	58	245	32	25

SECT. V. CAP. IX and X. See Caussinus, *De domo Dei*, Book 3, Ch. 12.

SECT. VI. CAP. IV. Scholium 3. Beware here of the error of Julius Schiller, who, in his *Cælum stellatum Christianum*, p. 23, says that Mars sometimes becomes as near the Earth as the Moon, and indeed even nearer, namely when it is at the 27th degree of Aquarius. He drew this from the *Systema Astronomicum* of Keckermann, Book 2, Ch. 5. For it is one thing to become nearer than the Sun, another to become nearer than the Moon.

[Margin: The approach of Mars to the Earth.]

CAP. XIV. At No. 2 add that Jupiter was occulted by the Moon in 1647 on January 20, namely by about a third part of the Moon, on its north side, when Procyon was at an altitude of 33° 56′ towards the west; and that Jupiter was left behind by the Moon when Regulus was at an altitude of 53° 43′ towards the west. So D. Vicenç Mut observed at Majorca, as he himself wrote to me, and as he later reported in his *Sol Alphonsinus*, p. 27. Again, in the same year on April 12 the Moon covered Jupiter with its northern part, with Regulus at an altitude of 57° 18′ towards the west, namely at 9^{h} 56^{m}. Jupiter emerged from the Moon's eclipsing with Regulus at an altitude of 49° 12′ towards the west,

or at 10^h 48^m. So Jupiter lay hidden under the Moon for 52^m; the same man observed this in the same place and wrote it in full to me.

[Margin: Jupiter occulted by the ☾ twice in the same year.]

But this astronomer's diligence and skill will be evident to posterity from the *Sol Alphonsinus* and from many other observations which, with the greatest candour of spirit and generosity, he has communicated to me, if either he himself publishes his own or I survive. The same holds for van Langren, who at Madrid in 1632 on February 5 saw Mars covered by that part of the Moon which lies between the spot of Balthasar and the Lake of Posidonius, when 178° 38′ was culminating. Then, after 57^m ⅓, he saw it come out from the spot of the Moon called Leototi. And in 1643 on August 15 he saw Mars covered by the Moon with degree 300° 33′ culminating, and come out with 308° 54′ culminating.

[Margin: In praise of D. Vicenç Mut and of van Langren.]

[Margin: Mars twice covered by the Moon.]

[Margin: For our names for these spots, see p. 205.]

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LAVS DEO.

Praise be to God.

Diagram. The printer's device, a woodcut, centred on the page between *Laus Deo* and the imprint. At the head, a child's face rests on a scrolled cartouche bearing the motto in capitals on three lines; leafy sprays and large shell-like volutes flank it, and a small grotesque mask hangs beneath it. Below is a large shield held by two winged, half-draped putti, one at each side, each standing on a scrolled bracket. The shield shows a storm: banked clouds across the top, rain falling in shafts, and a boat with a broken mast heeling among heavy waves,

with a low shore and small plants in the foreground. At the foot, within foliate scrollwork, is a small escutcheon patterned in black and white triangles. The motto is the only text on the block.

- Cartouche (top centre): FLVCTIBVS ET FREMITV ASSVRGENS BENACE MARINO — "rising, Benacus, with waves and a roar like the sea's"

BONONIÆ,
Bologna,
Typis Hæredis Victorij Benatij. MDCLI.
Printed by the Heir of Vittorio Benacci. 1651.
Superiorum permissu.
With the permission of the superiors.

ERRATA Corrigenda, nisi Auctor ipse calamo illa correxerit, vt optabat.

Errors to be corrected, unless the author himself has corrected them with his pen, as he wished.

Page	Col.	Line	Errors	Correct to
			Errors in the preface, the Chronicle, etc.	
4	·	35	ad	ab
7	·	28	inclitæ	inolitæ
17	·	37	sinixtra	sinistra
26	1	64	286. Eratosthenes	196. Eratosthenes
33	2	19	Vinctus	Vinetus
35	1	6	Attla	Affla
38	1	8	Drienes	Derienes
42	1	34	Hainztlius	Hainzelius
—	—	—	—	—
3	1	73	viam	vim
3	2	31	Astromi	Astronomi
3	2	62, 66	eius. iosoperimetris	cuius. isoperimetris
10	2	29	qui transeunt	qui non transeunt
39	1	22	semihoræ	bihorij
39	2	20, 24	56 ⅓ 1288	52. 288
43	1	31, 43	Epitricam. Isagogen	Epitritam. Isagoges
57	1	44	Ecliptica	Æquatore
59	1	68	est autem FB	est autem FD
61	2	58, 64, 66	53 ⅔	56 ⅔
63	2	22	53 ⅔ 6491	56 ⅔. 6497
63	2	39	8778	8278
72	1	16	ingressus	ingressus es
73	1	43	Bartholomæo	Bartolino
82	1	34	20. aut 25	40. aut 50
88	2	26	Langennus	van Langren
89	2	25	Cretaceus	Cretaceus 12

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110	2	50	1180 1120	1182 1/12. 1117
111	1	59	Mundo	Mundi
113	2	28	subtracta	addita
116	1	54	sensibilem	sensibilem reperit
119	2	41	31′. 50″. 30′. 30″.	31′. 56″. 30′. 50″.
121	2	29	3344	3375
123	1	38, 58	150 5/6. 14′. 50″.	1550 5/6 14′. 52″.
123	2	36	15′. 28″.	14′. 28″.
135	1	25	hoc	hos
141	2	71	182. 13.	182. 2.
153	1	15	178	187.
157	2	15	43 ½.	46 ½.
157	2	22	250. 7250.	253. 7327
160	1	26	aitq. si	aitq. se
177	1	34, 35	58″. 40‴.	59″. 20‴ ½.
214	2	69	20	28
217	1	46	vtrumq.	verumq.
219	2	72	58′. 0.	58′. 30″.
225	1	51, 63	53′. 66.24′.	53′. 37″. 66.24′.20″.
236	1	6	AHG, ita sec.	AH, ita sec. ang. AHG
238	1	57	mota est ♉	nota est ♉
240	2	72	πλήζους	πλήρους [Greek: plērous, "full"]
278	1	61	Eccentrici	Hypocycli
281	2	54	anomaliæ	anomaliam
286	2	50	plano	pleno
310	1	14	31. 1. 0.	13. 1. 0.
353	2	73	prætegmine	præsegmine
364	1	47	die 22.	die 21.
365	1	54	Frutini	Frusini
367	2	69	anno 8.	anno 9.
378	1	72	10. 5′.	15. 9′.
402	2	63	Αμνου′	Αμοὺν [Greek: Amoun]
418	1	32, 41	144000.	1440000.
419	1	78	144000. 0 3.	1440000. 3. 0.
465	1	60	versus	verus
466	2	76	occasum	occasuum
471	2	75	carnibus	cornibus
481	2	38	tradit ab Hygino l. 2.	tradit Hyginus lib. 1.
481	2	59, 62, 63	ὑπατῶν	ὑπάτων [Greek: hypatōn, "of the highest"]
522	2	31, 32	14312. 8162	14313. 8163
522	2	39, 40	14404. 8254	14405. 8255.
523	2	58	♌	♒
523	2	59	♒	♌
552	1	19	69	59
576	2	27	AO. 73852 ½. 952.	AQ. 70585. 910.
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606	1	44	gr. 1. 18′.	gr. 1. 8′.
642	1	50	insolitam	solitam
646	2	71, 72	131. 134.	331. 334.
647	1	15	☿	♃
647	1	35	inter esse	inter sese
663	2	21	Oppositionibus	Apparitionibus
676	2	72	in annum	post annum
687	2	7	55. 36.	65. 36.
741	2	58	4132. Iulianæ	Iulianæ 4132.

REGISTRVM

Register.

⁘A ⁘B ⁘C ⁘D ⁘E ⁘F ⁘G.

A B C D E F G H I K L M N O P Q R S T V X Y Z.

Aa Bb Cc. After p. 204 insert two leaves with figures of the Moon. Dd Ee Ff Gg Hh Ii Kk Ll Mm Nn Oo Pp Qq Rr Sſ Tt Vu Xx Yy Zz.

Aaa Bbb Ccc Ddd Eee Fff Ggg Hhh Iii Kkk Lll Mmm Nnn Ooo Ppp Qqq Rrr Sſſ Ttt Vuu Xxx Yyy Zzz.

Aaaa Bbbb Cccc Dddd Eeee Ffff Gggg Hhhh Iiii Kkkk Llll Mmmm Nnnn Oooo Pppp Qqqq Rrrr Sſſſ Tttt Vuuu Xxxx Yyyy Zzzz.

Aaaaa Bbbbb Ccccc.

All are duernions except Cc, Dd, Xxxx, which are ternions, and ⁘G, which is a unernion.

Blank endpaper: no printed matter.

Blank endpaper: no printed matter.

Inside of the back board with its pasted endpaper: no printed matter.

Outside of the back board, in mottled brown leather, with the raised bands of the spine at the right: no printed matter.